15.401 Recitation

6: Portfolio Choice

Learning Objectives

- Review of Concepts
  - Portfolio basics
  - Efficient frontier
  - Capital market line
- Examples
  - XYZ
  - Diversification
  - Sharpe ratio
  - Efficient frontier

Review: portfolio basics

- A portfolio is a collection of N assets ($A_1, A_2, \ldots, A_N$) with weights ($w_1, w_2, \ldots, w_N$) that satisfy
  - $\sum_{i=1}^{N} w_i = 1$
- Each asset A_i has the following characteristics:
  - Return: $\tilde{r}_i$ (random variable)
  - Mean return: $\bar{r}_i$
  - Variance and std. dev. of return: σ_i^2, σ_i
  - Covariance with A_j: σ_{ij}

Review: portfolio basics

- The return of a portfolio is

$$\tilde{r}_p = \sum_{i=1}^{N} w_i \tilde{r}_i$$

- The mean/expected return of a portfolio is

$$E(r_p) = \bar{r}_p = \sum_{i=1}^{N} w_i \bar{r}_i$$

- The variance of a portfolio is

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij}; \quad \sigma_p = \sqrt{\sigma_p^2}$$

- Note: $\sigma_{ii} \equiv \sigma_i^2$; $\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$

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Example 1: XYZ

	E(r)	Variance-Covariance		
		X	Y	Z
X	15%	0.090	0.125	0.144
Y	10%		0.040	-0.036
Z	20%			0.625

- What is the expected return and variance of a portfolio of …
  a. (X, Y) with weights (0.4, 0.6)?
  b. (X, Y, Z) with weights (0.2, 0.5, 0.3)?
  c. (X, Y, Z) with weights (1/3, 1/3, 1/3)?

Example 1: XYZ

- Answer:
  a. $E(r_p) = 12\%; \sigma_p^2 = 0.08880; \sigma_p = 29.80\%$
  b. $E(r_p) = 14\%; \sigma_p^2 = 0.10133; \sigma_p = 31.83\%$
  c. $E(r_p) = 15\%; \sigma_p^2 = 0.13567; \sigma_p = 36.83\%$

Example 1: XYZ

- What is the minimum possible variance of a portfolio with only Y and Z?

	E(r)	Variance-Covariance		
		X	Y	Z
X	15%	0.090	0.125	0.144
Y	10%		0.040	-0.036
Z	20%			0.625

Example 1: XYZ

- Answer:
  Let (w, 1–w) be the weights for (Y, Z), then
  $$\arg\min_{w} \left[w^2 \cdot 0.04 + 2w(1-w)(-0.036) + (1-w)^2 \cdot 0.625 \right]$$
- First-order condition:
  $$2w \cdot 0.04 + 2(1-2w)(-0.036) - 2(1-w) \cdot 0.625 = 0$$
  $$w^* = 0.8969$$
- The minimum variance portfolio is
  $$(0.8969, 0.1031)$$

Example 2: diversification

- Suppose that your portfolio consists of *N* equally weighted identical assets in the market, each of which has the following properties:
  - Mean = 15%
  - Std dev = 20%
  - Covariance with any other asset = 0.01
- What is the expected return and std dev of return of your portfolio if…
  - *N* = 2?
  - *N* = 5?
  - *N* = 10?
  - *N* = ∞?

Example 2: diversification

- Answer:
  - Expected return

$$E(r_p)=\sum_{i=1}^{N}\frac{1}{N}\cdot 0.15=0.15$$

  - Variance

$$\sigma(r_p)=\sum_{i=1}^{N}\frac{0.2^2}{N^2}+\sum_{i=1}^{N}\sum_{j\neq i}\frac{0.01}{N^2}=N\left(\frac{0.2^2}{N^2}\right)+N(N-1)\frac{0.01}{N^2}$$

$$=\frac{0.04}{N}+\left(1-\frac{1}{N}\right)0.01=0.01+\frac{0.03}{N}$$

Example 2: diversification

- Answer:
  - *N* = 2:

$$E(r_p)=15\%;\sigma_p^2=0.0250;\sigma_p=15.81\%$$

  - *N* = 5:

$$E(r_p)=15\%;\sigma_p^2=0.0160;\sigma_p=12.65\%$$

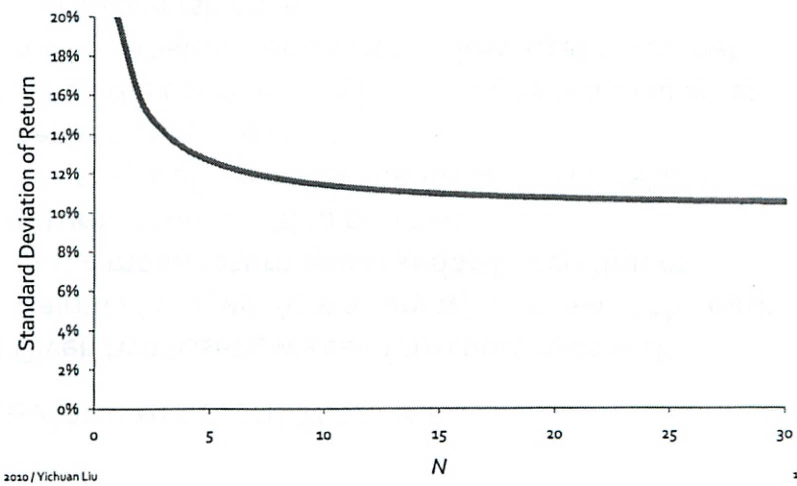
  - *N* = 10:

$$E(r_p)=15\%;\sigma_p^2=0.0130;\sigma_p=11.40\%$$

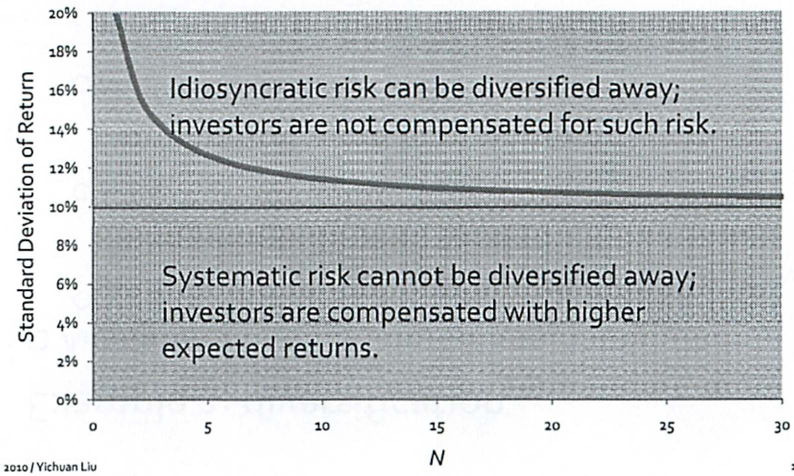
  - *N* = ∞:

$$E(r_p)=15\%;\sigma_p^2=0.0100;\sigma_p=10.00\%$$

Example 2: diversification



Review: diversification

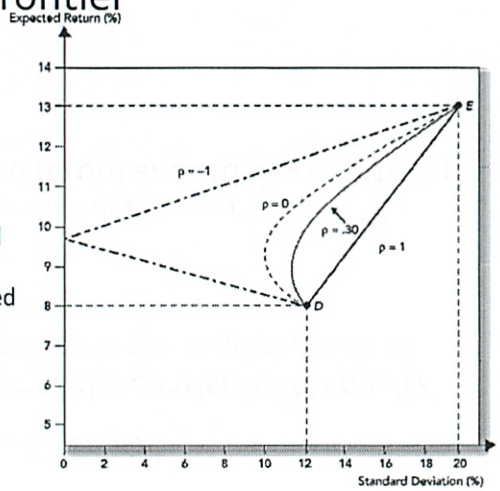


Review: efficient frontier

- Given two assets, we can form portfolios with weights (w, $1-w$). As we vary w, we can plot the **path** of the **mean return** and **standard deviation of return** of the resulting portfolio.
- The shape of the path depends on the correlation between the two assets.
- When the correlation is low, a large portion of asset return variation comes from idiosyncratic risk that can be diversified away.

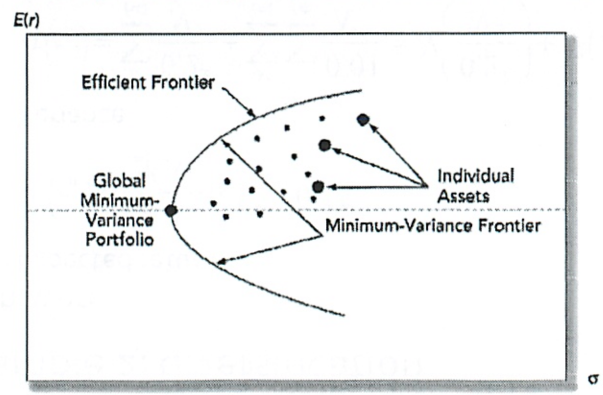
Review: efficient frontier

- $\rho = 1$
  perfectly correlated
  no risk reduction potential
- $-1 < \rho < 1$
  imperfectly correlated
  some risk reduction potential
- $\rho = -1$
  perfectly negatively correlated
  most risk reduction potential



Review: efficient frontier

- We can repeat the previous exercise for N assets:



Review: efficient frontier

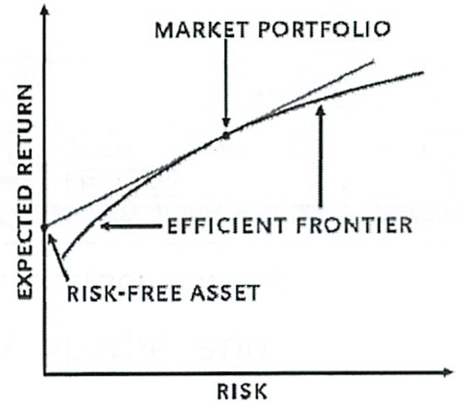
- The efficient frontier can be described by a function $\sigma^*(r_p)$, which minimizes the portfolio std dev given an expected return:

$$\sigma^*(r_p) \equiv \min_{\{w_i\}} \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}} \quad \text{s.t.} \quad \begin{cases} \sum_{i=1}^{N} w_i = 1 \\ \sum_{i=1}^{N} w_i \bar{r}_i = r_p \end{cases}$$

- Analytical solution for $\sigma^*(r_p)$ is possible but difficult to derive.

Review: capital market line

- Efficient frontier + risk-free asset = CML



Example 3: Sharpe ratio

- The Sharpe ratio measures the reward-risk tradeoff of an asset or a portfolio. It is defined as

$$S = \frac{\bar{r} - r_f}{\sigma}$$

- The higher Sharpe ratio, the more desirable an asset / a portfolio is. Suppose $r_f = 5\%$. What is the portfolio of (A, B) with the highest Sharpe ratio?

	E(r)	COV-VAR	
		A	B
A	15%	0.090	0.015
B	10%		0.040

Example 3: Sharpe ratio

- Answer:

$$\max_w S_p \equiv \max_w \frac{w r_A + (1-w) r_B - r_f}{\sqrt{w^2 \sigma_A^2 + 2w(1-w)\sigma_{AB} + (1-w)^2 \sigma_B^2}}$$

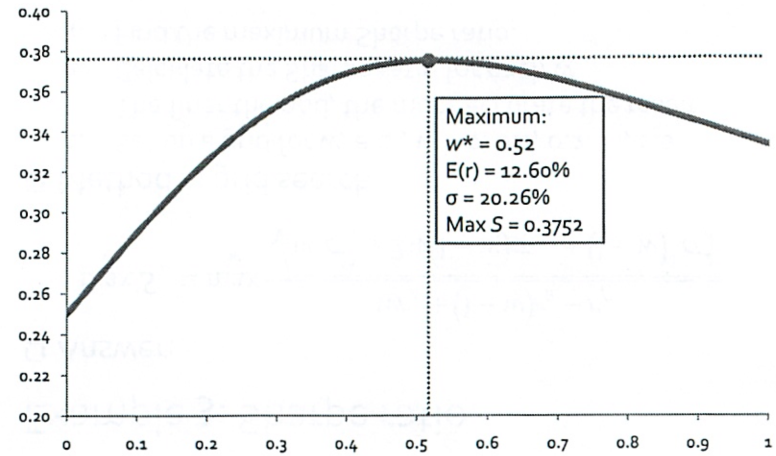
- Method 1: grid search
  1. Set up a grid for w, e.g., w = 0, 0.1, 0.2, ..., 1.0 The finer the grid, the more accurate the result
  2. Calculate the Sharpe ratio for each w
  3. Find the maximum Sharpe ratio.

Example 3: Sharpe ratio

❑ Method 1: grid search

w	$1-w$	$r_p - r_f$	σ_p	S_p
0	1	0.0500	0.2000	0.2500
0.1	0.9	0.0550	0.1897	0.2899
0.2	0.8	0.0600	0.1844	0.3254
0.3	0.7	0.0650	0.1844	0.3525
0.4	0.6	0.0700	0.1897	0.3689
0.5	**0.5**	**0.0750**	**0.2000**	**0.3750**
0.6	0.4	0.0800	0.2145	0.3730
0.7	0.3	0.0850	0.2324	0.3658
0.8	0.2	0.0900	0.2530	0.3558
0.9	0.1	0.0950	0.2757	0.3446
1	0	0.1000	0.3000	0.3333

Example 3: Sharpe ratio



Example 3: Sharpe ratio

❑ Method 2: Excel Solver

	A	B	C	D	E
1			E(r)	Asset A	Asset B
2				=B3	=B4
3	Asset A		0.15	0.09	0.015
4	Asset B	=1−B3	0.1	0.015	0.04
5					
6			$r_p - r_f$	σ_p	S
7			=f	=g	=C7/D7

f: SUMPRODUCT(B3:B4, C3:C4) − 0.05
g: SQRT(B3*D2*D3+B3*E2*E3+B4*D2*D4+B4*E2*E4)

Solver

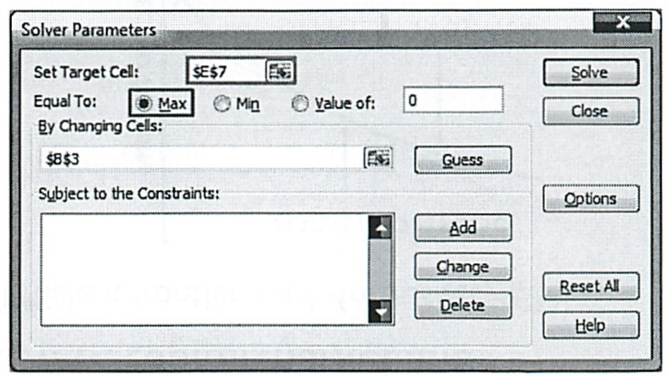
Set Target Cell:
E7

Equal To:
Max

By Changing Cell:
B3

Example 3: Sharpe ratio

❑ Method 2: Excel Solver dialog



Example 3: Sharpe ratio

- Method 2: Excel Solver

	A	B	C	D	E
1			E(r)	Asset A	Asset B
2				0.52	0.48
3	Asset A	0.52	0.15	0.09	0.015
4	Asset B	0.48	0.1	0.015	0.04
5					
6			$r_p - r_f$	σ_p	S
7			0.076	0.202583	0.375154

Example 3: Sharpe ratio

- Method 3: analytical solution
  - Full derivation:

$$\frac{\partial S}{\partial w} = \frac{(\bar{r}_A - \bar{r}_B)(\sigma_p^2)^{\frac{1}{2}} - \frac{1}{2}(\sigma_p^2)^{-\frac{1}{2}}(2w\sigma_A^2 + 2(1-2w)\sigma_{AB} - 2(1-w)\sigma_B^2)(\bar{r}_p - r_f)}{(\sigma_p^2)^{\frac{1}{2}}}$$

$$= \frac{(\bar{r}_A - \bar{r}_B)(w^2\sigma_A^2 + 2w(1-w)\sigma_{AB} + (1-w)^2\sigma_B^2) - (w\sigma_A^2 + (1-2w)\sigma_{AB} - (1-w)\sigma_B^2)(w\bar{r}_A + (1-w)\bar{r}_B - r_f)}{\sigma_p^2}$$

$$= 0$$

$$0 = (\bar{r}_A - \bar{r}_B)(w^2\sigma_A^2 + 2w(1-w)\sigma_{AB} + (1-w)^2\sigma_B^2) - (w\sigma_A^2 + (1-2w)\sigma_{AB} - (1-w)\sigma_B^2)(w\bar{r}_A + (1-w)\bar{r}_B - r_f)$$

$$= (\bar{r}_A - \bar{r}_B)(w^2\sigma_A^2 + 2w(1-w)\sigma_{AB} + (1-w)^2\sigma_B^2) - (w\sigma_A^2 + (1-2w)\sigma_{AB} - (1-w)\sigma_B^2)(w(\bar{r}_A - \bar{r}_B) + \bar{r}_B - r_f)$$

$$= (\bar{r}_A - \bar{r}_B)(w\sigma_{AB} + (1-w)\sigma_B^2) - (w\sigma_A^2 + (1-2w)\sigma_{AB} - (1-w)\sigma_B^2)(\bar{r}_B - r_f)$$

$$= [(\bar{r}_A - \bar{r}_B)\sigma_B^2 - (\sigma_{AB} - \sigma_B^2)(\bar{r}_B - r_f)] - [(\bar{r}_A - \bar{r}_B)(\sigma_B^2 - \sigma_{AB}) + (\sigma_A^2 - 2\sigma_{AB} + \sigma_B^2)(\bar{r}_B - r_f)]w$$

$$= [(\bar{r}_A - r_f)\sigma_B^2 - (\bar{r}_B - r_f)\sigma_{AB}] + [(\bar{r}_A - r_f)(\sigma_B^2 - \sigma_{AB}) + (\bar{r}_B - r_f)(\sigma_A^2 - \sigma_{AB})]w$$

$$w^* = \frac{(\bar{r}_A - r_f)\sigma_B^2 - (\bar{r}_B - r_f)\sigma_{AB}}{(\bar{r}_A - r_f)(\sigma_B^2 - \sigma_{AB}) + (\bar{r}_B - r_f)(\sigma_A^2 - \sigma_{AB})}$$

$$= 0.52$$

Example 3: Sharpe ratio

- Method 3: analytical solution
  - Result only:

    The general solution for the 2-asset Sharpe ratio maximization problem is

$$w^* = \frac{(\bar{r}_A - r_f)\sigma_B^2 - (\bar{r}_B - r_f)\sigma_{AB}}{(\bar{r}_A - r_f)(\sigma_B^2 - \sigma_{AB}) + (\bar{r}_B - r_f)(\sigma_A^2 - \sigma_{AB})}$$

Example 4: efficient frontier

- Given the risky assets A and B in the previous question, what is the efficient frontier?

	E(r)	COV-VAR	
		A	B
A	15%	0.090	0.015
B	10%		0.040

- Given 5% risk-free rate, what is the capital market line?

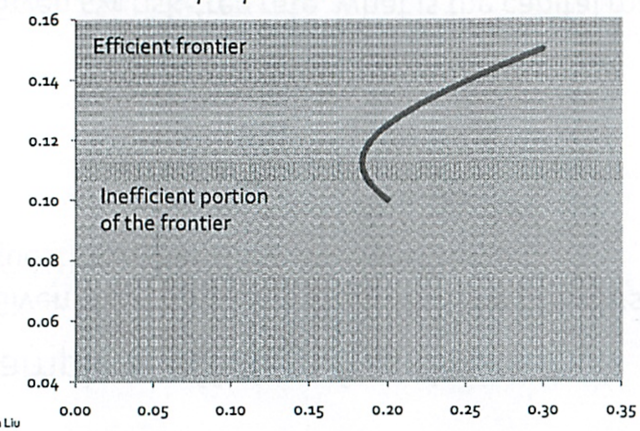
Example 4: efficient frontier

❑ Table from the previous question:

w	$1-w$	r_p	σ_p
0	1	0.1000	0.2000
0.1	0.9	0.1050	0.1897
0.2	0.8	0.1100	0.1844
0.3	0.7	0.1150	0.1844
0.4	0.6	0.1200	0.1897
0.5	0.5	0.1250	0.2000
0.6	0.4	0.1300	0.2145
0.7	0.3	0.1350	0.2324
0.8	0.2	0.1400	0.2530
0.9	0.1	0.1450	0.2757
1	0	0.1500	0.3000

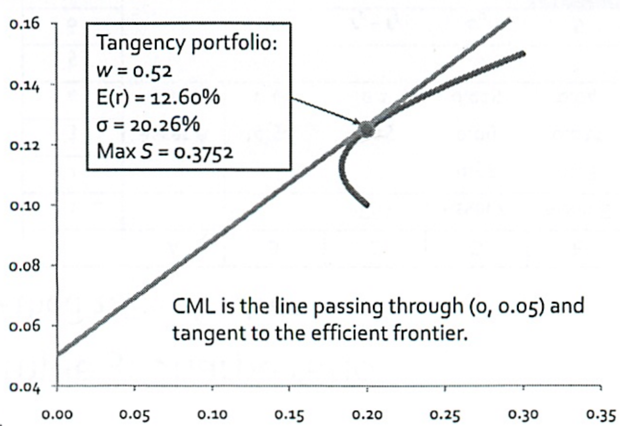
Example 4: efficient frontier

❑ Scatter plot of (r_p, σ_p) pairs:



Example 4: efficient frontier

❑ Capital market line:



Example 4: efficient frontier

❑ The moral of the story:

- The CML is tangent to the efficient frontier at the **tangency portfolio.**
- The tangency portfolio is the portfolio of risky assets that **maximizes the Sharpe ratio.**
- The slope of the CML is the maximum Sharpe ratio.
- Rational investors always hold **a combination of the tangency portfolio and the risk-free asset.** The proportion depends on investors' risk preferences.

Sneak Peak: CAPM

- The **tangency portfolio** is the **market portfolio.**
- An asset's **systematic risk** is measured by **beta,** which is defined as the **correlation** of its return and the market return, normalized by the variance of market return :

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2}$$

- Since investors are only compensated for **systematic risk,** asset return is an increasing function of beta:

$$E(\tilde{r}_i) = r_f + \beta_i(\tilde{r}_i - r_f)$$

15.401 Finance Theory I

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Lecture 9: Capital Asset Pricing Model (CAPM)

- The market portfolio
- Derivation of CAPM
- Implications of CAPM
- Understanding risk and return in CAPM
- Empirical tests of CAPM
- Extensions of CAPM

Readings:

- Brealey, Myers and Allen, Chapter 7, 8
- Bodie, Kane and Markus, Chapter 9

Portfolio theory analyzes investors' asset demand given asset returns.

1. Diversify to eliminate non-systematic risk.
2. Hold only the risk-free asset and the tangent portfolio.

How does investors' asset demand determine the relation between assets' risk and return in a market equilibrium?

A model to price risky assets:

$$E[r_i] = ?$$

The market portfolio is the portfolio of <u>all</u> risky assets traded in the market.

A total of n risky assets. The market capitalization of asset 'i' is

$$\text{MCAP}_i = (\text{price per share})_i \times (\#\text{of shares outstanding})_i$$

The total market capitalization of all risky assets is

$$MCAP_M = \sum_{i=1}^{n} MCAP_i$$

The market portfolio has the following portfolio weights:

$$w_i = \frac{\text{MCAP}_i}{\sum_{j=1}^{n} \text{MCAP}_j} = \frac{\text{MCAP}_i}{\text{MCAP}_M}$$

We denote the market portfolio by w_M

Starting point:

- Investors agree on the distribution of asset returns
- Investors hold efficient frontier portfolios
- There is a risk-free asset:
  - paying interest rate r_F
  - in zero net supply
- Demand of assets equals supply in equilibrium

Implications:

1. Every investor puts their money into two baskets:
   - the riskless asset
   - A **single** portfolio of risky assets, the tangent portfolio
2. All investors hold the risky assets in same proportions
   - they hold the same risky portfolio, the tangent portfolio
3. **The tangent portfolio is the market portfolio!**

In equilibrium, total asset holdings of all investors must equal the total supply of assets.

Example. There are only three risky assets, A, B and C. Suppose that the tangent portfolio is

$$\mathrm{w}_T = (w_A, w_B, w_C) = (0.25, 0.50, 0.25)$$

There are only three investors in the economy, 1, 2 and 3, with total wealth of 500, 1000, 1500 billion dollars, respectively. Their asset holdings (in billions of dollars) are:

Investor	Riskless	A	B	C
1	100	100	200	100
2	200	200	400	200
3	-300	450	900	450
Total	0	750	1500	750

In equilibrium, the total dollar holding of each asset must equal its market value:

Market capitalization of A = $750 billion
Market capitalization of B = $1500 billion
Market capitalization of C = $750 billion

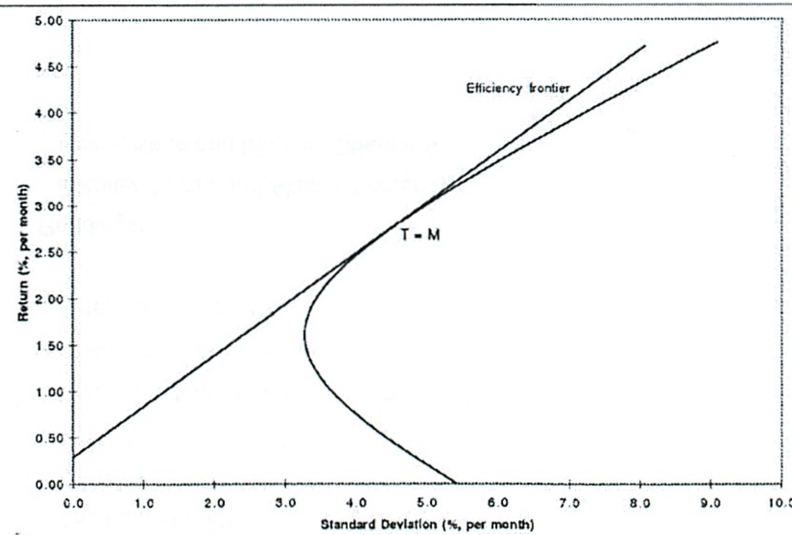
The total market capitalization is

$750 + $1500 + $750 = $3,000 billion

The market portfolio is the tangent portfolio:

$$\mathrm{w}_M = \left(\frac{750}{3000}, \frac{1500}{3000}, \frac{750}{3000}\right) = (0.25, 0.50, 0.25) = \mathrm{w}_T$$

The market portfolio is the tangent portfolio!



The marginal contribution of asset i to the market portfolio:

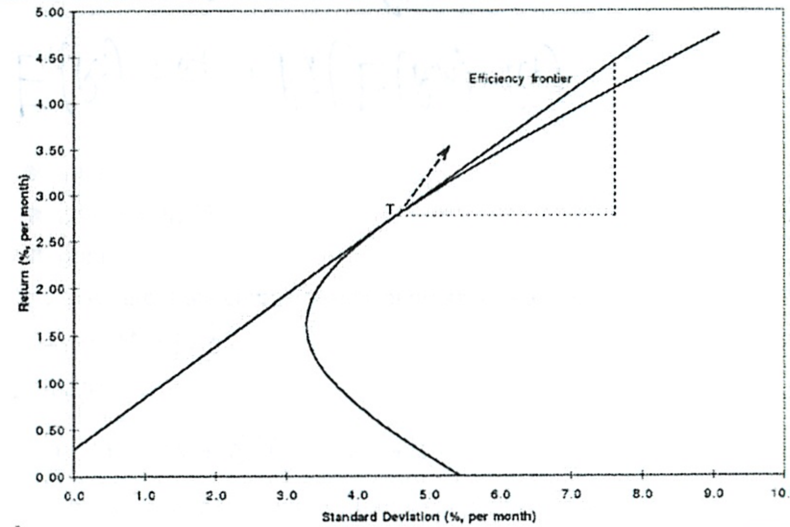
return: $\bar{r}_i - r_F$

risk: σ_{iM} / σ_M

For the market portfolio to be optimal, the return-to-risk ratio (RRR) of all risky assets must be the same:

$$\text{RRR}_i = \frac{\bar{r}_i - r_F}{(\sigma_{iM} / \sigma_M)} = \text{RRR} = \text{RRR}_M = \frac{\bar{r}_M - r_F}{\sigma_M}$$

Intuition: The RRR of a frontier portfolio cannot be improved.



Re-writing

$$\frac{\bar{r}_i - r_F}{(\sigma_{iM} / \sigma_M)} = \frac{\bar{r}_M - r_F}{\sigma_M}$$

we have

$$\boxed{\bar{r}_i - r_F = \frac{\sigma_{iM}}{\sigma_M^2}(\bar{r}_M - r_F) = \beta_{iM}(\bar{r}_M - r_F)}$$

where

$$\beta_{iM} = \sigma_{iM} / \sigma_M^2$$

is the beta of asset i with respect to the market portfolio.

This is the CAPM:

β_{iM} gives a measure of asset i's systematic risk

$\bar{r}_M - r_F$ gives the premium per unit of systematic risk

- The risk premium of an asset equals its systematic risk (β_{iM}) times the premium per unit of the risk ($\bar{r}_M - r_F$)

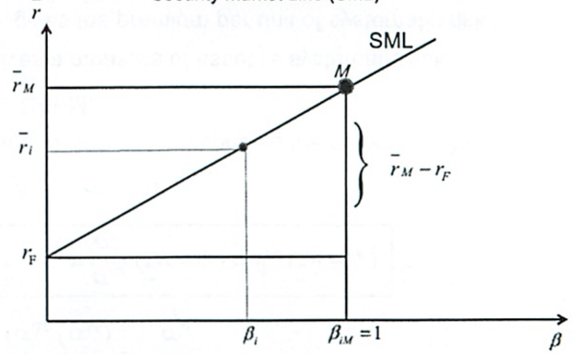
To the right are some beta estimates as examples produced by Bloomberg, Merrill Lynch, and Yahoo! Finance

Why do the companies indicated by arrows have a low or high beta?

What beta do you expect for a cinema?

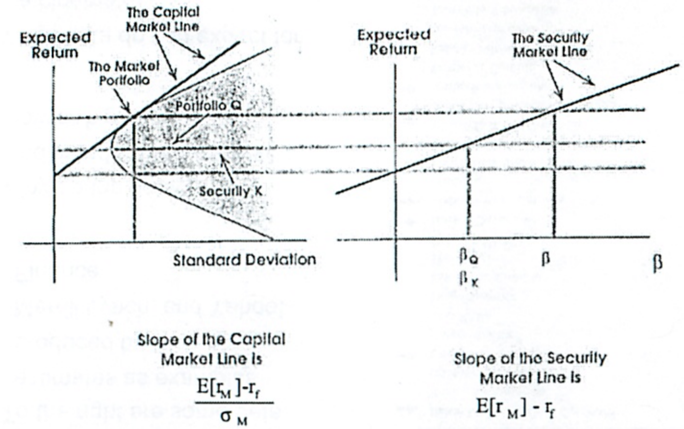
→	Battle Mountain Gold Company	.40
	Boeing Corporation	.90
	Bristol-Myers Squibb	.95
→	California Water Company	.45
	Caterpillar Inc.	1.20
	Coca-Cola	.95
	Dow Chemical	1.15
	Exxon Corporation	.65
	The Gap, Inc.	1.45
	General Electric	1.15
→	Harley-Davidson	1.65
→	Idaho Power Company	.65
	Intel Corporation	1.35
	Kaufman & Broad Home	1.65
	Kellogg	1.00
→	Merrill Lynch & Company	1.90
	Oshkosh B'Gosh (clothing mfg.)	.60
→	Outback Steakhouse	2.10
	Procter & Gamble	1.05
	Ralston Purina	.90
	Telefonos de Mexico	1.35
	Tootsie Roll Industries	.75
	Toys 'Я' Us	1.45
	Western Digital	1.85

The relation between an asset's premium and its market beta is called the Security Market Line (SML).



Given an asset's beta, we can determine its expected return.

The Capital Market Line and the Security Market Line:



Example. Suppose that CAPM holds. The expected market return is 14% and T-bill rate is 5%.

1. What should be the expected return on a stock with $\beta = 0$?
2. What should be the expected return on a stock with $\beta = 1$?
3. What should be the expected return on a portfolio made up of 50% T-bills and 50% market portfolio?
4. What should be expected return on stock with $\beta = -0.6$?

$$\bar{r} = 0.05 + (-0.6)(0.14 - 0.05) = -0.4\%$$

How can this be?

We can decompose an asset's return into three pieces:

$$\bar{r}_i - r_F = \alpha_i + \beta_{iM}(\tilde{r}_M - r_F) + \varepsilon_i$$

- $E[\varepsilon_i] = 0$
- $Cov[\tilde{r}_M, \varepsilon_i] = 0$.

So there are three characteristics of an asset's returns:

- Beta
- Sigma = SD (ε_i)
- Alpha

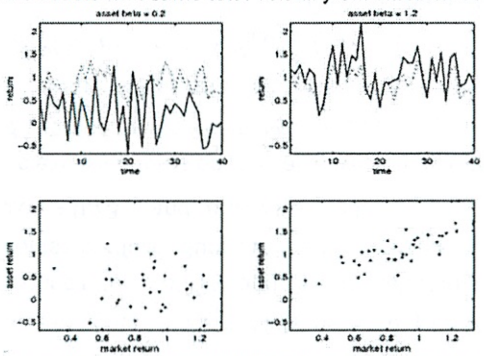
$$E(R_i) = R_f + \beta_i(E(R_m) - R_f)$$

$$\beta_i = \frac{\sigma_{im}}{\sigma_m} = \frac{\text{Cov stock to market}}{\text{St dev market}}$$

$$\bar{r}_i - r_F = \alpha_i + \boxed{\beta_{iM}}(\tilde{r}_M - r_F) + \varepsilon_i$$

- Beta measures an asset's systematic risk.

Two assets with same total volatility but different betas

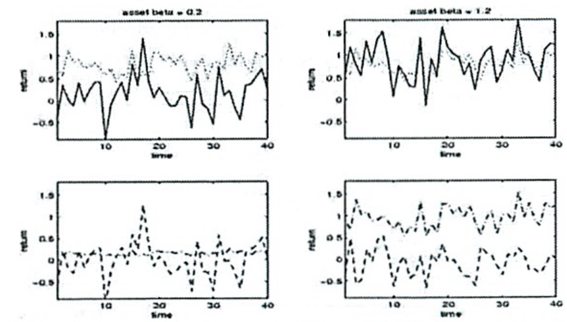


Market premium = 8%, market volatility = 25%, asset volatility = 40%.
Solid lines -- asset returns. Dotted lines -- market returns.

$$\bar{r}_i - r_F = \alpha_i + \beta_{iM}(\tilde{r}_M - r_F) + \boxed{\varepsilon_i}$$

- An asset's sigma measures its non-systematic risk.

Two assets with same total volatility but different betas



Market premium = 8%, market volatility = 25%, asset volatility = 40%.Solid lines -- asset returns. Dotted lines -- market returns. Dash-dot lines -- market component. Dashed lines -- idiosyncratic component

Example. Two assets with the same total risk can have very different systematic risks.

Suppose that $\sigma_M = 20\%$

Stock	Business	Market beta	Residual variance
1	Steel	1.5	0.10
2	Software	0.5	0.18

$$\sigma_1^2 = \beta_{1M}^2\sigma_M^2 + \sigma_{1\varepsilon}^2 = (1.5)^2(0.2)^2 + 0.10 = 0.19$$

$$\sigma_2^2 = \beta_{2M}^2\sigma_M^2 + \sigma_{2\varepsilon}^2 = (0.5)^2(0.2)^2 + 0.18 = 0.19$$

Percentage of systemic risk:

$$R_1^2 = \frac{(1.5)^2(0.2)^2}{0.19} = 47\%$$

$$R_2^2 = \frac{(0.5)^2(0.2)^2}{0.19} = 5\%$$

$$\tilde{r}_i - r_F = \boxed{\alpha_i} + \beta_{iM}(\tilde{r}_M - r_F) + \varepsilon_i$$

- According to CAPM, alpha should be zero for all assets
- Alpha measures an asset's return in excess of its risk-adjusted award according to CAPM

What to do with an asset with a positive alpha?

Check estimation error

Past value of α may not predict its future value

- Positive α may be compensating for other risks

....

Take 15 years (1995-2010) of monthly data on AT&T returns, S&P 500 returns and 1 month US interest rates.

Construct excess returns

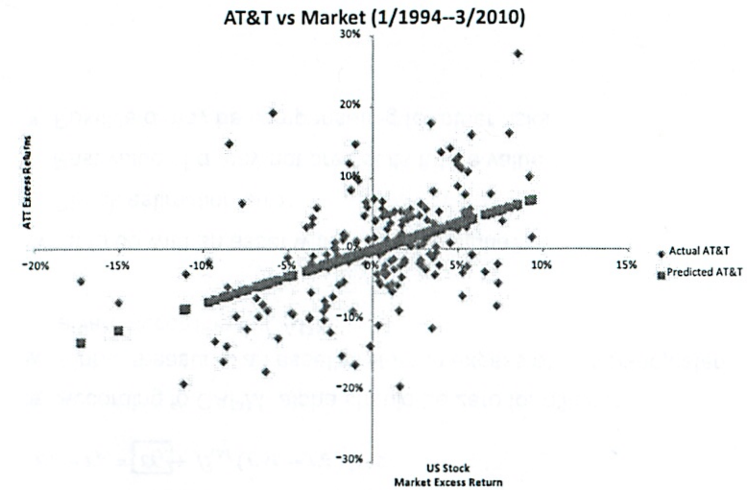
Run the regression, for instance using Excel:

- apply *Tools, Add-ins, Analysis ToolPak*
- use *Tools, Data Analysis, Regression*

The result is in the spreadsheet "Beta_Regression_ATT.xls"

Excel Regression output:

	Coefficients	*StandardErr*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-0.001	0.005	-0.112	0.798	-0.011	0.009
X Variable	0.740	0.106	6.981	0.000	0.535	0.967



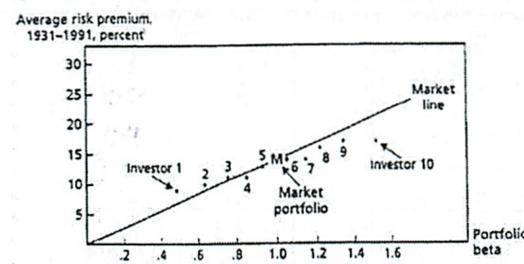
Example. Required rates of return on IBM and Dell

1. Use the value-weighted stock portfolio as a proxy for M.
2. Regress historic returns of IBM and Dell on the returns on the value-weighted portfolio. Suppose the beta estimates are
   $\beta_{\text{IBM,VW}} = 0.73$ and $\beta_{\text{Dell,VW}} = 1.63$.
3. Use historic excess returns on the value weighted portfolio to estimated average market premium: $\pi = \bar{r}_{\text{VW}} - r_F = 8.6\%$
4. Obtain the current riskless rate. Suppose it is $r_F = 4\%$
5. Applying CAPM: $\bar{r}_{IBM} = r_F + \beta_{\text{IBM,VW}}(\bar{r}_{\text{VW}} - r_F)$
   $= 0.04 + (0.73)(0.086) = 0.1028$

The expected rate of return on IBM (under CAPM) is 10.28%.

Similarly, the expected rate of return on Dell is 18.02%.

1 Long-run average returns are significantly related to beta:



(Source: Fisher Black, "Beta and return," *Journal of Portfolio Management*, 1993, 20(1), 8-18)

Dots show actual average risk premiums from portfolios with different betas.

high beta portfolios generated higher average returns

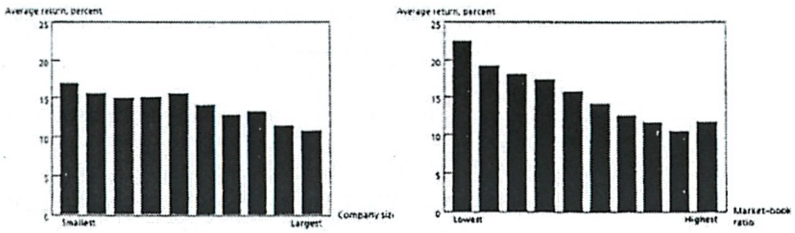
high beta portfolios fall below SML

low beta portfolios land above SML

a line fitted to the 10 portfolios would be flatter than SML

Empirical evaluation of CAPM

2. Factors other than beta seem important in pricing assets:



Source: G. Fama and K. French, "The Cross-Section of Expected Stock Returns" (1992).

Since the mid-1960s:

Small stocks outperformed large stocks

Stocks with low ratios of market-to-book value outperformed stocks with high ratios

The Value "Anomaly" in detail

Group all stocks each year in 10 portfolios, sorted on their Book-to-Market ratio (=BM deciles)

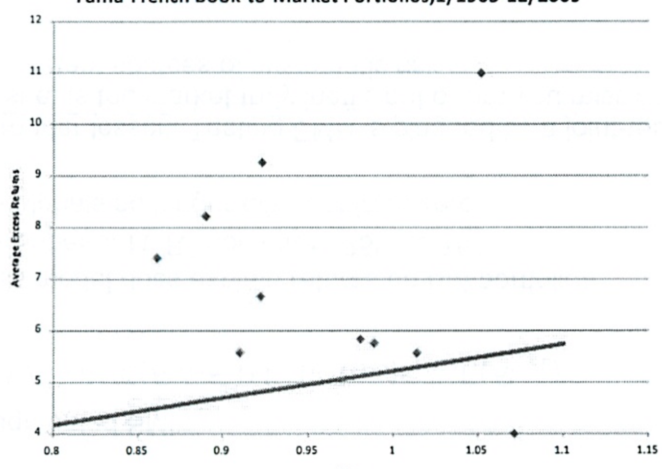
Average returns 1/1963-12/2009 data (564 months):
- 10th B/M decile: avg. annual return = 16.38%,
- 1st B/M decile: avg. annual return = 10.10%,
- **Value spread** = 16.38 - 10.10 = **6.37% per year**

CAPM Alpha 1/1963-12/2009 (564 months):
- 10th B/M decile (value stocks): α =5.57%
- 1st B/M decile (growth stocks): α = -1.86%

If you were a hedge fund manager what would you do?

The CAPM is Dead!

Fama-French Book-to-Market Portfolios,1/1963-12/2009



Fama-French Three-Factor Model

Fama and French (1993) argue that this evidence is not inconsistent with the Efficient Market Hypothesis (EMH).

Rather, it indicates that there is more than 1 source of systematic risk (exposure to market)

They add 2 new sources of systematic risk:
- Size factor R_{SMB}: return on a portfolio that goes long big stocks and short small stocks
- Value factor R_{HML}: return on a portfolio that goes long high B/M stocks and short low B/M stocks

Augmented SML:

$$E(R_i)-R_f=\beta_{im}[E(R_M)-R_f]+\beta_{is}E(R_{SMB})+\beta_{ih}E(R_{HML})$$

This model does explain returns on BM portfolios:
- Superior fit: R^2 goes from 25% to 75%
- Alpha's no longer different from zero

Broader lesson: Testing EMH is plagued by a **joint-test** issue: Is the market truly inefficient or are you missing important sources of systematic risk?

We can extend the market-risk model to include multiple risks:

$$\tilde{r}_i - r_F = \alpha_i + b_{iM}(\tilde{r}_M - r_F) + \cdots + b_{iN}(\tilde{r}_N - r_F) + u_i$$

where

$\tilde{r}_M$ and $\tilde{r}_N$ represent common risk factors

b_{iM} and b_{iN} define asset i's exposure to risk factors

u_i is part of asset i's risk unrelated to risk factors.

We then have

$$\boxed{\bar{r}_i - r_F = b_{iM}(\bar{r}_M - r_F) + \cdots + b_{iN}(\bar{r}_N - r_F)}$$

where

$\bar{r}_k - r_F$ is the premium on factor k

b_{ik} is asset i's loading of factor k

This model is called the Arbitrage Pricing Theory (APT) (Steve Ross)

Example. Suppose that there are two priced factors represented by:

return on the market portfolio $\tilde{r}_M$

return on Treasury bond portfolio $\tilde{r}_N$:

$$\tilde{r}_i - r_F = b_{iM}(\tilde{r}_M - r_F) + b_{iN}(\tilde{r}_N - r_F) + u_i$$

Suppose that

r_F	$\bar{r}_M - r_F$	$\bar{r}_N - r_F$
5%	8%	2%

APT implies that an asset's risk premium is given by

$$\bar{r}_i - r_F = b_{iM}(\bar{r}_M - r_F) + \cdots + b_{iN}(\bar{r}_N - r_F)$$

Suppose for assets A, B and C, we have

Asset	b_M	b_N
A	1.0	1.0
B	1.5	0.2
C	1.0	0.6

APT implies that individual assets have to offer returns of:

$\bar{r}_A$ = 0.05 + (1.0)(0.08) + (1.0)(0.02) = 15.0%

$\bar{r}_B$ = 0.05 + (1.5)(0.08) + (0.2)(0.02) = 17.4%

$\bar{r}_C$ = 0.05 + (1.0)(0.08) + (0.6)(0.02) = 14.2%

Suppose that $\bar{r}_A$ was instead 10% (and it has only factor risks).
We would then have an arbitrage:

a) Buy $100 of market portfolio
b) Buy $100 of bond portfolio
c) Sell $100 of asset A
d) Sell $100 of risk-free asset.

This portfolio has the following characteristics:
requires zero initial investment (an arbitrage portfolio)
bears no factor risk (and no idiosyncratic risk)
pays (13 + 7 – 10 - 5) = $5 surely

Thus, in absence of arbitrage, APT holds (hence its name)

The implementation of APT involves three steps:

1. Identify the factors
2. Estimate factor loadings of assets
3. Estimate factor premium(s)

Strength and Weaknesses of APT

1. The model gives a reasonable description of return and risk
2. Model itself does not say what the right factors are

Differences between APT and CAPM

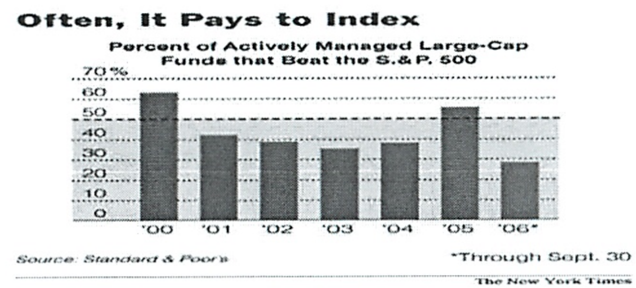
APT is based on the factor model of returns and "arbitrage"

CAPM is based on investors' portfolio demand and equilibrium

Implications from EMH for Securities Research:

- Why should I ever be a securities analyst?
- Do mutual fund managers who actively manage their portfolio (as opposed to holding a passive index) earn abnormal returns? Sadly not (as a group) …

after fees



The market portfolio

Derivation of CAPM

Implications of CAPM

Understanding risk and return in CAPM

Empirical tests of CAPM

Extensions of CAPM

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Capital Asset Pricing Model (CAPM)

- simple
- more simple than arbitrage pricing
- builds on portfolio theory
  1. Diversify
  2. Hold risk-free assets and tangent portfolio

* But how can you price risky assets?
  What is the expected return of a risky asset?
  Works for all risky assets
  - Market portfolio = all assets in market

so can get data

$MCAP_i = (\text{Price per share})_i \times (\text{\# of shares outstanding})_i$

"Market Capitalization"

$$MCAP_m = \sum_{i=1}^{n} MCAP_i$$

Then weight it

$$w_i = \frac{MCAP_i}{\sum_{j=1}^{n} MCAP_j} = \frac{MCAP_i}{MCAP_m}$$

②

Investors all see same thing

Diff risk assumptions

Lots of assumptions

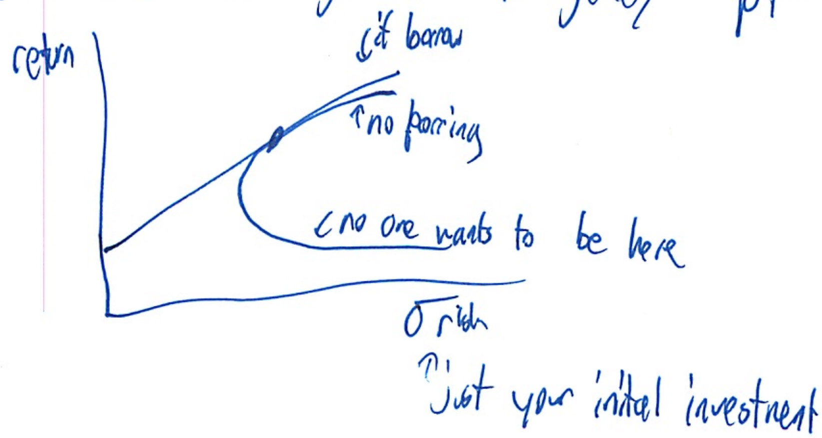
- 0 sum game
- efficent fronteir portfolio
- demand - supply
- risk-free asset
- investors ~~dis~~ agree on distribution of asset returns

Can also short riskless asset - ~~its~~ to hold ~~ing~~ more risky

Market cap. is sum of its market value

Market portfolio is the tangent portfolio

Remember people want to go to tangency pt



(3)

So marginal contribution of asset i to market portfolio

return: $\bar{r}_i - r_F$ ← some risk premium to buy risky investments per unit of systematic risk

risk: $\frac{\sigma_{iM}}{\sigma_M}$ ← var does not matter much, covar matters

So <u>Risk Return Ratio (RRR)</u>

$$RRR_i = \frac{\bar{r}_i - r_F}{\sigma_{iM}/\sigma_M} = RRR$$

↑ Must be same for all assets for portfolio to be optimal

Rewriting

$$\frac{\bar{r}_i - r_F}{\sigma_{iM}/\sigma_M} = \frac{\bar{r}_M - r_F}{\sigma_M}$$

So

$$\boxed{\bar{r}_i - r_F = \frac{\sigma_{iM}}{\sigma_M^2}(\bar{r}_M - r_F) = \beta_{iM}(\bar{r}_M - r_F)}$$

risk premium of asset

↑ $\beta_{iM} = \frac{\sigma_{iM}}{\sigma_M^2}$

↑ for asset i w/ respect to market
- asset i's systematic risk

(4)

$\sigma_{iM} = \rho_{iM} \times \sigma_i \times \sigma_m$

↑ cov dominantes
- does it move along w/ other stocks

β is measure of risk
- when you add another asset, what's its contribution?
- is it a big mover along w/ portfolio?
- or does it typically move in other direction?

CAPM - assumes β is stable

Can look up β or calculate easily

* its cov w/ market that matters *

Systematic risk matters
- individual stocks' risks <u>don't</u> matter

$\beta = 1$ = cov w/ market = exactly as risky as market
< 1 = less risky than market
- low systematic risk
- always buy the product - like utilities
> 1 = more risky than market
- affected by systematic risk - like econ
- like lux goods
$= 0$ does not move at all vs market

(5)

Tech industry usually > 1 since people can often defer purchases

Coca Cola .95 - cheap and addictive

Caterpiller 1.20 - large industrial equipment
- does not do well in recession

Putting something with high β in portfolio ↑ risk of portfolio

For studios β close to 0
- just good vs bad films
- not really econ conditions

For exhibitors
? do they do better in bad times
RGC = .92

Measurement period matters

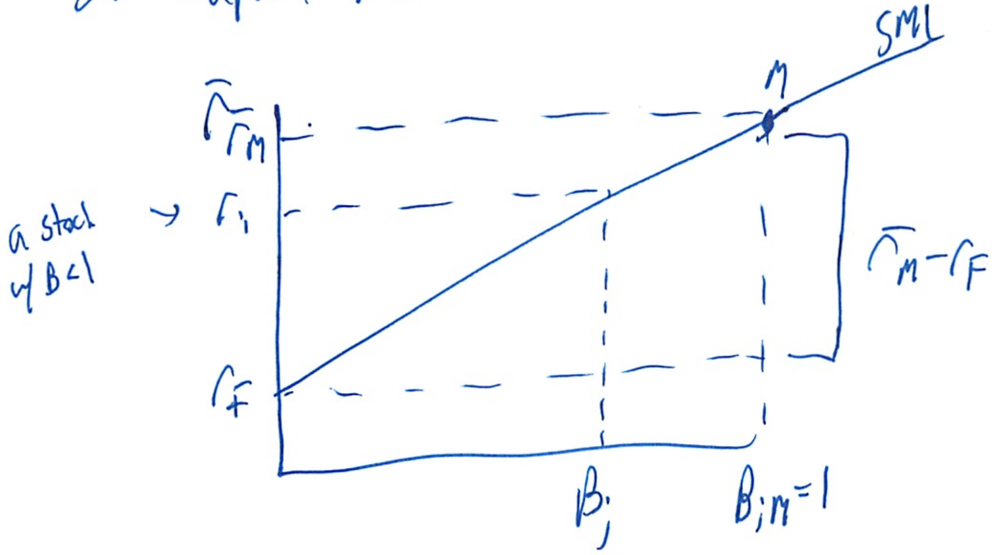
Find R^2 for regression Mon
- Says how accurate it is

GE ~ 1 since in everything

Merrill Lynch = 1.90 => most profits from risky assets
more mergers in bad econ

6

So capital market line called <u>Security Market Line</u>



* To shift β, buy or sell stocks

* β_{Port} is just weighted avg of pieces

So required return

$$= r_F + \beta_{iM}(\bar{r}_M - r_F)$$

- r_F ↑ risk free return
- β_{iM} ↑ amt of risk ↑ for stock or weighted avg of β for whole portfolio
- $(\bar{r}_M - r_F)$ ↑ risk premium

$$E(R_i) = R_f + \beta_i \cdot (E(R_m) - R_f)$$

$E(R_m)$ ↑ hardest to calculate

① Can have $\beta < 0$

- goes countercyclical
- But get - expected return

Can decompose assets' return into 3 pieces

- individual idosyncratic risk ε_i - diversify away
- systematic risk β_{im}
- Alpha α
  - should be 0, but not true in real life
  - return in excess of CAPM risk adjusted rate of return
  - Book: Chasing Alpha
  - what hedge funds are looking for

Problem Set 6: Portfolio Theory
(Due: Monday, April 25th, at 5:00 p.m.)

Unless specified as an "Excel Problem", all of these problems can (and should) be solved with a pencil, paper and a simple calculator. Do not use Excel (except to check your work, if you want). Show your work cleanly and write out the formulas that you used to solve the problems. Circle your final answers.

Problem 1

Suppose there are two assets, asset A and asset B, with the assets' mean returns and variances given by:

	E[r]	σ^2
Asset A	10%	$(0.15)^2$
Asset B	12%	$(0.10)^2$

And the assets have a correlation of $\rho = 0.2$.

1. If you could only hold either one of the assets but not both, which asset would you pick?

2. Calculate the portfolio mean return and standard deviation for weights from $w_1 = 0.0$ to $w_1 = 1.0$ in increments of w of 1/5 = 0.2. Draw the frontier in the (σ_P, $E[r_P]$) graph (i.e. σ_p on the horizontal axis, and $E[r_P]$ on the vertical axis).

3. Will anyone ever hold asset B? If so, why? (HINT: think about an investor who does not want more than 9.5% standard deviation)

Problem 2

Suppose there are 3 assets in the economy, assets 1, 2 and 3, with the assets' variances and covariances given by:

Variance/Covariance	Asset 1	Asset 2	Asset 3
Asset 1	$(0.1)^2$	0.012	0.0045
Asset 2	0.012	$(0.2)^2$	-0.003
Asset 3	0.0045	-0.003	$(0.15)^2$

1. What are the standard deviations σ_1, σ_2 and σ_3?

2. What are the correlations ρ_{12}, ρ_{13} and ρ_{12}?

3. What is the portfolio standard deviation of a portfolio with weight of 0.2 in the risk-free asset and weights 0.2, 0.4, 0.2, in the assets 1, 2 and 3 respectively?

Problem 3

Suppose there are two assets A and B with returns and variances given by:

	E[r]	σ^2
Asset A	15%	$(0.15)^2$
Asset B	12%	$(0.10)^2$

Suppose further that the assets have perfect negative correlation, $\rho = -1$.

1. Write out the variance of the portfolio, σ^2_P, with weights *w* and $(1-w)$ → there is no investment in the risk-free asset. Use the quadratic formula $(a-b)^2 = a^2+b^2-2ab$ to simplify.

2. Using the simplification from part 1, what is the lowest variance any portfolio consisting of A and B can achieve? What weights *w* and $(1-w)$ would give this variance?

3. Given your result from part 2, would you ever want to invest in a risk-free bond with expected return of 5%? Explain.

Problem 4

Suppose there are two countries, Country X and Country Y. Each consists of a very large number of stocks. Stocks in X return on average 8%, have a standard deviation of 35%, and the common correlation between stocks in Country X is $\rho_X = 0.49$. Stocks in Y return on average 10%, have a standard deviation of 30%, and the common correlation between stocks in Country Y is $\rho_Y = 0.64$.

1. Suppose that stocks from X and stocks from Y have a correlation of zero, that is $\rho_{XY} = 0$. Suppose you invest equally in only one stock from X and one stock from Y. What is the mean and standard deviation of this portfolio?

2. Consider only investing in stocks of country X. Given the common correlation, what is the variance of an equally weighted portfolio consisting of *n* stocks when *n* becomes very large (i.e. n → infinity)? What is its expected return?

3. Consider only investing in stocks of country Y. Given the common correlation, what is the variance of an equally weighted portfolio consisting of *n* stocks when *n* becomes very large (i.e. n → infinity)? What is its expected return?

4. Suppose now that you invest equally in country X and Y, but unlike in part 1, in each country you invest equally in a very large number of stocks. That is, take the resulting portfolios from parts 2 and 3 as your building block assets, and invest equally in the two. What is the new portfolio's expected return and variance?

Michael Plasmeier 15.401 - P-Set 6 4/24

9/10

1. 2 assets

a. If only 1 asset:

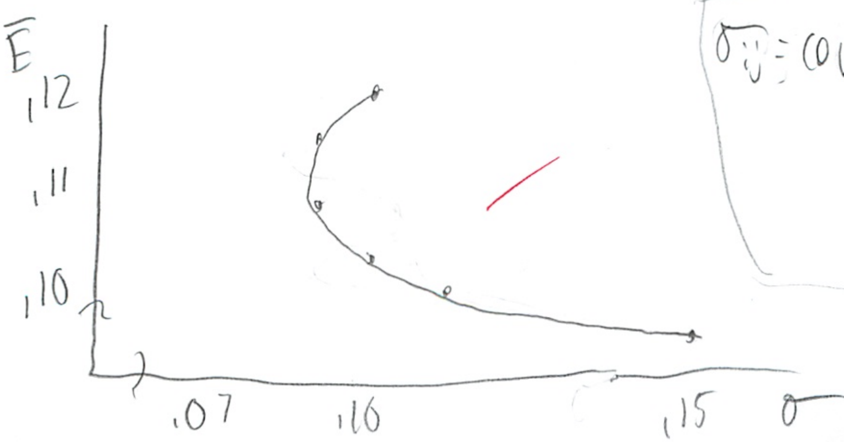
asset B since it has a higher return with less uncertainty ✓

b. Portfolio mean return and varience

W_A	W_B	$E(R_p)$	$\sigma_{R_p}^2 = var(R_p)$	σ_{R_p}
0	1	12%	$(.10)^2 = .01$	.10 ✓
.2	.8	11.6%	.0082	.0908 ✓
.4	.6	11.2%	.00864	.0929 ✓
.6	.4	10.8%	.0114	.1055 ✓
.8	.2	10.4%	.01576	.1255 ✓
1	0	10%	$(.15)^2 = .0225$	.15 ✓

$\sum_{i=0}^{N} w_i E(R_i)$

$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \sigma_{12}$



✓

$\sigma_{ij} = cov_{ij} = \rho_{ij} \sigma_i \sigma_j$ (← correlation)

$= .2 \cdot .10 \cdot .15$

$= .003$

②

c) Yes, as part of a portfolio to reduce systematic ✗ risk. For $\sigma < .095$ an investor could have ↳ idiosyncratic risk.

2% A and 80% B

2. a. What are St. Dev.

So this table is

	r_1	r_2	...	r_n
r_1	σ_1^2	σ_{12}	...	σ_{1n}
r_2	σ_{21}	σ_2^2	...	σ_{2n}
⋮	⋮	⋮	⋱	
r_n	σ_{n1}	σ_{n2}		σ_n^2

$\sigma^2 = var$

$\sigma_{ij} = \sigma_{ji}$

$\sigma_{ij} = cov_{ij} = \rho_{ij}\sigma_i\sigma_j$ (↙ correlation)

So square root of diagonal

A .1 ✓
B .2 ✓
C .15 ✓

b) What are correlations ρ_{ij}

ρ_{12}

$\sigma_{12} = \rho_{12}\sigma_1\sigma_2$

$.012 = \rho_{12} \cdot .1 \cdot .2$

$\rho_{12} = \frac{.012}{.1 \cdot .2} = .6$ ✓

(3)

ρ_{13}

$\sigma_{13} = \rho_{13}\,\sigma_1\,\sigma_3$

$.0045 = \rho_{13} \cdot .1 \cdot .15$

$\rho_{13} = \frac{.0045}{.1 \cdot .15} = .3$

ρ_{12}

Asked and aswered already :)

C. σ_p if .2 of #1, .4 of #2, .2 of #3, .2 of r.f.

risk free asset does not enter calculations

$\sigma_p{}^2 = (.2)^2 \cdot (.1)^2 + (.4)^2 \cdot (.2)^2 + (.2)^2 \cdot (.15)^2$
$+ 2(.2)(.4)(.012) + 2(.2)(.2)(.0045) + 2(.4)(.2)(-.003)$

$= .0095$

$\sigma_p = .09746$

(4)

3. Another portfolio

w = weight of A
(1-w) = weight of B

no R.F. asset

a. Write out Var

$Var(R_p) = \sigma_{R_p}^2 = w^2(.15)^2 + (1-w)^2(.10)^2 + 2(w)(1-w)(-1)(.15)(.10)$

$= w^2(.15)^2 + (1^2 + w^2 - 2w)(.10)^2 - .03(w - w^2)$

$= .0225w^2 + .01 + .01w^2 - .02w - .03w + .03w^2$

$= .1525w^2 - .05w + .01$

b) Minimize w. Take ∂, set = 0

$\frac{\partial var}{\partial w} = .1525 \cdot 2w - .05$

$= .305w - .05$

Set = to 0

$0 = .305w - .05$ —.25

$.05 = .305w$

$w = .163$

$(1-w) = .836$ ∝

Var at this point

$= .1525(.163)^2 - .05(.163) + .01$

$= .0059$

$\sigma = .0768$

Should be 0

(5)

c) The return of this portfolio is

$= W \cdot .15 + (1-w) \cdot .12$

$= (.163) \cdot .15 + .836 \cdot .12$

$= .124$

−.5

Number wise it seems uncertain, so I must have made a mistake somewhere

- wrote in notes if cov is < 0, then math blows up

But intuitvly - no since one stock will always go up while the other goes down -

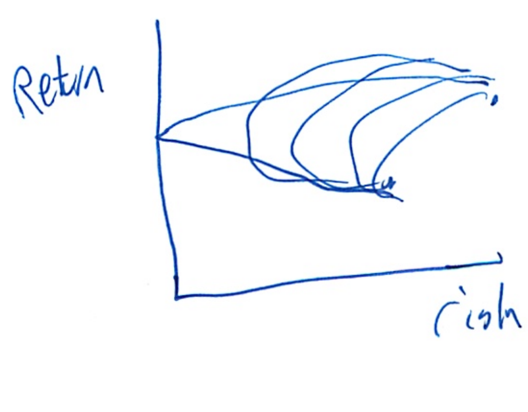
The wobbling around is the E[ ]

If had half + half then one would be

↑ 20% from E[ ] so say 18% return other ↓ 20% from E[ ] say 9.6% return

So would still earn a handsome reward

But there is an optimal place where you would have 0 risk



⑥

4. 2 countries X, Y

X: 8% = return, $\sigma = 35\%$, $\rho_x = .49$

Y: 10% 36% .64

a. Suppose $\rho_{xy} = 0$ and 1 stock from X, Y each

Mean = $.5 \cdot .08 + .5 \cdot .10 = .9$

↑ assuming invest evenly in each stock

$\sigma^2 = (.5)^2(.35)^2 + (.5)^2(.3)^2 + 2(.5)(.5)(0)(.35)(.3)$

$= .053125$

$\sigma = .2304$

b) What is var, E when $n \to \infty$ for X?

E[ ] is given = 8%

$\underset{\lim n \to \infty}{Var}$ = goes to avg cov

$\sigma_{i,j} = Cov_{i,j} = \rho_{i,j} \sigma_i \sigma_j$

$= .49 \cdot .35 \cdot .35$

$= .060025$

$Var = \sigma^2 = .003603$

(7) ..

c) Now for Y

$E[\,] = 10\%$ ✓

$\sigma_{ij} = cov_{ij} = \rho_{ij}\sigma_i\sigma_j$

$= .64 \cdot .3 \cdot .3$

$= .0576$ ✓

$Var = \sigma^2 = .00331776$

d) Suppose you invest in a lot of stocks from both

$E[\,] = .5 \cdot 8\% + .5 \cdot 16\% = 9\%$ ✓

↑ assume invest in both countries evenly

$\sigma^2 = (.5)^2(.0600)^2 + (.5)^2(.0576)^2 + 2(.5)(.5)(.64)(.0600)(.0576)$

$= .002836$ ← further reduced

$\sigma = .0532594$

✓ .25

Problem #1

$$E(R_p) = W_A \times R_A + W_B \times R_B$$
$$\sigma^2_p = W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2 + 2 W_A W_B \sigma_{AB}$$

$\rho_{AB} = 0.20$

$\sigma_A = .15 \qquad \sigma_B = .10$

$\therefore \sigma_{AB} = .20 \times .15 \times .10 = 0.0030$

$R_A = .10 \qquad R_B = .12$

(1) You would hold Asset B, as it dominates Asset A. B has both higher returns and lower standard deviation / variance.

(2) $E(R_p)$ and σ_p for weights $W_A = 0.0$ to $W_A = 1.0$, in increments of 0.20 requires calculation of $E(R_p)$ and σ_p at 6 different points, for example at $W_A = .40$ and $W_B = .60$:

$$E(R_p) = .40 \times .10 + .60 \times .12 = .1120$$
$$11.20\%$$

$$\sigma^2_p = .40^2 \times .15^2 + .60^2 \times .10^2 + 2 \times .40 \times .60 \times .0030$$
$$= .008640$$

$$\sigma_p = \sqrt{.008640} = .09295 = 9.295\%$$

Remainder of part 2 on Next page

Problem Set 6 Solutions
15.401

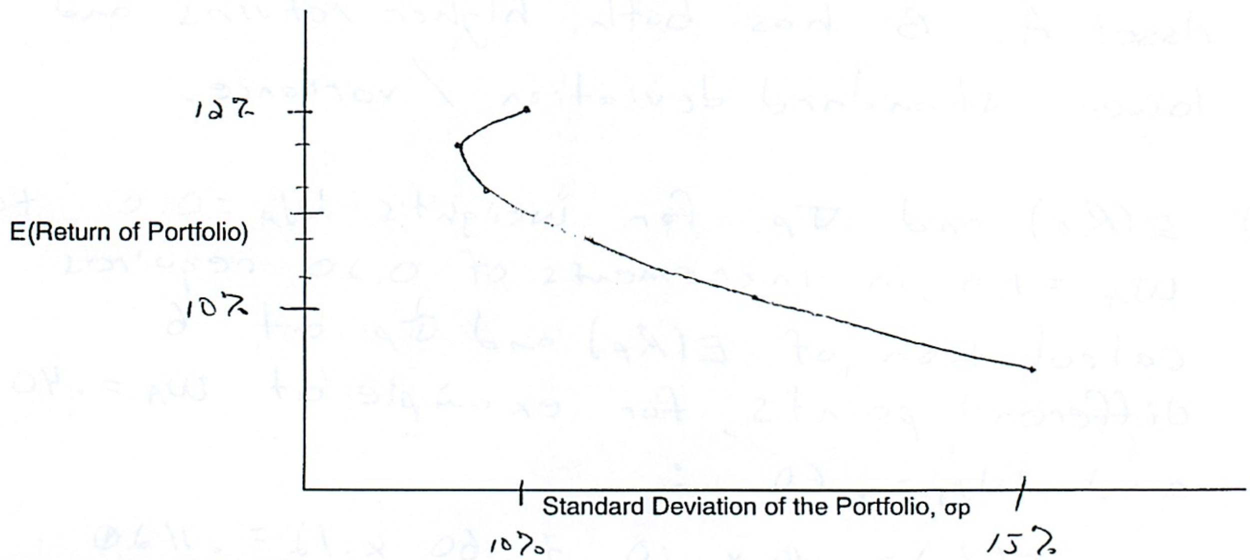
Problem #1 Data

	E(Ret)	Std Dev	Var
Asset A	0.1000	0.1500	0.0225
Asset B	0.1200	0.1000	0.0100
Correlation (A,B)	0.2000		
Covariance (A,B)	0.0030		

Problem #1 - Part 2 Solution

Portfolio Returns & Std Dev

Weight Asset A	0.00	0.20	0.40	0.60	0.80	1.00
Weight Asset B	1.00	0.80	0.60	0.40	0.20	0.00
E(Ret)	0.1200	0.1160	0.1120	0.1080	0.1040	0.1000
Variance	0.0100	0.0083	0.0086	0.0111	0.0158	0.0225
Std Dev	0.1000	0.0909	0.0930	0.1055	0.1255	0.1500



Problem #1 - Part 3 Solution

Yes, investors will still want to hold Asset A, in spite of Asset B being superior in isolation. Holding Asset A allows investors to reduce the risk, standard deviation of the portfolio below the 10% σ of Asset A alone. With the correlation between the 2 assets equal ot 0.20, holding A with B significantly reduces the standard deviation of the portfolio; well below the desired 9.5%.

PS 6 Solutions

Problem 2

Part 1

$\sigma_1 = .10$ ✓

$\sigma_2 = .20$ ✓

$\sigma_3 = .15$ ✓

Part 2

$\sigma_{12} = \rho_{12}\sigma_1\sigma_2$

$\sigma_{13} = \rho_{13}\sigma_1\sigma_3$

$\sigma_{23} = \rho_{23}\sigma_2\sigma_3$

$\therefore \quad \rho_{12} = \frac{\sigma_{12}}{\sigma_1\sigma_2} = \frac{.012}{.1 \times .2} = 0.60$ ✓

$\rho_{13} = \frac{\sigma_{13}}{\sigma_1\sigma_3} = \frac{.0045}{.1 \times .15} = 0.30$ ✓

$\rho_{23} = \frac{\sigma_{23}}{\sigma_2\sigma_3} = \frac{-.003}{.2 \times .15} = -0.10$ ✓

Part 3

$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2w_1w_2\sigma_{12} + 2w_1w_3\sigma_{13} + 2w_2w_3\sigma_{23}$

$\sigma_p^2 = .2^2 \times .1^2 + .4^2 \times .2^2 + .2^2 \times .15^2 + 2 \times .2 \times .4 \times .012 + 2 \times .2 \times .2 \times .0045 + 2 \times .4 \times .2 \times -.003$

$\sigma_p^2 = 0.0095$

$\sigma_p = \sqrt{0.0095} = .097468 = 9.747\%$ ✓

Problem 3

(1) $\sigma_p^2 = w^2\sigma_A^2 + (1-w)^2\sigma_B^2 + 2 \times w \times (1-w)\sigma_{AB}$

Students simplify this using quadratic equation

(2) What is the lowest variance portfolio consisting of A and B?

Zero reason = $\rho_{AB} = -1.00$

A cleverly constructed portfolio will result in zero variance

$w_A = 0.40$
$w_B = 0.60$ } for $\sigma_p^2 = \emptyset$

(3) You would never invest in a risk-free bond paying 5%. The 40/60 portfolio is also risk free, with Return of 13.2%, dominates the riskless bond.

(5)

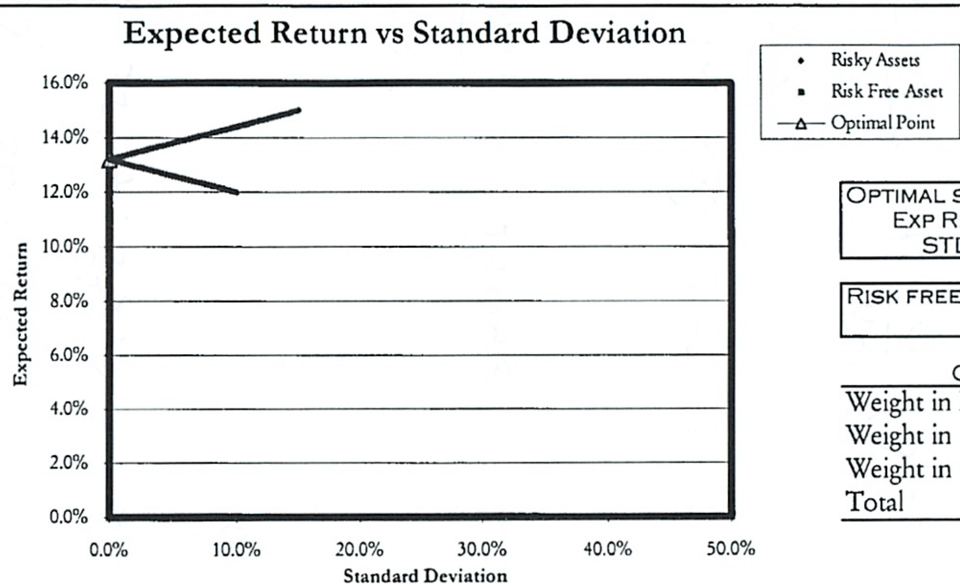
DIRECTIONS:

PLEASE INPUT THE VALUES IN THE CORRESPONDING TABLE TO REFLECT THE TWO RISKY ASSETS. ALL VALUES SHOULD BE IN DECIMAL FORM.
IF CELLS COME OUT RED, THEN THERE IS AN INCORRECT VALUE.

ASSET NAME	EXPECTED RETURN	STANDARD DEVIATION
ASSET A	0.15	0.15
ASSET B	0.12	0.1

RISK FREE RATE	0
RISK FREE % OF PORTFOLIO	0
CORRELATION BETWEEN STOCKS	-1

RESULTS:



Tangency Portfolio

Weight in Asset A	40.0%
Weight in Asset B	60.0%

OPTIMAL SOLUTION:

EXP RETURN	0.1320
STD DEV	0.0000

RISK FREE EQUATION=
0 + # DIV/0! (Sig)

OPTIMAL PORTFOLIO

Weight in Riskless Asset		0.0%
Weight in	Asset A	40.0%
Weight in	Asset B	60.0%
Total		100.0%

PS 6 Solutions

Problem 4

Part 1

$E(R_p) = .50 \times .08 + .50 \times .10 = .09 = 9\%$

$\sigma_p^2 = .5^2 \times .35^2 + .5^2 \times .30^2 + 2 \times .5 \times .5 \times \emptyset$

$\sigma_{xy} = \rho_{xy} \sigma_x \sigma_y$
$= 0.0 \times .15 \times .10$
$= \emptyset$

$\sigma_p^2 = .053125$

$\therefore \sigma_p = \sqrt{.053125} = .230489 = 23.05\%$

Part 2

Invest only in (X) n stocks, $n \to \infty$

$E(R_p) = 8\%$

$\sigma_{px}^2 = \frac{1}{n} \sigma_x^2 + \frac{n-1}{n} \sigma_{px\,px}$

$= \frac{1}{n} \times .35^2 + \frac{n-1}{n} \times .35 \times .35 \times .49$

$= \frac{1}{n} \times .1225 + \frac{n-1}{n} \times .060025$

$\frac{1}{n} \to \emptyset$ as $n \to \infty$; $\frac{n-1}{n} \to 1.0$ as $n \to \infty$

$\therefore \sigma_{px}^2 = .060025$

$\sigma_{px} = \sqrt{.060025} = 0.245$

24.5%

Problem 4

Part 3

Invest only in (Y) n stocks, $n \to \infty$

$E(R_P) = 10\%$

$$\sigma^2_{PY} = \frac{1}{n}\sigma^2_Y + \frac{n-1}{n}\sigma_{PYPY}$$

$$= \frac{1}{n} \times .30^2 + \frac{n-1}{n} \times .30 \times .30 \times .64$$

$$= \frac{1}{n} \times .09 + \frac{n-1}{n} \times .05760$$

$\sigma^2_{PY} = .05760$

$\sigma_{PY} = \sqrt{.05760} = .2400$

24.0%

Part 4

$E(R_P) = .5 \times 8\% + .5 \times 10\% = 9\%$ Same

$\sigma^2_P = .5^2 \times .245^2 + .5^2 \times .24^2 + 2 \times .5 \times .5 \times \emptyset$

$= .029406$

$\sigma_P = \sqrt{.029406} = .171482$

17.15%

Lower σ_P than in part one, as $\rho < 1.00$ within countries reduced σ within countries, and when combined in a cross-country portfolio, σ_P is lower.

4/25

15.401 – CAPM – April 25, 2011

Now that we've use 5 years of monthly stock markets returns data to calculate a beta coefficient of 1.06 for the Walt Disney Company, here are Disney betas from other sources, as well as beta coefficients for many firms in different industries (data pulled 4-24-2011):

Disney (Market Edge Research)	1.06
Disney (Thomson Reuters)	1.05
Disney (First Call)	1.10
Disney (Standard & Poor's)	1.11

PepsiCo	0.49	Coca Cola	0.55
Hewlett Packard	0.97	Apple	1.01
IBM	0.80	EMC	1.09
Network Appliance	1.38	Micron Technology	1.95
Ford Motor Co.	1.48		
Wal-Mart	0.41	Nordstrom	1.50
Colgate Palmolive	0.44	Proctor & Gamble	0.45
JP Morgan Chase	1.28	Bank of America	1.54
Citigroup	1.30		
Toll Brothers	1.17 (?)	Pulte Group	1.57
Vail Resorts	1.58	Marriott	1.52

Should be higher

1. Riskier assets on average earn higher returns

Arithmetic Mean return for last 85 years

Since ind in random walk process

	Mean (%)	Std(%)
Inflation	3.08	4.20
Treasury Bills	3.81	3.13
US Gov. Bonds (10 yr)	5.60	8.14
US Corp. Bonds (AAA)	6.49	7.04
S&P 500 stocks	11.74	20.52

Stock premium 11.74 − 5.60 = 6.14

Source: Global Financial Data and WRDS, Annual returns, 1925-2009.

15.401, Calculating CAPM

(3 min)

Loading data much eaiser today

~~Perm~~ Check for splits (facshr)

Calculate return to make sure return is correct

$$\text{~~Ret~~} = \frac{\text{this month's price} - \text{last month}}{\text{last month price}} + \frac{\text{dividend this month}}{\text{last month price}}$$

?are %

this month = "end" of month

last month = "start" of month

We care about 1 share - not whole company

Going to calc β, Cap-M

$$E(R_i) = R_f + \beta_i \left(E(R_m) - R_f\right)$$

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2}$$

$$\sigma_{im} = \rho_{im} \sigma_i \sigma_m$$

(2)

Covar function in excell

- b/w disney and market

same ans

Correlation b/w

σ of each

Calc var of market

- Varp not Var!

Beta = $\frac{\text{Cov(stock, market)}}{\text{Varp(stock)}}$

Disney = 1.05 - so a bit more risky than market

- goes a little higher when market ↑

lower ↓

~~Could~~ also ~~can Regression~~ using Data Analysis

Company - ~~ind~~ dep (Y)

Market - ~~dep~~ ind (X)

Beta given as slope

X Variable 1 Coefficents

Can then check t, P, f, adjusted R^2

(3)

Want adj R^2 < 20%

Daily data too much noise?

- Use monthly
- Bloomburg uses 2 years daily
- this is close of month price

Need to watch for diff people using diff methods

- kinda close

Want companies to be stable

Why Disney β close to 1?

- Theme Parks Volatile
- TV Fairly stable
- Movies Uncorrelated - Just to do w/ movie

HP?

- instrument biz
- for hospitals
- so close to 1

Ford?

- peoples purchases vary w/ time
- data from 2008 still in here

(4)

Right now $R_f = 3.39\%$

Looked up $\beta_{Disney} = 1.06$

Need estimate for market risk premium

- Hard to expect future returns for the market
- So proxy from market risk premium
  - See risk + return slide 19
  - From historical averages

$$E(R_i) = R_f + \beta_i \left(E(R_m) - R_f \right)$$

$$= 3.39\% + 1.06 \cdot 6.14$$

$$= 9.90\%$$

One factor model

- Only β is priced
- portfolio removes unique risk

Can do for any asset

- harder to find β if no pricing data

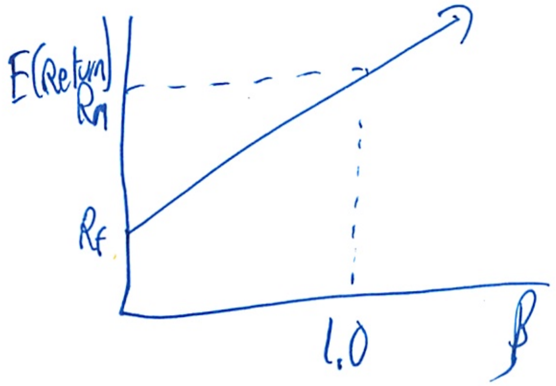
Note 2 diff R_f

$$= R_f + \beta_i \left(E(R_m) - R_f \right)$$

↑ today ↑ historical

5

Security Market Line



According to CAPM everything is on the SML
if above/below line people will buy/sell it till
it goes back to CAPM/SML line

In 2008, risk premium went up
- required returns of assets ↑
- so prices fell

Can't test conditions w/ expectations
- what is the market risk premium?
- 6% has been close

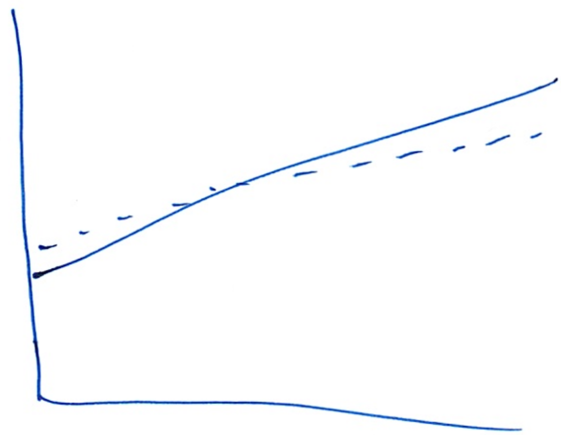
Problem w/ CAPM
- SML is not really what results have been

6

Low risk / low glamor - do a bit better than SML

High " / high " - do a little bit worse than SML

Slightly lower slope in real life



Arbitrage Pricing - Prices in other factors as well

- more next class

~~Math.~~ 15.401 Even more CAPM

4/27

Last time: Calculating CAPM β

looked at various stats

But returns vs market look all over the place

look at regression

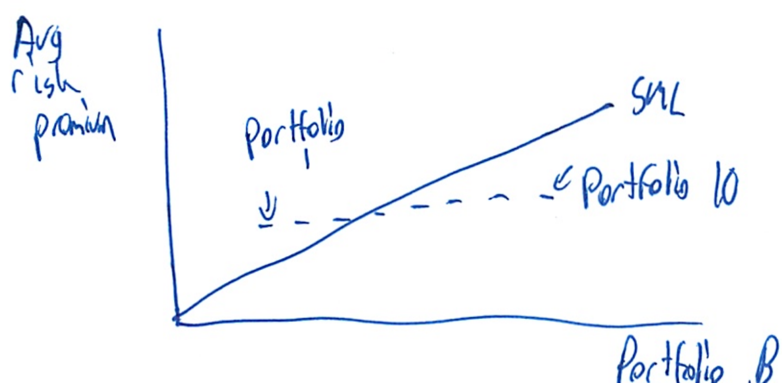
Works for everything ⇒ all asset prices

Calculating risk premiums

Is true Emperically

High β assets out perfor low beta assets

& But slope is lower than SML



Have to control for size

- Use $\ln(\text{Assets of firm})$

Also Market-to-book ratio

$$= \frac{\text{market value of equity}}{\text{book value of equity}}$$

(2)

$$= \frac{\$\ \text{price/share} \cdot \#\ \text{shares}}{\text{balance sheet} \rightarrow \$\ \text{common stock sold} + \text{retained earnings}}$$

= market capitalization

Since accounting (balance sheet) is <u>past</u>

investors care about future

So its like $\frac{\text{future expectations}}{\text{past}}$

Small firms have less history, more likly to grow

Firms w/ lower market to book have higher return

└ the ~~small~~ low glamor money making

People over value growth values it seems

Is the model wrong (we are not capturing something)

or are people/investors irrational

~~Bk~~ some α involved in value stocks

- that is the difference w/ the line
- ~~M~~ not investing in growth

(3) Growth stocks have to keep investing in business to reinvest

Value stocks can spin off to pay dividends / buy back stocks

Growth: believe future cash flows will be high enough to offset money too

Low β stocks outperform the market

Expectation so hard to know exactly

But we put in size and value (book-market)

Makes things more complex

Goodness of fit should ↑

Want to remove α to 0 for portfolio

Is the model wrong or the market

$$E[R_i] - R_f = \beta_{im}[E(R_m) - R_f] + \beta_{is} E(R_{SMB}) + \beta_{ih} E(R_{HML})$$

(R_{SMB} ↑ size; R_{HML} ↑ Value $\frac{\text{market}}{\text{book}}$)

Arbitrage Pricing Model

Can extend market share model to include multiple risks

$$\hat{r}_i - r_f = \alpha_i + b_{im}(\hat{r}_m - r_f) + \dots + b_{in}(\hat{r}_n - r_f) + u_i$$

- $\hat{r}_i$, r_f ↑ Common risk factors
- b_{im} ↑ asset i's exposure to risk factors
- u_i ↑ part of asset i's risk not included elsewhere

(4)

So example 2 priced factors
- return on market portfolio $\tilde{r}_M$
- " " Treasury bond " $\tilde{r}_N$

$$r_i - r_F = b_{iM}(\tilde{r}_M - r_F) + b_{iN}(\tilde{r}_N - r_F) + \tilde{u}_i$$

Get assets eq risk premium

Goes back to no arbitrage condition
- if asset is mispriced, people will arbitrage it

What drives stuff
- price of oil
- " " gold
- " " euro vs dollar

1. Identify factors
2. " loadings
3. " premiums

In theory far better - can factor in what affects price
- But hard to very actually do

(5)

Trying to pick α to beat the market in an actively managed ~~th~~ fund

- after fees

Is info in the stock?

Must outperform market and cover your fees

50% of people beat the market

?(can we say that)

~~Con~~

Asked: could be some bias constantly

Fund managers trying to find info not priced into stock

Must have news other people haven't heard

Hedge funds do outperform the market

- Very highly levered so returns magnified
- can invest in more types
- ~~ma~~ in order to invest must have a certain level of wealth

---

When funds say they beat the market - it means risk-adjusted ~~beat~~ beatin the market → $\beta_{\text{Mutual fund}} > 1$

⑥ Companies ~~capital~~ calc their + competitors cost of capital

Next week: apply to investment by companies

is the project a good idea

15.401 Recitation

7: CAPM

Learning Objectives

- Review of Concepts
  - CAPM
  - Beta and SML
  - Alpha
- Examples
  - The frontier
  - CML and SML

Review: efficient frontier

From Portfolio Choice…

- The CML is tangent to the efficient frontier at the **tangency portfolio**.
- The tangency portfolio is the portfolio of risky assets that **maximizes the Sharpe ratio**.
- The slope of the CML is the maximum Sharpe ratio.
- Rational investors always hold **a combination of the tangency portfolio and the risk-free asset**. The proportion depends on investors' risk preferences.

Review: CAPM

- Since each investor holds the **tangency portfolio** as part of his/her overall portfolio, the **market portfolio** must coincide with the tangency portfolio.
- Idea of CAPM: the contribution of a single risky asset to the risk of the market portfolio must be proportional to its risk premium.
- In other words, investors are compensated for exposure to **systematic risk**.
- **Idiosyncratic risk** is not compensated because they can be diversified away.

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Review: CAPM

- A measure of an asset's systematic risk is its beta:

$$\beta_i \equiv \frac{\operatorname{cov}(\tilde{r}_i, \tilde{r}_m)}{\operatorname{var}(\tilde{r}_m)} = \frac{\rho_{im}\sigma_i\sigma_m}{\sigma_m^2} = \rho_{im}\frac{\sigma_i}{\sigma_m}$$

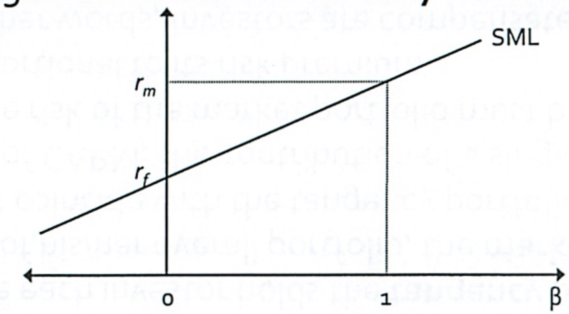
- Core result of CAPM:

$$\bar{r}_i = r_f + \beta_i(\bar{r}_m - r_f)$$

- Note:
  Market portfolio: $\beta_m = 1 \;\Rightarrow\; \bar{r}_m = r_f + (\bar{r}_m - r_f)$
  Risk-free portfolio: $\beta_f = 0 \;\Rightarrow\; \bar{r}_f = r_f + 0 \cdot (\bar{r}_m - r_f)$

Review: SML

- Graph of $\bar{r}_i = r_f + \beta_i(\bar{r}_m - r_f)$ in (beta, return) space is a straight line called the **Security Market Line**:



- If CAPM holds, every asset must be on the SML.

Review: testing CAPM

- CAPM does not hold exactly
- The regression

$$r_i = \alpha_i + \beta_i(r_m - r_f) + \varepsilon_i$$

  often give nonzero **alpha**.
- CAPM requires alpha to be zero for all assets
- CAPM may fail if factors other than beta affect asset returns, such as
  - Fama-French factors: market (beta), size, and book-to-market

Review: portfolio beta and alpha

- The beta of a portfolio is

$$\beta_p = \sum_{i=1}^{N} w_i \beta_i$$

- The alpha of a portfolio is

$$\alpha_p = \sum_{i=1}^{N} w_i \alpha_i$$

Example 1: the frontier

- The risk-free rate is 6%, the expected return on the market portfolio is 14%, and the standard deviation of the return on the market portfolio is 25%. Consider a portfolio with expected return of 16% and assume that it is on the efficient frontier.

  a. What is the beta of this portfolio?
  b. What is the composition of the portfolio?

Example 1: the frontier

- Answer:
  a. $\bar{r}_p = r_f + \beta_p(\bar{r}_m - r_f)$

     $0.16 = 0.06 + \beta_p(0.14 - 0.06)$

     $\beta_p = 1.25$

  b. Since the portfolio is on the efficient frontier, it is a combination of the risk-free asset (w) and the market portfolio (1-w):

     $0.16 = 0.06w + 0.14(1 - w)$

     $w = -0.25$

Example 2: CML and SML

- Using the properties of the capital market line (CML) and the security market line (SML), determine which of the following scenarios are consistent or inconsistent with the CAPM. Explain your answers.
- Let A and B denote arbitrary securities while F and M represent the riskless asset and the market portfolio respectively.

Example 2: CML and SML

- Scenario I:

Security	E[R]	β
A	25%	0.8
B	15%	1.2

- Answer: **inconsistent**
  Higher beta requires higher expected return

Example 2: CML and SML

- Scenario II:

Security	E[R]	σ[R]
A	25%	30%
M	15%	30%

- Answer: **inconsistent**
  A lies above the CML, which means that the market portfolio is inefficient.

Example 2: CML and SML

- Scenario III:

Security	E[R]	σ[R]
A	25%	55%
F	5%	0%
M	15%	30%

- Answer: **inconsistent**
  A lies above the CML, which means that the market portfolio is inefficient.

Example 2: CML and SML

- Scenario IV:

Security	E[R]	β
A	20%	1.5
F	5%	0
M	15%	1.0

- Answer: **consistent**
  Portfolio A lies on the SML

Example 2: CML and SML

- Scenario V:

Security	E[R]	β
A	35%	2.0
M	15%	1.0

- Answer: **inconsistent**
  The implied risk-free rate would be negative if A lies on the SML.

Part D Introduction to Corporate Finance

Chapter 10: Capital Budgeting

We have learned that:

- Business decisions often reduce to valuation of assets / CFs
- How to value assets using information from financial markets
- How to adjust for risk

In this section of the course, we apply the valuation tools to corporate financial decisions:

- Investment decisions (capital budgeting)
- Real options

15.401 Finance Theory I

Craig Stephenson

MIT Sloan School of Management

Lecture 10: Capital Budgeting

- NPV rule
- Cash flows from capital investments
- Discount rates
- Project interaction
- Alternative capital budgeting rules
- Real options

Readings:

Brealey, Myers and Allen, Chapters 5, 6, 9, 22

2/5

A firm's business involves capital investments (capital budgeting), e.g., the acquisition of real assets. The objective is to increase the firm's current market value. The decision reduces to valuing real assets, i.e., their cash flows.

Let the expected cash flow of an investment (a project) be:

$$\{CF_0, CF_1, \cdots, CF_t\}$$

Its current market value is:

$$NPV = CF_0 + \frac{CF_1}{1+r_1} + \cdots + \frac{CF_t}{(1+r_t)^t}$$

This is the increase in the firm's market value by investing in the project

Investment Criteria:

For a single project, accept it if and only if its NPV is positive

For many independent projects, accept all those with positive NPV

For mutually exclusive projects, accept the one with positive and highest NPV

In order to compute the NPV of a project, we need to analyze:

1. Cash flows,
2. Discount rates, and
3. Strategic options.

Main Points:

1. Use cash flows, not accounting earnings.
2. Use after-tax cash flows.
3. Use cash flows attributable to the project (compare firm value with and without the project):
   - Use incremental cash flows
   - Forget sunk costs: bygones are bygones
   - Include investment in working capital as a capital expenditure
   - Include the opportunity cost of using existing facilities

In what follows, all cash flows are attributable to the project.

CF = [Project Cash Inflows] − [Project Cash Outflows]
= [Operating Revenues]
− [Operating Expenses without depreciation]
− [Capital Expenditures]
− [Taxes]

Defining operating profit by:

Operating Profit = Operating Revenues
− Operating Expenses w/o Depreciation

Let τ be the ``effective'' tax rate. The income taxes paid are:

Taxes = (τ)[Operating Profit] − (τ) × [Depreciation]

Accounting depreciation affects CF by reducing the firm's tax bill.

CF = (1− τ)[Operating Profits] − [Capital Expenditures]
+ (τ)[Depreciation]

Example. **Accounting earnings vs. cash flows.** A machine purchased for $1,000,000 with a life of 10 years generates annual revenue of $300,000 and operating expense of $100,000. Assume that the machine will be depreciated over 10 years using straight-line depreciation. The corporate tax rate is 40%.

Date	Accounting Earnings Before Tax	Accounting Earnings After Tax	Cash Flow After-tax
0	0	0	- 1,000,000
1	300,000 - 100,000 - 100,000 = 100,000	(1-0.4)(100,000) = 60,000	(1-0.4) (300,000-100,000) + 40,000 = 160,000
2	100,000	60,000	160,000
3	100,000	60,000	160,000
4	100,000	60,000	160,000
5	100,000	60,000	160,000
6	100,000	60,000	160,000
7	100,000	60,000	160,000
8	100,000	60,000	160,000
9	100,000	60,000	160,000
10	100,000	60,000	160,000

Accounting earnings may not accurately reflect the actual CF timing.

Example. **Use after-tax cash flows.** Consider the following project (the cash flow is in thousands of dollars and tax rate is 50%):

Year	0	1	2	3	4	5
Investment	500					
Operating CF		0	100	300	300	300
- Depreciation		100	100	100	100	100
= Income		-100	0	200	200	200
- Tax		-50	0	100	100	100
= After-tax CF	-500	50	100	200	200	200
PV at 10%	-500	45.45	82.64	150.26	136.60	124.18

NPV = + $39.15.

Typically, there are timing differences between the accounting measure of earnings (Sales – Operating Costs) and cash flows.

Net Working Capital (NWC) = Inventory + A/R – A/P

Changes in Net Working Capital

Inventory: Cost of goods sold includes only the cost of items sold. When inventory is rising, the cost of goods sold understates cash outflows. When inventory is falling, cost of goods sold overstates cash outflows.

Accounts Receivable (A/R): Accounting sales may reflect sales that have not been paid for. Accounting sales understate cash inflows if the company is receiving payment for sales in past periods.

Accounts Payable (A/P) -- conceptually the reverse of A/R, representing payments to vendors. If the company is paying for goods or services received in past periods, accounting costs understate cash outflows.

Example. You run a chain of stores that sell sweaters. This quarter, you buy 1,000,000 sweaters at a price of $30.00 each. For the next two quarters, you sell 500,000 sweaters each quarter for $60.00 each. The corporate tax rate is 40%.

In million dollars, your cash flows are

Date	After Tax Profit	Inventory	Cash Flow
0	0	(1)(30) = 30	-30
1	(0.5)(60-30)(1-0.4) = 9	(0.5)(30) = 15	(0.5)(60) - (0.5)(60-30)(0.4) = 24
2	(0.5)(60-30)(1-0.4) = 9	0	(0.5)(60) - (0.5)(60-30)(0.4) = 24

Note:

Cash flow = Profit (after tax) – Change in Inventory

So far, we have shown that:

A project's discount rate (required rate of return or cost of capital) is the expected rate of return demanded by investors for the project

Discount rate(s) in general depend on the timing and risk of the project's cash flow(s)

Discount rates are usually different for different projects

It is in general incorrect to use a company-wide "cost of capital" to discount cash flows of all projects

What is the required rate of return on a project?

Simple case: single discount rate can be used for all cash flows of a project (the term structure of discount rates is flat)

General case: different discount rates for different cash flows

- the term structure of discount rates is not flat
- different pieces of cash flow at a given time have different risks

Use CAPM to estimate cost of capital (discount rate)

A project's required rate of return is determined by the project beta:

$$\bar{r}_{\text{project}} = r_F + \beta_{\text{project}} \left(\bar{r}_M - r_F \right)$$

What matters is the project beta, not the company beta!

What if project beta is unknown?

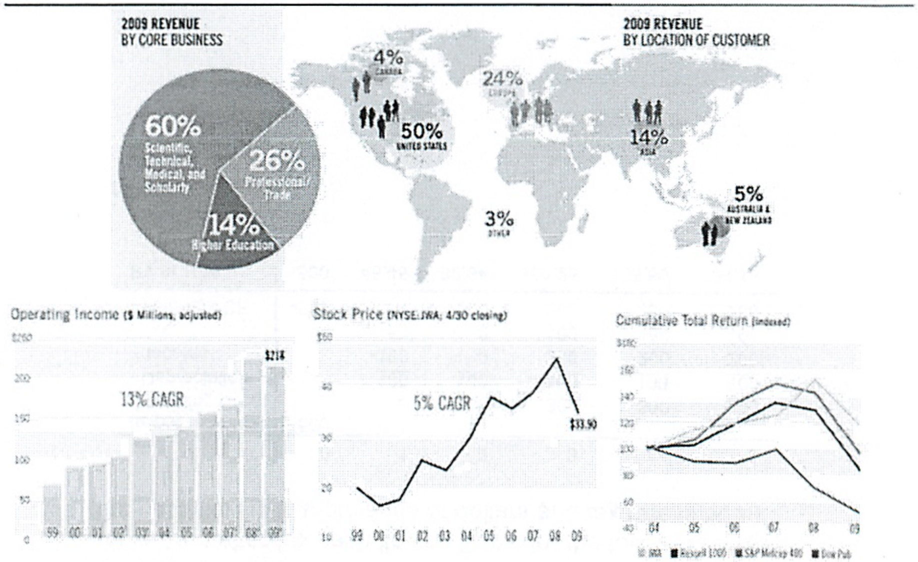
- Find a comparable "pure-play" company and use its beta
- Find comparable projects and use their cash flows to estimate beta
- Use fundamental analysis and judgment to guesstimate beta

Example. Bloomberg, a provider of financial data and analytics, is considering entering the publishing business (Bloomberg Press), and must evaluate the NPV of the estimated cash flow from this business. What cost of capital should it use for these NPV calculations?

Bloomberg should not use its own beta to discount Bloomberg Press cash flows

Bloomberg should use the beta of a publishing company (e.g., John Wiley & Sons)

What about using McGraw-Hill's beta?



Example (cont):

Beta of JW&S (from http://finance.yahoo.com): 0.80

Risk-free rate: 3%

Market risk premium: 6%

$$\bar{r}_{project} = r_F + \beta_{project}(\bar{r}_M - r_F)$$

$$\bar{r}_{project} = 0.03 + 0.80 \times 0.06 = 7.80\%$$

Use judgment in interpreting and adjusting these estimates

Estimates are merely approximations!

How good is the approximation?

Example. MSW Inc. is considering the introduction of a new product: Turbo-Widgets (TW).

TW were developed at an R&D cost of $1M over past 3 years

New machine to produce TW would cost $2M

New machine lasts for 15 years, with salvage value of $50,000

New machine can be depreciated linearly to $0 over 10 years

TW need to be painted; this can be done using excess capacity of the painting machine, which currently runs at a cost of $30,000 (regardless of how much it is used)

Operating costs: $40,000 per year

Sales: $400,000, but cannibalization would lead existing sales of regular widgets to decrease by $20,000

Net Working Capital (NWC): $250,000 needed over the life of the project

Tax rate: 34%

Opportunity cost of capital: 10%

Should MSW go ahead to produce TW?

1. Initial investment includes capital expenditure and NWC
2. R&D expense is a sunk cost
3. Depreciation is $2M/10 = $0.2M for first 10 years
4. Project should not be charged for painting-machine time
5. Project should be charged for cannibalization of regular widget sales
6. Salvage value is fully taxable since the book value at the end of year 10 is $0 (the machine cost has been fully depreciated)

The cash flows (in thousand dollars) are

Year	Cash Flow
0	- (2000+250) = -2250
1-10	(400-40-20)(1-0.34) + (200)(0.34) = 292.4
11-14	(400-40-20)(1-0.34) = 224.4
15	224.4 + (50)(1-0.34) + 250 = 507.4

NPV = - $57,617.

Often we have to decide on more than one project.

For mutually independent projects, apply NPV rule to each project

For projects dependent on each other (e.g., mutually exclusive), we have to compare their NPVs

Example. Potential demand for your product is projected to increase over time. If you start the project early, your competitors will catch up with you faster, by copying your idea. Your opportunity cost of capital is 10%. Denoting by FPV the project's NPV at the time of introduction, we have:

Year to Start	FPV	% Change in FPV	NPV
1	100	—	91
2	120	20	99
3	138	15	104
4	149	8	102

Before year 4, the return to waiting is larger that the opportunity cost of capital, 10%. As long as the growth rate of FPV remains below 10% after year 4, it is best to wait and introduce at the end of year 3.

In practice, investment rules other than NPV are also used:

- Payback Period
- Profitability Index (PI)
- Internal Rate of Return (IRR)
- And more ...

Firms use these rules because they were used historically and they may have worked (in combination with common sense) in the particular cases encountered by these firms.

These rules sometimes give the same answer as NPV, but in general they do not. We should be aware of their shortcomings and use NPV whenever possible.

The bottom line:

> The NPV rule dominates the alternative rules.

Payback period is the minimum length of time s such that the sum of net cash flows from a project becomes positive

$$CF_1 + CF_2 + \cdots + CF_S \geq -CF_0 = I_0$$

Decision Criterion Using Payback Period

For independent projects: Accept if s is less than or equal to some fixed threshold t*.

For mutually exclusive projects: Among all the projects having s ≤ t*, accept the one that has the minimum payback period.

Example. Let t* = 3. Consider the two independent projects with the following cash flows (in thousands):

	CF_0	CF_1	CF_2	CF_3	CF_4	CF_5	CF_6	t^*
Project 1	-100	20	40	30	10	40	60	4
Project 2	-100	10	10	80	5	10	10	3

Decision: Accept Project 2.

Payback period rule ignores cash flows after the payback period

It ignores the time value of money → discounting future cash flows

Example. (Cont) Suppose that the appropriate discount rate is a constant 10% per period. Then

$$NPV_1 = 39,315 \text{ and } NPV_2 = -7,270$$

But using the payback rule we accepted project 2 and not project 1!

Taking into account appropriate discounting, we have the discounted payback period, which is the minimum s so that

$$\frac{CF_1}{1+r} + \frac{CF_2}{(1+r)^2} + \cdots + \frac{CF_S}{(1+r)^S} \geq -CF_0$$

where r is the discount rate (cost of capital). (It still ignores the cash flows after the discounted payback period.)

A project's internal rate of return (IRR) is the number that satisfies

$$0 = CF_0 + \frac{CF_1}{(1+IRR)} + \frac{CF_2}{(1+IRR)^2} + \cdots \frac{CF_t}{(1+IRR)^t}$$

Decision Criterion Using IRR

For independent projects: Accept a project if its IRR is greater than some fixed IRR*, the threshold rate.

For mutually exclusive projects: Among the projects having IRR's greater than IRR* , accept the project with the highest IRR.

Example. Consider the following mutually exclusive projects:

	CF_0	CF_1	CF_2	CF_3	CF_4	CF_5	CF_6
Project 1	-100	20	40	30	10	40	60
Project 2	-100	10	10	80	5	10	10

Then, $IRR_1 = 21\%$ and $IRR_2 = 7\%$.

IRR rule leads to the same decisions as NPV if

1. Cash outflow occurs only at time 0
2. Only one project is under consideration
3. Opportunity cost of capital is the same for all periods
4. Threshold rate is set equal to opportunity cost of capital

1 Non-existence of IRR

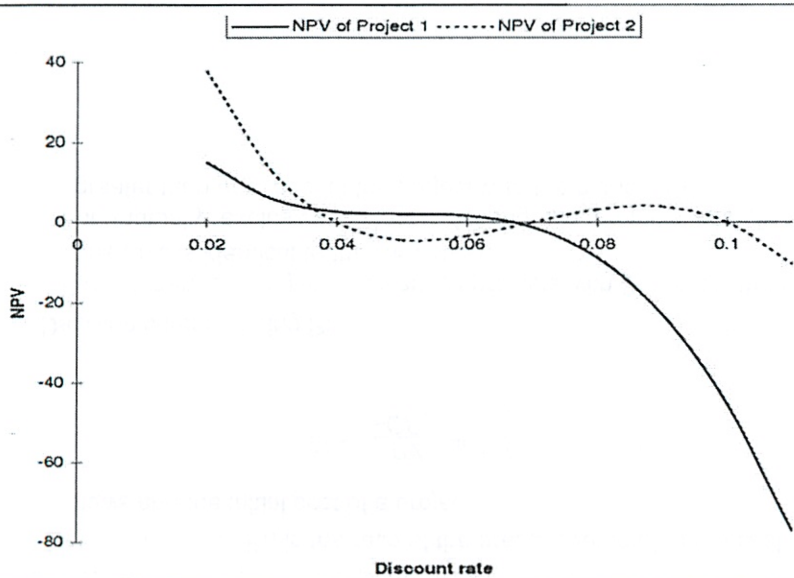
	CF_0	CF_1	CF_2
Project 1	-105	250	-150
Project 2	105	-250	150

No IRR exists for these two projects.

2. Multiple IRR's

	CF_0	CF_1	CF_2	CF_3
Project 1	-500,000	1,575,000	-1,653,750	578,815
Project 2	-500,000	1,605,000	-1,716,900	612,040

$IRR_1 = 7\%$ and $IRR_2 = \begin{cases} 4\% \\ 7\% \\ 10\% \end{cases}$



3. Project ranking using IRR for mutually exclusive projects:

a) Projects of different scale / size:

	CF_0	CF_1	IRR	NPV at 10%
Project 1	-10,000	20,000	100%	8,181.82
Project 2	-20,000	36,000	80%	12,727.27

A way around this problem is to use incremental CF:

See if lower investment (project 1) is a good idea

See if incremental investment (project 2) is a good idea.

	CF_0	CF_1	IRR	NPV at 10%
Project 1	-10,000	20,000	100%	8,181.82
Project 2	-20,000	36,000	80%	12,727.27
Project 2-1	-10,000	16,000	60%	4,545.45

b) Projects with different time patterns of cash flows:

CF_t	0	1	2	3	4	5	...	IRR	NPV at 10%
Project 1	-90	60	50	40	0	0	...	33.0%	35.92
Project 2	-90	18	18	18	18	18	...	20.0%	90.00
Project 2-1	0	-42	-32	-22	18	18	...	15.6%	54.08

Profitability index (PI) is the ratio of the present value of future cash flows and the initial cost of a project:

$$PI = \frac{PV}{-CF_0} = \frac{PV}{I_0}$$

Decision criterion using PI

For independent projects: Accept all projects with PI greater than one (this is identical to the NPV rule)

For mutually exclusive projects: Among the projects with PI greater than one, accept the project with the highest PI

Problems with PI

PI gives the same answer as NPV when

There is only one cash outflow, which is at time 0

Only one project is under consideration

PI scales projects by their initial investments. The scaling can lead to wrong answers in comparing mutually exclusive projects.

	CF_0	CF_1	IRR	NPV at 10%	PI at 10%
Project 1	-1,000	2,000	100%	818.18	1.82
Project 2	-2,000	3,600	80%	1,272.73	1.64
Project 2-1	-1,000	1,600	60%	454.55	1.45

Comparison of Methods Used by Large U.S. and Multinational Firms

	Large U.S. Firms	Multinationals	
	Percentage Using Each Method	Use as Primary Method	Use as Secondary Method
Payback Period	57%	5.0%	37.6%
IRR	76%	65.3	14.6
NPV	75%	16.5	30.0
Other	-	2.5	3.2

Historical Comparison of Primary use of Various Capital Budgeting Techniques

	1959	1964	1970	1975	1977	1979	1981
Payback Period	34%	24%	12%	15%	9%	10%	5.0%
IRR	19	38	57	37	54	60	65.3
NPV	-	-	-	26	10	14	16.5
IRR or NPV	19	38	57	63	64	74	81.8

Source: S. Ross, R. Westerfield, and B. Jordon, Fundamentals of Corporate Finance, McGraw-Hill Irwin, 2010, 9th ed.

1 Competitive Response:

- CF forecasts should consider the responses of competitors

2. Capital Rationing

3. Sources of Positive-NPV Projects:

- Short-run competitive advantage (right place at right time)
- Long-run competitive advantage
  - Patent
  - Technology
  - economies of scale, etc.
- Noise

In capital investment decisions, we often face situations involving strategic options.

Common and important options in capital investments include:

- The option to wait before investing
- The option to make follow-up investments
- The option to abandon a project
- The option to vary output or production methods → modify/manage

Two key elements in strategic options and their valuation are:

1. New information arrives over time
2. Decisions can be made after receiving new information.

Example. Real options in follow-up projects. In 1990, MC Inc. considers entering the PC business:

- R&D has come up with model A --- a new PC model
- Cash Flows of model A, if introduced, are as follows

	1990	1991	1992	1993	1994	1995
Investment ($M)	-450	-50	-100	-100	125	125
(R&D, plant, WC)						
Operating CF ($M)		140	159	259	185	
Net CF	-450	90	59	159	310	125

NPV at 20% is -$46 million. However,

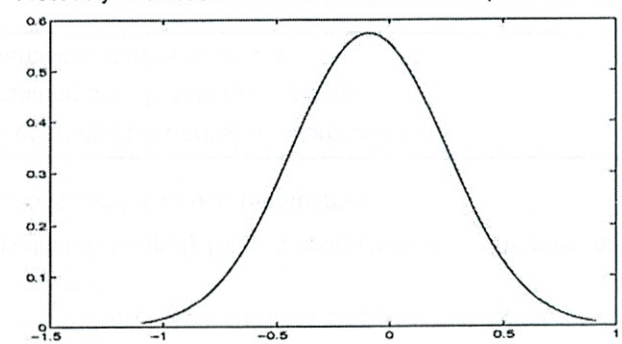
- Development and production of model A would allow MC Inc. to introduce model B in 1993
- Expected CFs from model B are twice that of model A
- In expectation, model B is a loser too
- But there are scenarios in which model B really pays off

Different Scenarios	PV of model B ($M)
Benchmark scenario	-92
Initial Investment reduced by 30%	178
Sales increase by 40%	368
Profit margin increases by 50%	302

Should MC Inc. start model A?

The expected value of model B is -$92 million. Could this prospect justify the $46 million sacrifice to enter the market with model A?

Probability Distribution of NPV in 1993 for Model B (in billions of dollars)



- Starting model B in 1993 is an option
- So long as MC can abandon the business in 1993, only the right-hand-side of the distribution is relevant
- NPV of the right-hand-side is huge even if the chance of ending up there is less than 50%

Assume:

Model B decision has to be made in 1993
Entry in 1993 with Model A is prohibitively expensive
MC has the option to stop in 1993 (possible loss is limited)
Investment needed for model B is $900M (twice that of A)
PV of operating profits from model B is $468 million in 1990
PV evolves with annual standard deviation of 35%

The opportunity to invest in model B is like a 3-year call option on an asset worth $468 million now with exercise price $900 million!

Using Black-Scholes formula:

Value of Call = $55 million

Total NPV of model-A ($M):

	A	A+B
DCF	-46	-99
Option value	55	
Total value	9	

- Naive DCF analysis tends to under-estimate the value of strategic options:
  - Timing of projects is an option (American call)
  - Follow-up on projects are options (American call)
  - Termination of projects are options (American put)
  - Expansion or contraction of production are options (conversion options)
- It is difficult to apply DCF to option valuation (the point of B-S!)
- Options can be valued (sometimes)

Think of strategic planning as a process of :
1. Acquiring and disposing of options
2. Exercising options optimally

NPV rule
Cash flows from capital investments
Discount rates
Project interaction
Alternative capital budgeting rules
Real options

15.401 ~~____~~ Capital Budgeting

15.402 in 2 days

Prof's old work

Rules to live by

1. Net Present Value = PV Inflows − PV outflows
2. Cash Flows not accounting flows
3. Incremental Cash Flows − IFF
4. If you accept, what cash flows are created?
5. Depeciation expense is a non-cash charge
6. Net working capital too ← Recievables, Inventory, ~~Payables~~ Payables
7. Good projects create value

Were looking at companies

Now looking at projects

Forcast cash flows, discount ~~the~~ risk of decision

$$\text{Net cash flows} = CF_0 + \frac{CF_1}{(1+r_1)} + \ldots + \frac{CF_t}{(1+r_t)^t}$$

②

Stock rises when company announceses ^unexpected capital budget

<u>NPV</u>

- "beatiful model"
- take all (+) projects
- if mutually exclusive - do one w/ higest PV

- need to look at

1. Cash flows
2. Discount rates
3. Strategic options

- all forecasts from various ~~companies~~ departments
    - do sensitivity analysis

- easy if adding another factory
- new projects difficult to estimate
- depends on competitors

(3)

Use <u>after tax</u> cash flows

Only care about the incremental/new cash flows

Forget sunk costs ~ bygones

Include investment in working capital as a capital expenditure
- recievables - you give loans to your custs
- inventory
- payables - custs lending you $

Include the opp. cost of using existing facilities
- like if could sell it, give sale price

Think of projects standing alone

Cash Flow = Project Cash Inflows − Project Cash Outflows

= Operating Revenues − Op Ex w/o depreciation − Capital Expenditures − Taxes

τ = effective tax rate

Taxes = τ · Op Profit − τ · Depreciation

(add depreciation back in)

(9)

Depreciation is tax shield
- it reduces your tax bill

Example

Cash Revenue	200
- Cash Exp	-100
- Dep Exp	- 50
Income b/f taxes	50
- Income taxes (40%)	-20
Net income	30
+ Dep. Ex	+50
After tax CF	80

-20 ← ↑ op profit - ↑ depreciation

$.4(200-100) - .4(50)$

$40 - 20$

20

+50 ← add back in

Depreciation sooner = tax break sooner
- Can manipulate to adjust employment

Net Working Capital (NWC) = Inv + A/R - AP

⑤

Example on slides

Project discount rate ≠ company-wide cost of capital

- differs for each biz line
- is the project more or less risky than rest of co?

$$\bar{r}_{project} = r_F + \beta_{project} (\bar{r}_m - r_f)$$

↑ not $\beta_{company}$

- if project β is unknown, try to find a comparable "pure play" co
- or make it up on your own

MIT Sloan School of Management

Finance Theory I | 15.401
Craig Stephenson | Spring 2011

Problem Set 7: CAPM and Capital Budgeting
(Due: Friday, May 6th, by 5:00 p.m.)

Unless specified as an "Excel Problem", all of these problems can (and should) be solved with a pencil, paper, and a simple calculator. Do not use Excel (except to check your work, if you want). Show your work cleanly and write out the formulas that you used to solve the problems. Circle your final answers.

Problem 1 (CAPM)

Here are some of the beta coefficients we discussed in class on April 25. Use these betas to answer all parts of Problem 1.

PepsiCo	0.49
Hewlett Packard	0.97
Micron Technology	1.95
Nordstrom	1.50
JP Morgan Chase	1.28

Part 1 – The current yield to maturity on 10-year Treasury bonds is 3.63%. Capital market history over the past 85 years shows the mean rate of return on the value weighted stock market exceeds the mean rate of return on long-term Treasury bonds by 6.10%. Given these data, what is the required rate of return for each of the 5 listed firms?

Part 2 – Assume inflation expectations increase interest rates uniformly across all maturities, and the yield on 10-year Treasury bonds increases to 5.25%. If there is no change in market risk premia, what is the required rate of return for each of the 5 listed firms?

Part 3 – Assume instead than an unanticipated information event increases investors' fears and worries, so the price of risk increases; investors require higher premiums to hold risky assets, and the price of risk increases by 20%. At the same time since investors are more worried about risky assets, the demand for risk-free assets increases, so the yield on 10-year Treasury bonds decreases to 3.30%. Under these conditions, what is the required rate of return for each of the 5 listed firms?

Part 4 – The new CEO of PepsiCo has decided that PepsiCo should remain a drinks and chips company, but chips now means potato chips and memory chips, and PepsiCo purchases Micron Technology. The market value of equity of PepsiCo is $109 billion, and Micron Technology's market value of equity is $12 billion. Going back to the data presented in Part 1, what is the required return of the "new" PepsiCo, which consists of drinks, snack chips, and silicon chips?

Problem 2 (Capital Budgeting)

Your company's operations and research departments have proposed a capital investment project code named "Marin," which they believe will generate significant amounts of revenue and cash flow over its useful life. Ops and R&D have already spent $5 million to define and develop the project, and they estimate the capital expenditures required to launch Marin will be $47 million, all paid at the beginning. Expected sales and operating costs are expected to be $20 million and $12 million, respectively, in the first year. The risk adjusted cost of capital for Marin is 9%, and the income tax rate is 25%. Your task is to decide whether the Marin project should be accepted or rejected.

Part 1 – Assume the capital expenditures are depreciated over 10 years using straight-line depreciation to zero salvage value, and the expected useful life of the project is also 10 years. If sales and operating costs both will not change during the project's life, should Marin be accepted or rejected?

Part 2 – Assume instead that the capital expenditures are depreciated over 5 years using straight-line depreciation to zero salvage value, and the expected useful life of the project remains 10 years. If sales and operating costs both will not change during the project's life, should Marin be accepted or rejected?

Part 3 – Assume the capital expenditures are depreciated over 10 years using straight-line depreciation to zero salvage value, but now the project is expected to last forever. If sales and operating costs both increase at a 5% annual rate forever, should Marin be accepted or rejected?

Part 4 – Assume the capital expenditures are depreciated over 10 years using straight-line depreciation to zero salvage value, and the project is expected to last forever. If sales increase at a 3% annual rate forever, but operating costs increase at a 6% annual rate forever, should Marin be accepted or rejected?

Michael Plasmeier   15.401 P-Set 7   5/1

7/10

1. CAPM

a) What is the required rate of return to beat the risk free asset?
(risk-adjusted)

$$r_i - r_f = \beta_{im}(\bar{r}_m - r_f)$$

(r_f: risk free return; $\bar{r}_m - r_f$: risk premium)

$$r_i = B_{im}(E[R_m] - r_f) + r_f$$

Pepsi $= .49(6.1\% - 3.63\%) + 3.63\%$

$= .048$

$= 4.84\%$

HP $= .97(6.1\% - 3.63\%) + 3.63\%$

$= 6.02\%$

Micron $= 8.44\%$ ✗ -.5

Nordstrom $= 7.33\%$

JP Morgan $= 6.79\%$

②

b) Assume r_f now = 5.25%

Pepsi = .49(6.1% − 5.25%) + 5.25%

= 5.66%

HP = .97(6.1% − 5.25%) + 5.25%

= 6.07%

Micron = 6.9%

Nordstrom = 6.52%

JPMorgan = 6.33%

✗ −.5

Why is it lower?

? Closer together, so less risk

? Says nothing about risk

- well smaller risk premium needed

- less add. return needed

(3)

c) Assume price of risk ↑ 20%

And r_f = 3.3%

risk premium = $\bar{r}_m - r_f$ + .2

~~6.1% - 3.3% + 20%~~

20% of that

= 1.2 (6.1% - 3.3%)

= 1.2 (2.8%)

= 3.36%

Pepsi = .49 (3.36%) + 3.3%

= 4.94%

HP = .97 (3.36%) + 3.3%

= 6.55%

Micron = 9.85%

Nordstrom = 8.34%

JP Morgan = 7.6%

-.5

(9)

d) Pepsi buys Micron

What is new required return?

Treat as portfolio of 2

$\sum$ of weighted assets

$W_{pepsi} = \frac{109}{109+12} = 90\%$ $\qquad \frac{12}{109+12} = 10\%$

β_{port} is weight avg of pieces

$\beta = .9 \cdot .44 + .1 \cdot 1.85$

$= .636$ ✓

So required return from a)

$= .636\,(6.1\% - 3.63\%) + 3.63\%$

$= 5.2\%$

(5)

2. Capital Budgeting

"Marvin"

\$ 5 million spent

\$ 47 " needed

Year 1 \$ 20 mill sales \$ 12 mill op cost

Cost of capital (risk adjusted) = 9%

Income tax rate = 25%

a) Assume Capex depreciated over 10 years

We don't care about depreciation - except for tax purposes

in millions

$$= -47 + \sum_{t=1}^{10} \frac{20 - (20 - 12 - (.1 \cdot (47+5))) .25 - 12}{(1+.09)^t}$$

$= \sim .151$ ✗

Not profitable slightly

⑥

b) Now can depreciate over 5 years

$$= -47 + \sum_{t=1}^{5} \frac{20 - (20 - 12 - .2 \cdot 52) .25 - 12}{(1+.09)^t} + \sum_{t=6}^{10} \frac{20 - (20-12) .25 - 12}{(1+.09)^t}$$

$= 1.61$ million ✗

project now profitable!

thanks to gov's new depreciation

c) Depreciation 10 years again

Lasts forever

Sales + costs ↑ 5% / year

$$= -47 + \sum_{t=1}^{10} \frac{20(1+.05)^t - (20(1+.05)^t - 12(1+.05)^t - .1 \cdot 52) .25 - 12(1.05)^t}{(1.09)^t}$$

$$+ \sum_{t=11}^{\infty} \frac{20(1+.05)^t - (20(1.05)^t - 12(1.05)^t) .25 - 12(1.05)^t}{(1.09)^t}$$

$= 118.843$ ✓

a good deal!

(7):

d) Now sales ↑ 3% forever

op cost ↑ 6% forever

$$= -47 + \sum_{t=1}^{10} \frac{20(1+.03)^t - (20(1.03)^t - 12(1.06)^t - 1.52).25 - 12(1.06)^t}{(1.09)^t}$$

$$+ \sum_{t=11}^{\infty} \frac{20(1.03)^t - (20(1.03)^t - 12(1.06)^t).25 - 12(1.06)^t}{(1.09)^t}$$

$= -99.157$

Bad deal!

15.401 - Section D

PS 7 Solutions

Problem 1 - Part 1

$R_f = 3.63\%$

$E(R_m) - R_f = 6.10\%$

	Beta Coeff
PepsiCo	0.49
H-P	0.97
MicronT	1.95
Nordstrom	1.50
JPM chose	1.28

Required Returns -

PepsiCo = 3.63% + 0.49 × 6.10% = 6.62%

H-P = 9.55%

Micron T = 15.53%

Nordstrom = 12.78%

JPM Chase = 11.44%

Only β varies across the 5 companies.

Problem 1 - Part 2

The Yield Curve shifts ↑ so $R_f = 5.25\%$, but MRP & Betas do not change. Now the required returns are:

PepsiCo = 5.25% + 0.49 × 6.10% = 8.24%

H-P = 11.17%

Micron T = 17.15%

Nordstrom = 14.40%

JPM Chase = 13.06%

Problem 1 - Part 3

Investors are more risk averse, so the price of risk, the MRP, $E(R_m) - R_f$ increases by 20%, $E(R_m) - R_f = 6.10\% \times 1.20 = 7.32\%$.
Also R_f falls to 3.30% as there is more demand for risk-free assets. The required Returns are:

PepsiCo $= 3.30\% + 0.49 \times 7.32\% = 6.89\%$

H-P = 10.40%

Micron T = 17.57%

Nordstrom = 14.28%

JPM chase = 12.67%

Problem 1 - Part 4

The beta of a portfolio is the weighted average beta of its component pieces. The "New" PepsiCo has a Beta of

$$= 0.49 \times \frac{\$109}{\$109 + \$12} + 1.95 \times \frac{\$12}{\$109 + \$12}$$

$$= .4414 + .1934$$

$$.6348$$

And the required return of the "New" PepsiCo is

$$3.63\% + 0.6348 \times 6.10\%$$

$$7.50\%$$

More risk, ↑ Beta, ↑ Required Return

Problem 2 - Part 1

Investment at Year 0 = -\$47,000,000

Revenue in Years 1→10 = \$20,000,000

Operating Costs Years 1→10 = -\$12,000,000

Depreciation Years 1→10 = $\frac{-\$47,000,000}{10}$ = -\$4,700,000

t_c = 25%

required return = 9%

- -

Project Cash Flows are:

Year 0 = -\$47,000,000

Years 1→10 = (\$20,000,000 - \$12,000,000) × (1 - .25)

+ (\$4,700,000) × .25

= \$6,000,000 + \$1,175,000

= \$7,175,000

The Net Present Value of this cash flow stream at a 9% required return is

-\$953,306

The NPV of this project is negative, so Project Marin should be rejected.

Problem 2 - Part 2

The only change from Part 1 is the depreciation expense is over 5 years - quicker / accelerated depreciation gives the tax benefit sooner, so NPV rises.

Depreciation Years 1 - 5 = $\frac{-\$47{,}000{,}000}{5} = \$9{,}400{,}000$

Project Cash Flows are:

Year 0 = - $47,000,000

Years 1 → 5 = ($20,000,000 - $12,000,000) × (1 - .25) + $9,400,000 × .25
= $6,000,000 + $2,350,000
= $8,350,000

Years 6 → 10 = ($20,000,000 - $12,000,000) × (1 - .25)
= $6,000,000

The Net Present Value of this cash flow stream at a 9% required return is

+ $646,627

The NPV of this project is positive, so Project Marin should be accepted.

Problem 2 - Part 3

The change from Part I is the project is expected to last forever, and both sales and operating costs increase at a 5% constant perpetual rate

Project Cash Flows are:

Year 0 = - $47,000,000

Years 1 → ∞ = ($20,000,000 - $12,000,000) x (1-.25) growing at 5% annual rate
= $6,000,000 growing

Years 1 → 10 = $4,700,000 x .25
= $1,175,000

PV initial investment = - $47,000,000

PV op CF 1 → ∞ = $\frac{\$6{,}000{,}000}{.09 - .05}$ = + $150,000,000

PV deprec 1 → 10 = + $7,540,747.80

Σ PV CF = NPV = ⊕ $110,540,747.80

The NPV of Project Marin is huge and positive (infinite life, growing CF), so the project should be accepted.

Problem 2 - Part 4

The change from Part 3 is sales grow at a 3% constant & perpetual rate, but costs grow faster at a 6% constant & perpetual rate. Still infinite life, still 10 years depreciation.

Project Cash Flows are:

Year 0 = - $47,000,000

Years 1 → ∞ (Revenue) = $20,000,000 x (1 - .25)
= $15,000,000 growing at 3% annual rate

Years 1 → ∞ (op Costs) = - $12,000,000 x (1 - .25)
= - $9,000,000 growing at 6% annual rate

Years 1 → 10 (depreciation) = $4,700,000 x .25
= $1,175,000

PV Initial Investment = - $47,000,000

PV Revenue = $+\dfrac{\$15{,}000{,}000}{.09 - .03} = +\$250{,}000{,}000$

PV Op Costs = $\dfrac{-9{,}000{,}000}{.09 - .06} = -\$300{,}000{,}000$

PV depreciation = + $7,540,747.80

Σ PV CF = NPV = - $89,459,252.20

The NPV of Project Marin is now huge and Negative, driven by rate of cost increases, the project should be rejected.

Example. MSW Inc. is considering the introduction of a new product: Turbo-Widgets (TW).

- TW were developed at an R&D cost of $1M over past 3 years
- New machine to produce TW would cost $2M
- New machine lasts for 15 years, with salvage value of $50,000
- New machine can be depreciated linearly to $0 over 10 years
- TW need to be painted; this can be done using excess capacity of the painting machine, which currently runs at a cost of $30,000 (regardless of how much it is used)
- Operating costs: $40,000 per year
- Sales: $400,000, but cannibalization would lead existing sales of regular widgets to decrease by $20,000
- Net Working Capital (NWC): $250,000 needed over the life of the project
- Tax rate: 34%
- Opportunity cost of capital: 10%

Should MSW go ahead to produce TW?

1. Initial investment includes capital expenditure and NWC
2. R&D expense is a sunk cost
3. Depreciation is $2M/10 = $0.2M for first 10 years
4. Project should not be charged for painting-machine time
5. Project should be charged for cannibalization of regular widget sales
6. Salvage value is fully taxable since the book value at the end of year 10 is $0 (the machine cost has been fully depreciated)

The cash flows (in thousand dollars) are

Year	Cash Flow
0	- (2000+250) = -2250
1-10	(400-40-20)(1-0.34) + (200)(0.34) = 292.4
11-14	(400-40-20)(1-0.34) = 224.4
15	224.4 + (50)(1-0.34) + 250 = 507.4

NPV = - $57,617.

Excel very good for this w/ complex problems

Ops + Research people always want their project approved

MSW case

Sunk costs don't count
- its been spent

Only $ that will spent if project is approved

At end of year 15 – sell for $50,000
Project is over

The R+D dept just made that up
- have to pay tax

Since it was fully depreciated – its a gain

If it was year 8, it would be a loss, since not fully depreciate

Straight-line depreciation = $\frac{\text{Cost}}{\text{Life}}$ = \$200,000/year for 10 years

Don't count the "extra" cost of running something already
- Doesn't change

No cost inflation in these #

②

Sales count - minus canabalization

- Happens often

If your competitor would have stolen the sales and you had to keep up, then its not canabalization

Discussed a lot in financial press

Net working capital - needed to be borrowed all the time - inventory, recivables, payables

Just borrow in year 0

If prices rise the value of this ↑

= AR + Inv - AP

For some sites NWC is negitive (airlines)

Finally paid off in last year

- less inv
- AR paid off

So just have it set aside in years between

Are we going to get all of the inv back

Not taxed - but discounted

Could put inflation

(3) Tax = 34%

Depreciation is just for tax purposes

Another tax break

All projects come to an end

Final year - still sell!

Year 0	Year 1-10	Year 11-14	Year 15
-2,000,000	400,000	+ 400,000	400,000
-250,000	-20,000	- 40,000	-20,000
Ø	-40,000	-20,000	-40,000
-2,250,000	340,000 op profit	340,000 op profit	340,000
	·(1-.34)	·(1-.34)	·(1-.34)
	224,400 net after tax cash inflow	224,400	224,400

Year 15:

+250,000 recovery

+50,000 sales price

- Ø Tax basis for depreciation
  - reduces gain

+50,000 gain on sale

·.34

-17,000

$200{,}000 \cdot .34 = 68{,}000$

depreciation tax shield "cash saved"

Discount to each year

(9) Can do on Excel

Need to be real careful what they say

- Mon Efficent Markets
- Wed Finish Efficent Markets + Other
- Fri No Classes, recitation

Often mutually ~~exclusive~~ ind

But sometimes interaction

Must consider what to do

Can delay

Other decision rules

- Payback Period
- Profitability Index (PI)
- Internal Rate of Return
- ...

People like to see Payback

- how quickly do I get my $ back

(5)

When is the project "paid off"

But could make big errors

- no time value of $

- ignores cash flow after payback

May deal w/ risk + uncertainty

But project may be wrong

Could use discounted payback period

- But people don't use

Internal Rate of Return

$$0 = CF_0 + \frac{CF_1}{(1+IRR)} + \frac{CF_2}{(1+IRR)^2} + \dots + \frac{CF_t}{(1+IRR)^t}$$

Looks for i so NPV = 0

"Investing at a __% return"

Noting but a search process

Can have multiple!

Have problems w/ scale and timing

Just choose higher NPV

(6)

Profitability Index

- never seen used
- if >1 then project good

Surveys of which methods people use

Company use all 3

Capital Rationing

- more attractive projects than funds to invest

SR Comp advantage - luck, timing

LR Comp advantage
- patents
- tech
- etc

Real Options

Often face strategic decisions

- can wait before investing
- can make follow-up investments
- can abandon
- can vary output → modify/manage

(7)

Don't just launch + watch!

Often lanch to get into market

Have to launch A to get B

After lanching A, can reevaluate B

- Cuts off dist for B
- Since can pull the plug if looks neg
- So good even though overall chance of sucess < 50%
  - Since can see what it will be ahead of time

Like Pixar movie sequals

Can put in Black-Sholes option pricing

- to find value of option
- the more time you have, the less uncertainty, the more its worth

Options are really embedded everywhere

You can truncate loser projects

15.401 Recitation

8: Capital Budgeting

Learning Objectives

- Review of Concepts
  - NPV
  - Payback Period
  - IRR
  - Profitability index
- Examples
  - Bart's Super-Widget*

* Bart's Super-Widget by Bart Raeymaekers

Review: capital budgeting

- Decision:
  - Accept or reject a project
  - Compare two projects
- Decision rule:
  - NPV, IRR, payback period, etc.
- Information
  - Cash flow projection
  - Risk projection
  - Tax regulation

Review: NPV

- **Net present value (NPV)** of a project is

$$NPV \equiv \sum_{t=0}^{\infty} \frac{CF_t}{(1+r_t)^t} > 0$$

- Decision should be based on **after-tax cash flow** instead of **accounting earnings.**
- Operating profit = operating revenue – operating expenses without depreciation
- CF = (1 – τ) x operating profit – capital expenditure + τ x depreciation

(τ is the tax rate)

Review: NPV

- Decision rule:
  - Independent projects: take all projects with NPV>0
  - Mutually exclusive projects: take projects with the highest NPVs
- The NPV rule **dominates all other rules** because it takes into account the **maximum amount of information,** including timing of all cash flows and risks, and makes the correct decision based on value creation.

Review: payback period

- The **payback period** is the minimum *T* such that

$$\sum_{t=1}^{T} CF_t \geq CF_0 = I_0$$

- T is the minimum number of period required to "recover" the initial investment, I_0.
- Decision rule:
  - Independent projects: take all projects with a payback period less than a fixed threshold *T**.
  - Mutually exclusive projects: take the project with the lowest payback period.

Review: payback period

- Pro:
  - Easy to calculate
- Con:
  - Ignores cash flows after the payback period
  - Ignores time value of money
- Discounted payback period: minimum T such that

$$\sum_{t=1}^{T} \frac{CF_t}{(1+r_t)^t} \geq CF_0 = I_0$$

  - Problem: still ignores cash flows after the payback period

Review: IRR

- The **internal rate of return (IRR)** is the discount rate that satisfies

$$0 = \sum_{t=0}^{\infty} \frac{CF_t}{(1+IRR)^t}$$

- IRR is the implied rate of return of the project.
- Decision rule:
  - Independent projects: take the projects with IRR > r*, where r* is the required rate of return.
  - Mutually exclusive projects: take the project with the highest IRR (provided it is greater than r*).

Review: IRR

- IRR gives the same decision as NPV if
  - Cash outflow occurs only at time 0
  - Only one project is under consideration
  - Required cost of capital is the same for all periods
  - Threshold rate is set to the required cost of capital
- Potential problem:
  - IRR may not exist
  - There may be multiple IRRs for a single cash flow.
  - IRR rule gives the wrong decision for mutually exclusive projects.

Review: profitability index

- The profitability index of a project is

$$PI = \frac{1}{I_0} \sum_{t=1}^{\infty} \frac{CF_t}{(1+r_t)^t}$$

- Decision rule:
  - Independent projects: take all projects with PI > 1.
  - Mutually exclusive projects: take the project with the highest PI.
- PI gives the same decision as NPV if
  - Cash outflow occurs only at time 0
  - There is only one project under consideration.

Example: Bart's Super-Widget

- **Project overview:**
  - Bart Co., a profitable widget maker, has developed an innovative new product called the Super-Widget. The company has invested $300,000 in R&D to develop the product and expects that it will capture a large share of the market.
- **Capital requirement:**
  - Bart Co. will have to invest $750,000 in new equipment. The machines have a useful life of 5 years, with an expected salvage value of $0.

Example: Bart's Super-Widget

- **Revenue projection:**
  - Over the next five years, unit sales are expected to be (5, 8, 12, 10, 6) thousand units.
  - Prices in the first year will be $480, and then will grow 2% annually.
- **Operating expenses:**
  - Sales and administrative costs will be $150,000/year.
  - Production costs will be $500/unit in the first year, but will decline by 8% every year thereafter.
  - The tax rate is 35% and the after-tax cost of capital is 12%.

Example: Bart's Super-Widget

❑ Revenue and Cost

	t=1	2	3	4	5
Revenue					
Units	5,000	8,000	12,000	10,000	6,000
Price/Unit	480	490	499	509	520
Total	2,400,000	3,916,800	5,992,704	5,093,798	3,117,405
Expenses					
SG&A	150,000	150,000	150,000	150,000	150,000
Cost/Unit	500	460	423	389	358
Total	(2,650,000)	(3,830,000)	(5,228,400)	(4,043,440)	(2,299,179)
Op. Profit	**(250,000)**	**86,800**	**764,304**	**1,050,358**	**818,226**

Example: Bart's Super-Widget

❑ Depreciation and Tax

	t=1	2	3	4	5
Op. Profit	(250,000)	86,800	764,304	1,050,358	818,226
Depreciation	(150,000)	(150,000)	(150,000)	(150,000)	(150,000)
EBIT	(400,000)	(63,200)	614,304	900,358	668,226
Taxes @35%	140,000	22,120	(215,006)	(315,125)	(233,879)
Net income	**(260,000)**	**(41,080)**	**399,298**	**585,233**	**434,347**

Example: Bart's Super-Widget

❑ Cash Flow

	t=0	1	2	3	4	5
Net income		(260,000)	(41,080)	399,298	585,233	434,347
CAPEX	(750,000)					
Cash flow	(750,000)	(110,000)	108,920	549,298	735,233	584,347
PV @ 12%	(750,000)	(98,214)	86,830	390,979	467,254	331,574
NPV	**$428,423**					

NPV @ 16% = 273,307

Example: Bart's Super-Widget

❑ Reminder:

- CF = after-tax operating income + depreciation tax shield – capital expenditure

  = $(1 - \tau)$ x operating income + τ x depreciation – capital expenditure
- The accounting net income is taxed even if it is negative.
- Depreciation is not a cash flow but reduces taxes.

15.401 Finance Theory I

Craig Stephenson

MIT Sloan School of Management

Lecture 11: Efficient Market Hypothesis

Did not attend lecture

Math afterward

The Efficient Market Hypothesis (EMH)

Implications of EMH

Supportive evidence to EMH

Challenges to EMH

Readings:

Brealey, Myers and Allen, Chapter 13

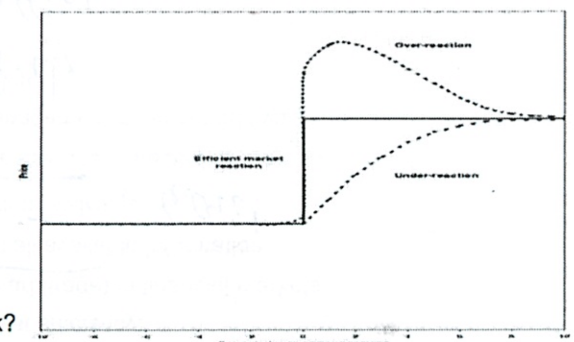
Bodie, Kane and Markus, Chapter 11

Example. Merck announces a new allergy drug to prevent hay-fever. How should Merck's share price react to this news?

Increase immediately to a new equilibrium level

Increase gradually to the new equilibrium level

First over-shoot and then settle back to new equilibrium level



What do you think?

Efficient Market Hypothesis: Market prices of securities reflect all available information about their value.

A precise definition of EMH needs to answer two questions:

1. What is "all available information"?
2. What does it mean to "reflect all available information"?

Answer:

1. All available information includes:
   - Past prices → Weak form
   - Public information (prices, news, ...) → Semi-Strong Form
   - All information including inside information → Strong Form
2. "Prices reflect all available information" means that financial transactions at market prices, using the available information, are zero NPV activities.

5/6

Implications of market efficiency:

No free lunch (no arbitrage) in financial markets

Prices fully reflect all available information

Prices follow random walks ← 6.042!

Trade-off between risk and expected return

"Active" asset management does not add value

Supposedly

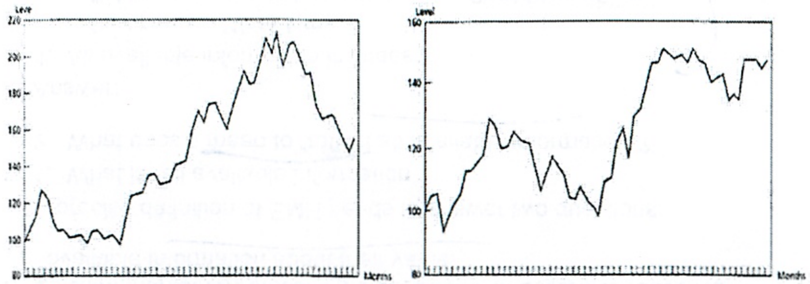
if efficient

1. Weak form of EMH is supported by the data

Technical trading rules are not consistently profitable.

S&P 500 Index (1980-1984) versus Coin-tossing

Source: R. Brealey and S. Myers, Principles of Corporate Finance.



Serial correlation in daily stock returns is close to zero

Serial Correlation of Daily Returns on Nine Stock Markets

Source: B. Solnik, "A Note on the Validity of the Random Walk for European Stock Prices." *Journal of Finance* (December 1973).

USA	0.03	UK	0.08
France	-0.01	Italy	-0.02
Germany	0.08	Holland	0.03
Belgium	-0.02	Switzerland	0.01
Sweden	0.06		

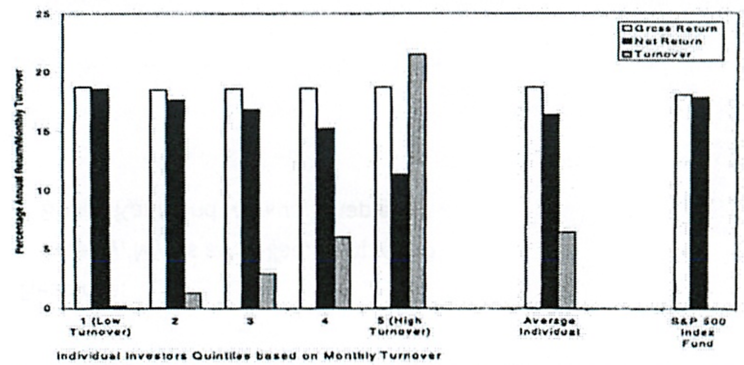


Figure 1. Monthly turnover and annual performance of individual investors. The white bar (black bar) represents the gross (net) annualized geometric mean return for February 1991 through January 1997 for individual investor quintiles based on monthly turnover, the average individual investor, and the S&P 500. The net return on the S&P 500 Index Fund is that earned by the Vanguard Index 500. The gray bar represents the monthly turnover.

(From B. Barber and T. Odean, Journal of Finance, 2000, 773-806.)

Example. Gender Issues in finance

	Single Men	Women	Difference
Average turnover	84.6%	50.6%	34.0%
Abnormal gross return	-0.89%	-0.35%	-0.54%
Abnormal net return	-2.90%	-1.45%	-1.45%

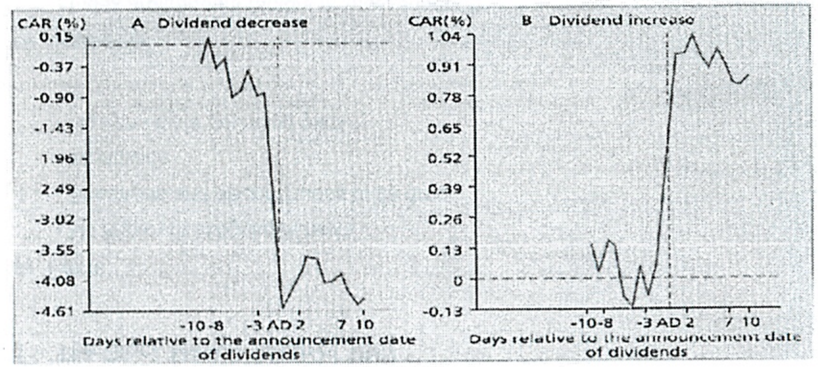
	Married Men	Women	Difference
Average turnover	73.3%	52.9%	23.4%
Abnormal gross return	-0.82%	-0.60%	-0.22%
Abnormal net return	-2.57%	-1.85%	-0.72%

(From B. Barber and T. Odean, Quarterly Journal of Economics, 2001, 261-292.)

2. Semi-strong form of EMH is generally supported by the data

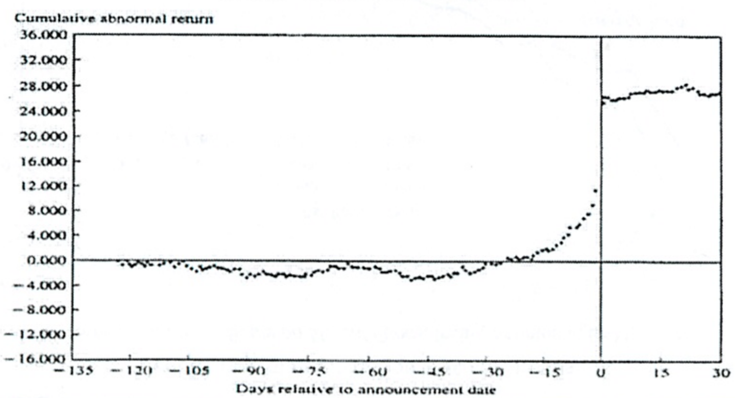
Prices react to news quickly (corporate actions, accounting changes ...

Cumulative Abnormal Returns (CAR) before and after Dividend Announcements



Cumulative Abnormal Returns (CAR) before and after Takeover Attempts: Target Companies

Source: A. Keown and J. Pinkerton, "Merger Announcements and Insider Trading Activity." *Journal of Finance* (1981).



3. Strong-form of EMH has mixed evidence:

Money managers cannot consistently outperform the market.

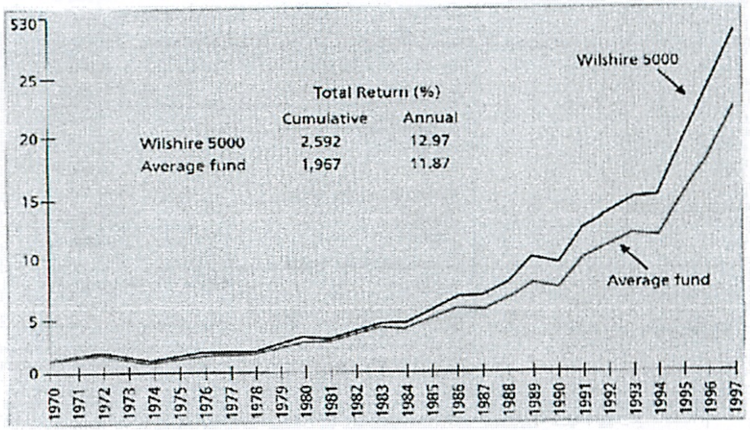
Mutual Fund Performance (Gross of Expenses)

Source: M. Jensen, "Risks, the Pricing of Capital Assets, and the Evaluation of Investment Performance." *Journal of Business* (April 1969).

10 years 1955 – 1964

Performance of Average Equity Mutual Funds

Source: J. Bogle, *Bogle on Mutual Funds*, Irwin (reprinted in BKM).



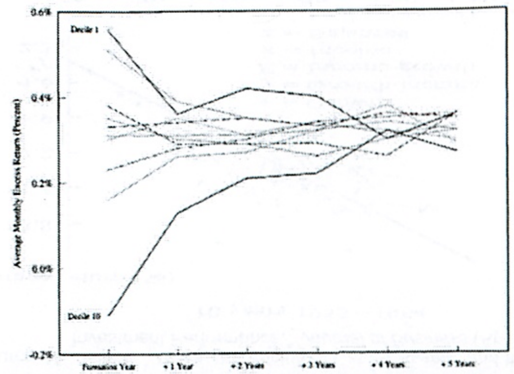


Figure 2. Post-formation returns on portfolios of mutual funds sorted on lagged one-year return. In each calendar year from 1962 to 1987, funds are ranked into equal-weight decile portfolios based on one-year return. The lines in the graph represent the excess returns on the decile portfolios in the year subsequent to initial ranking (the "formation" year) and in each of the next five years after formation. Funds with the highest one-year return comprise decile 1 and funds with the lowest comprise decile 10. The portfolios are equally weighted each month, so the weights are readjusted whenever a fund disappears from the sample.

Source: Carhart, Journal of Finance, 1997

Inside-trading is not profitable --- or is it?

Cumulative Abnormal Return (CAR) of Insider Trading

Source: L. Meulbroek, ``An Empirical Analysis of Illegal Insider trading." *Journal of Finance* (December 1992) .

Type of inside information	N	Insider holding period (# of trading days)	CAR over holding period (%)
Takeover related	145	12.5 (1.4)	29.9 (1.5)
Negative earnings	12	18.4 (7.6)	30.0 (4.7)
Positive earnings	3	21.3 (11.2)	3.3 (4.2)
Bankruptcy	10	26.4 (14.6)	73.9 (12.0)
Misc. good news	11	11.2 (7.7)	34.8 (6.9)
Misc. bad news	2	10.0 (7.0)	28.1 (2.5)
Total	183	13.7 (1.6)	32.2 (1.7)

Notes: The insider holding period begins with the first insider purchase or sale, and ends when the insider information becomes public. Standard errors are in parentheses.

1 The Stock Market Crash of 1987

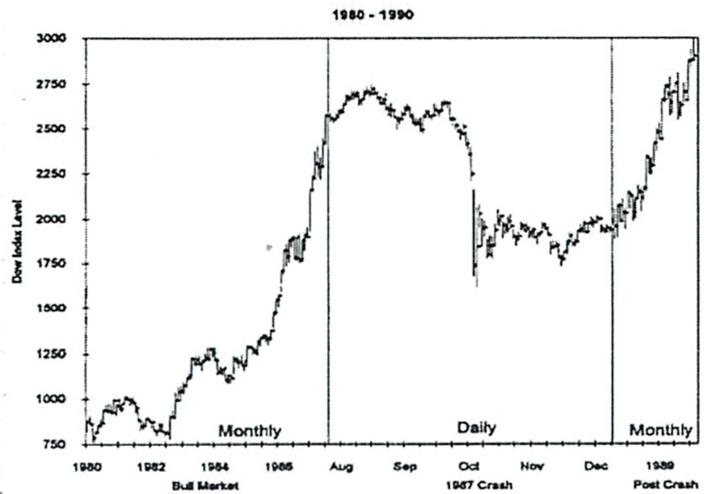
a) Facts:

- No apparent exogenous news
- Enormous and discontinuous price drop
- Worldwide
- No immediate bouncing back

b) Suspects:

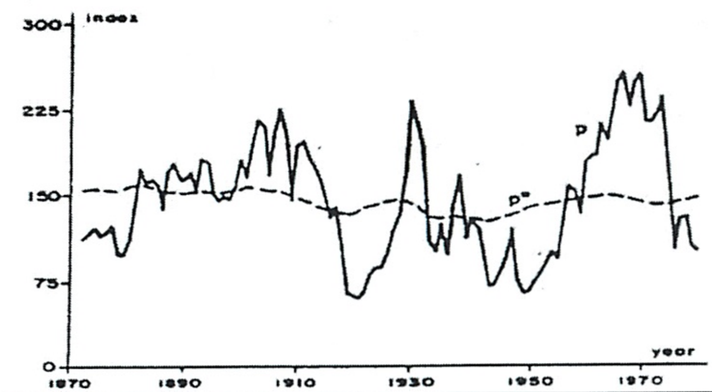
- Index arbitrageurs (actors or messengers?)
- Portfolio insurance
- Institutional selling

1987 Stock Market Crash --- U.S. Market

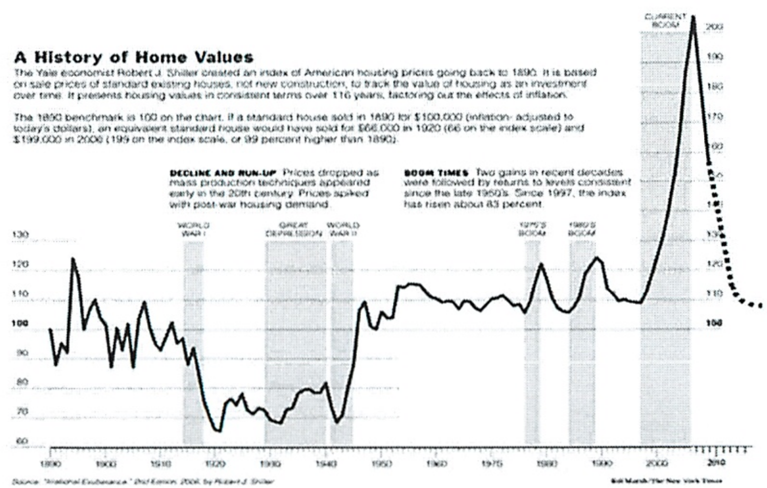


2 Smooth dividends but volatile prices (Shiller)

Real S&P Index p versus Ex Post Rational Price p^* (1871-1979)
Source: R. Shiller, "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?" *American Economic Review* (Vol. 71, 1981).



3 The Financial crisis of 2007 - 2009



- The Efficient Market Hypothesis (EMH)
- Implications of EMH
- Supportive evidence to EMH
- Challenges to EMH

15.401 Finance Theory I

Final Review Session

Marc Piette

Concepts

- Compounding/Discounting
- Bonds
- Forwards and Futures
- Stocks
- Options
- Portfolio Theory
- CAPM
- Capital Budgeting
- Real Options

Discounting, formulas

- Compounding and discounting, DCF formulas
  - (Not just at a constant or risk free rate anymore!)

DCF:

Perpetuity: $PV = \frac{C}{r}$

Growing Perpetuity: $PV = \frac{C}{r-g}$

First payment year 1 growth year 2

Annuity: $PV = \frac{C}{r}(1 - \frac{1}{(1+r)^t})$

Growing Annuity: $PV = \frac{C}{r-g}(1 - \frac{(1+g)^t}{(1+r)^t})$

Bonds, APR...

- Interest rates:

  Compounding:

  $(1 + r_{EAR}) = (1 + \frac{r_{APR}}{k})^k$

  Inflation: $(1+r_{real}) \times (1+i) = (1+r_{nom})$

- Bonds:

  $$B = \sum_{t=1}^{T} \frac{C}{(1+r_t)^t} + \frac{P}{(1+r_T)^T} = \sum_{t=1}^{T} \frac{C}{(1+y)^t} + \frac{P}{(1+y)^T}$$

  Duration

  $$D = \frac{1}{B} \sum \frac{CF_t}{(1+y)^t} \times t$$

  Modified duration

  $$MD = -\frac{1}{B} \times \frac{dB}{dy} = \frac{D}{1+y}$$

- Portfolio immunization: MD(A)xPV(A) - MD(L)xPV(L) = 0

Final Review 5/13

Forward rates

$$(1+r_s)^s(1+f_{s,t})^{t-s}=(1+r_t)^t$$

- Enter into a contract at time t=0
  - Loan is received at time t1
  - Repayment is done at time t2
- Expectation hypothesis:
  - $f_t=E[r_1(t)]$
  - forward rates predict future spot rates
  - Slope of the curve reflect market's expectation of future short-term rates
- Liquidity preference hypothesis:
  - $f_t=E[r_1(t)]$+liquidity premium
  - Risk premium is demanded by investors in long bonds

Stocks

- Valuation based
  - Expected dividends (Dividend discount model – DCF)
  - Discount rate for those dividends
- Gordon Model of constant growth
- Earnings/EPS,
- Payout ratio (div/earnings), plowback ratios (1-payout)
- Book value (cumulative retained earnings)
- Return on book equity (earnings/BV)
- P/E (P_0/EPS_1)
- PVGO
- What is the discount rate? (according to CAPM?)

Forwards and futures

- Pricing:
  - consider $F_t \approx H_t$ for now
  - 2 ways to buy asset for delivery at time T
    - Enter into a forward contract to buy at time T
    - Buy now and hold
  - No free lunch: buy and hold equivalent
    - Storage cost, convenience yield
  - Contango / Backwardation

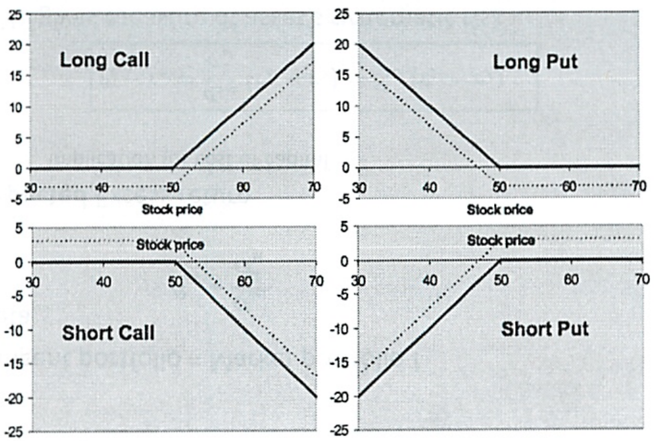
$$\begin{aligned} F \approx H &= S[1+r-(y-c)]^T \\ &= S(1+r-\tilde{y})^T \end{aligned}$$

$$\begin{aligned} F_T \approx H_T &= (1+r)^T S + \mathrm{FV}_T(\text{net storage costs}) \\ &= (1+r)^T S - \mathrm{FV}_T(\text{net convenience yield}) \end{aligned}$$

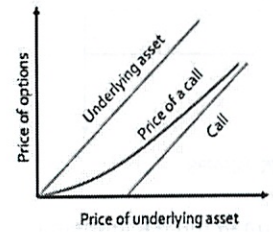
Options

- Call (option to buy)
- Put (option to sell)
- Strike price/Maturity
- European versus American
- Put-Call parity
  - $P + S = C + K/(1+r)^T = C + PV(K)$
- Binomial option pricing
- Black-Scholes

Options: payoff diagrams



Options (cont.)



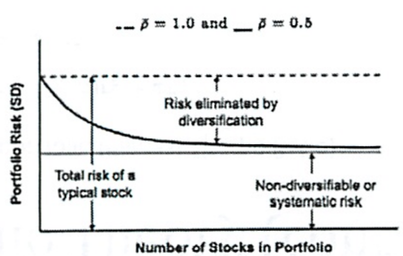
	Value of call	Value of put
Strike price (K)	Decrease	Increase
Price of underlying asset (S)	Increase	Decrease
Volatility of the underlying asset (σ)	Increase	Increase
Maturity (T)	Increase	Increase
Interest rate (r)	Increase	Decrease

Portfolio Theory

- Diversification
- Portfolio return, Portfolio variance

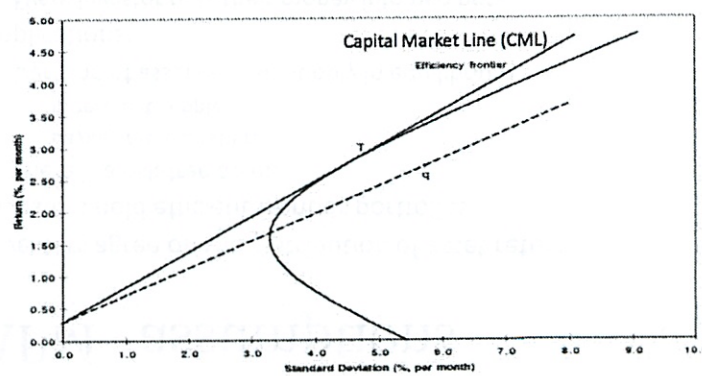
$$\bar{r}_p = w_1\bar{r}_1 + w_2\bar{r}_2 \qquad \sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12}$$

- Systematic versus idiosyncratic risk



Portfolio Theory (cont.)

- volatility-return tradeoff
- Portfolio frontier, Efficient frontier portfolio
- Capital market line



Portfolio Theory (cont.)

- Tangent portfolio (highest Sharpe Ratio)

$$\bar{r}_p = (1-x)r_F + x\bar{r}_q$$
$$\sigma_p^2 = x^2\sigma_q^2$$

- Sharpe Ratio

$$\text{Sharpe Ratio} \equiv \frac{\bar{r}_p - r_F}{\sigma_p}$$

CAPM - assumptions

- Investors agree on the distribution of asset returns
- Investors hold efficient frontier portfolios
  - There is a risk-free asset:
    - paying interest rate r_f
    - in zero net supply
  - Demand of assets equals supply in equilibrium
- Implications:
  - Every investor puts their money into two pots:
    - the riskless asset
    - a single portfolio of risky assets, the tangent portfolio
- All investors hold the risky assets in same proportions
  - they hold the same risky portfolio, the tangent portfolio
- The tangent portfolio is the market portfolio!

CAPM

- Tangent portfolio = Market portfolio !
- Beta

$$\beta_{im} = \frac{\sigma_{im}}{\sigma_m^2}$$

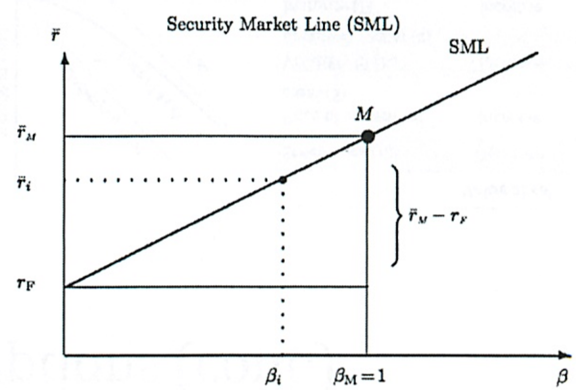
- **Expected** Excess return
  - Implication for cost of capital

$$\boxed{\bar{r}_i - r_F = \frac{\sigma_{iM}}{\sigma_M^2}(\bar{r}_M - r_F) = \beta_{iM}(\bar{r}_M - r_F)}$$

- Beta gives measure of asset's systematic risk
- $\bar{r}_m$ □ r_f gives premium per unit of systematic risk, the price of risk

CAPM (cont.)

- Security Market Line (SML)
  - Relationship between asset's premium and its market beta



CAPM (cont.)

- We can decompose and asset's return in 3 pieces

$$\tilde{r}_i - r_F = \alpha_i + \beta_{iM}(\tilde{r}_M - r_F) + \tilde{\varepsilon}_i$$

- Sigma (ε) : Asset's non-systematic risk
  - E[sigma] = 0
  - Correlation between sigma and market = 0
- Alpha: asset's return in excess of risk adjusted reward according to CAPM
- What to do with an asset with a positive alpha?
  - Check estimation error
  - Past value of α may not predict its future value
  - Positive α may be compensating for other risks
- Extension: Fama-French, APT (Arbitrage pricing theory) ...

Capital Budgeting

- Project selection
  - NPV (ex: $56M)
    - Analyze: CFs, discount rates, strategic options
    - What is the discount rate?
      - Use CAPM (find "comparables" to estimate "Beta")

$$NPV = CF_0 + \frac{CF_1}{1+r_1} + \cdots + \frac{CF_t}{(1+r_t)^t}$$

$$\bar{r}_{\text{project}} = r_F + \beta_{\text{project}}(\bar{r}_M - r_F)$$

- Cash flows after tax:
  - Operating profit = operating revenue – operating expenses without depreciation
  - CF = (1 – τ) x operating profit – capital expenditure + τ x depreciation
  (τ is the tax rate)

Capital Budgeting (cont.)

- IRR

$$0 = CF_0 + \frac{CF_1}{(1+IRR)} + \frac{CF_2}{(1+IRR)^2} + \cdots \frac{CF_t}{(1+IRR)^t}$$

- Payback period (ex: 5 years)
  - Smallest s such that

$$CF_1 + CF_2 + \cdots + CF_s \geq -CF_0 = I_0$$

  - Extension: discounted payback period
- Profitability index (ex: 2.3)

$$\text{PI} = \frac{PV}{-CF_0} = \frac{PV}{I_0}$$

- NPV improves on all other methods!

Real Options

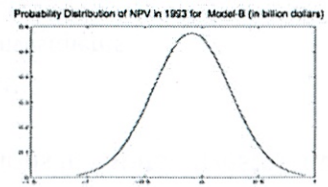
- Common and important options in capital investments include:
  - The option to wait before investing
  - The option to make follow-on investments
  - The option to abandon a project
  - The option to vary output or production methods.
- Two key elements in strategic options and their valuation:
  - New information arrives over time
  - Decisions can be made after receiving new information.

Real Options: example

- Manufacturer of A in 1990
  - NPV of A at 20% is $-46M in 1990
  - NPV of B is also very negative in 1990
  - Producing A would allow potential production of B in 1993
  - B production decision has to be made by 1993
- Entry in 1993 with A is prohibitively expensive
- Option to stop in 1993 (possible loss limited)
- Investment needed for B is $900M (twice that of A)
- PV of operating profits from B is $468 million in 1990

Real Options: example (cont)

- PV of B evolves with annual standard deviation of 35%
- Opportunity to invest in B is like a 3-year call option on an asset worth $468 million now with exercise price $900 million!
- Using Black-Scholes
  - Value of call: ~ $55M



- NPV A alone is $-46M
- Option to produce B if you produce A: $55M
- NPV of A taking into account option value: 55-46 = $9M!

Real Options (Cont.)

- Naive DCF analysis tends to under-estimate the value of strategic options:
  - Timing of projects is an option (American call)
  - Follow-on projects are options (American call)
  - Termination of projects are options (American put)
  - Expansion or contraction of production are options (conversion options).
- It is difficult to apply DCF for option valuation.
- Options can be valued (sometimes).

15.401 Final Review w/ Professor

5/13

(5 min late)

Forward rates

Stocks

- valuation
- payout ratio
- book ratio
- growth

Forwards + Futures pricing

- given carry, convience
- Contango/backwardation
- do what did in class + Problem set

Options

- binomial pricing
  - Call
  - Put
- strike price/maturity
- Put Call parity
- Replicating portfolio

(I've forgotten a lot of this stuff - need to review!)

(2)

- Options are everywhere

Could be asked to draw a payout diagram

- What happens to Call / Put prices in a variety thing

Portfolio

- Weighted avg of portfolio
- Calculating st dev
- do 2 asset portfolio quickly
- Correlation
  - risk form: diversifable and non diversifable
- Often can't diversifable
- when interest rates move, prices move

(Big part of exam)

Capital Market Line

Sharpe Ratio

$$\frac{\bar{r}_p - r_F}{\sigma_p}$$

(← portfolio; ← risk free)

(3)

CAPM assumption

- borrow + lend riskfree
- frictionless
- perfect info
- assets can be divided

Not asked derivation

CAPM

$\beta_{im} = \frac{\sigma_{im}}{\sigma_m^2}$ how correlated you are w/ the market

- systematic risk

$\bar{r}_m - r_f$ is risk premium for systematic risk

Why it matters:

- well diversified portfolio

Expected excess return

Is measure of risk vs market

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2} = \frac{\rho_{im}\,\sigma_i\,\sigma_m}{\sigma_m^2}$$

(4)

$$E[R_i] = R_f + \beta_i \times (R_m - R_f)$$

expected return of any asset

$$E[R_i] - R_f = \beta_i \times (R_m - R_f)$$

expected excess return

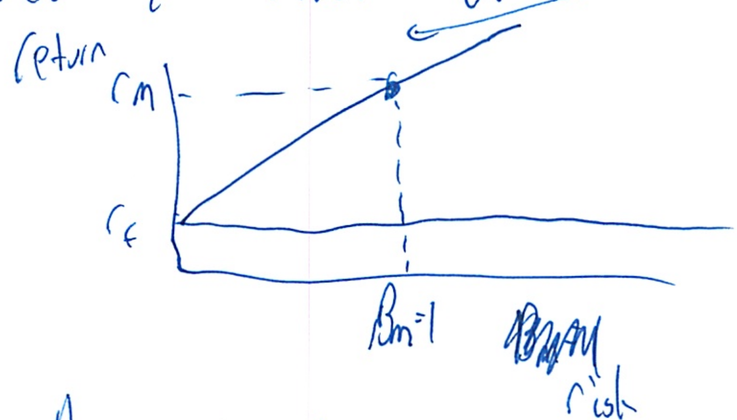
Came out of portfolio theory

Ideosyncratic risk disappears

But some factors common to everyone

- oil prices
- Forex
- interest rates

Security Market Line — want firms up here



As people buy it the price ↑, back to SML

⑤

Portfolio based on risk

high β = high risk

low β = low risk

Investors looking for α

$$\hat{r}_i - r_f = \alpha_i + \beta_i(\hat{r}_m - r_f) + \tilde{e}_i$$

α_i ↑ return in excess of risk adjusted rate of return

$\tilde{e}_i$ ↗ non systematic risk

low β stocks actually overperform SML

Arbitrage Pricing Theory (APT)

- CAPM w/ more factors
- Did not learn

$$\gamma_i = \frac{\sigma_{i,oil}}{\sigma^2_{oil}}$$ ← what are these factors

Capital Budgeting

Lays out cash flows for projects

$$CF = (1-\tau)\cdot \text{operating profit} - \text{Capex} + (\tau)\,\text{depreciation}$$

τ ↑ tax rate

(e)

Remember depreciation only counts ~~on taxes~~ for tax purposes

~~Have an IRR - internal rate of return~~

Salvage value + net working capital comes back at end

Are told depreciation method

- might not match project lenght

Calculate a NPV of a project

- discount the future cash flows back to present day

IRR is what makes NPV = 0

$$0 = CF_0 + \frac{CF_1}{(1+IRR)} + \ldots \text{etc}$$

internal rate of return

Have real options

- can wait
- do follow-on investment
- abandon a project
- vary output

New info arrives over time

Once costs sunk, forget

NPV tends to undervalue

(7)

There are options embedded everywhere

Being in a field with more options means have to be paid more

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Forwards + Futures arbitrage opportunity

- fixed income q/ left over
- like in P-set

Black Sholes - once it appeared become self fufilling propusy

σ of returns not visable

Investors belive markets will go to their model

$\frac{1}{3}$ or $\frac{1}{4}$ of prev ~~exap~~ exam

Two double sided cheat sheets