Problem Set # 5
14.02 Spring 2011
Due April 8

March 30, 2011

1 True/False [35 points]

Please state whether each of the following claims are True or False, and provide a brief justication for your answer. You may include graphs and equations to support your answer.

1. "A 20% increase in the nominal exchange rate between the US and the UK (defined as number of pounds per dollar) implies that US goods are now 20% more expensive relative to UK goods". [5 points]

2. "When Mexican investors buy $1,000 of US Government bonds, the financial account of the US balance of payments increases by $1,000". [5 points]

3. "If a US firm pays $1 million in dividends to foreign shareholders, then the financial account of the US balance of payments goes down by $ 1 million. " [5 points]

4. "If the exchange rate is 0.5 pounds per dollar, the one-year dollar interest rate is 2%, and the one-year pound interest rate is 5%, then the dollar is expected to depreciate vis a vis the pound in nominal terms." [5 points]

5. "Consider an economy that has a fixed nominal exchange rate with respect to the US dollar. If this economy has greater inflation than the US, then its real exchange rate (with respect to the US) will appreciate." [5 points]

6. "In the Mundell-Fleming model with flexible exchange rate of Chapter 20, an increase in firms' willingness to invest (i.e. firms invest more for every possible interest rate) results in a nominal depreciation of the exchange rate. " [5 points]

7. "In the Mundell-Fleming model of Chapter 20, if the exchange rate is fixed and there is a recession in the rest of the world, then the government needs to decrease the money supply." [5 points]

2 Open Economy IS [40 points]

Consider a world economy with only two countries. The home country is characterized by:

$$\begin{aligned} C &= c_0 + c_1 Y \\ I &= c_2 Y \\ \frac{IM}{e} &= mY \\ X &= \frac{x\tilde{Y}}{e} \end{aligned}$$

do they not know the ϵ key?

where $C, Y, \tilde{Y}, I, IM, X, e$ denote aggregate consumption, domestic output, foreign output, investment, imports, exports and the real exchange rate, respectively. Assume $c_0 > 0, 0 \leq c_1 \leq 1, c_2 > 0, 0 \leq m \leq 1, 1+m > c_1 + c_2, 0 \leq x \leq 1$.

The foreign country is characterized by

$$\begin{aligned} \tilde{C} &= \tilde{c}_0 + \tilde{c}_1 \tilde{Y} \\ \tilde{I} &= \tilde{c}_2 \tilde{Y} \\ \frac{\widetilde{IM}}{\tilde{e}} &= \tilde{m}\tilde{Y} \\ \tilde{X} &= \frac{\tilde{x}Y}{\tilde{e}} \end{aligned}$$

Assume that parameters in the foreign economy satisfy assumptions analogous to the ones for the domestic economy. We will assume throughout the question that the real exchange rate (e) is exogenously given.

Denote by G the amount of Government spending in the home country and by $\tilde{G}$ the amount of Government spending in the foreign country.

1. (5 points) Find the equilibrium level of output in the domestic goods market as a function of $e, G, \tilde{Y}$ and parameters. Find the multiplier (denote it by M). Find equilibrium in foreign goods market as a function of $\tilde{e}, \tilde{G}, Y$ and parameters. Find the multiplier in the foreign country (denote it by $\tilde{M}$).

2. (5 points) Does the Marshall Lerner condition hold for the particular functional forms of exports and imports functions assumed? Show why or why not.

3. (5 points) State a condition relating e and $\tilde{e}$.

4. (5 points) Write a condition relating IM and $\tilde{X}$ and a condition relating $\widetilde{IM}$ and X. Show that each condition imposes a restriction on the parameters.

5. (7 points) Solve for domestic output as a function of $e, G, \tilde{G}$ and parameters $(c_0, \tilde{c}_0, x, \tilde{x}, M, \tilde{M})$. Assume $Mx\tilde{M}\tilde{x} < 1$.

6. (6 points). How do G and $\tilde{G}$ affect output at home Y? What are the multipliers?

7. (7 points) Suppose the home country has decided to increase its Government budget deficit by 100 units (of the local good). Is it better to use these funds to expand domestic public spending ($\Delta G = 100$), or to give a grant to the Foreign country on the condition that the funds are fully used for Foreign public spending ($\Delta\tilde{G}$)? Which alternative has a greater impact on the output of the home country? State a precise mathematical condition that guarantees your answer.

3 Open Economy IS-LM [25 points]

Consider the Mundell-Fleming model of Chapter 20. Equilibrium in the goods market is given by

$$Y = C(Y - T) + I(Y, r) + G + X(Y^*, \varepsilon) - IM(Y, \varepsilon)/\varepsilon$$

where Y is output at home, Y^* is foreign output, T are taxes, $C(\cdot)$ is consumption, r is the real interest rate, $I(\cdot)$ is investment, G is government spending, X is exports, IM is imports, and ε is the real exchange rate. Equilibrium in the money market is given by the standard LM equation:

$$M/P = YL(i)$$

where i is the nominal interest rate. Assume that P and P^* are constant, with $P = P^* = 1$. Note that this implies zero inflation (actual, and expected). Assume further that the Marshall-Lerner condition holds.

1. State the uncovered interest parity condition. Provide intuition for this equation. Use E_t to denote the nominal exchange rate at time t, E^e_{t+1} to denote the expected nominal exchange rate at t+1 and i^*_t to denote the foreign interest rate. What assumption about local and foreign bonds is required for this condition to hold? [5 points]

2. Explain why nominal and real exchange rates must coincide given our assumptions. [3 points]

3. Using a IS-LM-UIP diagram, show the effects of a decrease in government spending. Assume that the economy has a flexible exchange rate, and that initially net exports are zero. State what would happen to output, the interest rate, the exchange rate, net exports, investment and consumption. [If the effect on any variable is undetermined, just state so]. [7 points]

4. Using a IS-LM-UIP diagram, show the effects of an increase in money supply. Continue to assume that the economy has a flexible exchange rate, and that initially net exports are zero. State what would happen to output, the interest rate, the exchange rate, net exports, investment and

consumption. [If the effect on any variables is undetermined, just state so]. [7 points]

5. Consider the scenario of government spending reduction described in part 3. What should the Government do to keep the exchange rate fixed? [3 points]

1. True or False

-5 ~~a)~~ True, we can look at changes - but not absolute ~~#s~~

b) ~~False~~ - ~~the net effect on the balance of payment table will be 0 always~~
- it always balances out

True - question asks about "financial" accounts. I am assuming this means capital accounts, which go up $1,000

-4 ~~c)~~ True. The $1 million dividend reduces the value of the company by $1 million. It also appears in the current account as - investment income.

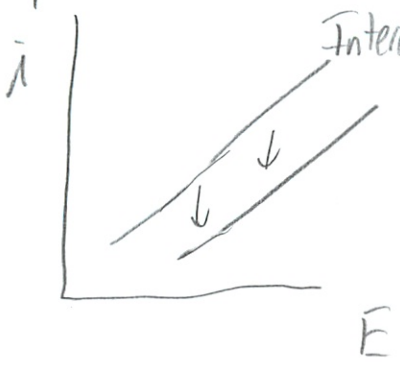
-4 ~~d)~~ $E = \frac{1+i}{1+i^*} \bar{E}^e$

True will depreciate if the foreign interest rate increases (not just is higher than) relative to the domestic rate

-4 ~~e)~~ $\epsilon = \frac{EP}{P^*}$ False. It will depreciate since you are increasing the denominator.
- has nothing to do w/ trying to match i

(2) · Mundell Fleming = model in these chapters

f.) r_1



False an increase in willingness to invest will lead to in increase/appreciation in E.

g.) The government must change i to keep rates pegged. If the world ↓ rates, then the gov needs to ↑ money supply to lower rates. ~~False~~ True -4

(3)

2. Open Econ IS

Home econ

$$C = c_0 + c_1 Y$$
$$I = c_2 Y$$
$$\frac{IM}{\epsilon} = mY$$
$$X = \frac{x\tilde{Y}}{\epsilon}$$

$c_0 > 0$
$0 \leq c_1 \leq 1$
$c_2 > 0$
$0 \leq m \leq 1$
$1 + m > c_1 + c_2$
$0 \leq x \leq 1$

Foreign Country

$$\tilde{C} = \tilde{c}_0 + \tilde{c}_1 \tilde{Y}$$
$$\tilde{I} = \tilde{c}_2 \tilde{Y}$$
$$\frac{\tilde{IM}}{\epsilon} = \tilde{m}\tilde{Y}$$
$$\tilde{X} = \frac{\tilde{x}Y}{\epsilon}$$

ϵ is exonegeous

a. Find equalibrium of domestic markets and multiplier

$$Y = C + I + G - \frac{IM}{\epsilon} + X$$
$$= c_0 + c_1 Y + c_2 Y + G - mY + \frac{x\tilde{Y}}{\epsilon}$$

So solve for Y

(4)

$$Y - c_1 Y - c_2 Y + m_y = c_0 + \bar{G} + x\frac{\tilde{Y}}{\epsilon}$$

$$Y[1 - c_1 - c_2 + m] = c_0 + \bar{G} + x\frac{\tilde{Y}}{\epsilon}$$

$$Y = \frac{c_0 + \bar{G} + x\frac{\tilde{Y}}{\epsilon}}{1 - c_1 - c_2 + m} \rightarrow (Y_1 = m(c_0 + x\tilde{Y}_e)) \text{ good}$$

The multiplier is $\frac{1}{1 - c_1 - c_2 + m} = M$

ans as a func of $G, \tilde{Y}, e, q$ parameters!

Foreign

$$\tilde{Y} = \tilde{C} + \tilde{I} + \tilde{G} - \frac{\tilde{IM}}{\epsilon} + \tilde{X}$$

$$\tilde{Y} = \tilde{C}_0 + \tilde{c}_1\tilde{Y} + \tilde{c}_2\tilde{Y} + \tilde{G} - \tilde{m}\tilde{Y} + \tilde{x}\frac{Y}{\epsilon}$$

$$\tilde{Y} - \tilde{c}_1\tilde{Y} - \tilde{c}_2\tilde{Y} + \tilde{m}\tilde{Y} = \tilde{C}_0 + \tilde{G} + \frac{\tilde{x}Y}{\epsilon}$$

$$\tilde{Y}[1 - \tilde{c}_1 - \tilde{c}_2 + \tilde{m}] = \tilde{C}_0 + \tilde{G} + \frac{\tilde{x}Y}{\epsilon}$$

$$\tilde{Y} = \frac{\tilde{C}_0 + \tilde{G} + \tilde{x}\frac{Y}{\epsilon}}{1 - \tilde{c}_1 - \tilde{c}_2 + \tilde{m}}$$

The multiplier is $\frac{1}{1 - \tilde{c}_1 - \tilde{c}_2 + \tilde{m}} = \tilde{M}$

(5)

b) Does the Marshall Lerner condition hold?

↳ Condition under which real depreciation leads to ↑ in net exports

Start with $NX = X - \frac{IM}{\epsilon}$

Assume balanced $(NX = 0)$

$\epsilon NX = \epsilon X - IM$

Now rates change $\Delta\epsilon$

$\epsilon(\Delta NX) = (\Delta\epsilon)X + \epsilon(\Delta X) - (\Delta IM)$

$$\frac{\epsilon(\Delta NX)}{\epsilon X} = \frac{(\Delta\epsilon)X}{\epsilon X} + \frac{\epsilon(\Delta X)}{\epsilon X} - \frac{\Delta(IM)}{\epsilon X}$$

$$\frac{\Delta NX}{X} = \frac{\Delta\epsilon}{\epsilon} + \frac{\Delta X}{X} - \frac{\Delta IM}{IM}$$

Change in trade balance — Sum of 3 terms must be positive

↳ We don't have # So how do know if it holds?

It does hold in real life

(6)

c) State a condition relating ϵ and $\hat{\epsilon}$

" How to do w/o using d) which comes later

Oh duh I was thinking about it earlier

$$\epsilon = \frac{1}{\hat{\epsilon}}$$

$$\hat{\epsilon} = \frac{1}{\epsilon}$$

(7)

d) Write a condition relating IM, $\widehat{X}$ and $\widehat{IM}$, X

Well one persons imports are another countries exports

IM in currency foreign country, X domestic

$$\frac{IM}{e} = \widehat{X}$$

$IM \# \widetilde{X}$

$$\widetilde{\frac{IM}{e}} = X$$

$\widetilde{IM} = X$

What does it mean that these conditions impose a restriction on the parameters?

$IM = \widetilde{X} \rightarrow m\,Ye = \frac{\widetilde{X}\,Y}{e} \rightarrow m = \widetilde{X}$

Similarly, $\widetilde{IM} = X \rightarrow \widetilde{m} = x$

$= 1$

(8)

f) Solve for domestic output

$$Y = \frac{1}{M}\left(C_0 + G + \frac{X\tilde{Y}}{\epsilon}\right)$$

$$\tilde{Y} = \frac{1}{\tilde{M}}\left(\tilde{C}_0 + \tilde{G} + \frac{\tilde{X}Y}{\tilde{\epsilon}}\right)$$

$$= \frac{1}{\tilde{M}}\left(\tilde{C}_0 + \tilde{G} + \tilde{X}Y\epsilon\right)$$

; does not used?

make it easier

$$Y = \frac{1}{M}\left(C_0 + G + \tilde{\epsilon} X \left(\frac{1}{\tilde{M}}\left(\tilde{C}_0 + \tilde{G} + \tilde{X}Y\epsilon\right)\right)\right)$$

$$= \frac{1}{M}C_0 + \frac{1}{M}G + \frac{1}{M}\tilde{\epsilon}X\frac{1}{\tilde{M}}\tilde{C}_0 + \frac{1}{M}\tilde{\epsilon}X\tilde{G}\frac{1}{\tilde{M}} + \frac{1}{M}\tilde{\epsilon}X\tilde{X}Y\epsilon\frac{1}{\tilde{M}}$$

$$Y\left[1 - \frac{1}{M}\epsilon\tilde{\epsilon}X\tilde{X}\frac{1}{\tilde{M}}\right] = \ldots$$

$$Y = \frac{\frac{1}{M}\left(C_0 + G + \tilde{\epsilon}X\left(\frac{1}{\tilde{M}}\left(\tilde{C}_0 + \tilde{G}\right)\right)\right)}{1 - \frac{1}{M}\frac{1}{\tilde{M}}\epsilon\tilde{\epsilon}X\tilde{X}}$$

Get rid of $\tilde{\epsilon}$

$$= \frac{\frac{1}{M}\left(C_0 + G + \frac{X}{\tilde{M}\epsilon}\left(\tilde{C}_0 + \tilde{G}\right)\right)}{1 - \frac{1}{M}\frac{1}{\tilde{M}}\frac{\epsilon}{\epsilon}X\tilde{X}}$$

See below -2

g)

g) How do $G, \tilde{G}$ affect home Y?

Both affect Y. G controls Y fairly directly, but is subject to a $\left(1 - \frac{1}{M}\frac{1}{\tilde{M}} \times \hat{x}\right)$ multiplier

↑G → increases Y

$\tilde{G}$ also affects Y, but more indirectly, $\frac{\times \tilde{G}}{M\epsilon}$ affects Y, also subject to the $\left(1 - \frac{1}{M}\frac{1}{\tilde{M}} \times \hat{x}\right)$ multiplier

How does it affect Y

⇒ ↑G and/or ↑$\tilde{G}$ ⇒ ↑Y

See silas -4

h) So gov has decided to ↑ deficit by 100 units, Should it spend these domestically or internationally?

? Do we have to see where budget deficit comes from?

- no not the gov

Well each is affected by multiplier, so does not matter.

So $\frac{1}{M}G$ vs $\frac{1}{M}\frac{\times \tilde{G}}{\tilde{M}\epsilon}$ So G vs $\frac{\times \tilde{G}}{\tilde{M}\epsilon}$

x is $0 \leq x \leq 1$

$\tilde{M} = \frac{1}{1 - \tilde{c}_1 - \tilde{c}_2 + \tilde{m}}$ and is always < 1 - good in denom

ϵ is some index ~ 1 can be > 1 or < 1

(10)

So with example values

$G = 100$

$x = .5$

$\hat{m} = .2$

$\epsilon = .5$ and 1.5

$$100 \quad \text{vs} \quad \frac{.5 \cdot 100}{.2 \cdot .5} \quad \text{vs} \quad \frac{.5 \cdot 100}{.2 \cdot 1.5}$$

$$100 \qquad 500 \qquad 166$$

Foreign will always be bigger

$$1 > \frac{<1 \cdot 1}{<1 \cdot \leq 1}$$

kinda handwavy - how would you say it better?

(11)

3 Open Econ IS-LM

$$Y = C(Y-T) + I(Y,i) + G + X(Y^*, \epsilon) - \frac{IM(Y,\epsilon)}{\epsilon}$$

$$\frac{M}{P} = YL(i)$$

$P = P^* = 1$ Zero inflation

Assume Marshall-Lerner condition holds

1. State uncovered interest parity condition

Uncovered if

$$E_t = \frac{1+i_t}{1+i_{t+1}^*} E_{t+1}^e \qquad (1+i_t) = (1+i_t^*) E_t / E_{t+1}^e$$

$\frac{1+i}{1+i_{t+1}}$ is the ratio of domestic to foreign interest rates,

E_{t+1}^e is the expected exchange rate after the time period is over.

This is because an investor can convert their dollars to foreign currency and invest in foreign assets. However they must then convert those foreign currencies back to dollars when they sell - So they care what the interest rate will be then

(12)

In addition, no arbitrage is allowed - so the foreign rate of return and expected interest rate must match.

$$(1+i_t) = (1+i^*_t)\left(\frac{E_t}{E^e_{t+1}}\right)$$

Of course i, i^* must be ≥ 0 and can't be in supper high ($\sim > 20\%$). Also no illegal transactions

Assumption: investors have access to local & foreign funds for year

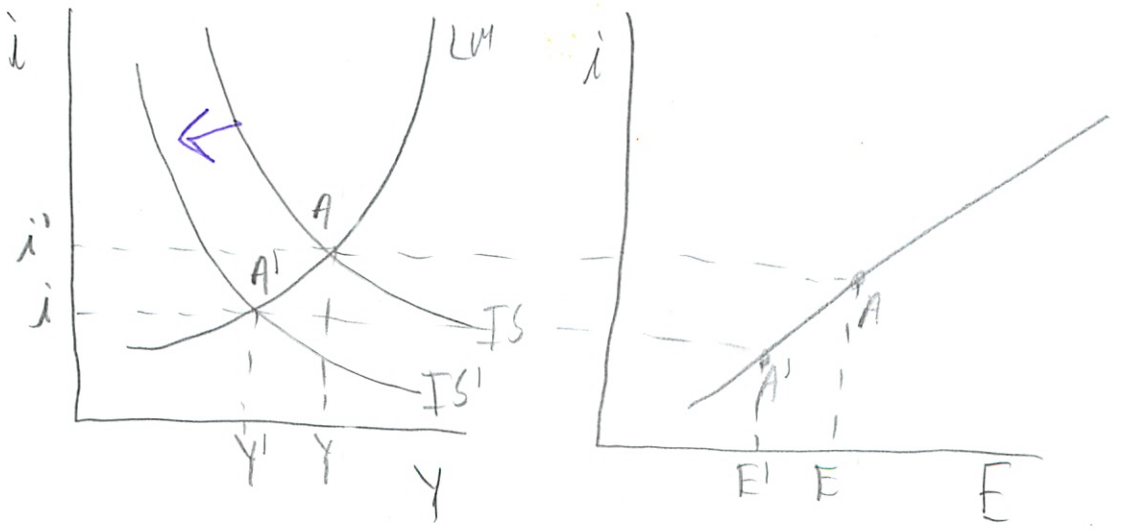
b) Nominal must = real interest rates since no inflation

since $P = P^* = 1 \longrightarrow \varepsilon = EP/P^* = E$

- for notational convienience

c) Using IS-LM-UIP diagram, show effects of ↓ G.

- flexible E
- start with $NX = 0$



(13)

The ↓G shifts IS inward. LM does not change.

New equlibium = A'

So ↓ G = ↓ output, ↓ i, depreiation ET

↓C since Y↓

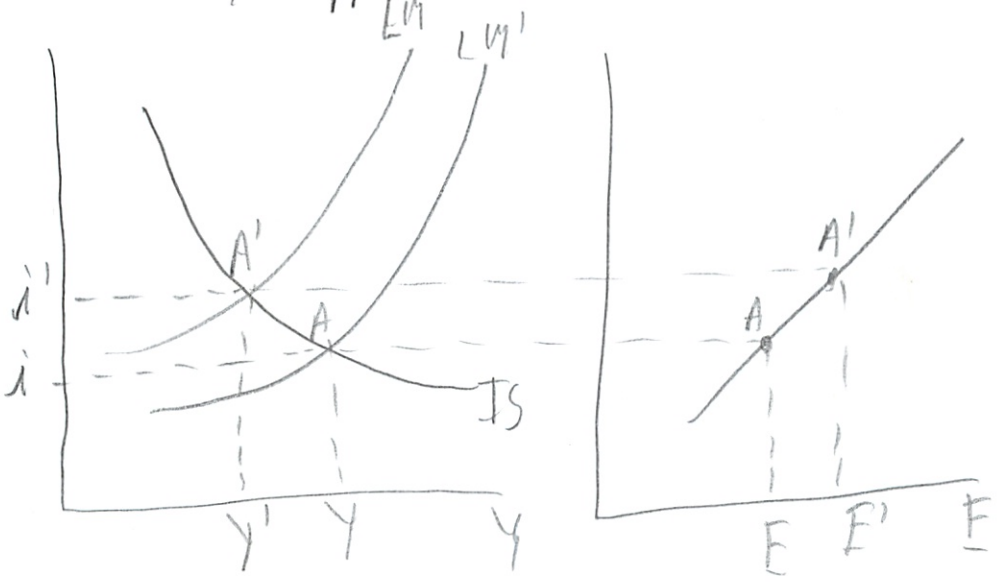
I is ambigeous like before

- output ↓ since ↓ income so less investment
- but ↓i means more investment ✓

-1 +NX ↓

- both ↓ output and depreciation leads to deteriation of trade balance

d) ↑ in Money supply



(14).

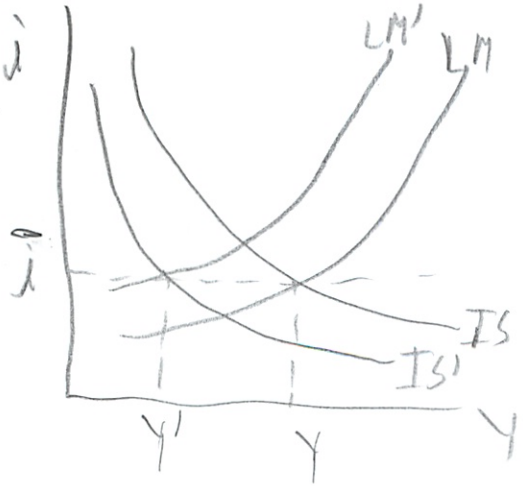
↑M means LM shifts upward. IS does not shift

↑ output means ↑ income means ↑C
↓ interest rate ↑
depreciation E↓
both trigger ↑I.

As i ↑ money demand ↑, leading to higher i
offsetting partially i↓ and depreciation
Net exports ↑ because US goods cheaper, other goods more expensive (IM↓) (×↑)

e) Consider G↓ in c). What could gov do to fix i

Well gov would want to do this if it had set a peg
It would have to set money supply - increase it
to keep interest rates the same.



Solutions

Problem Set # 5
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1 True/False [35 points]

Please state whether each of the following claims are True or False, and provide a brief justication for your answer. You may include graphs and equations to support your answer.

1. "A 20% increase in the nominal exchange rate between the US and the UK (defined as number of pounds per dollar) implies that US goods are now 20% more expensive relative to UK goods". [5 points]

   ANSWER. FALSE. For the US goods to be 20% expensive relative to UK goods we would need the real exchange rate to appreciate by 20%. A 20% nominal appreciation does not imply a 20% real appreciation since we are not told what happens to the US and UK price levels.

2. "When Mexican investors buy $1,000 of US Government bonds, the financial account of the US balance of payments increases by $1,000". [5 points]

   ANSWER. TRUE. See textbook page 387.

3. "If a US firm pays $1 million in dividends to foreign shareholders, then the financial account of the US balance of payments goes down by $ 1 million. " [5 points]

   ANSWER. FALSE. This transaction is recorded as investment income paid in the current account.

4. "If the exchange rate is 0.5 pounds per dollar, the one-year dollar interest rate is 2%, and the one-year pound interest rate is 5%, then the dollar is expected to depreciate vis a vis the pound in nominal terms." [5 points]

ANSWER. FALSE, the dollar is expected to appreciate

$$\begin{aligned}(1+i_t) &= (1+i_t^*)\frac{E_t}{E_{t+1}^e} \\ E_{t+1}^e &= \frac{1+i_t^*}{1+i_t}E_t = \frac{1.05}{1.02} * 0.5 = 0.514\,71 > 0.5\end{aligned}$$

5. "Consider an economy that has a fixed nominal exchange rate with respect to the US dollar. If this economy has greater inflation than the US, then its real exchange rate (with respect to the US) will appreciate." [5 points]

   ANSWER. TRUE. The real exchange rate is given by

   $$\varepsilon = EP/P^*$$

   This implies

   $$\gamma_\varepsilon = \gamma_E + \pi - \pi^*$$

   where γ_ε and γ_E denote the real rate of appreciation and the nominal rate of appreciation. Under fixed exchange rate, $\gamma_E = 0$, so that $\pi > \pi^*$ implies $\gamma_\varepsilon > 0$.

6. "In the Mundell-Fleming model with flexible exchange rate of Chapter 20, an increase in firms' willingness to invest (i.e. firms invest more for every possible interest rate) results in a nominal depreciation of the exchange rate. " [5 points]

   ANSWER. FALSE. It will result in a nominal appreciation of the exchange rate. The IS curve shifts up and to the right, increasing equilibrium output and the interest rate. The UIP thus implies a higher nominal exchange rate.

7. "In the Mundell-Fleming model of Chapter 20, if the exchange rate is fixed and there is a recession in the rest of the world, then the government needs to decrease the money supply." [5 points]

   ANSWER. TRUE. The recession in the rest of the world means that net exports decrease for the home country. This means the IS shifts down and to the left, thus reducing output and the interest rate. To keep the interest rate fixed at its old level (and thus the exchange rate fixed), the government needs to decrease the money supply, so as to make the LM curve shift up and to the left, to intersect the new IS curve at the old interest rate.

2 Open Economy IS [40 points]

Consider a world economy with only two countries. The home country is characterized by:

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The foreign country is characterized by

$$\begin{aligned} \tilde{C} &= \tilde{c}_0 + \tilde{c}_1 \tilde{Y} \\ \tilde{I} &= \tilde{c}_2 \tilde{Y} \\ \frac{\widetilde{IM}}{\tilde{e}} &= \tilde{m}\tilde{Y} \\ \tilde{X} &= \frac{\tilde{x}Y}{\tilde{e}} \end{aligned}$$

Assume that parameters in the foreign economy satisfy assumptions analogous to the ones for the domestic economy. We will assume throughout the question that the real exchange rate (e) is exogenously given.

Denote by G the amount of Government spending in the home country and by $\tilde{G}$ the amount of Government spending in the foreign country.

1. (5 points) Find the equilibrium level of output in the domestic goods market as a function of $e, G, \tilde{Y}$ and parameters. Find the multiplier (denote it by M). Find equilibrium in foreign goods market as a function of $\tilde{e}, \tilde{G}, Y$ and parameters. Find the multiplier in the foreign country (denote it by $\tilde{M}$).

   Answer:

$$\begin{aligned} Y &= c_0 + c_1 Y + c_2 Y + G + x\tilde{Y}/e - mY \\ Y &= \frac{1}{1 - c_1 - c_2 + m}\left[c_0 + x\tilde{Y}/e + G\right] \\ &\equiv M\left[c_0 + x\tilde{Y}/e + G\right] \end{aligned}$$

   where M is the multiplier. Similarly,

$$\begin{aligned} \tilde{Y} &= \frac{1}{1 - \tilde{c}_1 - \tilde{c}_2 + \tilde{m}}\left[\tilde{c}_0 + \tilde{x}Y/\tilde{e} + \tilde{G}\right] \\ &\equiv \tilde{M}\left[\tilde{c}_0 + \tilde{x}Y/\tilde{e} + \tilde{G}\right] \end{aligned}$$

2. (5 points) Does the Marshall Lerner condition hold for the particular functional forms of exports and imports functions assumed? Show why or why not.

   Answer: Yes, since net exports are decreasing in the real exchange rate e:

$$NX = \frac{x\tilde{Y}}{e} - mY$$

3. (5 points) State a condition relating e and $\tilde{e}$.

   Answer:

$$e = \frac{1}{\tilde{e}}$$

4. (5 points) Write a condition relating IM and $\tilde{X}$ and a condition relating $\widetilde{IM}$ and X. Show that each condition imposes a restriction on the parameters.

   Answer:

$$\begin{aligned} IM &= \tilde{X} \rightarrow mYe = \frac{\tilde{x}Y}{\tilde{e}} \rightarrow m = \tilde{x} \\ \widetilde{IM} &= X \rightarrow \tilde{m} = x \end{aligned}$$

5. (7 points) Solve for domestic output as a function of $e, G, \tilde{G}$ and parameters $(c_0, \tilde{c}_0, x, \tilde{x}, M, \tilde{M})$. Assume $Mx\tilde{M}\tilde{x} < 1$.

   Answer:

$$\begin{aligned} Y &= M\left[c_0 + x\tilde{M}\left[\tilde{c}_0 + \tilde{x}Y/\tilde{e} + \tilde{G}\right]/e + G\right] \\ &= M\left[c_0 + x\tilde{M}\tilde{c}_0/e + x\tilde{M}\tilde{x}Y + x\tilde{M}\tilde{G}/e + G\right] \\ &= \frac{M}{1 - Mx\tilde{M}\tilde{x}}\left[c_0 + x\tilde{M}\tilde{c}_0/e + x\tilde{M}\tilde{G}/e + G\right] \end{aligned}$$

6. (6 points). How do G and $\tilde{G}$ affect output at home Y? What are the multipliers?

   ANSWER. Increases in G and $\tilde{G}$ increase Y. The multipliers are

$$\begin{aligned} \frac{\partial Y}{\partial G} &= \frac{M}{1 - Mx\tilde{M}\tilde{x}} = \frac{1}{1 - c_1 - c_2 + m - \frac{xm}{1-\tilde{c}_1-\tilde{c}_2+x}} \\ \frac{\partial Y}{\partial \tilde{G}} &= \frac{M}{1 - Mx\tilde{M}\tilde{x}} x\tilde{M}/e = \frac{\frac{x}{1-\tilde{c}_1-\tilde{c}_2+x}/e}{1 - c_1 - c_2 + m - \frac{xm}{1-\tilde{c}_1-\tilde{c}_2+x}} \end{aligned}$$

7. (7 points) Suppose the home country has decided to increase its Government budget deficit by 100 units (of the local good). Is it better to use these funds to expand domestic public spending ($\Delta G = 100$), or to give a grant to the Foreign country on the condition that the funds are fully used for Foreign public spending ($\Delta \tilde{G}$)? Which alternative has a greater impact on the output of the home country? State a precise mathematical condition that guarantees your answer.

   Answer: The answer depends on parameters. If the home country gives the grant, the foreign country would expand public spending by $\Delta \tilde{G} = 100e$. The effect on Home's output would be greater than the effect of $\Delta G = 100$ provided

$$x\tilde{M} > 1$$

3 Open Economy IS-LM [25 points]

Consider the Mundell-Fleming model of Chapter 20. Equilibrium in the goods market is given by

$$Y = C(Y - T) + I(Y, r) + G + X(Y^*, \varepsilon) - IM(Y, \varepsilon)/\varepsilon$$

where Y is output at home, Y^* is foreign output, T are taxes, $C(\cdot)$ is consumption, r is the real interest rate, $I(\cdot)$ is investment, G is government spending, X is exports, IM is imports, and ε is the real exchange rate. Equilibrium in the money market is given by the standard LM equation:

$$M/P = YL(i)$$

where i is the nominal interest rate. Assume that P and P^* are constant, with $P = P^* = 1$. Note that this implies zero inflation (actual, and expected). Assume further that the Marshall-Lerner condition holds.

1. State the uncovered interest parity condition. Provide intuition for this equation. Use E_t to denote the nominal exchange rate at time t, E^e_{t+1} to denote the expected nominal exchange rate at t+1 and i^*_t to denote the foreign interest rate. What assumption about local and foreign bonds is required for this condition to hold? [5 points]

   ANSWER. The UIP is

$$1 + i_t = (1 + i^*_t) E_t / E^e_{t+1}$$

   This condition says that the return to holding domestic bonds must be equal to the return to holding foreign bonds. It is thus an arbitrage relation. For this condition to hold, we must assume that investors have access to both local and foreign bonds.

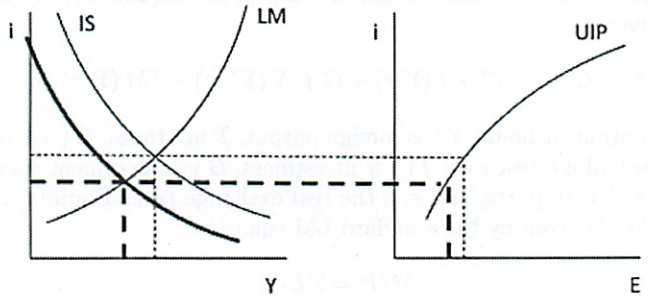
2. Explain why nominal and real exchange rates must coincide given our assumptions. [3 points]

   ANSWER. When $P/P^* = 1$ we have that

   $$\varepsilon = EP/P^* = E$$

3. Using a IS-LM-UIP diagram, show the effects of a decrease in government spending. Assume that the economy has a flexible exchange rate, and that initially net exports are zero. State what would happen to output, the interest rate, the exchange rate, net exports, investment and consumption. [If the effect on any variable is undetermined, just state so]. [7 points]

   ANSWER. The decrease in G shifts the IS curve down and to the left, thus reducing output and the interest rate. The UIP relation implies a lower exchange rate.

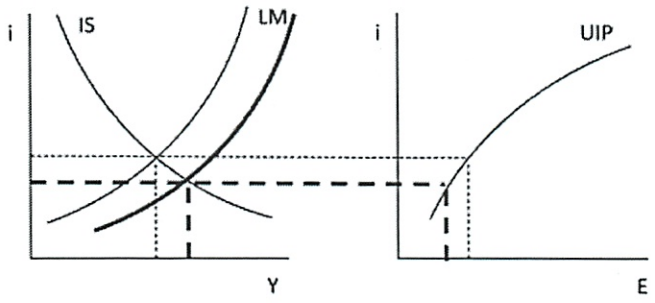


   Net exports increase (exports go up because of the lower real exchange rate, and the value of imports goes down because of the lower output, and the lower exchange rate, under the Marshall-Lerner condition). Investment is undetermined. Consumption goes down because outoput is lower.

4. Using a IS-LM-UIP diagram, show the effects of an increase in money supply. Continue to assume that the economy has a flexible exchange rate, and that initially net exports are zero. State what would happen to output, the interest rate, the exchange rate, net exports, investment and consumption. [If the effect on any variables is undetermined, just state so]. [7 points]

   ANSWER. The LM curve shifts to the right, thus increasing output and

reducing the interest rate. The exchange rate goes down.



The effect on net exports is undetermined: the nominal devaluation tends to increase net exports, but the increase in output tends to reduce net exports (as imports increase). Investment goes up as the interest rate is lower, and output is higher. Consumption goes up due to higher output.

5. Consider the scenario of government spending reduction described in part 3. What should the Government do to keep the exchange rate fixed? [3 points]

   ANSWER. The government has to decrase money supply, to make the LM intersect the new IS at the old interest rate.

14.02

(4 min late)

Output = demand for output

$$Y = C(Y_d) + I(Y, i) + G + NX\left(Y, Y^*, \frac{1+i}{1+i^*}, \bar{E}^e\right)$$

Net exports ↓ in $\bar{E}$
- Marshal Lerner condition we assume

Remember interest rate parity

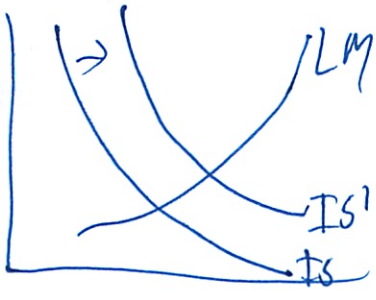
$$(1+i) = (1+i^*)\,\bar{E}^e$$

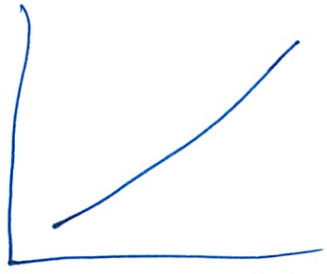
Now ↑ in i has 2 effects
- standard effect - decrease in I, ↓ Y
- plus new effect - ↑ in $\bar{E}$ → domestic goods more expensive
  → ↓ NX → ↓ domestic demand → ↓ domestic income ↓

LM curve stays the same

Effect of Fiscal Policy

G ↑





(2)

(Mara teaches straight out of the book)

Y goes ↑

i ↑

Since i ↑, E ↑ by interest rate parity condition

C ↑

I = I(Y, i) → Y ↑, i ↑ so ambigeous

NX ↓ , NX = NX(Y, Y*, E)

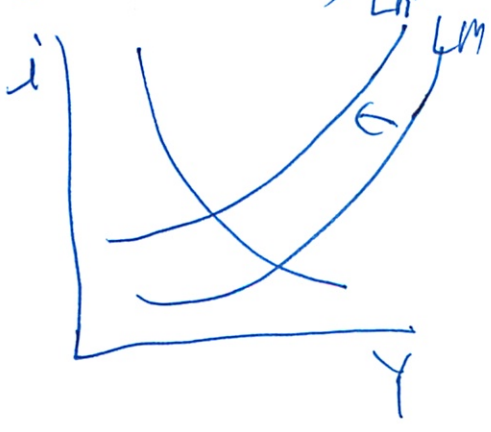
Y ↑, NX ↓ — since Y ↑, Demand ↑, so more imports

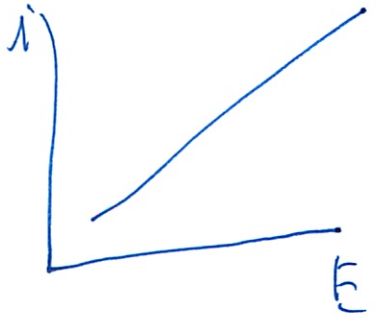
E ↑, NX ↓ (Marshall Lerner) - NX ↓ in real exchange rate ↑

Monetary Policy

Assume ΔM < 0

LM shifts inwards





Interest parity

(3)

Y goes ↓, i ↑

By interest-rate parity condition E ↑

IS-LM in open econ is <u>Mundell Fleming model</u>

- with that extra interest-rate parity condition

Fixed Exchange Rate

Interest parity must hold

Choose i such that $E_t = E^e_{t+1}$

$$(1+i_t) = (1+i^*_t)\frac{E_t}{E^e_{t+1}} = (1+i^*_t)\frac{\bar{E}}{\bar{E}} \rightarrow$$

$$i_t = i^*_t$$

Revisit LM

$$\frac{M}{P} = Y L(i^*)$$

Under a fixed rate gov must use monetary policy to target i. Can not set their own monetary policy!

Fiscal policy becomes an even more important tool

- hyper sensitive

Chap 21 Fragility of Fixed Ex Rate

i must reflect any changes in E^e to be credible

$$(1+i_t) = (1+i^*_t)\frac{E}{E^e_{t+1}}$$

After WW2 many countries moved to fixed system
Bretton Woods system

Consider a country w/ high public debt + short maturity

For every shekel it borrows, it has to pay back $1+i_t$ shekels
- if it depreciates currency, can pay back less

Then gov must ↑ i, E^e_{t+1} is smaller
Now even stronger incentive to depreciate

~~Because this~~

2nd channel

If a real exchange rate changes, under a fixed exchange rate, prices must be adjusted in order to get back to natural level of output. Recall that the real ex rate $= \epsilon = \frac{EP}{P^*}$

Say ϵ goes ↑, → NX ↓, econ in recession. To get out P has to adjust - but slow, but E changes quickly

(5)

Remember fixed exchange rate was supposed to cut down on volitality

Fiscal policy

Following special notes on Stellar
Not really from book
Exam Thur next remember

Gov can Tax, use up savings, or borrowing
(all types) (by selling bonds)

$$\text{deficit}_t = r B_{t-1} + G_t - T_t$$

(B_{t-1}: bonds outstanding)

Gov has to repay $(1+r) B_{t-1}$ on
Gov spends G_t
Gov's income T_t

So gov borrows $B_t = (1+r) B_{t-1} + G_t - T_t$

$$B_t - B_{t-1} = r B_{t-1} + G_t - T_t = \text{deficit}$$

Deficit = new gov borrowing

(2)

r = real interest rate $= i - \pi$

deficit = add borrowing vs last period

Suppose Gov cuts taxes 1 unit at year 0 (assume $B_{-1} = 0$ ∴ budget balanced before)

$$B_0 = G_0 - T_0 = 1$$

Repayment at year 1 (not continuing to borrow)

$$0 = B_1 = (1+r) B_0 + G_1 - T_1$$

$$1 + r = T_1 - G_1$$

So gov must ↑T or ↓G by $1+r$ after year 1

But what if gov does not want to repay in period 1 but after T years (assuming $T_t = G_t$ for $t \geq 1$)

$$B_1 = (1+r) B_0 = 1 + r$$

$$B_2 = (1+r) B_1 = (1+r)^2$$

$$\vdots$$

$$B_{T-1} = (1+r)^{T-1}$$

$$B_T = (1+r) B_{T-1} + G_T - T_T$$

$$T_T - G_T = (1+r)^T$$

(3)

The longer you wait to repay, the more you will pay!

Debt Stabilization

Assume tax cut = 1 in t=0, so $B_0 = 1$

and we want debt to remain at 1

$$B_1 = (1+r)B_0 + G_1 - T_1$$

$$1 + r = T_1 - G_1$$

↑ Ga must ↑T or ↓G by 1+r

(just pays interest)

Assuming $B_1 = B_0$

$$r B_0 = r = T_1 - G_1$$

$T_1 - G_1 > 0$ primary surplus is (+) and = r

↳ diff → T − G

If there is growth; then the extra we owe might be a smaller part of econ

Debt to GDP ratio

$$\frac{B_t}{Y_t} = \frac{(1+r)B_{t-1}}{Y_t} + \frac{G_t - T_t}{Y_t}$$

divide (2) by Y_t

(4)

↓ multiply and divide by Y_{t-1}. rearrange.

$$\frac{B_t}{Y_t} = (1+r)\frac{Y_{t-1}}{Y_t}\cdot\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

Note that $\frac{Y_{t-1}}{Y_t} = \frac{1}{1+g}$ ← debt to GDP ratio at $t-1$

g = growth rate of econ

Also approx. $\frac{1+r}{1+g} \approx 1 + r - g$ if r, g are fairly small

↑ use Taylor expansion from log function

Thus

$$\frac{B_t}{Y_t} \approx (1+r-g)\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

Assume $\approx$ means $=$

$$\frac{B_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}} = (r-g)\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

$(r-g)$ depends a lot on diff b/w r, g

≈ 1 if about same - debt increases quickly

< 1 then debt can shrink

$(g > r)$ Converges to 0 - even under balanced budget

> 1 $(g < r)$ gov must run primary surplus

(5)

If $g < r$ and gov wants to stabalize debt to GDP

- for example debt = 3% of output
- can borrow or repay debt

$$0 = \frac{B_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}} = (r-g)\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

$$\frac{T_t - G_t}{Y_t} = (r-g)\frac{B_{t-1}}{Y_{t-1}} > 0$$

assuming have ⊕ debt $B_{t-1} > 0$

If gov does not stabaline debt to GDP -
then the debt to GDP will grow exponentially

$$\frac{B_t}{Y_t} = (1+r-g)\frac{B_{t-1}}{Y_{t-1}}$$

$$\frac{B_{t+T}}{Y_{t+T}} = (1-r-g)^T \frac{B_t}{Y_t}$$

so $\lim_{T \to \infty} = \infty$

because $1+r-g > 1$

(6) Case $g > r$ debt to GDP converges to 0, even w/ balanced budget $G_t = T_t$

$$\frac{B_{t+T}}{Y_{t+T}} = (1+r-g)^T \frac{B_t}{Y_t}$$

$$\lim_{T \to \infty} = 0$$

14.02 Quantative Easing

4/8

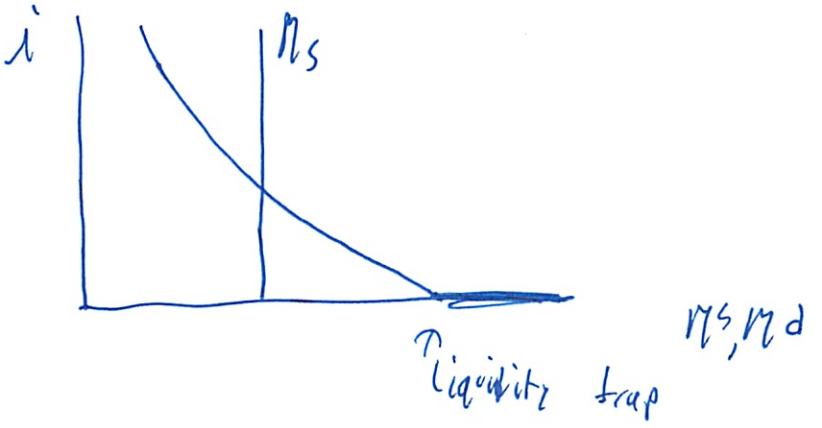
Monetary policy - even though fiscal policy chap 3 years would not have taught

$\frac{M}{P} = YL(i)$

$Y = C(Y-\bar{T}) + I(i, Y) + G$

So could ↑ C or ↑ M_S

But somewhere liquidity trap



Can't ↓ i below 0

- negitive i - people will just keep their money

$C = C_0 + C_1(Y-T) + C_2 W(i)$

↑ wealth: savings, etc

- value of money tomorrow, today
- use i to find PV

Works if inflation or not $i = r + \pi$

(2)

But G ↑

- takes time

And limits

- can't borrow anymore
- can't pay off

So Quantative Easing!

- there are many is

$i_1 = \frac{\$100 - \$P_{B_1}}{\$P_{B_1}}$ 1 year nominal interest rate

$i_2 = \frac{\$100 - P_{B_2}}{P_{B_2}}$ 2 year bond

Yields

- to compare nominal interest rates
- 1 year $Y_1 = i_1$
- 2 years $(1+Y_2)^2 = (1+i_1)(1+i_2^e)$ expected
- return should be =
  - 2 year bond
  - buy 1 year, then afterwards by another
  - no arbitrage

(3)

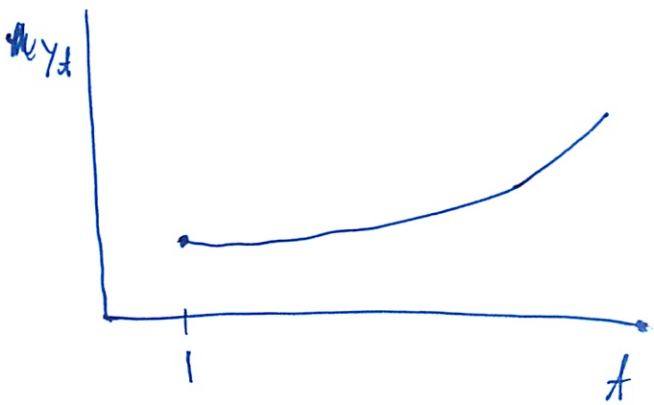
$(1 + {}_0i_2)$

$(1 + {}_0i_1)(1 + {}_1i_2^e)$

~~//~~

${}_t i_s = i_{t,s}$ = interest from t to s

What would y_2 have to be for interest rate to hold



Upward sloping usually:
1 year interest rate will be higher in future than today

Why upward sloping?

- Still a debate among economists
- liquidity premium
- Uncertainty
- economy will be growing

(4)

These are just gov bonds

Corp bonds too

- more risky

So now I (Y, i) means

$I(Y, Y_1, Y_2, Y_3, Y_4 \ldots$ also Corp Yields$)$

Wealth (i) → Wealth ($Y_1, Y_2, Y_3, \ldots$ also Corp)

i is the 1 month interest rate

But when i was ~0

But Fed said - could we lower other interest rates?

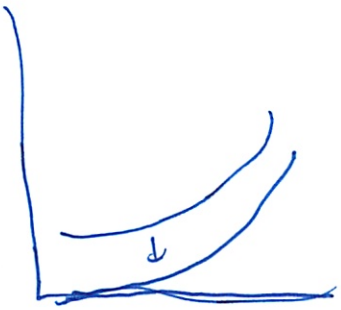
Remember fed buys bonds at the discount window

- Normally just 1 month or 1 year bonds
- But w/ QE buy bonds of all durations
- Demand ↑ → P↑ → i↓

$$\frac{100}{P_{B2}^{\$}} - 1$$

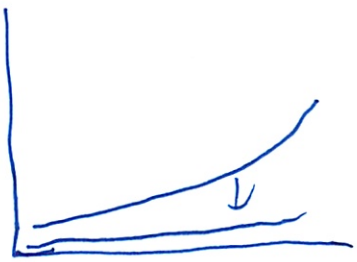
(5)

Normally



lower

But also decrease slope w/ QE



QE is controversal

- Ben Bernake worked on before being chairmen
- What is the downside?
  - default risk ~~from~~ from corp bonds
    - you should not own US bonds
  - more inflationary pressures
    - pumping in $
    - putting all yields below natural levels
  - no one has done this before
    - how much can gov change
    - will you overdo it or not enough

(6)

When $i \downarrow$, W?

- say in a 2 year world

$$-W_0 = (Y_1 - T_1) + (Y_2 - T_2)$$

can't just add
must discount to PV

$$= \frac{(Y_1 - T_1)}{(1 + o i_1)} + \frac{(Y_2 - T_2)}{(1 + o i_1)(1 + o i_2)}$$

+ consumption - but messes up formula

Called QE?

- Some people love, others hate

Chapter 21

HIGH DEBT

Why do economists worry so much when governments run large budget deficits and accumulate debt quickly? In answering this question, they argue that debt slows capital accumulation, puts at risk the stability of the economic system and makes it extremely difficult to conduct monetary policy.

In principle, a high government deficit is neither good nor evil. Deficits (and surpluses) can help to redistribute the burden of taxation over time. Deficits become a problem when they result in rapid accumulation of debt and also because, as we shall discuss, reducing a high debt, once it has been created, can take a long time, often many decades. This is why high debt will turn out to be the most long-lasting consequence of the 2007–2010 financial crisis.

This chapter is in three parts:

- Section 21.1 studies the budget constraint of government – namely the relationship between debt, deficit, government spending and taxes – and examines its consequences.
- Section 21.2 examines which factors determine the evolution over time – the dynamics – of the debt-to-GDP ratio, and what determines debt accumulation.
- Section 21.3 introduces a 'political theory' of debt that will help us understand some historical episodes of large build-ups and subsequent reductions in public debt.

21.1 THE GOVERNMENT'S BUDGET CONSTRAINT

Suppose that the government, starting from a situation of **balanced budget**, decides to cut taxes while keeping public expenditure unchanged, thereby creating a **budget deficit**. What will happen to debt over time? Will the government eventually be forced to increase taxes? If so, by how much: by more than its original cut? To answer these questions, let us start from a definition of the budget deficit. We can write the budget deficit in year t as:

$$deficit_t = rB_{t-1} + G_t - T_t \qquad [21.1]$$

All variables are in real terms, that is measured in units of real output, not in euros or pounds. B_{t-1} is public debt at the end of year $t-1$, i.e. at the beginning of year t. We define B as all the bonds and bills issued by the govrnemnt and held by the private sector (at home or abroad) but excluding those held by the central bank. r is the real interest rate, which for the time being we assume to be constant. rB_{t-1} represents the real interest paid on government bonds in circulation. G_t is government spending on goods and services in year t. T_t are taxes less transfers in year t.

◀ We do this because what matters, at least for this discussion, are the net sums payed by households to the state, that is the taxes households pay minus the what they receive from the state, such as unemployment benefits and pensions.

oh not for me

The budget deficit is equal to spending on goods and services, plus interest payments, minus taxes less transfers.

Equation (21.1) has two characteristics.

1. First, we measure interest in real terms – that is not considering the actual interest payments (which are calculated as the **nominal** interest rate times the stock of debt), but ***real* interest payments**, i.e. the *real* interest rate on existing debt. This is in fact the proper way to measure interest payments. In Brazil, for instance, in the early 1990s, when inflation was running at 50% per year, nominal interest payments were very large because the nominal interest rate was as high as 50%: but the real interest rate (the nominal minus expected inflation) was close to zero: debt was not really a burden for the state. However, the official measures of the deficit are based on actual expenditure and thus on nominal interest payments: as a result, they give a distorted picture of state finances. In Brazil, for instance, the official deficit was extremely high, but the deficit corrected for inflation was not as large. (To see how you can compute the deficit adjusted for inflation starting from the official measure of the deficit, look at the Focus box below).

◀ Remember there is a (sometimes big) difference between the official deficit and the deficit adusted for inflation.

if inflation > debt growth is that good?

2. To remain consistent with our definition of G as public spending on goods and services, we continue to assume that G includes neither interest payments nor transfers. Transfers are subtracted from T. The official measures of public spending include transfers, and they define revenues as taxes rather than as net taxes. This is simply an accounting convention: adding transfers to expenses or subtracting them from taxes obviously makes a difference when you compute G and T, but it is irrelevant when you compute the deficit.

Why I ignore transfers

When a government faces a budget deficit, it may ask the central bank to finance it. In this case, what the government technically does is to sell bonds to the central bank. Alternatively, it may sell the bonds directly to private investors.

◀ Recall that in our definition of public debt, this includes only the securities held by private investors, not those purchased by the central bank.

In Chapter 22, we will focus on the link between deficits, money creation (what happens when the central bank buys government bonds) and inflation. In this chapter, however, to keep things simple we assume that the only means of **deficit financing** is selling securities to private investors.

Why?

In this case, the budget constraint of the government simply says that the increase in government debt in year t must be equal to the deficit in year t:

$$B_t - B_{t-1} = deficit_t$$

Same as lecture

So if the government runs a deficit, government debt increases. If the government has a surplus, debt decreases. Using our definition of the deficit, we can write the government budget constraint as:

$$B_t - B_{t-1} = \underset{\text{interest payments}}{rB_{t-1}} + \underset{\text{primary deficit}}{(G_t - T_t)} \qquad [21.2]$$

FOCUS

How to compute the budget deficit corrected for inflation

Official measures of the budget deficit are constructed (omitting the time index) as the sum of nominal interest, iB, more government spending on goods and services, G, minus taxes net of transfers, T:

$$\text{official deficit} = iB + G - T$$

This is a measure of the change in *nominal debt*. If it is positive, the government is spending more than it collects, and must therefore issue new debt. If it is negative, the government reimburses part of the existing debt.

But this measure is not suitable to calculate the change in *real debt*, i.e. the variation in what the government will have to pay in terms of goods rather than money.

To understand why, suppose that the official measure of the deficit is equal to zero: the government does not issue or repay any securities. Suppose that inflation is positive and equal to 10%. In this case, at the end of the year, the real value of debt will be reduced by 10%. So, if we define – as we should – the deficit as the change in the real value of debt, the government recorded a budget surplus equal to 10% of the initial value of the debt.

In general: if B is the debt and inflation is π, the official measure of the deficit overstates the correct measure by an amount equal to πB. The correct, inflation-adjusted measure of the deficit is in fact equal to:

$$iB + G - T - \pi B = (i - \pi)B + G - T = rB + G - T$$

where $r = i - \pi$ is the real interest rate.[1] The correct measure of the deficit is thus equal to the real interest plus government spending minus taxes net of transfers, i.e. the measure that we use in the text.

The difference between the official and the proper measure is equal to πB. This means that the higher the rate of inflation (π) or the level of debt (B), the more imprecise is the official measure of the deficit. In countries where inflation and debt are both very high, the official measure could indicate a very large deficit, even in the presence of a decreasing real debt. For this reason, we should always take inflation into account whenever we derive conclusions on fiscal policy.

Figure 21.1 shows the official measure and the inflation-adjusted deficit for the UK since 1949. Both measures show a strong decline occurring after 1976, an improvement at the end of the 1980s, followed by a sharp deterioration in the mid-1990s, a significant improvement around 2000 and a further deterioration since 2006.

[1] Note that here r is the nominal interest rate minus actual inflation: it should be called the 'real effective interest rate' to distinguish it from the 'real interest rate' which we defined in Chapter 6 as the nominal interest rate less expected inflation.

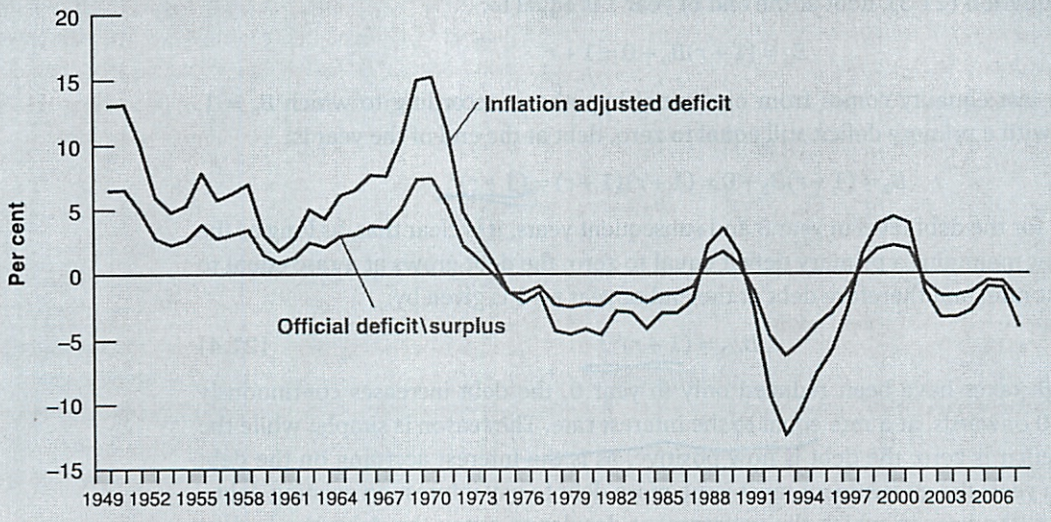


Figure 21.1

Official and inflation adjusted budget deficits for the UK, 1949–2006

Finally, if we move B_{t-1} to the right of the equation and we reorganise the terms, we get:

$$B_t = (1+r)B_{t-1} + G_t - T_t \qquad [21.3]$$

Debt at the end of year t is equal to $(1+r)$ times debt at the end of year $t-1$, plus the **primary deficit**, which is equal to the total defcit minus interst payments, that is $G_t - T_t$. This relation will be very useful in the next sections.

Current and future taxes

We now use equation (21.3) to run an experiment. We want to study the effect on the evolution of debt and future taxes of a tax cut in year 0. We start from a situation in which, up to year 0, the budget had always been in balance, so that debt is zero. In year 0, the government cuts taxes by 1 for one year (the amount is irrelevant, since we are computing everything in real terms; think of the tax cut as a reduction equivalent to 1 unit of output). Then, debt at the end of year 0, B_0, will be equal to 1. What happens next? Let us consider several cases.

- *Repayment in year 1* – suppose the government decides to repay the whole debt in year 1. From equation (21.3), the budget constraint in year 1 is given by:

$$B_1 = (1+r)B_0 + (G_1 - T_1)$$

If all debt is repaid in year 1 then, at the end of year 1, debt will be equal to zero: $B_1 = 0$. Replacing B_0 with 1 and B_1 with 0, the previous equation becomes:

$$T_1 - G_1 = (1+r)1 = 1+r$$

To repay the debt in year 1, the government must therefore create, in year 1, a **primary surplus** equal to $(1+r)$ units of goods. This can happen in two ways: by reducing spending or by increasing taxes. Here we assume that the adjustment is done through taxes, while spending remains unchanged. It follows that a tax cut of 1 in year 0 must be compensated by an increase in taxes by $(1+r)$ in year 1. The path of taxes and debt corresponding to this case is shown in Figure 21.2a (assuming $r = 10\%$). The coloured areas represent the deviations of taxes from their initial level, and grey areas represent the debt level.

- *Repayment after t years* – suppose now that the government decides to wait for t years before raising taxes to repay the debt. In this case, from year 1 to year t, the primary deficit will be equal to zero. Let's see what the effects are on the debt level at the beginning of year t (or at the end of year $t-1$). In year 1, the primary deficit is equal to zero. Then, in equation (21.3), debt at the end of year 1 is equal to:

Repayment means that the government buys back securities held by private investors.

$$B_1 = (1+r)B_0 + 0 = 1+r$$

where the last equality comes from our initial hypothesis according to which $B_0 = 1$. In year 2, with a primary deficit still equal to zero, debt at the end of the year is:

$$B_2 = (1+r)B_1 + 0 = (1+r)(1+r) = (1+r)^2$$

Solving for the debt level in year 3 and subsequent years, it is clear that, as long as the government maintains a primary deficit equal to zero, the debt grows at a rate equal to the interest rate, and therefore debt at the end of year $t-1$ is given by:

$$B_{t-1} = (1+r)^{t-1} \qquad [21.4]$$

Although taxes have been reduced only in year 0, the debt increases continuously from year 0 onwards, at a rate equal to the interest rate. The reason is simple: while the primary deficit is zero, the debt is now positive, as is the interest accruing on the debt itself. Each year, the government must issue more debt to pay interest on existing debt.

In year t, the year in which the government decides to repay the debt, the budget constraint is given by:

$$B_t = (1+r)B_{t-1} + (G_t - T_t)$$

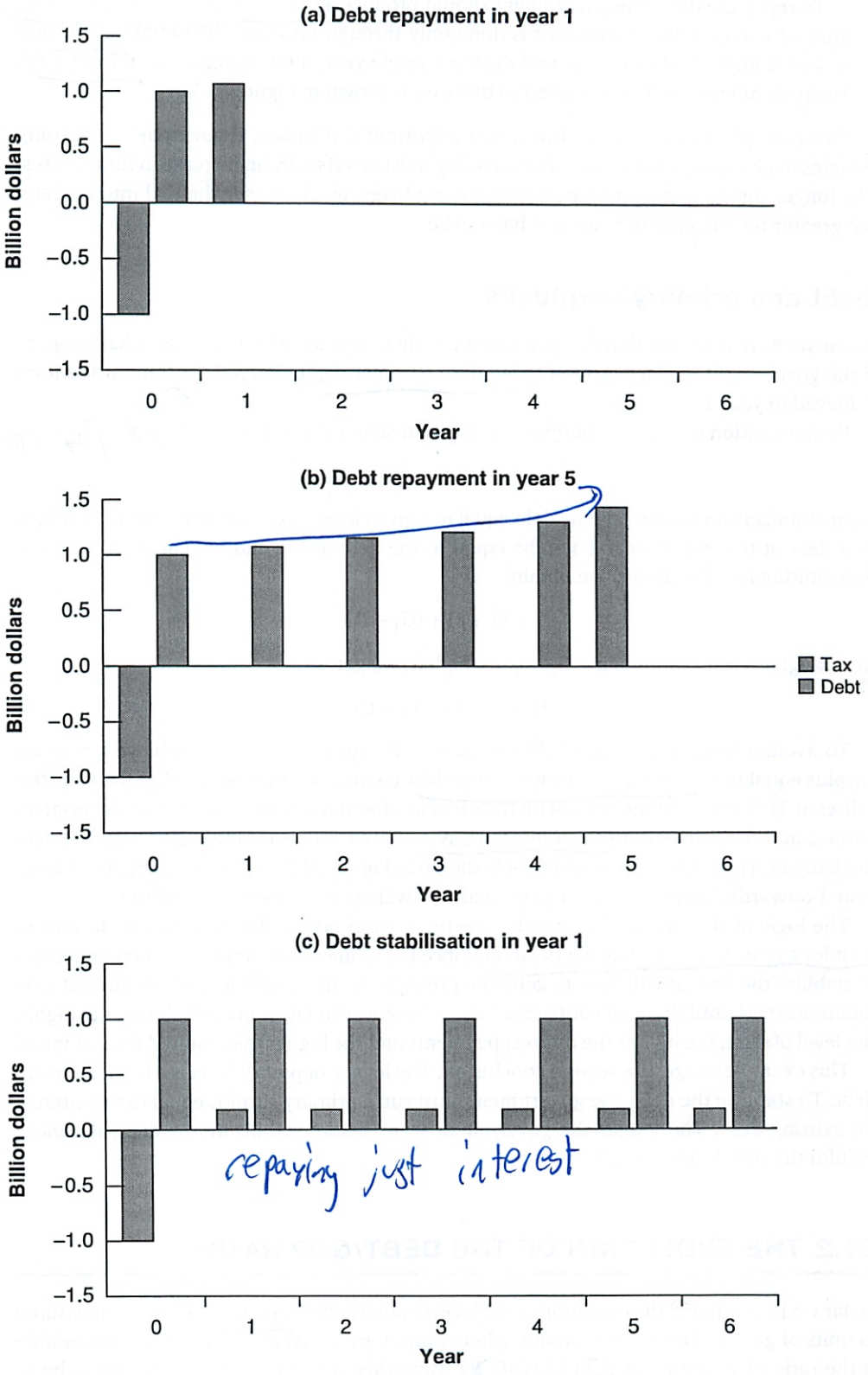


Figure 21.2

Tax reduction, debt repayment and debt stabilisation

(a) If debt is entirely repaid in year 1, a tax reduction equal to 1 in year 0 must be compensated by an increase in taxes by $(1 + r)$ in year 1.

(b) If debt is entirely repaid in year t, a tax reduction equal to 1 in year 0 must be compensated by an increase in taxes by $(1 + r)^t$ in year 1.

(c) If debt is stabilised from year 1 onwards, then taxes must be permanently higher by $(1 + r)$ from year 1 onwards.

If the debt is fully repaid in year t, B_t (the debt at the end of year t) will be equal to zero. Substituting B_t with zero and B_{t-1} with its expression from equation (21.4), we obtain:

$$0 = (1 + r)(1 + r)^{t-1} + (G_t - T_t)$$

By rearranging terms and moving $G_t - T_t$ to the left side, we get:

$$T_t - G_t = (1 + r)^t$$

To repay the debt, the government should produce a primary surplus equal to $(1+r)^t$ units of goods. If the adjustment is done only through taxation, the initial tax cut of 1 in year 0 implies, after t years and during a single year, a tax increase equal to $(1+r)^t$. The path of taxes and debt related to this case is shown in Figure 21.2b.

This example allows you to draw a first important conclusion. If government spending remains unchanged, a reduction of taxes today must be offset by an increase in future taxes. The longer the government waits before raising taxes, or the higher the real interest rate, the greater the increase in taxes will have to be.

Debt and primary surpluses

So far we have assumed that the government fully repays its debt. Let us see what happens if the government decides instead to stabilise the debt, i.e. to keep B constant at the level achieved in year 1.

◀ Stabilisation means that the government wants to keep the amount of existing debt constant.

From equation (21.3), the budget constraint in year 1 is given by:

$$B_1 = (1+r)\,B_0 + (G_1 - T_1)$$

Debt stabilisation means keeping debt equal to a given level, in our case the level of year 0, so that debt at the end of year 1 will be equal to the debt at the end of year 0, $B_1 = B_0 = 1$. Substituting into the above, we obtain:

$$1 = (1+r) + (G_1 - T_1)$$

Rearranging terms and moving $G_1 - T_1$ to the left, we get:

$$T_1 - G_1 = (1+r) - 1 = r$$

To avoid a further increase of debt in year 1, the government must achieve a primary surplus equal to the real interest on existing debt (remember that the level of the debt stabilises at 1). The government must do the same in subsequent years: every year, the primary surplus must be sufficient to cover interest payments, so as to leave the existing level of debt unchanged. The path of taxes and debt is shown in Figure 21.1c: the debt is equal to 1 from year 1 onwards. Taxes are higher from year 1 onwards, by an amount equal to r.

The logic of this argument extends directly to cases where the government decides to wait for t years before stabilising the debt. Since the moment when the government decides to stabilise the debt, it will have to generate primary surpluses sufficient to pay interest payments accrued until then. Of course the longer the government waits to stabilise, the higher the level of debt, the higher the interest payments and the higher the required level of taxes.

This example suggests a second conclusion. The legacy of past deficits is a higher current debt. To stabilise the debt, the government must run a primary surplus equal to the interest on existing debt. The longer the government waits before stabilising the debt, the more painful the stabilisation will be.

21.2 THE EVOLUTION OF THE DEBT/GDP RATIO

So far we have studied the evolution of the level of public debt in real terms (that is, measured in units of goods). But in an economy where output grows over time, the relevant variable is the ratio of government debt-to-GDP. By using this, we can tell if the debt is too high, where 'too' must be defined in relation to the ability of the government to repay the debt.

The government budget constraint in terms of GDP

To see how our previous conclusions change, let us move from equation (21.3) to an equation that expresses the evolution of debt-to-GDP ratio or **debt ratio**. To do this we

need to carry out some further steps. Let's divide both sides of equation (21.3) by real output, Y_t, to get:

$$\frac{B_t}{Y_t} = (1+r)\frac{B_{t-1}}{Y_t} + \frac{G_t - T_t}{Y_t}$$

Rewriting, on the right side, B_{t-1}/Y_t as $(B_t/Y_{t-1})(Y_{t-1}/Y_t)$ (in other words, multiplying the numerator and denominator by Y_{t-1}), the relationship becomes:

$$\frac{B_t}{Y_t} = (1+r)\left(\frac{Y_{t-1}}{Y_t}\right)\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

Now that each term is expressed in terms of GDP at time t, we can simplify this expression. By defining g, the growth rate of output, we have $Y_{t-1}/Y_t = 1/(1+g)$. Moreover, by approximating $(1+r)/(1+g)$ with $1+r-g$ (which is a good enough approximation if the real interest rate and the growth rate of output are relatively small numbers), we can rewrite the previous equation as:

$$\frac{B_t}{Y_t} = (1+r-g)\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t} \qquad [21.5]$$

Finally, moving B_{t-1}/Y_{t-1} to the left, we get:

$$\frac{B_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}} = (r-g)\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t} \qquad [21.6]$$

This equation tells us that the change in the debt ratio is equal to the sum of two terms:

- The first is the difference between the real interest rate and the rate of growth of GDP, multiplied by the debt ratio at the end of the previous period (and thus at the beginning of the current period). This term refers to interest payments, in real terms, corrected for the growth rate of real GDP. Depending on whether the real interest rate is higher or lower than the growth rate of real GDP, this term is a factor that increases or reduces the debt ratio. Therefore, r and g have opposite effects on the dynamics of the debt ratio.
- The second term is the ratio of the primary deficit to GDP. The primary balance relative to GDP has a positive or negative effect on the growth of debt, respectively, in the case of a deficit ($G_t - T_t > 0$) or a surplus ($G_t - T_t < 0$).

Let's now compare equation (21.6), which describes the evolution of the debt ratio, with equation (21.2), which describes the evolution of the debt level. The difference is the presence of $(r-g)$ in equation (21.6) rather than r in equation (21.2). The reason for this difference is simple. Suppose that the primary deficit is zero. In this case, the debt level will increase at a rate equal to the real interest rate, r. But, if GDP grows, the debt/GDP ratio will grow more slowly; it will grow at a rate equal to real interest rate minus the rate of growth of output, $r-g$. If the rate of growth of the economy is greater than the real interest rate, that is, if $r-g$ is negative, the debt ratio will not only grow more slowly, it will decline from year to year.

The debt ratio in the long run

Equation (21.6) – or, equivalently, equation (21.5) – allows us to analyse the change in the debt ratio from one year to the next. But we can look further ahead and ask if the debt ratio, given all the other variables, will tend to stabilise or will it instead tend to diverge, that is to shoot off to plus infinity or to zero.

Solving simple differential equations is not difficult, but to keep things simple here we will solve it only graphically.

At this point you could make a small (but very useful) investment to learn the basics of differential equations by reading the Focus box 'A qualitative solution of difference equations'. Otherwise you can skip directly to the next section.

FOCUS
A qualitative solution of difference equations

The simplest mathematical tool to study the 'dynamics' of a variable is a difference equation: an expression that relates a variable with its past values. In its simplest form, a difference equation can be written as:

$$y_t = A + \beta y_t - 1 \qquad [1]$$

where y_t is the value that variable y takes at time t. In equation [1] y_t depends on its past values and on an exogenous variable, A. β is, instead, a simple constant, which from now on we will call a parameter.

Since y_t depends on a single lagged value, equation [1] is called a first order difference equation.

We can study the qualitative properties of a difference equation by means of a chart. Equation [1], for example, can be represented graphically on a Cartesian plane, as shown in Figure 21.3.

The vertical axis shows variable y_t and the horizontal axis shows variable $y_t - 1$. The 45° line identifies the points where $y_t = y_t - 1$, whereas curve C represents equation [1] with a slope lower than 1 and a vertical intercept $A > 0$. Point y, the intersection between curve C and the 45° line, is, as we shall see, the equilibrium steady state.

From this graph, we can apply an iterative method. Indeed, if we choose a value of y_0 at time zero, shown on the horizontal axis, we can derive the value of y_1, the vertical axis through the curve C. The value of y_1 can then be shown on the horizontal line through the 45° line (having a slope equal to 1, this line identifies all the points for which the abscissa is equal to the order). The transfer of y_1 from the vertical to the horizontal axis can be done with the 45° line. Once the value of y_1 on the horizontal axis is identified, we can repeat the same reasoning, finding the value of y_2 on the vertical axis through the curve C. The iteration stops when there is no difference between two successive values of y_t. At this point, there is no more dynamics and the value of y_t coincides with that of steady state, y.

We would reach the same result if we started from an initial value of y greater than y_t such as y'_0. Again, as shown by the arrows in Figure 21.3, the value of y is reached over time. The only difference is that starting from an initial value $y_0 < y$ the dynamic equation [1] generates successively larger values of y_t until you reach stationary equilibrium. Instead, where $y'_0 > y$, the dynamic equation generates, moving from one period to another, smaller and smaller values, always leading to the value of y.

So, if the curve C has slope less than one, the equilibrium point y is reached regardless of the starting value of y_t. A balance that has these characteristics is called a stable equilibrium.

Things are different if the slope is greater than one, as shown in Figure 21.4. In this case, if we start from

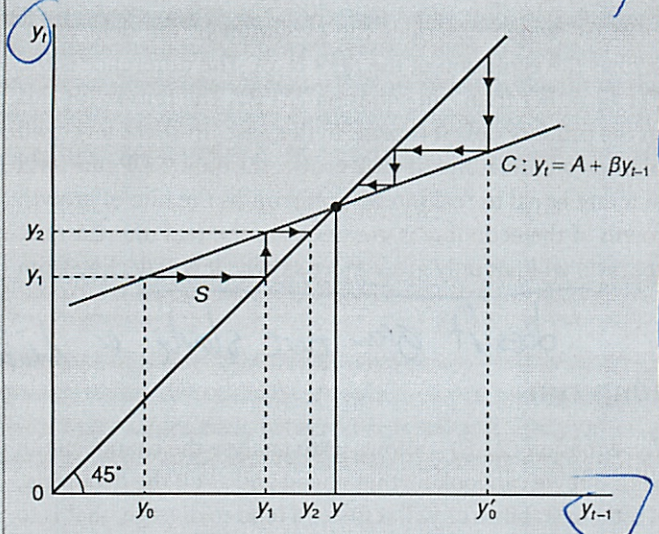


Figure 21.3
A stable equilibrium ($\beta < 1$)

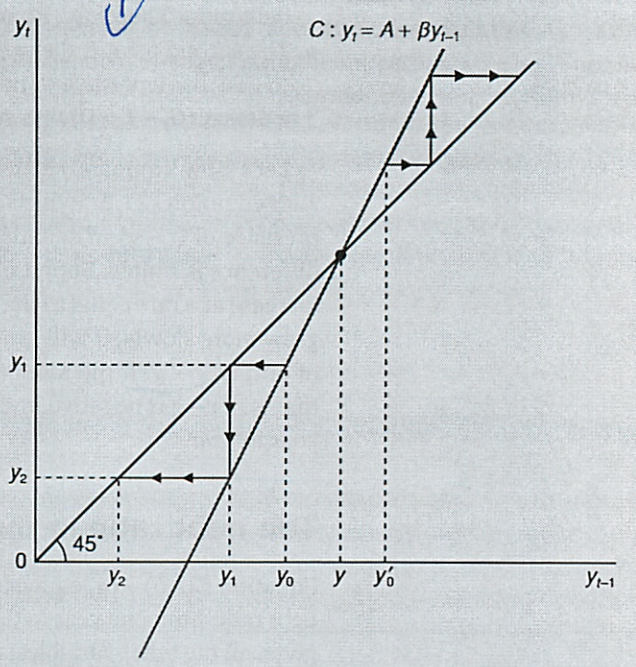


Figure 21.4
The unstable equilibrium ($\beta > 1$)

a starting value lower than y, the dynamic equation generates smaller and smaller values over time, gradually moving away from the point of equilibrium.

The same divergent trend occurs if you choose as the initial value $y_0' > y$. In other words, if the slope of the equation [1] is greater than one ($\beta > 1$), the steady-state equilibrium is unstable because, starting from different values of y, it gradually moves away from equilibrium. Equilibrium can be achieved only by choosing a value of y_0 equal to y.

The main rule to be drawn from this analysis is that the dynamic adjustment of a first-order difference equation toward equilibrium steady state is different depending on the value of β and of the initial value of y_t. In particular, the nature of the equilibrium steady state changes according to β: it is stable if $\beta < 1$, unstable if $\beta > 1$.

The Focus teaches you how to solve this simple difference equation:

$$y_t = \beta y_{t-1} + A$$

where y_t is a generic dynamic variable, β a parameter and A a constant, can be applied to the equation which describes how the debt/GDP ratio evolves through time:

$$\frac{B_t}{Y_t} = (1 + r - g)\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

where Y_t is the debt ratio, the parameter β is $1 - r - g$ and the exogenous variable A is $(G_t - T_t)/Y_t$. As we want to study the evolution of the debt ratio, given all other variables, we assume that the government runs primary deficits (or surpluses) in relation to GDP that are constant over time, namely that $(G_t - T_t)/Y_t$ is constant. To keep things simple we also assume that r and g are constant. To analyse the dynamics of the debt ratio a few graphs can help us. First, in Figure 21.5, we rewrite equation (21.5) representing the debt ratio at time t in terms of its value in the previous period. This equation is a line of slope $1 - r - g$ and intercept $(G_t - T_t)/Y_t$ (remember that we are assuming that the primary budget balance is a constant fraction of GDP). On the same figure, we also show the 45° straight line which allows us to project values from the vertical to the horizontal axis, as time goes by.

We have seen that the change in the debt-to-GDP ratio depends on whether the government runs primary surpluses or deficits, and on whether the real interest rate is higher or lower than the growth rate of GDP. Before we solve the equation graphically, you can already guess that two main cases can arise:

- *The normal case* – on most occasions, the growth rate of GDP is smaller than the real interest rate. Equation (21.5) is a straight line with slope greater than 1. What happens to the debt ratio in the long run? If the initial debt is positive (as it usually is), then the government needs primary surpluses to stabilise the debt ratio. The economic intuition is as follows. The interest rate is the rate at which the government accumulates debt because of interest due on the debt inherited from the past. If the government pays such interest by issuing new debt, rather than by means of primary surpluses, the debt ratio will continue to grow at a rate equal to the interest rate. Real GDP, however, grows at rate g – less than r – therefore the debt-to-GDP ratio increases over time. This is true even if the government maintains a primary budget balance and, even more so, if it runs primary deficits. In summary, if the growth rate is lower than the real interest rate, in the case of a positive debt inherited from the past and of primary deficits, the debt ratio increases, gradually diverging away from its equilibrium value. We can now ask what options does a government have if it wishes to stop this exponential growth in the debt ratio. The answer is simple: it must finance the servicing of the debt with adequate primary surpluses.
- *The more exotic case* – although less frequent, it can happen that the GDP growth rate exceeds the real interest rate. Equation (21.5) is then described by a straight line with slope lower than one ($1 + r - g < 1$). The straight line representing equation (21.5) is therefore has a lower slope than the 45° line, which has slope equal to 1. What happens

to the debt ratio in the long run? The debt-to-GDP ratio over time will converge to its steady state value, which we indicate with $\bar{b}$. The economic intuition is as follows: the interest rate determines the speed at which the debt grows if interest payments are financed by issuing new debt. The rate of output growth instead determines the speed at which GDP grows. So, if the primary budget is balanced, and the growth rate exceeds the interest rate, the debt ratio converges to zero. If the government continues running constant primary deficits, the debt ratio continues to converge, but at a value of the debt

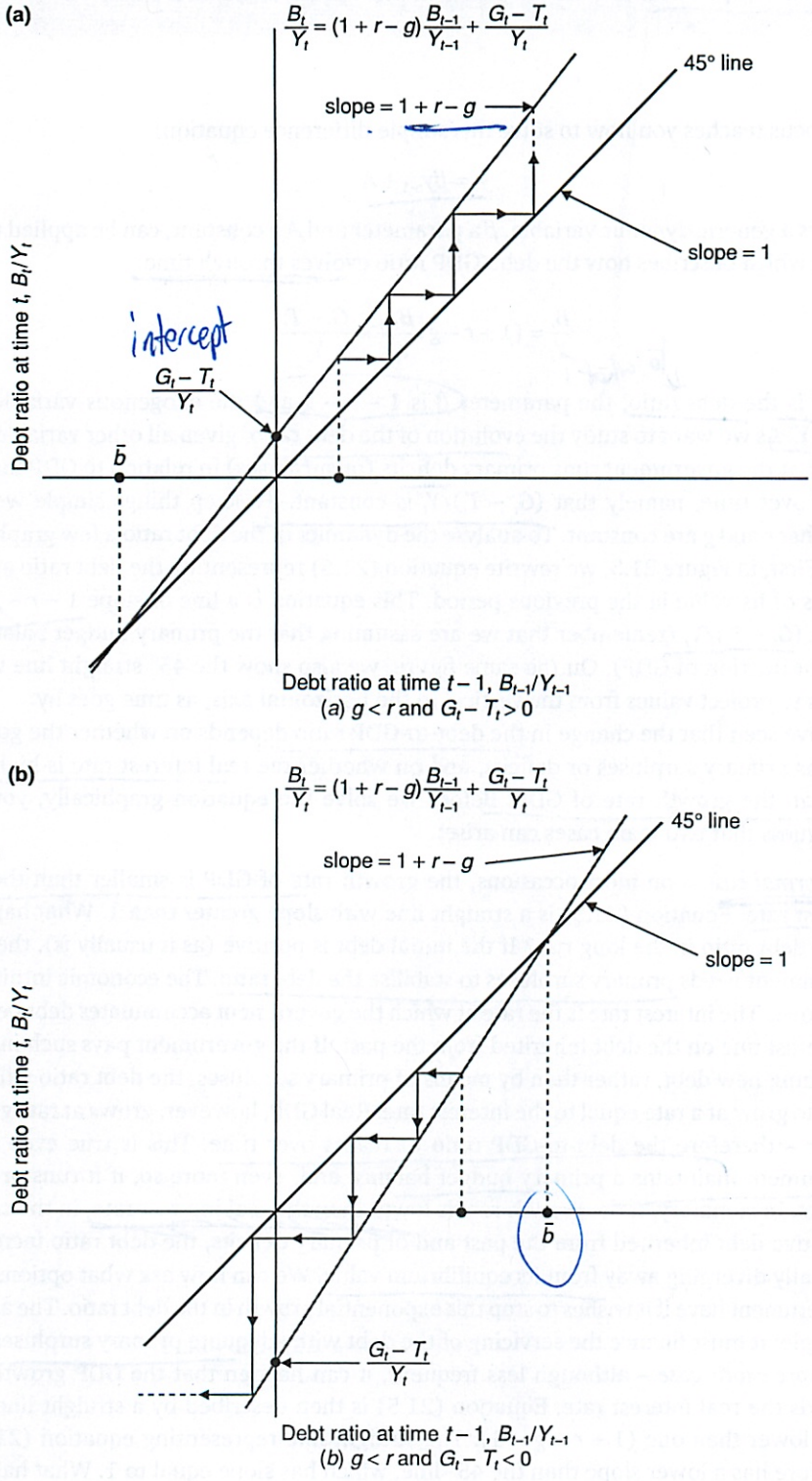


Figure 21.5

The dynamics of the debt to GDP ratio in the long run

(a) If $g < r$, and if the country has past debt and runs primary deficits ($G_t - T_t > 0$), then the debt ratio increases going farther away from equilibrium.

(b) Even if $g < r$, and if initial debt is positive, the debt ratio decreases over time if the government runs 'adequate' primary surpluses ($G_t - T_t < 0$).

(c) If $g > r$, the debt ratio converges to the equilibrium level despite the presence of primary deficits ($G_t - T_t > 0$).

(d) If $g > r$ and the government runs primary surpluses ($G_t - T_t < 0$), then the debt ratio always converges to its equilibrium level.

Since all variables are constant in the steady-state equilibrium, to find B/Y just ask, in equation (21.5), what if $B_t/Y_t = (B_{t-1}/Y_{t-1}) = \bar{b}$, to obtain:

$$\bar{b} = \frac{(G_t - T_t)/Y_t}{g - r}$$

which is positive: in equilibrium the government is a debtor.

In Figure 21.5 we describe all the possible cases arising, those with $g < r$ (cases a and b) and those with $g > r$ (cases c and d).

The evolution of the debt ratio in some European countries

The analysis conducted so far provides a useful guide to analysing the trend in the debt ratio of some European countries. The 1960s were a decade of strong growth in all countries, so strong that the average growth rate exceeded the real interest rate almost everywhere: $r - g$ was negative, and most countries succeeded in reducing the debt ratio without the need to generate large primary surpluses.

The 1970s, in contrast, were a period of much lower growth, but also of very low real interest rates (sometimes negative): $r - g$ on average was still negative, and this further reduced debt ratios. In the early 1980s (after the appointment of Paul Volker as Chairman of the Fed and the resulting shift in US monetary policy) the situation changed dramatically. Real interest rates increased and growth rates slowed down. To avoid an increase in the debt-to-GDP ratio, many countries should have created large surpluses. But this did not happen and, as shown in Table 21.1, the result was a sharp increase in debt ratios.

Table 21.1 shows the dramatic effect on debt and deficits of the use of fiscal policy during the 2007–2010 crisis. The primary balance turned from positive to negative in many European countries between 2008 and 2009 (in the UK, it had already been negative since 2002). The deficit (as a percent of GDP) increased to 11.6% in the USA and to almost 11% in the UK. The worst deterioration of the primary balance happened in Ireland, a country which run a (modest) surplus in 2007 and, in the next three years, registered a worsening primary deficit up to 12.5% of GDP. As a result, most European countries experienced a dramatic increase in their debt-to-GDP ratios. From 2007–2010, the debt ratio increased by several percentage points, 18% on average in the euro area. In EU27 outside of the euro area, the experiences of individual countries were very named. The debt ratio increased by 55 percentage points in the UK, up to 82% of GDP, but much less in countries which were less affected by the financial and economic crisis, such as Denmark and Sweden.

The dangers of a very high public debt

We have seen that the higher the level of public debt, the higher the primary surplus needed to kepp that debt level stable, i.e. to prevent it from growing. But large primary surpluses require high taxes (remember that so far we are assuming that G remains constant), which introduce distortions in the economy. The recent experience of some European countries with a debt ratio above 100% shows an additional cost: the risk of a vicious circle that can make it difficult or impossible to conduct monetary policy with the only objective being keep to inflation under control.

To understand why, let us return to equation (21.5), which describes the evolution of the debt-to-GDP ratio:

$$\frac{B_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}} = (r - g)\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

Consider a country with a high debt ratio, say 100%. Suppose that the real interest rate is equal to 3%, and that the growth rate is 2%. The first term on the right side of the

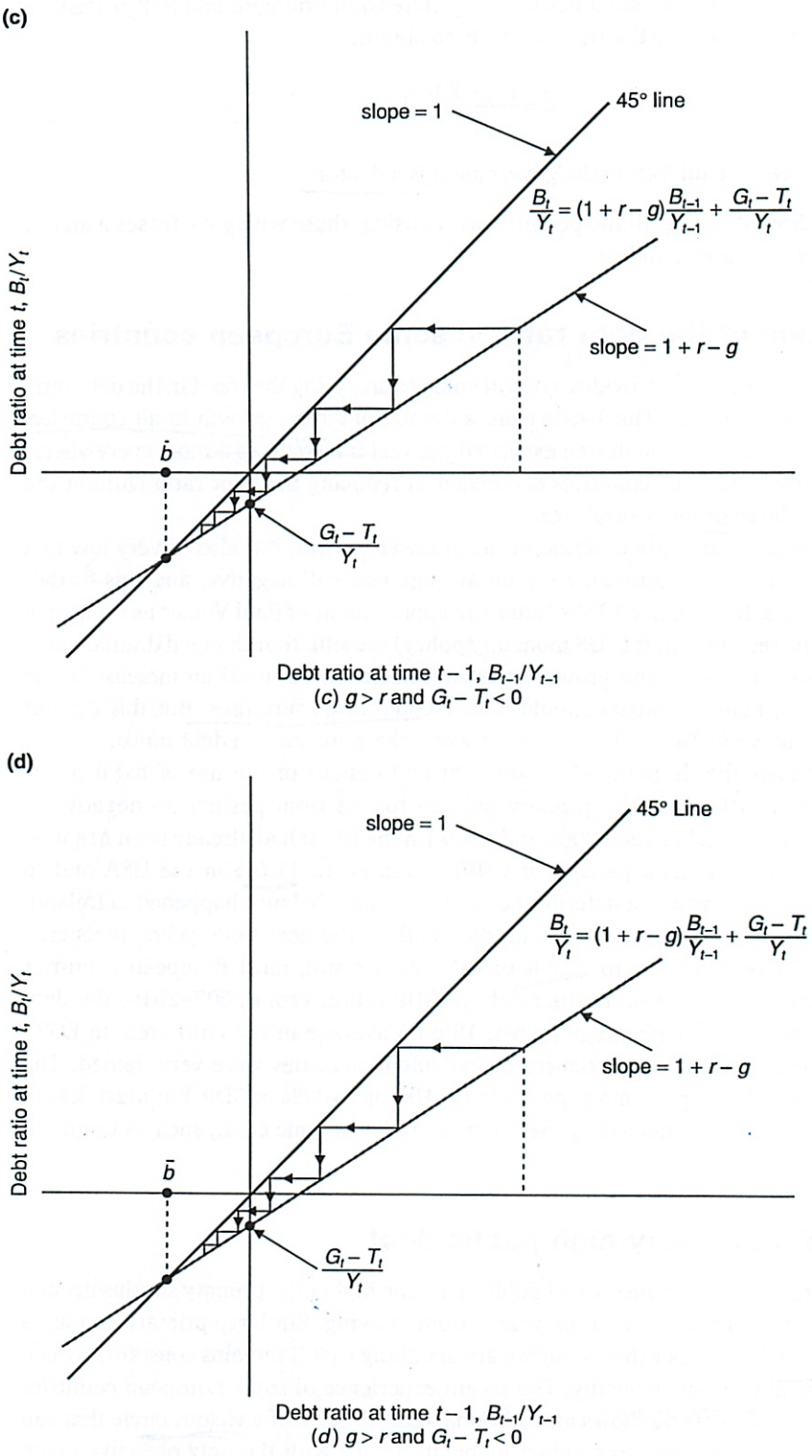


Figure 21.5
(*Cont'd*)

such that the reduction in the debt ratio due to the fact that the growth rate is greater than the interest rate exactly offsets the increase in the ratio due to primary deficits. In summary, if the growth rate of output is greater than the interest rate, whatever the value of the debt inherited from the past and even in the presence of primary deficits, the debt ratio always converges to its steady-state value. In the case, therefore, where g exceeds r, the government should not necessarily service the debt using primary surpluses, as constant deficits are compatible with a constant debt-to-GDP ratio.

just offsets
- but can still add to debt - makes sense - but could take a long, long, long time
. but takes a while to converge

Table 21.1 Primary balance, interest expenditure and gross debt in selected advanced countrie since 1992

	Primary balance					
	1992–2001	2002–2006	2007	2008	2009	2010
Germany	1	−0.4	2.6	2.6	−1.0	−2.9
Ireland	4.5	2.4	1.1	−6.1	−9.8	−12.5
Spain	0.5	2.5	3.8	−2.3	−6.9	−7.8
France	−0.25	−0.5	0.0	−0.6	−3.8	−4.0
Italy	4.05	1.4	3.5	2.4	0.2	0.1
Netherlands	3.25	1.1	2.6	3.2	−0.8	−3.4
Euro area	1.6	0.7	2.3	1.1	−2.3	−3.3
Denmark	4.5	4.8	6.1	5.0	0.1	−2.3
Sweden	1.55	2.6	5.6	4.2	−1.2	−2.5
UK	0.3	−1.0	−0.5	−3.1	−9.4	−10.8
USA	2.4	−0.9	0.2	−3.0	−9.5	−11.6
Japan	−1.4	−3.5	0.0	−0.3	−3.6	−5.2
	Interest expenditure					
	1992–2001	2002–2006	2007	2008	2009	2010
Germany	3.25	2.9	2.8	2.8	2.9	3.0
Ireland	4.15	1.2	1.0	1.1	2.3	3.2
Spain	4.25	2.1	1.6	1.6	1.6	1.9
France	3.25	2.7	2.7	2.8	2.8	3.1
Italy	9.3	4.9	5.0	5.1	4.7	4.8
Netherlands	4.9	2.5	2.2	2.2	2.6	2.7
Euro area	4.95	3.1	2.9	3.0	3.0	3.02
Denmark	5.25	2.3	1.5	1.4	1.7	1.6
Sweden	4.75	2.0	1.8	1.7	1.5	1.4
UK	3.05	2.0	2.2	2.3	2.2	3.00
USA	4.25	2.7	2.9	2.9	2.6	2.6
Japan	3.5	2.6	2.5	2.5	3.0	3.5
	Gross debt					
	2004	2005	2006	2007	2008	2009	2010
Germany	65.6	67.8	67.6	65.1	65.9	73.4	78.7
Ireland	29.4	27.5	24.9	25.0	43.2	61.2	79.7
Spain	46.2	43.0	39.6	36.2	39.5	50.8	62.3
France	64.9	66.4	63.7	63.8	68.0	79.7	86.0
Italy	103.8	105.8	106.5	103.5	105.8	113.0	116.1
Netherlands	52.4	51.8	47.4	45.6	58.2	57.0	63.1
Euro area	69.5	70.0	68.3	66.0	69.3	77.7	83.8
Denmark	44.5	37.1	31.3	26.8	33.3	32.5	33.7
Sweden	51.2	51.0	45.9	40.5	38.0	44.0	47.2
UK	40.6	42.3	43.4	44.2	52.0	68.4	81.7
USA	62.2	62.5	61.9	63.1	70.5	87.0	97.5
Japan	178.1	191.6	191.3	187.7	196.3	217.2	227.4

Notes: values are expressed as a percentage of GDP, 1992–2010; primary balance: net lending/borrowing excluding interest expenditure.

Source: European Commission–Economic Forecast, Spring 2009; IMF, *World Economic Outlook*, April 2009.

equation is equal to (3% – 2%) × 100% = 1% of GDP. Suppose further that the government generates a primary surplus of 1%, a level just sufficient to maintain a constant debt ratio – in this case, the right side of the equation is equal to 1% + (–1%) = 0%.

Suppose now that for some reason – for example, because of a political scandal which increases the perception of risk in the country – investors begin to demand higher returns for holding government bonds denominated in the currency of that country. Suppose further that the central bank wishes to defend the exchange rate (as we have discussed in Chapter 19), and in order to do so increases the interest rate from 3% to 6%. Finally, suppose that the higher interest rate triggers a recession, so that the growth rate falls to 0%. Let

us now turn to taxes: $r - g$ is equal to 6% – 0% = 6%. With an increase of $r - g$ to 6%, to maintain a constant debt ratio, the government should increase its primary surplus by five points, from 1% to 6%. It is at this point that the country may enter a vicious circle.

To increase the primary surplus, the government raises taxes, but tax hikes are unpopular; they generate even more political uncertainty and further increase the risk premium and, therefore, interest rates. The fiscal tightening induced by the first increase in interest rates then generates an even deeper recession, further reducing the rate of growth. The increase in the interest rate and the lower growth rate result in higher $r - g$, making it even more difficult to stabilise the debt ratio.

Alternatively, suppose that the government is unable or unwilling to increase the budget surplus by 5% of GDP. Debt increases, and with it the concern of financial markets: if the government was not able to change the primary surplus, the debt-to-GDP ratio would grow indefinitely. The result is that investors will ask for a further interest rate increase. The easiest way in which to exit the vicious circle is by resorting to monetary financing of the deficit (Chapter 23). But once investors realise it, the prospects of higher inflation push up interest rates futher, and so forth. These are not just intellectual speculations. In Italy, in the early 1990s, the 'Tangentopoli' scandals and frequent political crises triggered a series of speculative attacks on the Italian lira, making it necessary to increase interest rates, which resulted in large deficits and a growing debt ratio. It is therefore clear that countries with high debt should reduce it rapidly.

◀ In Chapter 23 we will see what could happen if the government decided to finance the deficit by printing new money.

How and at what speed should they reduce it? The answer is: through many years, perhaps decades, of primary surpluses. A good example is the UK in the 19th century. At the end of the wars against Napoleon, in the early 1800s, Britain had accumulated a debt ratio in excess of 200%. To reduce that ratio took almost a century: it was only by 1900 that the ratio that had dropped to 30%. The prospect of many decades of fiscal austerity is not encouraging. The result is that, when the level of debt is large, the idea of repudiating it may become attractive. The reasoning is very simple. To repudiate the debt – i.e. to cancel it, in whole or in part – is an attractive solution for the economy. The **debt repudiation** allows an immediate decrease in taxes, and hence in the distortions associated with them. It also reduces the risk of potential vicious circles. But repudiation carries with it a bigger problem, i.e. the problem of time inconsistency which we will discuss in Chapter 24. If the government fails to honour its promise to repay the debt, it will find it very difficult to borrow again in the future: following a default, financial markets will be very reluctant to buy new government securities. In other words, what seems the best today may not be so in the long term. The repudiation of the debt must be considered a last resort, and probably not a good solution even in situations of extremely high debt, exceeding even those of many countries today. In addition, debt repudation could produce a fall in consumption, as households see the value of their wealth, at least the portion invested in government bonds, reduced. (This is why governments, if they have the choice, prefer to default on securities issued abroad and held by non-resident investors, which of course has other consequences for the international reputation of the country.)

21.3 THE RETURN FROM A HIGH DEBT

At the end of the previous section, we saw that if the stock of public debt, as a ratio to GDP, reaches a very high level, the situation can escalate and lead to a debt crisis: for instance, the government finds it impossible to issue new debt except at extraordinary high interest rates. We can then ask why, in such a case, policy makers wait rather than immediately introducing adequate measures to adjust the budget. One of the reasons why corrective measures are often late and inadequate is that debt crises are largely unpredictable events, and governments tend to be short-sighted: until the crisis hits they are reluctant to admit that a crisis can burst. Therefore governments often do not perceive the urgency of an adjustment.

A second reason is that fiscal stabilisation is often the result of a political struggle between different groups, and between their political representatives. Various economic groups (the young and the old, entrepreneurs and rentiers, etc.) are likely to be affected differently by some of the measures that may be introduced to reduce a high debt. The various groups that make up a society will try to use their political pressure to defend their economic interests. To avoid losing political consensus, and thereby opening social conflicts, governments tend to delay the fiscal correction, allowing debt to grow to the point where it triggers a financial crisis.

How to reduce a high debt?

What are the options open to a government that wants to stabilise or reduce a high and growing debt ratio? There are only three ways to achieve this goal:

- Generate sufficient primary surpluses; to do so the government can cut spending and transfers or increase taxes.
- Resort to monetary financing by the central bank.
- Repudiate the debt, in whole or in part; this means that the government erases, at least in part, the existing debt, or introduces taxes on government securities that were not foreseen when investors had purchased those securities.

Generating primary surpluses is the most virtuous way to reduce a high debt. However, it is also the most difficult way. To cut spending is politically costly, and sometimes it is not socially viable: imposing new and higher taxes is an unpopular choice, and there is a limit to the tax burden beyond which the cost (especially the political cost) of collecting more taxes becomes too high. In this case, a tax plan consistent with the budget constraint becomes unfeasible.

Inflating the debt away can be done in different ways: Directly (by issuing money) or indirectly (through a decrease in the real value of debt, if the maturity is long enough).

The government can then convince the central bank to print money by purchasing government bonds. Given the initial level of debt and given the path of public spending, monetary financing reduces the tax levy required for debt relief. However, recourse to the issuing of money is not free of charge because, as we will see in Chapter 23, inflation is also a form, albeit peculiar, of taxation. An increase in the money supply increases inflation and reduces the real value of existing nominal debt and of the cash held by citizens. Inflation therefore acts like a tax – the 'inflation tax'. Inflation reduces the disposable income of households because it forces them to spend part of their income just to maintain the real value of the money stock or of the government bonds they wish to hold constant.

Finally, the government may decide to cancel all or part of the debt. Even if, as we have seen, repudiation is in some ways a good solution – it reduces the distortions caused by high tax rates – the result is the breaking up of the relationship of trust between the government and investors, who may no longer be willing to buy public debt. If this happens, the government loses the ability to run deficits in order to distribute over time the burden of an unexpected high expenditure.

Towards a 'political' theory of debt

At first glance, the three ways to finance the budget deficit and reduce debt accumulation – discussed above – may seem very different. In reality, these are different forms of taxation that differ only because they fall on different economic groups. Repudiation, for example, can be treated as a tax on wealth that affects those who hold securities. The inflation tax affects those who hold nominal assets not protected by inflation (money and bonds).

It becomes important, therefore, to take into account the fact that government debt and the economic policy choices needed to reduce it have significant effects on income distribution. This idea has been developed in a recent branch of the economic literature which lies on the border between economics and political science. This literature has

proposed a **political theory of government debt**. In particular, some economists argue that the choice of who should 'pay' for the reduction of a high debt is essentially a problem of redistribution of income and wealth between economic groups. When the different economic groups are represented politically it is also possible to establish a relationship between the degree of a country's political stability, distributional conflict and evolution of deficits and debt. Before showing how these ideas can help to understand the different solutions adopted in certain historical episodes of high public debt, we need to take a few more steps.

First, let us distinguish between a stable and an unstable political scenario. A political situation is stable if a political party has a solid majority and controls economic policy decisions. A political situation is unstable if each group has enough power to block a measure that is damaging some groups, but not enough to turn it around. In this case, fiscal adjustment may not be politically feasible. There is no way to resolve the dispute over which items of expenditure should be cut, or which taxes increased, and this prevents the government from reaching a decision, thus undermining its ability to reduce the deficit. In the end, the government must either monetise the debt or repudiate it. As we shall see, this assumption seems a good description of what happened in Germany and France in the first half of the 1920s.

If, instead, the political situation is stable, a political party has enough power to start a fiscal adjustment because it is strong enough to impose the burden of adjustment on others. This was the case in France in the mid-1920s and in the UK in the post-war period. The case of the USA after the Second World War is an example of a reduction in public debt that has not given rise to fights among different social and political groups. But this was only possible because of the high growth rates of income recorded in those years, which contributed to the reduction of the debt ratio.

Suppose that society can be divided into three groups:

- Rentiers (from the French '*rente*' which means annuity) hold wealth in the form of government bonds, i.e. securities which pay an annuity every year.
- Entrepreneurs hold wealth in the form of physical capital that produces profits.
- Workers own human wealth, which produces their salaries.

Each of these interest groups will seek to avoid the burden of adjustment and shift it onto someone else. Rentiers are opposed both to explicit debt repudiation and to an inflation tax, which reduces the real value of debt and, as such, is an implicit from of repudiation. If average debt maturity is short, and hence holders of securities are protected from inflation by the changes in interest rates, then rentiers prefer the inflation tax to debt repudiation. Rentiers also look favourably on various forms of tax increases, whether direct or indirect. Entrepreneurs are opposed to taxes on capital, but not to debt repudiation, nor to monetary financing or to taxes on consumer goods and income. Workers prefer taxes on wealth and capital, and the repudiation of debt, while being opposed to indirect taxes, particularly on consumer goods. They are affected by inflation if wages are not indexed, but they can be made better off by an increase in employment induced by a monetary expansion.

Four episodes of reduction of a high public debt

Historical experiences provides us with illuminating examples of how some countries have emerged from situations of high debt by taking very different solutions. We shall describe the experience of Germany, France and the UK at the end of the First World War and the case of the USA at the end of the Second World War.

Germany in the post-war period

Germany financed military spending during the First World War mainly through borrowing. During the war period, in fact, fiscal revenue accounted for a negligible fraction of

to buy government bonds. As a result, the government had to repay the bonds coming to maturity with monetary financing. In 1926 France was probably on the verge of hyperinflation.

At this point, Raymond Poincaré assumed the leadership of a new Conservative government and announced a drastic stabilisation program. The element that made this program different from previous attempts at fiscal adjustment was simply the greater political stability. The program was credible because the political opponents had been defeated. Inflation ended abruptly, even before the government had started the fiscal adjustment.

Only when Poincaré introduced a bill to shift the tax burden off bondholders did the demand for government bonds recover and inflation stop.

The UK in the post-war period

Even in the UK, the debt was very high at the end of the First World War: the debt-to-GDP ratio had reached 130% in 1919. The policies adopted in the UK, however, were very different from those in Germany and France. What distinguished the UK from Germany and France? The answer is simple: the degree of political stability. As we have seen, both in Germany and in France, the political situation at the end of the conflict was very unstable. In the UK, instead, except for two brief Labour governments, in 1924 and in 1930, the Conservative Party ruled continuously throughout the 1920s and 1930s. Democratic institutions were very solid and, despite very high unemployment, were never really threatened by the risk of a social revolt. This made it possible to introduce fiscal and monetary contractions, whose main objective was the stability of sterling and its return to its pre-war value – thus allowing a return to the gold standard. At the same time, the government produced budget surpluses in order to reduce the high public debt. The UK was one of the very few European countries where no expansionary fiscal policies were implemented to promote economic recovery.

Throughout the 1920s, and until the second half of the 1930s, fiscal surpluses, however, were not sufficient to reduce public debt. In this period, interest rates greatly exceeded the rate of growth of GDP. In 1923, the debt reached 170% of GDP and remained above 150% up to 1936. The debt-to-GDP ratio only started to decline in the second half of the 1930s, 15 years after the war.

Who bore the burden of debt reduction in the UK? Certainly not those who had bought government securities, since there was no form of repudiation, either explicit or implicitly through inflation. The burden of adjustment was borne primarily by taxpayers. Among them, those in the less wealthy classes were especially affected, because of an increasingly regressive tax system. For example, the introduction of taxes on specific products (tea, sugar, tobacco, milk, etc.) had a significant regressive effect.

The USA after the Second World War

The debt accumulated by the USA at the end of the Second World War was very close, in relation to GDP, to the debt ratio in the UK after the First World War. In both cases, moreover, the political situation was relatively stable. That is why both the UK and the US governments were able to start a fiscal adjustment without being forced to resort to repudiation. The USA, however, had greater success than the UK: 15 years after the end of the Second World War, the debt-to-GDP ratio was halved; in the UK, in contrast, 15 years passed before the debt ratio began to fall. So what distinguished the USA in the 1950s from the UK in the 1920s? During the period 1948–1968, the average growth rate of GDP in the USA was 4%, while real interest rates did not exceed 0.5%. Unlike the case of the UK, in the USA budget surpluses were accompanied by rapid output growth that exceeded the level of real interest rates. There is a simple but important lesson: it is easier to reduce a high debt when the economy is growing.

overall spending, and the resulting budget deficit was financed by issuing debt, especially short-term debt. But how did Germany plan to repay this debt? Like all the countries that took part in the conflict, she hoped to win the war and shift the debt burden onto the defeated countries. But Germany lost the war and at the end of the conflict found herself with a very high debt stock.

After the war, the German political situation was particularly unstable. Following from the military defeat, the old nationalistic regime, ruled by aristocrats and the military, collapsed. The Communist Party began to gain broad support but, rather than a communist revolution, what happened was the birth of a new democratic regime, the Weimar Republic. The political situation remained, however, quite unstable. The democratic regime was very weak, threatened both by the workers' unrest linked to the communist movement and, at the other extreme, by the forces of the old regime and new movements of far-right nationalists.

In the first half of the 1920s, the debt problem was aggravated by the high budget deficits accumulated by the Weimar government. In part, these deficits were related to the reparations Germany had to pay to the winners of the war, to France in partcular. In reality, reparations accounted for no more than one-third of the deficits in those years. The main reason for the deficits of the years 1920–1923 was a political impasse in fiscal policy. The proposal of drastic tax reforms had further weakened an already weak political situation, making it extremely difficult for the government to collect taxes. For example, the Socialists' proposal to levy an extraordinary tax on firms' capital and profits encountered violent opposition from nationalists and obviously from the entrepreneurs. Similarly, the proposal by entrepreneurs to raise income tax was rejected by the Socialists. The result was that no significant measure was introduced until 1922. The need to strike a compromise between the new and the old regimes had undermined the ability and willingness of the government to increase taxes. The political and fiscal policy impasse of these years left, as the only solution, monetisation which led, as we will see in the next chapter, to hyperinflation. One of effects of the German hyperinflation was the total cancellation of the debt that had existed at the end of the war. By the autumn of 1922, the debt did not exceed 5% of its real value in 1919. This dramatic reduction of wealth struck especially the middle class, which held the largest share of government debt. The reduction of wealth owned by the middle class worsened the income distribution, which is one of the reasons for the subsequent collapse of democratic institutions.

France in the post-war period

In the decade that followed the end of the First World War, the question of who should pay the cost of the debt issued to finance the conflict monopolised the political debate in France. The debt was a particularly difficult problem due both to its size – the public debt represented about 150% of GDP – and its composition – the short-term debt constituted 32% of the total. In the years 1919–1926, in France as well the political situation was very unstable: in a few years, Socialist and Conservative governments alternated one after another. But in the second half of the decade political instability decreased: in 1926 the right won the final fight and was able to form a stable conservative government headed by Raymond Poincaré.

At the beginning of the decade, there seemed to be an easy solution to the French public debt problem: have the Germans pay for it through reparations. It was only at the end of 1922, and after the occupation of the Ruhr, that the French began to understand that German taxpayers would not be able to pay. Then an endless debate began between the opposition, on the one hand, and the conservatives on the other. The left denounced the unfairness of the tax structure, maintaining that, although income taxes were very progressive, only 20% of tax revenue was collected through income taxes. The high incidence of indirect taxes meant that the tax burden fell mostly on the less wealthy. The left, therefore, proposed a unique and progressive tax. At the other extreme, the conservatives opposed progressive income taxes, proposing much more reliance on indirect taxes. The distributional conflict made the political situation increasingly volatile; the French franc was hit by speculation and inflation went up. In fact, the fear of a capital levy made the public unwilling

SUMMARY

- Governments, like households and individuals, can spend less or more than the amount of their revenues. When public spending exceeds taxes, a government runs a budget deficit. When public spending is lower than taxes, a government runs a budget surplus.
- In principle, a high government deficit is neither good nor evil. Deficits (and surpluses) can actually help to redistribute the burder of taxation over time. But deficits become a problem when they result in rapid accumulation of debt.
- To tell whether government debt is 'too' high, the relevant variable to look at is the ratio of government debt-to-GDP. Whether debt is 'too' high must be defined in relation to the ability of government to repay the debt.
- To stabilise the debt, the government must run a primay surplus equal to the interest on existing debt. The longer the government waits before stabilising the debt, the more painful the stabilisation wiil be.
- When $r > g$, the reduction of the debt ratio requires primary surpluses. When $r < g$, a country can reduce the debt ratio without the need to generate primary surpluses.
- The massive use of fiscal policy to help the economies of many European countries to face the recession of 2007–2010 resulted in a significant deterioration of primary balances that turned from positive to negative in most cases. These large budget deficits caused a dramatic increase in the debt ratio, which doubled in several countries.

KEY TERMS

balanced budget 437
budget deficit 437
nominal and real interest payments 437
deficit financing 437
primary deficit 439
primary surplus 439
debt stabilisation 441
debt-to-GDP ratio, or debt ratio 441
debt repudiation 449
political theory of government debt 451

QUESTIONS AND PROBLEMS

QUICK CHECK

1. *Using the information in this chapter, label each of the following statements true, false, or uncertain. Explain briefly.*

a. The seigniorage is equal to real money balances multiplied by the nominal interest rate.

b. During a hyperinflation, individuals increase the use of currency.

c. Given money balances, an increase in money growth causes an increase in seigniorage.

d. The net effect of money growth on seigniorage is certain and positive.

e. In the short term, increased rates of money growth cause a decrease of seigniorage through real money balances.

f. In the medium term, increasing rates of money growth generate a decrease in real money balances and an increase in seigniorage (at a decreasing rate).

g. In the long term, the government may finance the deficit with constant money growth rates.

h. A higher money growth leads to a steady increase in production.

i. A simple program to stabilise prices and wages can stop a process of hyperinflation.

j. The Olivera–Tanzi effect is the improvement in the deficit in the presence of high inflation.

2. *Consider an economy in which: the official budget deficit is 4% of GDP, the debt-to-GDP ratio is 100%, the nominal interest rate is 10% and the inflation rate is 7%.*

a. What is the relationship between the primary balance and GDP?

b. What is the balance adjusted for inflation as a percentage of GDP?

c. Suppose that production is down 2% compared to natural levels. What is the cyclically adjusted balance ratio to GDP adjusted for inflation and balance/GDP?

d. Suppose instead that the production is initially at its natural level, and that output growth remains constant at the normal rate of 2%. Does the debt ratio increase or decrease?

3. *Suppose that in a country's public debt, inflation and the rate of GDP growth are all equal to zero, and that the interest rate is 5%. In year t, the country recorded a deficit of 10% of GDP, and the year t+1 onwards eliminates the primary deficit.*

Calculate the deficit (as percentage of GDP) in years $t+1$ and $t+2$.

DIG DEEPER

4. *Consider the economy described in problem 2 and assume that there is a fixed exchange rate. Suppose further that financial investors fear that the debt level is too high, and that the government may have to devalue to stimulate production (and thus tax revenues) and reduce debt. Financial investors expect a devaluation of 10%. In other words, the expected exchange rate, E^e_{t+1}, decreases by 10% from its previous value.*

a. We recall the uncovered interest parity condition: if the foreign interest rate is and remains equal to 10%, what happens when the domestic interest rate decreases by 10%?

b. Suppose that domestic inflation remains unchanged. What happens to the real interest rate nationally? What will happen to the rate of growth?

c. What happens to the official budget deficit and to the deficit adjusted for inflation?

d. Suppose that the growth rate declines from 2% to 0%. What happens to the change in the debt/GDP? (We assume that the primary balance ratio to GDP remains unchanged, although the decrease in growth rate may reduce tax revenues.)

e. Were investors' fears justified?

5. *Consider the data of the previous year, however, assuming that the country has primary surpluses of less than 2% of GDP instead of 1% of GDP.*

a. Calculate the debt ratio in 2006, 2007 and 2008.

b. In light of the results, do you think that primary surpluses of 1% should be considered 'good' with a view to reducing the debt ratio.

EXPLORE FURTHER

6. *Consider an economy where the ratio of debt to GDP is 40%, the primary deficit is 4% of GDP, the rate of growth is 3% and the real interest rate is 3%.*

a. Using a spreadsheet, calculate the debt ratio after 10 years, assuming that the primary deficit remains at 4% of GDP each year as the economy grows at the normal rate of growth each year and that the real interest rate remains constant at 2%.

b. Suppose that the real interest rate increases to 5%, but that everything else remains as in part (a). Calculate the debt ratio after 10 years.

c. Suppose that the rate of growth falls to 1% and that the economy will grow at the normal rate each year. Everything else remains as in part (a). Calculate the debt ratio after 10 years. Compare your answer with that in (b).

d. Returning to the assumptions of part (a), suppose that the policy makers decide that a debt ratio above 50% is dangerous. Check that the immediate reduction of the primary deficit of 1% retained for 10 years will lead to a debt ratio of 50% after 10 years. What level of primary deficit will be necessary to maintain a debt ratio of 50%?

e. Following on from (d), suppose that policy makers wait five years before changing the tax policy. For five years, the primary deficit remains at 4% of GDP. What is the debt ratio after five years? Suppose that after five years economic policy makers decide to reduce the debt ratio to 50%. From year 6 to year 10, what is the constant primary deficit level which will lead to a debt ratio of 50% at the end of 10?

f. Suppose that the policy makers decide to enact the policy in part (d) or part (e). If these policies lead to a reduction in the rate of output growth for a time, how will they effect the level of primary deficit needed to achieve a debt ratio of 50% after 10 years?

g. What policy – the one in part (d) or the one in part (e) – do you think is more dangerous for the stability of the economy?

We invite you to visit the Blanchard page on the Prentice Hall website, at ***www.prenhall.com/blanchard*** *for this chapter's World Wide Web exercises.*

FURTHER READING

- Chapter 13 of the volume by **R. Farmer, *Macroeconomics*, Cincinnati, OH, South-Western College Publishing, 1998**, analyses the dynamics of debt and deficits in line with the discussion of this chapter.
- For an illuminating analysis of how various countries have emerged from situations of high debt, you can read **A. Alesina, 'The End of Large Public Debts,' in *High Public Debt: The Italian Experience*, edited by F. Giavazzi and L. Scares, Cambridge, Cambridge University Press, 1988.**

14.02
Notes on Fiscal Policy

Francesco Giavazzi

April 10, 2011

The price of goods today relative to tomorrow

- Assume the economy has a technology to transfer goods from today (period t) to tomorrow (period $t+1$). For instance one unit of corn used as seed and planted today yields $(1+r)$ units of corn tomorrow

$$y_{t+1} = (1+r)\, y_t$$

- Then the price of a unit of good at time $t+1$ relative to a unit of good at time t (*i.e.* the number of units of t good required to obtain 1 unit of $t+1$ good)

$$\frac{[\text{units of goods at time } t]}{[\text{units of goods at time } t+1]} = \frac{1}{(1+r)}$$

- Thus if a want to add up the two goods, the way to do it is

$$y_t + \frac{y_{t+1}}{(1+r)}$$

Consumption and wealth

- To start thinking about fiscal policy it is useful to move a step beyond the consumption function we used so far and assume that consumption also depends on a household's wealth

$$C = C\left(Y^{disp}, Wealth\right)$$

$$Wealth = W^{financial} + W^{housing} + PDV(Y^{disp})$$

- The first term is *financial wealth* (stocks and bonds), the second is the value of the family's house (because they can use it as "collateral" to borrow from a bank), the third is *human wealth*, the value of expected income (net of taxes) over a lifetime: if you attend an MBA you can go to the bank and ask for a loan anticpating you will land a job on Wall Street (we shall see in a minute what are the consequences if the bank refuses to lend you the money)

$$PDV(Y^{disp}) = \sum_{i=0}^{T} \frac{Y_{t+i} - T_{t+i}}{(1+r)^i}$$

Does it matter how a government finances G ?

- Assume thre are only two periods. The government's *intertemporal budget constraint*, i.e. its budget constraint over the two periods is

$$T_1 + \frac{T_2}{(1+r)} = G_1 + \frac{G_2}{(1+r)}$$

- The households' *intertemporal budget constraint* (assuming that both financial and housing wealth are zero) is

$$C_1 + \frac{C_2}{(1+r)} = (Y_1 - T_1) + \frac{(Y_2 - T_2)}{(1+r)}$$

The irrelevance of the government's financial policy

Assume now that households realize that the government is subject to an intertemporal budget constraint and consider two cases

1. The government budget is balanced in each period

$$T_1 = G_1, \quad T_2 = G_2$$

then

$$\begin{aligned} C_1 + \frac{C_2}{(1+r)} &= (Y_1 - T_1) + \frac{(Y_2 - T_2)}{(1+r)} \\ &= (Y_1 - G_1) + \frac{(Y_2 - G_2)}{(1+r)} \end{aligned}$$

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Assume now that households realize that the government is subject to an intertemporal budget constraint and consider two cases

1. The government budget is balanced in each period

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then

$$\begin{aligned} C_1 + \frac{C_2}{(1+r)} &= (Y_1 - T_1) + \frac{(Y_2 - T_2)}{(1+r)} \\ &= (Y_1 - G_1) + \frac{(Y_2 - G_2)}{(1+r)} \end{aligned}$$

2.

$$T_1 = 0, \quad G_1 = B, \quad T_2 = G_2 + B(1+r)$$

substituting we still get

$$C_1 + \frac{C_2}{(1+r)} = (Y_1 - G_1) + \frac{(Y_2 - G_2)}{(1+r)}$$

The irrelevance of the government's financial policy (cont.)

- From 1. and 2. we have seen that they way the government finances a given level of spending makes no difference. All that matters is $PDV(G) = G_1 + \frac{G_2}{(1+r)}$

See in class

- Assume G_1 increases to $G_1' > G_1$, while G_2 does not change
  - $\left(Y_1 - G_1'\right) + \frac{(Y_2 - G_2)}{(1+r)} = \left(C_1 + \frac{C_2}{(1+r)}\right)_{|G_1'} < \left(C_1 + \frac{C_2}{(1+r)}\right)_{|G_1} = (Y_1 - G_1) + \frac{(Y_2 - G_2)}{(1+r)}$
  - $\frac{d\left(C_1 + \frac{C_2}{(1+r)}\right)}{dG_1} < 0$ note that this has the *opposite sign* compared with what we have learned so far
  - Y will increase less than G_1 because consumption falls. Note that this crowding out of the effects of G happens through a very different channel compared with the crowding out which happens when i increases reducing investment

The "expansionary fiscal contraction" in Denmark, 1983-86

	$1979-82$	$1983-86$
G	**+ 4.0**	**0.0**
T	- 0.03	+ 1.3
$(G-T)$	+ 1.8	- 1.8
Δ *debt*	+10.2	0.0
Δ $Y^{disposable}$	+ 2.6	- 0.3
C	**- 0.8**	**+ 3.7**
I	- 2.9	+12.7
GDP	+ 1.3	+ 3.2

(SOURCE: Giavazzi, F. and M. Pagano, "Can Severe Fiscal Contractions Be Expansionary? Tales of Two Small European Countries", NBER Macro Annual, 1990).

The limits of Ricardian Equivalence

We will now show that the irrelevance of a government's financial policy—a result which is called *Ricardian Equivalence* after the economist David Ricardo who first wrote about it in the late XIII century—depends on a few strong assumptions

- The horizon of households corresponds to that of the government. In other words, people think they will pay all the taxes the government will eventually have to levy;
- People can freely borrow.

We now consider what happens if these conditions fail, namely if

1. Households' horizon is shorter than that of the government

The limits of Ricardian Equivalence

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- The horizon of households corresponds to that of the government. In other words, people think they will pay all the taxes the government will eventually have to levy;
- People can freely borrow.

We now consider what happens if these conditions fail, namely if

1. Households' horizon is shorter than that of the government
2. Households cannot freely borrow against their expected future income

1. Households' horizon is shorter than that of the government

- if people plan to be around in period 2
  - $\left\{ \begin{array}{c} T_1 = 0 \\ G_1 = B \\ G_2 = 0 \end{array} , T_2 = B(1+r) \right\}$
  - $C_1 + \frac{C_2}{(1+r)} = (Y_1 - G_1) + \frac{(Y_2 - G_2)}{(1+r)}$
- if people anticipate $T_2 = 0$, $T_3 = B(1+r)^2$ and think they will not be around in $t = 3$, then
  - $C_1 + \frac{C_2}{(1+r)} = Y_1 + \frac{Y_2}{(1+r)}$
- In this case

$$\frac{d\left(C_1 + \frac{C_2}{(1+r)}\right)}{dG_1} = 0$$

2. Liquidity Constraints (people cannot borrow on the expectation of higher future income)

- let (to keep the algebra simple)
  $\left\{ \begin{array}{cc} G_1 = G_2 = G & C_1 = C_2 = C \\ r = 0 & Y_1 = Y_2 = Y \end{array} \right\}$
- then the max achievable level of consumption is

$$C_1 + \frac{C_2}{(1+r)} = 2C = 2Y - 2G, \; C = Y - G$$

- now assume all taxes are levied in $t = 1$ $\left\{ \begin{array}{c} T_2 = 0 \\ T_1 = 2G \end{array} \right\}$

Liquidity Constraints (cont.)

- with $\left\{ \begin{array}{c} T_2 = 0 \\ T_1 = 2G \end{array} \right\}$
- in $t = 1 \left\{ \begin{array}{c} Y_1^{disp} = Y - 2G \\ C = Y - G \end{array} \longrightarrow C > Y_1^{disp} \right\}$
- in $t = 2 \quad C > Y_2^{disp} = Y$
- if households cannot borrow in $t = 1$ the max level of consumption cannot be achieved

The nominal and the real interest rate

- Remember our assumption that the economy has a technology to transfer goods from period t to period $t + 1$

$$y_{t+1} = (1 + r)\, y_t$$

- Now think that instead of goods, we wish to transfer Dollars from t to $t + 1$. Since the price of a unit of good in period t is P_t, with 1 Dollar you buy $1/P_t$ goods which at time $t + 1$ translate into $(1 + r)/P_t$ goods and $[(1 + r)/P_t]\, P_{t+1}$ dollars
  - $(1 + r)$: real interest rate
  - $(1 + i) = (1 + r)\, P_{t+1}/P_t$: nominal interest rate
  - $(1 + i) = (1 + r)\left(1 + \frac{P_{t+1} - P_t}{P_t}\right) = (1 + r)\,(1 + \text{inflation})$

must discount to PV

Real and Nominal Budget Deficit

- real budget deficit (*real* because measured in units of goods)

$$(\text{real deficit})_t = rB_{t-1} + G_t - T_t = B_t - B_{t-1}$$

$$rB_{t-1}: \text{ real interest payments}$$

$$G_t - T_t: \text{ primary deficit}$$

- *nominal* deficit (measured in current Dollars). Remember

$$(1+i) = (1+r)(1+\text{inflation})$$

then

$$(\text{nominal deficit})_t = i\$B_{t-1} + \$G_t - \$T_t = \$B_t - \$B_{t-1}$$

$$(\text{nominal deficit})_t - \text{ inflation } * B_{t-1} = (\text{real deficit})_t$$

The dynamics of the debt-GDP ratio

$$\frac{B_t}{Y_t} = (1+r)\frac{B_{t-1}}{Y_t} + \frac{G_t - T_t}{Y_t}$$

$$\frac{Y_t}{Y_{t-1}} = \frac{1}{1+g}$$

$$\frac{(1+r)}{(1+g)} \simeq 1 + r - g$$

$$\frac{B_t}{Y_t} = (1+r-g)\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

$$\left(\frac{B_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}}\right) = (r-g)\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

The cost of delaying paying for G

- delaying one period

$$\begin{aligned} T_1 &= 0, \quad G_1 = B \\ T_2 &= G_1 (1+r) \end{aligned}$$

- dealying t periods

$$\begin{aligned} T_1 &= T_2 = \ldots T_{t-1} = 0, \quad G_1 = B \\ T_t &= G_1 (1+r)^{t-1} \end{aligned}$$

Debt sustainability

$$\left(\frac{B_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}}\right) = (r-g)\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

$$\left(\frac{B_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}}\right) = 0, \quad i.e. \quad \frac{B_t}{Y_t} = b \text{ for all } t$$

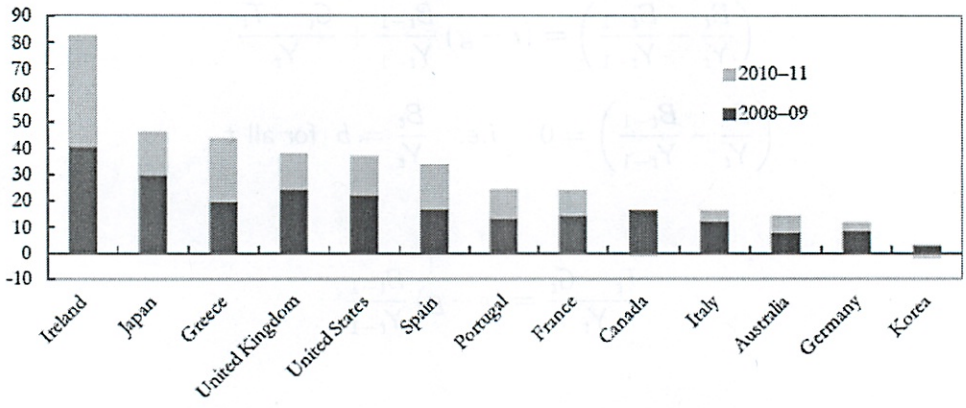
$\rightarrow$

$$\frac{T_t - G_t}{Y_t} = (r-g)\frac{B_{t-1}}{Y_{t-1}}$$

Debt dynamics after the Great Recession: Some numbers and graphs

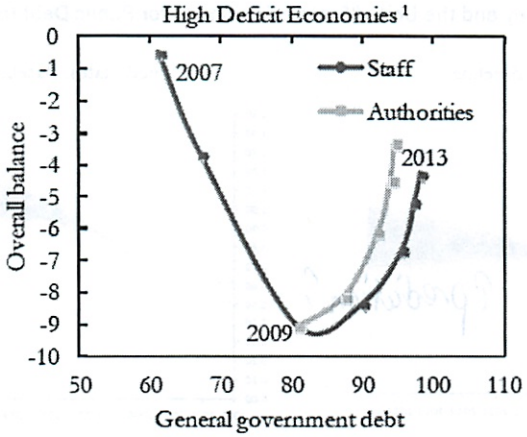
Figure 1.7. Selected Advanced Economies: Changes in Public Debt, 2008–11

(Percentage points of GDP)



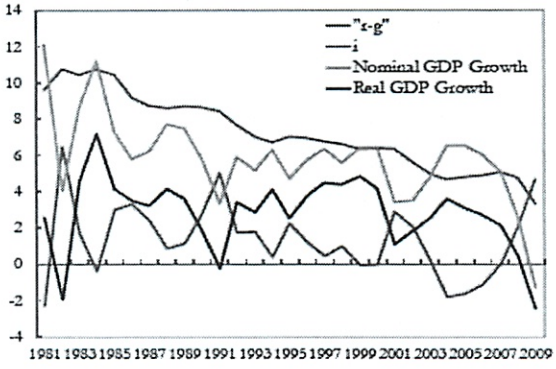
Source: October 2010 WEO.

Overall balance and general government debt; Percent of GDP



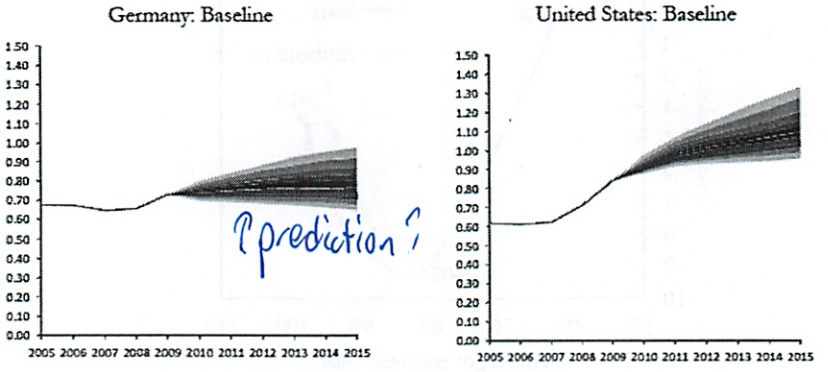
Sources: IMF staff calculations based on authorities' plans; and October 2010 WEO.

Figure A1.2. United States: Interest Rate-Growth Differential and Its Components
(Percent)



Source: IMF staff estimates.

Figure A4.2. Germany and the United States: Fan Charts for Public Debt-to-GDP Ratio



Or what is fanning out?

14.02 - What Happens to S and I when Taxes Change

Francesco Giavazzi

Of course the student in the back was 100% right ! Nothing happens when I is exogenous (and the economy is closed, *i.e.* it can not borrow or lend abroad). To get some action you need to assume that I is not exogenous (*i.e.* fixed).

Start from the beginning. The question was what happens to Total Savings when Taxes rise. The reason it could be ambiguous is that

$$\begin{aligned} Total\ Savings &= Private\ Savings + Public\ Savings \\ &= Private\ Savings + \ (T - G) \end{aligned}$$

when Taxes rise, Public Savings rise, but Private Savings fall (beacuse when taxes rise income falls). This is why the answer could in principle be ambiguous. In reality the answer is simple: Total Savings do not change if Investment is fixed (exogenous) because if $S = I$ and therefore

$$dS = dI = 0$$

And in fact if you compute $\frac{dS}{dT}$ at constant I

$$\frac{dS}{dT} = (1 - c_1)\frac{-c_1}{(1 - c_1)} + c_1 = 0$$

For the question to be interesting you need to allow Investment to change. For instance if

$$I = I(y) = d_0 + d_1 y$$

then

$$\frac{dI}{dT} = d_1 \frac{dy}{dT} = d_1 \frac{-c_1}{(1 - c_1)} < 0$$

in this case, since investment falls (because y falls), Savings will also fall, since in a closed economy it must always be true that $S = I$.

For the answer to be ambiguous you need to assume

$$I = I(i, y) = d_0 + d_1 y - d_2 i$$

in this case what happens to I when T rises is no longer certain. This is because y falls, pushing down I, but interest rates also fall (the economy moves to a different $IS - LM$ equilibrium), and this pushes investment up. Assume

$$\frac{M}{p} = f_1 y - f_2 i$$

then

$$\frac{di}{dT} = \frac{-c_1}{(1 - c_1 - d_1) f_1/f_2 + d_2}$$

and

$$\frac{dI}{dT} = d_1 \frac{dy}{dT} - d_2 \frac{di}{dT} = \frac{c_1}{(1 - c_1 - d_1) f_1/f_2 + d_2} (d_2 - d_1)$$

the sign then depends on the relative magnitude of d_2 and d_1.

This is it ! And sorry for the confusion.

Have a good Quiz !

14.02 Ricardian Equivalence

Review session Mon

Check regrade status request

Remember price today vs price tomorrow

$$Y_{t+1} = Y_t(1+r)$$

$$\frac{\text{Units of good at } t}{\text{" } t+1} = \frac{1}{(1+r)}$$

Because people could invest money at r

- When country can lend from abroad

So can add up units of goods across time

$$Y_t + \frac{Y_{t+1}}{(1+r)}$$

Just like 15.401 - financial markets

Remember wealth is important now

$C = C(y_{disp}, Wealth)$

$Wealth = W^{financial} + W^{housing} + PDV(y_{disp})$

- $W^{financial}$: stocks, bonds
- $W^{housing}$: housing - can borrow against
- $PDV(y_{disp})$: human wealth, value of expected income over life since you can borrow against future earnings

(2)

$$PDV(Y_{disp}) = \sum_{i=0}^{I} \frac{Y_{t+1} - T_{t+1}}{(1+r)^i}$$

I = # life span
T = # taxes
i = discounted to PV

How does gov finance G_i

* gov has inter temporal budget constraint

For 2 periods $T_1 + \frac{T_2}{(1+r)} = G_1 + \frac{G_2}{(1+r)}$

- must repay eventually
- through taxation
- debt as % of GDP goes to 0 if $g > r$
- but in nominal terms may need to repay eventually

Households also have intetemporal budget constraint

- assuming financial + housing wealth are 0

$$C_1 + \frac{C_2}{(1+r)} = (Y_1 - T_1) + \frac{(Y_2 - T_2)}{(1+r)}$$

If households know gov subject to intemporal budget constraint
- they are indiff to gov spending now + tomorrow
- since consumers can lend to gov

(3)

If gov's budget is balanced in each time period

$T_1 = G_1 \qquad T_2 = G_2$

then

$$C_1 + \frac{C_2}{(1+r)} = (Y_1 - T_1) + \frac{(Y_2 - T_2)}{(1+r)}$$ ← consumers intertemporal budget constraint

$$= (Y_1 - G_1) + \frac{(Y_2 - G_2)}{(1+r)}$$ ← rewrite

But what if gov Taxes 0 in current period, loans all $G_1 = B$

$T_1 = 0$

Then $T_2 = G_2 + B(1+r)$

Substitute in

$$C_1 + \frac{C_2}{(1+r)} = (Y_1 - G_1) + \frac{(Y_2 - G_2)}{(1+r)}$$

Has not changed

Since

$C_1 = Y_1$

$C_2 = Y_2 - T_2$

but $T_2 = G_2 + B(1+r)$

$= G_2 + B_1(1+r)$

but here we are assuming it borrows from international or that consumers can lend to foreigners at r

So we see when gov debt is financed makes no difference. All that matters is

$$PDV(G) = G_1 + \frac{G_2}{(1+r)}$$

Here opposite result from IS-LM!

$G\uparrow$ will $\downarrow C$

- since $\uparrow G$ will $\downarrow$ Income

Y will increase less than G_1 since C falls. Note that this crowding out of the effects of G happens through a very diff. channel compared to crowding out which happens when i increases, reducing investment

(5)

* People anticipate the higher taxes
  So they save more today, spend less
  Just like if taxes were higher today

Example from Denmark

Limitations

- gov's horizon = household's horizons
  - people think they will have to pay all future taxes
  - people can freely borrow

But what if these conditions fail?

- like if Household's horizon shorter
  - people will likely plan on dieing before gov
    - have no kids/don't care
    - they will spend more if they don't think taxes will be high in future
- Or if people can't borrow - liquidity constraints
  - then timing of taxation matters a lot
  - can't consume more than income today
  - lending as well
  - here matches IS-LM

14.02 Exam Review Section

4/13

Real ex rate ϵ

- price domestic vs foreign

(very few items in this unit - should really know this stuff!)

Nominal ex rate E

- units of foreign currency can buy 1 domestic currency
- appreciation - domestic goods more expensive $E \uparrow$
- depreciation

$\epsilon = \frac{E P_{car}}{P^*_{car}}$. * means foreign

~~Price of foreign goods~~

how much do domestic goods cost rel to foreign - if had 1 unit domestic - how many foreign goods could ya buy

- moves according to app/depreciation

GDP deflator - price of all goods produced domestically

- lets us know movements in real ex rate

GDP - value added domestically

GNP - " " by domestically owned production - anywhere in the world

(2)

Current account

$$CA = S - I$$

↑ how much save, in this period abroad

Since we have an open economy
- citizens can invest abroad
- foreigners can invest here

$$CA = NX + \text{Net return on assets} + \text{Net transfers}$$

$$= \text{Net capital flows}$$

↳ sum of all money flowing in + out

(+) we bought more foreign assets than foreigners bought domestically

---

IS Relation in Open Econ

$$Z = C + I + G - \frac{IM}{\epsilon} + X$$

$$IM = IM(Y, \epsilon)$$

(+) (+) ↳ price of domestic goods are in terms of foreign goods
- convert to domestic goods
- essential the relative price

(3)

$$X = X(Y^*, \mathcal{E})$$
$$\quad\;\; (+) \quad (-)$$

Equalibrium Output and Trade Balance

$Y = Z$

In equalibrium, might have trade deficit or surplus

Supply domestic goods — demand domestic goods

Like new IS relation

Depreciation, Trade Balance, Output

$$NX = X(Y^*, \mathcal{E}) - \frac{IM(Y, \mathcal{E})}{\mathcal{E}}$$

A real depreciation

$X \uparrow$

$IM \downarrow$

$\frac{1}{\mathcal{E}} \uparrow$

In theory its ambigias. But we assume Marshall-Lerner condition:

$$\frac{\partial NX}{\partial \mathcal{E}} < 0$$

depreciation leads to $\uparrow$ in NX

Domestic vs Foreign bonds

Interest rate parity condition

$$1 + i_t = (1 + i_t^*)\frac{E}{E^e_{t+1}}$$

Return on domestic bonds must = real return on foreign bonds

We assume $E^e_{t+1} = \bar{E}^e$

Putting Goods + Fin Markets Together

2 simplifications

- P, P^* constant so $\varepsilon = \frac{EP}{P^*}$ moves 1 to 1 w/ E
- No inflation/expected inflation $i = r$

So IS is:

$$Y = C(Y^d) + I(Y, i) + G + NX(Y, Y^*, E)$$

Substitution interest rate parity condition

$$Y = C(Y^d) + I(Y, i) + G + NX\left(Y, Y^*, \frac{1+i}{1+i^*}E^e\right)$$

Note LM curve is same - so can still find equilibrium like in closed

(5) all

Under assumption of flexible exchange rate regime

We preceed as in closed econ. (i vs Y) But to know what happens to NX - need to know E following change in i.

So do this w/ interest rate parity condition (i vs E)

- we did some examples

Fixed Ex Rate

- interest parity condition must hold
- Change i So $E_t = E^e_{t+1}$

Assume policy is creditable

$$1 + i_t = (1 + i^*_t)\frac{E_t}{E^e_{t+1}} = (1 + i^*_t)\frac{\bar{E}}{\bar{E}} \rightarrow i_t = i^*_t$$

Monetary policy Revisit LM

$$\frac{M}{P} = Y L(i)$$

Gov loses ability to conduct independent monetary policy!

Gov must chose M to make LM hold given i^*

But Gov gains on ability to do fiscal policy
- more effective

When $\Delta G > 0$:

- IS shifts outward - Y and i ↑
- But i must $= i^*$
- M ↑ until i is unchanged - equilibrium is w/ ↑ output (higher)

So fiscal policy is more effective
- necessitates ↑ in money supply

Fragility of Fixed Rate Regimes

For i to be creditable i^* must reflect any changes in expected exchange rate

$$1 + i_t = (1 + i^*_t)\frac{E}{E^e_{t+1}}$$

Consider country w/ high public debt of short maturity
- Since gov must repay $1 + i_t$ units for every unit borrowed
- So gov devalues its currency - makes every unit worth less
- So people expect lower E - so gov must pay higher i_t
- So even more incentive to depreciate!

And then need to wait for prices to adjust back to natural level of output
- takes a while

Fiscal Policy

- Gov can finance spending
  - taxes
  - selling bonds

$$deficit_t = r B_{t-1} + G_t - T_t$$

$$B_t = (1+r) B_{t-1} + G_t - T_t$$

$$B_t - B_{t-1} = r B_{t-1} + G_t - T_t = deficit_t$$

more of a flow variable

- Deficit = amt gov. borrows new
- If gov just rolls over

$$debt = (1+r)^t$$

- If gov stabalizes - must run primary surplus
  - just pays interest

$$B(T-G) = rB$$

Then care about Debt-to-GDP ratio
- If gov stabalizes this

8

$$0 = \frac{B_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}} \simeq (r-g)\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

Primary surplus if $r > g$

Otherwise could run a primary deficit

Debt crisis

Consider first case: gov must sustain a primary surplus

(skipping)

Ricardian Equivilance

If gov + households have same planning horizon
it does not matter if gov funds via T or B

Get dynamics diff from IS-LM

$G\uparrow$ will $\downarrow$ consumer wealth $\downarrow C$

Also $\downarrow T$ will $\downarrow C$