2.7 Graphs of Rational Functions
Day 2

10/10

Slant or Oblique Asymptote

when degree(top) > degree on bottom
↳ no horizontal asymptote

* When the degree(top) is exactly one more than degree on bottom → slant asymptote
↑ must do division

ex1 $\frac{x^2-x}{x+1}$

Domain{x ; x ≠ -1}
horiz asy = none (simple rule)
vert asy = $x+1=0$
$x=-1$
x-int : $x^2-x=0$
$x(x-1)=0$
$x=0$ ↘ $x=1$
(0,0) (1,0)

* Do division (2 methods)

y-int = $\frac{(0)^2-(0)}{(0)+1}$ $\frac{0}{1}$ = (0,0)

$$\begin{array}{r} x-2 \\ x+1 \overline{\big) x^2-x+0} \\ -(x^2+x) \\ -2x+0 \\ -(-2x-2) \\ +2 \end{array}$$

+2 → remainder

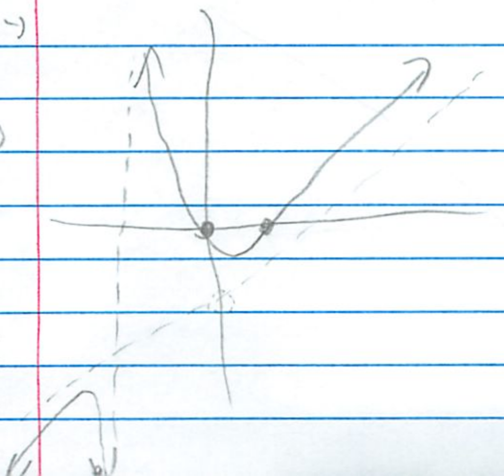
→ $\frac{x^2-x}{x+1} = x-2+\frac{2}{x+1}$

↑ if $x \to \infty$
$r(x) \to 0$
↑ remainder doesn't matter

$\boxed{x-2}$
↑ slant asymptote

test points

$x=-.999 \to$
(+)↑
$x=-1.001 \to$
(−)↓



Could also use synthetic division

$$\frac{x^2-x}{x+1} \qquad \begin{array}{r|rrr} -1 & 1 & -1 & 0 \\ & \downarrow & -1 & +2 \\ \hline & 1 & -2 & +2 \end{array}$$

$$\underset{\substack{\uparrow \\ \text{same value}}}{x-2} + \frac{2}{x+1}$$

ex 2 $f(x) = \frac{x^2-x-2}{x-1}$

Domain = $\{x : x \neq 1\}$

horiz asy = none

vert asy = $x-1=0$

$x=1$

x-int = x^2-x-2

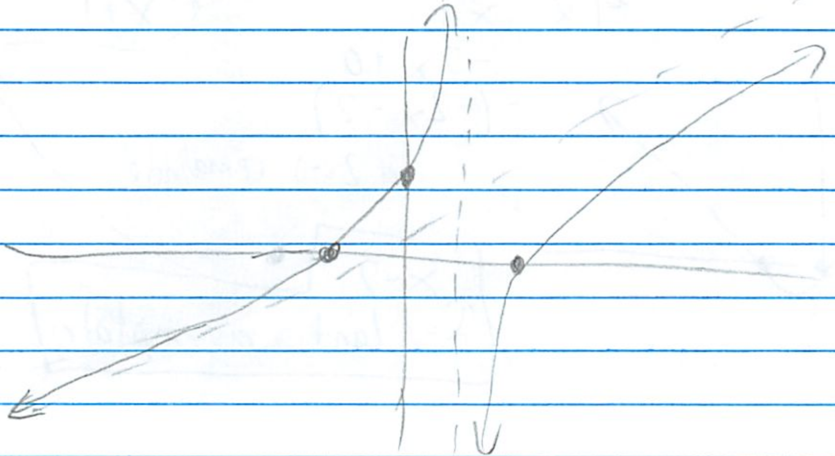
$(2,0)$ $(-1,0)$

y-int = $\frac{(0)^2-(0)-2}{(0)-1}$ $\frac{-2}{-1}$ $(0,2)$

$$\begin{array}{r|rrr} 1 & 1 & -1 & -2 \\ & \downarrow & 1 & 0 \\ \hline & 1 & 0 & -2 \end{array}$$

$x + \frac{-2}{x-1}$ ← slant asy = $y=x$

test points



p157

44. $\frac{1-x^2}{x}$

Domain = $\{x ; x \neq 0\}$

horiz asy = none

vert asy = $x = 0$

x-int = $1 - x^2 = 0$

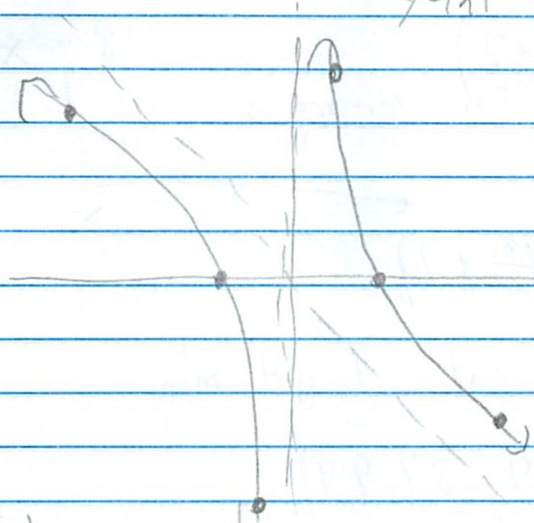
$(x-1)(x+1)$

$(1,0)\ (-1,0)$

y-int = $\frac{1-(0)^2}{(0)}$ $\frac{1}{0}$ = Und.

$$\begin{array}{r|rrr} 0 & -1 & 0 & 1 \\ & \downarrow & 0 & 0 \\ \hline & -1 & 0 & 1 \end{array}$$

$-x^2 + \frac{1}{x}$



$-5 \rightarrow$ 4.8

$5 \rightarrow -4.8$

$-.01 \downarrow$ -100

$.01 \downarrow$ 106

p158

49. $\frac{x^3+2x^2+4}{2x^2+1}$

Domain ; $\{x ; x \in \mathbb{R}\}$

horiz asy = none

vert asy = complex (no real)

x-int = $x^3+2x^2+4=0$

$\hookleftarrow$ graph + find zeros

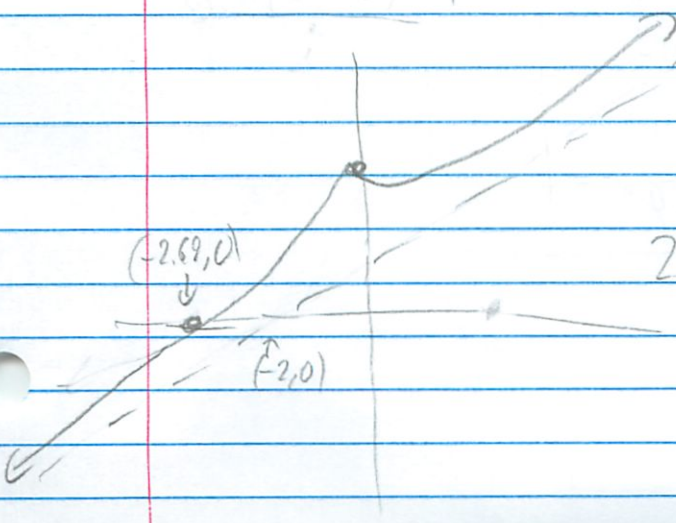
$(-2.59,0)$

y-int = $\frac{(0)^3+2(0)^2+4}{2(0)^2+1} = \frac{4}{1} = 4$

$(0,4)$

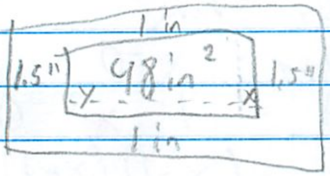
$$\begin{array}{r l} & \frac{1}{2}x + 1 \\ 2x^2+1 & \overline{\big)\, x^3 + 2x^2 + 0x + 4} \\ & -(x^3 + \frac{1}{2}x) \\ & \quad 2x^2 - \frac{1}{2}x + 4 \\ & \quad -(2x^2 + \quad 1) \\ & \quad\quad -\frac{1}{2}x + 3 \end{array}$$

$\leftarrow$ remainder



p156
ex6

Finding minimum area



$A = x \cdot y$

$\text{Area} = (\underset{\uparrow 1,5 \cdot 2}{x+3})(\underset{\uparrow 1 \cdot 2}{y+2})$ ← margins

$48 = xy$

$\frac{48}{x} = y$

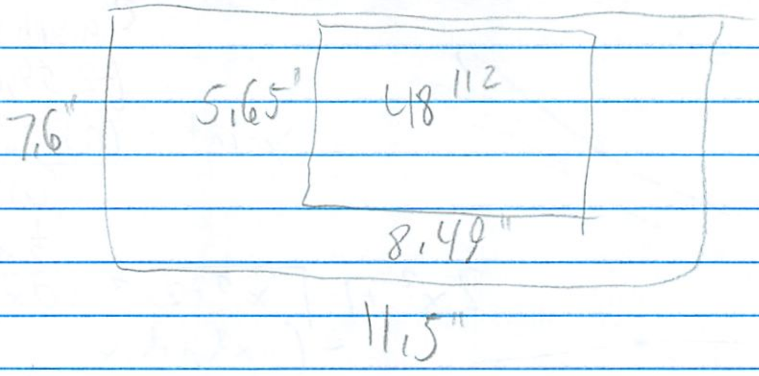
$\text{Area} = (x+3)\left(\frac{48}{x}+2\right)$

Plug into calc + get min

$(8,49,\ 87,94)$

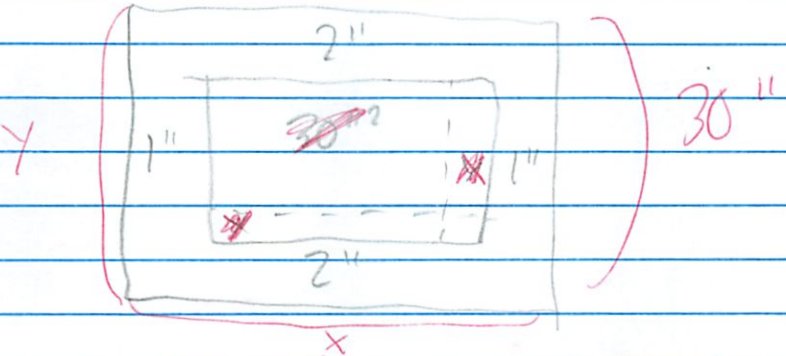
↑ lenght of page ↑ area of page

find $y = \frac{48}{8,49} = 5,65 = y$



p158

65. Another Page



~~$x \cdot y = 30$~~

~~$\frac{30}{x} = y$~~ → ~~$30'' = \left(\frac{30}{x+2}\right)\left(\frac{30}{x} + 4\right)$~~

$30'' = (x-2)(y-4)$

$x > 2$

~~a) ? type in calc + compare w/ given~~

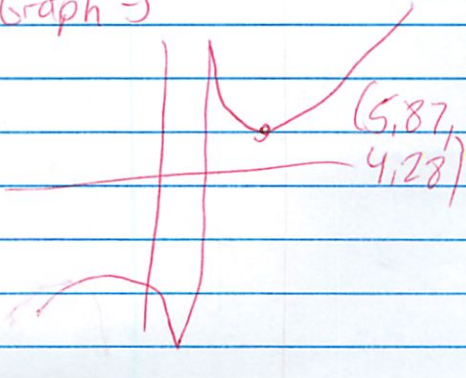
$y = \frac{30}{x-2} + 4$

$A = x\left[\frac{30}{x-2} + 4\right]$

$A = x\left[\frac{30 + 4(x-2)}{x-2}\right]$

$x\left[\frac{4x + 22}{x-2}\right]$

$\frac{x(2)[2x+11)}{x-2}$ ✓ → Graph →



2.7 Day 2 Slant Asymptotes

10/10

p 157

43. $\frac{2x^2+1}{x} =$

Domain $\{x ; x \neq 0\}$

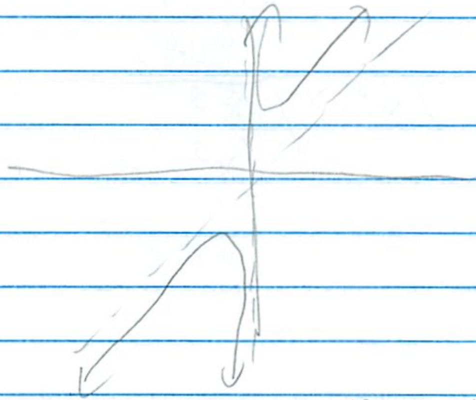
horiz asy = none

vert asy = $x = 0$

x-int = $2x^2+1 = 0$

i #s - und

y-int = $\frac{2(0)^2+1}{(0)} = \frac{1}{0}$ = und



$$\begin{array}{r} 2x \\ x\overline{)\,2x^2+1} \\ -(2x^2) \\ 1 \end{array}$$

p 158

45. $\frac{x^2}{x-1}$

Domain $\{x ; x \neq 1\}$

horiz asy = none

vert asy = $x-1=0$

$x=1$

x-int = $x^2=0$

$x=0$ $(0,0)$

y-int : $\frac{(0)^2}{(0)-1}$ $\frac{0}{-1}$ = $(0,0)$

$$\begin{array}{c|ccc} 1 & 1 & 0 & 0 \\ & \downarrow & 1 & 1 \\ \hline & 1 & 1 & 1 \end{array}$$

$x + 1 + \frac{1}{x-1}$

47. $\frac{x^3}{2x^2-8}$

$\hookrightarrow (x-2)(x+2)$

Domain $\{x ; x \neq 2, -2\}$

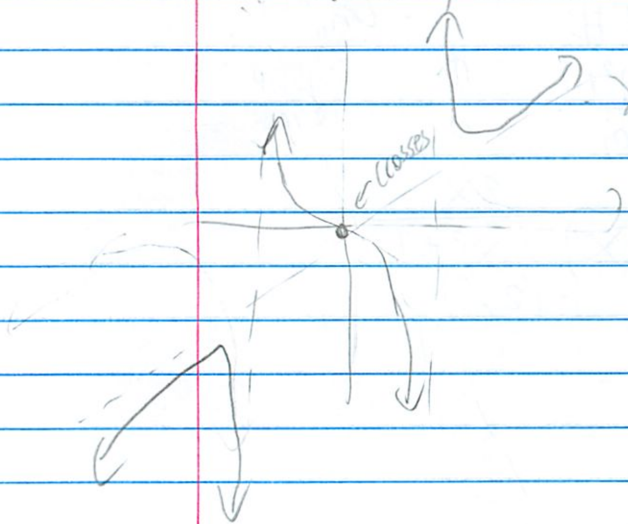
horiz asy = none

vert asy = 2, -2

x-int = $x^3=0$

$x=0$ $\quad (0,0)$

y-int = $\frac{(0)^3}{2(0)^2-8}$ $\quad \frac{0}{-8}$ $\quad (0,0)$



$$\begin{array}{r} \frac{1}{2}x \\ 2x^2-8 \overline{) x^3 \qquad} \\ -(x^3-4x) \\ -4x \end{array}$$

$\frac{1}{2}x + \frac{-4x}{2x^2-8}$

Find roots

53. $y = \frac{1}{x} - x$

$0 = \frac{1}{x} - x$

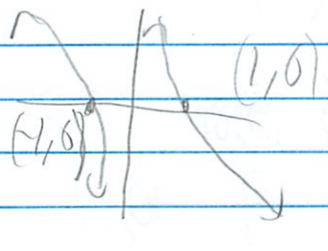
$x = \frac{1}{x}$

cross multiplied

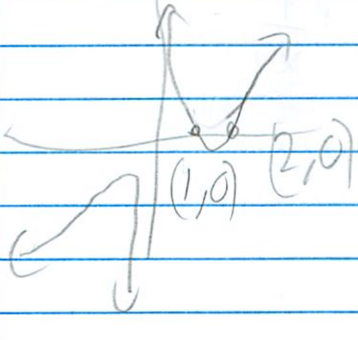
$x^2 = 1$

$x = \pm 1$

alegbrai

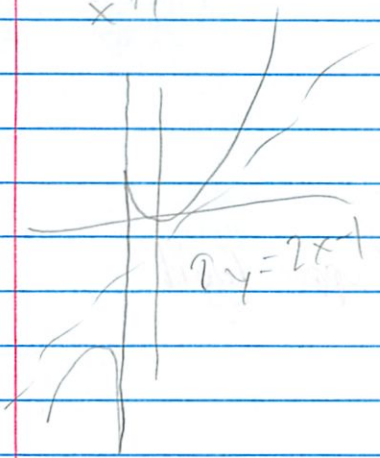


54. $y = x - 3 + \frac{2}{x}$



55. $\frac{2x^2+x}{x+1}$

Domain $\{x \neq -1\}$
horiz asy = none
vert asy $x = -1$

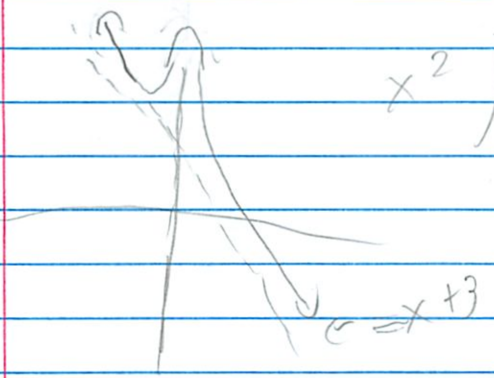


$$\begin{array}{r|rrr} -1 & 2 & 1 & 0 \\ & \downarrow & -2 & 1 \\ \hline & 2 & -1 & 1 \end{array}$$

$2x - 1 + \frac{1}{x+1}$

57. $\frac{1+3x^2-x^3}{x^2}$

Domain $\{x \mid x \neq 0\}$
horiz asy = none
vert asy = $x^2 = 0$
$x = 0$

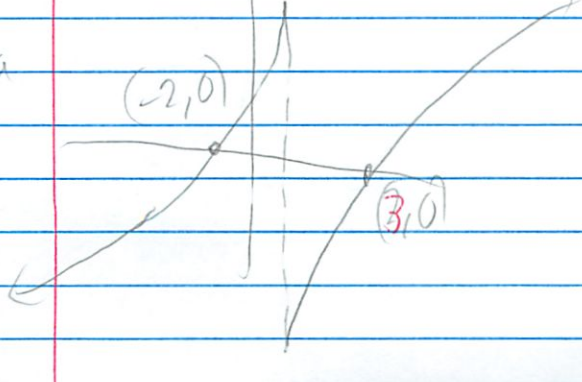


$$\begin{array}{r|l} & -x + 3 \\ \hline x^2 & -x^3 + 3x^2 + 0x + 1 \\ & -(-x^3) \\ & \quad 3x^2 + 0x + 1 \\ & \quad -(3x^2) \\ & \qquad\qquad 1 \end{array}$$

$-x + 3 + \frac{1}{x^2}$

61. $y = x - \frac{6}{x-1}$

algebra



$0 = x - \frac{6}{x-1}$

$0 = \frac{x(x-1)-6}{x-1}$ ↓ lcd

$0 = \frac{x^2-x-6}{x-1}$

$0 = \frac{(x-3)(x+2)}{x-1}$ → $x = 3$
$x = -2$

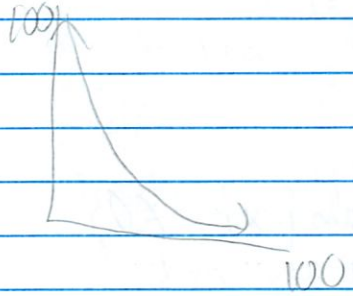
C4. $A = xy$

$500m^2 = xy$

$\frac{500m^2}{x} = y$

$D\{x : 0 < x$ ~~<500~~ $\}$

↑ no upper limit



$30 \times 16\frac{2}{3} = 500$

↑

y

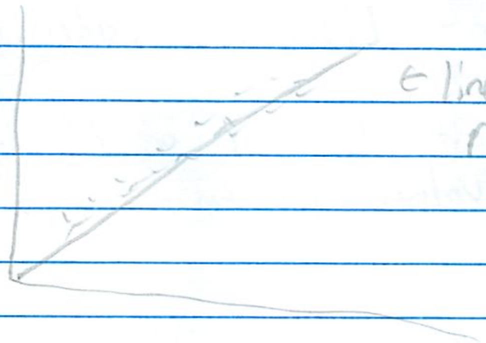
2.8 Quadratic Models

10/11

p161

1. Model data from p161 on calculator

a)



← linear

r should be close to |1|

b

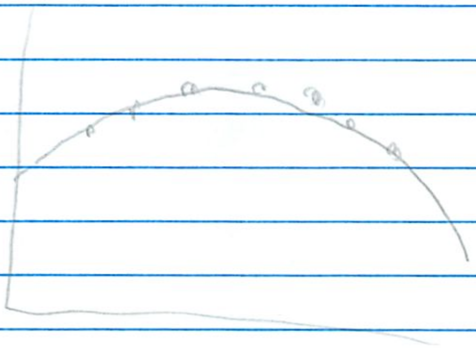
← quadratic

use r^2

↑ still close to |1|

p162

ex2 Plot speed vs milage (gallons / hr)



Quadratic: $y = -.008x^2 + .746 + 13.46$

↑ $r^2 = .9726$

Maximum: (45.5, 30.44)

↑ mph ↑ miles per gallon

3. Basketball - find zero

4. Book Spending - Linear or quadratic?

   - look at closet r or r^2 value
     - unless problem says

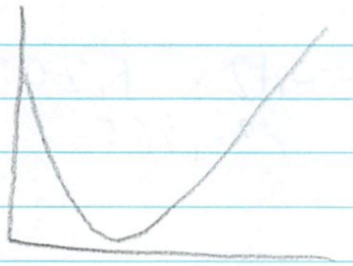
2,8/ Quadratic Data Models

HW

10/11

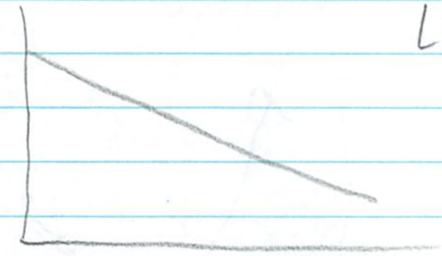
p165

1,



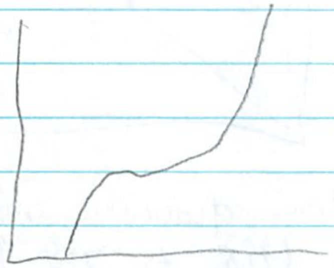
Quadratic

3,



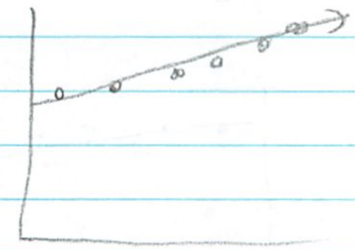
Linear

5,



Neither

7, Graph

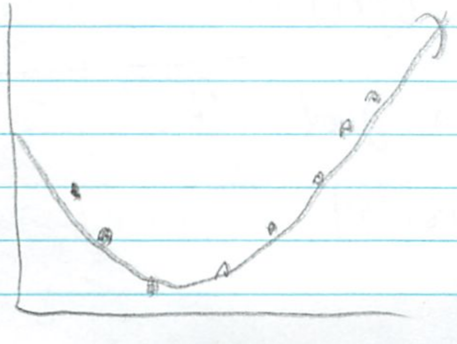


Linear

y = ,14 x + 2,24

r = ,985

9,



Quadratic

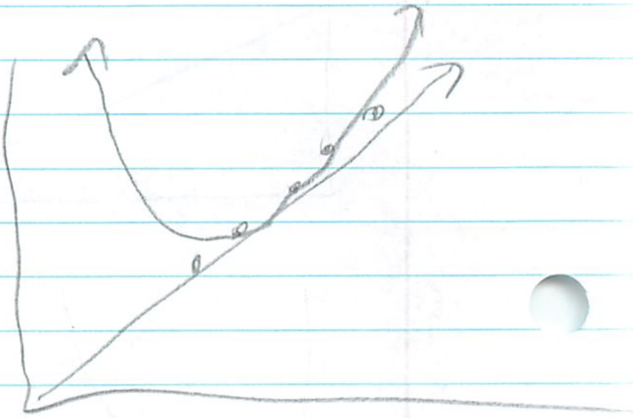
y = 5,55x² − 277,5 + 3478

r² = ,9999

11.

Quad
$y=-,12x^2+,208+7,53$
$r^2=,965$

13. Schools on tle Internet



$t=4=1994$

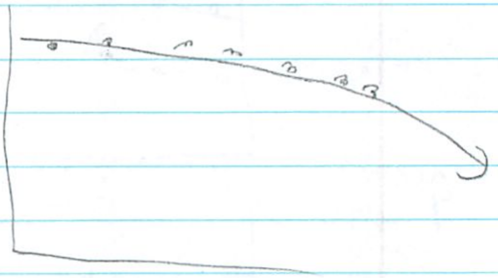
$y=2,47x^2-21,06+94,62$

No - it's bad
$r^2=,8491$
You can't predict using this quadratic model because it says 1998 + ,779 through year.

Not really - percentage can't go above 100%

p166

15. Hospitals
$t(0)=1960$

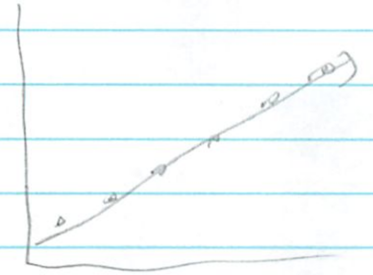


$y=-1,68x^2+40,625x+6903$
$r^2=,991$
Yes quadratic is a good fit

Maximum (12, 7148,55)
↑ 1972

p167

17. Linear or quadratic?



~~Linear~~

Quad actually better

$y = 2.47x + 1.13$

$r = .99496$

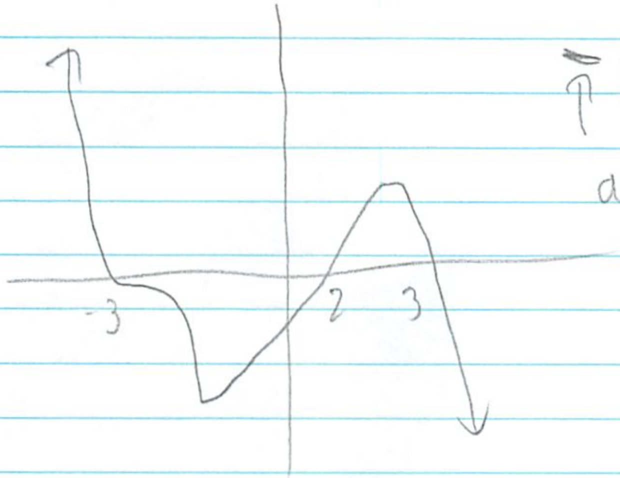
$y = .07x^2 + 1.7x + 2.7$

$r^2 = .99579$

2-2 Test Review

10/11

1.



$\uparrow (x+3)^3 (x-2) (x-3)$

$a = \ominus$

degree = 5

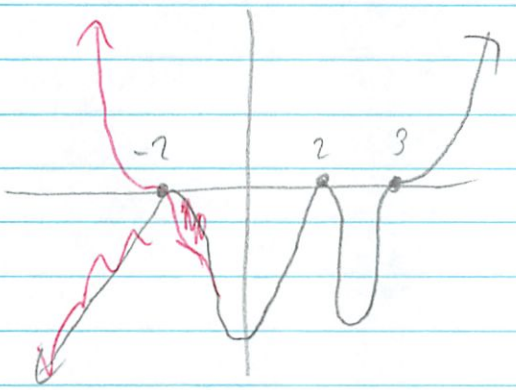
2. $f(x) = (x+2)^{\cancel{4}\,3} (x-2)^2 (x-3)^3$

n = ~~m~~ 8

copy error

$a = \oplus$

↓ ↑



3. Deterime polynomial of lowest degree w/ real co-efficent that has $-3+\sqrt{3}$ and -2 as zeros

$(x+2)(x-[-3+\sqrt{3}])(x-[-3-\sqrt{3}])$

$(x+2)[(x+3)^2-3]$

$(x+2)[x^2+6x+9-3]$

$(x+2)[x^2+6x+6]$

$x^3+6x^2+6x+2x^2+12x+12$

$x^3+8x^2+18x+12$

2-2 Test Topics

10/11

1. Sketch graph from polynomial
2. Get polynomial given zeros
3. Expressing functions as linear factors ← complex zeros
   - synthetic division to get zeros
4. Factor polynomials completely to get zeros,
5. Rational Functions
   - domain
   - asy
   - x + y int
   - graph
     - with added points

2-2/ Test Review
Part 2

10/12

1. Eval useing synthetic s-bstetion if $x = -3$

$f(x) = -2x^4 - 3x^3 + 4x - 2$

$\hookrightarrow f(-3) = -2(-3)^4 - 3(-3)^3 + 4(-3) - 2$ } show

$$\begin{array}{r|rrrrr} -3 & -2 & -3 & 0 & 4 & -2 \\ & \downarrow & 6 & -9 & 27 & -93 \\ \hline & -2 & 3 & -9 & 31 & \boxed{-95} \end{array}$$

p171

87. Find zeros

$f(x) = 3x(x-2)^2$

$x = 0$ $\quad$ $x = 2$

$t = 1$ $\quad$ $t = 2$

89. $(x+4)(x-6)(x-2i)(x+2i)$

$x = -4$ $\quad$ $x = 6$ $\quad$ $x = 2i$ $\quad$ $x = -2i$

$t = 1$ $\quad$ $t = 1$ $\quad$ $t = 1$ $\quad$ $t = 1$

90. $(t-8)(t-5)^2(t-3+i)(t-3-i)$

$x = 8$ $\quad$ $x = 5$ $\quad$ $x = 3+i$ $\quad$ $x = 3-i$

$t = 1$ $\quad$ $t = 2$ $\quad$ $t = 1$ $\quad$ $t = 1$

91. Find zeros + linear factors

$2x^4 - 5x^3 + 10x - 12$

$$\frac{\text{factors } (12)}{\text{factors } (2)} \quad \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1, \pm 2}$$

$\longrightarrow$

$\pm 1, \pm 2, \pm 3 \pm 4 \pm 6, \pm 12,$

$\pm \frac{1}{2}, \pm \frac{3}{2}$

$$\begin{array}{r|lllll} 2 & 2 & -5 & 0 & 10 & -12 \\ & \downarrow & \nearrow 4 & \nearrow -2 & \nearrow -4 & \nearrow 12 \\ & 2 & -1 & -2 & 6 & \textcircled{0}\ \checkmark \end{array} \qquad (x-2)$$

$2x^3 - x^2 - 2x + 6$

$$\begin{array}{r|llll} -1\frac{1}{2} & 2 & -1 & -2 & 6 \\ & \downarrow & \nearrow -3 & \nearrow 6 & -6 \\ & 2 & -4 & 4 & 0\ \checkmark \end{array} \qquad \begin{array}{c} \left(x+\frac{3}{2}\right) \\ (2x+3) \end{array}$$

$2x^2 - 4x + 4$

$$\frac{+4 \pm \sqrt{-16}}{4} \to \frac{4 \pm 4i}{4} \qquad \begin{array}{c} (x-[1+i]) \\ (x+[1+i]) \end{array}$$

$1 \pm i$

don't trust rooten

$(x-2)(2x+3)(x-[1+i])(x+[1+i])$

93, $x^3 - 7x^2 + 18x - 24$

$$\frac{\text{factors } (24)}{\text{factors } (1)} \qquad \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24}{\pm 1}$$

$$\begin{array}{r|llll} 4 & 1 & -7 & 18 & -24 \\ & \downarrow & \nearrow 4 & \nearrow -12 & \nearrow 24 \\ & 1 & -3 & 6 & 0\ \checkmark \end{array} \qquad (x-4)$$

$x^2 - 3x + 6$

$$\frac{+3 \pm \sqrt{-15}}{2}$$

← can't reduce just do that

$$f(x) = (x-4)\left(x - \left[\frac{3+\sqrt{15}\,i}{2}\right]\right)\left(x - \left[\frac{3-\sqrt{15}\,i}{2}\right]\right)$$

Linear Factors + X-intercepts

97. $x^3 + 6x^2 + 11x + 12$

$$\frac{\text{factors}(12)}{\text{factors}(1)} \qquad \frac{\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12}{\pm 1}$$

$$\begin{array}{r|rrrr} -4 & 1 & 6 & 11 & 12 \\ & \downarrow & -4 & -8 & -12 \\ \hline & 1 & 2 & 3 & 0 \; ☺ \end{array} \qquad (x+4)$$

$x^2 + 2x + 3$

$$\frac{-2 \pm \sqrt{-8}}{2}$$

$$\frac{-2 \pm \sqrt{4}\sqrt{2}\,i}{2}$$

$-1 \pm \sqrt{2}\,i$

$(x+4)(x - [-1 + \sqrt{2}i])(x - [-1 - \sqrt{2}i])$

! 99. $x^4 + 34x^2 + 225$ $\quad\downarrow$ remember $(x^2 \quad)(x^2 \quad)$

$$\frac{\text{factors}(225)}{\pm 1}$$

$(x^2 - 25)(x^2 - 9)$

$\downarrow \qquad \downarrow$

$x^2 = +25 \quad x^2 = +9$

$x = \pm 5i \quad x = \pm 3i$

$\uparrow ? \qquad \uparrow ?$

$(x - 5i)(x + 5i)(x - 3i)(x + 3i)$

$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$

???

101. Find polynomial w/ these zeros

$-2, -2, -5i, 5i$

$(x+2)^2 \underline{(x - 5i)(x + 5i)}$

$\swarrow$ don't forget $\qquad \downarrow$ extend

$(x+2)^2 (x^2 - 25i^2)$

$(x^2 + 4x + 4)(x^2 + 25)$

~~$x^4 + 4x^3 + 100x + 25x^2 + 100x + 100$~~ $\rightarrow$ ~~$x^4 + 4x^3 + 25x^2 + 200x + 100$~~

something wrong here

103. $1, -4, -3+5i, -3-3i$

$(x-1)(x+4)(x-[-3+5i])(x-[-3-5i])$

$(x^2-1x+4x-4)((x+3)^2-25i^2)$

One little thing screws the entire thing up →

$(x^2+3x-4)(x^2+6x+9+25)$

$(x^2+3x-4)(x^2+6x+34)$

$x^4+6x^3+34x^2+3x^3+18x^2+102x-4x^2-24x-136$

$x^4+9x^3+\cancel{42}x^2-\cancel{126}x-136$

48 + 78

p172

109. Find domain + asyemptotes

$f(x)=\frac{x-8}{1-x}$

Domain: $\{x: x \neq 1\}$

horiz asy: $\frac{1}{-1} = -1 = y$

vert asy: $x=1$

111. $\frac{2}{x^2-3x-18}$

$(x-6)(x+3)$

Domain = $\{x: x \neq 6, -3\}$

horiz asy = $y=0$

vert asy = $x=6$, $x=-3$

115. $\frac{4x^2}{2x^2-3}$

✓

Domain $(x: x \neq \cancel{\sqrt{\tfrac{3}{2}}}, -\cancel{\sqrt{\tfrac{3}{2}}}$ $\pm\frac{\sqrt{6}}{2}$

horiz = $\frac{4}{2} = 2 = y$

vert asy = $2x^2-3=0$

$2x^2=3$

$x^2=\frac{3}{2}$

$x=\pm\sqrt{\frac{3}{2}}$ → rationalize $\frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ $\frac{\sqrt{6}}{2}$

117. $\frac{2x-10}{x^2-2x-15}$

$(x-5)(x+3)$

Domain $\{x: x \neq 5, -3\}$

horiz = $y=0$

vert = $x=5$, $x=-3$

p172

119. $\frac{x-2}{|x|+2}$

Domain $\{x \neq 2, -2\}$
horiz asy = $\frac{1}{1}$, 1 $\frac{-1}{1}$, −1 ← yes there are 2
vert asy = ~~$x+2=0$~~ ~~$x=-2$~~ ~~$-x+2=0$~~ ~~$-x=-2$~~ ~~$x=2$~~

look at whole thing

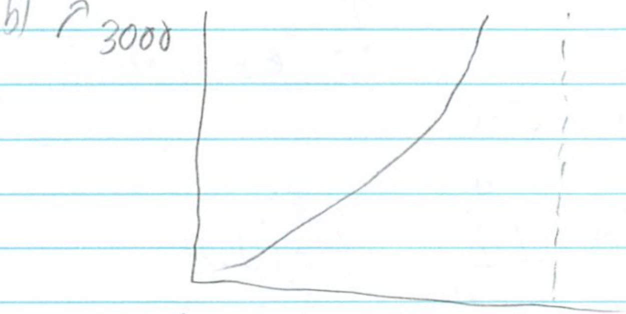
121 C = cost
p = percent

$C = \frac{528p}{100-p}$ $0 \leq p < 100$

p = 25 → \$ 176 millions
p = 50 → \$ 528 millions
p = 75 → \$ 1584 millions

that seems to be far above the higest rev point

b)



105 ← thats the domain

c) no - the domain says that you can never catch everything
plus it's a vertical asmyptote

123 Vertical Asymptotes
$f(x) = \frac{2x-1}{x-5}$

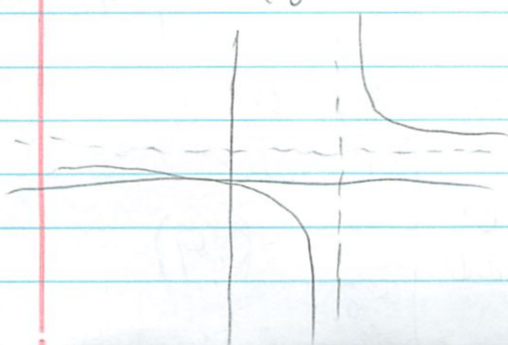
Domain $\{x \mid x \neq 5\}$
horiz asy = ~~none~~ $\frac{2}{1} = 2$
vert asy = $x - 5 = 0$
$x = 5$

↑ sure - should have seen that

$$\begin{array}{r|rr} 5 & 2 & -1 \\ & \downarrow & 10 \\ \hline & 2 & 9 \end{array}$$

$y = 2 + \frac{9}{x-5}$

+ test points
5.01 → 902
4.99 → −898



125. $\frac{2x}{x^2+4}$

Domain $\{x : x \in \mathbb{R}\}$

horiz asy = $y = 0$

vert asy = $x^2+4=0$

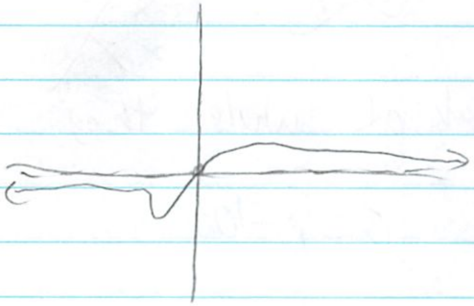
$x^2 = -4$

$x = \pm 2i$ ← non real

x-int = $2x = 0$

$x = 0$ $(0,0)$

y-int = $\frac{2(0)}{(0)^2+4}$ $\frac{0}{4} = (0,0)$



127. $\frac{x^2}{x^2+1}$

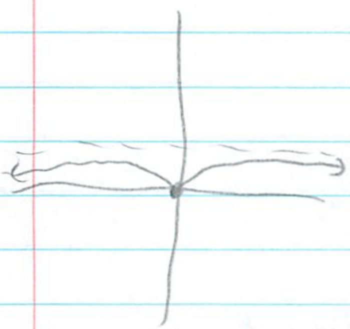
Domain $\{x : x \in \mathbb{R}\}$

horiz asy = $y = \frac{1}{1} = 1$

vert asy = $x^2+1=0$

$x^2 = -1$

$x = \pm 1i$ ← non real



x-int = $x^2 = 0$

$x = 0$

y-int = $\frac{(0)^2}{(0)^2+1}$ $\frac{0}{1} = (0,0)$

129. $\frac{2(x^2-16)}{x^2+2x-8} \rightarrow 2(x+4)(x-4)$

$\downarrow$ $(x-2)(x+4)$

Domain $\{x \mid x \neq 2, -4\}$

horiz asy = $\frac{2}{1} = 2$

vert asy = $x = 2$, $x = -4$

x-int = $2(x^2-16) = 0$

$2x^2 - 32 = 0$

$2x^2 = 32$

$x^2 = 16$

$x = \pm 4$

$(4,0)$

~~$(-4,0)$~~

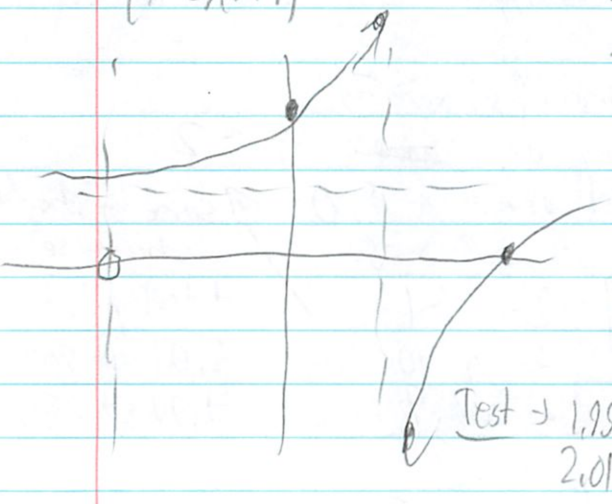
y-int = $\frac{2((0)^2-16)}{(0)^2+2(0)-8}$

$\frac{-32}{-8} = (0,4)$

$\frac{2\cancel{(x+4)}(x-4)}{\cancel{(x+4)}(x-2)}$

← simplify 1st

$\frac{2(x-4)}{(x-2)}$



Test → 1.99 = 402 (+) ↑

2.01 = -398 ↓

Slant Asymptotes

131, $\frac{2x^3}{x^2+1}$

Domain$\{x ; x \in \mathbb{R}\}$

horiz asy = none

vert asy = $x^2+1=0$

$x^2=-1$

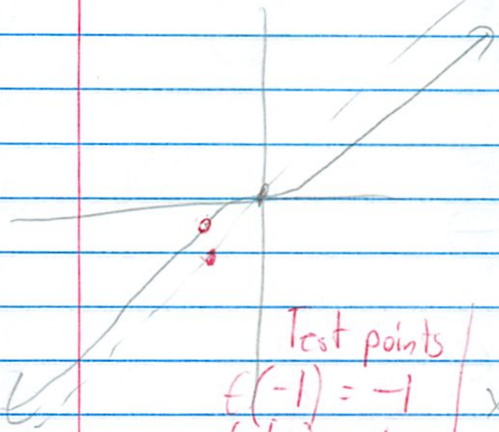
$x=\pm 1i$ ∈ nonreal

x-int = $2x^3=0$

$x^3=0$ $(0,0)$

$x=0$

y-int = $\frac{2(0)^3}{(0)^2+1}$ $\frac{0}{1}=(0,0)$



Test points

$f(-1)=-1$

$f(1)=1$

$$\begin{array}{r|l} & 2x \\ \hline x^2+1 & 2x^3+0x^2+0x+0 \\ & -(2x^3+\quad\quad 2x) \\ & \quad\quad -2x+0 \end{array}$$

$y=2x+\frac{-2x}{x^2+1}$

133, $\frac{x^2-x+1}{x-3}$

Domain $\{x ; x \neq 3\}$

horiz asy = none

vert asy = $x-3=0$

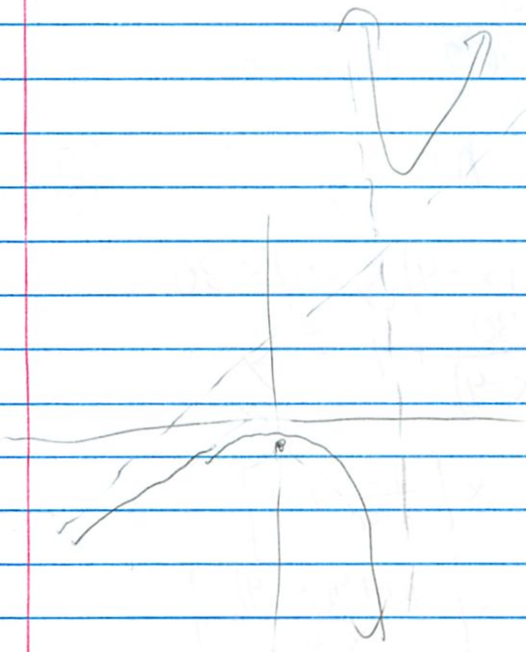
$x=3$

x-int = $x^2-x+1=0$

$\frac{+1\pm\sqrt{-3}}{2}$

non real

y-int = $\frac{(0)^2-(x)+1}{(0)^2-3}$ $\frac{1}{-3}$ $-\frac{1}{3}$

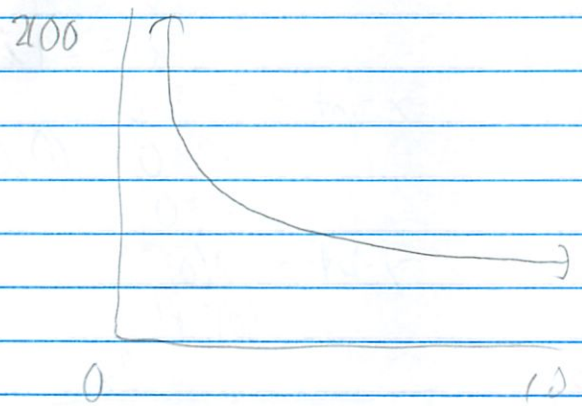


$$\begin{array}{r|rrr} 3 & 1 & -1 & 1 \\ & \downarrow & 3 & 6 \\ \hline & 1 & 2 & (7) \end{array}$$

$x+2+\frac{7}{x^2-x+1}$

in thousands

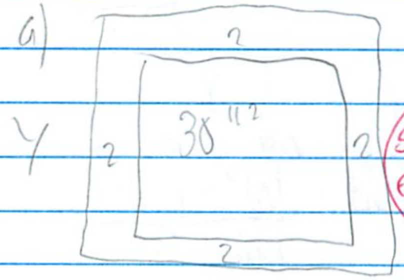
p135. $N = \frac{20(4+3x)}{1+.05t}$ $x \geq 0$ $x(0) \to 80$



b) $t(5) \to 72.381$
$t(10) \to 64.76$
$t(25) \to 60.19$
thousand fish

c) The maximum is an asymptote
at $t(0) =$ error
$t(0.01) = 7676.2$ thousand

136. a)



b) $A = xy$ $y = \frac{30}{}$

??

Solve for y $\to$ $(x-4)(y-4) = 30$

$\frac{30}{(x-4)} + 4 = y$

given $\to$ $\underset{\text{Page}}{A} = \frac{2x[2x+7)}{x-4}$

$\underset{\text{page}}{A} = x\left[\frac{30}{x-4} + 4\right]$

$A = x\left[\frac{30+4(x-4)}{x-4}\right]$

$A = x\left[\frac{14+4x}{x-4}\right]$

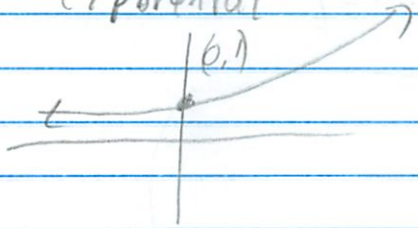
$A = 2x\left(\frac{7+2x}{x-4}\right)$

3.1 Exponential Log Function
Their Graphs

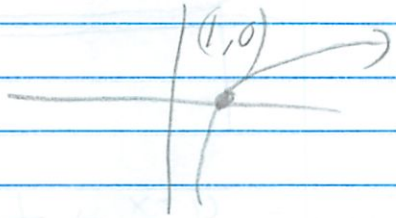
10/15

exponenital } functions ∈ non algermith functions
logarithmic

$f(0) \to 1$
exponental

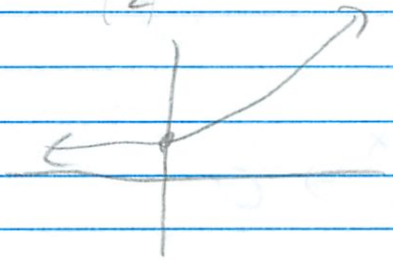


$f(1) = 0$
log

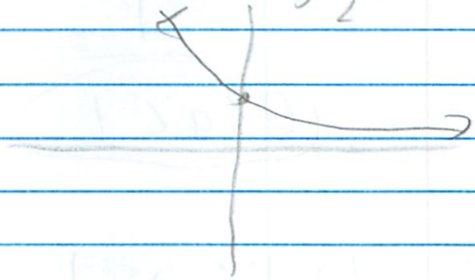


$f(x) = a^x$ $a \neq 1$ ∈ else you get constent function

$a > 1$ growth
$(2)^x$



$0 < a < 1$ decay
$(\frac{1}{2})^x$ } or 2^{-x}



Evaluate

$2^{-\pi} \to .11331473$
$8.6^{-\sqrt{2}} \to .047689$

x	-2	-1	0	1	2
2^x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
4^x	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16

∈ steeper

↑ always 1

$y = 2^x$
$y = 4^x$ ∈ steeper
↑ both growth functions

$\boxed{a > 1}$

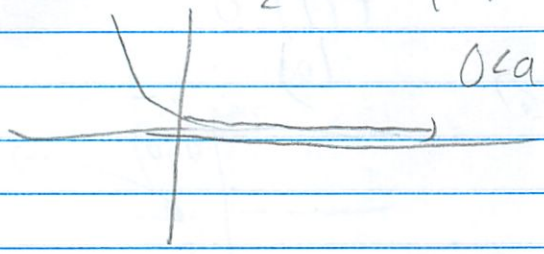
limit a^x
$x \to -\infty \to 0$

limit a^x
$x \to \infty \to \infty^+$

Declining Functions

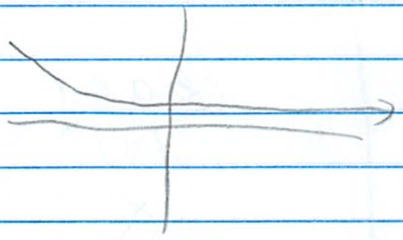
$y = a^{-x} \rightarrow \frac{1}{a^{x}}$

ex $f(x) = 2^{-x} \rightarrow \frac{1}{2^{x}} \rightarrow \left(\frac{1}{2}\right)^{2}$



ex 2 $f(x) = 4^{-x} \rightarrow \frac{1}{4x} \rightarrow \left(\frac{1}{4}\right)^{x}$

flatter

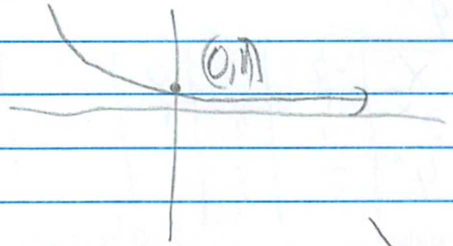


$\boxed{0<a<1}$ $\lim_{x \to \infty} a^{-x} \rightarrow 0$

p185

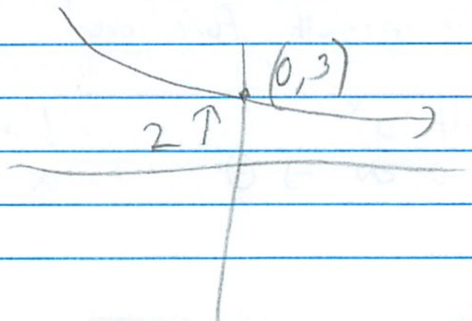
10. $\left(\frac{3}{2}\right)^{-x} \rightarrow \frac{1}{\left(\frac{3}{2}\right)^{x}} \rightarrow \left(\frac{1}{\frac{3}{2}}\right)^{x} \rightarrow \left(\frac{2}{3}\right)^{x}$

declining



Horiz asy = y=0 (x axis)

14. $\left(\frac{3}{2}\right)^{-x} + 2$
↑
outside



horiz asy = y=2

Rules

Domains always $\{x ; x \in \mathbb{R}\}$

Range $[0, \infty]$

y-int $(0,1)$

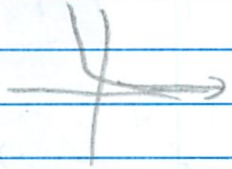
horiz asy $y=0$

Increasing

$\lim_{x \to -\infty} a^{x} \to 0$

Decreasing

$\lim_{x \to \infty} a^{-x} \to 0$



Properties of exponents

1. $a^{x} \cdot a^{y} = a^{x+y}$

$x^{2} \cdot x^{3} = x^{5}$

2. $\frac{a^{x}}{a^{y}} = a^{x-y}$

$\frac{a^{5}}{a^{10}} = a^{5-10}$

$a^{-5} \to \frac{1}{x^{5}} \to \left(\frac{1}{x}\right)^{5}$

3. $a^{-x} \to \frac{1}{a^{x}} \to \left(\frac{1}{a}\right)^{x}$

4. $a^{0} = 1$

5. $(ab)^{x} = a^{x} \cdot b^{x}$

$(xy)^{5} = x^{5} y^{5}$

6. $(a^{x})^{y} = a^{xy}$

$(a^{3})^{5} = a^{3 \cdot 5} = a^{15}$

7. $\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$

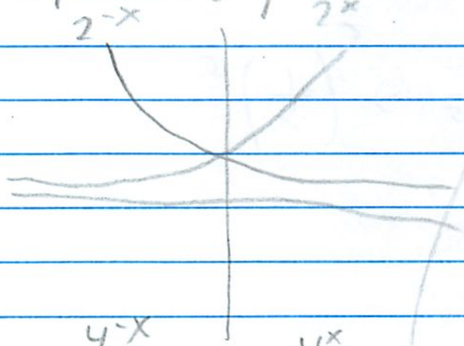
$\left(\frac{2}{3}\right)^5 \rightarrow \frac{2^5}{3^5} \rightarrow \frac{32}{243}$

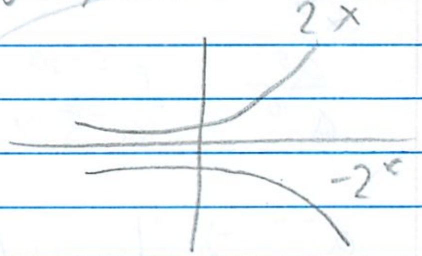
8. $|a^2| = |a|^2 = a^2$

P1 $\frac{2x^3y^4}{5(x^2)^6 z^{-1}} \rightarrow \frac{2x^3y^4z}{5x^{12}} \rightarrow \frac{2y^4z}{5x^9}$

p2 $\frac{[2x^2y^4]^0}{x^{-3}(y^2)^{-5}} \rightarrow \frac{1}{x^{-3}y^{-10}} \rightarrow x^3y^{10}$

Compare $f(x) = 2^{-x}$ to $g(x) = 2^x$

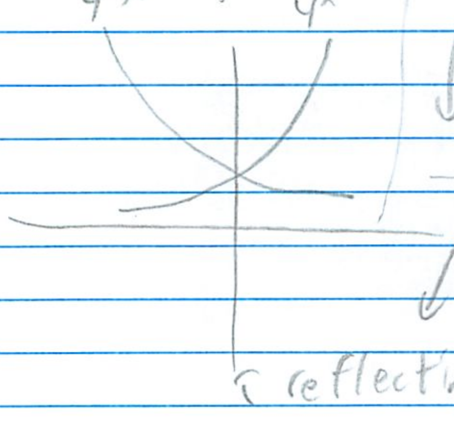




if ⊖ is outside function

$y = -2^x$ $\quad y = 2^x$

↓ will reflect around x axis



exponent ↓

if x is ⊖ within function ↙ will reflect around y axis

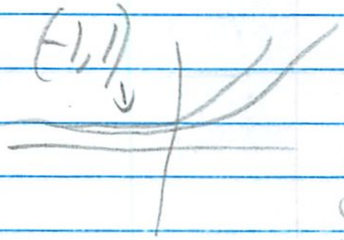
← reflecting around y axis

Shifts + Transformations

$f(x) = 3^x$

$g(x) = 3^{x+1}$ move ← 1

horiz asy = $y = 0$



looks like shifting up, but its ~~not~~ not

$h(x) = 3^x - 2$ ↓2

horiz asy = $y = -2$

3.1 Exponential Functions + Graphs
HW

10/15

p185 1. $f(x) = 3.4^{6.8} = 4117.03$

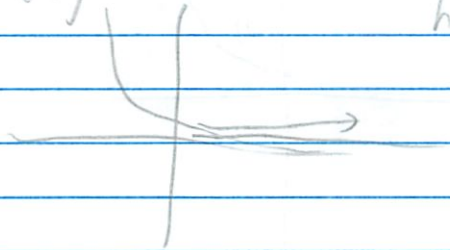
3. $f(x) = 5^{-\pi} = .00637$

4. $5000(2^{-1.5}) = 1767.7669$

5 $17^{2(\sqrt{5})} = 18297.85$

9. Graph

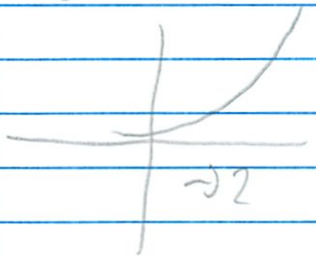
$\left(\frac{1}{5}\right)^x = 5^{-x}$



horiz asy = y=0

(0,1)

11. 5^{x-2}



horiz asy = y=0

(2,1)

y-int

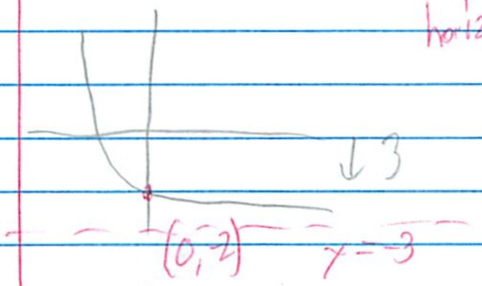
$5^{(0)-2}$

5^{-2}

$\left(\frac{1}{5}\right)^2$

$\frac{1}{25}$ $\left(0, \frac{1}{25}\right)$

13. $5^{-x} - 3$



horiz asy

y = -3

(0, -2)

$\left(\frac{1}{5}\right)^x - 3$

↑

decreasing

x-int

$0 = 5^{-x} - 3$

$3 = \left(\frac{1}{5}\right)^x$

take logs

Match

15, 2^{x-2} $\Rightarrow$ 2 → d

18. $2^{x}+1$ ↑ 1 → b

21, $f=\left(\frac{3}{5}\right)^{x}$ ← 4

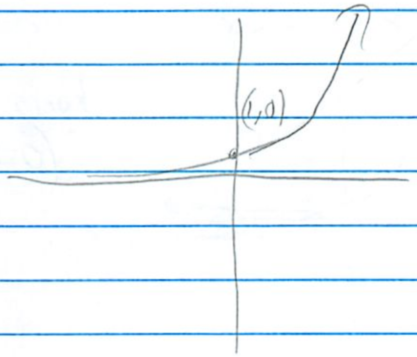
$g\left(\frac{3}{5}\right)^{x+4}$

23, $e^{9,2}$ = 9897,129

p186

29, $\left(\frac{3}{2}\right)^{x}$

x	y
-2	,16
-1	.4
0	1
1	2.5
2	6,25
3	15.625



3.1 Exponents + Their Functions
Day 2

10/17

Natural base of e

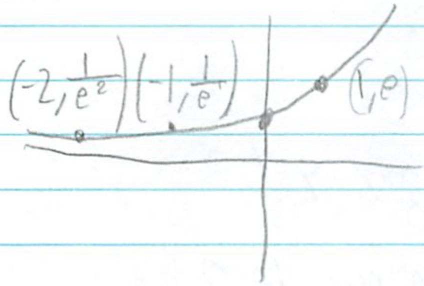
$e \approx 2.718281828$

↑ irrational # ← just a value

$\log_{10}$

$\log_e$ ← natural log ($\ln$)

e^x ← natural exponential function



$f(x) = e^x$

→ The number e develops from the expression $\left(1 + \frac{1}{x}\right)^x$

As $(x \to \infty) \to e$

related to compound interest

In x	$\left[1 + \frac{1}{x}\right]^x$	Calc values
10	$\left[1 + \frac{1}{10}\right]^{10}$	2.59
100	$\left[1 + \frac{1}{100}\right]^{100}$	2.7048
1,000	$\left[1 + \frac{1}{1000}\right]^{1000}$	2.71
10,000	$\left[1 + \frac{1}{10{,}000}\right]^{10{,}000}$	↓
10^5		e
10^6		↑ limit

$$\lim_{x \to \infty} \left[1 + \frac{1}{x}\right]^x \to e$$

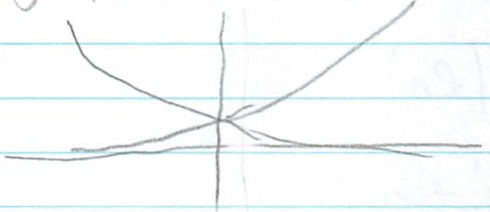
p186 p1

$g(7) = 100e^{.01(12)}$

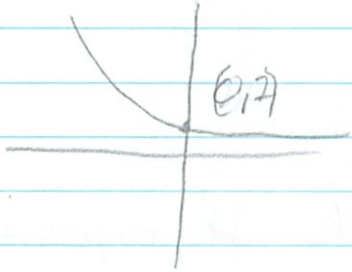
112,75

36. $f(x) = 2e^{-.5x}$

$g(x) = 2e^{.5x}$



46. $f(x) = 1 + e^{-x} \rightarrow 1 + \left(\frac{1}{e}\right)^x$



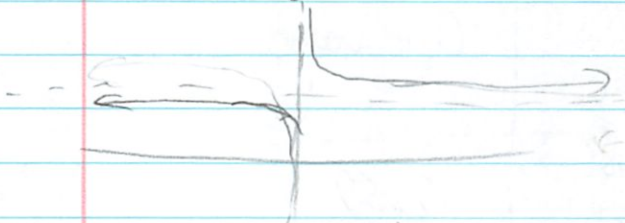
horiz asy y=1

$e^x \neq 0$

↑can never be 0

No vert asy

48. $g(x) = \frac{8}{1+e^{-.5/x}}$



Applications of Exponental Functions

Compounded Growth of an investment

↑at diff. intervals

using an intered rate + companding period

ex1 Principal = \$5,000
Invest at rate of 3%
Compounded every year for 4 years

$$\underbrace{\begin{array}{ll} 1: \ 5{,}000 \cdot 1.03 = & 5150 \\ 2: & 5304.5 \\ 3: & 5463.64 \\ 4: & 5627.54 \end{array}}$$

$$5000\left(1+\frac{.03}{\underset{\substack{\uparrow \\ \text{compounding} \\ \text{times per} \\ \text{year}}}{1}}\right)^{1\cdot\underset{\substack{\uparrow \\ \text{years}}}{4}} = \$5627.54$$

Compounded interest formula

$$A = P\left(1+\frac{r}{n}\right)^{n\cdot t}$$

A = amount in end
P = Principal
r = rate of interest
n = number of periods
t = number of years

quarterally $P\left(1+\frac{.03}{4}\right)^{4\cdot 4}$

monthly $n = 12$
semi-anually $n = 2$
daily - $n = 365$
continously $\left.P\left(1+\frac{.03}{x}\right)^{x\cdot t}\right\} e$

(if continously $A = Pe^{rt}$)

$$A = 5000e^{.03\cdot 4} = 5637.484$$

P1

P = $1000
r = 6%
t = 10 years

quarterly → $1000\left(1+\left(\frac{.06}{4}\right)\right)^{4\cdot 10}$

$1814,02

constinuosly → $1000e^{.06\cdot 10}$

$1822,12

Radioactive decay

radioactive elements have half-lives

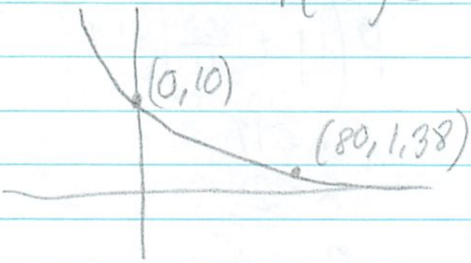
strontium $({}^{90}Sr)$ = halflife of 28 years

↑ atomic. mass

if 10 g is inital amount then after 28 years you have 5g left

$y = 10\left(\frac{1}{2}\right)^{t/28}$ intial amout → f(0) → 10

f(80) → 1,38 g

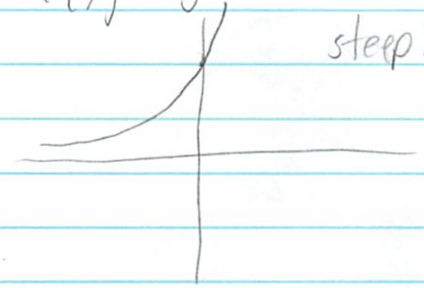


3.1 Exponential Functions
Day 2 HW

10/17

186 Graph

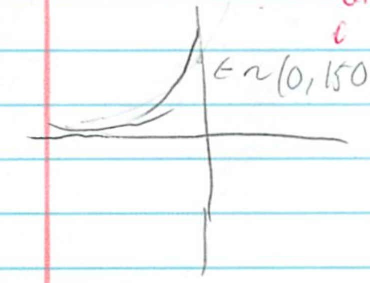
33. $f(x) = 3^{x+2}$



steepish

←2

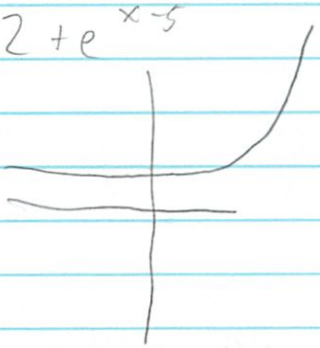
35. $3e^{x+4}$



around ← very steep

← 4

37. $2 + e^{x-5}$

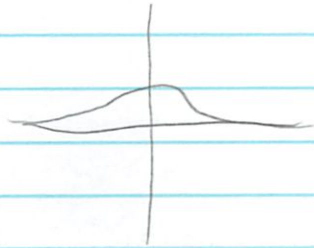


↑2

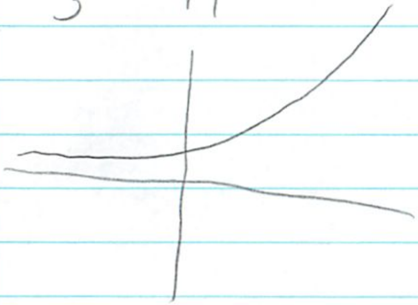
→5

31. 2^{-x^2}

horiz asy = none
vert asy = y=0
↑ no denominator



41. $3^{x-2}+1$



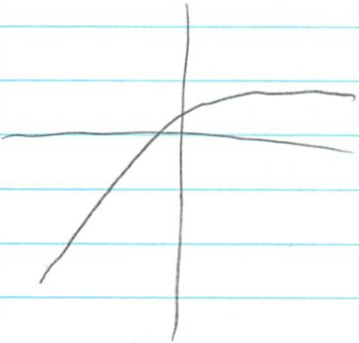
steepish

↑1

→2

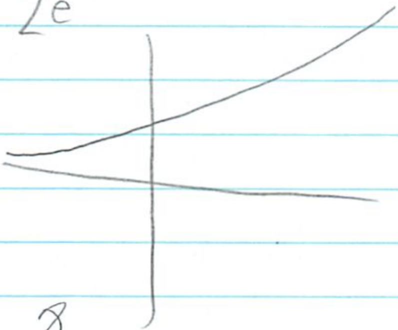
43. $2-e^{-x}$ ↑makes it go

declining



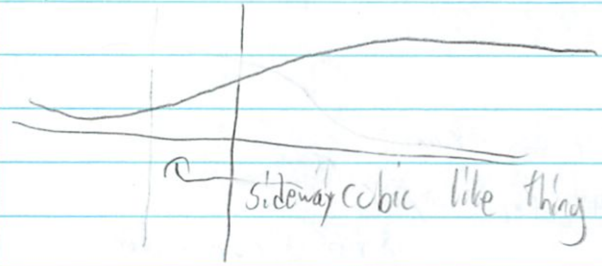
↑2

45. $2e^{.12t}$



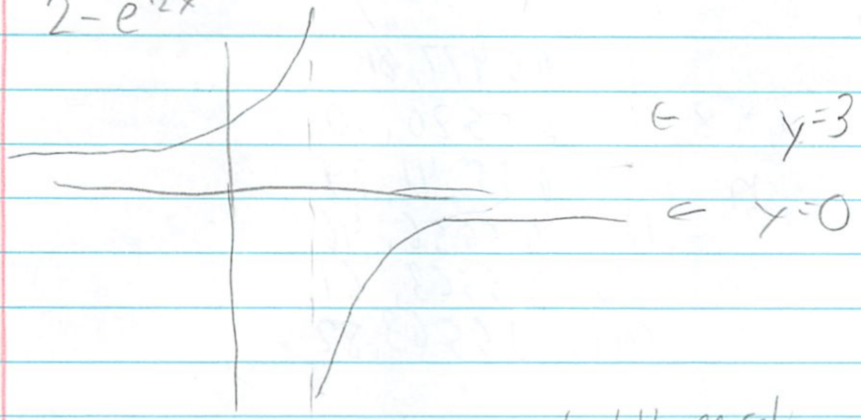
inclines slowly

47. $\frac{8}{1+e^{-.5x}}$



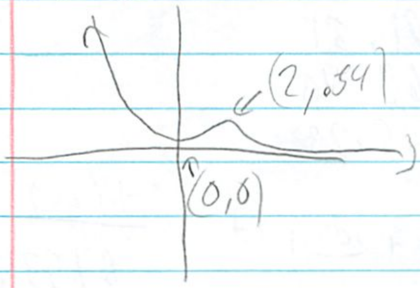
$y=0$
$y=8$ } look at table on calc

49. $\frac{6}{2-e^{.2x}}$

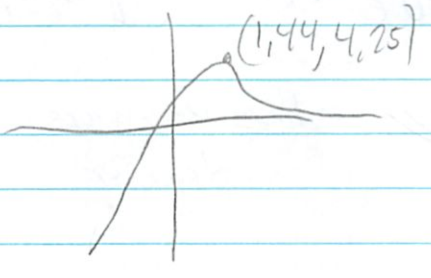


~ x=3.46 ← used table on calc

51. x^2e^{-x}



53. $x(2^{3-x})$



55. P= 2500
$r = 2.5\%$
t = 10 years

$\boxed{P\left(1+\frac{r}{n}\right)^{n\cdot t}}$ $\boxed{Pe^{rt}}$

1	= \$3200.21
2	\$3205.09
4	\$3207.57
12	\$3209.23
365	\$3210.04

Cont.
\$ 3210.06

57, P = 2500
r = 4%
t = 20 years

$$2500\left(1+\frac{.04}{n}\right)^{n\cdot 20} \quad\Big|\quad 2500e^{.04\cdot 20}$$

n

1	\$5477.81
2	\$5520.10
4	\$5541.79
12	\$5556.46
365	\$5563.61
cont.	\$5563.82

61, P = 12,000
r = 3.5%
n = continous

$\boxed{2500e^{.035t}}$

t:

1 →	\$12427.44
10 →	\$17028.81
20 →	\$24165.03
30 →	\$34291.81
40 →	\$48662.40
50 →	\$69055.23

64, P = \$500
r = 7%

		t(20)
annually	$500\left(1+\frac{.07}{1}\right)^{1t}$	\$1934.84
quarterly	$500\left(1+\frac{.07}{4}\right)^{4t}$	\$2003.96
continously	$500e^{.07t}$	\$2027.60 ← most

← lines very close togeter

Radioactive decay

65. $Q = \text{mass} \left({}^{226}\text{Ra} \right)$ in g

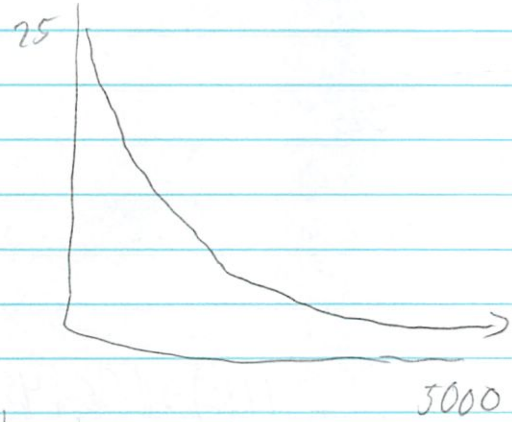
↑ half life- 1620 years

$t = 25\left(\frac{1}{2}\right)^{t/1620}$

a) $t(0) \to 25$ g

b) $t(1000) \to 16.30$ g

c)

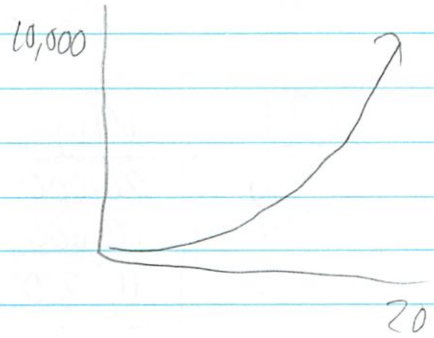


d) When 0g?

Never → its on asymptote

67 Bacteria

$P(t) = 100e^{.2197t}$



$P(0) \to 100$

$P(5) \to 299.97$

$P(10) \to 899.8$

68. Population Growth

$P(t) = 2500e^{.0293t}$

$t(0) = 2000$

$t(15) \to 3880$

$t(25) \to 5201$

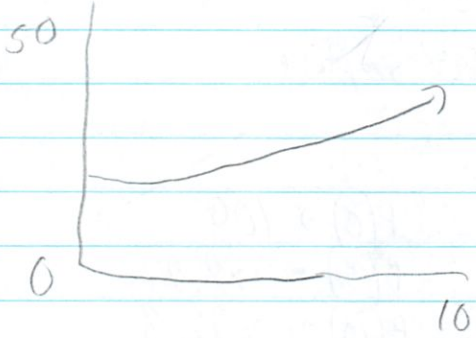
69. Inflation

4% for next 10 years

$C(t) = P(1.04)^t$

$P = 23.95$

$t(10) \to$ $35,45

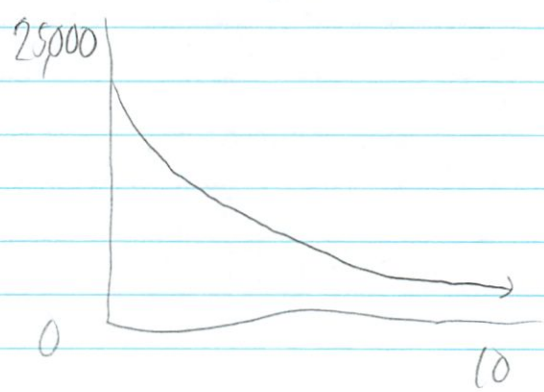


70. Depreciation

$V(t) = 20,000\left(\frac{3}{4}\right)^t$

↑ rounding rules

↑ round correclty



t	V(t)
0	20,000
1	15,000
2	11, 250
3	8,437,50
4	6328,13
5	4746.09
6	3559,57
7	2669.68
8	2002,26

$9 \to 1501,69$

$10 \to 1126,27$

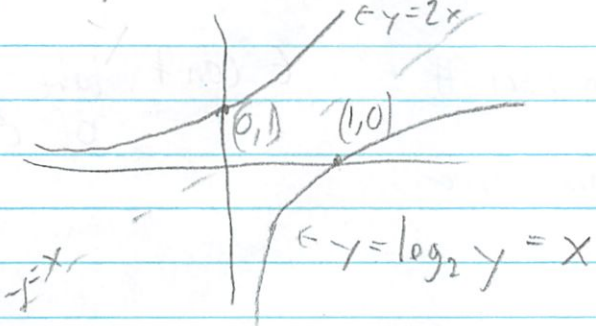
3.2 Logarithmic Functions

10/18

every exponental function $f(x) = a^x$ is a function and passess the horizontal line test means that its inverse is also a function

inverse of exponental function → logarithmic function
- * if $a^y = x$ then $y = \log_a x$
- * answer to a log is the exponent

ex1 $y = 2x$



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ex2 Evaluating logs

$f(x) = \log_2 x$ ↓ $x = 32$

$f(32) = \log_2 32$

↘ convert to exponential form

$2^y = x$ $y = 5$

$2^y = 32$ $2^5 = 32$

ex3 $y = \log_3 1 \Rightarrow 3^y = 1$

$y = 0$

ex4 $y = \log_4 2 \Rightarrow 4^y = 2$

$y = \frac{1}{2}$

P1 $y = \log_{10}\left(\frac{1}{100}\right) \rightarrow 10^y = \frac{1}{100}$

$y = -2$

p2 $y = \log_{16} \frac{1}{4}$ $16^{y} = \frac{1}{4}$

$y = -\frac{1}{2}$

→ ″ $16^{\frac{1}{2}} = 4$

$\sqrt{16} = 4$

so negitive

Calculator

$\boxed{\log}$ $\boxed{2}$ $\boxed{.}$ $\boxed{5}$ $\boxed{\text{Enter}}$

ex1 $\log 2.5 \to \log_{10}(2.5) \approx .3979$

↑ calc

p1 $\log(-2.6) \to$ non real # ← can't take real log of ⊖ #s

p2 $\log(0) \to$ domain error

Properties of Logs ✱ (Identies)

1. $\log_a 1 = 0 \to a^0 \to 0$
2. $\log_a a = 1 \to a^1 = a$
3. $\log_a a^x = x \to a^{\log_a x} = x \to a^x = a^x$
4. $a^{(\log_a x)} = x$
5. If $\log_a x = \log_a y \to x = y$

Solve for x or simplify

p1 $\log_4 4 = 1$

p2 $\log_5 5^x = x$

p3 $7^{(\log_7 14)} = 14$

p4 $28_2 2^{-1} = -1$

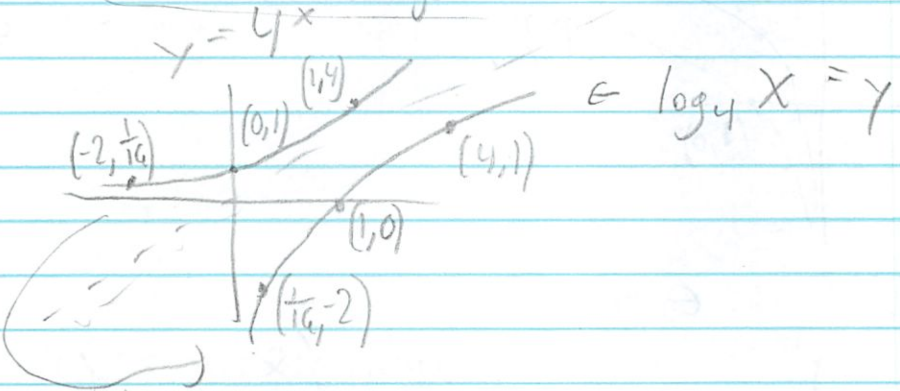
Convert to exponential or log form

p195

12. $9^{3/2} = 27 \rightarrow \log_9 27 = \frac{3}{2}$

14. $\log_e 4 = x \rightarrow e^x = 4$

Graphs of Logs



points flipped

$f(x) = \log_a x \leftarrow a > 0$

$x > 0$

Domain = $(0, \infty)$ or $\{x \mid x > 0\}$

Range = $(-\infty, \infty)$

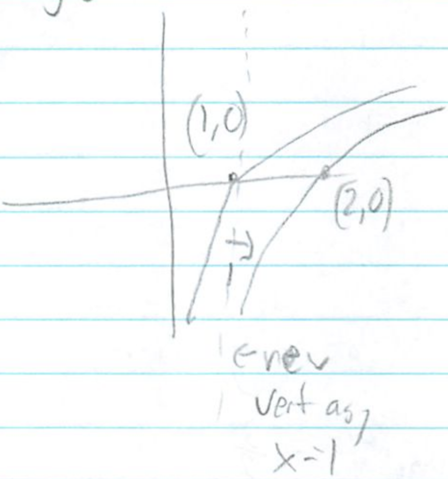
Intercept $(1,0)$

Increasing

Vert asy $\rightarrow x = 0$ (y-axis)

Graph Shifts (transformations)

ex1 $\log_{10}(x-1)$ → 1



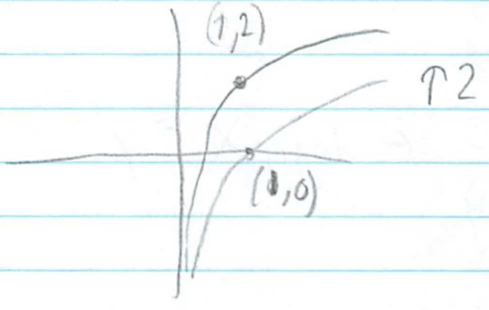
Domain → $x-1>0$

$x>1$

vert asy $x=1$

ex2 $h(x)=2+\log_{10}x$

↑2



vert asy still $x=0$

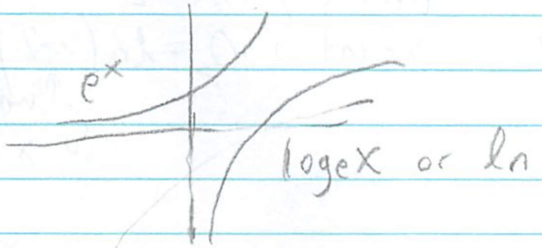
ex3 $\log_{10}{}^{x+7}$ ← 7

Domain $\{x : x>-7\}$

Vert asy $x=-7$

Natural logs → $\boxed{\ln}$ ← symbol
logs in base e

$\ln(x) \rightarrow \log_e x \rightarrow x > 0$



ex $\ln(18,31) \rightarrow 2,907$
$\boxed{LN}\ \boxed{(}\ \boxed{18}\ \boxed{.}\ \boxed{3}\ \boxed{1}\ \boxed{Enter}$

ex2 $\ln(-1) \rightarrow$ error non real

* Properties

1. $\ln(1) = 0 \rightarrow e^0 = 1$
   ↳ $\log_e 1 = 0$
2. $\ln(e) = 1 \rightarrow \log_e e = 1$
3. $\ln(e^x) = x \rightarrow \log_e e^x = x$
4. $e^{\ln(x)} = x \rightarrow e^{\log_e x} = x$

P1 $\ln\left(\frac{1}{e}\right) \rightarrow \log_e \frac{1}{e} \rightarrow \log_e e^{-1} \rightarrow -1$

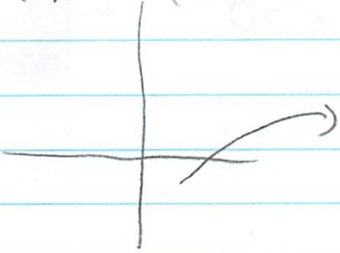
P2 $e^{\ln(5)} = 5 \rightarrow$

P3 $7 \cdot \ln(\underbrace{e^0}_{1}) \rightarrow 7\ln(\underbrace{1}_{0}) \rightarrow 7 \cdot 0 \rightarrow 0$

Big on test

↳ Finding Domains and Vert asy of log functions

ex1 $f(x) = \ln(x-2)$



Domain → $x-2>0$

$x>2$

Vert asy = $x=2$

x-int → $0 = \ln(x-2)$ ↓ $\ln(1)=0$

↱ want to be 1 b/c ↑

↳ $x-2=1$

$+2 \quad +2$

$x=3$

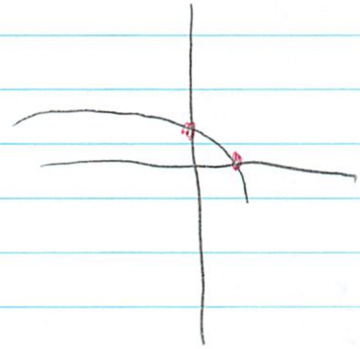
y-int → $\ln((0)-2)$

$\ln(-2)$

not possible

no y-int

ex2 $g(x) = \ln(2-x)$



Domain → $2-x>0$

$-x > -2$

$x \overset{\textcircled{s}}{\cancel{>}<} 2$

vert asy $x=2$

x-int → $0 = \ln(2-x)$

↳ $2-x=1$

$-x=-1$

$x=1$ $(1,0)$

y-int → $\ln(2-(0))$

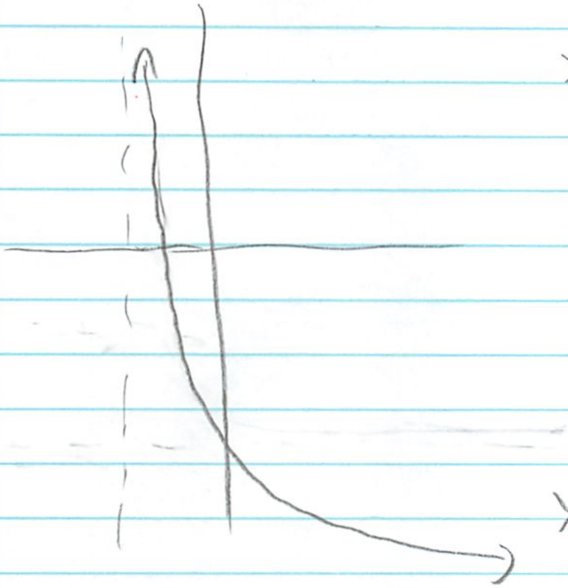
$\ln(2)$

~~not possible~~

type in calc ↳ $(0, .6931)$

p195

44. $-\log_3(x+2)-4$



Domain → $x+2>0$

$x>-2$

Vertasy = $x+2=0$

$x=-2$

x-int → $-\log_3(x+2)-4=0$

$-\log_3(x+2)=4$

$\div -1 \qquad \div -1$

$\log_3(x+2)=-4$

$3^{-4}=x+2$

$\frac{1}{81}=x+2$

$-\frac{161}{81}=x$

$(-1,987,0)$

y-int → $-\log_3((0)+2)-4$

$-\log_3(2)-4$

$-\left[\frac{\log 2}{\log 3}\right]-4$ ← calc solve

$(0,-4.6)$

3.2 Logarithmic Functions

HW

10/18

p 195 ✓

Write in exponential form

3 $\log_7 \frac{1}{49} = 3 \rightarrow 7^{\overset{3}{\cancel{x}}} = \frac{1}{49}$

5, $\log_{32} 4 = \frac{2}{5} \rightarrow 32^{2/5} = 4$

Write in logarithmic form

9, $5^3 = 125 \rightarrow \log_5 125 = 3$

11, $81^{1/4} = 3 \rightarrow \log_{81} 3 = \frac{1}{4}$

14, $10^{-3} = ,001 \Rightarrow \log_{10} ,001 = -3$

Evaluate

17, $\log_2 16 \rightarrow 2^4 = 16 \rightarrow \textcircled{4}$

18. $\log_{16} \frac{1}{4} \rightarrow 16^{?} = \frac{1}{4} \rightarrow \boxed{-\frac{1}{2}}$ ← ⊖ to get fractions

Calc Eval

23, $6 \log_{10} (14,8) \rightarrow 7,0216$

Solve for x

✓ 27, $\log_6 6^2 = x$

$2 = x$

29, $\log_8 x = \log_8 10^{-1}$

$x = 10^{-1}$

$x = .1$

Graph Shifts

33, $f(x) = e^x$

$g(x) = \ln x \rightarrow \log_e x$ } Inverse (y = X reflection)

37. $\log_{10}\left(\frac{x}{5}\right)$

Domain $\frac{x}{5} > 0$

$\cdot 5 \quad \cdot 5$

$x > 0$

vert asy $\frac{x}{5} = 0$

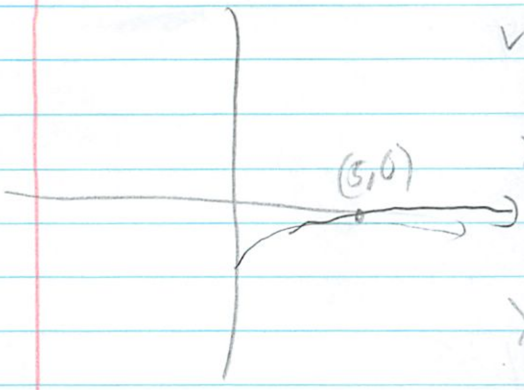
$x = 0$

x-int → $\frac{x}{5} = 1$

$x = 5$ $\quad (5,0)$

y-int $\log_{10}\left(\frac{0}{5}\right)$

$\log_{10}(0)$ $\quad$ ~~(0,0)~~ → error



39. $\log_4(x-3)$

Domain → $x - 3 > 0$

$x > 3$

vert asy → $x - 3 = 0$

$x = 3$

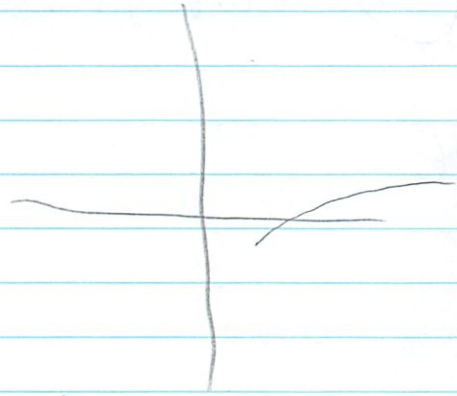
x-int → $x - 3 = 1$

$x = 4$

y-int → $\log_4((0)-3)$

$\log_4(-3)$ ← neg

error



41. $-\log_{10} x + 2$

Domain → $x > 0$

vert asy → $x = 0$

x-int → $-\log_{10} x + 2 = 0$

$-\log_{10} x = -2$

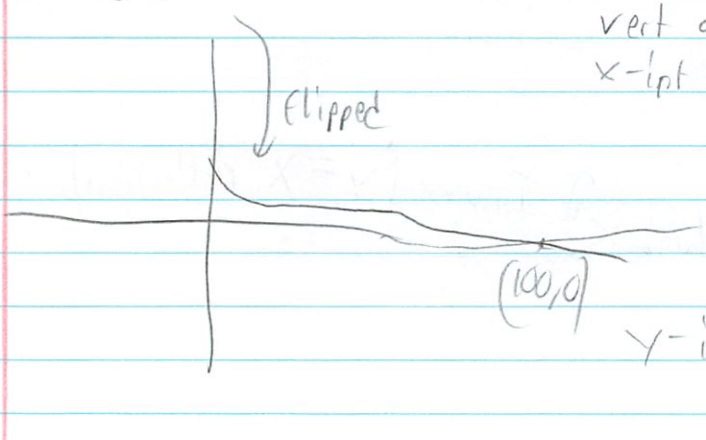
$\log_{10} x = 2$ } Δ to exponential form

$10^2 = x$

$100 = x$

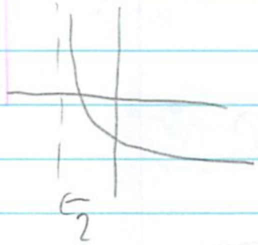
y-int → $-\log_{10}(0) + 2$

error



p196

47. Match graph
$-\log_3(x+2)$ → d
↑ flipped ←2

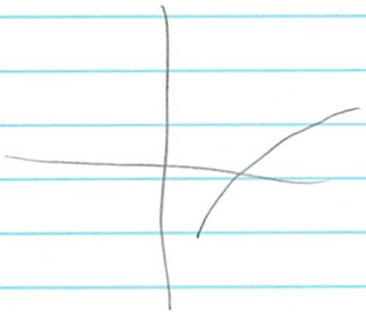


51. $\log_2 x$ ↑ y flipped
$y = \log_2 x$

55. Eval
$-\ln\left(\frac{1}{2}\right) = .6931$

59. $e^{\ln(1.8)}$ $e^{\log_e 1.8} = 1.8$

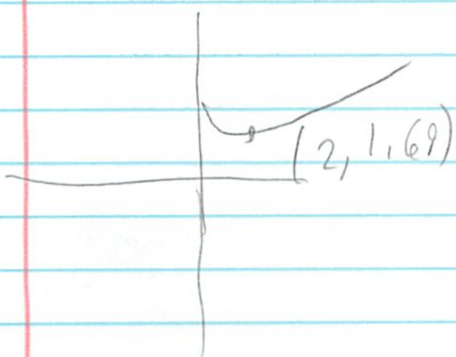
61. Graph
$\ln(x-1)$



Domain → $x-1>0$
$x>1$
Vert as, → $x-1=0$
$x=1$
x-int → $x-1=1$
$x=0$ $(0,0)$
y-int → $\ln((0)-1)$
$\ln(-1)$
non real

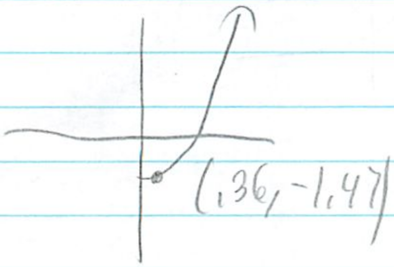
65. $\frac{x}{2} - \ln\left(\frac{x}{4}\right)$

domain



Domain → $x>0$
Decreasing $(0, 2]$
Increasing $(2, \infty)$

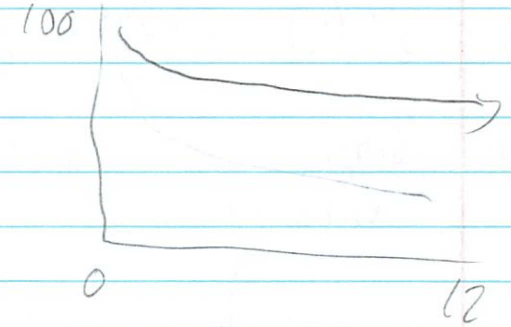
67. $4x \ln x$



Domain $x > 0$
Decreasing $(0, .37)$
Increasing $(.37, \infty)$

69. Human memory model
$f(t) = 80 - 17 \log_{10}(t+1) \rightarrow 0 \leq t \leq 12$

$f(0) \rightarrow 80$
$f(4) \rightarrow 68.118$
$f(10) \rightarrow 62.296$



p197

Sound intensity
73. $\beta\ 10 \log_{10}\left(\frac{Ix}{10^{-12}}\right)$

$x(1) \rightarrow 120$
$x(10^{-2}) \rightarrow 100$

No th # of decibles is $\frac{1}{5}$ as great
this is due to te model

3.3 Properties of Logs

10/19

Change of Base Formulas

-used to eval logs in other bases

$$\boxed{\log_a x = \frac{\log_{10} x}{\log_{10} a}}$$

↓ to type in calculator

ex1 $\log_4 25 = \frac{\log_{10} 25}{\log_{10} 4} = 2.322$

P1 $\log_2 12 = \frac{\log 12}{\log 2} = 3.585$

or $\log_2 12 = \frac{\ln(12)}{\ln(2)} = 3.585$

Properties of logs

a → (+)# ≠ 1

n → real #

If u + v are (+) real #s then

→ $\log_a(u \cdot v) = \log_a u + \log_a v$

ex $\log_3(8 \cdot 3) = \log_3(8) + \log_3(3)$

2.89

$\log_a\left(\frac{u}{v}\right) = \log_a u - \log_a v$

ex) $\log_4\left(\frac{3}{5}\right) = \log_4 3 - \log_4 5$

$-.368$

$\log_a u^n = n \cdot \log_a u$

ex) $\log_5 2^2 = 2 \log_5 2$

Write each log in terms of $\ln(2)$ + $\ln(3)$

P1 $\ln(6) \overset{\to \ln(2\cdot 3) \to}{=} \ln(2) + \ln(3)$

P2 $\ln\left(\frac{2}{27}\right) = \ln(2) - \ln(27)$

$\ln(2) - (\ln(3^3))$

$\ln(2) - 3\ln(3)$

p203

18. $\log_2(4^2 \cdot 3^4) = \log_2 4^2 + \log_2 3^4$

$2\log_2 4 + 4\log_2 3$

$\to 2\log_2 2^2 + 4\log_2 3$

$2\cdot 2\cdot \log_2 2 + 4\log_2 3$

$2\cdot 2\cdot 1 + 4\log_2 3$

$4 + 4\log_2 3$

20. $\ln \frac{6}{e^2} \to \ln(6) - \ln e^2$

$\ln(6) - \log_e e^2$

$\ln(6) - 2\log_e e$

$\ln(6) - 2\cdot 1$

$\ln(6) - 2$

Expanding log Expressions

ex1 $\log_4 5x^3y \to \log_4 5 + \log_4 x^3 + \log_4 y$

$\log_4 5 + 3\log_4 x + \log_4 y$

ex2 $\ln\left(\frac{\sqrt{3x-5}}{7}\right) \to \ln(3x-5)^{\frac{1}{2}} - \ln(7)$

$\frac{1}{2}\ln(3x-5) - \ln(7)$ ← leave if not evaluating

p203

40. $\ln\left(\sqrt{x^2(x+2)}\right) \to \ln\left(\sqrt{x^2}\right) + \ln\left(\sqrt{x+2}\right)$

$\ln(x) + \frac{1}{2}\ln(x+2)$

42. $\log_b\left(\frac{\sqrt{x}\ y^4}{z^4}\right)$ $\log_b(\sqrt{x}\ y^4) - \log_b(z^4)$

$\frac{1}{2}\log_b(x) + 4\log_b(y) - 4\log_b(z)$

Condensing Log Expression

ex1 $\frac{1}{2}\log_{10} x + 3\log_{10}(x+1)$

$\log_{10} x^{\frac{1}{2}} + \log_{10}(x+1)^3$

$\log_{10}\sqrt{x}(x+1)^3$

ex2 $2\ln(x+2) - \ln(x)$

$\ln((x+2)^2) - \ln(x)$

$\ln\left(\frac{(x+2)^2}{x}\right)$

* I always put ln() P with parthenses

ex3 $\frac{1}{3}\left[\log_2 x + \log_2(x-4)\right]$

$\frac{1}{3}(\log_2(x(x-4))$

$\log_2\sqrt[3]{x\cdot(x-4)}$

p204

62 $\frac{1}{2}\left[\ln(x+1) + 2\ln(x-1)\right] + 3\ln(x)$

$\frac{1}{2}\left[\ln(x+1) + \ln((x-1)^2)\right] + \ln(x^3)$

$\frac{1}{2}\left[\ln((x+1)(x-1)^2\right] + \ln(x^3)$

$\ln\left(\sqrt{(x+1)(x-1)^2}\right) + \ln(x^3)$

$\ln\left(\sqrt{(x+1)}(x-1)(x^3)\right)$

Find exact value w/o calc

204/70. $\log_5 \frac{1}{125} \rightarrow \log_5 125^{-1}$

$-\log_5 125$

$-\log_5 5^3$

-3

74. $\log_4 2 + \log_4 32 \rightarrow \log_4(64)$

$\log_4 4^3$

$3 \cdot \log_4 4$

$3 \cdot 1$

3

☆ On test →

$(625x^{10}y^2)^{\frac{1}{2}} - \left(\frac{1}{x^6y^2}\right)^{-\frac{1}{2}}$

$\sqrt{625x^{10}y^2} - \downarrow$

$25x^5y - \left(\frac{1}{x^6y^2}\right)^{-\frac{1}{2}}$

$25x^5y - \left(\frac{1^{-\frac{1}{2}}}{x^{6\cdot-\frac{1}{2}}y^{2\cdot-\frac{1}{2}}}\right)$

$25x^5y - \left(\frac{1}{x^{-3}y^{-1}}\right)$

$25x^3y - x^3y$

3.3 Properties of Logs

HW

10/21

p203

Rewrite

			common	natural
5.	$\log_a \frac{3}{10}$	→	$\frac{\log_{10} \frac{3}{10}}{\log_{10} a}$	$\frac{\ln(\frac{3}{10})}{\ln(a)}$

15. $\log_{15} 1460$ → $\frac{\log_{10} 1460}{\log_{10} 15} = 2.69$

Rewrite + Simplify

19 $\ln(5e^6)$ → $\ln(5) + \ln(e^6)$

①

$\ln(5) + 6 \log_e e$

$\ln(5) + 6$

Verify

21. $\log_5 \frac{1}{250} = -3 - \log_5 2$

$\log_5 250^{-1}$

$-\log_5 250$

$-\log_5 125 \cdot 2$

$-\log_5 125 - \log_5 2$

$-\log_5 5^3 - \log_5 2$

$-3 - \log_5 2$

Expand

33. $\ln(a^2 \sqrt{a-1})$ $a > 1$

$\ln(a^2) + \ln((a-1)^{\frac{1}{2}})$

$2 \ln a + \frac{1}{2} \ln(a-1)$

35 $\ln\left(\sqrt[3]{\frac{x}{y}}\right)$

$\frac{1}{3} \ln\left(\frac{x}{y}\right)$ ↓ 1 more

$\frac{1}{3} \ln(x) - \frac{1}{3} \ln(y)$

37. $\ln\left(\frac{x^2-1}{x^3}\right)$ $x>1$

$\ln(x^2-1) - \ln(x^3)$

diff 2 sq

$\ln(x+1) + \ln(x-1) - 3\ln(x)$

39. $\ln\left(\frac{x^4\sqrt{y}}{z^5}\right)$

$\ln(x^5\sqrt{y}) - \ln(2^5)$

$\ln(x^5) + \ln(\sqrt{y}) - 5\ln(2)$

$5\ln(x) + \frac{1}{2}\ln(y) - 5\ln(2)$

Condense

49. $2\log_2(x+3)$

✓ $\log_2(x+3)^2$

51. $\frac{1}{3}\log_3 7x$

✓ $\log_3 \sqrt[3]{7x}$

53. $\ln(x) - 3\ln(x+1)$

$\ln(x) - \ln((x+1)^3)$

$\ln\left(\frac{x}{(x+1)^3}\right)$

p 204.

57. $\ln(x) - 2[\ln(x+2) + \ln(x-2)]$

$\ln(x) - 2(\ln(x^2-4))$ $\leftarrow 2 \cdot -2 = -4 \neq -2$

$\ln(x) - \ln(x^2-4)^2$ simple math error

$\ln\left(\frac{x}{(x^2-4)^2}\right)$

59. $\frac{1}{3}\left[2\ln(x+3)+\ln(x)-\ln(x^2-1)\right]$

$\frac{1}{3}\left[\ln(x+3)^2+\ln(x)-\ln(x^2-1)\right]$

$\ln\left(\sqrt[3]{\frac{(x+3)^2\,x}{x^2-1}}\right)$

61. $\frac{1}{3}\left[\ln(y)+2\ln(y+4)\right]-\ln(y-1)$

$\frac{1}{3}\left[\ln(y)+\ln((y+4)^2)\right]-\ln(y-1)$

$\ln\left(\frac{\sqrt[3]{y(y+4)^2}}{y-1}\right)$

63. $y_1 = 2\left[\ln(8)-\ln(x^2+1)\right]$

$y_2 = \ln\left[\frac{64}{(x^2+1)^2}\right]$

they are the same thing

✓ 67. $\log_3 9 \rightarrow 3^2=9$

$\boxed{2}$

✓ 71. $\log_2(-4)$ - undef.

73. $\log_5 75 - \log_5 3$

$\log_5 (25\cdot 3) - \log_5 3$

$\log_5 25 + \cancel{\log_5 3} - \cancel{\log_5 3}$

$\log_5 25$ -

$\log_5 5^2$

2

or → $\log_5 \frac{75}{3}$

$\log_5 25$

$\log_5 5^2$

2

✓ 79. $\ln\left(\frac{1}{\sqrt{e}}\right) \rightarrow \ln(1) - \frac{1}{2}\ln(e)$

$0 - \frac{1}{2}\log_e e$

$-\frac{1}{2}$

81. $B = 10\log_{10}\left(\frac{I}{10^{-12}}\right)$

$10\log_{10} I - 10\log_{10} 10^{-12}$

✓ $10\log_{10} I - 10 \cdot -12$

$120 + 10\log_{10} I$

b)

I	B
10^{-4}	80
10^{-6}	60
10^{-8}	40
10^{-10}	20
10^{-12}	0
10^{-14}	-20

3.4 Solving Log + Exponential Functions

10/22

Rules

x = y

$a^x = a^y$

$a^{\log_a x} = x$

$\log_a a^x = x$

ex1 $2^x = 32 \to x = 5$

$\uparrow 2^5$

ex2 $\left(\frac{1}{3}\right)^x = 9$ $x = -2$

$\downarrow (3^{-1})^x = 3^2$ ← get same bases + exponents =

$(3^{-1})^{-2} = 9$

ex3 $\frac{1}{8} = 4^{6x+2}$

$(2^3)^{-1} = (2^2)^{6x+2}$ ← same bases

↓ take exponents

$-3 = 12x + 4$

$-7 = 12x$

$\frac{-7}{12} = x$

ex4 $\ln(x) - \ln(3) = 0$

$\ln(x) = \ln(3)$

$x = 3$

ex5 $e^x = 7$

$\cdot\ln \quad \cdot\ln$

$\ln(e^x) = \ln(7)$

$\log_e e^x = \ln(7)$

$x = \ln(7)$

ex6 $\log_e x = -3$ $\to$ or × each side by base e

$e^{-3} = x$ $e^{\log_e x} = e^{-3}$

$x = e^{-3}$

p213

24, $\left(\frac{3}{4}\right)^{x} = \frac{27}{64}$ $\quad x=3$

$\frac{3^x}{4^x} = \frac{3^3}{4^3} \rightarrow$

36, $\ln(3x+5) = 8$

$\log_e(3x+5) = 8$ $\quad \downarrow$ Δ to exponent

$e^8 = 3x+5$

$\frac{e^8-5}{3} = \frac{3x}{3}$

$\frac{e^8-5}{3} = x \xrightarrow{} 991,98$

Find # by typing in calc

44, $6^{5x} = 3000$

$\cdot\log_6 \quad \cdot\log_6$

$\log_6 6^{5x} = \log_6 3000$

$5x = \log_6 3000$

$5x = \frac{\log 3000}{\log 6}$

$x = \frac{\left(\frac{\log 3000}{\log 6}\right)}{5}$

→ or $\ln(6^{5x}) = \ln(3000)$

$\frac{5x\ln(6)}{\ln(6)} = \frac{\ln(3000)}{\ln(6)}$

$5x = \frac{\ln(3000)}{\ln(6)}$

$x = \frac{\left(\frac{\ln(3000)}{\ln(6)}\right)}{5}$

$=$

,8937

P1

$e^x = 72$

$\cdot \ln \quad \cdot \ln$

$\ln(e^x) = \ln(72)$

$x = \ln(72)$

4.277

P2

$\frac{3(2^x)}{3} = \frac{42}{3}$

$2^x = 42/3$

$2^x = 14$

$\log_2 2^x = \log_2 14$

$x = \log_2 14$

$x = \frac{\log 14}{\log 2}$ 3.807

P3

$\frac{4e^{2x}}{4} = \frac{40}{4}$

$e^{2x} = 10$

$\ln(e^{2x}) = \ln(10)$

$2x = \ln(10)$

$x = \frac{\ln(10)}{2}$ 1.151

50. $-14 + 3e^x = 11$

$+14 \qquad +14$

$\frac{3e^x}{3} = \frac{25}{3}$

$e^x = 25/3$

$\ln(e^x) = \ln(25/3)$ 2.120

$x = \ln(25/3)$

52. $4^{-3t} = .10$

$\log_4 4^{-3t} = \log_4 .10$

$-3t = \log_4 .10$

$-3t = \frac{\log 10}{\log 4}$

$t = \left(\frac{\log 10}{\log 4}\right) / -3$ $\quad -.554$

P4 $2(3^{2t-5}) - 4 = 11$

$\quad +4 \quad +4$

$\frac{2(3^{2t-5})}{2} = \frac{15}{2}$

$3^{2t-5} = 15/2$

$\log_3 3^{2t-5} = \log_3 15/2$

$2t-5 = \frac{\log 15/2}{\log 3}$

$2t = \left(\frac{\log 15/2}{\log 3}\right) + 5$

$t = \frac{\left(\frac{\log 15/2}{\log 3}\right) + 5}{2} = 3.417$

Solve an exponential equation in quadratic form

ex $e^{2x} - 3e^x + 2 = 0$

$(e^x - 2)(e^x - 1)$

when

you x you

add exponents

$e^x - 2 = 0$

$e^x = 2$

$\ln(e^x) = \ln(2)$

$x = \ln(2)$

.69314

$e^x - 1 = 0$

$e^x = 1$

$\ln(e^x) = \ln(1)$

$x = \ln(1)$

$x = 0$

$+ \quad \times$

56. $e^{2x} - 5e^{x} + 6 = 0$

$(e^{x} - 3)(e^{x} - 2)$

$\downarrow \qquad \downarrow$

$e^{x} - 3 = 0 \qquad e^{x} - 2 = 0$

$e^{x} = 3 \qquad e^{x} = 2$

$\ln(e^{x}) = \ln(3) \qquad \ln(e^{x}) = \ln(2)$

$x = \ln(3) \qquad x = \ln(2)$

1.099 .693

58. $\frac{525}{1+e^{-x}} = 275$

$\frac{525}{275} = \frac{275(1+e^{-x})}{275}$

$\frac{525}{275} = 1 + e^{-x}$

-1

-1

$\left(\frac{525}{275}\right) - 1 = e^{-x}$

$\log_e\left(\frac{525}{275} - 1\right) = \log_e e^{-x}$

$-\log_e\left(\frac{525}{275} - 1\right) = x$

$\boxed{\frac{-\log\left(\frac{525}{575} - 1\right)}{\log e}} = x$

$x = .095$

2/14

60. $$\left(16 + \frac{.878}{26}\right)^{3t} = 30$$

$$3t \ln\left(16 + \frac{.878}{26}\right) = \ln(30)$$

$$3t \quad \frac{\ln(30)}{\ln\left(16 + \frac{.878}{26}\right)}$$

$$\frac{3t}{3} = \frac{1.225}{3}$$

$$t = .408$$

P5 $$5 + 2\ln(x) = 4$$

$$-5 \qquad -5$$

$$\frac{2\ln(x)}{2} = \frac{-1}{2}$$

$$\log_e(x) = -\frac{1}{2}$$

$$e^{-\frac{1}{2}} = x$$

$$.607$$

P6 $$\log_3(5x-1) = \log_3(x+7)$$

$$5x - 1 = x + 7$$

$$+1 \qquad +1$$

$$5x = x + 8$$

$$-x \qquad -x$$

$$\frac{4x}{4} = \frac{8}{4}$$

$$x = 2$$

82, $4\log_{10}(x-6)=11$

$\log_{10}(x-6)=\frac{11}{4}$

$10^{11/4}=x-6$

$10^{11/4}+6=x$

568, 34

84, $\ln(\sqrt{x-8})=5$

$\cancel{\frac{1}{2}}\ln(x-8)^{1/2}=5$ ↙ don't move over

~~$\frac{1}{2}e^{5}=x-8$~~

~~$\frac{1}{2}e^{5}+8=x$~~

~~82,207~~

$e^{5}=(x-8)^{1/2}$

$e^{5\cdot 2}=x-8$

$e^{10}=x-8$

$e^{10}+8=x$

22,034.465

$e^{2x}-3e^{x}+2=0$

e^{x} $\ln(2)$ ← found x-int / zeros

$\ln(1)$

↑ can put in calc to confirm

3.4 Solving Exponential + Log Functions
HW

10/22

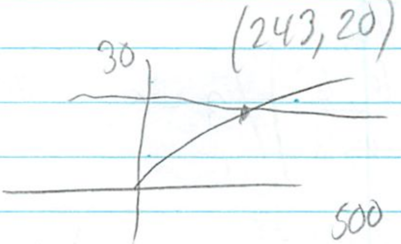
213

Find x-values

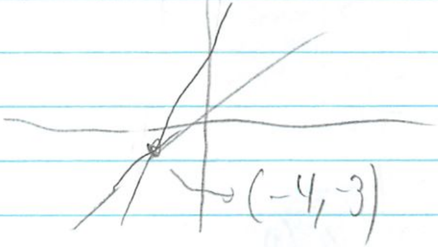
5. $\log_4(3x) = 3$

a) 21.3560 → $\log_4(3 \cdot 21.3560) = 3$ ✓
b) −4 → $\log_4(3 \cdot -4) =$ non real ⊗
c) $\frac{63}{3}$ → $\log_4(3 \cdot \frac{63}{3}) = 3$ ✓

13. Graph + Find Intercetion
$f(x) = 4\log_3 x$
$g(x) = 20$



15. $\ln(e^{x+1})$
$2x+5$



Solve for x

21. $\left(\frac{1}{8}\right)^x = 64$
$(8^{-1})^x = 64$ $x = -2$
$(8^{-1})^x = 8^2$

25. $e^x = 4$
$\log_e e^x = 4$
$x = 4$

35. $\ln(2x-1) = 5$
$\log_e 2x-1 = 5$
$e^5 = 2x-1$
$e^5+1 = 2x$
$\frac{e^5+1}{2} = x$

Simplify

39. $e^{\cancel{\ln}(5x+2)}$ Cancles out

$5x+2$

Solve

43. $8^{3x}=360$

$\log_8 360 = 3x$

$\frac{\log 360}{\log 8} = 3x$

$\left(\frac{\log 360}{\log 8}\right)/3 = x$ $\quad x = .9435$

49. $7-2e^{x}=5$

$-7 \qquad -7$

$\frac{-2e^{x}}{-2} = \frac{-2}{-2}$

$e^{x}=1$

$\log_e e^{x} = \ln(1)$

$x=0$

55. $e^{2x}-4e^{x}-5=0$

$(e^{x}-5)(e^{x}+1) \rightarrow e^{x}+1=0$

$e^{x}=-1$

$x=\ln(-1)$

error

$\downarrow$

$e^{x}-5=0$

$e^{x}=5$

$\log_e e^{x} = \ln(5)$

$x = \ln(5) \rightarrow 1.609$

57. $\frac{400}{1+e^{-x}} = 350$

$\frac{400}{350} = \frac{350\cdot(1+e^{-x})}{350}$

$8/7 = 1+e^{-x}$

$1/7 = e^{-x}$

$\ln(1/7) = -x$

$-\ln(1/7) = x$

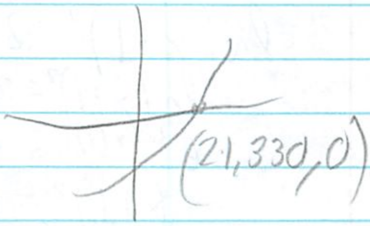
1.946

p214

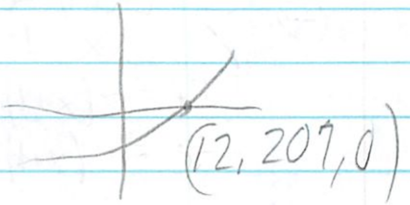
Find zero w/ calc

65. $\left(1+\frac{.065}{365}\right)^{365t}=4$ ∠general form

$\left(1+\frac{.065}{365}\right)^{365t}-4=7$



71. $e^{.09t}-3$



Solve

77. $-2+2\ln(3x)=17$

$\quad_{+2} \qquad\qquad _{+2}$

$\frac{2\ln(3x)}{2}=\frac{19}{2}$

$\ln(3x)=19/2$

$\frac{e^{19/2}}{3}=\frac{3x}{3}$

$x=\frac{e^{19/2}}{3}$ $\quad x=4453.24$

81. $7\log_4(.6x)=12$

$\log_4(.6x)=12/7$

$\frac{4^{12/7}}{.6}=\frac{.6x}{.6}$ $\quad x=17.945$

$\frac{4^{12/7}}{.6}=x$

83. $\ln(\sqrt{x+2})=1$

$e^{\ln(\sqrt{x+2})}=e^1$

$\sqrt{x+2}=e^1$

$x+2=e^2$

$\quad_{-2} \qquad _{-2}$

$x=e^2-2$ $\quad x=5.389$

85. $\ln(x+1)^2 = 2$

$\ln(x+1)^2 = 2$

$e^{\ln(x+1)^2} = e^2$

$(x+1)^2 = e^2$ ✓

$\pm(x+1) = \pm e$

⊖

$x = \pm e - 1$ → 1.718 / −3.718 ← Extraneous Domain

$+e = -(x+1) \rightarrow \pm e - 1 = x$

$-e = -(x+1) \rightarrow e - 1 = x$

doesn't even work

Graph on calc to verify ans

$\ln(-1+1)^2$

$\ln(0)$

ⓧ

~~$\ln(-2+1)$~~

~~$2\ln(-2+1)$~~

~~$2\ln(-1)$~~

ⓧ

even though it's square

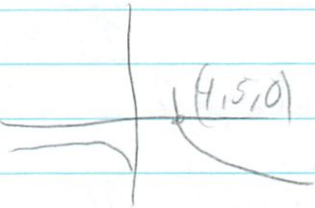
(?)

$x > -1$

215

Zero

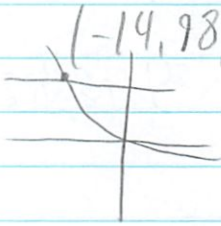
99. $\log_3 x + \log_3(x-3) = 1$



Intersection

105. $y_1 = 80$

$y_2 = 4e^{-.2x}$



3.4 Solving Log + Exponential Equations
Day 2

10/23

ex1 $\ln(x-2) + \ln(2x-3) = 2\ln(x)$ 1st Find domain

$x-2>0 \quad 2x-3>0 \quad x>0$

$x>2 \quad x>\frac{3}{2}$

find most restrictive subdomain

$x>2$

remove logs by putting all e^x

$(x-2)(2x-3) = x^2$

$2x^2-7x+6 = x^2$

$-x^2 \qquad -x^2$

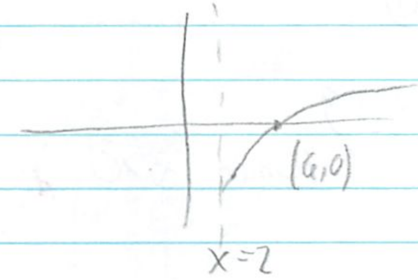
Set = to 0

$x^2-7x+6=0$

Factor

$(x-6)(x-1)$

$\boxed{x=6} \quad \cancel{x=1}$ not in domain - extranious



Graph to verify

ex2 $\log_3(5x-12) + \log_3(x) = 2$

Dm: $5x-12>0 \quad x>0$

$5x>12$

$x>\frac{12}{5}$

Domain $x>\frac{12}{5}$

$\log_3((5x-12)x) = 2$

$\log_3 5x^2-12x = 2$

↓ Δ to exponential form

$3^2 = 5x^2-12x$

$9 = 5x^2-12x$

$0 = 5x^2-12x-9$

$\rightarrow (x-3)(5x+3)$

$\boxed{x=3} \quad \cancel{x=-3/5}$ ← extranious

P1 $2\log_2 x - \log_2(2x-2) = 1$

$x > 0$ $2x-2>0$ Domain

$2x > 2$ $x < 1$

$x > \frac{2}{2}$

$x > 1$

$\log_2\left(\frac{x^2}{2x-2}\right) = 1$

$2^1 = \frac{x^2}{2x-2}$

$2(2x-2) = x^2$ ⌊ cross x to get quadratic

$4x-4 = x^2$

$-x^2+4x-4=0$ ←set = to 0

$(x-2)(x-2)$

$x=2$

✓ not extranious

P2 $5\log_3 x - 2\log_3 x = -3$

$\log_3\left(\frac{x^5}{x^2}\right) = -3$ ← Domain $x > 0$

$\log_3 x^3 = -3$

$3^{-3} = x^3$

$\frac{1}{27} = x^3$

$\sqrt[3]{\;} \quad \sqrt[3]{\;}$

$\frac{1}{3} = x$ ✓

P3

93 $2\log_2(4x) - \log_2(3x+1) = 1$

$4x > 0$ $\quad$ $3x+1 > 0$

$x > 0$ $\quad$ $3x > -1$

$x > -\frac{1}{3}$

Domain
$x > 0$

$\log_2\left(\frac{(4x)^2}{3x+1}\right) = 1$

$2^1 = \frac{(4x)^2}{3x+1}$

$2 = \frac{16x^2}{3x+1}$ ← cross multiply

$2(3x+1) = 16x^2$

$6x+2 = 16x^2$

$-16x^2+6x+2=0$ $\quad$ $\xrightarrow{/2} 2(-8x^2+3x+1)$

$(x+,213)(x-,588)$ ← dosn't come out nicely

$x = -,\cancel{213}$ $\quad$ $x = ,588$

extranious $\quad$ ✓ in domain

2/4

92. $\log_{10} 4x - \log_{10}(12+\sqrt{x}) = 2$

Dm: $4x > 0$ $\quad$ $12+\sqrt{x} > 0$ ← can't take $\sqrt{-}$, so x is (+)

$x > 0$ $\quad$ dosn't matter sub in 0

$\log_{10}\left(\frac{4x}{12+\sqrt{x}}\right) = 2$

$10^2 = \frac{4x}{12+\sqrt{x}}$

$100(12+\sqrt{x}) = 4x$

$1200 + 100\sqrt{x} = 4x$

$0 = 4x - 100\sqrt{x} - 1200$

$0 = 4(x - 25\sqrt{x} - 300)$

$\sqrt{x} = \frac{25 \pm \sqrt{(-25)^2 - 4(1)(-300)}}{2 \cdot 1}$ ← quad formula

$\sqrt{x} = 33,86$ $\quad$ $\sqrt{x} = -8,86$

2 2 $\quad$ 2 2

$\boxed{x = 1146,5}$

↑ can't do $\sqrt{-}$ so extranious

✓ get on calc

ex1 $\underbrace{\ln(x)}_{\text{intersection}} = x^2 - 2$ $\underbrace{\text{or}}$ $\ln(x) - x^2 + 2 = 0$

↑find zero

$(.1379, -1.98)$

$(1.56, .45)$

Doubling the Investment

have $500 deposited

pays 6.75% interest

compounded continuously $A = Pe^{rt}$

$A = 500e^{.0675 \cdot t}$

How long will it take to 2x $

$$\frac{1000}{500} = \frac{500e^{.0675 \cdot t}}{500}$$

doubling for triple use 3

$$2 = e^{.0675 \cdot t}$$

$$\frac{\ln(2)}{.0675} = \frac{.0675 \cdot t}{.0675}$$

$$\frac{\ln(2)}{.0675} = t \text{ to double} \qquad t = 10.27 \text{ years}$$

2.4 Solving Exponential + Log Equations

HW 2

10/23

224/ 8. Interest

$\$20{,}000$ investment

.0105 APY

? double

? after 10 years

$$2 = e^{.0105t}$$

$$\ln(2) = .0105t$$

$$\frac{\ln(2)}{.0105} = t$$

$$t = 66.014 \text{ years}$$

$$A = 20000e^{.0105 \cdot 10}$$

$$A = \$22{,}214.21$$

$$A = Pe^{rt}$$

$$\frac{20{,}000}{10{,}000} = \frac{10{,}000}{10{,}000}^{er(12)}$$

10. $\$10{,}000$

12 years to 2x

rate

$$2 = e^{r(12)}$$

$$\frac{\ln(2)}{12} = \frac{12r}{12}$$

$$r = \frac{\ln(2)}{12}$$

$$r = 5.77\%$$

10 years

$$A = 10{,}000e^{.0577 \cdot 10}$$

$$A = 17{,}817.97$$

214/ 87. Solve

$$\log_4 x - \log_4(x-1) = \frac{1}{2}$$

$$\log_4\left(\frac{x}{x-1}\right) = \frac{1}{2}$$

$$4^{\frac{1}{2}} = \frac{x}{x-1}$$

$$2 = \frac{x}{x-1} \quad \text{cross x}$$

$$2(x-1) = x$$

$$2x - 2 = x$$

$$-2x \qquad -2x$$

$$-2 = -x$$

$$\boxed{x = 2}$$

Remember to check for extranious

89. ~~$\ln(x+5) = \ln(x-1) - \ln(x+1)$~~

~~$e^{\ln(x+5)} = e^{\ln(x-1)} - e^{\ln(x+1)}$~~

~~$x+5 = x-1-x-1$~~

~~$x+5 = -2$~~

~~$x = -7$~~

$\ln(x+5) = \ln\left(\frac{x-1}{x+1}\right)$

$x+5 = \frac{x-1}{x+1}$

$(x+5)(x+1) = x-1$

$x^2+6x+5 = x-1$

$x^2 \; 5x+6 = 0$

$(x+2)(x+3)$

both extranious

90. ~~$\ln(x+1) - \ln(x-2) = \ln(x)$~~

~~$x+1-(x-2) = x$~~

~~$x+1-x+2 = x$~~

~~$3 = x$~~

215

109. Find time to double + triple

$r = .085$

$2 = e^{.085t}$	$3 = e^{.085t}$
$\ln(2) = .085t$	$\ln(3) = .085t$
$\frac{\ln(2)}{.085} = t$	$\frac{\ln(3)}{.085} = t$
8.155 years	12.924 years

110. $r = .12$

$2 = e^{.12t}$	$3 = e^{.12t}$
$\ln(2) = .12t$	$\ln(3) = .12t$
$\frac{\ln(2)}{.12} = t$	$\frac{\ln(3)}{.12} = t$
5.78 years	9.16 years

90. Redo

$\frac{x+1}{x-2} = x$

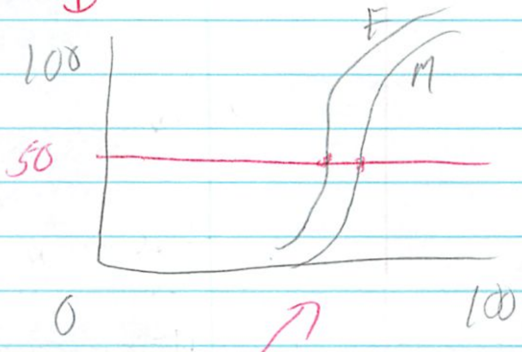
$x^2-2x = x+1$

$x^2-3x-1 = 0$ ← quad formula

$x = 3.3$

$x = -.30277$ ← extranious

logistic growth model
↓



Average hights

115. $m(x) = \frac{100}{1+e^{-.6114(x-69.71)}}$

$f(x) = \frac{100}{1+e^{-66607(x-64.51)}}$

b) horiz asy

$y=0$) limits on
$y=100$ percents

c) Find where $m(x)=50$

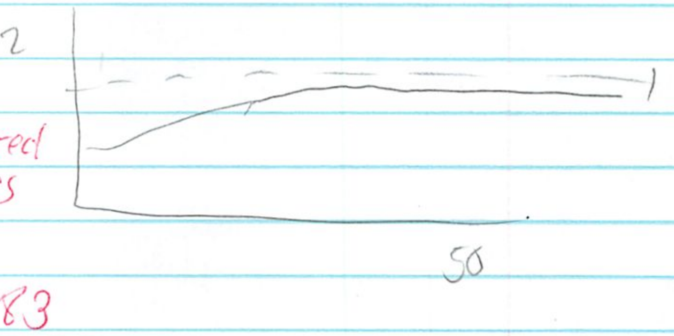
$y=50$

← at 50% is the average

hard →

males – 69.71 in
females – 64.51 in

116. Human Memory Model



a) $P = \frac{.83}{1+e^{-.2n}}$ ← porportion of correct responses

b) horiz asy $y = 1\ .83$

?! c) $P(60\%) = $? 4.79 trials → round 5 trials

?n ~~never it never goes above 1~~

↑ .60

↳ $.60 = \frac{.83}{1+e^{-.2n}}$

↓ cross multiply

3.4 5
More Pratice

10/24

1. $7^x = 5^{x-4}$

$\log_{10} 7^x = \log_{10} 5^{x-4}$

$\frac{x \log_{10}(7)}{x-4 \cdot \log_{10} 7} = \frac{(x-4)\log_{10}(5)}{x-4 \cdot \log_{10} 7}$

$\frac{x}{x-4} = \frac{\log 5}{\log 7}$

$\frac{x}{x-4} = .827$

$.827(x-4) = x$

$.827x - 3.3083 = x$

$-.827x \qquad -.827$

$\frac{-3.3083}{.173} = \frac{.173x}{.173}$

$\boxed{-19.1234}$

2. $6^{x+3} = 5^{x-1}$

$\ln(6^{x+3}) = \ln(5^{x-1})$

$\frac{x+3 \ln(6)}{x-1 \ln(6)} = \frac{x-1 \ln(5)}{x-1 \ln(6)}$

$\frac{x+3}{x-1} = \frac{\ln(5)}{\ln(6)}$

$\frac{x+3}{x-1} = .898$

↓ cross x

$.898(x-1) = x+3$

$.898x - .898 = x+3$

$-.898x \quad -3 \quad -.898x \quad -3$

$\frac{-3.898}{.101} = \frac{.101x}{.101}$

$\boxed{-38.310 = x}$

or

3. $\frac{1}{32} = 8^{4x+2}$

$\log_8 \frac{1}{32} = 4x+2$

$\frac{\log \frac{1}{32}}{\log 8} = 4x+2$

$-\frac{5}{3} = 4x+2$

$-2 \qquad -2$

$\frac{-\frac{11}{3}}{4} = \frac{4x}{4}$

$x = -\frac{11}{12}$

$2^{-5} = (2^{3})^{4x+2}$

$2^{-5} = 2^{12x+6}$

$-5 = 12x+6$

$-6 \qquad -6$

$\frac{-11}{12} = \frac{12x}{12}$

$\frac{-11}{12} = x$

4. $3 + 2 \cdot \boxed{3^{2x}} = 4 + \boxed{3^{2x}}$ ← treat as unit

$3 + 2 \cdot \boxed{3^{2x}} = 4 + 1 \cdot \boxed{3^{2x}}$

$-1 \cdot \boxed{3^{2x}} \qquad -1 \cdot \boxed{3^{2x}}$

$3 + 3^{2x} = 4$

$-3 \qquad -3$

$3^{2x} = 1$ ← just know

$x = 0$

~~$\log_{3} 1 = 2x$~~

~~$\frac{.477}{2} = \frac{2x}{2}$~~

~~$.2386 = x$~~

5. $6 + 4 \cdot \boxed{5^{3x}} = 8 - \boxed{5^{3x}}$

$+\boxed{5^{3x}} \qquad +\boxed{5^{3x}}$

$6 + 5 \cdot 5^{3x} = 8$

$-6 \qquad -6$

$\frac{5 \cdot 5^{3x}}{5} = \frac{2}{5}$

$5^{3x} = \frac{2}{5}$

$\log_5 \frac{2}{5} = 3x$

$\frac{\log 2/5}{\log 5} = 3x$

$-.569 = 3x$

$/3 \rightarrow -.190 = x$

Skipping 3.5

3.6 Exploring Data: NonLinear Models

10/25

model types

linear → $y = ax + b$

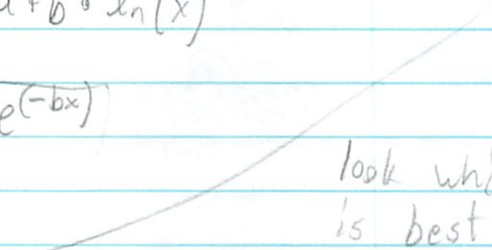
quadratic → $y = ax^2 + bx + c$

exponental → $y = a + b^x$

logarithmic → $y = y = a + b \cdot \ln(x)$

logistic → $y = \frac{c}{1 + ae^{(-bx)}}$

power → $y = a \cdot x^b$



look which is best

abs(r^2) → 1

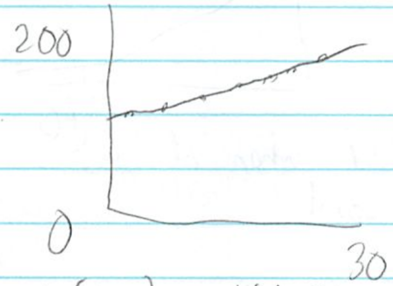
3/ Exponental + Log Functions Review

10/25

235/27. Election Data

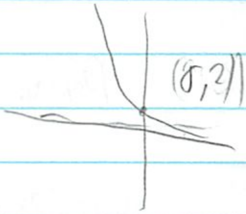
quadratic → $.01x^2+1.635x+94.858$ – $r^2=.9886$
exponental → $95 \cdot 1.02^x$ → $r^2=.9943$
power → $81.28 \cdot x^{.168}$ → $r^2=.8871$

exponental is best



$d(34) \to 166.34$

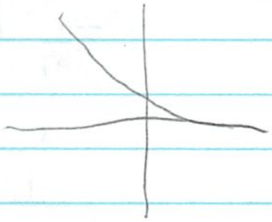
239/11. Graph by hand
$1+6^{-x}$ ← flipped
↑1 ↑ fairly narrow



Y=1 asy.

Decreasing $(-\infty,\infty)$

21. $4e^{-.5x}$



27. $\frac{10}{1+2^{-.05x}}$

y=0
y=10 } horiz asy

29. Interest

$r = 8\%$ continously

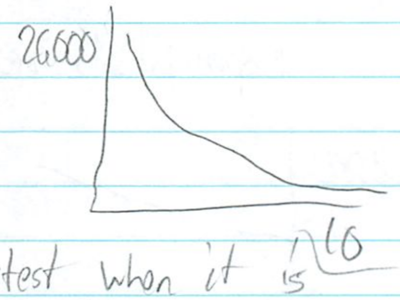
$A = Pe^{rt}$

$P = \$10,000$

t	1	10	20	30	40	50
A	10833	22255	49530	110232	245325	545982

31. Depreciation

$V(t) = 26,000\left(\frac{3}{4}\right)^t$



$V(2) = \$14,625$

c) It depreciates the fastest when it is tree - like the real word

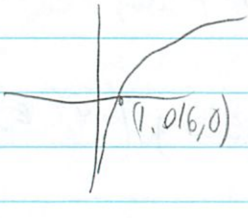
240

35. Write exporental in log form

$25^{3/2} = 125$

✓ $\log_{25} 125 = \frac{3}{2}$

43. $\log_2(x-1)+6$



Domain $x - 1 > 0$

$x > 1$

vert asy $x = 1$

x-int → $0 = \log_2(x-1)+6$

$-6 = \log_2(x-1)$

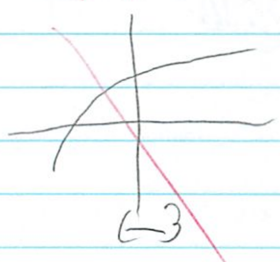
$2^{-6} = x-1$

$x = 2^{-6}+1$

1.016

c just around x

51. $\ln(x)+3)$



~~Domain $x+3>0$~~

~~$x>3$~~

~~vert asy $x=-3$~~

~~x int → $0 = \ln(x+3)$~~

~~$e^0 = x+3$~~

~~$0 = x+3$~~

~~$x = -3$~~

56. Home Mortgage

↓ monthly payment

$t = 12.542 \ln\left(\dfrac{x}{x-1000}\right)$ $x > 1000$

↑ lenght in years

?

a) $x = 1254.68 \rightarrow 20$ years ~~months~~

b) $20 \cdot 12 \cdot 1254.68 = ~~25,093.6~~$ $\boxed{\$301,123.20 \text{ total}}$

$- \; 150,000.00$

$\boxed{\$151,123.20 \text{ interest}}$

Expand

69. $\ln\left(\dfrac{x+3}{xy}\right) \rightarrow \ln(x+3) - \ln(xy)$

$\ln(x+3) - \ln(x) - \ln(y)$

241

Condense

76. $3\left[\ln(x) - 2\ln(x^2+1)\right] + 2\ln(5)$

$\ln\left(\dfrac{x}{(x^2+1)^2}\right)^3 + \ln(5^2)$

x ↓ can

$\ln\left(\left(\dfrac{x}{(x^2+1)^2}\right)^3 \cdot \overset{25}{\cancel{5^2}}\right)$ ⚠

$\ln\left(\dfrac{25x^3}{(x^2+1)^6}\right)$

82. Solve for x

$6^{x-2} = 1296$

$6^{x-2} = 6^4$

$x = 2$

84. $\log_x 243 = 5$

$x^5 = 243$

$\sqrt[5]{\;} \quad \sqrt[5]{\;}$

$x = 3$

Solve

93. $-4(5^x) = -68$

$\frac{}{-4} \quad \frac{}{-4}$

$5^x = 17$

$\log_5 17 = x$

$x = 1.76$

95. $e^{2x} - 7e^{x} + 10 = 0$

$(e^{x} - 5)(e^{x} - 2)$

$e^{x} - 5 = 0$ $e^{x} - 2 = 0$

$e^{x} = 5$ $e^{x} = 2$

$\log_e 5 = x$ $\ln(2) = x$

1.609 $.693$ ← check extranlins

no restrictions

← copy error

106. $\log_{10}(x+2) - \log_{10} x = \log_{10}(x+5)$

$\log_{10}\left(\frac{x+2}{x}\right) = \log_{10}(x+5)$

$\frac{x+2}{x} = (x+5)$ → $x+2 = x(x+5)$

~~$-x^2 - 2x = x+5$~~ cross x wrong

~~$-x^2 - 3x - 5 = 0$~~

~~No real solutions~~

$x+2 = 2x^2 + 5x$

$0 = x^2 + 4x - 2$

$\frac{-4 \pm \sqrt{24}}{2} \rightarrow \frac{-4 \pm 2\sqrt{6}}{2}$

$-2 \pm \sqrt{6} = x$

$-2 + \sqrt{6} = x$ $-2 - \sqrt{6} = 0$

$.4498 = x$ ⊗

~~??~~

105. $\log_{10}(x-1) = \log_{10}(x-2) - \log_{10}(x+2)$

$x > 2$ ← look at each

$x - 1 = \frac{x-2}{x+2}$

$x^2 + x + 2 = x - 2$

Combined wrong - stupid error - write it out →

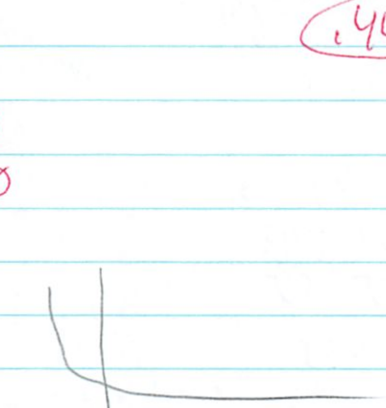
~~$x^2 - 4 = 0$~~ $x^2 = 0$

~~$(x+2)(x-2)$~~ $x = 0$

242

114. $y = 7 - \log_{10}(x+3)$ → d

↑7 ↑flipped ←3



123. Compound Interest

$\$ = 10{,}000$

continuously

doubles in 12 years

rate?

$2 = e^{r12}$

$ln(2) = r12$

$\frac{ln(2)}{12} = r$ 5.8%

t=1

$A = 10000e^{.058 \cdot 1}$

$A = 10594.63\$$

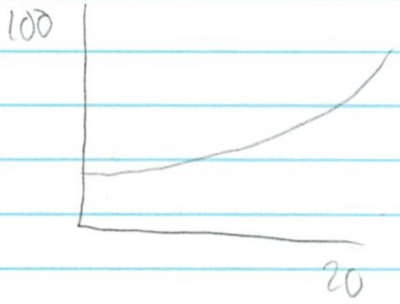
125 Typing Speed

$N = \frac{157}{1+5.4e^{-.12t}}$

$N=50 \rightarrow 50 = \frac{157}{1+5.4e^{.12t}}$

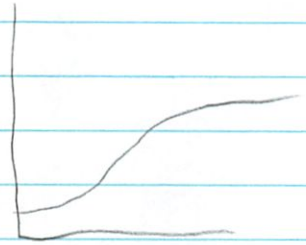
7.713

$N=75 \rightarrow$ 13.310 weeks



3.6 Non Linear Models of Data
Logistic Growth Model

10/25



some paplations have rapid growth in the begining followed by steady growth

$y = \frac{a}{1+be^{-rx}}$ has 2 horiz asy.

a = # of entire pouplation b-everybody else
r = rate of growth y: how long
x = time

used: spread of bacteria
college campus

ex1 College camps r = .8,
5,000 students ← contageous
1 student gets the flu
the spread of virus is modeled by

$y = \frac{5000}{1+4999e^{-.8t}}$

↑ # infected

college campus will close when >40% infected

a) how many students are enfected after 5 days
$t = 5 \to 54$ students

b) After how many days will it close
$y = .40 \cdot 5000$
2000 ← find intersect (10,14, 2000)
after 10.14 days
round↑ → 11 days

3/ Exponential + Log Functions

More Pratice

10/25

P1. Expand

$$\left(\frac{32x^5y^{-3}}{z^{-4}}\right)^{2/5}$$

$$\left(\frac{32x^5z^4}{y^3}\right)^{2/5}$$

Remember

$$(x^5)^{2/5} = x^{5\cdot .4} = x^2$$

$$\left(\frac{4x^2y^{8/5}}{y^{6/5}}\right)$$

P2.

$$\log_{10}\left(\sqrt[4]{\frac{x^8y^6}{z^3}}\right)$$

$$\log_{10}\left(\frac{x^8y^6}{z^3}\right)^{\frac{1}{4}}$$

$$\log_{10}\left(\frac{x^2y^{3/2}}{z^{3/4}}\right)$$

can expand

$$\log_{10}x^2 + \log_{10}y^{3/2} - \log_{10}z^{3/4}$$

farther

$$(2\log_{10}x + 3/2\log_{10}y) - 3/4\log_{10}z$$

Condense

P3

$$2\log_{10}(3x) + 4\log_{10}(-6x)$$

$$\log_{10}(3x)^2 + \log_{10}(-6x)^4$$

$$\log_{10}((3x)^2\cdot(-6x)^4)$$

go farther

$$\log_{10}(9x^2\cdot 1296x^4)$$

still combine coefficents + variables

$$\log_{10}(11664x^6)$$

4. $5 + 4 \cdot \boxed{2^{2x}} = 6 - 2 \cdot \boxed{2^{2x}}$

$$+2 \cdot 2^{2x} \qquad +2 \cdot 2^{2x}$$

$$5 + 6 \cdot 2^{2x} = 6$$

$$-5 \qquad -5$$

$$\frac{6 \cdot 2^{2x}}{6} = \frac{1}{6}$$

$$2^{2x} = \frac{1}{6}$$

$$\log_2 \tfrac{1}{6} = 2x$$

$$\frac{-2.58}{2} = \frac{2x}{2}$$

$$-1.29 = x$$

3 Test Topics

10/25

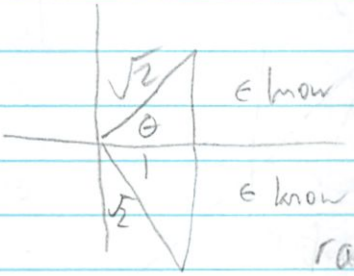
1- Simplify using properties of exponents
Sketch a log function
    intercepts, vert asy, domain
Condensing log expressions
expanding " "
Solving log + exponental equations
3 word problems

4-2 More Pratice on IS

11/1

1. $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\frac{1}{\sqrt{2}}$



← know 45°

← know not in range for $\cos^{-1}$

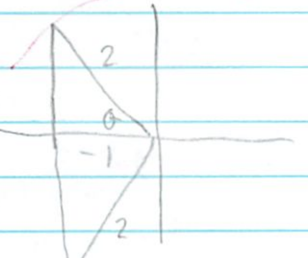
2. arc cos $\left(-\frac{1}{2}\right)$

convert radian → $= \frac{2\pi}{3}$

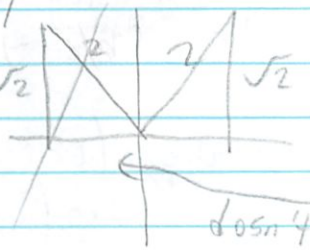
θ' = 60°

so θ = 120°

dosn't fit in range for $\cos^{-1}$ →



3. arcsin $\left(\frac{\sqrt{2}}{2}\right)$



arcsin range $= \frac{-\pi}{2}$ to $\frac{\pi}{2}$ so 1st + 4th quads

dosn't work

4. $\arccos\left(\cos\frac{7\pi}{2}\right) =$

ratio of sides

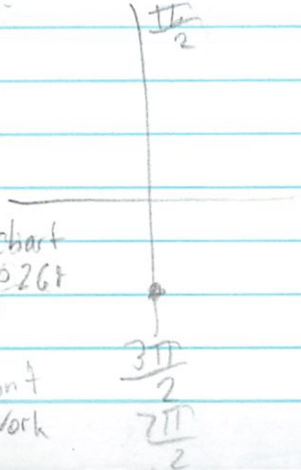
$\arccos\left(\cos\frac{3\pi}{2}\right)$ ← chart p267

$\arccos(0°)$

90° or $\frac{\pi}{2}$

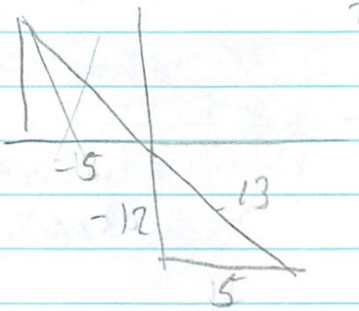
~~$-\frac{\pi}{2}$~~ ← don't

~~$\frac{3\pi}{2}$~~ ← work



inverse cos goes 0 → π

5. $\csc\left[\arctan \frac{-12}{5}\right)$



$\tan^{-1}$ range $-\frac{\pi}{2}$ to $\frac{\pi}{2}$
So 1st + 4th quad

$\csc = \frac{hyp}{opp}$, $h^2 = \sqrt{(-5)^2 + 12^2}$
$h = 13$

$\csc = \frac{13}{-12}$

6. Deterime a sin function given the amp, pd, and phase shift

pd = 4π
phase Δ = 2π →
amp = 2

$y = a \sin(bx+c) + d$ ↓ known
$y = 2\sin(bx+c) + 0$ ↓ ⇒
$\boxed{y = 2\sin(\frac{1}{2}x + \pi)}$ ↑ b/c →

b → $4\pi = \frac{2\pi}{b}$

$b = \frac{1}{2}$

c → $2\pi = \frac{c}{.5}$

$1\pi = c$

7. pd = 3π
phaseΔ = $\frac{\pi}{4}$ ←
amp = $3/2$
vert Δ = 2 ↓
sin function

$y = a\sin(bx+c) + d$
$y = 3/2 \sin(bx+c) - 2$

$\boxed{y = 3/2 \sin(2/3 x + \frac{\pi}{6}) - 2}$

b → $3\pi = \frac{2\pi}{b}$

$b = 2/3$

c → $\frac{\pi}{4} = \frac{c}{2/3}$

$\frac{\pi}{4} \cdot 2/3 = \frac{\pi}{6}$

4-Part 2

More Pratice After Test

11/5

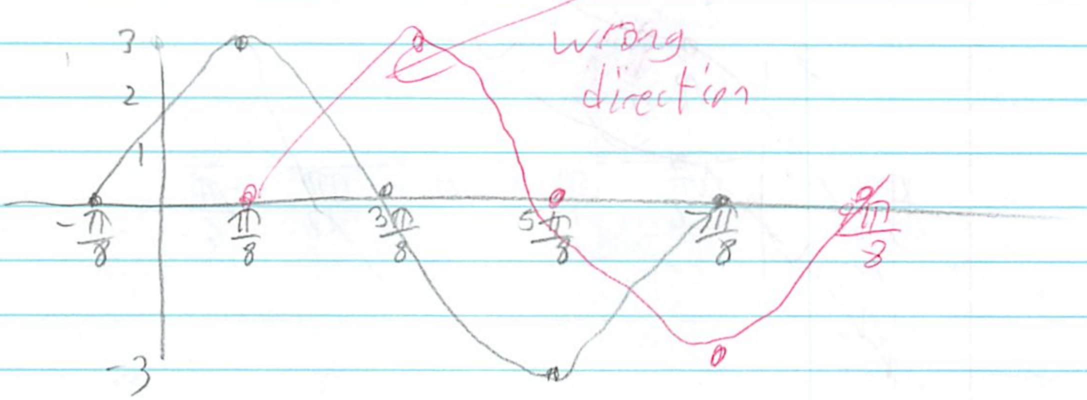
1. $y = 3\sin\left(2x - \frac{\pi}{4}\right)$

amp = 3 int = $\frac{\pi}{4}$

pd = $\frac{2\pi}{2} = \pi$

phase $\Delta = \frac{\pi}{4}/2 = \frac{\pi}{4} \cdot \frac{1}{2} = \frac{\pi}{8}$ →

vert $\Delta = 0$



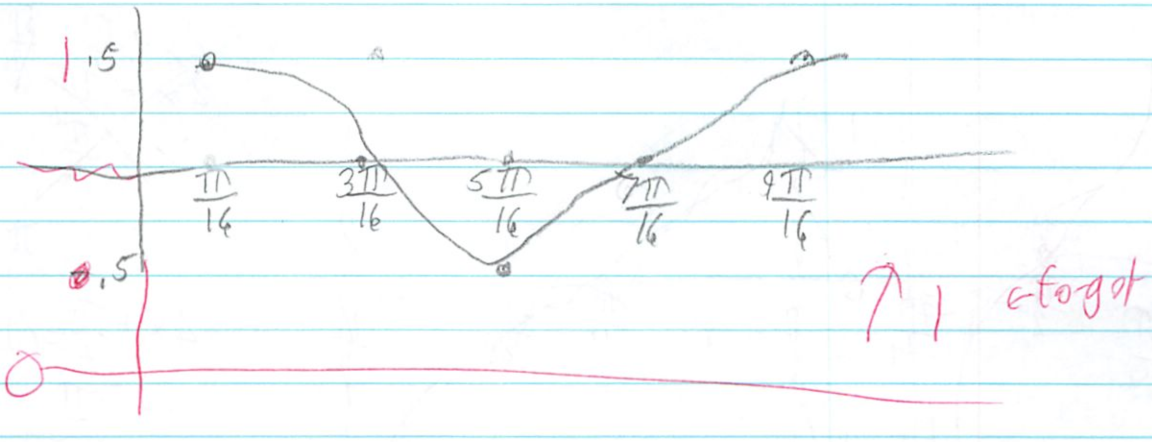
2. $y = \frac{1}{2}\cos\left(4x - \frac{\pi}{4}\right) + 1$

amp = $\frac{1}{2}$ int = $\frac{\pi}{8}$

pd = $\frac{2\pi}{4} = \frac{\pi}{2}$

phase $\Delta = \frac{\pi}{4}/4 = \frac{\pi}{4} \cdot \frac{1}{4} = \frac{\pi}{16}$ →

vert Δ = ↑1



Vert asy

$x - \frac{\pi}{6} = \frac{\pi}{2}$ $x - \frac{\pi}{6} = -\frac{\pi}{2}$

$x = \frac{2\pi}{3}$ $x = \frac{-\pi}{3}$

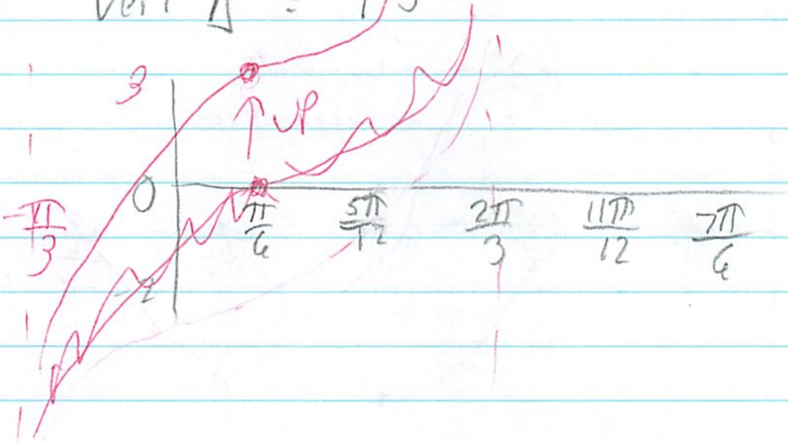
mid $= \frac{\frac{2\pi}{3}}{2} + \frac{-\pi}{3}$

$\frac{\pi}{6}$

$\pi/4 = \frac{\pi}{4}$

3. $2\tan\left(x - \frac{\pi}{6}\right) + 3$

amp = 2
pd = $\frac{\pi}{1} = \pi$
phase $\Delta = \pi/6 / 1$ $\frac{\pi}{6} \rightarrow$
vert $\Delta = \uparrow 3$



4. $y = \cot\left(2x + \frac{\pi}{2}\right)$

amp = 1
pd = $\frac{\pi}{2}$
phase $\Delta = \pi/2 / 2$ $\frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4} \leftarrow$
vert $\Delta = 0$

int = $\frac{\pi}{8}$

cot

$0, \pi, -\pi$

$2x + \frac{\pi}{2} = \cancel{\frac{\pi}{2}}$
$\cancel{2x = \frac{\pi}{4}}$
$\cancel{x = \frac{\pi}{2}}$

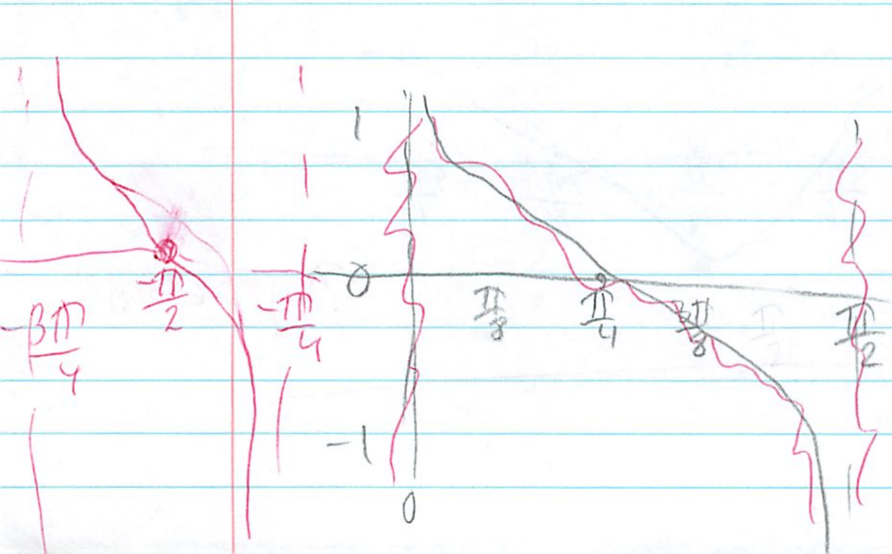
$2x + \frac{\pi}{2} = \cancel{-\frac{\pi}{2}}$
$\cancel{2x = 0}$
$\cancel{x = 0}$

$\frac{\pi}{4}$ ~~mid~~

$2x + \frac{\pi}{2} = 0$

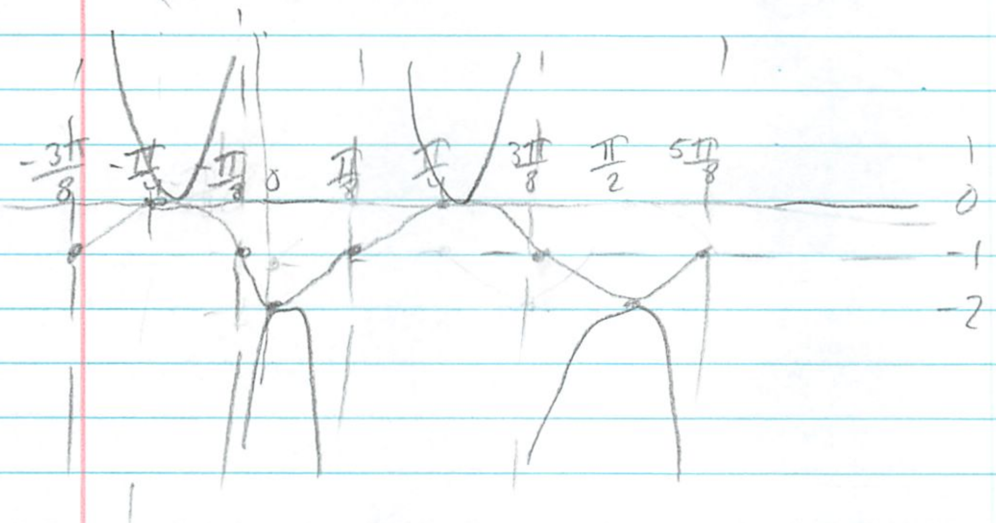
$x = \frac{-\pi}{4}$

$x = -3\pi/4$

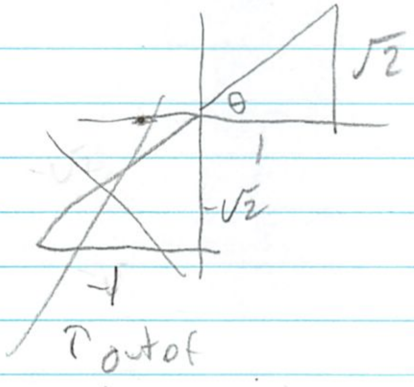


5. $y = \csc\left(4x + \frac{3\pi}{2}\right) - 1$

sin
amp = 1 $\qquad$ int = $\frac{\pi}{8}$
pd = $\frac{2\pi}{4} = \frac{\pi}{2}$
phase Δ = $\frac{3\pi}{2} / 4 = \frac{3\pi}{2} \cdot \frac{1}{4} = \frac{3\pi}{8} \leftarrow$
vert Δ = ↓ 1



6. $\csc\left(\arctan\left(\sqrt{2}\right)\right) \rightarrow \frac{\sqrt{2}}{1} \rightarrow$ between $\frac{-\pi}{2}, \frac{\pi}{2}$



↑ out of tan range

$\csc = \frac{hyp}{opp}$

$h = \sqrt{1^2 + 2}$
$h = \sqrt{3}$

$\frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{6}}{2}}$

6.3 Vectors In a Plane

10/28

area, time, and temperature can be represented by a value

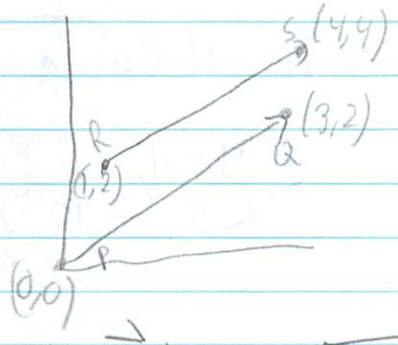
force + velocity have direction, so are vectors
- mangitude + direction

* vector = directed line segment

$\overrightarrow{PQ}$ is a directed line segment
- inital point is P
- terminal point is Q

- magnitude (lenghth) = $\|\overrightarrow{PQ}\|$
  - found using distance formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- if directed line segments have the same magnitude or lenght and direction then they are equivilant
- denoted by u, v, w

ex1 equivalent directed line segments

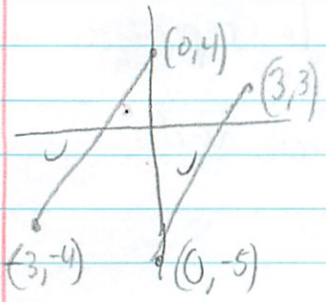


Magnitude
$\|\overrightarrow{PQ}\| = \sqrt{(3-0)^2 + (4-1)^2} = \sqrt{13}$ ⊜
$\|RS\| = \sqrt{(4-1)^2 + (4-2)^2} = \sqrt{13}$

Direction (slope)
$\frac{2-0}{3-0} = \frac{2}{3}$ ⊜ $\frac{4-2}{4-1} = \frac{2}{3}$
$\overrightarrow{PQ}$ $\overrightarrow{RS}$

4/17

2.

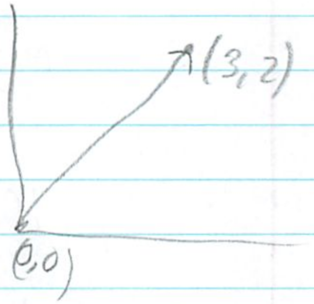


$\|u\| = \sqrt{(0--3)^2+(4--4)^2} = \sqrt{73}$
$\|v\| = \sqrt{(3-0)^2+(3--5)^2} = \sqrt{73}$ $=$

$u: \frac{4--4}{0--3} = \frac{8}{3}$ $v: \frac{3--5}{3-0} = \frac{8}{3}$ $=$

A directed line segments whose inital point is at (0,0) -> it is in standard position

Can represent the vector with the coordinates of its terminal point



- component form -> $\langle 3,2 \rangle$
$\langle 3-0, 2-0 \rangle$
$\langle 3,2 \rangle$

termonal - inital

If the inital point is (p_1, p_2) and the terminal point is $\langle q_1, q_2 \rangle$ then $\overrightarrow{PQ} =$
$\langle q_1 - p_1, q_2 - p_2 \rangle$
$\langle v_1, v_2 \rangle$ = vector v

$\langle q-p, q-p \rangle$

Magnitude $= \|v\| = \sqrt{(q_1-p_1)^2+(q_2-p_2)}$

ex2

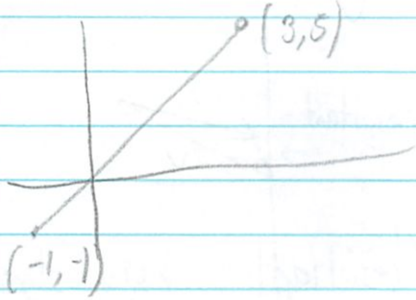
Component form

$\langle 4--1, -7-5 \rangle$
$\langle 5, -12 \rangle$ <- if it started at 0 it would go here

$\|v\| = \sqrt{5^2+(-12)^2}$
$\sqrt{169}$
13

p417

6.



Remember

$\langle x, y \rangle$

$\langle 5 - -1, 3 - -1 \rangle$

$~~\langle 6, 4 \rangle~~ = v \quad \langle 4, 6 \rangle$

$||v|| = \sqrt{6^2 + 4^2}$

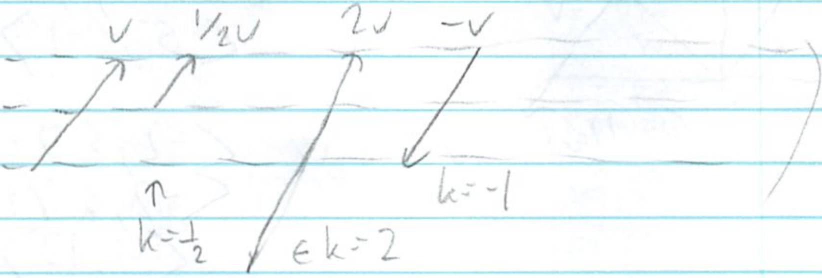
$\sqrt{52}$

7,211

Vector Operations

- scalar multiplication
- vector addition

Scalar quantity called k
↑ any constant



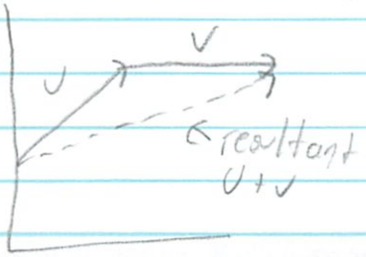
k is a multiplier

If k is ⊕, then k • v = ⊕
" " " ⊖, then k • v = ⊖ ← goes oppset direction

Vector Addition

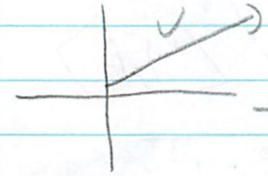
to add 2 vectors together, position them so that inital point of the first matches the terminal point of the 2nd vector

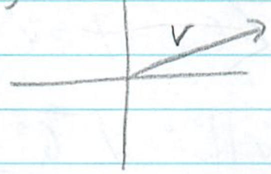
parallegram law →

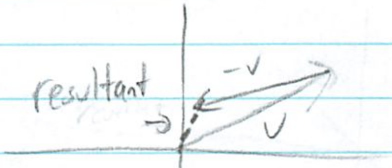


v and v are adjecent sides

sum(v+v) is formed from the middle of the parrallo gra ex3 $v + -v$ $v - v$



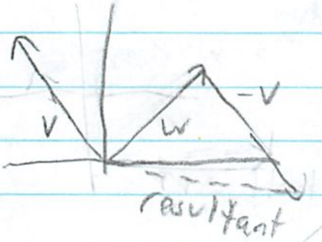




resultant is not always the diagonal

ex4 $v = \langle -2, 5 \rangle$
$w = \langle 3, 4 \rangle$

$2v: 2\langle -2, 5 \rangle$
$\langle -2 \cdot 2, 5 \cdot 2 \rangle$
$\langle -4, 10 \rangle$



$w - v: \langle 3, 4 \rangle - \langle -2, 5 \rangle$
$\langle 3 - -2, 4 - 5 \rangle$
$\langle 5, -1 \rangle$

$v + 2w: \langle -2, 5 \rangle + 2\langle 3, 4 \rangle$
$\langle -2, 5 \rangle + \langle 6, 8 \rangle$
$\langle 4, 13 \rangle$

6.3 Vectors in the Plane

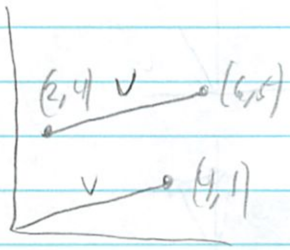
HW

10/29

417

Show $u = v$.

1.

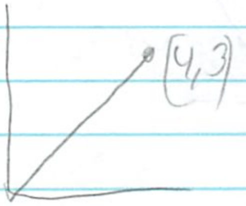


$||v|| = \sqrt{(4-0)^2+(1-0)^2} = \sqrt{17}$ ⊜

$||u|| = \sqrt{(6-2)^2+(5-4)^2} = \sqrt{17}$

v: $\frac{1-0}{4-0}$ $\frac{1}{4}$ v: $\frac{5-4}{6-2}$ $\frac{1}{4}$ ⊜

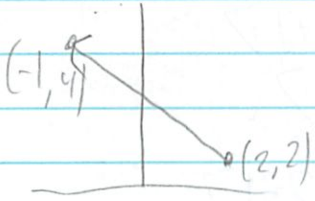
3.



$\langle 4,3 \rangle$

$||v|| = \sqrt{4^2+3^2} = \sqrt{25} = 5$

5.

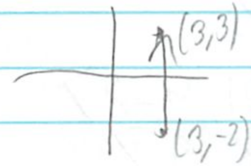


$\langle -1-2, 4-2 \rangle$

$\langle -3, 2 \rangle$

$||v|| = \sqrt{4^2+2^2} = \sqrt{20}$

7.



$\langle 3-3, 3--2 \rangle$

$\langle 0, 5 \rangle$

$||v|| = \sqrt{5^2} = 5$

9. $(\frac{5}{2}, 1)$ $(-2, -\frac{3}{2})$ ↓

$\langle -2 - \frac{5}{2}, -\frac{3}{2} - 1 \rangle$

$\langle -4.5, -2.5 \rangle$

$||v|| = \sqrt{4.5^2 + 2.5^2} = \sqrt{26.5}$

Tell

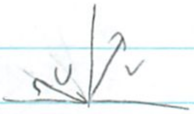
9-P

11. $(-3,-5)$ $(5,1)$ ↓

$\langle 5--3, 1--5 \rangle$

$\langle 8, 6 \rangle$

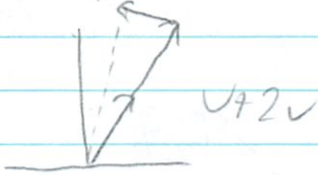
Sketch



13, $-v$

15 $u+v$

17, $u+2v$



418

Find $u+v$, $u-v$, $2u-3v$, $u+4v$

19. $u = \langle 4, 2\rangle$
$v = \langle 7, 1\rangle$

$u+v$) $\langle 4+7, 2+1\rangle$
$\langle 11, 3\rangle$

$u-v$) $\langle 4-7, 2-1\rangle$
$\langle -3, -1\rangle$

$2u-3v$) $\langle 8-21, 4-3\rangle$
$\langle -13, 1\rangle$

$u+4v$) $\langle 4+28, 2+4\rangle$
$\langle 32, 6\rangle$

21. $u = \langle -6, -8\rangle$
$v = \langle 2, 4\rangle$

$u+v$) $\langle -6+2, -8+4\rangle$
$\langle -4, -4\rangle$

$u-v$) $\langle -6-2, -8-4\rangle$
$\langle -8, -12\rangle$

$2u-3v$) $\langle -12-6, -16-12\rangle$
$\langle -18, 28\rangle$

$u+4v$) $\langle -6+8, -8+16\rangle$
$\langle 2, 8\rangle$

6.3 Vectors

More Notes

10/30

If both the inital point + the terminal point are at the origon - it is a zero vector $\langle 0,0 \rangle$

If the magnitude of the vector = 1, then the vector is a unit vector

If magnitude = 0, then it is a zero vector

Properties of Vector Addition + Scalar Multiplication

know property names

1. Communitive property
   $u + v = v + u$
2. Associative Property
   $(u+v) + w = u + (v+w)$
3. Additive Identity
   $v + 0 = v$
4. Additive Inverse
   $u + (-u) = 0$
5. Distributive Property
   $u(c+d) = uc + vd$
6. Distributive Property
   $c(u+v) = cu + cv$
7. Multiplicative Property
   $1 \cdot u = u$
8. $\underset{\uparrow \text{constant}}{\|c \cdot v\|} = \underset{\uparrow \text{abs value}}{|c|} \cdot \|v\|$

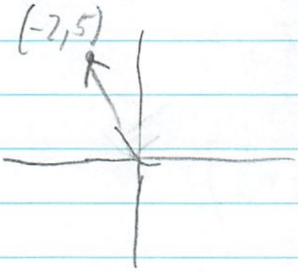
Unit Vector

useful to find a unit vector that has the same direction as another nonzero vector

u = unit vector

$u = \frac{v}{\|v\|}$ ← same direction, magnitude of 1

ex1 find the unit vector in the direction of $v = \langle -2,5 \rangle$



$\frac{\langle -2,5 \rangle}{\sqrt{29}} \to \frac{1}{\sqrt{29}} \cdot \langle -2,5 \rangle$

$\left\langle \frac{-2}{\sqrt{29}}, \frac{5}{\sqrt{29}} \right\rangle \to \boxed{\langle -.371, .928 \rangle}$

✓: That it has $\|u\| = 1$

$\sqrt{(-.371)^2 + (.928)^2} = \sqrt{\left(\frac{-2}{\sqrt{29}}\right)^2 + \left(\frac{5}{\sqrt{29}}\right)^2}$

$\sqrt{\left(\frac{29}{29}\right)}$

$\sqrt{1}$

1 ✓

p418

28. $v = \langle 3,-4 \rangle \to \frac{\langle 3,-4 \rangle}{\sqrt{25}}$ $\frac{1}{5} \cdot \langle 3,-4 \rangle$

carry the ⊖

$\boxed{\left\langle \frac{3}{5}, -\frac{4}{5} \right\rangle} \xrightarrow{✓:} \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} \to \sqrt{1} \to 1$ ✓

$\langle 1,0\rangle$ ← horiz component
$\langle 0,1\rangle$ ← vert component

standard unit vector and represented as
$\mathbf{i} = \langle 1,0\rangle$
$\mathbf{j} = \langle 0,1\rangle$

these vectors can be used to represent any vector

ex Write a linear combination of unit vectors to represent vector u,

inital → terminal
$(2,-5)$ $(-1,3)$

1. get in component form

$$\langle -1-2,\ 3--5\rangle$$
$$\langle -3,\ 8\rangle$$

2. Write linear combo

$$\boxed{-3\mathbf{i}+8\mathbf{j}}$$

44. $(-6,4) \rightarrow (0,1)$

$$\langle 0--6,\ 1-4\rangle$$
$$\langle 6,\ -3\rangle \rightarrow 6\mathbf{i}-3\mathbf{j}$$

ex6 vector operations

$u = -3\mathbf{i}+8\mathbf{j}$ — Find $2u-3v$
$v = 2\mathbf{i}-\mathbf{j}$

$2u)\ -6\mathbf{i}+16\mathbf{j}$ $3v)\ 6\mathbf{i}-3\mathbf{j}$

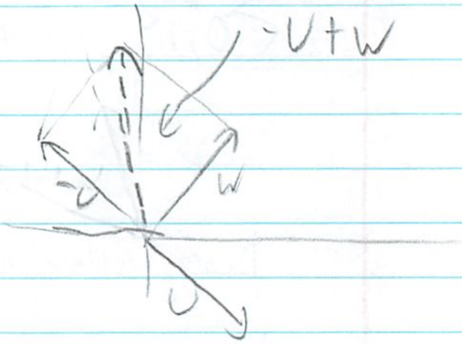
$-$

$-6\mathbf{i}-6\mathbf{i}$ $16\mathbf{j}+3\mathbf{j}$ ← keep track of signs

$-12\mathbf{i}+19\mathbf{j}$

4/18

48. $V = -U + W$ $\quad U = 2i - j$
$W = i + 2j$



$-(2i - j) + (i + 2j)$
$-2i + j + i + 2j$
$-i + 3j$

Find the vector V with the given magnitude + the same direction as U

36. magnitude = 3
direction = $\langle 4, -4 \rangle$

$\frac{\langle 4, -4 \rangle}{\sqrt{32}} = \left\langle \frac{4}{4\sqrt{2}}, -\frac{4}{4\sqrt{2}} \right\rangle$

Find unit vector $\rightarrow \left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$

and multiply by new magnitude $\quad 3\left\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle$

$\left\langle \frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}} \right\rangle \xrightarrow[\text{rationalize}]{} \left\langle \frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2} \right\rangle$

38. $\|v\| = 10$
$2i - 3j$ = direction

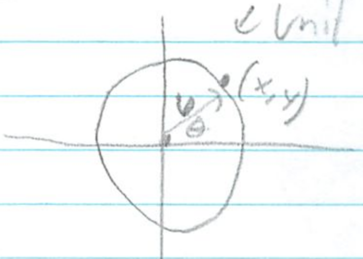
$\frac{\langle 2, -3 \rangle}{\sqrt{13}} = \left\langle \frac{2}{\sqrt{13}}, \frac{-3}{\sqrt{13}} \right\rangle$

$10\left\langle \frac{2}{\sqrt{13}}, \frac{-3}{\sqrt{13}} \right\rangle \rightarrow \left\langle \frac{20}{\sqrt{13}}, \frac{-30}{\sqrt{13}} \right\rangle \xrightarrow[\text{rationalize}]{} \left\langle \frac{20\sqrt{13}}{13}, \frac{-30\sqrt{13}}{13} \right\rangle$

$\frac{20\sqrt{13}}{13}i, -\frac{30\sqrt{13}}{13}j$

Direction Angles

$||v|| = 1$



$r = 1 \rightarrow ||v|| = 1$

$x = r\cos\theta$ $r\sin a$ $r = 1$ $x = \cos\theta$

$y = r\sin\theta$ $y = \sin\theta$

$$\langle x, y \rangle$$

$$\langle \cos\theta, \sin\theta \rangle$$

$$||v|| \langle \cos i + \sin j \rangle$$

$$||v|| \cdot \cos\theta\, i + ||v|| \sin\theta\, j$$

Directed Angle

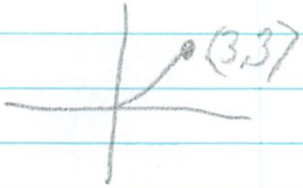
Find θ

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$v = 3i + 3j$

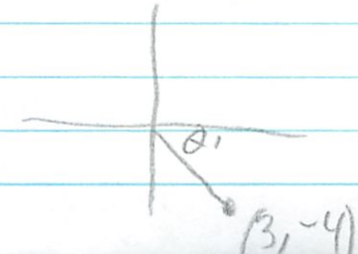
$\langle 3, 3 \rangle$



$\tan\theta = \frac{3}{3}$

$\tan^{-1}(1) = 45°$

$v = 3i - 4j$



$\tan\theta = \frac{4}{3} = 53.130 = \theta'$ ← ref angle

$360 - \theta' = 306.869°$

6.3 Vectors
Day 2 HW

10/30

p418

23. $u = i + j$
$v = 2i - 3j$

a) $u + v \rightarrow (i + j) + (2i - 3j)$
$3i - 2j$

b) $u - v \quad (i + j) - (2i - 3j)$
$i + j - 2i + 3j$
$-1i + 4j$

c) $2u - 3v \quad 2(i + j) - 3(2i - 3j)$
$2i + 2j - 6i + 9j$
$-4i + 11j$

d) $u + 4v \quad (i + j) + 4(2i - 3j)$
$i + j + 8i - 12j$
$9i - 11j$

Find unit vector

25. $\langle 6, 0 \rangle$

$\frac{\langle 6, 0 \rangle}{\sqrt{6^2}} \quad \left\langle \frac{6}{6}, \frac{0}{6} \right\rangle \quad \langle 1, 0 \rangle$

27. $\langle -1, 1 \rangle \quad \frac{\langle -1, 1 \rangle}{\sqrt{2}} \quad \left\langle \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle \quad \sqrt{\left(\frac{-1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$

$\sqrt{1} \rightarrow 1$

29. $\langle -24, -7 \rangle \quad \frac{\langle -24, -7 \rangle}{\sqrt{625} \rightarrow 25} \quad \left\langle \frac{-24}{25}, \frac{-7}{25} \right\rangle =$

31. $4i - 3j \quad \frac{\langle 4, -3 \rangle}{\sqrt{25} \rightarrow 5} \quad \left\langle \frac{4}{5}, \frac{-3}{5} \right\rangle$

33. $2j \quad \frac{\langle 0, 2 \rangle}{2} \quad \langle 0, 1 \rangle$

35. Find a vector in same direction w/ given magnitude

$\|v\| = 8$
$v = \langle 5, 6 \rangle$

$\left\langle \frac{5}{\sqrt{61}}, \frac{6}{\sqrt{61}} \right\rangle \cdot 8 = \left\langle \frac{40}{\sqrt{61}}, \frac{48}{\sqrt{61}} \right\rangle$

rationalize $\left\langle \frac{40\sqrt{61}}{61}, \frac{48\sqrt{61}}{61} \right\rangle$

37. $\|v\| = 7$
$v = 3i + 4j$

$\left\langle \frac{3}{5}, \frac{4}{5} \right\rangle \cdot 7 = \left\langle \frac{21}{5}, \frac{28}{5} \right\rangle$

39. $\|v\| = 8$
$v = -2i$

$\left\langle \frac{-2}{2}, 0 \right] \cdot 8 \quad \langle 8, 0 \rangle$

Write as i and j

41. $(-3, 1)$
$(4, 5)$ ↓

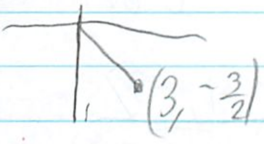
$4 - -3\, i + 5 - 1\, j$
$7i + 4j$

43. $(-1, -5)$
$(2, 3)$ ↓

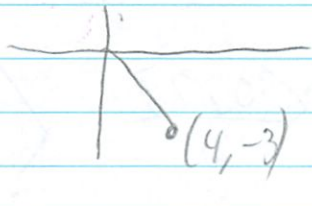
$2 - -1\, i + 3 - -5\, j$
$3i + 8j$

Find combonent form

45. $v = \frac{3}{2}(2i - j)$
$v = 3i - \frac{3}{2}j$



47. $v = (2i - j) + 2(i + 2j)$
$v = 2i - j + 2i + 4j$
$4i$
$-3j$



49. $\frac{1}{2}(3(2i-j)+(i+2j))$

$\frac{1}{2}(6i-3j+i+2j)$

$3i-1.5j+.5i+1j$

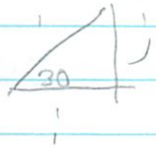
$3.5i-.5j$

51. Find magnitude + direction

$5(\cos 30 i+\sin 30 j)$

↑

$\|v\|=5$



angle = 30°

53. $v=6i-6j$

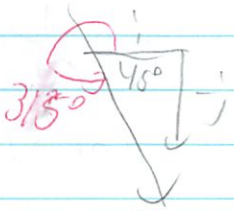
↑

to ↓ $\|v\|=\cancel{6}$

$\sqrt{6^2+6^2}$

$\sqrt{72}$

$6\sqrt{2}$



4th quad

↑

$\tan\left(\frac{y}{x}\right)=\frac{-6}{6}=-1$

$\tan^{-1}(-1)=45°=\theta'$

$360-\theta'=\boxed{315°}$

^

$\|v\|(\cos\theta)i+\|v\|(\sin\theta)j=$

$V=ai+bj$

418

56. Magnitude + Angle

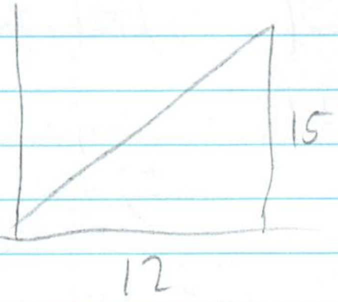
$v = 12i + 15j$

$\|v\| = \sqrt{12^2+15^2}$

$\sqrt{369}$

reduced $\sqrt{9} \cdot \sqrt{41}$

$3\sqrt{41}$



$\tan\theta = \frac{y}{x} = \frac{15}{12}$

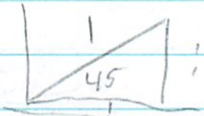
$\tan^{-1}(15/12) =$

$51.34°$

58. Find component form

$\|v\| = 1$

$\theta = 45°$



$\sin(45) = \frac{j}{1}$

$.707 = j$

$\cos(45) = \frac{i}{1}$

$.707 = i$

$\langle .707, .707 \rangle$

$1\langle \cos\theta, \sin\theta \rangle$

$\langle \cos 45, \sin 45 \rangle$

$\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$

↓ do like

6.3 Vectors Day 3

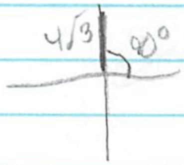
10/31

418

60. $\|v\| = 4\sqrt{3}$

$\theta = 90°$

$4\sqrt{3}\langle \cos 90, \sin 90 \rangle$



$4\sqrt{3}\langle 0, 1 \rangle$

$\langle 0, 4\sqrt{3} \rangle$

62. $\|v\| = 3$

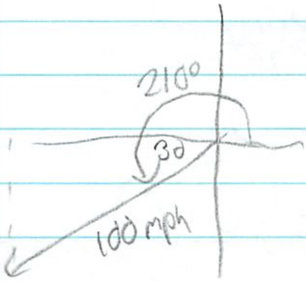
$\theta = 3i + 4j$ $\rightarrow$ $\frac{\langle 3, 4 \rangle}{5}$ $\left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$

$3\left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$

$\left\langle \frac{9}{5}, \frac{12}{5} \right\rangle$

ex8 Finding the component form of a vector

Find the component form of the vector that represents the velocity of an airplane decending at a speed of 100 mph at an angle of 30° below the horizontal



$v = \|v\| \cos\theta\, i + \|v\| \sin\theta\, j$ ← note - from 0

$100 \cos 210\, i + 100 \sin 210\, j$

$100\left(-\frac{\sqrt{3}}{2}\right) i + 100\left(-\frac{1}{2}\right) j$

$-50\sqrt{3}\, i - 50\, j$

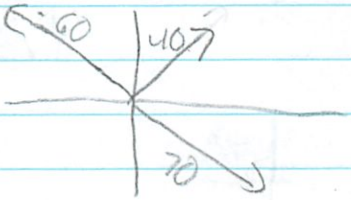
$\langle -50\sqrt{3}, -50 \rangle$

419

3 forces: u, v, w - looking for resultant force

74.

u = 70 lbs @ $-30°$
v = 40 " @ $45°$
w = 60 " @ $135°$



$u = 70\cos(-30)i + 70\sin(-30)j$

$70\ \frac{\sqrt{3}}{2}j + 70\ -\frac{1}{2}j$

$\frac{70\sqrt{3}}{2}i - 35j$

$\boxed{60.62i - 35j}$

$v = 40\cos(45)i + 40\sin(45)j$

$\boxed{28.28i + 28.28j}$

$w = 60\cos(135)i + 60\sin(135)j$

$\boxed{-42.43i + 42.43j}$

$u+v+w = \boxed{46.47i + 35.71j} \rightarrow \langle 46.47, 35.71 \rangle$

$\|u+v+w\| = \sqrt{(46.47)^2 + (35.71)^2}$

$\sqrt{3433.85}$

$\boxed{58.60 \text{ lbs.}}$

$\tan \frac{y}{x} = \frac{35.71}{46.47}$ $\boxed{37.54°}$

6.3 Vectors
Day 4

11/7

418/64.

Find the component form of sum of $U+V$

U; $\|u\| = 2$ | $V = \|v\| = 2$

$Q_u = 30^\circ$ | $Q_v = 90^\circ$

$2\cos 30 i + 2\sin 30 j$ | $2\cos 90 i + 2\sin 90 j$

$2 \frac{\sqrt{3}}{2} i + 2 \frac{1}{2}$ | $2 \cdot 0 i + 2 \cdot 1 j$

$\sqrt{3} i + j$ | $2j$

$+$

$\sqrt{3} i + 3j$

$\langle \sqrt{3}, 3 \rangle$

66. U; $\|u\| = 35$ | v; $\|v\| = 50$

25° | 120°

$35\cos 25 i + 35\sin 25 j$ | $50\cos 120 i + 50\sin 120 j$

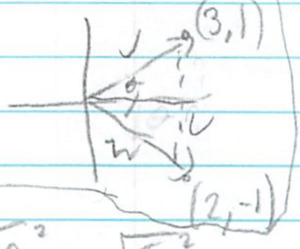
$31.70 i + 14.79 j$ | $-25 i + 43.30 j$

$25\sqrt{3}$

$6.7 i + 58.09 j$

$\langle 6.7, 58.09 \rangle$

68. $v = 3i + j$ - find the θ b/w - $w = 2i - j$



$$U^2 = v^2 + w^2 - 2vw \overbrace{\cos\theta}^{\text{what you want}}$$

$$\frac{U^2 - v^2 - w^2}{-2vw} = \cos\theta$$

need $\|v\| + \|w\|$

$$\frac{\sqrt{5}^2 - \sqrt{10}^2 - \sqrt{5}^2}{-2\sqrt{5}\sqrt{10}} \leftarrow \cos\theta$$

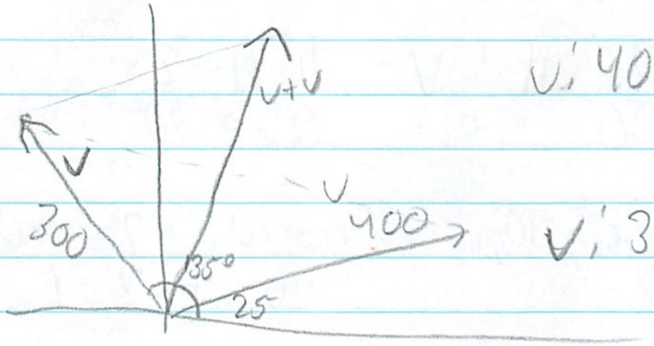
$$\frac{-10}{-2\sqrt{50}} = \frac{-10}{-10\sqrt{2}} \to \frac{-1}{\sqrt{2}}$$

$\sqrt{5}$

$\cos^{-1} \to 135^\circ$

$\|v\| = \sqrt{3^2 + 1^2} = \sqrt{10}$ $\|w\| = \sqrt{2^2 + 1^2} = \sqrt{5}$ $\|u\| = \sqrt{(3-2)^2 + (1--1)^2}$

70. Find resultant → Find magnitude + direction



u, 400 cos 25 i + 400 sin 25 j
362.52 i + 169.05 j

v, 300 cos 135 i + 300 sin 135 j
-212.13 i + 212.13 j

u+v = 150.39 i + 381.18 j

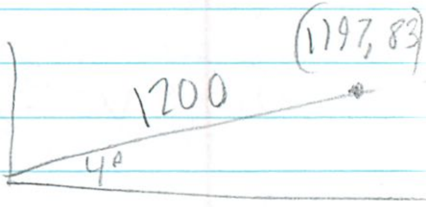
$\|u+v\| = \sqrt{(150.39)^2 + (381.18)^2}$
$\sqrt{167915}$
409.77

$\tan^{-1}\left(\frac{381.18}{150.39}\right) = 68.5°$
↑ inverse tan $\left(\frac{j}{i}\right)$

p419

76. A gun has velocity of 1200 ft/sec
4° angle to horizontal
Find horiz + vert

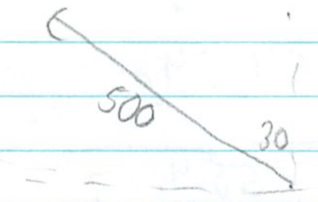
1200 cos 4 i + 1200 sin 4 j
1197.08 i + 83.71 j
<1197.08, 83.71>



Speed + direction

ex1 An airplane is travelling at 500 mph with a bearing (from N) of 330°
↑clockwise
at fixed altitude w/ some wind.
As the airplane reaches a certain point it gets a wind of 70 mph 45° NE
What are the resultant speed + direction of plane

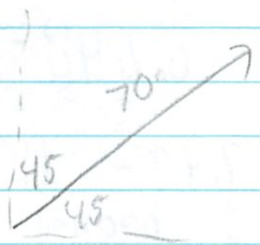
Plane
(ground speed)



velocity of plane alone = ground speed

$500 \cos 120° i + 500 \sin 120 j$
$500 \cdot -\frac{1}{2} i + 500 \frac{\sqrt{3}}{2} j$
$-250 i + 250\sqrt{3} j$

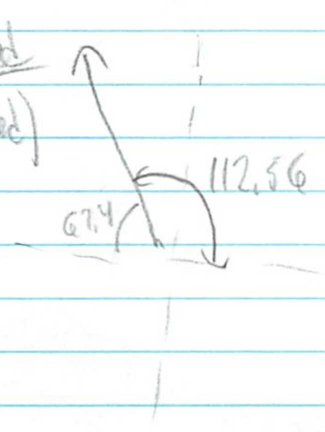
wind



$70 \cos 45 i + 70 \sin 45 j$
$70 \frac{\sqrt{2}}{2} i + 70 \frac{\sqrt{2}}{2} j$
$35\sqrt{2} i + 35\sqrt{2} j$

$-200.5 i + 482.51 j$

combined
(airspeed)



$\|v\| = \sqrt{200.5^2 + 482.5^2}$
$\sqrt{273006}$
522.50 mph

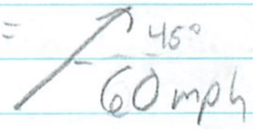
$\theta = \tan^{-1}\left(\frac{482.51}{-200.5}\right) = -67.44°$

$180 + -67.44 = 112.56°$

Bearing = 337.44°

420

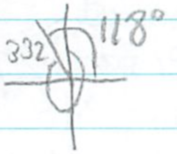
82. Jet = 580 mph Wind =



332° bearing

Jet $580 \cos 118 i + 580 \sin 118 j$

$-272,29 i + 512,11 j$



Wind $60 \cos 45 i + 60 \sin 45 j$

$\frac{60}{\sqrt{2}}$ $+42,23 i + 42.43 j$

u+v $-229,86 i + 554,54 j$

$\|u+v\| = \sqrt{229.86^2 + 554.54^2}$

$\sqrt{360350.23}$

600.29 mph

$\tan^{-1}\left(\frac{554.54}{-229.86}\right) = -67.48°$

$180 - 67.48 = 112.51°$

337.50° bearing

6.3/ Vectors
HW Day 4

11/7

418/55 Find magnitude + direction
$v = -2i + 5j$

✓

$\|v\| = \sqrt{(-2)^2 + 5^2}$
$\sqrt{29}$
~ 5.39

$\tan^{-1}\left(\frac{5}{-2}\right) = -68.20$
$111.80°$

57. Find component form
$\|v\| = 3$ @ $0°$

$\langle 3, 0 \rangle$

59. $\|v\| = 3\sqrt{2}$ @ $150°$

$[3\sqrt{2}\cos 150, 3\sqrt{2}\sin 150\rangle$
$\langle 3\sqrt{2} \cdot -\sqrt{3}/2, 3\sqrt{2} \cdot \frac{1}{2} \rangle$
$\left\langle \frac{-3\sqrt{6}}{2}, \frac{3\sqrt{2}}{2} \right\rangle$

61. $\|v\| = 2$
direction of $i + 3j$

$\tan^{-1}\left(\frac{3}{1}\right) = 71.56°$

$[2\cos 71.56, 2\sin 71.56\rangle$
$\langle .6324, 1.897 \rangle$

$\frac{2}{\sqrt{10}} i + \frac{6}{\sqrt{10}} j$

$\frac{2\sqrt{10}}{10} i + \frac{6\sqrt{10}}{10} j$

← don't need root form unless specifed

63. $\|u\| = 5$ $\quad$ $\|v\| = 5$

$60°$ $\quad$ $90°$

$5\cos 60\, i + 5\sin 60\, j$ $\quad$ $5\cos 90 + 5\sin 90\, j$

$5 \cdot \frac{1}{2} i + 5 \frac{\sqrt{3}}{2} j$ $\quad$ $5 \cdot 0\, i + 5 \cdot 1\, j$

$2.5 i + 2.5\sqrt{3} j$ $\quad$ $0 i + 5 j$

$2.5 i + 9.33 j$

$\|v+u\| = \sqrt{2.5^2 + 9.33^2}$

$\sqrt{93.2989}$

9.65

$\tan^{-1}\left(\frac{9.33}{2.5}\right) = 75°$

65. $\|v\| = 20$ $\quad$ $\|u\| = 50$

$45°$ $\quad$ $150°$

$20\cos(45) + 20\sin 45$ $\quad$ $50\cos 150 + 50\sin 150$

$14.14 i + 14.14 j$ $\quad$ $-43.30 i + 25 j$

$-29.16 i + 39.14 j$

$\|u+v\| = 48.81$

$\tan^{-1}\left(\frac{39.14}{-29.16}\right) = -53.31° \rightarrow 126.69°$

67. $v = i + j$

$w = 2(i - j)$

$w = 2i - 2j$

$||v|| = \sqrt{2}$

$||w|| = \sqrt{8} \rightarrow 2\sqrt{2}$

$||v H| =$

$(2-1)^2 + (-2-1)^2$

$\sqrt{10}$

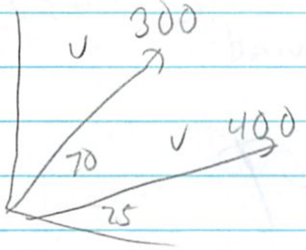
$$\frac{v^2 - v^2 - w^2}{-2vw} = \cos\theta$$

$$\frac{\sqrt{10}^2 - \sqrt{2}^2 - \sqrt{8}^2}{2\sqrt{10}\sqrt{8}}$$

$$\frac{10 - 2 - 8}{2\sqrt{80}}$$

$$\frac{0}{2\sqrt{80}} \rightarrow \cos^{-1}(0) \Rightarrow 90^\circ$$

69.



$300 \cos 70 i + 300 \sin 70 j$

$102.61 i + 281.91$

$400 \cos 25 i + 400 \sin 25$

$362.52 i + 169.05 j$

$465.13 i + 450.96 j$

$||v + v|| = 647.85$

$\tan\left(\frac{450}{465}\right) = 44.11^\circ$

419

73. 2000 @ 30°

$\langle 1732.05, 1000 \rangle$

900 @ -45°

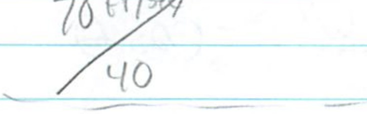
$\langle 636.396, -636.396 \rangle$

$\langle 2368.45, 363.6 \rangle$

$||r|| = 2396.197$

8.72°

75.



$70 \cos 40\, i + 70 \sin 40\, j$

$53.62\, i + 45\, j$

420

flipped function

81. 860 km @ 148° heading   plane $\langle \cancel{-729.32,\ 455.73} \rangle$

↳ −58° or 302° ↳ $\langle 455.73, -729.32 \rangle$

800 km @ 140° heading   plane + wind $\langle \cancel{612.84,\ 514.23} \rangle$

→ −50° or 310° ↳ $\langle 514.23, -612.84 \rangle$

plane + wind − plane = wind

$\langle \cancel{116.48,\ 58.5} \rangle$

↳ $\langle 58.5, 116.48 \rangle$ ↓ find magnitude + angle

$\|W\| = 130.35$ km/hr.

$\tan^{-1}\left(\frac{116.48}{58.5}\right) = 63.33°$

To

26.66 heading

6.4 Vectors + Dot Products

11/8

Vector addition + multiplication by a scalar
- each gets a new vector

3rd vector operation: dot product
- dot product yields a scalar quantity not a vector

Finding dot products

ex1 $u = \langle 4, 5 \rangle$ Find dot products $(u \cdot v)$

$v = \langle 2, 3 \rangle$ $4 \cdot 2 + 5 \cdot 3 = 23$

↑ scalar quanity

ex2 $u = \langle 2, -1 \rangle$ $2 \cdot 1 + -1 \cdot 2$

$v = \langle 1, 2 \rangle$ 0 ← are perpendicular

ex3 $u = \langle 0, 3 \rangle$ $0 \cdot 4 + 3 \cdot -2$

$v = \langle 4, -2 \rangle$ $0 + -6$

-6

Properties of Dot Products

u + v + w are vectors

1. $u \cdot v = v \cdot u$
   communitive property

2. $0 \cdot v = 0$
   zero product property

3. $u \cdot (v + w) = u \cdot v + u \cdot w$
   distributive property

4. $v \cdot v = \|v\|^2$ or $\sqrt{v \cdot v} = \|v\|$

5. $c(v \cdot v) = (c \cdot v) \cdot v$

associative property

Find dot product of 3 vectors

ex4 $u = \langle -1, 3 \rangle$ $(u \cdot v)\, w$

$v = \langle 2, -4 \rangle$ $-2 + -12 = -14\, w$

$w = \langle 1, -2 \rangle$ $-14 \langle 1, -2 \rangle$

429 $\langle -14, 28 \rangle$ ← back as a vector

$u = \langle 2, 2 \rangle$

$v = \langle -3, 4 \rangle$

$w = \langle 1, -4 \rangle$

6. $\|u\| - 2$

$\sqrt{8} \rightarrow 2.828 - 2$

$2\sqrt{2}$ $.828$ ← $2\sqrt{2} - 2$

8. $(w \cdot u)\, v$ ← $1 \cdot 2 + -4 \cdot 2$ remember dot product

$(-2 + -8)\, v$

$-6 \langle -3, 4 \rangle$

~~$\langle 4, -12 \rangle$~~

$\langle 18, -24 \rangle$

10. $4u \cdot v$

$4 \langle 2, 2 \rangle$

$\langle 8, 8 \rangle \cdot \langle -3, 4 \rangle$

$\rightarrow \langle -24 + 32 \rangle$ ← remember dot product

$8 \cdot -3 + 8 \cdot 4$ ⑧

↑

write it out until I can get it

Finding the Angle Between 2 Vectors

$$\cos Q = \frac{u \cdot v}{\|u\| \cdot \|v\|}$$

ex 1 $u = \langle 4, 3 \rangle$

$v = \langle 3, 5 \rangle$

$$\frac{\langle 4,3 \rangle \langle 3,5 \rangle}{\| 4,3 \| \; \| 3,5 \|}$$

$$\frac{4 \cdot 3 + 3 \cdot 5}{\sqrt{25} \cdot \sqrt{34}}$$

$$\frac{27}{5\sqrt{34}} \to \cos^{-1} \to \boxed{22,166^\circ}$$

429

18. Find θ

$\langle 4, 4 \rangle$

$\langle -2, 0 \rangle$

$$\frac{\langle 4,4 \rangle \langle -2,0 \rangle}{\| 4,4 \| \; \| -2,0 \|}$$

$$\frac{4 \cdot -2 + 4 \cdot 0}{\sqrt{32} + \sqrt{4}}$$

$$\frac{-8}{\underset{\hookrightarrow 8\sqrt{2}}{2\sqrt{32}}} \to \cos^{-1} \to \boxed{135^\circ}$$

24. $u = \cos\frac{\pi}{4} i + \sin\frac{\pi}{4} j \to \frac{\sqrt{2}}{2} i + \frac{\sqrt{2}}{2} j$

$v = \cos\frac{2\pi}{3} i + \sin\frac{2\pi}{3} j \to -\frac{1}{2} i + \frac{\sqrt{3}}{2} j$

$$\frac{\left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle \cdot \left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle}{1 \cdot 1}$$

$$\frac{-\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}}{1} \to \frac{-\sqrt{2}+\sqrt{6}}{4} \to \cos^{-1} \to 75^\circ$$

Orthogonal = Perpendicular

If vectors u and v are orthogonal
then $u \cdot v = 0$

ex 1 $\langle 2, -3 \rangle$ $\langle 6, 4 \rangle$

$2 \cdot 6 + -3 \cdot 4$

$12 + -12$

0 ✓ Orthogonal

430

38. $u = 8i + 4j \rightarrow \langle 8, 4 \rangle$

$v = -2i - j \rightarrow \langle -2, -1 \rangle$

↑ same slope so parallel

$8 \cdot -2 + 4 \cdot -1$

$-16 + -4$

-20 ← can't tell

6.4 Vectors + Dot Products

HW

11/8

429/ 3. Find dot product

✓

$u = 5i + j$

$v = 3i - j$

$\langle 5, 1\rangle \cdot \langle 3, -1\rangle$

$5 \cdot 3 + 1 \cdot -1$

$15 - 1$

14

7. ✓

$u = \langle 2, 2\rangle$

$v = \langle -3, 4\rangle$

$w = \langle 1, -4\rangle$

$(u \cdot v)\, w$

$2 \cdot -3 + 2 \cdot 4 (w)$

$2\langle 1, -4\rangle$

$\langle 2, -8\rangle$

13. Find magnitude

$20i + 25j$

$\leftarrow v \cdot v = \|v\|^2$

$20 \cdot 20 + 25 \cdot 25$

$400 + 625$

$1025 = \|v\|^2$

$\sqrt{1025} = \|v\|$ → $\boxed{5\sqrt{41}}$ reduce

or

$\sqrt{20^2 + 25^2}$

↓

$\sqrt{1025}$

17. Find θ

$u = \langle -1, 0\rangle$

$v = \langle 0, 2\rangle$

$$\frac{u \cdot v}{\|u\| \cdot \|v\|}$$

$$\frac{-1 \cdot 0 + 0 \cdot 2}{-1 \cdot 2}$$

$$\rightarrow \frac{0}{-2} \rightarrow 0 \rightarrow \cos^{-1} \rightarrow 90^\circ$$

19.

$u = 3i + 4j$

$v = -2i + 3j$

$$\frac{3 \cdot -2 + 4 \cdot 3}{\sqrt{25} \cdot \sqrt{13}}$$

$$\frac{6}{5\sqrt{13}} \rightarrow \cos^{-1} \rightarrow 70.56$$

23. $u = \cos\left(\frac{\pi}{3}\right)i + \sin\left(\frac{\pi}{3}\right)j \rightarrow \frac{1}{2}i + \frac{\sqrt{3}}{2}j$

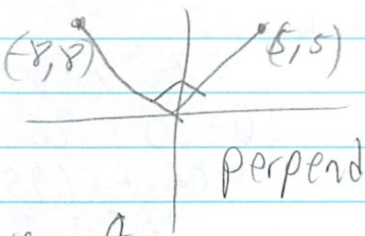
$v = \cos\left(\frac{3\pi}{4}\right)i + \sin\left(\frac{3\pi}{4}\right)j \rightarrow -\frac{\sqrt{2}}{2}i + \frac{\sqrt{2}}{2}j$

$$\frac{\frac{1}{2} \cdot -\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}}{\sqrt{1} \cdot \sqrt{1}}$$

$$\frac{.2588}{1} \rightarrow \cos^{-1} \rightarrow 75.00°$$

27. $u = 5i + 5j$

$v = -8i + 8j$



perpendiclar 90°

Find $u \cdot v$ w/ given θ

31. $\|u\| = 4$

$\|v\| = 10$

$\theta = \frac{2\pi}{3}$

$\rightarrow u \cdot v = \|u\|\|v\| \cos\theta$

$4 \cdot 10 \cdot \cos\frac{2\pi}{3}$

$4 \cdot 10 \cdot -\frac{1}{2}$

$\boxed{-20} = u \cdot v$

p430

33. $u = \langle -12, 30 \rangle$ $\quad -12 / \frac{1}{2} = -24$

$v = \langle 1/2, -5/4 \rangle$ $\quad 30 / -5/4 = -24$) porrallel

35 $u = \frac{1}{4}(3i - j) \rightarrow 3/4\, i - \frac{1}{4}j$ $\quad \frac{3}{4} \cdot 5 + -\frac{1}{4} \cdot 6$

$v = 5i + 6j$

$3.75 - 1.5$

2.25 ⊗

$2 \cdot 1 + -2 \cdot -1$

37. $u = 2i - 2j$ $\quad -2 + 2$

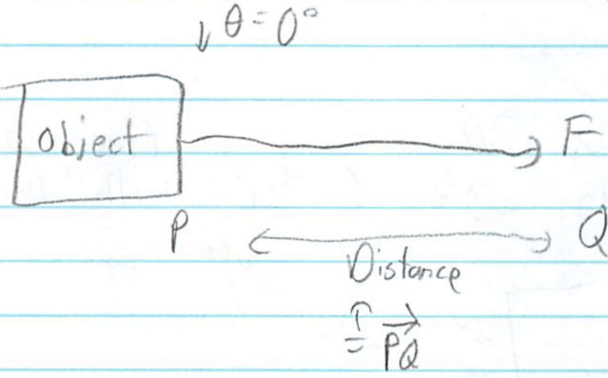
$v = -i - j$ $\quad 0$ ✓ Orthagonal

6.4 Vectors + Dot Products
Day 2

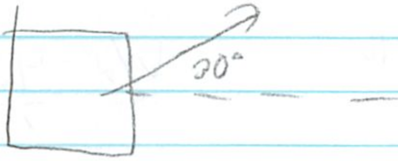
11/8

Work done on an object

Work (w) done by a constant force (F) along a line of motion of an object is given by $W_x = \underset{\substack{\downarrow \\ \text{Force}}}{\|F\|} \cdot \underset{\substack{\downarrow \\ \text{distance}}}{\|D\|} \cdot \underset{\substack{\downarrow \\ \text{angle}}}{\cos\theta}$



d = displacement $(\overset{\leftarrow \text{change}}{\Delta}$ in position)
that the object undergoes the influence of the force



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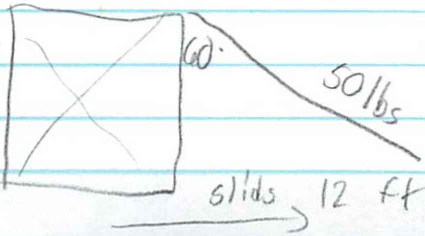
ex1. To close a barn sliding door, a person pulls on a rope with a constant Force of 50 lbs. at constant angle of 60°



Find work done to move 12 ft

$W = \|F\| \cdot \|D\| \cos\theta$

50 lbs · 12 ft · cos 60

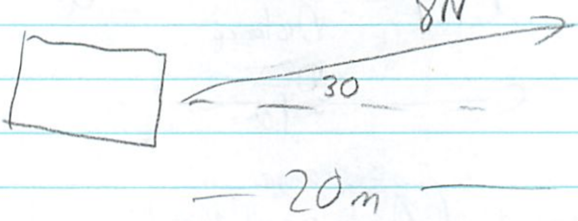
300 foot pounds

↓ basic unit of work = 1 Joule

- is work done by force of 1 N in moving an object in 1m

---

ex2 Move a box 20 m along a floor and the force makes an angle of 30° with the floor

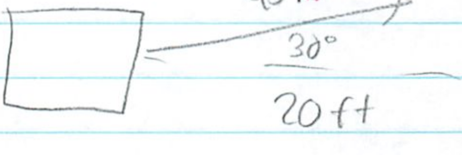


$W_x = \|F\| \cdot \|D\| \cos 30$

$W_x = 138,56$ J

431

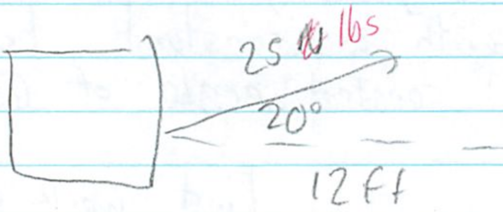
58.



$W_x = 45 \cdot 20 \cdot \cos 30$

779,42 ~~J~~ ft-lbs

60.



$W_x = 25 \cdot 12 \cdot \cos 20$

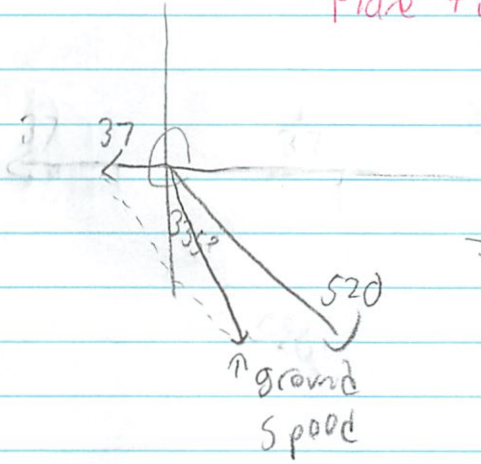
281,91 ~~N~~ ft-lbs

Airplane → Laws of Sin + Cos

Airplane 520 mph
115° bearing

Wind 37 mph
West → 180°

Find ground speed
↑ plane + wind



→

law of sines; $a^2 = b^2 + c^2 - 2bc \cos 25°$
- side lenght $a^2 = 520^{\circ} + 37^2 - 2(520)(37)\cos 25$
$a^2 = 236894$
$\boxed{a = 482 \text{ mph}}$

angle ; $\frac{\sin a}{37} = \frac{\sin 25}{487}$ cross multiply
$a = 1.84°$
$115 + 1.84 = \boxed{116.84° \text{ bearing}}$

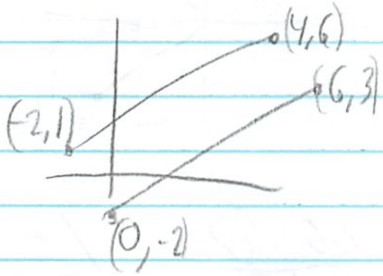
6/ Review

11/8

<u>445</u>

Show that $u = v$

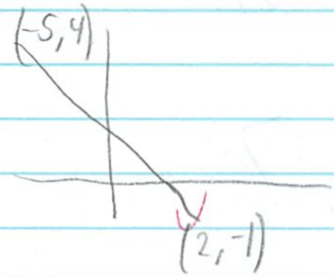
37.



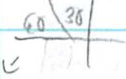
$||u|| = \sqrt{(4--2)^2 + (6-1)^2} = \sqrt{61}$
$||v|| = \sqrt{(6-0)^2 + (3--2)^2} = \sqrt{61}$

$\frac{6-1}{4--2} = \frac{5}{6}$ $\frac{3--2}{6-0} = \frac{5}{6}$ ✓

39 ✓



terminal - initial
~~$\langle -5\ 2, 4 - -1 \rangle$~~
$\langle 2 - -5, -1 - 4 \rangle$
$\langle 7, -5 \rangle$



43. $||v|| = 8$
$\theta = 120°$

Chart p 281

$8 \cos 120\, i + 8 \sin 120\, j$
$-4i + 8 \frac{\sqrt{3}}{2} j$
$-4i + 4\sqrt{3} j$

49. $u = 2i - j$
$v = 5i + 3j$

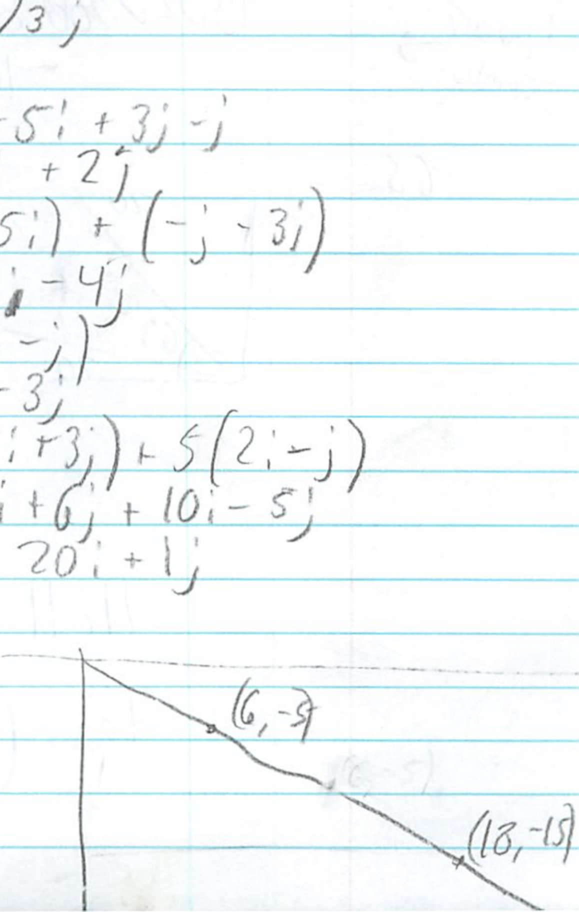
$u + v)$ $2i + 5i + 3j - j$
$7i + 2j$

$u - v)$ $(2i - 5i) + (-j - 3j)$
$-3i - 4j$

$3u)$ $3(2i - j)$
$6i - 3j$

$2v + 5u)$ $2(5i + 3j) + 5(2i - j)$
$10i + 6j + 10i - 5j$
$20i + 1j$

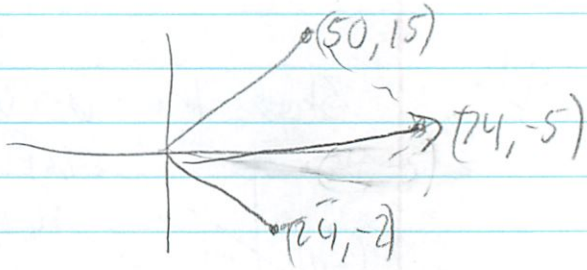
53. $3(6i - 5j)$ $\langle 18i - 15j)$



55. $4(6i - 5j) + 5(10i + 3j)$

$24i - 20j + 50i + 15j$

$74i - 5j$



59. Unit Vector $\rightarrow$ $\frac{v}{\|v\|}$ $\frac{\langle 5 -2 \rangle}{\sqrt{29}}$ $\left\langle \frac{5}{\sqrt{29}}, \frac{-2}{\sqrt{29}} \right\rangle$

$5i - 2j$

61. $(-8, 3)$ ↑I T $\langle 1 - -8, -5 - 3 \rangle$

$(1, -5)$ $\langle 9, -8 \rangle$

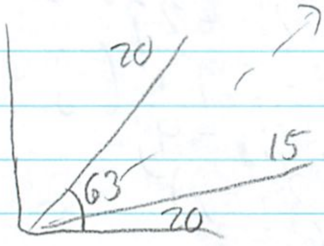
63. $v = -10i + 10j$ $\|v\| = \sqrt{200} \rightarrow 10\sqrt{2}$

↑ 2nd quad → $\tan^{-1}\left(\frac{10}{-10}\right) = -45°$ $\theta =$ ~~315°~~ $\leftarrow$ 135°

linear combo → $10\sqrt{2}$ ~~$\sqrt{200}$~~ $\cos 135\, i + \sqrt{\cancel{200}}\ 10\sqrt{2} \sin 135$

$-10i + 10j$

↑ points in 2nd quad so give 2nd quad angle

65. 

$20 \cos 63\, i + 20 \sin 63$

$9.08 i + 17.02 j$

$15 \cos 20 i + 15 \sin 20 j$

$14.10 i + 5.13 j$

$r = 23.18 i + 22.15 j$

$\|r\| = 32.06$

$\tan^{-1}\left(\frac{22.15}{23.18}\right) = 43.50$

446

11/9

67. Resultant Force

1- 250 lbs ⓐ 60°
2- 100 lbs ⓐ 150°
3- 200 lbs ⓐ -90°

p269 → 1) $250\cos 60\,i + 250\sin 60\,j$
$125\,i + 216.50\,j$

2) $100\cos 150\,i + 100\sin 150\,j$
$-86.60\,i + 50\,j$

3) $200\cos(-90)\,i + 200\sin(-90)\,j$
$0\,i - 200\,j$

sum: ~~34.4~~ 38.397 $i + 66.5\,j$ ← calc error

$\|sum\|$ = ~~74.87~~ 76.8 lbs

$\theta = \tan^{-1}\left(\frac{66.5}{\cancel{34.4}\ 38.397}\right)$ = ~~62.65°~~ 60°

71. Navigation

plane: 430 mph ⓐ 135° bearing → 315°
wind: 35 mph ⓐ N 30° E → 60°

$430\cos 315\,i + 430\sin 315$
$304.06\,i - 304.05\,j$

$35\cos 60\,i + 35\sin 60$
$17.5\,i + 30.31\,j$

sum: $321.56\,i - 273.74\,j$

↗ $\|plane + wind\|$ =
422.3 mph

$\theta = -40.41°$
130.41° bearing

72. plane: 724 km/hr @ 30° bearing → 60°
wind: 32 km/hr from West → 0°

$724\cos 60 i + 724\sin 60 j$
$362 i + 627 j$

$32\cos 0 i + 32\sin 0 j$
$32 + 0 j$

plane+wind $394 i + 627 j$

$\|\text{plane} + \text{wind}\| = 740.52$ km/hr

$\theta° = 57.86° \rightarrow 32.144°$
bearing

75. Dot product
$v = 6i - j$
$v = 2i + 5j$

$(6 \cdot 2) + (-1 \cdot 5)$
$12 + -5$
7

79. $4\langle -3, -4\rangle \cdot \langle 2 \cdot 1\rangle$
$\langle -12, -16\rangle \cdot \langle 2 \cdot 1\rangle$
$(-12 \cdot 2) + (-16 \cdot 1)$
$-24 + -16$
-40

81. Find θ
$v = \langle 2\sqrt{2}, -4\rangle$
$v = \langle -\sqrt{2}, 1\rangle$

error domain

$\frac{v \cdot v}{\|v\| \cdot \|v\|}$

$\frac{(2\sqrt{2} \cdot -\sqrt{2}) + (-4 \cdot 1)}{2\sqrt{6} \cdot \sqrt{3}}$

$\frac{-8}{~~\sqrt{128}~~} \rightarrow \cos^{-1}$ ~~cl~~ $\frac{4.2716}{\sqrt{24}}$

$\sqrt{72}$

8.49 → calc error

↳ 160.52

83. $v = \cos\frac{7\pi}{4} i + \sin\frac{7\pi}{4} j \rightarrow \frac{1}{\sqrt{2}} i - \frac{1}{\sqrt{2}} j$

$v = \cos\frac{5\pi}{6} i + \sin\frac{5\pi}{6} j \rightarrow -\frac{\sqrt{3}}{2} i + \frac{1}{2} j$

$$\frac{\left(\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{\sqrt{2}} + \frac{1}{2}\right)}{1 \cdot 1}$$

$-\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \rightarrow \frac{-.9659}{1} \rightarrow \cos^{-1} \rightarrow 164.99°$

$\frac{-\sqrt{6}-\sqrt{2}}{4}$

85. $4i + j$
$i - 4j$

$$\frac{(4 \cdot 1) + (1 \cdot -4)}{?}$$

$\frac{0}{?} \rightarrow 0 \rightarrow \cos^{-1} \rightarrow 90°$ perpendicular (orthogonal)

↑ make sure parentheses around top
Or just type it in

90. $v = \langle 8, -4 \rangle$
$v = \langle 5, 10 \rangle$

$(8 \cdot 5) + (-4 \cdot 10)$
$40 - 40$
0 Orthogonal

92. $v = \langle 15, 51 \rangle$
$v = \langle 20, -88 \rangle$

$(15 \cdot 20) + (51 \cdot -88)$
$300 + -4080$
not orth.

$\frac{51}{15} = 3.4 \quad \frac{-88}{20} = -4.4$
not parallel

neither

97 Work

18,000 lbs truck 48 in
→ 4 ft

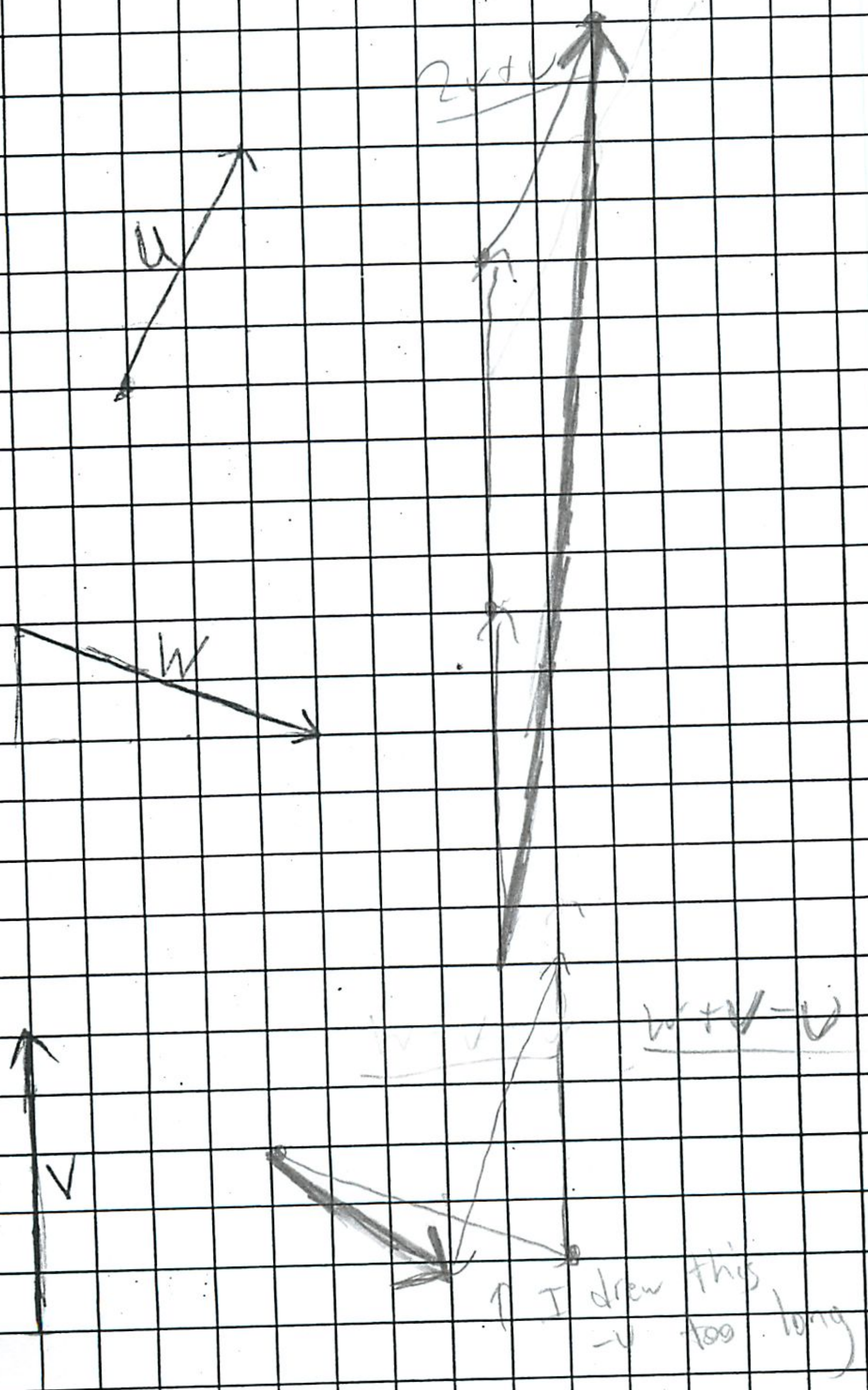
$W_x = m \cdot d \cdot \sin 90$
$W_y = 18000 \cdot 4 \cdot 1$
$W_y = 72{,}000$ ft-lbs

↑ gave problem of Work in y-direction

2v+u

Draw W+V-U

Where is the resultant vector?



↑ I drew this -u too long

6/ Review 2

11/12

1. $v = \langle 2,5 \rangle$ $\quad (-5-2,\ 6-5)$

terminal $\langle -5,6 \rangle$ $\quad (-7,\ 1)$ ← inital point

2. Find Work done by moving an object from x to y

$F = -3i - 4j$ $\quad W = \|F\| \cdot D \cdot \cos\theta$

$x = (7,3)$ I

$y = (1,-1)$ T $\quad \|F\| = \sqrt{(-3)^2 + (-4)^2}$

5

T − I

$(1-7, -1-3)$

$(-6, -4)$

$\|D\| = \sqrt{(7-1)^2 + (3--1)^2}$

$\sqrt{52}$

7.21

$\theta = \tan^{-1}\left(\frac{-4}{\cancel{-3}\ -6}\right) = \cancel{53.13}°$ 33.69

do the tan of the distance vector Not the Force ⟶

33.69

$W = 5 \cdot \sqrt{52} \cdot \cos(\cancel{53.13})$

$W = \cancel{21.633}$

30

7.1-7.3 Solving Systems of Equations

11/14

Combine

ex1

$2x+y=5$

$x-y=1$

$3x=6$

$\frac{3x}{3}=\frac{6}{3}$

$x=2$

↓combo ^ method
elimination

y/ $(2)-y=1$

$-y=-1$

$y=1$

or

2·2 2·1

$$\begin{bmatrix}2&1\\1&-1\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}5\\1\end{bmatrix}$$

↑coefficent ↑variables ↑answers

$$[A]^{-1}\cdot[B]=\begin{bmatrix}2\\1\end{bmatrix}$$

ex2

$x-y=2 \xrightarrow{\cdot 2} 2x-2y=4$

$-2x+2y=5$ ↙

$0=9$

↑ no solution → parallel lines

ex3

$\frac{3}{4}x+y=\frac{1}{8} \xrightarrow{\cdot -3} -\frac{9}{4}-3y=\frac{-3}{8}$

$\frac{9}{4}x+3y=\frac{3}{8}$ ↙

$0=0$

↑ same line, share ∞ # of points

3 variable

ex4

$x - 2y + 3z = 9$) $\boxed{y + 3z = 5}$

$-x + 3y = -4$

$2x - 5y + 5z = 17$

~~$-x + 3(-27) = -4$~~ ~~$2(-77) - 5(-27) + 5z = 17$~~

~~$-x + -81 = -4$~~ ~~$-154 + 135 + 5z = 17$~~

~~$+19 \quad +19$~~

~~$-x = 77$~~

~~$\boxed{x = -77}$~~ ~~$5z = 36$~~

~~$\boxed{z = 7.2}$~~

$-2x + 6y = -4$

$2x - 5y + 5z = 17$

$\boxed{y + 5z = 9}$

$-y - 3z = -5$

$y + 5z = 9$

$2z = 4$

$\boxed{z = 2}$ ←

$y + 5(2) = 9$

$y + 10 = 9$

$-10 \quad -10$

$\boxed{y = -1}$

$-x + 3(-1) = -4$

$-x - 3 = -4$

$+3 \quad +3$

$-x = -1$

$\boxed{x = 1}$

√, Enter as matrix
On calc

Get 1 fraction

ex5

$$\frac{2}{x-3} + \frac{-1}{x+2}$$

↓ Get LCD

$$\frac{2(x+2)}{(x-3)(x+2)} + \frac{-1(x-3)}{(x+2)(x-3)}$$

↓

$$\frac{2x + 4 - x + 3}{x^2 - x - 6}$$

↓

$$\frac{x + 7}{x^2 - x - 6}$$

Partial Fraction Decomposition

ex 6

$$\frac{x+7}{x^2-x-6} \rightarrow \frac{x+7}{(x+3)(x-2)} \rightarrow \frac{A}{x+3} + \frac{B}{x+2}$$

Factor denom

↑ linear factors that don't repeat

↓ has to be one degree less — in this case constant

↓ • LCD → $(x+3)(x-2)$

$$\frac{A\cancel{(x+3)}(x-2)}{\cancel{(x+3)}} + \frac{B(x+3)(x-2)}{(x+2)}$$

↓

$$x+7 = A(x+2) + B(x-3)$$

$$x+7 = Ax + 2A + Bx - 3B$$

group like terms ↓

$$x+7 = Ax + Bx + 2A - 3B$$

$$x+7 = x\underbrace{(A+B)}_{1} + 2A - 3B$$

A+B = coefficent of x

$$A+B = 1$$

$$\cdot{-2}\ \ 2A - 3B = 7$$

$$-2A + -2B = -2$$

$$\frac{-5B}{-5} = \frac{5}{-5}$$

$$\boxed{B = -1}$$

$$A + (-1) = 1$$

$$\boxed{A = 2}$$

$$\frac{x+7}{x^2-x+6} = \frac{2}{x+3} - \frac{-1}{x+2}$$

✓! ↑ y_1 $\overset{?}{=}$ ↑ $y2$

ex 7

$$\frac{7x-13}{x^2-2x-3} = \frac{7x-13}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$$

↑ LCD

• LCD = numerator → $7x - 13 = A(x+1) + B(x-3)$

$$Ax + A + Bx - 3B$$

$$Ax + Bx + A - 3B$$

$$x\underbrace{(A+B)}_{1} + A - 3B$$

$A+B=7$ ← coefficent

$\cdot{-1}$ $A - 3B = -13$ ← constant

$-A - B = -7$

$-4B = -20$

$A + (-5) = 7$

$$\boxed{A = 12}$$

write out answer as:

$$\boxed{\frac{7x-13}{x^2-2x-3} = \frac{2}{x-3} + \frac{5}{x+1}}$$

---

3 terms

$$\frac{4x^2+13x-12}{x^3+x^2-6x} \rightarrow \frac{4x^2+13x-12}{\underset{\uparrow}{x}(x^2+x-6)} \rightarrow \frac{4x^2+13x-12}{x(x+3)(x-2)}$$

LCD

$$\frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-2}$$

$\downarrow \cdot$ LCD

$$\frac{A\,\cancel{(x)}(x+3)(x-2)}{\cancel{x}} + \frac{B(x)\cancel{(x+3)}(x-2)}{\cancel{x+3}} + \frac{C(x)(x+3)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$A(x+3)(x-2) + B(x)(x-2) + C(x)(x+3)$$

$$A(x^2+x-6) + B(x^2-2x) + C(x^2+3x)$$

group terms

$$Ax^2 + Ax - 6A + Bx^2 - 2Bx + Cx^2 + 3Cx$$

$$Ax^2 + Bx^2 + Cx^2 + Ax - 2Bx + 3Cx - 6A$$

$$x^2(A+B+C) + x(A-2B+3C) - 6A$$

$-6A = -12$ ← compare to orignal numerator coefficent

$A - 2B + 3C = 13$

$A + B + C = 4$

$\boxed{A = 2}$

$(2) + B + C = 4$

$\boxed{B + C = 2}$

$(2) - 2B + 3C = 13$

$\boxed{-2B + 3C = 11}$

$\cdot 2$ $2B + 2C = 4$

$5C = 15$

$\boxed{C = 3}$

$B + 3 = 2$

$\quad -3 \quad -3$

$\boxed{B = -1}$