Skipped

15.401 Recitation

2b: Fixed-Income Securities

Learning Objectives

- Review of Concepts
  - Bond arbitrage
  - Duration/convexity
  - Immunization
- Examples
  - Duration
  - Bond arbitrage
  - True/false

Review: bond arbitrage

- Bond arbitrage is possible when its price is not equal to the PV of payments discounted at the spot rates
- Caveats:
  - the bond must have the same risk characteristics as the securities from which the spot rates are derived (e.g., riskless);
  - each coupon payment can be matched exactly by a spot rate;
  - it is possible to borrow/lend at all spot rates.
- General strategy:
  - Buy low, sell high

Review: bond arbitrage

- Detailed strategy:
  - Scale available payoff streams so that the net cash flow at $t = 1, 2, \ldots$ is exactly zero.
  - Adjust the signs so that the payoff at $t = 0$ is positive.

Multiplier	Asset	$t = 0$	$t = 1$	$t = 2$	...	$t = T$
m_A	A	$-m_A P_A$	$m_A C_{A1}$	$m_A C_{A2}$	...	$m_A C_{AT}$
m_B	B	$-m_B P_B$	$m_A C_{A1}$	$m_B C_{B2}$	...	$m_B C_{BT}$
...	...	...	...	...	...	...
m_N	N	$-m_N P_N$	$m_N C_{N1}$	$m_N C_{N2}$	...	$m_N C_{NT}$
+		π_0	0	0	...	0

9/12

Review: bond arbitrage

- Remarks:
  - Arbitrage strategy is not unique.
  - Given an arbitrage strategy (m_A, m_B, ..., m_N) with profit π_0, ($k \cdot m_A$, $k \cdot m_B$, ..., $k \cdot m_N$) is also an arbitrage strategy with profit $k \cdot \pi_0$.
  - A strategy where cash flows at t = 0, 1, ... T are all zero except at t = s > 0 (when it is positive) is also an arbitrage.
  - For the purpose of this course, we only consider the type of arbitrage strategies on the previous page.

Review: duration/convexity

- Duration and modified duration measure a bond's exposure to interest rate risk:

$$D = \frac{1}{P}\sum_{t=1}^{T}\frac{t \cdot C_t}{(1+y)^t} + \frac{T \cdot FV}{(1+y)^T}; \quad MD = \frac{D}{1+y}$$

- Since $MD = -\frac{1}{P}\cdot\frac{\partial P}{\partial y}$, a small change ($\Delta y$) in YTM will cause bond price to change by approximately $\Delta P \approx -P \cdot MD \cdot \Delta y$.
- The formula is **not accurate** for large changes in *y*.

Review: duration/convexity

- Convexity is...
  - the second derivative of P(y);
  - a measure of curvature of P(y);
  - the sensitivity of the duration to a change in the yield.

$$CX = -\frac{1}{P}\cdot\frac{\partial^2 P}{\partial y^2}$$

- A better approximation:

$$\frac{\Delta P}{P} \approx -\Delta y \cdot MD + \frac{(\Delta y)^2}{2}\cdot CX.$$

Immunization

- The duration of a portfolio with weight *w* on asset *X* and (1-*w*) on asset *Y* is [*w* × D(*X*) + (1–*w*) × D(*Y*)].
- Institutions such as banks, pension funds and insurance companies are highly exposed to interest rate fluctuations. They would like to insure or immunize against such fluctuations.
- Solution: structure the balance sheet so that V(Assets) × D(Assets) = V(Liabilities) × D(Liabilities).
- Continuous rebalancing is required for perfect immunization.

Example 1: duration

- Consider a 10-year bond with a face value of $100 that pays an annual coupon of 8%. Assume spot rates are flat at 5%.
  a. Find the bond's price and duration.
  b. Suppose that 10yr yields increase by 10bps. Calculate the change in the bond's price using your bond pricing formula and then using the duration approximation.
  c. Suppose now that 10yr yields increase by 200bps. Repeat your calculations for part (b).

Example 1: duration

- Answer:
  a. $P = \frac{8}{1.05} + \frac{8}{1.05^2} + \ldots + \frac{108}{1.05^{10}} = \$123.16$

  $D = \frac{1}{123.16}\left(\frac{8 \cdot 1}{1.05} + \frac{8 \cdot 2}{1.05^2} + \cdots + \frac{108 \cdot 10}{1.05^{10}}\right) = 7.54$

  b. Actual new price = $122.28.

  $\Delta P \approx -P \times \frac{D}{1+y} \times \Delta y = -123.16 \times \frac{7.54}{1.05} \times 0.001 = -\$0.88$

  $\Rightarrow P_{new} = \$122.28.$

  c. Actual new price = $107.02
  New price using duration approximation = $105.47

Example 2: bond arbitrage

- Find an arbitrage portfolio given the following riskless bonds:

Asset	$t=0$	$t=1$	$t=2$	$t=3$
A	-97	100		
B	-92		100	
C	-87			100
D	-102	5	5	105

Example 2: bond arbitrage

- Answer:

Multiplier	Asset	$t=0$	$t=1$	$t=2$	$t=3$
x	A	-97x	100x		
y	B	-92y		100y	
z	C	-87z			100z
w	D	-102w	5w	5w	105w
			0	0	0

- x = -0.05w
- y = -0.05w
- z = -1.05w

scale the cash flows so the net payoff is 0

100x = 5w

Example 2: bond arbitrage

- $\pi_0 = -97 \cdot (-0.05w) - 92 \cdot (-0.05w) - 87 \cdot (-1.05w) - 102w$
  $= -1.2w$
- Set $w = -1$
- Arbitrage strategy:
  - Long 0.05 A
  - Long 0.05 B
  - Long 1.05 C
  - Short 1 D
- Profit = 1.2

Example 3: true or false

- True or false:
  - Investors expect higher returns on long-term bonds than short-term bonds because they are riskier. Thus, the term structure of interest rates is always upward sloping.
  - To reduce interest rate risk, an overfunded pension fund, i.e., a fund with more assets than liabilities, should invest in assets with longer duration than its liabilities.

Example 3: true or false

- Answer:
  - False. The term structure depends on the expected path of interest rates (among other factors). For example, if interest rates are expected to fall, the term structure will be downward sloping.
  - False. To minimize interest rate risks, we want $MD(A) \times V(A) - MD(L) \times V(L) = 0$. If $V(A) > V(L)$, we want $MD(A) < MD(L)$. That means we should invest in assets with shorter duration.

15.401 – Finance Theory I
Spring 2011 – Sections C & D

Case #1
Due: February 28, 2011

RENSSELAER ADVISORS

Rensselaer Advisors (RA) manages portfolios for institutional investors (primarily corporate pension plans) and wealthy individuals. RA is located in the northern foothills of the Catskill Mountains, in the hamlet of Rensselaerville, NY. Despite its bucolic location, RA is linked real-time to all major markets and exchanges. In mid-2007 it had about $1.1 billion under management. Its management fees average 55 basis points (.55%), so RA's total revenue for 2007 will be about $(.0055)(1.1)(10^9) = \$5.5$ million.

It is now September of 2007 and RA has just landed a new client, Madison Mills, a conservative, long-established manufacturer of papermaking felt. Madison has established a defined-benefit[1] pension plan for its employees. The plan funding is $100 million, with $84.2 million in fixed-income securities (mostly U.S. Treasury bonds, notes and strips) and $15.8 million in equities.

Exhibit 1 shows the current composition of the fixed-income portfolio. You have been assigned to restructure and then manage this portfolio. The management fee will be 35 basis points per year.[2]

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[1] *Defined benefit* means that the company is committed to pay retirement income according to a formula. For example, annual retirement income could equal 40% of the employee's average salary in the five years prior to retirement. In a defined benefit plan, retirement income does not depend on the performance of the pension fund, provided, of course, that the assets in the fund are sufficient to cover pension benefits. If the assets are not sufficient, the company is required to contribute enough additional cash to cover the shortfall.

In *defined contribution* plans, the corporation contributes to the pension fund on behalf of its employees. Each employee has a claim on part of the fund, just as if the employee held shares in a mutual fund. Employees' retirement incomes depend on their balances in the fund at retirement. In this case, retirement income is directly linked to fund performance.

[2] Management fees are almost always lower for fixed-income portfolios than for equity portfolios.

Exhibit 2 shows Madison's obligations to its already-retired employees from 2008 to 2029. Each of these employees receives a fixed dollar amount each month. Total dollar payments from the fund decline as the employees pass away.

Madison seeks to cover most of the obligations shown in Exhibit 2 from the $81 million now invested in its fixed-income portfolio and from the portfolio's future earnings. If the fixed-income portfolio is insufficient to cover these obligations, Madison will have to draw down the equity portfolio and/or make additional contributions to the pension fund.[3]

After reviewing Madison's existing portfolio, you schedule a meeting with Hendrik van Wie, Madison's CFO. Mr. van Wie stresses Madison's conservative management philosophy and warns against "speculation." He also reiterates his hope that the fixed income portfolio will grow to cover most of the obligations shown in Exhibit 2, and he complains about the performance of the previous fixed-income portfolio manager.

Mr. Van Wie also asks for a detailed report explaining your investment strategy and the initial composition of the fixed-income portfolio. Please submit this report on February 28th, 2011, 5 p.m. Attach a printout of your starting portfolio (securities chosen and amount invested in each) to your report. Make sure that your report identifies the risks as well as the prospective returns from your investment strategy. Explain why your portfolio makes sense. Be explicit about Madison's (net) exposure to changes in interest rates if your strategy is followed.

In practice, RA would have thousands of fixed-income securities to choose from, but to keep this assignment manageable, restrict your portfolio to the securities listed in

[3] Madison hopes to cover retirement benefits for *current* employees with the $15.8 million equity portfolio, supplemented by future contributions to the pension fund.

Exhibit 3. You can sell any part or all of the positions in Exhibit 1 for the market values shown in the exhibit. You can invest any dollar amount[4] in any of the securities listed in Exhibit 3, provided that you've raised the necessary cash by sales from the existing portfolio. No borrowing or short-selling is allowed.

So how to start?
This is like a arbitrage thing??
Match durations to minimize risk
-which we want to do
-Calc duration of available assets
-real duration
What is settlement date?
Serial date? - start date
Key: match maturities
Want conservative so don't need too fancy
But arbitrage still fair game
Then balance what-ifs
We were never told exactly what we had to do!
Why are my duration #s so small?
< how about accrued principal?

[4] For simplicity, you can ignore accrued interest and the bid-asked spreads given in Exhibit 3. Just specify the amounts invested in each security, for example, "$5 million in the Mobil 8.625s of 2021."

Excel duration in terms of years?
Oh coupons need to be in % terms
This is taking a while - no clue what I am doing
What is CUSIP?
- Oh Id #
Have duration, whats next?
Its on p 519 in textbook!
Passive strategy
I don't really get which direction each thing moves

No yield
? solve for
Can you
- what is par/market value again?
Investopedia: 4 types of yields
- guessing its YTM
Market < Par - discount
So Par = redemption ✓
Redeem w/o call premium
Then what are yields?
Finally solved using webpage calculations
- Excel formula useless!

Need to now review current portfolio + make suggestions

Market =

For 100 Par, Ask is current MV of that bond

Market = Par · Ask

$$Par = \frac{Market}{Ask}$$

Can I calc return?

Ok done

Andy: Doesn't really know

Hardest part of the class

Exhibit 1 - Rensselaer Advisors

Madison Mills Pension Fund

Fixed-Income Portfolio on September 25, 2007

Security	Coupon	Maturity	Par Value	Market Value
Treasury				
Treasury Bill		12/27/07	$9,600,000	$9,510,969.60
Treasury Note	4.875	08/31/08	$5,100,000	$5,143,579.60
Treasury Note	4.375	08/15/12	$3,600,000	$3,635,847.69
Treasury Bond	5.375	02/15/31	$16,000,000	$17,053,301.37
TIPS	2.375	01/15/27	$6,279,125	$6,273,238.32
Treasury Strips		08/15/09	$2,273,221	$2,110,685.70
Treasury Strips		08/15/17	$14,400,000	$9,129,024.00
Government Agency				
Fannie Mae	6 5/8	11/15/30	$6,362,279	$7,518,672.23
Corporates				
Wal-Mart	6.875	08/10/09	$3,769,935	$3,922,605.59
Comcast	6.500	01/15/15	$6,502,082	$6,766,908.19
Chevron Texaco	9.750	03/15/20	$2,793,200	$3,793,596.22
Union Carbide	7.500	06/01/25	$9,506,955	$9,341,296.31
Total Portfolio Value				$84,199,724.82

Footnotes

Coupon	Annual coupon interest payments are received semi-annually Coupon rates for TIPS are stated in real terms; the actual coupon paid is adjusted for inflation.
Treasury Strips	All treasury strips are principal strips.

Exhibit 2 - Rensselaer Advisors

Madison Mills Pension Fund

Projected Benefits for Retired Employees

Year	Benefits
2008	$10,020,000
2009	$9,009,500
2010	$8,722,000
2011	$8,234,500
2012	$7,858,500
2013	$7,794,000
2014	$7,729,500
2015	$7,639,500
2016	$6,440,500
2017	$6,330,000
2018	$6,242,500
2019	$6,205,000
2020	$5,775,500
2021	$5,600,700
2022	$5,432,000
2023	$5,140,000
2024	$4,234,900
2025	$4,123,000
2026	$3,890,000
2027	$3,500,600
2028	$3,450,500
2029	$3,290,600
	$136,663,300

Exhibit 3 - Rensselaer Advisors

Madison Mills Pension Fund

Fixed-Income Securities Available for Purchase on September 25, 2007

Security	Coupon	Ask Price	B/A Spread	Maturity	Ticker	Rating	CUSIP	Yield
Treasury								
Treasury Bill		99.07	0.0026	12/27/07			912795B83	3.79
Treasury Bill		98.02	0.0040	03/27/08			912795D57	4.06
Treasury Note	4.875	100.69	0.0313	08/31/08			912828FR6	4.94
Treasury Note	4.000	100.00	0.0000	08/31/09			912828HB9	3.99
Treasury Note	4.125	100.16	0.0313	08/15/10			912828ED8	4.63
Treasury Note	4.375	100.75	0.0313	08/15/12			912828AJ9	4.20
Treasury Bond	4.750	101.06	0.0313	08/15/17			912828HA1	4.61
Treasury Bond	7.250	124.50	0.0313	08/15/22			912810EM6	4.91
Treasury Bond	6.375	117.91	0.0313	08/15/27			912810FA1	4.95
Treasury Bond	5.375	106.28	0.0313	02/15/31			912810FP8	4.92
Treasury Bond	4.750	97.81	0.0313	02/15/37			912810PT9	4.89
TIPS	2.375	100.19	0.1250	01/15/17			912828GD6	2.35
TIPS	2.375	99.91	0.1250	01/15/27			912810PS1	2.38
TIPS	3.375	120.22	0.1250	04/15/32			912810FQ6	2.29
Treasury Strips		96.51	0.0200	08/15/08			912833CU2	4.05
Treasury Strips		92.85	0.0500	08/15/09			918833CW8	3.97
Treasury Strips		81.60	0.0200	08/15/12			912833DC1	4.20
Treasury Strips		63.40	0.0200	08/15/17			912833KN9	4.66
Treasury Strips		47.56	0.0200	08/15/22			912833LJ7	5.05
Treasury Strips		37.14	0.0200	08/15/27			912833PE4	5.04
Treasury Strips		31.78	0.0200	02/15/31				4.96
Government Agency								
Fannie Mae	6.630	104.16	0.0313	09/15/09			31359MEY5	4.40
Federal Home Loan Bank	4.500	99.09	0.0313	11/15/12			3133MTZL5	4.70
Federal Home Loan Bank	5.250	102.50	0.0313	06/18/14			3133X7FK5	4.81
Fannie Mae	6.625	116.97	0.0938	11/15/30			31359MGK3	5.34
Corporates								
Time Warner	7.480	100.45	0.0600	01/15/08	TWX	BBB	887315AT6	5.84
Wal-Mart	6.875	103.62	0.2400	08/10/09	WMT	AA	931142BE2	4.82
Verizon	6.125	102.99	1.2000	01/15/13	VZ	A-	923474AA8	5.47
Altria	7.000	108.59	0.3420	11/04/13	MO	BBB	02209SAA1	5.33
Comcast	6.500	103.44	0.2970	01/15/15	CMCSA	BBB	20030NAB7	5.91
Hilton Hotels	7.500	112.03	0.9000	12/15/17	HLT	BB+	432848AS8	5.91
Chevron Texaco	9.750	135.68	0.0000	03/15/20	CVX	AA	881685AN1	5.71
American Airlines	9.750	91.20	0.9000	08/15/21	AMR	CCC	001765AP1	11.00
Mobil	8.625	131.44	2.3000	08/15/21	XOM	AAA	607059AT9	5.38
Union Carbide	7.500	97.07	0.0000	06/01/25	DOW	BB+	905581AR5	7.81
Ford Motor Credit	8.900	82.52	0.9000	01/15/32	F	CCC+	345370BV1	10.97

Footnotes

Coupon	Annual coupon interest payments are received semi-annually Coupon rates for TIPS are in real terms; the actual coupon paid is adjusted for inflation.
Ask Price	All prices listed in decimals.
TIPS	In order of maturity, the accrued principal on the TIPS is 1033, 1033, and 1174. How to deal w/?
Treasury Strips	All treasury strips are principal strips.
B/A Spread	The difference between the ask price and the bid price.

Yield = Yield to maturity = aka effective interest rate
- this is calculate assuming know current rates

1st draft
Not Handed In

2/27

Dear Mr. van Wie,

I have put together a portfolio based on our conversation. I have designed this portfolio to be low risk, through a *passive* management strategy. More specifically I am proposing a strategy of *immunization*. I have attempted to minimize interest rate risk by matching the dates of liabilities (payouts to employees) to the date of maturity of the assets.

If we were to not follow an immunization strategy and interest rates were to rise in the future, then our portfolio would suffer a capital loss, as the prices of bonds fall when interest rates rise. Slightly offsetting this, our reinvested coupons will grow at a faster rate. If this were to occur a long time then the portfolio might be unable to meet its obligations with the money set aside for those liabilities. If interest rates were to fall shortly before the required payout, then your portfolio would show a return.

Conversely, if interest rates were to fall, then our portfolio would be worth more, but we could not reinvest our coupons at the same rates as before. Here, if interest rates were to fall shortly before payout, then the portfolio might be unable to meet its obligations with the money set aside for those liabilities. If interest rates were to fall a long time before the required payout, then our portfolio would show a return.

The risk to the price of our assets is called the *price risk*, while the risk to the reinvestment rate of our coupons is called the *reinvestment rate risk*. By pursuing a strategy of immunization, we balance our price and reinvestment risk. Under this strategy we are able to lock in a rate, so we are not subject to fluctuations in the interest rate.

In addition, as interest rates change, our obligations change as well. However, under a strategy of immunization, the return on our exceed our obligations no matter which directions interest rates change. This is due to *convexity*. A coupon bond has greater convexity than the obligation which it funds.

Although this is a passive strategy, we must still continue to monitor your portfolio in order to *rebalance* it. As time progresses, the relative weights of the various payments change, causing the maturity to change. This is in addition to changes in duration due to interest rate changes. We must continue to make sure that the duration of our assets matches our liabilities in order to stay immunized.

I have reviewed your current portfolio. I first reviewed the credit worthiness of your corporate bonds. One bond, Union Carbide is just below the level I am comfortable recommending to our customers interested in low-risk investments. I would recommend selling it. In addition, in order to better match duration, I recommend the purchase of more long term bonds with the proceeds. In addition, you have a large amount of Treasury bonds coming due in 2031. I would recommend changing putting some of the money in stripes instead to lengthen the duration. In addition, I have added a Treasury Note due in 2010, with a 3 year duration to eliminate the large hole in your portfolio during that timeframe.

Otherwise I think you have a good match of durations, as well as a balance of risk in your portfolio. Although this strategy does not seek to maximize return, it gives you a good chance of meeting your future obligations.

-Michael Plasmeier

Proposed Pension Fund Pool

Madison Mills Pension Fund

Fixed-Income Portfolio on September 25, 2007

9/25/2007

Security	Coupon	Maturity	%	Par Value	Market Value	Ask Price	YTM	Duration
Current								
Treasury Bill		12/27/07		$9,600,000	$9,510,970	100.94%	3.69%	0.255556
Treasury Note	0.04875	08/31/08		$5,100,000	$5,143,580	99.15%	3.93%	0.918711
Wal-Mart	0.069	08/10/09		$3,769,935	$3,922,606	96.11%	4.60%	1.779792
Treasury Strips		08/15/09		$2,273,221	$2,110,686	107.70%	3.96%	1.888889
Treasury Note	0.04375	08/15/12		$3,600,000	$3,635,848	99.01%	4.15%	4.436515
Comcast	0.065	01/15/15		$6,502,082	$6,766,908	96.09%	5.81%	5.892813
Chevron Texaco	0.098	03/15/20		$2,793,200	$3,793,596	73.63%	5.70%	8.199261
Union Carbide	0.075	06/01/25		$9,506,955	$9,341,296	101.77%	7.68%	9.770032
Treasury Strips		08/15/17		$14,400,000	$9,129,024	157.74%	4.66%	9.888889
Fannie Mae	0	11/15/30		$6,362,279	$7,518,672	84.62%	5.26%	12.76041
Treasury Bond	0.05375	02/15/31		$16,000,000	$17,053,301	93.82%	4.90%	13.8238
TIPS	0.02375	01/15/27		$6,279,125	$6,273,238	100.09%	2.38%	15.52332
Transactions					$84,199,725			
S Treasury Bond	0.05375	02/15/31	-50%	-$8,000,000	-$8,526,651		4.90%	13.8238
S Union Carbide	0.075	06/01/25	-100%	-$9,506,955	-$9,341,296	101.77%	7.68%	9.770032
B Treasury Strips	0	08/15/27		$16,155,089	$6,000,000	37.14%	5.04%	19.88889
B Treasury Strips	0	02/15/31		$19,934,940	$6,335,324	31.78%	4.96%	23.38889
B T Note	0.0413	08/15/10		$ 5,523,989	$ 5,532,623	100.16%	4.07%	2.741322

Difference bw Buy and Sell	$0
Portfolio Value	$84,199,725

Exhibit 1 - Rensselaer Advisors

Madison Mills Pension Fund

Fixed-Income Portfolio on September 25, 2007

9/25/2007

Security	Coupon	Maturity	Par Value	Market Value	YTM	Duration
Treasury						
Treasury Bill		12/27/07	$9,600,000	$9,510,969.60	3.69%	0.255556
Treasury Note	0.04875	08/31/08	$5,100,000	$5,143,579.60	3.93%	0.918711
Treasury Note	0.04375	08/15/12	$3,600,000	$3,635,847.69	4.15%	4.436515
Treasury Bond	0.05375	02/15/31	$16,000,000	$17,053,301.37	4.90%	13.8238
TIPS	0.02375	01/15/27	$6,279,125	$6,273,238.32	2.38%	15.52332
Treasury Strips		08/15/09	$2,273,221	$2,110,685.70	3.96%	1.888889
Treasury Strips		08/15/17	$14,400,000	$9,129,024.00	4.66%	9.888889
Government Agency						
Fannie Mae	0	11/15/30	$6,362,279	$7,518,672.23	5.26%	12.76041
Corporates						
Wal-Mart	0.069	08/10/09	$3,769,935	$3,922,605.59	4.60%	1.779792
Comcast	0.065	01/15/15	$6,502,082	$6,766,908.19	5.81%	5.892813
Chevron Texaco	0.098	03/15/20	$2,793,200	$3,793,596.22	5.70%	8.199261
Union Carbide	0.075	06/01/25	$9,506,955	$9,341,296.31	7.68%	9.770032
Total Portfolio Value				$84,199,724.82		

Footnotes

Coupon	Annual coupon interest payments are received semi-annually Coupon rates for TIPS are stated in real terms; the actual coupon paid is adjusted for inflation.
Treasury Strips	All treasury strips are principal strips.

Exhibit 2 - Rensselaer Advisors

Madison Mills Pension Fund

Projected Benefits for Retired Employees

9/25/2007

Year		Benefits	Duration
1/1/2008	$	10,020,000	0.27
1/1/2009	$	9,009,500	1.27
1/1/2010	$	8,722,000	2.27
1/1/2011	$	8,234,500	3.27
1/1/2012	$	7,858,500	4.27
1/1/2013	$	7,794,000	5.27
1/1/2014	$	7,729,500	6.27
1/1/2015	$	7,639,500	7.27
1/1/2016	$	6,440,500	8.27
1/1/2017	$	6,330,000	9.28
1/1/2018	$	6,242,500	10.28
1/1/2019	$	6,205,000	11.28
1/1/2020	$	5,775,500	12.28
1/1/2021	$	5,600,700	13.28
1/1/2022	$	5,432,000	14.28
1/1/2023	$	5,140,000	15.28
1/1/2024	$	4,234,900	16.28
1/1/2025	$	4,123,000	17.28
1/1/2026	$	3,890,000	18.28
1/1/2027	$	3,500,600	19.28
1/1/2028	$	3,450,500	20.28
1/1/2029	$	3,290,600	21.28

Exhibit 3 - Rensselaer Advisors

Madison Mills Pension Fund

Fixed-Income Securities Available for Purchase on September 25, 2007

9/25/2007

Security	Coupon	Ask Price	B/A Spread	Maturity	Ticker	Rating	CUSIP	Yield	Actual Duration
Treasury									
Treasury Bill		99.07	0.0026	12/27/07			912795B83	3.79%	0.2548
Treasury Bill		98.02	0.0040	03/27/08			912795D57	4.06%	0.5041
Treasury Note	4.88%	100.69	0.0313	08/31/08			912828FR6	4.94%	0.9342
Treasury Note	4.00%	100.00	0.0000	08/31/09			912828HB9	3.99%	1.9342
Treasury Note	4.13%	100.16	0.0313	08/15/10			912828ED8	4.63%	2.8904
Treasury Note	4.38%	100.75	0.0313	08/15/12			912828AJ9	4.20%	4.8932
Treasury Bond	4.75%	101.06	0.0313	08/15/17			912828HA1	4.61%	9.8959
Treasury Bond	7.25%	124.50	0.0313	08/15/22			912810EM6	4.91%	14.8986
Treasury Bond	6.38%	117.91	0.0313	08/15/27			912810FA1	4.95%	19.9014
Treasury Bond	5.38%	106.28	0.0313	02/15/31			912810FP8	4.92%	23.4082
Treasury Bond	4.75%	97.81	0.0313	02/15/37			912810PT9	4.89%	29.4137
TIPS	2.38%	100.19	0.1250	01/15/17			912828GD6	2.35%	9.3151
TIPS	2.38%	99.91	0.1250	01/15/27			912810PS1	2.38%	19.3205
TIPS	3.38%	120.22	0.1250	04/15/32			912810FQ6	2.29%	24.5726
Treasury Strips		96.51	0.0200	08/15/08			912833CU2	4.05%	0.8904
Treasury Strips		92.85	0.0500	08/15/09			918833CW8	3.97%	1.8904
Treasury Strips		81.60	0.0200	08/15/12			912833DC1	4.20%	4.8932
Treasury Strips		63.40	0.0200	08/15/17			912833KN9	4.66%	9.8959
Treasury Strips		47.56	0.0200	08/15/22			912833LJ7	5.05%	14.8986
Treasury Strips		37.14	0.0200	08/15/27			912833PE4	5.04%	19.9014
Treasury Strips		31.78	0.0200	02/15/31				4.96%	23.4082
Government Agency									
Fannie Mae	6.63%	104.16	0.0313	09/15/09			31359MEY5	4.40%	1.9753
Federal Home Loan Bank	4.50%	99.09	0.0313	11/15/12			3133MTZL5	4.70%	5.1452
Federal Home Loan Bank	5.25%	102.50	0.0313	06/18/14			3133X7FK5	4.81%	6.7342
Fannie Mae	6.63%	116.97	0.0938	11/15/30			31359MGK3	5.34%	23.1562
Corporates									
Time Warner	7.48%	100.45	0.0600	01/15/08	TWX	BBB	887315AT6	5.84%	0.3068
Wal-Mart	6.79%	103.62	0.2400	08/10/09	WMT	AA	931142BE2	4.82%	1.8767
Verizon	6.13%	102.99	1.2000	01/15/13	VZ	A-	923474AA8	5.47%	5.3123
Altria	7.00%	108.59	0.3420	11/04/13	MO	BBB	02209SAA1	5.33%	6.1151
Comcast	6.50%	103.44	0.2970	01/15/15	CMCSA	BBB	20030NAB7	5.91%	7.3123

Hilton Hotels	7.50%	112.03	0.9000	12/15/17	HLT	BB+	432848AS8	5.91%	10.2301
Chevron Texaco	9.75%	135.68	0.0000	03/15/20	CVX	AA	881685AN1	5.71%	12.4795
American Airlines	9.75%	91.20	0.9000	08/15/21	AMR	CCC	001765AP1	11.00%	13.8986
Mobil	8.63%	131.44	2.3000	08/15/21	XOM	AAA	607059AT9	5.38%	13.8986
Union Carbide	7.50%	97.07	0.0000	06/01/25	DOW	BB+	905581AR5	7.81%	17.6959
Ford Motor Credit	8.90%	82.52	0.9000	1/15/2032	F	CCC+	345370BV1	10.97%	24.3233

Footnotes

Coupon	Annual coupon interest payments are received semi-annually Coupon rates for TIPS are in real terms; the actual coupon paid is adjusted for inflation.
Ask Price	All prices listed in decimals.
TIPS	In order of maturity, the accrued principal on the TIPS is 1033, 1033, and 1174.
Treasury Strips	All treasury strips are principal strips.
B/A Spread	The difference between the ask price and the bid price.

2/27

15.401D Finance Theory I (Section D)

Ideas to explore on Case 1

Author | Message

02/28/2011 00:01:40 **Subject:** Ideas to explore on Case 1 quote edit

Michael Plasmeier

Joined: 09/09/2009 13:01:54
Messages: 1

profile

What ideas should we explore in case 1.

After 1st draft

I was thinking of making a duration argument. Is anyone doing anything else?

02/28/2011 00:44:30 **Subject:** Re:Ideas to explore on Case 1 quote

Marc Piette

Joined: 09/09/2009 22:50:20
Messages: 1

The case covers the concepts you've seen in class so far:
- Present value (of assets and liabilities)
- Yield curve and spot interest rates
- Interest rate exposure and portfolio immunization (duration is a good place to look at)

The case states that your client warns against speculation:
- What type of securities does it then make sense to hold?
  - equities?
  - high or low rated bonds?

At a minimum you need to calculate the present values of your portfolio. To do so you need the spot interest rates. Feel free to calculate a few points and interpolate between. (linear is good enough - this is not an applied math class). You should then think of how to minimize your portfolio's interest rate exposure.

You can make simplifying assumptions. For example, present value calculations on bonds can be tricky in reality depending on when the next coupon payment happens. The goal is not to get exact values but for you to understand the things you need to look at.

Finally consider what will happen over time as the years go by. What does the portfolio manager need to do to satisfy the needs of his client. Can he plan those steps now or will they depend on market conditions?

Feel free to change your portfolio as you see fit. There is no single answer. This case is meant to make you think about the challenges that portfolio managers might face.

Finally, don't worry if you don't get all the fine points. The case is meant to be challenging.
I hope this helps,

Marc

profile

In class hints better

3/3

Dear Mr. van Wie,

I have put together a portfolio based on our conversation. I have designed this portfolio to be low risk, through a *passive* management strategy. More specifically I am proposing a strategy of *immunization*. I have attempted to minimize interest rate risk by matching the dates of liabilities (payouts to employees) to the date of maturity of the assets.

If we were to not follow an immunization strategy and interest rates were to rise in the future, then our portfolio would suffer a capital loss, as the prices of bonds fall when interest rates rise. Slightly offsetting this, our reinvested coupons will grow at a faster rate. If this were to occur a long time then the portfolio might be unable to meet its obligations with the money set aside for those liabilities. If interest rates were to fall shortly before the required payout, then your portfolio would show a return.

Conversely, if interest rates were to fall, then our portfolio would be worth more, but we could not reinvest our coupons at the same rates as before. Here, if interest rates were to fall shortly before payout, then the portfolio might be unable to meet its obligations with the money set aside for those liabilities. If interest rates were to fall a long time before the required payout, then our portfolio would show a return.

The risk to the price of our assets is called the *price risk*, while the risk to the reinvestment rate of our coupons is called the *reinvestment rate risk*. By pursuing a strategy of immunization, we balance our price and reinvestment risk. Under this strategy we are able to lock in a rate, so we are not subject to fluctuations in the interest rate.

In addition, as interest rates change, our obligations change as well. However, under a strategy of immunization, the return on our exceed our obligations no matter which directions interest rates change. This is due to *convexity*. A coupon bond has greater convexity than the obligation which it funds.

Although this is a passive strategy, we must still continue to monitor your portfolio in order to *rebalance* it. As time progresses, the relative weights of the various payments change, causing the maturity to change. This is in addition to changes in duration due to interest rate changes. We must continue to make sure that the duration of our assets matches our liabilities in order to stay immunized.

Of course, any low risk strategy seeks to invest in bonds over equities, and then only in highly-rated bonds. These bonds come from large, stable corporations and are generally rated BBB or above. The safest bonds are considered to be government bonds.

I have reviewed your current portfolio. I first reviewed the credit worthiness of your corporate bonds. One bond, Union Carbide is just below the level I am comfortable recommending to our customers interested in low-risk investments. I would recommend selling it.

The average duration of your liabilities is 7.16 years, while the average duration of your portfolio is 8.5 years. While you have a large amount money in 13 year Treasury bonds, you do not have much coverage in the 15-20 year span of your portfolio. I would recommend changing putting some of the money in stripes instead to lengthen the duration. Once I have done this I would look at the short term range timeframe of your portfolio. I have added a 3-year Treasury Note to fill in the duration hole in your portfolio during that timeframe. I have added some short term Treasuries in order to bring down the average duration of your portfolio. The average duration of my proposed portfolio is 7.18 years.

Although this strategy does not seek to maximize return, it gives you a good chance of meeting your future obligations.

-Michael Plasmeier

Proposed Pension Fund Pool

Madison Mills Pension Fund

Fixed-Income Portfolio on September 25, 2007

9/25/2007

Security	Coupon	Maturity	%	Par Value	Market Value	Ask Price	YTM	Duration	Weighted Duration
Current									
Treasury Bill		12/27/07		$9,600,000	$9,510,970	1.009	3.69%	0.256	$ 2,430,581
Treasury Note	4.88%	08/31/08		$5,100,000	$5,143,580	0.992	3.93%	0.919	$ 4,725,465
Wal-Mart	6.88%	08/10/09		$3,769,935	$3,922,606	0.961	4.60%	1.780	$ 6,981,422
Treasury Strips		08/15/09		$2,273,221	$2,110,686	1.077	3.96%	1.889	$ 3,986,851
Treasury Note	4.38%	08/15/12		$3,600,000	$3,635,848	0.990	4.15%	4.437	$ 16,130,493
Comcast	6.50%	01/15/15		$6,502,082	$6,766,908	0.961	5.81%	5.893	$ 39,876,125
Chevron Texaco	9.75%	03/15/20		$2,793,200	$3,793,596	0.736	5.70%	8.199	$ 31,104,687
Union Carbide	7.50%	06/01/25		$9,506,955	$9,341,296	1.018	7.68%	9.770	$ 91,264,768
Treasury Strips		08/15/17		$14,400,000	$9,129,024	1.577	4.66%	9.889	$ 90,275,904
Fannie Mae	6.63%	11/15/30		$6,362,279	$7,518,672	0.846	5.26%	12.760	$ 95,941,329
Treasury Bond	5.38%	02/15/31		$16,000,000	$17,053,301	0.938	4.90%	13.824	$ 235,741,440
TIPS	2.38%	01/15/27		$6,279,125	$6,273,238	1.001	2.38%	15.523	$ 97,381,474
Transactions					$84,199,725				
S Treasury Bond	5.38%	02/15/31	-80%	-$12,800,000	-$13,642,641		4.90%	13.824	$ (188,593,152)
S Union Carbide	7.50%	06/01/25	-100%	-$9,506,955	-$9,341,296	1.018	7.68%	9.770	$ (91,264,768)
B Treasury Strips	0.00%	08/15/27		$8,077,544	$3,000,000	0.371	5.04%	19.889	$ 59,666,667
B Treasury Strips	0.00%	02/15/31		$8,338,578	$2,650,000	0.318	4.96%	23.389	$ 61,980,556
B T Note	4.13%	08/15/10		$ 5,840,871	$ 5,850,000	1.002	4.07%	2.741	$ 16,036,731
B T Note	4.38%	08/15/12		$ 5,809,094	$5,852,665	1.008	4.21%	4.435	$ 25,958,638
B T Note	4.88%	08/31/08		$ 5,592,819	$5,631,272	1.007	4.12%	0.919	$ 5,173,388

Difference bw Buy and Sell	$0	Avg Durati	7.183
Portfolio Value	$84,199,724		

Exhibit 1 - Rensselaer Advisors

Madison Mills Pension Fund

Fixed-Income Portfolio on September 25, 2007

9/25/2007

Security	Coupon	Maturity	Par Value	Market Value	YTM	Duration	Weighted Duration
Treasury							
Treasury Bill		12/27/07	$9,600,000	$9,510,969.60	3.69%	0.255556	$2,430,581
Treasury Note	0.04875	08/31/08	$5,100,000	$5,143,579.60	3.93%	0.918711	$4,725,465
Treasury Note	0.04375	08/15/12	$3,600,000	$3,635,847.69	4.15%	4.436515	$16,130,493
Treasury Bond	0.05375	02/15/31	$16,000,000	$17,053,301.37	4.90%	13.8238	$235,741,440
TIPS	0.02375	01/15/27	$6,279,125	$6,273,238.32	2.38%	15.52332	$97,381,474
Treasury Strips		08/15/09	$2,273,221	$2,110,685.70	3.96%	1.888889	$3,986,851
Treasury Strips		08/15/17	$14,400,000	$9,129,024.00	4.66%	9.888889	$90,275,904
Government Agency							
Fannie Mae	0	11/15/30	$6,362,279	$7,518,672.23	5.26%	12.76041	$95,941,329
Corporates							
Wal-Mart	0.069	08/10/09	$3,769,935	$3,922,605.59	4.60%	1.779792	$6,981,422
Comcast	0.065	01/15/15	$6,502,082	$6,766,908.19	5.81%	5.892813	$39,876,125
Chevron Texaco	0.098	03/15/20	$2,793,200	$3,793,596.22	5.70%	8.199261	$31,104,687
Union Carbide	0.075	06/01/25	$9,506,955	$9,341,296.31	7.68%	9.770032	$91,264,768
Total Portfolio Value				$84,199,724.82			8.5017 Avg Duration

Footnotes

Coupon	Annual coupon interest payments are received semi-annually Coupon rates for TIPS are stated in real terms; the actual coupon paid is adjusted for inflation.
Treasury Strips	All treasury strips are principal strips.

Exhibit 2 - Rensselaer Advisors

Madison Mills Pension Fund

Projected Benefits for Retired Employees

9/25/2007

Year		Benefits	Duration	Rate	PV	Weighted Duration
1/1/2008	$	10,020,000	0.27	4.19%	$9,910,161	$2,660,810
1/1/2009	$	9,009,500	1.27	4.23%	$8,547,187	$10,865,466
1/1/2010	$	8,722,000	2.27	4.27%	$7,931,656	$18,014,638
1/1/2011	$	8,234,500	3.27	4.31%	$7,172,610	$23,463,278
1/1/2012	$	7,858,500	4.27	4.35%	$6,551,473	$27,982,869
1/1/2013	$	7,794,000	5.27	4.39%	$6,213,451	$32,769,568
1/1/2014	$	7,729,500	6.27	4.43%	$5,888,668	$36,945,345
1/1/2015	$	7,639,500	7.27	4.47%	$5,557,655	$40,426,232
1/1/2016	$	6,440,500	8.27	4.51%	$4,470,696	$36,990,414
1/1/2017	$	6,330,000	9.28	4.55%	$4,188,890	$38,859,132
1/1/2018	$	6,242,500	10.28	4.59%	$3,935,665	$40,445,700
1/1/2019	$	6,205,000	11.28	4.63%	$3,724,210	$41,996,846
1/1/2020	$	5,775,500	12.28	4.67%	$3,297,489	$40,482,330
1/1/2021	$	5,600,700	13.28	4.71%	$3,039,104	$40,357,641
1/1/2022	$	5,432,000	14.28	4.75%	$2,799,635	$39,977,260
1/1/2023	$	5,140,000	15.28	4.79%	$2,514,273	$38,416,719
1/1/2024	$	4,234,900	16.28	4.83%	$1,964,579	$31,982,277
1/1/2025	$	4,123,000	17.28	4.87%	$1,812,266	$31,319,933
1/1/2026	$	3,890,000	18.28	4.91%	$1,619,101	$29,600,707
1/1/2027	$	3,500,600	19.28	4.95%	$1,378,644	$26,583,285
1/1/2028	$	3,450,500	20.28	4.99%	$1,284,835	$26,059,279
1/1/2029	$	3,290,600	21.28	5.03%	$1,157,440	$24,636,037
SUM	$	136,663,300			$94,959,693.52	$680,835,766

7.169734239 Duration for portfolio

From Yield Curve

Exhibit 3 - Rensselaer Advisors

Madison Mills Pension Fund

Fixed-Income Securities Available for Purchase on September 25, 2007

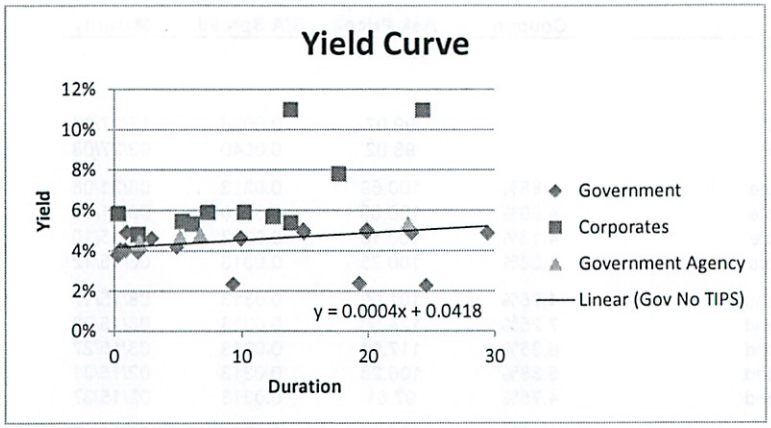
9/25/2007

Security	Coupon	Ask Price	B/A Spread	Maturity	Ticker	Rating	CUSIP	Yield	Actual Duration	Mac Duration	Mod Dur
Treasury											
Treasury Bill		99.07	0.0026	12/27/07			912795B83	3.79%	0.2548	0.2556	0.2508
Treasury Bill		98.02	0.0040	03/27/08			912795D57	4.06%	0.5041	0.5056	0.495502
Treasury Note	4.88%	100.69	0.0313	08/31/08			912828FR6	4.94%	0.9342	0.9187	0.896511
Treasury Note	4.00%	100.00	0.0000	08/31/09			912828HB9	3.99%	1.9342	1.8725	1.835878
Treasury Note	4.13%	100.16	0.0313	08/15/10			912828ED8	4.63%	2.8904	2.7401	2.678105
Treasury Note	4.38%	100.75	0.0313	08/15/12			912828AJ9	4.20%	4.8932	4.4359	4.344641
Treasury Bond	4.75%	101.06	0.0313	08/15/17			912828HA1	4.61%	9.8959	7.9763	7.79656
Treasury Bond	7.25%	124.50	0.0313	08/15/22			912810EM6	4.91%	14.8986	9.8973	9.660162
Treasury Bond	6.38%	117.91	0.0313	08/15/27			912810FA1	4.95%	19.9014	12.1551	11.86153
Treasury Bond	5.38%	106.28	0.0313	02/15/31			912810FP8	4.92%	23.4082	13.8097	13.47816
Treasury Bond	4.75%	97.81	0.0313	02/15/37			912810PT9	4.89%	29.4137	15.8968	15.51737
TIPS	2.38%	100.19	0.1250	01/15/17			912828GD6	2.35%	9.3151	8.3670	8.269658
TIPS	2.38%	99.91	0.1250	01/15/27			912810PS1	2.38%	19.3205	15.5233	15.34072
TIPS	3.38%	120.22	0.1250	04/15/32			912810FQ6	2.29%	24.5726	17.4281	17.23059
Treasury Strips		96.51	0.0200	08/15/08			912833CU2	4.05%	0.8904	0.8889	0.871246
Treasury Strips		92.85	0.0500	08/15/09			918833CW8	3.97%	1.8904	1.8889	1.852124
Treasury Strips		81.60	0.0200	08/15/12			912833DC1	4.20%	4.8932	4.8889	4.788334
Treasury Strips		63.40	0.0200	08/15/17			912833KN9	4.66%	9.8959	9.8889	9.663724
Treasury Strips		47.56	0.0200	08/15/22			912833LJ7	5.05%	14.8986	14.8889	14.5222
Treasury Strips		37.14	0.0200	08/15/27			912833PE4	5.04%	19.9014	19.8889	19.40001
Treasury Strips		31.78	0.0200	02/15/31				4.96%	23.4082	23.3889	22.82288
Government Agency											
Fannie Mae	6.63%	104.16	0.0313	09/15/09			31359MEY5	4.40%	1.9753	1.8802	1.839717
Federal Home Loan Bank	4.50%	99.09	0.0313	11/15/12			3133MTZL5	4.70%	5.1452	4.5687	4.463766
Federal Home Loan Bank	5.25%	102.50	0.0313	06/18/14			3133X7FK5	4.81%	6.7342	5.6939	5.560159
Fannie Mae	6.63%	116.97	0.0938	11/15/30			31359MGK3	5.34%	23.1562	12.7064	12.376
Corporates											
Time Warner	7.48%	100.45	0.0600	01/15/08	TWX	BBB	887315AT6	5.84%	0.3068	0.3056	0.296881
Wal-Mart	6.79%	103.62	0.2400	08/10/09	WMT	AA	931142BE2	4.82%	1.8767	1.7807	1.73876
Verizon	6.13%	102.99	1.2000	01/15/13	VZ	A-	923474AA8	5.47%	5.3123	4.5706	4.449037
Altria	7.00%	108.59	0.3420	11/04/13	MO	BBB	02209SAA1	5.33%	6.1151	4.9982	4.868456
Comcast	6.50%	103.44	0.2970	01/15/15	CMCSA	BBB	20030NAB7	5.91%	7.3123	5.8878	5.718747

Hilton Hotels	7.50%	112.03	0.9000	12/15/17	HLT	BB+	432848AS8	5.91%	10.2301	7.3628	7.151393
Chevron Texaco	9.75%	135.68	0.0000	03/15/20	CVX	AA	881685AN1	5.71%	12.4795	8.1975	7.969954
American Airlines	9.75%	91.20	0.9000	08/15/21	AMR	CCC	001765AP1	11.00%	13.8986	7.5210	7.12898
Mobil	8.63%	131.44	2.3000	08/15/21	XOM	AAA	607059AT9	5.38%	13.8986	9.0295	8.792936
Union Carbide	7.50%	97.07	0.0000	06/01/25	DOW	BB+	905581AR5	7.81%	17.6959	9.7213	9.35605
Ford Motor Credit	8.90%	82.52	0.9000	1/15/2032	F	CCC+	345370BV1	10.97%	24.3233	8.9803	8.513444

Footnotes

Coupon	Annual coupon interest payments are received semi-annually Coupon rates for TIPS are in real terms; the actual coupon paid is adjusted for inflation.
Ask Price	All prices listed in decimals.
TIPS	In order of maturity, the accrued principal on the TIPS is 1033, 1033, and 1174.
Treasury Strips	All treasury strips are principal strips.
B/A Spread	The difference between the ask price and the bid price.



15.401 Finance Theory I

Craig Stephenson

MIT Sloan School of Management

Lecture 4: Common Stocks

Introduction to stock markets
Discounted Cash Flow Model (DCF)
Modeling cash flows
EPS and ROE
Growth opportunities and growth stocks
P/E and PVGO

Readings:

Brealey, Myers and Allen, Chapter 4

Bodie, Kane and Markus, Chapter 18

Announcements

The mid-term exam will be held in class on Wednesday, March 9. it will cover lecture topics 2, 3, and 4:
- Present value,
- Fixed income, and
- Common stocks

The final exam will be held on Monday, May 16, from 1:30 p.m. to 4:30 p.m. in E51-325

Mark your calendars for these dates and times

Common stocks represent equity or ownership positions in a corporation.

Payments to common stocks are in the form of dividends:
- cash dividends
- stock dividends
- share repurchases

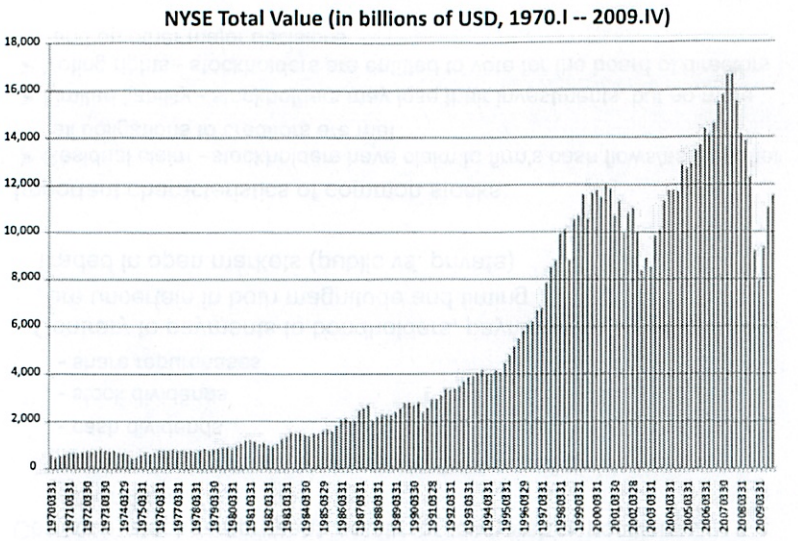
Contrary to payments to bondholders, payments to stockholders are uncertain in both magnitude and timing

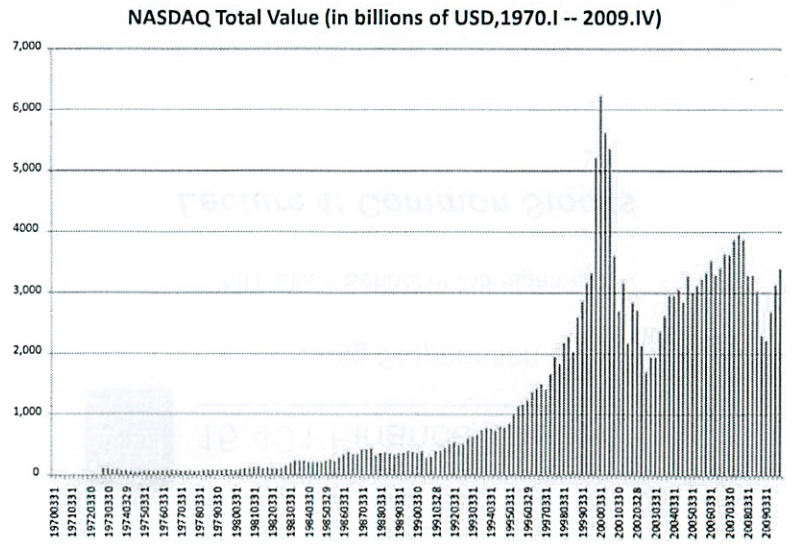
Traded in open markets (public vs. private)

Important characteristics of common stocks:
- Residual claim - stockholders have claim to firm's cash flows/assets after all obligations to creditors are met
- Limited liability - stockholders may lose their investments, but no more
- Voting rights - stockholders are entitled to vote for the board of directors and on other major decisions

2/12

1. Primary market - underwriting
   - Venture capital: A company issues shares to investment partnerships, investment institutions and wealthy individuals
   - Initial public offering (IPO): A company issues shares to the general public for the first time (i.e., going public)
   - Secondary (seasoned) offerings (SEO): A public company issues additional shares
   - Stock issuing to the public is usually organized by investment banks who act as underwriters
2. Secondary market (resale market) - Exchanges and OTC markets
   - Exchanges: NYSE Euronext, London, ECNs, …
   - OTC: NASDAQ
3. Trading in secondary markets
   - Trading costs: commission, bid-ask spread, price impact
   - Buy on margin
   - Long and short positions





Basic PV formula applies to the valuation of stocks. Need to know:

Expected future dividends

Discount rates for dividends

Notation:

P_t -- expected stock price at time t (ex-dividend)

D_t -- expected cash dividend at time t

r_t -- risk-adjusted discount rate for cash flow at time t

Dividend Discount Model (DCF)

Stock price is the present value of future dividends:

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r_t)^t} = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t}$$

Simplification: $r_t = r$.

Constant and Perpetual Growth

Dividends are expected to grow at a constant rate g in perpetuity:

$$D_{t+1} = (1+g) \times D_t$$

Then

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t} = \frac{D_1}{r-g}, \qquad r > g$$

This is the Gordon Model:

$$\boxed{P_0 = \frac{D_1}{r-g} = \frac{1+g}{r-g} D_0}$$

Example. Dividends are expected to grow at 6% per year and the current dividend is $1 per share. The expected rate of return is 20% The current stock price should be

$$P_0 = \frac{1.06}{0.20 - 0.06} \times 1 = \$7.57$$

Example. Determine the cost of capital (discount rate). Suppose the dividend yield for Duke Power is $D_0/P_0 = 0.052$. Estimates of long-run growth for Duke are:

Info Source	Value Line (VL)	I/B/E/S
Growth g	0.049	0.041

The cost of capital is given by

$$r = \frac{D_1}{P_0} + g = \frac{(1+g)D_0}{P_0} + g$$

Thus,

	Cost of Capital
VL	$r = (0.052)(1.049) + 0.049 = 10.35\%$
IBES	$r = (0.052)(1.041) + 0.041 = 9.51\%$

Cost of capital = Dividend yield + dividend growth

Example. Estimate the dividend growth rate. WSJ reported the following data on AT&T stock:

AT&T	DIV	YLD	P/E	High	Low	Last	Chg
	1.32	3.4	60	38.5	38.125	38.5	+.25

What is market's estimate of AT&T's dividend growth rate, if r = 12%?

Solving the valuation formula for g gives

$$g = \frac{r - D_0/P_0}{1 + D_0/P_0}$$

Since

$$P_0 = (38.5 + 38.125)/2 = 38.3125$$

$$D_0/P_0 = 1.32/38.3125 = 0.03445$$

We have

$$g = \frac{0.12 - 0.03445}{1.03445} = 8.27\%$$

Multi-stage growth

Firms evolve through different stages in their lifecycles. For example,

1. Growth stage - rapidly expanding sales, high profit margins, and abnormally high growth in earnings per share, many new investment opportunities, low dividend payout ratio.
2. Transition stage - growth rate and profit margin reduced by competition, fewer new investment opportunities, higher payout ratio.
3. Maturity stage - earnings growth, payout ratio and average return on equity stabilizes for the remaining life of the firm.

Example. In Example 1 (D_0 = $1 and r = 20%), suppose that the growth rate is 6% for the first 7 years and then drops to zero thereafter.

$$P_0 = \$6.49$$

Actual forecast of dividends involves many practical factors.

Terminology:

- Earnings (E or EPS): total profit net of depreciation and taxes
- Payout ratio: dividends/earnings = DPS/EPS = p
- Retained earnings Δ: (earnings - dividends)
- Plowback ratio: retained earnings Δ/earnings = b = 1 - p
- Book value (BV): cumulative retained earnings
- Return on book equity (ROE): earnings/BV

Example. Texas Western (TW).

Expected earnings $1.00 per share next year.

Book value is $10.00 now.

Plans an expansion to increase net book assets by 8% per year.

Return on new investment is 10%.

New investment is financed by retained earnings.

Cost of capital is 10%, same as rate of return on new investments.

Price TW's shares if

TW expands at 8% forever

TW's expansion slows down to 4% after year 5

Here,

Plowback ratio b = ($10)(0.08) / ($1) = 0.8

Payout ratio p = (1- 0.8)/(1) = 0.2

ROE = 10% = r (cost of capital)

Plowback ratio b = 0.8

Payout ratio p = 0.2

ROE = 10% = r

1. What-if there is no expansion, no investment? D = E = $1.0 and:

$$P_0 = \frac{E}{r} = \frac{1.0}{0.10} = \$10.00$$

2. If TW chooses expansion through investment:

$$\begin{aligned} g &= \text{ROE} \times b = (0.10)(0.8) = 0.08 \\ D_1 &= \text{EPS}_1 \times p = (1)(0.2) = 0.2 \\ P_0 &= \frac{D_1}{r-g} = \frac{0.2}{0.10 - 0.08} = \$10.00 \end{aligned}$$

Why are the values the same under these scenarios?

Earnings flow into investment, increasing book value, or dividends.
This increases future earnings, investment or dividends, and book value.

Begin with year 1 expected earnings

Determine the amount of earnings invested in expansion, vs. paid as dividends

Book value at the end of year 1 increases with investment/expansion

Year 2 expected earnings, calculated as book value x ROE increase with investment/expansion

Allocate year 2 earnings to investment/expansion or dividends

Book value at the end of year 2 increases with investment/expansion

Increasing year 3 expected earnings; repeat as specified in the example

3. 2-stage expansion. Forecast EPS, D, BVPS by year:

Year	0	1	2	3	4	5	6
EPS		1.00	1.08	1.17	1.26	1.36	1.47
Investment		0.80	0.86	0.94	1.00	1.08	0.59
Dividend		0.20	0.22	0.23	0.26	0.28	0.88
BVPS	10.00	10.80	11.66	12.60	13.60	14.69	15.28

$$P_0 = \sum_{t=1}^{5} \frac{D_t}{(1.1)^t} + \frac{1}{(1.1)^5}\frac{0.88}{(0.10-0.04)} = \$10.00$$

Growth opportunities are investment opportunities that earn expected returns higher than cost of capital.

Stocks of companies that have access to growth opportunities are considered growth stocks.

- The following may not be growth stocks
  - A stock with growing EPS
  - A stock with growing dividends
  - A stock with growing assets
- The following may be growth stocks
  - A stock with EPS growing slower than cost of capital
  - A stock with DPS growing slower than cost of capital

Example. ABC Software has the following data: Expected EPS next year is \$8.33; Payout ratio is 0.6; ROE is 25%; and, cost of capital r = 15%.

Thus,

$$\begin{aligned} D_1 &= p \times \text{EPS} = (0.6)(8.33) = \$5.00 \\ g &= b \times \text{ROE} = (0.4)(0.25) = 0.10 \end{aligned}$$

Following a no-growth policy (g=0, p=1), its value is

$$P_0 = \frac{D_1}{r-g} = \frac{\text{EPS}_1}{r} = \frac{8.33}{0.15} = \$55.56$$

Following the growth policy, its price is

$$P_0 = \frac{D_1}{r-g} = \frac{5.00}{0.15-0.10} = \$100$$

The difference of \$100 - \$55.56 = \$44.44 comes from the growth opportunities, which offers a return of 25%, higher than the cost of capital 15%.

Stock price has two components:

1. Present value of earnings under a no-growth policy
2. Present value of growth opportunities

$$P_0 = \frac{\text{EPS}_1}{r} + \text{PVGO}$$

Terminology:

- Earnings yield: E/P = EPS_1/P_0
- P/E ratio: P/E = P_0/EPS_1

(Note: In business media, E/P is often quoted EPS_0/P_0 rather than EPS_1/P_0. But finance is forward looking.)

Thus,

- If PVGO = 0, P/E ratio equals inverse of cost of capital

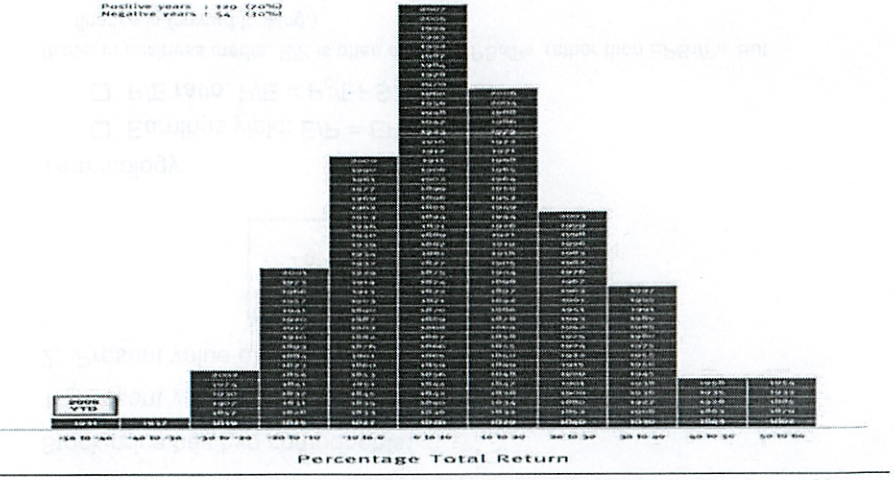
$$P/E = \frac{1}{r}$$

- If PVGO > 0, P/E ratio becomes higher:

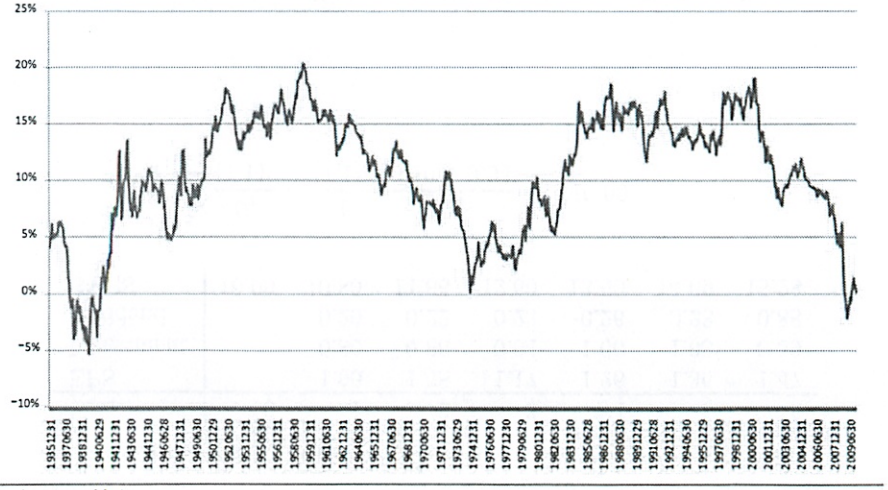
$$P/E = \frac{1}{r} + \frac{PVGO}{EPS_1} > \frac{1}{r}$$

- PVGO is positive only if the firm earns more than the cost of capital

Stocks are risky: in 1933 the return was +57.5%, but in 1931 the return was -44.4% (2008: -38.3%, 2009: 26.4%)



Geometric average of value-weighted NYSE/AMEX/NASDAQ nominal equity returns over the previous10 years (1935.12—2009.12):



Introduction to stock markets
Discounted Cash Flow Model (DCF)
Modeling cash flows
EPS and ROE
Growth opportunities and growth stocks
P/E and PVGO

15.401 Common Stock

2/28

Common Stock
- Very uncertain
- Cashflows: don't know when/how much

Look at their return on equity
- from their investment opportunity set
- when young + growing

Value in growth opportunities for biz

Have recidual claim - last in line

Voting rights for board

Limited Liability - can lose investment, but no more

Cash buybacks just another way to distribute cash

---

1. Family + friends
2. VC - not only give $, but connections
3. IPO - shares to the public
4. Secondary Seasoned Offerings - SEO - dilutes everyone's ownership
   - underwritten by investment banks
5. Secondary Markets - stock exchanges
   - much more liquid

(2) Same PV formula — Dividend Discount Model (DCF)

- for dividends
- but very uncertain

$$P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r_t)^t} = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t}$$

if $r_t = r$

- have to risk adjust as well

Constant and Perpetual Growth

$$D_{t+1} = (1+g) \cdot D_t$$

then $P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t} = \frac{D_1}{r-g}$ ← next dividend
- missed current dividend

$r > g$

If grows faster than rate of growth will be ∞

Economically impossible

Mathmatically impossible

$$P_0 = \frac{D_1}{r-g} = \frac{1+g}{r-g} D_0$$ Gordon Model

Hard part: how big is g, r

(3)

Formula works pretty well for utility
- driven by population of city
- thats the market growth

Look at Value Line (VL)
- lots of info on there
- predict growth rate
- information intermediary

I/B/E/S - another database researchers use

Cost of capital - the required return

$$r = \frac{D_1}{P_0} + g = \frac{(1+g)D_0}{P_0} + g$$

$r = (.052)(1.049) + .049 = 10.35\%$

- .052: dividend Yield $\frac{D_0}{P_0}$
- 1.049: growth % From VL
- .049: ↑

Or look at rate of return - find growth rate

$$g = \frac{r - D_0/P_0}{1 + D_0/P_0}$$

If you think growth rates will be less, sell or Short

(9)

Nestle grows faster than pop in world - how?

Why acquire companies?

- Consumer Products - hard to get shelf space
- loyalty - very strong

Need to get enough out of it to justify price

Companies that re-invest everything

- Should do when growing rapidly
- - have ~~a~~ lots of good investment opps
- Then when no more good investment opps
  - have free cash
  - Should be distributing to shareholders
  - a firm may come and buy up the company + vaccine out all of the cash

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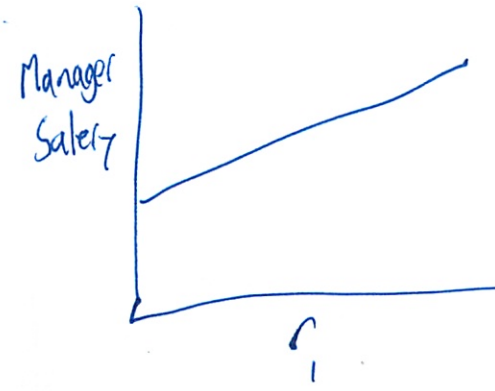


What variable best matches to manager salery

1. Size

(5)

Remember don't get first dividend
Do the growth of each dividend
Then discount back to PV

For further out CF - but only when constant perpetual

$$P_7 = \frac{D_8}{r-g}$$

↑ constant starts in time 7

Again estimating growth rate is the hard part

Terminology

Earnings (E, EPS) = total profit - depeciation - taxes

Payout ratio = $\frac{\text{dividends}}{\text{earnings}} = \frac{DPS}{EPS} = p$

Retained earnings Δ = (earnings - dividends)

Plow back ratio = $\frac{\text{retained earnings } \Delta}{\text{earnings}} = b = 1-p$

Book Value (BV) = total retained earnings, cumulative

one more

(Case examples on slides)
growth rate, dividend, Price, etc

(6)

Got same outcome if reinvest 80% of returns

ROE = Cost of capital so investors are indifferent to investing or not

(I should have been able to see that)

10% = risk-adjusted opportunity cost of capital

Go into cycle of increasing book value

At one point in time investment may fall

- Can't use Gordon model
- need to use table model - (see slides)

Again ROE = growth rate so investors are again indifferent to growth or dividends

---

Growth opportunities - investment expected rate of return > Cost of capital

If ROE > Cost of capital → invest in projects!

Stock Price has 2 components

$$P_0 = \underbrace{\frac{EPS_1}{r}}_{\text{PV earnings now}} + \underbrace{PVGO}_{\text{growth opp}}$$

(5)

Terms

$$\text{Earnings Yield} = \frac{E}{P} = \frac{EPS_1}{P_0}$$

$$P/E \text{ ratio} = \frac{P}{E} = \frac{P_0}{EPS_1}$$

- if high - then people looking at growth
- if low - then people look at growth unfavorably
  - more tuned to already predicted growth

If PVGO = 0 then

$$\frac{P}{E} = \frac{1}{r}$$ r = cost of capital

- just a perpitivity of required return

If PVGO > 0

$$\frac{P}{E} = \frac{1}{r} + \frac{PVGO}{Eps} > \frac{1}{r}$$

- Tech firms price so volitale as investors re evaluate PVGO

Why do stock prices move?

- ↑ required returns / discount rates = ↓ value
- ↓ PVGO
  - company
  - industry
  - economy

(8)

In .com bubble - people put a lot in PVCO of .coms

(Geometric avg - what is that - check?)

Case Hints

2/26

- Can be easy or hard
- Can liquidate entire portfolio
  - get value upon liquidation 82 million
- Calculate duration) of benefits
- The PV
- Interest rate should be
  - Conservative
  - So low
  - slightly upward sloping yield curve
- Get duration of whole obligation stream

Assets
- Can ~~Build~~ duration to immunitize (what I did)
  ~~Linear~~ set ← he would do

Other
- linear algebra software solution

(: So I got 70% right)

Could do all 30 day strips - reinvestment risk
Could do long investment - need to sell - price risk

Conservative, careful makes this interesting
Don't sell equities - not on spreadsheet
Don't bet on interest rates

(2)

Can look at coupons too

- large cash flows every year
  (I did not calc)
- pulls duration in
- cash flows in match those obligations

If you match it up perfectly in = out
then completly immunitized
don't need to worry about where interest rates go
(I was more correct than TA!)

Relooking at Case

2/28

Can calc some stff manually
Not just w/ Excel fn
- just avg duration?
But have to divide by bond price
- what is that?
The PV of cash flows
Does not give proper amt

Are my avg #s correct?
I made total duration longer!
I should also look at coupon rates

So just try something else for portfol.'s
Should use NPV for payouts curve
- Cool - got it
- For gav

Not doing year for year matching
Rather watching avg duration
I don't know if ~~simplist to~~ right thing to do
Just turn in

15.401 Finance Theory I

Craig Stephenson

MIT Sloan School of Management

Lecture 4: Common Stocks
Valuing an Ongoing Business
(CLB Partners, LLC)

VALUATION METHODOLOGIES

Book Value of Equity

Relative Values

Discounted Cash Flow

I. BOOK VALUE OF EQUITY APPROACH EXAMPLE

Forecasted Balance Sheet
Visions, Ltd.
(000s)

Cash	$500	Accts Payable	$2,520
Accts Rec.	800	Long Term Debt	7,000
Inventory	1,500	Common Shares	500
	2,800	Retained Earnings	2,380
Fixed Assets	9,600		
		Total Liab & Equity	$12,400
Total Assets	$12,400		

What is the book value of the company's equity?

II. RELATIVE VALUATION APPROACH

Comparable Companies
Comparable Transactions

Example

Suppose you want to estimate the value of ABC Manufacturing, a private company that makes transportation equipment. The company's most recent fiscal year annual report shows the company had $100 million in sales and $5 million of Net Earnings.

A review of public companies in similar business lines yields the following market information based on recent prices and the last twelve months of operating data:

($ In millions)	Revenue	Earnings	Market Cap	P/E
ABC Rail Products	249.5	11.8	196.0	16.61
Greenbriar Cos	424.8	17.0	182.9	10.76
Johnstown America	667.8	5.6	46.5	8.31
Westinghouse Air Brake	425.0	33.7	307.5	9.12
Average				**11.2**

RELATIVE VALUE APPROACH - ABC MANUFACTURING

Average Industry Peer Group P/E	11.2 x
ABC Earnings ($ million)	$5.0
Estimated Equity Value ($million)	$56.0

Pitfalls of Relative Value Approach

COMMON RELATIVE VALUE VARIABLES

Price/Earnings

Price/Cash Flow

Market Value Equity/Book Value Equity

Price/Sales

Others

III. DISCOUNTED CASH FLOW (DCF) VALUATION

DCF Definition:

An investment's value is equal to the present value of all future cash flows.

Four variables are critical inputs:

1. Relevant cash flow
2. The forecast period
3. Terminal value at end of the forecast period
4. Risk adjusted discount rate

FREE CASH FLOW TO ALL CAPITAL PROVIDERS

GENERAL FORMULA

$$V_o = \sum_{t=1}^{T} \frac{FCF_t}{(1+r)^t} + \frac{TV_T}{(1+r)^T}$$

FCF = Free cash flow available to all providers of capital

V_o = Total firm value today

TV_T = Total firm terminal value at period T

r = Risk adjusted discount rate for the company's cash flows

FORMAT FOR ESTIMATING FREE CASH FLOWS

Earnings before interest and tax (EBIT)

- Cash Tax on operating income

Earnings before interest, after tax (EBIAT)

± Investments in Working Investment (Accounts Receivable, Inventory, Accounts Payable, Accrued Expenses)

± Investments in Property, Plant and Equipment

+ Depreciation and Amortization

± Change in other assets and liabilities

Free Cash Flow (To ALL Capital Providers)

USING FREE CASH FLOW TO ALL PROVIDERS TO VALUE A COMPANY

- A company has two types of investors -- debt and equity.
- The total value of the firm (or enterprise value) is the value of the debt and the value of the equity (MV Firm = MV Liabilities + MV Owners' Equity).

FORECASTED INCOME STATEMENTS

Visions, Ltd.
Basic Assumptions

1. Sales and profits will grow at 11 percent annually for five years.
2. Sales and profits will grow at 7 percent annually after the fifth year.
3. The company's risk adjusted discount rate is 11.88%

FORECASTED INCOME STATEMENTS

Visions, Ltd.
(000s)

Year	1	2	3	4	5
Revenues	$10,000	$11,100	$12,321	$13,676	$15,181
Operating Expenses	6,420	7,126	7,910	8,780	9,746
Depreciation	1,600	1,776	1,971	2,188	2,429
EBIT	1,980	2,198	2,440	2,708	3,006
Interest Expense	980	1,088	1,207	1,340	1,488
Pre-tax Profit	1,000	1,110	1,232	1,368	1,518
Taxes (40%)	400	444	493	547	607
Profit	$600	$666	$739	$821	$911

FORECASTED CASH FLOW TO ALL PROVIDERS

Visions, LTD.
(000s)

Year	1	2	3	4	5
Revenues	$10,000	$11,100	$12,321	$13,676	$15,181
Operating Expenses	6,420	7,126	7,910	8,780	9,746
Depreciation	1,600	1,776	1,971	2,188	2,429
EBIT	1,980	2,198	2,440	2,708	3,006
- Taxes (40% of EBIT)	792	879	976	1,083	1,202
- W/C Investment	200	222	247	274	304
+ Depreciation	1,600	1,776	1,971	2,188	2,429
- F.A. Investment	2,500	2,775	3,080	3,419	3,188
Free Cash Flow	$88	$98	$108	$120	$741

Note: The rate of fixed asset investment growth decreased in year 5 in anticipation of the slowing sales growth rate in subsequent years.

WHY ESTIMATE TERMINAL VALUE ?

- After year 5 growth decreases to a 7% constant annual rate.
  - Using financial tools we can estimate the value of all cash flows past year 5.
  - This simplifies the problem of estimating the value of an infinitely-lived stream of cash flows.
- Terminal Value is this estimate

ESTIMATING TERMINAL VALUES - ALL CAPITAL PROVIDERS METHOD

- Constant Perpetual Growth Method (assumes constant growth)

$$\frac{\text{Free Cash Flow to all capital providers X (1 + g)}}{r - g}$$

- Relative Values - Asset or Enterprise Based Variables

ESTIMATING TERMINAL VALUE

Visions, Ltd.
(000s)

Estimated Free Cash Flow, year 5		$741
Estimated Growth Rate to year 6	*	1.07
Estimated Free Cash Flow, year 6		$793

$$\text{T.V. (year 5)} = \frac{\$793}{(.1188 - .07)}$$

$$\text{T.V. (year 5)} = \$16{,}243$$

FORECASTED FREE CASH FLOWS TO ALL PROVIDERS

Visions, Ltd.
(000s)

Year	**1**	**2**	**3**	**4**	**5**
Free Cash Flow	$88	$98	$108	$120	$741
Terminal Value					16,243
Total Free Cash Flow	88	98	108	120	16,984
PV Factor @ 11.88%	.8938	.7989	.7141	.6382	.5705
Present Value	$79	$78	$77	$77	$9,689
Total Present Value	$10,000				

ESTIMATING VALUE OF EQUITY

Visions, Ltd.
(000s)

Estimated Total Present Value of Free Cash Flow to All Providers	$10,000
Less: Value of Debt	7,000
Estimated Value of Equity	$3,000

(÷ number of shares = price per share)

VALUATION SUMMARY

- Various methodologies are tools, not absolute models
- Discounted Cash Flow valuation is the preferred tool

15.401 Stocks 3

- Midterm week from today 1 pg double sided cheat sheet
- Review Mon in class
- P-set over the weekend due Mon

$$P_0 = \frac{Div_1}{r-g} = \frac{Div_0(1+g)}{r-g}$$

price of any fin asset is PV of cash flows

$g = ROE \cdot b$ ← plowback ratio

Common stock cash flows last forever in theory

Constant perpetual growth

but growth rate is not perpetual

0	1	2	3	4	...	x
	1.00	1.50	2.16	2.73		$g = 6\%$

→ decreasing growth

$$P_{x-1} = \frac{D_1}{r-g}$$

← at some point hits constant perpetual rate

Everything is based off this ↳ other measures/formulas

If $g > r$ the whole thing melts anyway

$$r = \frac{P_1}{P_0} + g$$

↑ Stock owner 2 rewards: dividend yield, growth

(2)

Today: What happens inside a whole company
- book value of equity
- relative ~~values~~ valuation
- discanted cash flows

Book value of equity

Assets = Liabilities + Equity

So add up equity account
- Common Shares
- Retained Earnings

But Balance sheets are backward looking

As a stock holder you get the future, not the past

Relative Valuation

- private company
- Review similar publically traded companies
- Market Cap = Market Value of Equities = Price / Share • # Shares
- Find avg PE ratio • Earnings of company
- But assuming your company is avg
- Look at the company to see if that is true
  - Lots of judgement

(3)

But what if ⊖ earnings since too young

Price/Earnings
Price/Cash Flow
NV of
⋮
⋮ (missed)

Discounted Cash Flows

For the whole firm, not just dividends

$$V_0 = \sum_{t=1}^{T} \frac{FCF_t}{(1+r)^t} + \frac{TV_T}{(1+r)^T}$$

FCF = free cash flow available to all providers of capital
V_0 = Value of Firm today
TV_T = Terminal Value at Period T
- Young company could have fast growth

r = risk adjusted discount rate for cash flows

$TV_t = \frac{FCF_{t+1}}{r+g}$ ← year when FCF goes constant growth

(4)

Accounting for estimating cash flows

- not copying, see slides
- add depreciation back b/c interested in cash
As company grows need to reinvest

Split b/w the two types of investors: debt + equity
Assets = Liabilities + Equity

---

example 11% growth 1-5 years
- % growth >5 years - use perpetual formula

Have a balance sheet so add stuff back up
W/C = Working Capital = inventory, AR
F.A = Fixed Assets

Why estimate terminal value?
- makes calculations easier
- by assuming constant growth after some point in time

$$TV_5 = \frac{FCF_6}{(r-g)} \quad \text{or} \quad FCF_5 \cdot (1+g) = FCF_6$$

Collapse all future Cash flows (at perpetual growth)
└ still collapse at PV(TV_5) discount rate

Less Value of Debt
Remaining goes to common stock holders

(5)

~~Picked~~ ~~Divide by price per shar~~

Divide by # of shares to get price per share

All of these are different techniques

Do each

See how they compare

Examples

$P_0 = 1$

$g = .15$ year 1

$g = .12$ year 2

$g = .10$ year 3

$g = .08$ year 4

$g = .06$ year $5 \to \infty$

$$P_0 = \frac{Div_1}{r-g}$$

Lay out each one individually

next dividend starts at time 1 current divided already paid

$$\frac{1}{1.15} + \frac{2}{1.2880} + \frac{3}{1.4168} + \frac{4}{1.5301} + \frac{5}{1.6220} + \frac{5TV}{57.3090}$$

$$\frac{1.15}{1.09} + \frac{1.2880}{(1.09)^2} + \frac{1.4168}{(1.09)^3} + \frac{1.5301}{(1.09)^4} + \frac{1.6220}{(1.09)^5} + \frac{57.3090}{(1.09)^5} = 42.6182$$

$$P_5 = \frac{Div_6}{r-g} = \frac{1.6220 \cdot (1.06)}{.09 - .06}$$

$P_5 = 57.3090$ → still need to discount

(6)

Company announces new product people like

g↑ or g high longer

Interest rates ↑

Investors want biger returns

So price ↓

since r ↑ and denom ↑

Inflation is concern

Intrest rates ↑

See above

Risk ↓

Required return ↓

r ↓ denom ↓

So price ↑

Investors will only return in risk assets if they think they can get a higher return

Yesterday stock market as whole ↓ 2%

- Uncertainties over inflation, oil, ~~Kashi~~ Libra

Did not attend

15.401 Recitation

3: Common Stocks

Learning Objectives

- Review of Concepts
  - Discounted cash flow (DCF)
  - PVGO
- Examples
  - Flancrest Enterprises
  - CompuGlobalHyperMegaNet
  - Globex Corporation

Review: DCF

- The stock price today = sum of all expected future dividends discounted at the appropriate risk-adjusted rate.
- Constant dividend:

$$P_0 = \frac{D}{r}$$

- Growing dividend ($r > g$):

$$P_0 = \frac{D_1}{r - g}$$

Review: DCF

- Components of DCF:
  - ***D***: dividend forecast based on historical data and future prediction
  - ***r***: the discount rate = r_f (risk-free rate due to time value of money) + π (risk premium due to risk of dividend stream).
  - ***g***: growth rate based on…
    - return on equity (**ROE**): earnings / book value of equity
    - plowback ratio (**b**): retained earnings / total earnings
    - ***g* = ROE x b.**
    - Note: *g* must be the long-run growth rate.

3/4

Review: PVGO

- We can separate the value of a firm into its ongoing value and value of growth opportunities:

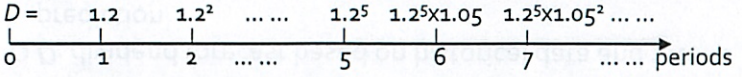
$$P_0 = \mathrm{V}_0 + \mathrm{PVGO} = \frac{EPS_1}{r} + \mathrm{PVGO}$$

- PVGO can be solved from the above equation, where P_0 is derived from DCF.
- Conversely, we can find the implied rate of return on a stock given market data:

$$r = \frac{D_1}{P_0} + g = \frac{D_0(1+g)}{P_0} + g$$

Example 1: Flancrest Enterprises

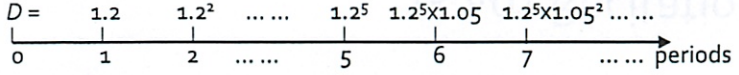
- Flancrest Enterprises recently paid a dividend of $1 per share. Its dividend is expected to grow at 20% for years 1-5. Afterwards, the growth rate will slow down to 5%. If the cost of capital for Flancrest Enterprises is 15%, what is the price of its stock today?



- What is the ex-dividend price of the stock at time 1? What is the rate of return of the stock in Year 1?

Example 1: Flancrest Enterprises

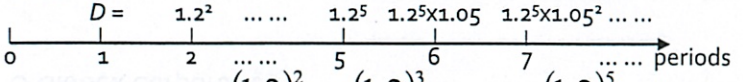
- Time 0:



- PV of Year 1-5: $\frac{1.2}{1.15} + \frac{(1.2)^2}{(1.15)^2} + \cdots + \frac{(1.2)^5}{(1.15)^5} = 5.6912$
- PV of Year 6-∞: $\frac{1.2^5 \cdot 1.05}{0.15 - 0.05} \cdot \frac{1}{(1.15)^5} = 12.9899$
- Price: $5.6912 + 12.9899 = \$18.68$

Example 1: Flancrest Enterprises

- Time 1:



- PV of Year 2-5: $\frac{(1.2)^2}{1.15} + \frac{(1.2)^3}{(1.15)^2} + \cdots + \frac{(1.2)^5}{(1.15)^4} = 5.3449$
- PV of Year 6-∞: $\frac{1.2^5 \cdot 1.05}{0.15 - 0.05} \cdot \frac{1}{(1.15)^4} = 14.9384$
- Price: $5.3449 + 14.9384 = \$20.28$
- Return: $\frac{\$20.28 + \$1.2}{\$18.68} - 1 = 15.00\%$

Example 2: CompuGlobalHyperMegaNet

- CompuGlobalHyperMegaNet (CGHMN) has an EPS of $2 last year. It has a payout ratio of 25% and ROE of 10%. If investors expect a return of 10% from the firm,
  - What is CGHMN's stock price?
  - What is CGHMN's PVGO?
  - What is CGHMN's P/E ratio?
- How would the answers change if
  - ROE = 12%?
  - ROE = 9%

Example 2: CompuGlobalHyperMegaNet

- (ROE = 10%)
  - $g = \text{ROE} \times b = 0.1 \times (1 - 0.25) = 0.075$

    $$P_0 = \frac{D_1}{r-g} = \frac{D_0 \cdot (1+g)}{r-g} = \frac{2 \cdot 0.25 \cdot (1+0.075)}{0.10 - 0.075} = \$21.50$$
  - $$\text{PVGO} = P_0 - \frac{EPS_1}{r} = 21.5 - \frac{2 \cdot 1.075}{0.10} = \$0.00$$
  - $$\text{PE}_0 = \frac{P_0}{EPS_1} = \frac{21.5}{2 \cdot 1.075} = 10$$

Example 2: CompuGlobalHyperMegaNet

- (ROE = 12%)
  - $g = \text{ROE} \times b = 0.12 \times (1 - 0.25) = 0.09$

    $$P_0 = \frac{D_1}{r-g} = \frac{D_0 \cdot (1+g)}{r-g} = \frac{2 \cdot 0.25 \cdot (1+0.09)}{0.10 - 0.09} = \$54.50$$
  - $$\text{PVGO} = P_0 - \frac{EPS_1}{r} = 54.5 - \frac{2 \cdot 1.09}{0.10} = \$32.70$$
  - $$\text{PE}_0 = \frac{P_0}{EPS_1} = \frac{54.50}{2 \cdot 1.09} = 25$$

Example 2: CompuGlobalHyperMegaNet

- (ROE = 9%)
  - $g = \text{ROE} \times b = 0.09 \times (1 - 0.25) = 0.0675$

    $$P_0 = \frac{2 \cdot 0.25 \cdot (1+0.0675)}{0.10 - 0.0675} = \$16.42$$
  - $$\text{PVGO} = P_0 - \frac{EPS_1}{r} = 16.42 - \frac{2 \cdot 1.0675}{0.10} = -\$4.93$$
  - $$\text{PE}_0 = \frac{P_0}{EPS_1} = \frac{16.42}{2 \cdot 1.0675} = 7.69$$

Example 3: Globex Corporation

- The dividend yield for shares of the Union Pacific Railroad is 1.9%. Security analysts are forecasting rapid growth in Globex's earnings per share (EPS), about 12.7% per year for the next three years. Does that imply an expected rate of return of 1.9 + 12.7 = 14.6%? Explain.

Example 3: Globex Corporation

- Answer:
  No.
  - EPS is only growing at 12.7% for the next three years, not forever. The expected rate of return can only increase by less than that amount.
  - There may be a cost to the rapid growth (e.g. part of the current earnings may be retained), so the rate of return is lowered further.

Cost of capital

= "risk free opp. cost" + risk premium

$$= \frac{D_1}{P_0} + g$$

$$= \frac{(1+g)D_0}{P_0} + g$$

= Divided yield + dividend growth

MIT Sloan School of Management

Finance Theory I — 15.401

Craig Stephenson — Spring 2011

Problem Set 3: Common Stocks

(Due: Monday, March 7th, 5:00 p.m.)

Unless specified as an "Excel Problem", all of these problems can (and should) be solved with a pencil, paper and a simple calculator. Do not use Excel (except to check your work, if you want). Show your work cleanly and write out the formulas that you used to solve the problems. Circle your final answers.

Problem 1

Stocks of Alpha Corp. are trading at $50 per share. Earnings and dividends are expected to be $3.6 and $2 per share next year, respectively; both will grow at 5% per year in subsequent years.

(a) What is the company's cost of capital?

(b) What is the company's PVGO?

(c) What is the company's ROE?

Problem 2

Beta Inc. is expected to pay a dividend of $0.5 per share one year from now. Every year afterward, the company will raise dividend by 30 cents. If company has a cost of capital of 20%, what is its stock price today?

Problem 3

Gamma International has just paid a dividend today, and its book value per share ($BVPS_0$) in year zero is 120. The payout ratio and ROE have been 20% and 25%, respectively, and they will remain the same for the next five years (ROE_t =.25 for t = 1, ... , 5). Thus EPS for year 1 is 120 x 25% = 30, and for future periods $EPS_t = BVPS_{t-1} \times ROE_t$. After that, payout ratio will increase to 80% forever. Gamma has a cost of capital of 18% and a stock price of $100.

(a) What is the company's expected long-run (constant) ROE?, i.e., what ROE_{6+} applies for t > 5? HINT: Build a small excel table as in the lecture notes (Chapter 4, page numbered 15), which looks like the table on the next page.

(b) Suppose that in absence of plowback, the company can only maintain a 20% ROE in the first five years (ROE_t = .20 for t = 1, … , 5) and 8% per year afterwards. (ROE_{6+} = .08). What is its PVGO (difference of share price in (b) vs. share price of $100)?

Year	0	1	2	3	4	5	6
EPS_t	-	$BVPS_0 \times ROE_1$	$BVPS_1 \times ROE_2$	...	...	...	...
Investment	-	$Plowback_1 \times EPS_1$	$Plowback_2 \times EPS_2$	...	...	...	...
Dividend	-	$Payout_1 \times EPS_1$	$Payout_2 \times EPS_2$	...	...	...	...
$BVPS_t$	$BVPS_0 = 120$	$BVPS_0 + Investment_1$	$BVPS_1 + Investment_2$	...	...	...	...

Problem 4

Epsilon Ltd. is a young firm looking to grow. Its ROE is forecasted to be as follows:

Year	1	2	3	4	5	6	7+
ROE (%)	5	10	12	19	25	18	14

The company has $50 per share in book equity today and a cost of capital of 15%.

(a) Suppose that Epsilon has a constant payout ratio of 50%, what is its stock price today?

(b) What is its PVGO?

(c) Suppose that the company can change its payout policy starting in Year 7 and that the policy will have no impact on ROE. What is the optimal payout policy and the maximized stock price?

(d) (Excel, optional) Suppose that the company has full control over its payout policy and that the policy will have no impact on ROE. What is the optimal payout policy and the maximized stock price?

Michael Plasmeier 15.401 P-set 3 3/6

7.75/10

1. Alpha Corp
   $50/share trading
   Year 1 $3.6 earning $2 dividends
   5% growth

a) Cost of capital
   - the required return

$$r = \frac{D_1}{P_0} + g$$

$$= \frac{2}{50} + .05$$

$$= 9.0\%$$ ✓

b) PVGO?
   Present value of growth opportunity
   = PV(growth) - PV(no growth)

$$= \frac{P_1}{r-g} - \frac{E}{r}$$

$$= \frac{2}{.09-.05} - \frac{3.6}{.09}$$ ✓

$$= 10$$

(2)

c) ROE:

$$\frac{\text{earnings}}{\text{book value}} = \frac{3.6}{50}$$

$$= .072$$ ✓ −.5

2. Beta Inc.

dividend .50/share in 1 year
dividend ↑ .30 each year
20% cost of capital
↳ risk free "opportunity cost" + risk premium
Share price: - not easy since divident not percentage

$$PV = \frac{.50}{(1+.20)} + \frac{.80}{(1.2)^2} + \frac{1.1}{(1.2)^3} + \dots$$ ✓

$$= \sum_{t=0}^{\infty} \frac{.50 + .30(t-1)}{(1+.20)^t}$$

$$= 10.2$$

3. Gamma Inc
Paid divident today
$BVPS_0 = 120$
Payout ratio = 70%
ROE = 25%
) for $t = 1, \dots, 5$

(3)

So $EPS_1 = 120 \cdot .25 = 30$

$EPS_t = BVPS_{t-1} \cdot ROE_t$

At 5, payout ratio = 80%

C = 18% (cost of capital)

P_0 = 100 (stock price)

a) What is the long-run ROE? ($t > 5$)

Build a table

Year	t	0	1
EPS	$BVPS_{t-1} \cdot ROE_t$	—	
Investment	$Plowback_1 \cdot EPS_1$	—	
Dividend	$Payat_1 \cdot EPS_1$	—	
$BVPS_t$	$BVPS_{t-1}$ + Investment t	120	

See printout

Oh EPS formula is round about way of finding

But how find EPS if don't know ROE?

$ROE = \frac{\text{earnings}}{\text{book value}}$ ← Have

← But what is their earnings? Is it C, the cost of capital?
- well that is what we're asking
- do we back it out from price?

Spreadsheet a1

(4)

I am guessing Find ROE by backing it out from the price

So our dividends are $5 for each of the months (discounted)

(The numbers must have been chosen so it works out - how would you do that?)

Anyway $25 of price is for first 5 years.

So $75 is for other years

Can find long run dividend growth rate

$$g_6 = \frac{r - D_5/P_5}{1 + D_5/P_5}$$

(D5 must be discounted, no third is pre discounted)

$$= \frac{.12 - 12.44/75}{1 + 12.44/75}$$

$$= 2.928\%$$ ✓ -.5

dividend = earnings · payout

but growth the same, so EPS grows 2.928%/year

But this leads to a (2) price of $142.04 (sheet)

(5)

Try discounting

$$\frac{12.44}{(1+.2)^5} = 4.999$$ ← much closer to 5

$$\frac{75}{(1+.2)^5} = 30.14$$

$$g = \frac{.2 - 4.999 / 30.14}{1 + 4.999/30.14}$$

Spreadsheet az

$= 2.922$ ← no leads to exact same result!

This is not the correct approach!

But is there any growth?

- .85

What do I know

$$\frac{75}{(1+.20)^5} = \sum_{t=6}^{\infty} \frac{D}{(1+.20)^t}$$

← ? or do I want $\frac{75}{(1+.2)^5}$ for year 5?
- no think I have it

Solve for d

Wolfram alpha not working - guess + check

$D = 37.33$

Plowback ratio of 86% means EPS $\cdot .8 = 37.33$

EPS = 46.6625 So $ROE_6 = \frac{E}{BV} = \frac{46.6625}{298.60} = .1562$

(6)

But won't this fall since BV continues to increase?
Dividend will increase too
But at what rate? g

$$\frac{75}{(1+2)^5} = \frac{D_6}{.20-.0292} \qquad D_6 = 31.87$$

Or should earnings grow at same rate as BV - so ROE never changes

At this D_6 $EPS = \frac{31.87}{.8} = 39.83$

$ROE = \frac{39.83}{298.60} = .133$

So which is right? test in spreadsheet

ROE .1562 gives 113.85 share value
.133 98.63

spreadsheet a3

So real ans is in middle

From guess + check .1354 so likely second model which assumes dividend growth was correct, but rounding errors also truncated to 100 years

→ over

So my initial method did not work because I tried to shoehorn in a dividend growth rate from g directly, rather than allow it to build.

I could also have been building "naturally" on top of that, causing errors

a

	0	1	2	3	4	5	6	7	8	9	10	11
ROE		25%	25%	25%	25%	25%						
Payout		20%	20%	20%	20%	20%	80%	80%	80%	80%	80%	80%
Plowback		80%	80%	80%	80%	80%	20%	20%	20%	20%	20%	20%
Cost Capital		20%	20%	20%	20%	20%	20%	20%	20%	20%	20%	20%
EPS		30	36	43.2	51.84	62.208						
Investment		24	28.8	34.56	41.472	49.7664						
Dividend		6	7.2	8.64	10.368	12.4416						
BVPS	120	144	172.8	207.36	248.832	298.5984	298.5984	298.5984	298.5984	298.5984	298.5984	298.5984
Discounted Dividend		5	5	5	5	5	0	0	0	0	0	0

12	13	14	15	16	17	18	19	20	21	22	23	24
80%	80%	80%	80%	80%	80%	80%	80%	80%	80%	80%	80%	80%
20%	20%	20%	20%	20%	20%	20%	20%	20%	20%	20%	20%	20%
20%	20%	20%	20%	20%	20%	20%	20%	20%	20%	20%	20%	20%
298.5984	298.5984	298.5984	298.5984	298.5984	298.5984	298.5984	298.5984	298.5984	298.5984	298.5984	298.5984	298.5984
0	0	0	0	0	0	0	0	0	0	0	0	0

25	26	27	28	
80%	80%	80%	80%	
20%	20%	20%	20%	
20%	20%	20%	20%	
298.5984	298.5984	298.5984	298.5984	
0	0	0	0	25

a2

	0	1	2	3	4	5	6	7	8	9	10	11
ROE		25%	25%	25%	25%	25%						
Payout		20%	20%	20%	20%	20%	80%	80%	80%	80%	80%	80%
Plowback		80%	80%	80%	80%	80%	20%	20%	20%	20%	20%	20%
Cost Capital		20%	20%	20%	20%	20%	20%	20%	20%	20%	20%	20%
						EPS Growtl	2.93%	2.93%	2.93%	2.93%	2.93%	2.93%
EPS		30	36	43.2	51.84	62.208	64.02945	65.90423	67.83391	69.82009	71.86442	73.96861
Investment		24	28.8	34.56	41.472	49.7664	12.80589	13.18085	13.56678	13.96402	14.37288	14.79372
Dividend		6	7.2	8.64	10.368	12.4416	51.22356	52.72339	54.26713	55.85607	57.49153	59.17489
BVPS	120	144	172.8	207.36	248.832	298.5984	311.4043	324.5851	338.1519	352.1159	366.4888	381.2825
Discounted Dividend		5	5	5	5	5	17.15467	14.71413	12.6208	10.82528	9.285204	7.964229

12	13	14	15	16	17	18	19	20	21	22	23	24
80%	80%	80%	80%	80%	80%	80%	80%	80%	80%	80%	80%	80%
20%	20%	20%	20%	20%	20%	20%	20%	20%	20%	20%	20%	20%
20%	20%	20%	20%	20%	20%	20%	20%	20%	20%	20%	20%	20%
2.93%	2.93%	2.93%	2.93%	2.93%	2.93%	2.93%	2.93%	2.93%	2.93%	2.93%	2.93%	2.93%
76.13441	78.36362	80.65811	83.01978	85.4506	87.95259	90.52784	93.1785	95.90677	98.71492	101.6053	104.5803	107.6424
15.22688	15.67272	16.13162	16.60396	17.09012	17.59052	18.10557	18.6357	19.18135	19.74298	20.32106	20.91606	21.52848
60.90753	62.6909	64.52649	66.41582	68.36048	70.36207	72.42228	74.5428	76.72541	78.97193	81.28423	83.66423	86.11392
396.5094	412.1821	428.3138	444.9177	462.0078	479.5984	497.7039	516.3396	535.521	555.264	575.585	596.5011	618.0296
6.831184	5.859335	5.025747	4.31075	3.697474	3.171447	2.720256	2.333254	2.00131	1.71659	1.472377	1.262906	1.083237

25	26	27	28	
80%	80%	80%	80%	
20%	20%	20%	20%	
20%	20%	20%	20%	
2.93%	2.93%	2.93%	2.93%	
110.7942	114.0382	117.3773	120.8141	
22.15883	22.80765	23.47545	24.16281	
88.63534	91.23058	93.90181	96.65126	
640.1884	662.996	686.4715	710.6343	
0.929128	0.796944	0.683566	0.586317	142.0461

a3

	0	1	2	3	4	5	6	7	8	9	10	to 100
ROE		25%	25%	25%	25%	25%	13.54%	13.54%	13.54%	13.54%	13.54%	
Payout		20%	20%	20%	20%	20%	80%	80%	80%	80%	80%	
Plowback		80%	80%	80%	80%	80%	20%	20%	20%	20%	20%	
Cost Capital		20%	20%	20%	20%	20%	20%	20%	20%	20%	20%	
EPS		30	36	43.2	51.84	62.208	40.43022	41.52507	42.64957	43.80452	44.99075	
Investment		24	28.8	34.56	41.472	49.7664	8.086045	8.305015	8.529915	8.760905	8.99815	
Dividend		6	7.2	8.64	10.368	12.4416	32.34418	33.22006	34.11966	35.04362	35.9926	
BVPS	120	144	172.8	207.36	248.832	298.5984	306.6844	314.9895	323.5194	332.2803	341.2784	
Discounted Dividend		5	5	5	5	5	10.832	9.271109	7.935142	6.791688	5.813006	100.17

Other rows hidden

(7)

4. Epsilon

Year	1	2	3	4	5	6	7+
ROE	5	10	12	19	25	18	14

$BVOE_0 = 50$

$r = 15\%$

So I did this in Excel because I thought it needed a table, like the previous problem, Still, I only used arathmetic calculations, no pre built functions

Excel posted to stellar since 108 pgs long

a) Suppose payout = 50%. What is Price?

? Make another table

See printout

Stock price = discounted dividends

So for years 1-6 Sum 35.67

and ~~years 7+~~ $P_6 = \frac{D_7}{r-g}$

does not work, still growing extend table

Sum = 105.13 = stock price ✓

b) PVGO?

$P_0 = \frac{EPS}{r} + PVGO$

? So what is earnings w/ no growth like previous problem

(9)..

b) Suppose in absence of plowback

20% ROE $t=1,\dots,5$

8% ROE $t>5$

PVGO difference share price in b vs share price of \$100:

"So all money paid to shareholders"

$D = E = ROE \cdot BV = 120 \cdot .20 = 24$

$ROE = \frac{\text{earnings}}{BV}$ ↗

$P_0 = \frac{EPS}{r} = \frac{24}{.18} = 133.3$ ✗

"But this is with certain constant assumptions" perpetual

$PV = \sum_{t=1}^{\infty} \frac{24}{(1+.18)^t} = 133.33$ ✓ yeah perpetual assumption

"starting at $t=1$ So wrong

$PV = \sum_{t=1}^{5} \frac{24}{(1+.18)^t} + \sum_{t=6}^{\infty} \frac{9.6}{(1+.18)^t}$

↑ 120.08

since Book value never changes because nothing is reinvested

$= 75.05 + 23.31 = 98.36$ So PVGO $= 100 - 98.36 = 1.64$

(9)

EPS = BV · ROE = diff each year

$$P0_0 = \sum_{t=0}^{\infty} \frac{BV \cdot ROE_t}{(1+r)^t}$$ ↓ Stays same

$$= 109.47$$

-1

PVGO = 105.13 − 109.47 = −4.34

¿So actually worse?

×

c) Suppose company will readjust payout in year 7. (w/o changing ROE) What should it do to maximize stock price?

The best policy is to pay out all of the money for a price of 109.84

d) What is best payout policy?

Reinvesting all of the money in first 6 years then paying all of it at after year 7

- ¿Should have chosen year at a time to find best

MIT Sloan School of Management

Finance Theory I
Craig Stephenson

15.401
Spring 2011

Problem Set 3: Common Stocks

(Due: Monday, March 7th, in class)

Unless specified as an "Excel Problem", all of these problems can (and should) be solved with a pencil, paper and a simple calculator. Do not use Excel (except to check your work, if you want). Show your work cleanly and write out the formulas that you used to solve the problems. Circle your final answers.

Problem 1

Stocks of Alpha Corp. are trading at \$50 per share. Earnings and dividends are expected to be \$3.6 and \$2 per share next year, respectively; both will grow at 5% per year in subsequent years.

(a) What is the company's cost of capital?

Solution:

$$\$50 = \frac{\$2}{r - 5\%} \Rightarrow r = 9\%.$$

(b) What is the company's PVGO?

Solution:

$$\text{PVGO} = \$50 - \frac{\$3.6}{9\%} = \$10 \text{ per share}$$

(c) What is the company's ROE?

Solution:

$$9\% = \text{ROE} \times \left(1 - \frac{\$2}{\$3.6}\right) \Rightarrow \text{ROE} = 20.25\%$$

Problem 2

Beta Inc. is expected to pay a dividend of \$0.50 per share one year from now. Every year afterward, the company will raise dividend by 30 cents. If company has a cost of capital of 20%, what is its stock price today?

Solution:

$$P = \frac{0.5}{(1+0.2)} + \frac{0.5+0.3}{(1+0.2)^2} + \frac{0.5+0.3\times 2}{(1+0.2)^3} + \cdots$$

$$= \left(\frac{0.5}{1+0.2} + \frac{0.5}{(1+0.2)^2} + \cdots\right) + \left[\left(\frac{0.3}{(1+0.2)^2} + \frac{0.3}{(1+0.2)^3} + \cdots\right) + \left(\frac{0.3}{(1+0.2)^3} + \frac{0.3}{(1+0.2)^4} + \cdots\right) + \cdots\right]$$

$$=\left(\frac{0.5}{1+0.2}+\frac{0.5}{(1+0.2)^2}+\cdots\right)+\left[\frac{.3}{1+.02}\left(\frac{1}{1+0.2}+\frac{1}{(1+0.2)^2}+\cdots\right)+\frac{.3}{(1+.2)^2}\left(\frac{1}{1+0.2}+\frac{1}{(1+0.2)^2}+\cdots\right)+\cdots\right]$$

$$=\frac{0.5}{0.2}+\left[\frac{0.3}{0.2\times(1+.2)^1}+\frac{0.3}{0.2\times(1+0.2)^2}+\frac{0.3}{0.2\times(1+0.2)^3}+\cdots\right]$$

$$=2.5+\frac{0.3}{0.2\times 0.2}$$

$$=2.5+7.5$$

$$=\$10.00\text{pershare}$$

Another way to see this is to observe that we have from the perpetuity formula:

$$\sum_{n=1}^{\infty}\frac{1}{(1+r)^n}=\frac{1}{r}.$$

Taking the derivative of this expression with respect to r gives:

$$\sum_{n=1}^{\infty}n\frac{1}{(1+r)^{n+1}}=\frac{1}{r^2}.$$

From this it follows that:

$$7.5=\frac{.3}{.2^2}=\frac{.3}{(1+.2)^2}+\frac{0.3\times 2}{(1+.2)^3}++\frac{0.3\times 3}{(1+.2)^4}+\ldots$$

Problem 3

Gamma International has just paid a dividend today. The payout ratio and ROE have been 20% and 25%, respectively, and they will remain the same for the next five years (i.e., five dividend payments). After that, payout ratio will increase to 80% forever. Gamma has a cost of capital of 18% and a stock price of $100.

(a) What is the company's expected long-run ROE?

Solution: The first five years:

Year	Starting Equity	Earnings	Dividend	Plowback	Ending Equity	PV(DIV)
0	100.00	25.00	5.00	20.00	120.00	
1	120.00	30.00	6.00	24.00	144.00	5.08
2	144.00	36.00	7.20	28.80	172.80	5.17
3	172.80	43.20	8.64	34.56	207.36	5.26
4	207.36	51.84	10.37	41.47	248.83	5.35
5	248.83	62.21	12.44	49.77	298.60	5.44

PV(first five years) $=\$26.30$ per share

Let e be the long-run ROE, then

$$\$100 = \$26.30 + \frac{\$298.60 \times e \times 0.8}{0.18 - e \times 0.2} \times \frac{1}{1.18^5}.$$

Thus $e = 11.13\%$.

(b) Suppose that in absence of plowback, the company can only maintain a 20% ROE in the first five years and 8% per year afterwards. What is its PVGO?

Solution:

$$\text{PVGO} = \$100 - \left[\frac{\$20}{1.18} + \frac{\$20}{1.18^2} + \cdots + \frac{\$20}{1.18^5} + \frac{\$100 \times 0.08}{0.18} \times \frac{1}{1.18^5}\right] = \$18.03 \text{ per share}$$

This solution is driven by the \$100 base from year zero. With no plowback, the \$100 BVPS never increases; all earnings are paid as dividends. Earnings = dividends = \$20 (\$100 x 20%) for years 1 through 5, and earnings = dividends = \$8 (\$100 x 8%) in years 6 and beyond.

Problem 4

Epsilon Ltd. is a young firm looking to grow. Its ROE is forecasted to be as follows:

Year	1	2	3	4	5	6	7+
ROE (%)	5	10	12	19	25	18	14

The company has \$50 per share in book equity today and a cost of capital of 15%.

(a) Suppose that Epsilon has a constant payout ratio of 50%, what is its stock price today?

Solution: The first six years:

Year	Starting Equity	ROE	Earnings	Payout Ratio	Dividend	Plowback	Ending E	PV(DIV)
0							50.00	
1	50.00	0.05	2.50	0.50	1.25	1.25	51.25	1.09
2	51.25	0.10	5.13	0.50	2.56	2.56	53.81	1.94
3	53.81	0.12	6.46	0.50	3.23	3.23	57.04	2.12
4	57.04	0.19	10.84	0.50	5.42	5.42	62.46	3.10
5	62.46	0.25	15.62	0.50	7.81	7.81	70.27	3.88
6	70.27	0.18	12.65	0.50	6.32	6.32	76.59	2.73

PV(first six years) = \$14.86 per share

$$P = \$14.86 + \frac{\$76.59 \times 0.14 \times 0.5}{0.15 - 0.14 \times (1 - 0.5)} \frac{1}{1.15^6} = \$43.84 \text{ per share.}$$

(b) What is its PVGO?

Solution: The first six years without plowback:

Year	Starting E	ROE	Earnings	Payout Ratio	Dividend	Plowback	Ending E	PV(DIV)
0							50.00	
1	50.00	0.05	2.50	1.00	2.50	0.00	50.00	2.17
2	50.00	0.10	5.00	1.00	5.00	0.00	50.00	3.78
3	50.00	0.12	6.00	1.00	6.00	0.00	50.00	3.95
4	50.00	0.19	9.50	1.00	9.50	0.00	50.00	5.43
5	50.00	0.25	12.50	1.00	12.50	0.00	50.00	6.21
6	50.00	0.18	9.00	1.00	9.00	0.00	50.00	3.89

And so

$$\text{PV(first six years without plowback)} = \$25.44 \text{ per share}$$

and

$$\text{No} - \text{growth } P = \$25.44 + \frac{\$50 \times 0.14}{0.15}\frac{1}{1.15^6} = \$45.61 \text{ per share.}$$

And so $PVGO = \$43.84 - \$45.61 = -\$1.78$ per share

(c) Suppose that the company can change its payout policy starting in Year 7 and that the policy will have no impact on ROE. What is the optimal payout policy and the maximized stock price?

Solution: Since $ROE < r$ starting in Year 7, it is optimal to set the payout ratio to 100%. Then

$$P' = \$14.86 + \frac{\$76.59 \times 0.14}{0.15}\frac{1}{1.15^6} = \$45.77 \text{ per share.}$$

And so, $PVGO = \$0.16$ per share. If possible, Epsilon would in fact like to liquidate all assets and have a single dividend payment of \$76.59 at the end of year 7. This is because the market return for equivalently risky assets is higher that the ROE that Epsilon is able to generate.

(d) (Excel, optional) Suppose that the company has full control over its payout policy and that the policy will have no impact on ROE. What is the optimal payout policy and the maximized stock price?

Solution: The optimal payout policy starting in Year 7 is the same as in (3). The optimal policy in the first six years is as follows:

Year	Starting E	ROE	Earnings	Payout Ratio	Dividend	Plowback	Ending E	PV(DIV)
0							50.00	
1	50.00	0.05	2.50	0.00	0.00	2.50	52.50	0.00
2	52.50	0.10	5.25	0.00	0.00	5.25	57.75	0.00
3	57.75	0.12	6.93	0.00	0.00	6.93	64.68	0.00

4	64.68	0.19	12.29	0.00	0.00	12.29	76.97	0.00
5	76.97	0.25	19.24	1.00	19.24	0.00	76.97	9.57
6	76.97	0.18	13.85	1.00	13.85	0.00	76.97	5.99

$$P^* = \$25.44 + \frac{\$76.97 \times 0.14}{0.15} \frac{1}{1.15^6} = \$46.61 \text{ per share}$$

Here we are assuming that from Year 7 the firm cannot liquiidate Notice that the first is re-investing the earnings even in the beginning when the ROE is less than the cost of capital. The reason is that the firm would like to have as many assets in place in years 5—6 to take advantage of the high ROE. Another potential option would be to pay out dividends early on but to raise capital when the ROE is high by issuing securities.

P-Set 3 help

Michael E Plasmeier

From: Craig Allen Stephenson <stephenc@MIT.EDU>
Sent: Sunday, March 06, 2011 5:45 PM
To: Michael E Plasmeier
Subject: Re: FW: [15.401D] Section D Problem Set 3

Hello Micheal

Yes, you know what the Year 5 End BVPS equals, so you need to determine the ROE for years 6 and beyond. This will give you the dividend in year 6 and beyond, allowing you to calculate the value of dividends 6 through infinity, as of year 5.

You know the current stock price of $100. You have calculated the dividends in years 1 through 5, so you can determine their present value. $100 minus this present value gives the the amount of the stock price which has to come from dividends 6 through infinity.

And remember dividends 6 through infinity give you the stock price in year 5, not year zero.

Good luck

Craig Stephenson

Quoting Michael E Plasmeier <theplaz@MIT.EDU>:

> I just realized that was the actual question we were supposed to
> answer. Are supposed to back it out the ROE t>6 from the price?
>
> -Michael
>
> From: Michael E Plasmeier
> Sent: Sunday, March 06, 2011 1:33 PM
> To: 'Craig Stephenson'
> Subject: RE: [15.401D] Section D Problem Set 3
>
> Hello,
>
> I am working on #3, but I still have a question. How do we know what
> their earnings are in year 6, if we do not have ROE? I saw somewhere
> in the notes that ROE = r (cost of capital). When is that true?
>
> -Michael
>
> From: Craig Stephenson [mailto:stephenc@MIT.EDU]
> Sent: Friday, March 04, 2011 6:02 PM
> To: Craig Allen Stephenson
> Subject: [15.401D] Section D Problem Set 3
>
>
> Problem set 3 for section D has been modified to help you in solving
> problem 3 in the set, and has been posted to Stellar. The problem
> istelf has not changed, but the wording has been modified to be more

> clear and give you additional guidance. No students had submitted
> their completed PS 3 in the original assignment, so this assignment
> was deleted from Stellar; you'll only see the modified problem set now
> for posting your assignment.
>
> Craig Stephenson
>
>
>

MITSloan MANAGEMENT

15.401 – Section D
Midterm Exam Review

Monday, March 7th, 2011

Agenda

- Exam logistics
- Practice questions
- Concepts

2

Exam logistics

- Exam takes place during our regularly scheduled class time (8:30 to 10:00) and place (E51-376)
- 1 8.5-11 inch formula sheet, prepared by you (can be written or typed)
- 1 calculator (graphing calculator is okay, just don't program anything in it)
- Covers everything in the course through common stocks
- Format: Some multiple choice questions, some problems

3

Best practices

- Arrive on time
- Make life easy on graders
  - Write clearly and legibly
  - Circle final answer
  - Use one area for thought process / scratch, then write the short, clean version with simple formulas and answer under the question
- For multiple choice questions, explanation is more important than answer
- State your assumptions - this doesn't mean we will accept them, but it can only help :-)
- If multiple interpretations are possible, err on the side of the simplest one
- Don't spend all your time on one problem. Make sure you get something down (even basic formulas) for everything. 80 minutes isn't very long.
- If you have time, double check your calculations. We will try to give partial credit when possible... but do you know what's better than partial credit? Full credit.

4

Agenda

- Exam logistics
- Practice questions
- Concepts

5

Practice Question 1(a)

1a) TRUE, FALSE or IT DEPENDS. The cost of capital of a firm decreases with the amount of cash it has in hand.

FALSE. The cost of capital is the expected return that is foregone by investing in a project rather than investing in comparable securities. Absent market frictions and capital constraints, the cost of capital does not depend on the amount of cash that a firm has on hand.

6

Practice Question 1(b)

1b) TRUE, FALSE, or IT DEPENDS. Bonds with higher coupon rates have more interest rate risk.

FALSE. Keeping everything else constant, bonds with higher coupon rates have lower duration and lower interest rate risk. Bonds with higher coupons have lower duration because a larger share of the cash flow happens sooner:

$$D=\frac{1}{P}\left[\sum_{i=1}^{i=T}\left(\frac{C_i}{(1+y)^i}*i\right)+\frac{FV}{(1+y)^T}*T\right]$$

7

Practice Question 2

2) Your car dealer offers you a loan for part of the purchase price of a new car, citing an annual percentage rate (APR) of 8.5%. What is the effective annual rate of such a loan (recall that an auto loan typically requires monthly payments)

$$r_{EAR}=\left(1+\frac{r_{APR}}{N}\right)^N-1$$

$$r_{EAR}=\left(1+\frac{8.50\%}{12}\right)^{12}-1=8.84\%$$

8

Practice Question 3

3) The Wall Street Journal gives the following prices for STRIPS (with a principal of 100):

Bond	Maturity Year	Price
A	1	95.92
B	2	92.01
C	3	87.00

(a) Determine the 1-, 2-, and 3-year spot interest rates

$$(1+r_T)^T=\frac{FV}{P}$$

$$r_1=\frac{100}{95.92}-1=4.25\%$$

$$r_2=\left(\frac{100}{92.01}\right)^{\frac{1}{2}}-1=4.25\%$$

$$r_3=\left(\frac{100}{87.00}\right)^{\frac{1}{3}}-1=4.75\%$$

9

Practice Question 3(b)

3b) Compute the annual forward rate from year 2 to year 3

$$(1+r_2)^2(1+r_{23})=(1+r_3)^3$$

$$r_{23}=\frac{(1+r_3)^3}{(1+r_2)^2}-1$$

$$r_{23}=\frac{(1+4.75\%)^3}{(1+4.25\%)^2}-1=5.76\%$$

10

Practice Question 3(b)

3b) Suppose a 2-year coupon bond with a principal of 100 and annual coupon payments of 4.25% is traded at $100.25. Is there an arbitrage opportunity?

What is a fair price for this bond given the market spot rates?

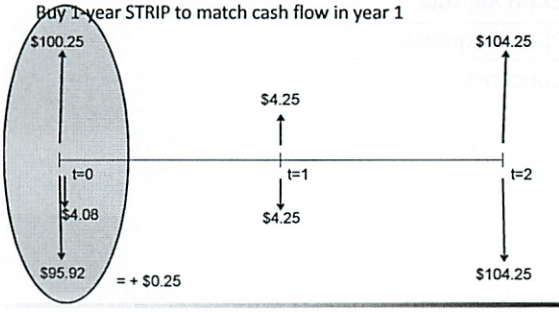
$$P=\frac{4.25}{1+r_1}+\frac{104.25}{(1+r_2)^2}=\frac{4.25}{1+4.25\%}+\frac{104.25}{(1+4.25\%)^2}=100$$

The 2-year bond is too expensive at $100.25, so we should sell it.

11

Practice Question 3(b)

Sell the 2-year coupon bond because it is too expensive
Buy 2-year STRIP to match cash flow in year 2
Buy 1-year STRIP to match cash flow in year 1



12

Practice Question 4(a)

4) You have a leasing liability of $2 million a year for 3 years. The first payment is exactly one year from now. The term structure is currently flat at 5%.

a) Compute the present value of your leasing liability

$$PV = \frac{\$2m}{.05}\left(1 - \frac{1}{(1+.05)^3}\right) = \$5{,}446{,}496$$

13

Practice Question 4(b)

4) You have a leasing liability of $2 million a year for 3 years. The first payment is exactly one year from now. The term structure is currently flat at 5%.

b) Compute the duration and modified duration of the liability

$$D = \frac{1}{5.4464}\left(\frac{2}{1.05}\times 1 + \frac{2}{1.05^2}\times 2 + \frac{2}{1.05^3}\times 3\right) = 1.9675$$

$$D^* = \frac{D}{1+y} = \frac{1.9675}{1.05} = 1.8738$$

14

Practice Question 4(c)

4) You have a leasing liability of $2 million a year for 3 years. The first payment is exactly one year from now. The term structure is currently flat at 5%.

c) Suppose that the interest rate goes down by 0.1%. How does the value of the liability change?

$$\Delta P = -P \times D^* \times \Delta y$$

$$\Delta P = -\$5{,}446{,}496 \times 1.8738 \times (-0.001) = \$10{,}205.62$$

15

Practice Question 5

5) SW Co. expects earnings of $1.25 per share next year, out of which $0.50 will be paid out as dividends. Earnings and dividends are expected to grow at the same constant rate g each year afterwards. SW shares are now traded at $20. The cost of capital for SW Co. is 10%.

a) What is the expected growth rate of earnings, *g*?

$$P_0 = \frac{D_1}{r-g}$$

$$\$20 = \frac{\$0.50}{10\% - g}$$

$$g = 7.5\%$$

16

Practice Question 5(b,c)

b) What is the ROE for SW Co?

$$p = \frac{D_1}{EPS_1} = \frac{\$0.50}{\$1.25} = 40\%$$

$$g = ROE \times (1-p) = ROE \times b$$

$$ROE = \frac{g}{b} = \frac{7.5\%}{60\%} = 12.50\%$$

b) Is SW Co a growth company?

Yes. It has a return on equity (12.50%) greater than its cost of capital (10%)

17

Agenda

- Exam logistics
- Practice Questions
- Concepts

18

Quick Review

- Present value/Discounting/Compounding
  - Formulas for:
    - Perpetuity with/without growth
    - Annuity with/without growth
  - Interest rates
    - APR versus EAR
    - Inflation
    - Forward rates
  - Expectation hypothesis (EH)
    - Forward rate = expected future spot rates? No

19

Quick Review

- Bonds
  - Bond pricing formula (DCF)
  - Yield-to-maturity
  - Duration
  - Modified Duration
  - Portfolio immunization

20

Quick Review

- Stocks
  - Valuation (PV of expected dividends)
  - Terminology:
    - Plowback/payout ratio
    - EPS
    - P/E ratio – Earnings yield
    - PVGO
    - Growth stock (ROE > r)
  - Gordon Model (assumptions)
  - ...

21

MITSloan MANAGEMENT

22

15.401

(5 min late)
Exam Wed
1 formula sheet
1 calculator
Show your work
State your assumptions
Err on side of simple
Show which answer you want if you wrote multiple

Problems

1. Does cost of capital change w/ cash on hand?
   No - only depends on biz model
   About your risk
   Volitality of biz itself
2. Bonds w/ ? coupon rates have more interest rate risk.
   Fake - its actually the other way around.
   You get your money sooner
   (Remember diff b/w interest rate + inflation)
   If think interest rates will ↑, go short-term
   ↓ long-term

(2)

3. Find EAR from APR

$$r_{EAR} = \left(1 + \frac{r_{APR}}{N}\right)^N - 1$$

What they are actually charging $\frac{8.5}{12}$ each month

4. Stripes are 1 cash flow

Can find spot interest rates

$$r_1 = \frac{100}{95.92} - 1 = 4.25$$

$$r_2 = \left(\frac{100}{92.01}\right)^{1/2} - 1 = 4.25$$

Also forward rates

$$(1+r_2)^2 \; (1+r_{23}) = (1+r_3)^3$$

↑ does not matter that $r_1 = r_2$ in this case

↑ this is what ya are looking for

Is geometric avg of previous rates

Then watch compounding

Actually - STrip - no coupons

$(92.01) \cdot (1+i)^2 = 100$ for 2 year

If coupons paid semiannual

- a) think of it in terms of periods - divide interest rate!
- b) have fractions there - 30 for year count

3)

5. Abitrage opportunity?

Calculate PV by discounting cash flow

If this is under the price the bond is too expensive
↑ the "fair price" ↑ listed

So buy ~~a~~ 2 strips and sell the bond
(study slides more)

6. Compute PV of your liability

- annuity
- simple calculation

7. Calculate Duration

- another simple Calculation
(Review formula/ put on sheet)
- Modified duration lets you estimate small changes in interest rate

$$\Delta P = -P \cdot D^* \cdot \Delta y$$

↑ original ↑ duration (-.0001)
"a .1% drop in rates

- ~~Be~~ If want interest rates ↑ rich can ~~go~~ be long
If short - minimizing price rich
Immunize assets to liabilities

(4)

8. Earnings + Dividends — read carefully!
   - don't start till next year!
   - Solving for g = growth rate of earnings
   - ~~Solving~~ What the needed cost of capital must be
   - Finding ROE $= \frac{g}{\text{Plowback}}$
     - did not do on p-set
   - If ROE < Cost of Capital, pay a dividend of everything
     - liquidate the company, pay it out
     - no one does

(get more intuitive on this)

No forwards + futures this year

PV / Discounting / Compounding

: whole list on slides

Simple example of immunization

Calculating PVGO for apple

$\$360 = \frac{29.97}{.09} + PVGO$ ← earnings projection

- $360: stock price
- .09: later in class, capital asset pricing model

(5)

PVGO = $105

Perpetual Growth model

- Only when growth is perpetual and constant

Can come early to start early

Grades are only for section D

$P_0 = \frac{D_1}{r-g} = \frac{D_0(1+g)}{r-g}$ Gordon Dividend Growth model

- must be constant + perpetual

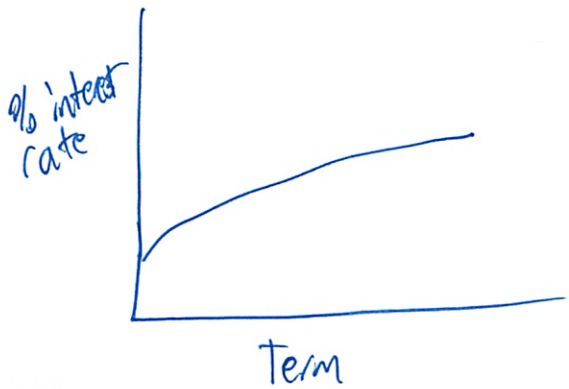
If g varies w/ time

$P_6 = \frac{Div_7}{.13-.06}$

Then discount this

Then get a price

Modify growth rate to see what it does to stock price



(6)

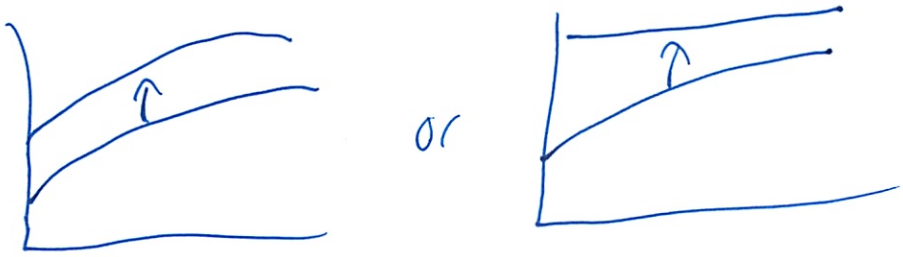
If were CFO financing asset, duration = 10

Borrow 7 day commercial paper

Now rates are almost 0

If interest rates go ↑

- inflation from oil + food



Taking a big risk if interest rates go up