Michael Plasmeier theplaz@mit.edu F10

staple!

14.01 Fall 2010
Problem Set 3
Due in class on October 1st

1. (10 points) A government considers two policy options, (i) a cash grant of \$100 and (ii) a certificate good for \$100 worth of food. Is a wealthy person more likely than a poor person to prefer the cash grant? Explain using graphs.

2. (23 points) Suppose there are exactly two consumers (Albie and Bubbie) who demand strawberries. Suppose that Albie's demand for strawberries is given by

$$q_a(p) = p^{\alpha} f_a(I_a)$$

and Bubbie's demand is given by

$$q_b(p) = p^{\beta} f_b(I_b)$$

where I_a and I_b are Albie and Bubbie's incomes, and $f_a(\cdot)$ and $f_b(\cdot)$ are two unknown functions.

(a) (5 points) Find Albie and Bubbie's (own-price) elasticities of demand, $\epsilon_{q_a,p}$ and $\epsilon_{q_b,p}$. Use the sign convention that $\epsilon_{y,x} = \frac{\partial y}{\partial x}\frac{x}{y}$.

(b) (5 points) Suppose that $\alpha > 0 > \beta$. Are strawberries a Giffen good for Albie? Are strawberries a Giffen good for Bubbie?

(c) (13 points) Are strawberries an inferior good for Albie? Are strawberries an inferior good for Bubbie? Assume that these demands arise from utility maximization given linear budget constraints. 4 Hint: This question should not require much/any algebra beyond (b).

4. (25 points) Sophia considers peaches and nectarines to be perfect substitutes. She spends \$5 a month on these fruits. Initially, peaches are \$1 a pound and nectarines are \$1.25 a pound. Then, the price of peaches increases to \$1.50 a pound. Her income allocated to fruit does not change, however.

(a) (5 points) How does consumption change when the price of peaches changes?

(b) (5 points) Show with the aid of a graph how utility changes when the price changes.

(c) (5 points) How much must her budget increase in order to return to the original utility level?

(d) (5 points) Derive and graph the demand curve for nectarines assuming that income is \$5 and peach price is \$1.

(e) (5 points) Derive and graph the Engel curve for nectarines (under the assumption that the price of peaches is \$1.50 a pound and the price of nectarines is \$1.25 a pound).

5. (42 points) Xiaoyu spends all her income on statistical softwares (S) and clothes (C). Her preferences can be represented by the utility function: $U(S, C) = 4ln(S) + 6ln(C)$.

(a) (6 points) Compute the marginal rate of substitution of softwares for clothes. Is the MRS increasing or decreasing in S? How do we interpret this?

(b) (6 points) Find Xiaoyu's demand functions for softwares and clothes, $Q_S(p_S, p_C, I)$ and $Q_C(p_S, p_C, I)$, in terms of the price of softwares (p_S), the price of clothes (p_C) and Xiaoyu's income (I).

(c) (6 points) Draw the Engel curve for softwares.

(d) (6 points) Suppose that the price of softwares is $p_S = 2$, the price of clothes is $p_C = 3$ and Xiaoyu's income is $I = 10$. What bundle of softwares and clothes (S, C) maximize Xiaoyu's utility?

(e) (6 points) Suppose the price of softwares increases to $p_S = 4$. What bundle of softwares and clothes does Xiaoyu demand now?

(f) (6 points) Given the price increase, how much income does Xiaoyu need to remain as happy (have the same utility) as she was before the price change? What bundle of softwares and clothes would Xiaoyu consume if she had that additional income given the new prices.

(g) (6 points) Going back to the situation in part (e) ($p_S = 4$ and $I = 10$) decompose the total change of softwares and clothes demanded into substitution and income effects. In a clearly-labeled diagram with softwares on the horizontal axis show the income and substitution effects of the increase in the price of softwares.

Michael Plasmeier

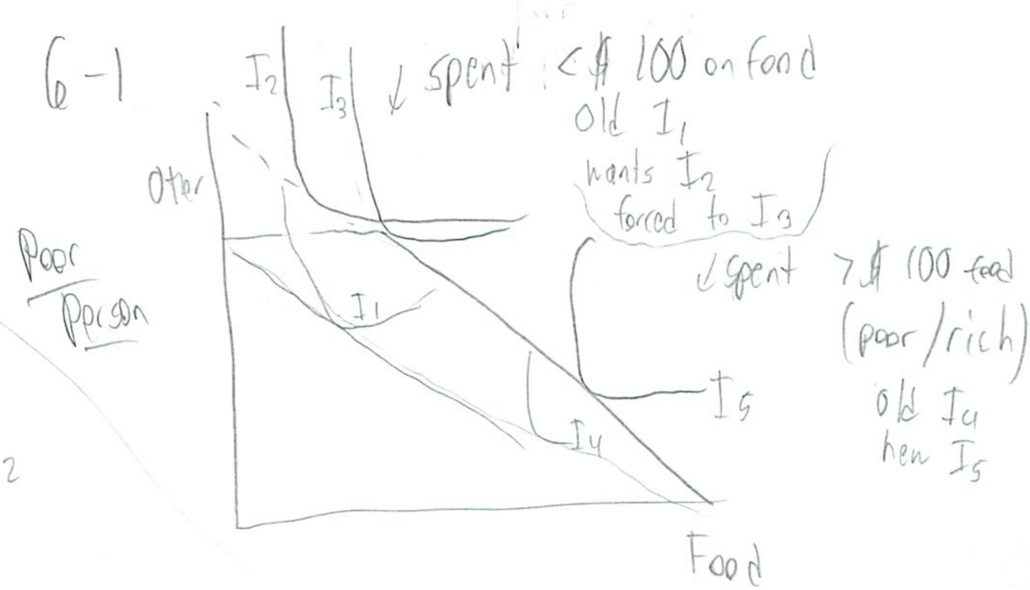
P-Set 3

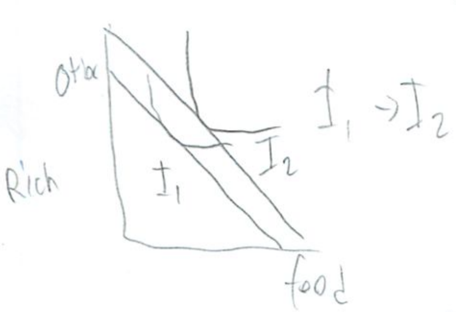
51

9/30

1. A wealthy person would not really care. I am assuming that they spend more than $100 on food already. If they got a certificate they would merely use that to pay for existing food purchases. They would then use the leftover $100 to spend on whatever they prefer, just as if they would have gotten cash in the first place. They would probably just prefer cash as it would be easier. The poor person would be in the same situation if they spent over $100 on food. If they spent less, than they would be losing utility if they were forced to spend that $ on food. So in short, everyone prefers cash except for the paternalistic gov.

See figure 6-1





(2)

2. Alice demand for Strawberries $q_a(p) = p^{\alpha} f_a(I_a)$

Bubbie " " " $q_b(p) = p^{\beta} f_b(I_b)$

I_a, I_b incomes

$f_a(), f_b()$ unknown functions

a) Find own price elasticities $\epsilon_{q_a,p}$ $\epsilon_{q_b,p}$

Note $\epsilon_{y,x} = \frac{\partial y}{\partial x} \frac{x}{y}$

$\cancel{\frac{\Delta Q/Q}{\Delta Y/Y}}$ $\cancel{= \Delta P/P}$ ~~I think I wrote it wrong in notes~~

$$\epsilon_{q_a,p} = \frac{\partial q_a}{\partial p}\frac{p}{q} = \alpha p^{\alpha-1} \underbrace{f_a(I_a)}_{\text{constant}} \cdot \frac{p}{q_a(p)}$$

$$= \frac{\alpha p^{\alpha-1} \cancel{f_a(I_a)} \cdot p}{p^{\alpha} \cancel{f_a(I_a)}}$$

$$= \frac{\alpha p^{\alpha-1} \cdot p}{p^{\alpha}} = \frac{\alpha \cancel{p^{\alpha}}}{\cancel{p^{\alpha}}} = \alpha$$

$$\epsilon_{q_b,p} = \frac{\partial q_b}{\partial p}\frac{p}{q} = \beta p^{\beta-1} f_b(I_b) \cdot \frac{p}{q_b(p)}$$

③

$$\frac{\beta p^{\beta-1} \cancel{f_p(I_0)} \cdot p}{p^{\beta} \cancel{f_b(I_b)}}$$

$$\frac{\cancel{\beta} \cancel{p}^{\beta}}{\cancel{p}^{\beta}}$$

β ✓

b) Suppose $\alpha > \partial > \beta$, Griffin good? -5

✗ No: Income effect must be bigger than substitution efect

c) Inferior good?

~~$\beta < 0$ so Strawberries are inferior to Bubbie~~

$$\gamma = \frac{\Delta Q/Q}{\Delta Y/Y} = \frac{dQ}{dI} \cdot \frac{I}{Q}$$

-10

$\gamma < 0$ = inferior

$\gamma > 0$ = normal

$$\gamma = p^{\alpha} \frac{df(I_a)}{dI_a} \cdot \frac{I_a}{p^{\alpha} f(I_a)} = \frac{df(I_a)}{dI_a} \cdot \frac{I_a}{f(I_a)}$$

$$\ldots = \frac{df(I_b)}{dI_b} \frac{I_b}{f(I_b)}$$

need to solve for

(4)

4. Sophia → peaches + nectoriers are substitute

$I = \$15$

$P = \$1$

$N = \$1.25$

$P_n = \$1.5$

a) Well at first she spends all her money on peaches (5 units) because they are perfect substitutes. When the price spikes on peaches, she just switches all to nectarines (4 units purchased)

b)

c) She is now getting 4 units of citrus when she had 5. She needs a \$1.25 budget hike.

d) Peach

-3

perfectly elastic

Nectorines

(5)

e) Engall Curve

As income goes ↑, will be 0 always demanded - since buy Peaches

6

5. Xiaoyu buys S, C

$$U(S,C) = 4\ln(S) + 6\ln(C)$$

a) Compare MRS of software for clothes

$$-\frac{MU_x}{MU_y} = \frac{-MU_s}{MU_c}$$

~~Solve for U in terms of C~~

$$\cancel{U = 4\ln(S) + 6\ln(C)}$$

$$\cancel{\frac{-4\ln(S)}{6} = \ln(C)}$$

$$\cancel{e^{\wedge} \qquad e^{\wedge}}$$

$$\cancel{e^{\frac{-4\ln(S)}{6}} = C}$$

-1

$$MRS = -\frac{\frac{\partial U_s}{\partial U}}{\frac{\partial U_c}{\partial U}} = -\frac{\frac{4}{S}}{\frac{6}{C}} = \frac{4/S}{6/C} \quad \frac{4}{S} \cdot \frac{C}{6} = \cancel{\frac{4C}{6S}}$$

↑ want denominator to = 1

- but where on S?

marginal utility > 0

decreasing - well in normal cases (like this) MRS decreases as one consumes more goods

- can have variables in
  - but what does that mean?

(7)

b. Find her demand function $Q_s(P_s, P_c, I)$

$Q_c(P_s, P_c, I)$

P_s = price software

P_c = price clothes

I = income

$Q_s = I - Q_c$

$Q_c = I - Q_s$

$Q_s + Q_c = I$

$U = U_c(Q_c) + U_s(I - Q_c)$

cross price elasticities

$$\epsilon = \frac{\%\text{ change demand } A}{\%\text{ change price } B}$$

$\boxed{\text{math never taught}}$

- but what does that give you?
  - slope of demand curve

but how does that figure to demand curve
  - say function of

$$Q_s = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q} + f(\epsilon_{P_s, P_c})$$

$Q_c = I - Q_s$

web: hint: Lagrangian

end of last recitation

↓

(8) key Qs ; At each price how many to buy
- but multidimensional is throwing me off

Look at OCW notes
- very helpful!

Ok I got the optimize → but how to get demand curve! ?

Budget $I = P_c Q_c + P_s Q_s$

$MRS = \frac{4 Q_c}{6 Q_s} = \frac{P_c}{P_s}$

So $Q_c = \frac{I - P_s Q_s}{P_c}$?, so how does that help?

$Q_s = \frac{I - P_c Q_c}{P_s}$

-3

?? totally confused

Oh Expenditures formula

$$E = E(p_1, p_2, \bar{U}) \qquad \boxed{3.29}$$

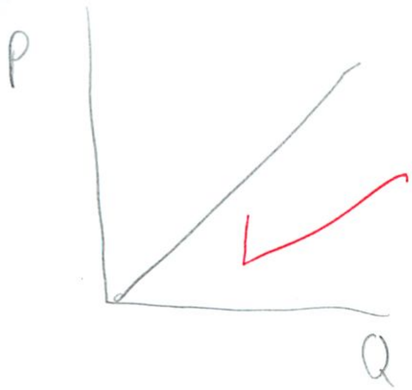
- dut that is to minimize expenditures

$$\min_{q_1 q_2} E = p_1 q_1 + p_2 q_2$$

$$\bar{U} = U(q_1, q_2)$$

(4)

c) Engall Curve for software



d) Suppose $P_s = 2$ $I = 10$
$P_c = 3$

What bundel maximizes utility

$$U = U_c(Q_c) + U_s(I - Q_c)$$
$$= U_s(Q_s) + U_c(I - Q_s)$$

fill in + maximize

-4

(10) $L | x, y, \lambda = U(x,y) + \lambda\left(P_s \cdot Q_s + \left(I - (Q_s \cdot P_s)\right)\right)$

↑ the add utility
you get if income ↑
1 unit

$= 4\ln(Q_s) + 6\ln(QC) + \lambda\left(Q_s \cdot P_s + \left(I - (Q_s \cdot P_s)\right)\right)$

$\underline{\partial Q_s}$ $= \dfrac{4}{Q_s} + 6\ln(Q_c) - \lambda P_s = 0$

$\underline{\partial Q_c}$ $= 4\ln(Q_s) + \dfrac{6}{Q_c} - \lambda P_c = 0$

$$\begin{cases} \dfrac{4}{Q_s} + 6\ln(Q_c) = \lambda P_s \\ 4\ln(Q_s) + \dfrac{6}{Q_c} = \lambda P_c \end{cases}$$

λ
$$\frac{\frac{4}{Q_s} + 6\ln(Q_c)}{P_s} = \frac{4\ln(Q_s) + \frac{6}{Q_c}}{P_c}$$

now put in #
$$\frac{\frac{4}{Q_s} + 6\ln(Q_c)}{2} = \frac{4\ln(Q_s) + \frac{6}{Q_c}}{3}$$

that is not making things simple

Uccc - why not coming together
no full examples given

(11)

e) Suppose $p_s = 4$ What bundle now?

Readjust formuals

f) Can't do w/o solving b

g) "

-18

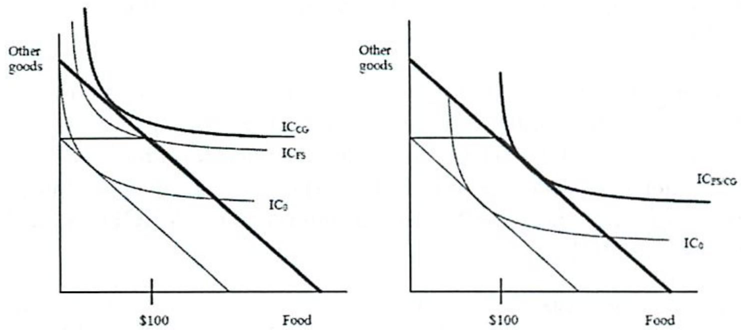
14.01 Fall 2010

Problem Set 3

Due in class on October 1st

1. (10 points) A government considers two policy options, (i) a cash grant of \$100 and (ii) a certificate good for \$100 worth of food. Is a wealthy person more likely than a poor person to prefer the cash grant? Explain using graphs.

Recall that a budget constraint line (BC) will shift out by \$100 for an individual with \$100 additional income. For one receiving \$100 in food stamps, the BC shifts out by \$100 along one segment only; this new BC also has a kink because one cannot spend more on other goods than the maximum affordable pre-policy. Now consider two cases, shown in Figures 1A and 1B:



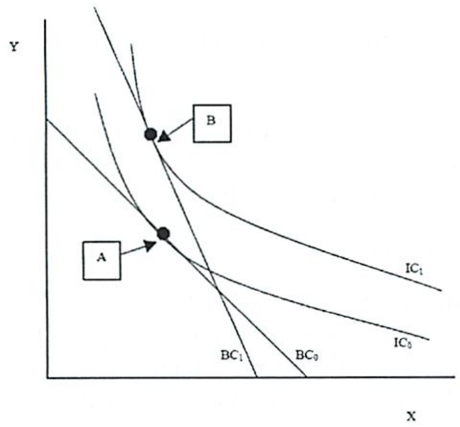
An individual with indifference curves (IC) as shown in Fig. 1A prefers \$100 in cash to \$100 in food stamps. The dotted BCs and ICs are those under the cash grant program, and the solid-line BCs and ICs are those under the food stamp program. Note that under the food stamp program the IC will shift out from IC0 to ICFS; under the cash grant policy, the IC would shift out from IC0 to ICCG, a higher (and thus more desirable) IC. Someone with ICs as shown in Fig. 1B will be indifferent between cash grant and food stamp policies–both policies shift IC0 by the same amount (the new IC being ICFS/CG). So which graph is representative of a poor individual, and which represents a wealthy one? Fig. 1A represents someone whom, when given \$100 cash, still chooses to spend less than \$100 total on food. Such an individual will prefer the cash to food stamps because it allows the individual to take full advantage of the subsidy while continuing to spend less than \$100 on food. Fig. 1B represents someone spending more than \$100 on food when given \$100 cash. This type of person is indifferent between the two policies. If one believes that wealthy individuals are likely to spend more than \$100 on food, and poor individuals are more likely to spend less than \$100 on food (since their limited income must cover other necessities (e.g. housing)), then the poor person will prefer the cash over the food stamps. The wealthy person is indifferent.

Note that as the budget line shifts farther and farther out into the northeast direction (indicating greater wealth), the percentage of the budget line to the left of 100 units of food falls. Thus the likelihood of a consumer being concerned with a preference of cash over food stamps becomes smaller and income rises.

2. (10 points) Julia maximized her utility subject to her budget constraint. Then prices changed. After the change, she became better off. Therefore we can conclude that the new bundle costs more at the old prices than the old bundle did. True or false? Explain.

The figure below shows the situation described above. At original prices and income, Prudence maximizes utility by consuming at point A, where MRS (from indifference curve IC0) equals the price ratio (from budget constraint BC0). We know that she becomes better off after the price change; therefore, her new consumption choice, point B, must be located on a higher indifference curve, IC1. If she is maximizing utility subject to her budget constraint, the new bundle is located where MRS (from indifference curve IC1) equals the price ratio (of a new budget constraint, sketched as BC1).

The question's statement is true–the new bundle costs more at the old prices than the old bundle did. At old prices, the slope of any budget constraint must equal that of BC0. A inward parallel shift of BC0 indicates the taking away of income, and an outward parallel shift of BC0 indicates the addition of income. A parallel shift of BC0 that hits point B will require an outward shift, indicating that the new bundle costs more at old prices than the old bundle did. Note that this holds regardless of where point B is drawn on the new indifference curve IC1.



3. Suppose there are exactly two consumers (Albie and Bubbie) who demand strawberries. Suppose that Albie's demand for strawberries is given by

$$q_a(p) = p^\alpha f_a(I_a)$$

and Bubbie's demand is given by

$$q_b(p) = p^\beta f_b(I_b)$$

where I_a and I_b are Albie and Bubbie's incomes, and $f_a(\cdot)$ and $f_b(\cdot)$ are two unknown functions.

(a) Find Albie and Bubbie's (own-price) elasticities of demand, $\epsilon_{q_a,p}$ and $\epsilon_{q_b,p}$. Use the sign convention that $\epsilon_{y,x} = \frac{\partial y}{\partial x}\frac{x}{y}$.

$$\epsilon_{q_a,p} = \frac{\partial q_a}{\partial p}\frac{p}{q_a(p)} = [\alpha p^{\alpha-1} f_a(I_a s)]\frac{p}{p^\alpha f_a(I_a)} = \alpha$$

and similarly, $\epsilon_{q_b,p} = \beta$.

(b) Suppose that $\alpha > 0 > \beta$. Are strawberries a Giffen good for Albie? Are strawberries a Giffen good for Bubbie?

$\epsilon_{q_a,p} = \alpha \geq 0$ which means that strawberries are Giffen for Albie. $\epsilon_{q_b,p} = \beta < 0$, regular for Bubbie.

(c) Are strawberries an inferior good for Albie? Are strawberries an inferior good for Bubbie? Assume that these demands arise from utility maximization given linear budget constraints. 4 Hint: This question should not require much/any algebra.

Strawberries must be inferior for Albie, since Giffen goods must be inferior. The substitution effect means a higher price for strawberries encourage Albie to buy less of them. Thus if he buys more, the income effect must make him buy more (i.e., it is inferior), and outweigh the substitution effect.

For Bubbie, strawberries could be normal or inferior. If inferior, it's just that the substitution effect outweighs the income effect.

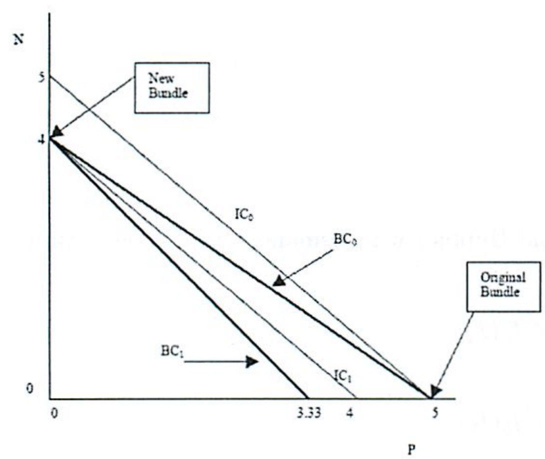
4. (25 points) Sophia considers peaches and nectarines to be perfect substitutes. She spends \$5 a month on these fruits. Initially, peaches are \$1 a pound and nectarines are \$1.25 a pound. Then, the price of peaches increases to \$1.50 a pound. Her income allocated to fruit does not change, however.

(a) (5 points) How does consumption change when the price of peaches changes?

Intuitively, since peaches and nectarines are perfect substitutes for Sophia, she will spend all of her income on whichever good is cheapest. Initially peaches were cheaper, so she would have purchased I/P_P of them. But after the price of peaches increases (making nectarines the relatively cheaper fruit), Sophia will switch to buying I/P_N nectarines instead. Therefore, her original bundle (P, N) is $(5, 0)$ and her new bundle is $(0, 4)$.

(b) (5 points) Show with the aid of a graph how utility changes when the price changes.

The figure below sketches the changes. Since peaches and nectarines are perfect substitutes, the utility function will have a linear form (such as U(P,N)=P+N). Indifference curves are therefore linear. As the price changes she shifts to a lower indifference curve and thus her utility falls.



(c) (5 points) How much must her budget increase in order to return to the original utility level?

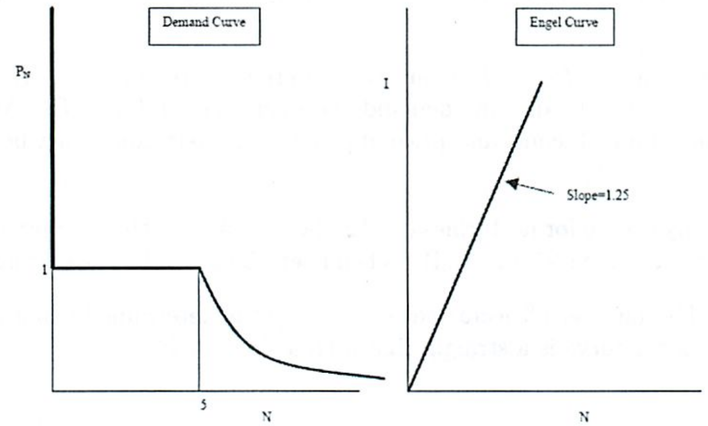
In order to achieve her old level of utility (not necessarily her old bundle), she needs enough income to increase total consumption of fruit to five units. Nectarines are the cheaper good, and by purchasing one additional nectarine she will return to her original indifference curve. Thus she needs an additional \$1.25 if she is to return to her original utility level.

(d) (5 points) Derive and graph the demand curve for nectarines assuming that income is \$5 and peach price is \$1.

Sophia consumes I/P_N nectarines so long as $P_N < P_P$: she is indifferent as to how she divides income between peaches and nectarines if $P_N = P_P$; finally, she demands no nectarines if $P_N > P_P$. We can now graph the demand curve for nectarines (note: Income and price of peaches are held constant when sketching the nectarine demand curve.)

(e) (5 points) Derive and graph the Engel curve for nectarines (under the assumption that the price of peaches is \$1.50 a pound and the price of nectarines is \$1.25 a pound). When nectarines are the cheaper good, Sophia

will spend all her income on them. The number of nectarines she will buy is determined using $N = I/P_N$. Rearranging gives $I = P_N N$, so the Engel curve is a straight line with a slope of P_N.



5. (30 points) Xiaoyu spends all her income on statistical softwares (S) and clothes (C). Her preferences can be represented by the utility function: $U(S, C) = 4ln(S) + 6ln(C)$.

(a) (4 points) Compute the marginal rate of substitution of softwares for clothes. Is the MRS increasing or decreasing in S? How do we interpret this?

We have that $MRS = \frac{\frac{4}{S}}{6C} = \frac{2}{3}\frac{C}{S}$. Hence, the MRS is decreasing in S. As Xiaoyu moves down an indifference curve consuming more softwares and fewer clothes she is ready to give up fewer clothes for extra softwares.

(b) (6 points) Find Xiaoyu's demand functions for softwares and clothes, $Q_S(p_S, p_C, I)$ and $Q_C(p_S, p_C, I)$, in terms of the price of softwares (p_S), the price of clothes (p_C) and Xiaoyu's income (I).

At an interior optimum, $MRS = \frac{p_S}{p_C} \Longrightarrow \frac{2}{3}\frac{C}{S} = \frac{p_S}{p_C}$. Hence $C = \frac{3}{2}\frac{p_S}{p_C}S$. Substituting for C into the budget constraint we get that $p_S S + p_C \frac{3}{2}\frac{p_S}{p_C}S = I$, which means that $Q_S(p_S, p_C, I) = S = \frac{2}{5}\frac{I}{p_S}$. Hence $Q_C(p_S, p_C, I) = \frac{3}{5}\frac{I}{p_C}$.

(c) (2 points) Draw the Engel curve for books. The Engel curve is a linear function with zero intercept and a slope of $\frac{5}{2}p_S$.

(d) (3 points) Suppose that the price of softwares is $p_S = 2$, the price of clothes is $p_C = 3$ and Xiaoyu's income is $I = 10$. What bundle of softwares and clothes (S, C) maximize Xiaoyu's utility?

Using the demand functions from part b) we get that $S = \frac{2}{5}\frac{10}{2} = 2$ and $C = \frac{3}{5}\frac{10}{3} = 2$.

(e) (3 points) Suppose the price of softwares increases to $p_S = 4$. What bundle of softwares and clothes does Xiaoyu demand now? Using the demand functions from part b) we get that $S' = \frac{2}{5}104 = 1$ while $C = 2$ as before.

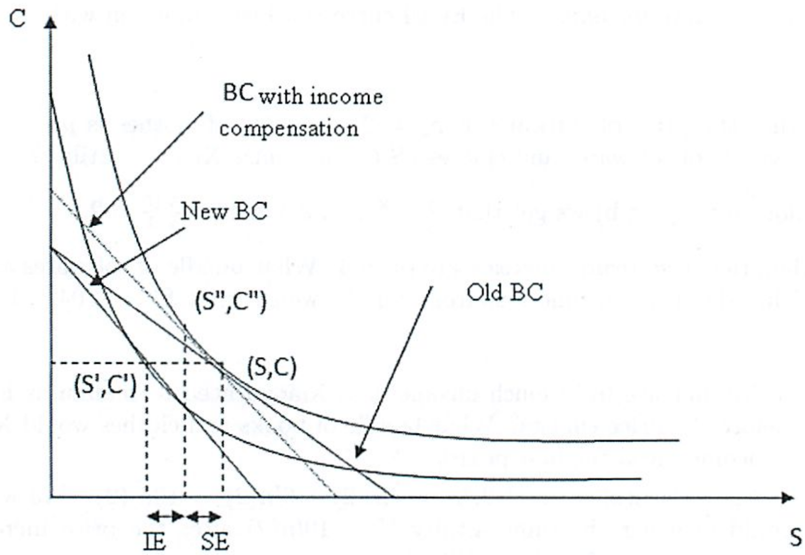
(f) (6 points) Given the price increase, how much income does Xiaoyu need to remain as happy (have the same utility) as she was before the price change? What bundle of books and clothes would Xiaoyu consume if she had that additional income given the new prices.
Prior to the price change Xiaoyu's utility is: $U(2,2) = 4ln(2) + 6ln(2) = 10ln(2)$. We want to find the income level I^* which would give her the same uitility $U = 10ln(2)$ after the price increase. Plugging Xiaoyu's demand functions form part b) into her utility function we get that:
$U(S,C) = 4ln(\frac{2}{5}\frac{I}{p_S}) + 6ln(\frac{3}{5}\frac{I}{p_C}) = 4ln(c) + 6ln(3) + 10ln(I) - 10ln(5) - 4ln(p_S) - 6ln(p_C)$.
Now, I^* solves: $U(2,2) = 10ln(2) = 4ln(2) + 6ln(3) + 10ln(I) - 10ln(5) - 4ln(4) - 6ln(3) \Longrightarrow 6ln(2) = 10ln(I^*) - 10ln(5) - 4ln(4) \Longrightarrow ln(I^*) = \frac{3ln(2)+5ln(5)+2ln(4)}{5} = ln(5\dot{2}^{\frac{7}{2}})$. Hence $I^* = 2^{1.4}5 = 13.1951$. If Xiaoyu had I^* she would consume $S'' = 1.3195$ and $C'' = 2.639$.

(g) (6 points) Going back to the situation in part (e) ($p_S = 4$ and $I = 10$) decompose the total change of books and clothes demanded into substitution and income effects. In a clearly-labeled diagram with books on the horizontal axis show the income and substitution effects of the increase in the price of books.

The total effect is the difference between the bundles consumed after and before the price change, i.e. $(S', C') - (B, C) = (-1, 0)$, i.e. the total effect is a decrease in the consumption of books by 1 and no decrease in the consumption of clothes.

The substitution effect (SE) is the difference between the bundles consumed and before the price change staying on the same indifference curve, i.e. $(S'', C'') - (S, C) = (-0.6805, .639)$. Hence, the pure substitution effect is a decrease in the consumption of books by .6805 and an increase in the consumption of clothes by .639.

The income effect (IE) is the difference between (S', C') and (S'', C''), i.e. a decrease of .3195 in softwares and a decrease of .639 in clothes.


C
BC with income compensation
New BC
Old BC
(S",C")
(S,C)
(S',C')
IE
SE
S

Massachusetts Institute of Technology

Department of Economics

14.01 Principles of Microeconomics

Exam 1

Tuesday, October 5th, 2010

Last Name (Please print): Plasmeier

First Name: Michael

Kerberos ID: theplaz

Instructions. Please read carefully.

The exam has a total of 100 points. Answers should be as concise as possible. This is a closed book exam. You are not allowed to use notes, equation sheets, books or any other aids. You are not allowed to use calculators. You must write your answers in the space provided between questions. DO NOT attach additional sheets of paper. This exam has 18 pages (13 pages + 5 blank pages for scratch work)

Circle Your Section/Recitation:

Please circle the section or recitation, which you are attending below. The marked exam will be returned to you in the section or recitation that you indicate.

MWF 9AM	F 10AM
MWF 11AM	F 11 AM
MWF 1PM	F 1PM
MWF 2PM	F 3PM

DO NOT WRITE IN THE AREA BELOW:

Question 1 13/16	Question 2 10/10
Question 3 21/23	Question 4 20/25
Question 5 20/26	
Total 84/100	

pretty good
a few pts off in each

B

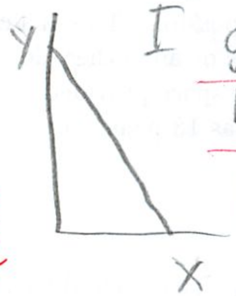
Mean 74 -flat distribution
-A/B division small

1. True/False/Uncertain Questions (16 points)

In this section, write whether each statement is True, False or Uncertain. You should fully explain your answer, including diagrams where appropriate. Points will be given based on your explanation.

(a) (4 points) A consumer finds two goods to be perfectly substitutable. Claim: The optimal bundle for this consumer will always be a corner solution.

True. If the goods are perfect substitutes, their is no preference one way or another so the person would just buy which ever is cheaper. If they were the exact same price, I guess he would pick some random number of each - but their is no provision in Economics for this randomness, so assume 1 is always cheaper



Not true all the time

$U(x,y) = x + y$

$P_x = P_y = 1$

$I = 10$

- any combo of x, y feasable

knife edge result

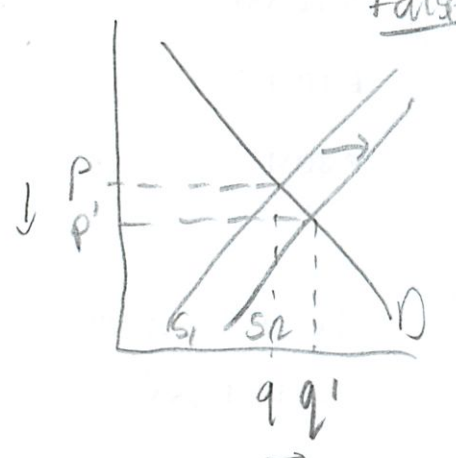
−1

(b) (4 points) Innovations in the production of batteries lead to a rightward shift in the market supply for hybrid cars while demand stays the same. Since this leads to a decrease in the equilibrium price and an increase in the equilibrium quantity, demand is more inelastic at the new equilibrium.

False

$\epsilon = \dfrac{\Delta Q}{\Delta P} \cdot \dfrac{P}{Q}$

demand even more elastic there



- Depends on slope of demand
- If the curve becomes steeper it will become more inelastic

- If the curve becomes flatter more elastic

- If ~~linear~~, no change) this is more likely to happen so elastic prob.

if demand linear elasticity stays the same

(c) (4 points) A consumer has selected an optimal bundle of two goods that includes some of each good. The price of one good increases. Claim: her utility is lower after the price increase compared to before it.

-2

True, as long as it is not a corner solution, where utility would be same.

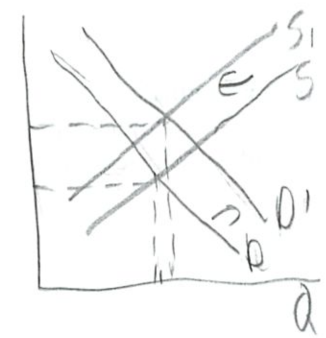
MRS ≥ MRT

Consumer already has their optimal bundle. If the price of a normal good increases, demand for it will decrease. This will lead to a substitution to a different good as well as making the person feel poorer, decreasing consumption, thus utility (in this case)

Counter example - part a consume 1 good - utility remains same

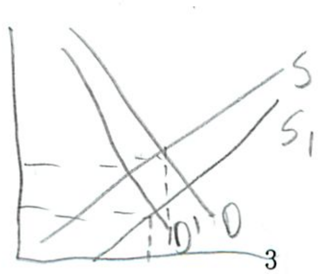
(d) (4 points) When market demand and supply shift in opposite directions we can unambiguously say how the equilibrium price and quantity change.

D ↑
S ↓



False, you need to know the relative effects of each change. Depending on which is larger will determine which way Q will change. Price moves knowable.

D ↓
S ↑



$P + \frac{1}{2}Q = 5$ $\quad$ $P = 20 - Q$

$\frac{1}{2}Q = 5 - P$ $\quad$ $P + Q = 20$

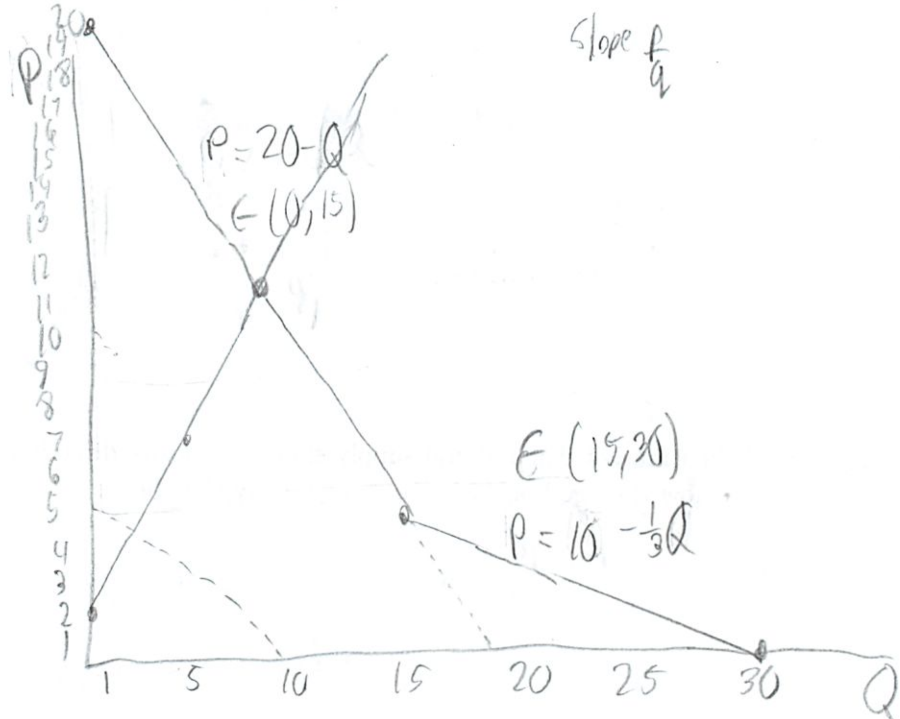
2. Market demand for frozen yogurt (10 points) $Q = 10 - 2P$ $\quad$ $Q = 20 - P$

Market surveys show that there are two types of consumers for frozen yogurt. The first type like frozen yogurt and have an inverse demand curve of $P = 5 - \frac{1}{2}Q$. The second type are crazy about frozen yogurt and have an inverse demand curve of $P = 20 - Q$. In the town of Smallville there are only 2 consumers: one of them likes frozen yogurt and the other is crazy about frozen yogurt.

(a) (5 points) Using the individual demand curves above derive the market demand for frozen yogurt in Smallville. Plot the market demand curves.

never really looked at this

P	Q_1	Q_2	ΣQ	S
0	10	20	30	2
1	8	19	27	3
2	6	18	24	4
3	4	17	21	5
4	2	16	18	6
5	0	15	15	7
6	0	14	14	8
7	0	13	13	9
⋮		12	12	10
9		11	11	11
20	0	0	0	22



(b) (5 points) Suppose that the market supply for frozen yogurt in Smallville is given by $Q^S = 2 + P$. Find the equilibrium price and quantity. How much does each consumer buy at the equilibrium price? (Hint: Check the equilibrium price and quantity you get on a graph.)

$S = 2 + P$

$2 + P = 20 - P$ $\quad$ $Q = 2 + P = 20 - P$

$2 - 20 = -P - P$ $\quad$ $= 11 \quad = 11$

$-18 = -2P$

$P = 9$

Lovers buy 11 "crazy"

Likers buy none ☹

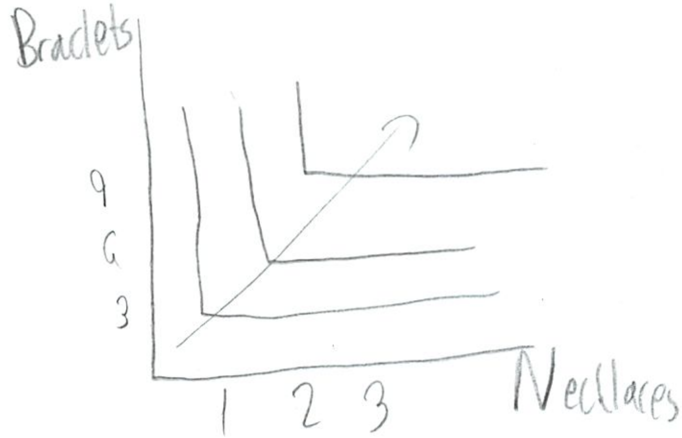
3. Consumer preferences and optimal allocations (23 points)

Mary is starting a jewelry collection. She wants to own matched sets of three bracelets and one necklace that can be worn together, and she doesn't want to own any bracelets or necklaces that are not in a matched set of this size.

(a) Draw Mary's indifference curves and write her utility function. Put bracelets on the y axis and necklaces on the x axis. Assume she receives utility of 3 utils from each matched jewelry set she owns.

$U(B,N) = \cancel{3}\min(B, 3N)$

Y



(b) (5 points) Currently, Mary has 32 dollars to spend. The price of necklaces is $p_n = 2$ and the price of bracelets is $p_b = 2$. What is the optimal allocation of necklaces and bracelets for Mary? How much utility does she achieve from this allocation?

can have non int B, N

$B \cdot P_B + N \cdot P_N = I$ $\qquad 3N = B$

$B \cdot 2 + N \cdot 2 = 32$

$3N \cdot 2 + N \cdot 2 = 32$ $\qquad B \cdot 2 + 4 \cdot 2 = 32$

$6N + 2N = 32$ $\qquad B \cdot 2 = 24$

$8N = 32$ $\qquad B = 12$

$N = 4$

$U = 3 \cdot 4 = 12$ "utility units"

(c) (4 points) Due to a shortage of gold, the price of necklaces increases to $p_n = 10$. What is the new allocation of necklaces and bracelets at this price level, and what utility does Mary obtain?

$B \cdot P_B + N \cdot P_N = I$

$B \cdot 2 + N \cdot 10 = 32$

$3N \cdot 2 + 10N = 32$

$6N + 10N = 32$

$16N = 32$

$N = 2$

$3N = B$

$B \cdot 2 + 2 \cdot 10 = 32$

$B \cdot 2 = 12$

$B = 6$

$U = 2 \cdot 3 = 6$ "utility units"

(d) (4 points) Luckily, Mary's parents value their daughter's utility, and are willing to give her enough income to ensure that she has the same utility she did prior to the price change. How much extra money do they have to give her?

$B \cdot 2 + N \cdot 10 = I$

$N \cdot 3 = 12$

$N = 4$

$3N = B$

$3 \cdot 4 = B$

$B = 12$

$2B + 4 \cdot 10 = I$

$2 \cdot 12 + 4 \cdot 10 = I$

$24 + 40 = I$

$I = 64$

'no cross price elasticity since she requires $3N = B$

extra \$ = 64 − 32

= \$32

(e) (5 points) Mary has a sister Lily who doesn't like wearing matched sets of jewelry and has different preferences. Her utility function is nb^2. If she started with the same jewelry budget as Mary of 32 dollars and then faced the same price shock, what would be the decrease in her utility when the price of necklaces increases from \$2 to \$~~4~~ 10?

10 correction

$U = nb^2$

$B \cdot P_B + N \cdot P_N = I$

$B \cdot 2 + N \cdot 2 = 32$

MRS = MRT

$$\frac{\frac{\partial U}{\partial B}}{\frac{\partial U}{\partial N}} = \frac{P_B}{P_N}$$

$$\frac{2n\cancel{b}}{b^{\cancel{2}}} = \frac{2}{2}$$

$\frac{2n}{b} = 1$

$B = 2N$

$2N \cdot 2 + N \cdot 2 = 32$

$4N + 2N = 32$

$6N = 32$

$N = 5\frac{1}{3}$

$B = 2N$

$B = 2 \cdot 5\frac{1}{3}$

$B = 10\frac{2}{3}$

$U = 5\frac{1}{3}(10\frac{2}{3})^2$

≈ 500

$\frac{2N}{B} = \frac{2}{10}$

$2B = 20N$

$B = 10N$

$10N \cdot 2 + N \cdot 10 = 32$

$20N + 10N = 32$

$N \approx 1$

$B = 10(1) = 10$

$U' = (1)(10)^2$

≈ 100

utility would fall from ≈ 500 to ≈ 100

(f) (3 points) Mary and Lily's parents are going to give a gift of equal monetary value to both sisters. They are trying to decide whether to give cash or give jewelry. Which sister is more likely to prefer cash? Please explain intuitively and/or graphically; there is no need for algebra in this section.

Lilly has a much higher for Jewelry because it is exponential where as Marry's utility function has a minimum, If Marry got 2 bracelets, she would not care (no change in utility). Lilly would, depending on how many bracelets she has, perhaps double or triple her utility, Perhaps there is something else which Marry would like better with the cash.

Correct answer ~~reason~~ but not quite right

-1

TA: pretty hard

cash more substitutable

Lilly can substitue away easier

20/25

4. Labor markets and labor supply shocks (25 points)

Consider the labor market in the country of Widgetland. The demand for labor is given by:

$$L^D = 34 - 4w$$

where w is the wage rate.
Labor supply is:

$$L^S = \overline{L} + 2w$$

where $\overline{L}$ is the number of people in the country willing to work at a wage of zero.

5/5

(a) (5 points) Suppose that $\overline{L} = 10$. Find the equilibrium wage and equilibrium demand for labor. Is demand for labor elastic or inelastic at the equilibrium wage?

Equallibrium $L^D = L^S$

$34 - 4w = 10 + 2w$

$34 - 10 = 2w + 4w$

$24 = 6w$

$w = 4$ ✓

$L = 10 + 2(4)$

$L = 18$ ✓

$L = 34 - 4(4)$

$L = 18$

$\frac{\partial L}{\partial w} \cdot \frac{w}{L} = -4 \cdot \frac{4}{18}$

$= \frac{-16}{18} = -\frac{8}{9}$ ✓

<1 so it is slightly inelastic at this wage
- about what you would expect

Suppose that there is a sudden influx of migrant labor, which increases the number of people willing to work at a wage of zero to $\overline{L} = 16$. For the remainder of the problem set $\overline{L} = 16$.

5/5

(b) (5 points) Compute the new market equilibrium. What happens to the equilibrium wage rate?

$34 - 4w = 16 + 2w$

$34 - 16 = 2w + 4w$

$18 = 6w$

w went down $\rightarrow$ $w = 3$

$L = 16 + 2(3)$

work done went up $\rightarrow$ $L = 22$

$L = 34 - 4(3)$

$L = 22$

$\frac{\partial L}{\partial w} \cdot \frac{w}{L} = -4 \cdot \frac{3}{22}$

$= \frac{-12}{22}$

more inelastic

1/5

(c) (5 points) In reality an increase in population should affect the demand for labor as well as the supply. Explain how the equilibrium wage and labor demanded will change compared to the market equilibrium in part (a) if demand for labor were to increase as well.

- So the qtd. of labor demanded did change from 18→22

Are you asking about a shift in the demand curve, due to a stronger economy or other factors. If so, then the equation would be different and there would be a new equilibrium

See solutions

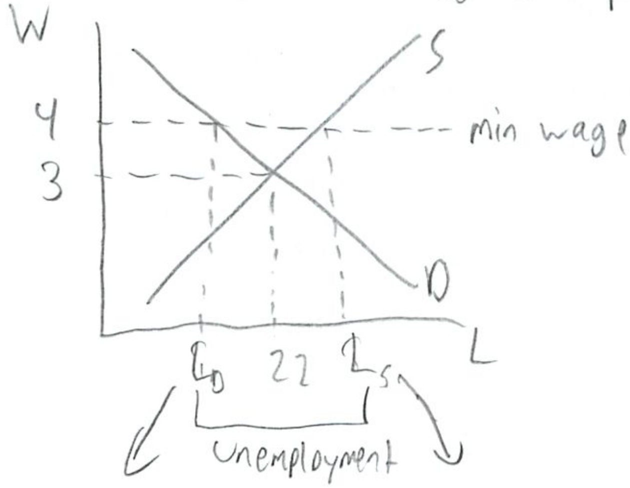
ie $64-4w = 16+2w$
$48 = 6w$
$w=7$ ← wage rate shot up

$L = 16+2(7)$
$L=30$ ← amt of labor demanded increased as well

5/5

(d) (5 points) The government in Widgetland becomes worried about the upcoming election and decides to appease voters by imposing a minimum wage of $\underline{w} = 4$. What happens in the labor market as a result? What is the demand and supply for labor now? Include a graph in your explanation.

There would be unemployment



$L_D = 34-4(4)$
18

$L_S = 16+2(4)$
$L_S = 24$

24 - 18

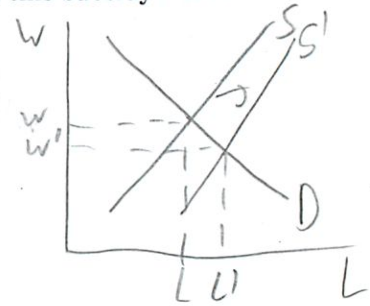
6 laborers would want to work, but cant so they are unemployed.

think of if gov make apples \$1 cheaper
- supply shifts out
- or is it something far more?

? like a tax suppliers

4/5

(e) (5 points) The government is unhappy with the results of the minimum wage law and repeals it. Instead it introduces a subsidy of $\tau = 1$ dollar on labor that is paid to workers. What happens to the equilibrium wage and labor used as a result of this subsidy? How much do workers get in equilibrium?



firms → $D = 34 - 4w$ ✓

$S = 16 + 2(w+1) = 18 + 2w$

?

$34 - 4w = 18 + 2W$

$34 - 18 = 2w + 4w$

$16 = 6w$

$2\frac{2}{3} w = \frac{8}{3}$ ✓

wages fall slightly

$L = 34 - 4(2\frac{2}{3})$

$23\frac{1}{3}$

$L = 18 + 2(2\frac{2}{3})$

$18 + \frac{16}{3}$

$23\frac{1}{3}$

slightly more people working

Firms pay wages $1\frac{2}{3}$

Gov pays 1

Workers receive $3\frac{2}{3}$

Same as graph (+6)

5. Income and substitution effects (26 points)

Glenn's utility function for goods X and Y is represented as $U(X,Y) = X^{0.2}Y^{0.8}$. Assume his income is \$100 and the prices of X and Y are \$10 and \$20, respectively.

(-1) (a) Express his marginal rate of substitution (MRS) of good Y for good X. As the amount of X increases relative to the amount of Y along the same indifference curve, does the absolute value of the MRS increase or decrease? Explain (4 points)

See sol

MRS

$$-\frac{\frac{\partial U}{\partial y}}{\frac{\partial U}{\partial x}} \quad \frac{.8 y^{-.2} x^{.2}}{.2 x^{-.8} y^{.8}} \quad -\frac{.8 \frac{x^{.2}}{y^{.2}}}{.2 \frac{y^{.8}}{x^{.8}}} \quad -.8 \frac{x^{.2}}{y^{.2}} \cdot \frac{1}{.2} \frac{x^{.8}}{y^{.8}}$$

$$-4\frac{X}{Y}$$

true

Yes? if we were to hold y fixed increasing x would increase the absolute value of the MRS

(b) What is his optimal consumption bundle (X^*, Y^*), given income and prices of the two goods? (5 points)

$$P_x \cdot X + P_y \cdot Y = I$$

$$10 \cdot X + 20 \cdot Y = 100$$

MRS = MRT

$$-4\frac{X}{Y} = \frac{20}{10}$$

$$4X \cdot 10 = 20Y$$

$$2X = Y$$

$$10 \cdot X + 20 \cdot 2X = 100$$

$$10X + 40X = 100$$

$$X = 2$$

$$X = \frac{20Y}{40}$$

$$X = \frac{1}{2}Y$$

$$10 \cdot \frac{1}{2}Y + 20 \cdot Y = 100$$

$$5Y + 20Y = 100$$

$$Y = 4$$

$$(2, 4)$$

(c) How will this bundle change when all prices double and income is held constant? When all prices double AND income doubles? (5 points)

$$-4\frac{X}{Y} = -\frac{40}{20}$$

$$4X \cdot 20 = 40Y$$

$$20X = 10Y$$

$$2X = Y$$

$$20 \cdot X + 40 \cdot 2X = 100$$

$$X = 1$$

$$20 \cdot \frac{1}{2}Y + 40 \cdot Y = 100$$

$$10Y + 40Y = 100$$

$$Y = 2$$

So just half of everything

$$20X + 40 \cdot 2X = 200$$

$$X = 2$$

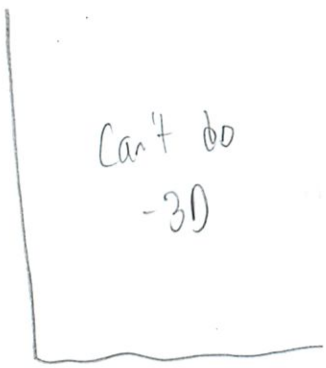
$$20 \cdot \frac{1}{2}Y + 40Y = 200$$

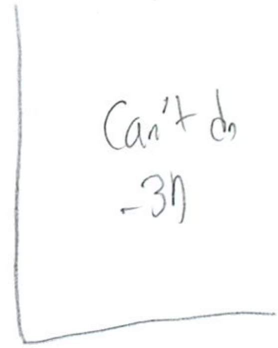
$$Y = 4$$

So in this case nothing changes

(d) Derive the demand curve for good X and the demand curve for good Y as a function of prices assuming income is \$100. (4 points)

-3





$$P_x \cdot X + P_y \cdot 2x = 100$$
$$X \cdot (P_x + 2P_y) = 100$$
$$X = \frac{100}{P_x + 2P_y}$$

See Sol

$$P_x \cdot \tfrac{1}{2}y + P_y \cdot y = 100$$
$$Y\left(\tfrac{1}{2}P_x + P_y\right) = 100$$
$$Y = \frac{100}{\tfrac{1}{2}P_x + P_y}$$

Now a government subsidy program lowers the price of Y from \$20 per unit to \$10 per unit.

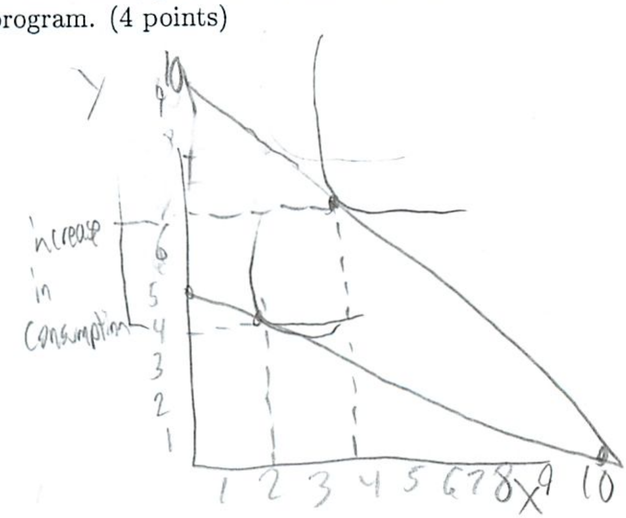
here we go again! - well just do lower price

(e) Calculate the change in good Y consumption resulting from the program. In a clearly labeled diagram with Y on the y-axis and X on the x-axis, graphically show the change in consumption of good Y resulting from the program. (4 points)

-2

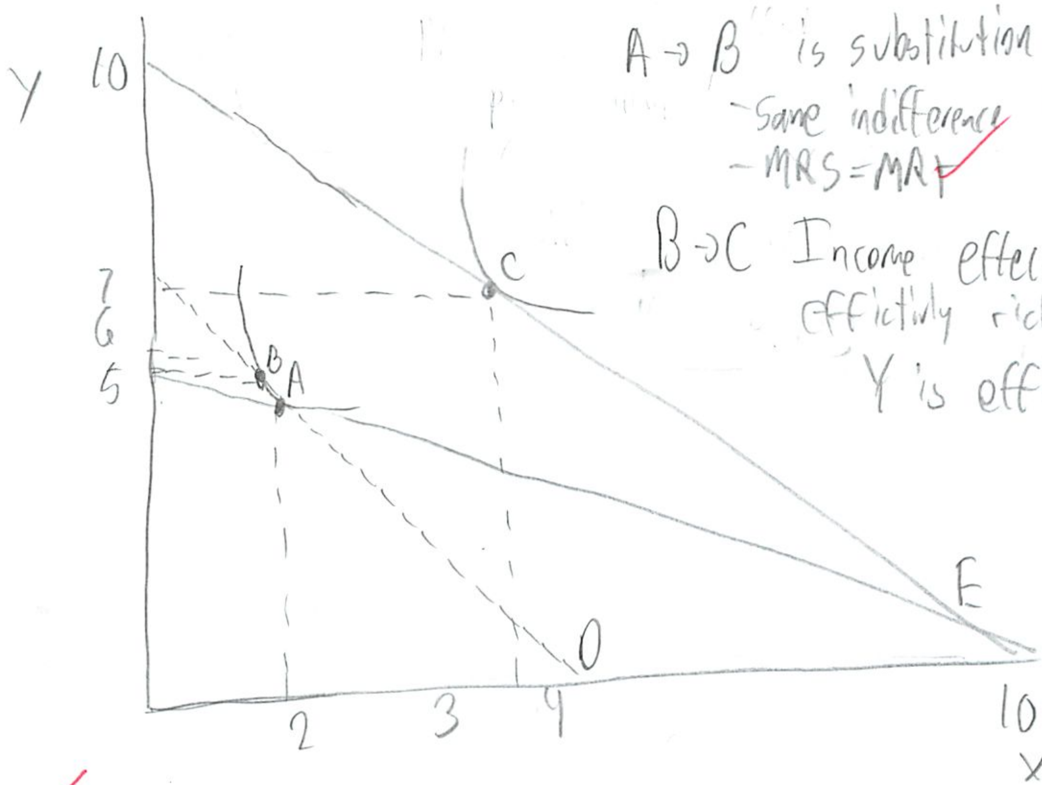
$$X = \frac{100}{10+2(10)} = \frac{100}{30} = \frac{10}{3} = 3\tfrac{1}{3}$$
$$= 3\tfrac{1}{3}$$

$$Y = \frac{100}{\tfrac{1}{2}(10)+10} = \frac{100}{15} = \frac{10}{1.5} = \frac{20}{3}$$
$$= 6\tfrac{2}{3}$$



(f) In a clearly labeled diagram with Y on the y-axis and X on the x-axis, graphically show the change in consumption attributable to the separate income and substitution effects for good Y only. Explain the intuition of the income and substitution effects. No calculations are required for this part. (4 points)

effect of them having different ↓ prices



A → B is substitution effect
- Same indifference curve
- MRS = MRT

- new budget line D

B → C Income effect due to being effictivly richer because Y is effictivly cheaper

D ∥ E
∥ is parallel to

(g) How much does the program cost the government?

He purchases 8 Y goods → So it cost them $80.

END OF EXAM

done w/ 15 min to ✓

BLANK PAGE

BLANK PAGE

BLANK PAGE

BLANK PAGE

BLANK PAGE

14.01 Fall 2010: Midterm 1 Solution Set

October 13, 2010

1. True/False/Uncertain Questions

Write whether each statement is True, False or Uncertain. You should fully explain your answer, including diagrams where appropriate. Points will be given based on your explanation.

1. A consumer finds two goods to be perfectly substitutable. Claim: The optimal bundle for this consumer will always be a corner solution.

   False. The optimal bundle will be a corner solution if the prices of the two goods are different; if the prices are the same, then any allocation is optimal.

2. Innovations in the production of batteries lead to a rightward shift in the market supply for hybrid cars while demand stays the same. Since this leads to a decrease in the equilibrium price and an increase in the equilibrium quantity demand is more inelastic at the new equilibrium.

   Uncertain. This is true in the case of linear demand since elasticity of demand is increasing in price along a linear demand curve. However, this is false in the case of a constant elasticity demand curve.

3. A consumer has selected an optimal bundle of two goods that includes some of each good. The price of one good increases. Claim: her utility is lower after the price increase compared to before it.

   True. If the consumer was indifferent to one of the goods, she would have selected a bundle of only the other good. Clearly, she obtains utility from both goods; thus when the price of either increases and her budget constraint shifts in, she will be on a lower indifference curve.

4. When market demand and supply shift in opposite directions, we can unambiguously say how the equilibrium price and quantity change.

   False. When demand and supply shift in the same direction we can unambiguously say how the equilibrium price changes (equilibrium price increases if supply shifts to the left and demand shifts to the right and vice versa). However, the effect on equilibrium quantity is ambiguous; it could either increase or decrease in both cases.

2. Market demand for frozen yogurt

Market surveys show that there are two types of consumers for frozen yogurt. The first type like frozen yogurt and have an inverse demand curve of $P = 5 - \frac{1}{2}Q$. The second type are crazy about frozen yogurt and have an inverse demand curve of $P = 20 - Q$. In the town of Smallville there are only 2 consumers: one of them likes frozen yogurt and the other is crazy about frozen yogurt.

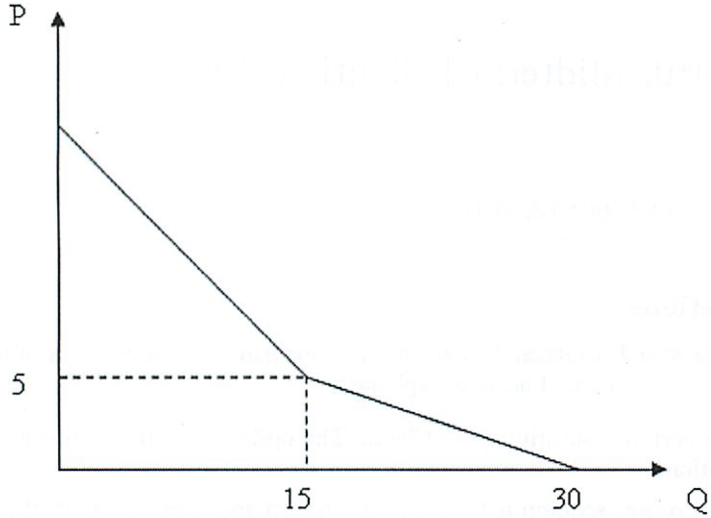
1. Using the individual demand curves above derive the market demand for frozen yogurt in Smallville. Plot the market demand curves.

   The demand of the first group is

$$Q_1^D = \begin{cases} 10 - 2P & , P \leq 5 \\ 0 & , P > 5 \end{cases}$$

   and the demand of the second group is

$$Q_2^D = 20 - P$$



Summing the two demand curves we get a market demand curve of

$$Q_M^D = Q_1^D + Q_2^D = \begin{cases} 30 - 3P & , P \leq 5 \\ 20 - P & , P > 5 \end{cases}$$

The inverse market demand curve is then:

$$P = \begin{cases} 20 - Q & Q \leq 15 \\ 10 - \frac{1}{3}Q & Q > 15 \end{cases}$$

2. Suppose that the market supply for frozen yogurt in Smallville is given by $Q^S = 2 + P$. Find the equilibrium price and quantity. How much does each consumer buy at the equilibrium price? (Hint: Check the equilibrium price and quantity you get on a graph)

   The market demand curve is piecewise linear. However, since at $p = 5$ $Q^S = 7 < 15 = Q^D$ *it follows that supply would intersect demand to the left of the kink. Hence,* $Q^S = Q^D$ *or* $2 + P = 20 - P$ *and hence* $P^* = 9$*. This means that* $Q^* = 11$*. Only the crazy consumer buys at the equilibrium price.*

3. Consumer preferences and optimal allocations

Mary is starting a jewelry collection. She wants to own matched sets of three bracelets and one necklace that can be worn together, and she doesn't want to own any bracelets or necklaces that are not in a matched set of this size.

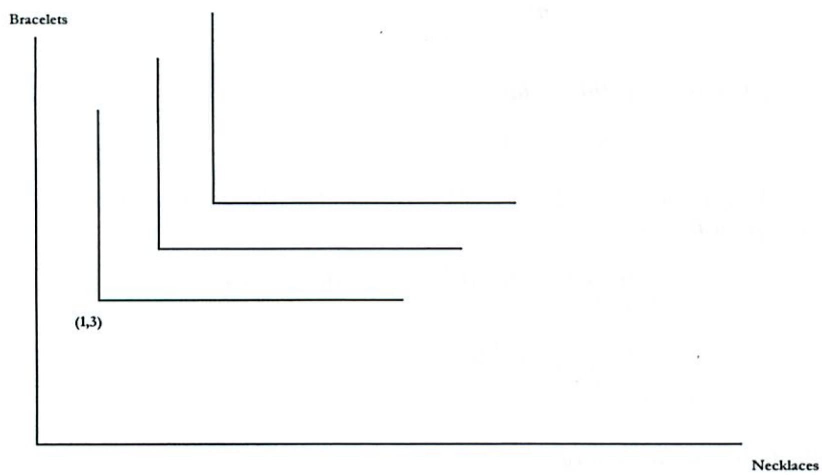
1. Draw Mary's indifference curves and write her utility function. Put bracelets on the y axis and necklaces on the x axis. Assume she receives utility of 3 utils from each matched jewelry set she owns.

   U(b,n)=3 min (n, $\frac{1}{3}$*b) or U(b,n)=3min (3n, b)*

2. Currently, Mary has 32 dollars to spend. The price of necklaces is $p_n = 2$ and the price of bracelets is $p_b = 2$. What is the optimal allocation of necklaces and bracelets for Mary? How much utility does she achieve from this allocation?

   We can find the optimal allocation by finding the intersection of the lines $b = 3n$ *and* $2n + 2b = 32$*. Substituting in to the budget constraint, this yields the following.*

$$\begin{aligned} 2n + 6n &= 32 \\ n &= 3 \end{aligned}$$



We have $n = 4$, $b = 12$, $u = 12$.

3. Due to a shortage of gold, the price of necklaces increases to $p_n = 10$. What is the new allocation of necklaces and bracelets at this income level, and what utility does Mary obtain? What is the proportional decrease in her utility?

   Re-solve the problem with a new budget constraint. This yields

$$10n + 6n = 32$$

   We have $n = 2$, $b = 6$, $u = 6$.

4. Luckily, Mary's parents value their daughter's utility, and are willing to give her enough income to ensure that she has the same utility she did prior to the price change. How much extra money do they have to give her?

   Mary's parents have to give her sufficient income to buy two more matched sets at the new prices. This requires the purchase of two necklaces and six bracelets, at a cost of $2 * 10 + 6 * 2 = 32$. They have to double her income, increasing it by \$32, in order to maintain her at the same utility level.

5. Mary has a sister Lily who doesn't like wearing matched sets of jewelry and has different preferences. Her utility function is nb^2. If she started with the same jewelry budget as Mary of 32 dollars and then faced the same price shock, what would be the decrease in her utility when the price of necklaces increases from \$2 to \$10?

   First, solve for Lily's original optimal allocation. We set the ratio of marginal utilities equal to the ratio of prices.

$$\begin{aligned} \frac{b^2}{2nb} &= \frac{2}{2} \\ b &= 2n \end{aligned}$$

   Substituting into the budget constraint, this yields

$$2(2n) + 2n = 32$$

This yields $n = \frac{16}{3}$, $b = \frac{32}{3}$, *and* $u = \frac{16}{3}(\frac{32}{3})^2 = \frac{512}{27}$
When prices increase, we now re-solve the problem.

$$\begin{aligned} \frac{b^2}{2nb} &= \frac{10}{2} \\ b &= 10n \end{aligned}$$

Substituting into the budget constraint, this yields

$$2(10n) + 10n = 32$$

This yields $n = \frac{16}{15}$, $b = \frac{160}{15}$, *and* $u = \frac{16}{15}(\frac{160}{15})^2 = \frac{2560}{225}$. *To calculate the decrease in utility, subtract the second level of utility from the first.*

$$\begin{aligned} &= (\frac{16 * 16^2 * 4}{27} - \frac{16 * 16^2 * 100}{27 * 5^3})/(\frac{16 * 16^2 * 4}{27}) \\ &= \frac{16 * 16^2(4 * 5^3 - 100)}{27 * 5^3} \\ &= 485.4 \end{aligned}$$

Lily experiences a 80% decline in her utility.

6. Mary and Lily's parents are going to give a gift of equal monetary value to both sisters. They are trying to decide whether to give cash or give jewelry. Which sister is more likely to prefer cash? Please explain intuitively and/or graphically; there is no need for algebra in this section.

   Mary is more likely to prefer cash, because she only receives utility from a gift that is given as a matched set and no utility from any other type of gift. You can portray this graphically by showing a graph of the two utility functions and indicating that for Mary, any increase in the quantity of one good or the other keeps her on the same indifference curve, while for Lily, any increase will move her to a new indifference curve. The below graph shows the result of a gift of necklaces only to both girls. Mary is on the same utility curve, but Lily has increased utility. In this case and others like it, Mary is more likely to prefer cash.

4. Labor markets and labor supply shocks

Consider the labor market in the country of Widgetland. The demand for labor is given by:

$$L^D = 34 - 4w$$

where w is the wage rate.
Labor supply is:

$$L^S = \overline{L} + 2w$$

where $\overline{L}$ is the number of people in the country willing to work at a wage of zero.

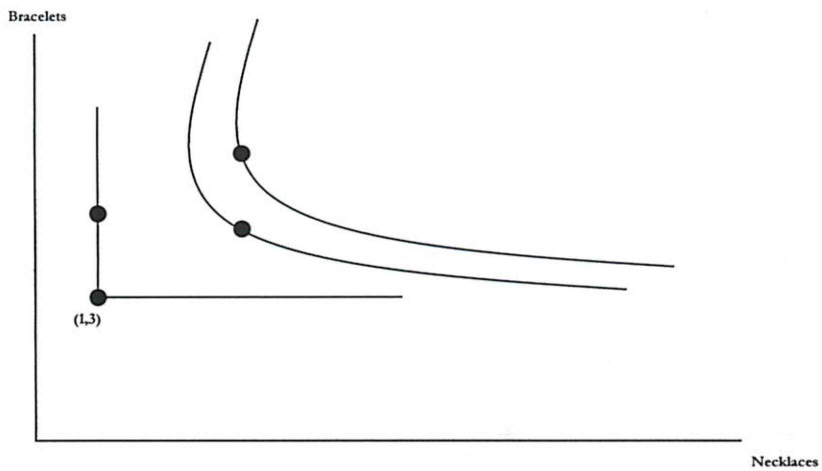
1. Suppose that $\overline{L} = 10$. Find the equilibrium wage and equilibrium demand for labor. Is demand for labor elastic or inelastic at the equilibrium wage?

   To find equilibrium wage we set $L^S = L^D$ *and hence* $34 - 4w = 10 + 2w$. *Hence,* $w^* = 4$. $L^* = 18$. *The elasticity of demand at the equilibrium wage is*

$$\epsilon_D = \frac{dL^D}{dw}\frac{w^*}{L^*} = -4\frac{4}{18} = -\frac{16}{18} = -\frac{8}{9}$$

   Hence, demand for labor is inelastic at w^*.

   Suppose that there is a sudden influx of migrant labor, which increases the number of people willing to work at a wage of zero to $\overline{L} = 16$. For the remainder of the problem set $\overline{L} = 16$.



2. Compute the new market equilibrium. What happens to the equilibrium wage rate?

   In the new equilibrium $34 - 4w = 16 + 2w$. *Hence,* $w^* = 3$. $L^* = 22$. *Hence the equilibrium wage rate decreases.*

3. In reality an increase in population should affect the demand for labor as well as the supply. Explain how the equilibrium wage and labor demanded will change compared to the market equilibrium in part (a) if demand for labor were to increase as well.

   A shift in demand in the same direction as the shift in supply increases the equilibrium demand for labor unambiguously. However, the change in the wage rate is ambiguous as the wage rate could either increase or decrease.

4. The government in Widgetland becomes worried about the upcoming election and decides to appease voters by imposing a minimum wage of $\underline{w} = 4$. What happens in the labor market as a result? What is the demand and supply for labor now? Include a graph in your explanation.

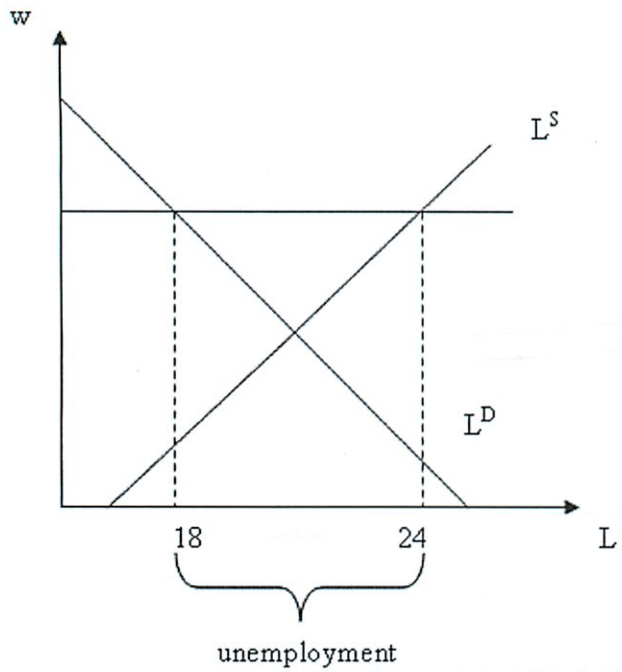
   Since $\underline{w} > w^*$ *it follows that the minimum wage will be binding. At the minimum wage demand for labor is* $L^D = 18$ *whereas supply for labor is* $L^S = 24$. *Hence, as a result of the minimum wage there is unemployment of* $L^S - L^D = 6$.

5. The government is unhappy with the results of the minimum wage law and repeals it. Instead it introduces a subsidy of $\tau = 1$ dollar on labor that is paid to workers. What happens to the equilibrium wage and labor used as a result of this subsidy? How much do workers get in equilibrium?

   A subsidy behaves like a negative tax. Hence, it effectively shifts supply down by τ. *The new equilibrium wage is determined where labor supply with the subsidy equals labor demand, i.e.* $L^S(p + \tau) = L^D$ *or* $16 + 2(w + \tau) = 34 - 4w$. *Hence,* $w^* = \frac{16}{6} = \frac{8}{3}$. *Hence workers get paid* $w^* + \tau = \frac{8}{3} + 1 = \frac{11}{3}$.

Income and substitution effects

Glenn's utility function for goods X and Y is represented as $U(X, Y) = X^{0.2}Y^{0.8}$. Assume his income is \$100 and the prices of X and Y are \$10 and \$20, respectively.



1. Express his marginal rate of substitution (MRS) between goods X and Y. As the amount of X increases relative to the amount of Y along the same indifference curve, does the MRS increase or decrease? Explain.

   $MU_X = 0.2(Y/X)^{0.8} and MU_Y = 0.8(X/Y)^{0.2}$
   $MRS_{Y for X} = -MU_X/MU_Y = -Y/4X$
   The MRS (in absolute value) gets smaller as the amount of X increases relative to Y. In other words, the more X (and less Y) one has, the less of Y one is willing to give up in order to obtain an additional unit of X.

2. What is his optimal consumption bundle (X^*, Y^*), given income and prices of the two goods?

   At the optimal consumption bundle, the MRS is equal to the ratio of prices. That is, $MRS_{Y for X} = -P_X/P_Y$. Plugging in our prices and the results from (a), $-Y/4X = -10/20 \Longrightarrow -Y/4X = -1/2 \Longrightarrow 2Y = 4X \Longrightarrow Y = 2X$
   The budget constraint must hold as well: $I = P_X X + P_Y Y \Longrightarrow 100 = 10X + 20Y$
   We now have two equations and two unknowns. Solve for (X^*, Y^*): $100 = 10X + 20(2X) \Longrightarrow 100 = 50X \Longrightarrow X^* = 2$
   $Y = 2X = 2(2) \Longrightarrow Y^* = 4$

3. How will this bundle change when all prices double and income is held constant? When all prices double AND income doubles?

   The utility function does not change and therefore the formula for MRS does not change. Prices change, but the ratio $-P_X/P_Y$ does not. Therefore, from $MRS_{Y\ for\ X} = -P_X/P_Y$ we still get $Y = 2X$. The budget constraint changes to $100 = 20X + 40Y$. Solving as before (with two equations, two unknowns), we get $(X^*, Y^*) = (1, 2)$. If all prices and income change proportionally, the optimal bundle does not change.

4. Derive the demand curve for good X and demand curve for good Y.

Solve for X^* and Y^* as before, except this time do not plug in explicit values for P_X, P_Y and I.
$MRS_{Y \text{ for } X} = -P_X/P_Y \implies -Y/4X = -P_X/P_Y \implies P_Y Y = 4P_X X$
Budget constraint: $I = P_X X + P_Y Y$
Now let's solve for X^* by substituting out Y. $I = P_X X + 4P_X X = 5P_X X \implies X^*(P_X; I) = I/(5P_X)$.
Holding income constant at $100 *gives us a demand curve of* $X^*(P_X; I = 100) = 20/P_X$

We can solve for Y^* by substituting out X: $P_Y Y = 4P_X X \implies Y = (4P_X X)/P_Y \implies Y = (4P_X[I/(5P_X)])/PY$ $Y^*(P_y; I) = 4I/(5P_y)$
Holding income constant at $100 *gives us a demand curve of* $Y^*(P_y; I = 100) = 80/P_y$

Now a government subsidy program lowers the price of Y from $20 per unit to $10 per unit.

5. Calculate and graphically show the change in good Y consumption resulting from the program.

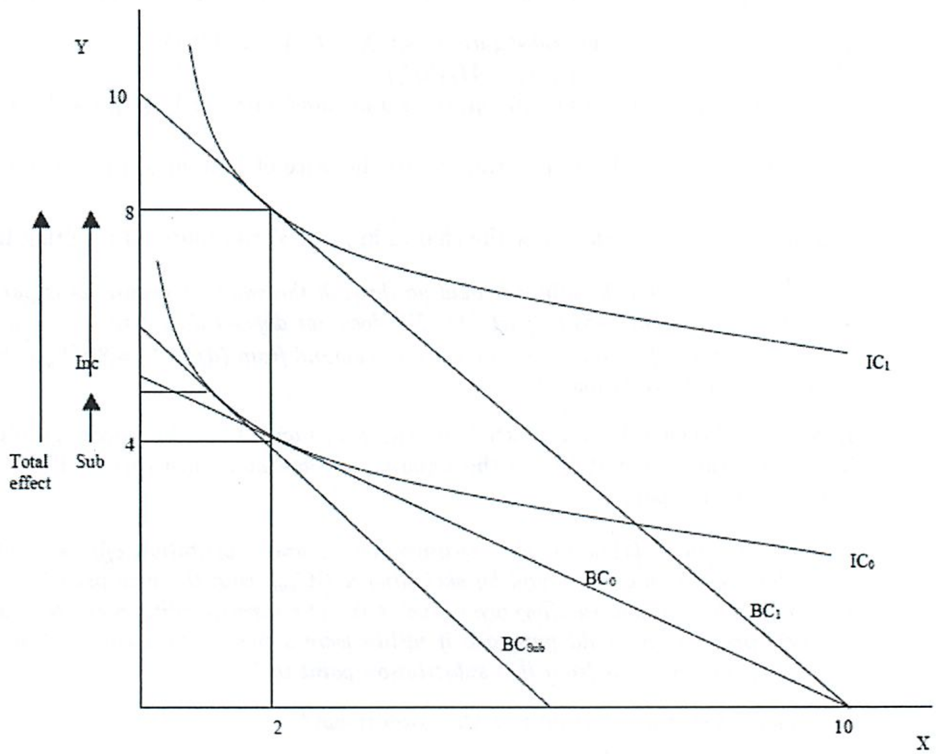
To calculate the new bundle we could go through the same procedure as is part (b) or simply use the demand equations derived in part (d). X^* does not depend on P_y so $X^{*'} = 2$.
Y^* is a function of its own price and, using demand from (d), $Y^{*'} = 80/P_y = 80/10 = 8$. *This change is depicted in figure below.*

6. In a clearly labeled diagram with Y on the y-axis and X on the x-axis, graphically show the change in consumption attributable to the separate income and substitution effects. No calculations were required for this part.

Refer to the figure below for the separate income and substitution effects. Note that the substitution effect for good Y may be found by sketching a BC_{sub} with the new price ratio (parallel to BC_1) but tangent to the original indifference curve $IC0$. The corresponding point tells us what a person facing the new price ratio would purchase if utility were somehow held constant at its original level. The income effect brings us from this substitution point to $Y^{*'}$.

7. How much does the program cost the government?

For each unit of Y purchased, the government pays out $10. *Since* 8 *units of Y are currently purchased, the cost will be* 8($10) = $80.



Y
10
8
4
Inc
Total effect
Sub
IC1
IC0
BC0
BC1
BCSub
2
10
X

Lecture 8

~~[scribbled]~~ 10/4

Application: Substitution + Income effects

- Child labor
  - ~~..~~ if working ⇒ don't go to school
  - never get out of cycle
  - does free trade ↑ child labor?
    - Gruber: ambigeous, may not be true
- figure 8-1
  pre 1989 - all rice produced in Vietnam could not sell outside
  1997 - quota totally disappered
- produces were able to sell larger Q at larger price
- what did it to to child labor?
  - if kids are not working, they are in school
  - shifts out demand to D2 ⇒ seems like more child labor
  - but farmers are richer, so they send their kids to school
    - kids pulled off market
  - must be large contraction in supply of child's labor for net child labor to go ↓

* Must think through all effects?
  - what happened?

Look what really happened?
  - large PP near ports (low transport costs)
    - child labor went down
  - did not do much inland (high transit costs)

(2) Producer Theory

- not on tomorrow's exam
- supply curve
- suppliers actually make up the price
- 2x as many lectures

Producer: black box

inputs → $\boxed{\text{firm}}$ → outputs

goal: maximize profits

= revenue - expenses

key: efficient production

---

today: firm production function

for today 2 inputs firms use

L labor

k capital

output: q units of production

$$q = f(L, k)$$

q = firm's output

Q = market's output

③

variable vs fixed inputs

↑ hours easily changed

↑ size of the building

Short run vs long run

↑ Some outputs are fixed

↑ period in which **all** inputs are variable

e tomorrow

e 10 years

but don't know where line is

Short Run Production

- labor is variable
- capital is fixed

How many workers should I hire?

- depends on marginal product of labor
  - add. output from 1 more unit of labor at given level k — constant

$$MP_L = \frac{\Delta q}{\Delta L}\Big|_{\bar{k}}$$

- typically diminishing marginal product
- every worker helps, but less + less

~~- each worker~~ capital is fixed

- ie there is only 1 shovel

4

Long Run Production

- all inputs are variable
- firm must choose L and k
  - trade off
- [Same mechanics as Utility]
- $q = \sqrt{k \cdot L}$ ← demo/example
- figure 8-3

Isoquants - just like indifference curves

- Marginal Rate of Technical Substitution (MRTS)

$$\left.\frac{\Delta k}{\Delta L}\right|_{\bar{q}}$$

- figure 8-5

Here quantity is important
- ordinality + carnality important
- MRTS varries

Returns to Scale

- important concept
- what if we ↑ all inputs proportionally
  - ie double ~~all~~ L and k
  - or ↓

(5)

answer: depends on production process

- Constant → $f(2L, 2k) = 2f(L,k) = 2q$

double firm, twice as much stuff

- IRS (↑) → $f(2L,2k) > 2f(L,k)$
increasing
$> 2q$

- Decreasing RS (↓) → $f(2L, 2k) < 2f(L,k)$
$< 2q$

Figure
- 8-6a constant

- economists don't think of ↑ returns to scale is true
  - they look at mature firms
  - who would have expanded their already

10/4

Figure 8-1: Rice export quotas in Vietnam

Market for Rice Produced in Vietnam

Lecture 8

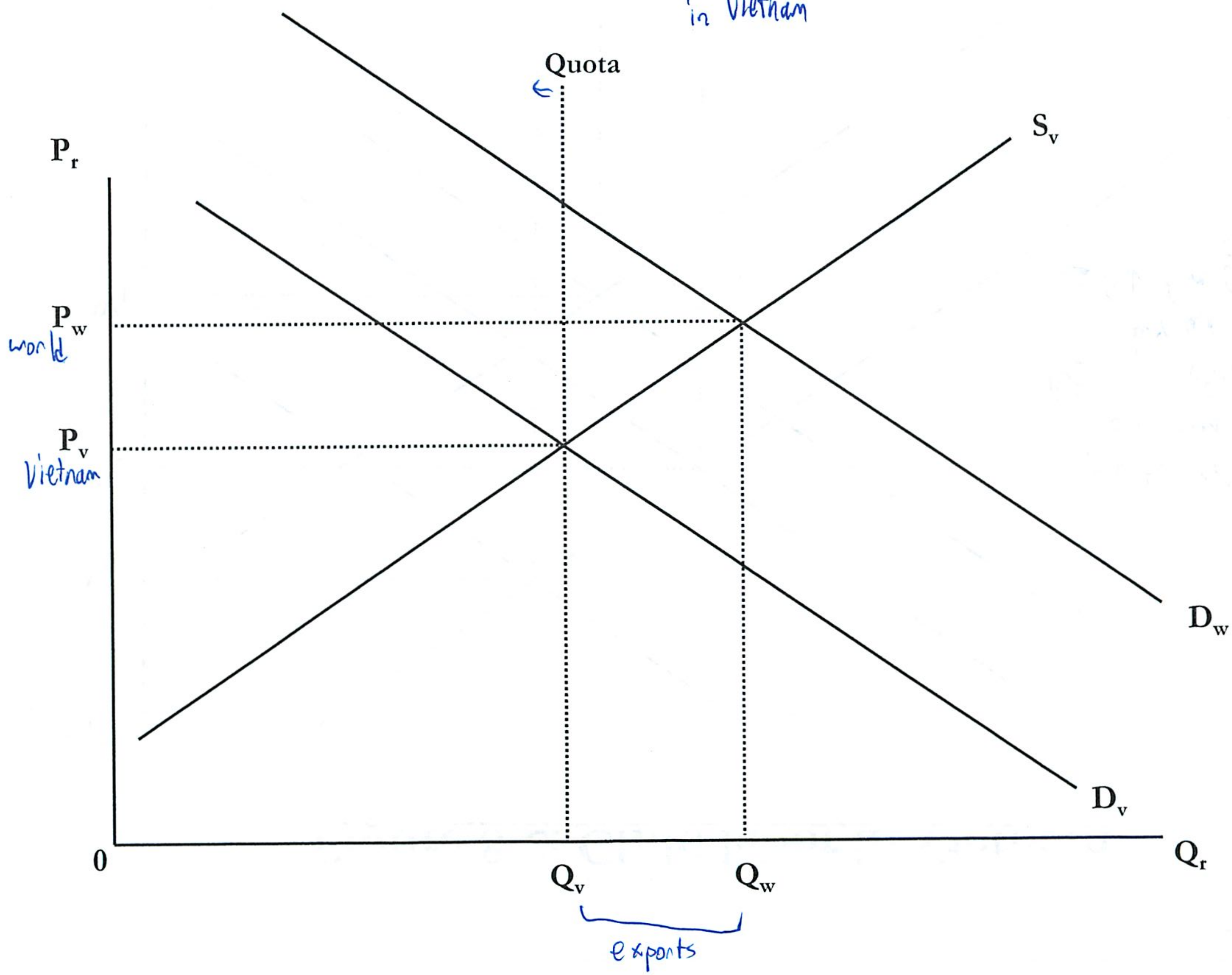
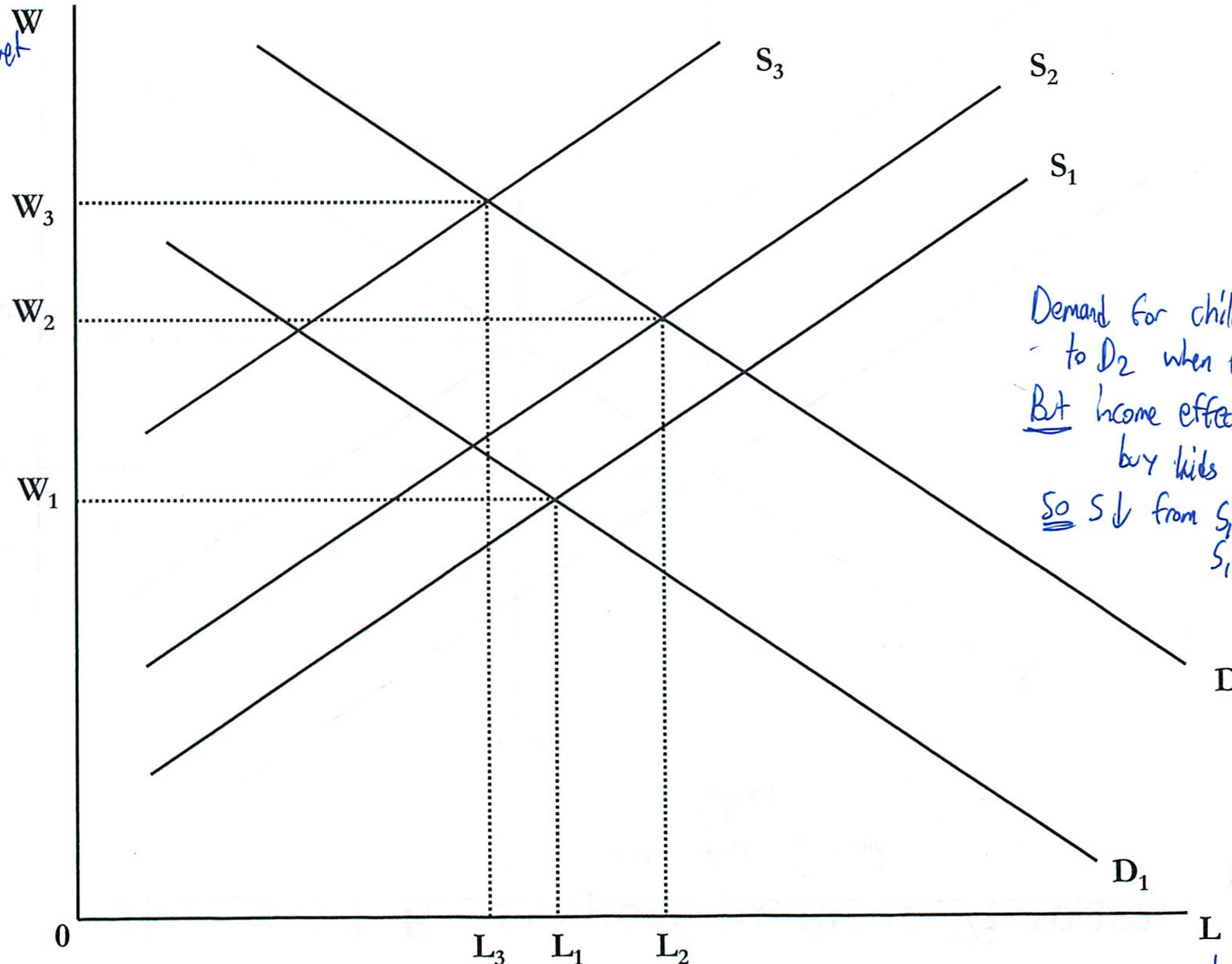


Figure 8-2: Child labor in Vietnam

world market



labor

Demand for child labor shifts to D_2 when world market

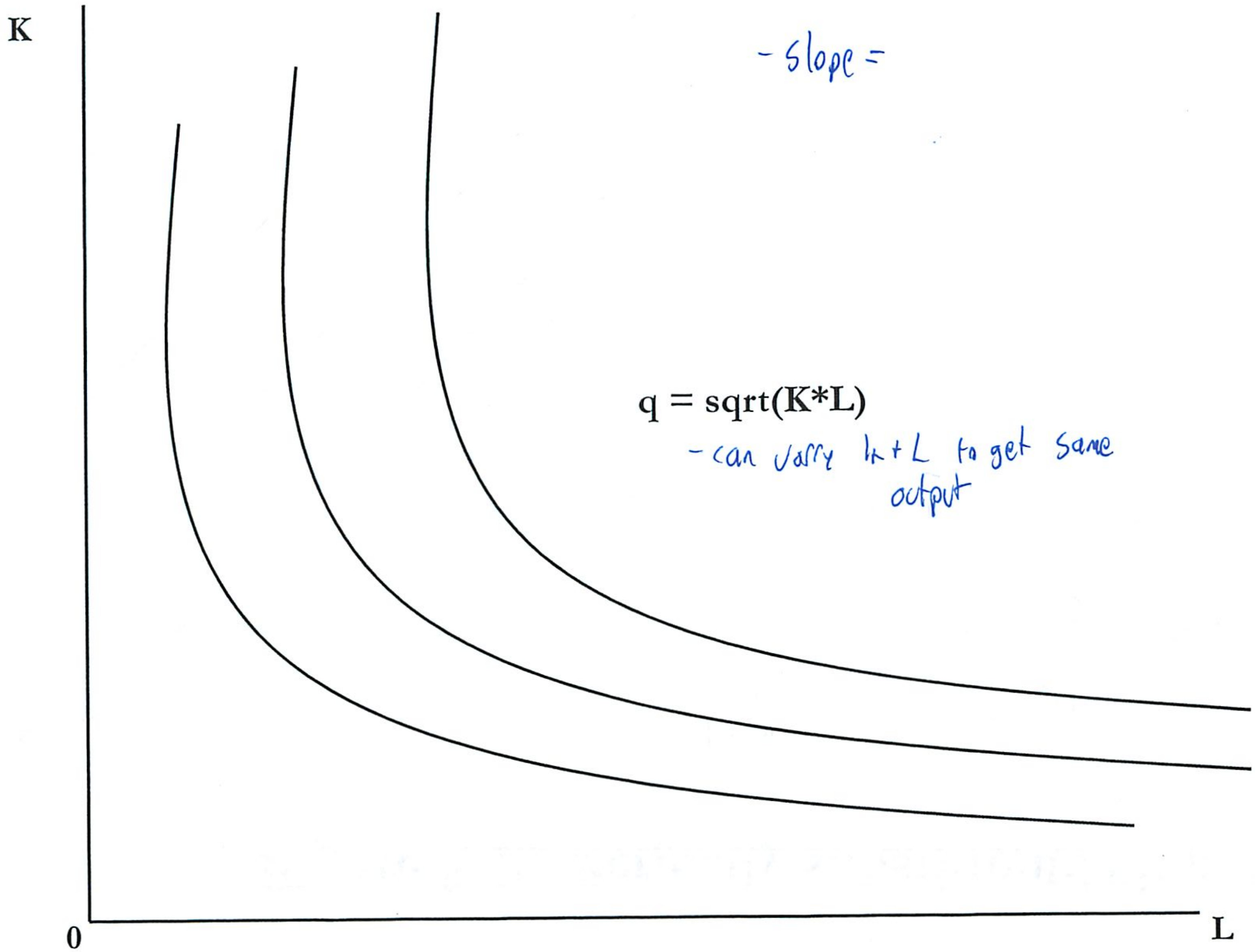
But income effect - farmers will buy kids an edu

So S ↓ from $S_1 \to S_2$ ∈ still an ↑

$S_1 \to S_3$ ∈ less child labor

Figure 8-3: Isoquants

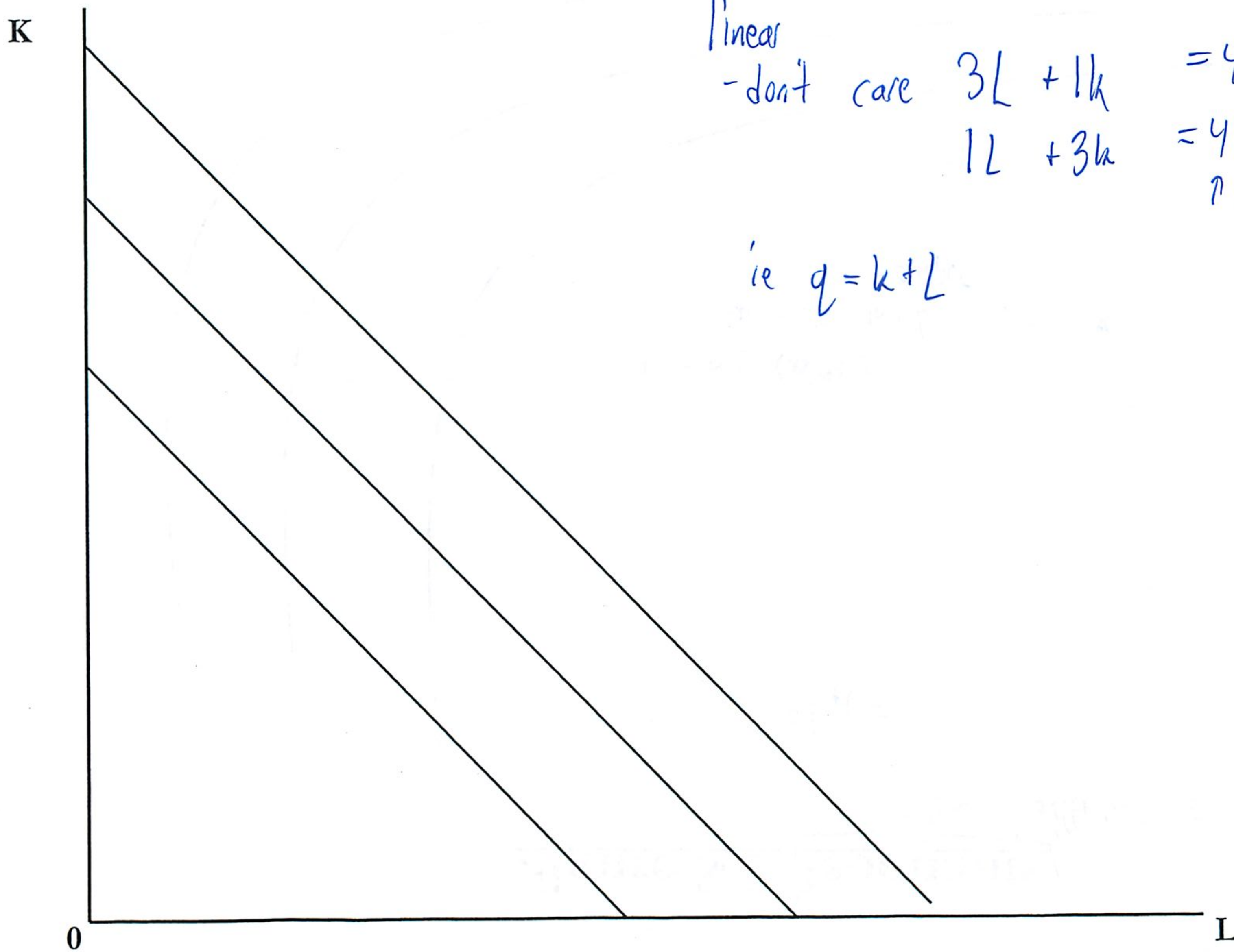
- like indifference curves



- slope =

- can vary K + L to get same output

Figure 8-4a: Perfectly substitutable inputs



linear
- don't care $3L + 1k = 4$
$1L + 3k = 4$
↑ all the same

ie $q = k + L$

Figure 8-4b: Non-substitutable inputs

"Leontieff"

Cereal + cereal boxes

need both

given 1 input → need a certain amt of other good

which you have least of matters

$q = \min(k, L)$

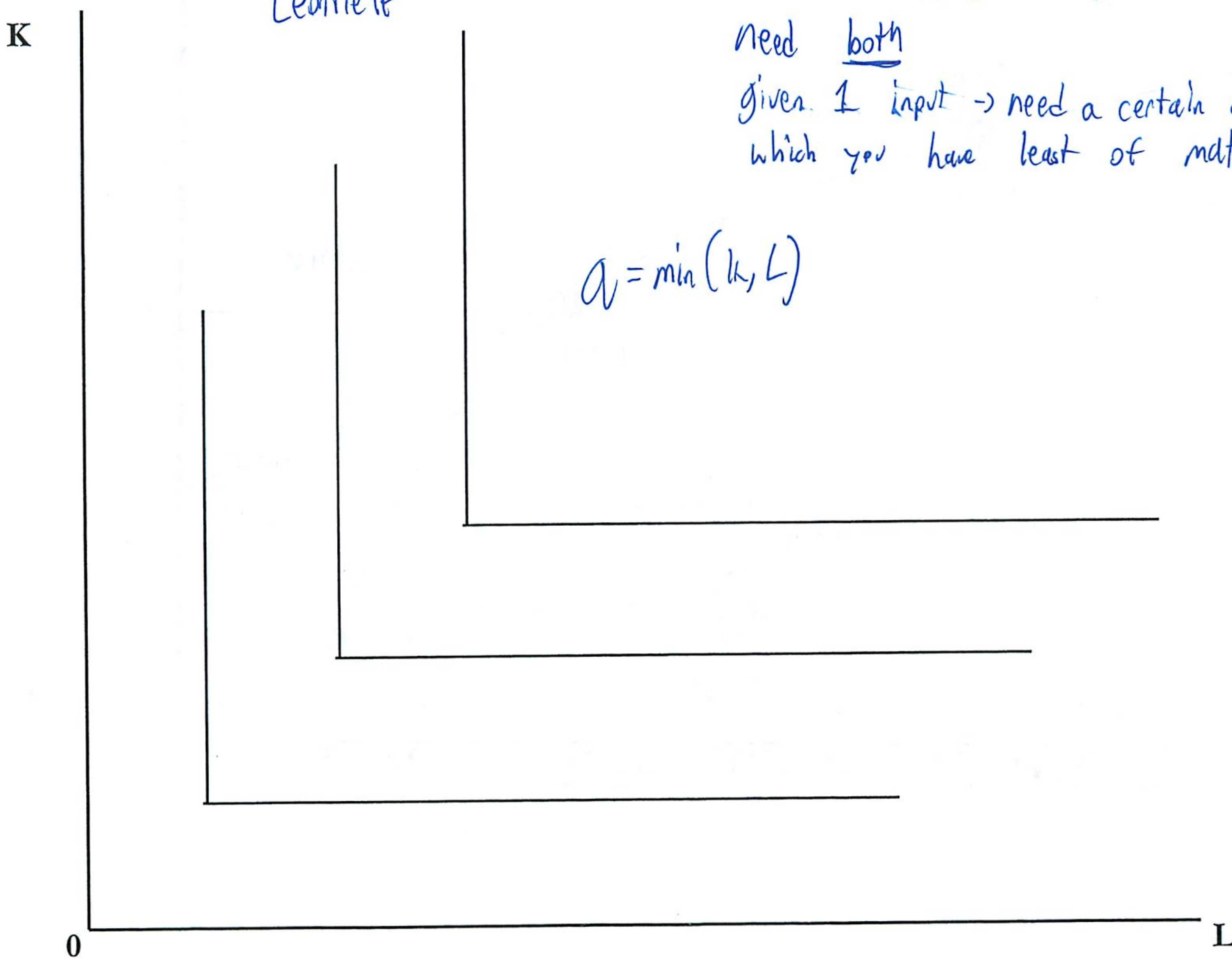


Figure 8-5: Isoquants and MRTS

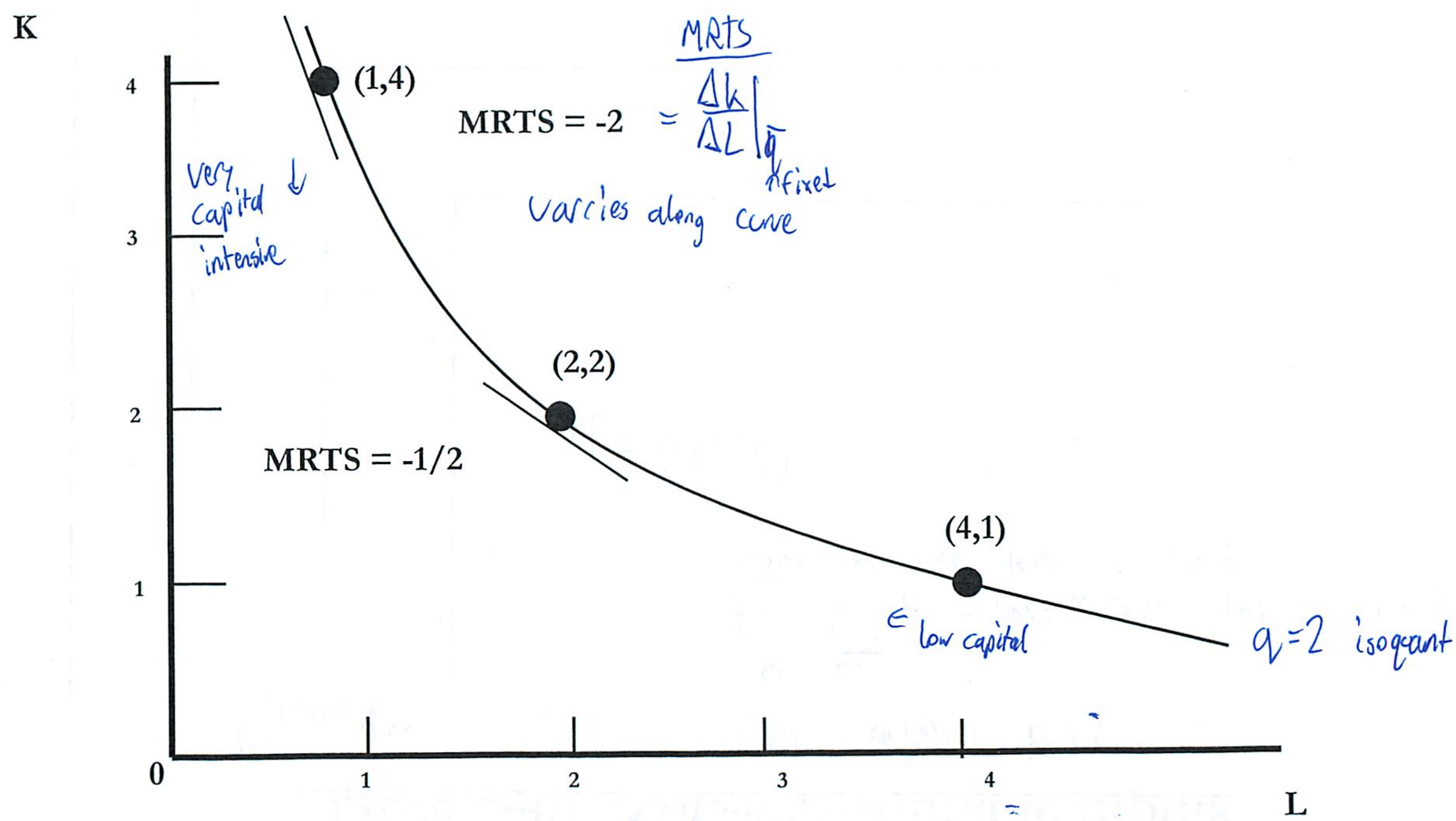


Figure 8-6a: Isoquants and $\boxed{\text{constant returns}}$ to scale

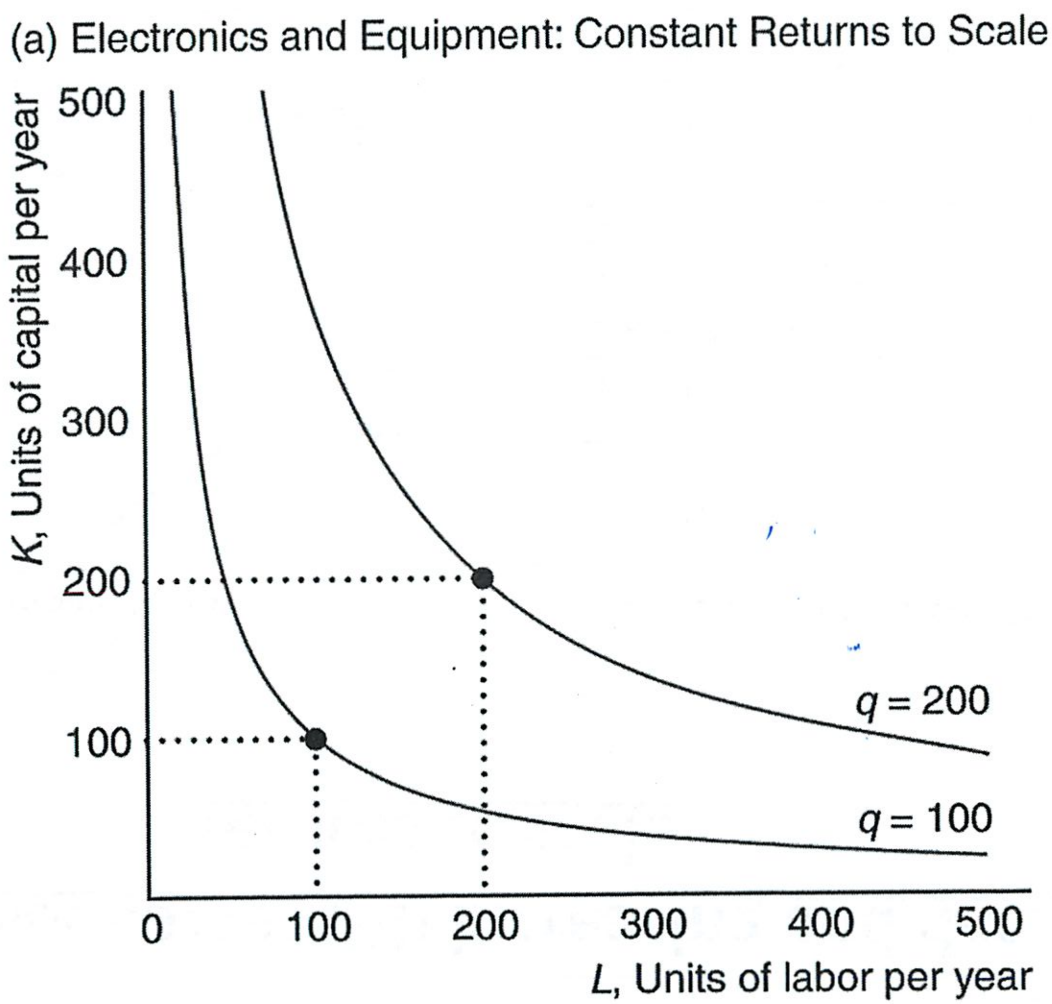
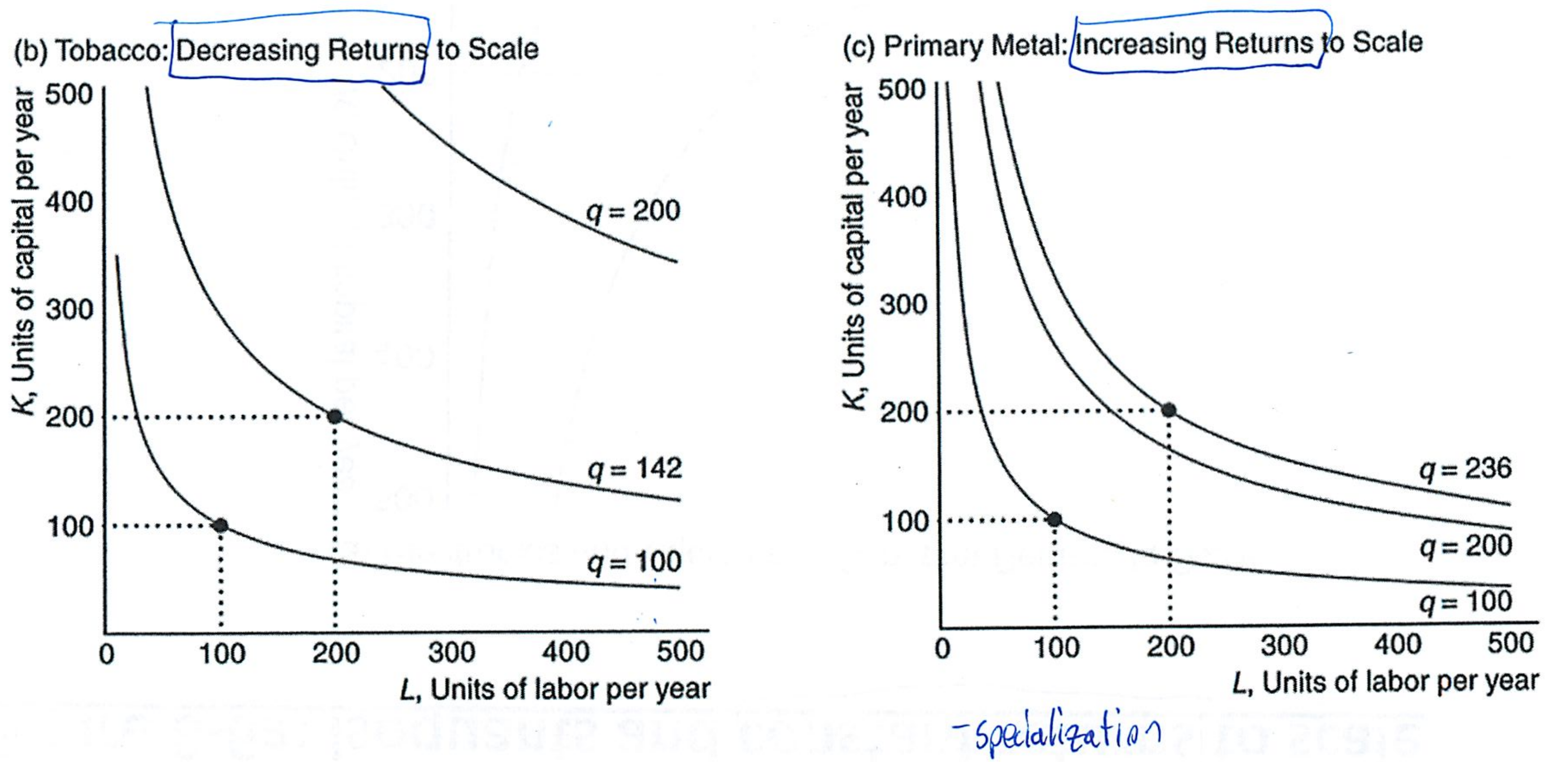


Figure 8-b: Isoquants with increasing and decreasing returns to scale



Lecture 9 10/6

- Malthus(?) → would be massive starvation
  - Food production would be flat
  - demand every increasing

- * aggregate production is about k and l and aggregate ~~producto~~ productivity
  $\cancel{A}$ $Q = A \cdot f(k, l)$
- we use ~~in puts of land~~ use resouces more effictivly
  - tractors, etc
  - weed killers
- Food production is ↑ 40% since 1950
- productivity > MRP
  marginal rate of ~~productivity~~ production
- What determins the overall standard of living in society
  - depends on society's productivity
- productivity → no more hard work of machines
  - actual growth does factor that in
- less savings in US → so less capital
  ~ 30% US
  - ~~Jap~~ 20% Japan
- drop in productivity growth in 1980s
- if society is more productive
  - its like free money
  - US bought stuff

(2)

Europe bought more leisure w/ that
- and less stuff

Costs

- Prof: most boring part

$\pi = R - C$ ← maximization
↑ profit

- fixed costs - cost of inputs can not vary in short run
- variable costs - can vary in short run
- total = fixed + variable
- marginal cost $= \dfrac{\Delta \text{cost}}{\Delta \text{output}}$

- avg. cost $= \dfrac{\text{cost}}{\text{output}}$

---

get costs from production function

$C(q) = f(wL + rk)$

↑ wage rate ← rental rate - $ per use of capital

↑ harder to think about

for now think all machines are rented

- want "flow" calculation

(3)

$FC = r\bar{k}$

fixed capital in short run

$VC = w \cdot L(q)$) short run total costs

$= r\bar{k} + w \cdot L(q)$

$MC = \frac{\Delta c}{\Delta q} = w \cdot \frac{\Delta L}{\Delta q}$

marginal product of labor

= wage/marginal product of labor

- the more productive the next worker is, the lower the additional cost

---

Long Run

- firms can choose input mix
  - to maximize production efficiency
  - aka minimizing cost
- isoquants
  - combos of capital + labor that deliver same output
  - choose the one that minimize cost
- draw isocost lines
  - figure 9-1

④

- Slope $= -\frac{W}{r}$

- how do we know how much $ they have to spend?
  - more complex, next week
- about opportunity costs
- ~~knowing isocosts and~~
- economically efficent input combo determined
  tangency w/ isocost and ~~production function~~ isoquant
  - figure 9-2

$MSRT = \frac{MP_L}{MP_C}$ ← marginal product

SMRTS

$$MSRT = \frac{MP_L}{MP_C} = \frac{W}{r}$$

← price ratio.

· rewrite

$$\frac{MP_L}{W} = \frac{MP_k}{r}$$

for each $, get same amt
of production w/ both labor + capital
efficent pt is where they are =

(5)

$$MP_L = \frac{\frac{1}{2}K}{\sqrt{K \cdot \ell}} \qquad MP_K = \frac{\frac{1}{2}L}{\sqrt{k \cdot l}}$$

$$\frac{k}{l} = MRTS$$

$$\frac{k}{l} = \frac{w}{r} = \frac{1}{2}$$

? use half as much capital as labor

Firms harder
- no budget constraint
- it has to decide, determine how much money it can spend

- 9-4a figure -
- ~~...~~ underlies the long run cost curve

---

as firms produce more
- mix depends on production level
- q (total cost) comes from market competion
- issue: Fixed vs sunk costs

fixed ??? [missed what he said

? fixed even in the long run
Once you produce, gone
Can never change

10/6

Lecture 9

Figure 9.1: Isocost Lines

Isocost Equation

$$K = \frac{\overline{C}}{r} - \frac{w}{r} L$$

Initial Values

$\overline{w} = \$5$ wage rate

$r = \$10$ rental cost

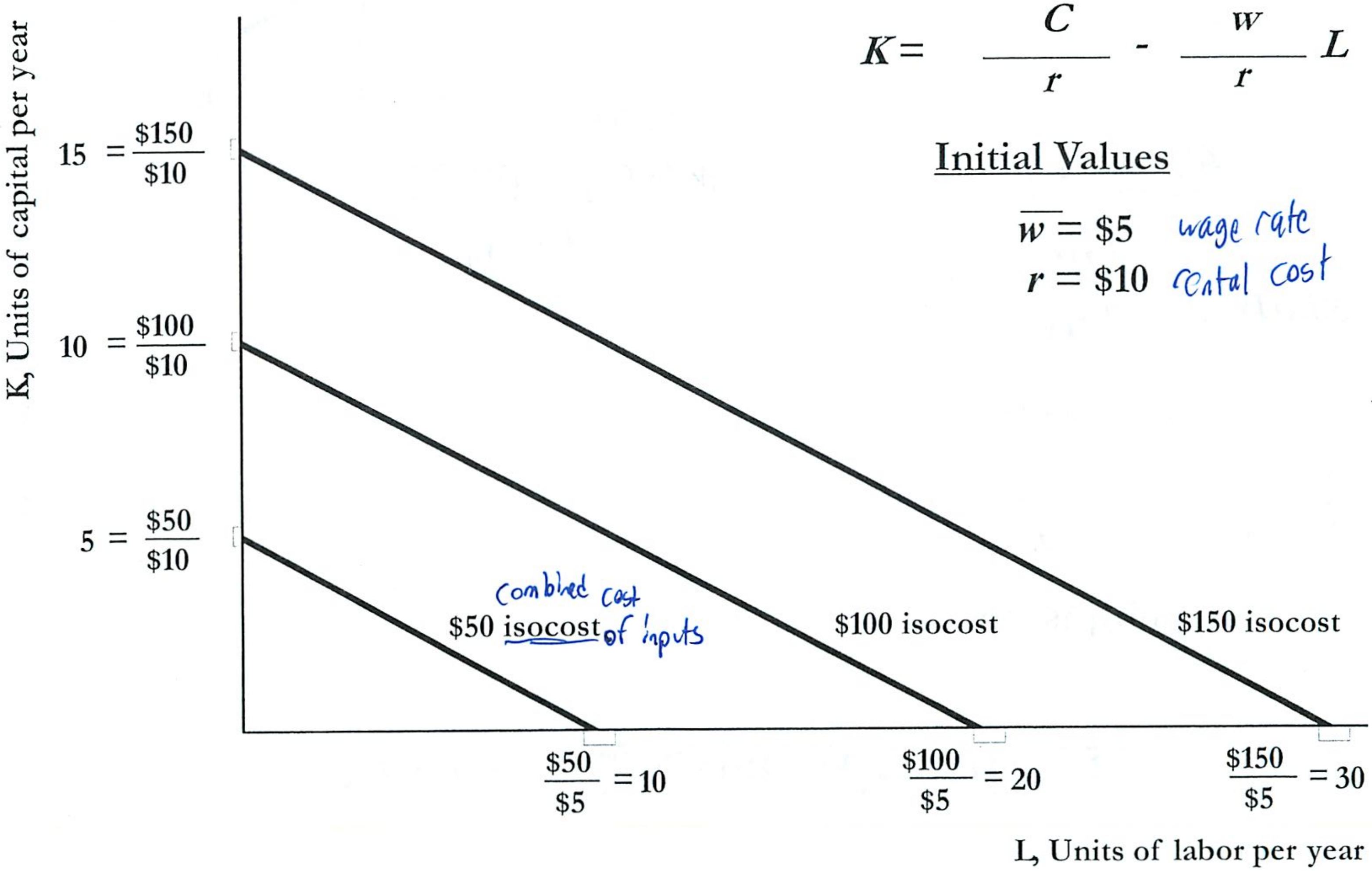


Figure 9-2: Cost Minimization

Isocost Equation

$$K = \frac{\overline{C}}{r} - \frac{w}{r} L$$

Isoquant Slope

$$-\frac{MP_L}{MP_K} = MRTS$$

$q = \sqrt{5 \cdot 2.5}$

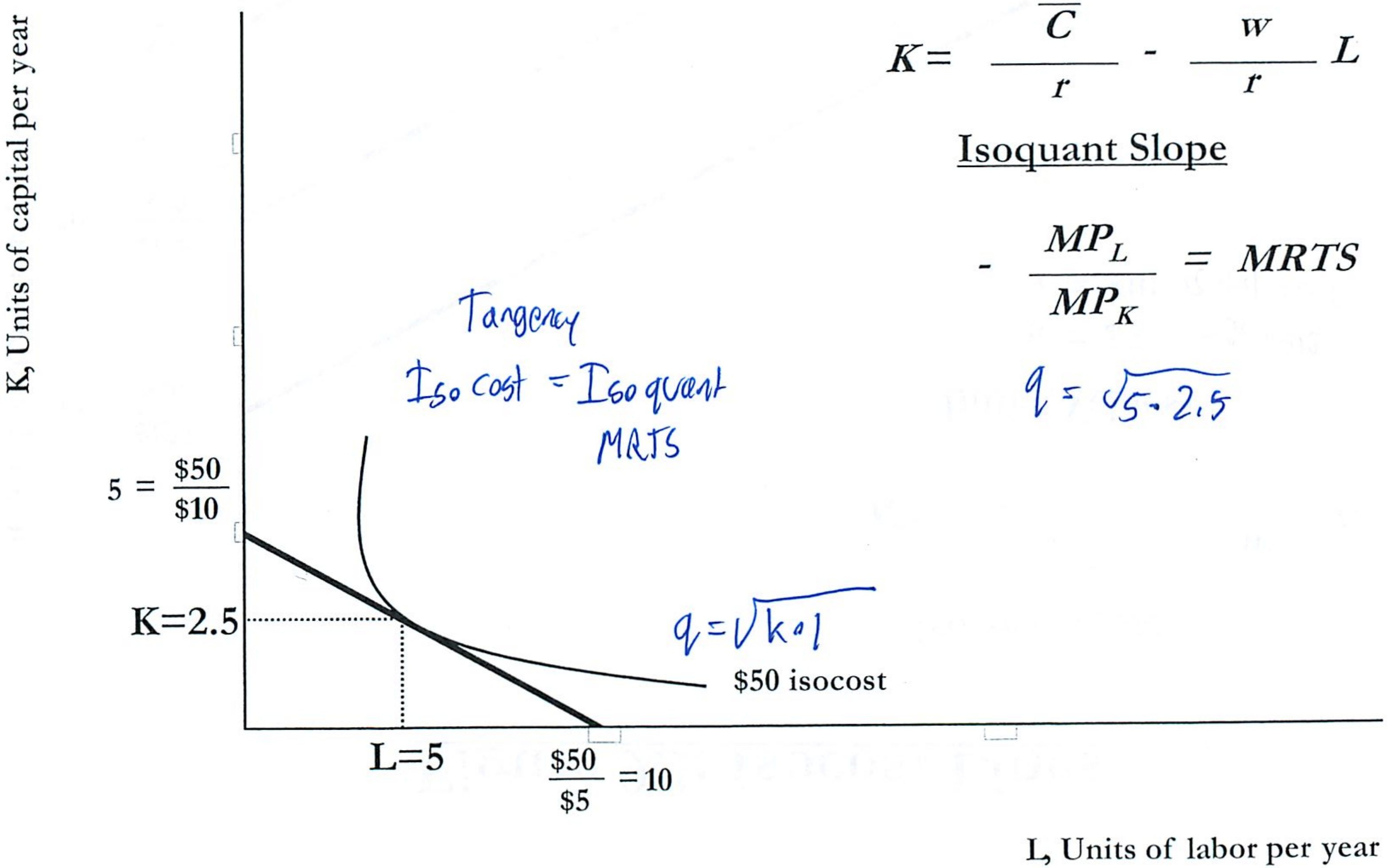
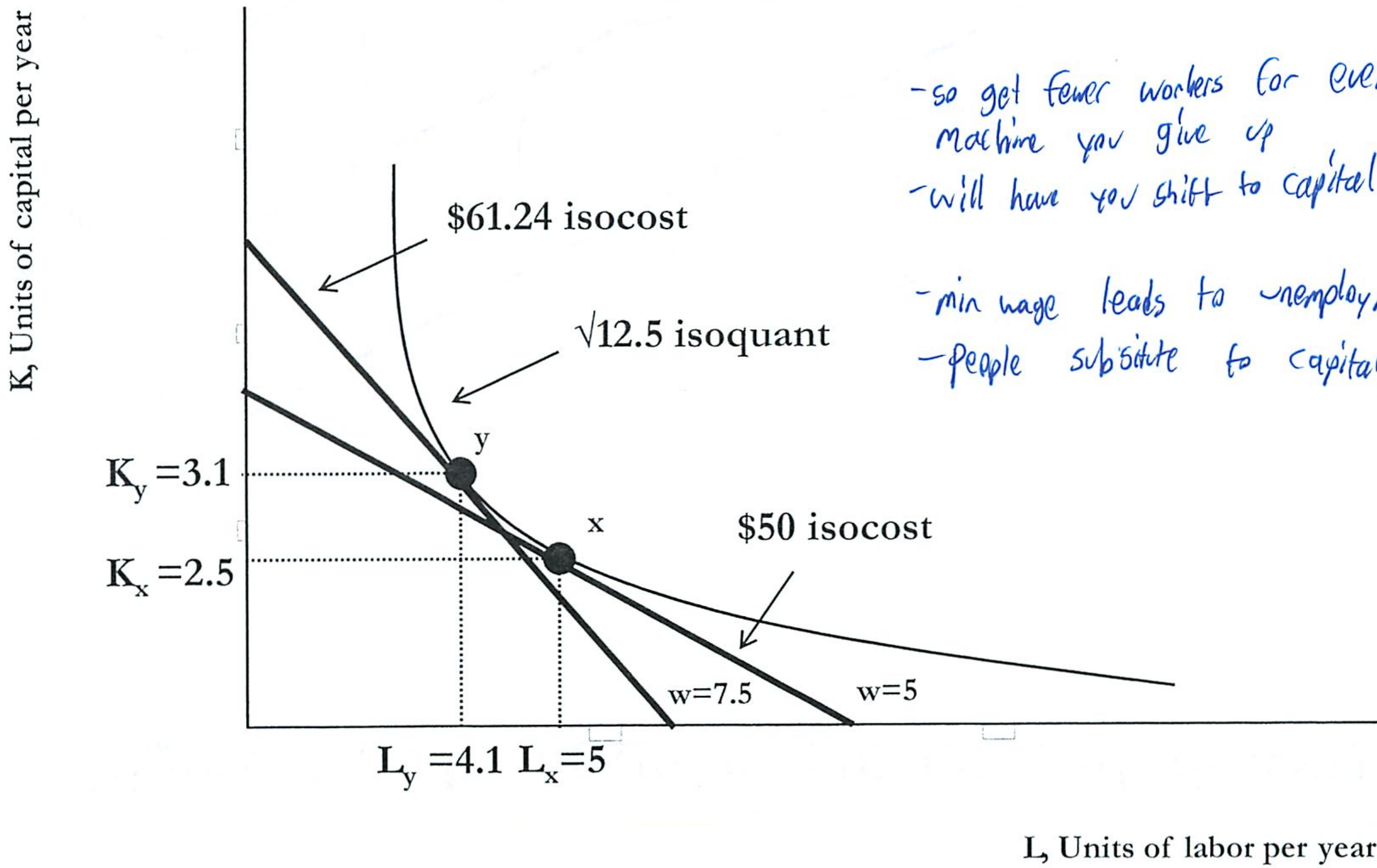


Figure 9-3: Cost Minimization with an increase in wages



wages go ↑ due to new min wages

- so get fewer workers for every machine you give up
- will have you shift to capital

- min wage leads to unemployment
- people subsitute to capital

Figure 9-4a: Long-run expansion path (linear)

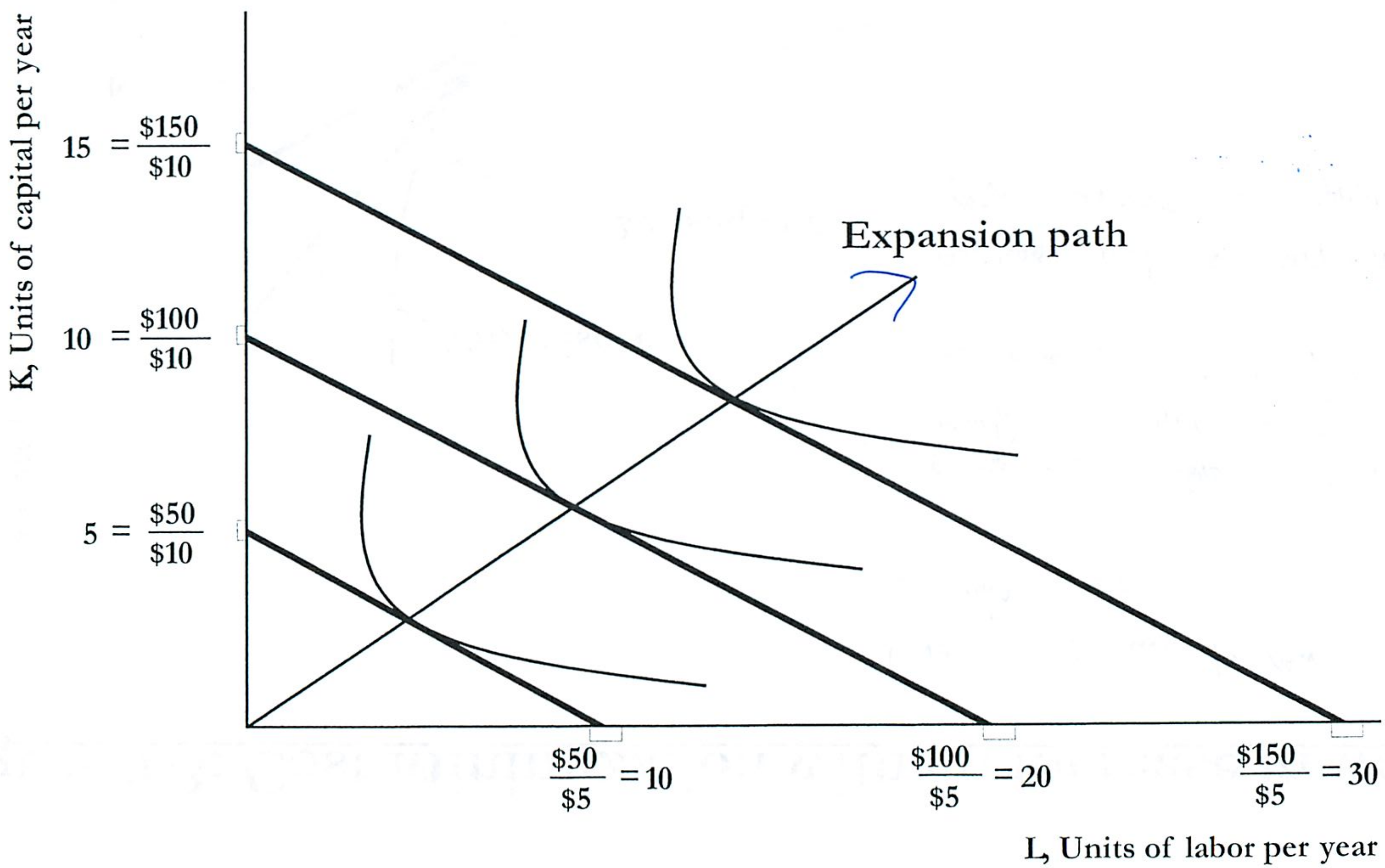
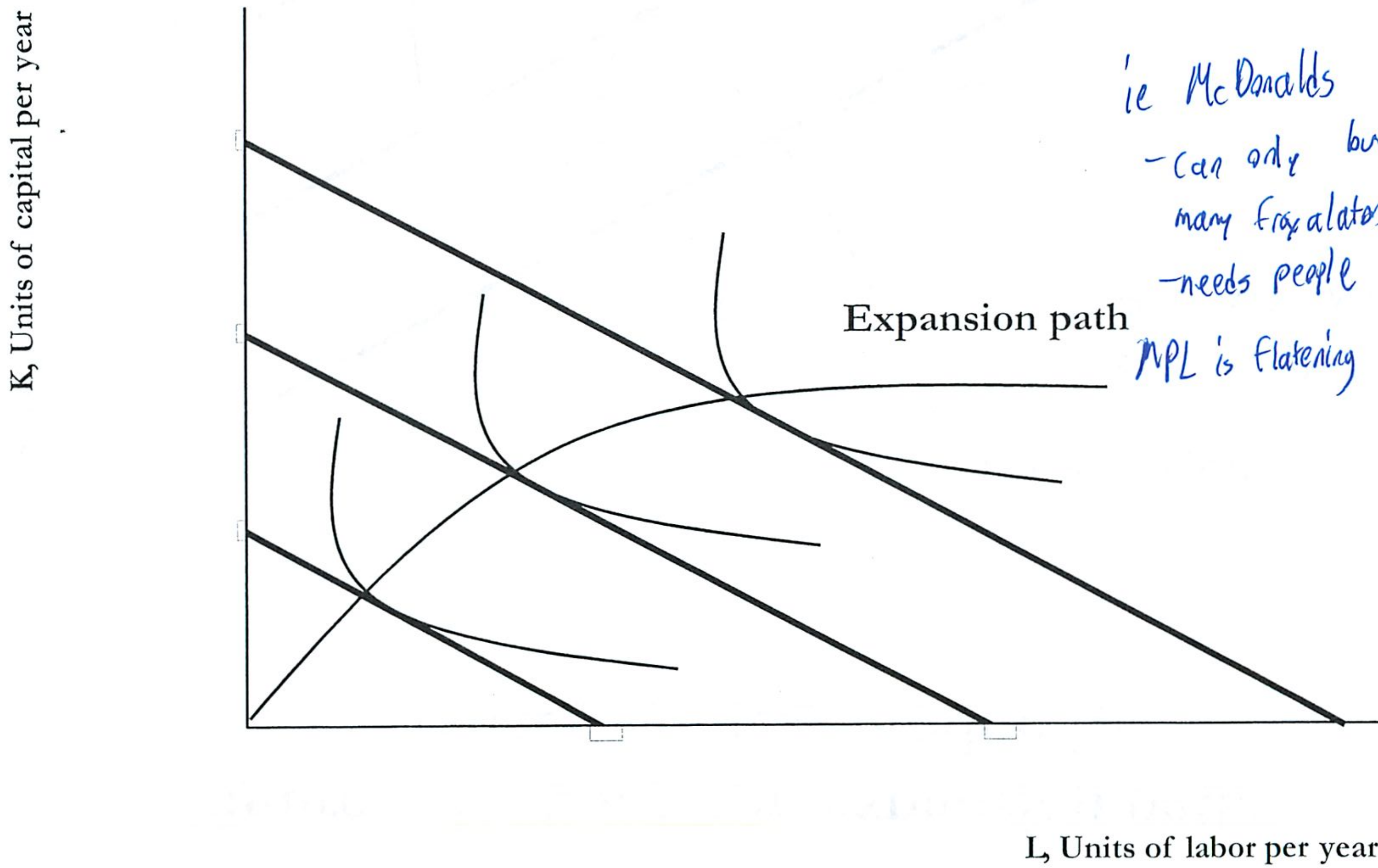
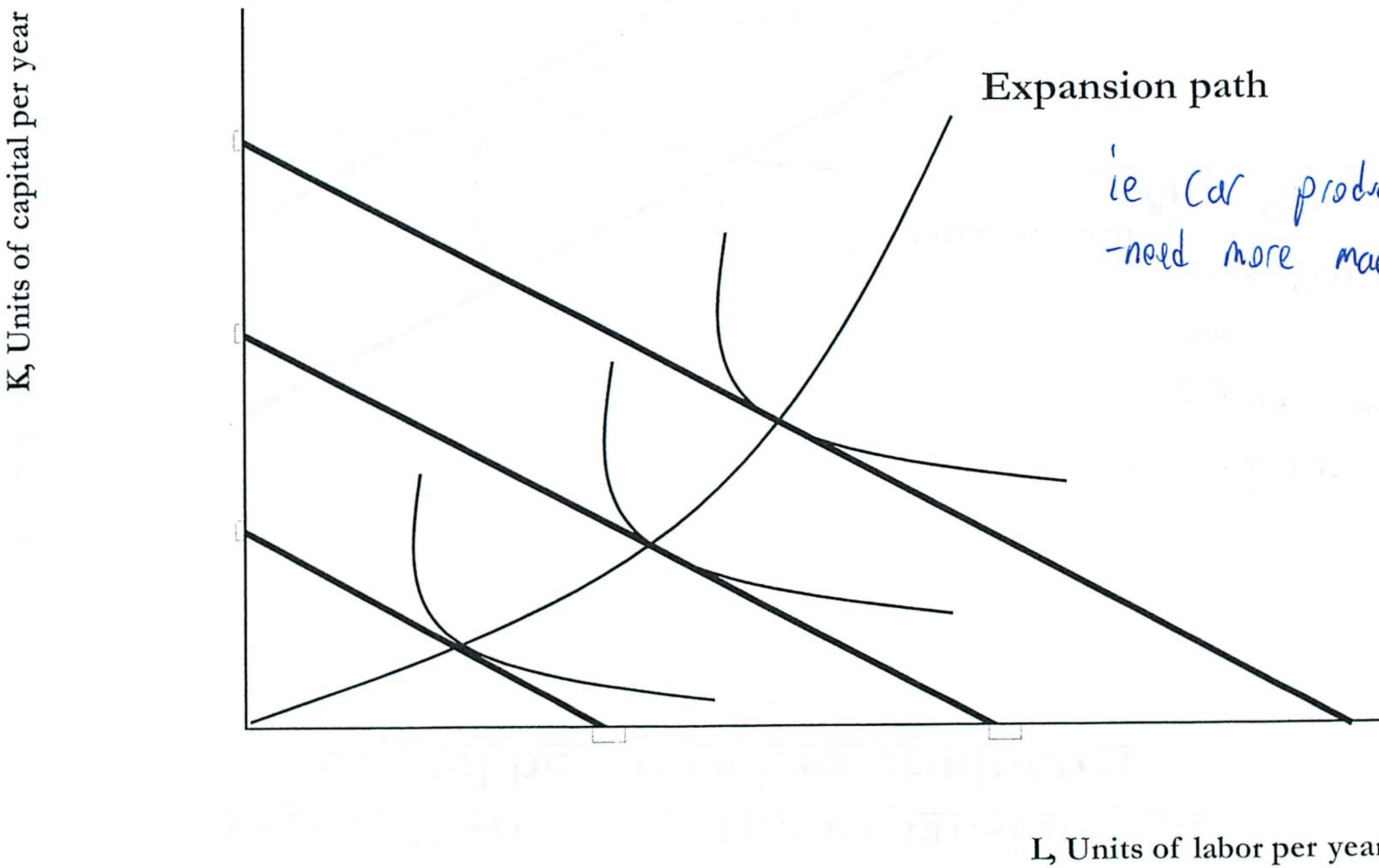


Figure 9-4b: Long-run expansion path: capital becomes less productive



ie McDonalds
- can only buy so many fryalators
- needs people
MPL is flatening

Figure 9-4c: Long-run expansion path: labor becomes less productive



Recitation 5

10/7

1. Production function examples
2. Costs
   - short run
   - long run
   - short run vs long run costs
3. Economoies of scale + scope

---

1. Production functions

a) $Q = L^{1/2} k^{1/2}$ Cobb douglas

$Q = L^{\alpha} k^{\beta}$

$1 > \alpha > 0$

$1 > \beta > 0$

- Marginal product of labor
- " " " capital
- " rate of technical substitution
  - substitute labor for capital to hold output constant

$$-MP_L = \frac{\Delta Q}{\Delta L} = \frac{\partial Q}{\partial L} = \alpha L^{\alpha - 1} k^{\beta}$$

$$= \alpha \frac{Q}{L}$$

$$-MP_c = \frac{\Delta Q}{\Delta k} = \frac{\partial Q}{\partial k} = \beta L^{\alpha} k^{\beta - 1}$$

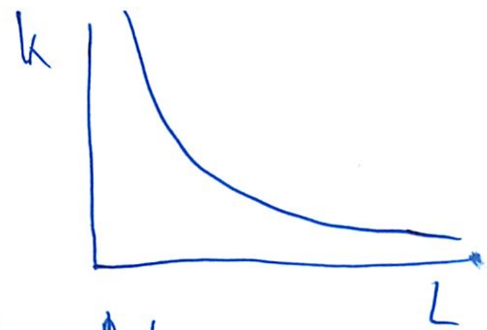
$$= \beta \frac{Q}{k}$$

②

↙ slope of isoquant

$$MRTS = -\frac{dk}{dL} = \frac{MP_L}{MP_k} \leftarrow \text{prices} = \frac{\alpha Q/L}{\beta \frac{Q}{k}} = \frac{\alpha}{\beta}\frac{k}{L}$$

↑ will = at optimal pt



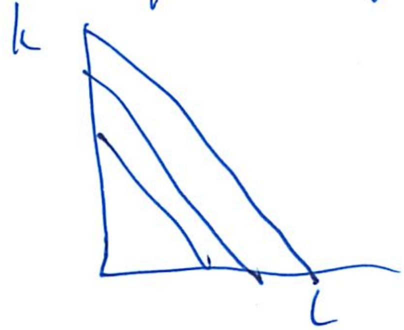
b) $Q = aL + bk$

$MP_L = a$ ← no diminishing productivity of labor / capital

$MP_k = b$

$MRTS = \frac{a}{b}$ ← so fixed tradeoff linear

isoquants straight



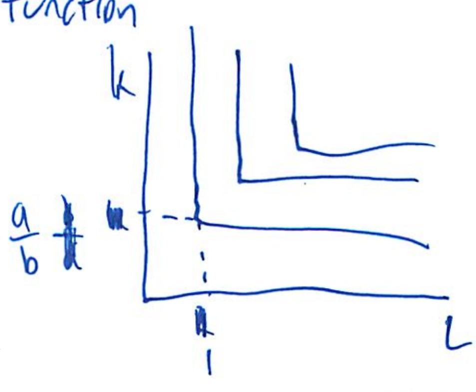
perfect substitutes

c) $Q = \min\{aL, bk\}$

Leontief production function

$aL = bk$

$\frac{k}{L} = \frac{a}{b}$



$$MRTS = \begin{cases} \infty & \text{for } L = 1 \\ 0 & \text{for } L > 1 \\ \text{und} & \text{for } L < 1 \end{cases}$$ here

③

2. Costs

- are always forward looking in econ
- opportunity cost of using the resources

$$C = \underbrace{r \cdot k} + w \cdot L$$

accounting treats this totally different

even if own plant, could lease it out, so always rent

We only care about future expenses

- sunk cost - once you incur the expense you can't sell it
- economics does not care about them
- the cost of something is what you could sell it for

Firms want to maximize profits

$$\pi = P \cdot Q - c(Q)$$

produce maximum outputs at minimum costs

a) Short run costs

$\bar{k}$ - capital is fixed

total cost $= C = r \cdot K + w \cdot L$

Fixed cost $= r \cdot K$

Variable cost $= w \cdot L$

(4)

Marginal cost $= \frac{\Delta C}{\Delta Q} \approx \frac{dC}{dQ}$

Average fixed cost $= \frac{FC}{Q}$

Average variable cost $= \frac{VC}{Q}$

Average cost $= \frac{TC}{Q} = AFC + AVC$

$Q = L^{1/2} k^{1/2}$

$k = 10$

$C(Q) = r \cdot k + w \cdot L(Q)$

$Q = L^{1/2}\sqrt{10}$

$L(Q) = \frac{Q^2}{10}$

$r = \$1 \qquad w = \$5$

$C(Q) = 1 \cdot 10 + 5 \cdot \frac{Q^2}{10}$

$= 10 + \frac{1}{2} Q^2$

short run total cost

$FC = \$10$

$VC = \frac{1}{2} Q^2$

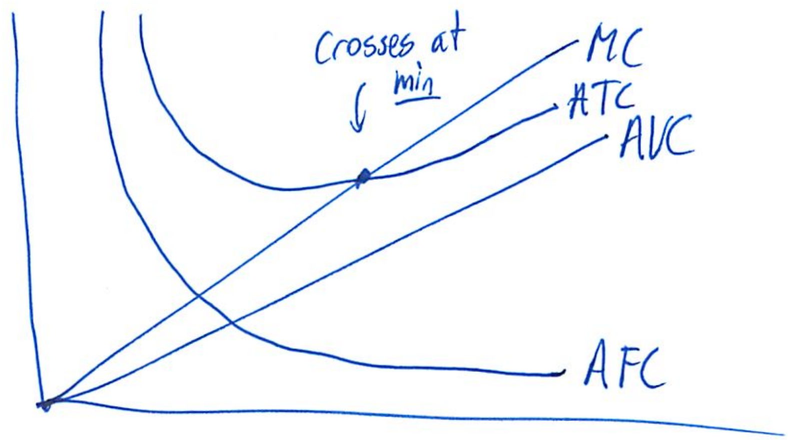
$MC = Q$

(5)

$AFC = \frac{10}{Q}$

$AVC = \frac{1}{2}Q$

$ATC = \frac{10}{Q} + \frac{1}{2}Q$



$$\min_Q A\cancel{V}C = \min_Q \frac{C(Q)}{Q}$$

F.O.C first order condition = first derivative

$$= \frac{\frac{dC(Q)}{dQ} \cdot Q - C(Q)}{Q^2} = 0$$

determines minimum

$$= \cancel{\frac{\quad Q}{Q^2}}$$

$$= MC \cdot Q - C(Q) = 0$$

$$MC = \frac{C(Q)}{Q} = ATC$$

$MC(Q) = ATC(Q)$

$Q = \frac{10}{Q} + \frac{1}{2}Q$

$Q = 2\sqrt{5}$

Becomes expensive to produce more

So vary capital → long run

(left 15 min early)

Chap 6 Production

10/11

Only doing what we did not cover in class + highlights

The Supply side → producers

theory of the firm
- cost minimizing production

	Consumers		Producers
1.	Consumer Pref	→	Production Tech
2.	Budget Constraint	→	Cost Constraints
3.	Given that, max. satisfaction	→	Input Choices

all gets represented in the production function

6.1 Tech of Production

inputs = factors of production
- labor
- materials
- capital

various sub cats

Production function
- indicates highest output q from every combo of inputs

$$q = F(K, L)$$

max q

just doing capital + labor here

* is a flow (ie per year)

(2) - is for a given technology
- new tech allows you to override it
* what is technically feasable when the firm operates efficiently

Short Run vs Longrun

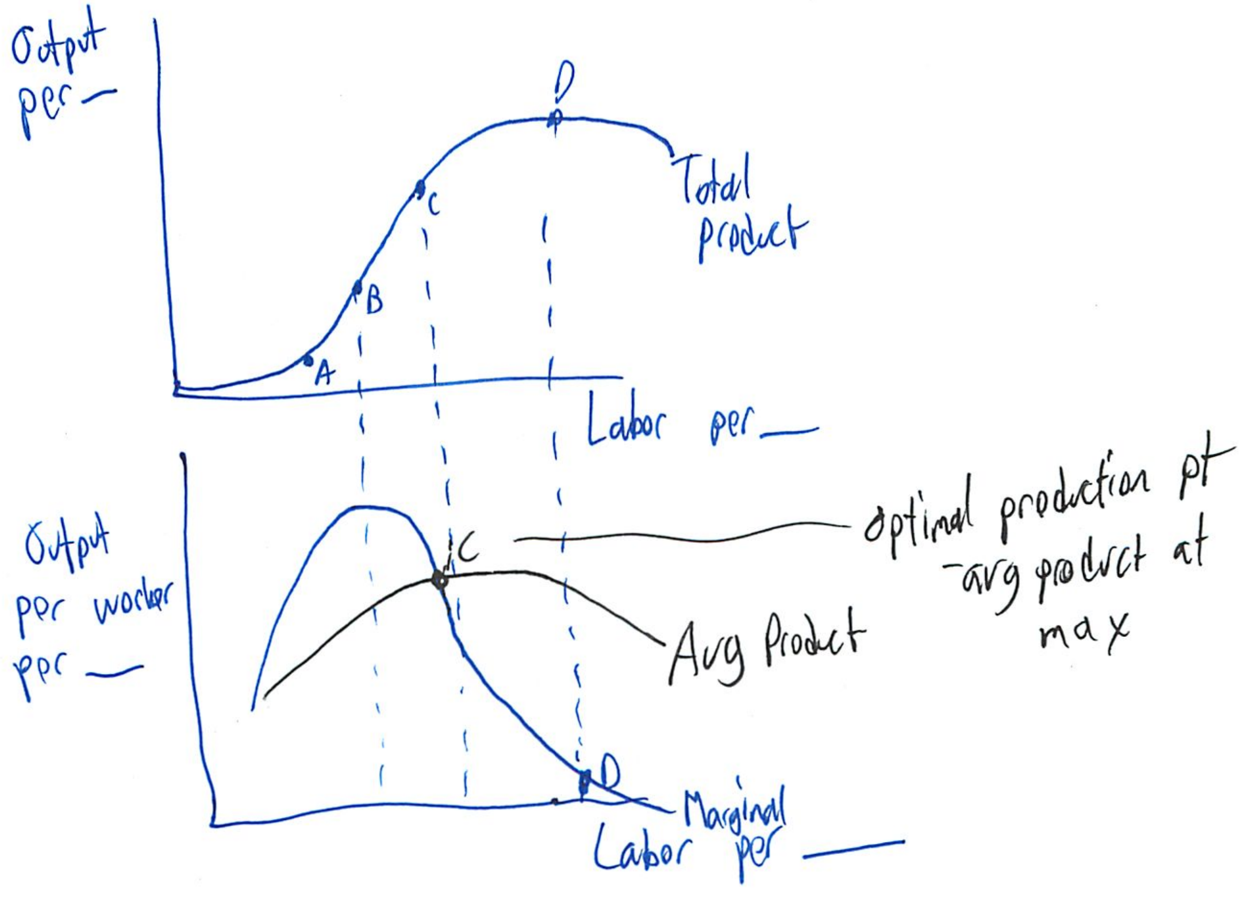
Short - one factor fixed/can't be changed
long - all variable

Short Run

Look at costs on both an incremental/marginal basis + average basis

$\frac{\Delta q}{\Delta L}$ — change in output from 1 add. labor

$\frac{q}{L}$



3

Note how graphs related

- total product slope (+) = (+) marginal product
- marginal > average = average slope (+) / increasing up to pt C
- marginal < avg = average slope (−) / decreasing past pt C
- marginal = avg when avg is max pt C

Graph meanings

- Avg product = slope of line from origin to corresponding pt on total product curve
- Marginal Product = slope of total product at that pt

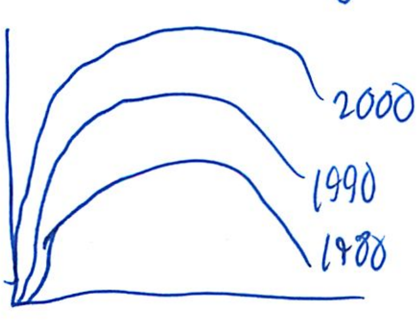
Law of Diminishing Marginal Returns

- pt exists when adding more resources will make output actually decrease pt D ← extream case

well resulting additions to outpt (pt C)
← more general

* Don't care about that quality of labor ↓ (ie most skilled laborers hired 1st)

And its for a given tech



~~peak~~ total product shifts upward over time

* no negitive long run implications to growth *

* "free" wealth

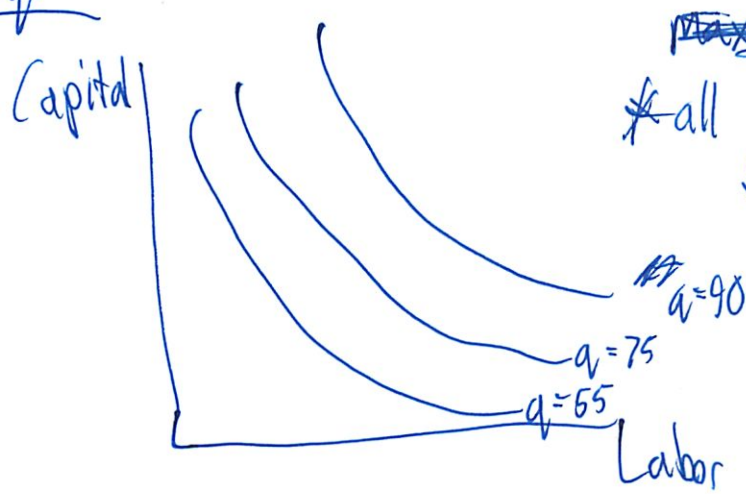
④ Labor Productivity

- avg product of labor for econ as a whole
- determines real st. of living that can be achieved
  * Consumers can only increase satisfaction in the long run by increasing the amt they produce
- Stock of capital - how much is avalible
- tech change - more efficient, higher quality

6.3 Production w/ 2 Variable Inputs (Long Run)

- all factors are variable
- firms choose b/w labor + capital

Isoquant



~~max that can be produced~~

* all possible combos that yield same output *

(at each pt - max that can be produced w/ that combo)

number of isoquants on 1 graph = isoquant map

5

<u>Input Flexability</u> - firms can change b/w labor + capital if the prices of each change

<u>Diminishing Marginal Returns</u> -

- for both labor + capital
- in long + short run

the more workers/capital you add, the less ~~each~~ each add. one adds to production

<u>Substitution Among Inputs</u>

MRTS = marginal rate of technical substitution

- of labor for capital

- amt of capital ↓, when add 1 unit labor for same output
- like consumer's MRS

$$= - \frac{\Delta K}{\Delta L} \text{ for fixed } q$$

- changes depending on where you are (if isoquant is not linear, right?)

diminishing MRTS

- ~~A~~ MRTS falls as we move down along an isoquant
- (isoquants are convex)
- tells us productivity of any 1 input is limited
- need a mix of <u>both</u> inputs

⑥

Add. output from ~~#~~ extra units labor

$= MP_L \, (\Delta L)_{\text{units extra labor}}$

marginal product of labor

add. output per unit of add. labor

Reduction in output from ↓ use of capital

$= (MP_K)\,\Delta K$

As we move along isoquant

$(MP_L)(\Delta L) + (MP_K)(\Delta K) = 0$

Rearrange terms

$$\frac{MP_L}{MP_K} = \frac{-\Delta K}{\Delta L} = MRTS$$

---

2 special cases

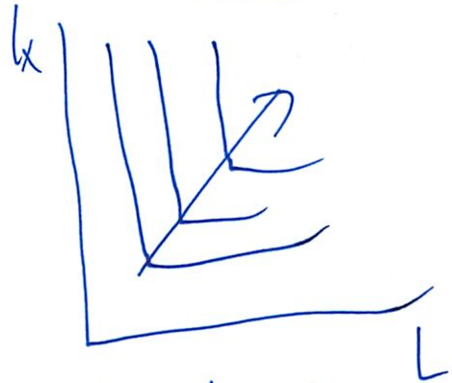
Perfect Substitutes

labor + capital are perfect substitutes to each other

ie: Musical instruments

(2)

Fixed Proportions Production Function



impossible to make substitutions amoung inputs

- each output needs certain level of labor & capital

example : one person (and only 1) needed to operate jack hammer

or a recipe for food items

6.4 Returns to Scale

firms must also consider factors of production (inputs) when they want to increase output

~~scale~~

could change scale of operation by increasing all inputs of production in proportion

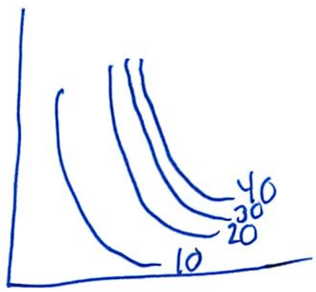
↳ what happens to output?

- return to scale → double input = double output ie: consulting
- increasing returns to scale - double input = more than double output
  - ie an assembly line
  - one large company better than many small company
- decreasing returns to scale - difficulties in running large operation actually produces less output than 2 small orgs together

(8)

Describing Returns to Scale

- ~~Does~~ not nessirally uniform ~~among~~ over all possible levels of output



↑ isoquants are moving closer togeter

- Varries greatly among firms + industries
- manufacturing more than service industries

c

Matt's help

2. $f(l,k) = 2 l^{1/4} k^{1/4}$

$$MRTS = \frac{\frac{1}{4} \cdot 2 l^{-3/4} k^{1/4}}{\frac{1}{4} \cdot 2 l^{1/4} k^{-3/4}} = \frac{k}{l}$$

Oh forgot that

The did MP_L, MP_K lot!

$$\frac{k}{l} = -\frac{w}{r}$$

← and he compared derived values - you can do that!

$$k = \frac{w}{r} l$$

← now can solve for one

- My big issue
- would never have figured out

$$f^4 = 16 l^2 \frac{w}{r}$$

$$TC = rk + wl = 2rl$$

↙ and now get l from it

what I did originally by plugging in k

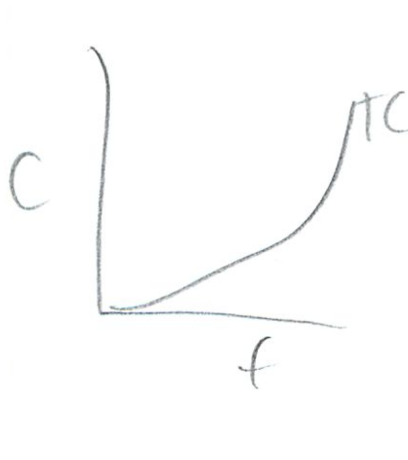
but in terms of f, ~~r~~ w only!

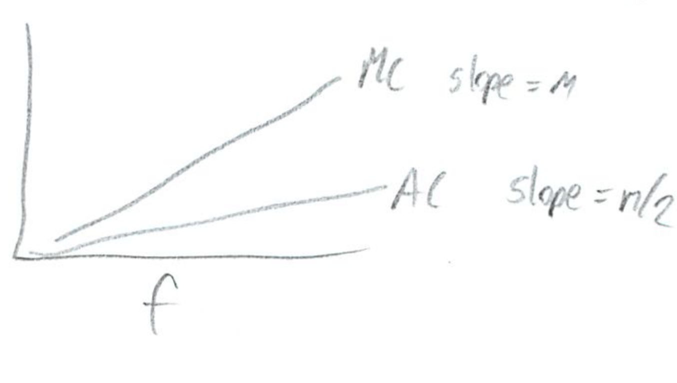
$$\frac{f^2}{4} \sqrt{\frac{r}{w}} = l$$

$$TC = \frac{1}{2} \sqrt{rw} f^2$$

$$\frac{\partial c}{\partial f} = \sqrt{rw} f = MC$$

$$\frac{c}{f} = \frac{1}{2} \sqrt{rw} f = AC$$





Q1
6

Cost

total
Cost $= wL + rk$

$$C = w\left(\frac{Q}{2k^{1/4}}\right)^4 + r\left(\frac{Q}{2L^{1/4}}\right)^4$$

Now replace k, L w/ Q

$$= \frac{wQ^4}{16k} + \frac{rQ^4}{16L}$$

→ try to get Q

$$= \frac{wQ^4L + rQ^4k}{16kL}$$

total C $= \dfrac{Q^4(wL + rk)}{16kL}$

$$AC = \frac{\frac{Q^4(wL+rk)}{16kL}}{Q}$$

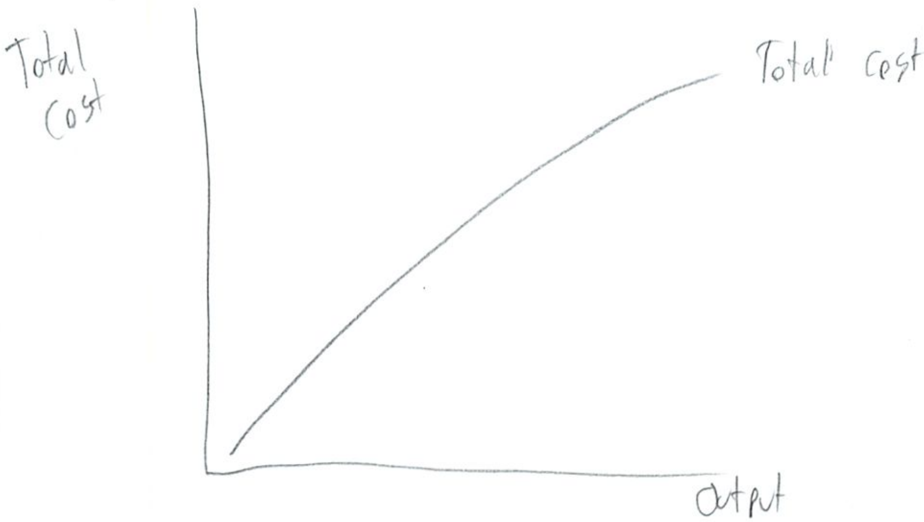
$$= \frac{Q^3(wL+rk)}{16kL}$$

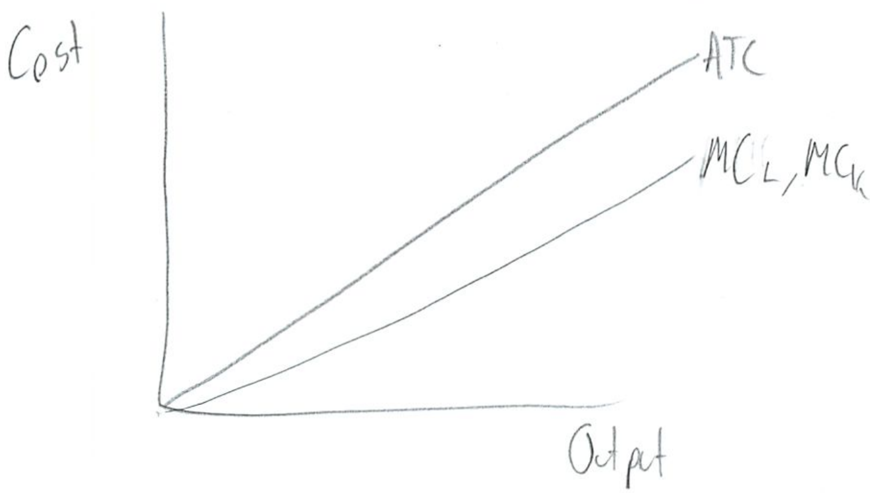
old 9

Cut

$$MC_L = \frac{\partial Q}{\partial L} = 2 \cdot \frac{1}{4} L^{-3/4} \cdot K^{1/4}$$

$$MC_K = \frac{\partial Q}{\partial K} = 2 \cdot \frac{1}{4} K^{-3/4} \cdot L^{1/4}$$





Chap 7 Cost of Production

10/11/12

~~Only doing stuff~~ not covered in class

now factoring price in to determine firm's production

all along isoquant is the same output

- now need to find the point that is least costly

7.1 Measuring Cost

We are interested in economic cost

- cost of utilizing resources in production
- ie the opportunity cost — future

not the accounting cost

- current/past cost for depreciation/tax purposes

Opportunity Cost

- cost of forgone/alternative uses of resources
- ie firm owns office building + pays no rent + morgage
  - but is opportunity cost of not ~~lending~~ renting it to someone else

Sunk Cost

- expenditure that has been made + can not be recovered
- always ignore ~~for~~ when making future economic decisions
- opportunity cost = 0 since no alternative
- just consider future costs from now

2

Fixed + Variable Costs

Fixed (FC)

- does not varry w/ output
- can only be eliminated by going out of business

- insurance
- management employees
  ↑some
-

Variable Cost (VC)

- varries as output varries
-

- raw materials
- factory labor

depends on time horizion

- Short - most costs are fixed
  contracts in place
- long (several years) most costs can be negiotated

Sunk + Fixed Costs are different

- but most people think of them the same
- we will too in most cases

Marginal + Avg Costs

Marginal / Incremental

- extra cost from 1 more output
- equal to the extra variable cost
- [fixed costs don't change here, remember]

$$MC = \frac{\Delta VC}{\Delta q} = \frac{\Delta TC}{\Delta q}$$

③ Avg Total Cost (ATC)

- aka avg economic cost
- $\frac{\text{firms total cost}}{\text{quantity}}$
- per unit cost of production
- made up of Avg Fixed Cost and Avg Variable Cost

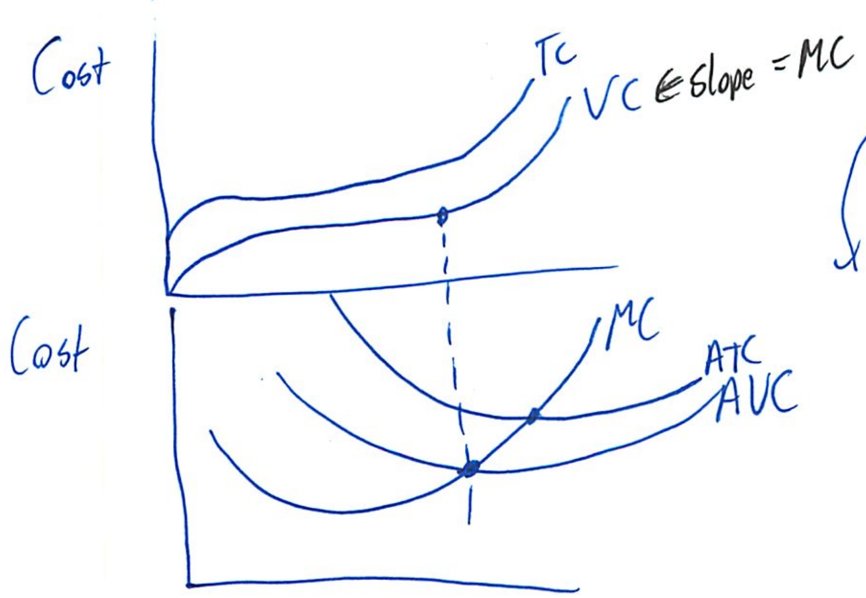
  AFC: $\frac{FC}{q}$ AVC: $\frac{VC}{q}$

7.2 Costs in Short Run

- remember costs change depending on time period
  - ↳ what is fixed and what is variable
- short term: only labor can change
  not capital
- and remember decreasing marginal return on the labor

$$MC = \frac{\Delta VC}{\Delta q} = \frac{W \Delta L}{\Delta q} = \frac{W}{MP_L}$$

? only labor matters



= inflection in VC

MC = minimum AVC

MC = min ATC

(4)

total costs are a flow } q per year
outputs are a flow }

avg/MC are in $/unit

C = Cost

AC = ATC

7.3 Costs in the Long Run

- firm can change all inputs
- treat capital always as if it was rented
  - ↳ since costs are usually amortized "economic depreciation"
  - allows us to think a year at a time
- if did not buy item could collect interest on the cash
  - ↳ User cost of capital - economic depreciation

User Cost of Capital = Economic Depreciation + Interest Rate × Value of Capital

rate! r = Depreciation rate + interest rate

Cost Minimizing Input Choice

- how to select inputs to minimize cost at given outputs
- depends on price of inputs

(5)

Price of Capital r

- large initial expenditures
- including on specialized equipment which would be considered "sunk"
- flow: dollars per year
  - ↳ the user cost of capital

Rental Rate of Capital

- ↳ cost/year for renting a unit of capital
- should = the user cost
- since they could rent what they own out instead of using it themselves

---

Iso Cost line

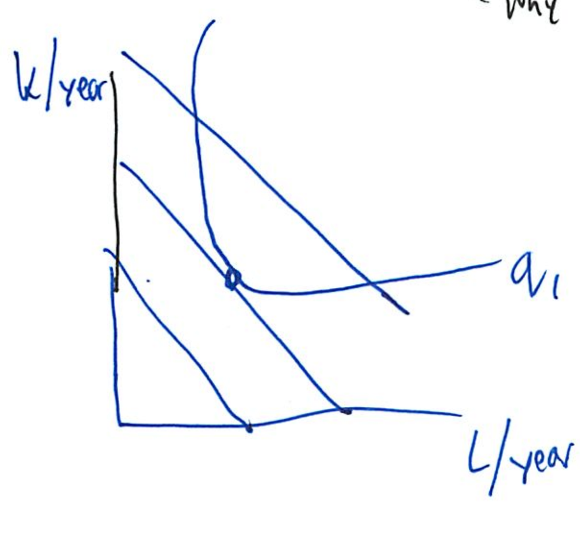
- all possible combos of the input purchasable at same cost

$$C = wL + rK$$

$$K = \frac{C}{r} - (w/r)L$$

← rewrite to get eq of straight line

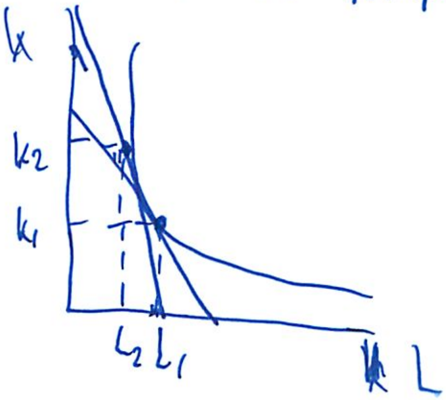
- why do I never think of things in terms of that!



← want to produce at q_1
want isoquant to minimize total cost

(6)

When cost of an input changes



change mix of ~~required~~ desired capital + labor

$$MRTS = -\frac{\Delta K}{\Delta L} = \frac{MP_L}{MP_K} = \frac{W}{r}$$

$$\frac{MP_L}{W} = \frac{MP_K}{r}$$

? add. output from spending an extra $ for labor

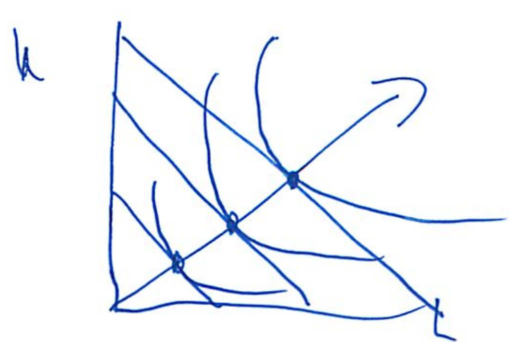
Want them to be =

need practice w/ the Ets

Cost Minimization w/ Varying Output levels

- lets see how costs depend on output level
- Give a input cost

  $C = 10L + 20K$
- can draw multiple isocost lines
- point of tagency w/ max units that can be produced at that cost



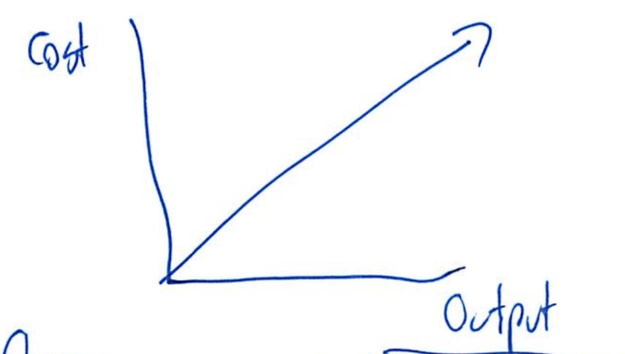
(7)

line though all the pts of tangency
= expansion ~~curve~~ path
= Combo of labor + capital that firm will choose to minimize cost at each output level

Slope $\frac{\Delta k}{\Delta L}$

Expansion Path and Long Run Costs

expansion path = long run cost curve



1. Choose an output level represented by isoquant
   Then find tangency isoquant to isocost
2. From given isocost find minimum cost of producing that output level
3. Graph

* minimum long run cost of producing each level of output

Straight line = Constant economies of scale

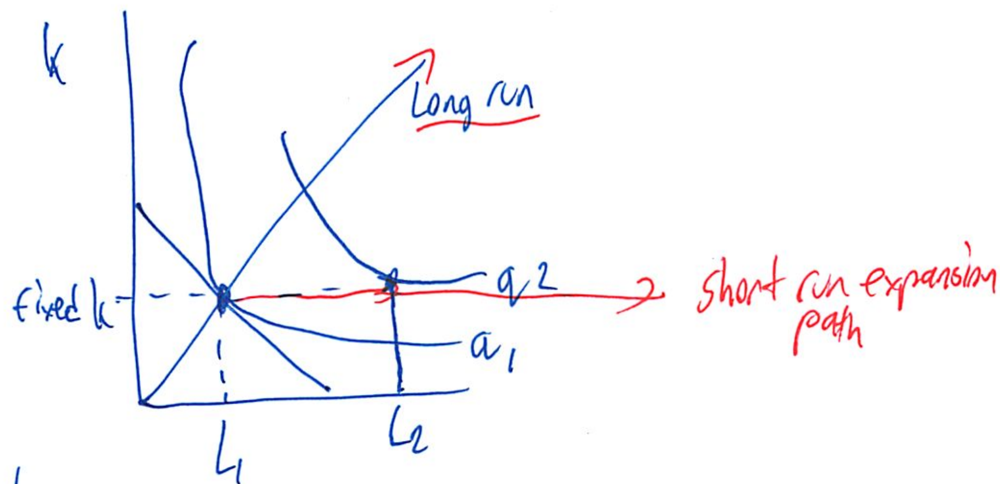
8

7.4 Long Run vs Short Run Cost Curves

Short run U
long run U or ↗

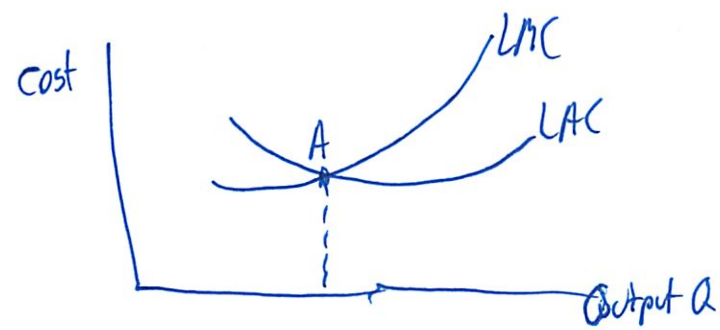
Why different shapes?
inflexibility of short run production

- lower costs in long run since flexibility



Long run avg costs

Can ↓ capital to ↓ costs in long run



Shape ~~LR~~ LMC + LAC based on scale + inputs

if constant scale → LAC would be flat
increasing returns to scale → LAC falls
decreasing " " " " rises

Shape Long run Avg cost curves based on scale
- not diminishing marginal returns on inputs

9

LMC can be determined from LAC
↳ measures change in LTC as output ↑

LMC below LAC when LAC falling
" above " " " rising
* " intercect " when LAC minimum *

When LAC is constant they are =
^LAC ∩ LMC

10/15

Economies + Diseconomies of Scale

- average cost of production likely to decline to a point
- because
  1. Workers can specialize as more workers are added.
  2. More flexibility
  3. Purchasing power on inputs
  4. (not in the book) Tech ~~stu~~ has a very low marginal cost
     - email server costs same 100 worker, 5,000 workers
     - or a little more
- but only ~~effect~~ economies of scale up to a point
  1. In short run factory space makes it more difficult to get work done (this seems weird)
  2. Managing firm gets more complex and inefficent.
     (this I think is good)

10

3. Bulk purchasing up to a limit, Plus may be shortage of key inputs, push prices on getting more of them up.

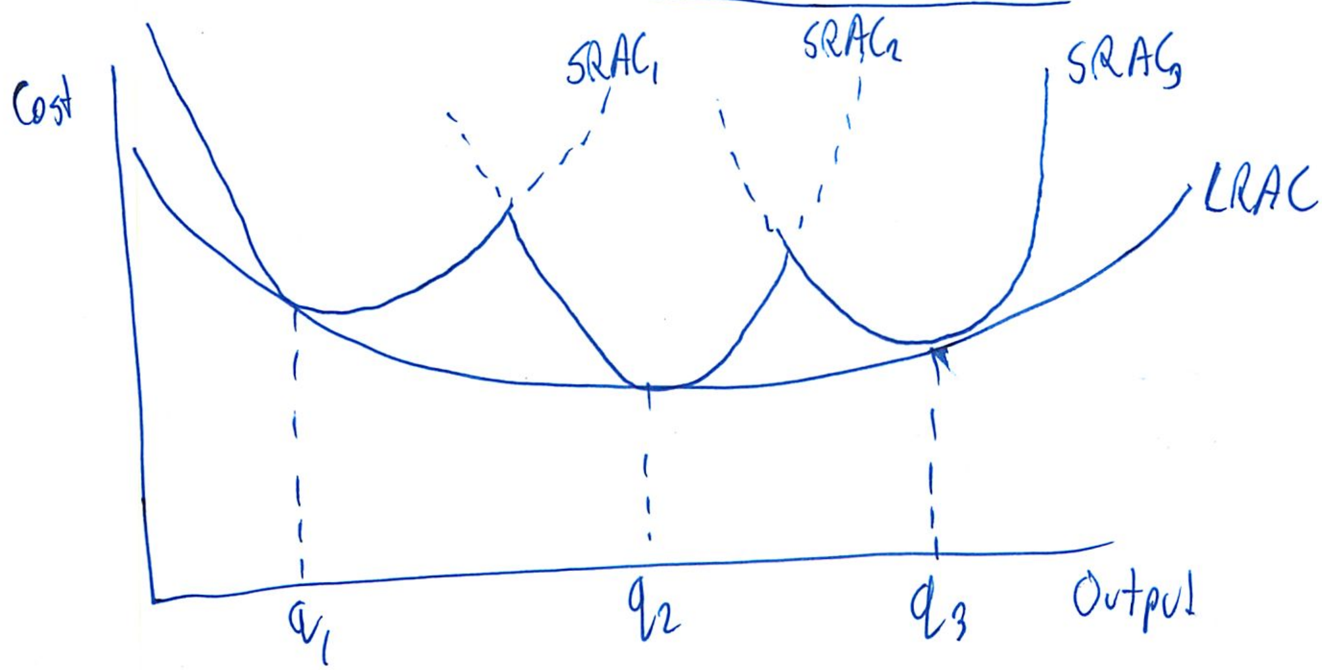
So expansion price is no longer a straight line

- oh only if input proportions change
- then can't talk about economies of scale

economies of scale - doubling output for less than twice cost

$$E_c = \frac{\Delta C}{C} / \frac{\Delta Q}{Q} = \frac{\frac{\Delta C}{C}}{\frac{\Delta Q}{Q}} = \frac{MC}{AC}$$

Relationship Between Short-Run and Long-Run Cost



3 short run costs. Can only move between them in the long run. Firm follows solid SRAC lines which averages to LRAC - but can only do the steps

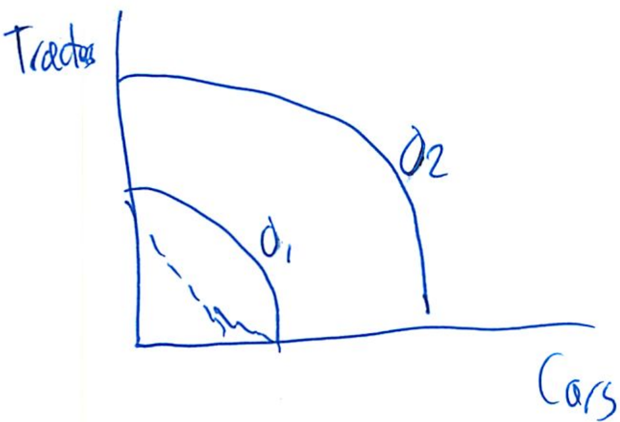
Challenge is to know when to switch from SR_1 to SR_2
at each SR it will produce at minimum point
So firm's will jump when it opens a new factory
output

(skipping some minor details like LRAC)

7.5 Production w/ 2 Outputs Economies of Scope

- many firms produce more than 1 product w/ similar inputs (or not)
- chicken farm can do eggs or chicken meat
- either linked like that or joint marketing advantages or factories that could be used for something else

Product Transformation Curve



negitive slope (must trade off 1 good for other)

O_2 is doubling inputs - which doubles output
So constant returns to scale
for both products

dotted line would be producing one or other would not be a gain or a loss.

But usually bowed outward since company has advantage in producing both.
- ie shared management

12

Economies + Diseconomies of Scope

- economies of scope - joint output of a single firm is greater than the output that can be achieve by two different firms each producing a single product (with its equivilant production inputs allocated to these seperate firms)
- no direct relationship b/w economies of scale and economies of scope
- degree of ~~econom~~ economies of scope

$$SC = \frac{C(q_1) + C(q_2) - C(q_1 q_2)}{C(q_1 q_2)}$$

(looks like (e.041 probablity)

$C(q_1 q_2)$ = joint cost

- w/ economies of scope < $C(q_1 + q_2)$
  - less than sum of individual costs

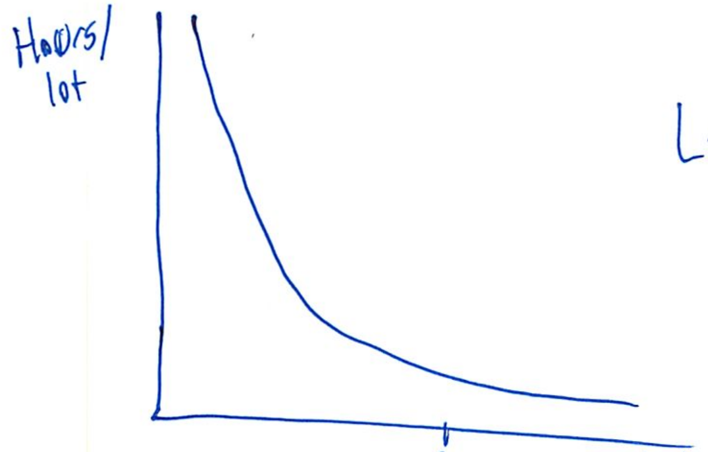
greater the SC, the more economies of scope

SC<0 (ie ⊖) means diseconomies of scope

13 7.6 Dynamic Changes in Cost: The Learning Curve

- as firm gets older, management + labor become better at producing same output more efficently
  - in addition to all the other effects I've talked about
1. Workers get more adept w/ experience
2. Management gets better at scheduling
3. Engineers get more daring in designing stuff
4. Suppliers know more about their business

learning curve

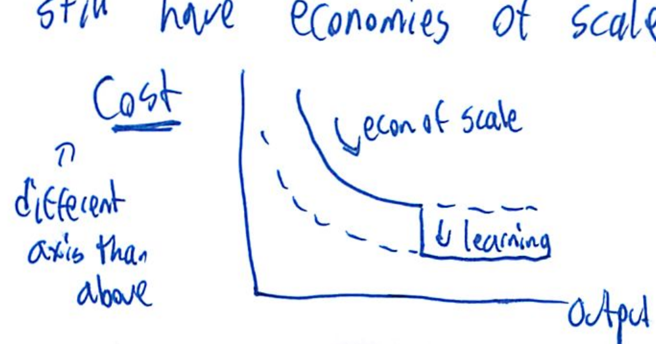


$$L = A + BN^{-\beta}$$

N = # units
A, B, β constants

(did not write about values for A, B, β)

- here once firm gets passed 20 outputs effect disappears
- can still have economies of scale



14

7.7 Estimating + Predicting Cost

must predict how cost will change as output changes

- called a cost function

- need relationship b/w variable cost + output
  - [usually do a ~~least~~ least square relationship to data]

$VC = \beta q$

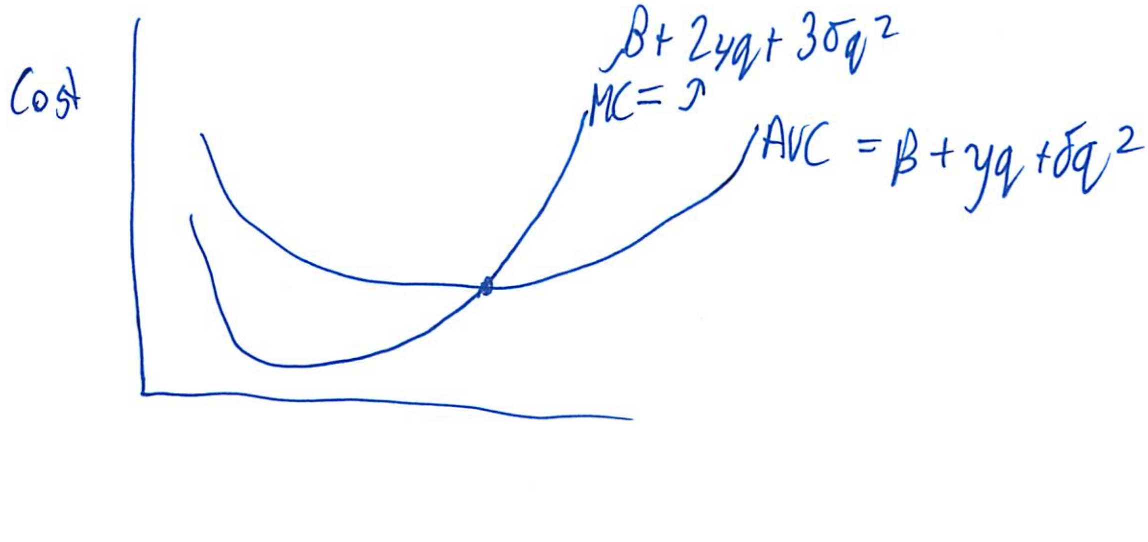
- easy to use
- linear
- only if MC is constant

$VC = \beta q + \gamma q^2$

- quadratic
- U shaped
- relates variable cost to output + output squared

$VC = \beta q + \gamma q^2 + \delta q^3$

- cubic
- U shaped marginal and average cost curves



(101) (15)

Can be difficult to measure in real life

- produce different models
- Cost data from accounting data (no opp. cost)
- how to allocate overhead

Scale economies index

(why is this here
- book disorganized)

- if scale of economies

$$SCI = 1 - E_c$$

Lecture 10 Competition

Level of outcome is not predetermined
- must adjust due to market
- Its as if Consumer's income level would not be predetermined

Perfect Competition

- when firms are price takers on output + input markets
- no action they take can affect this ↗
- when: perfectly elastic demand for goods
  perfectly elastic supply of inputs

4 conditions

1. Identical Products
   - percieved by consumers to be identical
2. Consumers need full info on all prices
3. Low/no transaction/shopping costs
   - consumers go to cheapest one
4. Free entry + exit of the market for firms

- ie: open air markets
- in reality no market perfectly competitive
- Peter Diamond won prize for search costs
  - buyers + sellers pay to find each other
  - Job market
  - both vacancies + unemployed people at same time
  - since charastics don't match

2

Friction leads to natural rate of unemployment
~~twor~~ - changes over time
~4% ~7% ~How much is it today??
- big question

But in our perfect competion - Peter Diamond does not exist

Firm vs Market Demand

Even if a given firm's demand is elastic
Does not imply ~~a~~ demand of whole market is elastic

ie every ~~marke~~ ~~#~~ ~~a~~ torist-crap seller competes on price
but is a total demand of everyone for
torist-crap

Residual demand

$D(p)$

$D^r(p) = D(p) - S^o(p)$

$$\frac{dD^r}{dp} = \frac{dD}{dp} - \frac{dS^o}{dp}$$

$\ominus\ominus$ $\ominus$ $\oplus$

very negitive #

firm very dependent on price

check on this

(3)

Assume

$q = \frac{Q}{n}$ ← all firms same

no # of firms

Then

$Q^0 = (n-1)q$

$\varepsilon_i = n\varepsilon - (n-1)\eta$

market elasticity of demand

elasticity of supply

So if $\varepsilon = -1$, $\eta = 1$ } $\varepsilon_i = -199$

Profit Maximization

- every decision producers make is to maximize profits in short run

- what are profits? revenue - cost

↳ economic cost → opportunity cost - what could have done w/ cash

- value of your time included - what you make elsewhere

accounting cost

↳ cashflow costs

- what lay out in cash

π = profits

$\pi(q) = R(q) - C(q)$

$\frac{dR}{dq} = \frac{dC}{dq}$ ← firm choose q

so marginal revenue = marginal cost

④

- in competitive market $MR = price$
  handed to them

so $\boxed{P = MC}$
   ↳ profit maximizing condition

Figure 10-2A

* are in short run
  - no firm entry and exit
  - ignore condition #4
  - fixed costs are sunk
    - can't leave
    - unless you ~~at~~ are losing so much $ can not cover fixed costs you already paid

---

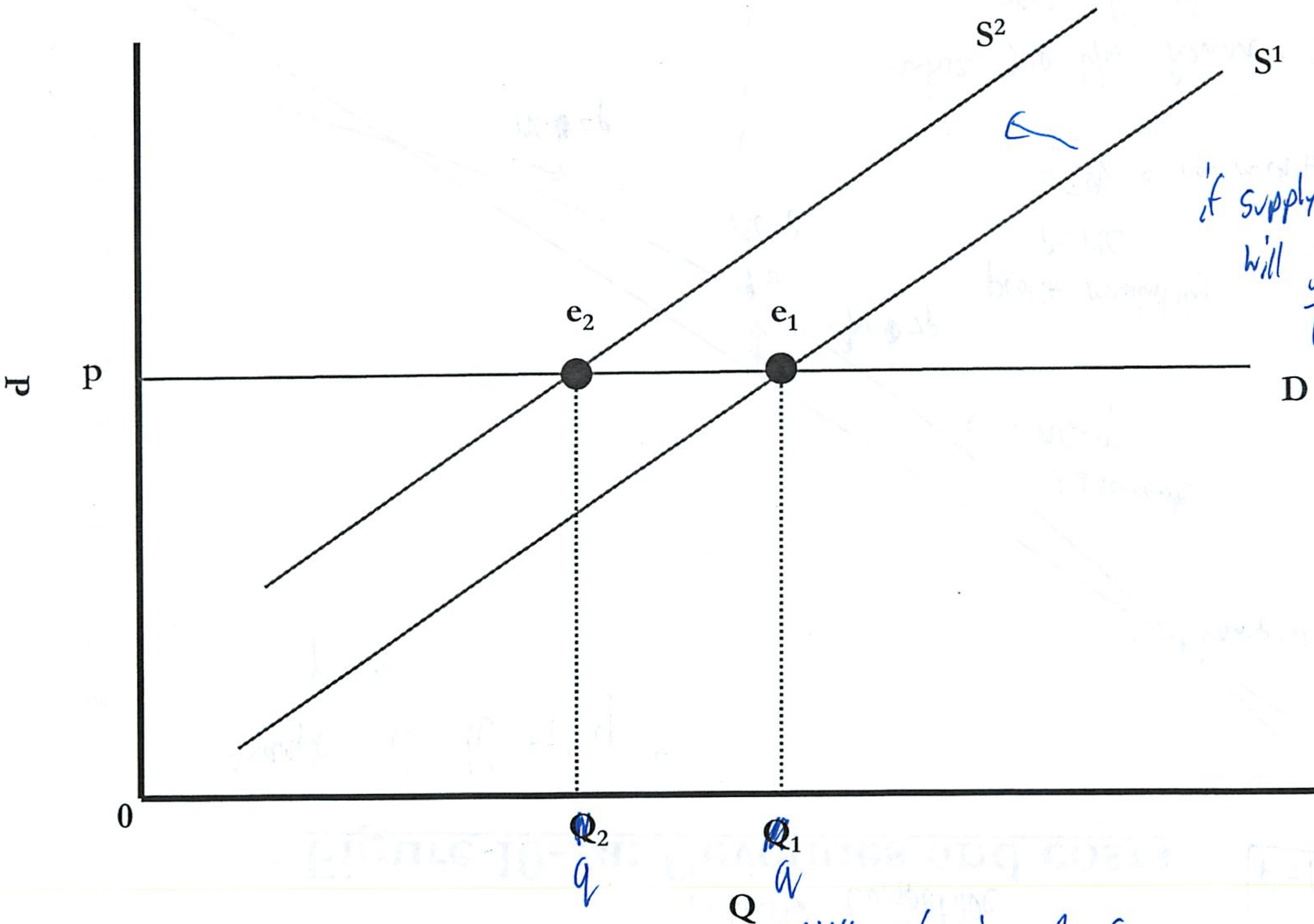
<u>long run</u>

- can also decide to shut the firm down
- if it losses $ by contining to produce
- can decide not to sink more fixed cost

10/13

Lecture 10 Competition

Figure 10-1: Supply shift with perfectly elastic demand



if supply ↓, then they will just reduce Q. They can't change price

little q's since 1 firm - not the market

Perfectly Competitive

Figure 10-2a: Revenues and costs

$\boxed{P = MC}$

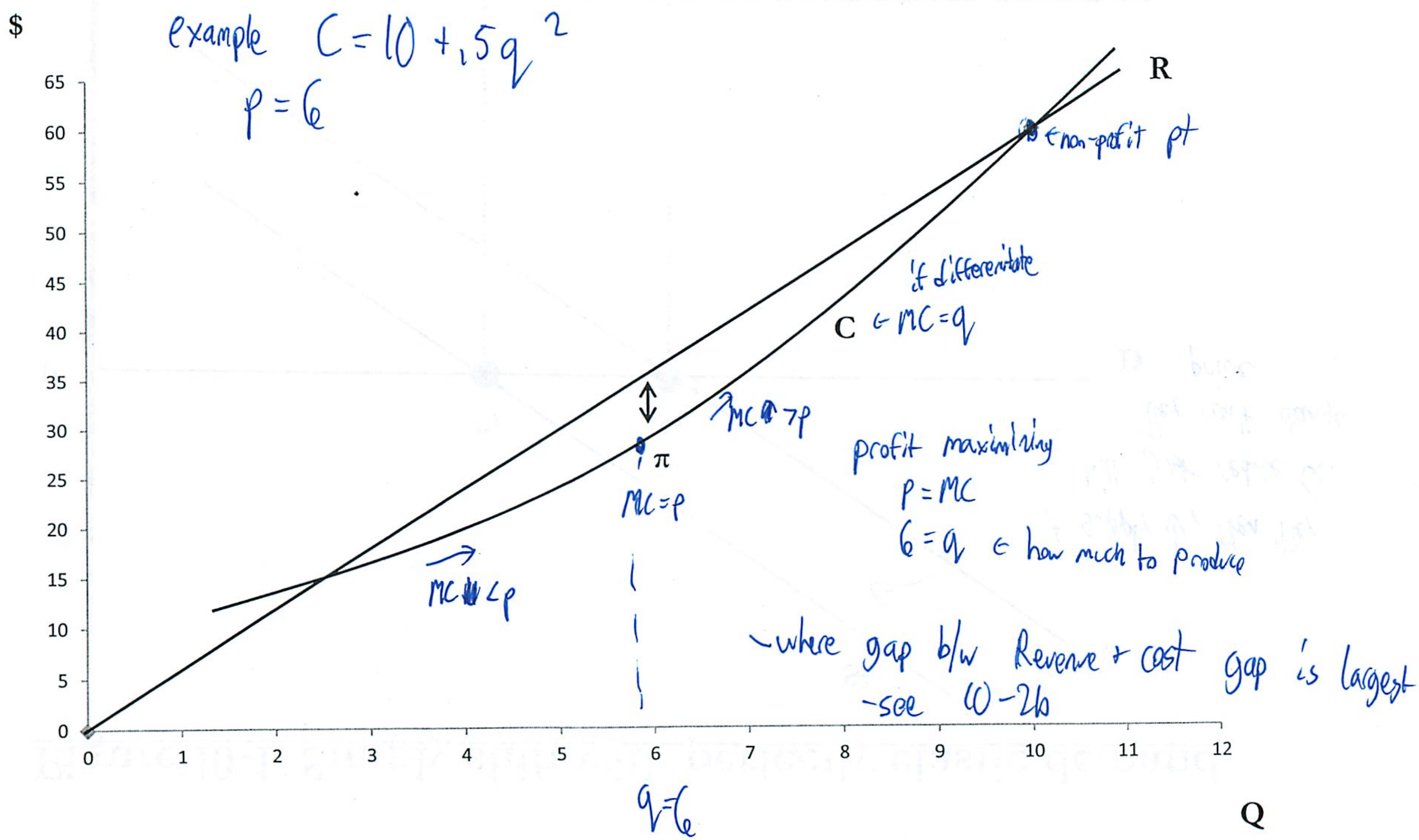
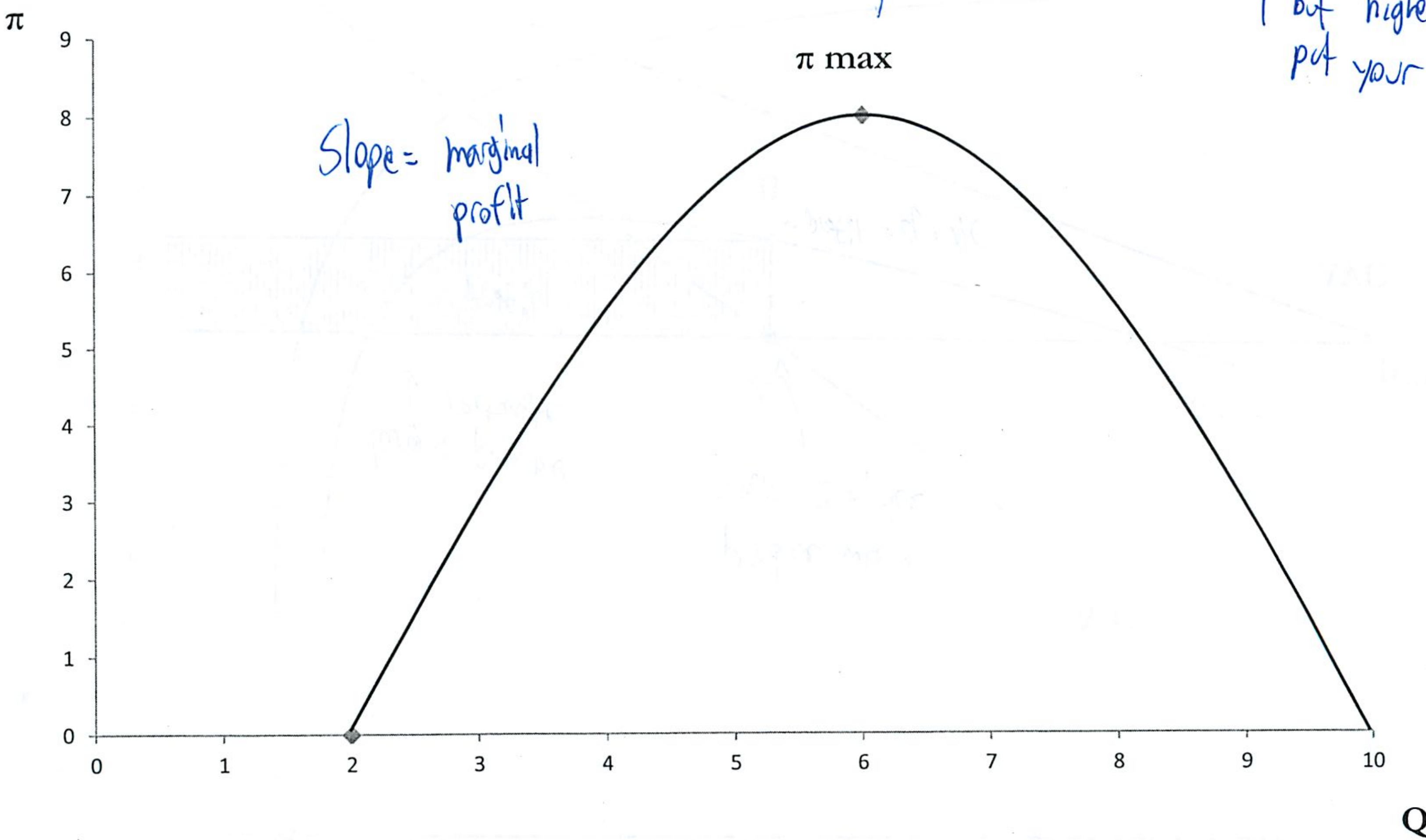


Figure 10-2b: Profit maximization

you do make ~~a~~ more profit here

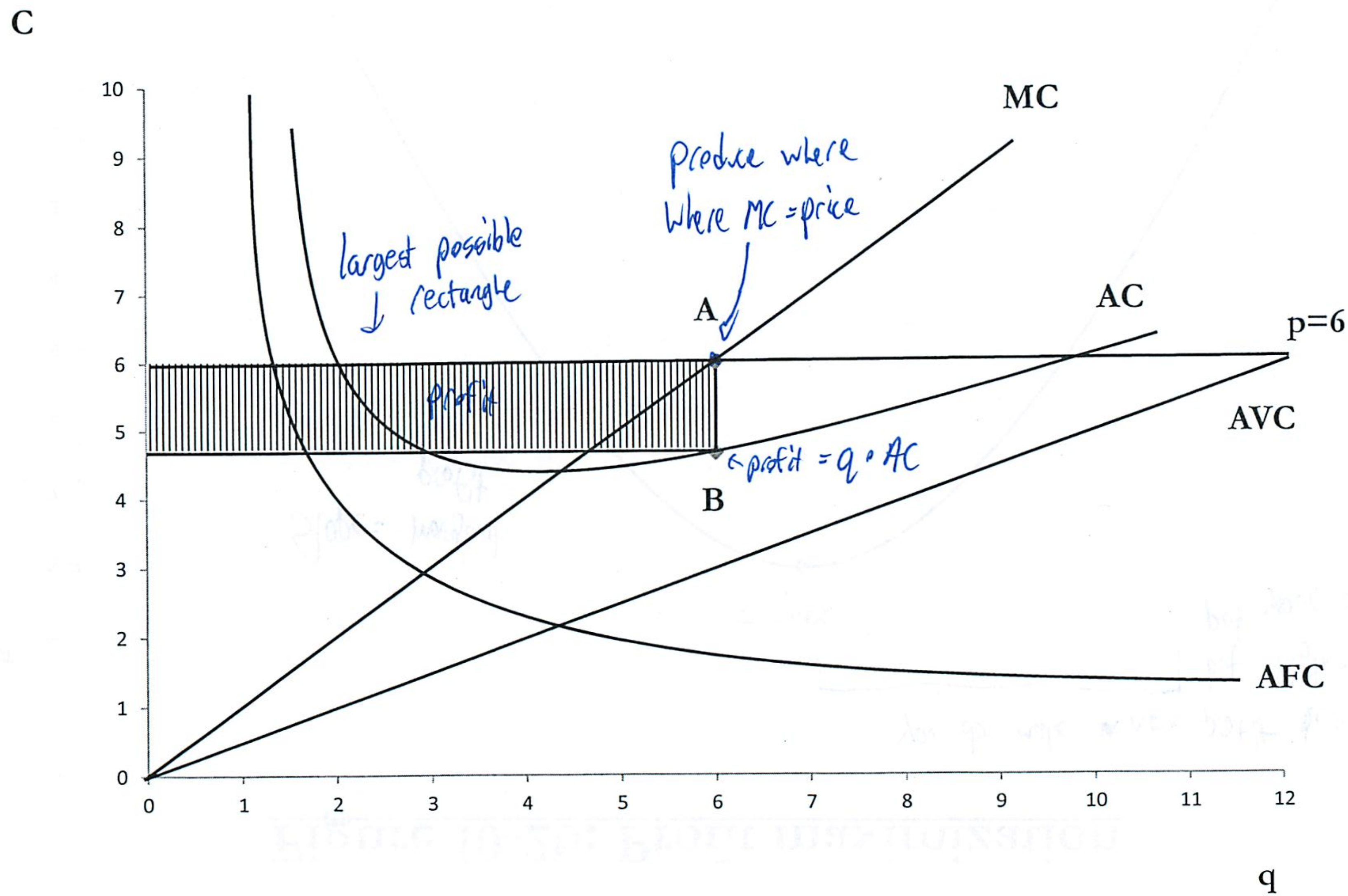
but higher opportunity cost
put your $ elsewhere



Slope = marginal profit

Perfectly Competive

Figure 10-3: Cost curves for C=10+.5q^2



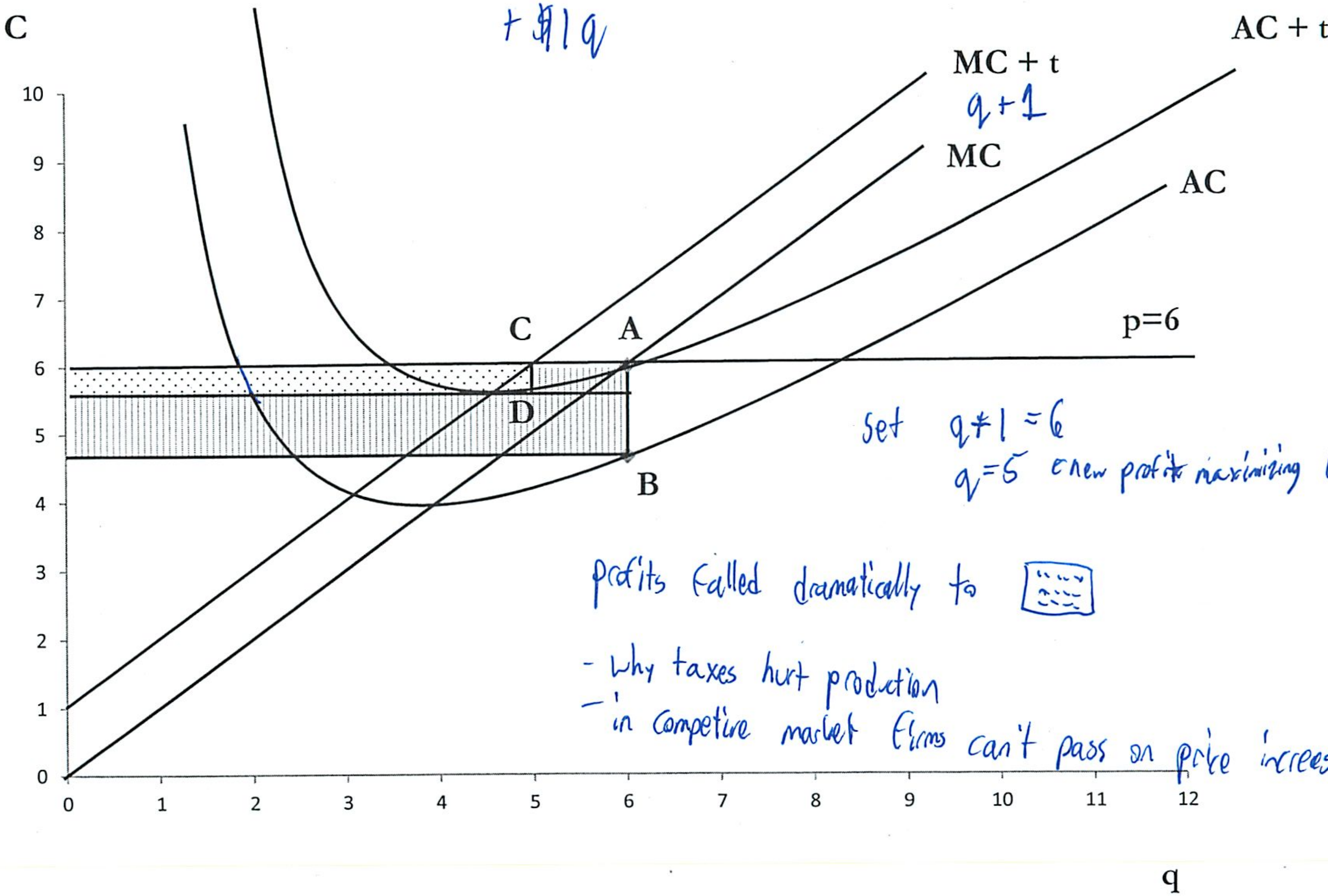
Perfectly Competive Cost shock

Figure 10-4: Cost curves with tax added

$C = 10 + .5q^2 + tq$

$+ \$1q$

\$1 on every unit produced



$q + 1$

set $q + 1 = 6$

$q = 5$ ← new profit maximizing level

profits falled dramatically to

- why taxes hurt production
- in competive market firms can't pass on price increase

Recitation

10/14

Economies of scale / Returns

- constant returns to scale

$$Q = f(L, k)$$

$$2Q = f(2L, 2k) = 2f(L, k)$$

neither efficient or inefficent

- increasing returns to scale

$$f(2L, 2k) > 2f(L, k)$$

- decreasing returns to scale

$$f(2L, 2k) < 2f(L, k)$$

Cost Function

$C(Q)$

long run

If economies of scale

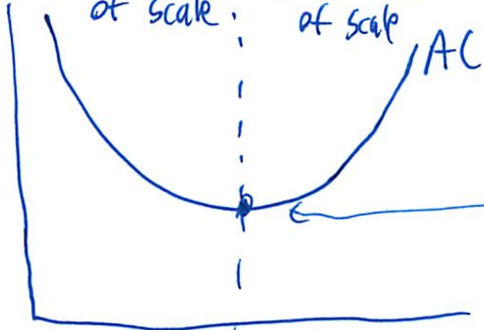
$$C(2Q) < 2C(Q) \rightarrow AC(Q) \downarrow$$

dis economies of scale

$$C(2Q) > 2C(Q) \rightarrow AC(Q) \uparrow$$

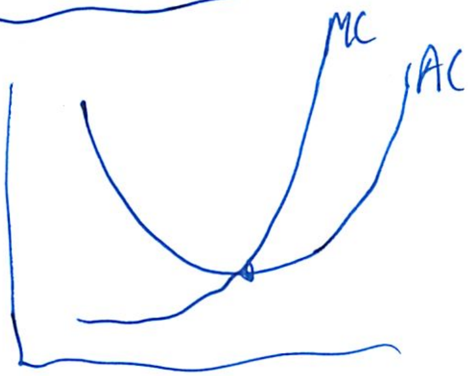
2

increasing returns to scale	decreasing returns to scale	production
economies of scale	diseconomies of scale	cost



minimum efficent scale
- in long run
- free entry + exit
- variable inputs

efficent to ↑ production   efficent to ↓ production



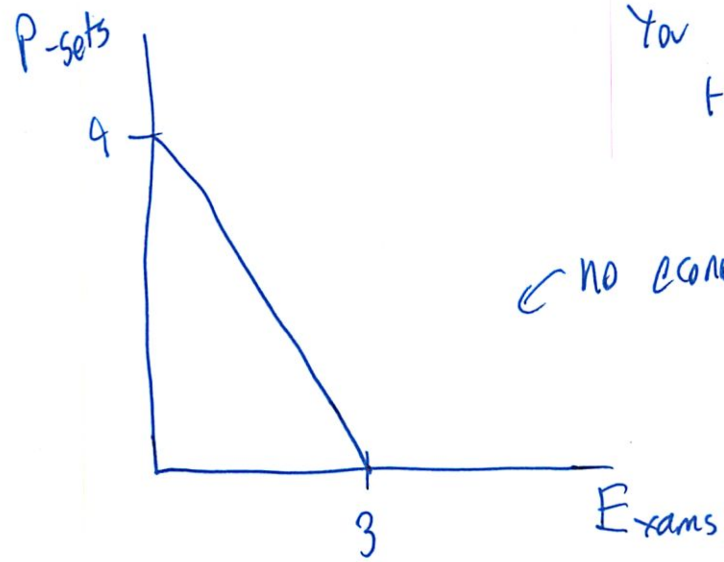
at best pt MC = AC

maximize revene
MC = MR
competitive market
price = that P = MC = MR

Economies of Scope

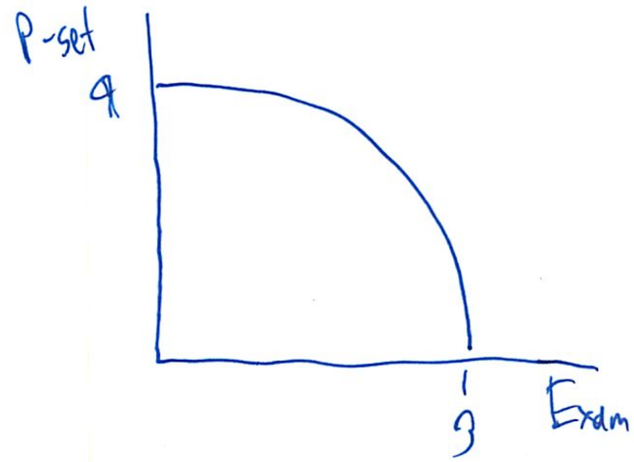
Firm can produce more than 1 type of good
- w/ same inputs

(3) Student in 1401



You have 1 unit of labor
How combine?

← no economies of scale



But if you do P-set,
You will do better on exam
- have economies of scope

$$C(q_1,0) + (0,q_2) > C(q_1,q_2)$$

producing goods at same time is better than doing one or other

④ PSet 4

#2, 3, 4 were same problem
- will do one

2. $Q = f(L, k) = 2\,l^{1/4} k^{1/4}$

- Find long run total, average cost
- "conditional demand for labor and capital"
- w, r solve for letters

$C = wL + rk$

conditional demand based on how much output

$f(2L, 2k) = 2\,(2l)^{1/4}(2k^{1/4}) = \sqrt{2}\; 2\,l^{1/4} k^{1/4} < 2f(L,k)$

So decreasing ~~returns~~ returns ~~to~~ to scale

$MRTS = \dfrac{-dk}{dL} = \dfrac{-w}{r} = \dfrac{-MU_L}{MU_k}$

$\dfrac{MP_L}{MP_k} = \dfrac{w}{r}$ ← optimal demand for labor and capital

5)

$$MP_L = \frac{1}{4} \cdot 2 \cdot L^{-3/4} K^{1/4}$$

$$MP_K = \frac{1}{4} \cdot 2 \cdot L^{1/4} K^{-3/4}$$

star indicates optimal demand

$$MRTS = \frac{MP_L}{MP_K} = \frac{K^*}{L^*} = \frac{w}{r}$$

want

$$L^*(Q, r, w) \qquad K^*(Q, r, w)$$

$$Q = 2L^{*\,1/4} K^{*\,1/4}$$

$$K^* = \frac{w}{r} L^*$$

$$Q = 2L^{*\,1/4} \left(\frac{w}{r} L^*\right)^{1/4}$$

$$= 2L^{*\,1/2} \left(\frac{w}{r}\right)^{1/4}$$

$$L^*(Q, r, w) = \frac{Q^2}{4\left(\frac{w}{r}\right)^{1/2}} \qquad K^*(Q, r, w) = \frac{w}{r} \frac{Q^2}{4\left(\frac{w}{r}\right)^{1/2}}$$

$$= \left(\frac{w}{r}\right)^{1/2} \frac{Q^2}{4}$$

Plug back in

$$C(Q, r, w) = r \cdot K^* + wL^*$$

$$= r\left(\frac{w}{r}\right)^{1/2} \frac{Q^2}{4} + w \frac{Q^2}{4\left(\frac{w}{r}\right)^{1/2}}$$

$$= \text{Simplify} = (rw)^{1/2} \cdot \frac{Q^2}{2}$$

$$MC = \text{add. cost} = \frac{\partial C}{\partial Q} = (rw)^{1/2} Q$$

(e) $AC = \frac{C}{Q} = (r \cdot w)^{1/2} = \frac{Q}{2}$

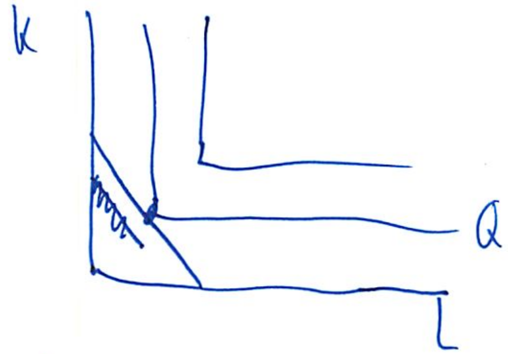
Economies of scale?

$AC(Q) \downarrow$ ~~diseconomies~~ economies

$AC(Q) \uparrow$ diseconomies ~~← increasing function of output~~

---

4. $Q = \min\{L, K\}$



MRTS is undefined

But much easier since always producing at the kink

$L^* = K^*$

$Q = L^*$

Chap 8 Profit Maximization + Competitive Supply

10/16

A cost curve gave the minimum cost to produce at each level of output

$\boxed{\text{But what level of output should be produced at??}}$

this chap only perfectly competive markets

- easy entry + exit and switching b/w suppliers
- all same product (commodities) - homogeneous
- each producer small relative to the size of the market.
  - ie <u>price taking</u> - firm takes price as given
  - consumers too must take the listed price
  - no quantity discounts
  - ~~do~~ no individual ~~$~~ player can affect price

When is a market highly competitive?

- agriculture
- aircrafts } not
- drugs
  - lots of R+D costs
  - products not perfect sustitutes (drugs esp)
  - high entry + exit costs
- firm must be able to break into market
- and sell assets + leave if losing $
- more if demand is elastic
- but no single factor

② 8.2 Profit Maximization

- firms intend to maximize profit
- ~~assum~~ firms act rationnaly

common assumption
but is it actually true?

- textbook says
  - small firms whose owners own them think about it
  - large firms not in touch w/ their shareholders
- I think its the exact opposet
  - Small firm owner may chose to take a day off
    - chosing leasure over profits
  - large firms see relentless quarterly earnings pressure
- oh book gets to that
  - large firms tend to maximize market value of firm
    - not nessarrly long term profits, which is in firm's best interst
  - and in recent years that may have been factor
    - Dot.com
    - lots of Bankrupcies
    - large CEO pay checks
    - board should replace bad managers then

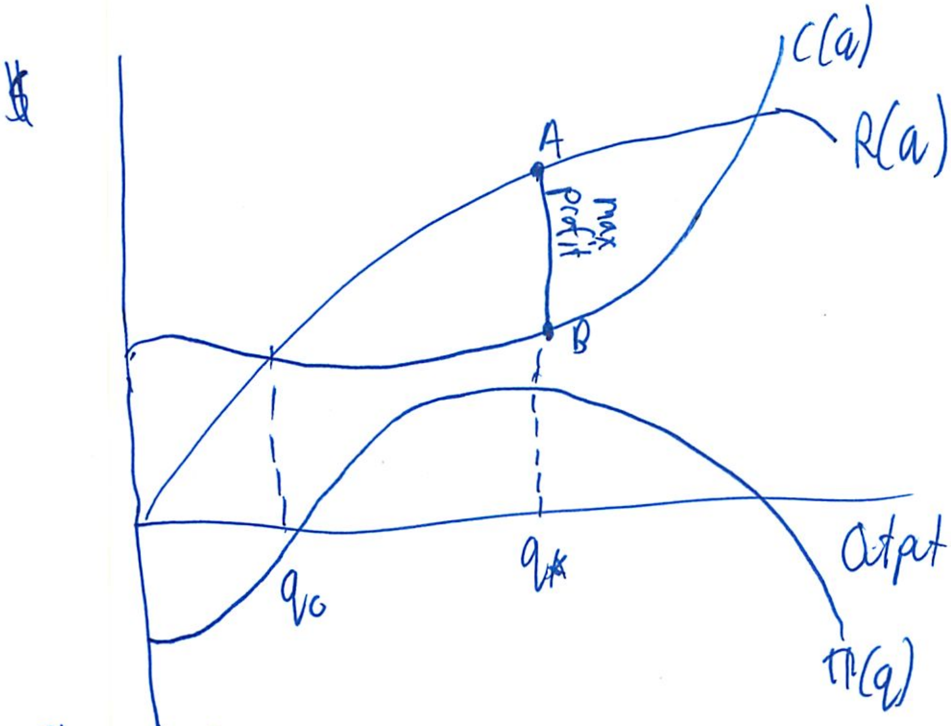
↓

③ # 8.3 Marginal Revenue, Marginal Cost, Profit Maximization

profit = total revenue − total cost

↑ π ← (why in all world does π = profits)

$\pi(q) = R(q) - C(q)$



(I like this graph)

Slope of revenue =

Marginal revenue = ~~cost~~ change in revenue from 1 unit ↑ in output

$C(q)$ some value when $q = 0$ because of fixed cost

$q*$ ~~[illegible]~~ $MC(q) = MR(q)$

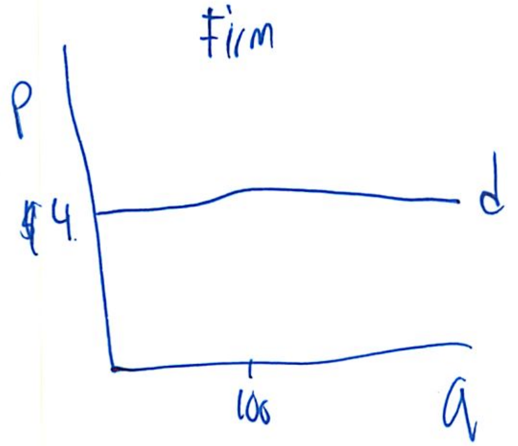
- above costs rise faster than revenues

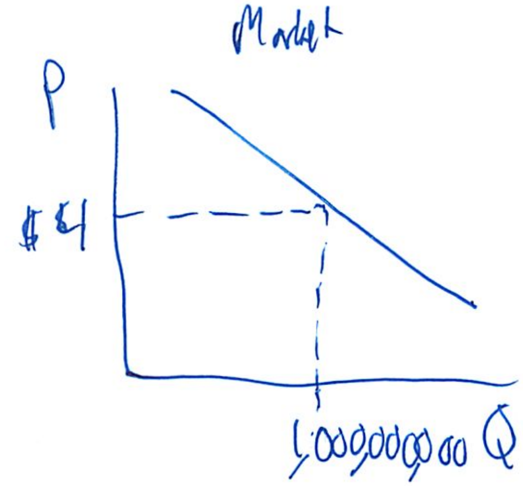
$$\frac{\Delta \pi}{\Delta q} = \underset{\uparrow \atop MR}{\frac{\Delta R}{\Delta q}} - \underset{\uparrow \atop MC}{\frac{\Delta C}{\Delta q}} = 0$$

(4) # Demand and Marginal Reveme for a Competive Firm

- how much output the firm decides to sell will have no effect on the market price of the product
  - ↳ since price taker
  - price fixed

notation	
	Q, D = market
	q d = firm





Marginal + Avg revenue constant

* demand = ~~margin~~ price = marginal revenue = avg revenue *

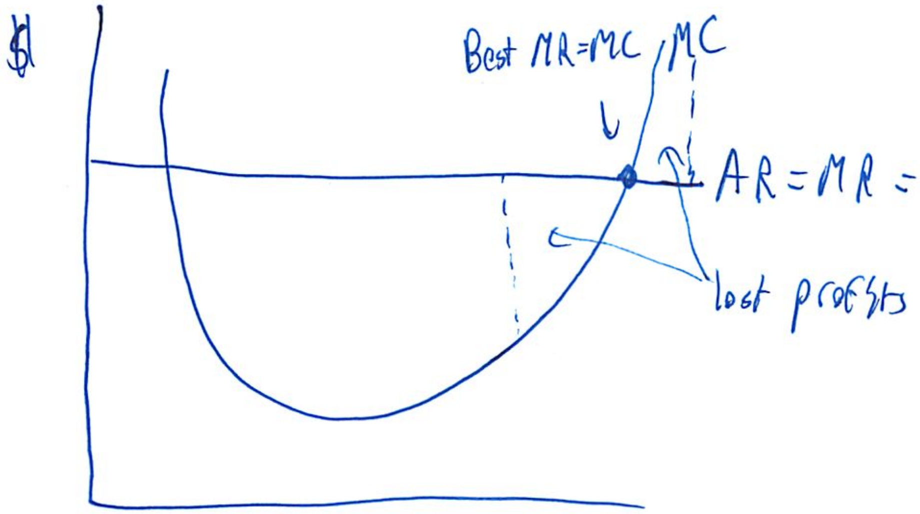
Profit Maximization by a Competive Firm

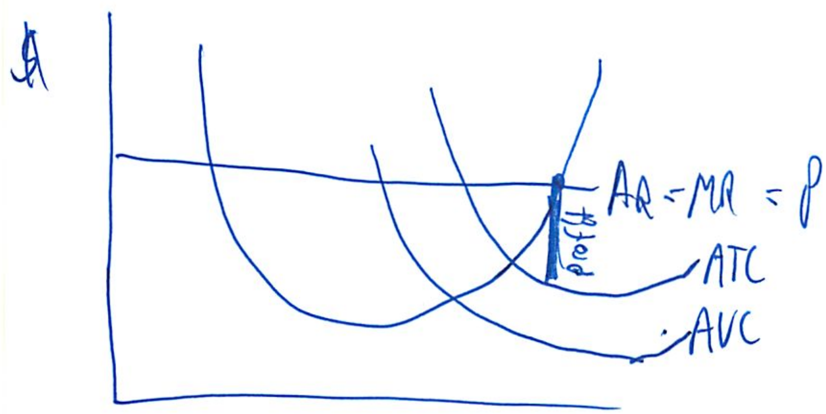
$$MC(q) = MR = P$$

⑤

8.4 Choosing Output in Short Run

Short-Run Profit Maximization by a Competive firm





may not be drawn best

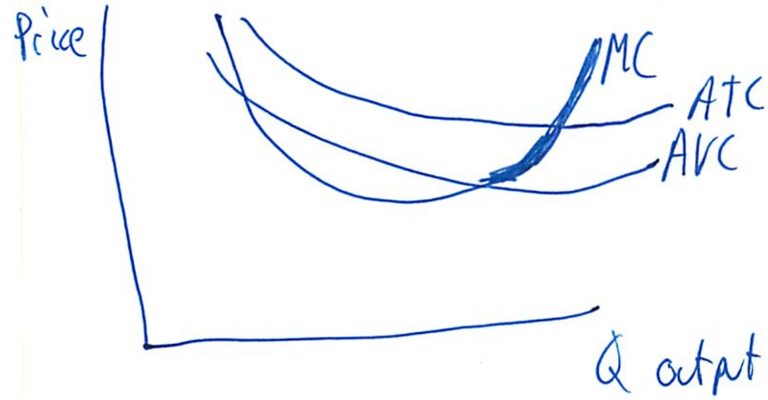
* MR=MC where MC is rising

Short Run profits of a competive firm

If price < avg. economic AVC cost of production would be in best interest to shutdown
- may make more $ later
- or can't shut down due to labor, etc issues
  - must pay those costs even it shuts down
- so produces ~~even~~ if price > AVC

⑥ 8.5 Competive Firm's Short Run Supply Curve

- supply curve for a firm tells us how much output it will produce at every possible price
- * the portion of the firm's marginal cost curve which is greater than avg economic cost / AVC

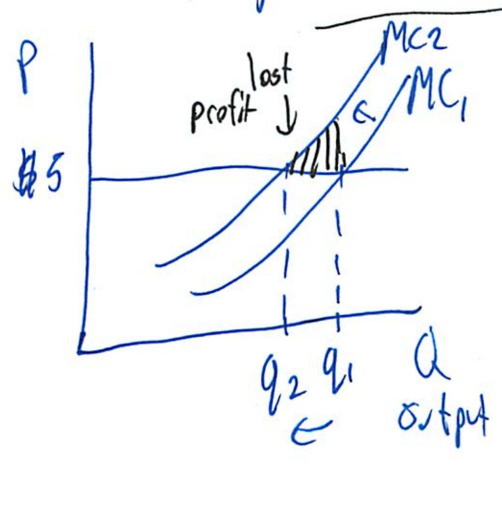


- remember slope upwards, since MC ↑

So increase in the market price will get firms to ↑ production

Firm's Response to an Input Price Change

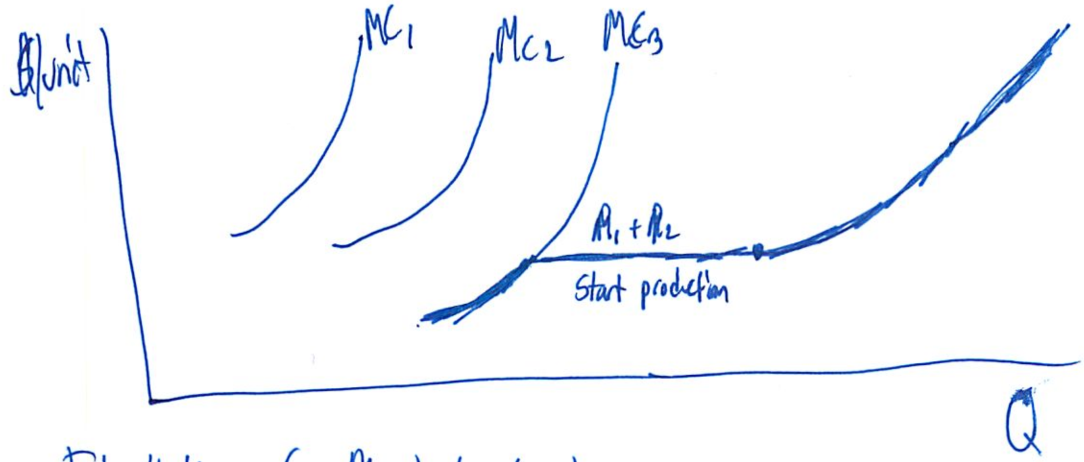
- firm changes output so MC = price
- often product market price changes same time price of inputs change



if don't ↓ production

7 8.6 Short Run Market Supply Curve

- amt of output industry will produce at each possible price
- sum of quanties supplied by each individual firm



Elasticity of Market Supply

- actually not as simple as adding individual supply curves
- since as prices rise firms try to ↑ output
- the added output increases demand for input
- possibly jacking up input prices
- and thus shifting MC inward

$$E_s = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}}$$

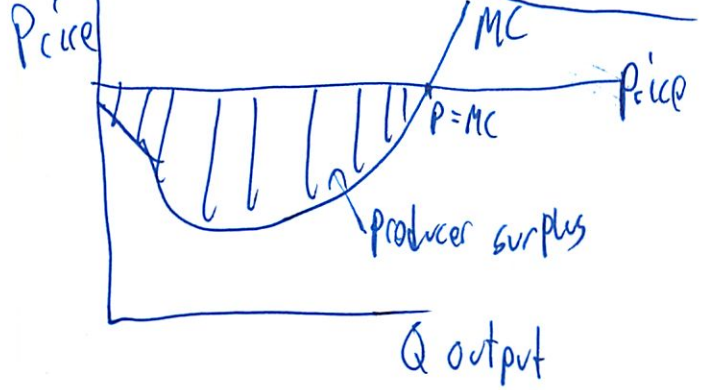
% change in Q
based on a 1% change in price

- perfectly inelastic supply - new factories must be built
- " elastic " - marginal cost is constant

(8)

Producer Surplus in the Short Run

- firms earn a surplus on all but ~~that~~ the last unit of output
  ? where MC = MR
- sum/integral of area above producer supply curve
  below market price



Firm

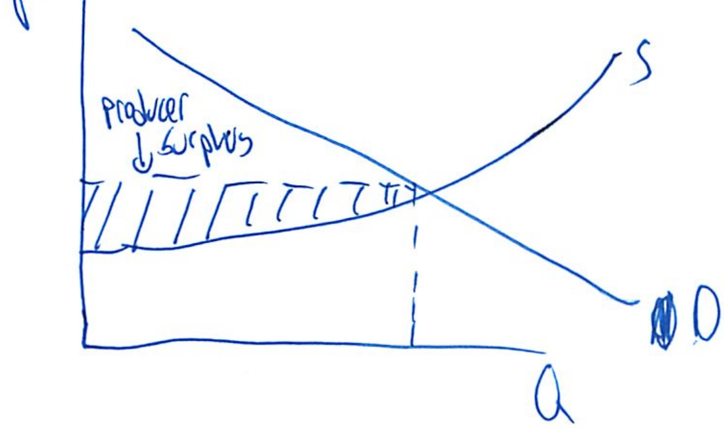
- also difference Revenue - total variable cost
- closely related but not = to profit

$$PS = R - VC$$

$$\text{Profit} = \pi = R - VC - \underline{FC}$$

So in short run when fixed ~~cost~~ cost is (+)

$$PS > \text{Profit}$$

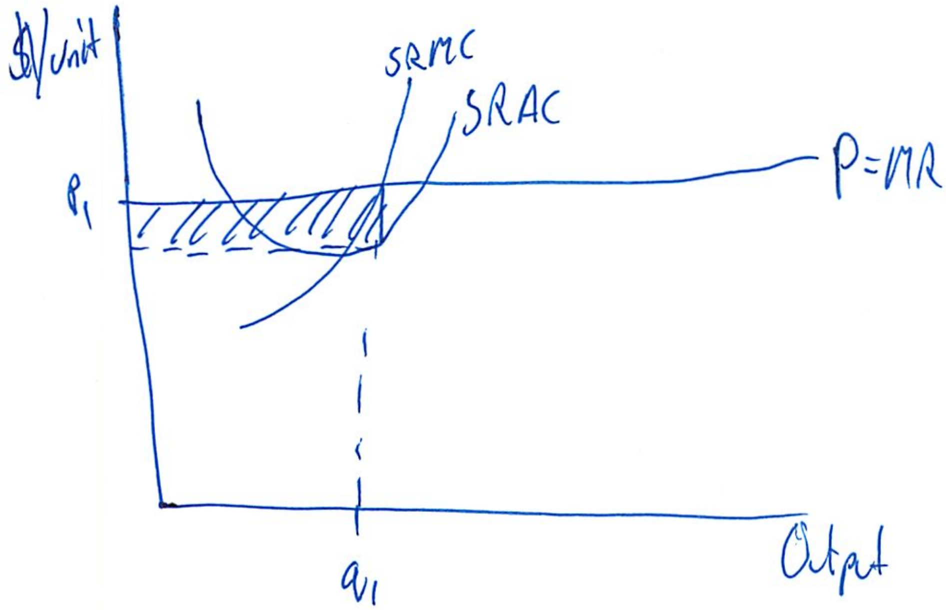


Market

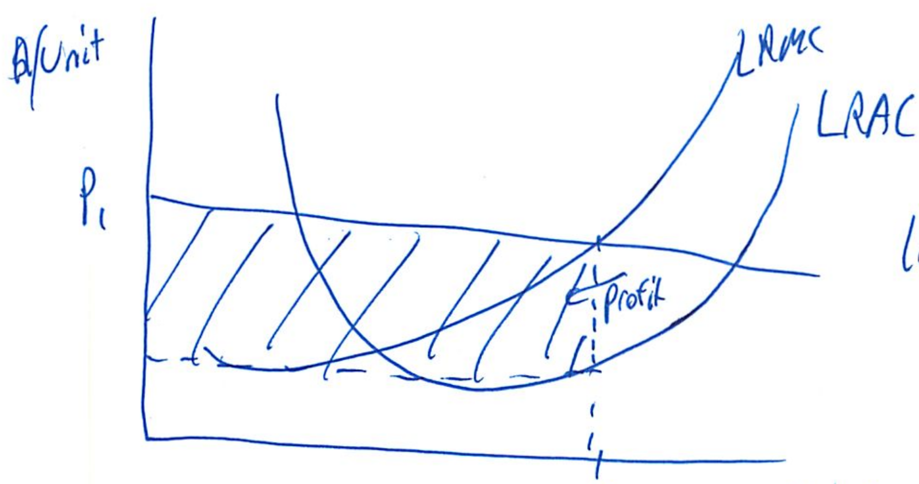
④ 8.7 Choosing Output in ~~Short~~ Long Run

- all outputs variable
- can enter/exit industry
- remember still talking about perfect competition

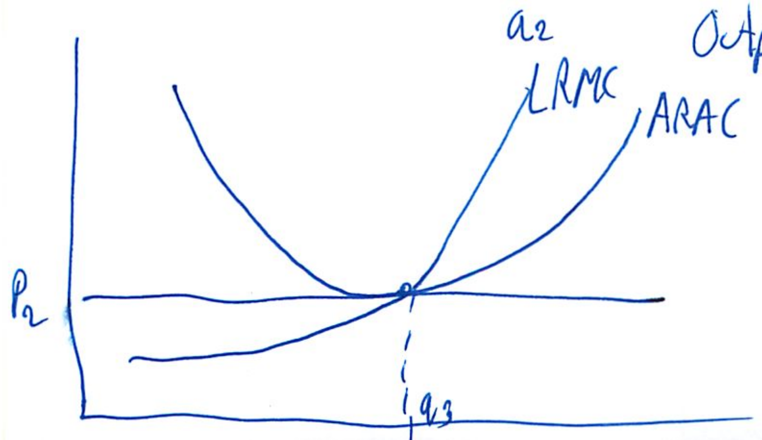
Long Run Profit Maximization



short run



long run



~~@~~ market price drops
now firm breaking even
- producing lower q

(10)

long run output : where long run Marginal cost = price
the price, the higher the profit of a firm

Long Run Competitive Equalibrium

- for long run equalibrium ~~#~~ Companies must want not to enter/exit

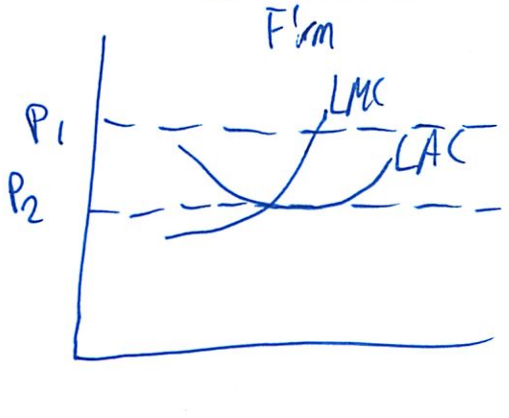
~~Accounting Profit ≠ Economic Profit~~

economic profit $\pi = R - wL - rk$

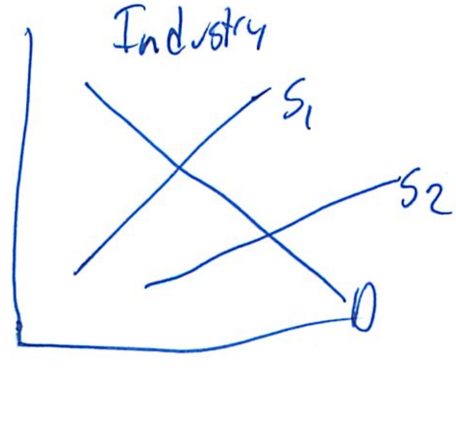
Zero Economic Profit

- Means firm is earning a normal/competitive return on investment
- firms oppurtunity cost is investing the money elsewhere
- in competive markets long run will be no economic profits
  - simply = to a competive rate of return
- Since if their is a high economic profit firms will enter the market
- if negitive economic profit, firms will drop out to spend their ~~$~~ elsewhere

Firm



Industry



(11)

In long run competitive equalibrium

1. All firms are maximizing profit
2. No firms will want to enter/exit
3. Zero economic profit
4. Price so that $Q_S = Q_D$

So why do firms enter a market hoping to earn 0 economic return?
- Since it is a competive return
- no incentive to go elsewhere

Firms have identical costs
- firms will exit industry if price drops

Firms have different costs - if firm has a patent to make production more efficent it could sell it at an equalibrium price + exit the industry

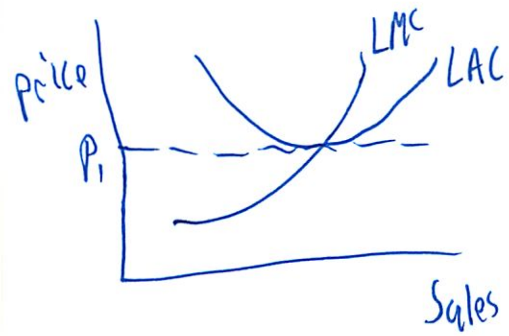
Oppurtunity Cost of land - land's price should reflect its oppurtunity cost, could be used for other purpose

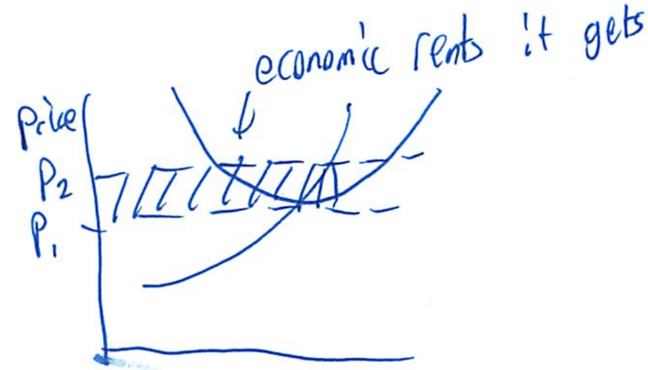
Economic rent - what firms will pay for something - min cost to buy
- like special land or skills
- (don't fully understand)
- ? what you get by selling asset to most efficent user of it?

(12)

Producer Surplus in the Long Run

- producer surplus on its outputs = economic rents from inputs
- in a non competitive market it would = economic profit





* this rent is an opportunity cost it could sell to another firm

thus no economic profit

8.8 The Industry's LR Supply Curve

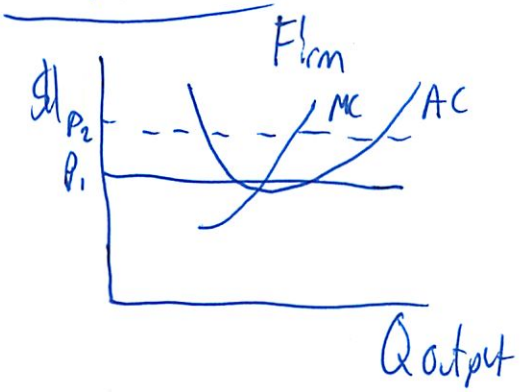
Since firms enter + exit at different prices can't just add up supply curves

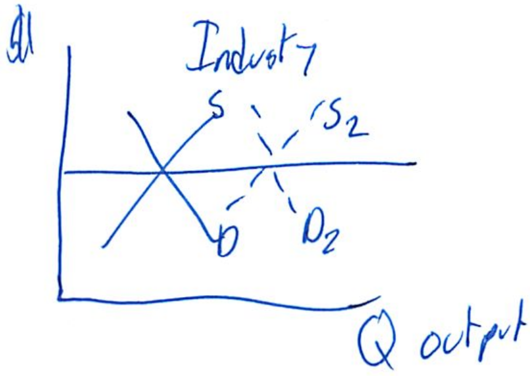
depends on input prices, economies of scale

tech not considered
- all firms have same tech
- no inventions

price of input does not depend on industry size

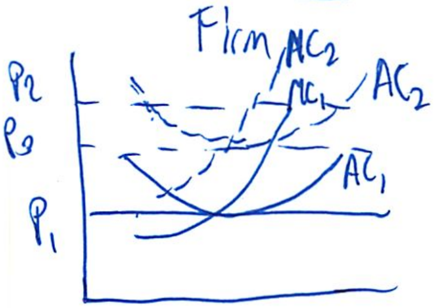
(13) Constant Cost

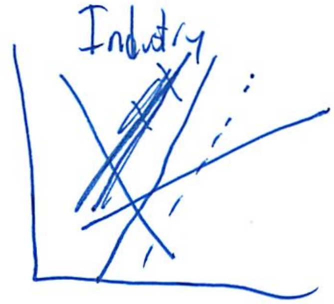




price = min. LR avg cost of production
- at higher price would be profit, market entry, ↑ short term supply, ↓ price pressure

Increasing Cost Industry





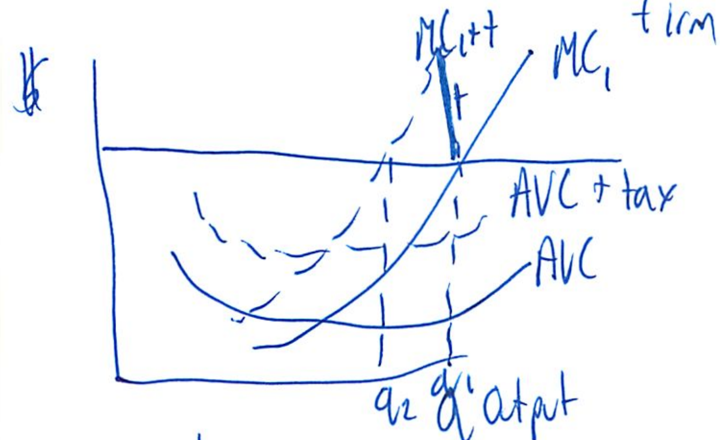
↑ bad pic

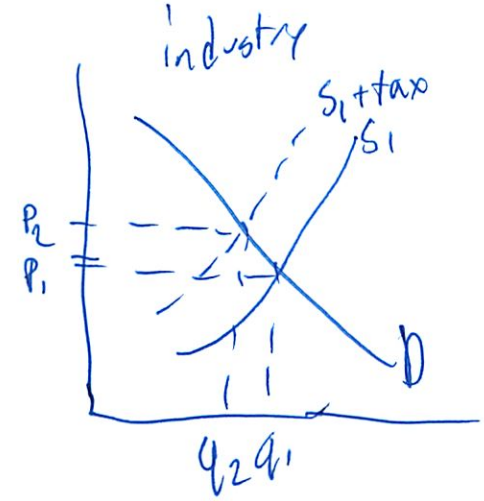
long run supply curve upwards sloping

(14)

Effects of a Tax

- ~~changes~~ tax on 1 input of production
- affects input mix selection
- tax on output of only 1 firm
- causes firm to ↓ output





q drops

p samp $S_2 = S_1 + tax$

Long Run Elasticities of Supply

- Same as short run
- Constant cost: elasticity ∞
  - ↳ small ↑ price = large ↑ output
- increasing cost: positive but finite
  industries can adjust, so elasticity larger here than w/ decreasing cost
- Magnitude depends on which input cost
  - widly avalible : more elastic

Michael Plasmeier 10/15

14.01 Fall 2010

Problem Set 4

Due in class on Octobers 15th

78

1. For each of the following production functions,

$$(a) F(L, K) = LK^3$$
$$(b) F(L, K) = L + 3K$$
$$(c) F(L, K) = (min\{L, K\})^{\frac{1}{3}}$$

(i) (2 points for each production function) sketch a representative isoquant,

(ii) (3 points) calculate the marginal product for each input, and indicate whether each marginal product is diminishing, constant, or increasing,

(iii) (2 points) calculate the marginal rate of technical substitution for each function,

(iv) (2 points) also indicate whether the function exhibits constant, increasing, or diminishing returns to scale.

2. Consider the production function $f(l, k) = 2l^{\frac{1}{4}}k^{\frac{1}{4}}$. $= q$ Cost =

(a) (15 points) Find the associated (long-run) total, average, and marginal cost curves.

L could do C, w as variable

(b) (6 points) Sketch the total cost curve on one set of axes, and average and marginal cost curves on another.

$MRST = \frac{C}{w} = \frac{MP_L}{MP_k}$

3. Suppose the process of producing corn on a farm is described by the function

$$q = 8K^{1/3}(L - 40)^{\frac{2}{3}}$$ Correction

k in terms of L

where q is the number of units of corn produced, K the number of machine hours used, and L is the number of person-hours of labor. In addition to capital and labor, the farmer needs to pay a \$15 transportation fee to deliver corn to downtown. So the total cost can be written as:

Cost function $Ck + wL$

$$TC = 15q + rK + wL$$

where wage rates is w and the rental rate of machines is r.

get rid of k + L replace w f

(a) (8 points) Suppose in the short run, the machine hours rented are fixed at $K = 8$, and its rental rate $r = 64$, and wage rate $w = 16$. Derive the short run total costs and average costs as a function of output level (q).

(b) (6 points) Suppose the farm wants to produce 64 units of corn, i.e., $q = 64$, based on the answer to (a), what.s the total short run cost?

(c) (10 points) In the long run, the farm can change its capital level. By minimizing cost subject to the production function, derive the cost-minization demands for K and L as a function of output (q), wage rates (w) and rental rates of machines (r).

4. You run a cost-minimizing firm with production function $f(l, k) = [min\{l, k\}]^{\frac{1}{3}}$, where l is labor and k is capital. Assume that you are a price-taker in the input markets: you pay w for each unit of labor you hire

and r for each unit of capital (where w and r are set exogenously), and face no costs other than those from labor and capital.

(a) (15 points) Assuming that you can freely choose both labor and capital (i.e., the "long-run problem"), derive expressions for your cost-minimizing conditional input demands, $l_*(r, w, Q)$ and $k_*(r, w, Q)$. Confirm that the conditional input demand functions are "homogeneous of degree zero" in w and r; that is

Same

Set k, l =
for optimization

$$l_*(tr, tw, Q) = l_*(r, w, Q) \text{and}$$
$$k_*(tr, tw, Q) = k_*(r, w, Q)$$

for all $t > 0$.

$l = k = f^3$

no change, need min amt

(b) (8 points) What will happen to your conditional demand for labor if there is an increase in the wage rate, assuming that r and Q remain the same? Explain in one sentence why your answer makes intuitive sense.

adding does not change f
adding less L

(c) (5 points) Use your answers from (a) to write down an expression for your total cost function $TC(r, w, Q)$. Is this function "homogeneous of degree one" in w and r; that is does $TC(tr, tw, Q) = tTC(r, w, Q)$?

$TC = wl + rk = (w + r)f^3 = TC$

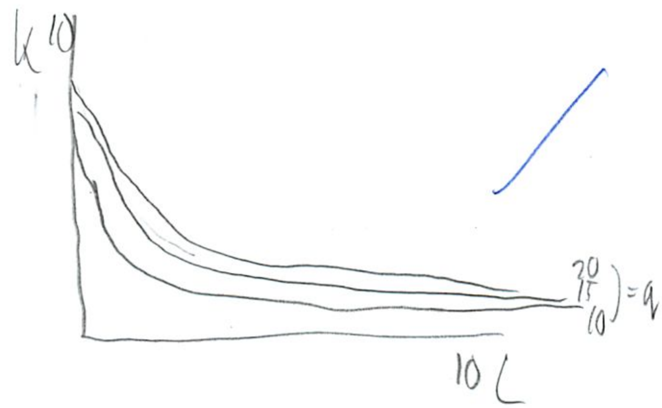
$TC(tr, tw, Q) = (tw + tr) f^3 z + (w + r) f^3 = t \cdot TC$

Yes homogeneous of degree 1 in w and r

14.01 P-Set 4

10/15

1. Show isoquants



$Q = Lk^3$

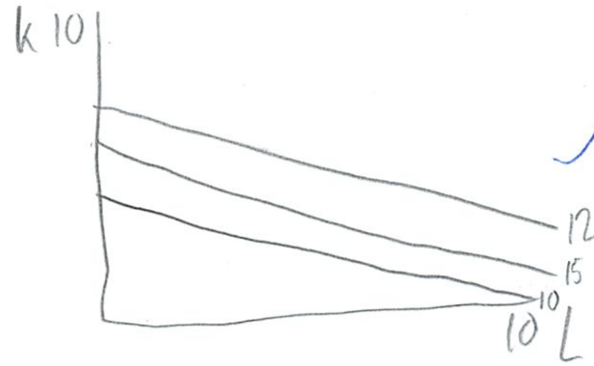
Solve for k

$k = \sqrt[3]{\frac{Q}{L}}$

give a q and graph

$Q = 10$ $Q = 20$

$Q = 15$

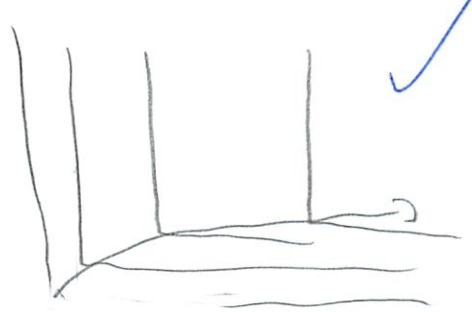


$Q = L + 3k$

$k = \frac{Q - L}{3}$

$Q = 10, 15, 20$

don't have intuition just graph



$Q = \min\{L, k\}^{1/3}$

?

b. Marginal Product of each input

$\frac{\Delta Q}{\Delta L}$ $\frac{\Delta Q}{\Delta k}$

depends of course where you are

Slope of total product

finally getting the basic algebra

(2)

$\frac{\Delta Q}{\Delta L} = \cancel{\frac{15 - 10}{1 - .25}}$ change in out from 1 extra L

Capital fixed - say 5 units

At 5 units of Labor

$Q = 5 \cdot 5^3 = 625$

Go to 6 units $= 125$

$Q = 6 \cdot 5^3 = 750$

could you also do

$\frac{\partial Q}{\partial L} = 1 \cdot k^3$ increasing

$\frac{\partial Q}{\partial L} = 3k^2 L$ increasing

$k = 5 \quad L = 5 \quad Q = 625$

$k = 6 \quad L = 5 \quad Q = 1080 \quad = 455$

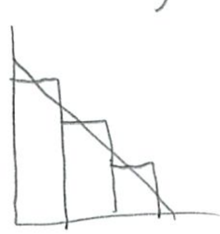
; but it changes at every point

~~and does how derivitive fit into this~~

~~not matching real life estimate~~

- it does - test w/ $3(5.5)^2 5$

'remember drawing boxes in math!



- calculus refresher

(3)

b. $\frac{\partial Q}{\partial L} = 1$ Constant

$\frac{\partial Q}{\partial k} = 3$ increasing

-2

C. $\cancel{\frac{\partial Q}{\partial L} L^{1/3} = \frac{1}{3} L^{-2/3}}$ ~~decreasing~~ zero

$\cancel{\frac{\partial Q}{\partial k} k^{1/3} = \frac{1}{3} k^{-2/3}}$ ~~decreasing~~

-2

But with min

0 - need more of both

0 an increase in one does nothing w/o the other

~~Why are all my answers increasing~~ - it is usually decreasing

Why are some increasing?

iii. Calculate MRTS

$= -\frac{\Delta k}{\Delta L} = \frac{MP_L}{MP_k}$ = slope of isoquant

derivitive of k= function

a) $\frac{k^3}{3k^2 L}$ ← confirm $\frac{d}{dL} \sqrt[3]{\frac{Q}{L}} = -\frac{Q}{3L^2\left(\frac{Q}{L}\right)^{2/3}}$ sub in for Q $-\frac{Lk^3}{3L^2\left(\frac{Lk^3}{L}\right)^{2/3}}$

$= -\frac{Lk^3}{3L^2 k^2}$

$= \frac{k^3}{3Lk^2}$ ✓ checks out but kinda hairy

b) $\frac{1}{3}$

c) $\cancel{\frac{\frac{1}{3}L^{-2/3}}{\frac{1}{3}k^{-2/3}}} = \cancel{\frac{L^{-2/3}}{k^{-2/3}}} = \cancel{\frac{k^{2/3}}{L^{2/3}}} = \cancel{\frac{\sqrt[3]{k^2}}{\sqrt[3]{L}}}$

$\frac{0}{0}$ ← Undefined Can't sub one for the other

-2

(4)

iv) Returns to Scale

- rate of output as inputs increased porportionally
- how different from MP_L?
  - all factors of production

a) Increasing → exponentially ✓

b) Increasing → by factor of ~3 −2

c) If double all inputs → min does not matter, but here the 1/3 power → decreasing ✓

1 hr for first av
- "easy" usually
its figuring out the algebra

(5)

2. $f = 2l^{1/4}k^{1/4}$

a. Find long run total, avg, marginal cost curves

-but how in all possible - other chap

~~-can do product curves~~

-no costs listed

↳ could do in terms of r, w

$$MRTS = \frac{MP_L}{MP_k} = \frac{r}{w}$$

$$Q = 2L^{1/4}k^{1/4}$$

$$k^{1/4} = \frac{Q}{2L^{1/4}}$$

$$k = \left(\frac{Q}{2L^{1/4}}\right)^4$$

$$L^{1/4} = \frac{Q}{2k^{1/4}}$$

$$L = \left(\frac{Q}{2k^{1/4}}\right)^4$$

$$MC_k = \frac{\partial Q}{\partial k} = 2 \cdot \frac{1}{4} k^{-3/4} L^{1/4}$$

$$MC_L = \frac{\partial Q}{\partial L} = 2 \cdot \frac{1}{4} L^{-3/4} k^{1/4}$$

⑥

$$MRTS = \frac{MC_L}{MC_K} = \frac{w}{r} = \frac{2 \cdot \frac{1}{4} L^{-3/4} k^{1/4}}{2 \cdot \frac{1}{4} k^{-3/4} L^{1/4}} = \frac{k}{L}$$

? even substitution

$$\frac{k}{L} = \frac{w}{r}$$

$$k = \frac{Lw}{r}$$

$$Q^4 = \frac{16 L^2 w}{r} \rightarrow$$

solve for L

$$Q^4 r = 16 L^2 w$$

$$L = \sqrt{\frac{Q^4 r}{16 w}}$$

$$L = \frac{Q^2}{4} \sqrt{\frac{r}{w}}$$

$$TC = rk + wL$$

$$= \frac{rLw}{r} + wL$$

$$= 2Lw$$

$$= 2\left(\frac{Q^2}{4}\sqrt{\frac{r}{w}}\right) w$$

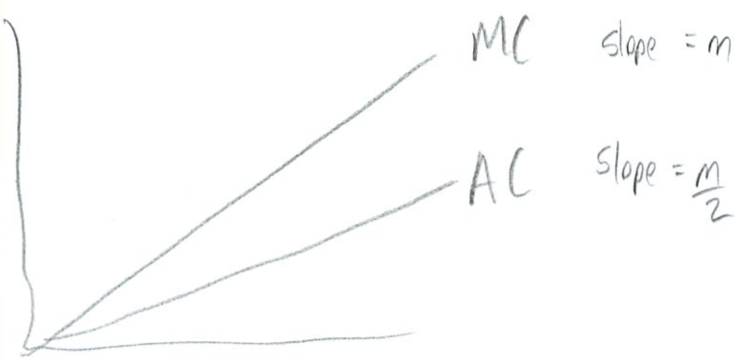
reduce!

$$= \frac{Q^2 w}{2} \sqrt{\frac{r}{w}}$$

(7)

$$\frac{\partial C}{\partial Q} = \frac{1}{2} \cdot 2 Q w \sqrt{\frac{r}{w}} = Q w \sqrt{\frac{r}{w}} = m$$

$$\frac{C}{Q} = \frac{\frac{Q 2 w}{2} \sqrt{\frac{r}{w}}}{Q} = \frac{Q w}{2} \sqrt{\frac{r}{w}} = \frac{m}{2}$$



8

3. Corn production on a farm

$$q = 8k^{1/3}(L-40)^{2/3}$$

↑ units produced

W = wage

r = rental

$$TC = 15q + rk + wL$$

a) Short run cost

$k = 8$

$r = 64 \quad w = 16$

$$\text{Total cost} = 15\left(8k^{1/3}(L-40)^{2/3}\right) + rk + wL$$

$$15\left(8(8)^{1/3}(L-40)^{2/3}\right) + 64 \cdot 8 + 16 \cdot L$$

$$15\left(16(L-40)^{2/3}\right) + 512 + 16L$$

$$240(L-40)^{2/3} + 512 + 16L$$

$$ATC = \frac{TC}{q} = \frac{15\left(8(8)^{1/3}(L-40)^{2/3}\right) + 64 \cdot 8 + 16 \cdot L}{8(8)^{1/3}(L-40)^{2/3}}$$

$$= \frac{240(L-40)^{2/3} + 512 + 16L}{16(L-40)^{2/3}}$$

$$TC(q) = 15q + 64 \cdot 8 + 16 \cdot L$$

$$15q + 512 + 16L$$

-2

$$ATC = \frac{15q + 512 + 16L}{q}$$

this type like better

(9)

wolfram Alpha

$$MP_L = \frac{\partial Q}{\partial L} \rightarrow \cancel{8k^{1/3} \cdot 4(5L)^{2/3}} = \frac{16\sqrt[3]{k}}{3\sqrt[3]{L-40}}$$

$$\frac{\partial Q}{\partial k} = \frac{8(L-40)^{2/3}}{3k^{2/3}}$$

wolfram alpha

$$\frac{\frac{16\sqrt[3]{k}}{3\sqrt[3]{L-40}}}{\frac{8(L-40)^{2/3}}{3k^{2/3}}} = \frac{w}{r} = \frac{2k}{L-40}$$

↑ much better
- must have been easier way to have done this

Solve for k, L ↑ right

$$\frac{w(L-40)}{r} = 2k$$

$$\boxed{k = \frac{w(L-40)}{2r}}$$

$$w(L-40) = 2kr$$

$$L-40 = \frac{2kr}{w}$$

$$\boxed{L = \frac{2kr}{w} + 40}$$

the magic of algebra

(10)

b) Suppose she wants 64 q

she can only adjust labor

$$64 = 8(8)^{1/3}(L-40)^{2/3}$$

$$64 = 16(L-40)^{2/3}$$

$$4 = (L-40)^{2/3}$$

$$^{3/2} \qquad ^{3/2}$$

$$8 = L-40$$

$$L = 48$$

Short run best total cost

$$TC = 15(64) + 64 \cdot 8 + 16 \cdot 48$$

$$2240$$

✓

c) Long Run can change capital level

Find cost-minimization demands for k, L as functions of q, w, r

↳ I thought this was producer theory? I see demand of inputs

So we want to choose long run input level

Firms minimize cost when

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$

— 10

(11)

4. $f = [\min\{l, k\}]^{1/3}$

l = labor = L
k = capital = k

w = wage
r = rental rate

exogenously = set outside system

Firm is a price taker in input market
normal assumption

a) Find amt l and k you would use
- this was like problem just did

$$\frac{MP_L}{MP_k} = \frac{w}{r}$$

But MP_L on a min function in 0

Is that what they are talking about in all that text?

web "homogeneous of degree zero" - property of a function that if you scale all arguments by same proportion, value of function does not change

"homogeneous of degree 1" = constant returns to scale
? why could they not just say that!

So it says that changing inputs produces no additional outputs ???

$L = K = f^3 = Q^3$

(12)

b) What will happen to your conditional demand for labor if their is an increase in wage rate, assuming r and Q remain the same? Explain why makes sense.

No change, Need a corresponding increase in K to make use of the new labor.

(13)

c) Using ans from A find $TC(r, w, Q)$

Is it homogeneous in degree 1?

↳ aka $TC(tr, tw, Q) = t\,TC(r, w, Q)$

$TC = wL + rK = (w+r)f^3 = TC$

$TC(tr, tw, Q) = (tw + tr)f^3 = t(w+r)f^3 = TC$

Yes → homogeneous in degree of 1 ✓