Will be $\frac{1}{4} - \frac{1}{3}$ exam 1
- which was actually fairly early
- PV, fixed income, Stocks

The new stuff is
Forwards + futures
Options
Portfolio theory
CAPM
Capital Budgeting

Old Stuff

key ideas: opportunity cost of capital
Role of fin markets
no arbitrage

1. How to value assets
2. How to finance project
3. What to payout

Need to __value__ a key

* It's the bottom line of a biz decision

Markets value assets

1. Investors prefer more to less

2. Investors are risk adverse

3. Money paid in future is worth less than today

4. Markets are competive - no arbitrage!

Need to pay risk premium

$$PV = \frac{CF_t}{(1+r)^t}$$

(Don't confuse myself with all the formulas, I am able to derive)

Higher r is $PV\downarrow$

FV is how much $1 today is worth

$$FV = (1+r)^T$$

Annuity - easier calcs

(I have all this on my cheat sheet)

$$PV = A \cdot \frac{1}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

↑
the money
paid per year

even more complex w/ growth

③

Perpetuity $= \dfrac{A}{r}$

↑ forever

w/ growth $= \dfrac{A}{r-g}$

Compounding

APR = annual rate

Interest per period $\dfrac{APR}{k}$

Actual annual rate (since compounding) = EAR ≠ APR

Say, 5% APR compounded semiannually

Period rate $= \dfrac{5\%}{2} = 2.5\%$

So if invest $10,000, have

$$10,000 + (1+.025)(1+.025) = 10,506.25$$

$$r_{EAR} = (1+.025)^2 - 1 = 5.0625\%$$

$$r_{EAR} = \left(1 + \dfrac{r_{APR}}{k}\right)^k - 1$$

Continuous $e^{.05} =$

④

(Need to practice also)

Nominal — in that year

The discount rate is inflation or risk-adj discount rate

$$r_{real} = \frac{1 + r_{nominal}}{1 + i} - 1 \approx r_{nominal} - i$$

Fixed Income Securities

promises of fixed amts of $ at fixed dates

Bonds
- gov
- municipal
- corp
- asset backed securities

Have a fixed term

Coupon - pay $ out each year (or semi-annual)

Currency matters

If borrower is a credit risk matters

Seniority

Covenents

Other provisions
- we are not responsible for

⑤

<u>Spot</u> interest rate - changes over time

(I need to learn this discount/premium thing)

WP

<u>discount bonds</u> bond bought at price lower than face value

↑ maturity repayment

Often no coupons
value rises as time gets closer to maturity
(oh is it the coupon value which controls it?)

<u>Premium bond</u> - higher than par value

⁻often cause interest rates higher than market rates

* but in relation to <u>$ paid at maturity</u> *

(so the two are not very similar at all)
⁻and I think terms used differently than L5, 5A

Can find spot rates from zero coupon bonds

$$B_t = \frac{1}{(1+r)^t}$$

<u>Coupons</u> - regular payments

⁻add up PV of coupons + maturity

⁻use spot rate as discount rate?

⑥

Can calculate Yield to Maturity (YTM)

$$B = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t} + \frac{P}{(1+y)^T}$$

↑ Coupons   ↑ maturity

YTM is like effective interest rate over life of bond

--- is weighted as to when cash is paid at

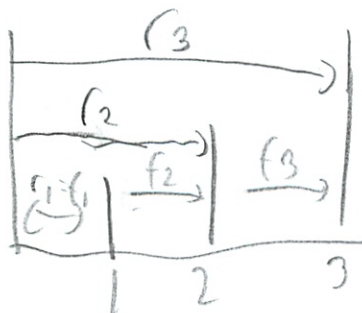---

Forward interest rate

like between t_1 and t_2

$$(1+r_t)^t = (1+r_{t-1})^{t-1} + (1+f_t)$$

↑ weird terminology

r_t is rate from now to t

f_t is rate from $t-1$ to t

(all on my cheat sheet)

I like $_0 r_2$ notation better

Expectation Hypothesis

forward rates predict future spot rates

↑ short-term

$$f_t = E[r_1(1)]$$

also add in a Liquidity Premium

- investors prefer short bonds to long bonds
- so must pay a liquidity premium on long bonds

long term rates is geometric average of short term rates

- we did not really use

Have interest rate risk as bond prices change over time

Can calculate duration

Macaulay duration — weighted avg term to maturity

$$D = \sum_{t=1}^{T} \frac{PV(CF_t)}{B} \cdot t$$

Modified duration relative price change w/ respect to a unit change in yield

$$MD = -\frac{1}{B}\frac{\Delta B}{\Delta y} = \frac{D}{1+y}$$

(I need to review this more)

<u>Convexity</u> measure of curvature of bond price as f(yield)

$$CX = \frac{1}{2} \frac{1}{B} \frac{\Delta^2 B}{\Delta y^2}$$

<u>Immunitization</u> - duration matching

(that was the case - need to review)

<u>Inflation risk</u> - (you have it, not much more I think you can say

<u>Default premium</u> = Promised YTM - Expected YTM

<u>Risk premium</u> = Expected YTM - Risk free YTM

(I have no clue why these are split

Notes <u>duration</u> - true duration of the bond
effective maturity of period
just matches if no coupons
if coupons shorter

<u>modified duration</u> - calc what a change in interest rate
does to bond price
a range estimation tool esp when increments are small

<u>immunitization</u> - wasn't this matching maturity of assets w/ maturity
of liabilites

S+L crisis was issue w/ immunitization and duration

if i↑ cost of borrowing exceeds return on portfolio

need to be able to reinvest at right rate to make future payments

Manage price risk vs reinvestment risk

when i↑   prices ↓   but can reinvest at higher rate
     i↓   ↑   value ↓   value liabilites ↑

Can try to make $ by trying to guess i

Can protect from inflation w/ TIPS

Stock Markets

- Market orders - now
- Limit orders - at a price

- Bid - price a dealer will buy from you
- Ask -   "   "   "   " Sell to   "

Exchanges becoming automated

Dark pools - prices and identies private

Prices are *supposedly* determined using fundemental anaylsis
  - Present value of cash flow
      - Using required return
  - Are both common + unique factors for stocks

ETFs, Index funds, Mutual Funds = far stock holders

Hedge Funds - Not far

Shorts theadge funds - really hatted

HFT is new - split second arbitrage opportunites

Stocks are
  - residual claims
  - limited liability
        - up to value of investmant

  - voting rights

---

discounted dividends

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r_A)^t}$$

↑The rate at that line
often assume constant

⑪

Dividends are expected to grow at g in perpituity

$$P_0 = \frac{D_1}{r-g} = \frac{1+g}{r-g} D_0$$

Cost of capital

$$r = \frac{D_1}{P_0} + g$$

$= $ Dividend yield + dividend growth

Usually is ~~soor~~ growth is in multiple stages

1. Growth
2. Transition
3. Maturity

Earnings ("EPS") = total profit net of depreciation + taxes
                    ?after?

Payout $= \dfrac{\text{dividend}}{\text{earnings}}$

RE $= \Delta(\text{earnings} - \text{dividends})$

Plowback $= \dfrac{RE\Delta}{\text{earnings}} = 1 - \text{payout}$

(BV) Book Value $=$ cumulative RE

Return on book equity (ROE) $= \dfrac{\text{earnings}}{BV}$

⑫

So after each year, company decides how much to reinvest
Are also growth opportunities

$g =$ return $-$ cost of capital

$P_0 = \dfrac{D_1}{r-g} = \dfrac{5.00}{.15 - .10} = 100$

cost of capital ↗   ↗ growth opp has 25% return
                      $-$ must be $>$ cost of capital

So stock price has two factors

$P_0 = \dfrac{EPS}{r} + PVGO$

⌊_____⌋   ⌊_____⌋
earnings   growth
no
growth

So if $PVGO = 0$

$\dfrac{P}{E} = \dfrac{1}{r}$

↗ price to earnings

if $PVGO > 0$

$\dfrac{P}{E} = \dfrac{1}{r} + \dfrac{PVGO}{EPS_1} > \dfrac{1}{r}$

Other ways to value : Equity calculation

Actually Finding book value of equity

+ Total PV of all free cashflows
- Value of debt
= estimated value of equity

↑ have a terminal value
one year
so easier

Assets = Liabilities + Equity
    └ Common shares
      Retained earning

↑ Backwards looking view

___

Relative valuation - look at similar companies

Market Cap = Market Value of Equities = $\frac{Price}{share}$ • # shares

    ↑ So value of all shares outstanding

Assumes corp is average

Discount Cash Flow for whole firm
        ↑
       Free)

___

If inflation ↑ investors want more returns, price ↓

(14)

Arbitrage opp

- Calc PV
- if under price listed, bond is too expensive
  ↑
  this pv price
- Buy a strip + sell/short the bond

Dividend - always assume has happened

Solving for g = growth rate of earnings

If ROE < cost of capital, liquidate co, pay it out

(Write this on cheat sheet instead for Unit 2 -So have during exam!)
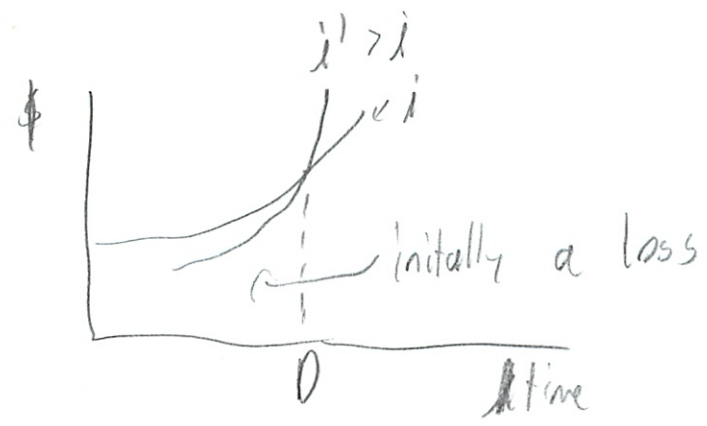
Review
- duration
- convexity
- immunization

Price vs reinvestment risk

- So offsets

if owner of bonds and $i \uparrow$

  bonds worth less

  but reinvested coupons grow faster offsetting loss

want to cancle out

balances value coupons (reinvestment) and value bond (price) [sale]

(I finally get this when had not before!)

Will do unit 2 on cheat sheet

Forwards + Futures we start w/
Make sure write down all formulas!
The black slides notes not clear

C risk - this is the less quantative section

Confused what points are on capital allocation line

what would this be: ↙ leverage

← what is this   ⊏ WP: this is 100% risky assets

This is 100% risk Free

Can crank down for less returns w/ less risk

Oh right want to move ↖ ⊛   ← what I forgot

which is tangent to CML

So this is 50% risky, 50% risk free
at ↓ ~~E[r]~~ ↓E[r]
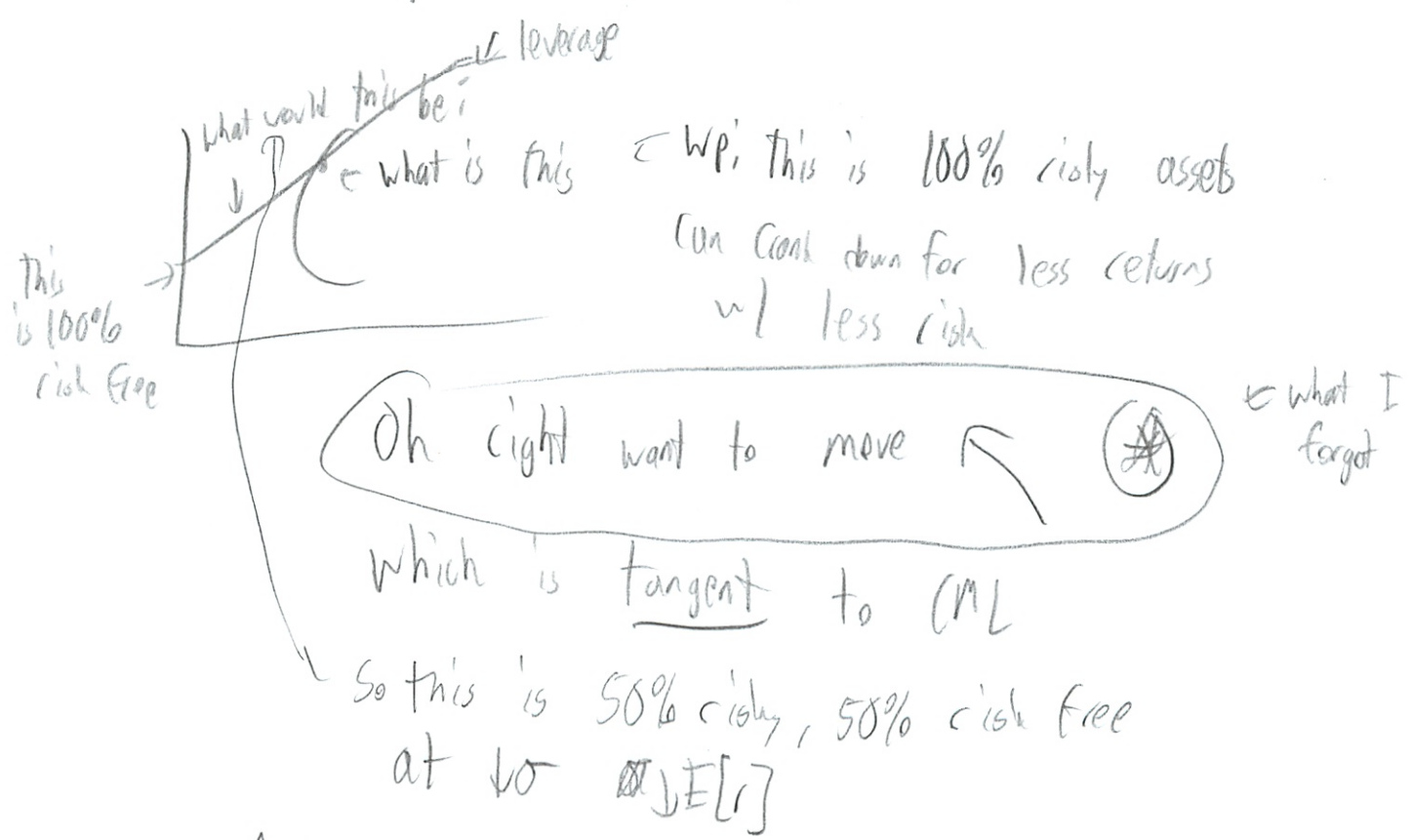
(Oh I get this now)

(17)

SML ≠ CML

r

c — [graph: upward sloping line, x-axis labeled β]

$E[r]$ — [graph: upward sloping line with "risk free" labeled, x-axis labeled σ]

β
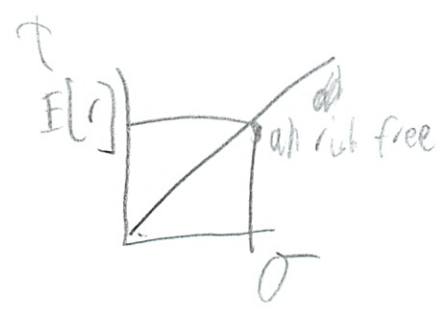
asset premium
vs Beta

How to find β again?

Remember exam likely focuses on calculating stuff

Forwards + Futures

Forward - custom
Future - standard

Forwards

Contract to buy/sell a given amt of commodity on given date at given price
called forward price

long person in possesion to buy commodity
Short   Sell

- custom, over the counter
- Counter party risk - no guarentee

Futures

- Standardized size, grade, quant, maturity
- clearing house
- mark to market - updated every day
- margin acct to cover losses

Are considered derivitive securities
Linear payoffs

No need to loan $1
No need to stor $F \approx H = S(1+r)^T$ +storage
                      ↑   ↑   ↑
But no arbitrage means it is Consistent   forward future spot price

Contango - futures prices ↑ w/ maturity
Backwardation - " " ↓ " "
  - if net convience yield > i
  $\hat{y} = y - c$, benefit from owning
  ↑ % holding cost
  Sometimes c)   ~annual rates!

$F \approx H = S[1+r-(y-c)]^T$
  $= S[1+r-\hat{y}]^T$ - can calc r - solve for

If financial, no costs, sometimes dividend
  $F = S(1+r-d)^T = H$
    ↑ annual dividend yield
  $d = [1+r - (F/S)^{12/3}]$
    ↑ 3 month i ↑ # of dividends

Futures price may react faster!
And easier to get in since little $ upfront

[15.401 cheat Sheet 2]

Interest Rate Futures Example

7% annual coupon, semiannual
At par
5% APR
6 month forward price
$F = S(1+r-y)^T = 100(1+.025-.035)$
  $= 99$

Hedging w/ Forwards   ↙ spot price   year 5

Long pos in act oil $S(5)$
Short pos oil forward $F - S(5)$
Net   posi oil forward F

Derivatives = Forwards, Futures, Options
  want to lock in a rate
  are 0 sum games - someone wins + loses
  basis risk - hedging w/ contract that does not match exposure

#Contracts = $\dfrac{amt}{amt\ per\ contract}$

Options opportunity to buy at set price
Call - right to buy asset
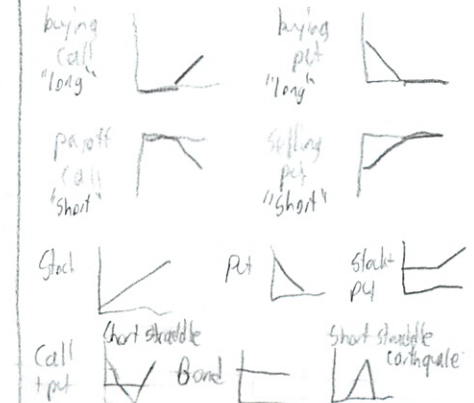put - right to sell asset
European - only on expiration date ←us
American - any date before ex date
  - basically series of Eu options

S = underlying asset price
k = exercise/strike price
T = expiration/maturity date

Payoff call option never negative
  $CF_T(call) = max[S_T - k, 0]$

Put-call parity ↗ strike ↙ put ↙ asset price
$C + \dfrac{k}{(1+r)^T} = P + S$

Option price ↑ as volatility ↑ (σ)

Binomial Option pricing   prob useless   5/13

$S_0 \begin{cases} S_{up} \\ S_{down} \end{cases}$ $i = 10\%$ either way $50 \begin{cases} 75 \\ 25 \end{cases}$

Find portfolo a shares of the stock
week t b dollars in riskless bond

$75a + 1.1b = 25$ ← option Call or stock value ⟶ Solve a, b   vs original
$25a + 1.1b = 0$ ← option payoff   or can from previous
  ↑ intrest
$a = .5$ ← hedge ratio/delta
buy .5 shares stock, sell 11.36 of bond $b = -11.36$
$P_U = (.5 \cdot 50) - 11.36 = 13.64$
Find PV of this replicating portfolio
$\Delta = \dfrac{Call\ up - Call\ down}{Stack\ up - Stock\ down}$ ← payoff - value from up vs down
  $= \dfrac{75-50}{75-25}$ $\dfrac{25-0}{75-25} = \frac{1}{2}$

$B = \dfrac{Stock\ up \times Call\ down - Stock\ down \times Call\ up}{(Stock\ up - stock\ down) \times (1+r)}$ bond price $\dfrac{75 \cdot 0 + 25 \cdot 25}{(75-25)(1.1)}$ 11.3

$P = \Delta \cdot Stock + B = 13.64$

Black Scholes - period smaller to ∞

$C(S/k, T) = SN(x) - kR^{-T}N(x - \sigma\sqrt{T})$
$x = \dfrac{ln\left(\dfrac{S}{kR^T}\right)}{\sigma\sqrt{T}} + \dfrac{1}{2}\sigma\sqrt{T}$ ← stock borrowed   I don't like this one

T = units in years
$R = 1+r$ the riskless rate of return
σ = Volatility of annual returns underlying asset
$N(\ )$ = CDF (from table) = options delta
C = option's price = call premia

but cost H; must subtract from payoff to get profit
warrent - call option on firm's stock
Convertable bond - bond + call on stock
  exercise price related to conversion ratio
callable bond - bond + call on bond

$C = SN(d_1) - ke^{-rT}N(d_2)$
$d_1 = \dfrac{ln(S/k) + (r + \frac{1}{2}\sigma^2)(T)}{\sigma\sqrt{T}}$

$d_2 = d_1 - \sigma\sqrt{T}$ $e = 2.7183$
  ↑ I like this version better
$P = C + \dfrac{k}{e^{rT}} - S$

At the money - strike price = market price
in the money - worth exercising   vs out of
  (call strike < market)   the money

Risk + Return

$D_1 = $ RV future dividend
$P_0 = $ current price $P_1 = $ RV future price

$$\tilde{r}_1 = \frac{D_1 + P_1 - P_0}{P_0} = \frac{D_1 + P_1}{P_0} - 1$$

$E[\tilde{r}_1] = $ expected return $\boxed{Var = \sigma^2}$

excess return $\tilde{r}_1 - r_F$

Risk Premium $E[\tilde{r}_1] - r_F = \pi$ } predict future

$\bar{r} = E[\tilde{r}]$

$\sigma^2 = E[(\tilde{r} - \bar{r})^2]$ $\sigma = \sqrt{\sigma^2}$

Estimators

$\hat{r} = \frac{1}{T}\sum_{t=1}^{T} r_t$ } measuring past

$\hat{\sigma}^2 = \frac{1}{T-1}\sum_{t=1}^{T}(r_t - \hat{r})^2$

$Cov(\tilde{r}_i, \tilde{r}_j) = \sigma_{iv} = E[(\tilde{r}_i - \bar{r}_i)(\tilde{r}_j - \bar{r}_j)]$
$= \rho_{ij}\sigma_i\sigma_j$

$Corr(\tilde{r}_i, \tilde{r}_j) = \rho_{ij} = \frac{E[(\tilde{r}_i - \bar{r}_i)(\tilde{r}_j - \bar{r}_j)]}{\sigma_i \sigma_j}$

$\beta_{ij} = \frac{\sigma_{ij}}{\sigma_j^2}$

$\rho = -1$ ✗ $\rho = 0$ $\rho = .5$ $\rho = 1$ ✗

return ↑ ↑ investors want | diff securities at diff risks require diff returns

risk

How is the dist of the RV,
Is it skewed towards +
Kurtosis - does it have fat tails?
what does the peak look like?

need to rebalance every year

Portfolio Theory - collection of assets

Value - just weighted avg of pieces
if $v = 0$ have arbitrage portfolio
know cov w/ each stock

	r_1	r_2	$\cdots$	r_n	
r_1	σ_1^2	σ_{12}	$\cdots$	σ_{1n}	cov
r_2	σ_{21}	σ_2^2	$\cdots$	σ_{2n}	$\sigma_{ij} = \sigma_{ji}$
$\vdots$	$\vdots$			$\vdots$	
r_n	σ_{n1}	σ_{n2}		σ_n^2	

↑ diagonal var

Var is not as clear as mean $\boxed{Var = n\sigma^2}$

$$\sigma_p^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2\sigma_{12}$$

$$= W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + W_3^2\sigma_3^2 + 2W_1W_2\sigma_{12} + 2W_1W_3\sigma_{13} + 2W_2W_3\sigma_{23}$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} W_i W_j \sigma_{ij}$$

Optimal balances (common $E(R_p) - .5A \cdot var(R_p)$
                                          ↑ risk aversion
desirable - indifferent / utility curve
Feasible - efficient frontier

Risk comes in 2 types
- unique → diversifiable
- systemic → non diversible

Investors hold frontier portfolios
- linear combo tangent portfolio + risk free
$$\bar{r}_p = (1-x)r_F + x\bar{r}_q$$
↑ % in risky assets
$$\sigma_p^2 = x^2\sigma_q^2$$

Sharpe ratio $\frac{\bar{r}_p - r_F}{\sigma_p}$ higher better | find best possible / all combos

corr = .5
corr = 0 -1 is the best

Find mean-variance frontier portfolio
Solve the optimization problem - don't need to

Simple Test for different weights

$E[]$ ⌐ ICM Extended if short sales allowed
        Risk
σ

Adding more stocks moves up + left
(So this ↓ diversible risk)
Stocks

Can also invest in risk free asset
- like portfolio of portfolios
Capital Market Line
T = best portfolio

$E(r_m)$ ⌐ Levered risky assets
        All market risky
        Some risk free
        all risk free
$\sigma_m - \beta = 1$
all points have same sharpe ratio

CAPM Capital Asset Pricing Model

Market capitalization $= \frac{Price}{Share} \cdot \#$ share

Risk-return ratio RRR $= \frac{\bar{r}_i - r_F}{?} = \frac{\bar{r}_m - r_F}{\sigma_m}$

$\beta_{im} = \frac{\sigma_{im}}{\sigma_m^2} = \frac{Cov(r_i, r_m)}{Var(market)} \cdot \frac{\sigma_m/\sigma_m}{?}$ measures systemic risk 1 = like market

---

Securities Market Line   assets premium vs beta

Given an asset's beta, can find $E[]$
Everything is on line or people buy/sell

$\hat{r}_i - r_F = \alpha + \beta_{im}(\tilde{r}_m - r_F) + \varepsilon_i$
          extra bonus   ↑ unique risk
        "chasing alpha"

Fama-French add in more factors

= Size
- Value Book to market

Arbitrage Pricing Theory APT
- add in more risk factors

$E(R_i) = R_f + \beta_{im}(E(R_m) - R_f)$

$\sigma_{im} = \beta_{im}\sigma_i\sigma_m$

To find β Market vs Company profits for the period

Capital Budgeting

find NPV of project
rate of return affected by risk
$r_{Project} = r_F + \beta(\bar{r}_m - r_F)$
CF = Project cash in - project cash out
= Op revenue - op ex (no dep) - capex - taxes
= $(1-J)$ Op profits - capex + (J) dep
          ↑ tax rate   ↑ tax credit

dep not in NPV, but tax break

Net Working Capital $= Inv + AR - AP$
- must set aside at start
- get it back at end

Include canibalization and new/marginal costs
Salvage value is taxable

Payback period - something some people look at
$IRR \quad 0 = \sum_{t=1}^{T}\frac{CF_t}{(1+IRR)^t}$ Pick projects with $IRR >$ cost of capital (if projects ind)

update w/ new info + sink costs
have choices (real options) what to do
- can calc value w/ Black scholes

Effecient Market Hypothesis Market price of
Securities include all available info
- no arbitrage
- active management adds nothing

Column 1

Value of asset = value of cash flows

OPP. cost capital = expected rate of return (time + risk) of other investment

$$PV = \frac{CF_1}{(1+r)^t}$$

0 = now
1 = in one year

$$PV_{annuity} = A \cdot \frac{1}{r}\left[1 - \frac{1}{(1+r)^t}\right]$$
amt each year

$$FV_{annuity} = (1+r)^T \cdot PV\ annuity$$
- starts in year one

$$PV_{annuity\ w/growth} = A\left[\frac{1}{1+r} + \frac{1+g}{(1+r)^2} + \cdots + \frac{(1+g)^{T-1}}{(1+r)^T}\right]$$

$$= \begin{cases} A \cdot \frac{1}{r-g}\left[1 - \left(\frac{1+g}{1+r}\right)^T\right] & r \neq g \\ A \cdot \frac{1}{1+r} & r = g \end{cases}$$

$$PV_{perpetuity} = \frac{A}{r}$$

$$PV_{perpetuity\ w/growth} = \frac{A}{r-g} \quad r > g$$

ARR = 5% compounded monthly
each month $\frac{5\%}{12} = .41\%$

at end of each year $100(1+.41)(1+.41)\cdots$

$$EAR = \left(1 + \frac{APR}{k}\right)^k - 1$$

$$(Real\ CF)_t = \frac{(Nominal\ CF)_t}{(1+\lambda)^t}$$

$$r_{real} = \frac{1+r_{nominal}}{1+\lambda} - 1$$

λ = inflation from CPI

r = inflation risk, default, risk premium

prob weighted avg   promised→expected   expected→default free

Column 2

Yield curve

Continuous compounding
$$\lim_{n\to\infty}\left(1 + \frac{APR}{N}\right)^N - 1 = e^{APR} - 1$$

Zero Coupons $B_t = \frac{1}{(1+f_t)^t}$
- good way to figure out spot rates

Coupons
$$B = \sum_{t=1}^T C_t \cdot B_T + P_t \cdot B_T$$
final coupon w/ principal
$$= \frac{C_1}{1+f_1} + \cdots + \frac{C_{T-1}}{(1+f_{T-1})^{T-1}} + \frac{(T+P)}{(1+f_T)^T}$$
↑ spot rates

YTM = effective interest
$$P = \frac{C}{1+f_T} + \frac{C}{(1+f_T)^2} + \cdots$$
P found normal way   spot rate to maturity

Forward Rates
$$(1+f_2)^2 = (1+f_1)\cdot(1+f_2)$$
$$f_t = \frac{B_{t-1}}{B_t} - 1 = \frac{(1+r_t)^t}{(1+r_{t-1})^{t-1}} - 1$$
bond price
$$_2f_3 = \frac{(1+f_3)^3}{(1+f_2)^2} - 1$$

To borrow $100 from year 3 to year 4
1. Buy 100 of 3 year bonds at $B_3 = P_3$
2. Sell 4 years of P_3

	0	3	4 Buy
Purchase	-80	100	
Sale	+80		-102
Bank makes	$\frac{102}{100}$	-1	

Longer bonds riskier - charge risk premium = liquidity pref.
$$f_t = E[r_1(t)]$$

Duration weighted avg term to maturity
$$D = \sum_{t=1}^T \frac{PV(CF_T)}{B}\cdot t = \frac{1}{B}\sum_{t=1}^T \frac{CF_t}{(1+y)^t}\cdot t$$
B = bond price
$$MD = -\frac{1}{B}\frac{\Delta B}{\Delta y} = \frac{D}{1+y} \quad y = r$$
-measure of volatility
$$\Delta P = -P \cdot D^* \cdot \Delta y$$

Column 3

Convexity (curvature of price as f(yield))
$$\left(X = \frac{1}{2}B\frac{\Delta^2 B}{\Delta V^2}\right)$$

debenture - not asset backed

immunization - price risk - prices fall as i↑
  - vs reinvestment rate risk - coupons will grow at fast rate as i↑

but 30 bonds now worth much less. obligations fall too

arbitrage sell/short the overpriced asset buy the underpriced asset, profit
underpriced: selling below PV
Use revenue from underpriced asset at each period to pay back bond purchase price - return (borrowed $ w/ a short) At each time period

When interest rates go ↑, bond prices ↓ ↓duration since ↓ yield cc

Duration price = Old price - (Change YTM)·Old duration

Bid - price to buy

Ask - price to sell

Stocks - Discount cash flows

Constant + Perpetual growth - Gordon model
$$P_0 = \frac{D_1}{r-g} = \frac{1+g}{r-g}D_0$$
r = cost of capital = discount rate = req return
$$= \frac{D_1}{P_0} + g = \frac{(1+g)D_0}{P_0} + g$$
g = growth of dividends
$$= \frac{r - D_0/P_0}{1 + D_0/P_0}$$

Earnings (E or EPS) = total profits - taxes - dep

Payout Ratio = dividends/earnings = p

Retained Earnings Δ = earnings - dividends

Plowback Ratio = $\frac{retained\ earnings\ \Delta}{earnings}$ = b = 1-p

Book Value = cumulative retained earnings

Return on book value of equity (ROE) = $\frac{earnings}{BV}$

$$D_1 = p \cdot EPS$$
$$g = b \cdot ROE$$
$$P_0 = \frac{EPS}{r} + PVGO$$
↑now   ↑expected future

If PVGO = 0

$$P/E = \frac{1}{r}$$

PVGO > 0

$$P/E = \frac{1}{r} + \frac{PVGO}{EPS} > \frac{1}{r}$$

residual claims

voting rights

limited liability

reinvest when ROE > Cost of Capital

Earnings yield $= \frac{E}{P} = \frac{EPS}{P_0}$ [Otherwise dividend all/liquidate]

P/E high — people looking at growth

$$V_0 = \sum_{t=1}^{T} \frac{FCF_t}{(1+r)^t} + \frac{TV_T}{(1+r)^T}$$

FCF = free cash flow

V_0 = value today

TV = terminal value = w/ Gordon model [found]

3 other ways to value a company

1. Book value of equity
   assets — liab [no formula!]

2. Relative valuation
   - w/ industry

3. Discounted Free cash Flows of Firm

inflation ↑, people want bigger returns ↓

risk ↓, required return ↓

$$V_0 = \sum_{t=1}^{T} \frac{FCF_t}{(1+r)^t} + \frac{TV_T}{(1+r)^T}$$

$$r = ROE \cdot \left(1 - \frac{dividend}{earnings}\right)$$

growth company; ROE > Cost capital [or PVGO > 0]
Otherwise R+D bad idea!

Hints

arbitrage opportunity bond
- calc P_v
- if price listed under, bond expensive
- buy a strip/short the bond

Dividend — don't count t=0 dividend
   Solve for g — growth rate

If ROE ~ cost of capital
   liquidate Co + pay out to shareholders

For stock holders — ETFs, Index Funds, Mutual Funds
Not For — Hedge Funds
Hated — Short Hedge,
Fast — HFT

---

discount bond — bought at price below
   face value/maturity payment
   often also means no coupons
   Value ↑ as time passes

Premium bonds — higher than par value
   if pay higher interest than market

duration = [weighted] avg maturity

price vs reinvestment risk

{ reinvest coupons ↑ ↑ when i↑
  sale price of bonds ↓ when i↑ }

if i↑, initially drop of sale price
leads to loss, until some point ①
where coupon reinvestment benefit
more than makes up for it — and starts
being profitable
this time: pt ① So, no risk
   ↑i? ↓i

Constant growth [✓earnings ✓reinvest rate] [5/15]
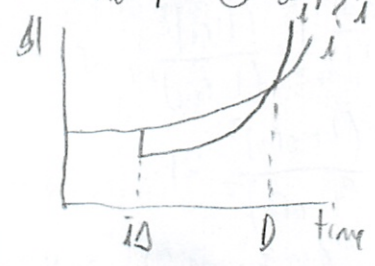
$$P_0 = \frac{Div_1}{r-g} = \frac{E_1 \cdot (1-b)}{r - b \cdot ROE}$$

[↑req rate of return] [↑return of investment project]

So Div = $E_1(1-b)$
   $g = b \cdot ROE$

$$FCF_6 = (1+g) FCF_5$$

$$P_0 = V_0 + PVGO = \frac{EPS}{r} + PVGO$$

$$r = \frac{D_1}{P_0} + g = \frac{D_0(1+g)}{P_0} + g$$

= dividend yield + dividend growth

FCF — after Cap ex

---

longer durations = more price sensitive

Holding Period Return — % Value of asset has grown

$$HPR_n = \frac{Income + (P_{n+1} - P_n) [capital gains]}{P_n}$$

Annualized

$$HPR = \left(\frac{D + (P_{n+1} - P_n)}{P_n} + 1\right)^{1/T} - 1$$

Last Name: _____ First Name: _____

M.I.T. ID# _____ Section: _____

(Verdelhan: 2:30-4 = A, 4-5:30 = B; Manso: 2:30-4 = C; Milbradt: 8:30-10:00 = D, 10:00-11:30 = E)

5/15

15.401 Final Exam

Spring 2010

Please make sure that your copy of the exam contains 30 pages (including this one).

- Please write your name and MIT ID number on every page.

- Check your section number.

- The exam lasts 180 minutes. It consists of ten questions. Please answer all of them.

- Credit for each question is equal to the amount of time you should spend on it (1 point = 1 minute). Therefore, do not agonize over a 10-point question without having tackled a 20-point question.

- You are allowed two $8\frac{1}{2}$" ×11" sheets of formulas and one calculator.

- Answer these questions without consulting anyone.

- Use the space provided. If more space is needed, use the two extra pages provided at the end of the exam.

- Be neat and show your work. Answers without work receive no credit. Wrong answers with partially correct work may receive partial credit.

Good luck!

Name: _____ MIT ID#: _____

15.401 Final Examination 2010 Grade Sheet

1. _____ / 30

2. _____ / 15

3. _____ / 20

4. _____ / 15

5. _____ / 20

6. _____ / 20

7. _____ / 10

8. _____ / 20

9. _____ / 15

10. _____ / 15

Total _____ / 180

1. (30 points) True, false or "it depends"? Give a brief explanation for each answer.

   (a) (5 points) "If the corporate tax rate decreases from 30% to 25%, future after-tax cash-flows from a project always increase."

   True
   —depreciation benefit is reduced

   Wait - if project losing $ + depreciation is credit not deduction

   False ↗   *They had as a credit*

   (b) (5 points) "A firm has invested $10M in a project. If the present value of future cash flows from the project is less than $10M, then the manager of the company should abandon the project."

   False   10M sunk
        If < 0 - cancel   ✓

(c) (5 points) "According to the CAPM, if a security's beta is negative, then its expected return must be negative."

No   B- means neg correlated   w/ market ✓ true

E[] is lower than risk free rate

✓

E(market) can never be negative
Portfolio
r ↑ since its a portfolio ya pick↑

(d) (5 points) "According to the Expectations Hypothesis of the term structure, the 1-year rate today equals the expected one-year rate next year."

No, says nothing about it

✓

Use cheat sheet

(e) (5 points) "An airline company knows it will need 1000 gallons of fuel for next year. Because the company has no cash available today, it cannot use a forward contract to hedge the price risk of fuel."

Forward — no cash need

False ✓

(f) (5 points) "The value of an at-the-money option is zero."

? what is at-the-money
add to cheat sheet

Guessing False ~~off~~ — ~~can know~~

True if currently expiring

otherwise could move in the $ ← what I thought

Name: _____ MIT ID#: _____

2. (15 points)   Bring calculator

Jack and Jill just had a baby (today) and have decided to start saving money for the baby's college education. Every year on the baby's birthday, Jack and Jill will make a deposit to a savings account that yields 4% annually. The first deposit will be next year (when the baby turns 1) and the last deposit will be when the baby turns 17. College will require four annual tuition payments with the first one coming on the day the baby turns 18. The annual cost of college tuition was $40,000 in the year that just finished, and experts expect this cost to rise at 5% annually for the foreseeable future. How much money do Jack and Jill need to save each year in order for their savings to completely pay for their child's education?

Wish had VA

FV annuity for closed form

First calc college

$\underline{40,000}$ FV

$$40,000(1+.05)^{18} + 40,000(1.05)^{19} \cdots \quad 20 \quad 21$$

$$96264 \qquad + \ 101\,078 \ + \ 106131 \ + \ 111438$$

(ending

$$= 414911 \quad \text{year } 18$$

,would be 40000

414911   in each year dollars

                              No in PV $?

$$414911 = A \frac{1}{.04}\left[1 - \frac{1}{(1.04)^{17}}\right]$$

$$= A \cdot 12.166^{6}$$

$$A = 34105 \quad \text{seems too much?}$$

Yeah way wrong

Answer

$$\frac{C_{savings}}{r}\left(1 - \frac{1}{(1+r)^{17}}\right) = \frac{C_{tuition}(1+g)^{18}}{r-g}\left(1 - \left(\frac{1+g}{(1-r)}\right)^4\right)\frac{1}{(1+r)^{17}}$$

C had
PV ~~of~~ ~~stream~~ of savings

So what did they do here?
The PV of annuity w/ growth

I should have PVed both <del>~~~~</del>
Conciously set at to compare apples to apples

Money still grows in college! ← think 2 errors
↑ in bank

So costs grow slower

Can't calc costs in each year and PV down
  ~ PV is in > 40,000 too

$C_{savings}$ = 15,851
Total cost of school = 192,835 at PV

Name: _____ MIT ID#: _____

3. (20 points)

Cald have wrote More on

Pearson Mobile is expected to earn $2.5 per share next year. In the past, the company invested 10% of its earnings into R&D projects that typically generated ROE = 10%, and paid out the rest as dividends. Pearson's stock is currently trading at $22.50.

(a) What is the company's cost of capital? *? how find*

— cost of borrowing

10%?

$$P = \frac{E_1 \cdot (1-b)}{r - b \cdot ROE}$$ *←what model did they use?*

$$22.5 = \frac{2.5 \cdot .9}{r - .1 \cdot 10\%}$$

$$= 11\%$$

Where is that on cheat sheet?
— oh previous unit
(looked up before)

— = required return

$$E_s = R_f + \beta (R_m - R_f)$$
↑ expected return

$$k = \frac{Dividend}{Price} + Growth$$

"Constant Growth Model"

$$P_o = \frac{Div_1}{r-g} = \frac{\$}{}$$

Oh so thats how find dividend
Put in (1-b)·E for dividend
Solve backwards

b·ROE = growth rate

(b) Pearson Mobile has recently hired a new management team who has a track-record of focusing on technological development. Your broker advises you to buy more Pearson stocks, as Pearson will soon get a competitive edge against its industry peers by investing more heavily into R&D. How should you react to his recommendation?

Well if ya think the tech will pay off and other people have not realized it yet than go buy some

← would have taken too

I gnore since ROE = 10%

C = 11%

so R+D is bad idea

↑ pay attention

$\frac{C}{3}$ (should have written explitly) this was highly unlikely

(c) Calculate Pearson's PVGO. Is Pearson a growth firm? Justify your answer.

~~No~~ ~~State~~ ~~Negitive~~   No since $r > ROE$

$$P_0 = V_0 + PVGO$$

$$r$$

$$\sum FCF \text{ over time period}$$

$$(2.5 \cdot .90)$$ $\leftarrow$ they used $\dfrac{E_1}{r}$

$\wr$ perpituity

$$\dfrac{2.5 \cdot .9}{.11}$$ why all — not just FCF ~~doh~~ oh FCT

~~G?can you use this?~~   ~~must be before~~

No after

CAPX

$$22.56 = 20.45 + PVGO$$

$$PVGO = 2.05$$

$$\neg .227$$

I think I am right

(d) Toady in a press conference, Person's top executives announce that the management team chooses an optimal plowback ratio. How much can you expect the stock price to jump immediately after the announcement?

What is optimal

- Based on # in quiz — optimal is 0 ✓

Stock price ↑ as managers not wasing it on
R + D — more $ for investors to vaccure ret

✓ no Playpack

$$P_{max} = \frac{E_1}{r} = \frac{2.5}{11} = 22.727$$

$$\Delta p = P_{max} - P_0 = 22.77 - 22.5 = .0227$$

Oh I could have caculated
I assumed they did not want it here

4. (15 points) **forwards + futures**

The current price for silver is \$20/ounce. Suppose that there is no storage cost or convenience yield. The 6-month forward price is \$20.5/ounce.

↙ useless info

(a) Suppose that the 6-month interest rate is expected to go down by 1% (EAR) in 3 months. What is the 6-month spot interest rate (EAR) now?

"based off above data"

does not affect forward rate

$$F = S(1+r)^{.5}$$

$$r = \left(\frac{F}{S}\right)^2 - 1 = 5.0625\%$$

Oh since no forac conviencie yield its prty on interest rate ploy

The red hering threw me off

$$F^2 = S^2(1+r)$$
$$F^2 = S^2 + S^2 r$$
$$S^2 r = F^2 - S^2$$
$$r = \frac{F^2}{S^2} - \frac{S^2}{S^2} \quad \frac{}{11}$$
$$= \left(\frac{F}{S}\right)^2 - 1$$

(b) At time 0, Jim enters into a forward contract to sell 1000 ounces of silver in 9 months. After 3 months have passed, silver turns out to be in high demand and the spot price goes up to $30/ounce. The 6-month interest rate is now 4% (effective annual rate). Jim is concerned about the price of silver going up further and he decides to exit the contract that he entered into 3 months ago. After closing out his position, Jim sees that he has made a loss of $9,657.44. What was the forward price of silver in the 9-month contract that Jim originally entered into?

[handwritten annotations:] does not matter? · reduce to APR? · Or compare apples to apples

$.5 \leftarrow$ 6 months left to go!

New $\quad F = S(1+r)^{.25}$ year

$F = 30(1+.04)^{.25}$

$= 30.59 \cdot 1000 \text{ oz} = 30895$

Price he had $\cancel{\text{from}} \ 30295 - 9675.44 \sim$

$= 20638 = F$

$20638 = S(1+.04)^{.75} \leftarrow$ time period ↓

(when is "now") ✓

$S = 20038 \ /1000 = 20.04$

Close! Twould my method have worked

✗ divide loss by 6 months

$\hat{F} - F$

$\dfrac{}{(1+.04)^{.5}} \cdot 1000^{12} = -9657.44 \leftarrow$ why do this?

Convert to future value — but why? · *"mark to market"*

(c) Back at time 0 (when Jim originally entered into his 9-month contract), the spot price for soy beans was $1/bushel and the 9-month storage cost was 2% (not annualized). Soy beans have no convenience yield. Making use of the information in part (b), calculate the 9-month forward price for soy beans (from the perspective of time 0).

do we care?

$$F = S(1 + r - (y - c))^T$$

$$= 1(1 + .04 - (0 - .02))^{.75}$$

$$= 1.04$$

Need to back out 9 month rate from 9 month.
· forward price of silver

↑I thought they were not using

from part b

$$\hat{F} = S(1 + r_{9m})$$

$$r_{9m} = \frac{\hat{F}}{S} - 1 \quad \leftarrow \text{no need to adjust}$$
— already right period

$$= \frac{20.7454}{20} - 1 \quad \text{(need to get from prev qu)}$$

$$= 3.727\%$$

I think I am not comfortable w/ getting rate from future price —did we ever do?

✓ Since already 9 months don't multiply by anything!

$$1(1 + r_{9m} + c) = 1 + 3.727 + 2$$

13

5. (20 points) *new question!* ?

The yield curve is flat at 5% (EAR) per year and only shifts up and down over time. All bonds have face value of $100 and are fairly priced.

(a) What is the price and duration of a three-year bond with annual coupons of 6%?

$$PV = \frac{6}{(1.05)} + \frac{6}{(1.05)^2} + \frac{106}{(1.05)^3}$$

$$= 102.72 \checkmark$$

$$Duration = \frac{1}{102.72}\left(\frac{6}{1.05} \cdot 1 + \frac{6}{(1.05)^2} \cdot 2 + \frac{106}{(1.05)^3} \cdot 3\right)$$

$$= 2.835 \checkmark$$

(b) A three year bond with monthly coupons trades in the market for the same price as the bond in (a). Explain qualitatively how its duration compare to the bond in (a) and why. (You are not required to calculate its price or duration).

Duration shorter since more $ up front

(c) A ten-year bond with annual coupons of 4% last year becomes a nine-year bond today. Suppose that the price sensitivity of the latter to interest rate risk is actually higher than the former. What can you say (qualitatively) about the change in the yield curve over the past year? Explain your answer.

Is the sensilivty different — or the i'?

— yeah — shorter more volatile

no slope — _flat_

i_9 is higher now than it used to be?

Should ↓ duration

But price sensitivity going ↑ suggests duration ↑

(longer more price sensivity)

but yield curve is flat

did not really notice yield curve is flat

$o ↓ duration only be because ↓ Yield

(↓ value coupon payments → ↓ duration)

but that is i↑

Online I see duration varies inversly w/ yield rate
emailed in

(d) Two years ago, the yield curve was ⓕlat at 7%. At that time, you purchased a 30-year bond with 7% annual coupons at par. The yield curve has since declined. One year ago it was at 6% and it is now at 5%. If you have re-invested your coupons at the available spot rates, what is your annualized holding period return for the past two years?

i enagh # to do

So say 100 each year coupon

$$\frac{100}{(1+.06)} + \frac{200}{(1.05)}$$

Rat spot rate

← kinda made up on the spot

$= 284.85$

$$284 = \frac{300}{(1+x)^2}$$ No

$284(1+x)^2 = 300$

$(1+x)^2 = 1.05$

2.63%

Price today of original bond

$$P = \frac{7}{.05}(1-1.05^{-28}) + \frac{100}{1.05^{-28}} = 129.796$$

lst year coupon $7 \cdot 1.06 = 7.42$

Jut get $7

$$HPR = \left(\frac{7 + 7.42 + 129.796}{100}\right)^{.5} - 1 = 20.09$$

$\sqrt{}$ or $\frac{1}{t}$ for annuallizel

2 years

Totally diff than what I had

T holding period return ← never learned

6. (20 points)

There are two risky assets A and B in the market with the following risk and return characteristics summarized in the table below. The risk-free rate is $r_f = 2\%$.

Asset	$\mathbb{E}(r)$	Cov.	A	B
A	6%	A	0.010	0.006
B	14%	B		0.040

(a) What is the expected return and the standard deviation of returns of a portfolio with 60% in A, 15% in B and 25% in the risk free asset?

$$E[r] = .6 \cdot .06 + .15 \cdot .14 + .25 \cdot .02$$

$$= .062$$

$$6.2\% \checkmark$$

↓ rf does not count

$$\sigma^2 = (.6)^2 (.01) \times \text{already var} + (.15)^2 (.04) \times + 2(.6)(.15).006$$

$$\sigma = .0019 \quad 7.5$$

(b) Is the portfolio described in part (a) on the Capital Market Line (CML)? (Hint: Think of Sharpe ratios.)

Would not have thought of

$$\frac{r_p - r_f}{\sigma_p} \qquad \frac{6.2 - 2}{7.5} = .56$$

↑ what does this tell you

higher is better

what is CML?

Can calc sharp ratio for every asset possible combo

better

Can see that B

$$\frac{14 - 2}{\sqrt{(.04)}} = .06$$

✓ Optimal

(c) Suppose now a market portfolio exists (ignore assets A & B). Consider a mean-variance efficient portfolio that has weight 50% on the market portfolio. What is the correlation between the returns of this portfolio and the market returns?

as opposed to cf

??

Mean-var portfolio should be on CML
— weighted combo market portfolio + risk free bond

So All var from market portfolio position

So 1 correlation always

tricky

Name: _____ MIT ID#: _____

8. (20 points)

The annual interest rate is 10%. The stock price of Midas Internet Technology (MIT) is $50. The share price is to evolve according to the binomial tree shown below over the next two years.

```
                                              → $70
                            $60
                           ╱     ╲
                          ╱       → $50      Option makes
              $50                             it when price
                          ╲       → $50             ↓
                            $40
                           ╲     ╲
                                  → $30
```
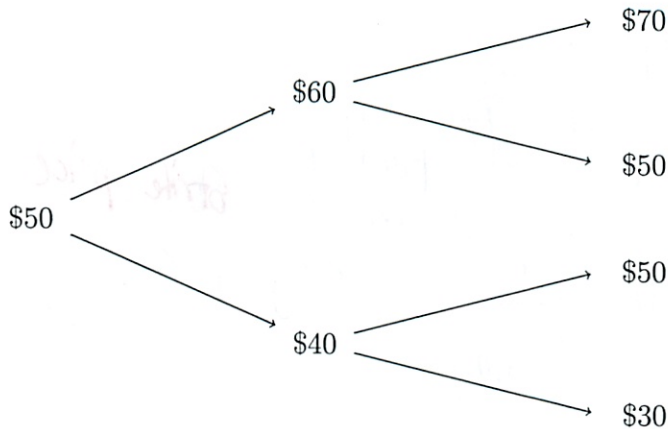
(a) What is the price of an European put option on MIT with a strike price of $60 and a time-to-maturity of two years?

Not practiced at this

Assume 60 in year 1

A = shares MIT
B = $ in risk free

$$70 A + 1.1 B = 0$$ ← they did it this way
$$50 A + 1.1 B = 10$$

Solve, $A = -.5$ $B = 31.81$

$$P_U = (-.5 \cdot 60) + 31.81 = 1.81$$

P_{Down}

$$50 A + 1.1 B = 10$$) why 10 and 30
$$30 A + 1.1 B = 30$$ not 0 and 10 — its vs 40

$$A = -1 \quad B \overset{22}{=} 54.54$$

$$P_d = (-1 \cdot 40) + 54.54 = 14.54$$

vs original

7. (10 points)

   Suppose the term structure of interest rates is flat at 5%. Two-year European call option
   and put option on Giggle Inc. with a strike price of $100 trade for $40 and $30, respectively.
   Determine the share price of Giggle Inc.

Binomial?

No ~~eu~~ put-call parity: strike price

$$40 + \frac{k \quad \swarrow \text{what is } k}{(1+.05)^2} = 30 + S$$

$$40 + \frac{100}{(1+.05)^2} = 30 + S$$

$$S = 100.70 \quad \checkmark$$

No year 0

$$60A + 1.1B = 1.81$$
$$40A + 1.1B = 14.54$$

$$A = -.63 \quad B = 36.36$$

$$P = (-.6364 \cdot 50) + 36.36 = \boxed{4.54}$$

Try solving by hand

$$70A + 1.1B = 0$$
$$50A + 1.1B = 10$$

$$1.1B = 10 - 50A$$

$$B = \frac{10}{1.1} - \frac{50}{1.1}A$$

$$70A + 1.1\left(\frac{10}{1.1} - \frac{50A}{1.1}\right) = 0$$

$$70A + 10 - 50A = 0$$

$$20A = -10$$

$$A = -.5$$

$$B = \frac{10}{1.1} - \frac{50}{1.1}\left(-\frac{1}{2}\right)$$

$$= 31.8181 \qquad \checkmark \quad \text{did it}$$

Name: _____ MIT ID#: _____

(b) What is the price of an American put option on MIT with a strike price of $60 and a time-to-maturity of two years?

Did not do American pricing

23

9. (15 points)

Go Sushi is a new chain of fast food restaurants. The company franchises its stores. Today each franchise has a market value of $400,000. This is the price an owner of a franchise can obtain if sells his franchise to another person. One year from now each of the company's franchises will have a market value of either $720,000 or $240,000. The probability that the market value of each franchise in a year will be $720,000 is 75%, and the probability that the market value of each franchise will be $240,000 is 25%. This is shown below:

$$75\% \longrightarrow \$720,000$$

Now

$$\$400,000$$

$$25\% \longrightarrow \$240,000$$

Go Sushi offers two plans for buying franchises. In Plan A, the company simply sells a franchise today for $400,000 and the buyer takes the risk that the franchise will only be worth $240,000 in one year. In Plan B, the company sells the franchise today at a higher price than $400,000 but guarantees that the company will be prepared in one year to buy back the franchise at a price of $400,000. Under Plan B the franchise owner is not obligated to sell the franchise back to the company in one year at $400,000, but he can choose to do so if he wants. Assume that the guarantee is transferable when a franchise is sold. This means that if someone sells a franchise purchased under plan B, the guarantee is transferred to the new owner. Assume one can borrow or lend $100 today for a riskless amount of $108 delivered in one year (i.e., that the riskless interest rate is 8%). Given the information provided, what is the market value (today) of a franchise purchased under plan B (which includes the guarantee)?

Example

$$\frac{75-0}{75-25} = \frac{75}{50} = \frac{3}{2}$$

should be $\frac{1}{2}$ $\frac{25-0}{50}$

$$\frac{25 \cdot 0 + 25 \cdot 25}{(75-25)(1+.08)} = 11.0736$$

Try again

$C_{up} = 0$ ← 320,000 $S_{up} = 720,000$

$C_{down} = 160,000$ S_{down} 240,000

$$\Delta = \frac{0 - 160,000}{(720,000 - 240,000)} = -\frac{1}{3}$$ ← $\frac{2}{3}$

$$B = \frac{720 \cdot 160 - 240 \cdot 0}{(720-240)(1.08)} = 222,222$$

$$P = -\frac{1}{3}(400,000) + 222,222$$

$$= 88,888$$ ← 488,888
must have done something silly

Oh good state
is +320,000
bad is 0
stock price 400,000

Now I see their
original method
better

Option

— but how w/ uneven payout ? ↙ even if not exercised!

$$a = \frac{CF_u - CF_d}{S_u - S_d} = \frac{720,000 - 400,000}{720,000 - 240,000} = \frac{2}{3} \quad \text{why this here?}$$

$$b = \frac{1}{1+r}\left(\frac{CF_d S_u - CF_u S_d}{S_u - S_d}\right)$$

$$= \frac{1}{1.08}\left(\frac{400,000 \times 720,000 - 720,000 \times 240,000}{720,000 - 240,000}\right)$$

$$= 222,000$$

$$MV = aS + b = \frac{2}{3}\,400,000 + 222,222 = 488,889$$

* probabilities unimportant *

10. (15 points)

Suppose you are considering pricing two assets, A and B. The table below summarizes the expected return of the market and the covariance of future returns of A, B and the market M. The risk-free rate is $r_f = 2\%$

$$\beta_A = \frac{Cov(r_a, r_m)}{\sigma_m^2} = \frac{.04}{.04} =$$

Asset	$\mathbb{E}(r)$	Cov.	M	A	B
M	6%	M	0.040	0.040	0.036
A	?	A		0.090	0.000
B	?	B			0.160

(red annotation: Oh b is not β how stupid can you be —)

$$\beta_B = \frac{.036}{.04} = .9$$

(a) What are the expected rates of return of asset A and B if the CAPM holds?

(red: have same RRR? No)

$$E(R_A) = R_f + \beta_{AM}(E[R_M] - R_f)$$

$$= 2\% + .04(6\% - 2\%)$$

$$= 2\% + .0016 = 2.16\% \quad 6\%$$

$$E(R_B) = 2\% + .96(6\% - 2\%)$$

$$= 2.144\% \quad 5.6\%$$

(red: Seems low)

(c) Suppose the covariance between the returns of asset A and B changes from 0.000 to 0.030 (but everything else remains the same). What is the correlation between the returns of A and B now? Will this change any of the expected return relationships derived in (a)?

What do on table now

$$Corr = \rho$$

$$\sigma_{ij} = \rho_{ij} \; \sigma_i \; \sigma_j$$

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} = \frac{.030}{\sqrt{.09} \sqrt{.16}} = .25 \checkmark$$

$E[\;]$ does not depend on this $\checkmark$

Systemic risks not change

(b) What is the riskier asset in an expected return sense, A or B? Does this square with the assets' standard deviations? Explain.

B has lower return, lower than market

Look at A,A on table

Yes B is riskier

A is riskier - higher β so higher $E(r)$

Contrary to σ, as B higher σ

More systemic risk in A than B

- Diversification makes systemic risk shine through

Last Name: _____ First Name: _____

M.I.T. ID#: _____ Section: _____

(Verdelhan: 2:30-4 = A, 4-5:30 = B; Manso: 2:30-4 = C; Milbradt: 8:30-10:00 = D, 10:00-11:30 = E)

15.401 Final Exam

Spring 2010

Please make sure that your copy of the exam contains 30 pages (including this one).

- Please write your name and MIT ID number on every page.

- Check your section number.

- The exam lasts 180 minutes. It consists of ten questions. Please answer all of them.

- Credit for each question is equal to the amount of time you should spend on it (1 point = 1 minute). Therefore, do not agonize over a 10-point question without having tackled a 20-point question.

- You are allowed two $8\frac{1}{2}" \times 11"$ sheets of formulas and one calculator.

- Answer these questions without consulting anyone.

- Use the space provided. If more space is needed, use the two extra pages provided at the end of the exam.

- Be neat and show your work. Answers without work receive no credit. Wrong answers with partially correct work may receive partial credit.

Good luck!

1

Name: _____ MIT ID#: _____

15.401 Final Examination 2010 Grade Sheet

1.	_____ / 30
2.	_____ / 15
3.	_____ / 20
4.	_____ / 15
5.	_____ / 20
6.	_____ / 20
7.	_____ / 10
8.	_____ / 20
9.	_____ / 15
10.	_____ / 15
Total	_____ / 180

2

1. (30 points)

(a) (5 points) "If the corporate tax rate decreases from 30% to 25%, future after-tax cash-flows from a project always increase."

Answer:

False. After tax cash flows are given by

$$\text{Op Income} * (1 - \tau) + \text{Depr} * \tau - \text{Capex}$$

A lower tax rate will increase the after-tax operating income, but will also decrease the effects of the depreciation tax shield. Depending on the relative magnitude of the operating income and depreciation, the after tax cash flows may not increase. The following alternative answer is also correct. If the operating income from the project is negative (a loss), then decreasing the tax rate will result in less of a tax deduction and hence more negative after tax cash flows.

Common Mistakes: The most common error we saw here was people who simply said that after-tax cash flows might go up or down because a multitude of variables other than the tax rate might also be changing at the same time. This is does not constitute an explanation of what is wrong with the question sentence, unfortunately.

(b) (5 points) "A firm has invested $10M in a project. If the present value of future cash flows from the project is less than $10M, then the manager of the company should abandon the project."

Answer:

False. The $10M already invested in the project is a sunk cost and should not be considered when deciding to continue or abandon the project. The manager should abandon the project if the present value of future cash flows is less than 0. Also correct, rarely seen addition: If some portion of the original $10M investment can be salvaged (sold off), then the manager should abandon the project if the present value of future cash flows is less than the salvage value of the investment.

Common Mistakes: A large proportion of the class got this wrong because they responded as if the $10M investment was still something that the firm was considering undertaking, not an investment that had already been made in the past.

Title or question at top (rotated): 1. (30 points) True, false or "it depends"? Give a brief explanation for each answer.

(c) (5 points) "According to the CAPM, if a security's beta is negative, then its expected return must be negative."

Answer:

False. According to CAPM, if a securities beta is negative, then its expected return must be lower than the risk free rate.

Common Mistakes:

Example: "Under the CAPM, beta measures the covariance of a security with the overall market. So if the market were to go down, this security would go up, leading to a potentially positive return."

Though this answer defines beta correctly (for the most part), it is not a valid answer to this question. The question is asking about expected return, not a return on one particular observation. The expected return on the market portfolio can never be negative (or even below the risk free rate).

(d) (5 points) "According to the Expectations Hypothesis of the term structure, the 1-year rate today equals the expected one-year rate next year."

Answer:

False. According to the Expectations Hypothesis of the term structure, the 1 to 2 year forward rate today equals the expected one-year rate next year.

(e) (5 points) "An airline company knows it will need 1000 gallons of fuel for next year. Because the company has no cash available today, it cannot use a forward contract to hedge the price risk of fuel."

Answer:

False. A forward contract is an agreement to purchase or sell something (in this case fuel) at some point in the future at a price agreed upon today. The airline company would not require cash today, they would require it in one year when it comes time to actually purchase the fuel. (This assumes that the counterparty of the airline allows this forward contract to proceed without margin, which is the convention we have used for forwards in this class.)

Common Mistakes:

Example: "False. The airline could obtain cash by borrowing money from a bank, shorting a bond, etc. in order to purchase the forward."

While it is of course true that companies have many ways of obtaining cash, the point of the question was to recognize that no cash is needed to enter into a forward contract.

(f) (5 points) "The value of an at-the-money option is zero."

Answer:

While the value of an at-the-money option would be zero if it is currently expiring, if there is time left to expiration then the option will have a positive value as there is a possibility that the underlying security will move before expiry and the option will finish "in the money."

Common Mistakes:

Many people assumed that the option in question was at expiration. Unfortunately, the question does not say the option is specifically at expiration (so you should consider all possible scenarios, as the example answer does) and more importantly, we are generally interested in the value of options before expiry. At expiration, the value of the options is fairly obvious.

2. (15 points)

Jack and Jill just had a baby (today) and have decided to start saving money for the baby's college education. Every year on the baby's birthday, Jack and Jill will make a deposit to a savings account that yields 4% annually. The first deposit will be next year (when the baby turns 1) and the last deposit will be when the baby turns 17. College will require four annual tuition payments with the first one coming on the day the baby turns 18. The annual cost of college tuition was $40,000 in the year that just finished, and experts expect this cost to rise at 5% annually for the foreseeable future. How much money do Jack and Jill need to save each year in order for their savings to completely pay for their child's education?

Answer:

$$\frac{C_{savings}}{r}\left(1 - \frac{1}{(1+r)^{17}}\right) = \frac{C_{tuition} \times (1+g)^{18}}{r-g}\left(1 - \left(\frac{1+g}{1+r}\right)^4\right)\frac{1}{(1+r)^{17}}$$

$$\frac{C_{savings}}{0.04}\left(1 - \frac{1}{(1+0.04)^{17}}\right) = \frac{\$40K \times (1+0.05)^{18}}{0.04 - 0.05}\left(1 - \left(\frac{1+0.05}{1+0.04}\right)^4\right)\frac{1}{(1+0.04)^{17}}$$

$$\frac{C_{savings}}{0.04}\left(1 - \frac{1}{(1+0.04)^{17}}\right) = \$192,835$$

$$C_{savings} = \frac{\$192,835 \times 0.04}{\left(1 - \frac{1}{(1+r)^{17}}\right)} = \$15,851$$

Common Mistakes:

• Many people neglected to multiply the $40K tuition figure by 1.05^{18} on the right side of the above equation. This would calculate the cost as if tuition starts at $40K in year 18 and then only starts growing during college.

• Many people calculated the future value of tuition in each of the four years, summed them together and then discounted that value by 18 years. This gives you an answer that is pretty close, but it is a significant mistake. You can never take the sum of cash flows that occur at different times.

• Some people wrote $r = g$ and then used the formula $A/(1+r) * T$. You need to be careful to understand the meaning of the different rates in the question. The four percent savings account gives you r, the cost of capital. The g in the growing annuity formula is the rate at which the college tuition is growing. They are not the same.

3. (20 points)

Pearson Mobile is expected to earn $2.5 per share next year. In the past, the company invested 10% of its earnings into R&D projects that typically generated ROE = 10%, and paid out the rest as dividends. Pearson's stock is currently trading at $22.50.

(a) What is the company's cost of capital?

Answer:

Set up the Constant Growth Model, and solve for cost of capital:

$$P = \frac{E_1 \times (1-b)}{r - b \times ROE}$$

$$22.5 = \frac{2.5 \times 0.9}{r - 0.1 \times 10\%}$$

$$\boxed{r = 11\%}$$

Common Mistakes:

- The question clearly states "Pearson Mobile is expected to earn $2.5 per share **next year**". Some people misread the timing and calculated $E_1 = 2.5 \times (1 + g)$.

(b) Pearson Mobile has recently hired a new management team who has a track-record of focusing on technological development. Your broker advises you to buy more Pearson stocks, as Pearson will soon get a competitive edge against its industry peers by investing more heavily into R&D. How should you react to his recommendation?

Answer:

You should ignore the broker's recommendation, because $ROE = 10\%$ is lower than $r = 11\%$, the stock price can only collapse further if the management indeed chooses to invest more intensively into R&D.

We also accept answers that claim the broker cannot have any inside information because EMH implies all available information has been priced in.

(c) Calculate Pearson's PVGO. Is Pearson a growth firm? Justify your answer.

Answer:

$PVGO = P_0 - \frac{E_1}{r} = 22.5 - 22.727 = \boxed{-0.227}.$

Pearson is not a growth company since $ROE < r$, $PVGO < 0$.

(d) Today in a press conference, Pearson's top executives announce that the management team chooses an optimal plowback ratio. How much can you expect the stock price to jump immediately after the announcement?

Answer:

The optimal investment policy is to set $b = 0$ and payout all the dividends.

$$P_{max} = \frac{E_1}{r} = \frac{2.5}{11\%} = 22.727$$

$$\Delta P = P_{max} - P_0 = 22.72 - 22.5 = \boxed{0.227}$$

We also accepted answers that claim the price will not move because EMH implies available information has been priced in, the market has anticipated the policy change.

4. (15 points)

The current price for silver is \$20/ounce. Suppose that there is no storage cost or convenience yield. The 6-month forward price is \$20.5/ounce.

(a) Suppose that the 6-month interest rate is expected to go down by 1% (EAR) in 3 months. What is the 6-month spot interest rate (EAR) now?

Answer:

Interest rate movement does not affect current forward rate. Let 6-month EAR be r. Then

$$F = S(1+r)^{0.5}$$
$$r = \left(\frac{F}{S}\right)^2 - 1 = 5.0625\%$$

Common Mistakes:

- Some students are misled by the useless information "...expected to go down by 1%", which is meant to test whether they understand that the pricing of a forward is through no arbitrage rather than expectation of future prices.
- Some students are still not clear what is meant by EAR and confuse it with APR.

11

(b) At time 0, Jim enters into a forward contract to sell 1000 ounces of silver in 9 months. After 3 months have passed, silver turns out to be in high demand and the spot price goes up to \$30/ounce. The 6-month interest rate is now 4% (effective annual rate). Jim is concerned about the price of silver going up further and he decides to exit the contract that he entered into 3 months ago. After closing out his position, Jim sees that he has made a loss of \$9,657.44. What was the forward price of silver in the 9-month contract that Jim originally entered into?

Answer:

The 6-month forward price 3 months later is

$$F = S(1+r)^{0.5}$$
$$= 30 \times (1+4\%)^{0.5}$$
$$= 30.5941$$

where S, F and r denotes the spot price, forward price and spot interest rate 3 months later.

Jim's forward contract, which was entered at time 0 to sell at forward price $\hat{F}$, incurs a loss. And if we mark to market, the loss is

$$\frac{\left(\hat{F} - F\right)}{(1+4\%)^{0.5}} \times 1000 = -9657.44$$
$$\hat{F} = -9.65744 \times (1+4\%)^{0.5} + F$$
$$= 20.7454$$

Common Mistakes:

- Some students forget to divide by $(1+4\%)^{0.5}$ when evaluating the loss
- Some student raise $(1+4\%)$ not to the power of 0.5 (6 months). Some raise to power of 1 and some to 0.75.
- Some students use the new spot price S (\$30) instead of the new 6-month forward price F(\$30.59) to evaluate the loss

12

(c) Back at time 0 (when Jim originally entered into his 9-month contract), the spot price for soy beans was $1/bushel and the 9-month storage cost was 2% (not annualized). Soy beans have no convenience yield. Making use of the information in part (b), calculate the the 9-month forward price for soy beans (from the perspective of time 0).

Answer:

Going back to time 0, we can infer the 9-month interest rate (not annualized) from the forward price from part(b).

$$\hat{F} = S(1 + r_{9M})$$
$$r_{9M} = \hat{F}/S - 1$$
$$= 20.7454/20 - 1$$
$$= 3.727\%$$

9-month forward price for soy bean is given by standard formula

$$\$1 \times (1 + r_{9M} + c) = 1 + 3.727\% + 2\%$$
$$= \$1.057/\text{bushel}$$

Common Mistakes:

- Some students do not realize that the ONLY way to back out 9 month interest rate is through 9-month forward price of silver. They attempt to get it from term structure, which does not contain sufficient information. (Incorrect 9-month forward price from part(b) is not penalized again as long as you use the wrong number in the correct way)

- Some students subtract storage cost instead of adding it.

- Some students forget that the storage cost is for 9 month (not annualized).

---

5. (20 points)

The yield curve is flat at 5% (EAR) per year and only shifts up and down over time. All bonds have face value of $100 and are fairly priced.

(a) What is the price and duration of a three-year bond with annual coupons of 6%?

Answer:

$$P = \frac{6}{1.05} + \frac{6}{1.05^2} + \frac{106}{1.05^3} = 102.72$$

$$D = \frac{1}{102.72}\left(\frac{6 \times 1}{1.05} + \frac{6 \times 2}{1.05^2} + \frac{106 \times 3}{1.05^3}\right) = 2.8358$$

(b) A three year bond with monthly coupons trades in the market for the same price as the bond in (a). Explain qualitatively how its duration compare to the bond in (a) and why. (You are not required to calculate its price or duration).

Answer:

If coupons are paid monthly, more are paid out earlier than annual payment. Since duration is a weighted average of payment times of discounted cash flows, earlier payments lead to lower duration.

Common Mistakes:

• Some students think since each monthly coupon is small and carries less weight, the duration increases.

(c) A ten-year bond with annual coupons of 4% last year becomes a nine-year bond today. Suppose that the price sensitivity of the latter to interest rate risk is actually higher than the former. What can you say (qualitatively) about the change in the yield curve over the past year? Explain your answer.

Answer:

Holding YTM constant, the duration of a 9 year coupon bond is less than a 10 year one since CFs are nearer. This should decrease duration. But that price sensitivity goes up suggests duration actually goes up. Since the question states very clearly "the yield curve is flat and only shifts up and down over time", the decrease in duration can be only explained by a decrease in yield. This discounts less heavily the distant coupons and principal payment and increase duration.

Common Mistakes:

• Students forget to analyze the impact on duration when a 10 year coupon bond becomes a 9 year one. Although this does not change the conclusion, it is important not to miss this effect and make sure it does not lead to other possibilities.

• Some students argue around the slope of term structure (the question says " yield curve is flat over time") or convexity effect.

(d) Two years ago, the yield curve was flat at 7%. At that time, you purchased a 30-year bond with 7% annual coupons at par. The yield curve has since declined. One year ago it was at 6% and it is now at 5%. If you have re-invested your coupons at the available spot rates, what is your annualized holding period return for the past two years?

Answer:

Price today:

$$P = \frac{7}{0.05}(1 - 1.05^{-28}) + \frac{100}{1.05^{28}} = 129.796$$

The first year coupon is re-invested at 6%, which gives

$$7 \times 1.06 = 7.42$$

You also just received a coupon of $7.

$$HPR = \left(\frac{7 + 7.42 + 129.796}{100}\right)^{0.5} - 1 = 20.09\%$$

Common Mistakes:

- Some students do not understand what "holding period return" means.
- Some students do not "annualize" properly.

6. (20 points)

There are two risky assets A and B in the market with the following risk and return characteristics summarized in the table below. The risk-free rate is $r_f = 2\%$.

(a) What is the expected return and the standard deviation of returns of a portfolio with 60% in A, 15% in B and 25% in the risk free asset?

Answer:

Asset	E(r)	Cov.	A	B
A	6%	A	0.010	0.006
B	14%	B		0.040

Expected return $= 2\% \times (1 - 60\% - 15\%) + 6\% \times 60\% + 14\% \times 15\%$

$= 6.2\%$

Risk-free asset doesn't have volatility. Any term related to risk-free asset is 0.

Var $= (60\%)^2 \times 0.01 + (15\%)^2 \times 0.04 + 2 \times 60\% \times 15\% \times 0.006$

$= 0.00558$

$\sigma = (0.00558)^{0.5} = 7.5\%$

Common Mistakes:

- Some students interpret the diagonal elements of covariance matrix as standard deviations. (They are variances not standard deviations)

(b) Is the portfolio described in part (a) on the Capital Market Line (CML)? (Hint: Think of Sharpe ratios.)

Answer:

$$\text{SR of this portfolio} = \frac{6.2\% - 2\%}{7.5\%} = 0.56$$

$$\text{SR of asset B} = \frac{14\% - 2\%}{(0.04)^{0.5}} = 0.6$$

This portfolio does not have maximum SR. Therefore, it is not on the CML.

Common Mistakes:

- Some students assume this portfolio is the market portfolio.
- Some students attempt to solve for maximum SR portfolio, which is time-consuming and prone to mistakes. However, if they solve it correctly, full credit is given.

(c) Suppose now a market portfolio exists (ignore assets A & B). Consider a mean-variance efficient portfolio that has weight 50% on the market portfolio. What is the correlation between the returns of this portfolio and the market returns?

Answer:

The mean-variance efficient portfolio should be on CML, which is a weighted combination of market portfolio and risk-free bond. Since risk-free bond does not have any risk, the co-movement between a mean-variance efficient portfolio with the market portfolio all comes from the portion on market portfolio. This suggests that the correlation with market portfolio is 1.

Common Mistakes:

- Some students forget about the fact that all MV efficient portfolio are on CML, which consists of bond and market portfolio.
- The question is asking for correlation not covariance. Even though only 50% is on market portfolio, the correlation is still 1.

Name: _____ MIT ID#: _____

7. (10 points)

Suppose the term structure of interest rates is flat at 5%. Two-year European call option and put option on Giggle Inc. with a strike price of $100 trade for $40 and $30, respectively. Determine the share price of Giggle Inc.

Answer:

The put-call parity gives $40 + 100/(1 + 5\%)^2 = 30 + S$. Therefore, $S = \$100.70$.

Common Mistakes:

A few students forgot the ^{2} term, and a few others switched 40 and 30.

Name: _____ MIT ID#: _____

8. (20 points)

The annual interest rate is 10%. The stock price of Midas Internet Technology (MIT) is $50. The share price is to evolve according to the binomial tree shown below over the next two years.
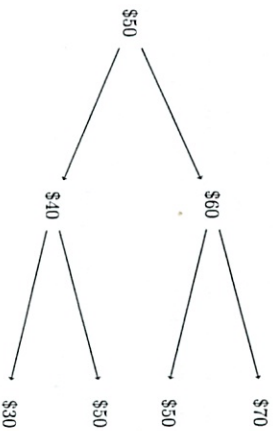
(a) What is the price of an European put option on MIT with a strike price of $60 and a time-to-maturity of two years?

Answer:

Assume the stock price goes up to $60 in Year 1. The number of shares of MIT (A) and the amount invested in the risk-free asset (B) in the replicating portfolio must satisfy

$$70A + 1.1B = 0$$
$$50A + 1.1B = 10$$

that's why wrong

Solving this system of equations, we get $A = -0.5$ and $B = 31.8181$. Therefore, the price of the option in this node is

$$P_u = (-0.5 \times 60) + 31.8181 = 1.8181$$

Assume the stock price goes down to $40 in Year 1. The number of shares of MIT (A) and the amount invested in the risk-free asset (B) in the replicating portfolio must

well got that wrong

satisfy

$$50A + 1.1B = 10$$
$$30A + 1.1B = 30$$

Solving this system of equations, we get $A = -1$ and $B = 54.5454$. Therefore, the price of the option in this node is

$$P_d = (-1 \times 40) + 54.5454 = 14.5454$$

.

Finally, in Year 0, the number of shares of MIT (A) and the amount invested in the risk-free asset (B) in the replicating portfolio must satisfy

$$60A + 1.1B = 1.8181$$
$$40A + 1.1B = 14.5454$$

Solving this system of equations, we get $A = -0.6364$ and $B = 36.3636$. Therefore, the price of the option in Year 0 is

$$P = (-0.6364 \times 50) + 36.3636 = \boxed{4.5454}$$

(b) What is the price of an American put option on MIT with a strike price of $60 and a time-to-maturity of two years?

Answer:

Note that the holder of the option could exercise the option in Year 1, if doing so is in his/her interest, and will indeed do so if the share price drops to $40 from $50. Therefore, the value of this American put option in Year 1 is $1.8181 in the up node and $20 in the down node.

The number of shares of MIT (A) and the amount invested in the risk-free asset (B) in the replicating portfolio must satisfy

$$60A + 1.1B = 1.8181$$
$$40A + 1.1B = 20$$

Solving this system of equations, we get $A = -0.9091$ and $B = 51.2382$. Therefore, the Year 0 price of the American option is

$$P = (-0.9091 \times 50) + 51.2382 = \boxed{5.7832}$$

Common Mistakes:

- Some people simply claimed that "American options have the same value as European options, because early exercise does not bring any additional advantage." This claim is true only under certain conditions.

- Some people correctly identified that it is optimal to exercise the option when the stock price goes down to $40, but incorrectly used $0 as the payoff from the "up" node. This is wrong. If you do not exercise the option at the "up" node when stock price is $60, you still have the right to exercise it in the next period, hence the option has a positive value in the "up" node.

- Some people misread the question and priced a call option.

9. (15 points)

Go Sushi is a new chain of fast food restaurants. The company franchises its stores. Today each franchise has a market value of $400,000. This is the price an owner of a franchise can obtain if he sells his franchise to another person. One year from now each of the company's franchises will have a market value of either $720,000 or $240,000. The probability that the market value of each franchise in a year will be $720,000 is 75%, and the probability that the market value of each franchise will be $240,000 is 25%. This is shown below:

$720,000

$400,000

$240,000

Go Sushi offers two plans for buying franchises. In Plan A, the company simply sells a franchise today for $400,000 and the buyer takes the risk that the franchise will only be worth $240,000 in one year. In Plan B, the company sells the franchise today at a higher price than $400,000 but guarantees that the company will be prepared in one year to buy back the franchise at a price of $400,000. Under Plan B the franchise owner is not obligated to sell the franchise back to the company in one year at $400,000, but he can choose to do so if he wants. Assume that the guarantee is transferable when a franchise is sold. This means that if someone sells a franchise purchased under plan B, the guarantee is transferred to the new owner. Assume one can borrow or lend $100 today for a riskless amount of $108 delivered in one year (i.e., that the riskless interest rate is 8%). Given the information provided, what is the market value (today) of a franchise purchased under plan B (which includes the guarantee)?

Answer:

Under plan B, the payoff to the franchise owner is $720,000 in the good state. In the bad state the payoff would be $400,000, because the franchise owner would want to exercise his right to sell the franchise back to the company. We can find the market value (today) of the franchise under Plan B by replicating its payoffs with a position in the underlying asset (a Plan A franchise) and a position in the riskless asset. We have

$$a = \frac{CF_u - CF_d}{S_u - S_d} = \frac{\$720,000 - \$400,000}{\$720,000 - \$240,000} = \frac{2}{3}$$

$$
\begin{aligned}
b &= \frac{1}{1+r}\left(\frac{CF_d S_u - CF_u S_d}{S_u - S_d}\right) \\
&= \frac{1}{1.08}\left(\frac{\$400,000 \times \$720,000 - \$720,000 \times \$240,000}{\$720,000 - \$240,000}\right) \\
&= \$222,222
\end{aligned}
$$

and therefore the market value of the franchise under Plan B is

$$MV = aS + b = \frac{2}{3}\$400,000 + \$222,222 = \boxed{\$488,889}$$

Common Mistakes:

- Some people used the probabilities 0.75 and 0.25 to calculate the expected payoff. This is incorrect. You should recall from Options lecture notes and problem sets that the state probabilities are irrelevant for option prices.
- Some people decomposed the real option payoff into $400,000 + C, where C is a call option that pays $320,000 in the "up" state and $0 in the "down" state. Indeed, the call option can be priced by

$$a = \frac{\$320,000 - \$0}{\$720,000 - \$240,000} = \frac{2}{3}$$

$$b = \frac{1}{1+r}\left(\frac{CF_d S_u - CF_u S_d}{S_u - S_d}\right)$$
$$= \left(\frac{0 \times \$720,000 - \$320,000 \times \$240,000}{\$720,000 - \$240,000}\right)$$
$$= -148148$$

$$C = aS + b = \frac{2}{3}\$400,000 - \$148148 = \$118,519$$

$$MV = 400,000 + C = \$518,519$$

This is incorrect because the risk-less $400,000 will be received one year later, therefore must be discounted as well. The correct solution is

$$MV = \frac{400,000}{1.08} + C = \$370,370 + \$118,519 = \$488,889$$

Name: _____   MIT ID#: _____

10. (15 points)

Suppose you are considering pricing two assets, A and B. The table below summarizes the expected return of the market and the covariance of future returns of A, B and the market M. The risk-free rate is $r_f = 2\%$

Asset	E(r)	Cov.	M	A	B
M	6%	M	0.040	0.040	0.036
A	?	A		0.090	0.000
B	?	B			0.160

(a) What are the expected rates of return of asset A and B if the CAPM holds?

Answer:

The betas are

$$\beta_A = \frac{Cov(r_A, r_m)}{\sigma_M^2} = \frac{.04}{.04} = 1$$

$$\beta_B = \frac{Cov(r_B, r_m)}{\sigma_M^2} = \frac{.036}{.04} = .9$$

Expected return are

$$r_A = r_f + \beta_A(r_m - r_f) = .02 + 1(.06 - .02) = \boxed{6\%}$$

$$r_B = r_f + \beta_B(r_m - r_f) = .02 + .9(.06 - .02) = \boxed{5.6\%}$$

Name: _____ MIT ID#: _____

(b) What is the riskier asset in an expected return sense, A or B? Does this square with the assets' standard deviations? Explain.

Answer:

Asset A is the riskier asset as it has a higher beta and thus a higher expected return. This is contrary to the standard deviation's of the assets, as B clearly has higher standard deviation of returns than A. The reason is that the systematic risk in A's returns is higher than in B's returns, and diversification implies only systematic risk will be priced in a universe of many stocks.

29

Name: _____ MIT ID#: _____

(c) Suppose the covariance between the returns of asset A and B changes from 0.000 to 0.030 (but everything else remains the same). What is the correlation between the returns of A and B now? Will this change any of the expected return relationships derived in (a)?

Answer:

The correlation between A and B is now

$$\rho_{AB} = \frac{.03}{.3 \times .4} = \boxed{0.25}$$

A change in the covariance between A and B should not imply any changes in the asset pricing relation, because the assets' systematic risks are not changed, hence the expected returns also stay constant.

Common Mistakes:

- Some people could not read the covariance table. See "Portfolio Theory" lecture notes.

- Some people calculated the slope of CML and attempted to find asset returns by their Sharpe Ratios. However, CAPM only states that the market portfolio is on the CML, but **not** the individual assets. According to CAPM, individual assets are on the SML.

30

Last Name: _____ First Name: _____

M.I.T. ID# _____ Section: _____

(Stomper: 1-2:30 = A, 2:30-4 = B; Wang: 10:20-11:50 A.M. = C, 1-2:30 = D)

15.401 Final Exam

A. Stomper and J. Wang

Fall 2009

Please make sure that your copy of the exam contains 22 pages (including this one).

- Please write your name and MIT ID number on every page.

- Fill in your section number.

- The exam lasts 180 minutes. It consists of 9 questions. Please answer all of them.

- Allocate your time optimally.

- You are allowed two $8\frac{1}{2}$" $\times$ 11" sheets of formulas and one calculator.

- Answer these questions without consulting anyone.

- State explicitly any additional assumptions you feel needed in order to answer a question.

- Use the space provided. If more space is needed, use the other side of the page.

- Be neat and show your work. Answers without work receive no credit. Wrong answers with partially correct work may receive partial credit.

Good luck!

Name: _____ MIT ID#: _____

15.401 Final Examination 2009 Grade Sheet

1.	_____ /	40
2.	_____ /	20
3.	_____ /	20
4.	_____ /	15
5.	_____ /	15
6.	_____ /	15
7.	_____ /	20
8.	_____ /	20
9.	_____ /	15
Total	_____ /	180

1. (40 points) True, false or "it depends"? Give a brief explanation for each answer.

    (a) (5 points) "Other things equal, bonds paying higher coupons have higher interest rate risks."

    (b) (5 points) "A firm's PVGO (the present value of growth opportunities) is positive as long as the expected returns on its future investments are positive."

(c) (5 points) "A call option on a stock is always worth more than the stock itself since the option has only an upside."

(d) (5 points) "Call options are worth more than put options on the same asset with the same exercise price and maturity because they have unlimited upsides."

(e) (5 points) "Investors can achieve a riskless return if they can diversify in infinitely many stocks."

(f) (5 points) "By the CAPM, no project a firm will find can yield an expected return above the Security Market Line."

(g) (5 points) "Emerging markets must yield higher returns than mature markets because they are riskier."

(h) (5 points) "Projects with higher IRR (internal rate of return) are always preferred because they yield higher returns."

2. (20 points)

The Wall Street Journal gives the following prices for the STRIPS:

Maturity (Years)	1	2	3
Price (% of Par Value)	95.2381	90.7029	86.3838

Your liabilities consist of two payments: $30 million in year 2 and $30 million in year 3.

(a) (5 points) Calculate the 1, 2, and 3 year spot rates.

(b) (5 points) Calculate the PV of your liabilities.

(c) (5 points) Calculate the Modified Duration of the liabilities.

(d) (5 points) Based on your answers to the last question, by how much will the value of your liabilities change if interest rates increase by 1 basis point $= (1/100)\%$?

3. (20 points) Sunshine Technology has book value per share of $100 at year 0 (after year 0 dividend is paid) and enjoys a ROE of 12% per year. Sunshine Technology currently has a payout ratio of 50% and plans to maintain that ratio in the future.

   (a) (5 points) What is the expected dividend per share at year 1?

   (b) (5 points) A competing firm in the same business is publicly traded, with a beta of 1.2. The riskfree rate is 4% and the risk premium on the market portfolio is 5%. What should be the cost of capital for Sunshine Technology?

(c) (5 points) What should Sunshine Technology's ex-dividend share price be at year 0?

(d) (5 points) Is Sunshine Technology a growth company? If so, what is its PVGO.

4. (15 points) In January, the term structure is flat, one-month interest rate is $r = 0.6\%$ (annualized). Oil futures are quoted as follows,

Maturity Month	Feb	Mar	Apr
Oil Futures ($ per barrel)	80.1192	80.4397	?

(a) (5 points) What is the current net convenience yield on oil?

(b) (5 points) Can you infer the spot and 3-month futures prices, assuming the net convenience yield is the same as above?

Name: _____ MIT ID#: _____

(c) (5 points) Dan bought 10,000 barrels of 2 month oil futures in January. In February, oil futures are quoted as below:

Maturity Month	Mar	Apr	May
Oil Futures ($ per barrel)	83.415	83.8321	84.2512

What is Dan's profit/loss in February?

12

5. (15 points) The current price of a non-dividend paying stock is $800 and the riskfree interest rate is 5%. A 1-year European call on the stock with a strike price of $815 costs $75 and a put with the same strike price and maturity costs $45.

   (a) (9 points) How can you use the call and put options and the stock to construct a risk-free payoff?

(b) (6 points) Is there an arbitrage opportunity? If so, please describe specifically how you could profit from the arbitrage opportunity.

6. (15 points)

The current price of stock "Green Light" (GL) is \$100. In the next two periods, it follows the following binomial process:

```
                                  ┌─ 225
                        ┌─ 150 ─┤
                        │         └─ 135
               100 ─┤
                        │         ┌─ 135
                        └─ 90 ──┤
                                  └─ 81
```

The riskfree interest rate is zero. An investment bank offers you an insurance for each \$100 dollar invested in the stock: The insurance pays nothing if you make money on your stock investment over the two periods, and it pays the loss if you lose money. For example, if the stock value increases to \$225, you receive nothing while if the stock value drops to \$81, you receive 100-81 = \$19.

(a) (10 points) Describe the payoff of this insurance policy at the end of two periods. What is the cost of this insurance for the investment bank?

(b) (5 points) Describe the trading strategy the bank may use to replicate the payoff of this insurance policy.

7. (20 points) Assume the market portfolio has an expected return of $r_m = 6\%$ and standard deviation $\sigma_m = 10\%$. There are only two investors in this market: Ashley and Brad. Each investor has $5 trillion of wealth. They both hold efficient frontier portfolios. The riskfree interest rate is $r_f = 2\%$. Ashley and Brad can only borrow/lend from/to each other (i.e., their holdings of the riskfree asset sum to zero).

(a) (5 points) Brad says that his portfolio has a Sharpe ratio of $1/2$. Can that be true? Please provide a short explanation.

(b) (6 points) Ashley is investing 150% of her wealth in the market portfolio and shorting 50% of the riskfree asset. Calculate the expected return, standard deviation, and the Sharpe ratio of Ashley's portfolio.

(c) (4 points) Given Ashley's holdings, can you infer the portfolio weights of Brad? What is the Sharpe ratio of his portfolio?

(d) (5 points) Is the Sharpe ratio of Brad's portfolio different from Ashley's? Can you explain why? Which one of them is more risk averse?

8. (20 points) Stock A and B have standard deviations of 20% and 10%, respectively. Their correlations with the market portfolio are 0.6 and 0.3. The market portfolio has standard deviation of 15%.

   (a) (6 points) What are the betas of A and B?

   (b) (7 points) Expected returns of A and B are 6.8% and 3.2%. Plot the security market line (SML) and find the market risk premium, riskfree rate and expected return of the market portfolio.

(c) (7 points) Mr. Sharpe holds a efficient frontier portfolio. He chooses to hold a portfolio with 20% in the riskfree asset and the rest in stock A and B. What is the expected return and standard deviation of his portfolio?

Name: ———————————————— MIT ID#: ————————————————

9. (15 points) NiceCraft is a local furniture manufacturer and it plans to expand its business by selling its products to customers who live further away. NiceCraft faces a tax rate of 34% and a cost of capital of 10%.

   (a) (6 points) With local customers only, its expected pre-tax profit is $1 million per year forever. What is the current market value of NiceCraft.

(b) (9 points) NiceCraft can build an outlet to increase sales to far away customers. The building cost is $1.5 million. The expected pre-tax profit from increased sales is another $1 million per year forever. NiceCraft's new outlet could be depreciated linearly to $0 over 3 years. Should NiceCraft carry out this expansion? Explain.

Last Name: _____ First Name: _____

M.I.T. ID# _____ Section: _____

(Stomper: 1-2:30 = A, 2:30-4 = B; Wang: 10:20-11:50 A.M. = C, 1-2:30 = D)

15.401 Final Exam

A. Stomper and J. Wang

Fall 2009

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Good luck!

1

Name: _____ MIT ID#: _____

15.401 Final Examination 2009 Grade Sheet

1. _____ / 40

2. _____ / 20

3. _____ / 20

4. _____ / 15

5. _____ / 15

6. _____ / 15

7. _____ / 20

8. _____ / 20

9. _____ / 15

Total _____ / 180

2

1. (40 points)

(a) (5 points) True, false or "it depends"? Give a brief explanation for each answer.

"Other things equal, bonds paying higher coupons have higher interest rate risks."

Answer:
False. Interest rate risk is measured by duration, which gives the sensitivity of bond price to yield changes. Bond with higher coupon has shorter duration as more cash flows are paid out earlier. So higher coupon implies lower interest rate risk.

(b) (5 points) "A firm's PVGO (the present value of growth opportunities) is positive as long as the expected returns on its future investments are positive."

Answer:
False. Positive PVGO is equivalent to ROE>r (return on future investments > cost of capital). ROE>0 is not sufficient.

(c) (5 points) "A call option on a stock is always worth more than the stock itself since the option has only an upside."

Answer:
A call option is worth less than a stock at time 0 because the final payoff of a stock is greater than that of a call option for all realizations of the final stock price.
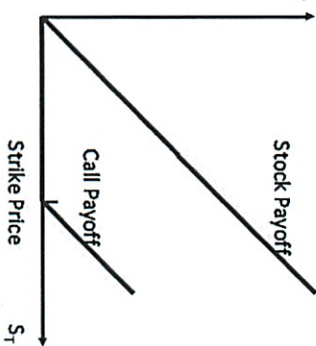
(d) (5 points) "Call options are worth more than put options on the same asset with the same exercise price and maturity because they have unlimited upsides."

Answer:
False or depends. Put-call parity gives

$$C + \frac{K}{1+r} = P + S$$

Hence,

$$C - P = S - \frac{K}{1+r}$$

This shows that whether a call is worth more than a put (for the same exercise price and maturity) depends on the current stock price relative to the strike price (discounted at the appropriate risk-free rate). If stock price is very low relative to the strike price, a put option is worth more than a call option.

(e) (5 points) "Investors can achieve a riskless return if they can diversify in infinitely many stocks."

Answer:
False or depends. They can do that only if there is NO systematic risk. In general, it may not be true in the presence of systematic risk, which can't be reduced through diversification.

(f) (5 points) "By the CAPM, no project a firm will find can yield an expected return above the Security Market Line."

Answer:
False. Expected return of a publicly traded asset must be on the SML by CAPM. However, a firm may have projects, which are NOT accessible (i.e. not publicly traded) to the public. In fact, any project with positive NPV yields expected return higher than that demanded by the market. The latter, rather than the former, is on the SML by CAPM.

(g) (5 points) "Emerging markets must yield higher returns than mature markets because they are riskier."

Answer:
False. Though emerging markets might have greater total risk (measured by standard deviation) than mature markets, a large portion of the risk could be idiosyncratic, which, by CAPM, does not require higher return to compensate for.

(h) (5 points) "Projects with higher IRR (internal rate of return) are always preferred because they yield higher returns."

Answer:
False. Project choices should be based on NPV, which does NOT yield the same conclusion as IRR. A project with higher IRR might have lower NPV due to a smaller scale, which IRR ignores. In addition, IRR cannot distinguish the sign of NPV. Two projects with opposite cash flows (therefore, opposite NPVs) have the same IRR.

2. (20 points)

The Wall Street Journal gives the following prices for the STRIPS:

Maturity (Years)	1	2	3
Price (% of Par Value)	95.2381	90.7029	86.3838

Your liabilities consist of two payments: $30 million in year 2 and $30 million in year 3.

(a) (5 points) Calculate the 1, 2, and 3 year spot rates.

Answer:
Spot rates:

$$r_1 = \frac{100}{95.2381} - 1 = 5\%$$

$$r_2 = \left(\frac{100}{90.7029}\right)^{\frac{1}{2}} - 1 = 5\%$$

$$r_3 = \left(\frac{100}{86.3838}\right)^{\frac{1}{3}} - 1 = 5\%$$

The term structure is flat.

(b) (5 points) Calculate the PV of your liabilities.

Answer:

$$PV_L = \frac{30}{(1+r_2)^2} + \frac{30}{(1+r_3)^3} = 53.1260$$

(c) (5 points) Calculate the Modified Duration of the liabilities.

Answer:
From part (1), the term structure is flat, $y = r = 5\%$.

$$D_L = \frac{2 \times \frac{30}{(1+5\%)^2} + 3 \times \frac{30}{(1+5\%)^3}}{\frac{30}{(1+5\%)^2} + \frac{30}{(1+5\%)^3}} = 2.4878$$

$$MD_L = \frac{D}{1+y} = \frac{2.4878}{1+5\%} = 2.3693$$

(d) (5 points) Based on your answers to the last question, by how much will the value of your liabilities change if interest rates increase by 1 basis point = (1/100)%?

Answer:
Interest rates increases by 1 basis points,

$$\frac{\Delta PV_L}{PV_L} = -MD_L \times \Delta y$$

$$\Delta PV_L = -MD_L \times PV_L \times \Delta y$$

$$= -2.3810 \times 53.1260 \times \frac{1}{100} \times 0.01$$

$$= -0.0126(\$mil)$$

3. (20 points) Sunshine Technology has book value per share of $100 at year 0 (after year 0 dividend is paid) and enjoys a ROE of 12% per year. Sunshine Technology currently has a payout ratio of 50% and plans to maintain that ratio in the future.

(a) (5 points) What is the expected dividend per share at year 1?

Answer:

$$E_1 = BV_0 \times ROE = 100 \times 12\% = \$12$$
$$D_1 = E_1 \times p = 12 \times 50\% = \$6$$

(b) (5 points) A competing firm in the same business is publicly traded, with a beta of 1.2. The riskfree rate is 4% and the risk premium on the market portfolio is 5%. What should be the cost of capital for Sunshine Technology?

Answer:

By CAPM,

$$r = r_f + \beta(r_M - r_f)$$
$$= 4\% + 1.2 \times 5\%$$
$$= 10\%$$

(c) (5 points) What should be Sunshine Technology's ex-dividend share price be at year 0?

Answer:
Growth rate of dividend is

$$g = ROE(1 - p) = 12\% \times 50\% = 6\%$$

$$P_0 = \frac{D_1}{r - g}$$
$$= \frac{6}{10\% - 6\%}$$
$$= \$150$$

(d) (5 points) Is Sunshine Technology a growth company? If so, what is its PVGO.

Answer:
Yes, it is since ROE > r.

$$PVGO = P_0 - \frac{E_1}{r}$$
$$= 150 - \frac{12}{10\%}$$
$$= \$30$$

PVGO is $30/share.

4. (15 points) In January, the term structure is flat, one-month interest rate is $r = 0.6\%$ (annualized). Oil futures are quoted as follows,

Maturity Month	Feb	Mar	Apr
Oil Futures ($ per barrel)	80.1192	80.4397	?

(a) (5 points) What is the current net convenience yield on oil?

Answer:
Given the annualized risk-free rate, assume the annualized net-convenience-yield is $\bar{y}$, the futures prices are

$$H_{1,2} = S_1(1 + r - \bar{y})^{\frac{1}{12}}$$
$$H_{1,3} = S_1(1 + r - \bar{y})^{\frac{2}{12}}$$

Hence the net convenience yield is determined by

$$\frac{H_{1,2}}{H_{1,3}} = \frac{80.4397}{80.1192} = 1 + 0.4\%$$

$$(1 + r - \bar{y})^{\frac{1}{12}} = 1 + 0.4\%$$
$$(1 + r - \bar{y})^{\frac{1}{12}} = 1 + 0.4\%$$
$$1 + 0.6\% - \bar{y} = (1 + 0.4\%)^{12} = 1.04907$$
$$\bar{y} = 1 + 0.6\% - (1 + 0.4\%)^{12}$$
$$\bar{y} = -4.307\%$$

(b) (5 points) Can you infer the spot and 3-month futures prices, assuming the net convenience yield is the same as above?

Answer:
The spot and three-month futures prices should be

$$S_1 = \frac{H_{1,2}}{(1 + r - \bar{y})^{\frac{1}{12}}}$$
$$= \frac{80.1192}{1 + 0.4\%}$$
$$= 79.8$$
$$H_{1,4} = S_1(1 + r - \bar{y})^{\frac{3}{12}} = S_1\left[(1 + r - \bar{y})^{\frac{1}{12}}\right]^3$$
$$= 79.8 \times (1 + 4\%)^3$$
$$= 80.7614$$

(c) (5 points) Dan bought 10,000 barrels of 2 month oil futures in January. In February, oil futures are quoted as below:

Maturity Month	Mar	Apr	May
Oil Futures ($ per barrel)	83.415	83.8321	84.2512

What is Dan's profit/loss in February?

Answer:
Dan's contract matures in March. In February, it is priced at $83.415, which is higher than the entering price $ 80.4397 (two-month future prices in January). Dan has made money.

Futures contract is marked to market, therefore the PnL is booked as

$$PnL = 10,000 \times (83.415 - 80.4397) = \$29753$$

In this class, however, we do not distinguish forwards vs. futures. Following Problem Set 4, Question 2 (a very similar problem on Forwards), the following answer is also accepted: First, calculate the implied 1-month interest rate r' (annualized) in February. Assume the new spot price is S_2, then the futures prices should be

$$H_{2,3} = S_2(1 + r' - \bar{y})^{\frac{1}{12}}$$
$$H_{2,4} = S_2(1 + r' - \bar{y})^{\frac{2}{12}}$$

$$(1 + r' - \bar{y})^{\frac{1}{12}} = \frac{H_{2,4}}{H_{2,3}} = \frac{83.8321}{83.415} = 1 + 0.5\%$$

$$1 + r' - \bar{y} = (1 + 0.5\%)^{12} = 1.06168$$
$$r' = (1 + 0.5\%)^{12} - 1 + \bar{y}$$
$$= 1.06168 - 1 - 0.04307$$
$$r' = 1.86\%$$

His profit in February is

$$PnL = 10,000 \times \frac{H_{2,3} - H_{1,3}}{(1 + r')^{\frac{1}{12}}}$$
$$= 10,000 \times \frac{83.415 - 80.4397}{(1 + 1.86\%)^{\frac{1}{12}}}$$
$$= 29707$$

5. (15 points) The current price of a non-dividend paying stock is $800 and the riskfree interest rate is 5%. A 1-year European call on the stock with a strike price of $815 costs $75 and a put with the same strike price and maturity costs $45.

(a) (9 points) How can you use the call and put options and the stock to construct a risk-free payoff?

Answer:
Long: 1 put and 1 stock
Short: 1 call
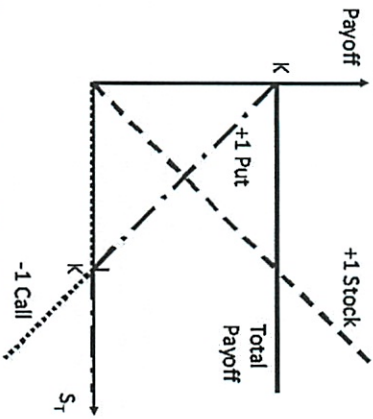
This gives a risk-free payoff of $815 at maturity.

(b) (6 points) Is there an arbitrage opportunity? If so, please describe specifically how you could profit from the arbitrage opportunity.

Answer:

There is an arbitrage.
Put-call parity suggests that

$$P_0 + S_0 = C_0 + \frac{K}{1+r}$$

$$P_0 + S_0 - C_0 = \frac{K}{1+r}$$

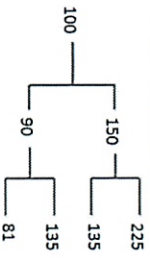Plugging in the prices, we have

$$P_0 + S_0 - C_0 = 45 + 800 - 75$$
$$= \$770 < \frac{815}{1.05} = \frac{K}{1+r}$$

So put-call parity fails. To construct an arbitrage, we should buy low and sell high. Specifically, at time 0, we should:

1. Borrow $770 at 5%
2. Long 1 stock
3. Long 1 put
4. Short 1 calls

6. (15 points)

The current price of stock "Green Light" (GL) is $100. In the next two periods, it follows the following binomial process:

```
100 ─┬─ 150 ─┬─ 225
     │       └─ 135
     └─ 90  ─┬─ 135
             └─ 81
```

The riskfree interest rate is zero. An investment bank offers you an insurance for each $100 dollar invested in the stock: The insurance pays nothing if you make money on your stock investment over the two periods, and it pays the loss if you lose money. For example, if the stock value increases to $225, you receive nothing while if the stock value drops to $81, you receive 100-81 = $19.

(a) (10 points) Describe the payoff of this insurance policy at the end of two periods. What is the cost of this insurance for the investment bank?

Answer:

In the second period, the payoff of the insurance in the "uu", "ud", "du", "dd" nodes are {0, 0, 0, 19} respectively.

Back to period 1. In the "up" node, the insurance is worth zero. In the "down" node, the insurance has a positive value. Replicate its payoff with a_d shares of stock, and b_d dollars in the bond:

$$135a_d + b_d = 0$$
$$81a_d + b_d = 19$$
$$\Rightarrow a_d = -\frac{19}{54} = -0.3519$$
$$b_d = \frac{19 \times 135}{54} = 47.5$$
$$V_d = a_d S_d + b_d = -\frac{19}{54} \times 90 + 47.5 = 15.83$$

Back to period 0. The insurance is worth $V_u = 0$ in the "up" node and $V_d = 15.83$ in the "down" node. Replicate its payoff with a shares of stock and b in the bond:

$$150a + b = 0$$
$$90a + b = 15.83$$
$$\Rightarrow a = -0.2638$$
$$b = -150a = 39.575$$
$$V = aS + b = -0.2638 \times 100 + 39.575$$
$$= 13.195$$

15

(b) (5 points) Describe the trading strategy the bank may use to replicate the payoff of this insurance policy.

Answer:
At period 0, the bank needs to short −0.2638 shares of stock and long 39.575 in the riskless bond;
At period 1, if the stock goes up to $150, just unwind the entire position; if the stock goes down to $90, then adjust the portfolio to short −0.3519 shares of stock and long 47.5 in the riskless bond.

16

7. (20 points) Assume the market portfolio has an expected return of $r_m = 6\%$ and standard deviation $\sigma_m = 10\%$. There are only two investors in this market: Ashley and Brad. Each investor has $5 trillion of wealth. They both hold efficient frontier portfolios. The riskfree interest rate is $r_f = 2\%$. Ashley and Brad can only borrow/lend from/to each other (i.e., their holdings of the riskfree asset sum to zero).

(a) (5 points) Brad says that his portfolio has a Sharpe ratio of 1/2. Can that be true? Please provide a short explanation.

Answer:
The highest Sharpe ratio attainable in this market is

$$SR_{max} = \frac{r_m - r_f}{\sigma_m} = \frac{6\% - 2\%}{10\%} = 0.4 < \frac{1}{2}$$

Brad's claim is outside of the CML, he must be lying.

(b) (6 points) Ashley is investing 150% of her wealth in the market portfolio and shorting 50% of the riskfree asset. Calculate the expected return, standard deviation, and the Sharpe ratio of Ashley's portfolio.

Answer:
Ashley invests with $w_A = 1.5$ in the market portfolio, and $1 - w_A = -0.5$ in the riskfree asset.

$r_A = w_A \times r_m + (1 - w_A) \times r_f = 1.5 \times 6\% - 0.5 \times 2\% = 8\%$

$\sigma_B = w_A \times \sigma_m = 1.5 \times 10\% = 15\%$

$SR_A = \frac{r_A - r_f}{\sigma_A} = \frac{8\% - 2\%}{15\%} = 0.4$

Note that $SR_A = SR_{max}$ since Ashley lies on the CML.

(c) (4 points) Given Ashley's holdings, can you infer the portfolio weights of Brad? What is the Sharpe ratio of his portfolio?

Answer:
Assume Brad invests w_B in the market portfolio and $1 - w_B$ in the riskfree asset. In aggregate, their holdings of the riskfree asset sums to zero, and their holdings of the market portfolio coincides with the market portfolio.

$w_A \times 5 + w_B \times 5 = 10$

$(1 - w_A) \times 5 + (1 - w_B) \times 5 = 0$

Therefore $w_B = 0.5$, Brad holds $5 \times 0.5 = 2.5$ trillion in the market portfolio and the riskfree asset each.
Brad also lies on the CML, so his Sharpe ratio should be equal to the maximal Sharpe ratio in the market,

$$SR_B = SR_{max} = 0.4$$

(d) (5 points) Is the Sharpe ratio of Brad's portfolio different from Ashley's? Can you explain why? Which one of them is more risk averse?

Answer:
Brad has the same Sharpe ratio as Ashley because they both lie on the CML. Brad is more risk-averse because he chooses to put more weight on the riskfree asset.

8. (20 points) Stock A and B have standard deviations of 20% and 10%, respectively. Their correlations with the market portfolio are 0.6 and 0.3. The market portfolio has standard deviation of 15%.
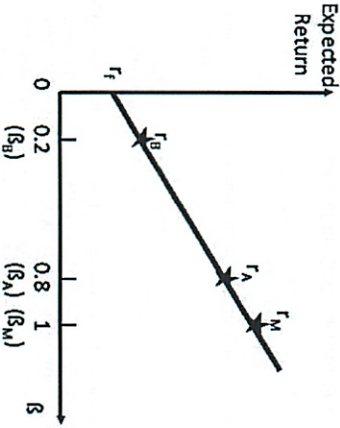
(a) (6 points) What are the betas of A and B?

Answer:

$$\beta_A = \frac{cov(\tilde{r}_A, \tilde{r}_M)}{var(\tilde{r}_M)} = \frac{\rho_{A,M}\sigma_A\sigma_M}{\sigma_M^2}$$

$$= \frac{\rho_{A,M}\sigma_A}{\sigma_M} = \frac{0.6\,(20\%)}{15\%} = 0.8$$

$$\beta_B = \frac{\rho_{B,M}\sigma_B}{\sigma_M} = \frac{0.3\,(10\%)}{15\%} = 0.2$$

(b) (7 points) Expected returns of A and B are 6.8% and 3.2%. Plot the security market line (SML) and find the market risk premium, riskfree rate and expected return of the market portfolio.

Answer:

Market risk premium is the gradient of the line:

$$r_M - r_f = \frac{r_A - r_B}{\beta_A - \beta_B} = \frac{6.8\% - 3.2\%}{0.8 - 0.2} = 6\%$$

Hence, by CAPM

$$r_f = r_A - \beta_A(r_M - r_f) = 2\%$$
$$r_M = 2\% + 6\% = 8\%$$

(c) (7 points) Mr. Sharpe holds a efficient frontier portfolio. He chooses to hold a portfolio with 20% in the riskfree asset and the rest in stock A and B. What is the expected return and standard deviation of his portfolio?

Answer:
The other 80% must be in the market portfolio. Hence, the portfolio expected return is

$$0.2\,(2\%) + 0.8\,(8\%) = 6.8\%$$

The portfolio standard deviation is

$$0.8\,(15\%) = 12\%$$

9. (15 points) NiceCraft is a local furniture manufacturer and it plans to expand its business by selling its products to customers who live further away. NiceCraft faces a tax rate of 34% and a cost of capital of 10%.

(a) (6 points) With local customers only, its expected pre-tax profit is $1 million per year forever. What is the current market value of NiceCraft.

Answer:
After-tax profit per year is

$$(1 - 0.34) \times 1 = 0.66 \text{ million}$$

The perpetuity formula gives the market value

$$\frac{0.66}{10\%} = 6.6 \text{ million}$$

(b) (9 points) NiceCraft can build an outlet to increase sales to far away customers. The building cost is $1.5 million. The expected pre-tax profit from increased sales is another $1 million per year forever. NiceCraft's new outlet could be depreciated linearly to $0 over 3 years. Should NiceCraft carry out this expansion? Explain.

Answer:
We should consider the **incremental cash flows.**

PV of tax shield is

$$\frac{0.34(1.5/3)}{1+10\%} + \frac{0.34(1.5/3)}{(1+10\%)^2} + \frac{0.34(1.5/3)}{(1+10\%)^3} = 0.423 \text{ million}$$

PV of incremental after-tax profit

$$\frac{(1-0.34)\,1}{10\%} = 6.6 \text{ million}$$

NPV of incremental CFs is

$$6.6 + 0.423 - 1.5 = 5.52 \text{ million} > 0$$

It should be carried out.

$$125 \begin{cases} 156.25 \\ 100 \end{cases} \qquad 31.25 \qquad \text{(testing an ans here 1st)}$$

$$0$$

$156.25A + 1.1B = 31.25$

$100A + 1.1B = 0$

$$B = \frac{-100A}{1.1}$$

$156.25A - 100A = 31.25$

$56.25A = 31.25$

$A = .555$

$B = -50.50$

Going w/ much more realistic

$.555(125) - 50.50 = 18.87$

$$80 \begin{cases} 100 & 20 \\ 64 & 0 \end{cases}$$

$100A + 1.1B = 20$

$64A + 1.1B = 0$

$$B = \frac{-64A}{1.1}$$

$36A = 20$

$A = 5/9$

$B = -32$

$5/9(80) - 32 = 12.12$

$$100 \begin{cases} 18.87 \\ 12.12 \end{cases}$$

$125A + 1.1B = 18.87$

$80A + 1.1B = 12.12$

$$B = \frac{12.12 - 80A}{1.1}$$

$125A + 12.12 - 80A = 18.87$

$45A = 6.75$

$A = .15$

$B = .109$

$.15(100) - .109 = 14.891$

Verify

$$\frac{18.87 - 12.12}{125 - 80} = .15 \checkmark$$

$$\frac{125 \cdot 12.12 - 80 \cdot 18.87}{(125 - 80) \, 1.1} = .109 \quad \checkmark \text{ same}$$

then switched to the wrong Binomial
theorm

(had it right - darn)

What price ya subtract from is strike price