18.02 Practice Problems for Exam 2 (75 mins; Exam 2 is 50 mins.)

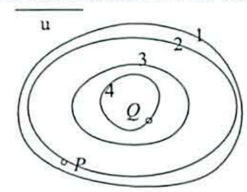
1. (20) For the function $w = y(1+x) + \sin(xy)$,

a) Write an approximate formula showing how Δw depends on Δx and Δy, at the point $(0,1)$.

b) At the point $(0,1)$, is w more sensitive to x or y? (give reason)

c) Find the directional derivative $\left.\dfrac{dw}{ds}\right|_{\mathbf{u}}$ at the point $(0,1)$ in the direction of the vector $3\mathbf{i} - 4\mathbf{j}$.

d) Starting at the point $(0,1)$, what is the minimal distance you could travel to increase the value of w by .2? (show work or indicate reasoning)



2. (15) Level curves for $w = f(x,y)$ are shown; **u** is a unit distance.

a) At P, draw in the vector $(\nabla f)_P$. b) At Q, estimate $\left(\dfrac{\partial w}{\partial x}\right)$.

c) Mark a point R where $f(R) = 3$ and $w_y = 0$.

3. (25) A wooden rectangular drawer with a capacity of one cubic foot is to be constructed. The wood costs \$1/sq.ft. for the bottom and the back, \$2/sq.ft. for the two sides, and \$3/sq.ft. for the front; there is no top. Let x be the end width, y the side width, and z the height, and C the total cost. What values for x, y, z minimize the total cost?

a) Show this leads to minimizing $C = xy + 2/x + 4/y$.

b) Find the minimizing values for x, y, z.

c) Use the second derivative test to show it is actually a minimum.

d) Give one of the equations for the Lagrange multiplier method, and use it to determine the value of the multiplier λ corresponding to the minimum.

4. (10) Where does the tangent plane to $x^2 + 2y^2 + 3z^2 = 12$ at the point $(1, 2, -1)$ intersect the y-axis?

5. (15) Let $w = w(x,y)$, and let r, θ be the usual polar coordinates.

a) Express $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial \theta}$ in terms of w_x, w_y, r and θ.

b) $\nabla w = 2\mathbf{i} + 3\mathbf{j}$ at $(1,1)$; evaluate $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial \theta}$ at this point.

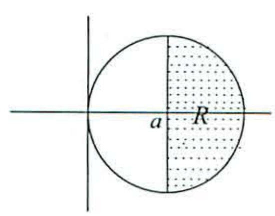
6. (15) Let $w = xy + xz + yz$, where the variables x, y, z are not independent, but constrained by a relation $y = f(x,z)$. Express $\left(\dfrac{\partial w}{\partial y}\right)_z$ in terms of x, y, z and the formal partial derivatives f_x and f_z. You can use either method: the chain rule or differentials.

7. (10) Find the volume of the region in space lying under the graph of $z = x^2 + y^2$ and over the triangle in the xy-plane having vertices at $(0,0)$, $(1,0)$, $(0,1)$.

8. (20) Let R be the upper half of the circular disc of radius a centered at the origin. Express the average distance of a point in R from the x-axis by an iterated integral in

(a) rectangular coordinates (b) polar coordinates (c) evaluate the two integrals

9. (10) Evaluate $\displaystyle\int_0^1 \int_x^1 \frac{dy\,dx}{\sqrt{1+y^2}}$ by changing the order of integration.



10. (15) Using polar coordinates and taking density $\delta = 1$,

a) set up an iterated integral giving the moment of inertia about the y-axis of the pictured shaded semicircular region R of radius a. *Don't evaluate it.*

b) Calculate the moment of inertia about the y-axis of the entire circular disc.

Definite integral formulas:

$$\int_0^{\frac{\pi}{2}} \sin^n x\,dx = \int_0^{\frac{\pi}{2}} \cos^n x\,dx = \frac{(n-1)!!}{n!!} A_n\,; \quad A_n = \begin{cases} 1, & n \text{ odd integer } \geq 3; \\ \pi/2, & n \text{ even integer } \geq 2; \end{cases} \quad n!! = n(n-2)(n-4)\cdots.$$

Exam 2 Pratice

1. $w = y(1+x) + \sin(xy)$

(a) $w_x = y + y\cos(xy)$; $= 2$ at $(0,1)$

$w_y = 1 + x + x\cos(xy)$, $= 1$ at $(0,1)$.

$\therefore \Delta w \approx 2\Delta x + \Delta y$.

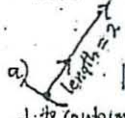
b) to x, since coeff. of Δx is bigger.

c) $\frac{dw}{ds}\Big|_{\hat{u}} = \nabla w \cdot \hat{u} = \langle 2,1\rangle \cdot \frac{\langle 3,-4\rangle}{5} = \frac{2}{5}$

d) $\frac{\Delta w}{\Delta s} \approx \frac{dw}{ds}\Big|_{\hat{u}} = |\langle 2,1\rangle| = \sqrt{5}$

(go in dir $\hat{u}$ = dir ∇w to get most rapid increase) $\therefore \Delta s = \frac{\Delta w}{\sqrt{5}}$

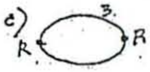
Ans: $\approx .1$ or $\frac{.2}{\sqrt{5}}$ $\qquad \approx \frac{.2}{2.2}$

2. a) 

$|\nabla w|_P = \frac{\Delta w}{\Delta s} = \frac{1}{1/2} = 2$

$\perp$ to contour line (twice length of $\hat{u}$)

b) $\frac{\partial w}{\partial x}\Big|_{Q} \approx \frac{\Delta w}{\Delta x} = \frac{-1}{1/2} = -2$

c) 

either point (where tan. line is vertical)

3. 

$C = xy + xz + 2\cdot 2yz + 3\cdot xz$ $\qquad xyz = 1$

a) $\therefore C = xy + 4yz + 4xz$
$= xy + \frac{4}{x} + \frac{4}{y}$

or using: $\left[xy + \frac{2}{x} + \frac{4}{y}\right]$

b) $C_x = y - \frac{4}{x^2} = 0$
$C_y = x - \frac{4}{y^2} = 0$

soln: $x = y$ (by symmetry) $\therefore x^3 = 4$
$x = \sqrt[3]{4}$
$y = \sqrt[3]{4}$
$z = 1/4^{2/3}$

$C_x = y - \frac{2}{x^2} = 0$
$C_y = x - \frac{4}{y^2} = 0$
$\therefore x = 1$, $y = 2$, $z = 1/2$

c) $C_{xx} = \frac{8}{x^3} = 2$
$C_{yy} = \frac{8}{y^3} = 2$
$C_{xy} = 1$

2nd der. test: in both cases, $AC - B^2 = 3$, $A > 0$ $\therefore$ minimum

$A = C_{xx} = \frac{4}{x^3} = 4$
$C = C_{yy} = 8/y^3 = 1$
$B = C_{xy} = 1$

$\rightarrow$

4. $x^2 + 2y^2 + 3z^2 = 12$, $(1,2,-1)$

tan. plane has normal

$(\nabla w)_{(1,2,-1)} = \langle 2x, 4y, 6z\rangle_{(1,2,-1)}$
$= \langle 2, 8, -6\rangle$
or $\langle 1, 4, -3\rangle$

$\boxed{x + 4y - 3z = 12}$ (since it goes through $(1,2,-1)$)

intersects y-axis where $x = z = 0$, $\therefore \boxed{y = 3}$.

5. a) $w_r = w_x\cos\theta + w_y\sin\theta$ $\quad \begin{pmatrix} x = r\cos\theta \\ y = r\sin\theta \end{pmatrix}$

$w_\theta = -w_x r\sin\theta + w_y r\cos\theta$

b) $(x,y) = (1,1) \Rightarrow r = \sqrt{2}$, $\theta = \pi/4$

$\therefore w_r = 2\cdot\frac{\sqrt{2}}{2} + 3\cdot\frac{\sqrt{2}}{2} = \frac{5\sqrt{2}}{2}$

$w_\theta = -2\cdot 1 + 3\cdot 1 = 1$

6. $\left(\frac{\partial w}{\partial y}\right)_z = w_x\left(\frac{\partial x}{\partial y}\right)_z + w_y\left(\frac{\partial y}{\partial y}\right)_z + w_z\left(\frac{\partial z}{\partial y}\right)_z$ (with $\left(\frac{\partial y}{\partial y}\right)_z = 1$, $\left(\frac{\partial z}{\partial y}\right)_z = 0$)

$1 = f_x\left(\frac{\partial x}{\partial y}\right)_z + f_z\underbrace{\left(\frac{\partial z}{\partial y}\right)_z}_{0}$

$\therefore \left(\frac{\partial w}{\partial y}\right)_z = w_x\cdot\frac{1}{f_x} + w_y = \frac{(y+z)}{f_x} + (x+z)$

Differentials:

$dw = w_x dx + w_y dy + w_z dz$
$dy = f_x dx + f_z dz$ $\quad$ eliminate dx

$dw = w_x\left(\frac{dy}{f_x} - \frac{f_z}{f_x}dz\right) + w_y dy + w_z dz$

$= \left(\frac{w_x}{f_x} + w_y\right)dy + \left(-\frac{f_z}{f_x} + w_z\right)dz$

$\uparrow = \left(\frac{\partial w}{\partial y}\right)_z$

d) Lagrange:

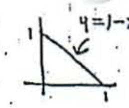
$C_x = \lambda g_x$: $y + 4z = \lambda yz$
$C_y = \lambda g_y$: $x + 4z = \lambda xz$
$C_z = \lambda g_z$: $4(x+y) = \lambda xy$
$x = y = \sqrt[3]{4}$ $\quad 8\cdot 4^{1/3} = \lambda 4^{2/3}$
$\lambda = 2\cdot 4^{2/3}$

or:

$y + 4z = \lambda yz$
$x + 2z = \lambda xz$
$4x + 2y = \lambda xy$
$x = 1$, $y = 2$ $\quad 8 = 2\lambda$, $\lambda = 4$

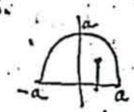
$\leftarrow$

7. $\int_0^1\int_0^{1-x}(x^2+y^2)\,dy\,dx$



Inner: $x^2y + \frac{1}{3}y^3\Big]_0^{1-x}$
$= x^2(1-x) + \frac{1}{3}(1-x)^3$

Outer: $\frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{-1}{12}(1-x)^4\Big]_0^1 = \frac{1}{12} - \left(\frac{-1}{12}\right)$
$= \frac{1}{6}$

8. 

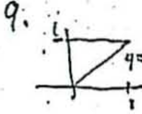
$\int_{-a}^{a}\int_0^{\sqrt{a^2-x^2}} y\,dy\,dx \Big/ \frac{\pi a^2}{2}$

$\frac{1}{\pi a^2/2}\int_0^{\pi}\int_0^{a} r\sin\theta\cdot r\,dr\,d\theta$

Evaluating (2nd form): $= \frac{2}{\pi a^2}\cdot\int_0^{\pi}\sin\theta\,d\theta\cdot\int_0^a r^2\,dr$
$= \frac{2}{\pi a^2}\cdot 2\cdot\frac{1}{3}a^3 = \frac{4a}{3\pi}$

1st: Inner: $\frac{1}{2}y^2\Big]_0^{\sqrt{a^2-x^2}} = \frac{1}{2}(a^2 - x^2)$

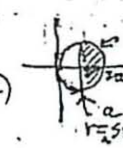
outer: $\frac{1}{2}\left(a^2x - \frac{1}{3}x^3\right)\Big]_{-a}^{a} = \frac{2}{3}a^3$

9. 

$\int_0^1\int_x^1 \frac{dy\,dx}{\sqrt{1+y^2}}$

$= \int_0^1\int_0^y \frac{dx\,dy}{\sqrt{1+y^2}}$

Inner: $\frac{y}{\sqrt{1+y^2}}$ $\quad$ Outer: $\sqrt{1+y^2}\Big]_0^1$
$= \sqrt{2} - 1$

10. a) 

$\int_{-\pi/4}^{\pi/4}\int_{a\sec\theta}^{2a\cos\theta} r^2\cos^2\theta\cdot r\,dr\,d\theta$

b) $2\int_0^{\pi/2}\int_0^{2a\cos\theta} r^3\cos^2\theta\,dr\,d\theta$

Inner: $2\cdot\frac{1}{4}r^4\cos^2\theta\Big]_0^{2a\cos\theta} = 2\cdot\frac{1}{4}\cdot 16a^4\cos^6\theta$

Outer: $8a^4\cdot\int_0^{\pi/2}\cos^6\theta\,d\theta = 8a^4\cdot\frac{5\cdot 3}{6\cdot 4\cdot 2}\cdot\frac{\pi}{2}$
$= \frac{5\pi}{4}a^4$

0/31

18.02 Exam Pratice

Lets start reviewing topic list

Last exam 66/90

Vector
- find angle
- matrix
- area of triangle
- unwinding tape

don't remember, but won't really be on

need for final

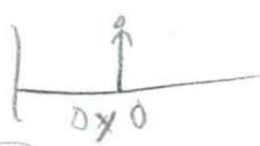
Last time got average, neet to do this well again!

Function of several variables

$f(x, y) = xy + x$

distance from origin $\sqrt{x^2 + y^2 + z^2}$

from y axis

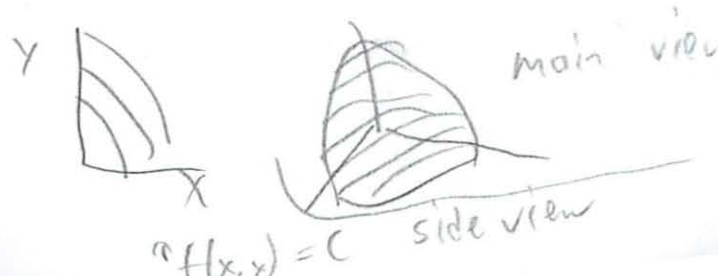


$\sqrt{x^2 + z^2}$

$$\boxed{\sqrt{x^2} = |x|}$$

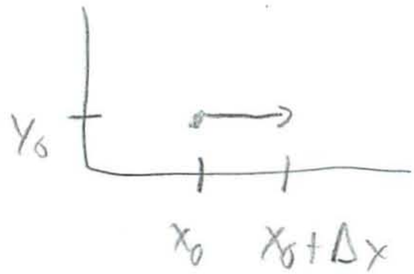
← don't forget abs value

Can't graph normally
- its a 3D surface



like a topo map

(2)



$\Delta w = w_1 - w_0$

$w = (x_0, y_0)$

$$\left(\frac{\partial w}{\partial x}\right)_0 = \lim \frac{\Delta w}{\Delta x}$$

at that pt

blah, blah, blah

basically keep one fixed

$$\left(\frac{\partial}{\partial x}\right)_y x^4 y^2$$

deriv of x keeping y constant ? or where evaling at ???

$4x^3 \; 2y \; \frac{\partial w}{\partial x}$

By how much does temp change as you go from one pt to other

$$\Delta w = \frac{\partial w}{\partial x} \Delta x + \frac{\partial w}{\partial y} \Delta y$$

? why both

-well $\Delta w \uparrow$ = $\cancel{\frac{\Delta w}{\Delta x} \cdot \Delta x}$ $\uparrow$

3)

$Y = x^2 y$

$x = 3m$

$y = 1m$

$m = 2xy \Delta x + x^2 \Delta y$

plug in

$2 \cdot 3 \cdot 1 + 3^2$

$6 \Delta x + 9 \Delta y$

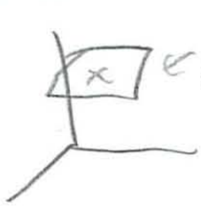
$\Delta v = (15)$

total possible error

y has biggest error

← slope of function at that pt

$\frac{\delta w}{\delta y}$ (y_0, w_0)



← plane

$a(x - x_0) + b(y - y_0) + c(w - w_0)$

$w - w_0 = A(x - x_0) + B(y - y_0)$

why I am doing this even ?

LA real questions for limitied

don't really need theory

④

$A =$ Slope in $\cancel{x}\ \hat{\jmath}$ dir

$$w - w_0 = A(x - x_0)$$

$$= \frac{\partial w}{\partial x}(x - x_0) + \frac{\partial w}{\partial y}(y - \cancel{x_0})$$

For example have sphere

 section piece

$$\boxed{\frac{dw}{ds} = \left(\frac{\partial w}{\partial x}\right)\frac{dx}{ds} + \left(\frac{\partial w}{\partial y}\right)\frac{dy}{ds}}$$ approx formula

[so same basic thing in 3D

$$\frac{dw}{ds} = \left\langle \frac{\partial w}{\partial x}, \frac{\partial w}{\partial y} \right\rangle \cdot \left\langle \frac{dx}{ds}, \frac{dy}{ds} \right\rangle$$

–I don't really get this

tez = right ?

∇ gradient
– level curves
3D deriv
(remember from physics)

5

Can convert to polar

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* key think of polar map
have to ask your self how much will temp change
as I move?

Think I am just going to flip through fairly quick +
do review
- and research as doing it

Min + Max when $w - w_0 = 0$

if $\frac{\partial f}{\partial x} = 0$

$\frac{\partial f}{\partial y} = 0$ at $P = 0$

Remember the box optimization problem
and least square line

2nd deriv

$AC - B^2$ ⊖ saddle
⊕ more → A or C ⊕ min
↳ A or C ⊖ max

Know how to do
- will prob have review plan

(6)

So have function

find W_x and w_y

set = to

solve for x, y these are critical pts.

and then $AC - B^2$.

$A = f_{xx}$ f_{xx}

$B = f_{yy}$ f_{yx}

$C = f_{xy}$ f_{yy}

(−) saddle

(+) → A (+) min

A (−) max

Lagrange multipler

Variables are constrained

oh this is w/ λ

Need to pratice one

$C = 4xz + 2yz + 3xy$ * constraint g

$xyz - 3 = 0$

$$\begin{cases} C_x = 4z + 3y = \lambda yz \\ C_y = \\ C_z = \end{cases}$$

Don't hack - be symetric - stare out

Solve for λ

7

Set = to

Now solve

$$\frac{24}{xyz} = 8$$

well before

$$\frac{4}{y} = \frac{2}{x} = \frac{3}{z}$$

3

Perp to level curve

Chain rule ~~Don't~~

$$\frac{dw}{dx}, \frac{dx}{dt}$$

(Don't think will do well on this test - material much more abstract this time

$w = w(x, y)$

$w = w(x(t), y(t))$

like if that is temp of ant at that pt and that time

$$\boxed{\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt}} = \underbrace{\frac{dw}{dt} = \vec{\nabla} w \cdot \frac{d\vec{r}}{dt}}_{\text{temp} \cdot \text{motion}}$$

(8)

Helix $\langle a\cos t, a\sin t, t \rangle$

Alright wasted an ~~unit~~ hr unfocused

Pratice test

1. $w = y(1+x) + \sin(xy)$

Write approx formula showing how Δw on $\Delta x + \Delta y$ at pt $(0,1)$

so what was it

$\Delta w = y(1+1) + \cos(xy) \cdot y \, \Delta x + 1(1+x) + \cos(xy) \Delta x \Delta y$

$\cancel{2} y + y\cos xy \Delta x + \cancel{2} + x + x\cos(xy) \Delta y$

$\cancel{2} \cdot 1 + 1 \cdot \cos 0 \, \Delta x + 2 + 0 + 0 \, \Delta y$

$2 + 1 \Delta x + 2\Delta y$

$\cancel{\Delta w = 4}$ ~~why does $\Delta x \Delta y$ drop out?~~

does not leave

$\Delta w = 2\Delta x + \Delta y$

differeiated wrong
- grrr
- should have distributed
- but did ~~chain rule right~~ ~~product rule~~ chain rule

$x(y)$

$x'(y) + y'$

compare

$y + xy$

$dx = y + y = 2y$ ← same answer!

$dy = 1 + x = 1 + x$

(9)

No duh derivitive of constant = 0

Don't make really stupid mistakes on question 1.

b) At pt is more sensitive to x or y

Δx because has large coefficient will varry more

perfect

c) Find the directional derivitive $\frac{dw}{ds}\Big|_{\hat{u}}$ at pt (0, 1) in dir $3\hat{i} - 4\hat{j}$

- forget this completely - what is directional deriv.

is it $\frac{\partial w}{\partial s}$

- lecture 10 (w/ gradient)

$$\frac{dw}{ds} = \frac{\partial w}{\partial x} \cdot \frac{dx}{ds} + \frac{\partial w}{\partial y} \cdot \frac{dy}{ds}$$

$$= \vec{\nabla} w \cdot \hat{u}$$

direction of the gradiant

$$\vec{\nabla} w \cdot (3\hat{i} - 4\hat{j})$$

$$\langle 2, 1 \rangle \cdot \frac{(3, -4)}{5}$$

← magnitude $\sqrt{3^2 + 4^2}$

right? now what?

(10) Remember how to do a dot product

$$(2,1)\cdot\left(\frac{3}{5},-\frac{4}{5}\right)$$

$$2\cdot\frac{3}{5}+1\cdot-\frac{4}{5}$$

$$\frac{6}{5}-\frac{4}{5}=\frac{2}{5}$$ ✓

d) Starting at pt $(0,1)$ what is min distance you could travel to ↑ the value by w by .2?

- I remember that we did this
- but forget how

$$\frac{\Delta w}{\Delta s}=\left.\frac{dw}{ds}\right|_{\hat{v}}$$ ~~oh~~ yeah what I found above

$$=|\langle 2,1\rangle|=\sqrt{5}$$

Lecture 11 (did exact problem before

- walk in dir $\vec{\nabla} r$ - but how far to go?

$$\langle 2,1\rangle$$

$$|\nabla\vec{w}|=\sqrt{5}$$ ← magnitude of gradient $\sqrt{x^2+y^2+z^2}$

set = to $=\left|\frac{dw}{ds}\right|_{\nabla w}$

SS

$$\frac{\Delta w}{\Delta s}=\frac{2}{\Delta s}$$

1)

$\frac{2}{\Delta s} = \sqrt{5}$

$\Delta s = \frac{.2}{2.2} = .1$

$\Delta s = \frac{\Delta w}{\sqrt{5}}$

So ~~go~~ go in that dir some distant

~~Δ~~ Δw = is the the amt temp should change by

~~$\sqrt{13}$~~ $|\Delta w|$ is the magnitude

So ~~$\frac{\sqrt{2}}{\sqrt{3^2+4^2}}$~~ $\frac{.2}{\sqrt{3^2+4^2}}$

n the direction vector we should travel

So basically the distance we should travel $= \Delta s$

$\Delta s = \frac{\Delta w}{\sqrt{3^2+4^2}}$ ← amt we want temp to change

← dir we are going

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reviewing test w/ answers works best

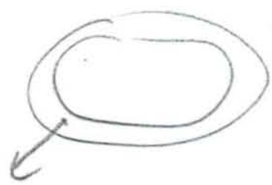
2

2. Level curves are shown u = unit distance

- good I like these conceptual

a) draw in $(\nabla f)_p$

- so go perpendicular down steepest path



No more complex than that

$|\nabla w|_p = \frac{\Delta w}{\Delta s} = \frac{1}{1/2} = 2$

oh I remember you go Δw which is one line
then you see how far you traveled Δs
but why is it going uphill
- guess I was wrong wants to go uphill
(must be thinking physics)

b) At Q estimate $\frac{\partial w}{\partial x}$

- so change in w as x changes

So some $\frac{\Delta w}{\Delta s}$ as go along x axis $\longrightarrow$

$\frac{1}{3/4} = \frac{4}{3}$ well $\frac{-1}{1/2} = -2$

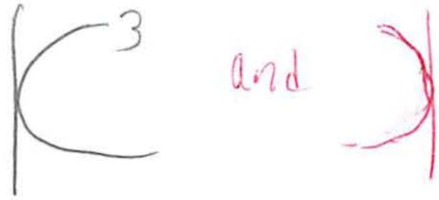
So use $\ominus$ sign going downhill + mis ~~cent~~ judged u lenght

(13)

Mark a point R where $f(R) = 3$ and $w_y < 0$

on 3 curve

where vertical tangent



✓ got one right

3. Where is that board problem

- convential + lagrange
- need to clarify exactly how it is done

$V = 1\ ft^3 = x \cdot y \cdot z$

2 sides

$\$ = 1 \cdot xy + 1\,xz + 2 \cdot 2\,yz + 3\,xz$

$\$ = xy + \cancel{2}\,4\,yz + 4\,xz = C$

So figured this out - now how to minimize?

well they just what you to see it simplifies in part A

~~$C = xy + \frac{yz}{4} + \frac{xz}{4}$~~ completly wrong

but how in all world does convert w/ constraint

$x = \frac{1}{yz}$ $\quad yz = \frac{1}{x}$

$y = \frac{1}{xz}$ $\quad xz = \frac{1}{y}$

$z = \frac{1}{xy}$ $\quad xy = \frac{1}{z}$

OPPS
blank

(15)

$\cancel{C = xy + \frac{1}{4y} + \frac{1}{4y}}$

$= xy + \frac{4}{x} + \frac{4}{y}$

So gotta work that constraint in somewhere

b) now find minimization

start w/ partial divs

$w_x = 1y + 4\cancel{\ln(x)} + 0$

$w_y = x \cdot 1 + 0 + 4\cancel{\ln(y)}$

not ln this is not integration
- get the simple rules down!

$w = xy + 4x^{-1} + 4y^{-1}$ ← write like this

$w_x = 1 \cdot y + 4 \cdot -1\, x^{-2} + 0$ ✓ better

$w_y = x \cdot 1 + 0 + 4 \cdot -1\, y^{-2}$

Now it is where both $= 0$

- find crit pts

$0 = y - \frac{4}{x^2}$ $(+2, 1)$ $(-2, 1)$

$0 = x - \frac{4}{y^2}$ $(1, +2)$ $(1, -2)$

well ∞ values ~~(0, 0)~~

$(4, \frac{4}{16})$ $(4, \frac{1}{4})$

well stick to integers

I gues

(16)

So is that ans

they say $x = 4$ by symmetry

$x^3 = 4$ - I don't see how they get this

$x = \sqrt[3]{4}$

d) In any case can prove now

$C_{xx} =$ well $C_x = y - \frac{4}{x^2} = y - 4x^{-2}$

$C_{xx} = 0 - -2 \cdot 4\, x^{-3}$

$8x^{-3}$

$C_{xy} = 1 - 0$

$C_{yy} = 0 - -2 \cdot 4 y^{-3}$ $\quad x - \frac{4}{y^2} = x - 4y^{-2}$

$8y^{-3}$

$AC - B^2$

$8x^{-3} \cdot 8y^{-3} - 1$

need to plg in pts
well found supposly in c

$-$ saddle
$\oplus \to A \ominus$ max
$\oplus$ min

AC

$8(\sqrt[3]{4})^{-3} \cdot 8(\sqrt[3]{4})^{-3} - 1$

$2 \cdot 2 - 1^2$

$\oplus$ ~~saddle~~ and a $\oplus$ so min ✓

~~##~~ 17

That problem is a disaster - making too many mistakes - not focusing

d) Now w/ Lagrange

- what do I set λ = to?

definie
$$\begin{cases} C_x = \lambda g_x \\ C_y = \lambda g_y \\ C_z = \lambda g_z \end{cases}$$

$$\begin{cases} y + 4z = \lambda yz \\ x + 4z = \lambda xz \\ 4(x+y) = \lambda xy \end{cases}$$ need $xyz = 1$ 4 eq, 4 unknowns

So what is this?

~~$xyz4$~~

Oh duh C_x is partial deriv w/ respect to x

What is g_x, g_y, g_z? sides

* constraint partial derivitives $xyz = 1$

Now ~~so~~ solve for λ

$$\lambda = \frac{y+4z}{yz} = \frac{x+4z}{xz} = \frac{4(x+y)}{xy}$$

? what next
should be able to get

(18)

$$\frac{\cancel{x}y}{\cancel{x}z} + \frac{4z}{yz} = \frac{x}{xz} + \frac{4z}{xz} = \frac{4x}{xy} + \frac{4y}{xy}$$

$$\frac{1}{z} + \frac{4}{y} = \frac{1}{z} + \frac{4}{x} = \frac{4}{y} + \frac{4}{x}$$

now some how

$$\frac{1}{z} = \frac{4}{x} = \frac{4}{y}$$

$xyz = 1$

; but how is that useful

~~$\frac{1}{?} = \frac{4}{z} = \frac{4}{?}$; guess~~

~~; this was suppose to help~~

multiply together

$$\frac{16}{xyz} \quad \frac{16}{1} = 16$$

$$r/3 = \frac{16}{3}$$

$$\frac{1}{z} = 16/3 \rightarrow z = \frac{3}{16} =$$

$$\frac{4}{x} = 16/3 \qquad x = 4 \cdot \frac{3}{16}$$

$$\frac{4}{y} = \frac{16}{3} \qquad y = 4 \cdot \frac{3}{16}$$

; really weird ans

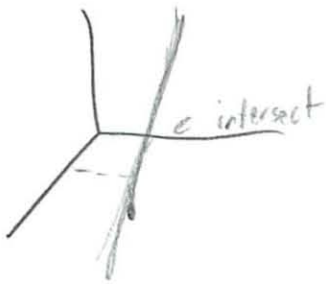
- so is ~~then~~ what they got

- don't really get why ;?

(19)

4. Where does the tangent plane $x^2 + 2y^2 + 3z^2 = 12$ at $(1, 2, -1)$ intersect y axis ?

- well I know distance from



tan. plane normal to

$$(\vec{\nabla} w)_{(1,2,-1)} = \langle 2x, 4y, 6z \rangle$$

find gradiant at that pt of the function

- but its a plane ???

plug pt in

$$\langle 2(1), 4(2), 6(-1) \rangle$$

$$\langle 2, 8, -6 \rangle$$

where it is located

reduce $\langle 1, 4, -3 \rangle$

$$x + 4y - 3z = ?\ 12$$

same ans

~~intersects when $y = 0$~~

~~now solve~~

$\cancel{x + 4(0) - 3z = 12}$

$\cancel{x - 3z = 12}$

intersects y when $x, z = 0$

$$0 + 4y - 3(0) = 12$$

$$4y = 12$$

$y = 3$ ①

(20)

Really should know that
Should do this test all over again tmo morning w/o looking

5. Let $w = w(x, y)$ let r, θ be polar

a) express $\frac{\partial w}{\partial r}$ $\frac{\partial w}{\partial \theta}$ in terms of w_x w_y r θ

? is this a chain rule problem ??

change in variable

$x = r\cos\theta$ $r = \sqrt{x^2+y^2}$

$y = r\sin\theta$ $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$W_r = W_x\cos\theta + W_y\sin\theta$

$W_\theta = -W_x r\sin\theta + W_y\cos\theta$

where in all world did they get this From?

From Lecture 14 chain rule

$w = \sqrt{x^2+y^2}$ $\frac{\partial w}{\partial x} = \frac{x}{\sqrt{x^2+y^2}}$

$\frac{\partial w}{\partial r} = \frac{x}{w}\cos\theta + \frac{y}{w}\cdot\sin\theta$

which $= 1$ since $w = r$

$\cos\theta\cdot\cos\theta + \sin\theta\sin\theta$

$= 1$

always ? or just if $w = \sqrt{x^2+y^2}$

here it is undefined

(2)

So we had

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} \cancel{\Delta x}^{\cos\theta} + \frac{\partial w}{\partial y} \cancel{\Delta y}^{\sin\theta}$$

don't know

(well w_x)

$w_x \cos\theta + w_y \sin\theta$ ✓

but then they want w_θ

~~but~~ let me look at my generic formula

$$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial v}$$

same w_x

w_y

but what is

$$\frac{\partial x}{\partial \theta}$$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$x = r\cos\theta$ ← so take deriv of this $r\cos\theta \cdot 1$

$y = r\sin\theta$ ← $r\sin\theta \cdot 1$

$-$ w_x ~~cos~~ $r\cos\theta + w_y\, r\sin\theta$

neg for some reason - otherwise good

JB

(2) Just need to remember this!

$$\frac{\partial w}{\partial x}\frac{\partial x}{\partial v}+\frac{\partial w}{\partial y}\frac{\partial y}{\partial v}$$ ← new

old new old

Then its not so hard - but don't really know when to use what

b) $\nabla w = 2\hat{\imath} + 3\hat{\jmath}$ at $(1,1)$

eval at pt

- lets see if I can do this

$$\frac{\partial w}{\partial r} = w_x \cos\theta + w_y \sin\theta$$

what is w → the vector the gradiant?

well gradiant given $\langle 2, 3 \rangle$ point we just need to know

$w_x = 2$
$w_y = 3$ ← right

$2\cos\theta + 3\sin\theta$

how do we find θ?

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

← so do use pts

$\tan^{-1}\left(\frac{1}{1}\right)$

$45°$

(23)

$2\cos 45^\circ + 3\sin 45^\circ$

$2 \cdot \frac{1}{\sqrt{2}} + 3\,\frac{1}{\sqrt{2}}$

$\frac{2}{\sqrt{2}} + \frac{3}{\sqrt{2}}$

$\frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$ ✓ got it, figured it out

$\frac{\partial w}{\partial \theta} = -w_x\, r\cos\theta + w_y\, r\sin\theta$

$-2 \cdot \sqrt{13}\,\frac{1}{\sqrt{2}} + 3\sqrt{13}\,\frac{1}{\sqrt{2}}$ ← copied pts wrong

$\boxed{\begin{array}{l} r = \sqrt{2^2 + 3^2} \\ \cancel{\sqrt{18}}\ \sqrt{13} \end{array}} = \sqrt{2}$

$\frac{-2\sqrt{13}}{\sqrt{2}} + \frac{3\sqrt{13}}{\sqrt{2}}$

$\frac{3\sqrt{13} - 2\sqrt{13}}{\sqrt{2}}$

$\frac{\sqrt{13}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{26}}{2} = \frac{3\sqrt{3}}{2}$

Key $-2 \cdot 1 + 3 \cdot 1 = 1$
— copied pts wrong

Cool I finally figured out — hopefully can remember

1

(24) 6. Let $w = xy + xz + yz$ where not independent (constrained by a function) $y = f(x, z)$

Express $\left(\frac{\partial w}{\partial y}\right)_z$ in terms of x, y, z and f_x f_z

Chain rule or differentials
(forget distinction

(Lecture 15 Chain Rule for Non independent variables)

1. Sub 2 into to remove it

Well let me just copy this one

$$\left(\frac{\partial w}{\partial y}\right)_z = W_x \left(\frac{\partial x}{\partial y}\right)_z + W_y \left(\frac{\partial y}{\partial y}\right)_z + \left(\frac{\partial z}{\partial y}\right)_z$$

$\uparrow 1$ $\uparrow 0$

So take partial deriv of original function
well they are just writing out chain rule
then one section will be 1 and one 0

$\frac{\partial y}{\partial y}$ ← same $\left(\frac{\partial z}{\partial y}\right)_z$ ← same

, but still don't add #s

$$1 = f_x \left(\frac{\partial x}{\partial y}\right)_z + f_y + 0$$

← don't really get

$$W_x \cdot \frac{1}{f_x} + W_y$$

here we actually add values

$$\underline{(1y + 1z) \cdot \frac{1}{f_x}} + x + z$$

25

Ok that does not seem that hard
- just do steps

Differentials

$$w = w(x,y,z)$$

$$dw = w_x\,dx + w_y\,dy + w_z\,dz$$

$$dy = f_x\,dx + f_z\,dz \quad \text{(eliminate } dx\text{)}$$

$$dw = w_x\left(\frac{dy}{f_x} - \frac{f_z}{f_x}\,dz\right) + w_y\,dy + w_z\,dz$$ diff for every function - take deriv of

$$= \underbrace{\left(\frac{w_x}{f_x} + w_y\right)}_{= \left(\frac{\partial w}{\partial y}\right)_z} dy + \left(-\frac{f_z}{f_x} + w_z\right) dz$$

Review

→ you can add tem
multiply by scalar function

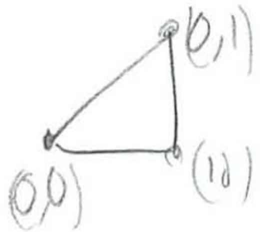
$$dw = \underset{\frac{\partial w}{\partial x}}{f(x,y)}\,dx + \underset{\frac{\partial w}{\partial y}}{g(x,y)}\,dy$$

; then just distribute
want part before dy (coefficent of)

(26)

Now moving into familiar teritory

7. Find volume under $z = x^2 + y^2$



$$\iint_R x^2 + y^2 \, dA$$

$$\int_0^x \int_0^1 x^2 + y^2 \, dx \, dy$$

y x

$$\int_0^1 x^2 + y^2 \, dx$$

$$\frac{x^3}{3} + y^2 x \Big|_0^1$$

$$\frac{1}{3} + y^2$$

$$\int_0^x \frac{1}{3} + y^2 \, dy$$

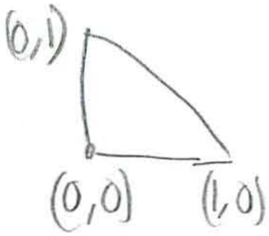
$$\frac{1}{3} y + \frac{y^3}{3} \Big|_0^x$$

$$\frac{x}{3} + \frac{x^3}{3}$$

$$\frac{x^4}{3}$$

I drew the triangle wrong! went far too fast!

27)



$$\iint_R x^2+y^2 \, dA$$

$$\int_0^{1-x} \int_0^1 x^2+y^2 \, dx \, dy$$

$y \quad x$

$$\int_0^1 x^2+y^2 \, dx$$

$$\frac{x^3}{3}+y^2x \Big|_0^1$$

$$\frac{1}{3}+y^2$$

$$\int_0^{1-x} \frac{1}{3}+y^2 \, dy$$

$$\frac{1}{3}y+\frac{y^3}{3} \Big|_0^{1-x}$$

$$\frac{1}{3}(1-x)+\frac{(1-x)^3}{3} \qquad (1-x)(1-x)(1-x)$$

$$\frac{1}{3}-\frac{1}{3}x+(1$$

Well shouldn't it come out to a #?

They did other way around -
- it should still come out same though
- but when 1-x on inside it gets replaced by a #

(28) Let me try their way (Checked notes and they always put functions inside)

$$\int_0^1 \int_0^{1-x} x^2+y^2 \, dy \, dx$$

$$\int_0^{1-x} x^2+y^2 \, dy$$

$$x^2y+\frac{y^3}{3}\Big|_0^{1-x}$$

$$x^2(1-x)+\frac{(1-x)^3}{3}$$

$$\int_0^1 x^2(1-x)+\frac{(1-x)^3}{3} \, dx$$

Should simplify and then have chain rule
-not easy either

$$\int_0^1 x^2-x^3+\frac{(1-x)^3}{3} \, dx$$

$$\frac{x^3}{3}-\frac{x^4}{4}+\frac{1}{3}\cdot 3\cancel{(1-x)}\cdot$$ ← no u substitution!

$$-\frac{1}{12}(1-x)^4 \Big|_0^1$$

then can do it

Let me try that u sub (remember how

$$\int \frac{(1-x)^3}{3} \, dx \qquad u=1-x \qquad u=(1-x)^3$$

$$du=0-1 \qquad du=3(1-x)\cdot -1$$

$$-3+x$$

29

Its eaiser than you think

pull out $\frac{1}{3}$

$$\frac{1}{3}\int (1-x)^3 \, dx$$

$$-\frac{(1-x)^4}{4}$$

$u = 1-x$

$du = -1\, dx$

somehow this works

$$-u^3 \, du$$

$$-\frac{u^4}{4}$$

$$-\frac{(1-x)}{4}$$ and it does work!

so was able to work backwords to get it to work - just need to be able to do this on a test

---

8. Let R be upper half of circular disk radius a

express average distance from x axis in rect + polar

(30)

$D = |y|$ ✓ → but they don't do abs value

duh since only top half of circle

Need to remember these pratical formulas

- well this is not one of them
- I think just write it

$$\int_{y\,0}^{x^2+y^2}\int_{x\,-r}^{r} y\,dx\,dy$$

? ~~start~~ based on last problem do it opposet way

$$\int_{-r}^{r}\int_{0}^{x^2+y^2} y\,dy\,dx$$

$$\int_{0}^{x^2+y^2} y\,dy$$ ← is this right?

$$\frac{y^2}{2}\Big|_{0}^{x^2+y^2}$$

- ~~can just write~~ r

$\sqrt{r^2+r^2}$ $\sqrt{a^2-x^2}$

well $a = r$ ↑ want $y =$

$x^2+y^2=a^2$

so ~~can~~ write

I knew that felt wrong

$$\int_{-r}^{r} r^2\,dy$$

$$\frac{r^2 y}{1}\Big|_{-r}^{r}$$

$$r^2 r - [r^2 \cdot -r]$$

$$r^3 + r^3$$

$$r^6$$

(31)

$$\int_{-a}^{a}\int_{0}^{\sqrt{a^2-x^2}} y\,dy\,dx$$

$$\int_{0}^{\sqrt{a^2-x^2}} y\,dy$$

$$\frac{y^2}{2}\Big|_{0}^{\sqrt{a^2-x^2}}$$

$$\frac{\left(\sqrt{a^2-x^2}\right)^2}{2}$$

$$\frac{1}{2}\int_{-a}^{a} a^2-x^2\,dx$$

$$\frac{1}{2}\cdot a^2x-\frac{x^3}{3}\Big|_{-a}^{a} \quad ✓$$

$$\frac{1}{2}\left[a^2a-\frac{a^3}{3}-\left[a^2\cdot -a-\frac{(-a)^3}{3}\right]\right]$$

$$\frac{1}{2}\left[a^3-\frac{a^3}{3}-\left[-a^3+\frac{a^3}{3}\right]\right]$$

$$\frac{1}{2}\left[a^3-\frac{a^3}{3}+a^3-\frac{a^3}{3}\right]$$

$$\frac{1}{2}\left[2a^3-\frac{2a^3}{3}\right]$$

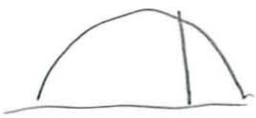
$$a^3-\frac{a^3}{3} \quad \text{aka } \frac{2}{3}a^3 \; ✓$$

So don't mess up inital step!

(32)

Now polar - should be easier

- well not that vert line



- actually $= r\sin\theta$

$$\int\int y \; r \, dr \, d\theta$$

$\theta \quad r$

$$\int_0^{\pi}\int_0^{a} r\sin\theta \; r \, dr \, d\theta \cdot \frac{1}{\frac{\pi a^2}{2}}$$ ← why in all world

Oh ~~...~~ does avg distance needs to be $\cdot$ area

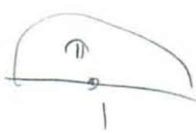
$$\int_0^{a} r^2 \sin\theta \, dr$$

$$\left.\frac{r^3 \sin\theta}{3}\right|_0^{a}$$

$$\int_0^{\pi} \frac{a^3 \sin\theta}{3} \, d\theta$$

$$\left.\frac{1}{3} \cdot a^3 \cos\theta \cdot 1 \right|_0^{\pi}$$

$$\frac{1}{3} \cdot a^3 \cos\pi \cdot 1$$



$$\frac{a^3}{3} \cdot 2 \cdot \frac{2}{\pi a^2}$$

↑ where from

$$\frac{4a}{3\pi}$$

(33)

But then other problem shows

$$\frac{\int_{-a}^{a}\int_{0}^{\sqrt{a^2-x^2}} y\,dy\,dx}{\frac{\pi a^2}{2}}$$

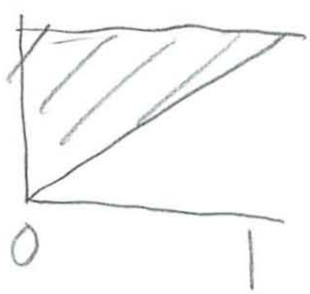
but they don't use when evaluating

wtf?

---

9. Eval $\int_0^1 \int_x^1 \frac{dy\,dx}{\sqrt{1+y^2}}$ change order of integration

I remember this draw pic



$\int_0^1 \int_0^Y$ ___ $dx\,dy$

*make sure to switch value

? how change original function again ?

don't -right ?

$$\int_0^1 \int_0^Y \frac{dx\,dy}{\sqrt{1+y^2}}$$

now do it

---

don't know if they spring Jacobian - should do such a problem

(34)

$$\int_0^1 \int_0^y \frac{1}{\sqrt{1+y^2}}\, dx\, dy$$

$$\int_0^y (1+y^2)^{-1/2}\, dx$$

$u = 1+y^2\ dx$

? just constant

$du =$

$$x(1+y^2)^{-1/2} \Big|_0^y$$

$$\int_0^1 y(1+y^2)^{-1/2}\, dy$$ ⊘ right here

$u = 1+y^2$

$\frac{1}{2} du (u)^{-1/2}$ ~~du~~ $du = 2y\, dy$

$\frac{1}{2} du = y\, dy$

$$\cancel{\tfrac{1}{2}} \cdot \frac{u^{1/2}}{\cancel{\tfrac{1}{2}}}$$

$u^{1/2}$ ✓ got that

$(1+y^2)^{\cancel{2}\,1/2} \Big|_0^1$ ← copy error turned $\frac{1}{2} \to 2$ which was wrong

$(1+1^2)^{1/2} - 1^2$

~~3~~

$\sqrt{2} - 1$

Nope not what they found

35

need like a 5min break every hr

Jacobian quick review

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$$

$$dA = \left| \frac{\partial(x,y)}{\partial(u,v)} \right| du\,dv$$

so basically this is how change is corresponding w/ before results

$$\iint_R f(x,y)\,dx\,dy = \iint g(u,v) \left| \frac{\partial(x,y)}{\partial(u,v)} \right| du\,dv$$

δ = density

Shorthand notation

Mass = Area · density $\qquad \iint_R \delta\, f(x,y)\,dA$

Moment$_x$ = $\cancel{y}$ · Mass

y = x · Mass

Center of mass $\bar{x} = \dfrac{\text{Moment}_y}{\text{Mass}} = \dfrac{x \cdot \text{mass}}{\text{mass}}$

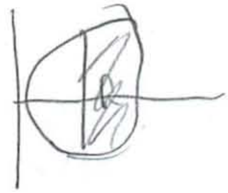
Moment of Inertia$_x$ = y^2 · Mass

y = x^2 · mass

z = r^2

36

10. Polar coord, $\delta = 1$



Moment of ieriat around y axis

$$\iint_R x^2 \cdot 1 \cdot dA$$

$$\int\int \cancel{x^2}$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^a (r\cos\theta)^2 \, r \, dr \, d\theta$$

↑ here one I think

r, no offset needed

$$\left[r^2\cos^2\theta + a\right] r \, dr \, d\theta$$

b) Now whole disk

$$\int_0^{2\pi} \int_0^a \left[r^2\cos\theta^2 + a\right] r \, dr \, d\theta$$

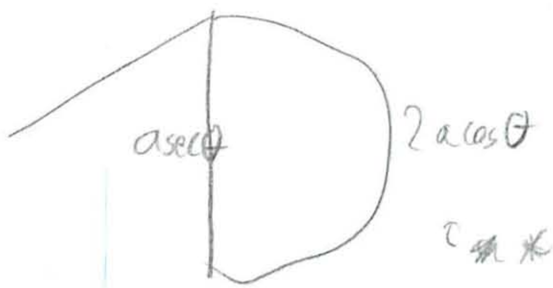
⋮ and do again

(37)

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int_{a\sec\theta}^{2a\cos\theta} r^2 \cos^2 \theta \, r \, dr \, d\theta$$

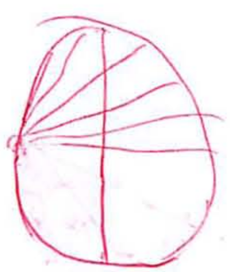
I guess they want you to go directly here

- make that mistake before
- not like previous scotch tape problem



think of

b) $2 \cdot \int_0^{\pi/2} \int_0^{2a\cos\theta} \cos^2\theta \, dr \, d\theta$



oh actually does reach everywhere

38

4/1

Day of Test - 2 hrs to go
- should try Recitation problems again
- learnd from pratice test a lot

approx formual

$\Delta w = \frac{\partial w}{\partial x} \Delta x + \frac{\partial w}{\partial y} \Delta y$

plug in pts if have them
(this test would be ~~hard~~ hard if open note)

deriv of constant = 0 for last time

larger coeffient more sensitive

directional deriv

$\nabla w \cdot \hat{u}$

$\langle 2, 1 \rangle \cdot \frac{\langle 3, -4 \rangle}{\sqrt{3^2 + 4^2}}$

dot product

$\left(2 \cdot \frac{3}{\sqrt{5}}\right) + \left(1 \cdot \frac{-4}{\sqrt{5}}\right)$

Min distance will change

$\Delta s = \frac{\Delta w}{\sqrt{5}}$ ← how far temp/etc will change
Δs ← distance
$\sqrt{5}$ ← magnitude

Level curve

perp to increasing #s

$\frac{\Delta w}{\Delta x}$ ← amt temp moving (lines
← distance move on paper

~~[illegible]~~ $W_y = 0$

Board problem (oh boy)
- set up fine
- they wanted you to write it simplified and 2 variables
(have to sub in constraint

minimize w/ ~~[illegible]~~ differeintation (do it right!)
- set $= 0$ ($w_x = w_y = 0$)

Solve for crit pts

test $AC - B^2$
⊖ saddle
⊕ → A ⊖ max
A ⊕ min

Now w/ λ

differeinitate C w/ x, y, z
" g (constraint)

$dC_x = \lambda \, dg_x$

(10) ~~A~~ 4th eq constraint

4 eq + 4 unknowns

solve for λ = something

then you do something I don't really remember

when get $\frac{16}{xyz}$ ← know what that is

I think I got this wrong

Let's look at Recitation

He divided it up into cases

$\boxed{x, y \neq 0,\ x=0,\ y=0}$

then easy to solve

$x^2 = \lambda - 1$

$y^2 = \lambda - 1$

$x^2 + y^2 = (\lambda - 1) + (\lambda - 1) = \ldots$

then solve for λ

x, y

n can have lots of possible values for each

Still don't get

- why cases here and not before
- and don't get some of the previous algebra

1) where does plane intersect Y axis

- take gradiant
- plug in pt
- reduce
- write in eq = 12 (same as before)
- y axis → x, z = 0

solve y = 3

Now the chain rule problem

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial r}$$

normal partial deriv

now w/ respect to r

$r = \sqrt{x^2 + y^2}$

$x = r\cos\theta$

$\frac{\partial x}{\partial r} = \cos\theta$

write as W_x and it was given here $\langle 2, -3 \rangle$

W_x

θ can be found $= \tan^{-1}\left(\frac{y}{x}\right)$

$r = \sqrt{1^2 + 1^2} = \sqrt{x^2 + y^2}$

② In 3D

$$\left(\frac{\partial w}{\partial y}\right)_z = w_x\left(\frac{\partial x}{\partial y}\right)_z + w_y \underset{\uparrow \atop 1}{\left(\frac{\partial y}{\partial y}\right)_z} + w_z \underset{\uparrow \atop 0}{\left(\frac{\partial z}{\partial y}\right)_z}$$

$$w_x\left(\frac{\partial x}{\partial y}\right)_z + w_y$$

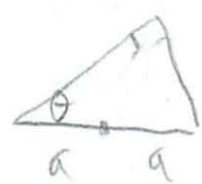
then I don't know what they did

- isn't it just last problem, but easier

Finding volume of shape I can do

- as long as I draw it correctly
- put the functions inside - when function of other variable

$$V = \cancel{\pi} \int_0^{2a\cos\theta}$$



- figured out simple chain rule - good

$\leftarrow \sqrt{a^2-x^2}$

Since ~~M~~ $a^2 = x^2 + y^2$

solve for <u>y</u>

Changing integration

(43) And the actual value

- well when to u and v I did change it to be intoms of that - but ~~the~~ thats changing variables, not order

I can't find any change order - but ~~it is what~~ I'm ~~W~~ pretty sure you don't touch the actual value

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1 hr to go - look at recitation problems

- Confirmed reversing limits = no change

To go to new variables use "chain rule type thing"

$f(x,y) - f(u,v)$

Well chain rule simplified

$$= x_u f_x + y_u f_y$$

$$x_u w_x + y_u w_y$$

$$= \underset{\uparrow}{w_x} x_u + \underset{\uparrow}{w_y} y_u$$

given in gradiant

(44)

$$\frac{\partial f}{\partial v} = 3xv + 1yv$$

and that is what they wanted - in terms of those variables - all makes sense now

b) then they define u ✓

now can find

$\frac{\partial x}{\partial u}$ $\frac{\text{old}}{\text{new}}$ $u = \frac{x+y}{2} = \frac{x}{2} + \frac{y}{2}$

$= \frac{1}{2}$

then plug values $(1,2)$ in

~~Nope~~ $\boxed{\text{No}}$ $\frac{\partial x}{\partial u}$

· differentiate x w/ respect to u

Solve for $x = u + v$

now differentiate w/ respect to $u = 1$

Don't actually need point, do we?

(45) These reciation problems look a lot eaiser than tey once did

Except the La Grange

Ok 10:16 AM

Shower now ten test at 11