Last Name: _____ First Name: _____

M.I.T. ID# _____ Section: _____

(10:30–12:00pm = C, 1:30–2:00pm = D)

15.401 Midterm Exam

A. Verdelhan

Fall 2010

Please make sure that your copy of the exam contains 21 pages (including this one).

- Please write your name and MIT ID number on every page.

- Check your section number.

- The exam lasts 80 minutes. It consists of five questions. Please answer all of them.

- Credit for each question is suggestive of the amount of time you should spend on it (1 point = 1 minute). Therefore, do not agonize over a 10-point question without having tackled a 20-point question.

- You are allowed one $8\frac{1}{2}$" ×11" sheet of formulas and one calculator.

- Answer these questions without consulting anyone.

- Use the space provided. If more space is needed, use the extra space provided at the end of the exam.

- Be neat and show your work. Answers without work receive no credit. Wrong answers with partially correct work may receive partial credit.

Good luck!

Name: _____ MIT ID#: _____

15.401 Midterm Examination 2010 Grade Sheet

1. _____ / 24

2. _____ / 16

3. _____ / 13

4. _____ / 15

5. _____ / 12

Total _____ / 80

1. (24 points) True, false or "it depends"? Give a brief explanation for each answer.

    (a) (4 points) "The return from a one-year CD (Certificate of Deposit) that offers a 10% APR with semiannual compounding is higher than the return from a one-year CD that offers a 10% EAR."

    (b) (4 points) "If the net convenience yield of a future contract is positive (storage costs are low and convenience yields are high for example), futures price will be higher than the spot price adjusted for the time value of money."

(c) (4 points) "Assume a flat term structure of interest rates. Rising interest rates will shorten the duration of a coupon bond."

(d) (4 points) "According to Bloomberg, the yield curve on October 26, 2010 is upward-sloping (spot interest rates increase with maturity). This implies that investors expect the Federal Reserve to raise short term interest rates in the future."

(e) (4 points) "If next year's expected dividend payment of companies A and B are the same and the expected dividend growth of company A is higher than the expected dividend growth of company B, then company A must have a higher price than company B."

(f) (4 points) "If an investor purchases a two-year bond (with annual coupon payments), holds it to maturity and reinvests all coupons at the appropriate forward rates determined today, then she will receive an annualized return equal to today's two-year spot rate."

2. (16 points) Three institutional investors, A, B and C, hold portfolios of risk-free bonds. The value of these portfolios and their payoffs are as follows:

Portfolio	Value ($)	Payoffs
A	667,572	$700,000 in two years
B	772,929	$300,000 in one year, $500,000 in two years
C	45,614	$550,000 in one year, -$550,000 in three years

(a) (4 points) Infer the one-, two-, and three-year spot rates from the information given above.

(b) (4 points) Company D expects a cash inflow of $100,000 at the end of Year 1 but has no need for it until the end of Year 3. What can it do to lock in an interest rate between years 1 and 3 (assume forward contracts are available)? How much will it have three years from now?

(c) (8 points) Trader E observes that a two-year risk-free bond with a face value of $100 and an annual coupon rate of 3% is priced at par (face value) in the market. Is there an arbitrage opportunity for this trader? If so, detail an arbitrage strategy using this bond and portfolios A, B, and C that yields a positive profit of $10,000 today (assume these portfolios can be bought and sold in the market).

3. (13 points) MANE Co. is a New England supermarket chain with a cost of capital of 10%. The company is public with 1 million shares outstanding and the investment cost of opening a new supermarket is $6 million. Each of its current supermarket stores has a return over investment of 12% per year. Today $(t = 0)$, the market expects that earnings per share (EPS) for next year $(t = 1)$ will be $10.

   (a) (4 points) At $t = 1$, MANE Co. expects to have the possibility to invest in one new supermarket to be opened in the coming year $(t = 2)$. What should be the plowback ratio b at $t = 1$ to finance this new store investment?

(b) (3 points) If MANE Co. is able to keep expanding its business at the same pace as in $t = 1$, so that both the plowback ratio b and the return over investment remain constant forever, what is the present value of its growth opportunities (PVGO)?

(c) (3 points) CNBC just announced that Kwik-E-Mart, an international supermarket powerhouse, will enter the New England market immediately. This is expected to drive the return over investment for all the stores of MANE Co. down to 8% per year permanently. Keeping the plowback ratio from part a, what is the PVGO now?

(d) (3 points) Based on part c, is it a good idea for MANE Co. to open new super-markets? Why?

4. (15 points) You manage a pension fund, and your liabilities consist of three payments as follows:

Payment	Amount	Maturity
1	$5M	5 year
2	$7M	10 year
3	$12M	15 year

The current yield curve of riskless bonds is flat at 6%.

(a) (3 points) What is the present value (PV) of your liabilities?

(b) (3 points) What is the duration of your liabilities?

(c) (4 points) If interest rates increase by 25 basis points (100 basis points = 1%), by how much approximately would the value of your liabilities change? (Note: assume that the whole curve increases by 25 basis points.)

(d) (5 points) Suppose that your pension fund is fully funded: the plan has a cash amount equal to the PV of its future liabilities. The fund can invest in a 1-year zero coupon Treasury bond and a 30-year zero coupon Treasury bond. What is the fraction of the cash you would like to invest in each of these two bonds so that the pension fund is immunized from small interest rate variations?

5. (12 points) You own the right to exploit a copper mine. The followings are the forward prices for copper at the end of October 2010, along with the corresponding annualized interest rates:

Date (End of month)	Jan 2011	October 2011	Jan 2012
Forward Price (US dollars / pound)	3.7860	3.7915	3.7625
Spot rates	0.13%	0.18%	0.22%

The spot price of copper is $3.7750 per pound.

(a) (4 points) Compute the net convenience yield (as an effective annual rate) of copper for each of these maturities using the market information provided above.

(b) (4 points) Your expert engineer estimated that the production of copper should be 369,890 million pounds at the end of January 2012 (this is the only date with production), and that your costs are given by the following table:

Date (End of month)	Jan 2011	October 2011	Jan 2012
Cost (Millions of US Dollars)	323,864.80	323,864.80	539,774.67

Your opportunity cost of capital is 12%.

What is the present value of your business if you fully hedge the risk of copper price fluctuations using forward prices?

(c) (4 points) Assume that you decided to fully hedge the risk of copper price fluctu-
ations. If at the end of January 2012 the spot price of copper is $3.40 per pound,
how much would have you gained or lost at that time?

Extra space

Extra space

Last Name: _____ First Name: _____

M.I.T. ID# _____ Section: _____

(10:30-12:00pm = C, 1-2:30pm = D)

15.401 Midterm Exam

A. Verdelhan

Fall 2010

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- You are allowed one $8\frac{1}{2}'' \times 11''$ sheet of formulas and one calculator.

- Answer these questions without consulting anyone.

- Use the space provided. If more space is needed, use the two extra pages provided at the end of the exam.

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Good luck!

1

Name: _____ MIT ID#: _____

15.401 Midterm Examination 2010 Grade Sheet

1. _____ / 24

2. _____ / 16

3. _____ / 13

4. _____ / 15

5. _____ / 12

Total _____ / 80

2

1. (24 points) True, false or "it depends"? Give a brief explanation for each answer.

(a) (4 points) "The return from a one-year CD (Certificate of Deposit) that offers a 10% APR with semiannual compounding is higher than the return from a one-year CD that offers a 10% EAR."

True. The return corresponding to a 10% APR with semiannual compounding is

$$\left(1 + \frac{10\%}{2}\right)^2 - 1 = 10.25\%,$$

which is higher than 10%.

(b) (4 points) "If the net convenience yield of a future contract is positive (storage costs are low and convenience yields are high for example), futures price will be higher than the spot price adjusted for the time value of money."

False. Positive net convenience yield makes buying the asset at $T = 0$ relatively more attractive than entering into a futures contract with maturity T. Thus, the future price will be lower than the spot price adjusted for the time value of money. Recall the formula for futures price:

$$F_T = S(1 + r)^T - FV(\text{net convenience yield}).$$

(c) (4 points) "Assume a flat term structure of interest rates. Rising interest rates will shorten the duration of a coupon bond."

True. Duration of a bond is the present-value weighted average maturity of all the cash flows from the bond. Higher interest will reduce the present value of cash flows with longer maturity (coupons and principal that are due far into the future) more than those that have short maturity. As a result, the weights on those long maturities will fall, and those on the short maturities will rise, causing the weighted average maturity to drop.

Comments: Many students put that it is FALSE that rising interest rates will shorten the duration of a coupon bond. It is true that the value of the bond will decrease but the sum of the present value weighted maturity of cash flows will decrease to a greater extent as longer maturities are more affected by rising interest rates.

(d) (4 points) "According to Bloomberg, the yield curve on October 26, 2010 is upward-sloping (spot interest rates increase with maturity). This implies that investors expect the Federal Reserve to raise short term interest rates in the future."

False. According to the liquidity preference hypothesis, even if the short term interest rates are expected to remain constant, the yield curve can still be upward-sloping when investors demand higher (liquidity) premium for holding longer term bonds than shorter term bonds.

(e) (4 points) "If next year's expected dividend payment of companies A and B are the same and the expected dividend growth of company A is higher than the expected dividend growth of company B, then company A must have a higher price than company B."

False. According to the Gordon formula,

$$P_t = \frac{D_{t+1}}{r - g}$$

The two companies have the same dividends for next year. Even though the growth rate g is higher for company A, if company B has a sufficiently low discount rate r, its stock price can be higher than company A's.

Comments: One common mistake was to assume that the cost of capital was the same for both companies.

(f) (4 points) "If an investor purchases a two-year bond (with annual coupon payments), holds it to maturity and reinvests all coupons at the appropriate forward rates determined today, then she will receive an annualized return equal to today's two-year spot rate."

True. When buying a two-year coupon bond, one receives cash flows at the end of year 1 and 2. However, if coupon in year 1 is reinvested at the forward rate f_2 today (earning an interest rate of f_2 for year 2), all the cash flows from the bond will be received at the end of year 2, and there will be no uncertainty involved. This effectively converts the two-year coupon bond into a two-year zero-coupon bond, which should earn an annual return equal to the two-year spot rate.

Comments: Most common mistake was to say FALSE since the rates at reinvestment would be different without realizing that the exercise assumes the forward rates are determined TODAY.

2. (16 points) Three institutional investors, A, B and C, hold portfolios of risk-free bonds. The value of these portfolios and their payoffs are as follows:

Portfolio	Value ($)	Payoffs
A	667,572	$700,000 in two years
B	772,929	$300,000 in one year, $500,000 in two years
C	45,614	$550,000 in one year, -$550,000 in three years

(a) (4 points) Infer the one-, two-, and three-year spot rates from the information given above.

Solution: Let us find these three spot rates:

- $r_2 = \left(\frac{700,000}{667,572}\right)^{1/2} - 1 = 2.40\%$

- $772,929 = \frac{300,000}{1+r_1} + \frac{500,000}{(1+r_2)^2} \implies r_1 = 1.32\%$

- $45,614 = 550,000\left(\frac{1}{1+r_1} - \frac{1}{(1+r_3)^3}\right) \implies r_3 = 3.42\%$

(b) (4 points) Company D expects a cash inflow of $100,000 at the end of Year 1 but has no need for it until the end of Year 3. What can it do to lock in an interest rate between years 1 and 3 (assume forward contracts are available)? How much will it have three years from now?

Solution: Company can use a forward contract to lock in the interest rate between year 1 and year 3. The forward rate is $f_{1,3} = \left[\frac{(1+r_3)^3}{1+r_1}\right]^{1/2} - 1 = 4.49\%$. Three years from now, company D will have: $100,000 \times (1 + f_{1,3})^2 = \$109,173.80$.

7

(c) (8 points) Trader E observes that a two-year risk-free bond with a face value of $100 and an annual coupon rate of 3% is priced at par (face value) in the market. Is there an arbitrage opportunity for this trader? If so, detail an arbitrage strategy using this bond and portfolios A, B, and C that yields a positive profit of $10,000 today (assume these portfolios can be bought and sold in the market).

Solution: The value of the bond, $V = \frac{3}{1+r_1} + \frac{103}{(1+r_2)^2} = 101.19$, exceeds its price, so it is undervalued. This bond is cheap. An arbitrage strategy involves buying this bond and shorting (e.g selling) fairly priced bonds.

For each cheap bond we buy, we expect to make a profit of 101.19-100=1.19. In order to make a total profit of $10,000, we thus need to buy 8,403 of these cheap bonds.

This implies a cash flow of 3*8,403=$25,209 at date 1 and 103*8,403=$865,509 at date 2. We will received these amounts. We need to offset these cash flows. We are going to use portfolios A and B to do that.

If we sell a fraction 25,209/300,00 = 0.084 of portfolio B, we will have to pay 25,209 at date 1. This offsets our first cash flow. But it will imply a payment of 0.084*500,000=$42,000 at date 2.

We thus still have to offset a positive cash flow of 865,509- 42,000 = $823,509 at date 2. We can do this by selling some shares of portfolio A. We will sell 823,509/700,000= 1.18 shares of portfolio A.

Let us check our result: we will receive 1.18 times the value of portfolio A (e.g 667,572) and 0.084 times the value of portfolio B (e.g 772,929). We will pay 100 times the number of bonds (8,403). The difference is our profit, e.g $10,000.

8

3. (13 points) MANE Co. is a New England supermarket chain with a cost of capital of 10%. The company is public with 1 million shares outstanding and the investment cost of opening a new supermarket is $6 million. Each of its current supermarket stores has a return over investment of 12% per year. Today ($t = 0$), the market expects that earnings per share (EPS) for next year ($t = 1$) will be $10.

(a) (4 points) At $t = 1$, MANE Co. expects to have the possibility to invest in one new supermarket to be opened in the coming year ($t = 2$). What should be the plowback ratio b at $t = 1$ to finance this new store investment?

Solution: Total earnings for year 1 is $EPS_1 \times$ shares outstanding $= \$10 \times 1mm = \$10mm$. To finance the investment of $6mm, the plowback ratio has to be:

$$b = 6/10 = 60\%.$$

Comments: Some students assumed that $EPS = \$10$ happen at $t = 0$, thus they get the answer of 6million/$(1 + 10\%)/((EPS * $ number of outstanding shares$) = 54.5\%$

(b) (3 points) If MANE Co. is able to keep expanding its business at the same pace as in $t = 1$, so that both the plowback ratio b and the return over investment remain constant forever, what is the present value of its growth opportunities (PVGO)?

Solution: The growth rate will be

$$g = ROE \times b = 12\% \times 60\% = 7.2\%.$$

DPS for year 1 is $EPS_1 \times (1 - b) = \$4$. The value of the stock will be

$$P_0 = \frac{DPS_1}{r - g} = \frac{4}{10\% - 7.2\%} = \$142.86.$$

Thus,

$$PVGO = P_0 - \frac{EPS_1}{r} = 142.86 - \frac{10}{10\%} = \$42.86.$$

(c) (3 points) CNBC just announced that Kwik-E-Mart, an international supermarket powerhouse, will enter the New England market immediately. This is expected to drive the return over investment for all the stores of MANE Co. down to 8% per year permanently. Keeping the plowback ratio from part a, what is the PVGO now?

Solution: First we need to figure out the initial book value using information based on the old ROE:

$$BVPS_0 = \frac{EPS_1}{ROE} = \frac{10}{12\%} = \$83.33.$$

Next, based on the new and lower ROE, the expected EPS for year 1 should be revised from $10 to

$$EPS_1 = 83.33 \times 8\% = \$6.67.$$

Keeping the plowback ratio unchanged at $b = 0.6$, the new growth rate will be

$$g = 8\% \times 60\% = 4.8\%,$$

and the new stock price will be

$$P_0 = \frac{EPS_1 \times (1-b)}{r-g} = \frac{6.67(1-0.6)}{10\% - 4.8\%} = \$51.31.$$

Thus, the new PVGO will be

$$PVGO = P_0 - \frac{EPS_1}{r} = 51.31 - \frac{6.67}{10\%} = -\$15.39.$$

Comments: Most students ignored the impact of reduced ROE towards EPS1, and assumed EPS1 is unchanged at $10, so that the only change is the growth rate g. But as ROE changes, both g and the expected EPS1 should change as well.

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(d) (3 points) Based on part c, is it a good idea for MANE Co. to open new super-markets? Why?

Solution: It is not a good idea to open new supermarkets since the PVGO is negative. This happens because ROE is lower than the cost of capital; hence investors would be better of investing their money elsewhere than in new supermarkets of MANE Co.

12

4. (15 points) You manage a pension fund, and your liabilities consist of three payments as follows:

Payment	Amount	Maturity
1	$5M	5 year
2	$7M	10 year
3	$12M	15 year

The current yield curve of riskless bonds is flat at 6%.

(a) (3 points) What is the present value (PV) of your liabilities?

Solution: Let us compute the present value of each payment:

- $PV(1) = 5/1.06^5 = 3.73629M$
- $PV(2) = 7/1.06^{10} = 3.9088M$
- $PV(3) = 12/1.06^{15} = 5.0072M$

The present value of all liabilities is thus: PV of liabilities = $PV(1) + PV(2) + PV(2)$ + $PV(3) = 12.6522$. You can report it as a negative number since these are liabilities.

(b) (3 points) What is the duration of your liabilities?

Solution: The duration of each payment is simple:

- Duration of payment 1: 5 years,
- Duration of payment 2: 10 years,
- Duration of payment 3: 15 years.

The duration of all liabilities is the weighted average of all these durations: $5 * 3.73 + 10 * 3.9 + 15 * 5.0/(3.73 + 3.9 + 5) = 10.502239$ years.

(c) (4 points) If interest rates increase by 25 basis points (100 basis points = 1%), by how much approximately would the value of your liabilities change? (Note: assume that the whole curve increases by 25 basis points.)

Solution: Let us use the duration formula: $dB/dy = -B * D/(1+y)$, where y denotes the yield, D the duration and B the value of the bond (here the present value of total liabilities).

$$[12.6522 * 10.502239/(1 + 0.06)] * 0.0025 = 0.3133887$$

Recall that if yields increase, bond prices decrease. Same reasoning here: if interest rates increase, bond prices decrease. If interest rates increase, the present value of all liabilities will decrease. If interest rates increase by 25 basis points, our liabilities would decrease by $0.3134M to $12.3389M.

Comments: A common mistake estimating the change was to confuse duration and modified duration on the formula. Furthermore some students did not calculate the actual value change only the percentage change. Finally some students refer to the value change as a percentage change, i.e 0.3134% instead of $0.3134M.

(d) (5 points) Suppose that your pension fund is fully funded: the plan has a cash amount equal to the PV of its future liabilities. The fund can invest in a 1-year zero coupon Treasury bond and a 30-year zero coupon Treasury bond. What is the fraction of the cash you would like to invest in each of these two bonds so that the pension fund is immunized from small interest rate variations?

Solution: If the plan is fully funded, you have $12.6522 millions. The duration of the 1-year TBill is 1 year. The duration of the 30-year T bond is 30 years. A combination of both has a duration of $[x*1 + (12.6522 - x)*30]/12.6522$, where x denotes the dollar amount invested in the 1-year TBill. Let us find x such as asset and liability durations match:

$$[x*1 + (12.6522 - x) * 30]/12.6522 = 10.502239$$

We obtain $x = \$8.5065M$. To immunize the portfolio, we want to allocate $8.5065M (67%) in 1-year TBills and $4.1457M in 30-year bonds (33%).

5. (12 points) You own the right to exploit a copper mine. The followings are the forward prices for copper at the end of October 2010, along with the corresponding annualized interest rates:

Date (End of month)	Jan 2011	October 2011	Jan 2012
Forward Price (US dollars / pound)	3.7860	3.7915	3.7625
Spot rates	0.13%	0.18%	0.22%

The spot price of copper is $3.7750 per pound.

(a) (4 points) Compute the net convenience yield (as an effective annual rate) of copper for each of these maturities using the market information provided above.

Solution: Using $F_{0,T} = S_0(1 + r_f - \hat{y})^T$ we have $\hat{y} = 1 + r_f - \left(\frac{F_{0,T}}{S_0}\right)^{\frac{1}{T}}$
Then at 3m -1.04%, 12m -0.26%, and 15m 0.49%

Comments: Some students forgot to take the power of T or that T should be in years not months.

(b) (4 points) Your expert engineer estimated that the production of copper should be 369,890 million pounds at the end of January 2012 (this is the only date with production), and that your costs are given by the following table:

Date (End of month)	Jan 2011	October 2011	Jan 2012
Cost (Millions of US Dollars)	323,864.80	323,864.80	539,774.67

Your opportunity cost of capital is 12%.

Solution: What is the present value of your business if you fully hedge the risk of copper price fluctuations using forward prices?

$$PV_{cost} = \frac{323864.80}{(1+12\%)^{\frac{1}{12}}} + \frac{323864.80}{(1+12\%)^{\frac{11}{12}}} + \frac{539774.67}{(1+12\%)^{\frac{14}{12}}} = 1,072,461.61M \ USD$$

$$PV_{benefir} = \frac{F_{0,15}Production}{(1+r_f)^{\frac{14}{12}}} = \frac{376.25}{100}\frac{369890}{(1+0.22\%)^{\frac{14}{12}}} = 1,387,893.37M \ USD$$

$$NPV = 315,431.76$$

Comments: The cash flow from the hedged copper production should be discounted at the risk-free rate while production cost should be discounted at the cost of capital.

(c) (4 points) Assume that you decided to fully hedge the risk of copper price fluctuations. If at the end of January 2012 the spot price of copper is $3.40 per pound, how much would have you gained or lost at that time?

Solution: Profit per contract is $F - S = 3.7625 - 3.4000 = 0.3625$.

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20

Name: _____ MIT ID#: _____

Extra space

Last Name: _____ First Name: _____

M.I.T. ID# or S.S.#: _____

Section C (MW 1:00-2:30pm): _____ Section D (MW 10:00-11:30am): _____

15.401 Midterm Exam

Professor Joslin

Spring 2008

Instructions:

- Please fill in your name, ID number, and check your section number.

- The exam lasts 80 minutes. It consists of six questions. Please answer all of them. Questions 4(d) and 6 are bonus questions.

- Credit for each question is noted below. The maximum score is 100 points. Bonus points increase your score to a maximum of 100 points.

Question	Points Possible	Points Received
1	20	
2	20	
3	20	
4	25	
5	15	
Bonus	10 (3/7)	
Total	100	

- You are allowed one $8\frac{1}{2}$" ×11" sheet of formulas and one calculator.

- Answer these questions without consulting anyone.

- Use the space provided. If more space is needed, use the other side.

- Be neat and show your work. Except where stated, answers without work receive no credit. Wrong answers with partially correct work may receive partial credit.

- Allocate your time optimally.

Good luck!

1. **(20 points, 2 points each)** Circle True or False. No explanations are necessary.

   (a) True or False. A problem with the discounted cash flow model is that while it takes into account the timing of cash flows, it does not take into account their riskiness.

   (b) True or False. Shareholders of a firm in general would like to have more money. They may also prefer money at certain times. Firm management should take shareholders' desire for money at certain times into account when deciding investment and dividend policy.

   (c) True or False. A certain coupon bond has a modified duration of 7.8 years. A hedge fund has a long position in the coupon bond worth $10 million. A short position of $5 million of 4 year zero coupon bonds would provide a reasonably good hedge against small changes in the level of interest rates.

   (d) True or False. The default premium for investment-grade corporate bonds is usually lower than the default premium for junk bonds.

   (e) True or False. If the yield curve is upwards sloping (i.e. spot rates are higher for higher maturities), it must be that it is expected that interest rates will rise in the future.

   (f) True or False. Equity is a residual claim in the sense that equity holders have claim to to a firm's assets and cash flows only after debt holders are paid.

   (g) True or False. The DCF model for valuing stocks applies only if the buyer intends to hold the stock long enough to collect all future dividends from the firm.

   (h) True or False. Depreciation is import to consider in valuing an investment project because it results in a cash flow through a tax credit.

   (i) True or False. The present value of growth opportunities (PVGO) can only be positive only if a firm earns more than the cost of capital. That is, only if the return on equity (ROE) is higher than the cost of capital.

   (j) True or False. Both a 2-year and 10-year treasury will have the same return in any given year because they are both riskless fixed income securities.

1

2. **(20 points)**

On a job interview, you were handed the following quotes on U.S. Treasuries:

Bond	Maturity (years)	Coupon Rate	Yield to Maturity
1	1	5%	4.5%
2	2	5%	5.0%
3	3	0%	5.5%

Assume that the par value is $100 and coupons are paid annually, with the first coupon payment coming in exactly one year from now. The yield to maturity is also quoted as an annual rate. You are then asked the following questions:

(a) **(8 Points)** What should be the price of a bond with a maturity of 3 years and coupon rate of 5%, given the above information? (Note: this is a different bond than the 3-year bond in the table.)

(b) **(4 points)** What should be the 1-year forward rate between years 2 and 3?

(c) **(4 points)** What is the modified duration of a bond portfolio with 30% invested in bond 1 and 70% invested in bond 3?

(d) **(4 points)** How much would the value of the portfolio in (c) change if the yields of all bonds increased by 0.15%?

3. **(20 points)** A consol bond is bond which pays perpetual coupon payments and no maturity. Suppose that the term structure is flat at 10%. Also, assume that there is no risk of default.

(a) **(10 points)** Compute the price of a consol bond promising annual payments of £1. Assume that the first payment is exactly 1 year from now.

(b) **(5 points)** If interest rates rise by 1 basis point to 10.01%, will the value of the consol bond rise or fall?

(c) **(5 points)** Compute the new value when interest rates are 10.01%.

4. **(25 points)** Texas Western (TW) has a current book value per share of $10.00. During the next four years, Texas Western has a return on equity of 20%. Texas Western plans to maintain a payout ratio of .50. After year 4, TW will lose its competitive advantage and the firm's ROE will drop to 5%. Firm management has decided in this case the best thing to do will be to liquidate the firm and payout the entire firm value as a dividend. That is, the dividend in Year 4 will be equal to all of the earnings in year 4 plus the BVPS in Year 3, there will be no further cash flows or investments from TW. The cost of capital for TW is 10%.

   (a) **(10 points)** Complete the following table:

Year	0	1	2	3	4
EPS	-				
Investment	-				
Dividend	-				
BVPS	10				
PV of Dividend	-				

   Note that in Year 4, the firm will have a negative investment (since it is selling assets, not buying them) equal to the BVPS at the beginning of the year.

   (b) **(5 points)** Compute the share price of TW at $t = 0$.

(c) **(10 points)** Suppose that TW increases the payout ratio to .7 and the ROE remains at 20%. Will the value of TW increase or decrease? Justify your answer. You may use computations to support your answer, but for full credit you must explain your answer intuitively.

(d) **(3 bonus points)** Do you agree or disagree with management's decision to liquidate the firm when the ROE drops to 5%? Explain why.

5. **(15 points)** XYZ Software is considering investing in developing a new software program. This will require XYZ to invest $500 million in year 0 to build a new development center. For tax purposes, this investment can be depreciated over 10 years with straight line depreciation. The software is anticipated to generate pre-tax profits of $100 million every year for the next 10 years. Additionally, beginning in year 1, $50 million will be required in working capital which will be fullly recovered at the end of life of the software in year 10. XYZ's tax rate is 40%.

(a) **(7 points)** Fill in the table below which summarizes the cash flows and accruals of XYZ in years 1 to 10.

Year	Investment	Working Capital	Operating Revenue	Depreciation	Taxes	After-tax Cash Flow
0						
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						

(b) **(8 points, 2 points each)** Suppose that XYZ is considering another project and after analysis, have concluded the following information:

	Original Project	New Project
IRR	8.5%	9.7%
NPV	$100.8M	$39.8M
Payback Period	7	6

- According to the IRR criterion, which project should be done?

- According to the NPV criterion, which project should be done?

- According to the Payback Criterion, which project should be done?

- Which project would you recommend XYZ to invest in?

6. **(Bonus: 7 points)** XYZ Financial Innovations plans to introduce a new financial contract. The contract allows one, for a price, to commit to buy a coupon bond in the future at a fixed price. The contract is as follows:

- At initiation $(t = 0)$, purchasers of the contract pay a premium, p.

- XYZ becomes obligated to sell a coupon bond in exactly 1 year for $100. Specifically, the purchasers of the contract must pay XYZ $100 in year 1 and at that time XYZ must in turn deliver a 2-year coupon bond which pays annual coupon payments of $5 with principal of $100.

- This is an obligation – in one year, the contract purchases must pay $100 and XYZ must deliver the coupon bond.

XYZ is willing to both buy and sell this contract at the given premium p. Suppose the term structure of spot interest rates is flat at 3%. (The spot yield is 3% for all maturities.)

Assume also that all cash flows are riskless.

(a) **(1 point)** Create a table with the cash flows at $t = 0, 1, 2, 3$ for a long position in XYZs contract as well as a 1-, 2-, and 3-year zero coupon bonds.

(b) **(1 point)** Construct a portfolio of zero coupon bonds positions (long & short) that has the same cash flows as XYZs contract in years 1,2, and 3.

(c) **(2 point)** What should p be?

(d) **(1 point)** Suppose that XYZ is willing to both buy and sell the contract for $1. In exactly 4 words, describe how you can make money based on this price.

(e) **(2 point)** Describe explicitly how you can construct an arbitrage.

15.401 Midterm Exam Solution

Professor Joslin

Spring 2008

1. (20 points, 2 points each) Circle True or False. No explanations are necessary.

(a) True or False. A problem with the discounted cash flow model is that while it takes into account the timing of cash flows, it does not take into account their riskiness.

False. The discount rate used in NPV takes care of riskiness of cashflow.

(b) True or False. Shareholders of a firm in general would like to have more money. They may also prefer money at certain times. Firm management should take shareholders' desire for money at certain times into account when deciding investment and dividend policy.

False. Shareholders can sell the stock when they need money, so the managers need not be concerned.

(c) True or False. A certain coupon bond has a modified duration of 7.8 years. A hedge fund has a long position in the coupon bond worth $10 million. A short position of $5 million of 4 year zero coupon bonds would provide a reasonably good hedge against small changes in the level of interest rates.

False. You're shorting too little as MD of the 4-year bond is lower than 7.8.

(d) True or False. The default premium for investment-grade corporate bonds is usually lower than the default premium for junk bonds.

True. The default premium is high if the default rate is high. Since junk bonds default more than investment grade bonds, the default premium of junk bond is higher.

(e) True or False. If the yield curve is upwards sloping (i.e. spot rates are higher for higher maturities), it must be that it is expected that interest rates will rise in the future.

False. Long maturity yields reflect both expectations of future interest rates and risk premia.

(f) True or False. Equity is a residual claim in the sense that equity holders have claim to a firm's assets and cash flows only after debt holders are paid.

True. Stockholders are last in line to claim the firm's assets.

(g) True or False. The DCF model for valuing stocks applies only if the buyer intends to hold the stock long enough to collect all future dividends from the firm.

False. The DCF model does not need such an assumption. If you invest in short term, you can resell your stock at the value of the future dividends. Therefore the value of the stock to short-term and long-term investors are the same.

(h) True or False. Depreciation is import to consider in valuing an investment project because it results in a cash flow through a tax credit.

True. That's exactly how it works.

(i) True or False. The present value of growth opportunities (PVGO) can only be positive only if a firm earns more than the cost of capital. That is, only if the return on equity (ROE) is higher than the cost of capital.

True. That's the only way the firm has positive NPV investments

(j) True or False. Both a 2-year and 10-year treasury will have the same return in any given year because they are both riskless fixed income securities.

False. This is almost never true.

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2. (20 points)

On a job interview, you were handed the following quotes on U.S. Treasuries:

Bond	Maturity (years)	Coupon Rate	Yield to Maturity
1	1	5%	4.5%
2	2	5%	5.0%
3	3	0%	5.5%

Assume that the par value is $100 and coupons are paid annually, with the first coupon payment coming in exactly one year from now. The yield to maturity is also quoted as an annual rate. You are then asked the following questions:

(a) **(8 Points)** What should be the price of a bond with a maturity of 3 years and coupon rate of 5%, given the above information? (Note: this is a different bond than the 3-year bond in the table.)

Solution:

First, we compute the spot rates. The one and three year spot rates are $r_1 = 4.5\%$ and $r_3 = 5.5\%$, since the corresponding bonds in the table are zero coupon bonds. For the given 2-year coupon bond, we can compute the price of the bond from the definition of YTM as:

$$B_2 = \frac{\$5}{1+.05} + \frac{\$105}{(1+.05)^2}$$
$$= \$100$$

But the price is also computed as:

$$B_2 = \frac{\$5}{1+r_1} + \frac{\$105}{(1+r_2)^2}$$

Solving $B_2 = \$100$ gives $r_2 = 5.0126\%$.

Thus the price of the 5% 3-year coupon bond is

$$price = \frac{5}{1+r_1} + \frac{\$5}{(1+r_2)^2} + \frac{\$105}{(1+r_3)^3}$$
$$= \$98.7382$$

(b) **(4 points)** What should be the 1-year forward rate between years 2 and 3?

Solution:

$$f_{2,3} = \frac{(1+r_3)^3}{(1+r_2)^2} - 1 = 6.4817\%$$

(c) **(4 points)** What is the modified duration of a bond portfolio with 30% invested in bond 1 and 70% invested in bond 3?

Solution:

$$MD(\text{bond } 1) = \frac{1}{1+.045}$$
$$= 0.9569 \text{ years}$$

$$MD(\text{bond } 3) = \frac{3}{1+.055}$$
$$= 2.8436 \text{ years}$$

$$MD(\text{portfolio}) = .30 \cdot 0.9569 + .70 \cdot 2.8436$$
$$= 2.2776 \text{ years}$$

(d) **(4 points)** How much would the value of the portfolio in (c) change if the yields of all bonds increased by 0.15%?

Solution:

That means the portfolio will decrease in value by $0.15\% \cdot 2.2776 = 0.3416\%$.

Grading Comments: This is the same as the practice midterm so many got it right. Some students us bond 2's YTM as the 2-year spot rate, which is incorrect (although the number will be similar)

3. **(20 points)** A consol bond is bond which pays perpetual coupon payments and no maturity. Suppose that the term structure is flat at 10%. Also, assume that there is no risk of default.

(a) **(10 points)** Compute the price of a consol bond promising annual payments of £1. Assume that the first payment is exactly 1 year from now.

Solution:

This is a perpetuity that pays £1 each period. Therefore the value today is
$£1/.10 = £10$

(b) **(5 points)** If interest rates rise by 1 basis point to 10.01%, will the value of the consol bond rise or fall?

Solution:

As interest rates rise, the value fall. This is because the payments are fixed and there is more discounting.

(c) **(5 points)** Compute the new value when interest rates are 10.01%.

Solution:

The new price is $£1/.1001 = £9.99$

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4. **(25 points)** Texas Western (TW) has a current book value per share of $10.00. During the next four years, Texas Western has a return on equity of 20%. Texas Western plans to maintain a payout ratio of .50. After year 4, TW will lose its competitive advantage and the firm's ROE will drop to 5%. Firm management has decided in this case the best thing to do will be to liquidate the firm and payout the entire firm value as a dividend. That is, the dividend in Year 4 will be equal to all of the earnings in year 4 plus the BVPS in Year 3, there will be no further cash flows or investments from TW. The cost of capital for TW is 10%.

(a) **(10 points)** Complete the following table:

year	0	1	2	3	4
EPS	-	2	2.2	2.42	2.662
Investment	-	1	1.1	1.21	-13.31
Dividend	-	1	1.1	1.21	15.972
BVPS	10	11	12.1	13.31	0
PV of Dividend	-	0.909	0.909	0.909	10.909

Note that in Year 4, the firm will have a negative investment (since it is selling assets, not buying them) equal to the BVPS at the beginning of the year.

Solution:
Numbers entered above.

(b) **(5 points)** Compute the share price of TW at $t = 0$.

Solution:
Share price $= \sum PV(\text{dividends}) = \$13.64$

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(c) (10 points) Suppose that TW increases the payout ratio to .7 and the ROE remains at 20%. Will the value of TW increase or decrease? Justify your answer. You may use computations to support your answer, but for full credit you must explain your answer intuitively.

Solution:
The value of the firm will go down since the ROE is greater than cost of capital. It is better for the firm to invest more and payout less when the ROE is greater than the cost of capital.

(d) (3 bonus points) Do you agree or disagree with management's decision to liquidate the firm when the ROE drops to 5%? Explain why.

Solution:
Agree. Once the ROE drops, the assets are only returning 5% whereas we can make 10% with similar risk in the market. That is, we are better off investing the capital in other firms rather than keeping it in TW.

Grading Comments: Most student did part (a) and (b) correctly. For part (c), some student say that the value decreases because the dividend growth ratio decrease, without mentioning the ROE > r.

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5. **(15 points)** XYZ Software is considering investing in developing a new software program. This will require XYZ to invest $500 million in year 0 to build a new development center. For tax purposes, this investment can be depreciated over 10 years with straight line depreciation. The software is anticipated to generate pre-tax profits of $100 million every year for the next 10 years. Additionally, beginning in year 1, $50 million will be required in working capital which will be fully recovered at the end of life of the software in year 10. XYZ's tax rate is 40%.

(a) **(7 points)** Fill in the table below which summarizes the cash flows and accruals of XYZ in years 1 to 10.

Year	Investment	Working Capital	Operating Profit	Depreciation	Taxes	After-tax Cash Flow
0	-500					- 500
1		-50	100	50	20	30
2			100	50	20	80
3			100	50	20	80
4			100	50	20	80
5			100	50	20	80
6			100	50	20	80
7			100	50	20	80
8			100	50	20	80
9			100	50	20	80
10		+50	100	50	20	130

Solution:
Values are added. Note: here working capital is recorded as the change in working capital, but you also may list the accumulated WC.

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(b) **(8 points, 2 points each)** Suppose that XYZ is considering another project and after analysis, have concluded the following information:

	Original Project	New Project
IRR	8.5%	9.7%
NPV	$100.8M	$39.8M
Payback Period	7	6

- According to the IRR criterion, which project should be done?

Solution:
New project.

- According to the NPV criterion, which project should be done?

Solution:
Original project.

- According to the Payback Criterion, which project should be done?

Solution:
New project.

- Which project would you recommend XYZ to invest in?

Solution:
Original project.

6. **(Bonus: 7 points)** XYZ Financial Innovations plans to introduce a new financial contract. The contract allows one, for a price, to commit to buy a coupon bond in the future at a fixed price. The contract is as follows:

- At initiation ($t = 0$), purchasers of the contract pay a premium, p.

- XYZ becomes obligated to sell a coupon bond in exactly 1 year for $100. Specifically, the purchasers of the contract must pay XYZ $100 in year 1 and at that time XYZ must in turn deliver a 2-year coupon bond which pays annual coupon payments of $5 with principal of $100.

- This is an obligation – in one year, the contract purchases must pay $100 and XYZ must deliver the coupon bond.

XYZ is willing to both buy and sell this contract at the given premium p. Suppose the term structure of spot interest rates is flat at 3%. (The spot yield is 3% for all maturities.)

Assume also that all cash flows are riskless.

(a) **(1 point)** Create a table with the cash flows at $t = 0, 1, 2, 3$ for a long position in XYZ's contract as well as a 1-, 2-, and 3-year zero coupon bonds.

Solution:

security	year 0	year 1	year 2	year 3
XYZ	-p.	-100	5	105
1-year zero	-$100/1.03	+$100	0	0
2-year zero	-$100/1.03^2	0	+$100	0
3-year zero	-$100/1.03^3	0	0	+$100

(b) **(1 point)** Construct a portfolio of zero coupon bonds positions (long & short) that has the same cash flows as XYZ's contract in years 1,2, and 3.

Solution:
Short 1 unit of 1 year zero. Buy 0.05 unit of 2 year zero. Buy 1.05 unit of 3 year zero. (assume zeros have face value of 100)

Grading comments: Almost everyone did this question correctly.

(c) **(2 point)** What should p be?

We can compute p by discounting the cash flows:

$$p = \frac{-100}{1 + .03} + \frac{\$5}{(1 + .03)^2} + \frac{\$105}{(1 + .03)^3}$$

$$= \$3.7154$$

(d) **(1 point)** Suppose that XYZ is willing to both buy and sell the contract for $1. In exactly 4 words, describe how you can make money based on this price.

Solution:

Buy low, sell high.

(e) **(2 point)** Describe explicitly how you can construct an arbitrage.

We should buy the XYZ security (since it is relatively underpriced at \$1) and sell the replicating portfolio (since it is relatively overpriced). Selling the replicating portfolio (since it is relatively overpriced). Selling the replicating portfolio means to buy \$100 face value of one-year zeros, short \$5 face of 2-year zeros, and short \$105 face of 3-year zeros. This gives as \$3.7154 to put in over pocket with no future cash flows.

Grading Comments: A lot of students did not try this problem. For those who tried, some did not subtract the PV of the year 1 payment when computing the premium p.

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