15.401 Cheat Sheet 1

Value of asset = value of cash flows

OPP. cost capital = expected rate of return (time + risk) of other investments

$PV = \frac{CF_t}{(1+r)^t}$

0 = now

1 = in one year

$PV_{annuity} = A \cdot \frac{1}{r}\left[1 - \frac{1}{(1+r)^t}\right]$

A ↑ amt each year

$FV_{annuity} = (1+r)^T \cdot PV_{annuity}$

- starts in year one

$PV_{annuity\ w/growth} = A\left[\frac{1}{1+r} + \frac{1+g}{(1+r)^2} + \dots + \frac{(1+g)^{T-1}}{(1+r)^T}\right]$

$= \begin{cases} A\cdot\frac{1}{r-g}\left[1-\left(\frac{1+g}{1+r}\right)^T\right] & r\neq g \\ A\cdot \frac{1}{1+r} & r=g\end{cases}$

$PV_{perpetuity} = \frac{A}{r}$

$PV_{perpetuity\ w/growth} = \frac{A}{r-g} \quad r > g$

APR = 5% compounded monthly

each month $\frac{5\%}{12} = .41\%$

at end of each year \$100(1+.41)(1+.41)...

$r_{EAR} = \left(1+\frac{r_{APR}}{k}\right)^k - 1$

$(Real\ CF)_t = \frac{(Nominal\ CF)_t}{(1+\lambda)^t}$

$r_{real} = \frac{1+r_{nominal}}{1+\lambda} - 1$

λ = inflation from CPI

r = inflation risk, default risk premium

prob weighted avg — promised − expected — expected default free

Yield curve i

time

Continuous Compounding

$\lim_{n\to\infty}\left(1+\frac{APR}{N}\right)^N - 1 = e^{APR} - 1$

Zero Coupons $B_t = \frac{1}{(1+{}_0r_t)^t}$

- good way to figure out spot rates

Coupons $B = \sum_{t=1}^{T} C_t \cdot B_T + P\cdot B_T$ (final coupon w/ principal)

$= \frac{C_1}{1+{}_0r_1} + \dots + \frac{C_{T-1}}{(1+r_{T-1})^{T-1}} + \frac{C_T+P}{(1+r_T)^T}$

↑ spot rates

YTM = effective interest

$P = \frac{C}{1+{}_0r_T} + \frac{C}{(1+{}_0r_T)^2} + \dots$

P found normal way; ↑ spot rate to maturity

Forward Rates

$(1+{}_0r_2)^2 = (1+{}_0r_1)\cdot(1+{}_1f_2)$

$f_t = \frac{B_{t-1}}{B_t} - 1 = \frac{(1+r_t)^t}{(1+r_{t-1})^{t-1}} - 1$

${}_2f_3 = \frac{(1+{}_0r_3)^3}{(1+{}_0r_2)^2} - 1$

To borrow \$100 from year 3 to year 4
1. Buy 100 of 3 year bonds at $B_3 = P_3$
2. Sell 4 years of $\frac{P_3}{B_4}$

	0	3	4
Purchase	-80	100	
Sale	+80		-102

Bank makes $\frac{102}{100} - 1$

Longer bonds riskier - charge risk premium = liquidity pref.

$f_t = E[r_1(t)]$

Duration weighted avg term to maturity

$D = \sum_{t=1}^{T} \frac{PV(CF_t)}{B}\cdot t = \frac{1}{B}\sum_{t=1}^{T} \frac{CF_t}{(1+y)^t}\cdot t$

$MD = -\frac{1}{B}\frac{\Delta B}{\Delta y} = \frac{D}{1+y} \quad y = r$

- measure of volatility

$\Delta P = -P \cdot D^* \cdot \Delta y$

Convexity curvature of price as f(yield)

$CX = \frac{1}{2}\frac{1}{B}\frac{\Delta^2 B}{\Delta y^2}$

debenture - not asset backed

immunization - price risk - prices fall as i↑
- vs reinvestment rate risk - coupons will grow at fast rate as i↑

but 30 bonds now worth much less! obligations fall too

arbitrage sell/short the overpriced asset, buy the underpriced asset, profit

underpriced: selling below PV

use revenue from underpriced asset at each period to pay back bond purchase price - return (borrowed \$ w/ a short)

At each time period

Duration price Old price − (Change YTM)·old duration

Bid - price to buy

Ask - price to sell

Stocks - Discount cash flows

Constant + Perpetual growth - Gordon model

$P_0 = \frac{D_1}{r-g} = \frac{1+g}{r-g}D_0$

r = cost of capital = discount rate = req return

$= \frac{D_1}{P_0} + g = \frac{(1+g)D_0}{P_0} + g$

g = growth of dividends

$= \frac{r - D_0/P_0}{1 + D_0/P_0}$

Earnings (E or EPS) = total profits − taxes − dep

Payout Ratio = dividends / earnings = p

Retained Earnings Δ = earnings − dividends

Plowback Ratio = $\frac{\text{retained earnings } \Delta}{\text{earnings}}$ = b = 1 − p

Book Value = cumulative retained earnings

Return on book value of equity (ROE) = $\frac{Earnings}{BV}$

$D_1 = p \cdot EPS$

$g = b \cdot ROE$

$P_0 = \frac{EPS}{r} + PVGO$

↑now ↑expected future

If $PVGO = 0$

$P/E = \frac{1}{r}$

$PVGO > 0$

$$P/E = \frac{1}{r} + \frac{PVGO}{EPS} > \frac{1}{r}$$

residual claims

voting rights

limited liability

reinvest when ROE > cost of capital

Earnings yield $= \frac{E}{P} = \frac{EPS_1}{P_0}$ otherwise dividend all, liquidate

P/E high - people looking at growth

$$V_0 = \sum_{t=1}^{T} \frac{FCF_t}{(1+r)^t} + \frac{TV_T}{(1+r)^T}$$

FCF = free cash flow

V_0 = value today

TV = terminal value = found w/ Gordon model

3 ways

1. Book value of equity
   assets - liab
2. Relative valuation
   - w/ industry
3. Discounted cash flows

inflation ↑, people want bigger returns

risk ↓, required return ↓

$$r = ROE \cdot \left(1 - \frac{\text{dividend}}{\text{earnings}}\right)$$

growth companies, ROE > cost capital

15.401 Studying

This is midterms already? — only 1
Semester is barreling by!
Well he said this one is esp early
Session 11 of 26

Opp cost of capital
All markets link economic units
no arbitrage — one price
Value of assets = value of cash flows
Connect those w/ $ to those w/ ideas

I see where EAR comes from now!

---

inflation: market seems to ignore inflation
- but its there behind the scenes!
- investors need a return over inflation
- if under that, the investors will just lose real $

But projected future cash flows for companies should include inflation

---

match periods as well!

Seems easier this time around — b/c I learned it!
redo P-set afterward to test!

②

Look at the silly mistakes I have made

- not adding decimal points for rates

- going $\sum_{0}^{20}$ not $\sum_{0}^{19}$

- also first one I mostly did Σ not main formula

---

fixed income securities

Oh ~~the~~ 501's discount and preimum

Finding spot rates

Practice some arbitrage!

- Can't figure at year by year

Called <u>convergence trading</u>

Selling/shorting might be confusing me

Its not really covered well in literature

Short the bonds

Contract pays $300 year 3

So how many bonds to face year 3

? or start at begining? - no they start w/ C

(3)

Price of C = ~~258.94~~ 97.27

B ~~183~~ 97.19

A ~~96.~~ 96.15

#

Re look at recitation notes

not helpful

~~Get 300~~

Need to raise \$ to buy our contract

will get 100 in year 1 ~~so how many~~ so how much can we short 1 to get 100 in year 1

Sell \$100 worth for 96.158 today

Or sell ___ worth for 100 today?

$$100 = \frac{\quad}{(1.04)}$$

No in year 1 need to pay ___ .

Or perhaps formula would help

Do recitation example

(4)

	$t=0$			
A	-97	100		
B	-92		100	
C	-87			100
D	-102	5	5	105

So calc true price of D with rates from above

$97 = \frac{100}{(1+r)}$ $\sim 3\%$

$92 = \frac{100}{(1+r_2)^2}$ 4.25%

$87 = \frac{100}{(1+r_3)^3}$ 4.75%

So true price of D is

$\frac{5}{(1+.03)} + \frac{5}{(1.0425)^2} + \frac{105}{(1.0475)^3} = 100.08$

So overpriced

Sell / short D – but how much? — profit opp = 1.191

Say 1 D

This means I get \$102 today
but have to pay \$5 $t=1$
\$5 $t=2$
\$105 $t=3$

(5) Remember multiple strategies
(may be confusing me too)

So with the 102 need to buy of other

So need $5 in year 1

So buy .05 of ~~year~~ A - costs $.05 \cdot 97 = 4.85$

.05 B $.05 \cdot 92 = 4.6$

1.05 C $1.05 \cdot 87 = 91.35$

100.8

So $102 - 100.8 = \$1.20$ profit today!

Figured it out! Now can I do other way? on other problem

So contract is underpriced. That means sell other stuff in order to buy 1 contract.
↑say

Contract will give $100 in year 1. Need to ~~m~~ Use it to pay off short - want to short as much as possible

$\frac{0}{-96.15}$ $\frac{1}{100}$

So sell 1 A - make 96.15

2 B $2 \cdot 97.14$

3 C $3 \cdot 97.27$

582.34

but contract costs 500 - dam

Oh the bonds have coupons - screws stuff up

(6) I think you need a matrix - style sol
which would not be on exam
Going to skip

So made it through review slides pretty well
P-set qu were hard
- I may not remember ans
Prob will make lots of silly mistakes
No more practice now