

Problem Set # 2
14.02 Spring 2011
Due Feb 25

Feb 11

1 True/False [30 points]

Please state whether each of the following claims are True or False, and provide a brief justification for your answer. You may include graphs and equations to support your answer.

1. "The demand for money can decrease if the price of bonds falls" [6 points]
2. "In the IS-LM model, monetary policy can be an effective instrument to increase the level of output in the short run" [6 points]
3. "In an economy in which banks are not allowed to make loans (i.e. they are used only for security reasons), total money supply equals the amount of central bank money" [6 points]
4. "Consider a given increase in taxes (fiscal contraction). Both the model of the goods market studied in chapter 3, and the IS-LM model studied in chapter 5 predict the same effect (quantitative) on output." [6 points]
5. "Taking as given the level of nominal income, an increase in the money supply will result in a decrease in the interest rate" [6 points]

- must look up - but understand

2 Financial Markets [20 points]

Suppose that people hold a fixed proportion of their money in currency - denote this proportion by c , and assume $c = 1/4$. Suppose also that banks are required to hold 20% of their deposits in reserves ($\theta = 1/5$). The supply of money (M) in this economy is 2,000 billion dollars.

1. Calculate the amount of currency (CU), reserves (R) and deposits (D) in this economy. [5 points]

2. What is the demand for central bank money (denote by H^d)? [5 points]
3. Suppose that the demand for money is given by $M^d = 200/i$ (in billion dollars). Find the equilibrium interest rate. [5 points]
4. Suppose the Fed creates 100 billion of central bank money. What is the total increase in the money supply? What is the new interest rate? [5 points]

3 IS-LM [50 points]

Consider the following IS-LM model

$$\begin{aligned} C &= c_0 + c_1 Y_d \\ I &= i_0 + i_1 Y - i_2 i \\ M^d/P &= d_1 Y - d_2 i \end{aligned}$$

where Y_d is disposable income ($Y_d = Y - T$). Note that output and the interest rate are non-negative variables.

1. Find the combinations of Y and i that keep the goods market in equilibrium. Express Y as a function of i . This is the IS. [10 points]
2. What assumption on the parameters of this model is required to have an equilibrium in the goods market? (Hint: you also need this assumption to have a multiplier larger than unity) [4 points]
3. For a given interest rate, how much would output change in response to a 1 dollar increase in Government spending (G)? [4 points]
4. For a given interest rate, how much would output change in response to a 1 dollar decrease in Taxes (T)? Why is this different from part 3? [3 points]
5. The statement of the problem gives us a demand for real money balances. What condition, not stated in the problem set up, do we need in order to find the equilibrium in financial markets? Find the combinations of Y and i that keep financial markets in equilibrium. This is the LM. [7 points]
6. Use the IS and LM to find the equilibrium levels of output (call it Y^*) and the interest rate (call it i^*). You can assume that the parameters are such that $Y^* > 0$ and $i^* > 0$. [8 points]
7. Show a sketch of the IS and LM curves on the usual (Y, i) space. Show the effect of an increase in taxes on the IS and LM lines. What happens to equilibrium output and the interest rate? [7 points]

6 sentence fragment

8. Using your answer to point 6, what is the effect of a 1 dollar increase in Government spending (G) on the equilibrium level of output (Y^*)? How does this number compare to your answer to part 3? Try to provide some intuition. [7 points]

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True-False

1. If bond prices fall that means interest rates are predicted to go up (or have gone up in our short run world. Higher i means a lower demand for money as more people buy bonds.

True.



2. True. Expansionary monetary policy will expand output



3. $M^d = C^d + R^d$
 TO

True. No reserves means currency = central bank money

4. Fiscal contraction



②

True since the goods market in Chap 3 is simply
the IS part in Chap 5. The LM curve does not shift

5. True



Financial Markets

$C = 1/4$ = proportion of money in currency

$\theta = 1/5$ = amt banks must hold in reserve

$M = 2000$ billion

$M^s = M^d$ right?

← can you equate this
or is a wrong.

a. $C U^P = C M^d$

$$= 1/4 \cdot 2,000 \text{ billion}$$

$$= 500 \text{ billion}$$

✓ $D^D = (1 - c) M^d$

$$= (1 - 1/4) \cdot 2,000 \text{ billion}$$

$$= 1,500 \text{ billion}$$

③

$$R = \theta D$$

$$= \frac{1}{5} \cdot 1,500 \text{ billion}$$

$$= 300 \text{ billion}$$

$$b. H^d = C U^d + R^d$$

$$= 500 \text{ billion} + 300 \text{ billion}$$

$$= 800 \text{ billion}$$

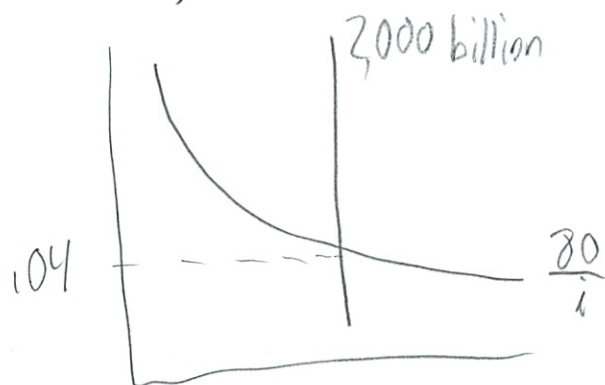
$$c. M^d = \frac{200}{\bar{i}} \text{ billion}$$

$$H^d = c M^d + \theta(1-c) M^d$$

$$= [c + \theta(1-c)] M^d$$

$$-5 \quad = \left[\frac{1}{4} + \frac{1}{5}(1 - \frac{1}{4}) \right] \frac{200}{\bar{i}}$$

$$= \frac{80}{\bar{i}} \text{ billion}$$



$$\bar{i} = 4\%$$

(4)

$$d. M' = M + 100 \text{ billion}$$

$$M' = 2,000 \text{ billion} + 100 \text{ billion} \\ = 2,100 \text{ billion}$$

→ Solve for i

$$\left\{ 2,100 \text{ billion}, \frac{80}{i} \right\}$$

$$i = 3.8\%$$

IS-LM

$$C = C_0 + c_1 Y_d$$

$$I = i_0 + i_1 Y - i_2 i$$

$$M^d/P = d_1 Y - d_2 i$$

$$Y_d = Y - T$$

a. Find equilibrium $Y(i), i$ (IS)

$$Z = C + I + G$$

$$Y = C_0 + c_1 (Y - T) + i_0 + i_1 Y - i_2 i$$

Solve for Y

$$Y = C_0 + c_1 Y - c_1 T + i_0 + i_1 Y - i_2 i$$

5

$$Y - c_1 Y - \dot{\lambda}_1 Y = C_0 - c_1 T + \dot{\lambda}_0 - \dot{\lambda}_2 \dot{\lambda} + G$$

$$Y(1 - c_1 - \dot{\lambda}_1) = C_0 - c_1 T + \dot{\lambda}_0 - \dot{\lambda}_2 \dot{\lambda} + G$$

✓ $Y(\dot{\lambda}) = \frac{1}{1 - c_1 - \dot{\lambda}_1} [C_0 - c_1 T + \dot{\lambda}_0 - \dot{\lambda}_2 \dot{\lambda} + G]$

? What next

b. What assumptions have been made?

Well the constants

-2 $C_0, c_1, \dot{\lambda}_0, \dot{\lambda}_1, \dot{\lambda}_2, T$

Boad assumes that increase in output is less than 1 for 1 increase in demand

6

C. For a given i , how would Y change with $G' = G + 1$?
The dollar would go through the multiplier

✓ $\frac{1}{1 - c_1 - i_1}$ and equal $\frac{1}{1 - c_1 - i_1}$

d. For given i , how would Y change with $T' = T - 1$?

$$Y'(i) = \frac{1}{1 - c_1 - i_1} [C_0 - C_1 T' + i^0 - i^2 i + G]$$

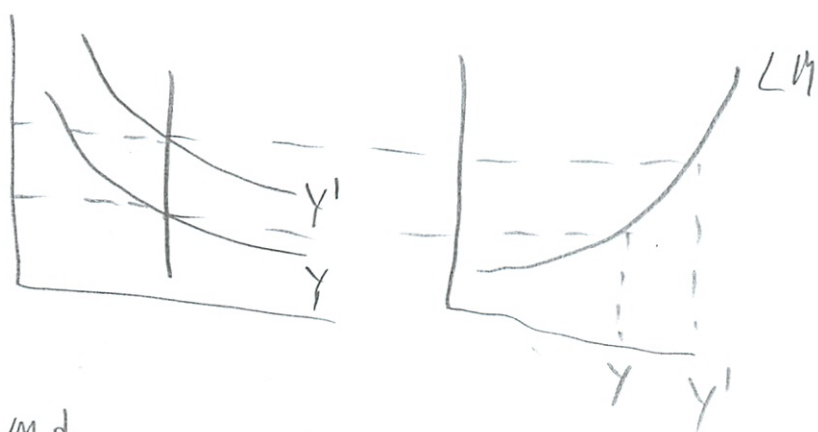
✓ Here T is also multiplied by c_1 in addition to the multiplier, so the drop in output would be smaller than the gain in G .

This is because people will spend c_1 anyway no matter what taxes are, only the c_2 component is affected by the higher taxes.

⑦

e. The LM equilibrium is set by the current income level (known) and by the gov's money supply (unknown). This gives us the interest rate

$$\frac{M_d}{P} = d_1 Y - d_2 \bar{i}$$



(book never went over formula)

$$\frac{M_d}{P} = d_1 \left(\frac{1}{1 - c_1 - i_1} \right) [C_0 - c_1 T + i_0 - i_2 \bar{i} + G] - d_2 \bar{i}$$

Or solve for \bar{i} :

$$d_2 \bar{i} = d_1 Y - \frac{M_d}{P}$$

$$\bar{i} = \frac{d_1}{d_2} Y - \frac{M_d}{d_2 P}$$

8. Find equilibrium Y^* i^*

Substitute LM into IS, solve for Y , put into LM, solve for i
- did above
other way around

$$Y(i) = \frac{1}{1-c_1-i_1} \left[C_0 - C_1 T + i_0 - i_2 \left(\frac{d_1}{d_2} Y - \frac{M^d}{d_2 P} \right) + G \right]$$

$$\frac{M^d}{P} = d_1 \left(\frac{1}{1-c_1-i_1} \left[C_0 - C_1 T + i_0 - i_2 \left(\frac{d_1}{d_2} Y - \frac{M^d}{d_2 P} \right) + G \right] \right) - d_2 i$$

$$i = \frac{d_1}{d_2} \left(\frac{1}{1-c_1-i_1} \left[C_0 - C_1 T + i_0 - i_2 \left(\frac{d_1}{d_2} Y - \frac{M^d}{d_2 P} \right) + G \right] \right) - \frac{M^d}{d_2 P}$$

Solve for Y

$$i + \frac{M^d}{d_2 P} = \frac{d_1}{d_2} \left(\frac{1}{1-c_1-i_1} \left[C_0 - C_1 T + i_0 - i_2 \left(\frac{d_1}{d_2} Y - \frac{M^d}{d_2 P} \right) + G \right] \right)$$

$$\frac{d_2}{d_1} \left(i + \frac{M^d}{d_2 P} \right) = \frac{1}{1-c_1-i_1} \dots$$

$$\frac{d_2(1-c_1-i_1)}{d_1} \left(i + \frac{M^d}{d_2 P} \right) = C_0 - C_1 T + i_0 - i_2 \left(\frac{d_1}{d_2} Y - \frac{M^d}{d_2 P} \right) + G$$

this can't be right!

9)

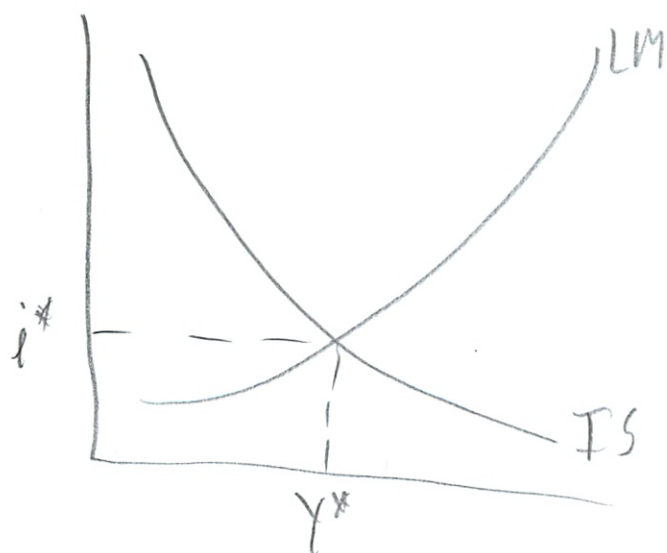
$$\frac{d_2(1-c_1-i_1)}{d_1} \left(i + \frac{M_d}{d_2 P} \right) - G - C_0 + C_1 T - i_0 = - \frac{i_2 d_1 y}{d_2} + \frac{i_2 M_d}{d_2 P}$$

$$y^* = - \frac{d_2}{i_2 d_1} \left[\frac{d_2(1-c_1-i_1)}{d_1} \left(i + \frac{M_d}{d_2 P} \right) - G - C_0 + C_1 T - i_0 + \frac{i_2 M_d}{d_2 P} \right]$$

Can place this in LM to get i^*

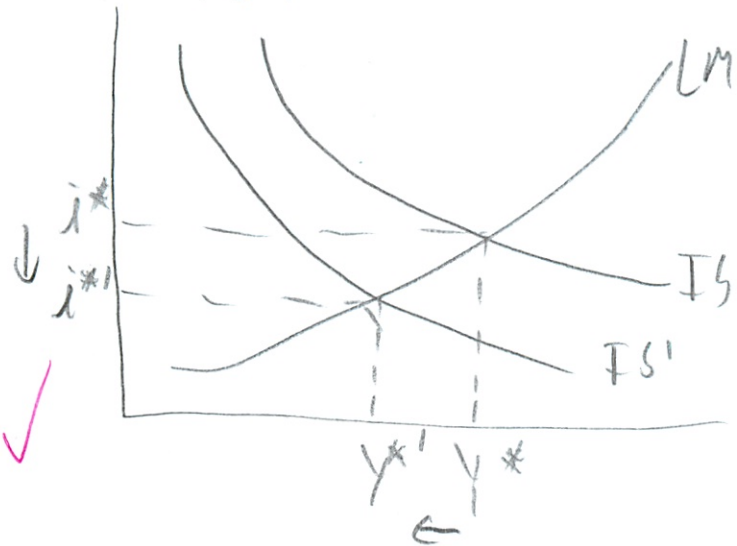
$$i^* = \frac{d_1}{d_2} y^* - \frac{M_d}{d_2 P}$$

g) Sketch

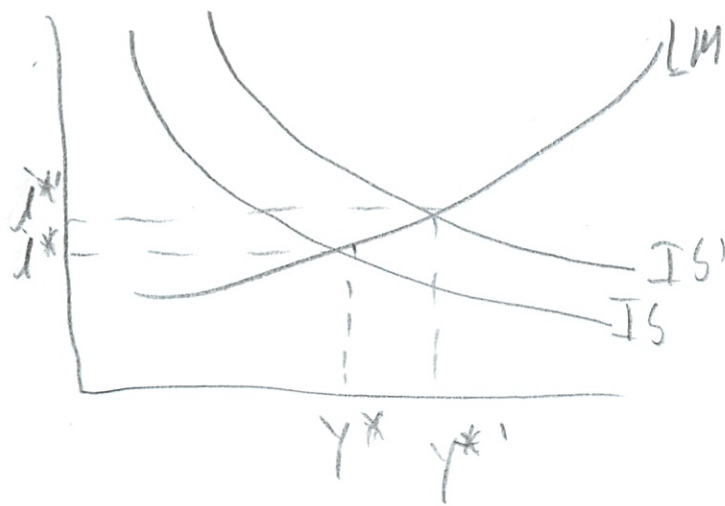


(10)

Taxes ↑



h) $G \uparrow$



-7 Yes this is the same number as before because the IS curves moves out like before.

There are just more constants affecting the number this time

Solutions to Problem Set # 2

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1 True/False [30 points]

Please state whether each of the following claims are True or False, and provide a brief justification for your answer. You may include graphs and equations to support your answer.

1. "The demand for money can decrease if the price of bonds falls" [6 points]

ANSWER. TRUE. A decrease in the price of bonds makes money relatively less attractive.

2. "In the IS-LM model, when the interest rate is greater than zero, monetary policy can be an effective instrument to increase the level of output in the short run" [6 points]

ANSWER. TRUE. When the interest rate is positive (i.e. we are not in a liquidity trap), an increase in money supply shifts the LM curve down, thus increasing the level of output.

3. "In an economy in which banks are not allowed to make loans (i.e. they are used only for security reasons), total money supply equals the amount of central bank money" [6 points]

ANSWER. TRUE. In this case ($\theta = 1$), the money multiplier is 1, so that $H = M$.

4. "Consider a given increase in taxes (a fiscal contraction). Both the model of the goods market studied in chapter 3, and the IS-LM model studied in chapter 5 predict the same quantitative effect on output." [6 points]

ANSWER. FALSE. While it is true that qualitatively both models predict a decrease in output, the statement is nevertheless false. This is because quantitatively the IS-LM model predicts a smaller decrease in output (as long as the LM curve is upward sloping). In the standard model of the

goods market studied in chapter 3, the interest rate is fixed. When we allow the interest to adjust (in chapter 5), output will fall by less.

5. "Taking as given the level of nominal income, an increase in the money supply will result in a decrease in the interest rate" [6 points]

ANSWER. FALSE. This claim is not true in the case of the liquidity trap. If the initial interest rate is zero, and people already hold enough money for transactions, then an increase in money supply will have no effect on the interest rate.

2 Financial Markets [20 points]

Suppose that people hold a fixed proportion of their money in currency - denote this proportion by c , and assume $c = 1/4$. Suppose also that banks are required to hold 20% of their deposits in reserves ($\theta = 1/5$). The supply of money (M) in this economy is 2,000 billion dollars.

1. Calculate the amount of currency (CU), reserves (R) and deposits (D) in this economy. [5 points]

ANSWER.

$$\begin{aligned} CU &= cM = 500 \\ D &= (1 - c)M = 1500 \\ R &= \theta D = 300 \end{aligned}$$

2. What is the demand for central bank money (denote by H^d)? [5 points]

ANSWER.

$$\begin{aligned} H^d &= CU^d + R \\ &= 800 \end{aligned}$$

3. Suppose that the demand for money is given by $M^d = 200/i$ (in billion dollars). Find the equilibrium interest rate. [5 points]

ANSWER.

$$\begin{aligned} M^d &= M^s \\ 200/i &= 2000 \\ i &= 0.1 = 10\% \end{aligned}$$

4. Suppose the Fed creates 100 billion of central bank money. What is the total increase in the money supply? What is the new interest rate? [5 points]

ANSWER. As shown in the book, money supply is

$$\frac{1}{c + \theta(1 - c)} H$$

so that the increase will be $\frac{1}{c + \theta(1 - c)} \times 100 = 250$ billion (the multiplier is 2.5). The new money supply is 2250 billion, and thus the interest rate will be

$$\begin{aligned} 200/i &= 2250 \\ i &= 200/2250 = 8.88\% \end{aligned}$$

3 IS-LM [50 points]

Consider the following IS-LM model

$$\begin{aligned} C &= c_0 + c_1 Y_d \\ I &= i_0 + i_1 Y - i_2 i \\ M^d/P &= d_1 Y - d_2 i \end{aligned}$$

where Y_d is disposable income ($Y_d = Y - T$). Note that output and the interest rate are non-negative variables.

1. Find the combinations of Y and i that keep the goods market in equilibrium. Express Y as a function of i . This is the IS. [10 points]

ANSWER.

$$\begin{aligned} Y &= C + I + G \\ Y &= c_0 + c_1(Y - T) + i_0 + i_1 Y - i_2 i + G \\ Y &= \frac{1}{1 - c_1 - i_1} [c_0 - c_1 T + i_0 - i_2 i + G] \end{aligned}$$

2. What assumption on the parameters of this model is required to have an equilibrium in the goods market? (Hint: you also need this assumption to have a multiplier larger than unity) [4 points]

ANSWER.

$$c_1 + i_1 < 1$$

3. For a given interest rate, how much would output change in response to a 1 dollar increase in Government spending (G)? [4 points]

ANSWER. Output would increase by

$$\frac{1}{1 - c_1 - i_1}$$

4. For a given interest rate, how much would output change in response to a 1 dollar decrease in Taxes (T)? Why is this different from part 3? [3 points]

ANSWER. Output would increase by

$$\frac{c_1}{1 - c_1 - i_1}$$

This is lower than in part 3 because consumers will save a fraction $1 - c_1$ of the tax cut rather than spend it.

5. The statement of the problem gives us a demand for real money balances. What condition, not stated in the problem set up, do we need in order to find the equilibrium in financial markets? Find the combinations of Y and i that keep financial markets in equilibrium. This is the LM. [7 points]

ANSWER. We need the supply of money: $M^s = M$. Combining supply and demand we find the LM to be

$$\begin{aligned} d_1 Y - d_2 i &= \frac{M}{P} \\ Y &= \frac{1}{d_1} \frac{M}{P} + \frac{d_2}{d_1} i \end{aligned}$$

6. Use the IS and LM to find the equilibrium levels of output (call it Y^*) and the interest rate (call it i^*). You can assume that the parameters are such that $Y^* > 0$ and $i^* > 0$. [8 points]

ANSWER.

$$\frac{1}{d_1} \frac{M}{P} + \frac{d_2}{d_1} i = \frac{1}{1 - c_1 - i_1} [c_0 - c_1 T + i_0 + G] - \frac{i_2}{1 - c_1 - i_1} i$$

$$\left[\frac{d_2}{d_1} + \frac{i_2}{1 - c_1 - i_1} \right] i = \frac{1}{1 - c_1 - i_1} [c_0 - c_1 T + i_0 + G] - \frac{1}{d_1} \frac{M}{P}$$

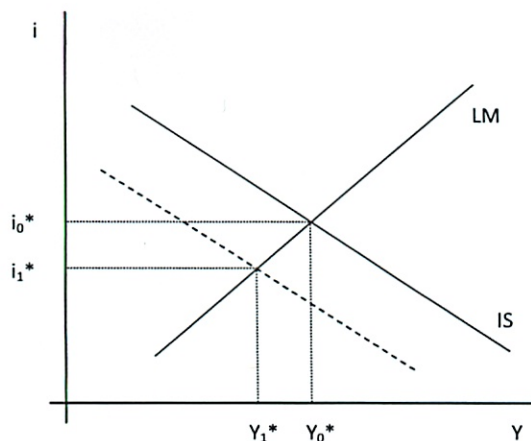
$$i^* = \frac{1}{\frac{d_2}{d_1} + \frac{i_2}{1 - c_1 - i_1}} \left\{ \frac{1}{1 - c_1 - i_1} [c_0 - c_1 T + i_0 + G] - \frac{1}{d_1} \frac{M}{P} \right\}$$

$$Y^* = \frac{1}{d_1} \frac{M}{P} + \frac{1}{1 + \frac{d_1 i_2}{d_2 (1 - c_1 - i_1)}} \left\{ \frac{1}{1 - c_1 - i_1} [c_0 - c_1 T + i_0 + G] - \frac{1}{d_1} \frac{M}{P} \right\}$$

7. Show a sketch of the IS and LM curves on the usual (Y, i) space. Show the effect of an increase in taxes on the IS and LM lines. What happens to equilibrium output and the interest rate? [7 points]

ANSWER. The IS curves shifts in, so both output and the interest rate

go down.



8. Using your answer to point 6, what is the effect of a 1 dollar increase in Government spending (G) on the equilibrium level of output (Y^*)? How does this number compare to your answer to part 3? Try to provide some intuition. [7 points]

ANSWER. The new "multiplier" is

$$\frac{1}{1 + \frac{d_1}{d_2} \frac{i_2}{1 - c_1 - i_1}} \frac{1}{1 - c_1 - i_1}$$

which is smaller than the multiplier found in part 3, as $\frac{1}{1 + \frac{d_1}{d_2} \frac{i_2}{1 - c_1 - i_1}} < 1$.

We should think of the multiplier in part 3 as a "partial equilibrium" multiplier, that gives us the effect of a dollar increase in G when the interest rate is not able to adjust. However, in "general equilibrium" the interest rate will tend to increase when the IS shifts outwards. If the interest rate did not increase, the money market would not be in equilibrium (money demand goes up as income increases, but money supply is fixed). The interest rate needs to increase. This is why output increases by less than in part 3.

Multipliers in the IS-LM model

$$\frac{dY}{dT} = \frac{-c_y}{(1 - c_y - I_y) + I_i \frac{L}{Y L_i}}$$

$$\frac{dY}{dG} = \frac{1}{(1 - c_y - I_y) + I_i \frac{L}{Y L_i}}$$

$$\frac{dY}{d(M/P)} = \frac{1}{(1 - c_y - I_y) \frac{Y L_i}{I_i} + L}$$

14.02

2/22

(15 min late)

- Applications of IS-LM

IS-LM

IS $Y = C(Y-T) + I(Y, i) + G$

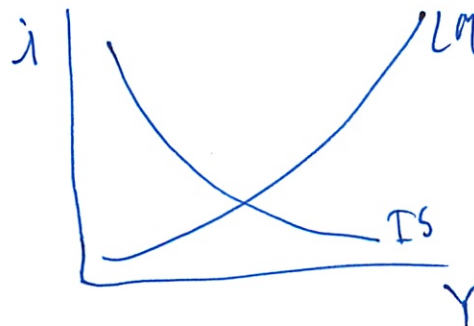
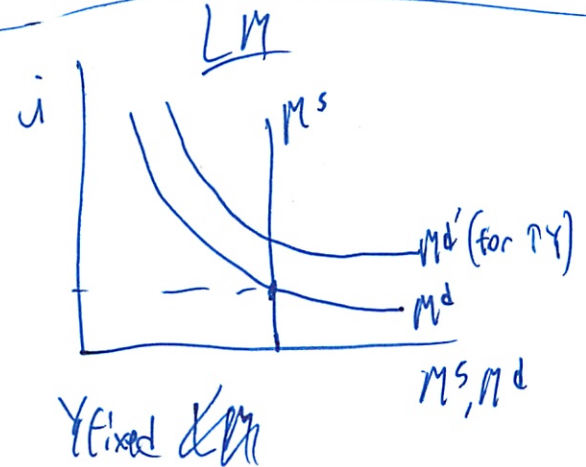
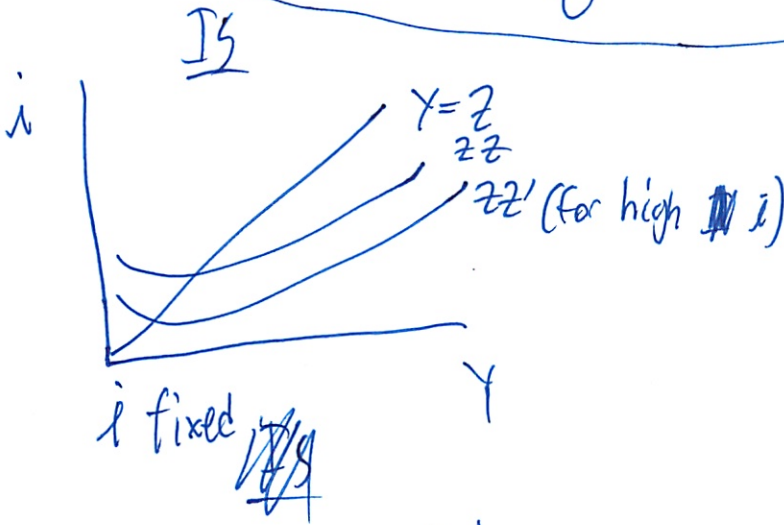
⊕ ⊕ ⊖

Exog $T, \bar{G}, \bar{M}, \bar{P}$

LM $\frac{M}{P} = Y \cdot L(i)$

⊖

End Y, C, I, i, L



Any point on IS is equilibrium of ^{goods} ~~the~~ market

" " " LM " " " financial market

Only one point that is equilibrium on both

Model can only answer for changes in exogenous

(2)

$$i = \frac{100 - P_B}{P_B} = \frac{100}{P_B} - 1$$

$$LM: \frac{M_d}{P} = Y_0 L(i)$$

$$M^s = M$$

Solving algebraically

- need $I()$ fn

- given in P-set

$$I(Y, i) = a + bY - ci$$

- Now can solve b/c linear eq

$$Z = C(Y - T) + I(Y, i) + G$$

When shifting demand curve, we're not shifting equilibrium

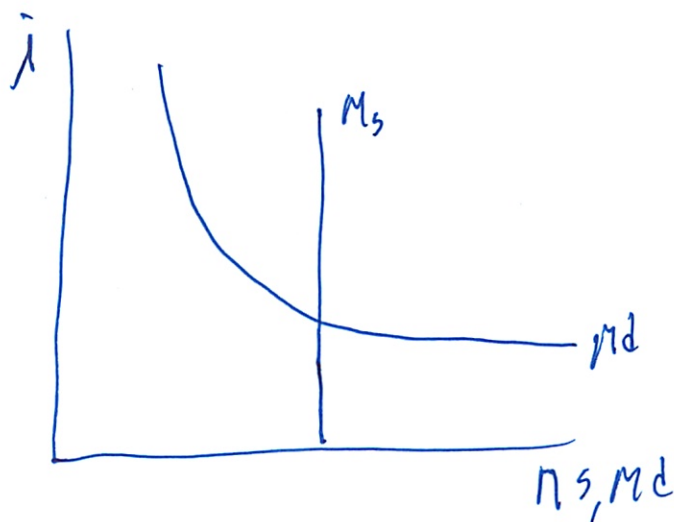
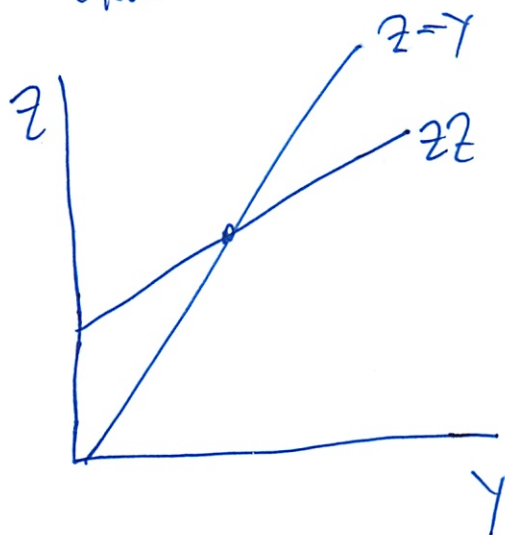
IS shifts $\downarrow \uparrow$

* make sure get it right

LM shifts $\rightarrow \leftarrow$

(coming in late confused me - but ~~same~~ same as was in textbook)

③ Start clean



Then gov says i big stimulus package, fiscal expansion, $G \uparrow$

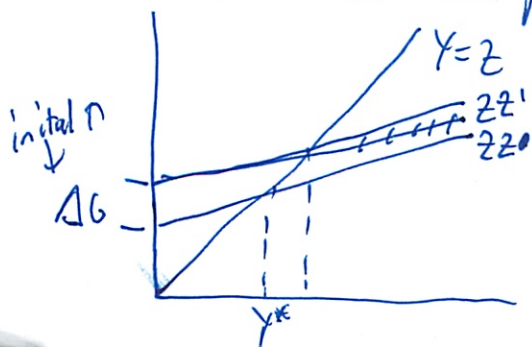
$$\underline{I_s Z = C + I + G}$$

$$Z = Y$$

So ZZ curve will shift up ^{or} ~~and~~ down

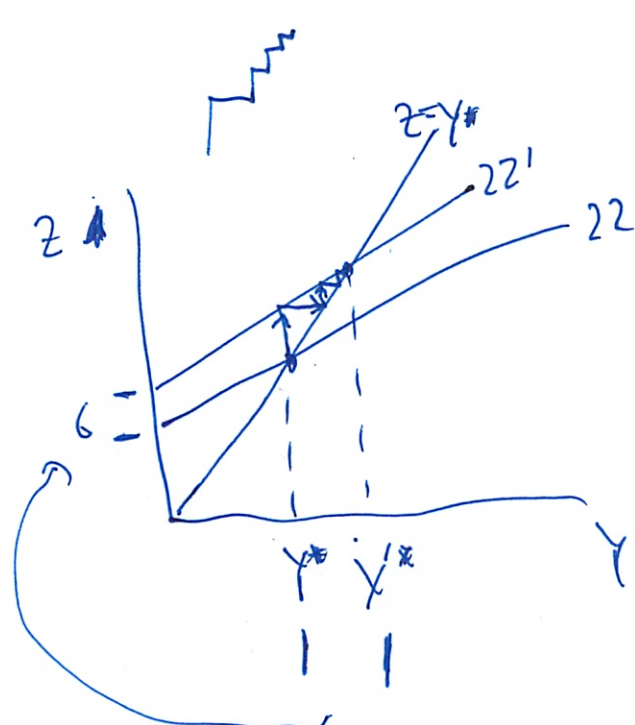
i is fixed for now - on the IS chart

For same level of output, there is more demand



4

Multiplier effect between ~~C~~ ~~I~~ C and I



That distance is bigger than that

← ratio $\frac{\text{this}}{\text{that}}$ is multiplier
↑ or reversed

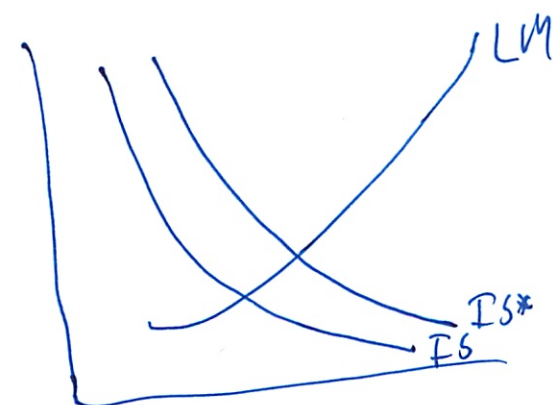
In LM model,

No change

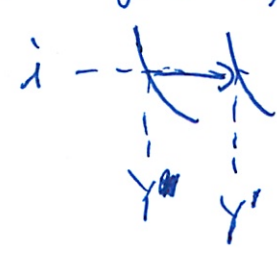
Y is fixed

Shift along curve

Together

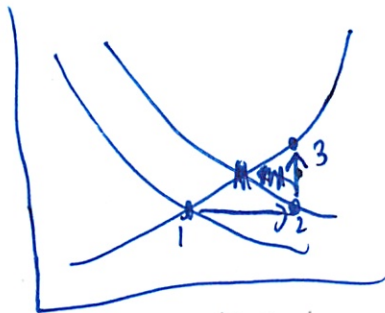


At a given i , greater output



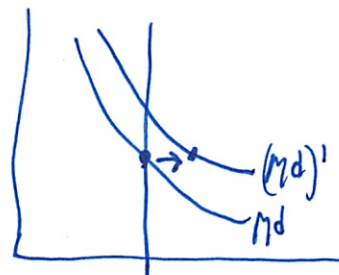
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Then shift to new equilibrium on LM curve



or it would help
if I labeled my graph!

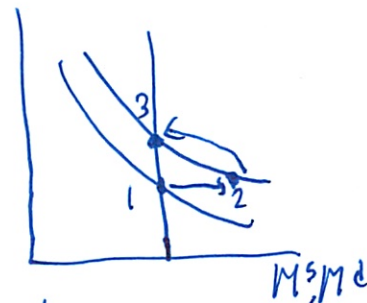
For any i , when $Y \uparrow$, $M_d \uparrow$



So i adjusts

Move along money demand curve

Back to equilibrium



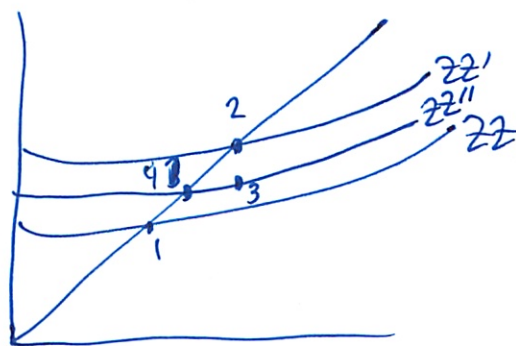
Output did not change on that last part

- any point on curve has same output

But now IS market is not on equilibrium

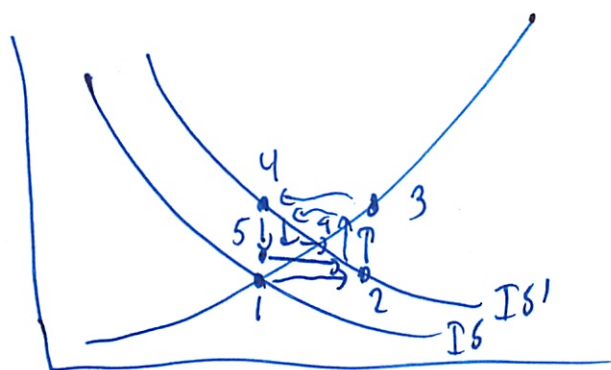
When $i \uparrow$, $I \downarrow$

So $Z \downarrow$

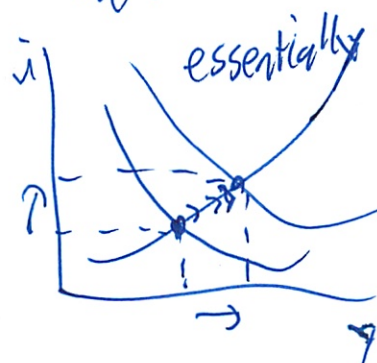


④

So now out of equilibrium in LM
So repeat



Feedback effect to new equilibrium



The LM dampened the impact of just the IS \uparrow
from extra gov spending

Suppose to happen 'instantaneously'
But not in real life

2006

Total Pop 301 million

Non inst civilian pop 228 mill
- people who can work

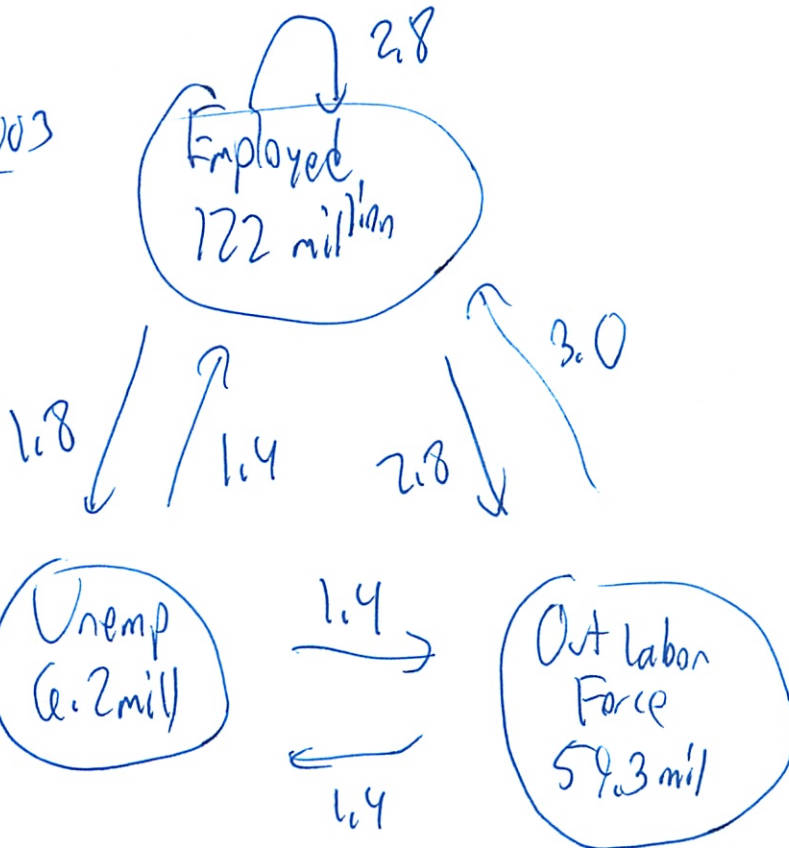
In Labor force ~~151.4~~ 151.4 mill
↳ have a job or actively looking

Out of labor force 72.4 million

Employed
144 million

Unemployed
7.0 mil

1996-2003



Flows ~~pretty~~ pretty large

②

Flows volatile

Flows determine health of labor market

Europe; much harder to hire or fire

- smaller flows in Europe

What is the optimal level of unemployment?

- need a buffer

- takes time to move from job to job

Graphs of 'inflows + outflows from unemployment

Remember also that people leaving workforce

Graph of # as % of last month's #
Employment

25-30% of people every 3 months flow into + out of economy

Huge spike in unemployment

But almost no change in inflows to employment

(Slides over)

Wage Determination

$$\frac{w}{\text{wage}} = F(u)$$

↑
unemployment
rate

↓ See below

3

$$u = \frac{U}{L}$$

$U \leftarrow \# \text{ people unemployed}$
 $L \leftarrow \# \text{ people in labor force}$
 \uparrow
unemployment rate

$$E = \# \text{ people employed} = L - U$$

z = catch-all variable = anything else that affects wages

- level of unemployment benefits

- if high, can hold out longer, so wage is higher
~~rate higher~~

- republicans: moral hazard \rightarrow decreases incentive to work

- Unions/collective bargaining

- wages generally higher

- demand for goods

- more labor

p^e = expected price level

- of goods + services for living

$$W = p^e F(u, z)$$

\uparrow
nominal wage
level

9

Now try to endogenize price level

LS was exogenous in LS-IM model

Production Function

$$Y = A \cdot E^{\alpha} \cdot K^{1-\alpha} \quad (A=1)$$

↑
production
or GDP

↑
productivity

↑
employment

↑
Capital
takes time
to adjust
K_t

- ignore for now

$$Y = A \cdot E$$

Price setting

$$P = W$$

Price driven to cost in world w/ perfect competition

But there are barriers to entry

So are profits μ = Markup

$$P = W(1 + \mu)$$

Next time equilibrium P, W

14.02

2/25

Need to let Nina know day in advanced if have any special requests to go over stuff

Question from Last Exam

- Solving IS-LM w/ Liquidity Trap

$$P = 1$$

$$\frac{M^d}{P} = Y - r$$

$$C = 1 + .5Y$$

$$I = 1 - .5r$$

$$G = \bar{G}$$

$$Y = C + I + G$$

$$\frac{M^s}{P} = \frac{\bar{M}}{P}$$

$$\frac{M^d}{P} \leq \frac{M^s}{P} \quad \text{with} \quad \frac{M^d}{P} = \frac{M^s}{P} \quad \text{if } r \geq 0$$

$$r = i - \pi^e$$

$$\pi^e = 0$$

a. Explain min value r can take

0, can't go below 0

Otherwise just hold cash, not bonds

②

1.
b Derive the IS curve

$$Y = C + I + G$$

$$Y = 1 + .6Y + 1 - .5r + \bar{G}$$

Solve for Y

$$Y(1 - .5) = 2 - .5r + \bar{G}$$

$$Y = \frac{1}{1 - .5} 2 - .5r + \bar{G}$$

$$Y = -4 + r + \frac{\bar{G}}{.5} \quad \leftarrow \text{so is } \bar{G}$$

r is exogenous here

Answer in terms of r

- ~~Wait~~ $r = 4 - Y + 2\bar{G}$

TA: Even better ans in terms of Y

3)

c) Derive the LM curve

$$M^d = PY L(i)$$

$$\frac{M^d}{P} = Y - r$$

exogenous here

$$\frac{M^s}{P} \text{ is fixed}$$

$$r = \bar{i} - 0$$

$$\frac{M^d}{P} = Y - \bar{i}$$

$$\text{So } \frac{M^d}{P} = \frac{M^s}{P} = \frac{\bar{M}}{P}$$

$$\frac{\bar{M}}{P} = Y - \bar{i}$$

$$\text{Watch } PY = P \cdot Y$$

$$\frac{\bar{M}}{P} = P \cdot Y - \bar{i}$$

or not given above

$$\rightarrow r = Y - \bar{M}$$

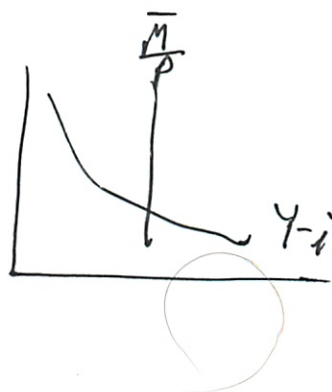
I had
but not
in terms of r

Oh piecewise

$$r = \begin{cases} 0 & \text{if } Y \leq \bar{M} \\ Y - \bar{M} & \text{otherwise} \end{cases}$$

← did not handle special case

$P=1$ as well



4

d) What are the equilibrium interest rate + output level in econ? Equilibrium r to be positive?

So solve IS LM

Set $r =$ to each other

$$Y - Y + 2\bar{G} = \begin{cases} 0 & \text{if } Y \leq M \\ Y - M & \text{otherwise} \\ & \text{if } M < Y \end{cases}$$

$$Y + 2\bar{G} = 2Y - \bar{M}$$

What solving for

$$\begin{matrix} i \\ \downarrow \\ Y \end{matrix}$$

$$2Y = Y + 2\bar{G} + \bar{M}$$

$$Y = 2 + \bar{G} + \frac{\bar{M}}{2}$$

Solve for r as well separately

For r to be + $M < Y$?

Interest rate positive if $\bar{M} < Y + 2\bar{G}$

Equilibrium rate

$$r^* = 2 + \bar{G} - \frac{\bar{M}}{2}$$

$$Y^* = 2 + \frac{\bar{M}}{2} + \bar{G}$$

⑤

Set r or $Y =$ to each other
! do both separately!

Otherwise $r = 0$

$$Y = Y + Z\bar{G}$$

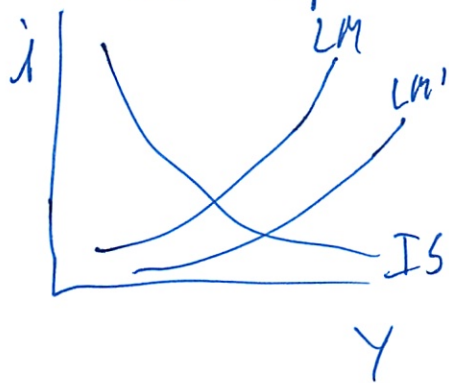
? this when $\bar{M} \geq Y + Z\bar{G}$

$$M \geq Y$$

so plug Y in
and have $r = 0$

So I did not take it far enough

e) Suppose gov ~~mon~~ expansionary monetary policy
(Interesting to see - be able to solve this on my own!)



Does not work when interest rates $= 0$

Nowhere to go past that, the Liquidity trap
 \bar{M} was exogenous above

Above eq showed that \bar{M} no longer part of Y^*
above

(ok that's what we covered in recitation)
(will take lighter notes)

$\frac{3}{4}$ quits $\frac{1}{4}$ layoff

50,000 worker separations / day

unemployment $\sim 2-3$ months avg in US

high flows

nonemployment rate - broader measure

people don't quit job when high unemployment

and people have higher prob of losing jobs

$< 15\%$ have collective bargaining

~~Wage~~ $>$ reservation wage
wages in all countries

^{wage}
depends on employment conditions

depends on bargaining power

- which depends on replacement cost

(nice simple explanation)

when unemployment high, easier to find replacement, so wages \downarrow

(2)

paying good \rightarrow higher productivity = efficiency wage
and \downarrow turnover

$$W = p^e F(u, z)$$

\uparrow expected price level
 \uparrow unemployment rate
 \uparrow catchall

- unemp insurance, \uparrow the wage
emp protection

min wage props all other wages up

Workers care about amt of goods can buy w/

$$Y = AN \quad \leftarrow \text{production function}$$

\uparrow output
 \uparrow employment
 \uparrow productivity

assumes constant returns
to add. labor

plus firms use capital + raw materials

Simplify by setting $A=1$ as baseline

$$Y = N$$

so marginal cost is W , the wage

In perfect market $P = W$

But firms add a markup

$$P = (1 + \mu) W$$

③

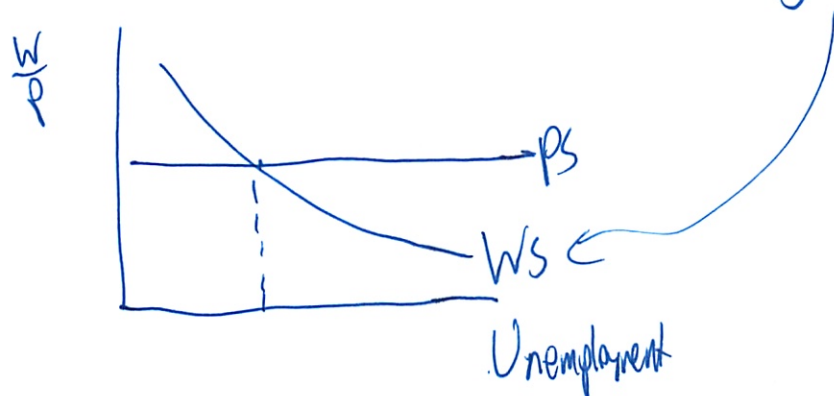
6.5 Nat Rate Unemployment

- Use actual P , not P^e for rest of chap

$$W = P F(u, z)$$

$$\boxed{\frac{W}{P} = F(u, z)} = WS$$

$\uparrow u_{emp} = \downarrow \text{wage} \Leftarrow \text{wage-setting relation} = WS$



$$\frac{P}{W} = 1 + \mu$$

$$\boxed{\frac{W}{P} = \frac{1}{1 + \mu}} = PS \quad \text{implied real wage}$$

\uparrow in markup of firm must \uparrow wages or real pay \downarrow

Equilibrium unemp rate

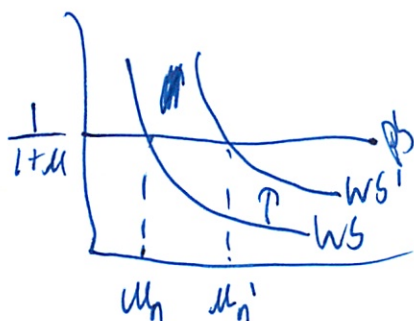
$$F(u_n, z) = \frac{1}{1 + \mu}$$

\uparrow natural rate of unemployment

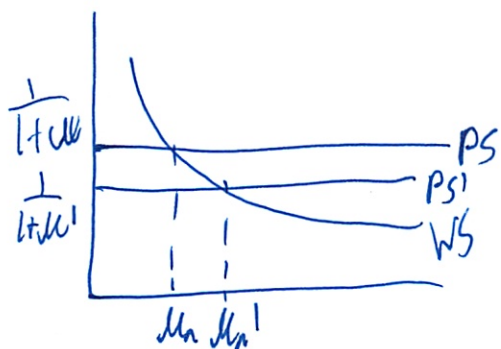
④

Called natural, but is based on gov policy

Like if \uparrow unemp benefits



Or if $p \uparrow$ since \downarrow antitrust regs



natural level of employment

$$u = \frac{U}{L} = \frac{L-N}{L} = 1 - \frac{N}{L}$$

$N = \text{employment}$

$$N = L(1-u)$$

$$N_n = L(1-u_n)$$

natural level of output

$$Y_n = N_n = L(1-u_n)$$

$$F\left(1 - \frac{Y_n}{L}, z\right) = \frac{1}{1+u}$$

the equilibrium

5

Assumptions made

- equilibrium in labor market

- $p^e = p$

- but that might not be true in short run

- need to use eq from previous chaps effect

- but expectations not off base for a long time

- goes back to medium-run level

(Perhaps read too fast)

(Not really seeing big picture)

* natural rate of unemp = unemp rate such that

real wage chosen in wage setting =
real wage implied by price setting

$$W = P F(u, z)$$

$$\frac{W}{P} = F(u, z)$$

$$F(u_n, z) = \frac{1}{1+u_n}$$

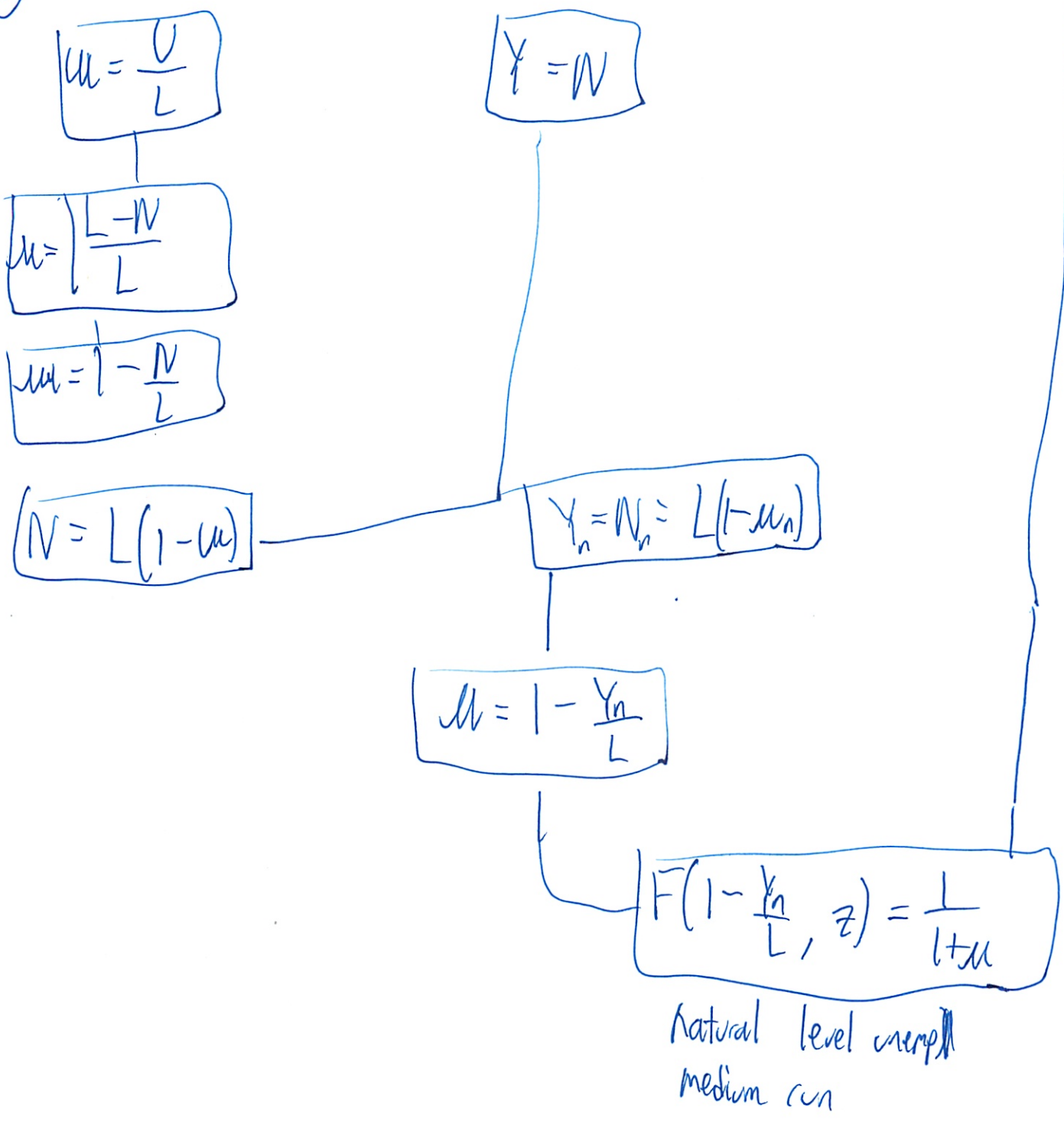
$$P = (1+u)w$$

$$\frac{P}{W} = 1+u$$

$$\frac{W}{P} = \frac{1}{1+u}$$



6



(This eq chart was very helpful!
I should make it for other concepts!)
(And how this fits in to other stuff)
(Should publish charts)

AS-AD Chap 7 Reading

2/25

put together short + medium run output

↑
Chap 5

↑
Chap 6

goods +
financial

labor

get
aggregate supply and aggregate demand
↑ from chap 6 ↑ from chap 5

7.1 Aggregate Supply

$$W = P^e F(u, z)$$

$$P = (1 + u) W$$

here don't assume $P = P^e$

First remove W from eq

$$P = P^e (1 + u) F(u, z)$$

↑ constants ↑

$$u = \frac{U}{L} = \frac{L - N}{L} = 1 - \frac{N}{L} = 1 - \frac{Y}{L}$$

since $Y = N$ - one unit of output
req 1 extra worker

↑ Labor force

so

$$P = P^e (1 + u) F\left(1 - \frac{Y}{L}, z\right)$$

②

\uparrow in output = \uparrow in price level

- Since \uparrow output = \uparrow E

\uparrow E = \downarrow U = \downarrow M

\downarrow M = \uparrow ~~W~~ W

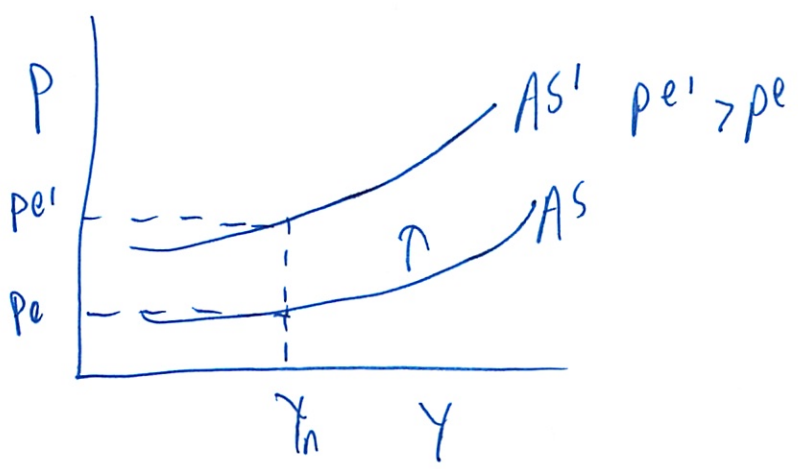
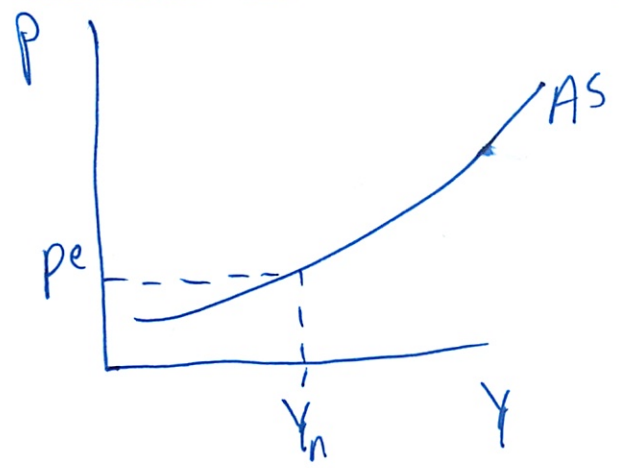
\uparrow W = \uparrow P

\uparrow in p^e = \uparrow P actual

- one for one

1. If wage setters expect P level \uparrow , will set \uparrow W

2. This \uparrow W leads to an \uparrow costs, so \uparrow P



(3)

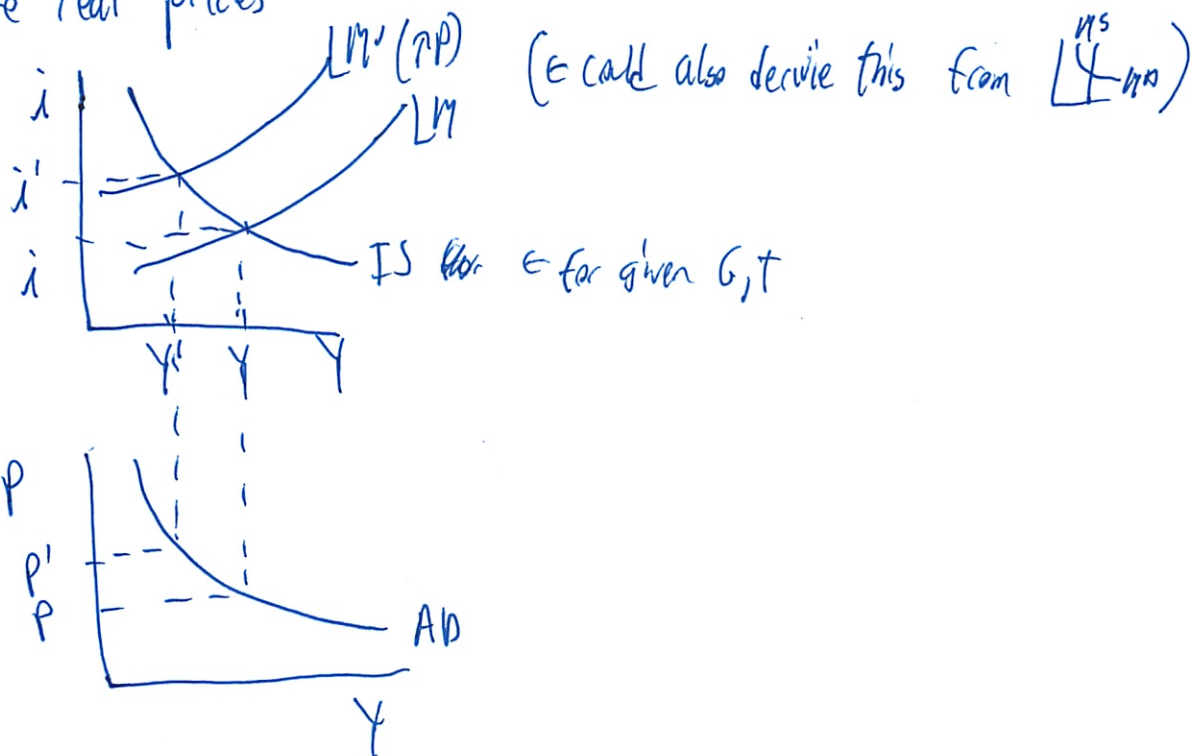
7.2 Aggregate Demand

$$Y = (Y - T) + I(Y, i) + G$$

Equilibrium _{goods} → output = demand for goods

$$\frac{M}{P} = Y L(i)$$

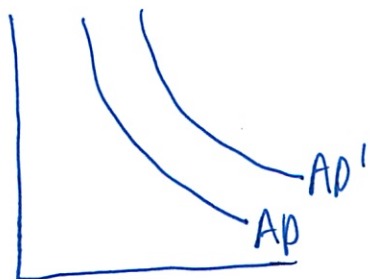
Equilibrium financial → ~~output~~ = supply = demand for M
note real prices



(In HS we will up the other way)

Any variable other than P , shifts either IS or LM

↑ G



4

$$Y = Y\left(\frac{M}{P}, G, T\right)$$

⊕ ⊕ ⊖

* 7.3 Equilibrium Short Run + Medium Run

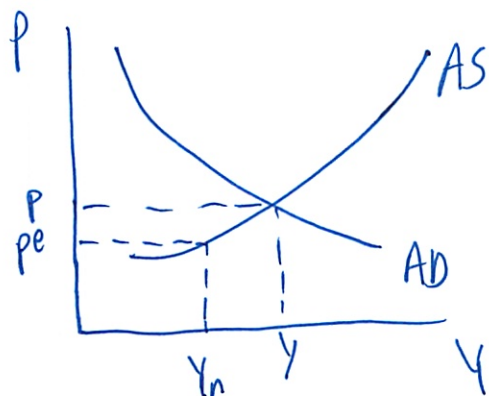
AS	$P = P^e (1 + \mu) F\left(1 - \frac{Y}{L}, Z\right)$
AD	$Y = Y\left(\frac{M}{P}, G, T\right)$

for given $\underbrace{P^e}_{AS}, \underbrace{M, G, T}_{AD}$

get Y, P

P^e - is based on position of AS

→ SR - can take P^e as given



no reason why $Y = Y_n$
 - depends on $AS(P^e)$ and $AD(M, T, G)$

↑ $Y > Y_n$ here, but can be other way

* In SR, no reason why $Y = Y_n$

⑤ (So how does what we see in real world affect this?)

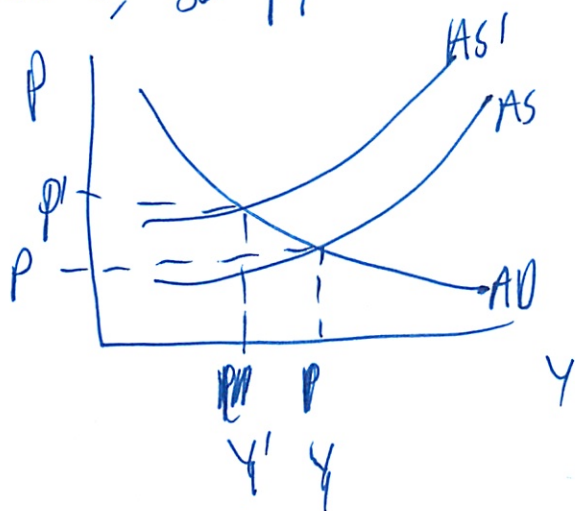
Medium Run (how does this work in real world)

If $Y > Y^n$, then $P > P^n$, so ~~wage~~ ~~up~~ ~~we~~ ~~plu~~

$$W < W^e$$

So next they can, wage setters $\uparrow W$

So AS \uparrow , so $P \uparrow$



Shift continues

Until $Y = Y^n$

So $P = P^e$

And W stop \uparrow

(Don't remember learning in this)

So in medium run ~~the~~ $Y = Y^n$

Works in symmetrical case

⑥ Remember $AD \hookrightarrow$, $AS \uparrow$

7.4 Effects of Monetary Expansion

$\uparrow M$ to M'

$$Y = Y\left(\frac{M}{P}, G, T\right)$$

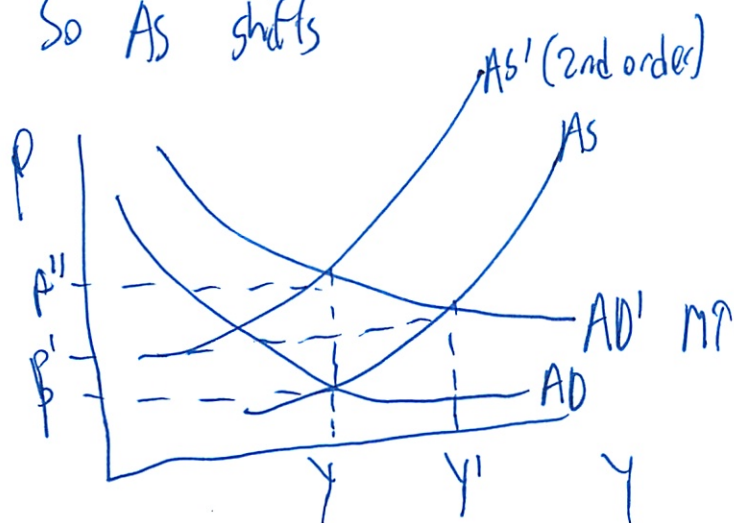
P Given

$$\uparrow M = \uparrow \frac{M}{P} = \uparrow Y$$

Then over time price expectation comes into play

P higher than W settles expected

So AS shifts



* Price level in proportion w/ nominal M *

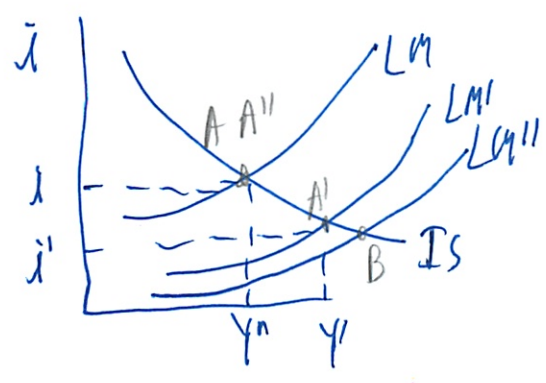
- Since real M is held steady

⑦

Behind the Scenes

look at what happens to i in LS-IM model

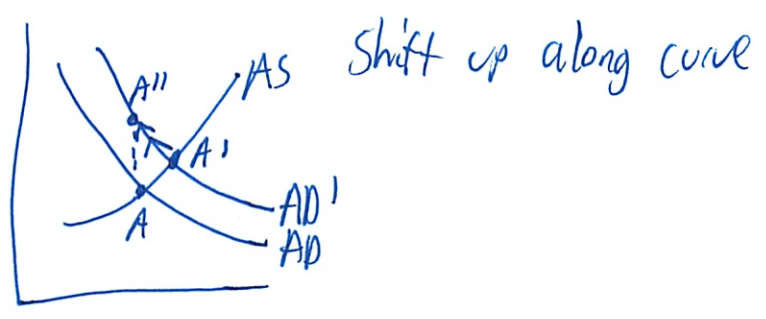
draw first



two effects: \uparrow in M so from $LM \rightarrow LM''$
 $\uparrow P$ offsets above so $LM'' \rightarrow LM'$ } so $LM \rightarrow LM'$ net
 even in short run

Over time since $Y > Y_n$, P continues to \uparrow , so LM shifts back up
 Eventually LM returns where it was, Y_n back, i back, but higher P

draw second



this is called neutrality of money

only temporary

takes about 4 years according to Taylor model

(They should demo a real model + source code to us)

(reading this chap much closer)

⑧ 7.5 Decrease in budget deficit

$$G \downarrow \rightarrow G'$$

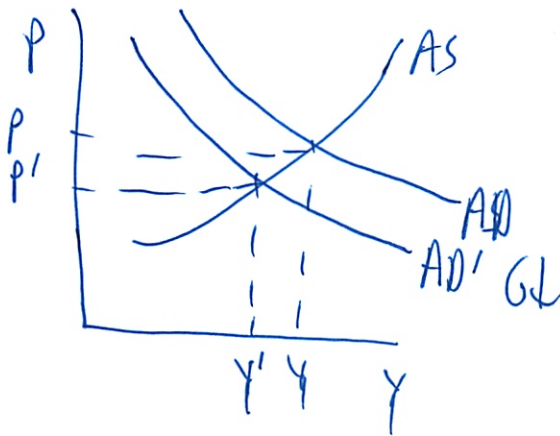
T unchanged

Output initially $= Y^n$

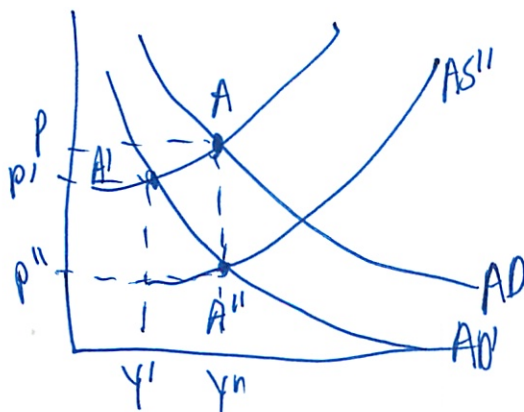
$\downarrow G =$ shift in AD

For given p , lower Y

Short run



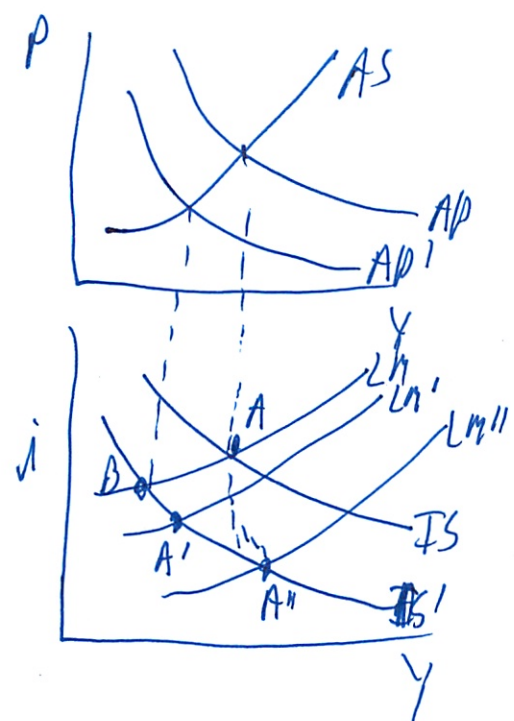
Over time $Y < Y^n$ so AS curve shifts \downarrow moving along AD
Until output back at Y^n



P, Y are now lower

- look at underlying IS-LM model
(should practice building up)

9) Deficit Reduction, Output, Interest Rate



As gov ↓ deficit $IS \rightarrow IS'$

If price would not change $A \rightarrow B$

But since $P \downarrow$, $\frac{M}{P} \uparrow$, ~~expansion~~ so $LM \rightarrow LM'$
 - offsetting some of the impact

So we move from $A \rightarrow A'$

- Y is lower, i is lower

- But don't know ^{investment} ~~interest rate~~ since $\downarrow Y = \downarrow I$ but $\downarrow i = \uparrow I$

So long as $Y < Y^n$, P continues to ↓, $\frac{M}{P}$ continues to ↑ (Point A'') on LM''

At A'' output back to normal, but $i \downarrow$
 (C is same, I must be higher
 - but the amt of the $\downarrow G$!

⑩ Summarize

SR $\rightarrow Y \downarrow, I \text{ may } \downarrow$

w/o change in monetary policy

gov should $\uparrow M$ enough to offset

MA $\rightarrow Y = Y^n, i \downarrow, I \uparrow$

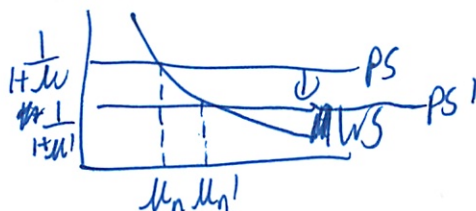
All of this also applies to measures to \uparrow private saving

7.6 Changes in Price of Oil (\uparrow in μ [markup])

but how does it fit into AS-AD model?

Since we assumed output was only ~~price~~ based on labor
We will roll into μ - the markup

$$\frac{W}{P} = \frac{1}{1+\mu}$$



\uparrow in μ is an downward shift of PS

\hookrightarrow the lower the real wage implied by price setting

So real wage \downarrow , means \uparrow Unemployment, means $\frac{W}{P} \downarrow$, means $\downarrow \mu_n$

So $Y^n \downarrow$

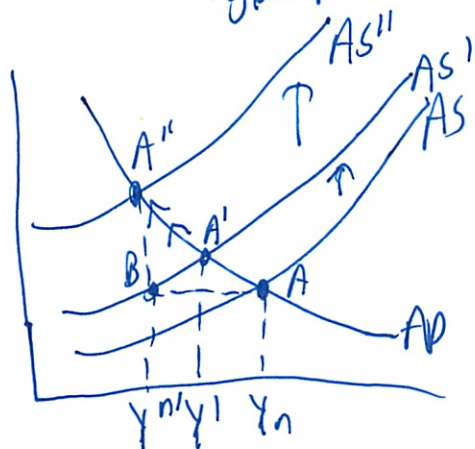
11

Dynamics of adjustment

$$Y^n \rightarrow Y_n'$$

$$P = P^e(1 + \mu) F\left(1 - \frac{Y}{L}, z\right)$$

$\mu \uparrow = \text{given } P^e$, so $AS \uparrow$ to point B



Assume AD Does not shift

- but may ↓
- lower I^e
- oil producers don't consume as much as oil buyers
- may also ↑
- so assume no shifts

Econ moves along AD from $A \rightarrow A'$

Y' is still $> Y_n'$

So effect continues

matches 1970s

stagflation = negative growth + high inflation

(12)

7.7 Conclusions

- diff in SR, MR effects is why some economists disagree
 - which is more important?
- Gives a general way to think about output fluctuation/biz cycle
 - econ always hit by shocks
 - carried through by propagation mechanisms
- here we assumed constant nominal M
- next chaps about money growth

1 Diamond-Mortensen-Pissarides

- We start with flows in page 115 of the textbook. We simplify by assuming no one is out of the labor force (or that we only study transitions between employed and unemployed, even if other flows also matter).
- We will think of everything happening within a year, but we could have assumed any time interval.
- Define the following variables:

u = unemployment rate

v = vacancies

h = hires

s = separation rate

- Vacancies are simply available jobs that have not been filled yet. Note that the labor market described by the model in the textbook does include vacancies as a variable.
- Hires are the number of vacancies filled this year and the separation rate is the number of workers who lose their jobs this year.
- The key variable will be "market tightness", defined as v/u . When markets are "tight" (i.e. v/u is high), there are a lot of vacancies and not many unemployed.
- The key assumption that we are going to make is that it is difficult to match workers with job. By "difficult" we mean that matching is time consuming and/or is costly. We model that process using a matching function. We assume the following form:

$$h = m\sqrt{uv},$$

where m is a parameter that controls the quality of the matching (the higher m , the more hires, h). The matching function captures the intuitive idea that if there are more vacancies or more unemployed people, there will be a higher number of hires in the economy. A great advantage of the matching function is to identify that the macroeconomic variables u and v are the main variables that determine h , without having to explicitly figure out how each individual job is being filled and what each individual person does to find a job.

- The matching function has "constant returns to scale" (which has empirical support), meaning that when both vacancies and unemployment double, the number of hires also doubles.

- The separation rate s indicates the number of workers who lose jobs per year. We will assume that this separation rate is exogenous (that is, given). So we carefully model how many jobs are created through the matching function, but we have nothing to say about how many jobs are destroyed or why.
- Now we analyze the two sides of the market, first unemployment dynamics and then vacancies dynamics (by "dynamics" we mean "evolution over time"). From now on, we normalize the total size of the labor force to 1 (how would the analysis below change if the labor force were L instead of 1?).

1. Unemployment dynamics are given by:

$$u_{t+1} - u_t = s(1 - u_t) - h_t.$$

The left-hand side, $u_{t+1} - u_t$, is the change in the unemployment rate from last year to this year. When the change in unemployment is zero, we say that unemployment is at its steady state (it is no longer changing over time)

In steady state $u_{t+1} - u_t = 0$, and so

$$h^{ss} = s(1 - u^{ss}),$$

where we use the superscript ss to denote that variables have reached their steady state. Plugging in the matching function gives a relation between the unemployment rate u and vacancies v :

$$m\sqrt{u^{ss}v^{ss}} = s(1 - u^{ss}).$$

We call this relationship the Beveridge curve after William Beveridge (1879-1963). It is downward sloping if we plot it in a graph where the x-axis is u and the y-axis is v (we call this "uv space"). See figure.

2. Vacancies dynamics:

$$v_{t+1} - v_t = \frac{u_t}{v_t} - \frac{1}{x}$$

where x is a variable that shifts the underlying demand for labor (more on this below).

In steady state, $v_{t+1} - v_t = 0$ and

$$v^{ss} = xu^{ss}$$

which traces a ray from the origin in "uv space". See figure.

Let's focus now on the steady-state only, forgetting about "dynamics". What determines x ? We can assume that the unemployment wage (e.g. value of leisure + unemployment benefits - in page 120 of the textbook

it is called the reservation wage) is exogenously given by b and that each firm produces an exogenously given level of output y . Then the wage w is determined by the relative bargaining power β of workers and firms (see page 121 of the book for a discussion on bargaining) and the market tightness

$$w = \beta b + (1 - \beta)y + \frac{v}{u}.$$

If $\beta = 0$, workers appropriate all output. If $\beta = 1$, employers get all the surplus. Now think intuitively, why does market tightness affect the wage? If there are a lot of unemployed people and not too many vacancies available, what should happen to the wage?

We can use our understanding of how the wage is determined to define the factors that shift the demand for labor (why does the demand for labor depend on the wage?)

$$x = w - \beta b + (1 - \beta)y$$

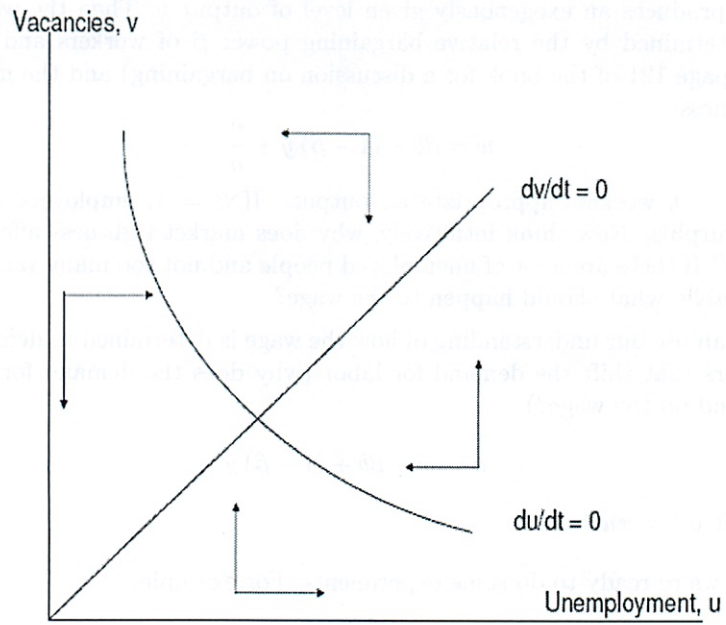
to get $v^{ss} = xu^{ss}$.

- Now we're ready to do some experiments. For example,

$$\text{when } x \uparrow \text{ then } u \downarrow \text{ and } v \uparrow$$

Another interesting thing to look at is h/u , which is the exit rate from unemployment or, equivalently, u/h which is the average duration of unemployment (why is that?). In this case, when $x \uparrow$ we have $u/h \downarrow$

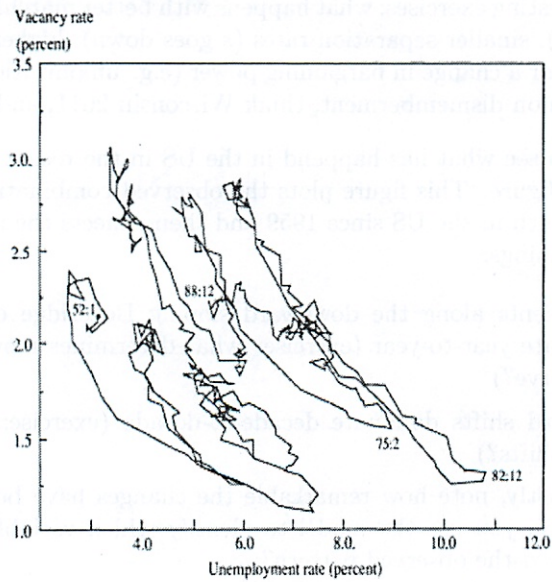
- Other interesting exercises: what happens with better matching technology (m goes up), smaller separation rates (s goes down), higher productivity (y goes up) or a change in bargaining power (e.g. unionization, and β goes down, or union dismemberment, think Wisconsin 2011, and β goes up)?
- We can also see what has happened in the US in the recent past. Look at the second figure. This figure plots the observed combination of u and v for each month in the US since 1959 and then connects them by lines. We note three things:
 1. Movements along the downward sloping Beveridge curve seem to dominate year-to-year (exercise: what determines movements along this curve?)
 2. Outward shifts dominate decade-to-decade (exercise: what creates these shifts?)
 3. And lastly, note how remarkable the changes have been in the crisis. Can you use the model to identify which variables could have produced the observed pattern?



Olivier Jean Blanchard and Peter Diamond

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Figure 8. Beveridge Curve, 1952-88



14.02 Diamond Labor model

2/28

- flows are critically important
 - matching b/w employers + employees
 - friction in labor market
 - Vacancies and unemployment exist simultaneously
 - odd from a supply + demand POV
 - Matching workers to jobs is ~~is~~ long + difficult
 - Steady state of vacancies and unemployment
-

u = unemployment %

v = vacancies = available jobs

h = hires = vacancies filled

s = separation rate = quit or fire

Over a certain period
of time

$\frac{v}{u}$ = market tightness

- high = tight = lots of vacancies w/ few unemployed

$h = m\sqrt{uv}$ matching function

- difficult/costly/tough to find vacancies

m = quality of matching, matching technology

↳ the higher the m , the higher the h

homogeneous of degree 1 in uv

↳ constant returns to scale

②

So if double vacancies + unemployment \rightarrow # of hires also double

If scale model only to look at a state - ~~get~~ ~~can~~ still works

Separation rate = exogenous (given)

Look at dynamics (evolution over time)

- Normalize labor force size to 1

$$\underbrace{u_{t+1} - u_t}_{\text{Change in unemployment rate}} = \underbrace{s(1 - u_t)}_{\substack{\text{Separated} \\ \text{people}}} - \underbrace{h_t}_{\substack{\text{people} \\ \text{hired}}}$$

When $= 0$ = steady state

$$\hookrightarrow h^{ss} = s(1 - u^{ss})$$

\rightarrow separation rate = % of pop that is employed

Matching fn - equate.

$$m \sqrt{u^{ss}} v^{ss} = s(1 - u^{ss})$$

Beveridge curve

- ~~downward~~ downward sloping



Vacancies dynamics

$$V_{t+1} - V_t = \frac{u_t}{V_t} - \frac{1}{\chi}$$

χ is variable that shifts the underlying demand for labor

③

In steady state $V_{t+1} - V_t = 0$

So $V^{ss} = X U^{ss}$



$\frac{1}{X}$ is related to how many new vacancies you want to create

Wage

$$W = \beta b + (1 - \beta) y + \frac{V}{U}$$

- high wage - corps don't want to hire

b = reservation wage = wage people get if unemployed

↳ exogenously given

↳ basically unemployment benefits

y = Output per worker

β = relative bargaining power

$\beta = 0$ workers get to appropriate all output

$\beta = 1$ employers get all surplus

$\frac{V}{U}$ = tightness - more outside options

(assume $y > b$ - has to be for it to work

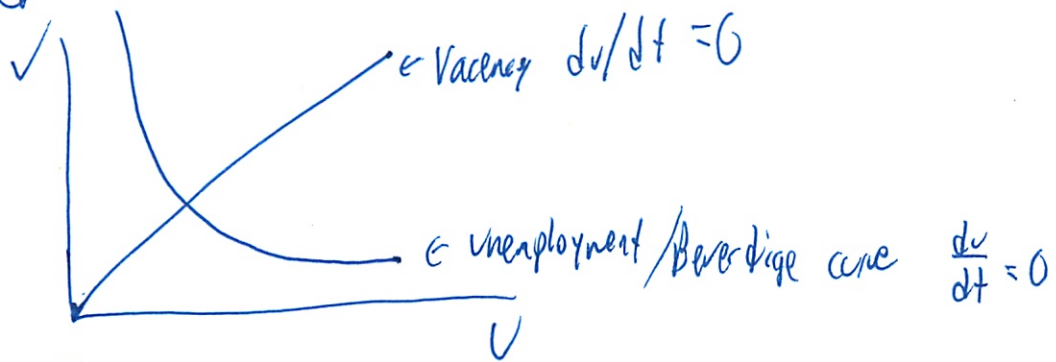
Nina thinks should be inside β

So X

$$X = W - \beta b - (1 - \beta) y$$

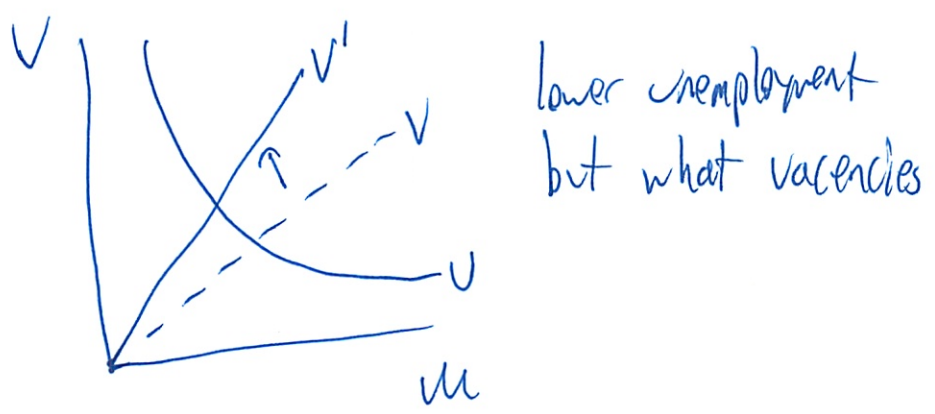
to get $V^{ss} = X U^{ss}$

④

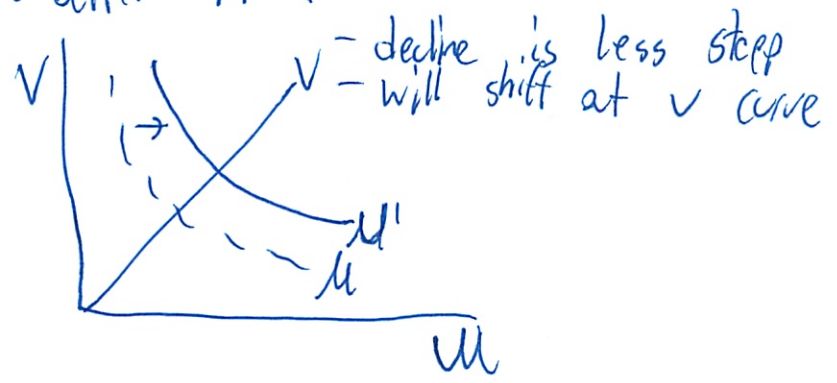


Now can do all sorts of comparative statics

What if $w \uparrow$



With better $m \uparrow$



Random
Don't know if
duplicate?

If not ~ 2/28

from very start of year

From nominal to real GDP and the Chain Index

You can compute the change in real GDP from year t to year $t + 1$ in two alternative ways

$$\frac{Y_{t+1}}{Y_t} = \frac{P_t Y_{t+1}}{P_t Y_t}$$

or

$$\frac{Y_{t+1}}{Y_t} = \frac{P_{t+1} Y_{t+1}}{P_{t+1} Y_t}$$

the two ways of computing it are obviously identical.

But now let there be two goods in the economy, Y_1 and Y_2 . Then the two ways of computing the change in real GDP from year t to year $t + 1$ are no longer identical:

$$\left(\frac{Y_{t+1}}{Y_t} \right)' = \left(\frac{P_{1,t} Y_{1,t+1} + P_{2,t} Y_{2,t+1}}{P_{1,t} Y_{1,t} + P_{2,t} Y_{2,t}} \right)$$

$$\left(\frac{Y_{t+1}}{Y_t} \right)'' = \left(\frac{P_{1,t+1} Y_{1,t+1} + P_{2,t+1} Y_{2,t+1}}{P_{1,t+1} Y_{1,t} + P_{2,t+1} Y_{2,t}} \right)$$

if you divide thorough $\left(\frac{Y_{t+1}}{Y_t} \right)'$ by $P_{1,t}/P_{2,t}$ and $\left(\frac{Y_{t+1}}{Y_t} \right)''$ by $P_{1,t+1}/P_{2,t+1}$ you

can verify that the two ways of computing the change in real GDP from year t to year $t + 1$ are equal only if $P_{1,t}/P_{2,t} = P_{1,t+1}/P_{2,t+1}$. Since the relative price of goods changes over time, this condition will in general not be satisfied. Thus the two expressions will give you two different

changes in real GDP. The chain index addresses the problem by simply defining the change in GDP as the weighted average of the two

$$g_{(01/00)} = .5 \left[\left(\frac{Y_{t+1}}{Y_t} \right)' + \left(\frac{Y_{t+1}}{Y_t} \right)'' \right]$$

Finally it is customary to compute the change in real GDP using an index that is (arbitrarily) set to be equal to 100 in a "base" year, say the year 2000:

$$\text{chain index}_{2000} = 100$$

$$\text{chain index}_{2001} = 100 * g_{(01/00)}$$

TABLE B-7. Chain-type price indexes for gross domestic product, 1960-2009

[Index numbers, 2005=100, except as noted; quarterly data seasonally adjusted]

Year or quarter	Gross domestic product	Personal consumption expenditures			Gross private domestic investment					
		Total	Goods	Services	Total	Fixed investment				Residential
						Total	Nonresidential			
							Total	Structures	Equipment and software	
1960	18.604	18.606	29.144	13.581	26.607	25.530	33.978	11.516	54.445	12.962
1961	18.814	18.801	29.253	13.827	26.533	25.449	33.793	11.446	54.146	12.983
1962	19.071	19.023	29.404	14.090	26.548	25.465	33.789	11.537	53.678	13.003
1963	19.273	19.245	29.648	14.306	26.463	25.391	33.784	11.636	53.581	12.901
1964	19.572	19.527	29.971	14.573	26.613	25.545	33.955	11.801	53.558	13.003
1965	19.928	19.810	30.286	14.846	27.037	25.981	34.342	12.143	53.607	13.372
1966	20.493	20.313	30.953	15.277	27.592	26.528	34.854	12.580	53.749	13.857
1967	21.124	20.824	31.499	15.786	28.320	27.271	35.741	12.973	54.940	14.339
1968	22.022	21.636	32.597	16.468	29.378	28.367	36.999	13.621	56.416	15.100
1969	23.110	22.616	33.860	17.326	30.770	29.767	38.527	14.518	57.965	16.144
1970	24.328	23.674	35.152	18.287	32.072	31.047	40.348	15.473	60.119	16.666
1971	25.545	24.680	36.208	19.285	33.671	32.611	42.246	16.664	61.905	17.632
1972	26.647	25.525	37.135	20.103	35.077	34.009	43.673	17.863	62.651	18.703
1973	28.124	26.901	39.350	21.078	36.972	35.888	45.355	19.247	63.716	20.359
1974	30.669	29.703	44.261	22.868	40.648	39.422	49.733	21.910	68.414	22.460
1975	33.577	32.184	47.837	24.836	45.666	44.361	56.591	24.534	78.523	24.547
1976	35.506	33.950	49.709	26.558	48.190	46.932	59.719	25.741	83.143	26.124
1977	37.764	36.155	52.363	28.560	51.805	50.616	63.905	27.973	88.063	28.759
1978	40.413	38.687	55.576	30.779	56.020	54.891	69.078	30.675	92.731	32.281
1979	43.773	42.118	60.832	33.353	61.099	59.866	73.606	34.238	98.610	35.902
1980	47.776	46.641	67.644	36.905	66.826	65.468	80.098	37.421	107.032	39.789
1981	52.281	50.810	72.669	40.558	73.154	71.551	87.832	42.567	114.661	43.036
1982	55.467	53.615	74.650	43.712	76.899	75.468	92.670	45.927	119.155	45.340
1983	57.655	55.923	75.997	46.433	78.706	75.349	91.843	44.757	119.406	46.380
1984	59.623	58.038	77.435	48.850	77.256	75.790	91.621	45.147	118.264	47.714
1985	61.633	59.938	78.677	51.053	78.047	76.744	92.340	46.219	118.221	48.944
1986	63.003	61.399	78.309	53.378	79.737	78.579	93.908	47.106	120.064	50.594
1987	64.763	63.589	80.827	55.413	81.263	80.036	94.753	47.863	120.750	53.079
1988	66.990	66.121	82.958	58.127	83.120	82.111	95.857	49.895	122.256	54.913
1989	69.520	68.994	86.150	60.844	85.107	84.099	98.890	51.848	123.766	56.680
1990	72.213	72.147	89.678	63.812	86.747	85.808	100.783	53.522	125.389	58.011
1991	74.762	74.755	91.870	66.586	87.981	87.082	102.341	54.491	127.178	58.771
1992	76.537	76.954	92.578	69.240	87.672	86.831	101.488	54.502	125.681	59.486
1993	78.222	78.643	93.786	71.299	88.673	87.838	101.540	56.103	124.408	61.890
1994	79.667	80.265	94.740	73.205	89.628	89.023	102.029	58.089	123.695	64.069
1995	81.533	82.041	95.625	75.370	90.840	90.060	102.247	60.601	122.265	66.403
1996	83.083	83.826	96.676	77.479	90.455	89.817	101.054	62.141	119.323	67.828
1997	84.554	85.395	96.563	79.817	90.120	89.589	99.775	64.516	115.768	69.557
1998	85.507	86.207	95.106	81.695	89.109	88.756	97.597	67.490	110.641	71.412
1999	86.766	87.536	95.603	83.515	88.989	88.700	96.173	69.559	107.406	74.151
2000	88.648	89.777	97.520	85.824	89.954	89.751	96.219	72.298	106.114	77.415
2001	90.654	91.488	97.429	88.428	90.748	90.553	95.788	76.087	103.603	80.994
2002	92.113	92.736	96.430	90.807	91.118	90.924	95.363	79.292	101.454	83.002
2003	94.099	94.622	96.380	93.692	92.411	92.301	95.355	82.174	100.267	86.953
2004	96.769	97.098	97.867	96.687	95.632	95.541	96.834	88.441	99.697	93.296
2005	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000	100.000
2006	103.263	102.746	101.508	103.411	104.371	104.419	103.534	112.922	100.194	106.081
2007	106.221	105.502	102.789	106.364	106.677	106.718	106.209	121.275	100.715	107.513
2008	108.481	109.031	106.150	110.582	107.355	107.551	107.897	125.207	101.455	105.779
2009 ^P	109.754	109.252	103.632	112.221	106.458	106.114	107.510	122.759	102.010	100.687

AS-AD model

Endogenize Price level

Combine IS-LM and labor market

Good exercise for exam

Review Labor market (from Fri) (Not ~~the~~ Diamond Model)- wage setting ^{↳ how prices/wages set}

- price setting

$$W = p^e F(\mu, z)$$

⊖ ⊕

↳ Shows frictions

$$P = W(1 + \mu)$$

↳ remember ^{markup} Production

Y = need labor + capital

$$= F(L, K)$$

↳ fixed in medium run

$$= A \cdot L^\alpha$$

↳ Labor

↑ productivity

- capital

- edu level

- technology

- management/org skills

~~Q~~ ~~APB~~

$$Y = L \text{ effective units of Labor}$$

↳ if higher unemp comp, wages ↑
Since more negotiating power

Assumed $P = p^e$

for short run - not thinking about future

- also in medium run - since wages set,
Complex mechanism

- in SR may be different

$$S_0 \quad \frac{W}{P} = \frac{1}{1 + \mu}$$

$$\frac{W}{P} = F(\mu, z)$$

$$\left. \begin{array}{l} \frac{W}{P} = \frac{1}{1 + \mu} \\ \frac{W}{P} = F(\mu, z) \end{array} \right\} \rightarrow \frac{1}{1 + \mu} = F(\mu, z)$$

↑ natural/structural
level of unemp.
(if $P = p^e$, non inflationary eq)

~~Exogenous~~
exod p^e, z, μ, U

↳ for 2 eq, need 2 unknown
end W, P, Y
- not a law
- but usually true

(still not clear on how to reach steady state - the math)

②

Now combine in $Y=L$

$$Y = L = N \cdot (1 - u)$$

↑ ↑ ↑
People Labor Unemployment
Employed Force Rate

└─┬─┘
i got backward
on P-set
- need to fix

$$u = 1 - \frac{Y}{N}$$

Combine w/ other eq

$$\frac{1}{1+\mu} = F\left(1 - \frac{Y^N}{N}, z\right)$$

↑
natural level of output
when $P=P_e$

✱

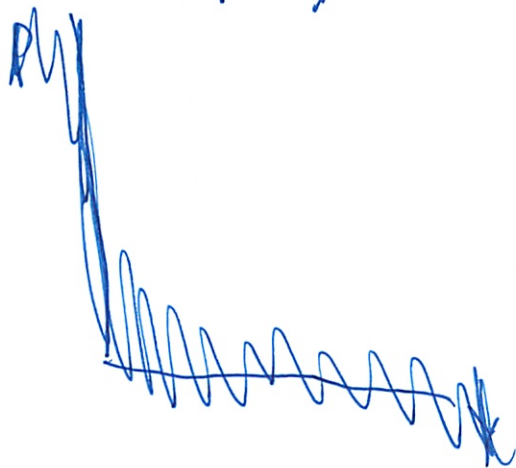
L means no inflationary/deflationary pressure
 z , markup determines this

- in real world both of these are not constant
- μ low when output is high ~~μ~~ ~~μ~~ ~~μ~~ $\mu = \mu(Y)$
- would make a nice exam question

Now combine is the $LG-IM$ model to get AS-AD model

will require all 3
to be in equilibrium

goal to endogenize price level



Chandra Prasad

Phag phn

PS $\frac{P}{W} = 1 + \mu$ AS
WS $W = P^e F(v, z)$ $\rightarrow P^e F(v, z) = \frac{P}{1 + \mu}$ \leftarrow expresses v as F_n of y
 $U = 1 - \frac{y}{N}$ $\rightarrow P = P^e(1 + \mu)$
 $F(1 - \frac{y}{N}, z)$

IS $Y = C(Y, T) + I(i, Y) + G$
LM $\frac{M}{P} = L(i, Y)$

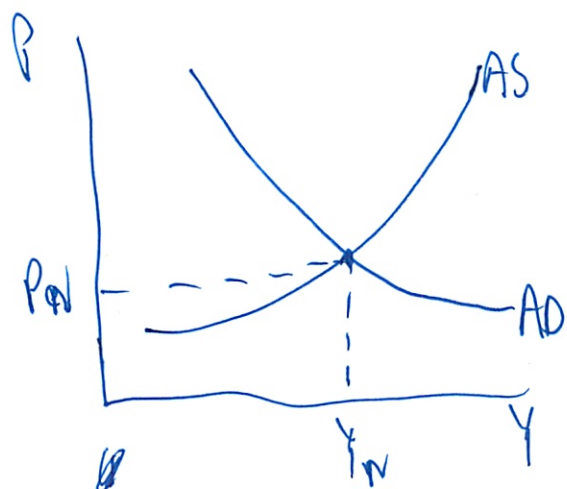
← solve for i

plug into

AD $Y = Y\left(\frac{M}{P}, G, T\right)$
↑ complicated
fun, just call Y

4

	exog	end
AS	P^e, μ, Y, N, z	P
AD	M, P, G, T	Y
AS+AD	μ, N, z, G, T, M	P, Y



higher output = higher price
each pt is = librium in labor market

When P_T, Y_L
each pt is = librium in goods + labor markets

Solve for 2 unknowns

Need fns F, Y

$$\Delta E = 0$$

$$\Delta U = 0$$

$$s > 0$$

$$f > 0$$

$$U_t = U_{t-1} = U^* \quad \leftarrow \text{why? Oh at steady state!}$$

$$E_t = E_{t-1} = E^* \quad \leftarrow \text{That's what makes it special!}$$

$$\frac{E^*}{U^*} = \frac{f}{s} \quad \leftarrow \text{then plug in}$$

$$E + U = L$$

2 eq
2 in E^*, U^*

$$E^* + U^* = L$$

solve for in terms of E, s, L

$$\text{So } E^* + U^* = L$$

$$\Delta U_t = U_t - U_{t-1} = s E_{t-1} - f U_{t-1}$$

$$\Delta E_t = f U_{t-1} - s E_{t-1}$$

TA said this might be wrong
but he did not really seem to know/be sure

At steady state $\Delta U_t = 0$ which is in problem

Problem Set # 3
14.02 Spring 2011
Due March 4

February 25, 2011

1 True/False [40 points]

Please state whether each of the following claims are True or False, and provide a brief justification for your answer. You may include graphs and equations to support your answer.

1. "An economy with a low rate of separations will have a low rate of unemployment". [5 points]
2. "In recessions, the reservation wage of the workers tends to decrease". [5 points]
3. "Expectations matter in the AS-AD model because wages are set before the price level is known". [5 points]
4. "It is not possible to have an increase in the rate of unemployment when the number of employed people is going up." [5 points]
5. "A tax on firing workers can result in less hiring." [5 points]
6. "In the AS-AD model, when output is above its natural level, wage setters revise their expectations downwards so that output decreases continuously towards its natural level" [5 points] *-1, true Sect 7.4*
7. "In the medium run, a decrease in public spending affects the AD curve, and leaves the AS curve unchanged" [5 points]
8. "According to the empirical evidence, the Beveridge curve holds at any time frequency." [5 points]

2 AS-AD [30 points]

Consider an economy with a labor force of size L . Let N denote the employment level. The production function is

$$Y = N$$

The price and wage setting relations are:

$$\begin{aligned} P &= (1 + \mu) W \\ W &= P^e (1 + z - u) \end{aligned}$$

where u denotes the unemployment rate, z denotes "other" unemployment benefits, and the rest of the variables are as in class. The consumption and investment functions are

$$\begin{aligned} C &= c_0 + c_1 (Y - T) \\ I &= b_1 Y - b_2 i \end{aligned}$$

Finally, money demand is given by

$$\frac{M^d}{P} = m_0 + m_1 Y - m_2 i$$

1. Derive the AS relation. Show your derivations clearly. [7 points]
2. Why is the AS curve upward sloping? Provide your intuition [3 points]
3. Find the natural level of output (Y_n). Draw the AS relation in the (Y, P) space. What is the value of price level when $Y = Y_n$? Locate this point in the graph. [7 points]
4. Derive the AS relation (solve for Y as a function of P and parameters). [7 points] *AO*
5. Suppose that the economy is in its medium run equilibrium ($Y = Y_n$, $P = P^e$). Suppose that taxes T are increased. In the short run, what happens to equilibrium price and output? What happens to the expectations of wage setters as time goes by? Explain how is the new medium run equilibrium, and in particular, what happens to the interest rate [6 points]

3 Labor market dynamics [30 points]

This question explores the dynamics of labor market flows - that is, the flows into and out of the pools of employed and unemployed people. We have seen in class a model designed to address these issues, but we have focused on the long run (we studied the steady state). In this question we focus on the economy's transition

workers. As a result, the rate of job separation increases (assume that the rate of job finding remains constant). What will happen to the unemployment rate in the long run? Describe (in words, one line will suffice) what will happen to the unemployment rate in the transition to the new steady state. [3 points]

10. Assume, again, the economy has reached its original steady state (i.e., the one derived in point 4). Suppose that there is a change in labor market regulations which decreases the costs of hiring workers. As a result, the rate of job finding increases (assume that the rate of job separation remains constant). What will happen to the unemployment rate in the long run? Describe (in words, one line will suffice) what will happen to the unemployment rate in the transition to the new steady state. [3 points]
11. Assume, again, the economy has reached its original steady state (i.e., the one derived in point 4). Suppose that there is a demographic change that increases the size of the labor force in this economy (assume that the rates of job separation and job finding remain constant). What will happen to the *level* of unemployment in the long run? What will happen to the rate of unemployment in the long run? Describe (in words, briefly) what will happen (in the short-run) to both the level of unemployment and the rate of unemployment after the demographic change. [3 points]

to this steady state. To simplify the analysis, we abstract from the dynamics for vacancies, and concentrate exclusively on the dynamics of unemployment (and employment).

Let L be the labor force in the economy. Agents can be employed or unemployed. Time is discrete and denoted by subscript t . Let E_t and U_t denote total employment and unemployment at time t , respectively. Assume that the labor force is constant, and equal to L in every period. Suppose that the evolution of U_t and E_t is given by

$$\Delta U_t \equiv U_t - U_{t-1} = sE_{t-1} - fU_{t-1} \quad (1)$$

Assume the economy starts with $U_0 > 0$, and $E_0 > 0$. Assume also that $s > 0$, $f > 0$ and $s + f < 1$.

1. What do the parameters s and f represent? [2 points]
2. Explain in words the logic behind equation (1). [2 points]
3. Find an equation for ΔE_t in terms of U_{t-1} and E_{t-1} . [2 points]
4. Find a steady state for this economy. That is, find the level of unemployment (U^*) and employment (E^*) such that once the economy reaches these levels it stays there forever. [Hint: take (1) and the equation obtained in part 3, and impose $\Delta E_t = \Delta U_t = 0$] [3 points]
5. Use equation (1) to find an expression for U_t as a function of U_{t-1} and parameters. [2 points]
6. Now we will fully characterize transitional dynamics in this model. Use the equation derived in point 4 to obtain an expression for U_t as a function of parameters and time t only. To do this you need to substitute recursively the expression derived in point 5. [Hint: (i) the first step of this procedure consists of evaluating the expression obtained in part 5 for both $t = 1$ and $t = 2$; then substitute the expression obtained for U_1 into the expression obtained for U_2 ; this way you obtain an expression for U_2 as a function of U_0 ; the idea is to repeat this procedure t times to obtain an expression for U_t (ii) note that for $r \in (0, 1)$ we have that $\sum_{i=0}^n r^i = \frac{1-r^{n+1}}{1-r}$] [5 points]
7. Check that your result from part 6 is consistent with your result from part 4. That is, take the limit as $t \rightarrow \infty$ on the expression you got in part 6 and verify that U_t converges to the steady state level found in point 4. [2 points]
8. In point 7 you have shown that the economy will converge in the long run steady state (U^*, E^*). Show that if $U_0 < U^*$ then $U_t > U_{t-1}$, while if $U_0 > U^*$ then $U_t < U_{t-1}$. [3 points]
9. Assume the economy has reached the steady state. Suppose that there is a change in labor market regulations which decreases the costs of firing

1a. False. One does not necessarily imply the other. For example this could be true but there could also be many people coming into the labor force.

2 b. ~~Unsure~~ There is nothing inherent about a recession to cause people who felt that the wage they could get out in the labor force was not worth going to work. You could make arguments either way. For example: your spouse was laid off, so you will now take whatever job you could get. (Conversely the extended unemployment benefits in a recession could allow you to stay unemployed for longer. After thinking about it, I think the answer might be closer to true.

People are willing to accept wages because jobs are harder to find.

c. True. The model says $W = P^e F(u, z)$. If the wage setters expect prices to rise, then they will insist on higher wages.

d. False. Again, people could be entering the labor force at a rate faster than people are being employed.

2)

e) Yes, it can. If companies feel disincentivized to remove less productive workers then they can not hire a worker that is a better fit for the position. Companies rarely fire someone in order to replace them, but it can (and does, on some scale) happen.

f) ~~FFF~~

~~False - when output is above equilibrium, wage setters do not adjust wages. - it's not wages that get set it's AS~~

~~Plus, wages rarely go down for political reasons, nominal~~

~~But is AS \rightarrow output? Yes~~

~~So is actually true~~

~~Section 7.4 has process but in reverse~~

False wrong direction. When output is higher than its natural level, prices are higher, so wage setters raise wages,

~~Money is progressively worth less till real money supply~~
out of scope is back to natural levels

3

g) False. We can see in Figure 7-9 that AS will shift downward as long as output is below the natural level of output.

h) ~~True~~, See bottom figure on 2/28's notes on the Diamond model

It held from 1952 - 88

look @ year to year frequency

3

(4) :

2. AS-AD model

Labour force = L

N = employment level

Production function $Y = N$

$$P = (1 + \mu) W$$

$$W = P^e (1 + z - u)$$

\uparrow \uparrow
other $r_{unemp \text{ rate}}$

$$C = c_0 + c_1 (Y - T)$$

$$I = b_1 Y - b_2 i$$

$$\frac{M^d}{P} = m_0 + m_1 Y - m_2 i$$

1. Derive AS relation

$$P = (1 + \mu) W$$

$$= (1 + \mu) P^e (1 + z - u)$$

$$= (1 + \mu) P^e (1 + z - (1 - \frac{Y}{L}))$$

$$P = (1 + \mu) P^e (z - \frac{Y}{L})$$



$$u = \frac{U}{L} = \frac{L - N}{L} = 1 - \frac{N}{L} = 1 - \frac{Y}{L}$$

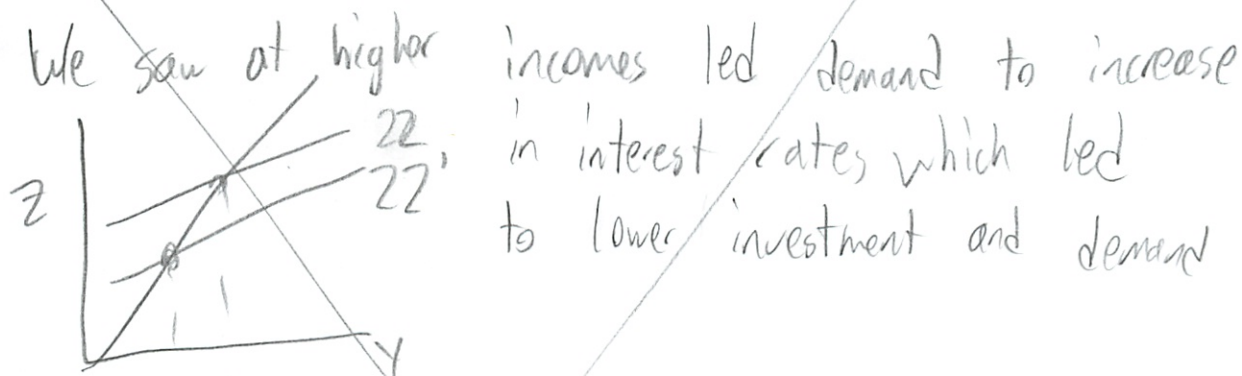
5

b) Why is AS upward sloping?

An increase in output leads to an increase in price level

We saw some of this in the IS model where we saw output increase as demand increased.

Trace Back



This produced the downward sloping IS curve.



How to tie interest rates to increase in price level?

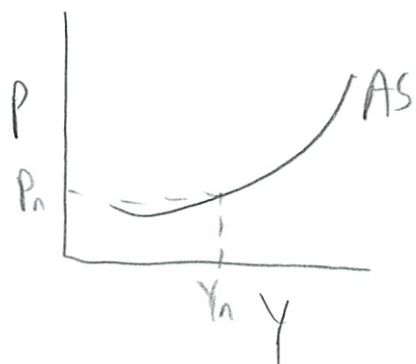
Alternatively, micro econ, does this in terms of scarcity, scale up.
 \uparrow greater P for greater demand

(6) : oh right employment

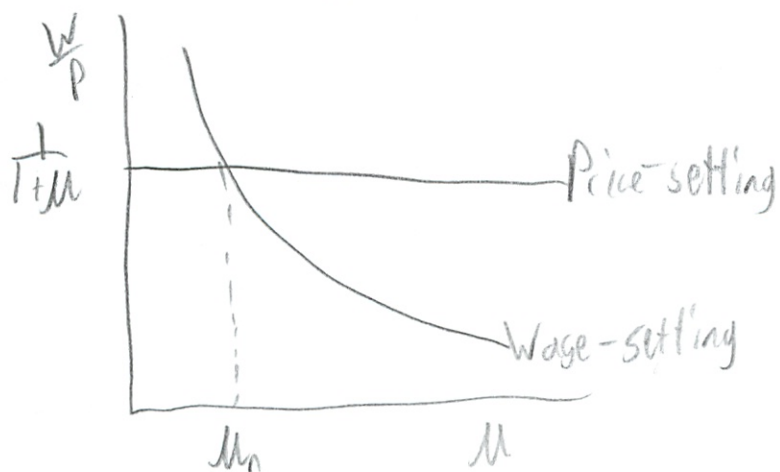
This is because in order to have higher output (since $Y = N$) you need higher employment. Higher employment = lower unemployment ($U = L - N$), (with L)

So wages are higher. Higher wages need to be funded with higher prices. (This is the inflationary spiral?)

C. Find Y_n .



You need to find the value from the wage-setting, price setting graph



⑦:

$$W = P^e (1 + z - \mu)$$

Assume $P^e = P$

$$\frac{W}{P} = 1 + z - \mu$$

Not beverage curve

$$P = (1 + \mu) W$$

$$\frac{P}{W} = 1 + \mu$$

$$\frac{W}{P} = \frac{1}{1 + \mu}$$

$$\frac{W}{P} = 1 + z - \mu_n = \frac{1}{1 + \mu}$$

Solve for μ_n

$$\mu_n = -\frac{1}{1 + \mu} + 1 + z$$

$$N_n = L(1 - \mu_n)$$

$$= L\left(1 - \left[1 + z - \frac{1}{1 + \mu}\right]\right)$$

$$= L\left(1 - 1 - z + \frac{1}{1 + \mu}\right)$$

$$Y_n = N_n = L\left(-z + \frac{1}{1 + \mu}\right)$$

✓

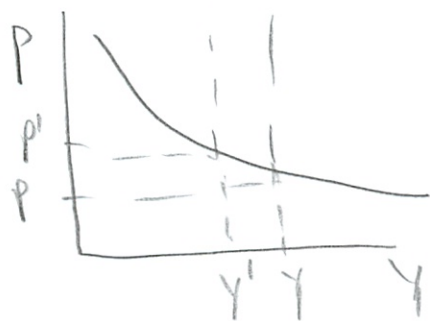
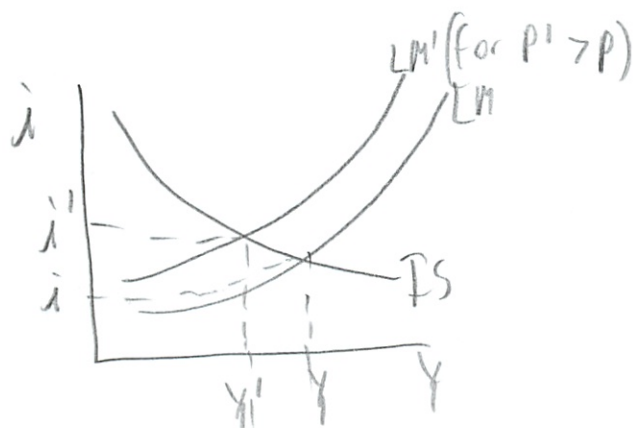
(8):

d. Find AS relation

Do you mean AD, since we found AS above?

$$Y = C + I + \bar{G}$$

$$= C_0 + c_1(Y - T) + b_1 Y - b_2 i + \bar{G}$$



$$Y = C_0 + c_1 Y - c_1 T + b_1 Y - b_2 i + \bar{G}$$

$$Y(1 - c_1 - b_1) = C_0 - c_1 T - b_2 i + \bar{G}$$

$$Y = \frac{C_0 - c_1 T - b_2 i + \bar{G}}{1 - c_1 - b_1}$$

forgot what I did in P-set 2,

1 set $Y =$

④

$$m_1 Y = \frac{M^d}{P} - m_0 + m_2 i$$

$$Y = \frac{\frac{M^d}{P} - m_0 + m_2 i}{m_1}$$

substitute
out interest
rate

$$Y = \frac{C_0 - C_1 T - b_2 i + G}{1 - C_1 - b_1} = \frac{\frac{M^d}{P} - m_0 + m_2 (i)}{m_1}$$

It says solve for Y as fn of P , parameters

3

(10):

e) Suppose econ in medium run equilibrium

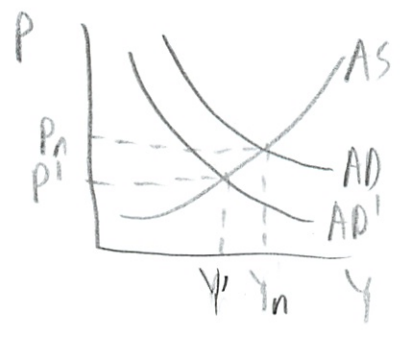
$$Y = Y_n \quad P = P^e$$

Suppose $T \uparrow$

Like \downarrow in G from 7.5 but slightly different

The increase in T , decreases C ,

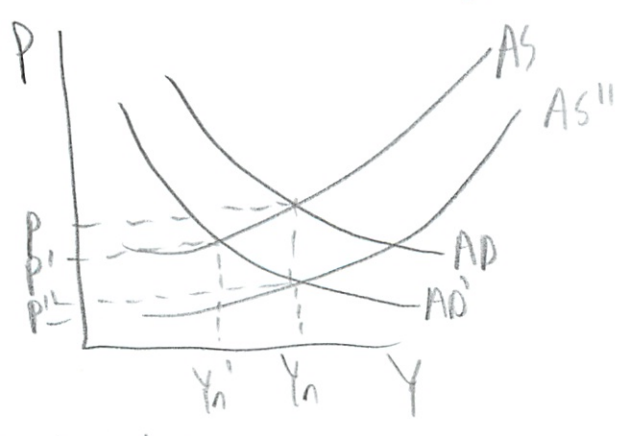
This shifts the AD curve to the left



At this point output and prices are lower,

But now output decreases since we are below our natural Y_n so AS curve shifts down

Until we reach Y_n again




Output is back at Y_n , but price level and interest rates are lower.

(11):

Prices are lower because of P-set 3 # 1f.

As output decreases, by definition ($Y=N$) employment falls also, leading to higher unemployment, which leads to lower wages. Companies have a lower labor bill so Prices can fall.

Note that in real life this rarely happens as politically lowering wages is very hard and companies are hesitant to lower prices



11b:

3. Labor market dynamics / flows

Did steady state in class

Now doing transition to steady state

Look at just at unemployment, not vacancies

L = Labor force =

↳ either employed or unemployed

$$L = E_t + U_t$$

$$\Delta U_t = U_t - U_{t-1} = \underset{\substack{\uparrow \\ s > 0 \\ f > 0}}{s} E_{t-1} - \underset{s+f < 1}{f} U_{t-1} \quad (1)$$

1. What do parameters s, f represent?

are those the flows?

s = % of people employed last year who leave their job this year
= separation rate

f = % of people unemployed last year who get a job this year

2. Explain eq (1)

This is the change in unemployment in the current year.

It can be this year's unemployment minus last year's unemployment (the traditional definition)

(12)

The 3rd component says that ^{the change in} unemployment is also equal to the % of people employed last year who become unemployed this year (ie the people added to unemployment roles) minus the percent of people unemployed last year who gain employment (ie the people who leave unemployment)

C. Find eq for ΔE_t

$$\Delta E_t = f U_{t-1} - s E_{t-1}$$

D. Find a steady state for the economy

$$\Delta E_t = \Delta U_t = 0 \quad \text{TA: don't forget } E^* + U^* = L$$

$$s E_{t-1} - f U_{t-1} = f U_{t-1} - s E_{t-1} = 0$$

TA When steady state $U_t = U_{t-1} = U^*$

$$E_t = E_{t-1} = E^*$$

Solve for U^*, E^*
in terms of f, s, L

Also don't forget

$$U^* + E^* = L$$

$$s E^* - f U^* = f U^* - s E^* = 0$$

$$s E^* - f(L - E^*) = f(L - E^*) - s E^* = 0$$

$$s E^* - f L + f E^* = f L - f E^* - s E^* = 0$$

(12b)

$$\frac{2sE^*}{2} + \frac{2fE^*}{2} = \frac{2fL}{2}$$

$$sE^* + fE^* = 2fL$$

$$E^*(s+f) = 2fL$$

$$E^* = \frac{2fL}{s+f}$$

algebraic error?

$$U^* = L - E^*$$

2

$$L - \frac{2fL}{s+f} = \frac{s}{s+f}L$$

Eq

is this right - more get these
math relations - should be top of mind

(13)

- Not related to d, right?

e. Find U_t as fn U_{t-1} , L , f , s

$$U_t - U_{t-1} = s E_{t-1} - f U_{t-1} + U_{t-1}$$

$$U_t = s E_{t-1} + (1-f) U_{t-1}$$

$$U_t = s (L - U_{t-1}) + (1-f) U_{t-1}$$

$$U_t = sL - sU_{t-1} + U_{t-1} - f U_{t-1}$$

$$U_t = sL + (U_{t-1})(1-s-f)$$

f. Now characterize dynamics

Find expression for U_t as function of param t onlySo substitute recursively for e

$$U_t = sL + (U_{t-1})(1-s-f)$$

$$U_1 = sL + (U_0)(1-s-f)$$

$$U_2 = sL + (U_1)(1-s-f)$$

$$= sL + (sL + U_0(1-s-f))(1-s-f)$$

Can you do more?

Find general equation.

$$U_t = sL + U_0(1-s-f)^t \text{ not true}$$

(14)

$$V_2 = sL + (1-s-f)sL + (1-s-f)U_0(1-s-f)$$

$$\begin{aligned} V_3 &= sL + (V_2)(1-s-f) \\ &= sL + (sL + (sL + U_0(1-s-f))(1-s-f))(1-s-f) \\ &= sL + sL(1-s-f) + sL(1-s-f)^2 + U_0(1-s-f)^3 \end{aligned}$$

$$V_t = \sum_{i=0}^t sL(1-s-f)^i + U_0(1-s-f)^t$$

So this is where 2nd hint fits in

$$\text{for } r \in (0,1) \text{ we have } \sum_{i=0}^{\infty} r^i = \frac{1-r^{n+1}}{1-r}$$

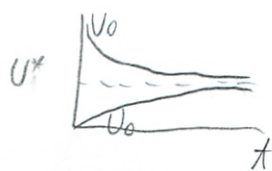
$$V_t = \frac{1 - (sL(1-s-f))^{t+1}}{1 - (sL(1-s-f))} + U_0(1-s-f)^t$$

But how does this fit in to part d?

$$U^* = L - \frac{2fL}{s+f}$$

V_t will approach the lim U^*
and then no longer change

But how show mathematically?



(15)

g) Check b by taking $\lim_{t \rightarrow \infty}$ from t

$$\lim_{t \rightarrow \infty} \frac{1 - (sL(1-s-t))^{t+1}}{1 - (sL(1-s-t))} + U_0 (1-s-t)^t$$

algebra

$$\text{Should} = U^*$$

$$= L - \frac{2tL}{s+t}$$

doesn't it depend on $\#$

I tried same $\#$ and got L so wrong

How would do w/ $\#$
algebraically

(66)

W) In g showed will converge to a steady state.

If $U_0 < U^*$ then $U_t > U_{t-1}$, while

if $U_0 > U^*$ then $U_t < U_{t-1}$

I don't know how they want us to show this, with a proof?

But we know it will converge to a steady state U^*

So if $U_0 < U^*$ then U will get bigger each year. That is what it means to converge

Same for $U_0 > U^*$ getting smaller

Say $U^* = 5$ $U_0 = 3$

$$U_1 = sL + U_0(1-s-t)$$

U depends what the parameters are. But where do we determine the parameters?

See solution!

(17)

i) Assume econ has reached steady state.

Assume change in labor market regulation to ↓ Cost of firing a worker, so separation rate (s) increases
What happens to unemployment in the long run?

The new steady state unemployment will be higher,

So unemployment will rise each year in transition to this

Why do steady states change

j) Back to original steady state.

Now cost of hiring ↓ so $f \uparrow$

The new steady state unemployment will be lower, meaning unemployment will decrease each year.

k) Back to steady state. $L \uparrow$

So the unemployment rate does not change but the absolute numbers of unemployed rise.

But L is in our equation because s is a percentage of L .

(18)

Does this make sense?

S will stay same - same % of people will separate,
but raw # will \uparrow

So yes, rate of unemployment same, but raw number larger
yes

\rightarrow In the short run, there will be a jump in # unemployed,
and that # gradually drops as people find jobs \rightarrow converges
to steady state

Problem Set # 3
14.02 Spring 2011
Due March 4

February 23, 2011

1 True/False [40 points]

Please state whether each of the following claims are True or False, and provide a brief justification for your answer. You may include graphs and equations to support your answer.

1. "An economy with a low rate of separations will have a low rate of unemployment". [8 points]

ANSWER. FALSE. The separations rate tells us about the flows into the pool of unemployed. But there are other flows into this pool (the people who is out of the labor force and then decides to start working but cannot find a job), and there are flows out of this pool, namely the rate of hires. The unemployment rate will be affected by all of these forces, so the statement is false.

2. "In recessions, the reservation wage of the workers tends to decrease". [8 points]

ANSWER. TRUE. In recessions, firms tend to hire less and fire more workers. This tends to increase the unemployment rate, and make the prospect of being unemployed worse (the probability of finding a job is lower). This decreases the reservation wage (so that agents tend to accept lower wages).

3. "Expectations matter in the AS-AD model because wages are set before the price level is known". [8 points]

ANSWER. TRUE. See textbook.

4. "It is not possible to have an increase in the rate of unemployment when the number of employed people is going up." [8 points]

ANSWER. FALSE. The unemployment rate is

$$u = \frac{U}{U + E} = \frac{1}{1 + E/U}$$

so as long as the number of unemployed people (U) goes up by a higher percentage than the number of employed people (E), the unemployment rate (u) will indeed go up.

5. "A tax to firing workers can result in less hiring." [8 points]

ANSWER. TRUE. Firms will be less willing to hire new workers, when the cost of firing is higher.

6. "In the AS-AD model, when output is above its natural level, wage setters revise their expectations downwards so that output decreases continuously towards its natural level" [8 points]

ANSWER. FALSE. When $Y > Y_n$, actual price exceeds expected price, $P > P^e$. Thus, wage setters will revise their expectations *upwards*. The rest of the statement is correct.

7. "In the medium run, a decrease in public spending affects the AD curve, and leaves the AS curve unchanged" [8 points]

ANSWER. FALSE. It affects both curves. In the short run, it only affects the AD. But then P^e adjusts, which shifts AS until $Y = Y_n$ again the medium run.

8. "According to the empirical evidence, the Beveridge curve holds at any time frequency." [8 points]

ANSWER. FALSE. As the class notes show, the relation between the vacancy rate and the unemployment rate is only negative when one looks at the yearly frequency.

2 AS-AD [30 points]

Consider an economy with a labor force of size L . Let N denote the employment level. The production function is

$$Y = N$$

The price and wage setting relations are:

$$\begin{aligned} P &= (1 + \mu)W \\ W &= P^e (1 + z - u) \end{aligned}$$

where u denotes the unemployment rate, z denotes "other" unemployment benefits, and the rest of the variables are as in class. The consumption and investment functions are

$$\begin{aligned} C &= c_0 + c_1(Y - T) \\ I &= b_1Y - b_2i \end{aligned}$$

Finally, money demand is given by

$$\frac{M^d}{P} = m_0 + m_1Y - m_2i$$

1. Derive the AS relation. Show your derivations clearly. [7 points]

ANSWER. Price and wage setting imply

$$P = (1 + \mu) P^e (1 + z - u)$$

Then, we need the definition of the unemployment rate, and the fact that the labor force is $L = N + U$:

$$u = \frac{U}{L} = \frac{L - N}{L} = 1 - \frac{N}{L}$$

Putting everything together, and noting that $Y = N$

$$P = (1 + \mu) P^e \left(z + \frac{Y}{L} \right)$$

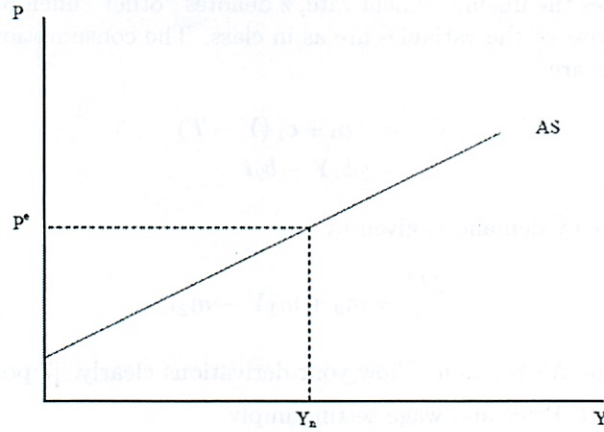
2. Why is the AS curve upward sloping? Provide your intuition [3 points]

ANSWER. A higher Y requires a higher level of employment, which implies a lower unemployment rate. This increases worker's bargaining power, which in turn increases the wage. Thus, firms set a higher price.

3. Find the natural level of output (Y_n). Draw the AS relation in the (Y, P) space. What is the value of price level when $Y = Y_n$? Locate this point in the graph. [7 points]

ANSWER. When $P = P^e$

$$Y_n = \left(\frac{1}{1 + \mu} - z \right) L$$



Clearly, when $Y = Y_n$, the price level is $P = P^e$.

4. Derive the AS relation (solve for Y as a function of P and parameters). [7 points]

ANSWER. Lets first derive the IS curve

$$Y = c_0 + c_1(Y - T) + b_1Y - b_1i + G$$

or

$$Y = \frac{1}{1 - c_1 - b_1} [c_0 - c_1T - b_1i + G]$$

Now, the LM curve

$$\frac{M}{P} = m_0 + m_1Y - m_2i$$

We need to eliminate i from this system of two equations, to obtain a relation between Y and P .

$$i = \frac{m_0}{m_2} + \frac{m_1}{m_2}Y - \frac{1}{m_2} \frac{M}{P}$$

Plugging this into the IS

$$Y = \frac{1}{1 - c_1 - b_1} \left[c_0 - c_1T - b_1 \left(\frac{m_0}{m_2} + \frac{m_1}{m_2}Y - \frac{1}{m_2} \frac{M}{P} \right) + G \right]$$

$$Y = \frac{1}{1 - c_1 - b_1} \left[c_0 - c_1T - b_1 \frac{m_0}{m_2} - b_1 \frac{m_1}{m_2}Y + \frac{b_1}{m_2} \frac{M}{P} + G \right]$$

$$Y = \frac{1}{1 + \frac{b_1 m_1}{1 - c_1 - b_1} \frac{1}{m_2}} \frac{1}{1 - c_1 - b_1} \left[c_0 - c_1T - b_1 \frac{m_0}{m_2} + \frac{b_1}{m_2} \frac{M}{P} + G \right]$$

5. Suppose that the economy is in its medium run equilibrium ($Y = Y_n$, $P = P^e$). Suppose that taxes T are increased. In the short run, what happens to equilibrium price and output? What happens to the expectations of wage setters as time goes by? Explain how is the new medium run equilibrium, and in particular, what happens to the interest rate [6 points]

ANSWER. From the expression above, the AD shifts to the left. Thus, both output and the price level decrease in the short run. As time goes by, wage setters revise their expectations downwards, which makes the AS to shift downwards. This continues until a new medium run equilibrium is reached when $Y = Y_n$, and $P = P^e$ is lower than the original $P = P^e$.

3 Labor market dynamics [30 points]

This question explores the dynamics of labor market flows - that is, the flows into and out of the pools of employed and unemployed people. We have seen in class a model designed to address these issues, but we have focused on the long run (we studied the steady state). In this question we focus on the economy's transition to this steady state. To simplify the analysis, we abstract from the dynamics for vacancies, and concentrate exclusively on the dynamics of unemployment (and employment).

Let L be the labor force in the economy. Agents can be employed or unemployed. Time is discrete and denoted by subscript t . Let E_t and U_t denote total employment and unemployment at time t , respectively. Assume that the labor force is constant, and equal to L in every period. Suppose that the evolution of U_t and E_t is given by

$$\Delta U_t \equiv U_t - U_{t-1} = sE_{t-1} - fU_{t-1} \quad (1)$$

Assume the economy starts with $U_0 > 0$, and $E_0 > 0$. Assume also that $s > 0$, $f > 0$ and $s + f < 1$.

1. What do the parameters s and f represent? [2 points]

ANSWER. s is the separations rate, and f is the hiring rate.

2. Explain in words the logic behind equation (1). [2 points]

ANSWER. The level of unemployment at time t consist of those people who were unemployed at $t-1$ (U_{t-1}), plus those who lost their jobs (sE_{t-1}), minus those who found a job (fU_{t-1}).

3. Find an equation for ΔE_t in terms of U_{t-1} and E_{t-1} . [2 points]

ANSWER. Since $E_t + U_t = L$ we have that

$$\Delta E_t + \Delta U_t = 0$$

$$\Delta E_t = fU_{t-1} - sE_{t-1}$$

4. Find a steady state for this economy. That is, find the level of unemployment (U^*) and employment (E^*) such that once the economy reaches these levels it stays there forever. [Hint: take (1) and the equation obtained in part 3, and impose $\Delta E_t = \Delta U_t = 0$] [5 points]

ANSWER. Let E and U denote steady state levels. Plugging $U = U_t = U_{t-1}$ in equation (1)

$$\begin{aligned} 0 &= sE - fU \\ 0 &= s(L - U) - fU \end{aligned}$$

$$\begin{aligned} sL &= (s + f)U^* \\ \frac{U^*}{L} &= \frac{s}{s + f} = \frac{1}{1 + f/s} \\ \frac{E^*}{L} &= 1 - \frac{s}{s + f} = \frac{f}{s + f} = \frac{1}{\frac{s}{f} + 1} \end{aligned}$$

5. Use equation (1) to find an expression for U_t as a function of U_{t-1} and parameters. [4 points]

ANSWER. Use equation (1) together with the fact that $E_t + U_t = L$

$$\begin{aligned} U_t &= U_{t-1} + sE_{t-1} - fU_{t-1} \\ &= U_{t-1} + s(L - U_{t-1}) - fU_{t-1} \\ &= sL + U_{t-1}(1 - s - f) \end{aligned}$$

6. Now we will fully characterize transitional dynamics in this model. Use the equation derived in point 4 to obtain an expression for U_t as a function of parameters and time t only. To do this you need to substitute recursively the expression derived in point 5. [Hint: (i) the first step of this procedure consists of evaluating the expression obtained in part 5 for both $t = 1$ and $t = 2$; then substitute the expression obtained for U_1 into the expression obtained for U_2 ; this way you obtain an expression for U_2 as a function of U_0 ; the idea is to repeat this procedure t times to obtain an expression for U_t (ii) note that for $r \in (0, 1)$ we have that $\sum_{i=0}^n r^i = \frac{1-r^{n+1}}{1-r}$] [5 points]

ANSWER. Let $a \equiv 1 - s - f$. Then

$$\begin{aligned}
 U_t &= sL + aU_{t-1} \\
 U_1 &= sL + aU_0 \\
 U_2 &= sL + aU_1 = sL + a(sL + aU_0) \\
 &= sL(1 + a) + a^2U_0 \\
 U_3 &= sL + sL(a + a^2) + a^3U_0 \\
 &= sL(1 + a + a^2) + a^3U_0 \\
 U_t &= sL \left(\sum_{i=0}^{t-1} a^i \right) + a^t U_0 \\
 U_t &= sL \left(\frac{1 - a^t}{1 - a} \right) + a^t U_0 \\
 U_t &= \frac{s}{1 - a} L + a^t \left(U_0 - \frac{s}{1 - a} L \right) \quad (2) \\
 U_t &= \frac{s}{s + f} L + (1 - s - f)^t \left(U_0 - \frac{s}{s + f} L \right) \quad (3)
 \end{aligned}$$

7. Check that your result from part 6 is consistent with your result from part 4. That is, take the limit as $t \rightarrow \infty$ on the expression you got in part 6 and verify that U_t converges to the steady state level found in point 4. [4 points]

ANSWER. Note that $\lim_{t \rightarrow \infty} (1 - s - f)^t = 0$ since $0 \leq 1 - s - f \leq 1$.

$$\begin{aligned}
 \lim_{t \rightarrow \infty} U_t &= \frac{s}{s + f} L + \lim_{t \rightarrow \infty} (1 - s - f)^t \left(U_0 - \frac{s}{s + f} L \right) \\
 &= \frac{s}{s + f} L = U^* \text{ from point 6.}
 \end{aligned}$$

8. In point 7 you have shown that the economy will converge in the long run steady state (U^*, E^*) . Show that if $U_0 < U^*$ then $U_t > U_{t-1}$, while if $U_0 > U^*$ then $U_t < U_{t-1}$. [5 points]

ANSWER. Use equation (2) to get

$$U_t - U_{t-1} = (a - 1) a^{t-1} \left(U_0 - \frac{s}{1 - a} L \right)$$

By assumption, we know $(a - 1) < 0$. Hence $U_0 - \frac{s}{1 - a} L < 0$ implies $U_t - U_{t-1} > 0$. This corresponds to convergence "from below".

9. Assume the economy has reached the steady state. Suppose that there is a change in labor market regulations which decreases the costs of firing workers. As a result, the rate of job separation increases (assume that the

rate of job finding remains constant). What will happen to the unemployment rate in the long run? Describe (in words, one line will suffice) what will happen to the unemployment rate in the transition to the new steady state. [5 points]

ANSWER. The economy will converge to a new and higher steady state. That is, unemployment will start to increase immediately, and continue to increase until it "converges" to the new steady state.

10. Assume, again, the economy has reached its original steady state (i.e., the one derived in point 4). Suppose that there is a change in labor market regulations which decreases the costs of hiring workers. As a result, the rate of job finding increases (assume that the rate of job separation remains constant). What will happen to the unemployment rate in the long run? Describe (in words, one line will suffice) what will happen to the unemployment rate in the transition to the new steady state. [5 points]

ANSWER. The economy will converge to a new and lower steady state. That is, unemployment will start to decrease immediately, and continue to decrease until it "converges" to the new, lower steady state.

11. Assume, again, the economy has reached its original steady state (i.e., the one derived in point 4). Suppose that there is a demographic change that increases the size of the labor force in this economy (assume that the rates of job separation and job finding remain constant). What will happen to the *level* of unemployment in the long run? What will happen to the rate of unemployment in the long run? Describe (in words, briefly) what will happen (in the short-run) to both the level of unemployment and the rate of unemployment after the demographic change. [5 points]

ANSWER. The level of unemployment in the long run (U^*) will be higher (we have seen it is linear in L). The unemployment rate, however, will remain constant in the long run. Dynamics: the level of unemployment (U_t) increases every period until the new, higher steady state. The rate of unemployment (U_t/L) decreases at the moment of the shock, and then starts to increase every period, converging to the previous steady state.

Section

3/4

Skipped - Dining Meeting