

Announcements

1. This week tutorials on inventory
2. Sport Obermeyer case is due on March 28 just after the break (office hours will be scheduled over SIP week)

Inventory Lecture (1)



Trade-off:

Inventory Cost Vs. Capacity Vs. Service Level

From the Trenches...

Too much:

- "Liz Clairborne experiences unexpected earnings decline as a consequence of higher-than-expected excess inventories" *WSJ*, July 1993.
- "On Tuesday, the network-equipment giant Cisco provided the grisly details behind its astonishing \$2.25 billion inventory write-off in the third quarter" *News.com*, May 2001.

Too little:

- "IBM struggles with shortages in ThinkPad line due to ineffective inventory management" *WSJ*, 1994.
- "Since 1990 we have designated the Department of Defense's management of its inventory, including spare parts, as high risk because [...] its management systems and procedures were ineffective.", *Army Inventory: Parts Shortages Are Impacting Operations and Maintenance Effectiveness*, US General Accounting Office report, August 2001.

Why Inventory Costs Money

See notes

⇒ Typical per annum inventory holding cost

Why Hold Inventory? How Much?

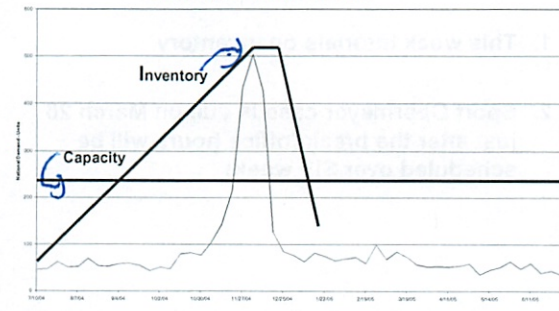
| Type of Inventory | Decision Tool |
|--|--------------------------|
| Buffer/Decoupling | Build-up diagrams |
| Seasonal/Anticipation <i>Scooter from booth</i> | Build-up diagrams |
| Cycle stock <i>lunch hr</i> | Today (EOQ) |
| Safety stock <i>econ of scale</i> | Today (Newsvendor) |
| Pipeline <i>must order in advanced</i> | Little's Law (Next time) |

next lecture

Christmas tree

Anticipation Stock

Capacity-Inventory Tradeoff

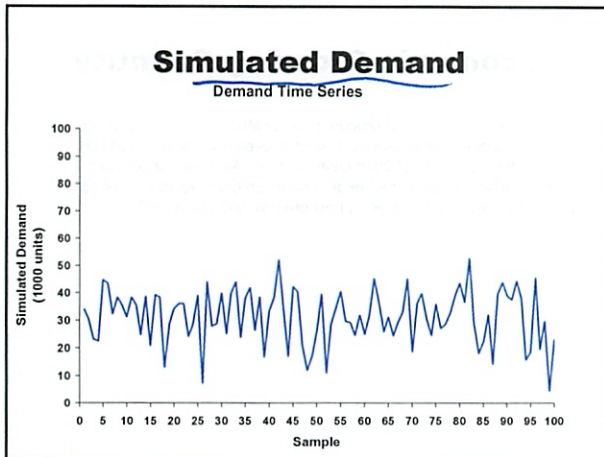


Outline

- This lecture: basic trade-offs and models
Newsvendor model: safety inventory
Economic order quantity: cycle inventory
— *EOQ*
- Next lecture: replenishment models
Periodic Review / Order-up-to policy
Continuous Review / Reorder point policy

The Newsvendor Problem

Preparing for the Christmas sales season, Tree Inc. has to decide how many Christmas trees to purchase. The selling season starts December 1, but due to long supply lead times Tree Inc. must decide how many trees to purchase before the beginning of the selling season. The purchasing price is \$15 per tree, and the per tree selling price is \$105. At the end of the season Tree Inc. can salvage excess inventory for \$5 per tree. Unfortunately the exact demand for Christmas trees is uncertain. The marketing department anticipates an average demand of 30K trees with a standard deviation of 10K. To support the decision, they simulated the demand distribution, assuming it follows Normal(30;10) distribution (measured in 1000 units). How many trees should Tree Inc. order?



? Survey
Past season

Newsvendor Model Parameters

- q = Order Quantity (units) *decision*
 - $c = \$15$ = Unit Cost (\$)
 - $r = \$105$ = Unit Revenue (\$)
 - $b = \$5$ = Unit Salvage Value (\$)
- parameters*
($r > c > b$)
- D = Demand (units) - random variable, uncertain value, follows Normal(30,10)

for problem
to be
interesting

Supply-Demand Mismatch

IF $d > q$
(demand > quantity ordered)

Opportunity cost (Lost-Sales):
 $(r - c) \times (d - q) =$
 $\$90 \times (d - q)$

IF $q > d$
(quantity ordered > demand)

Disposal cost (Salvage cost):
 $(c - b) \times (q - d) =$
 $\$10 \times (q - d)$

Objective:

minimize expected opportunity + disposal cost:
 $E[(r - c) \times \max\{D - q, 0\} + (c - b) \times \max\{q - D, 0\}] =$
 $E[90 \times \max\{D - q, 0\} + 10 \times \max\{q - D, 0\}]$

Newsvendor Model

- One time decision under uncertainty, entire supply arrives before the selling season
 - Trade-off:
 - Ordering too much (waste, salvage value < cost) versus
 - Ordering too little (excess demand is lost)
 - Examples:
 - Restaurant;
 - Fashion;
 - High Tech;
- ➡ Key tool to determine/evaluate safety inventory

Newsvendor Example

Based on forecasts and marketing studies you are expecting a total lifecycle demand $N(60,000;20,000)$ for a new product due to launch in the future. The product has a gross margin of \$750 and a net liquidation/disposal cost (for unsold inventory) of \$250. Because of long lead-times you must commit orders to supplier for the entire product life-cycle now. How much should you order?

Economic Ordering Quantity

A PC assembly operation procures its 128Mb memory chips at \$45 each (purchase + shipment cost) from a foreign vendor; in addition each order also costs \$500 in customs fees. Assuming a constant demand of 400 chips per week and an inventory holding cost of 45% per dollar investment per year, how often would you order?

Economic Order Quantity Model

- Set order size for repetitive ordering process with fixed ordering costs (order in batches)
- Trade-off:
 - Batch size too large (too much average inventory) versus
 - Batch size too small (too much ordering cost)
- Examples:
 - Change-over costs (e.g., first 10 items must be scraped);
 - Transportation/Shipment costs...

➡ Key tool to determine/evaluate cycle inventory

EOQ Model Parameters

- Q = Order Quantity *decision*
- $D = 400$ = Demand Rate (units/time)
- $C = 45$ = Purchasing Cost (\$/unit) *parameters*
- $F = 500$ = Fixed Order Cost (\$)
- $H = 45$ = Inventory Holding Cost rate (%/\$ investment/time)

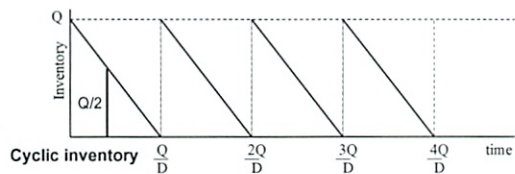
Assumptions:

- constant, deterministic demand
- instantaneous replenishment

EOQ Model Derivation

• Inventory Cost $C \cdot H \cdot \frac{Q}{2}$; Order Cost $F \cdot \frac{D}{Q}$;

• Total Cost $V(Q) = F \cdot \frac{D}{Q} + C \cdot H \cdot \frac{Q}{2}$



EOQ Formula

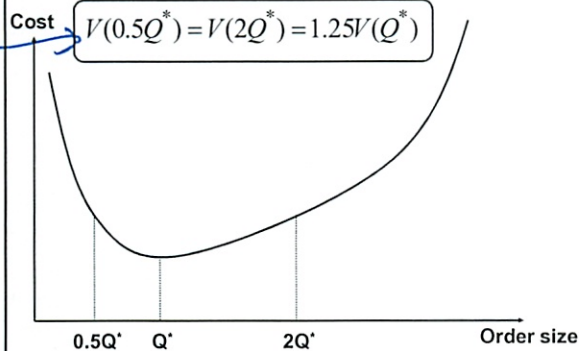
• Set first derivative to 0: $\frac{\partial V}{\partial Q} = -\frac{DF}{Q^2} + \frac{CH}{2} = 0$

• This yields:

$$Q^* = \sqrt{\frac{2DF}{CH}}$$

$$V(Q^*) = \sqrt{\frac{FDCH}{2}} + \sqrt{\frac{FDCH}{2}} = \sqrt{2FDCH}$$

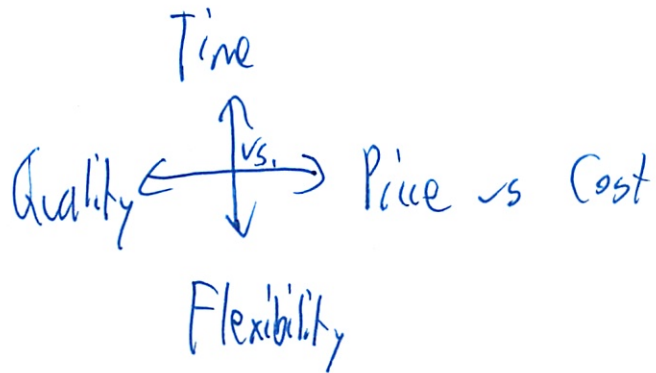
EOQ - Robustness



Inventory Lecture Wrap-Up

1. Functions of inventory: seasonal, cyclic, safety stock
2. EOQ (cyclic stock) and Newsvendor (safety stock) models

Sport Obermeyer case due March 28
- today's lecture is all you need



Inventory Cost vs Capacity vs Service level

Why does inventory cost \$?

- perishable / obsolescence

- ~~risk of loss~~

- shrinkage (risk of loss)

 - theft

 - fire

 - damage

- capital to buy held up (opportunity cost)

- warehouse costs

- management labor

- IT management systems

②

- Workers Perception / morale

↳ from the Goal

Hold diff goals in inventory

Holding cost

- Divide ~~the~~ total cost to per unit per unit of time

- % ~~per~~ ~~of~~ of the value of the item

- retail ~ 30%

- or per year

Why inventory chart on slide

Christmas tree

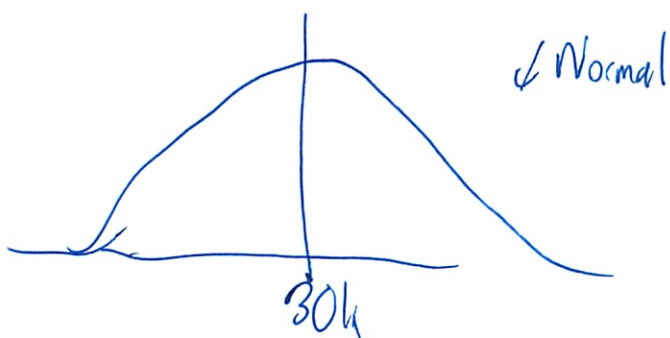
c = unit cost = 15

b = salvage value = 5

$D = N(30k, 10k)$ = demand

r = revenue = 105

Q = quantity to order = ?



③

So sell a tree $105 - 15 = +90$

not use a tree $5 - 15 = -10$

So what is the key here?

What are you trying to optimize?

$$\pi = \text{sold} \cdot 90 - \text{unsold} \cdot 10$$

but what other equations?

At each point ask where $E[\text{revenue}] = E[\text{loss}]$



E of normal is mean

But it's prob that demand is there or less - CDF

What is CDF of a normal again?

$$P(\text{sale}) \cdot 90 - (1 - P(\text{sale})) \cdot 10 = 0$$

~~then~~ solve

$$P(\text{sale}) = .1$$

So look on normal table for .1

4)

One student: want steady revenue so under sell market
- go under 30k

Others: 90 - 95% e-what I did, Prof agrees
- ~~When~~ so 50-55k & where get that?

$D > q$

- one situation
- did not order enough
- opp. cost of leaving \$ on table

$$(D - q)(r - c)$$

$D < q$

$$(q - D)(c - b)$$

If D 's known, just order D

But uncertainty/variability

Want opportunity costs to =

If we have q^* , should we order $q+1$?

$$E[\text{profit}] = P[D \leq q](b - c) + P[D > q](r - c)$$

⑤ $E[\text{profit}] > 0$

If ④, order more

Repeat till $E[\text{profit}]$ goes to 0

The q that gives us this is the solution

$$p[D \leq q](b-c) + (1-p[D \leq q])(r-c) = 0 \quad \text{what I had}$$

$$p[D \leq q] = \frac{r-c}{(r-c) + (c-b)}$$

Then go to normal table - to find st dev from the mean

- then multiply by mean

? forgot it gave you
last st dev - but
makes sense

U = understocking cost

O = overstocking costs

$$p[D \leq q] = \frac{U}{U+O}$$

overstocking cost

* Incremental analysis
- marginal



$$q^* = \mu + k\sigma$$

? from table
multiple of σ

(c)

So put in #

$$\frac{D > a}{5 - 90}$$

$$\frac{D < a}{(a - 0)(c - b)} = \frac{90}{90}$$

$$P[0 \leq q] = \frac{u}{u + 0} = \frac{90}{100} = .9$$

$k \rightarrow$ # ~~step~~ st devs from mean \star inverse CDF

$$= 1.29 = \{ \text{off table at } .9 \}$$

$$q^* = \mu + k\sigma$$

$$= 30k + \underbrace{1.29 \cdot 10k}_{\text{"Safety stock"}}$$

$$= 43k$$

Example 2

750 = u understacking cost - lost revenue

$$250 = 0$$

$$\frac{u}{u + 0} = .75$$

⑦

$$P[D \leq q^*] = .75$$

↳ table $\rightarrow k = .68$

$$q^* = 60k + .68 \cdot 20k = 73.6k$$

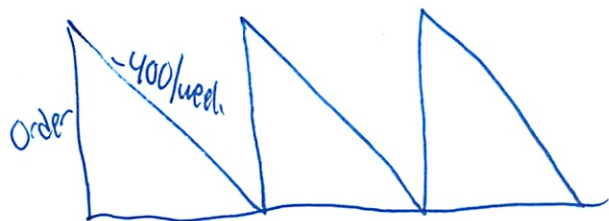
Also for u build in effects on future revenue if you tell them
"sorry sold out"

larger variation = larger safety stock
this is all 1 time ordering

Economic ordering quantity

How often / how many to order?

Once you order it arrives right away



D = Demand

Q = quantity / order

f = fixed ordering cost

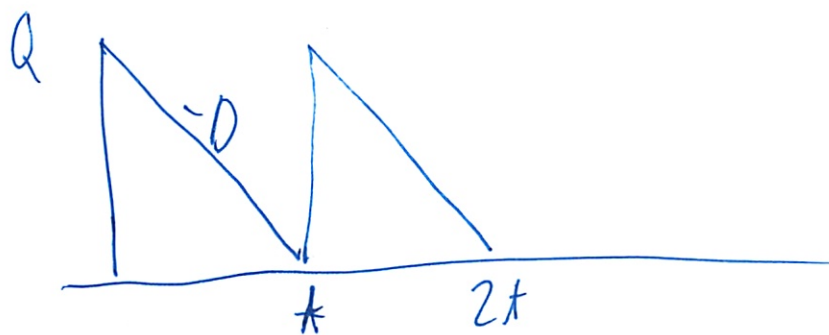
H = inventory holding cost

C = purchasing cost / unit

8

(I am starting to realize how to find ans myself!
Yeah algebra!)

How much are we going to order / how often



Trade off

- batch size too large: high holding costs

- " " " small: too ~~much~~ much ordering costs

Need to calc holding cost

$$\text{base} = \frac{Q}{D}$$

~~area~~ so is area of triangle
base

$$= \frac{Q \cdot \frac{Q}{D} \cdot \frac{1}{2}}{\frac{Q}{D}} = \boxed{\frac{Q}{2}} = \text{avg inventory in system}$$

(9)

So Value of inventory is $C \cdot \frac{q}{2}$

So holding cost is

$$h \cdot C \cdot \frac{q}{2}$$

Fixed ordering Cost

~~FI~~ (total)

$F \cdot \# \text{ of orders } \cancel{\text{per period per}}$

$$= F \cdot \frac{D}{q}$$

So Total Cost

$$V(Q) = \frac{FD}{q} + hC \frac{q}{2}$$

Find the minimum

Take derivative

$$\frac{dV'}{dq} = -FDq^{-2} + \frac{hC}{2}$$

$$\text{set} = 0$$

10

Find q^* that solves problem

$$q^* = \sqrt{\frac{2FD}{CH}}$$

With our #

$$= \sqrt{\frac{2 \cdot 500 \cdot 20,000}{45 \cdot 45}} \leftarrow D \text{ should be per year}$$

\uparrow is .45 since % Since F is per year

≈ 1000 quantity to order

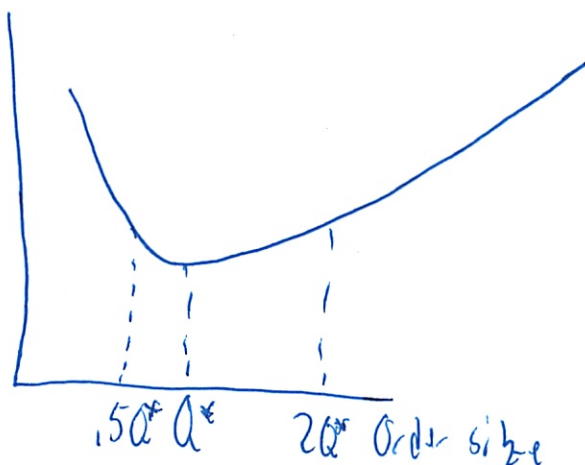
Order

$\frac{q}{D}$ often

$$\frac{1000}{400} = 2.5 \text{ weeks}$$

\uparrow per week

Can graph

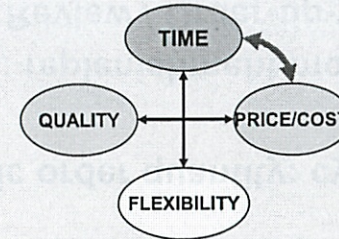


If you order half as much, your revenue only \downarrow 1.25
double \uparrow as you should

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3. Mid-class online survey coming soon

Inventory Lecture (1)



Trade-off:

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Solutions

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Why Inventory Costs Money



Typical per annum
inventory holding cost

Financial Inventory Metrics

$$\text{Inventory Turns} = \frac{\text{COGS} \leftarrow \text{Earnings or P \& L}}{\text{Inventory Value} \leftarrow \text{Balance sheet}}$$

$$\text{Inventory Cost / Unit} = \frac{\text{Inventory Value} \times \text{Holding Cost}}{\text{COGS}} = \frac{\text{Holding Cost}}{\text{Inventory Turns}}$$

Example: 10k filings, 2002 (\$M)

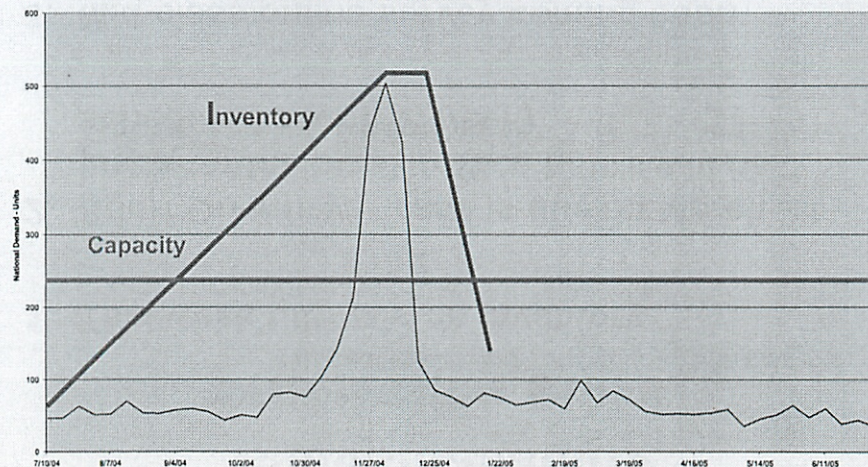
| | Wal Mart Stores Inc. | Kmart Corp. |
|-----------|----------------------|-------------|
| Inventory | \$22,749 | \$4,825 |
| C.O.G.S | \$171,562 | \$26,258 |

Why Hold Inventory? How Much?

| Type of Inventory | Decision Tool |
|-----------------------|--------------------------|
| Buffer/Decoupling | Build-up diagrams |
| Seasonal/Anticipation | Build-up diagrams |
| Cycle stock | Today (EOQ) |
| Safety stock | Today (Newsvendor) |
| Pipeline | Little's Law (Next time) |

Anticipation Stock

Capacity-Inventory Tradeoff



Outline

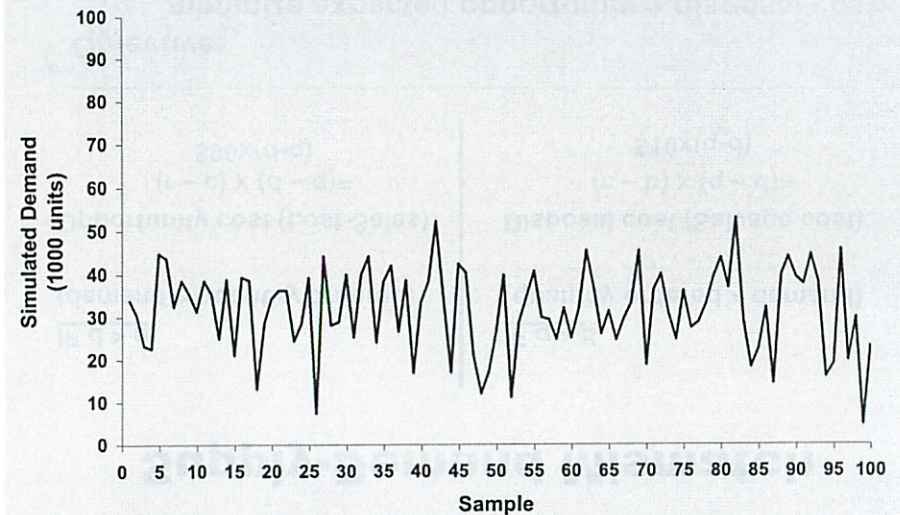
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Newsvendor model: safety inventory
Economic order quantity: cycle inventory
- Next lecture: replenishment models
Periodic Review / Order-up-to policy
Continuous Review / Reorder point policy

The Newsvendor Problem

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Simulated Demand

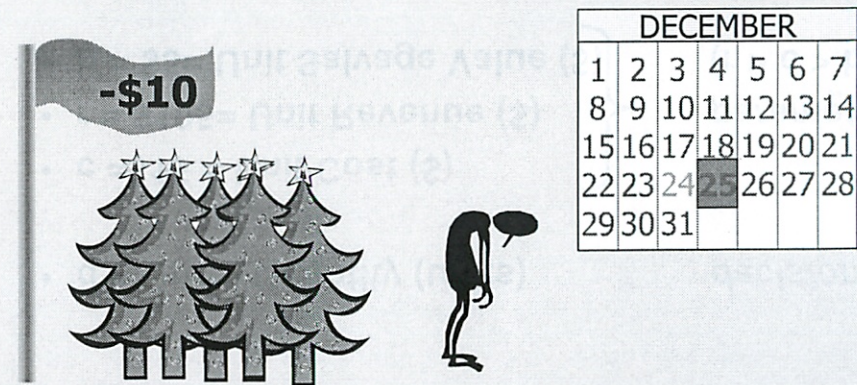
Demand Time Series



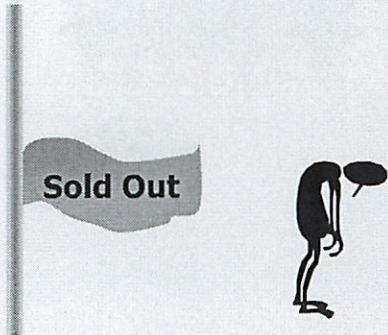
Christmas Tree Problem



Ordering Too Many....



...Versus Ordering Too Few!



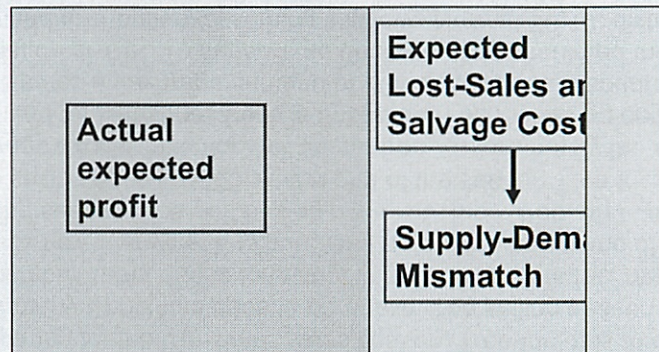
| DECEMBER | | | | | | |
|----------|----|----|----|----|----|----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| 22 | 23 | 24 | 25 | 26 | 27 | 28 |
| 29 | 30 | 31 | | | | |

Newsvendor Model Parameters

- q = Order Quantity (units) *decision*
 - $c = \$15$ = Unit Cost (\$)
 - $r = \$105$ = Unit Revenue (\$)
 - $b = \$5$ = Unit Salvage Value (\$)
- parameters*
($r > c > b$)
- D = Demand (units) - *random variable, uncertain value, follows Normal(30,10)*

Supply-Demand Mismatch

Crystal Ball Profit = $(r-c)E[D]$



Minimize supply-demand mismatch
(= Maximize expected profit)

Supply-Demand Mismatch

IF $d > q$
(demand > quantity ordered)

Opportunity cost (Lost-Sales):
 $(r - c) \times (d - q) =$
 $\$90 \times (d - q)$

IF $q > d$
(quantity ordered > demand)

Disposal cost (Salvage cost):
 $(c - b) \times (q - d) =$
 $\$10 \times (q - d)$

Objective:

minimize expected opportunity + disposal cost:
 $E[(r-c) \times \max\{D-q, 0\} + (c-b) \times \max\{q-D, 0\}] =$
 $E[90 \times \max\{D-q, 0\} + 10 \times \max\{q-D, 0\}]$

Newsvendor Model

- One time decision under uncertainty, entire supply arrives before the selling season
 - Trade-off:
 - Ordering too much (waste, salvage value < cost) versus
 - Ordering too little (excess demand is lost)
 - Examples:
 - Restaurant;
 - Fashion;
 - High Tech;
- ➡ Key tool to determine/evaluate safety inventory

Solution Derivation

Incremental Analysis: $q \rightarrow q+1$:

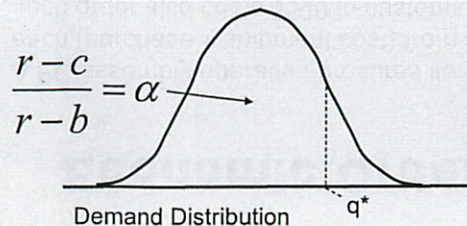
| | • IF $d > q$ (demand > order qty) | • IF $d \leq q$ (demand < order qty) |
|--------------------------------|---|---|
| Δ Mismatch: | $-(r - c)$ | $(c - b)$ |
| $E[\Delta \text{ Mismatch}] =$ | $-P(D > q) \cdot (r - c) + P(D \leq q) \cdot (c - b)$ | |

As long as the expected Δ Mismatch is negative, it is lucrative to increase q to $q + 1$!!!

Newsvendor Formula

$$\underbrace{P(D \leq q^*)}_{\text{In-Stock Probability}} = \frac{r - c}{r - b} = \frac{r - c}{\underbrace{(r - c)}_{\text{cost of under-stocking}} + \underbrace{(c - b)}_{\text{cost of over-stocking}}} = \frac{u}{u + o}$$

$$q^* - E[D] = \text{safety stock}$$



Remark: If D is Normal(μ, σ),

$$q^* = \mu + k \cdot \sigma \text{ with}$$

| | | |
|-------------------|---------------|------------|
| $\alpha = 95\%$ | \rightarrow | $k = 1.64$ |
| $\alpha = 99\%$ | \rightarrow | $k = 2.32$ |
| $\alpha = 99.9\%$ | \rightarrow | $k = 3.09$ |

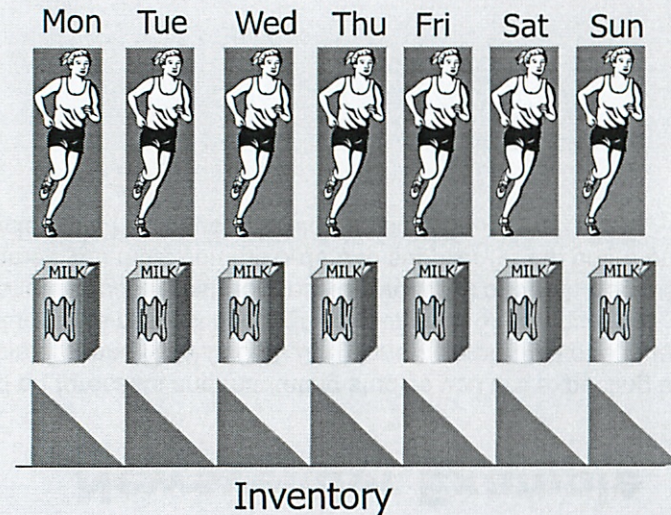
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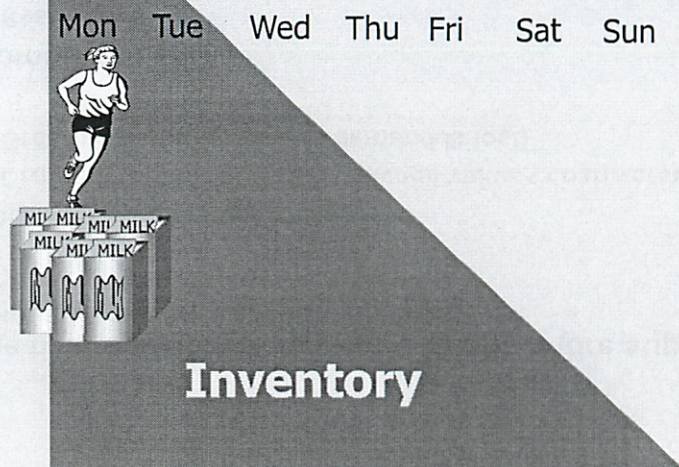
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Running to the Store a Lot...



Vs. Running to the Store a Little



Economic Order Quantity Model

- Set order size for repetitive ordering process with fixed ordering costs (order in batches)
- Trade-off:
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- Examples:
 - Change-over costs (e.g., first 10 items must be scrapped);
 - Transportation/Shipment costs...

➡ Key tool to determine/evaluate cycle inventory

EOQ Model Parameters

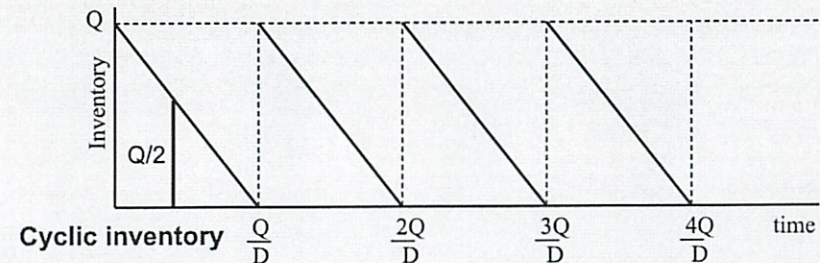
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- $D = 400$ = Demand Rate (units/time)
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- $F = 500$ = Fixed Order Cost (\$)
- $H = 45$ = Inventory Holding Cost rate (%/\$ investment/time)

Assumptions:

- constant, deterministic demand
- instantaneous replenishment

EOQ Model Derivation

- Inventory Cost $C \cdot H \cdot \frac{Q}{2}$; Order Cost $F \cdot \frac{D}{Q}$;
- Total Cost $V(Q) = F \cdot \frac{D}{Q} + C \cdot H \cdot \frac{Q}{2}$



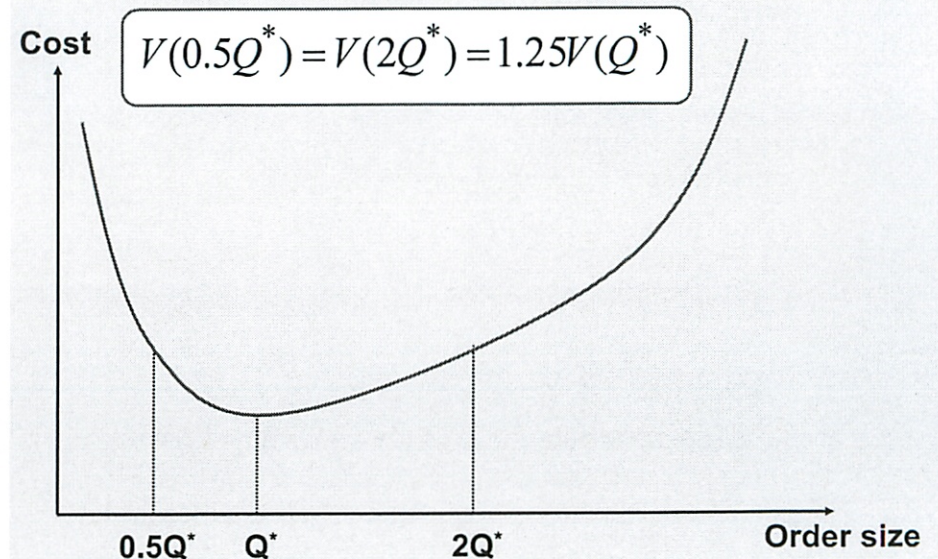
EOQ Formula

- Set first derivative to 0: $\frac{\partial V}{\partial Q} = -\frac{DF}{Q^2} + \frac{CH}{2} = 0$
- This yields:

$$Q^* = \sqrt{\frac{2DF}{CH}}$$

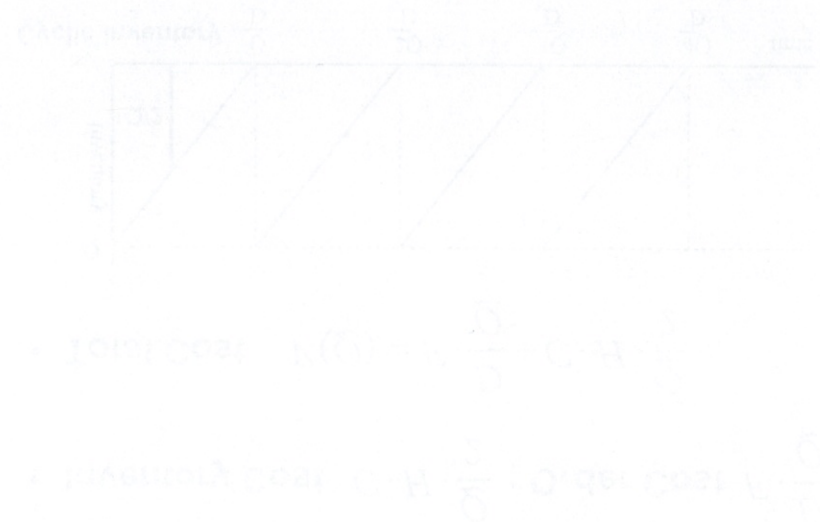
$$V(Q^*) = \sqrt{\frac{FDCH}{2}} + \sqrt{\frac{FDCH}{2}} = \sqrt{2FDCH}$$

EOQ - Robustness



Inventory Lecture Wrap-Up

1. Functions of inventory: seasonal, cyclic, safety stock
2. EOQ (cyclic stock) and Newsvendor (safety stock) models



Inventory 2

3/10

Announcements

1. Tutorials this week about inventory management
2. Next class is on Monday, March 29
3. Sport Obermeyer is due first class after break!
4. Online mid-class survey is posted

Inventory Lecture (2)

Review



Trade-off:

Inventory Cost Vs. Capacity Vs. Service Level

Outline

- Quick review on last lecture
- Replenishment policies:
 - Periodic Review, Order-up-to policy
 - Continuous Review, Reorder point policy
- Implications for supply-chain design

News vendor Formula

Under Ordering (r-c) VS. Over Ordering (c-b)

$$\underbrace{P(D \leq q^*)}_{\text{In-Stock Probability}} = \frac{r-c}{r-b} = \frac{r-c}{\underbrace{(r-c)}_{\text{cost of under-stocking}} + \underbrace{(c-b)}_{\text{cost of over-stocking}}} = \frac{u}{u+o}$$

$q^* - E[D] = \text{safety stock}$

Forecasting

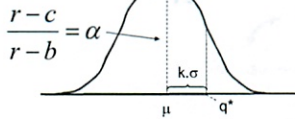
"Order just enough so that the probability of having enough inventory is $u / u+o$ "

today: periodic, but uncertain demand

News vendor Formula

D is Normal(μ, σ)

P(Have Enough Inventory)



Want: $P(D \leq q^*) = \alpha$

Solution:

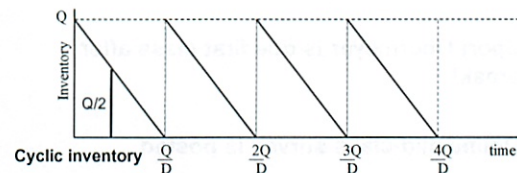
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$\alpha = 95\% \rightarrow k = 1.64$
 $\alpha = 99\% \rightarrow k = 2.32$
 $\alpha = 99.9\% \rightarrow k = 3.09$

EOQ Model Derivation

• Inventory Cost $C \cdot H \cdot \frac{Q}{2}$; Order Cost $F \cdot \frac{D}{Q}$;

• Total Cost $V(Q) = F \cdot \frac{D}{Q} + C \cdot H \cdot \frac{Q}{2}$



$F = \text{fixed cost per order}$

economies of scale

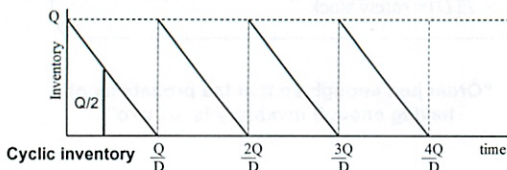
Combine uncertainty, lead time

EOQ Model

Fixed Ordering Cost (F) VS. Holding Costs (HC)

$$Q^* = \sqrt{\frac{2DF}{CH}}, V(Q^*) = \sqrt{2FDCH}$$

Constant rate demand and instantaneous replenishment



Multi-Period Models

• Dynamic ordering over time under uncertainty

• Periodic Vs. Continuous Review

• Backlogged demand Vs. Lost-Sales

• Objective:

Minimize holding and transportation costs under service level constraints

Minimize overall expected cost (holding, ordering, backlogging/lost-sales)

assumptions

- no lead time
- constant, stochastic demand

don't lose any customers

Penalty for losing cost
- hard to find in real life

Periodic Review Policy

• Order-up-to S policy:

Every review period, check the inventory position IP , and order Q such that:

$$Q + IP = S \quad \text{Base Stock Level}$$

• In this system:

$$IP = I_h + I_p$$

Inventory On Hand Inventory in Pipeline

An Order-Up-To S Policy

⇒ "order back to S every review period"

Set S as the newsvendor solution:

$$P(\text{DDLTRP} \leq S) = \alpha$$

where:

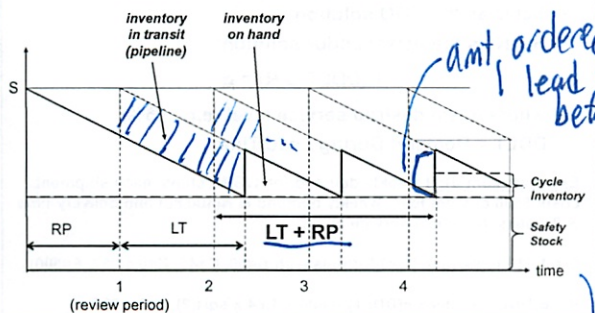
- α is the desired service level (e.g., 95%)

- $\text{DDLTRP} =$
Demand During Lead-Time and Review Period

General Picture

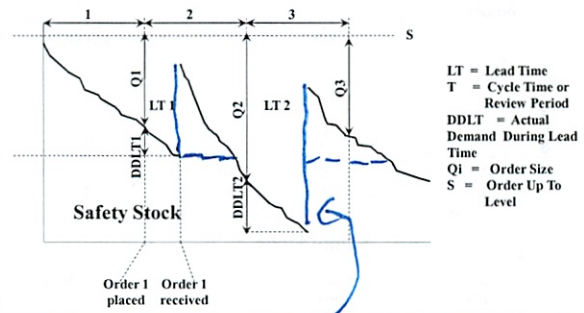
Order-Up-To Policy

⇒ "order back to S every review period"



Periodic Review System

⇒ "order back to S every review period"



Periodic Review Parameters

Main idea: set target level S such that:

$$P(\text{DDLTRP} \leq S) = \alpha \quad (\text{ex: } 95\%)$$

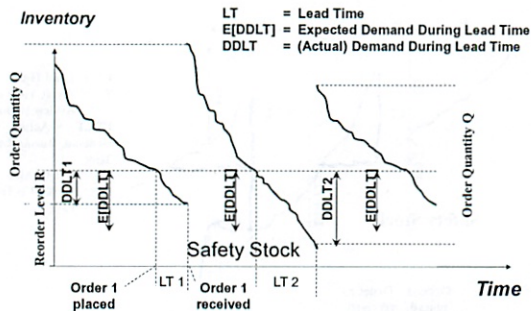
- Target Level: $S = E[\text{DDLTRP}] + k\sigma[\text{DDLTRP}]$
- Safety Stock: $SS = k\sigma[\text{DDLTRP}]$
- Cycle Stock: $CS = E[\text{DDRP}] / 2$
- Pipeline Inventory: $PS = E[\text{DDLT}]$
- Total Stock: $TS = S - CS$

Periodic Review Example

A PC assembly operation procures its 128Mb memory chips at \$45 each (purchase + shipment cost) from a foreign vendor by placing weekly orders. Assume a normally distributed weekly demand $N(400, 80)$, a delivery lead-time of 2 weeks, and an inventory holding cost of 45%. What periodic review order-up-to S policy would you use to achieve a 95% service level? What is the Annual inventory cost?

Continuous Review System

⇒ "order Q whenever inventory reaches R "



(R,Q) Parameters

⇒ "order Q whenever inventory reaches R "

- Set Q as the EOQ solution
- Set R as the newsvendor solution:

$$P(\text{DDLT} \leq R) = \alpha$$

where α is a desired service level (e.g., 95%)

DDLT = Demand During Lead Time

Example (cont'd): if weekly demand for 128Mb chips, each shipment costs \$500 custom fees, weekly demand is $N(400, 80)$ and delivery time is 2 weeks, for a 95% service level:

$Q = 1,013$ units (use EOQ formula with $D=40$, $C=45$, $H=0.45/52$, $F=500$)

$$R = E[\text{DDLT}] + 1.64 \times \sigma[\text{DDLT}] = 800 + 1.64 \times \sqrt{2} \times 80 = 986$$

Inventory (2) Lecture Wrap-Up

- EOQ model to evaluate cyclic inventory
- Use basic models to develop heuristics for multi-period models
- Continuous and discrete replenishment policies (safety stock formulas)

#15.761 Inventory 2

3/10

Tutorial this week - 4:00 - 5:30

Next class 3/29

- next week off

Do survey

this first year teaching

DATA

85-100 very well

70-85 missed 1 or 2 important things

≤ 70 missing conceptual things

See slides

Periodic-review system

lead time = 2 periods



- orders placed
- demand served
- pipeline inventory arrives

ordering cost = c

holding " = h

backlog = p

- "order not available" cost

(2)

inventory position = On hand + in pipeline - back ordered
~~else~~ \uparrow Oh on hand = true inventory - back order \uparrow so don't count twice

Decision

- when/how much to order at each period
- length of period

So want simple, easy to implement

Order up to S policy

$$Q + IP = S$$

- at end of each period S = base stock level

$$IP = \underset{\substack{\uparrow \\ \text{on hand}}}{I_n} + \underset{\substack{\uparrow \\ \text{in pipeline}}}{I_p}$$

Want to meet a service level ~~else~~

- choose S to meet goal

- like 90% of costs served by inv in stock

Assuming things are not perishable here

- What is S to meet this

- Assuming S in any period has same dist.

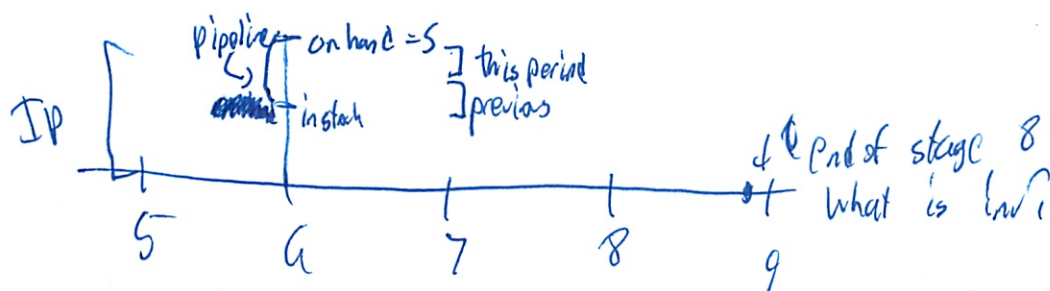
③

Suppose want 100% service level

Fact: $I_{\text{inv on hand}} = S + \text{DDLRR}$

↳ Demand during lead time and review period

Assume lead time = 2



Want $S - \text{DDLRR} > 0$ to have perfect service

Want $P(\text{DDLRR} < S) = \alpha$

↳ Some percentage of service level like 95% = .95

Demand is equally distributed
Same as news vendor problem
(slides)

9

Cycle stock = fluctuation in demand in review period

Pipeline = expected inventory in lead time

Example PC memory chips

LT = 2 weeks

$D = N(400, 80)$
/week

Want service level 95%

Says nothing of length of review period
- decide it yourself!

$F = 500$

$D \approx 20,000/\text{year}$ mean

$H = .45$

$C = 45$

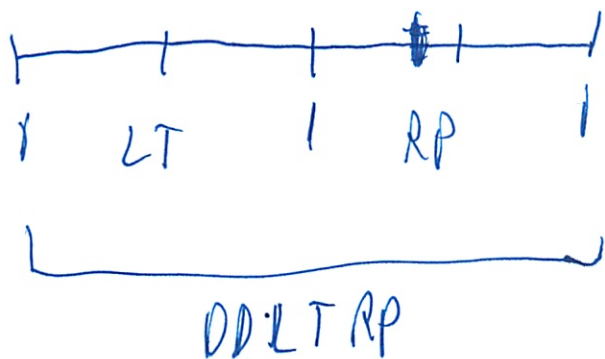
Step 1: Use EOQ to set RP (review period)

$$Q^* = \sqrt{\frac{20F}{CH}} \approx 1,000 \text{ units}$$

$$RP = \frac{Q}{D} = \frac{1000}{400} = 2.5 \text{ weeks}$$

Round review period - can do this in real life
 $RP \approx 2 \text{ weeks}$

5



$$Z \sim N(\underbrace{4000}_{\substack{\uparrow \\ \text{weeks}}}, \underbrace{\sqrt{4}}_{\substack{\uparrow \text{mean} \\ \uparrow \text{weeks}}} \cdot \underbrace{80}_{\substack{\uparrow \text{std dev}}})$$

$$\alpha = 95\%$$

So see $k = 1.65$ on normal table

$$So S = \rightarrow 1600 + \underbrace{1.65 \cdot \sqrt{4} \cdot 80}_{\text{safety stock}} = 1862$$

(this is ^{based off} newsvendor model)

If lead time \downarrow , then safety stock will shrink as well

Wal-Mart did this over past decade

If \downarrow SLA then can cut costs

Or want same SLA $\downarrow \sigma$ (variation) can cut costs

Can change S for seasonality

Assumes constant price

(6)

Similar: Continuous Review System

No periods

Just order when below reorder level r

Assumes we can know our inventory level constantly

- ie PC tracks

- not manual count on shelf

Choose r so not likely to have 0 inv

(R, Q) policy

- what above is called

Why is this better?

R, Q - riskier, less profitable

high fixed ordering cost

less safety stock

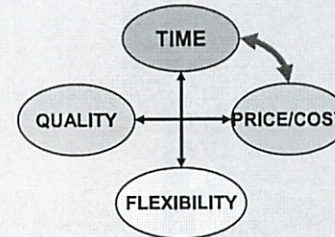
- well follow level closer

Use when can see continuous inventory

Announcements

1. Tutorials this week about inventory management
2. Next class is on Monday, March 28
3. Sport Obermeyer is due first class after break!
4. Online mid-class survey will posted soon

Inventory Lecture (2)



Trade-off:

Inventory Cost Vs. Capacity Vs. Service Level

Outline

- Quick review on last lecture
- EOQ model
- Replenishment policies:
 - Periodic Review, Order-up-to policy
 - Continuous Review, Reorder point policy
- Implications for supply-chain design

Why Hold Inventory? How Much?

| <i>Type of Inventory</i> | <i>Decision Tool</i> |
|--------------------------|----------------------|
| Buffer/Decoupling | Build-up diagrams |
| Seasonal/Anticipation | Build-up diagrams |
| Cycle stock | EOQ |
| Safety stock | Newsvendor |
| Pipeline | Little's Law |

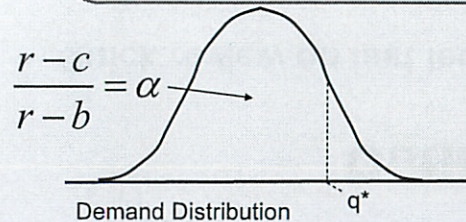
Solutions

Newsvendor Formula

Under Ordering (r-c) VS. Over Ordering (c-b)

$$\underbrace{P(D \leq q^*)}_{\text{In-Stock Probability}} = \frac{r-c}{r-b} = \frac{r-c}{\underbrace{(r-c)}_{\text{cost of under-stocking}} + \underbrace{(c-b)}_{\text{cost of over-stocking}}} = \frac{u}{u+o}$$

$q^* - E[D] = \text{safety stock}$



Remark: If D is Normal(μ, σ),

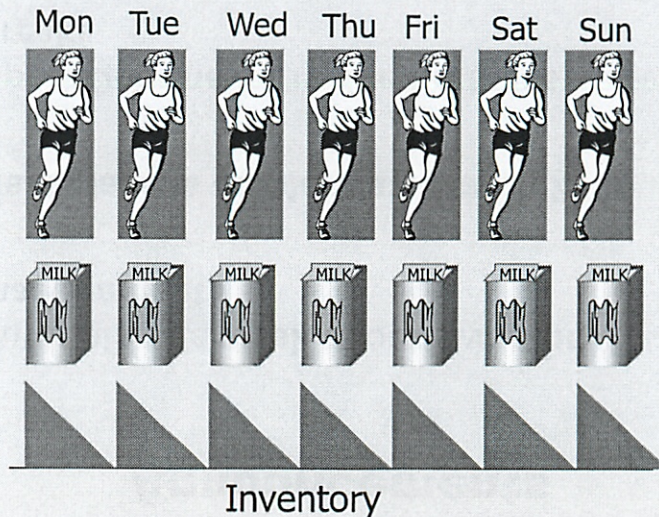
$$q^* = \mu + k \cdot \sigma \text{ with}$$

| | |
|-------------------|------------------------|
| $\alpha = 95\%$ | $\rightarrow k = 1.64$ |
| $\alpha = 99\%$ | $\rightarrow k = 2.32$ |
| $\alpha = 99.9\%$ | $\rightarrow k = 3.09$ |

Economic Ordering Quantity

A PC assembly operation procures its 128Mb memory chips at \$45 each (purchase + shipment cost) from a foreign vendor; in addition each order also costs \$500 in customs fees. Assuming a constant demand of 400 chips per week and an inventory holding cost of 45% per dollar investment per year, how often would you order?

Running to the Store a Lot...



Vs. Running to the Store a Little



Economic Order Quantity Model

- Set order size for repetitive ordering process with fixed ordering costs (order in batches)
 - Trade-off:
 - Batch size too large (too much average inventory) versus
 - Batch size too small (too much ordering cost)
 - Examples:
 - Change-over costs (e.g., first 10 items must be scraped);
 - Transportation/Shipment costs...
- ➡ Key tool to determine/evaluate cycle inventory

EOQ Model Parameters

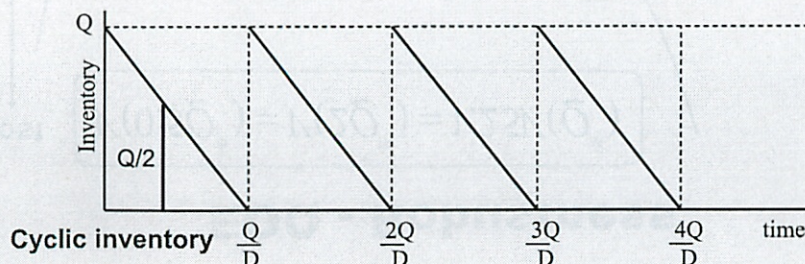
- Q = Order Quantity *decision*
- $D = 400$ = Demand Rate (units/time)
- $C = 45$ = Purchasing Cost (\$/unit) *parameters*
- $F = 500$ = Fixed Order Cost (\$)
- $H = 45$ = Inventory Holding Cost rate (%/\$ investment/time)

Assumptions:

- constant, deterministic demand
- instantaneous replenishment

EOQ Model Derivation

- Inventory Cost $C \cdot H \cdot \frac{Q}{2}$; Order Cost $F \cdot \frac{D}{Q}$;
- Total Cost $V(Q) = F \cdot \frac{D}{Q} + C \cdot H \cdot \frac{Q}{2}$



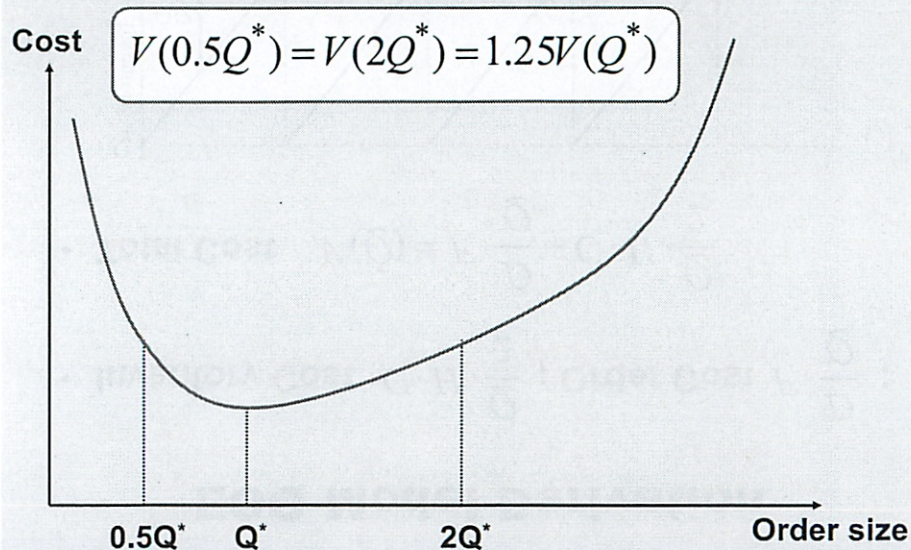
EOQ Formula

- Set first derivative to 0: $\frac{\partial V}{\partial Q} = -\frac{DF}{Q^2} + \frac{CH}{2} = 0$
- This yields:

$$Q^* = \sqrt{\frac{2DF}{CH}}$$

$$V(Q^*) = \sqrt{\frac{FDCH}{2}} + \sqrt{\frac{FDCH}{2}} = \sqrt{2FDCH}$$

EOQ - Robustness

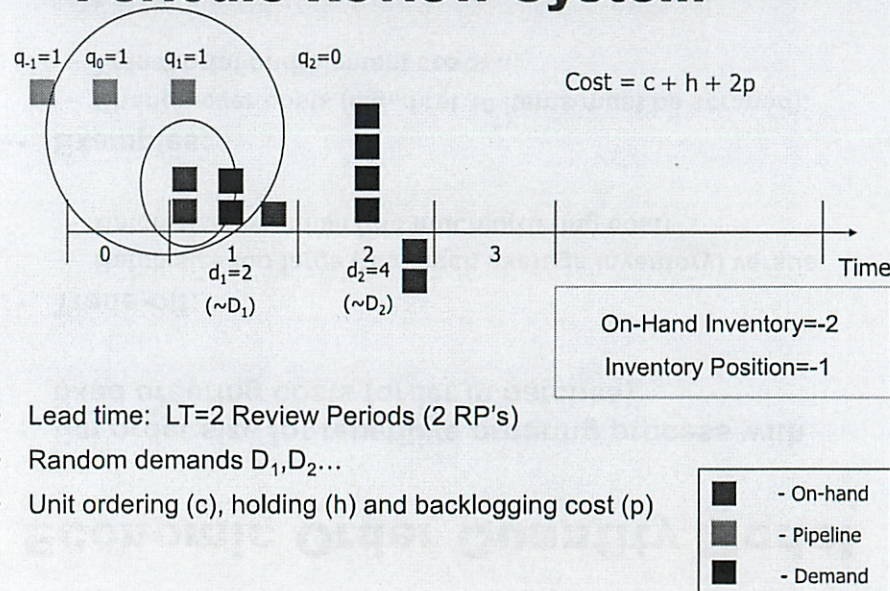


Multi-Period Models

- Dynamic ordering over time under uncertainty
- Periodic Vs. Continuous Review
- Backlogged demand Vs. Lost-Sales
- Objective:
Minimize holding and transportation costs under service level constraints

Minimize overall expected cost (holding, ordering, backlogging/lost-sales)

Periodic-Review System



Periodic Review Policy

- Order-up-to S policy:
Every review period, check the inventory position IP, and order Q such that:

$$Q + IP = S \quad \leftarrow \text{Base Stock Level}$$

- In this system:

$$IP = I_h + I_p$$

Inventory On Hand Inventory in Pipeline

An Order-Up-To S Policy

➡ “order back to S every review period”

Set S as the newsvendor solution:

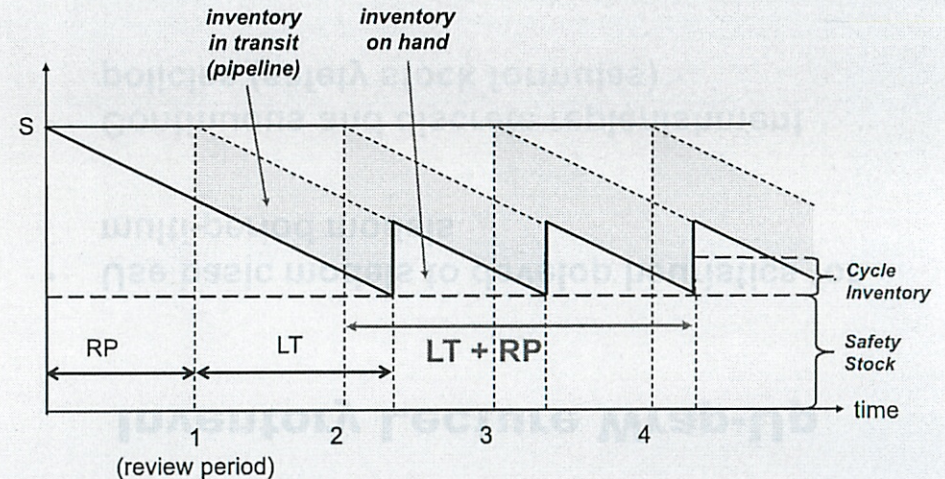
$$P(\text{DDLTRP} \leq S) = \alpha$$

where:

- α is the desired service level (e.g., 95%)
- DDLTRP = Demand During Lead-Time and Review Period

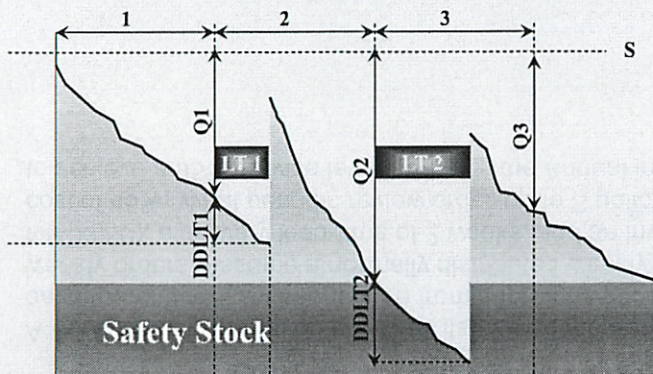
Order-Up-To Policy

➡ “order back to S every review period”



Periodic Review System

➡ “order back to S every review period”



LT = Lead Time
T = Cycle Time or Review Period
DDLTP = Actual Demand During Lead Time
Q_i = Order Size
S = Order Up To Level

Periodic Review Parameters

Main idea: set target level S such that:

$$P(\text{DDLTRP} \leq S) = \alpha \quad (\text{ex: } 95\%)$$

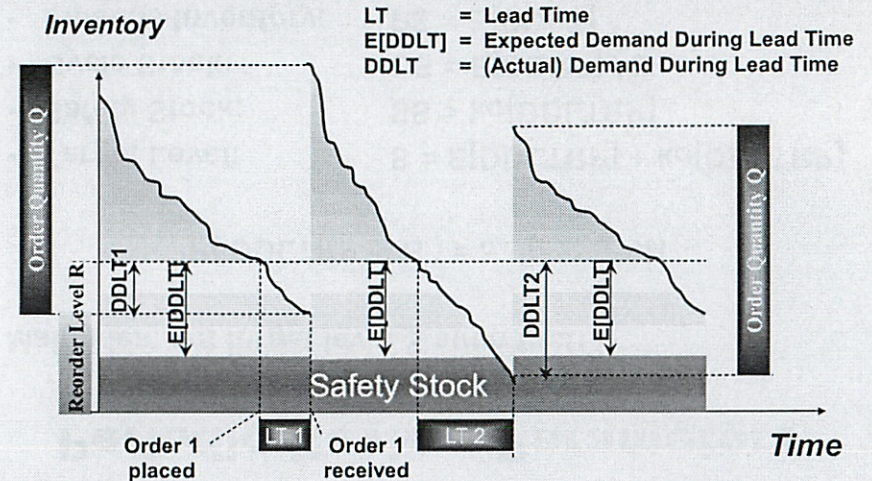
- | | |
|-----------------------|---|
| • Target Level: | $S = E[\text{DDLTRP}] + k\sigma[\text{DDLTRP}]$ |
| • Safety Stock: | $SS = k\sigma[\text{DDLTRP}]$ |
| • Cycle Stock: | $CS = E[\text{DDRP}] / 2$ |
| • Pipeline Inventory: | $PS = E[\text{DDLT}]$ |
| • Total Stock: | $TS = S - CS$ |

Periodic Review Example

A PC assembly operation procures its 128Mb memory chips at \$45 each (purchase + shipment cost) from a foreign vendor by placing weekly orders. Assume a normally distributed weekly demand $N(400, 80)$, a delivery lead-time of 2 weeks, and an inventory holding cost of 45%. What periodic review order-up-to S policy would you use to achieve a 95% service level? What is the Annual inventory cost?

Continuous Review System

➔ "order Q whenever inventory reaches R "



(R, Q) Parameters

➔ "order Q whenever inventory reaches R "

- Set Q as the EOQ solution
- Set R as the newsvendor solution:

$$P(DDLT \leq R) = \alpha$$

where α is a desired service level (e.g., 95%)

DDLT = Demand During Lead Time

Example (cont'd): if weekly demand for 128Mb chips, each shipment costs \$500 custom fees, weekly demand is $N(400, 80)$ and delivery time is 2 weeks, for a 95% service level:

$Q = 1,013$ units (use EOQ formula with $D=40$, $C=45$, $H=0.45/52$, $F=500$)

$R = E[DDLT] + 1.64 \times \sigma[DDLT] = 800 + 1.64 \times \text{sqrt}(2) \times 80 = 986$

Inventory Lecture Wrap-Up

- Use basic models to develop heuristics for multi-period models
- Continuous and discrete replenishment policies (safety stock formulas)
- Inventory pooling effect

HP

15.761 HP

3/29

Case Sport Obermeyer due 3/31

The Goal report due April 7!

Simulation game

- data available 4/1

- cons 4/10 - 4/15

Lunches scheduled for feedback

(I have a lot to say!)

- email Kanaka

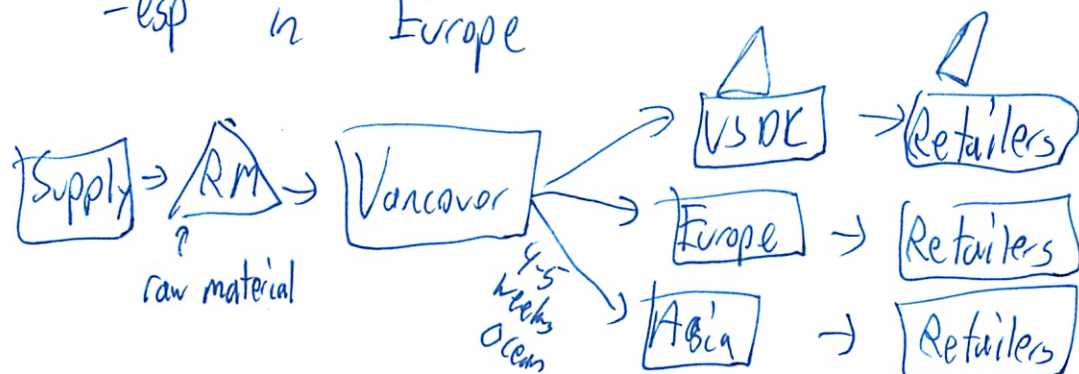
Recitation on multi-period inventory

Online survey

HP Supply Chain

Factory in Vancouver doing well

But problems with rest of supply chain
- esp in Europe



②

Vancouver

- efficient
- just-in-time
- Mant to order
- almost no inventory
- "kanban"
- short cycle time

But DCs have problems

- forecasting
 - shortages
 - excess
- different models
- localizations

6 week + lead time

- ocean shipment

disagreement on safety stock

What are carrying costs?

arbitrary pick success rate (LIFR) @ 98%

forecasting is done by marketing + country managers in DC

- care about sales
- don't care about inventory carrying cost
- not revenue
- just care about customers

③

Inventory costs sent to Vancouver

- people doing ~~more~~ forecasting not accountable for inventory costs

Each part of org has diff objective

- fight w/ each other

Incentives not aligned

Solutions:

- improve forecasting
- air shipment for Europe
 - lead time
- plant in Europe
- carrying even more inventory
- do localization in DC
 - delay of differentiation
 - making workers unhappy might be part of the job
- standardization
 - throw plug adapter in every box
- be more quantitative

Review

Set S such that

$$P(DDLTRP \leq S) = 98\%$$

Target level = $S = E[DDLTRP] + k\sigma[DDLTRP]$

\uparrow demand for lead time + replenishment period

Safety stock = $SS = k\sigma[DDLTRP]$

\uparrow from normal table
of std. devs

Cycle stock = $CS = \frac{E[DDRP]}{2}$

\leftarrow demand over lead time
Little's Law

Pipeline inventory = $PS = E[DDLT]$

Total stock = $TS = S - CS = SS + CS + PS$

X_1, X_2, \dots, X_n are RVs

X_i = demand in week i

$$Y = X_1 + X_2 + \dots + X_n$$

$$E[Y] = n \cdot E[X]$$

$$\text{st dev}[Y] = \text{sqrt}[n] \cdot \text{st dev}[X]$$

5

Look at each countries ~~on~~ # to calc safety stock
Use the model

$$k = 2.054 \text{ for a } 98\% \text{ LIFR}$$

$$LT = 5 \text{ weeks} = 35 \text{ days}$$

$$RP = 7 \text{ days} \quad \text{) 42 days}$$

review period

Assuming Demand is normal

Divide $\frac{\text{mean}}{30}$ $\frac{\text{st dev}}{\sqrt{30}}$ to get mean, st dev for each day

$$DDLT_{RP} \sim N(42 \cdot 14, \sqrt{42} \cdot 37) \quad \text{for whole review period } N(14, 37)$$

$$\underbrace{2.054}_k \cdot \underbrace{\sqrt{42} \cdot 37}_\sigma = \text{safety stock}$$

$$\begin{aligned} \text{Cycle stock} &= \text{demand over review period} = \frac{E[DDLT]}{2} \\ &= \frac{7 \cdot 14}{2} \end{aligned}$$

$$\begin{aligned} \text{Pipeline stock} &= \text{demand on lead time} = E[DDLT] \\ &= 35 \cdot 14 \end{aligned}$$

⑥ $DORP \sim N(7.14, \sqrt{7.37})$

Now can start talking about costs

Assumed holding cost = 58%

Just add up stocks and multiply by .5 • COGS
~~total inventory~~

Then calc what it Localization in DLs

And what it air?

- higher shipping cost

- shorter lead time

So much smaller PS

and smaller SS

So run the # and see what the costs are

SS - Since $\sqrt{\quad}$ in there it went down by more than half
Pipeline - linear change
- start change

If localize combine (add) means ~~not~~

- just one model

St dev - go back to original data to calculate

7

Distribution system w/ regional warehouses

$$N(10k, 2k)$$

95% service level

$$DLTRP \sim N(2 \cdot 10k, \sqrt{2} \cdot 2k)$$

one regional
center

thousand

$$SS = k \cdot \sigma$$

one center

$$= 1.64 \cdot \sqrt{2} \cdot 2k$$

or for whole system

Each has its own safety stock

But if one central warehouse?

$$DLTRP \sim N(8 \cdot 10k, \sqrt{8} \cdot 2k)$$

so we

are having
8 weeks

Since just adding

Safety stock cut in half

"risk pooling"

actually

Don't forget costs to do localization
or trucking further from other DCs

Announcements

1. Sport Obermeyer case report is due Wednesday, March 30 (beginning of class)
2. Goal report is due Wednesday, April 6
3. Mid Class survey is still available. Lunches are scheduled to get direct feedback
4. Simulation game: Data Available on April 1; Game April 10-15
5. Recitations this week on Multi-Period inventory (continuous and periodic-review)

Hewlett-Packard Case

1. The HP supply chain and distribution system
2. What are the causes of the inventory / service crisis?
3. Target inventory levels for European options
4. Recommendations and Implementation Plan

HP Supply Chain

Causes for the Inventory/Service Crisis

Solution

Periodic Review Parameters

A good heuristic is to set target level S such that:

$$P(DDLTRP \leq S) = 98\%$$

- **Target Level:** $S = E[DDLTRP] + k\sigma[DDLTRP]$
- **Safety Stock:** $SS = k\sigma[DDLTRP]$
- **Cycle Stock:** $CS = E[DDRP] / 2$
- **Pipeline Inventory:** $PS = E[DDLT]$
- **Total Stock:** $TS = S - CS$

Probability Review

- Let X_1, X_2, \dots, X_N be i.i.d random variables (e.g. X_i = demand in week i) and

$$Y = X_1 + X_2 + \dots + X_N$$

- Then $E[Y] = N \times E[X]$ and $Std[Y] = Sqrt(N) \times Std[X]$

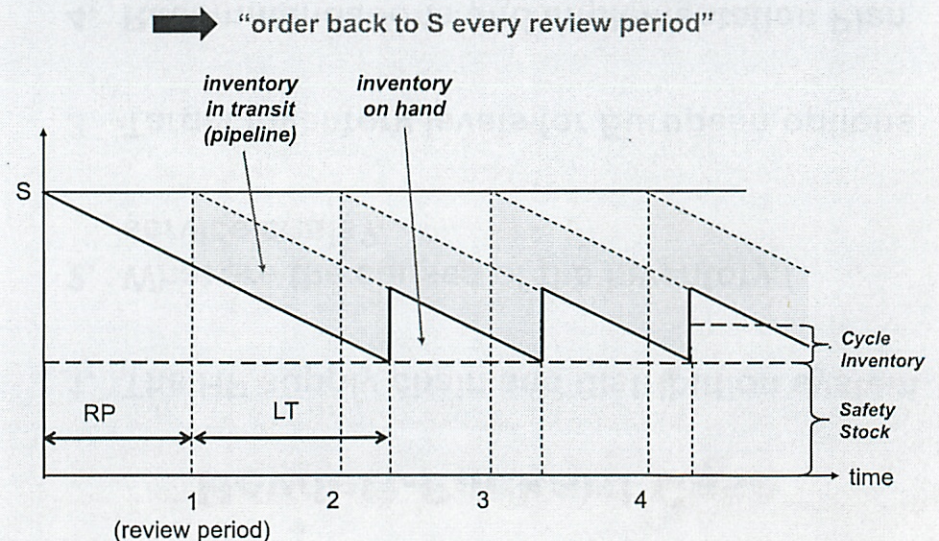
- Without independence and $Y = X_1 + X_2$:

$$\text{Std}[Y] = \text{Sqrt}(\text{Var}[X_1] + \text{Var}[X_2] + \text{Cov}(X_1, X_2))$$

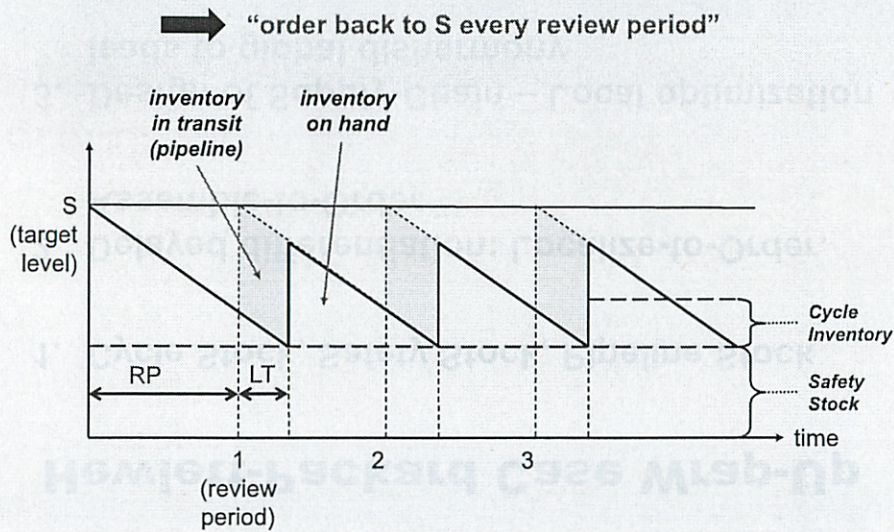
Sea Vs. Air Shipment

| | Demand Rate | | | | | | | | Stock | | | | |
|----------|-------------|--------|---------|--------|---------------------|-------|----------|-------|-----------|---------------|----------|------------|--------|
| | Per Month | | Per Day | | Average Stock Level | | | | Target | Cost Per Year | | | |
| | Mean | Stddev | Mean | Stddev | Safety | Cycle | Pipeline | Total | Inventory | Shipment | Total | % Profit | |
| AA (Sea) | 420 | 204 | 14 | 37 | 496 | 49 | 490 | 1035 | 1084 | \$ 273,177 | \$5,042 | \$ 278,219 | 41.80% |
| AA (Air) | 420 | 204 | 14 | 37 | 242 | 49 | 42 | 333 | 382 | \$ 87,869 | \$55,462 | \$ 143,331 | 21.54% |

Long Lead Times



Short Lead-Times

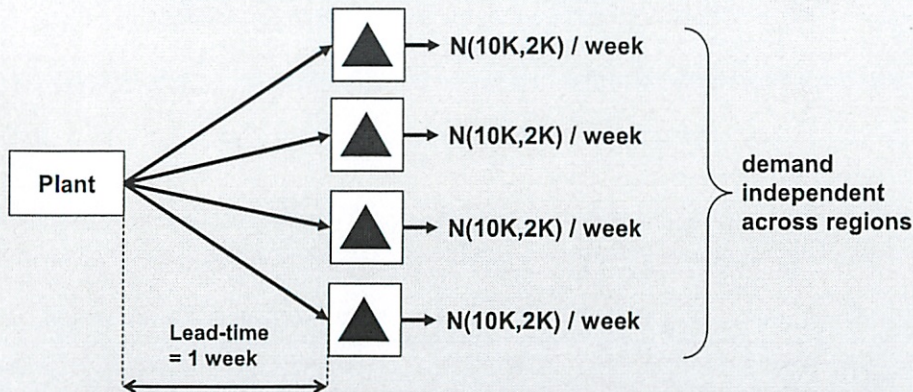


Localization

| | Demand Rate | | | | Average Stock Level | | | | Stock Target | Cost Per Year | | | |
|-----------------|-------------|---------|------|-------|---------------------|-------|----------|-------|--------------|---------------|-----------|--------------|----------|
| | Per Month | Per Day | Mean | Stdev | Safety | Cycle | Pipeline | Total | | Inventory | Shipment | Total | % Profit |
| Localization US | 23109 | 770 | | | 22689 | 2696 | 26960 | 52345 | | \$13,819,206 | \$277,303 | \$14,096,509 | 38.51% |
| Localization DC | 23109 | 6244 | 770 | 1140 | 15173 | 2696 | 26960 | 44829 | 47525 | \$11,834,860 | \$277,303 | \$12,112,163 | 33.09% |

Distribution System Example

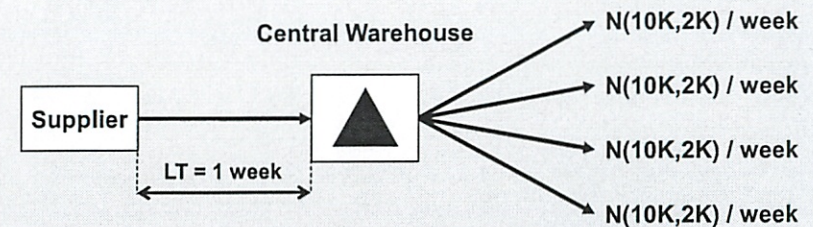
Regional Warehouses



Assuming order-up-to S weekly review policy in each warehouse (95% service level); How much safety stock should there be in this distribution system?

$$ss = 4 \times 1.64 \times \text{std}[\text{Reg. DDLTRP}] = 4 \times 1.64 \times \text{std}(2) \times 2000 = 18554$$

Central Warehouse



With an order-up-to S weekly review policy in the central warehouse (95% service level), How much safety stock should there be now?

$$ss = 1.64 \times \text{std}[\text{Total DDLTRP}] = 1.64 \times \text{std}(8) \times 2000 = 9277$$

Recommendations

Implementation Plan

Hewlett-Packard Case Wrap-Up

1. Cycle Stock, Safety Stock, Pipeline Stock
2. Delayed differentiation: Localize-to-Order, Assemble-to-Order
3. Design of Supply-Chain – Local optimization leads to global disharmony
4. Changing the Supply-Chain management

Case: Sport Obermeyer

Case Analysis – Team Assignment

Note to Students:

Hand in one paper copy of the write-up for each student group at the beginning of the class on **28 Mar** (29 Mar for Sections B & C). Your paper should provide answers to the specific case questions listed below. The answers must be less than 4 pages in length (excluding appendices) with font size of 12. Every graph or table/spreadsheet showing the results of computations must be accompanied by both a clear description of what all numbers shown represent qualitatively, and an exhaustive explanation of how they are computed, including relevant mathematical formulas or algorithms.

Our general policy for this class is that when preparing cases and assignments you should not receive any related input from anyone who has already participated in a faculty-led discussion of the same material, be it at Sloan or another school. When preparing any graded assignment you may not consult or use material not already included in the course packet or posted on the course webpage, unless this has been explicitly authorized by the instructor.

Case Questions:

1. Using the sample data given in Exhibit 10, make a quantitative recommendation for how many units of each style Wally should make during the initial phase of production. Assume that all of the ten styles in the sample problem are made in Hong Kong (minimum order quantity 600 units per style, provided any quantity of a style is ordered), and that Wally's initial production commitment must be at least 10,000 units because of capacity constraints later in the season.

WARNING: THERE IS NO 'RIGHT' ANSWER HERE. THE MODELS WE HAVE LOOKED AT WILL NOT COVER ALL YOUR BASES ON THIS PROBLEM. BE QUANTITATIVE BUT CREATIVE; IMAGINE IT'S YOUR COMPANY ON THE LINE.

2. What operational changes would you recommend to Wally to improve performance?

News vendor

Value of sale

~~Value~~ Loss of disposal

Balance expected value of each

$$E[\text{revenue}] = E[\text{loss}]$$

Where profit curve peaks

Want opp costs to =

Much simplified scope for #1

- just Hk

- only 10%

- just need to place half order now

- can order more after show

Nice even base

Wisdom of crowd - take avg

$$E[\text{profit}] = \$27 \quad \text{- just given, correct!}$$

Disposal \$9

? Said sold below wholesale

So lost profit as well - no that is in other cost

This is just they sell for ~\$102 and paid \$112.50

So not tricky

② No tricks on timing - 2 main orders from China

So don't need to worry about lead times, capacity, etc

So calc for each

On prices different

24% profit
8% loss



(3)

$$P = \frac{U}{U+O}$$

$U = 24\%$ • whole sale price
 $O = 8\%$ • wholesale price

k = Normal table for P

$$Q = \mu + k \cdot \sigma$$

So table is st dev of ^{people's} estimation - not demand!

~~So use~~

- No case says use st dev

- Actually case says st dev of sales = $2x$
st dev of committees

K should be # st devs from mean

- which is variation

WP: inverse CDF



4

Close - but does not match class

That is q ,

- are we calling h
(should be read)

We find Forecast for ~~20,000~~ 32,000

- elsewhere 20,000 is capacity

Only has to order 10,000 new

- so discount evenly

- or just the "safe" values

Half - min (600 or half)

If over 10,000 - not a problem

All about justifying.

Michael Plasmeier
Michael Nackoul
15.761
March 31, 2011

Question 1:

In the past, the Buying Committee of Sport Obermeyer Ltd. made decisions by arriving at a consensus after spending several hours in a meeting. However, this year Wally Obermeyer asked each member to write down their own forecast. From this data, it is our job to instruct Wally how many of each of the 10 parka styles he should order for next year.

We choose a “wisdom of the crowds” approach to processing each person’s forecast. The wisdom of the crowds theory holds that various data points will assemble into roughly a normal distribution, through the law of large numbers.

Because Wally reported that the Buying Committee’s forecasts were usually off by a factor of two times the standard deviation, we used two times the standard deviation of each member’s estimates in our model to reflect the additional uncertainty.

We based our analysis off the “newsvendor” model. This model is most appropriate when all of the stock needs to be ordered before the season begins. Although Wally does not need to order all of his stock at once, he needs to order well before the season starts.

This model seeks to balance the cost of liquidating excess inventory with the lost revenue of running out and missing a sale. Although Obermeyer is able to liquidate unsold inventory at the end of the year, they do so at a loss. The overstocking cost o was given as 8% of the wholesale price. The understocking cost u is their gross margin – which is 24% of the wholesale price.

We are looking for the point where, given our sales forecast, the next marginal unit ordered will no longer make us money, but instead cost us money to liquidate. We want to set the probability of running out to the understocking cost per item over the sum of the understocking and overstocking costs per item.

$$P[d \leq q] = \frac{u}{u + o}$$

We then calculate the CDF of the probability and then take the inverse. We use the inverse CDF (k) to find the amount of standard deviations we need to order away from the original predicted forecast and get the forecast for the season. The forecast is the mean of each person’s estimate, plus k times the standard deviation of the estimates, adjusted for the additional uncertainty.

$$q = d_{\text{forecasted}} + k * 2\sigma$$

The final task for our group was to determine how much to order at this time based on the forecasted demand for the season. The minimum order for each particular style was 600 units, with a total that had to be at least 10,000 units. This is only the first order that Sport Obermeyer has to place. We decided to put in a good base before the Las Vegas show. After the trade show, the forecast is updated and it becomes more accurate (exhibit 5 in case), and another order can be placed.

We decided to order half of our forecast of each item at this time. We believe that this would allow us to react to changes at the Las Vegas show. Further refinement of the estimate might result in a reduction in the forecast, but we do not believe that the reduction will be as deep as half of the item's current forecast + 600 (so a new order can be placed).

Our team thinks that this will give us enough cushion to give us a head start on the next round of orders without overstocking.

Question 2:

Under the current model, Sport Obermeyer has some operational flaws that could be greatly improved upon. There are three main goals where Sport Obermeyer should work to improve efficiency. Foremost, Sport Obermeyer's biggest problem stems from the uncertainty in forecasting demand for their products. They could either estimate better or reduce lead times. Next, they could take other steps to have lower labor and shipping charges at the same level of service. Finally, they could try to find higher prices for liquidated goods. Here are some things that could be done in order to accomplish these goals:

Improving Forecasting

- Run focus groups with consumers to see preferences
- Post products on their website or Facebook page and track consumer comments
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 - Since the Designs are finalized in September of the previous year, allowing customers to pre-order could give a much better detailed forecast to the proportion of designs that need to be ordered
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- Ask for more real-time sales data from retailer for knowing last-year's data AND to reorder earlier
 - Install a computer program that could track the sales of the previous year instantly to get a better feel for how the market is operating.

Cut Costs and Lead Times

- Bring Lo Village Plant online
 - Have Obersport strictly oversee operations in the new Lo Village plant
 - Develop quality managers that could run the plant efficiently
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- Suggest to Alpine that they open plants in China
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- Use common fabrics in multiple items
 - Could aggregate stock to concentrate uncertainty (like the Amex Travel Call Center)
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- Have less styles of items to aggregate demand as well
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- could save the overhead of running a US warehouse and enable orders to ship directly to retailer

Liquidated Goods at a Higher Value

- Become better at liquidating excess product at the end of the year
- Perhaps work with a deal site like Groupon or Woot to sell excess inventory at higher prices than they are receiving now
 - Online Bargaining Sites will help sell the bulk of the excess at a good price for both sides
 - May also help attract new customers to Sport Obermeyer
- Sell inventory the next season at full price
- Sell excess inventory at new locations
 - Help to test new markets to see where Sport Obermeyer should expand

| Style | Wholesale Price | Avg Forecast | St Dev Sales (2x Buying Committee St Dev) | Understocking Cost % | Overstocking Cost % | Understock Cost | Overstock Cost | Prob[D<=q] | CDF | Inverse CDF | Amount to Buy during Season | Order Max (Half or 600) for first Order |
|-----------|-----------------|--------------|---|----------------------|---------------------|---------------------------|--------------------------|------------|--------------|-------------|-----------------------------|---|
| Given | Given | μ | σ | Lost profit | Money lost | =wholesale * understock % | =wholesale * overstock % | =u/(u+o) | Normal Table | k=1/CDF | = $\mu+k*\sigma$ | max(half, 600) |
| Gail | 110 | 1017 | 388 | 24% | 8% | 26.4 | 8.8 | 0.75 | 0.773373 | 1.293038 | 1518.7 | 759 |
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| Sum | | | | | | | | | | | 32191.8 | 16095 must be >10000 |

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17/25

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Wrong k value calculated
 so optimal order quantity is too high

$$q = d_{\text{forecasted}} + k * 2\sigma$$

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★ See Figure 1 for orders

This results in over-ordering styles with high forecast variability, which is risky. Better to defer much of these styles orders to round 2 when you have much better demand info.

In light of much better demand info for round 2, better to order 10k minimum in round 1.

As such, you cannot order more than 12k in round 1 due to 3k/month limit.

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- Sell inventory the next season at full price
- Sell excess inventory at new locations
 - Help to test new markets to see where Sport Obermeyer should expand

+3

could potentially sell in S. America where the
northern hemispheres' summer is winter.
could avoid discounting altogether.

9/10

presentation: 1.5/2

Figure 1

| Style | Wholesale Price | Avg Forecast | St Dev Sales (2x Buying Committee St Dev) | Understocking Cost % | Overstocking Cost % | Understock Cost | Overstock Cost | Prob[D<=q] | CDF | Inverse CDF | Amount to Buy during Season | Order Max (Half or 600) for first Order |
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| | | | | | | | | | | | Should be ~26k | |

Goal due 4/7

MP Guest lecture 4/13 11:30-1 E51-345

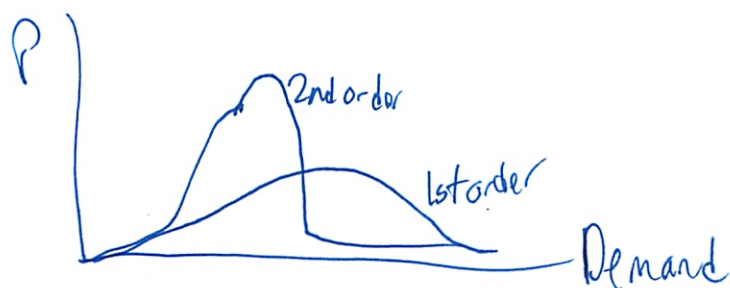
Forecasting

- understanding tradeoffs
- matching supply + demand
- place 50%

(by going 2 st dev we may have ordered too much!)
fell in trap

- Customs issue
- first order - only rough estimates
 - can't use previous year - styles change

- speculative 2-6 / 93
- Reactive 6-9 / 93



②

6,000 SKU

80% change each year

Sell to retailers

Forecast how accurate forecast it will be

Further up in supply chain forecast becomes less accurate

Forecasting is difficult

Usually people just want mean

But also need to consider σ

What are guiding principles

- No more than 10,000 so ~~want~~ could order more later
- Use info
- Be prepared
- Prioritize most profitable styles
- Order more accurate items now
- Tradeoff: profit vs risk

What is risk?

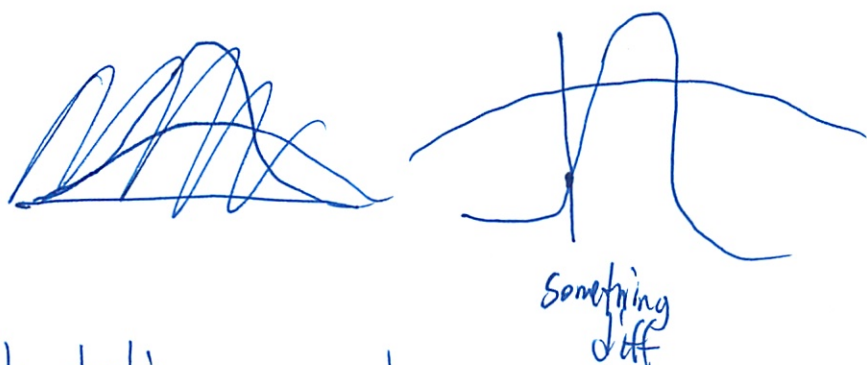
- st dev
- next time have to order 600

③

Prof: much more worried about not selling the product
- can always order more

Risk

- depends on dist



- not taking margins into account
- no risk on lead time
- smaller order less risky
 - less can lose
 - (hmm complex...)
- Coefficient of variation - standardizes variation scales
 - normalizes
- higher price - markdown more painful
 - so price does not take into account

(4)

Newsvendor

Too little

$$d > Q$$

↑ demand ↑ orders

Stockout risk

$$24\% \cdot (d - Q) \cdot \text{whole sale price}$$

Too much

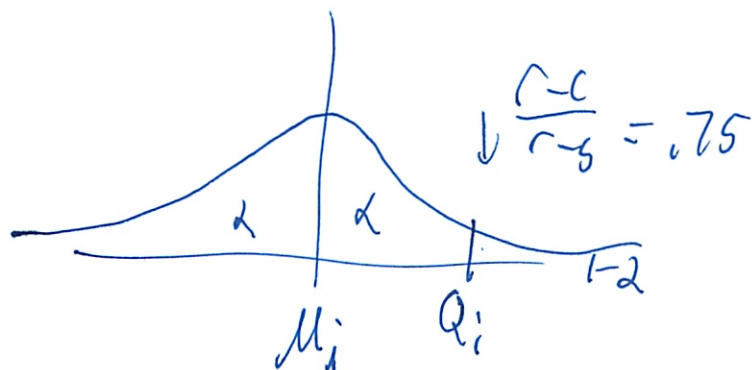
$$d < Q$$

Liquidation risk

$$8\% \cdot (Q - d) \cdot \text{wholesale price}$$

Choosing a percentile ← (oh that's what it is!)
- amt of stock going to order = $P(D \leq q)$

- assume demand $D = N(\mu, \sigma)$



$$Q = \mu_i + k \sigma_i$$

5

Come up guidelines

- these are your problems

- this is how I am thinking about your problems

No one right ans

Tough to solve optimally

Simple heuristics are almost always pretty good

Can find k such that $\sum Q_i = 10,000$

- with max 600 or $\mu + k\sigma_i$

? don't do that - violating risk principle!

His
Solution
well
Heuristic



(I don't get this)

$$P(D_i > 1200)$$

high as possible

$$D_i = \mu_i + z \sigma_i$$

$N(0,1)$

$$= P(\mu_i + z \sigma_i > 1200)$$

$$= P(z > \frac{1200 - \mu_i}{\sigma_i})$$

6) want ~~the~~ $\frac{1200 - \mu_i}{\sigma_i}$ as low as possible

$$SF = \frac{\mu_i - 1200}{\sigma_i} \leftarrow \text{want as high as possible}$$

Safety factor

$$SF = 0 \text{ if } 600 < \mu_i < 1200$$
$$= \frac{(\mu_i - 1200)}{\sigma_i} \text{ if } \mu_i > 1200$$

this is ~~the~~ for this solution

Choose n products w/ largest SF

$$Q_i = \mu_i + k \sigma_i \text{ w/ } k \text{ such that } \sum Q_i = 10,000$$

Choose n such that smallest $Q_i = 600$

Second Order

$$LV_{\text{Demand}} = 1.75 - Q_i + k \sigma_i$$

Since just 80%

Opp changes

↑ capacity w/ factory
air ship (↓ lead time)

⑦

Send samples out early to vendors

Sell to Chile - other seasonal time

Lower min order quantity

Nice picture w/ changes

Uniform zipper

- have inventory

Profit up 3% \rightarrow 5% (66% \rightarrow)