SLOAN SCHOOL OF MANAGEMENT
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Finance Theory I – 15.401

Craig A. Stephenson Spring 2011 – Section D

Finance Theory I provides a rigorous introduction to the fundamentals of modern financial economics and financial analysis, and their applications to business problems involving valuation, risk and return, corporate investment decisions, and basic security analysis and investment management. The four major sections of the course are: (A) an introduction to the financial challenges faced by firms and households, and the principles of modern finance which are useful in tackling these challenges; (B) valuation of stocks, bonds, forwards, futures, and options; (C) risk analysis including portfolio theory, mean-variance optimization, and the Capital Asset Pricing Model; and (D) applications of finance theory to corporate finance, including capital budgeting and capital investment decisions. 15.401 is a prerequisite for all finance electives.

Class Schedule
The class meets two times each week; every Monday and Wednesday, from 8:30 a.m. to 10:00 a.m., in E51-376.

Recitations
Our course TA will hold weekly recitations during which class material will be reviewed and additional problems and exercises will be presented. Recitations will be on Fridays from 8:30 a.m. to 10:00 a.m. in E51-145.

Course Website
The website for our section is: http://stellar.mit.edu/S/course/15/sp11/15.401D/ All teaching materials including announcements, chapter and topic notes, problem sets, and office hours will be posted to this site. Solutions to problem sets will be posted no later than one week after the sets are due; they will not be distributed in paper form in class. The course calendar is also available on this site.

Teaching Assistant
The TA for section D is Marc Piette (marc.piette@sloan.mit.edu).

Office Hours
Craig Stephenson: 10:00 a.m. to 11:30 a.m. Mondays, E62-659, and by appointment.
Marc Piette: 3:00 p.m. to 4:00 p.m. Wednesdays, and by appointment. Please contact him through e-mail or phone at 646-732-8539 if you will attend his Wednesday office hours, or to make an appointment.

Administrative Assistant
Hannah Lee, E62–611, (617) 253–9747, solhee@mit.edu

Course Requirements and Grading
Course requirements include regular attendance and participation in class, seven problem sets, two case write-ups, and the midterm and final exams. The following weights will be used to determine each student's course grade:

5%	Class Participation
25%	Problem Sets and Case Study Write-Ups
25%	Midterm Exam
45%	Final Exam

Problem sets and case write-ups must be turned in by 5:00 p.m. on the stated due date. Place your problem sets and case write-ups in the 15.401 Section D box located next to Hannah Lee's work station in E62-611.

The closed-book in-class midterm exam will be on March 9th, and the closed-book final exam will be given on the MIT-scheduled final exam date (May TBA) — please reserve these dates and schedule your interviews and travel plans accordingly.

Course Materials
Textbook: R. Brealey, S. Myers, and F. Allen, *Principles of Corporate Finance*, 10th edition, Irwin/McGraw Hill.
Class Notes and Recitation Notes: Notes will be available on the course website. They contain material not found in Brealey, Myers, and Allen, and provide additional perspectives on the major themes of the course.
Problem Sets and Case Assignments: Problem sets and case assignments will be available on the course website.

Supplemental Readings (not required)
Z. Bodie, A. Kane, and A. Marcus, *Investments*, 8th edition, Irwin/McGraw Hill, 2009. BK&M focus exclusively on capital markets, and present a more rigorous and thorough analysis of investments than Brealey, Myers, and Allen.
B. Malkiel, *A Random Walk Down Wall Street*, 2007. This best-selling introduction to investing is now in its 9th edition and is as popular as ever, due to its entertaining style and thoughtful advice. Malkiel is a good entry into financial markets, particularly for those without a strong financial background.
P. Bernstein, *Capital Ideas*, Free Press, 1993: Bernstein is one of the most well-respected and influential practitioners in the financial industry, and the founding editor of the *Journal of Portfolio Management*. This is a lively and well written account of the most important ideas in academic finance, many of which were developed at MIT in the 1960's and 1970's.

Keeping Current
The financial world is constantly changing and evolving, and I strongly encourage you to follow financial and macroeconomic news in the Financial Times, the Wall Street Journal, or The Economist. I try to relate the course's topics to current financial practice, and if you find a particularly interesting article that is relevant to our work, please send it to me.

Exams
The midterm and final exams will test the depth of your understanding of the key class concepts. They will not test your ability to memorize facts, or use your calculator. Because of this, you may find them challenging exercises. When preparing for the exams, you should

review the slides together with your own class notes, the handouts, the required readings, the problem sets, the sample exams, and supplemental or suggested readings.

You are allowed one double-sided page of notes at the midterm exam and two double-sided pages of notes at the final exam. The sheets must be no larger than 8.5 X 11. There are no restrictions on the content of the formula sheets. No laptops, cell phones, or personal digital assistants are allowed while writing the exams. The final exam is cumulative, and students can request their graded final exams after the end of the semester.

Problem Sets and Cases

There will be 7 problem sets over the course of the semester. Problem sets may contain 1 Excel question, emphasizing a practical implementation of an important financial concept. The problem sets are graded on a 10 point scale. *Late problem sets will not be accepted.* You are encouraged to work in groups on the problems, but you must hand in your own copy. You also need to acknowledge any help you received on the first page of your work. The exam questions will have the same format as the problem sets (without any Excel questions); but the problem sets will be slightly easier. At your option, the grade on any problem set may be replaced by the exam scores.

There will also be 2 cases during the semester. The cases will require you to apply the skills and tools you have learned in Finance Theory 1 to a timely real-world business and financial situation.

Sloan Values

You are responsible for upholding Sloan's code of conduct, which mandates zero tolerance for cheating and plagiarism. For more details on Sloan's academic policies, please read the Document "Classroom Values @ MITSloan," which is attached as pages 7 through 10 of this syllabus.

Course and Recitation Schedule –This is an approximate schedule for the course; some material may take more or less time than is allocated here.

Session	Date	Topic	Assignment Due
1	02-Feb	Introduction to Finance	
	04-Feb	No recitation session today	
2	07-Feb	Present Value 1	
3	09-Feb	Present Value 2	
	11-Feb	Recitation: Present Value	
4	14-Feb	Fixed Income Securities 1	Problem Set 1: Present Value
5	16-Feb	Fixed Income Securities 2	
	18-Feb	Recitation: Fixed Income 1	
6	22-Feb (Tues)	Fixed Income Securities 3	
7	23-Feb	Common Stocks 1	Case 1
	25-Feb	Recitation: Fixed Income 2	
8	28-Feb	Common Stocks 2	Problem Set 2: Fixed Income
9	02-Mar	Common Stocks 3	
	04-Mar	Recitation: Common Stocks	
10	07-Mar	Mid-Term Review	Problem Set 3: Common Stocks
11	09-Mar	In-Class Mid-Term Exam	
	11-Mar	Recitation: Mid-Term Exams returned	
12	14-Mar	Forwards & Futures 1	
13	16-Mar	Forwards & Futures 2	
	18-Mar	Recitation: Forwards & Futures	
14	28-Mar	Options 1	
15	30-Mar	Options 2	
	01-Apr	Recitation: Forwards & Futures	
16	04-Apr	Options 3	Problem Set 4: Forwards & Futures
17	06-Apr	Introduction to Risk & Return	
	08-Apr	Recitation: Options	
18	11-Apr	Portfolio Theory 1	Problem Set 5: Options
19	13-Apr	Portfolio Theory 2	
	15-Apr	Recitation: Portfolio Theory	
20	20-Apr	CAPM 1	Problem Set 6: Portfolio Theory
	22-Apr	Recitation: CAPM	
21	25-Apr	CAPM 2	
22	27-Apr	CAPM 3	
	29-Apr	Recitation: CAPM	
23	02-May	Capital Budgeting 1	Problem Set 7: CAPM
24	04-May	Capital Budgeting 2	
	06-May	Recitation: Capital Budgeting	Case 2
25	09-May	Market Efficiency	
26	11-May	Final Review	
	13-May	Recitation: Final Review	
	XX-May	Final Exam	

Course Outline

Chapters listed below refer to the main course textbook, Brealey, Myers, and Allen (10th edition).

PART A. INTRODUCTION

Feb 02 Introduction to Finance and Course Overview – Chapter 1

- Financial decisions of households and corporations
- Approaches to valuing financial and real assets
- Opportunity cost of capital
- The role of financial markets
- Unifying principles of finance

Feb 07 & 09	Present Value – Chapter 2 - Present value (PV) and net present value (NPV) - Discount rates and the time value of money - Compound interest - Annuity and perpetuity formulas - Real vs. nominal cash flows
PART B.	**VALUATION**
Feb 14, 16 & 22	Fixed Income Securities – Chapters 3, 23 & 24 - Fixed-income markets - Term structure of interest rates - Properties of bond prices and market conventions - Interest rate risk - Inflation risk & credit risk
Feb 23 & 28, Mar 02	Common Stocks – Chapter 4 - Discounted cash flow (DCF) model - EPS, P/E, PVGO
Mar 07	Mid-Term Review
Mar 09	Mid-Term Examination (in-class, closed-book)
Mar 14 & 16	Forwards & Futures – Chapter 26 - Definitions of forwards and futures - Arbitrage pricing relationships - Using forwards and futures to hedge risks
Mar 28 & 30, Apr 04	Options – Chapters 20 & 21 - Definition of options - Basic properties of options - Binomial and Black-Scholes option pricing models
PART C.	**RISK & RETURN**
Apr 06	Introduction to Risk & Return – Chapters 7 & 8 - Historical asset returns - Risk / return trade-off
Apr 11 & 13	Risk Analytics / Portfolio Theory – Chapters 7 & 8 - Measures of risk - Diversification - Systematic and idiosyncratic risks - Portfolio optimization - Efficient risk / return trade-offs
Apr 20, 25 & 27	Capital Asset Pricing Model (CAPM) – Chapters 7 & 8 - The CAPM and linear risk / return trade-offs - Applications of the CAPM - Empirical evidence and extensions of the CAPM

PART D.	**CORPORATE FINANCE**
May 02	Capital Budgeting 1 – Chapters 5, 6 & 9 - Capital budgeting criteria - NPV rule, cash flow calculations & discount rates
May 04	Capital Budgeting 2 – Chapter 22 - Project interactions - Real options
May 09	Market Efficiency – Chapter 13 - Origins of the Efficient Markets Hypothesis - Implications and empirical tests of the EMH - Recent developments
May 11	Final Review
May XX	Final Examination (in-class, closed book)

MITSloan MANAGEMENT

Classroom Values@MITSloan

The MIT Sloan Mission *http://mitsloan.mit.edu/about/mission.php*

The mission of the MIT Sloan School of Management is to develop principled, innovative leaders who improve the world and to generate ideas that advance management practice.

Values@MIT Sloan
https://sloanpoint.mit.edu/administration/values/Pages/default.aspx

The MIT Sloan Mission statement (above) provides the context for core values that express who we are at our best. These core values include ***integrity, respect, collaboration, innovation,*** and ***positive impact.*** We invite all members of our community – students, staff, faculty, alumni – to practice these values in all the ways we work together, both inside and outside of the classroom.

MIT Sloan Policy on Classroom Behavior

In order to create a productive learning environment and to ensure mutual respect it is essential that the norms and rules of classroom etiquette and behavior reflect the highest standards. It is also important that these norms be consistently enforced by the faculty across all classes. Although in the final analysis each faculty member is responsible for his or her own classroom, there are significant negative consequences for other faculty and for the School if rules are not consistent and are not enforced. Therefore it is the policy of the MIT Sloan School that

- Students are expected to arrive promptly on time and to stay for the entire class.
- Faculty are expected to begin and end class on time.
- Laptops and e-readers not be open in the classroom except with explicit permission of the faculty (e.g., when used to deliver an e-course pack or otherwise used as part of the instructional program or when required by students because of physical or other challenges)
- Cellphones and PDAs are not be used or permitted to ring in the classroom.
- Students are expected to attend all classes.

It is expected that faculty will articulate how these rules apply in their class as well as how the rules will be enforced.

MIT Sloan Career Development Office Recruiting Policy

Students are required to schedule campus interviews outside of scheduled class times and to make every attempt to schedule second round interviews and site visits outside of class times. Classes missed for such activities are *not* excused absences and may count against your participation grade.

ACADEMIC HONESTY – INTEGRITY IN PRACTICE

As a member of the MIT Sloan academic community, you are expected to uphold the highest standards of academic integrity. Violations of academic integrity include, but are not limited to, cheating, plagiarism, unauthorized collaboration, and facilitating academic dishonesty. Please see the document Academic Integrity at the Massachusetts Institute of Technology: A Handbook for Students for further discussion of this topic. These standards are also discussed below, specifically regarding plagiarism, individual work, and team work.

It is your responsibility to make yourself aware of MIT's rules of academic integrity and to adhere to them. When students are found to have violated academic standards, disciplinary action will result. Possible consequences include grade reduction, an F grade, a transcript notation, delay of graduation, or expulsion from MIT.

This discussion of academic integrity below is not exhaustive, and there may be areas that remain unclear to you. ***If you are unsure whether some particular course of action is proper, it is your responsibility to consult with your professor and/or teaching assistant for clarification.***

Plagiarism

Plagiarism occurs when you use another's intellectual property (words or ideas) and do not acknowledge that you have done so. Plagiarism is a very serious offense. If it is found that you have plagiarized -- deliberately or inadvertently -- you will face serious consequences, as indicated above.

The best way to avoid plagiarism is to cite your sources - both within the body of your assignment and in a bibliography of sources you used at the end of your document.

Materials gathered through research via the Internet must be cited in the same manner as more traditionally published material. Lack of such citation constitutes plagiarism.

To review rules of citation: http://libguides.mit.edu/content.php?pid=80743&sid=598642

Individual Assignments

Many assignments in MIT Sloan courses are expected to be done individually. The information below outlines what is meant by "individual" work. These rules should be observed unless otherwise defined by the instructor.

When you are asked to do ***individual*** work, you are expected to adhere to the following standards:

- Do not copy all or part of another student's work (with or without "permission").
- Do not allow another student to copy your work.
- Do not ask another person to write all or part of an assignment for you.
- Do not work together with another student in order to answer a question, or solve a problem, or write a computer program jointly.
- Do not consult or submit work (in whole or in part) that has been completed by other students in this or previous years for the same or substantially the same assignment.
- Do not use print or internet materials directly related to a case/problem set unless explicitly authorized by the instructor.
- Do not use print or internet materials without explicit quotation and/or citation.
- Do not submit the same, or similar, piece of work for two or more subjects without the explicit approval of the two or more instructors involved.

Please note that many classes will require a combination of team work and individual work. *Be sure that you follow all the guidelines for individual work when a faculty member identifies an assignment as an individual one.*

Team Assignments

When you are asked to ***work in teams***, there is a broad spectrum of faculty expectations. Three general types of appropriate collaboration on team assignments are described below. The instructor will indicate in the syllabus what his/her expectations are. If there is any uncertainty, it is the student's responsibility to clarify with the professor or TA the type of team work that is expected.

Type 1 collaboration (Appropriate for 15.402 problem sets and cases):
The professor states that collaboration is allowed, but the final product must be individual. An example of this might be a problem set.

- You are allowed to discuss the assignment with other team members and work through the problems together.
- What you turn in, however, must be your own product, written in your own handwriting, or in a computer file of which you are the sole author.
- Copying another's work or electronic file is not acceptable.

Type 2 collaboration:
The professor states that collaboration is encouraged but that each person's contribution to a given deliverable does not have to be substantial (allowing groups to take a "divide and conquer" approach). An example of this might be a brief progress report that is part of a more extensive collaboration (as a whole, the more extensive collaboration may be Type 3).

- Each team member is encouraged to contribute substantially to the team assignment, however, the team may choose to assign one or more team members to prepare and submit the deliverable on behalf of the team.
- Regardless of how work is shared or responsibilities are divided among individual team members, each member of the team will be held accountable for the academic integrity of the entire assignment. If, for example, one member of the team submits plagiarized work on behalf of the team, the entire team will be subject to sanctions as appropriate.
- The team may not collaborate with other students outside of the team unless the professor explicitly permits such collaboration.

Type 3 collaboration:
The professor states that collaboration is expected and that each team member must contribute substantially to the deliverable. An example of this might be the 15.311 OP project.

- Each team member must make a substantial contribution to the assignment. It is not, for example, acceptable to divide the assignments amongst the team members (e.g., part of the team completes the OP Project while the rest of the team prepares a team case for DMD), though the team may divide the work of any one assignment to complete it as they deem appropriate.
- The team may not collaborate with other students outside of the team unless the professor explicitly permits such collaboration.

If you are unsure whether some particular form of interaction is proper in individual or team work, it is your responsibility to consult the instructor and/or teaching assistant for clarification and guidance.

15.401 Finance Theory I

Craig Stephenson
MIT Sloan School of Management

Lecture 1: Introduction

15.401 Finance Theory I

Lecture Notes

Craig Stephenson
MIT Sloan School of Management

Practical information about the class

- Rules of engagement
- How to get the most out of this course
- Course overview

Introduction

- What is finance?
- Opportunity cost of capital
- Role of financial markets
- No arbitrage

Part A Introduction

Chapter 1: Introduction to Finance

Chapter 2: Present Value

Lectures on Mondays and Wednesdays
- 8:30am to 10:00am in E51-376

Recitations on Fridays - 8:30 am to 10:00 am in E51-145

Class Website:

http://stellar.mit.edu/S/course/15/sp11/15.401D/

Classroom Civility
- The class starts on Sloan-time, and ends on Sloan-time.
- Turn off laptops, cell phones, iphones, and blackberries.
- No side conversations.

My office hours:
- Monday, 10:00--11:30 am
- in my office E62-659
- Other times by appointment (stephenc@mit.edu)

Stellar's Forum

Teaching assistant:
- 15.401 D: Marc Piette, marc.piette@sloan.mit.edu
- See class website for contact info & office hours
- Also answer questions on Stellar's Forum

Four Parts

A. Introduction
- Lecture 1: Introduction to finance (framework of financial analysis)
- Lecture 2: Present value (principles of asset valuation)

B. Valuation
- Lecture 3: Fixed income securities
- Lecture 4: Common stocks
- Lecture 5: Forwards and futures
- Lecture 6: Options

C. Risk
- Lecture 7: Risk and return (measuring risk)
- Lecture 8: Portfolio theory (managing risk)
- Lecture 9: The Capital Asset Pricing Model (incorporating risk into valuation methods)

D. Corporate Finance Applications
- Lecture 10: Capital Budgeting (capital investment decisions)
- Lecture 11: Real Options

Final Lecture: Market Efficiency (putting it all together)
- Do financial markets always work well in discovering prices?
- Where does money come from in financial markets?
- How should finance theory be used in practice?

Questions we would like to answer in this course:

1. How to value assets?
2. How do corporations make financial decisions?
   - Investments: What projects to invest in?
   - Financing: How to finance a project?
     - Selling financial assets / securities / claims (debt, stocks, ...)
   - Payout: What to pay back to shareholders?
     - Paying dividends, buyback shares, ...
   - Risk management: What risks to take or to avoid and how?
3. How do households make financial decisions?

January 1926 to December 2009

Statistics	Market Return	T-Bill Return
Mean	0.91%	0.33%
Volatility	5.45%	0.29%
Minimum	-29.01%	-0.93%
Maximum	38.37%	2.13%
Total Return*	$2,097	$27

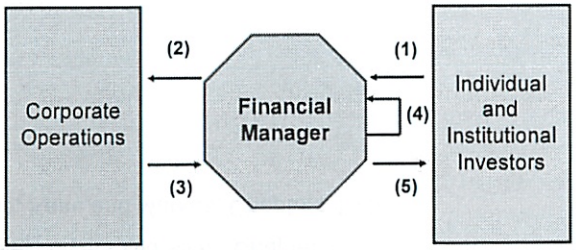
Note: *Based on a $1.00 initial investment in January 1926. Market return from CRSP, value-weighted returns, including dividends, NYSE/AMEX/NASDAQ.

"Perfect" Asset Allocation

Rebalancing Base	Total Return
Annual	$253,062
Quarterly	$78,989,897
Monthly	$71,835,987,263

Note: If stock returns are negative, invest in T-Bill

Corporate financial decisions



(1) Cash raised from investors by selling financial assets (claims on CF)

(2) Cash invested in real assets (tangible and intangible)

(3) Cash generated by operations

(4) Cash reinvested

(5) Cash returned to investors (debt payments, dividends, etc.)

Finance is about the bottom line of business activities

A business activity is a process of acquiring and disposing assets

- Real / financial
- Tangible / intangible

All business activities reduce to two functions:

- Grow wealth (create value)
- Manage wealth to best meet economic needs

Financially, a business decision starts with the valuation of assets

- "You can't create and manage what you can't measure"

Valuation is the central issue of finance

Value is an objective measure --- determined by financial markets

Opportunity cost of capital – investors have alternatives

Role of financial markets – linking economic units

No arbitrage – the law of one price

Readings:

Brealey, Myers and Allen, Chapter 1

Bodie, Kane and Markus, Chapters 1 - 3

Course requirements

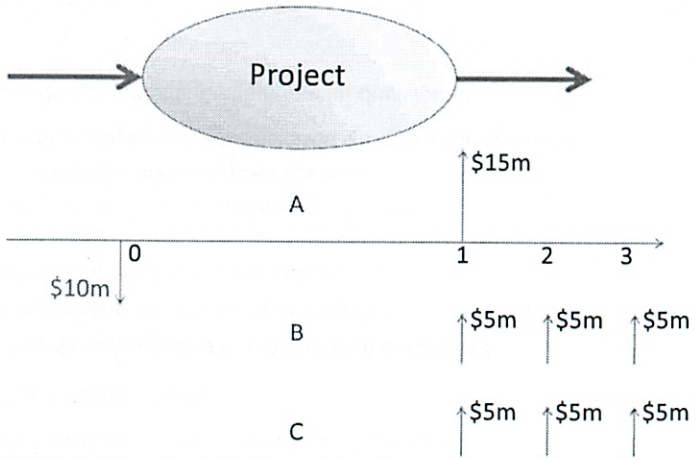
Lectures and readings (attendance and participation = 5%)

7 problem sets and 2 case write-ups (25%)

Midterm exam (25%)

Final exam (45%)

How to make a business decision?



Some helpful suggestions

Do readings ahead of time (skim textbook chapters in advance)

Take copious notes during lectures (lecture notes are not complete)

Review the lectures afterwards with your study group

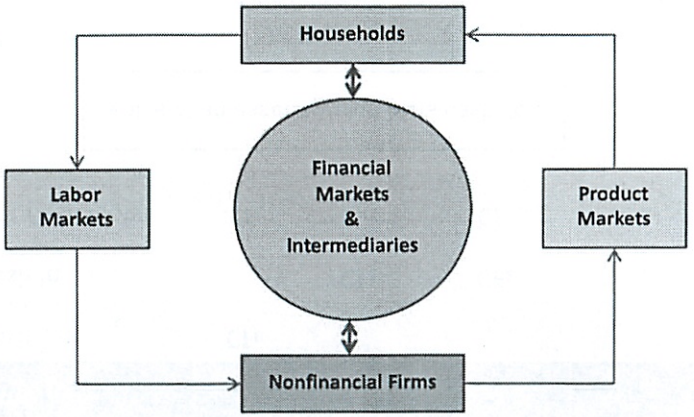
Work on problem sets --- "Finance is not a spectator sport"

Ask questions!!!

Finance is Based on Simple Axioms:

1. **Investors prefer more to less**
   A: $100 now, or
   B: $200 now
2. **Investors are risk averse**
   A: $100 now, or
   B: $200 now with 50% chance, nothing otherwise
3. **Money paid in the future is worth less than the same amount today**
   A: $100 now, or
   B: $100 in a year
4. **Financial markets are competitive; no arbitrage**

Financial markets at the center of universe



DIY: Example of an arbitrage opportunity:

- Return Parasol & Co: -10% if rain, +20% if sun
- Return Umbrella & Co: +20% if rain, -10% if sun
- You can lend and borrow at 4% interest rate
- How can you make a *sure* profit with *zero initial cash outlay*?

Financial markets - where financial assets are traded

- Money markets: Short-term debt securities
  - Short-term government, bank and corporate debt (T-bills, CDs, CPs, ...)
- Capital markets: Long-term securities
  - Government and corporate bonds, asset-backed securities, ...
  - Stocks, ...
- Derivatives: Securities with payoffs tied to other prices
  - Forwards and futures, options, ...

Financial Intermediaries - Own mostly financial assets

- Banks, insurance companies, S&Ls, ...
- Mutual funds, hedge funds, private equity, ...

Nonfinancial firms - Own mostly real assets

Households - Own both real and financial assets

Two important characteristics of a cash flow:

1. Time

Which one do you prefer? --- Time value of money

2. Risk

Which one do you prefer? --- Risk premium

Time and risk are two key elements in finance

A firm's cash flows

(1) Cash raised from investors by selling financial assets
(2) Cash invested in real assets (tangible and intangible)
(3) Cash generated by operations
(4) Cash reinvested
(5) Cash returned to investors (debt payments, dividends, etc)

Management decisions --- manage cash flow (1), (2), (4), (5)

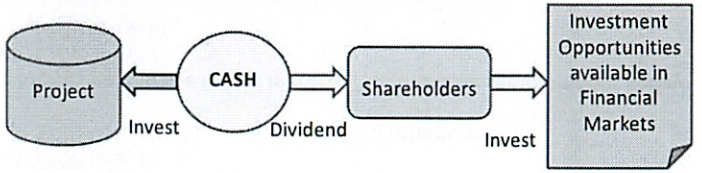
Investment: (2) ⇒ (3) (valuing real assets)
Financing and payout: (1), (4), (5) (valuing financial assets)
Risk management: (1) and (5) (value financial assets)

Objective: Create maximum value for shareholders

Sound business decisions rely on how to value assets

Valuation of a project

- A firm can always give cash back to shareholders
- A shareholder can invest in financial markets



Opportunity cost of capital is the expected rate of return offered by equivalent (in time and risk) investments in financial markets

Market valuation of the project (i.e., its CF):

- Good if it offers a return higher than its cost of capital
- Bad if it yields a return lower than its cost of capital

Time	0	1	2	....
Cash Out	CF_0			...
Cash In		CF_1	CF_2	....
Net Cash Flows	$-CF_0$	CF_1	CF_2	....

Value of an asset = Value of its cash flow




- Harry Markowitz (1990): Optimal portfolio selection. "Don't put all your eggs in one basket."
- William Sharpe (1990): Capital Asset Pricing Model. In equilibrium, riskier assets have higher returns
- Robert "Bob" Merton, Myron Scholes (1997): No arbitrage and pricing of derivatives (options)

- Evaluating a business boils down to valuation of its assets
- An asset is defined by its cash flow (CF)
- Two important characteristics of a CF: time and risk
- Value of assets (CFs) are determined by financial markets
- Opportunity cost of capital
- Financial markets
- No arbitrage

15.401 First Day

- interdepartmental Section "D" (good I think) - w/ MBAs
- Syllabus on Stellar

Portfolio analysis in the 50s

TA: Marc + Piette

Prof: Craig Stepenson
teaches at Babson
part time here MW 9-12

7 P sets

Midterm, Final - large part of grade

Has corp work background
- did IPO for Dell

keep up w/ changes in the industry

1 cheat sheet midterm
2 " 's final

don't break his train of thought

dates are fairly set

no recitation this fri

(2)

theory + practice
do the problems

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* Connect those who have money to those who have ideas

amsterdam was first stock exchange

law of one price → no arbitrage

variability/uncertainty is always there

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tons of data available for fin markets
and lots of people look at it to make $
value creation + appropriation

How do you create employment?

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key themes

- time value of $
  - want $ sooner
- risk premium
  - want extra $ if larger risk
- opportunity cost of capital
  - what can I get elsewhere

(3) too low rate of return

price will fall till rate of return ~~is~~ matches market

Financial markets
- money markets
- capital market
- derivatives

financial intermediaries
- own mostly financial assets
- banks
- hedge funds
- etc

non financial firms

households - have both financial + real assets

Borrow + lend 4%

Parasol Corp -10% rain +20% sun

Umbrella +20 -10%

So

Borrow $1000 Now

Repay $1040 1 period

Invest half in each

(4)

P

500 → R → 450
500 → S → 600

U

500 → R → 600
500 → S → 450

Total

R 450 + 600 − 1040 = 10 profit (π)

S 600 + 450 − 1040 = 10

This is not an equalibium market

So what happens?

- interest rate goes up
- price of P + U goes up
- So return disappers

me;
(not rate of return since company can't invest at same rate of return)
prof – well actually that as

15.401 Finance Theory I

Craig Stephenson

MIT Sloan School of Management

Lecture 2: Present Value

Present value

Future value

Special cash flows

Compounding

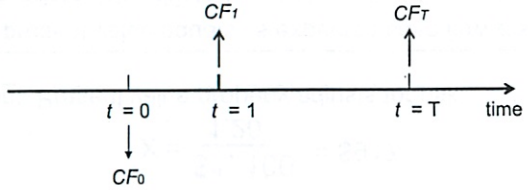
Nominal versus real → cash flows and discount rates

Extensions

Readings:

Brealey, Myers, and Allen, Chapter 2

Visualizing cash flows (assets):



Example. A drug company has developed a flu vaccine and needs to choose between two strategies:

Strategy A: To bring to market in 1 year, invest \$1B (billion) now and returns \$500M (million), \$400M and \$300M in years 1, 2 and 3, respectively

Strategy B: To bring to market in 2 years, invest \$200M in years 0 and 1, and returns \$300M in years 2 and 3

How to value / compare the two strategies (i.e., their CFs)?

Example. How much is a sure cash flow of \$1,100 in one year worth now?

Market: Traded safe assets offer a 5% annual return

A potential buyer of the sure CF also expects a 5% return (cost of capital). Let the price she is willing to pay be X. Then

$$X(1 + 0.05) = \$1,100$$

Thus,

$$X = \frac{\$1,100}{1.05} = \$1,048$$

which is the CF's present value, i.e., its current market value

Observation: Present value properly adjusts for time

Example. How much is a risky cash flow in one year with a forecasted value of $1,100 worth now?

Market: Traded assets of similar risk offer 20% annual return

A potential buyer of the risky CF also expects a 20% return. Let the price be X. Then

$$X(1 + 0.20) = \$1,100$$

Thus, the present value of the risky CF is

$$X = \frac{\$1,100}{1.20} = \$917$$

Observation: Present value properly adjusts for risk

An asset's present value equals its expected cash flow discounted at the appropriate risk adjusted cost of capital (discount rate)

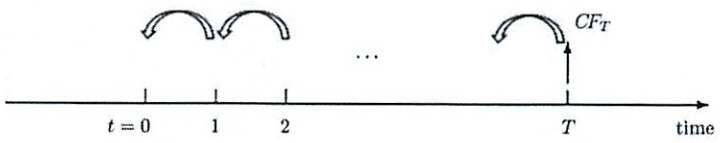
Expected rate of return on traded assets

(1/1926 — 8/2010, nominal, monthly returns, annualized, in %)

Asset	Mean	St. Dev.	Premium
Short term government debt	3.7	0.9	0.0
Long term government debt	5.5	6.4	1.8
Long term corporate debt	6.3	5.6	2.6
Large stocks	10.4	17.8	6.7
S&P500	11.0	19.2	7.3
Small stocks	17.4	35.6	13.7

Risk-free interest rate --- time value of money

Extra return of risky assets over the safe asset --- risk premium

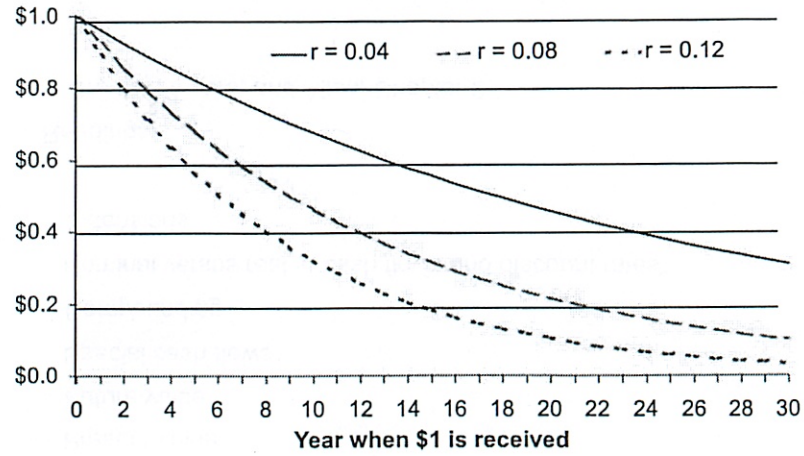


$$PV(CF_T) = \frac{CF_T}{(1+r)^T}$$

Example. (A) $10M in 5 years or (B) $15M in 15 years. Which is better if r = 5%?

$$PV_A = \frac{\$10}{1.05^5} = \$7.84; \quad PV_B = \frac{\$15}{1.05^{15}} = \$7.22$$

PV of $1 Received In Year t



$$PV(CF_1, CF_2, \ldots, CF_T) = \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \cdots + \frac{CF_T}{(1+r)^T}$$

Solution to Example. Flu Vaccine. Assume that r = 5%

Strategy A:

Time	0	1	2	3
Cash Flow	-1,000	500.0	400.0	300.0
Present Value	-1,000	476.2	362.8	259.2
			Total PV	98.2

Strategy B:

Time	0	1	2	3
Cash Flow	-200	-200.0	300.0	300.0
Present Value	-200	-190.5	272.1	259.2
			Total PV	140.8

Firm should choose strategy B, and its value would increase by $140.8 M

How much will $1 today be worth in one year?

Current interest rate is, say, 4%

$1 investable at a rate of return $r = 4\%$

FV in 1 year

$$FV = 1 + r = \$1.04$$

FV in T years $FV = \$1 \times (1 + r) \times \cdots \times (1 + r)$

$$= (1 + r)^T$$

Example. Bank pays an annual interest of 4% on 2-year CDs and you deposit $10,000. What is your balance two years later?

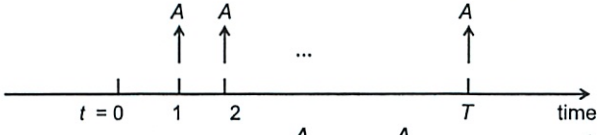
$$FV = \$10,000 \times (1 + 0.04)^2 = \$10,816$$

The Last Real Estate Bargain in New York City....

- The Dutch West India Company dispatched the first permanent settlers to Manhattan Island in 1624. They established Fort Amsterdam, which grew into the town of New Amsterdam as more settlers arrived. In 1626, the fledgling town's governor, Peter Minuit, bought Manhattan—meaning "Island of Hills"—from the Canarsie tribe for 24 dollars' worth of beads and trinkets.
- Locals sometimes cite this transaction as one of the last real estate bargains in New York.

- How big of a bargain was it? At 6% interest rate? At 7%?

Annuity A constant and equal cash flow for T periods (starting in period 1)



$$PV = \frac{A}{1+r} + \frac{A}{(1+r)^2} + \cdots + \frac{A}{(1+r)^T}$$

$$(1+r) \times PV = A + \frac{A}{(1+r)} + \cdots + \frac{A}{(1+r)^{T-1}}$$

$$r \times PV = A - \frac{A}{(1+r)^T}$$

$$PV\ (\text{Annuity}) = A \times \frac{1}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

$$FV\ (\text{Annuity}) = (1+r)^T \times PV\ (\text{Annuity})$$

Which car can you afford:

or

- You have no available cash.
- You can afford a payment of $632 per month.
- You can borrow at an interest rate of 1% per month.
- You want to have paid the loan in full in 48 month.

Ans.: PV = A * PV factor
PV factor $=(1-1/1.01^{48})/0.01=37.974$

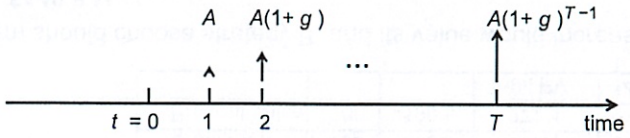
PV = A * PV factor = $632 * 37.974 = $24,000

Example. An insurance company sells an annuity of $10,000 per year for 20 years. Suppose r = 5%. What should the company sell it for?

$$PV = \$10,000 \times \frac{1}{0.05} \times \left(1 - \frac{1}{1.05^{20}}\right) = \$10,000 \times 12.46$$

$$= \$124,622.10$$

Annuity with constant growth rate g (growing annuity)



$$PV \text{ (Annuity with growth)} = A \times \left[\frac{1}{1+r} + \frac{1+g}{(1+r)^2} + \cdots + \frac{(1+g)^{T-1}}{(1+r)^T}\right]$$

$$= A \times \begin{cases} \frac{1}{r-g}\left[1-\left(\frac{1+g}{1+r}\right)^T\right] & \text{if } r \neq g \\ \frac{T}{1+r} & \text{if } r = g \end{cases}$$

Example. Saving for retirement - Suppose that you are now 30 and need $2 million at age 65 for your retirement. You can save each year an amount that grows by 5% each year. How much should you start saving now, assuming that r = 8%?

$$A = \frac{\$2,000,000}{308.977} = \$6,472.97$$

Example. You just won the lottery and it pays $100,000 a year for 20 years. Are you a millionaire? Suppose that r = 10%.

$$PV = \$100{,}000 \times \frac{1}{0.10}\left(1 - \frac{1}{1.10^{20}}\right) = \$100{,}000 \times 8.514$$
$$= \$851{,}356$$

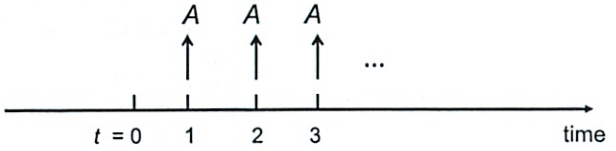
What if the payments last for 50 years?

$$PV = \$100{,}000 \times \frac{1}{0.10}\left(1 - \frac{1}{1.10^{50}}\right) = \$100{,}000 \times 9.915$$
$$= \$991{,}481$$

How about forever - a perpetuity?

$$PV = \frac{\$100{,}000}{0.10} = \$1{,}000{,}000$$

Perpetuity An annuity with infinite maturity



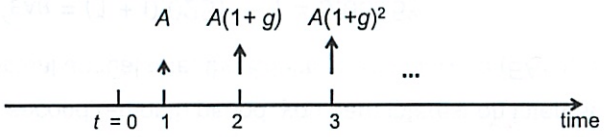
$$\text{PV (Perpetuity)} = \frac{A}{r}$$

Suppose that maintenance of your grave costs $100 every year, forever.

The interest rate is 5% per year.

How much money should you leave the trustee of your grave?

Perpetuity with constant growth rate g



$$\text{PV (Perpetuity with growth)} = \frac{A}{r-g}, \quad r > g$$

Example. Super Growth Inc. will pay an annual dividend next year of $3. The dividend is expected to grow 5% per year forever. For companies of this risk class, the expected return is 10%. What should be Super Growth's price per share?

$$PV = \frac{3}{1.10} + \frac{3(1+0.05)}{1.10^2} + \frac{3(1+0.05)^2}{1.10^3} + \cdots = \frac{3}{0.10 - 0.05}$$
$$= 60$$

Interest may be credited/charged more often than annually

- Bank accounts: daily
- Loans and leases: monthly
- Bonds: semi-annually

For the same quoted interest rate, the effective annual rate may differ

Why?

Typical quote convention:

- Annual Percentage Rate (APR)
- k periods of compounding
- Interest per period is APR/k
- Actual annual rate differs from APR

10% Compounded Annually, Semi-Annually, Quarterly, and Monthly

Month	$1,000	$1,000	$1,000	$1,000
1				$1,008
2				$1,017
3			$1,025	$1,025
4				$1,034
5				$1,042
6		$1,050	$1,051	$1,051
7				$1,060
8				$1,069
9			$1,077	$1,078
10				$1,087
11				$1,096
12	$1,100	$1,103	$1,104	$1,105

Example. Bank of America's one-year CD offers 5% APR, with semi-annual compounding. If you invest $10,000, how much money will you have at the end of one year? What is the actual annual rate of interest you earn?

- Quoted APR of 5% is not the actual annual rate
- It is only used to compute the 6-month interest rate:

  $(5\%)(1/2) = 2.5\%$
- Investing $10,000, at the end of one year you have:

  $\$10{,}000(1+0.025)(1+0.025) = \$10{,}506.25$
- In the second 6-month period, you earn interest on interest
- The actual annual rate, the Effective Annual Rate (EAR), is

$$r_{EAR} = (1 + 0.025)^2 - 1 = 5.0625\%$$

Let r_{APR} be the APR and k be the number of compounding intervals per year. In one year, one dollar invested today yields:

$$\left(1 + \frac{r_{APR}}{k}\right)^k$$

Effective annual rate, r_{EAR} is given by

$$(1 + r_{EAR}) = \left(1 + \frac{r_{APR}}{k}\right)^k$$

or

$$r_{EAR} = \left(1 + \frac{r_{APR}}{k}\right)^k - 1$$

Example. Suppose $r_{APR} = 5\%$.

Here,

$e \approx 2.71828$

k	Value of $1 in a year	r_{EAR}
1	1.050000	5.0000%
2	1.050625	5.0625%
12	1.051162	5.1162%
365	1.051268	5.1267%
8,760	1.051271	5.1271%
⋮	⋮	⋮
∞	$e^{0.05} = 1.051271$	5.1271%

Example. Fixed rate mortgage calculation in the U.S.

20% down payment, and borrow the rest from bank using the property as collateral

Pay a fixed monthly payment for the life of the mortgage

Have the option to prepay

Suppose that you bought a house for $500,000 with $100,000 down payment and financed the rest with a thirty-year fixed rate mortgage at 8.5% APR compounded monthly

The monthly payment M is determined by

$$\$400{,}000 = \sum_{t=1}^{360} \frac{M}{[1 + (0.085/12)]^t}$$

$$= \frac{M}{(0.085/12)} \left\{1 - \frac{1}{[1 + (0.085/12)]^{360}}\right\}$$

$$= M \times \frac{(0.9212)}{(0.085/12)}$$

$$M = \$3{,}075.65$$

Effective annual interest rate (EAR):

$$[1 + (0.085/12)]^{12} - 1 = 1.08839 - 1 = 8.839\%$$

Monthly payments

t (month)	Principal	Interest	Sum	Remaining P.
1	242.37	2833.33	3075.7	399,757.63
2	244.08	2831.62	3075.7	399,513.55
3	245.81	2829.89	3075.7	399,267.74
⋮	⋮	⋮	⋮	⋮
120	561.29	2514.42	3075.7	354,415.49
121	565.26	2510.44	3075.7	353,850.23
⋮	⋮	⋮	⋮	⋮
240	1309.27	1766.43	3075.7	248,068.95
241	1318.54	1757.16	3075.7	246,750.41
⋮	⋮	⋮	⋮	⋮
359	3032.60	43.10	3075.7	3,054.07
360	3054.07	21.63	3075.7	0.00

Total monthly payment is the same for each month
The percentage of principal payment increases over time
The percentage of interest payment decreases over time

Nominal vs. real CFs

Example. Inflation is 4% per year. You expect to receive $1.04 in one year, what is this CF really worth next year?

The inflation adjusted or real value of $1.04 in a year is

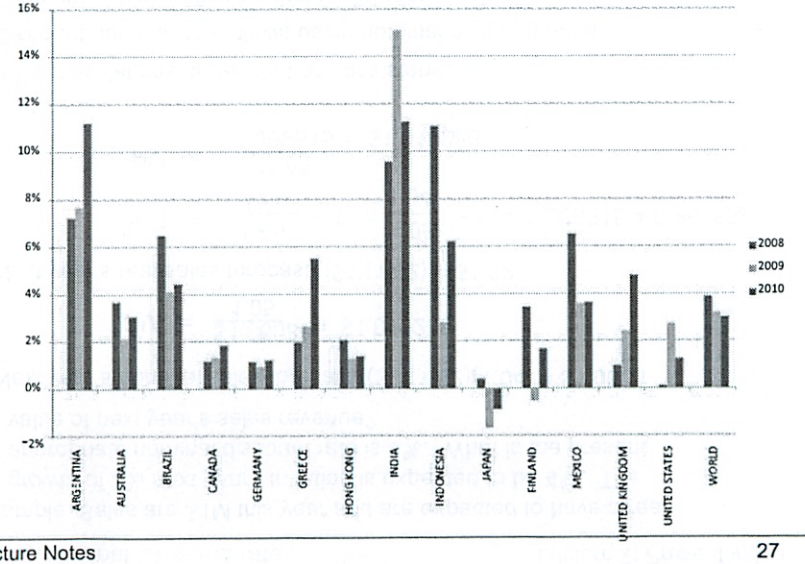
$$\text{Real CF} = \frac{\text{Nominal CF}}{1 + \text{inflation}} = \frac{\$1.04}{1 + 0.04} = \$1.00$$

Nominal cash flows → expressed in actual-dollar cash flows
Real cash flows → expressedinconstantpurchasingpower

At an annual inflation rate of i, we have

$$(\text{Real CF})_t = \frac{(\text{Nominal CF})_t}{(1+i)^t}$$

Annual Inflation rates in 2008, 2009, and 2010 (up to July)



Nominal vs. real rates of return

Nominal rates of return → prevailing market rates
Real rates of return → inflation adjusted rates

Example. $1.00 invested at a 6% interest rate grows to $1.06 next year. If inflation is 4% per year, then the real value is

$1.06/1.04 = $1.01923

The real return is 1.923%.

$$r_{\text{real}} \quad \frac{1 + r_{\text{nominal}}}{1 + i} - 1 \approx r_{\text{nominal}} - i$$

Example. Sales are $1M this year and are expected to have a real growth of 2% next year. Inflation is expected to be 4%. The appropriate nominal discount rate is 5%. What is the present value of next year's sales revenue?

Next year's nominal sales forecast: ($1)(1.02)(1.04) = $1.0608

$$PV = \frac{\$1.0608}{1.05} = \$1.010286$$

Next year's real sales forecast: ($1)(1.02) = $1.02

$$r_{real} = \frac{1+r_n}{1+i} - 1 = \frac{1.05}{1.04} - 1 = 0.009615 = 0.9615\%$$

$$PV = \frac{\$1.02}{1.009615} = \$1.010286$$

For PV calculations, treat inflation consistently

Discount nominal cash flows using nominal discount rates

Discount real cash flows using real discount rates

Taxes

Currencies

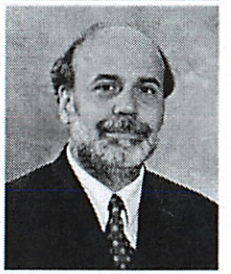
Term structure of interest rates

Forecasting cash flows

Choosing the right discount rate (risk adjusted)

$$PV(CF_1, CF_2, \ldots, CF_T) = \frac{E[CF_1]}{(1+r_1)} + \frac{E[CF_2]}{(1+r_2)^2} + \ldots + \frac{E[CF_T]}{(1+r_T)^T}$$

Ben Bernanke
- Chairman of the Board of Governors of the United States Federal Reserve
- Chairman of the Federal Open Market Committee (FOMC), which sets the federal funds rate (the rate that commercial banks charge on overnight loans among themselves)



Last change was Dec 16, 2008 when the target for the federal funds rate was lowered to 0 → 0.25%

Next meeting of the FOMC: March 15, 2011

Stock market usually rallies after *unexpected* rate cut announcements. When The Chairman speaks, Wall Street listens.

Present value

Future value

Special cash flows

Compounding

Nominal versus real cash flows and discount rates

Extensions

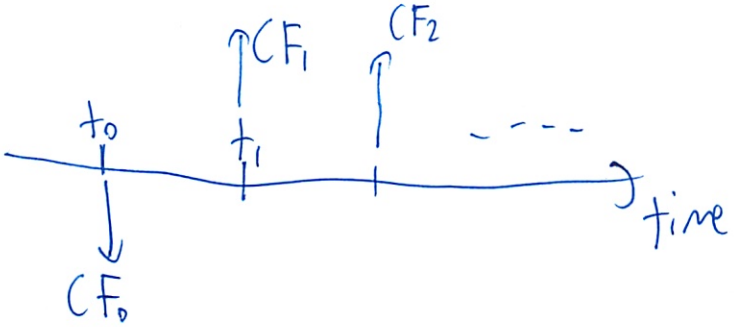
15.401 Lecture 2

Present Value

Key meaning of finance

spend $ today → get $ tomorrow

* need to compare cash flows through time



Money has more value tomorrow
- since can invest $1 today
- opportunity cost

Cost of capital – next best investment at same risk

At 5% interest rate, in 1 year will have

$$X(1+.05) = 1{,}100$$

So what would you pay for it today

$$X = \frac{1100}{(1+.05)} = 1047$$

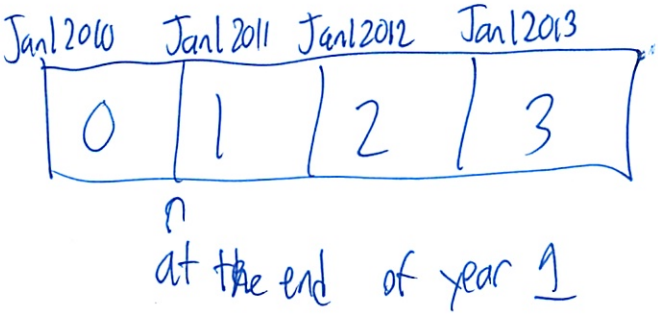
20% risky

$$X(1+.2) = 1100$$

$$X = \frac{1100}{1.2} = 917$$

(2)

Time period can be anything
↳ need to match rate to period → daily rate
monthly rate
annual rate (usual)



$$PV(CF_0, CF_1, \ldots, CF_T) = CF_0 + \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \ldots + \frac{CF_T}{(1+r)^T}$$

But lots of uncertainty
Risk + uncertainty not in cash flows
Risk adjust the discount/interest rate
- do sensitivity analysis around expectations
- at risk in Excel

Chart of mean returns
As go down, increasing risk
So get a risk premium

(3)

Short term gov securities is the base rate

"Treasuries + _____"

↑premium

Inflation is built into the mean returns as well

Need to make sure this is embeded in future cash flow projections

Risk on long term gov bonds is inflation risk - not default risk

Inflation risk + default risk built into other securities

$r = 5\%$

\$10 in 5 years vs \$15 in 15 years

$$\frac{10}{(1.05)^5} = 7.8353 \qquad \frac{15}{(1.05)^{15}} = 7.2153$$

↑better offer

Will do risk when do option pricing

↑Or just increase rate if this one risky

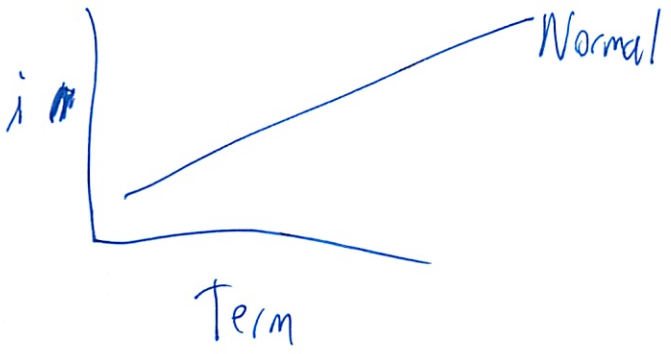
this stays same if low risk

Use rate from table

$$\frac{10}{(1.10)^5} = \ldots$$

(4)

Yield curve

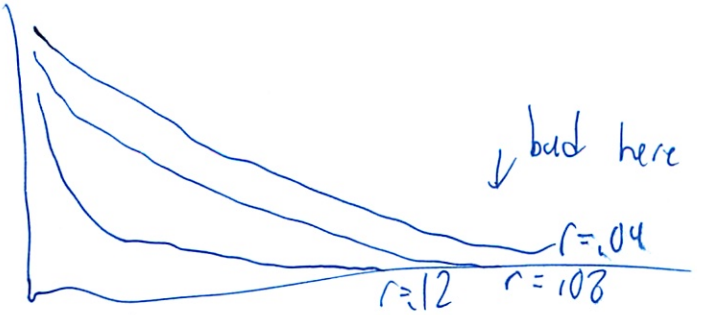


So what we calculated was PV of what we get
That would be what people would normally be willing to pay
If price lower or higher can do NPV w/ purchase price (the timeline)
(I am starting to understand more)

$$7.2153 \times (1.05)^{15} = 15$$

"Is value - just the PV discounted to today

Can chart diff rates effects on PV



↓ bad here at end — but does get really small at the end - companding

(5)

FV is reverse of PV

$FV = 10000 \cdot (1.04)^2 = 10,816$

↑ pays 4% in 2 years

NYC was bought in 1624 for $24

↑ in 1624 dollars

$24 \cdot (1.06)^{384} = 125,217,060,205$

$24 \cdot (1.07)^{384} = 4,608,736,440,146$

But you can't invest at 6%

To know how much $24 is - need CPI index today

P set due in 1 week

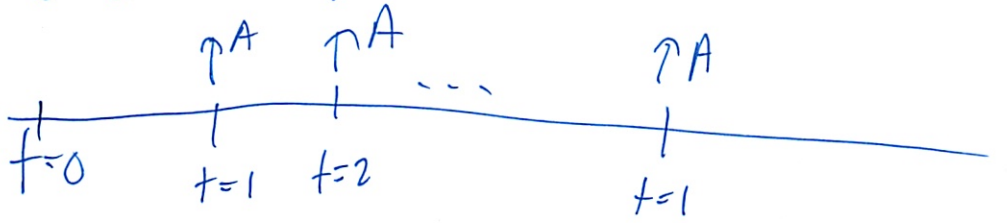
Plus case using CPI - apply both rate of return - and CPI

(Ok so that is how inflation matters - must subtract $ lost due to CPI - so treasury rate must be above predicted future CPI) ← my guess

6

Annuity

- series of equal cash flows



- must be every year

$$PV = \frac{A}{1+r} + \frac{A}{(1+r)^2} + \cdots \frac{A}{(1+r)^T}$$

$(1+r) \cdot PV =$ [illegible]

(missed info; see slides)

annuity can also be you paying - mortgage
- car loan
- etc

Sell annuity 10,000/year 20 years $r = 5\%$

$$PV = 10{,}000 \cdot \frac{1}{.05} \cdot \left(1 - \frac{1}{1.05^{20}}\right) = 10{,}000 \cdot 12.46$$

$$= 124{,}622.10$$

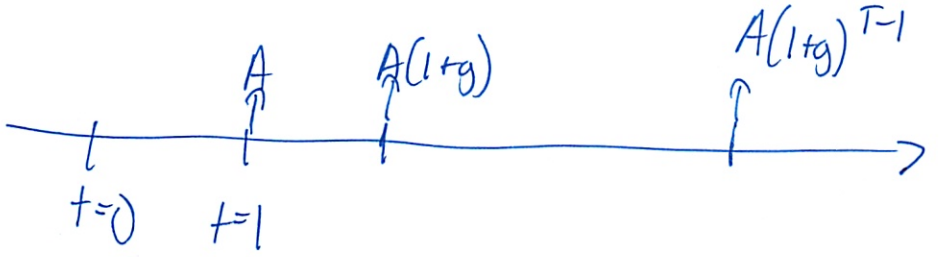
Remainder grows each year at 5%

But then also inflation

↳ so need annuity w/ growth

(7)

Each year inflation extracts purchasing power decreaase



$$PV = A \cdot \left[\frac{1}{1+r} + \frac{1+g}{(1+r)^2} + \cdots + \frac{(1+g)^{T-1}}{(1+r)^T} \right]$$

$$= A \begin{cases} \frac{1}{r-g}\left[1 - \left(\frac{1+g}{1+r}\right)^T\right] & \text{if } r \neq g \\ \frac{1}{1+r} & \text{if } r = g \end{cases}$$

Example w/ saving for retirement want $20 mill on 65th bday
- your deposit grows at 5%/year
- $r = 8\%$

How big must first deposit be to reach 30th bday goal amount?

$$PV = A \cdot \left[\frac{1}{.08-.05} - \frac{1}{.08-.05} \cdot \left(\frac{1.05}{1.08}\right)^{35} \right]$$

$$= 20.897 \cdot A$$

(8)

30 — r — 65

2,000,000 FV

need A

t = 35

r = .08

$2{,}000{,}000 \cdot \frac{1}{(1.08)^{35}} = 135,629.09$

~~xxxxx~~

$35629.09 = A \cdot 20.897566$

$A = 6472.97$

Special cashflows; Lotto

Do you take the annuity or the lump sum

Perpuity - very voltile $= \frac{A}{r}$

bad deal when other investments > 15%

(Still somewhat unclear on this, better than was, still need to do hw)

15.401 #3

(slides from 2 cont)

recitation 8:30 - info on stellar

Perpetuity $= \frac{A}{r}$

Valuing very simple

- very volitle
- VS: only perfered stock was common
- not much anymore
- Now VC - financing thing

Example maintence tomb $100/year

5%/year

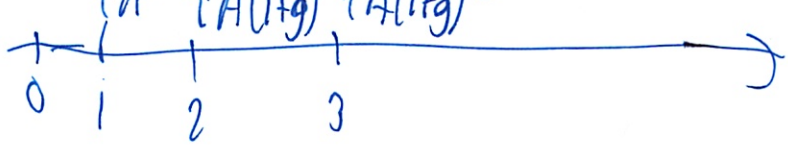
Need to leave $\frac{100}{.05} = \$2{,}000$

But not adjusted for inflation!

Purchasing power will ↓ over time

Perpituity w/ growth

$\frac{A}{(r-g)}$ $r > g$



(2)

Dividend growth model / Dividend discount model
- Gordon model
- to value common stock

Most have constant perpetual growth

g% growth in income each year

Example

$r = .10$
$g = .05$
$A = 3$

$$PV = \frac{A}{r-g} = \frac{3}{.10-.05} = 60$$

$g = .07$ $\quad PV = \frac{3}{.1-.07} = 100$

↑ would be stock price

$g = .09$ $\quad PV = 300$

$g = .11$ $\quad PV = -300$ Does not work $g > r$

rate of return approaching the growth rate

Cash flow is growing so fast

requires constant + perpetual growth rate - P+G

in new company g may be fast - but will slow down

(3)

Grave example w/ inflation

But do endow fellowship instead - less morbid

Cost = $80,000 this year in 2011

Want this purchasing power forever

$r = .13$ rate of return of stock portfolio ← to make result odd

$g = .06$ growth in cost of living

$$PV = \frac{80000}{.13 - .06} = 571,428$$

If r lower → need more

If g higher → need more

This would be conservative ↷ would fear this is true

~~[illegible]~~

This is the retirement fear

- running out of $

(4)

Compounding

Bonds

Europe - pay interests once a year

US Bonds - twice a year

loans - semi monthly

Chart

	Annually	Quarterly	Monthly

quoted APR

CD = 5% interest rate - just the quoted rate

need effective

$$R_{EAP} = \left(1 + \frac{R_{APR}}{k}\right)^k - 1$$

↑ effective annual rate

limit is continuous

best is continuous - earn interest on interest

(5)

Mortgage example

- need to adjust APR to actual month payment
  - monthly rate $\frac{APR}{12}$
  - raise to t the # of periods
  - get EAR

Both are on the board at the bank

Credit Card Example

22.6% APR

Compounded monthly

$$EAR = \left(1 + \frac{.226}{12}\right)^{12} - 1$$

$$= .2509$$

You are most broke your first month of work

Plus if you don't pay minimum → rate jumps

2 Previously on all your cards

Now tell people how many years if pay minimum

(6)

Moral think in periods

Amortization Table

Montly payment from table

Total cost - 3075 · 360

One extra payment a year really helps
- towards principal

Nominal vs real CFs

- always in nominal terms
- Use both nominal or real cash flows/interest rates
  - match

$$\text{Real CF} = \frac{\text{Nominal CF}}{(1+\text{inflation})^t} \quad \frac{1.04}{1+.04} = 1.00$$

Maintained your value

People always calculate nominally
- easier

Problems when rate of return < inflation
Plus taxes on gain - so actually lost value!

6

Some CPI / inflation here (or issued Fri?)

Inflation data available online

$$r_{real} = \frac{1 + r_{nominal}}{1 + i} \rightarrow \approx r_{nominal} - i$$

(but not exactly!

Must discount by inflation!

(What I was wondering on mon)
(Makes a lot more sense now)

Growth example

2% growth - is in units

Table of this in an M+A deal

Extensions

- taxes - come out of nominal return
- currencies
- term structure of interest rates
- forcasting cash flow
- choosing right risk adjusted discount rate

(8)

If inflation is too high
- Bank return taxed
- So buy real assets → gold, platnam

Ben Bernake
- sets targets for fed funds rate
- buys assets to meet that
- only changes market actions if its a suppise
  - already anticipated

Next class
Valuing other assets

#Recitation Fri
P-Set Mon - submit online or box

15.401 Recitation

1: Present Value

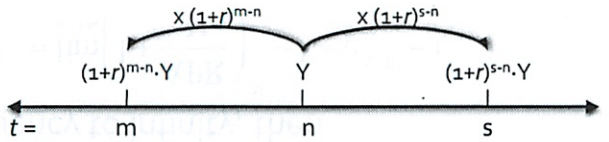
Marc Piette

Learning Objectives

- Review of Concepts
  - Compounding/discounting
  - PV/FV
  - Annuities and perpetuities
- Examples
  - CD
  - Auto loan
  - Project planning

Review: Compounding / Discounting

- We can…
  - move money forward in time by **compounding**.
  - move money backward in time by **discounting**.



- Note:
  - Only relative time matters
  - Multiplying by $(1+r)^{m-n}$ = dividing by $(1+r)^{n-m}$.

Review: APR vs. EAR

- **Annual percentage rate** (APR) vs. **equivalent annual return (EAR):**

$$\text{EAR} = \left(1 + \frac{\text{APR}}{N}\right)^{N} - 1 \quad (N = \text{comp. freq.})$$

- Note:
  - **always** use the EAR when compounding and discounting
  - Due to interest compounding, the EAR is higher than the APR whenever the compounding frequency is higher than once a year.

Continuous Compounding (optional)

- Given a fixed APR, higher compounding frequency leads to higher EAR. Suppose we take compounding frequency to infinity, then

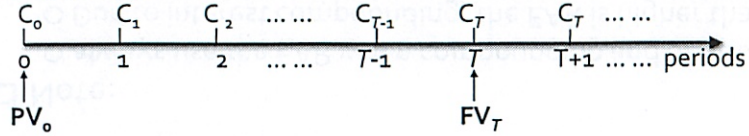
$$\mathrm{EAR}_{\infty} = \lim_{n\to\infty}\left(1+\frac{\mathrm{APR}}{N}\right)^{N} - 1 = e^{\mathrm{APR}} - 1.$$

$(e = 2.71828183...)$

- The continuously compounded EAR is the highest possible EAR for a given APR.

Review: PV / FV

- Cash flow:



- Present value (PV):

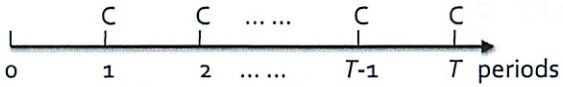
$$\mathrm{PV}_0 = \mathrm{C}_0 + \frac{\mathrm{C}_1}{(1+r)^1} + \frac{\mathrm{C}_2}{(1+r)^2} + \cdots$$

- Future value (FV) :

$$\mathrm{FV}_T = \mathrm{C}_0(1+r)^T + \mathrm{C}_1(1+r)^{T-1} + \cdots + \mathrm{C}_T(1+r)^0 + \mathrm{C}_{T+1}(1+r)^{-1} + \cdots$$

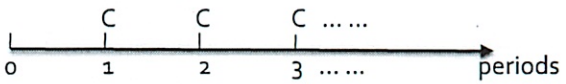
Review: Annuity/Perpetuity

- Annuity:



$$\mathrm{PV}_0 = \frac{\mathrm{C}}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

- Perpetuity:

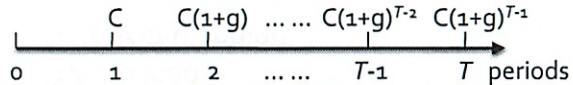


$$\mathrm{PV}_0 = \frac{\mathrm{C}}{r}$$

Simple formula
- but gives me something slightly different
- ? w/ periods

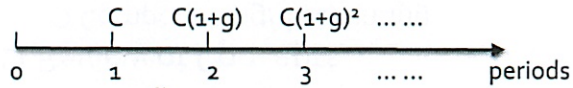
Review: Growing Annuity/Perpetuity

- Growing Annuity:



$$\mathrm{PV}_0 = \frac{\mathrm{C}}{r-g}\left[1 - \frac{(1+g)^T}{(1+r)^T}\right].$$

- Growing Perpetuity ($r > g$):



$$\mathrm{PV}_0 = \frac{\mathrm{C}}{r-g}.$$

Example 1: CD

- You can invest $10,000 in a CD offered by your bank. The CD matures in 5 years and the bank quotes you a rate of 4.5%. How much will you have in 5 years, if the 4.5% is
  a) EAR
  b) Quarterly APR
  c) Monthly APR

Example 1: CD

- Answer:

a) $10,000 \times (1.045)^5 = \$12,461.82$

b) $r_{EAR} = \left(1 + \frac{0.045}{4}\right)^4 - 1 = 4.576\%$

$10,000 \times (1.04576)^5 = \$12,507.51$

c) $r_{EAR} = \left(1 + \frac{0.045}{12}\right)^{12} - 1 = 4.594\%$

$10,000 \times (1.04594)^5 = \$12,517.96$

Example 2: Auto Loan

- You would like to buy a new car for $22,000. The dealer requires a down payment of $10,000 and offers you 6% APR financing (compounded monthly) for 5 years for the remaining balance. What is your monthly payment?

Example 2: Auto Loan

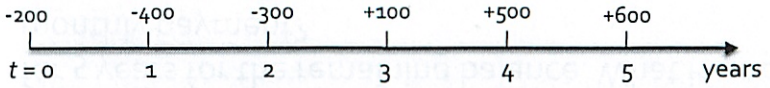
- Answer: let C be the monthly payment, then

$$22000 = \frac{C}{0.06/12}\left[1 - \frac{1}{(1+0.06/12)^{12\times 5}}\right] + 10000.$$

$C = \$231.99.$

Example 3: Project Planning

- GeneriCorp is considering whether or not to expand into a new market. The company faces the following cash flow (in $million) if it decides to expand:



- A committee appointed by the CEO determined that the appropriate discount rate is 9%. Should the company take on the expansion project?

Example 3: Project Planning

- Answer:

$$\text{NPV} = -200 - \frac{400}{1.09} - \frac{300}{1.09^2} + \frac{100}{1.09^3} + \frac{500}{1.09^4} + \frac{600}{1.09^5}$$
$$= \$1.91\text{m}.$$

- Positive NPV = take the project; though NPV is dangerously close to zero.

Review: Nominal vs. Real Interest Rate

- Nominal-real interest rate conversion:

$$1 + r_{\text{real}} = \frac{1 + r_{\text{nominal}}}{1 + i}$$

- Nominal-real cash flow conversion:

$$C_{\text{real}} = \frac{C_{\text{nominal}}}{1 + i}$$

- When you discount or compound,
  - Either use the **nominal** cash flow and the **nominal** interest rate
  - Or use the **real** cash flow and the **real** interest rate
  - **Do not** mix and match

Example 4: Scholarship Fund

- You would like to establish a scholarship fund that will help outstanding students with financial difficulties pay their college tuition.
  - Starting today, you hope to give 50 students $20,000 each in today's money (i.e., adjusted for inflation) every year.
  - The effective nominal interest rate is 5%/yr.
  - Inflation is 2%/yr.
- How much money do you need now if you want the fund to last forever?

Example 4: Scholarship Fund

- Answer:
  - Method 1: nominal amount + nominal interest rate

$$1\text{m} + \frac{1\text{m} \times 1.02}{1.05 - 1.02} = 35\text{m}.$$

  - Method 2: real amount + real interest rate

$$r_{\text{real}} = \frac{1.05}{1.02} - 1 = 2.9412\% \quad \Rightarrow \quad 1\text{m} + \frac{1\text{m}}{0.029412} = 35\text{m}.$$

  - Note: same answer!
- You need $35 million today.

15.401 Recitation

(5 min late)

APR = yearly rate

$$EAR = \left(1 + \frac{APR}{N}\right)^N - 1$$

N = Compounding rate
# of Compounds per year

---

example

APR 6% 6 months

- yearly rate, need to reduce if loan less than 1 year but not quoted

$$\frac{6\%}{2} = \frac{6\% \cdot 6}{12} = 3\%$$

$$EAR = 100 \cdot (1 + 3\%)^2$$

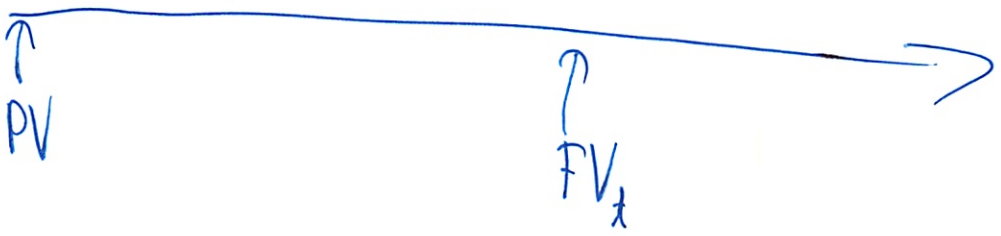
EAR > APR

Continuous Compounding

$$EAR_\infty = \lim_{n \to \infty} \left(1 + \frac{APR}{N}\right)^N - 1 = e^{APR} - 1$$

The highest possible EAR

(2) PV/FV



Annuity

$$PV_0 = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

essentially 1 perpetuity minus another

Perpetuity

$$PV_0 = \frac{C}{r}$$

Payments start at year 1 -
- need to add +C if start year 0 payment
- ie after 1 year

Growing Annuity

$$\frac{C}{r-g}\left[1 - \frac{(1+g)^T}{(1+r)^T}\right] = PV_0$$

$r < g$ can be valid for an annuity

(3)

Growing Perpetuity $(r > g)$

$$PV_0 = \frac{C}{r-g}$$

If $r < g$ then payments would tend to ∞

Example 1

10,000 in a CD
5 years
4.5%

a) EAR = $10000 \cdot (1.045)^5$

b) Quarterly APR - change into EAR
- or. $\frac{}{4}$ for APR

$$r_{ear} = \left(1 + \frac{.045}{4}\right)^4 = 0.04$$

$10000 \cdot (1.04576)^5$

(4)

balt) 4.5% quarterly

$$3 \text{ month rate} = \frac{4.5\%}{4}$$

$$10000 \cdot \left(1 + r_{3\text{month}}\right)^{20}$$

Better to do other way - to analize
Easier to compare

Example 2 Auto Loan

Car 22,000
10,000 Down
6% APR
monthly compounding
5 years

Use the anuitty formula

$$22000 = \frac{C}{.06/12}\left[1 - \frac{1}{(1+.06/12)^{12\cdot 5}}\right] + 1000$$

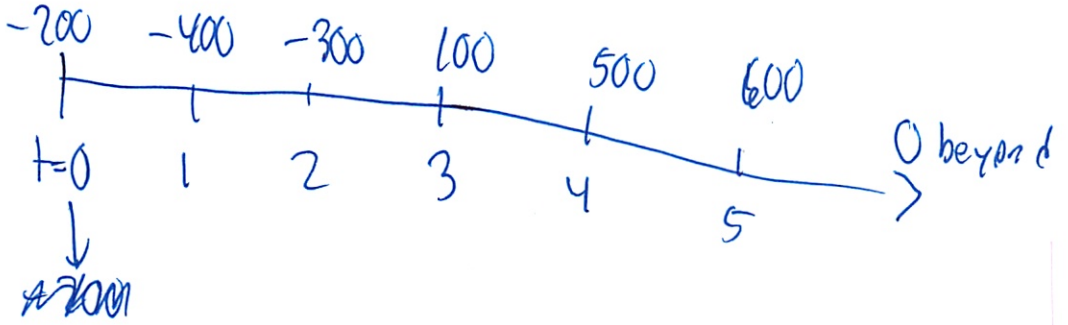
$$C = 231.99$$

(5)

This one is done monthly (1 period = 1 month

Payments start in month 1

Example 3 Project Planning



~~-200~~

Discount rate = 9%

Then look if PV is > 0

$$-200 - \frac{400}{(1+r)} - \frac{300}{(1+r)^2} + \frac{100}{(1+r)^3} + \frac{500}{(1+r)^4} + \frac{600}{1.095}$$

$= 1.41$ million

Just barely profitable

But what is the risk

Generally uncertainty built into discount rate

6

Nominal vs Real Interest Rate

~~Nominal - real conversion~~

Interest rate

$$1 + r_{real} = \frac{1 + r_{nominal}}{1 + i}$$

Cash flow

$$C_{real} = \frac{C_{nominal}}{1 + i}$$

i = inflation

$r_{real} < r_{nominal}$

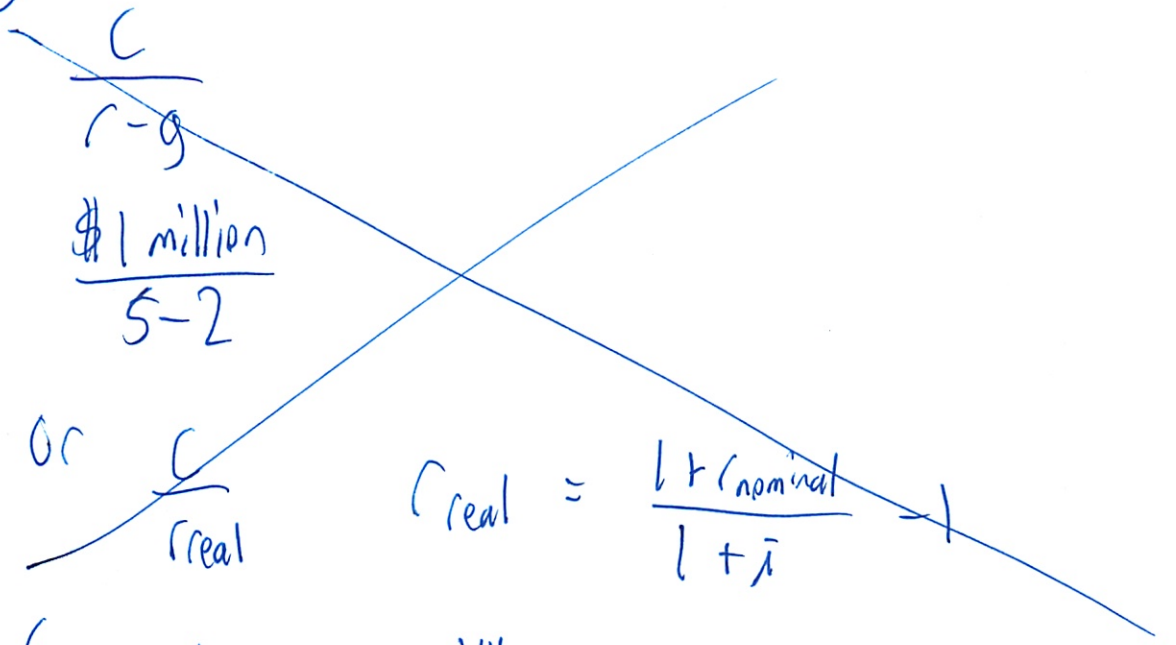
Everyone quotes nominal rates
- since inflation is uncertain

Don't mix + match!

Example 4 Scholarship

50 students $20,000 each/year
Rate is 5%/year
Inflation is 2%/year
How much $ put in for last forever?

(7)



So need 35 million

method 1 $1m + \frac{1m \cdot 1.02}{1.05 - 1.02} = 35m$

method 2 $r_{real} = \frac{1.05}{1.02} - 1 = 2.9412\%$

$1m + \frac{1m}{.029412} = 35m$

MIT Sloan School of Management

Finance Theory I
Craig Stephenson

15.401
Spring 2011

Problem Set 1: Present Value

(Due: Monday, February 14th,by 5:00 p.m.)

Ignore inflation, tax consequences and transaction costs throughout this problem set.

Unless specified as an "Excel Problem", all of these problems can (and should) be solved with a pencil, paper and a simple calculator. Do not use Excel (except to check your work, if you want). Show your work cleanly and write out the formulas that you used to solve the problems. Circle your final answers.

Warm-up (no need to hand in)

1. Compute the EAR for 20% APR compounded semiannually, quarterly, monthly, daily and hourly. EAR stands for effective annual rate. APR stands for annual percentage rate. See lecture slides for definitions.
2. Compute the present value of 50 annual payments of $20, 000 (first occurring in one year) discounted at 10% per year.

Answer:

1. 21.00%, 21.55%, 21.94%, 22.13%, 22.14%
2. $198, 296.29

Question 1

Michael Schumacher (or Dale Earnhardt Jr. if you are unfamiliar with Mr. Schumacher) has hired you as a financial advisor. He just received two offers to star in advertising campaigns. He can't do both and wants to select the better financial offer. The first one, L'Oreal, is a $15 million offer for $3 million a year for 5 years. The second one, Head & Shoulders, is a $16 million offer of $2 million a year for four years and $8 million in year 5.

What is your advice? (Hint: assume a range of discount rates, say 8% -16%)

Question 2

Frank just graduated from Sloan and got a prestigious private equity job in the City. He is short of cash but wants to live the high life. He finds a great deal for a sports car -a barely used, top-end Aston Martin Rapide for $140, 000. He's become somewhat obsessed with the car and has his eye set on it. Unfortunately, MBA life was expensive and he doesn't have much money aside for the down payment. Fortunately, HSBC is offering a special loan for car buyers that involves a 12% down payment and an interest rate of 5% (APR, monthly) for 20 years. Frank can easily cover the down payment with his sign-on bonus but he is worried that the monthly payment will be too high since he'll also be spending a lot on rent for his bachelor pad (and let's face it, fuel won't be cheap). Given his income, he'll be able to spend only $1, 000 a month to repay the loan.

Can he afford the Aston Martin Rapide, or should he go for something cheaper like a Chevy Volt?

Question 3

It is March 2009. Small businesses in the US are reeling from the recession and many have already gone bankrupt. TreeHouse Inc. is the last US maker of wood furniture, based in Morganton, NC. Business is particularly hard, as competition from the emerging economies is driving prices down at the same time as construction in the US has collapsed. The company has two emergency plans and must choose one of them in order to survive:

1. Close and sell the West Coast manufacturing plant, which will generate a salvage value of $6 million today and save $2 million annually in cost and expenses over the next 10 years. Because of the drop in capacity, the company will lose $2 million in revenue the first year, and the loss will increase by 10% per year over the following 9 years as the economy recovers.

2. Outsource R&D from the Morganton headquarters to IDEO, which will generate $7 million in cash today. Revenue will not be affected for five years but will suffer a reduction of $3.5 million in Years 6-10 due to lack of competitiveness.

The company's cost of capital (i.e. the risk adjusted discount rate it should use) is 12% per year. Which emergency plan should the company undertake? (Assume that revenues and costs occur at the end of each year.)

Question 4

Howard just went to Caesar's Palace in Las Vegas for the weekend. He participated in a prestigious poker tournament and won a prize for "$20 million". The CEO of Harrah's, Gary Loveman, gives him three options for receiving the money:

- Option 1 is $1 million a year for 20 years (with the first payment coming today)
- Option 2 is an immediate payment of $12 million
- Option 3 is $70, 000 a month for the rest of his life (starting at the end of this month)

Howard has access to a savings account with a 6% APR (compounded monthly).

1. Which is better, Option 1 or Option 2?

2. How many more years would Howard have to expect to live to make him select Option 3? (Hint: ignore possible risks in longevity and just use expectations)

Question 5 (Excel Problem -Optional)

Download the monthly S&P 500 prices from January 1950 until today. To do this, go to Yahoo Finance at www.finance.yahoo.com, click Investing → S&P 500 → Historical Prices → Monthly → Get Prices then at the bottom of the page click "Download to Spreadsheet".

1. What is your best estimate for next month's return?
2. What would have been your annualized return over the period if you had invested at the start of the dataset?
3. In what month did the lowest monthly return occur? What happened?

Return is gain

do later

Michael Plasmeier 15.401 P-Set 1 2/13

9.5/10

0a. $EAR = \left(1 + \frac{APR}{N}\right)^N - 1$ N = # of compounds per year

APR = 20%

Annual ($N=1$)

$$\left(1 + \frac{.20}{1}\right)^1 - 1 = 20\%$$ same

Semi annually ($N=2$)

$$\left(1 + \frac{.20}{2}\right)^2 - 1 = 21\%$$

Quarterly ($N=4$) = 21.5506%

Monthly ($N=12$) = 21.9391%

Daily ($N=365$) = 22.1336%

Hourly ($N=365 \cdot 24$) = 22.14%

Continuous

$$e^{.20} - 1 = 22.1403\%$$

(2)

b. Compute PV of 50 payment 20,000, 10% discount rate

$$\frac{20,000}{1+.1} + \frac{20,000}{(1+.1)^2} + \cdots + \frac{20,000}{(1+.1)^{50}}$$

$$\sum_{n=1}^{50} \frac{20,000}{(1+.1)^n} = 188,296$$

1. PV_1 8% rate $= \sum_{n=1}^{5} \frac{3,000,000}{(1+.08)^n}$ $= 11,978,100$ ✓

PV_2 8% $= \sum_{n=1}^{4} \frac{2,000,000}{(1+.08)^n} + \frac{8,000,000}{(1+.08)^5}$ $= 12,068,919$ ✓

Best

12% rate $= 10,814,300$

$= 10,614,110$

Best

16% rate $= 9,822,880$ ✓

$= 9,405,265$ ✓

Best ✓

It depends where he forcasts the future interest/discount rate at. Intersects at 9.12% so he should choose offer 2 when $r < 9.12\%$ or offer 1 when $r > 9.12\%$

(5)

2. 140,000 purchase
   12% down - covered
   5% APR
   monthly, 20 years
   budget 1,000/month
   Can he afford?

Down $140,000 \cdot .12 = 16,800$

Loaned $140,000 - 16,800 = 123,200$

$$EAR = \left(1 + \frac{.05}{12}\right)^{12} - 1 = 5.116\%$$

$$123,200 = \sum_{t=1}^{12 \cdot 20} \frac{M}{\left(1 + \frac{.05}{12}\right)^{12 \cdot 20}} *$$

← (Did we go over this formula in class?)

$$= \frac{M}{\left[\left(\frac{.05}{12}\right)\left(1 - \frac{1}{\left(1 + \left(\frac{.05}{12}\right)^{12 \cdot 20}\right)}\right)\right]}$$

↙ could take inverse of to solve for

↓ formula was right, - why not ans

$M = \cancel{513.33}$ 813.07

Yes he can afford it, He could almost buy 2.

(4)

3. $PV_1 = 6{,}000{,}000 + \sum_{n=1}^{10} \frac{2{,}000{,}000}{(1+.12)^n} - \frac{2{,}000{,}000}{.12-.10}\left[1 - \frac{(1+.10)^{10}}{(1+.12)^{10}}\right]$ ✓

↑ Oh duh for rates

$= 9{,}264{,}967$

$PV_2 = 7{,}000{,}000 - \sum_{n=6}^{10} \frac{3{,}500{,}000}{(1+.12)^n}$

$= -159{,}064$ ✓

4. His discount rate is his EAR

$\left(1 + \frac{.106}{12}\right)^{12} - 1 = 6.16778\%$

$PV_1 = \sum_{n=0}^{\cancel{20}\ 19} \frac{1{,}000{,}000}{(1+.0616778)^n} = 12{,}315{,}315$ ← Better

↑ first payment is today

$PV_2 = 12{,}000{,}000$ -.5

b. $12{,}315{,}315 = \sum_{n=1}^{\ell} \frac{90{,}000}{\left(1+\frac{.06}{12}\right)^n}$ ✓ Solve for ℓ

↑ that amount every month (was very confused over)

$\ell = 416.391$

$= 34\ 2/3$ more years

(5)

5. a) I would guess it would be similar to this month
- close to $40
without accounting for news (Egypt)

b)

MIT Sloan School of Management

Finance Theory I — 15.401
Konstantin Millbradt — Spring 2011

Problem Set 1 Solution : Present Value

Question 1

To understand the NPV of each deal, we discount the cash flows over the 5 year period. The question doesn't clarify whether the first payment would be today or a year from now - in this solution we assume that it will arrive a year from now, but either solution would be acceptable.

The table below gives the NPV for each value of the discount rate:

Discount rate	L'Oreal NPV $ million	Head & Shoulders NPV $ million	Comment
8%	11.98	12.07	Head&Shoulders preferrable
9%	11.67	11.68	Equivalent
10%	11.37	11.31	L'Oreal preferrable
11%	11.09	10.95	L'Oreal preferrable
12%	10.81	10.61	L'Oreal preferrable
13%	10.55	10.29	L'Oreal preferrable
14%	10.30	9.98	L'Oreal preferrable
15%	10.06	9.69	L'Oreal preferrable
16%	9.82	9.41	L'Oreal preferrable

For discount rates below 10%, Head & Shoulders is a better offer; L'Oreal is better for discount rates above 10%.

Our advice to Michael Schumacher: if you anticipate high interest rates in the next 5 years, you should take the L'Oreal offer. if you anticipate low interest rates in the next 5 years, you should take the Head&Shoulders offer.

Question 2

The monthly payment is $140000 \times 0.88 \times \left[\frac{1}{0.05/12}\left(1-\frac{1}{(1+0.05/12)^{240}}\right)\right]^{-1} = \$813.07$. Good news - Frank can afford the Aston Martin Rapide!

Question 3

Plan A: NPV$= 6 + \frac{2}{0.12}\left(1-1.12^{-10}\right) - \frac{2}{0.12-0.10}\left(1-\left(\frac{1.10}{1.12}\right)^{10}\right) = 0.812$ million

Plan B: NPV$= 7 - \frac{1}{1.12^5}\frac{3.5}{0.12}\left(1-1.12^{-5}\right) = -.15$ million

Plan A is better than Plan B. Actually, plan B would be a money-losing project...

Question 4

First, let's find the effective annual rate (compounded yearly): $r_{year} = (1 + \frac{r_{month}}{12})^{12} - 1$, and $r_{year} = 6.16\%$

1. Option 1: $NPV1 = (1 + r_{year}) * \frac{1}{r_{year}} * (1 - \frac{1}{r_{year}^{20}}) = \$12.01\text{million}$ (note the first payment is coming today and not in a year's time, which explaines the $(1 + r_{year})$ factor). Option 1 and 2 are almost the same in terms of NPV, so the question is who Howards trusts more to deliver on their promise - Caesar's Palace or his bank.

2. Let's calculate the NPV depending on the number of months n Howards receives the payment: expressed in \$ million, with $r = \frac{0.06}{12} = 0.005$,

$$NPV3 = \frac{0.07}{r} * (1 - \frac{1}{(1+r)^n})$$

The equation we're trying to solve is $NPV3 = \$12$ million. It simplifies into:

$$(1+r)^n = \frac{1}{1 - \frac{12*r}{0.07}}$$

We could either try different values of n, or solve with logarithms: using the property $log(r^n) = n * log(r)$ and taking a log on each side of the equation (which could be simplified further)

$$n = \frac{log(\frac{1}{1-\frac{12*r}{0.07}})}{log(1+r)}$$

Finally, $n = 390.15$ months or 32 years and a half.

Part B Valuation

Chapter 3: Fixed-Income Securities

Chapter 4: Common Stocks

Chapter 5: Forwards and Futures

Chapter 6: Options

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And 2/22 - slide 39

We have learned that:

- Business decisions often reduce to valuation of assets / CFs
- Two elements are important in valuing a CF: time and risk
- Value of CFs is determined in financial markets

From the market, we can learn

- How time affects value → time value of money
- How risk affects value → risk premium

In particular,

- Prices in the bond market reveal the time value of money
- Prices in the stock market reveal the risk premium

In this part of the course, we study the market's valuation of

- Bonds
- Stocks
- Forwards and futures
- Options

15.401 Finance Theory I

Craig Stephenson

MIT Sloan School of Management

Lecture 3: Fixed-Income Securities

- Fixed income securities
- Overview of fixed-income markets
- Term structure of interest rates
- Discount bonds and coupon bonds
- Forward interest rates
- Expectation hypothesis
- Interest rate risk
- Inflation risk
- Default risk

Readings:

- Brealey, Myers and Allen, Chapters 3, 23, 24
- Bodie, Kane and Markus, Chapters 14, 15, 16
- Salomon Brothers, "Understanding Duration and Volatility"

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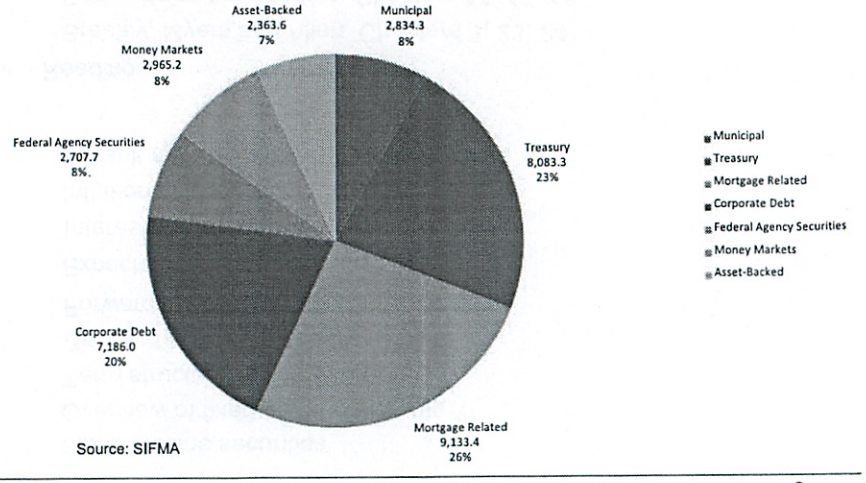
Fixed-income securities are financial claims with promised cash flows of fixed amounts paid at fixed dates.

Classification of fixed-income securities:

1. Treasury securities:
   - U.S. Treasury securities (bills, notes, bonds)
   - Bunds, JGBs, U.K. Gilts ...
2. Federal agency securities:
   - Securities issued by federal agencies (FHLB, FNMA ...)
3. Corporate securities:
   - Commercial paper (CP)
   - Medium-term notes (MTNs)
   - Corporate bonds ...
4. Municipal securities (Munies)
5. Mortgage-backed securities (MBS)
6. Asset backed securities (ABS), ...

US Bond Market (2010 Q1, $ billions)

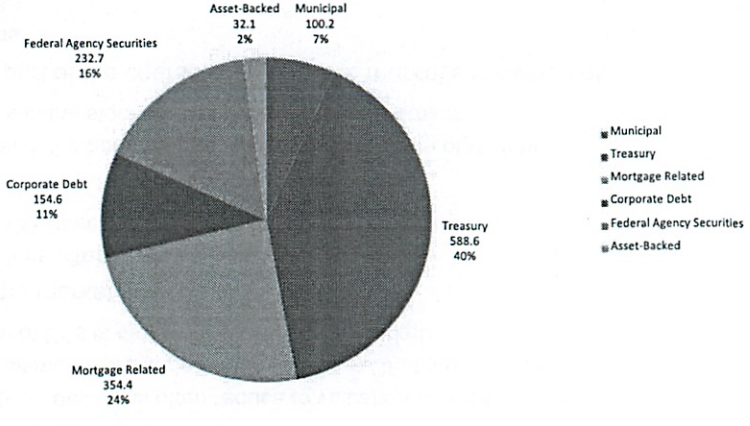


Source: SIFMA

Outstanding U.S. bond market debt ($ billions)

	Municipal	Treasury	Mortgage Related	Corporate Debt	Federal Agency Securities	Money Markets[3]	Asset-Backed[4]	Total
1996	1,261.6	3,666.7	2,486.1	2,126.5	925.8	1,393.9	404.4	12,265.0
1997	1,318.7	3,659.5	2,680.2	2,359.0	1,021.8	1,692.8	535.8	13,267.8
1998	1,402.7	3,542.8	2,955.2	2,708.5	1,302.1	1,977.8	731.5	14,620.6
1999	1,457.1	3,529.5	3,334.3	3,046.5	1,620.0	2,338.8	900.8	16,227.0
2000	1,480.5	3,210.0	3,565.8	3,358.4	1,853.7	2,662.6	1,071.8	17,202.8
2001	1,603.6	3,196.6	4,127.4	3,836.4	2,157.4	2,587.2	1,281.2	18,789.8
2002	1,763.0	3,469.2	4,686.4	4,132.8	2,377.7	2,545.7	1,543.2	20,518.0
2003	1,876.8	3,967.8	5,238.6	4,486.4	2,626.2	2,519.9	1,693.7	22,409.4
2004	2,000.2	4,407.4	5,862.0	4,801.7	2,700.6	2,904.2	1,827.8	24,503.9
2005	2,192.1	4,714.8	7,127.7	4,965.8	2,616.0	3,433.7	1,955.2	27,005.3
2006	2,363.5	4,872.3	8,452.8	5,344.6	2,651.3	4,008.8	2,130.4	29,823.7
2007	2,580.1	5,081.5	8,931.4	5,946.8	2,933.3	4,172.0	2,472.4	32,117.5
2008	2,635.3	6,082.2	8,897.3	6,205.1	3,207.8	3,791.7	2,671.8	33,491.2
2009	2,801.1	7,610.5	9,187.7	6,849.0	2,729.7	3,127.8	2,429.0	34,734.8
2010-Q1	2,843.3	8,083.3	9,133.4	7,186.0	2,707.7	2,965.2	2,363.6	35,273.5

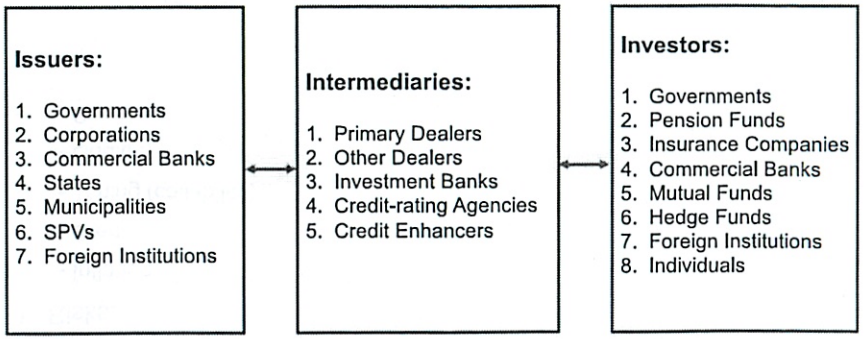
U.S. bondmarket issuance, 2010Q2($billions)



Source: SIFMA

Issuance in U.S. bond markets ($ billions)

	Municipal	Treasury	Mortgage-Related	Corporate Debt	Federal Agency Securities	Asset-Backed	Total
1996	185.2	612.4	492.6	343.7	277.9	168.4	2,080.2
1997	220.7	540.0	604.4	466.0	323.1	223.1	2,377.3
1998	286.8	438.4	1,143.9	610.7	596.4	286.6	3,362.7
1999	227.5	364.6	1,025.4	629.2	548.0	287.1	3,081.8
2000	200.8	312.4	684.4	587.5	446.6	281.5	2,513.2
2001	287.7	380.7	1,671.3	776.1	941.0	326.2	4,383.0
2002	357.5	571.6	2,249.2	636.7	1,041.5	373.9	5,230.4
2003	382.7	745.2	3,071.1	775.8	1,267.5	461.5	6,703.8
2004	359.8	853.3	1,779.0	780.7	881.8	651.5	4,424.3
2005	408.2	746.2	1,966.7	752.8	669.0	753.5	5,296.4
2006	386.5	788.5	1,987.8	1,058.9	747.3	753.9	5,722.9
2007	429.3	752.3	2,050.3	1,127.5	941.8	509.7	5,810.9
2008	391.3	1,037.3	1,339.7	707.2	984.5	139.5	4,602.1
2009	409.6	2,185.5	1,957.2	901.8	1,117.0	146.2	6,717.2
2010-Q2	100.2	588.6	354.4	154.6	232.7	32.1	1,462.6



1. Issuer:
   - US Treasury / US Government
   - States, municipalities, and agencies
   - Corporations
   - Foreign governments (sovereign bonds)

2. Term (number of years to maturity):
   - Short (less than 1 yr)
     - T-bills, CD's, TD's, Commercial paper
   - Long (more than 1yr)
     - T-bonds, corporate bonds
     - Consols

3. Price vs. par value (par = face value)
   - par bond
   - discount bond
   - premium bond

4. Coupon
   - Coupon rate: total annual interest payment per dollar of face value (par value x coupon interest rate = coupon payment)
   - Period (usually semi-annual)
   - Fixed or variable (floaters and inverse floaters)
   - Nominal or inflation-indexed (TIIS / TIPS)
   - Possibly no coupons (zero-coupon bonds)

5. Currency
   - Yankee bonds, Samurai bonds
   - Eurobonds

6. Credit risk
   - Risk free
   - Defaultable

7. Seniority and security
   - Senior, subordinated senior, junior…
   - Secured by properties and equipment, other assets of the issuer, income-stream, etc
   - Sinking fund provisions (sinkers)

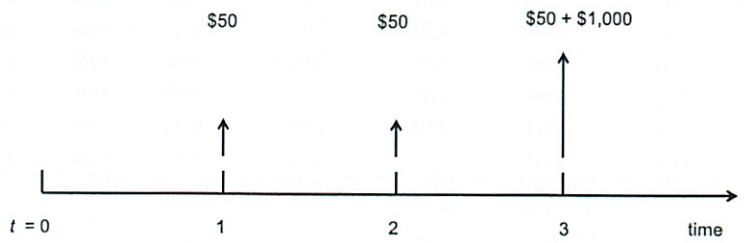
8. Covenants
   - Restrictions on additional debt issues, dividends, and other corporate actions.

9. Option provisions
   - Callability: After a certain period, issuer has the right to pay back the loan before it matures.
   - Putability: After a certain period, bondholder has the right to demand payment of the loan before maturity.
   - Convertibility: After a certain period, bondholder has the right to exchange the bond for stocks of the issuer.

Cash flow:

1. Maturity
2. Principal
3. Coupon

Example. A 3-year bond with par/face value of $1,000 and annual coupon payment of 5% has the following cash flow:



Valuation:

1. Time value
   - Interest rates
2. Risks:
   - Inflation
   - Credit
   - Timing (callability)
   - Liquidity
   - Currency …

Our objective here is to value riskless cash flows

Given the rich set of fixed-income securities traded in the market, their prices provide the information needed to value riskless cash flows at hand

In the market, this information on the time value of money is given in many different forms:

1. Spot interest rates
2. Prices of discount bonds (e.g., zero-coupon bonds or strips)
3. Prices of coupon bonds
4. Forward interest rates

Spot interest rate r_t is the current (annualized) interest rate for maturity date t

r_t is for payments only on date t

r_t is different for each different date t

Example. Spot interest rates on 2005.08.01:

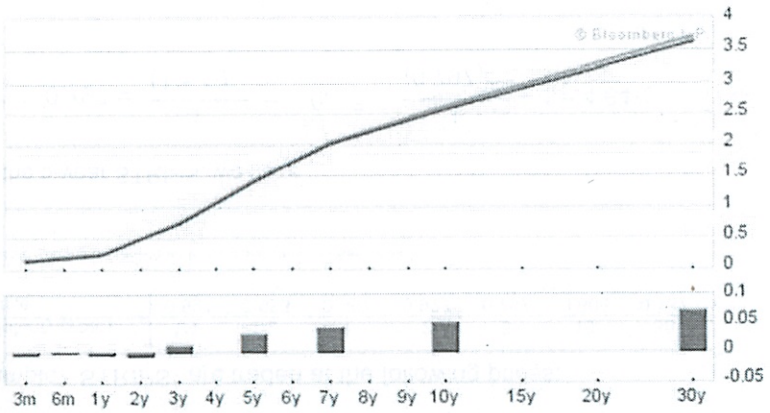
Maturity (year)	1/4	1/2	1	2	5	10	20	25	25.5	(longest)
Interest Rate (%)	3.29	3.61	3.87	3.97	4.06	4.41	4.65	4.57	4.61	

The set of spot interest rates for different maturities

$$\{r_1, r_2, \ldots, r_t, \ldots\}$$

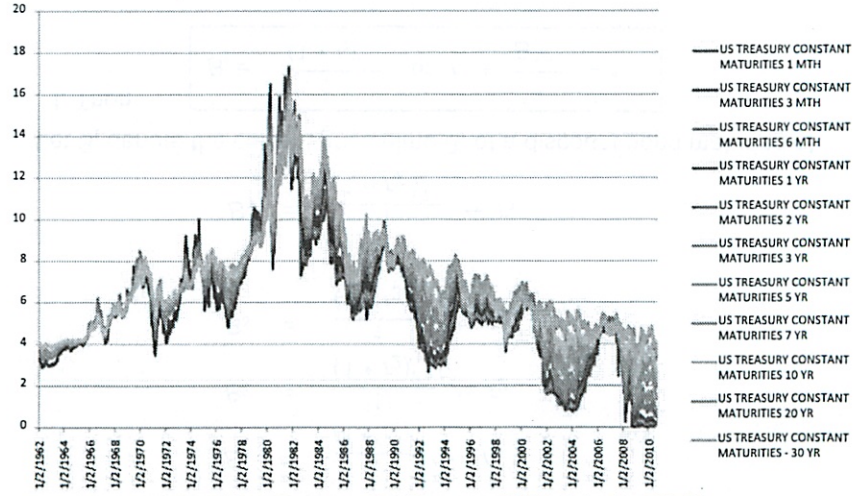
gives the term structure of interest rates, which refers to the relation between spot rates and their maturities

September 2, 2010 (Bloomberg)



http://online.wsj.com/mdc/public/page/mdc_bonds.html?mod=mdc_topnav_2_3000

History of U.S. term structure of interest rates (1/1/1962—9/1/2010)



http://fixedincome.fidelity.com/fi/FIHistoricalYield

A discount bond (zero coupon bond) with maturity date t is a bond which pays \$1 only at t

Example. STRIPS* are traded at the following prices:

Maturity (year)	1/4	1/2	1	2	5	10	30
Price	0.991	0.983	0.967	0.927	0.797	0.605	0.187

* Separate Trading of Registered Interest and Principal Securities

For the 5-year STRIPS, we have

$$0.797 = \frac{1}{(1+r_5)^5} \Rightarrow r_5 = \frac{1}{(0.797)^{1/5}} - 1 = 4.64\%$$

Prices of discount bonds provide information about spot interest rates

$$B_1 = \frac{1}{(1+r_1)} \leftrightarrow r_1$$

$$B_2 = \frac{1}{(1+r_2)^2} \leftrightarrow r_2$$

$$B_3 = \frac{1}{(1+r_3)^3} \leftrightarrow r_3$$

$$B_T = \frac{1}{(1+r_T)^T} \leftrightarrow r_T$$

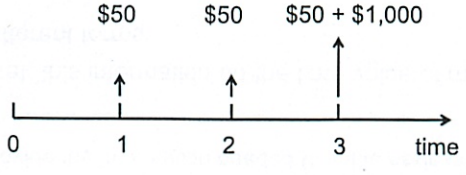
Let B_t denote the current price (time 0) of a discount bond maturing at t. Then

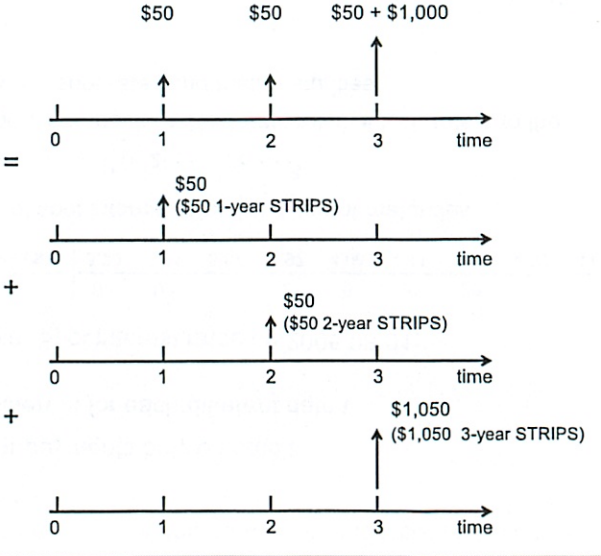
$$\boxed{B_t = \frac{1}{(1+r_t)^t} \quad \text{or} \quad r_t = \frac{1}{B_t^{1/t}} - 1}$$

A coupon bond pays a stream of regular coupon payments and the par or principal value at maturity.

A coupon bond is a portfolio of discount bonds.

Example. A 3-year bond with a \$1,000 par value and 5% annual coupon.





Suppose that the discount bond prices are as follows

t	1	2	3	4	5
B_t	0.952	0.898	0.863	0.807	0.757

What should the price of the coupon bond be?

$$\text{Price} = (\$50)(0.952) + (\$50)(0.898) + (\$1{,}050)(0.863)$$

$$= \$998.65$$

What if not?

The price of a coupon bond is given by

$$B = \sum_{t=1}^{T} (C_t \times B_t) + (P \times B_T)$$

$$= \frac{C_1}{1+r_1} + \cdots + \frac{C_{T-1}}{(1+r_{T-1})^{T-1}} + \frac{C_T + P}{(1+r_T)^T}$$

Yield-to-maturity of a bond, denoted by y, is given by

$$B = \sum_{t=1}^{T} \frac{C_t}{(1+y)^t} + \frac{P}{(1+y)^T}$$

Given its maturity, the principle and the coupon rate, there is a one to one mapping between the price of a bond and its YTM

Example. Current 1- and 2-year spot interest rates are 5% and 6%, respectively. The price of a 2-year Treasury coupon bond with a par value of \$100 and a coupon rate of 6% is

$$B = \frac{\$6}{1+0.05} + \frac{\$106}{(1+0.06)^2} = \$100.0539$$

Its YTM is 5.9706%:

$$\$100.0539 = \frac{\$6}{1+0.059706} + \frac{\$106}{(1+0.059706)^2}$$

Note the difference between YTM definition and bond pricing formula

So far, we have focused on spot interest rates: rates for a transaction between today, 0, and a future date, t

Now, we will study forward interest rates: rates for a transaction between two future dates, for instance, t_1 and t_2

For a forward transaction to borrow money in the future:

Terms of the transaction are agreed on today, t = 0

Loan proceeds are received on a future date t_1

Repayment of the loan occurs on date t_2

Note:

Future spot rates can be different from current corresponding forward rates – forward rates are forecasts of future spot rates

Example. As the CFO of a U.S. multinational, you expect to repatriate \$10 M from a foreign subsidiary in 1 year, which will be used to pay dividends 1 year later. Not knowing the interest rates in 1 year, you would like to lock into a lending rate one year from now for a period of one year. What should you do?

The current interest rates are:

time to maturity t (years)	1	2
spot interest rate r_t	0.05	0.07

Strategy:

Borrow \$9.524M now for one year at 5% (\$10.0 1.05)

Invest the proceeds \$9.524M for two years at 7%

Outcome (in million dollars):

Year	0	1	2
1-yr borrowing	9.524	−10.000	0
2-yr lending	−9.524	0	10.904
Repatriation	0	10.000	0
Net	0	0	10.904

The locked-in 1-year lending rate 1 year from now is 9.04%

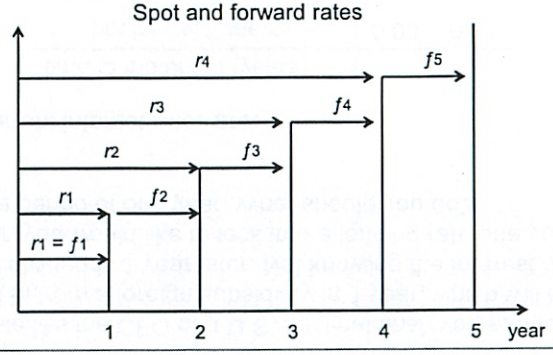
The forward interest rate between time t - 1 and t is

$$(1 + r_t)^t = (1 + r_{t-1})^{t-1}(1 + f_t)$$

or

$$f_t = \frac{B_{t-1}}{B_t} - 1 = \frac{(1 + r_t)^t}{(1 + r_{t-1})^{t-1}} - 1$$

Spot and forward rates



Example. Suppose that discount bond prices are as follows:

t	1	2	3	4
B_t	0.9524	0.8900	0.8278	0.7629
r_t	0.05	0.06	0.065	0.07

A customer wants a forward contract to borrow \$20M three years from now for one year. Can you (a bank) quote a rate?

$$f_4 = 8.51\%$$

What should you do today to lock-in these cash flows?

1. Buy 20,000,000 of 3 year discount bonds, costing
   (\$20, 000, 000)(0.8278) = \$16, 556, 000
2. Finance this by selling 4 year discount bonds of amount
   \$16,556,000/0.7629 = \$21,701,403
3. This creates a liability in year 4 in the amount \$21,701,403

Cash flows from this strategy (in million dollars):

Year	0	1—2	3	4
Purchase of 3-year bonds	-16.556	0	20.000	0
Sale of 4-year bonds	16.556	0	0	-21.701
Total	0	0	20.000	-21.701

The interest for this future investment is given by:

$$\frac{\$21, 701, 403}{\$20, 000, 000} - 1 = 8.51\%$$

What determines the term structure of interest rates?

1. Expected future spot rates
2. Risk of long bonds

Models of interest rates:

Expectations Hypothesis

Liquidity Preference

Dynamic Models (Vasicek, Cox-Ingersoll-Ross, ...)

Expectations Hypothesis: Forward rates predict future spot rates

$$f_t = E[r_1(t)]$$

Implications:

The slope of the term structure reflects the market's expectations of future short-term interest rates

Liquidity Preference Hypothesis: Investors regard long bonds as riskier than short bonds

$$f_t = E[r_1(t)] + \text{Liquidity Premium}$$

Implications:

Long bonds on average receive higher returns than short bonds

Forward rates on average "over-predict" future short-term rates

Term structure reflects

a) expectations of future interest rates, and

b) risk premium demanded by investors in long bonds

Another version of the EH: The long term rate is a geometric average of current and expected future short rates.

$$(1+r_t(2)) = [(1+r_t(1))(1+E_{t+1}(1))]^{1/2}$$

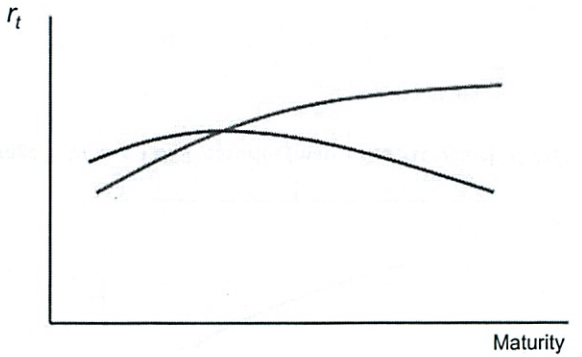
Recall the definition of the forward rate:

$$(1+r_t(2)) = [(1+r_t(1))(1+f_t(1))]^{1/2}$$

Compare with *prediction* of EH theory

Therefore, under the EH theory, the forward rate equals the expected future 1-year interest rate.

Term structure contains information about future interest rates



Average rates of return on US Treasuries, 1952/1 - 2009/12.

(Source: CRSP)

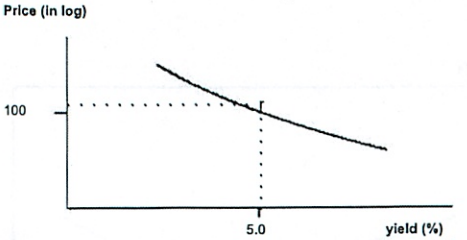
	Long Term	Short Term
Nominal	6.58%	5.35%
Real	2.93%	1.70%

Average inflation: 3.65%

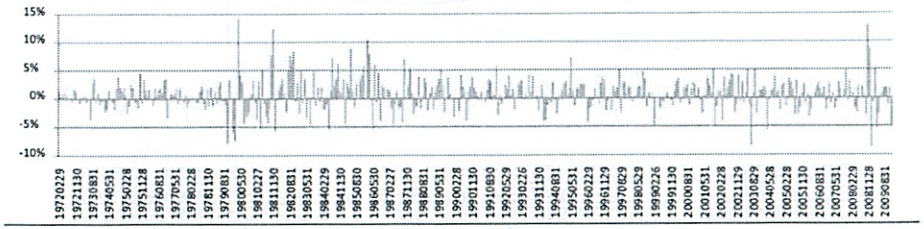
Long term = maturities > 60 months and ≤ to 120 months

Short term = maturities < 6 months

As interest rates change (stochastically) over time, bond prices also change. The value of a bond is subject to interest rate risk.



Average Returns on US Treasuries (maturities >10 years), 1972/1--2009/12



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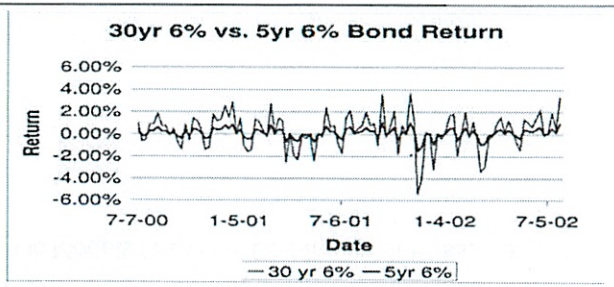
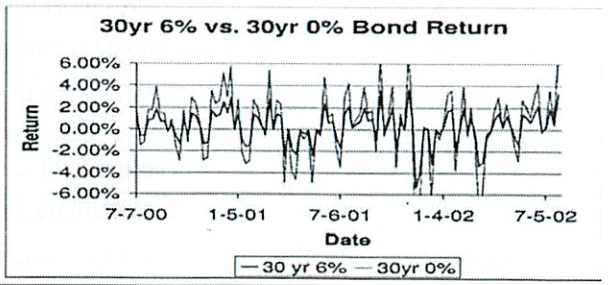




Duration and Modified Duration (assume a flat term structure at $r_t = y$)

Macaulay duration is the weighted average term to maturity

$$D = \sum_{t=1}^{T} \frac{PV(CF_t)}{B} \times t = \frac{1}{B} \sum_{t=1}^{T} \frac{CF_t}{(1+y)^t} \times t$$

A bond's interest rate risk can be measured by its relative price change with respect to a unit change in yield:

$$MD = -\frac{1}{B}\frac{\Delta B}{\Delta y} = \frac{D}{1+y}$$

This is called a bond's modified duration or volatility.

Example. Consider a 4-year T-note with face value $100 and 7% coupon, selling at $103.50, yielding 6%.

t	CF	PV	$t \cdot$ PV(CF)
1	3.5	3.40	3.40
2	3.5	3.30	6.60
3	3.5	3.20	9.60
4	3.5	3.11	12.44
5	3.5	3.02	1510
6	3.5	2.93	17.59
7	3.5	2.85	19.92
8	103.5	81.70	653.63
		103.50	738.28

- For T-notes, coupons are paid Semi-annually
- Use 1/2 year (6 months) as time unit

Duration: D=(738.28)/103.50=7.13

Modified duration (volatility): MD = D/(1 + y) = 7.13/1.03 = 6.92

If the semi-annual yield moves up by 0.1%, the bond price decreases roughly by 0.692%

The duration of a portfolio is the weighted average of the durations of the constituents

What is the duration of a zero-coupon bond?

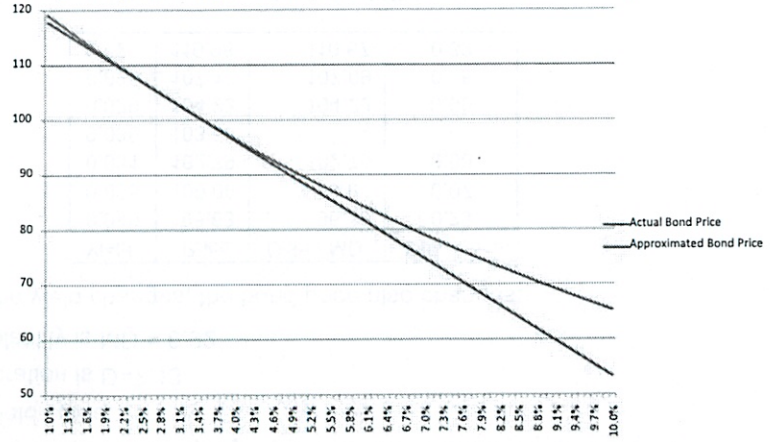
What must be true for the duration of a coupon bond?

What happens to the duration of a coupon bond if (all else equal) the coupon rate increases?

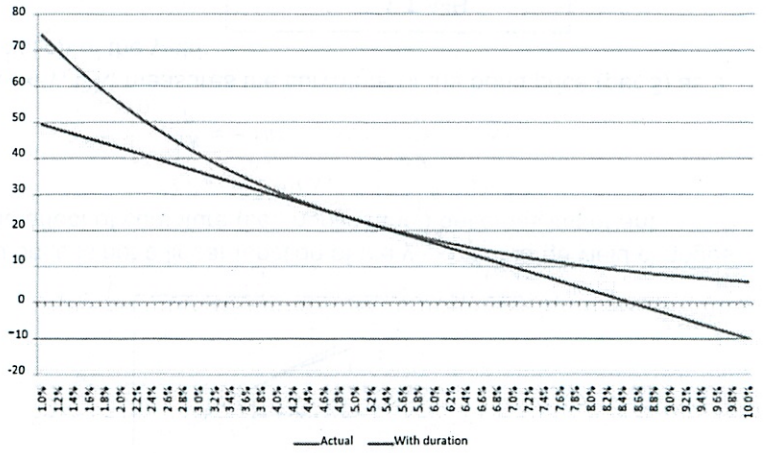
What happens to the duration of the bond if (all else equal) the YTM increases?

The duration of a perpetuity is: (1+y)/y

Same example: 4-year T-note, face value $100, 7% coupon, 6% yield (semi-annual coupons and 6 month time units)



Another example: 30-year zero-coupon bond, y=5%



Example (Continued) 4-year T-note with 7% coupon and 6% flat yield curve.

Duration is D=7.13

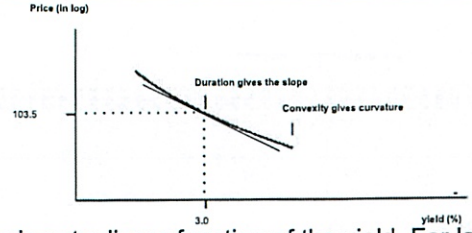
Volatility is MD = 6.92

As the yield changes, the bond price also changes:

Yield	Price	Using MD	Difference
0.040	96.63	96.35	0.29
0.035	100.00	99.93	0.07
0.031	102.79	102.79	0.00
0.030	103.50	-	-
0.029	104.23	104.23	0.00
0.025	107.17	107.09	0.08
0.020	110.98	110.67	0.32

For small yield changes, pricing by MD is accurate

For large yield changes, pricing by MD is inaccurate



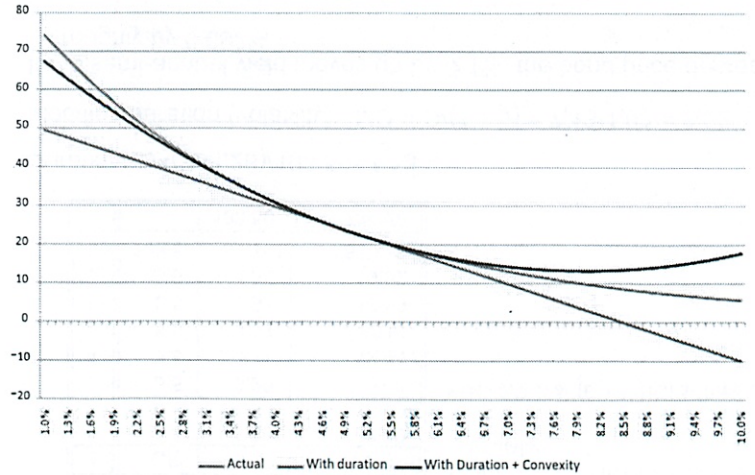
Bond price is not a linear function of the yield. For large yield changes, the effect of curvature (i.e., nonlinearity) becomes important.

$$(\Delta B) = \frac{\Delta B}{\Delta y}(\Delta y) + \frac{1}{2}\frac{\Delta^2 B}{\Delta y^2}(\Delta y)^2 + \ldots$$

$$\frac{\Delta B}{B} \approx -MD \quad (\Delta y) + CX \quad (\Delta y)^2$$

Convexity, CX, measures the curvature of the bond price (per $) as a function of the yield:

$$CX = \frac{1}{2}\frac{1}{B}\frac{\Delta^2 B}{\Delta y^2}$$



Zero-coupon bond, y=5%, T=30.

Actual and approximated prices (using duration, or duration and convexity adjustments).

Duration matching means to make the duration of assets and liabilities equal.

Then, the sensitivity to interest-rate changes is:

$$\Delta P \cong \frac{D^{assets}}{1+y} P^{assets} \Delta y - \frac{D^{liabilities}}{1+y} P^{liabilities} \Delta y = 0$$

Interest rate changes makes the values of assets and liabilities change by the same amount: The portfolio is *immunized*.

Michael Lewis in his book Liar's Poker described S&L members as part of the "3-6-3 club": Borrow money at 3 percent, lend it out at 6 percent, and be on the golf course every afternoon by three o'clock.

S&L's had predominantly short-term deposits (short duration liabilities) and long-term mortgage loans (long duration assets).

William Poole, former president FRB St. Louis: "The decline of the savings institutions [in the 80s] was a consequence of rising nominal interest rates combined with duration mismatch."

The exact same argument is being made for the financial crisis of 2008

General Motor s pension fund had

- liabilities with duration of about 15 years
- assets (bonds) with duration of about 5 years
- Problem: Duration mismatch!

Price risk: When the interest rate falls:

- the value of the bonds increases, but
- the present value of the liabilities increases more!

Reinvestment risk:

- At the new interest rate, the assets could not be reinvested to make the future payments.

See Excel chart

Suppose:

- GM pension fund must pay $100M in 15 years
- the current market interest rate is 6% at all maturities
- the fund's current assets equals $100M / 1.06^{15} = $41.73M

The pension fund wants to invest in 1-year and 30-year zero-coupon bonds

- What are the prices of these bonds (face value is $100)?
- How many securities should the fund buy?

Right after the fund buys the bonds, the interest rate rises to 7%

- what is the new value of the fund's bond position?
- what is the new present value of the fund's liabilities?

	Maturity	Portfolio weight	PV bonds @ y = 6%	Bond price @ y = 6%	Nbr of bonds (mil.)	FV 15 years @ y = 6%	Bond price @ y = 7%	PV bonds @ y = 7%	FV 15 years @ y=7%
L	15		41.73	41.73	1.00	1.00	36.25	36.25	100
A	1	0.517	21.58	94.34	0.23	51.72	93.46	21.38	58.99
A	30	0.483	20.14	17.41	1.16	48.27	13.14	15.20	41.93
A	Total	1.00	41.73			100		36.58	100.9

Most bonds give nominal payoffs. In the presence of inflation risk, real payoffs are risky even when nominal payoffs are safe.

Example. Suppose that inflation next year is uncertain ex ante, with equally possible rate of 10%, 8% and 6%. The real interest rate is 2%.

The 1-year nominal interest rate will be (roughly) 10%.

Consider the return from investing in a 1-year Treasury security:

Year 0 value	Inflation rate (%)	Year 1 nom. payoff	Year 1 real payoff
1000	0.10	1100	1000
1000	0.08	1100	1019
1000	0.06	1100	1038

Fixed-income securities have promised payoffs of fixed amount at fixed times. Excluding government bonds, other fixed-income securities, such as corporate bonds, carry the risk of failing to pay interest and principal value as promised.

Default risk (credit risk) refers to the risk that a debt issuer fails to make the promised payments (interest or principal).

Bond ratings by rating agencies (e.g., Moody's and S&P) provide indications of the likelihood of default by each issuer.

Description	Moody's	S&P
Gilt-edge	Aaa	AAA
Very high grade	Aa	AA
Upper medium grade	A	A
Lower medium grade	Baa	BBB
Low grade	Ba	BB

Investment grade: Aaa -- Baa by Moody's or AAA -- BBB by S&P
Speculative (junk): Ba and below by Moody's or BB and below by S&P

Example. Suppose all bonds have par value $1,000 and
10-year Treasury strip is selling at $463.19, yielding 8%
10-year zero issued by XYZ Inc. is selling at $321.97
Expected payoff from XYZ's 10-year zero is $762.22

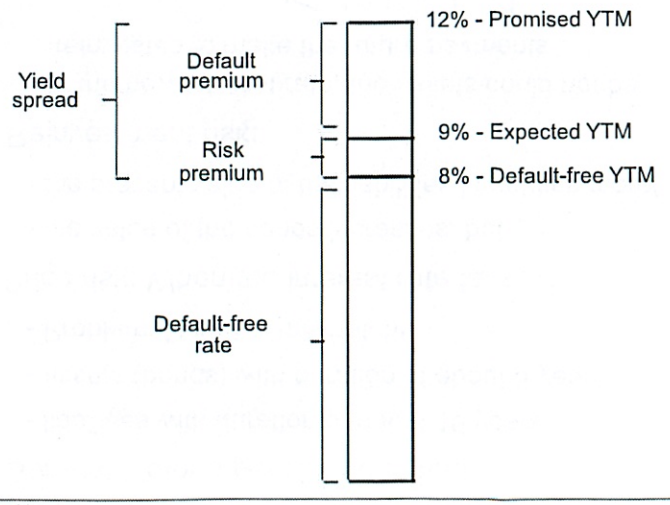
The XYZ bond: Promised YTM $= \left(\frac{1000.00}{321.97}\right)^{1/10} - 1 = 12\%$

Expected YTM $= \left(\frac{762.22}{321.97}\right)^{1/10} - 1 = 9\%$

and Default Premium = Promised YTM − Expected YTM
= 12% − 9% = 3%
Risk Premium = Expected YTM − Default-free YTM
= 9% − 8% = 1%

Promised YTM: the yield if default does not occur
Expected YTM: the probability-weighted average of all possible yields
Default premium: the difference between promised yield and expected yield
Bond risk premium: the difference between the expected yield on a risky bond and the yield on a risk-free bond of similar maturity and coupon rate

Yield-to-maturity for a risky bond



Average Annual Credit Loss Rates by Letter Rating, 1982-2008[1]

Year	Aaa	Aa	A	Baa	Ba	B	Caa-C	Inv-Grade	Spec-Grade	All Rated
1982	0.000	0.000	0.164	0.204	1.787	1.476	14.818	0.136	2.293	0.666
1983	0.000	0.000	0.000	0.000	0.431	2.981	19.907	0.000	1.808	0.456
1984	0.000	0.000	0.000	0.183	0.421	3.430	50.590	0.049	1.686	0.468
1985	0.000	0.000	0.000	0.000	0.563	2.981	0.000	0.000	1.374	0.377
1986	0.000	0.000	0.000	0.638	0.967	5.499	12.640	0.151	2.675	0.903
1987	0.000	0.000	0.000	0.000	1.015	2.294	7.454	0.000	1.574	0.561
1988	0.000	0.000	0.000	0.000	0.689	3.396	15.645	0.000	1.960	0.744
1989	0.000	0.339	0.000	0.334	1.706	4.900	14.048	0.162	3.256	1.319
1990	0.000	0.000	0.000	0.000	2.147	9.837	37.053	0.000	6.284	2.245
1991	0.000	0.000	0.000	0.172	3.099	7.980	24.632	0.041	5.935	1.856
1992	0.000	0.000	0.000	0.000	0.155	4.665	14.517	0.000	2.619	0.712
1993	0.000	0.000	0.000	0.000	0.356	2.840	16.766	0.000	1.931	0.514
1994	0.000	0.000	0.000	0.000	0.112	1.874	2.435	0.000	0.959	0.279
1995	0.000	0.000	0.000	0.000	0.374	2.215	4.764	0.000	1.530	0.481
1996	0.000	0.000	0.000	0.000	0.000	0.509	5.138	0.000	0.610	0.195
1997	0.000	0.000	0.000	0.000	0.083	0.849	6.424	0.000	0.890	0.300
1998	0.000	0.000	0.000	0.069	0.472	2.206	6.570	0.022	1.840	0.702
1999	0.000	0.000	0.000	0.054	0.781	3.078	11.266	0.022	3.336	1.351
2000	0.000	0.000	0.000	0.268	0.654	4.421	15.294	0.104	4.830	1.974
2001	0.000	0.000	0.130	0.150	1.027	7.463	24.723	0.104	7.952	3.045
2002	0.000	0.000	0.117	0.863	1.041	3.216	19.822	0.356	5.569	2.131
2003	0.000	0.000	0.000	0.000	0.555	1.205	12.217	0.000	2.978	1.013
2004	0.000	0.000	0.000	0.000	0.183	0.399	5.502	0.000	1.124	0.390
2005	0.000	0.000	0.000	0.079	0.000	0.552	2.788	0.029	0.781	0.299
2006	0.000	0.000	0.000	0.000	0.081	0.515	2.662	0.000	0.759	0.279
2007	0.000	0.000	0.000	0.000	0.000	0.000	2.746	0.000	0.429	0.163
2008	0.000	0.341	0.220	0.301	0.700	1.314	9.620	0.264	2.733	1.241
Average	0.000	0.025	0.023	0.124	0.719	3.041	13.335	0.053	2.582	0.913
Max	0.000	0.341	0.220	0.863	3.099	9.837	50.590	0.356	7.952	3.045
Min	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.429	0.163

1. Data as percent and based on issuer-weighted average default rates and issuer-weighted average senior unsecured bond recovery rates

- Fixed-income securities
- Overview of fixed-income markets
- Term structure of interest rates
- Discount bonds and coupon bonds
- Forward interest rates
- Expectation hypothesis
- Interest rate risk
- Inflation risk
- Default risk

15.401 Lecture

(1 min late)

Part B Valuation

- Key: Biz decision reduced to value of assets + CF_s
  - Time and risk
  - Value determined by the markets

Fixed Income Securities

- large market but not very exciting
- known cash flows at a known date

Bills - <1 year zero coupon

~~Notes~~ - auctioned every week <1 year

Notes - 1-7 years

Bonds - >7 years) original issue date

30 year bond is used as a benchmark

Bonds repaid by
- borrowing more
- printing money
- taxing

(2)

Securities issued by federal agencies Fannie + Freddie
- 2ndary markets
- now quasi-private banks
- get all their money at once
- legally not backed, but knew fed would protect

Commercial Paper = like a T-Bill
- < 270 days
no interest

Medium-Term Notes

Corporate bonds

Municipal Securities (Munies)

Morgage Backed Securities (MBS)
- get a little bit each month

Asset backed securities (ABS)
- other types of assets

35 trillion $ a year ago

Morgaged backed larger than Treasury

But Treasury was 40% of 2010Q2 issuance

(3)

Banks cautious since have very little capital

Assets		Liab + Equities	
Cash	$10	Liab	$91
Loans	$50		
Securities	$25	Equity	$9
Plant	$45		
	$100		$100

Asset backed securities fell off
- Even during good times banks would not loan them $

Much harder to borrow when econ is bad
esp from banks

Underwrite - help you decide terms for a bond issue
know buyers

Investment banks are of all sizes
- Some only do 1 function

(Review the bond stuff)

Face value = $1000

Semi-annual

(4)

debenture - not asset backed

Subordinated - back of the line, but before stock holders

Must be registered
- Tax bill

discount → currently selling at less than par
(I like this definition better)

Coupon is also fixed
- sometimes variable Floater

Samurai bond - Non Japan - company but in Japanese market

Yankee " " US " " " US "
- like Honda
- avoid exchange rate volatility + exposure
- risk management
- or if one market is strong

Sinking - recovers a little at a time
- (confirm)

Covenant - set max dividends, refuse further lending

Callability - issuer can pay back early
- no choice

Put-ability - gives bond holder to get paid par value

Convertability - exchange bond for stock

5

Always value in options
- fairly new

Managers work for stockholders, not bond holders

Investment bankers set up these options

Cash flow

(simpler than I thought - confused it - actually fairly simple)

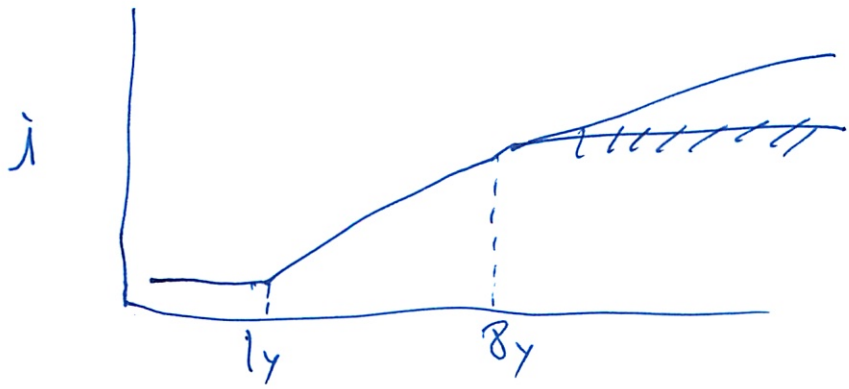
Simply cash flow calculations

Risks
- inflation
- credit
- timing (callability)
- liquidity - hard to trade
- currency

Spot - today

Forward - future expected number

look at term structure of interest rate chart + curve



usually upward sloping

(6)

Short rates more volatile

Was inverted in the 1980s

- in inflationary environment now, think it won't last (I could have figured this out)

Zero coupon bond

- discount bond
- only pays at maturity
- need to accrue tax every year
  + pay
  - so mostly low tax individuals
  - or tax exempt institutions
- don't need to worry about cash flows too much
  - easy to calculate
- don't need to worry about reinvesting your cash flow (getting a complete picture w/ here 15.50%, 14.02%, 15.48%)

$$B_t = \frac{1}{(1+r_t)^t} \quad \text{or} \quad r_t = \text{(missed slide)}$$

Good for pension orgs

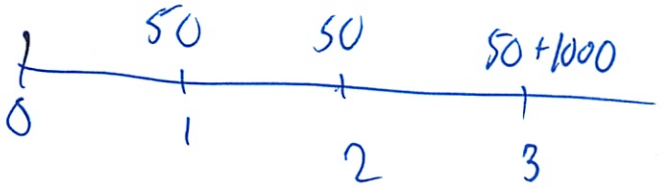
But if you need to cash out - you don't get much

Monday: strips

Coupon bonds

- the regular payout
- and periodic ~~pay~~ "coupons"

3 year bond ฿1,000 par 5% coupon



Oh so coupon at end too

Can decompose
- Cut off coupons into different categories

$$B = \sum_{t=1}^{T} (C_t \cdot B_t) + (P \cdot B_t)$$

$$= \frac{C_1}{1+r_1} + \cdots + \frac{C_{T-1}}{(1+r_{T-1})^{T-1}} + \frac{C_T + P}{(1+r_T)^T}$$

Add them up to get current price

(2)

Yield to maturity

Interest rates change over time
Can have different ones
Find price
Solve back to find effective interest rate

$$\$100.0539 = \frac{\$6}{1 + or_1} + \frac{\$106}{(1 + or_2)^2}$$

* key

6% is coupon rate

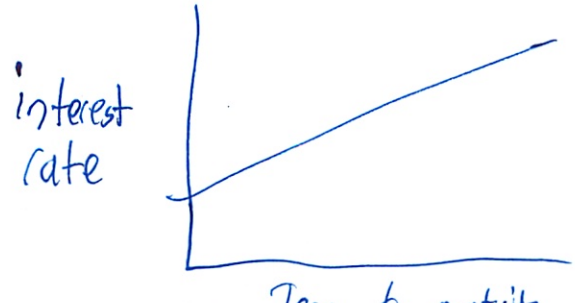
What interest rate does it solve for $100.0539 = \frac{6}{1+YTM} + \frac{106}{(1+YTM)^2}$

Iterative - guess + check is the old way to do it

Yield curve upward sloping

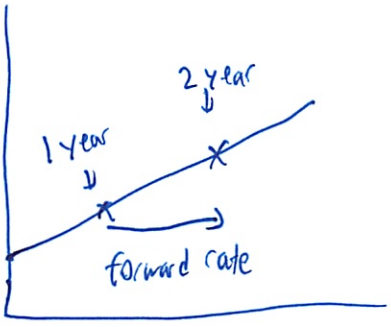
Spot rates - deal today

Forward rate - anticipated future spot rates
- terms agreed upon today to
- loan starts at t_1
- repaid to



(3)

$$(1+{}_0r_2)^2 = (1+{}_0r_1)\times(1+{}_1r_2)$$



$$(1+.04)\times(1+{}_1r_2 = .07)$$

1 year spot rate

in a year forcasted 1-year rate

Can solve for what 2 year spot rate is

$${}_0r_2 = (1.11280)^{1/2} - 1$$

$$= 5.497\%$$

But generally you calculate the ${}_1r_2$ rate

Forcasting interest rate is quite hard

But can be profitable

$$(1+{}_0r_2)^2 = (1+{}_0r_1)\times(1+{}_1r_2)$$

(spot 2, spot 1, forward)

${}_0r_2 = 4\%$

${}_0r_1 = 5\%$

So

${}_1r_2 = 13.15\%$

(4)

Forcast of spot rates in a year

Is by the market "unbiased"

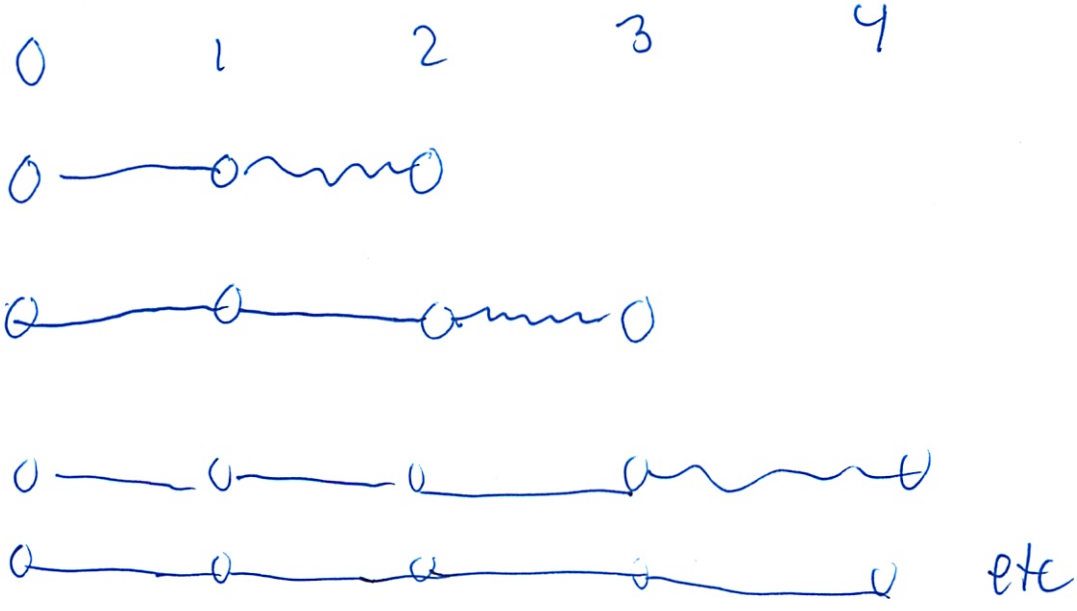
General formula

$$r = \frac{(1+r)}{(1+r)} - 1$$

So for ~~2R3~~ $2R_3$

$$2r_3 = \frac{(1+{}_0r_3)^3}{(1+{}_0r_2)^2} - 1$$

Do part by part to find out interest rates each year



etc

Can lock in yields or borrowing rates

Example Foreign Company

Can borrow the PV of $10 mill reciept and then invest it

Then when it comes in repay loan

Now have 2 years of intrest at that rate you locked in

If interest rates will rise at year 1, you will do worse in second year

When market is complete people can make any trade they want like

---

Example 2 Can the bank quote you a rate?

Yes calculates the rates using formulas on p 4

This middle position is the minimal risk.
Can just sit + wait and take the risk.
Or they can lock in their position

- Buy 3 year bonds
- Finance by selling 4 year bonds

Determinations

1. Expected future spot rates
2. Risk of long bonds

(6)

Expectations Hyp - just geometric average over time periods - just future expectation

$$f_t = E[r(t)]$$

Liquidity Pref - long term is more risk, so needs to be a risk preimum for going long term

$$f_t < E[r(t)] + \text{liquidity preimum}$$

We've been doing the pure Expectations

For Liquidity pref we need to back at the preimum

Another way to write this (slide 34)

Ons

Value of bonds based on interest rate

As interest ↑, price of bond ↓

- smaller PV of cash flow

(7)

Long term return of treasuries
- lost $ for investors sometimes

Long-term bonds are more voltile than short-term bonds

Bonds without coupons are ~~less~~ more voltile
- the higer the coupon the less voltile
- b/c you get your money sooner, so less exposure to interest rates

know interest rates ↓
- long-term
- no coupon
) best

interest rate ↑
- short term
- lots of coupons
) best

Duration

measure of effective maturity

0 coupon its just the stated maturity

with coupons you get some money sooner, so effective duration is shorter

(8)

B = bond price

$$D = \sum_{t=1}^{T} \frac{PV(CF_t)}{B} \cdot t = \dots$$

Shows sensitivity of bond to interest rate changes

Did not attend

15.401 Recitation

2a: Fixed-Income Securities

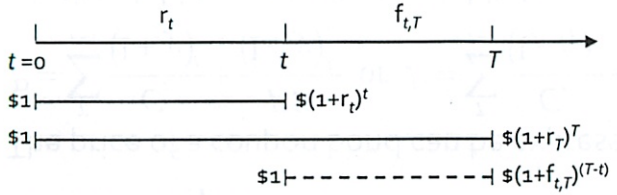
Learning Objectives

- Review of Concepts
  - Spot/forward interest rates
  - YTM and bond pricing
- Examples
  - Spot/forward
  - YTM and price
  - Rate of return

Review: spot/forward interest rates

- **Spot rate** (r_t) is the interest rate for the period (0, t).
- **Forward rate** ($f_{t,T}$) is the interest rate for the period (t, T) determined at time 0.
- No arbitrage implies $(1+r_t)^t \times (1+f_{t,T})^{(T-t)} = (1+r_T)^T$.

? notation mix up



Review: zero-coupon bond

- The spot rates are implied in the prices of zero-coupon (pure discount) bonds.
- We can calculate r_t given the price of a t-period zero-coupon bond:

$$P = \frac{FV}{(1+r_t)^t} \Leftrightarrow r_t = \left(\frac{FV}{P}\right)^{\frac{1}{t}} - 1$$

(FV = face value)

- After we find r_t and r_T, we can calculate $f_{t,T}$.

8/12

Review: coupon bond

- The price of a coupon bond can be expressed as:

$$P=\sum_{t=1}^{T}\frac{C_t}{(1+y)^t}+\frac{FV}{(1+y)^T} \text{ or } P=\sum_{t=1}^{T}\frac{C_t}{(1+r_t)^t}+\frac{FV}{(1+r_T)^T}$$

- y is the yield-to-maturity (or yield). It is equal to the rate of return on the bond if
  - it is bought now at price P and held to maturity, and
  - all coupons are reinvested at rate y.
- y is not a spot rate.
- There is a one-to-one mapping between y and P.

when is it not =?

Example 1: spot and forward rates

- Yields on three Treasury notes are given as follows:

Maturity (yrs)	Coupon rate (%)	YTM (%)
1	0	5.25
2	5	5.50
3	6	6.00

a. What are the prices of the 1-year, 2-year and 3-year notes with face value = $100?
b. What are the spot interest rates for year 1, 2 and 3?
c. What is the implied forward rate for year 2 to year 3?

Yes it was effective interest rate if it changes over time

when was yield taught in class?

Example 1: spot and forward rates

- Answer:

a. $P_1=\frac{100}{1+5.25\%}=\$95.01$

$P_2=\frac{5}{1+5.50\%}+\frac{105}{(1+5.50\%)^2}=\$99.08$

$P_3=\$100$

b. $r_1=5.2500\%;\ r_2=5.5063\%;\ r_3=6.0359\%$

c. $f_{2,3}=\frac{(1+r_3)^3}{(1+r_2)^2}-1=7.1032\%$

exactly what is given except for rounding error?

Example 2: YTM and price

- What is the price of a ten-year 5% treasury bond (face value = $100, annual coupon payments) if the yield to maturity is...
  - 4%?
  - 5%?
  - 6%?
- When is the price above/at/below par?

Example 2: YTM and price

- Answer:

$$P(FV, r, y) = \sum_{t=1}^{T} \frac{FV \cdot r}{(1+y)^t} + \frac{FV}{(1+y)^T}$$

$$= FV\left[\frac{r}{y}\left(1 - \frac{1}{(1+y)^T}\right) + \frac{1}{(1+y)^T}\right].$$

  - 4%: $108.11
  - 5%: $100.00
  - 6%: $ 92.64
- Price is above/at/below par when YTM is lower than/equal to/higher than the coupon rate.

Example 3: Rate of Return

- Suppose that you bought a 2-year STRIP (face value = $100) a year ago, and the interest rates at the time were as follows:

Years	Spot rate (%)
1	2.5
2	3
3	5

  - You sell your STRIP right now, and the yield curve happens to be the same as a year ago. What is the annualized return on your investment?
  - What is the annualized return if you sell it next year?

Example 3: Rate of Return

- Answer:
  - Purchase price = $100/(1.03)^2 = \$94.26$
  - Current price = 100/1.025=$97.56
  - Sell now: realized return = 3.5024% per year
  - Sell next year: return = 3% per year (for sure)

Example 3: Rate of Return (revisited)

- Suppose that five years ago today, you bought a 6% ten-year treasury bond (face value = $100, annual coupon payments) at a yield of 3.5% per year.
- Since then, you have deposited the coupons in a bank at 2% per year.
- Today you sell the bond at a yield of 5% per year.
- What is the annualized return on your investment?

Example 3: Rate of Return (revisited)

- Answer:
  - Cumulative value of deposited coupons = 31.22
  - Selling price today = 104.33
  - Total payoff = 135.55
  - Purchase price = 120.79
  - Annualized realized return = 2.3328%
- Follow-up question:
  - Why is the realized return so low?

15.401

Final Exam May 16 1:30-4:30PM

Arbitrage

Sell the overvalued asset - recieve $
Buy the undervalued asset - pay $
Profit now Reciepts > Payments

↓Sold
Short Pos ↘
Long Pos ↗ Square
↑Bought

Net Position
What you owe is same as what you own

But if it is just your expectation
Will have to be done at the same time
(Guess looks online)

Measures

Duration - true duration of the bond

effective maturity of all the value

Again for 6 months periods
- divide interest in half and coupon in half
- periods count up

"true maturity"
just the maturity if 0 coupon, otherwise shorter

(2)

Can calculate interest rate volatility

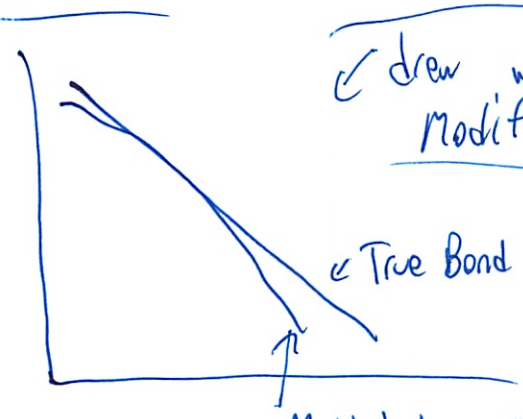
Modified duration $\frac{D}{1+\frac{r}{2}}$

r = interest rate now

If interest rate ↑ 1%, Bond prices ↓ MD%

6.92% here

Long duration means prices will change a lot
When interest rates change, get a new MD though
Immunization - can pick liabilities to reduce interest rates
But must activily manage



Modified duration - above linear relation - slope of line

6.92 is

When yield changes have MD change
Small increments accurate

(3)

Bond price is true price

MD is accurate at small increments

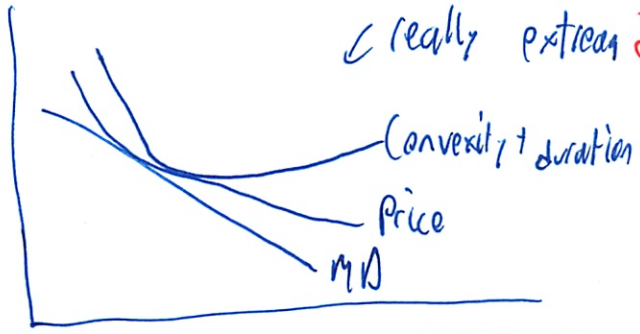
Convexity gives curvature

$$X = \frac{1}{2}\frac{1}{B}\frac{\Delta^2 B}{\Delta y^2}$$

Duration of portfolio is weighted avg of durations of the parts

Can do price sensitivities

How exposed am I to interest rate risks?



Immunization / Matching

Set portfolio to match duration of assets + liabilities

$$\Delta P \simeq \frac{D^{assets}}{1+y} P^{assets} \Delta y - \frac{D^{liab}}{1+y} P^{liab} \Delta y = 0$$

So both move together

And no risk

(4)

Savings + Loans used to be very important

True duration of a 30 year ~~morage~~ morgage is ~7 years
Since early repayment

Lending long and borrowing short
So losing liquidity
Need to roll over your liabilities frequently
If interest rates ↑ your cost of borrowing exceeds return on your portfolio
S+L biz model fundamentically mismatched duration

Also GM's pension fund
You know how much you have to pay out
But if the PV of bonds does not increase enough
You can't reinvest at the right rate to make future payments
Duration issue!
(Oh just realized these slides were already posted as part of 1/15 class!)
Seems like price risk
Manage price risk vs reinvestment risk

(5)

owe 100 M in 15 years

6% now

$$\text{Assets} = \frac{100\text{M}}{1.06^{15}} = 41.73\text{ M}$$

Then interest rates go up to 7%. What happens?

- See Excel spreadsheet

Two assets - a 1 year or a 30 year bond

So buy about half so you match the future PV

And the future duration!

When 7% interest rate ↑

- Obligations fell
  - 1 year bond falls a little
  - 30 year falls a lot

But this all matches

And we made a little money too

- Price risk - prices fell
- Reinvestment risk - can reinvest at a higher rate
  - The 1 years will be reinvested more later
  - at the new 7% rate

Offset by price drop on 30 year

But still net even and ever made some $

(when interest rate ↑)

(6)

Our recession
- interest rates fell
- Value of liabilities ↑
- Value of assets ↑ but when reinvest at higher rates

Now
If want 5 year assets buy a lot more short term bonds
But if interest rates ↓, value liabilities ↑, value assets ↑ (esp 30 year) but when reinvest prices ↑
So lose $ net
But if interest rates ↑ than make a lot more $

Easist portfolion management
- buy assets that mature exactly when liabilities do

Or can try to make $
- buy guessing on interest rates

Depends what you want to do
Want to shorten duration of assets if think interest rate ↑
But risky - can contribute less
- or a lot more

Inflation risk

- in nominal $
- so inflation ↓ real return
- so need to think what will inflation look like
- can protect by TIPS - Treasury Inflation Protected Security
- or some companies is some countries

Default Risk

- Bond ratings
- predictive risk of default
- econometric models
- do they lead reality or follow truth?
- more after the fact
- Investors move faster than bond agencies
- many pension funds can only invest in "investment grade"

- discount the expected payoff with expectation that they will pay = default preimum
- risk preimum = default preimum - Treasury

default preim - added risk of default
risk preimum - for company
default risk - inflation

(8)

CA has a default preimum now

Simultaneous troubled budget and higher borrowing rates

Markets work faster for companies than gov

- Since gov have a bunch of ways to meet obligations
- Only goes up when they realize they won't do it

Avg credit loss by letter chart

Slices come forward in recessions

15.401 - Fixed Income - Lecture 3
Immunization example on slides 51 & 52 of Lecture

Input Block	
Obligation of fund (FV liability)	$100,000,000
Obligation duration in years	15
Current YTM - all maturities	6.0%
FV of assets	$100,000,000
Current asset value (PV)	$41,726,506
Face value of zero coupon bonds	$100
Maturity of short bonds in years	1
Current price of 1 year zero	$94.340
Maturity of long bonds in years	30
Current price of 30 year zero	$17.411
What-if factor for interest rates?	7.0%

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
							What-if Analysis at			7.0%
	Maturity	Portfolio Weight	Bond Price at 6%	Number of Bonds in Millions	(3) x (4) PV Bonds at 6%	FV in 15 Years at 6% Millions	Bond Price at What-if Interest Rate	Number of Bonds in Millions	PV Bonds at What-if Interest Rate	FV in 15 Years at w-i int rate Millions
Liabilities	15		$41.727	1.0000	$41.727	$100.000	$36.245	1.0000	$36.245	$100.000
Asset 1	1	51.71%	$94.340	0.2287	$21.575	$51.707	$93.458	0.2287	$21.374	$58.971
Asset 2	30	48.29%	$17.411	1.1574	$20.151	$48.293	$13.137	1.1574	$15.204	$41.949
Σ Assets		100.00%			$41.727	$100.000			$36.578	$100.920
Dur Assets	15.0051									

15.401 - Fixed Income - Lecture 3
Immunization example on slide 50 of Lecture

Input Block	
Obligation of fund (FV liability)	$100,000,000
Obligation duration in years	15
Current YTM - all maturities	6.0%
FV of assets	$100,000,000
Current asset value (PV)	$41,726,506
Face value of zero coupon bonds	$100
Maturity of short bonds in years	1
Current price of 1 year zero	$94.340
Maturity of long bonds in years	30
Current price of 30 year zero	$17.411
What-if factor for interest rates?	5.0%

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
							What-if Analysis at			5.0%
	Maturity	Portfolio Weight	Bond Price at 6%	Number of Bonds in Millions	(3) x (4) PV Bonds at 6%	FV in 15 Years at 6% Millions	Bond Price at What-if Interest Rate	Number of Bonds in Millions	PV Bonds at What-if Interest Rate	FV in 15 Years at w-i int rate Millions
Liabilities	15		$41.727	1.0000	$41.727	$100.000	$48.102	1.0000	$48.102	$100.000
Asset 1	1	85.91%	$94.340	0.3800	$35.849	$85.914	$95.238	0.3800	$36.190	$75.237
Asset 2	30	14.09%	$17.411	0.3376	$5.878	$14.086	$23.138	0.3376	$7.811	$16.238
Σ Assets		100.00%			$41.727	$100.000			$44.001	$91.476
Dur Assets	5.0849									

15.401 After Class HW

2/22

b) ↓ calc from a

$$97.19 = \frac{3}{1.04} + \frac{103}{(1+_0r_2)^2}$$

↑ always goes back from start

$$97.2768 = \frac{4}{1.04} + \frac{4}{(1+_0r_2)^2} + \frac{104}{(1+_0r_3)^3}$$

↑ from above

↑ solve for

MIT Sloan School of Management

Finance Theory I — 15.401
Craig Stephenson — Spring 2011

Problem Set 2: Fixed Income

(Due: Wednesday, February 23rd, by 5:00 p.m.)

Assume that all bonds have face value of $100 and have annual coupon payments.

Unless specified as an "Excel Problem", all of these problems can (and should) be solved with a pencil, paper and a simple calculator. Do not use Excel (except to check your work, if you want). Show your work cleanly and write out the formulas that you used to solve the problems. Circle your final answers.

Question 1

does not say FV, r, find from

Suppose we have prices of two bonds trading in the market: one-year (zero coupon) bonds are trading at $96 and two-year 7% coupon bonds are trading at $101. Moreover, suppose we know that three-and four-year forward rates are 5% and 8% (f3 =0.05 and f4 =0.08), respectively. Thus, for example, one can lock in a loan to borrow money in 2 years that is to be repaid in 3 years at a rate of 5%.

(a) Calculate the one-and two-year spot rates (r1 and r2).
(b) Calculate the two-year forward rate, f2.
(c) Calculate the three-and four-year spot rates (r3 and r4).

Question 2

We have the following information on the bonds traded in the market.

	Maturity	Coupon Rate	YTM
A	1 year	0%	4%
B	2 years	3%	4.5%
C	3 years	4%	5%

(a) Calculate the prices of the three bonds.
(b) Calculate the spots rates, r1, r2, and r3.
(c) Suppose that a financial contract paying $100 in year 1, $200 in year 2, and $300 in year 3 trades at $500. Is there an arbitrage? If so, describe arbitrage strategy in detail (for example, x shares of bond A, y shares of bond B, etc) and demonstrate that this strategy yields an arbitrage profit.

↑ What you think it is worth?
If worth more than price can buy
And fund by selling overvalued assets

(So sell stuff) - not really - I should read online

Question 3

Consider a three-year 6% coupon bond with yield-to-maturity of 4%.

(a) Calculate the price of the bond.

(b) Calculate its Macaulay duration and modified duration.

(c) Suppose the YTM were to increase to 4.01%. Use the duration formula to approximate the new bond price and compare it to the exact price under the new YTM.

(d) Now suppose the YTM were to increase to 4.5%. Use the duration formula to approximate the new bond price and compare it to the exact price under the new YTM.

9.25/10

1. a. One-year spot rates $_0r_1$

Just the one-year zero coupon bond rate

$$r_t = \left(\frac{FV}{P}\right)^{\frac{1}{t}} - 1$$

$$\left(\frac{100}{96}\right)^{1/1} - 1$$

$$_0r_1 = 4.1\%$$ ✓

b. Two year rate $_0r_2$

$$101 = \frac{7}{(1+{}_0r_1)^1} + \frac{107}{(1+{}_0r_2)^2}$$ ✓

$$_0r_2 = 6.53$$

(Before had to guess + check)

c) Two year forward rate $_1r_2$

$$(1+{}_0r_2)^2 = (1+{}_0r_1)\cdot(1+{}_1r_2)$$

$$(1.0653)^2 = (1.041)\cdot(1+{}_1r_2)$$

$$_1r_2 = 9.81\%$$

~

(2)

d) Three year spot rate ${}_0r_3$

$$(1+{}_0r_3)^3 = (1+{}_0r_1)\cdot(1+{}_1r_2)\cdot(1+{}_2r_3)$$

$$= (1.041)(1.0901)(1.05)$$

$$= 1.19153$$

${}_0r_3 = 6.02\%$ ✓

e) Four year spot rate

$$(1+{}_0r_4)^4 = \ldots (1+{}_3r_4)$$

1.08

${}_0r_4 = 6.508\%$ ✓

(3)

2. a. Calculate price of $100 FV

A) $P = \frac{100}{(1+.04)^1}$

$= 96.15$ ✓

B) Coupons annual - says up top

$P = \frac{3}{(1+.045)} + \frac{103}{(1.045)^2}$

$= 97.1910$ ✓

C) $P = \frac{4}{(1+.05)^1} + \frac{4}{(1.05)^2} + \frac{104}{(1.05)^3}$

$= 97.2768$ ✓

b. Calc the spot rates

a) $0r_1$) That year's yield to maturity

$96.15 = \frac{100}{(1+r_1)}$

$r = 4.00\%$ ← same as above ✓

(4)

b) $_0r_2$

$$97.1910 = \frac{3}{(1+{}_0r_1)^1} + \frac{103}{(1+{}_0r_2)^2}$$

c) $_0r_3$

$_0r_2 = 4.51\%$ ✓

$$97.2768 = \frac{4}{(1.04)} + \frac{4}{(1.0451)^2} + \frac{104}{(1+{}_0r_3)^3}$$

$_0r_3 = 5.03\%$ ✓

c) Is this arbitrage?

If no arbitrage

$$(1+{}_0r_2)^2 = (1+{}_0r_1)\cdot(1+{}_1r_2)$$

$$_2r_3 = \frac{(1+{}_0r_3)^3}{(1+{}_0r_2)^2} - 1$$

← don't think had to do

$$= 6.08\%$$

$$_1r_2 = \frac{(1+{}_0r_2)^2}{(1+{}_0r_1)} - 1$$

$$= 5.02\%$$

(5)

PV =

Contract

$$\frac{100}{(1+dr_1)} + \frac{200}{(1+dr_2)^2} + \frac{300}{(1+dr_3)^3}$$

$= \$538.194$

so undervalued at $\$500$

Arbitrage

Sell the overpriced asset - recieve $

Buy the underpriced asset - pay money

Sell/short C to raise $

$97.2768 \cdot Q = 500$

$Q = 5.14$

Sell/short 5.14 bonds of C ... × −.5

Recieve $500

Buy the special contract for $500

After year 1, recieve $100

Pay back $4 \cdot 5.14 = \$20.56$ in coupons

Coupons complicate things - hope correct

After year 2, recieve $200

Pay back $4 \cdot 5.14 = \$20.56$ in coupons

After year 3, recieve $300

Pay back $104 \cdot 5.14 = 534.56$

Over →

PV of profit

$$= \frac{100 - 20.56}{1.04} + \frac{200 - 20.56}{(1.041)^2} + \frac{300 - 534.56}{(1.0503)^3}$$

hope did right

$= \$39.50$

Are there any other, better opportunities

Just says find one

- hope correct

(6)

3. Three year 6% coupon YTM 4%

a. $PV = \frac{6}{(1.04)} + \frac{6}{(1.04)^2} + \frac{106}{(1.04)^3}$

Price is the PV w/ no risk

$= 105.550$ ✓

b. Macaulay duration = weighted avg term to maturity of the cash flows from a bond. The weight of each cash flow is divided by the price. Measure of volatility w/ respect to interest rates

$$= \frac{\sum_{t=1}^{n} \frac{t \cdot C}{(1+y)^t} + \frac{n \cdot M}{(1+y)^n}}{\text{Current price}}$$

Called the effective maturity in class

$$= \frac{\frac{1 \cdot 6}{(1.04)} + \frac{2 \cdot 6}{(1.04)^2} + \frac{3 \cdot 6}{(1.04)^3}}{105.550}$$

$= 2.83813$ ✓

(7)

Modified duration - expresses measurable change in value of a security in respone to change in interest rates

$$= \frac{\text{Macaley Duration}}{\left(1 + \frac{YTM}{n}\right)}$$

(# coupon periods/year

$$= \frac{2.83813}{\left(1 + \frac{.104}{2}\right)} \quad -.25$$

$$= 2.78248$$

c) Suppose YTM = 4.01%

What is duration price and exact price?
- in class 2/22

New exact price = 105.521 ✓

New Macualy duration = 2.83811

New Modified duration = 2.78246] don't need

duration price $(105.550 - (4.01 - 4)) \cdot 2.78248 = 105.523$

105.550 → old price; 2.78248 → old duration; 105.523 → close

(8)

d. YTM = 4.5

New price = 104.123

New Macauly duration = 2.83696] don't need

New Modified duration = 2.78133]

Duration price = 105.550 − (4.5 − 4.) · 2.78248 =

= 104.189

not as close

15.401 Section D Spring 2011 – Solution to Problem Set #2

Question #1

Part A

1 year spot rate = r_1 = \$100/\$96 – 1 = 0.041667 = 4.17%

2 year spot rate = r_2

Year 0	Year 1	Year 2
\$101	\$7	\$107

$\$101 = \$7/(1.0417) + \$107/(1 + r_2)^2$

$\$94.28 = \$107/(1 + r_2)^2$

$r_2 = .065324 = 6.53\%$

Part B

The 2 year forward rate, f_2, = $(1.0653)^2/1.0417 - 1 = 0.089435 = 8.94\%$

Part C

The 3 year spot rate:

$(1 + r_3)^3 = (1.0417) \times (1.0894) \times (1.05)$

$r_3 = (1.191569)^{1/3} - 1 = 0.060164 = 6.02\%$

The 4 year spot rate:

$(1 + r_4)^4 = (1.0417) \times (1.0894) \times (1.05) \times (1.08)$

$r_4 = (1.286895)^{1/4} - 1 = 0.065089 = 6.51\%$

Question #2

Part A

Bond A price = \$100/1.04 = \$96.1538

Bond B price = $\$3/1.045 + \$103/(1.045)^2 = \$97.1910$

Bond C price = $\$4/1.05 + \$4/(1.05)^2 + \$104/(1.05)^3 = \$97.2768$

Part B

Spot rate from year zero to year 1, $r_1 = 4\%$ (stated)

Spot rate from year zero to year 2, r_2:

$$\$97.1910 = \$3/1.04 + \$103/(1+r_2)^2$$

$$\$94.3064 = \$103/(1+r_2)^2$$

$$.045076 = 4.5076\% = r_2$$

Spot rate from year zero to year 3, r_3:

$$\$97.2768 = \$4/1.04 + \$4/(1.045076)^2 + \$104/(1+r_3)^3$$

$$\$89.7683 = \$104/(1+r_3)^3$$

$$.050276 = 5.0276\% = r_3$$

Part C

The contract payoffs are:

Year 1	$100
Year 2	$200
Year 3	$300

And the contract trades for $500. Is arbitrage possible? Is there a mispricing of the contract? The package trades for $500, compared to the "true" prices of the individual component pieces, which are:

$\$100/1.04$	$= \$96.1538$
$\$200/(1.045076)^2$	$= \$183.1194$
$\$300/(1.050276)^3$	$= \$258.9469$

And sum to $538.2201. So yes, an arbitrage opportunity exists. The contract as a whole trades for $38.2201 less than the individual components.

Arbitrage requires you to buy the cheaper financial contract and short or sell a portfolio of the component bonds that replicates the contract. The payoff chart of the 3 bonds and the contract is:

	Year 1	Year 2	Year 3
Bond A	$100		
Bond B	$3	$103	
Bond C	$4	$4	$104

	Year 1	Year 2	Year 3
Contract	$100	$200	$300

Solving for the number of bonds sold which precisely match the contract – you sold the bonds so you must pay the bonds' cash flows and you bought the contract, so you receive the contract cash flows – the result is:

2.8826 of Bond C

1.8297 of Bond B

0.8297 of Bond A

If you sell these proportions of the 3 bonds you will receive $538.22, generating an arbitrage profit of $38.22 above the cost of the contract you purchased. The cash flows received from the contract exactly cover the cash payments you owe on the bonds sold.

Question #3

Parts A and B

	Year 1	Year 2	Year 3
Cash Flow	$6	$6	$106
Present value factor at 4% for t periods	0.961538	0.924556	.888996
P.V. Cash Flow	$5.7692	$5.5473	$94.2336
x Weighted by t	1	2	3
Weighted P.V. C.F.	$5.7692	$11.0946	$282.7008

A. The price of the bond is the Σ present value of cash flows = $105.5501.

B. The Σ present value of the cash flows, weighted by t = $299.5646.

The duration of the bond is $299.5646/$105.5501 = 2.8381 periods, years in this case

The modified duration of the bond is 2.8381/(1+.04) = 2.7290 periods, years in this case

Part C

Suppose market interest rates, YTM, increase to 4.01%. What happens to the bond's price?

Precisely, changing the discount rate to 4.01% and applying this to the bond's cash flows, the new price is $105.5213.

Using modified duration, the change in price is:

-2.7290 x .0001 = -0.0002729

And the new price will be:

$105.5501 x (1 – 0.0002729) = $105.5213 → exactly the same as above. For small changes in YTM, modified duration gives very accurate changes in bond prices.

Part D

Suppose market interest rates, YTM, increase to 4.50%. What happens to the bond's price?

Precisely, changing the discount rate to 4.50% and applying this to the bond's cash flows, the new price is $104.1234.

Using modified duration, the change in price is:

-2.7290 x .005 = -0.013645

And the new price will be:

$105.5501 x (1 – 0.013645) = $104.1099 → not exactly the same as above. For larger changes in YTM, modified duration produces less accurate changes in bond prices.

15.401 Finance Theory I

Craig Stephenson

MIT Sloan School of Management

Lecture 4: Common Stocks - Equity Markets

Early Markets

- Amsterdam Bourse, founded in 1602, was the first market to trade shares on ventures. The first share traded was the Verenigde Oostindische Compagnie
- Trading begins in London in 1698. Stock dealers are expelled from the Royal Exchange for rowdiness and trade in the streets and coffee house of the City, particularly in Jonathan's Coffee House in Change Alley
- London Stock Exchange formally opens in 1801

Types of Orders

- Market Order
  - Buy or sell right now at the current market price
    - Guaranteed Execution, not guaranteed Price
- Limit Order
  - Buy or sell if and only if the market reaches your price
    - Not guaranteed execution, but if executed, guaranteed price
- Order Information
  - Stock, number of shares, Buy or Sell, *Price, Time*
    - You can either specify Price or Time, but not both
      - Price → Limit Order
      - Time (now) → Market Order

Types of Investors

- Retail – 39% of all listed corporate stock in the United States
  - You and I
  - Generally less than 3,000 shares
- Institutional – 61% of all listed corporate stock in the U.S.
  - Usually more shares per trade (trend is opposite)
  - Block trades are large ≥ 10,000 shares
- We'll look at investor types in more detail later

2/24

Types of Stock Quotes

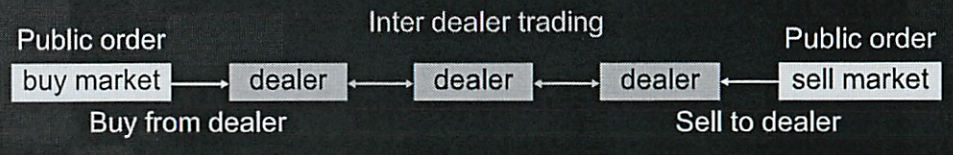
- Bid
  - Price at which a dealer/individual will buy the stock (from you)
- Ask
  - Price at which a dealer/individual will sell the stock (to you)
- Bid-Ask Spread
  - Difference between the highest bid and lowest ask in any market
  - Dealers buy at their bid and sell at their ask earning the spread
- Quote
  - The Bid Price and amount and the Ask Price and amount
    - $16.65 for 500 shares Dell – Dealer or Individual Ask
    - $16.64 for 300 shares Dell – Dealer or Individual Bid

Types of Markets

- Major Markets
  - Where you list and who governs you
    - NASDAQ
    - NYSE Euronext
- Other Markets
  - Just trades your stock
    - Regional exchanges – Pacific, Midwest
    - Electronic Communication Networks – ECNs
    - Dark Pools

Dealer Markets vs. Auction Markets

Pure dealer market



Pure auction market

NASDAQ

- "National Association of Security Dealers Automated Quotation" system, the world's first electronic stock market → floorless
  - Created in 1971
- NASDAQ is a Dealer Market
  - Most Public Orders are filled by Dealers
    - Similar to an automobile dealer
  - No physical location, dealers are linked electronically

NASDAQ, Instinet, and OMX

- July of 2006 NASDAQ purchases Instinet Group Inc.'s electronic trading network
- Instinet owns an Electronic Communications Network, or ECN, where buyers and sellers trade predominately NASDAQ-listed shares
- May of 2007 NASDAQ purchases Sweden's OMX AB Exchange, allowing investors to globally trade cash equities, derivatives, debt and ETFs
- NASDAQ OMX Group hopes to increase its position in global trading (30% of U.S. cash equities trading volume in 2008)

NYSE

- New York Stock Exchange
  - 1792, The Buttonwood Agreement, signed by 24 brokers and merchants, agreeing to trade securities, 3 government bonds and 2 bank stocks
- NYSE is an Auction Market, at least in perception
  - Most Public Orders interact with other public orders
  - Like an art or estate auction
  - Physical Location in New York with electronic links
    - All orders come to a single location at 11 Wall Street

"Old" NYSE

- Orders flow to the floor where trades are executed
- Floor brokers, both house and independent (1,366), execute trades
- Specialists, 7 firms and over 400 individuals, facilitate trades and provide liquidity

NYSE Becomes NYSE Group
NYSE Group Becomes NYSE Euronext

- NYSE merges with Archipelago Exchange, a leading ECN, or Electronic Communications Network, in April 2005
- NYSE Group merges with Euronext N.V. (Amsterdam, Brussels, Paris, and London; cash and derivatives) in April 2007
- The new group, NYSE Euronext, allows investors to globally trade cash equities and derivative securities, its goal is to create a more efficient market in execution, information, and technology
- NYSE Euronext acquires the American Stock Exchange in October 2008, and today has 4 platforms in the U.S.; NYSE, NYSE Arca, NYSE Amex, and ArcaEdge
- NYSE Euronext hopes to increase its position in world wide stock trading (42% of global cash equities trading volume in 2008)

NYSE Evolves to a Hybrid Market

- 99% of all orders now reach the trading post through SuperDot, an electronic order routing system
- In the Fall of 2006 restrictions were increased on maximum electronic order size from 1099 shares to 1 million shares
- By early 2007 3,618 NYSE securities were traded almost exclusively electronically
- Instead of overseeing trades in a few stocks, specialists handle more stocks by supervising computer programs which execute thousands of trades a minute
- Average order execution time decreases from 9 seconds to 3/10 of a second
- An indicator of this transition away from the floor towards electronic execution is the number of specialist clerk keystrokes have decreased from 45 million per day to less than 20 million per day in early 2007
- The trading floor is lonelier, with fewer specialist staffers and floor brokers

"New" NYSE

- Designated Market Makers convene a physical auction for fewer than 1 in 30 trades
- Floor brokers ($n \approx 650$) execute 4 of 30 trades
- Leaving 25 of 30 trades executed electronically between computer systems of dealers, brokers, and institutional investors

February 2011 NYSE is Sold

- On February 15, 2011, Deutsche Boerse AG purchases NYSE Euronext in a $9.53 billion all-stock transaction
- This deal follows:
  - Singapore Exchange Ltd.'s earlier and still active bid for ASX, Ltd. (Australia)
  - London Exchange Group Plc's deal to buy Canada's TMX Group Inc.
  - Hong Kong Exchanges & Clearing Ltd. stating in early February it's open to "alliances, partnerships, and other relationships"
- Global exchanges trading cash equities, derivatives, and other financial assets, efficiently and instantaneously

Electronic Communication Networks - ECNs

- Fully automated open electronic limit order books
- Instinet and NYSE Arca are examples
- Individual investors can view the limit order book and enter buy and sell orders
- The ECN automatically matches orders for execution
- www.nysearca.com/marketdata/book_info.asp

The Limit Order Book - DIS

DIS The Walt Disney Company

Bid				Ask			
ID	Price	Size	Time	ID	Price	Size	Time
ARCA	25.47	10300	10:12:42	ARCA	25.48	9200	10:12:40
ARCA	25.46	7100	10:12:23	ARCA	25.49	6400	10:12:16
ARCA	25.45	5900	10:11:57	ARCA	25.50	5700	10:12:33
ARCA	25.44	5600	10:12:03	ARCA	25.51	6150	10:12:18
ARCA	25.43	2800	10:12:29	ARCA	25.52	3400	10:11:45
ARCA	25.42	3100	10:11:31	ARCA	25.53	4500	10:12:28
ARCA	25.41	2300	10:12:14	ARCA	25.54	1600	10:12:02
ARCA	25.40	2400	10:12:40	ARCA	25.55	2900	10:12:37
ARCA	25.39	3300	10:11:58	ARCA	25.56	1800	10:11:48
ARCA	25.38	1200	10:12:34	ARCA	25.57	2100	10:12:12
ARCA	25.37	1700	10:12:24	ARCA	25.58	1800	10:11:51
ARCA	25.36	2200	10:11:32	ARCA	25.59	1500	10:12:06

The Limit Order Book - FUN

FUN Cedar Fair, LP

Bid				Ask			
ID	Price	Size	Time	ID	Price	Size	Time
ARCA	10.45	500	9:35:18	ARCA	10.48	300	9:52:40
ARCA	10.44	800	9:32:34	ARCA	10.50	100	9:57:21
ARCA	10.42	200	9:49:56	ARCA	10.53	1100	9:36:54
ARCA	10.40	1400	9:41:28	ARCA	10.54	700	3:56:47
ARCA	10.37	100	3:43:12	ARCA	10.57	200	3:59:34
ARCA	10.35	400	3:38:04	ARCA	10.61	2500	9:58:35
ARCA	10.34	300	3:44:51	ARCA	10.63	500	3:30:06
ARCA	10.33	100	9:44:37	ARCA	10.64	300	10:01:28
ARCA	10.31	600	3:32:49	ARCA	10.66	100	3:49:13

Other Electronic Markets

- BATS Exchange
  - All electronic match of buyers & sellers
  - Launched in 2006, became a national securities exchange in November 2008 (SEC)
    - Clear own trades, national best bid & offer reg.
  - BATS Europe too
  - Members must be registered broker/dealers
  - 3rd largest U.S. cash equities market

Other Electronic Markets

- Direct Edge
  - All electronic match of buyers & sellers
  - Became a national exchange in March 2010
  - 9.6% U.S. cash equities market share in February 2010 (vs. 10.7% for BATS)
  - 6% market share of FTSE 100 securities

Dark Pools (sounds kind of scary)

Dark Pools In Reality

- Dark Pools are crossing networks that provide liquidity that is not displayed on order books
- They account for ≈ 10% of equity trades in U.S. markets
- Trading parties are totally anonymous and traded securities and prices are not reported until after execution
- Dark Pools are owned by financial intermediaries and institutional investors who also trade in the pools – even the NYSE itself has launched its own Dark Pool to (try to) lure back block trades
- So why would these institutions want anonymous trades?

Markets in Year 2011

- Multiple stock markets exist and compete
  - NYSE, NASDAQ, LSE, Mumbai, Tokyo, etc.
- But trading platforms are converging
- New trading mechanisms are being launched
- Away from auction markets and dealers
- Towards electronic order input, flow, and execution
- Towards integrated markets where cash equities and derivatives can be traded in one locale
- On Friday, July 11, 2008, NASDAQ OMX Group traded 1.726 billion shares of NYSE-listed stocks, vs. 1.723 billion shares of NYSE-listed stocks traded on NYSE

How are Prices Determined?

- Fundamental Analysis
  - look at financial results and expectations
  - determine the firm's expected future cash flows
  - firm value = present value of expected cash flows
    - Price = Σ [Cash Flow$_t$ / (1 + required return)^t]
- Information changes expectations and prices
  - Market factors (β): oil prices ↓ 3% on July 16, 2008 driving AMR ↑ 32%
  - Unique factors (α): American Airlines cancels 900 flights on April 9, 2008 (FAA airworthiness compliance) driving AMR ↓ 11%
- Investor psychology vs. fundamentals and information?

Types of Shareholders Stock Prices and Volatility

- The composition of a firm's shareholders impact its stock price movement
  - Individuals vs. Institutions
  - Types of Institutions
    - Mutual Funds
    - Exchange Traded Funds and Index Funds
    - Pension Funds
    - Hedge Funds
  - Long holders vs. Shorts
  - High frequency traders

Individuals vs. Institutions

Institutional holdings = 61% in aggregate, but vary widely by specific firm

Walmart Stores	39%	Pfizer, Inc.	70%
Exxon Mobil	49%	Research in Motion	70%
Proctor & Gamble	58%	Amazon.com	70%
Starbucks	62%	Toll Brothers	78%
Southwest Airlines	64%	Starwood Hotels	90%
Johnson & Johnson	65%	Harley-Davidson	90%

Favorite Stockholders

ETFs and Index Funds

- Exchange Traded Funds (ETFs) and Index Mutual Funds hold 12% of the total assets managed by registered investment companies
- ETFs and Index Mutual Funds turnover their holdings infrequently, as they are constructed to track designated indices

Mutual Funds

- Median mutual fund turnover is 63% per year
- Since ETFs and Index Funds are not actively managed, median turnover for actively managed mutual funds is ≈ 70% per year
- Mutual fund turnover, not surprisingly, varies widely by specific fund

Mutual Fund Turnover

Fidelity NASDAQ Composite Index	3%
Vanguard 500 Index	6%
Vanguard Wellington	30%
Fidelity Magellan	67%
Fidelity Blue Chip Growth	107%
Vanguard Growth Equity	222%
Fidelity Select Pharmaceuticals	240%
Fidelity Dividend Growth	249%

Not So Favorite Stockholders

Hedge Funds

- Median hedge fund turnover is 102% per year, meaning the median hedge fund turns over its entire portfolio every year
- 102% is a median, the data is skewed right, so mean turnover > 102%
- A recent study of hedge funds finds more turnover is associated with superior return performance

Source: "How Smart are the Smart Guys? A Unique View from Hedge Fund Stock Holdings," John Griffin and Jin Xu

Institutional Effect

- ↑ institutional turnover implies more trades of large blocks ↑ stock price volatility
- ↑ ETF and index fund holdings ↓ turnover
- ↑ mutual fund holdings ↑ turnover
- ↑ hedge fund holdings ↑↑ turnover

Really Not Favorite Shareholders

Shorts

Short interest varies by firm, here is a sample:

Walmart Stores	0.7%	Starbucks	5.5%
Exxon Mobil	0.7%	Caterpillar	8.0%
Proctor & Gamble	1.0%	American Airlines	9.2%
Bank of America	1.1%	Toll Brothers	9.7%
Research in Motion	3.2%	Starwood Hotels	15.4%
Pfizer	4.0%	Harley-Davidson	19.9%
Amazon.com	4.7%	Palm	30.1%

Shorts and Hedge Funds

- Hedge funds are more likely to be short (sometimes they are right, sometimes they are not)
- Hedge funds trade more frequently
- Hedge funds trade in large blocks
- A tough combination to any firm with large institutional short interest as we saw in the 2008 meltdown:
  - Bear Stearns
  - Lehman
  - Fannie & Freddie
  - Others

Fast and Predatory (?) Shareholders

(254 mph / 408 kmph, model #1 of 300, $1.9 million list price)

High Frequency Trading (HFT)

- Also called Algorithmic Trading
- Super fast computers scan multiple equity markets for tiny price differentials, and trade on these in milli-seconds or micro-seconds
- HFT accounts for 56% of all U.S and 38% of all European equity trades
- Exchanges facilitate HFT through co-location – placing HFT computers right next to the Exchanges' own systems
- Closer → faster
- Note that exchanges profit from HFT

High Frequency Trading II

- HFT's claim to benefit equity markets by providing liquidity
- HFT's are suspected of "quote stuffing," sending unusually heavy traffic to some exchanges to slow down execution
- While at the same time trading in faster exchanges
- This "Latency Arbitrage" is possible due to differential speed of execution – HFT's are able to trade in advance of small price movements earning profits

15.401: Stocks 1

2/28

(4 min late)

Stock markets allocate $ to people w/ investment opportunities

Amsterdam was first exchange
- for ship trips

In London people would meet in coffee houses

Market - now, set time not price

Limit - set price but not time

Now mostly institutions

Bid - price at which dealers/individuals will buy

Ask " " " " " sell

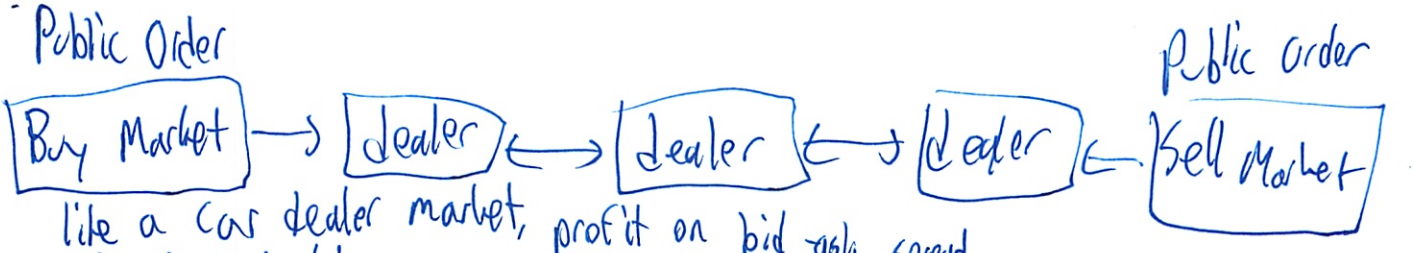
Bid-Ask Spread - profit for dealer
- Small for widely traded stocks
- highly fluid
- market conditions drive
- change prices for diff customers
  - if someone has more info, you charge them more

Corps list on one exchange
- Some multiple
- Minimum requirements to be listed
- Used to be hiracacy of exchanges
  - But not any more

(2)

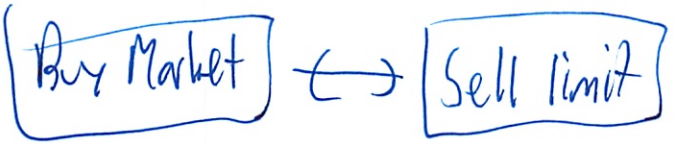
Can also participate in other markets
- regional
- Electronic Communication Network (ECN)
- Dark Pools
- all compete for order flow

Pure Dealer Market



like a car dealer market, profit on bid ask spread

Pure Auction Market



Order brought to desk where that stock is traded

Specialist - buyer of last resort
- sit on a lot of capital

Every exchange's nightmear - that orders don't get executed

NYSE

Used to be very profitable when price was regulated

headed to floor less

(3)

NASDAQ

- started in the 1971
- linked by phone
- dealers

Bid-ask spreads got too small

So dealers quit

So they joined an ECN

Executes if buyers + sellers want same price

So Nasdaq bought Instinet

Wants market share, order flow

NYSE

- started w/ Buttonwood Agreement
- location at 11 Wall St
- trades used to always go to floor
- bought an ECN in 2005
  - They say competive advantage is that they have multiple mechanisms to ensure liquidity
- and merged w/ Euronext
- Want best platforms for people to trade
- floor is noticably emptier
- specialits are now Designated Market Makers
- oversee a few trades
- most done online

4

Germans bought NYSE

Everyone else looking for a deal
↑exchanges

ECN - just matches buyers + sellers at same price

Can see limit order book

Used to be an actual book

Thinly traded
- order book is not as big/long
- larger bid-ask spreads

BATS
- Kanas City
- electronic
- started 4 years ago
- allows NYSE orders to be traded
- brokers/dealers only
- 10% marketshare

Direct Edge
- started last year
- 10% market share as well

(5)

Dark Pools

- No order book
- Parties are anonymous
- Prices not shown until after execution
- Owned by financial intermediaries
- People like the anonmitey
  - Don't want to share what they know
- No price discovery
- SEC doesn't want it to take over

→ In 2008 Nasdaq traded more NYSE-shares than the NYSE

Information drives prices

Shareholders

Shares ownership by institution varies

ETFs don't trade that frequently

- Buy shares in the basket
- Have shares of basket

Mutual Funds

- Shares not tradable

Unfav. shareholders

Hedge Funds

- trade all the time
- Stock price very volitatle

Institutional effects on volitatilty

(4)

Really unfav shareholders

Shorts

Hedge funds - likley to be short

- freq
- large blocks

High Freq. Trading / Algorthmic Trading

- minute price differenentials
- 56% of trades in US.
- exchanges profit
- allow them to co-locate
- they think they provide liquidity
- beat others to the punch
- need to net out transaction ~~costs~~ costs