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15.401 Midterm Exam – Craig Stephenson

Spring 2011

Instructions:

- Please fill in your name and ID number.
- The exam lasts for 80 minutes, allocate your time efficiently.
- The exam consists of six questions, please answer all of them.
- Credit for each question is proportional to the time recommended.

Question	Points (minutes)
1	20
2	10
3	10
4	10
5	15
6	15
Total	80

- You are allowed one 8 ½ by 11 sheet of formulae and one calculator.
- Answer the questions without consulting anyone.
- Use the space provided. If more space is needed, use the blank pages at the end of this document.
- Be neat and show your work. Answers without work receive no credit. Wrong answers with partially correct work may receive partial credit.

Good luck!

20

1. Multiple Choice Questions and Problems (5 questions worth 4 points each)

A risk-free security pays $200 after one year, $400 after two years, and $600 after three years which is its final payment. The risk-free (annualized) interest rate is 5%. What is the current price of the security?

a. $1,204.01
b. $1,123.23
c. $1,071.59
d. $998.61
e. None of the above

✓

The PV $\frac{200}{(1.05)} + \frac{400}{(1.05)^2} + \frac{600}{(1.05)^3}$

1071.59

You own 1,000 risk-free 5% 3-year coupon bonds with a face value of $1,000 that pay annual coupons. You wish to sell all of these cash flows as zero coupon bonds with face value of $1,000. How many 1-year, 2-year, and 3-year zero coupon bonds can you create and sell from the bonds you own?

a. 0 1-year zeros, 0 2-year zeros, 1,000 3-year zeros
b. 1,000 1-year zeros, 1,000 2-year zeros, 1,000 3-year zeros
c. 50 1-year zeros, 50 2-year zeros, 50 3-year zeros
d. 50 1-year zeros, 50 2-year zeros, 1,050 3-year zeros ← coupons also in last year
e. 50 1-year zeros, 50 2-year zeros, 1,000 3-year zeros

✓

Have 1000 · 1000

1) 50,000 - 50 1 year 0s
2) 50,000 - 50 2 year 0s
3) 1,050,000 1050 3 year 0s

A stock pays an expected dividend of $3 next year. Stock analysts predict the dividends to grow at a rate g = 4% forever. You perform your own analysis of the discount rate and determine r = 10% is correct given the risk of the stock. If you believe the analysts, what do you think is the correct price of the stock according to the dividend discount model (the Gordon model)?

a. $30
b. $50
c. $52
d. $55
e. $60

$\frac{D}{r-g}$ $\frac{3}{.10-.04}$

✓

You are the CEO or a former market leader in the smart phone industry. Your company invests most of its earnings in research and development (R&D), but in spite of this plowback of earnings, your company's return on equity (ROE) is less than the discount rate (also called the cost of capital) the market applies to your company. What would happen to the price of your company's stock if you decide to increase the plowback ratio and invest more $ at the same ROE?

a. Stock price will decrease
b. Stock price will stay the same
c. Stock price will increase
d. There is not enough information to answer this question

Invest more and get same ROE?
Investors not happy
Dividends ↓

✓

$P_0 = \frac{EPS}{r} + PVGO$

$D_1 = p \cdot EPS$

$g = b \cdot ROE$ ← but higher growth earnings will go up next term

ROE = 4
Discount = 5
Plow back

A bond with a current price of $960 has a modified duration of 5.00 years. Using this modified duration, how much will the price of the bond decrease if the yield to maturity (YTM) of the bond increases by 0.1%?

a. The bond price will not change
b. The bond price will decrease by 0.05% (a decrease of $0.48)
c. The bond price will decrease by 0.50% (a decrease of $4.80)
d. The bond price will decrease by 5.00% (a decrease of $48.00)
e. The bond price will actually increase

↑ rate ↓, price ↑

$$\Delta P = -P \cdot D^* \cdot \Delta y$$

$$= -960 \cdot 5.00 \cdot .001$$

$$= -4.80$$

2. Annuity Problem

You have one year left before you graduate from College, and you have found a great apartment in Brookline where you want to live for this year. The monthly rent is $1,800, payable at the end of each month, and you just signed a 12-month lease. How much money do you need to deposit in the bank at the beginning of your lease to be able to make every rent payment (and have no money left over after the 12th payment is made)? The annual percentage rate (APR) paid by the bank on your deposit is 3.6% per year.

Learn interest on deposits

- What is the effective annual rate earned on your funds deposited at the bank?
- How much money do you need to deposit at the beginning of your lease to make all 12 payments?

10

$r_{ear} = \left(1 + \frac{3.6\%}{12}\right)^{12} - 1 = 3.66\%$

$\frac{3.6\%}{12} = .3\%$ rate per month

$PV = 1{,}800 \cdot \frac{1}{.003}\left[1 - \frac{1}{(1+.003)^{12}}\right]$

1 month, .003 rate per month

✓

$= 21{,}184.63$

Sanity check: Should be a little less than less than $12 \cdot 1800$

$= 21600$

- So sanity checked ✓

3. Forward Interest Rates Problem $_0r_1 = 3\%$ $_0r_2 = 5\%$

The 1-year zero coupon interest rate is currently 3.0% and the 2-year zero coupon interest rate is 5.0%. These zero coupon bonds have a face value of $1,000.

• What is the forward interest rate from year 1 to year 2?

• Can you build a 2-year 4.0% coupon bond from these zero coupon bonds (i.e., can you construct the cash flows from a $1,000 face value bond which pays a 4.0% annual coupon for 2 years) if you can purchase fractions of the zero coupon bonds? If this is possible, how much would it cost to build this portfolio of zero coupon bonds?

arbitrage?

$$_1r_2 = \frac{(1+{}_0r_2)^2}{(1+{}_0r_1)^1} - 1 = \frac{(1+.05)^2}{(1+.03)} - 1$$

$$= \boxed{7.04\%} \quad \checkmark$$

← not connected to rest of problem

Cash flow for this bond

10

1) -40
2) -1040

$$PV = \frac{40}{(1+.03)^{?r_1}} + \frac{1040}{(1.05)^{?r_2}} = 982.15$$

So would need payout of 40 in year 1, so need 4% of a 1 year
Need payout 1040 in year 2, so need 1.04% of a 2 year
So Yes can build

This would cost

$$PV_1 = \frac{1000}{(1.03)} = 970.87 \quad PV_2 = \frac{1000}{(1.05)^2} = 907.03$$

$$.04 \cdot 970.87 + 1.04 \cdot 907.03 = \$982.15 \quad \checkmark$$

So no arbitrage

~~3.5% / period~~

4. Coupon Bonds Problem

2 per year - no perpetuity??

You are analyzing a 4-year 7.0% coupon bond which pays semi-annual coupons. The bond has a face value of $1,000, and the quoted yield to maturity (YTM) on the bond is 7.60%. 3.8% / period

• What is the market price of this bond? Is this a premium or discount bond?

PV = cash flows

$$= \frac{70}{(1.038)^{35}} + \frac{70}{(1.038)^{35}} + \frac{70}{(1.038)^3} + \frac{70}{(1.038)^4} + \frac{70}{(1.038)^5} + \frac{70}{(1.038)^6} + \frac{70}{(1.038)^7} + \frac{1070}{(1.038)^8}$$

5

$$= \frac{70}{(1.076)^{1.5}} + \frac{70}{(1.076)^1} + \frac{70}{(1.076)^{1.5}} + \frac{70}{(1.076)^2} + \frac{70}{(1.076)^{2.5}} + \frac{70}{(1.076)^3} + \frac{70}{(1.076)^{3.5}} + \frac{1070}{(1.076)^4}$$

$$= 1222.60$$

vs 1217.23 other method (rounding differences)

It is selling for more than its face value, so it is selling at a premium

zero coupon = discount

5. Common Stock Problem

Hudson Company just paid a dividend (D_0) of $4.00. The company's dividends are expected to grow at 15% per year for years 1 and 2, and then 12% per year in years 3 and 4. In years 5 and beyond, the dividends are expected to grow at a 7% annual rate. The appropriate cost of capital (discount rate) for stocks of Hudson's risk class is 10%.

• What is the current market price of Hudson's common stock?

$P_5 = \frac{D_6}{r-g}$

$= PV(\text{cash flows})$

15

$= \frac{4.6}{(1+.1)} + \frac{5.29}{(1+.1)^2} + \frac{5.92}{(1+.1)^3} + \frac{6.63}{(1.1)^4} + \frac{7.100}{(1.1)^5} + \frac{\frac{7.597}{.1-.07}}{(1.1)^5}$

↑ include

$= 21.93 + 157.23$

$= 179.18$

6. **Immunization Problem**

You manage a client's defined benefits pension fund for its retired workers. This plan has very few surviving beneficiaries, and you expect payments will be made to retirees for only 3 more years. The obligations payable to the beneficiaries are:

Year	1	2	3
Payments Due	$12,000,000	$6,000,000	$3,000,000

The pension fund is currently completely funded, that is, the value of its assets equals the value of its liabilities. The interest rate structure (term structure) is flat – the interest rate on all maturities is 4.0%.

(Cost of capital)

- What is the present value of the plan's liabilities?
- What is the (Macaulay) duration of the plan's liabilities?
- You want to be certain that the pension fund stays completely funded. You can invest in both 1-year and 10-year zero coupon bonds. How much of your portfolio, stated in percentages, must you invest in each of these zero coupon bonds?

$$PV_{Liab} = \frac{12,000,000}{(1.04)} + \frac{6,000,000}{(1.04)^2} + \frac{3,000,000}{(1.04)^3}$$

$$= 19,752,787.89 \quad \checkmark$$

$$D = \frac{1}{B} \sum_{t=1}^{T} PV(CF_t) \cdot t$$

15

$$= \frac{1}{19752787.89} \left[\frac{12,000,000}{(1.04)} \cdot 1 + \frac{6,000,000}{(1.04)^2} \cdot 2 + \frac{3,000,000}{(1.04)^3} \cdot 3 \right]$$

$$= \frac{1}{19752787.89} \left[30634103.32 \right]$$

$$= 1.55 \quad \checkmark$$

Want to match duration to stay funded

x = % in 1 year bonds

$$1.55 = X \cdot 1 + (1-X) \cdot 10$$

Solve for x

$X = 93.88\%$ = % 1 year bonds ✓

$(1-X) = 6.1111\%$ = % 10 year bonds

75/80

15.401 Mid-term Exam – Craig Stephenson

March 9, 2011

Here are the solutions to the March 9 mid-term exam. The directions on the cover page asked you to show your work and stated answers without work would receive a loss of credit. This was applied to the multiple choice questions; correct answers without any work shown resulted in point loss.

The results of the exam were:

Mean	66.7 points out of 80 possible
Median	69.0 points out of 80 possible
n	47 students took this exam

A reasonable dividing line for the exams is the median score of 69 points. Scores above this a likely A's, and those below are likely not.

Last name:

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Spring 2011

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Good luck!

(1) **Multiple Choice Questions and Problems (5 questions worth 4 points each)**

A risk-free security pays $200 after one year, $400 after two years, and $600 after three years which is its final payment. The risk-free (annualized) interest rate is 5%. What is the current price of the security?

a. $1,204.01
b. $1,123.23
(c.) $1,071.59
d. $998.61
e. None of the above

r = 5%

1	2	3
$200	$400	$600
×.9524	×.9070	×.8638
$190.48	$362.80	$518.28

Σ = 1,071.56 → $1,071.5905 precisely

You own 1,000 risk-free 5% 3-year coupon bonds with a face value of $1,000 that pay annual coupons. You wish to sell all of these cash flows as zero coupon bonds with face value of $1,000. How many 1-year, 2-year, and 3-year zero coupon bonds can you create and sell from the bonds you own?

a. 0 1-year zeros, 0 2-year zeros, 1,000 3-year zeros
b. 1,000 1-year zeros, 1,000 2-year zeros, 1,000 3-year zeros
c. 50 1-year zeros, 50 2-year zeros, 50 3-year zeros
(d.) 50 1-year zeros, 50 2-year zeros, 1,050 3-year zeros
e. 50 1-year zeros, 50 2-year zeros, 1,000 3-year zeros

1 Bond:

1	2	3
$50	$50	$1050

1000 Bonds:

1	2	3
$50,000	$50,000	$1,050,000
n = 50	n = 50	n = 1,050

(9) A stock pays an expected dividend of $3 next year. Stock analysts predict the dividends to grow at a rate g = 4% forever. You perform your own analysis of the discount rate and determine r = 10% is correct given the risk of the stock. If you believe the analysts, what do you think is the correct price of the stock according to the dividend discount model (the Gordon model)?

a. $30
b. $50
c. $52
d. $55
e. $60

$$P_0 = \frac{D_1}{r-g}$$

$$= \frac{\$3.00}{.10-.04}$$

$$= \$50.00$$

You are the CEO or a former market leader in the smart phone industry. Your company invests most of its earnings in research and development (R&D), but in spite of this plowback of earnings, your company's return on equity (ROE) is less than the discount rate (also called the cost of capital) the market applies to your company. What would happen to the price of your company's stock if you decide to increase the plowback ratio and invest more $ at the same ROE?

a. Stock price will decrease
b. Stock price will stay the same
c. Stock price will increase
d. There is not enough information to answer this question

ROE < r

∴ Not meeting required return

Value loss

Stock price ↓

(Δ) this

A bond with a current price of $960 has a modified duration of 5.00 years. Using the modified duration, how much will the price of the bond decrease if the yield to maturity (YTM) of the bond increases by 0.1%?

a. The bond price will not change
b. The bond price will decrease by 0.05% (a decrease of $0.48)
(c.) The bond price will decrease by 0.50% (a decrease of $4.80)
d. The bond price will decrease by 5.00% (a decrease of $48.00)
e. The bond price will actually increase

$$\Delta P = -\text{Mod Dur} \times \Delta Y$$

$$= -5.00 \times 0.001$$

$$= -0.0050 \quad (-0.5\%)$$

of $960 Bond Price

↓

$-\$4.80$

Price falls by $4.80

(2) **Annuity Problem**

You have one year left before you graduate from College, and you have found a great apartment in Brookline where you want to live for this year. The monthly rent is $1,800, payable at the end of each month, and you just signed a 12-month lease. How much money do you need to deposit in the bank at the beginning of your lease to be able to make every rent payment (and have no money left over after the 12th payment is made)? The annual percentage rate (APR) paid by the bank on your deposit is 3.6% per year. APR monthly.

- What is the effective annual rate earned on your funds deposited at the bank?

(No Space line)

- How much money do you need to deposit at the beginning of your lease to make all 12 payments?

(1) EAR $\left(1 + \frac{.036}{12}\right)^{12} - 1 = .036600$, 3.66%

(2) Annuity = $1,800

n = 12 periods

i = 0.3% = 0.003 per period

FV = Ø

PV = $? = $21,184.63109 ✓

$$PV = \$1,800 \times \frac{1}{.003} \times \left(1 - \frac{1}{(1.003)^{12}}\right)$$

$$= \$1,800 \times 333.333\overline{3} \times .035308$$

$$= \$21,184.63092$$ ✓

3. Forward Interest Rates Problem

The 1-year zero coupon interest rate is currently 3.0% and the 2-year zero coupon interest rate is 5.0%. These zero coupon bonds have a face value of $1,000.

• What is the forward interest rate from year 1 to year 2?

• Can you build a 2-year 4.0% coupon bond from these zero coupon bonds (i.e., can you construct the cash flows from a $1,000 face value bond which pays a 4.0% annual coupon for 2 years) if you can purchase fractions of the zero coupon bonds? If this is possible, how much would it cost to build this portfolio of zero coupon bonds?

1 year spot rate = 3.0% (stated)

2 year spot rate = 5.0% (stated)

Solution part 1:

$((1.05)^2 / 1.03) - 1 = 0.070388 = 7.04\%$

Solution part 2:

	Year 1	Year 2
You want	$40	$1040

Buy $40 / $1000 = 0.04 of the 1 year zero

Buy $1040 / $1000 = 1.04 of the 2 year zero

$0.04 \times \$1000 / 1.03 =$	$38.8350	
$1.04 \times \$1000 / 1.05^2 =$	$943.3107	
Total cost	$982.1457	or $982.15

④ **Coupon Bonds Problem**

You are analyzing a 4-year 7.0% coupon bond which pays semi-annual coupons. The bond has a face value of $1,000, and the quoted yield to maturity (YTM) on the bond is 7.60%.

• What is the market price of this bond? Is this a premium or discount bond?

PMT = $1,000 × .07 ÷ 2 = $35

FV = $1,000

N = 4 × 2 = 8

YTM = 7.60% ÷ 2 = 3.80% = .038

PV = $? = $979.6339 , $979.63 ✓

Trading at a discount < $1000 Face value

$$PV = \$35 \times \frac{1}{.038} \times \left(1 - \frac{1}{(1.038)^8}\right) + \$1000 \times \frac{1}{(1.038)^8}$$

$$= \$237.6044 + \$742.0295$$

$$= \$979.6339 , \$979.63$$

$D_0 = \$4.00$

(5) Common Stock Problem

$\$4.00$

Hudson Company just paid a dividend (D_0) of ~~$3.00~~. The company's dividends are expected to grow at 15% per year for years 1 and 2, and then 12% per year in years 3 and 4. In years 5 and beyond, the dividends are expected to grow at a 7% annual rate. The appropriate cost of capital (discount rate) for stocks of Hudson's risk class is 10%.

$r = 10\%$

- What is the current market price of Hudson's common stock?

15% → D_1 $\$4.60$ → 15% → D_2 $\$5.29$ → 12% → D_3 $\$5.9248$ → 12% → D_4 $\$6.6358$ → 7% → D_5 $\$7.1003$

$\times .9091 = \$4.1819$ $\quad .8264 = \$4.3717$ $\quad \times .7513 = \$4.4513$ $\quad \times .6830 = \$4.5323$

PV $Div_1 - Div_4 = \$17.5372$ ✓

$$P_4 = \frac{Div_5}{r-g} = \frac{\$7.1003}{.10 - .07} = \$236.6767$$

$$PV\ P_4 = \$236.6767 \times \frac{1}{(1.10)^4} = \$161.6534$$

∴ Current Stock Price is

$\$17.5372 + \$161.6534$

$\$179.1906$

↓

$\$179.19$

⑥ **Immunization Problem**

You manage a client's defined benefits pension fund for its retired workers. This plan has very few surviving beneficiaries, and you expect payments will be made to retirees for only 3 more years. The obligations payable to the beneficiaries are:

(No space)

Year	1	2	3
Payments Due	$12,000,000	$6,000,000	$3,000,000

The pension fund is currently completely funded, that is, the value of its assets equals the value of its liabilities. The interest rate structure (term structure) is flat – the interest rate on all maturities is 4.0%.

- What is the present value of the plan's liabilities? $19,752,600 → $19,752,787.89 precisely
- What is the (Macaulay) duration of the plan's liabilities?
- You want to be certain that the pension fund stays completely funded. You can invest in both 1-year and 10-year zero coupon bonds. How much of your portfolio, stated in percentages, must you invest in each of these zero coupon bonds?

	1	2	3
	$12,000,000	$6,000,000	$3,000,000
✓	× .9615	× .9246	× .8890
Σ = $19,752,600	$11,538,000	$5,547,600	$2,667,000
	× 1	× 2	× 3
Σ = $30,634,200	$11,538,000	$11,095,200	$8,001,000

$$\text{DUR} = \frac{\$30,634,200}{\$19,752,600} = 1.5509 \text{ years} \quad ✓$$

Desired Duration Asset Portfolio = 1.5509 years

Dur 1-year Zero = 1.00
Dur 10-year Zero = 10.00

Let x = weight of 1-year Zero

$$1x + 10(1-x) = 1.5509$$
$$x + 10 - 10x = 1.5509$$
$$8.4491 = 9x$$
$$.9388 = x$$

✓ 93.88% of Portfolio in 1-year Zero
12% of Portfolio in 10-year Zero