

## Final Exam Review Problems Categorized Into Eight Categories (listed chronologically)

Search: SP09 #4.2, FA09 #6B, SP10 #8, Alt #7.1

Probability: SP09 #6, FA09 #5, SP10 #7, Alt #1

Circuits: SP09 #2, FA09 #4, SP10 #4, SP10 #5, SP10 #6

Control Systems: SP09 #7\*, FA09 #3\*

System Functions: SP09 #5, SP09 #7\*, FA09 #3\*, SP10 #3

State Machines: FA09 #2, SP10 #2

Classes: SP09 #3, SP09 #4.1, FA09 #1, SP10 #1

Procedures: SP09 #1

\* implies overlap, as all of the problems that seem to focus on different control & feedback systems rely on system functions for analysis.

Topics

"Kendras" + Evans' Comedy Hr"

~~W~~

Strategies

- read everything
  - 5-10 min
- start w/ problems easy to solve and short - no long writing at code
- skip over hard problems
- Ask proctors questions if you don't understand
- know the concepts / building blocks

Search


- Fall 09 #6

Heuristic - admissible, must underestimate

n-1 - is good  
 will always be less  
 can't have n-1 trips

n-2 - is always  $\leq$  the proper value

# of trips  $\approx$  oh did not think



②  
n-3 will sometimes be more  
So n-2 is ~~the~~ best

Roads

- none of roads buried
- state
  - your current loc
  - your current car
  - but only 1 place to swap car → loc A is only place
    - excess info may hurt you
    - says need to keep
    - but keep it in for now if unsure
- don't need car cost because doing simple search
  - not doing A-star or heuristic

(loc, car)  
( 's', 'sports' )

c) Goal test

- what pass in?
  - its a procedure
  - need a lambda
  - lambda loc: ~~loc~~ loc[0] = 'D'
- ← since state, and this is 1st component of loc*

3) aka

```
def goalTest(self, state):  
    return state[0] == 'D'
```

Can write either way

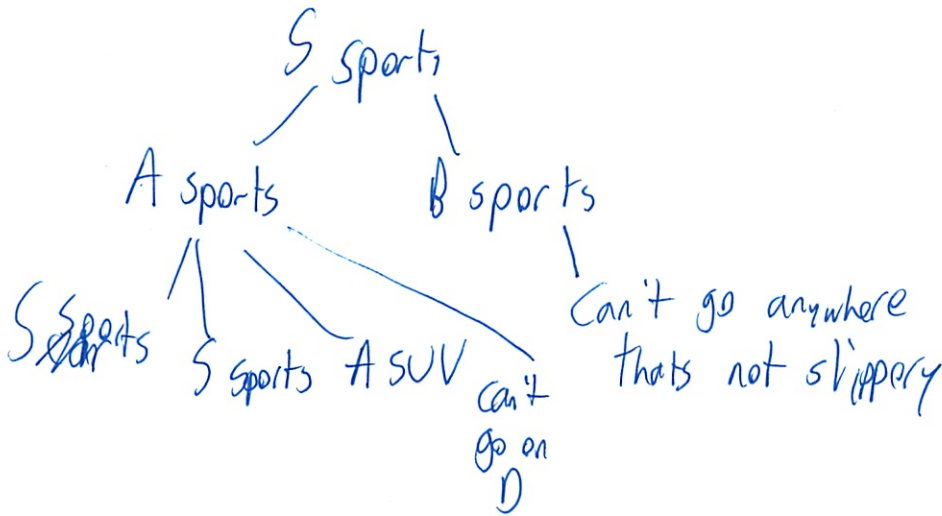
d) Now do the tracing



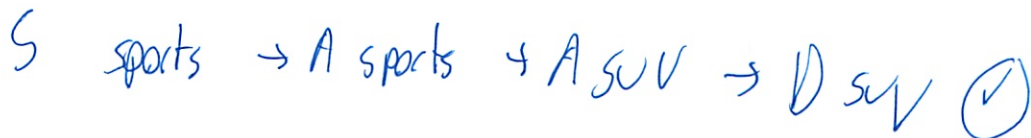
? the 0 means first path

e) Now breath 1st

- guaranteed to get shortest # of hops
- and good since uniform cost (cost does not matter)



What will it do



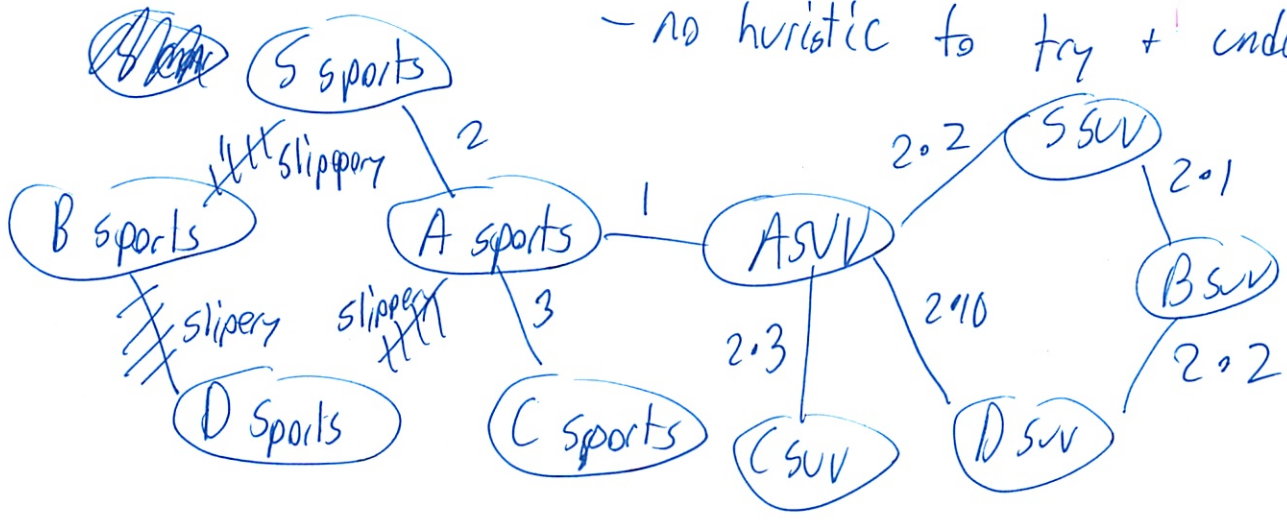
4

### e) Uniform Cost search

- all paths are same cost

- ~~no here costs matter~~ Use the <sup>given</sup> costs - costs matter

- no heuristic to try + under estimate



$$2 + 1 + 4 + 2 + 4 = 13$$

Draw the paths

S sports is a different state from SSUV

- here might add cost to previous state

### Depth First

- build code not to have it go backwards

- it picks the first item in the list of neighbours

### f) Admissible heuristic - anything that underestimates

$$2n - 1$$

better heuristics get closer to actual value

5) Uniform Cost looks down every path

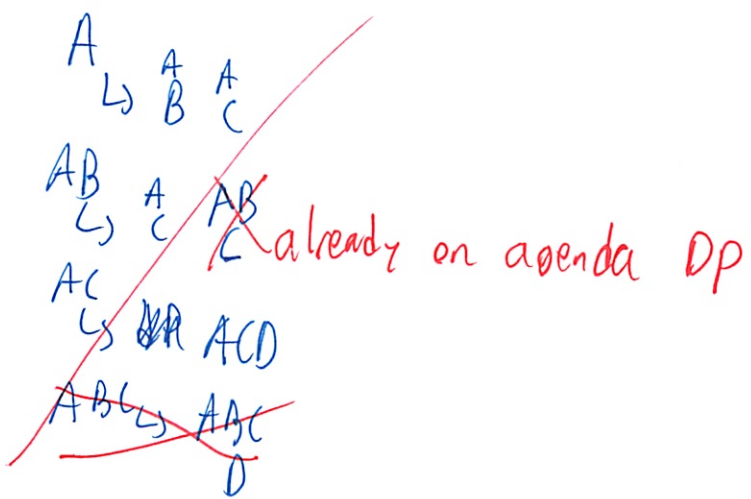
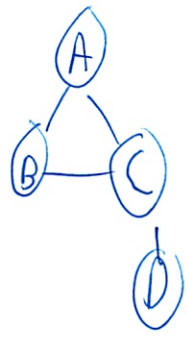
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### DF No DP

- ~~not~~ guaranteed to find a path
- are some edge cases - if unsure, ask
- but not guaranteed to be shortest
- states can be visited multiple times
- agenda is short

### DF w/ DP

- yes will find path
- not necessarily finds best path
- will not visit same state multiple times



but then stack

6

A  
↳ A B C  
    ↑ use last

A  
C ↳ A B AC  
    D  
AB ↳ AC D   won't add ABC since DP

BF no DP

- guaranteed to find path
- " " " shortest path
- will visit pts again
- agenda will grow long

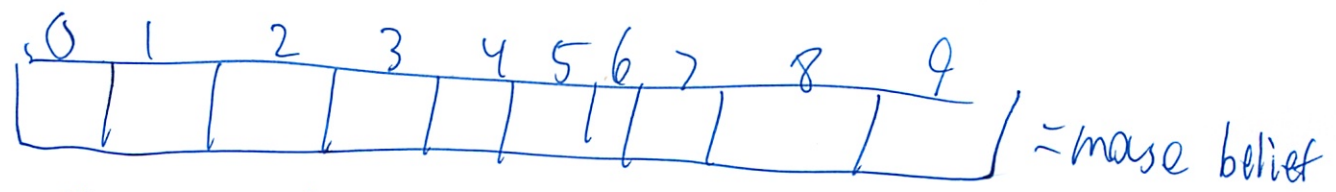
BF w/ DP

- will not visit pt again

All will find a solution - assuming pruning rules

Probability

Cat + Mouse - extra problems packet

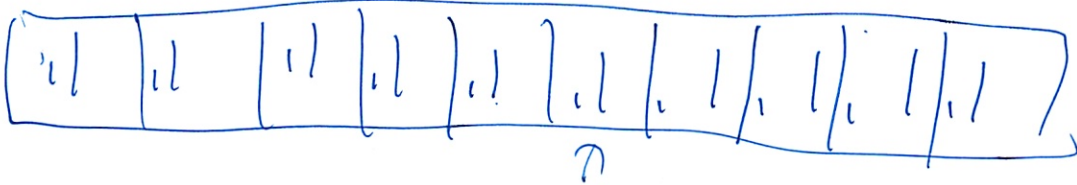


like our robot in the hallway

1)

Cat Loc  $\{0, \dots, 9\}$  - knows where it is

start dist



assume  
Cat  
in 5

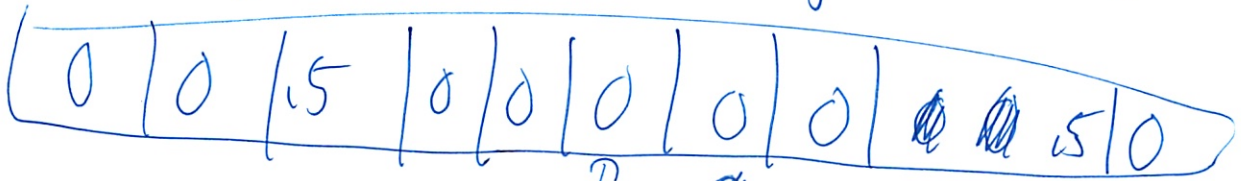
20,

Obs - hear noise

Cat hears "3"

so can be loc ~~2~~, 8

- (assuming cat has perfect hearing - can always do noise)



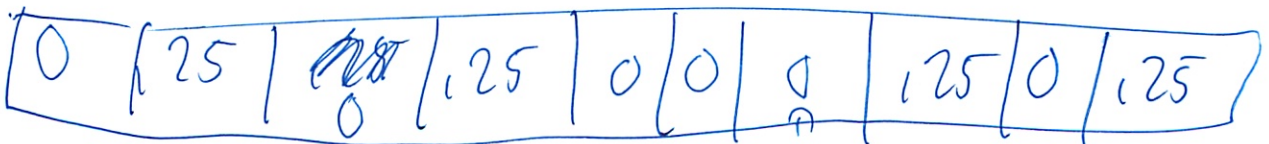
(post  
~~pre~~ normalized)

21. Now transition update

- mouse  $\xrightarrow{15.5}$

- cat move is given - so don't have to track

- says cat moves  $\downarrow$



new cat

(I think I got this)

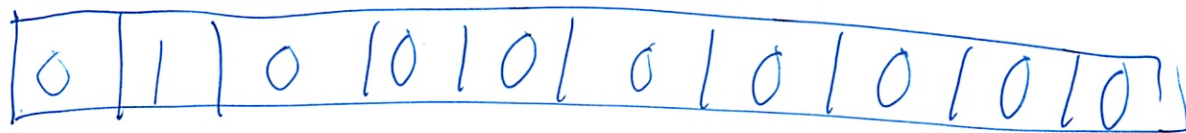


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need to review systems  
circuits on own

22. Now cat hears again

- hears 5



↑ know where mouse is

Don't look at last 2 problems

- old code

- must write in | get next value

- SM abstraction

23. def \_\_init\_\_(self, catLoc, H=10):

self.startState = ([l, o/H for l in range(~~len(H)~~)])

self.H = H

↳ in range(H), catLoc

↑ should prob use a PDist

Could have noise in exam

will likely be more work than this

Fall 04 #5

Conditional probabilities

- I got this

- but did not do it formally, even though I thought about it

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$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

$$P(S=w|O=n) = \frac{P(O=n|S=w)P(S=w)}{P(O=n)}$$

$$= \frac{.1 \cdot .4}{\sum = .28} = \frac{4}{28}$$

$$P(S=s|O=n) = \frac{.3 \cdot .5}{\sum = .28} = \frac{15}{28}$$

$$P(S=c|O=n) = \frac{.9 \cdot .1}{\sum = .28} = \frac{9}{28}$$

$$\sum = .1 \cdot .4 + .3 \cdot .5 + .9 \cdot .1 = .28$$

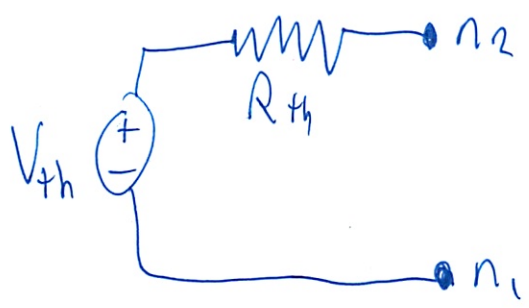
Not doing B

10

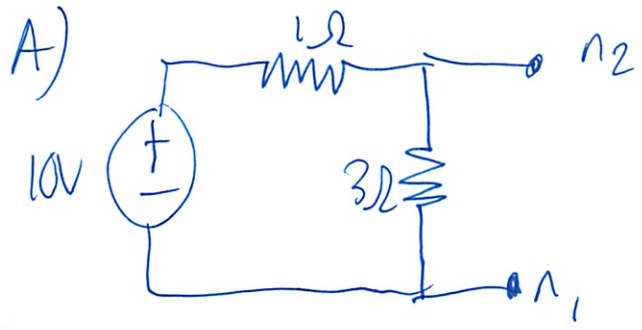
Circuits

- 2nd exam
- Thevinence failed big time on exam 2
- lots of Op-amp problems

Thevinin



Match any other circuits to this circuit



1st what is the voltage drop <sup>voltage</sup> just b/w pins

- voltage divider

$$V_{th} = \frac{3}{4}(10V) = 7.5V$$

2nd short the wires  $n_1 \rightarrow n_2$

find eq resistance

$$R_{th} = \frac{V_{th}}{I_{short\ circuit}} \quad \left. \vphantom{\frac{V_{th}}{I_{short\ circuit}}} \right\} \text{ w/ current}$$

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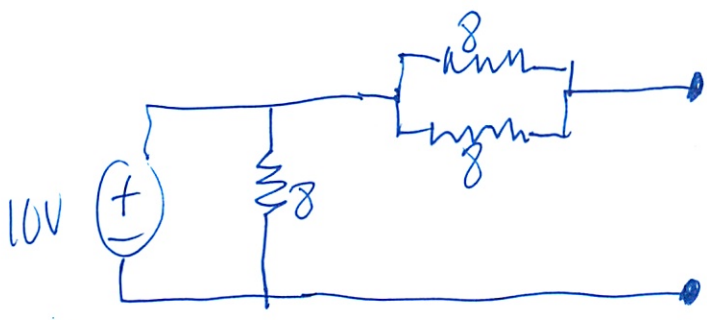
$$I_{sc} = 1$$

$V = IR$   
10      1      only the 1Ω resistor in use

$$I = \frac{10}{1}$$

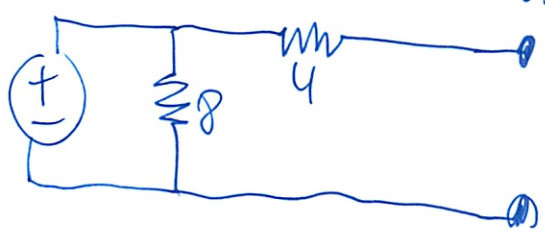
$$R_{th} = \frac{7.5}{10} = .75$$

B



First thought: this looks ugly  
find ea resistance

- usually ends up helpful  
- but not always need to hide



$$V_{th} = 10V$$

- no current drawn through wire - ignore 4Ω
- 8V not used either
- if did voltage divider =  $\frac{8}{8}$

$$R_{th} = \text{add } \cancel{\text{short}} \text{ short circuit wire}$$

$$= \frac{V_{th}}{I_{sc}}$$

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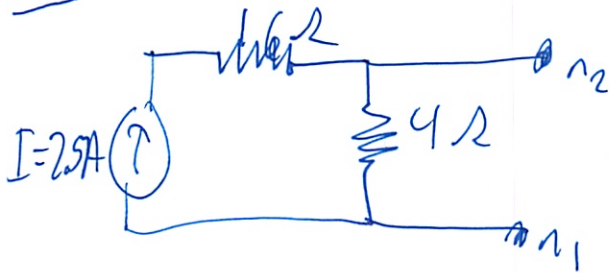
$$I_{sc} =$$

$$V = IR$$

$$I = \frac{V}{R} = \frac{10}{4} = \frac{5}{2}$$

$$R_{th} = \frac{10}{\frac{10}{4}} = 4 \Omega$$

C

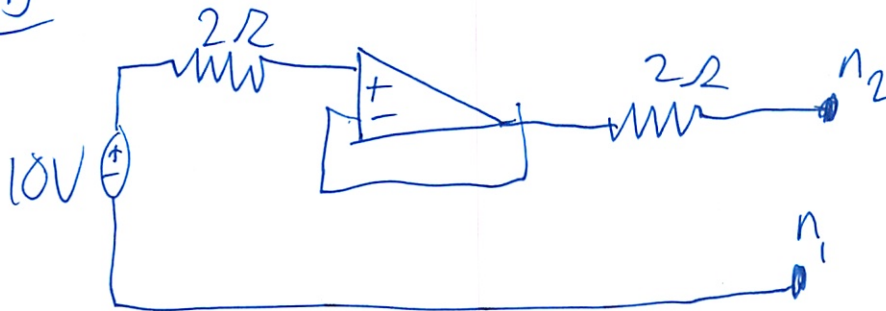


$$V_{th} = IR = 2.5 \cdot 4 = 10V$$

$$R_{th} = \frac{10}{2.5} = 4 \Omega$$

2.5 given

D



$$V_{th} = \text{Op amp fed } 10V$$

ignore <sup>first</sup>  $2 \Omega$

$$= 10V$$

- if op amp was limited to  $5V = 5V$

⑬

$R_{th}$  = first 2 amp still ignore

Just 2nd resistor

$$= 2 \Omega$$

$$= \frac{10}{5}$$

Norton almost same as Thevenin

$$R_{th} = R_{Norton}$$

Just need to use current instead

and  $V = IR$

Current Divider

Bathing  $Q_v$

$$V_+ - V_- = 1.02 (T_n - T_r)$$

$$\Delta T = 1$$
$$\Delta V = 5$$

$k' = ?$  What gain do you need

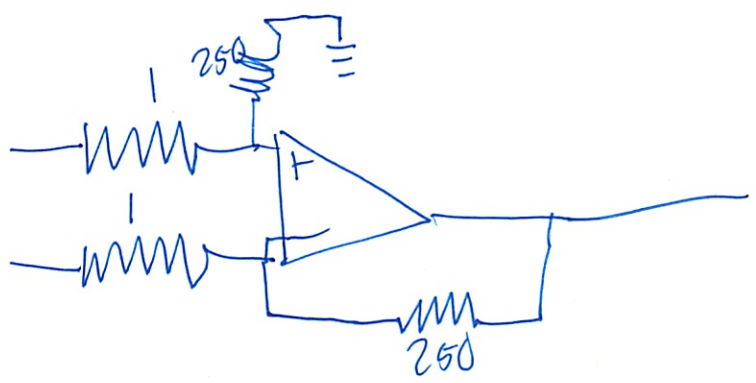
$$k' (V_+ - V_-) = 1.02 (T_n - T_r) \quad \leftarrow \text{so eq holds}$$

$$k' = 50 \times 5 = 250$$

← 5 times stronger  
makes it 1 to 1 cation - get rid of .02

(14)

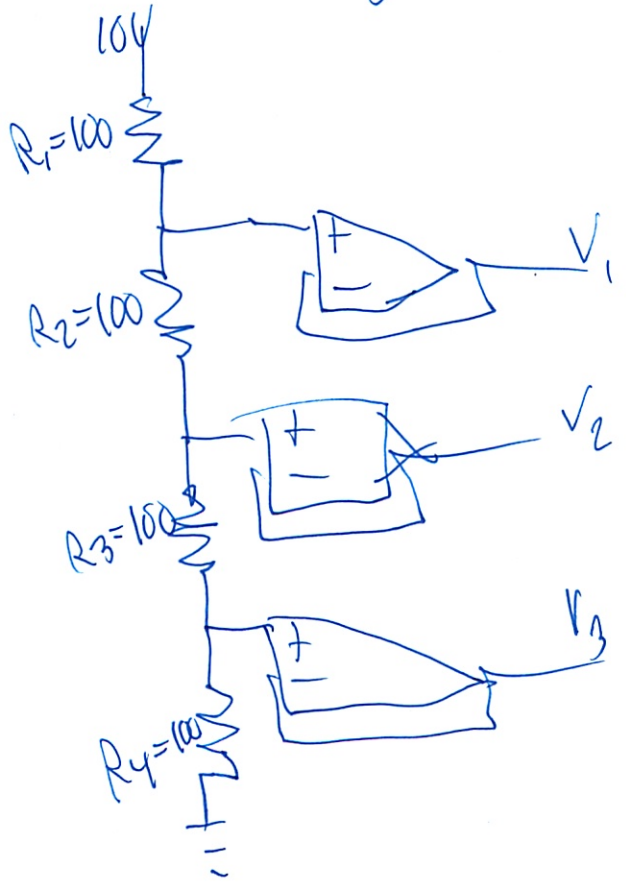
Then just use a voltage subtractor from the book/notes



Gain defined as what again?  
- relative resistor values (see above)

Fall 07 # 4

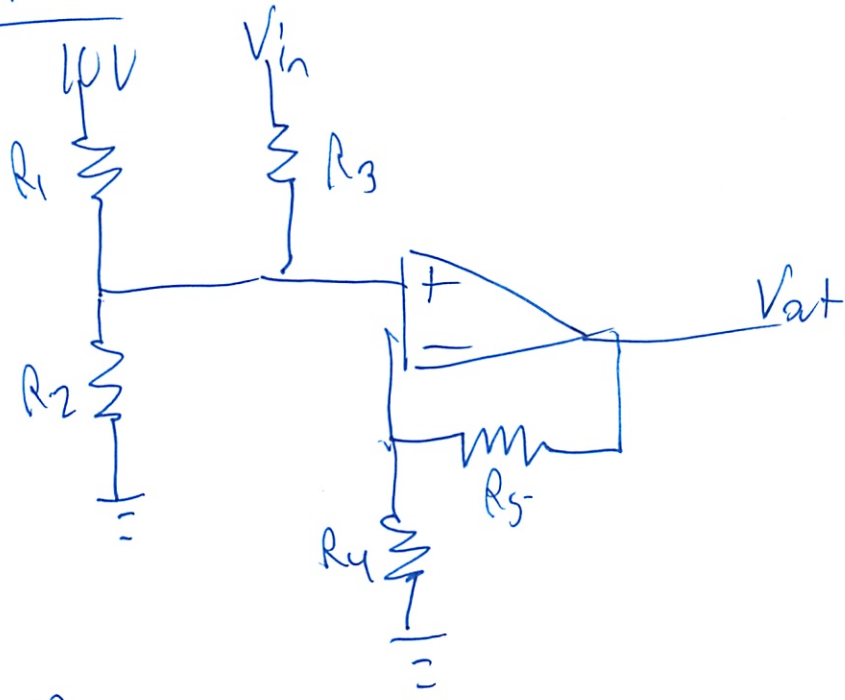
- ugly looking circuit thing



(15)

Part B

key items  
 $V=IR$   
 $V_+ = V_-$



A)  $V_+$        $i_2 = i_1 + i_3$

~~$V_+ = 0$~~   
 if ground was not 0 - this would be something different

$$\frac{V_+ - 0}{R_2} = \frac{10 - V_+}{R_1} + \frac{V_{in} - V_+}{R_3}$$

Now solve for  $V_+$  (algebra)

$$R_1 R_3 V_+ = R_2 R_3 (10 - V_+) + R_1 R_2 (V_{in} - V_+)$$

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B) Now that know  $V_+$ , want  $V_{out}$

$$i_4 = i_5$$



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$$\frac{V_t - 0}{R_4} = \frac{V_{out} - 0}{R_4 + R_5}$$

$R_4 + R_5$   
req resistance

$$V_o = \frac{R_4 + R_5}{R_4} V_t$$

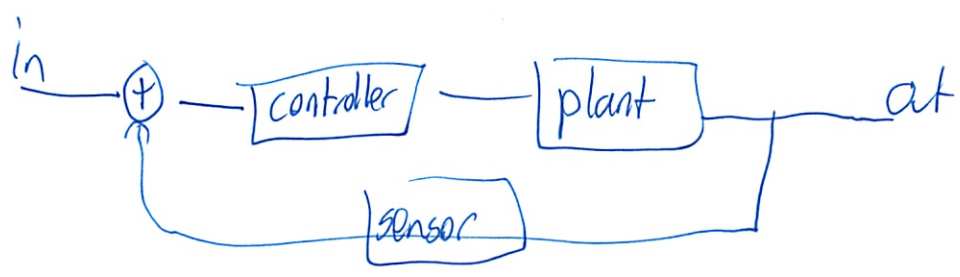
c) Analyze circuit

- Simple
- Just show no way to ~~have~~ invert a voltage source
- It can't do this

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## Control Systems

Simplest

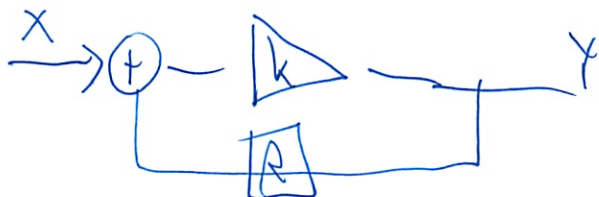


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# System Functions

Fall 09 ← best exam (one I was doing)

~~→~~  
~~→~~



change variables to make easier

Turn into an equation

$$y[n] = x[n] \cdot k + y[n-1] \cdot k$$

*so is purely n*                      *also goes through gain*

Want to analyze how behaves w/ unit signal

$$y[0] = .5 x[0] + y[-1] \cdot .5$$

*always 1*                      *starts at rest*

$$y[0] = .5 \cdot 1 + .5 \cdot 0 = .5$$

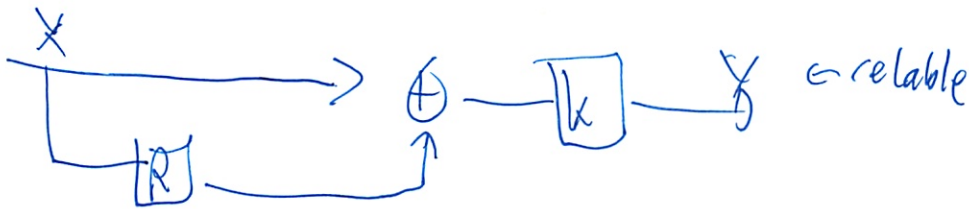
$$y[1] = .5 \cdot 1 + .5 \cdot .5 = \frac{3}{4}$$

$$y[2] = .5 \cdot 1 + .5 \cdot \frac{3}{4} = \frac{7}{8}$$

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So assume for 100 will  $\sim 1$

B) Now for other circuit



$$Y[n] = x[n] \cdot k + x[n-1] \cdot k$$

- no feedback

- feed forward add

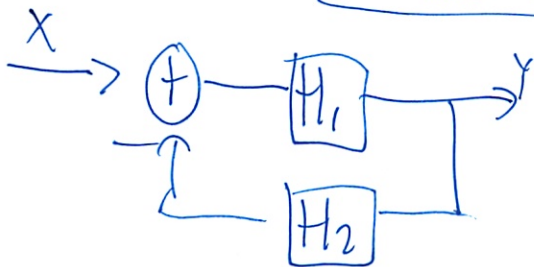
$$Y[0] = 1 \cdot 0.8 + 0 \cdot 0.8 = 0.8$$

$$Y[1] = 1 \cdot 0.8 + 1 \cdot 0.8 = 1.6$$

$$Y[n] = 1 \quad 1 \quad 1 \quad 1 \quad = 1.6$$

↓ always

Block's Formula - Feedback Subtract



$$\frac{Y}{X} = \frac{H_1}{1 + H_1 H_2}$$

$$H_1 = \frac{A}{B}$$

$$H_2 = \frac{C}{D}$$

19) C if sensor B

$$y[n] = kx[n] + kx[n-1] \quad \leftarrow \text{diff eq}$$

$$Y = kX + kXR \quad \leftarrow \text{Opp eq}$$

$$\frac{Y}{X} = \frac{k + kR}{1} \quad \leftarrow \text{system function}$$

D) Start fresh

$$\frac{R}{.25R^2 - R + .25R + 1}$$

$$R = \frac{1}{2}$$

and then the converge/diverge rules

---

3. How to pick right gain

- Dominate pole (magnitude) - inc if complex  $\sqrt{r^2 + i^2}$

- Oscillation does not matter

- Dom pole ~~not~~

## 6.01 Recread Notes

12/12

- Spent most time on circuits in my notes
  - Need to learn Thornton + Norton
  - for op amps - just practice + it will come back
  - do those problems we did in review section again
- 

## UC search

by total path cost

priority queue

test for goal - when take out of agenda

Guaranteed to find shortest

Oh - I was right current cost is included in agenda

Midterm 1 Spring 2010 #7



System Function

↓ That's how I did before  
Same mistake!

$$T[n] = C I[n] + k_1 C I[n-1] - k_2 C T[n-1]$$

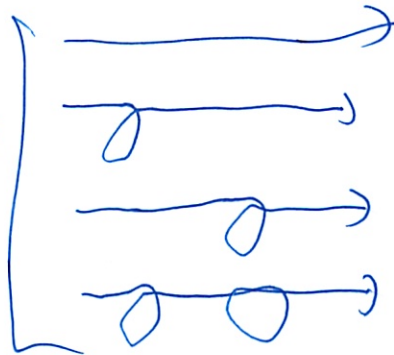
$$T = C I + k_1 C I R - k_2 C T R$$

$$T + k_2 C T R = C I + k_1 C I R$$

$$T(1 + k_2 C R) = I(C + k_1 C R)$$

$$\frac{T}{I} = \frac{C + k_1 C R}{1 + k_2 C R}$$

Possibilities



$$\frac{T}{I} = \frac{C}{(1 - k_1 R)(1 + k_2 R)}$$

Diff eq

$$T[n] = \underbrace{(k_1 - k_2) T[n-1]}_{\text{actually had this except merged}} + \underbrace{k_1 k_2 T[n-2]}_{\text{plus possibility it goes around both}} + C I[n]$$

Midterm 1 Fall 09 #5

$$Y[n] = k_1 k_2 x[n-1] - k_2 y[n-2] - k_1 k_2 y[n-2]$$

$$Y = k_1 k_2 XR - k_2 YR^2 - k_1 k_2 YR^2$$

$$Y + k_2 YR^2 + k_1 k_2 YR^2 = k_1 k_2 XR$$

$$Y(1 + k_2 R^2 + k_1 k_2 R^2) = k_1 k_2 XR$$

$$\frac{Y}{X} = \frac{k_1 k_2 R}{k_1 k_2 R^2 + k_2 R^2 + 1}$$

$$\frac{k_1 k_2 R}{1 + k_2 R^2 (1 + k_1)}$$

✓ what I had  
 (see I can do it)

b)  $k_1 = 1$   $k_2 = -2$  start at rest, unit input

$$Y[n] = 1 \cdot -2 x[n-1] - -2 y[n-2] - 1 \cdot -2 y[n-2]$$

$$= -2x[n-1] + 2y[n-2] + 2y[n-2]$$

$$Y[0] = -2 \cdot 0 + 2 \cdot 0 + 2 \cdot 0 = 0 \quad \checkmark$$

$$Y[1] = -2 \cdot 1 + 2 \cdot 0 + 2 \cdot 0 = -2 \quad \checkmark$$

↑ is actual last input?? No

$$Y[2] = -2 \cdot 0 + 2 \cdot 0 + 2 \cdot 0 = 0$$

$$Y[3] = -2 \cdot 0 + 2 \cdot -2 + 2 \cdot -2 = -8$$

what in all world did I do wrong?  
 Oh is unit just 1 at x=1 not everywhere  
 (yes)

Now try finding for the equivalents - well need to say if eq

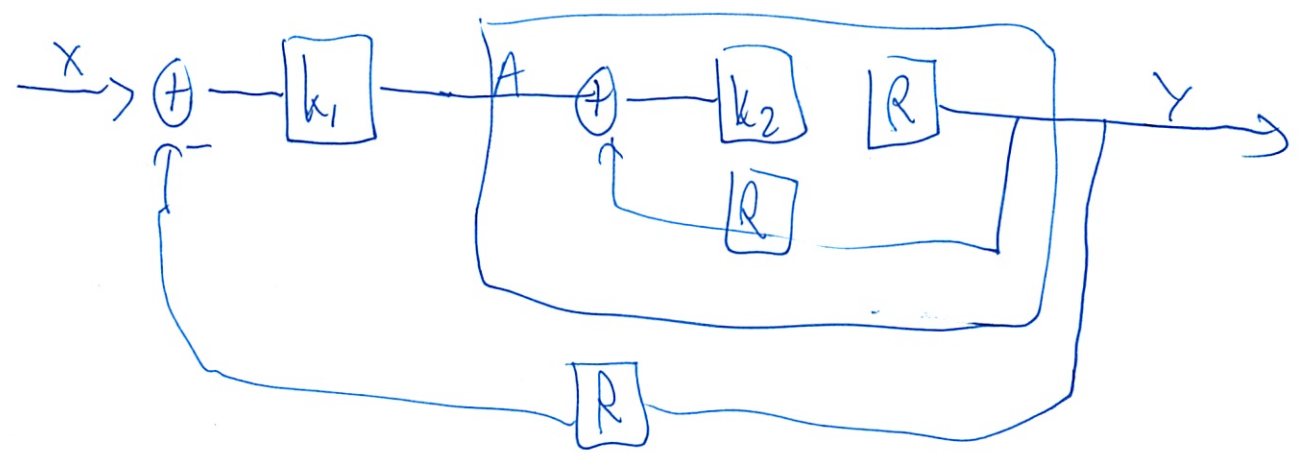
$$Y[n] = k_1 k_2 x[n-1] + k_2 y[n-1] + k_1 k_2 y[n-2]$$

not eq ✓

$$Y[n] = k_1 k_2 x[n-1] + k_2 y[n-2] + k_1 k_2 y[n-2]$$

✓ same ✓

Now copy to the OTH where built original one piece at a time



So boxed section

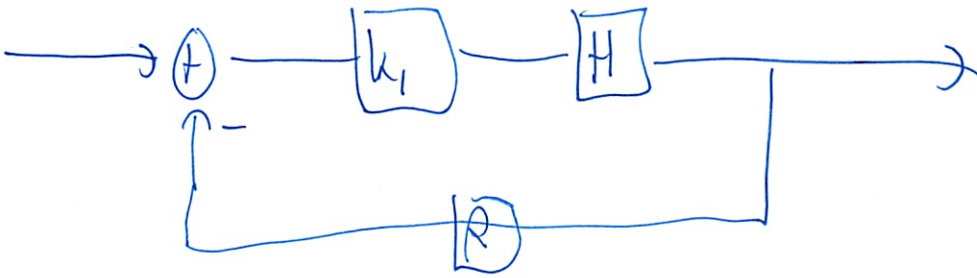
$$H_1 = \frac{Y}{A} \quad H_1 = k_2 R$$

$$H_2 = R$$

$$\frac{H_1}{1 + H_1 H_2} = \frac{k_2 R}{1 + (k_2 R)(R)} = \frac{k_2 R}{1 + k_2 R^2}$$



Add the other part



$$H_1 = \frac{k_1 k_2 R}{1 + k_2 R^2}$$

$$H_2 = R$$

$$\frac{Y}{X} = \frac{k_1 k_2 R}{1 + k_2 R^2} \left( 1 + \left( \frac{k_1 k_2 R}{1 + k_2 R^2} \right) (R) \right)$$

try our own to simplify

$$\frac{\frac{k_1 k_2 R}{1 + k_2 R^2}}{1 + \frac{k_1 k_2 R^2}{1 + k_2 R^2}}$$

$$\frac{k_1 k_2 R}{1 + k_2 R^2} \cdot \left( 1 + \frac{1 + k_2 R^2}{k_1 k_2 R^2} \right)$$

$$\frac{k_1 k_2 R}{1+k_2 R^2} + \frac{\cancel{(k_1 k_2 R)} \cancel{(1+k_2 R^2)}}{\cancel{(1+k_2 R^2)} \cancel{(k_1 k_2 R^2)}}$$

$$\frac{k_1 k_2 R + \frac{1}{R}}{1+k_2 R^2} \quad \text{need common denom}$$

$$\frac{k_1 k_2 R}{1+k_2 R^2} + \frac{1+k_2 R}{1+k_2 R^2}$$

$$\frac{k_1 k_2 R + 1 + k_2 R}{1+k_2 R^2}$$

⊗ flipped-ish again  
 - just like my first  
 final review problem

Now check

$$\frac{k_1 k_2 R}{k_1 k_2 R^2 + k_2 R^2 + 1}$$

So what did we do in OH

$$\frac{Y}{X} = \frac{k_1 k_2 R}{1+k_2 R^2}$$

$$1 + \left( \frac{k_1 k_2 R}{1+k_2 R^2} \right) \left( \frac{R}{1} \right) \quad \checkmark \text{ had that}$$

$$\frac{k_1 k_2 R}{1 + k_2 R^2}$$

$$1 + \frac{k_1 k_2 R^2}{1 + k_2 R^2} \quad \checkmark \text{ had}$$

then they multiplied both sides by  $1 + k_2 R^2$

$$\frac{k_1 k_2 R}{(1 + k_2 R^2)} (k_1 k_2 R^2)$$

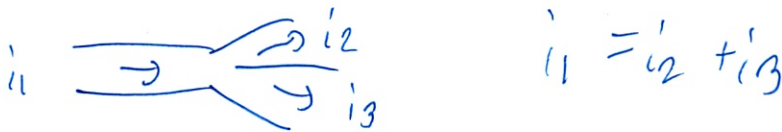
oh so much better  
- why don't I just do it that way?  
- why does my value screw up?

$$\frac{k_1 k_2 R}{\cancel{k_1 k_2 R^2} R^2 (k_1 k_2 + k_2) + 1}$$

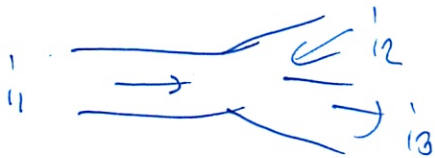
Now let me try mine again

# Circuit review of lectures

- current flows in loops
- incompressible
- Kirchoff Current Law - sum of currents into  $p = 0$



$$i_1 = i_2 + i_3$$



$$i_2 + i_1 = i_3$$

- Difference in voltage is what matters



- Kirchoff's Voltage Law - sum of voltage in closed loop = 0

- What was my fav method again?

## NVCC

1. Label each node
2. Make voltage variables at each node
3. Set one node to ground / 0
4. Make current variables for each component  
table directions
5. Write  $V_+ - V_- = i_k R$  equations

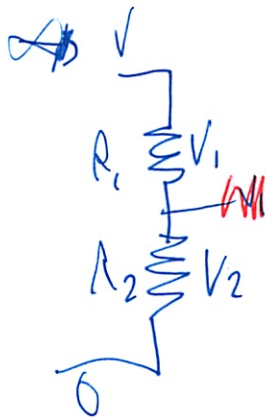
Resistor  $V_+ - V_- = i_k R$

V source  $V_+ - V_- = V$

I same  $i_k = I$

(Now let me try practice again)

### Voltage Divider



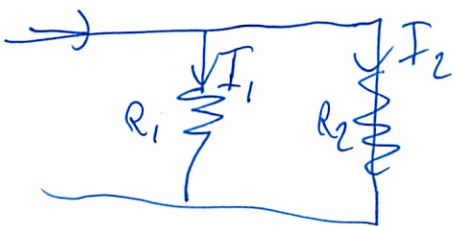
$$I = \frac{V}{R_1 + R_2}$$

$$V_1 = R_1 I = \frac{R_1}{R_1 + R_2} V$$

$$V_2 = R_2 I = \frac{R_2}{R_1 + R_2} V$$

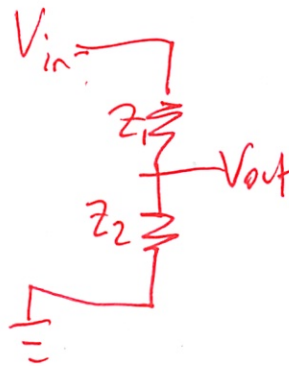
Current same  
 actually is this right?  
 ↳ perhaps for drop

### Current divider



$$I_1 = \frac{V}{R_1} = \frac{R_2}{R_1 + R_2} I$$

$$I_2 = \frac{V}{R_2} = \frac{R_1}{R_1 + R_2} I$$



$$V_{out} = \frac{Z_2}{Z_1 + Z_2} V_{in}$$

↳ near bottom

# 6.01 Final Exam: Fall 2009

Name:

Practice 12/12/10

Enter all answers in the boxes provided.

You may use any paper materials you have brought.

You may use a calculator.

Open book

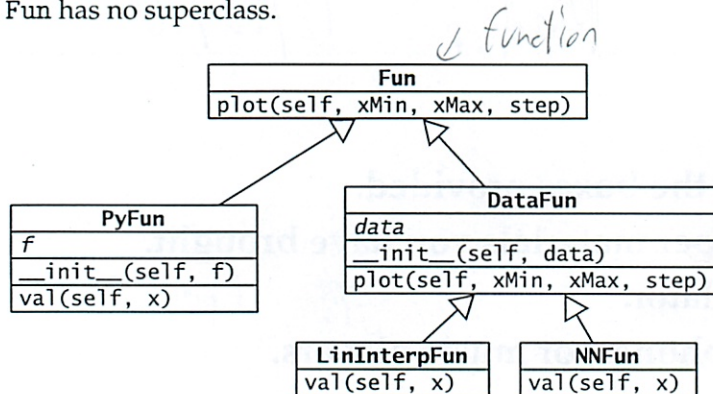
No computers, cell phones, or music players.

For staff use:

1.	/20
2.	/10
3.	/20
4.	/20
5.	/10
6.	/20
total:	/100

## 1 Object-Oriented Programming (20 points)

We will build a set of classes for representing functions (that is, mathematical functions, mapping real numbers to real numbers) in Python. Here is a diagram of the class hierarchy. Each box corresponds to a class, with its name shown in bold. Class attributes are shown in italics and methods are shown in regular text with their arguments. Arrows point upward to the superclass of a class. Fun has no superclass.



Every instance of a subclass of **Fun** that we create will have two methods: `val`, which takes an `x` value and returns the associated `y` value; and `plot`, which takes a description of the minimum and maximum `x` values for plotting, and the spacing between plotted points, and makes a plot. Different functions will be represented internally in different ways. In this problem, we will implement the **Fun**, **PyFun**, **DataFun**, and **LinInterpFun** classes; the **NNFun** class is in the diagram to illustrate where we might put a subclass of **DataFun** that uses a different interpolation strategy.

Any que

↳ next pg

A. The class Fun is an *abstract superclass*. It won't be useful to make an instance of it, but we can put the plot method, which is shared among its subclasses, in it. The plot method should create two lists:

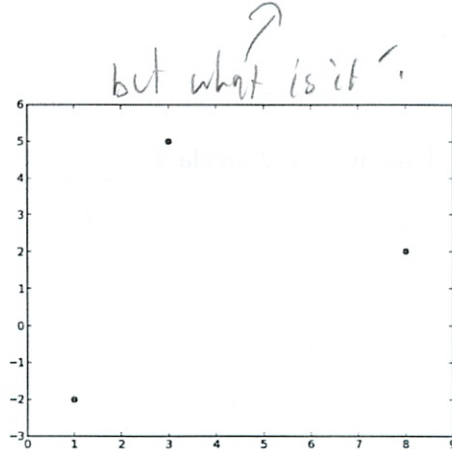
- xVals is a list whose first value is xMin, and whose subsequent values are xMin + step, xMin + step + step, etc., stopping so that the last value is as large as it can be, but less than xMax. step will typically be a float.
- yVals is a list of the y values of the function, corresponding to the x values in xVals. You can use the val(self, x) method, which is required to be defined for any actual instance of a subclass of Fun.

could do range but forget so implement

Then, it can call the global function makePlot on those lists. Assume makePlot is already defined.

So, for example,

makePlot([1, 3, 8], [-2, 5, 2]) *that is not a range though*  
would make the plot on the right.



but what is it?

I don't know why I did not get - just did not understand q

Implement the plot method. Recall that Python's range only works with integer step size.

```

class Fun:
    def plot(self, xMin, xMax, step):
        xVals = myRange(xMin, xMax, step)
        yVals = []
        for x in xVals:
            yVals.append(self.val(x))
    def myRange(min, max, step):
        ans = [min]
        while cur + step <= max:

```

no not a constant range step

makePlot(xVals, yVals)

ans makes a lot more sense

xVals = []

yVals = []

x = xMin

while x <= xMax:

xVal.append(x)

yVal.append(self.val(x))

cur = ans[-1]

ans.append(cur + step)

return ans

→ x += step

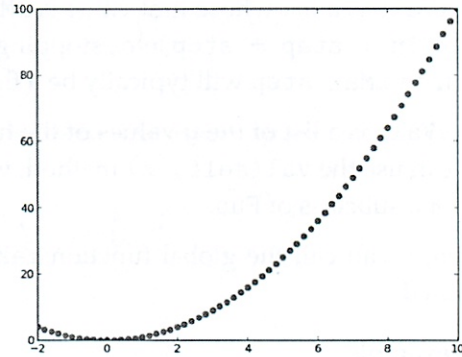
makePlot(xVals, yVals)

←



B. The PyFun class is a subclass of Fun. It represents a function just using a Python procedure of a single argument. It should operate as follows:

```
>>> t1 = PyFun(lambda x: x**2)
>>> t1.val(2)
4
>>> t1.val(-3)
9
>>> t1.plot(-2, 10, 0.2)
```



Implement the PyFun class.

```
def __init__(self, f):
    self.f = f
def val(self, x):
    return self.f(x)
```

Tduh

C. The DataFun class is a subclass of Fun, representing functions using a list of  $(x, y)$  data points, and using an interpolation rule to evaluate the function at values of  $x$  that don't appear in the data set. The data points are not sorted in any particular way. Different subclasses of DataFun will provide different interpolation strategies by implementing different `val` methods. DataFun is an abstract class, because it does not itself provide a `val` method. It will provide useful `__init__` and `plot` methods, however.

- The `__init__` method should take a list of  $(x, y)$  data pairs as input and store it in an attribute called `data`.
- The `plot` method should first plot the function with regularly-spaced  $x$  values, using the `plot` method from the parent class, Fun; then it should plot the actual data points stored in the `data` attribute.

Implement the DataFun class. (example of atpt?)

```
def __init__(self, data):
    self.data = data
```

to call superclass:

```
def plot(self, xMin, xMax, step):
```

```
    self.plot(xMin, xMax, step):
```

XVals = [ ]

```
    for x in XVals:
```

```
        YVals.append(self.val(x))
```

```
    make plot(XVals, YVals)
```

and do the m, Range thing - we'd

↑ they did list comprehension

and y is in data

```
make plot ([ x for (x,y) in self.data ],
           [ y for (x,y) in self.data ])
```

This part of the problem is worth 5 points; we suggest that you do it only after you have finished the rest of the exam.

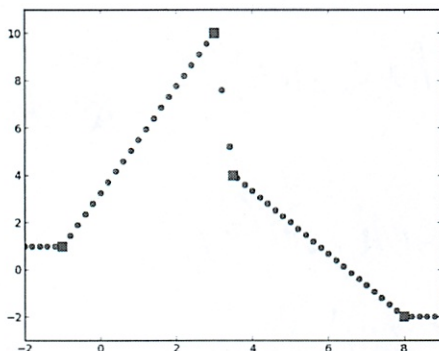
- D. The `LinInterpFun` class is a subclass of `DataFun`. Its only method is `val`, which performs linear interpolation. That is, given an  $x_i$  value as input, it should look through the data to find the  $x_{lo}$  value that is, of all  $x$  values less than or equal to  $x_i$ , closest to  $x_i$ , and the  $x_{hi}$  value that is, of all  $x$  values greater than or equal to  $x_i$ , closest to  $x_i$ . Let  $y_{lo}$  be the  $y$  value associated with  $x_{lo}$  and  $y_{hi}$  be the  $y$  value associated with  $x_{hi}$ . The method should return the linear interpolation value of  $y$  for input  $x_i$ , which is:

$$y = y_{lo} + (x_i - x_{lo}) \frac{(y_{hi} - y_{lo})}{(x_{hi} - x_{lo})} .$$

If the query  $x_i$  is lower than any  $x$  value in the data, then the method should return the  $y$  value associated with the smallest  $x$  value. Values of  $x_i$  that are higher than any  $x$  value in the data should be handled analogously. You can assume that all numbers are floats; do not assume that the data are sorted in any way.

Here is an example plot made from an `LinInterpFun` instance. The large squares are the actual stored data points.

```
t3 = LinInterpFun([(3, 10), (-1, 1), (8, -2), (3.5, 4)])  
t3.plot(-2, 10, 0.2)
```



Write your answer in the box on the next page.

range ([start], stop, [step])

## 2 State machines (10 points)

Consider the following program

```

def thing(inputList):
    output = []
    i = 0
    for x in range(3):
        y = 0
        while y < 100 and i < len(inputList):
            y = y + inputList[i]
            output.append(y)
            i = i + 1
    return output

```

wTF - bad code

every 3 values i - no 0 to 2  
 - ? does this ever increment - yes past 100

A. What is the value of

```
thing([1, 2, 3, 100, 4, 9, 500, 51, -2, 57, 103, 1, 1, 1, 1, -10, 207, 3, 1])
```

[1, 3, 6, 106, 4, 13, 513, 51, 49, 106] ✓

B. Write a single state machine class MySM such that MySM().transduce(inputList) gives the same result as thing(inputList), if inputList is a list of numbers. Remember to include a done method, that will cause it to terminate at the same time as thing.

Orig - now need to fix

```

class MySM(sm):
    def getNextValue(self, state, inp):
        i = state[0] ← actually don't need
        y = state[1]
        x = state[2]
        if x == 3:
            return self.done(state) ← i don't really know (look up)
            i = i + 1
        else:
            if i > len(inputList):
                return self.done(state)
            else:
                if y > 100:
                    x = x + 1
                    return ((i, y, x), y)
                else:
                    y = y + inp
                    if y < 100:
                        return ((i, y, x), y)

```

state = (i, y, x)  
 output = value to append

define done separately ← did not understand  
 def done(self, state)  
 (x, y) = state  
 return x >= 3

it will track

Y = Y + inp  
 if y < 100!  
 return ((i, y, x), y)

- So done check is run after every getNextValue if returns true

Should also have class heading

class LinInterpFun(DataFun):

Implement the LinInterpFun class.

find hi+lo

```

def val(self, x):
    xLo = self.findLow(x)
    xHi = self.findHigh(x)
    yLo = self.findValue(xLo)
    yHi = self.findValue(xHi)
    return yLo + (x - xLo) * ((yHi - yLo) / (xHi - xLo))
    
```

can use min() max()

again use both x, y

I don't know how to do sort

```

def sort(self):
    tuple

def findLow(self, xi):
    best = None
    for x in self.data:
        if x <= xi:
            best = x
    return best

def findHigh(self, xi):
    best = None
    
```

```

for x in self.data:
    if x >= best and x > xi:
        best = x
return best
    
```

```

def findValue(self, xi):
    for x in self.data:
        if x == xi:
            return self.data[x][1]
    
```

Run through on paper

Output [ ]

$i = 0$

$y = 0$

$y = 1$

Output = [ 1 ]

$i = 1$

$y = 3$

Output [ 1, 3 ]

$i = 2$

~~Output~~

$y = 6$

Output [ 1, 3, 6 ]

~~Output~~  $i = 3$

$y = 106$  ← was under 100 when enter

Output [ 1, 3, 6, 106 ]

$i = 4$

← ??  $x = 1$

$y = 0$

$y = 4$

think get it now

[ ..., 4, 13, 513

last two

[ ..., 51, 49, 106]

not defined in notes

C. Recall the definition of `sm.Repeat(m, n)`: Given a terminating state machine `m`, it returns a new terminating state machine that will execute the machine `m` to completion `n` times, and then terminate.

`sm.Repeat(MyNewSM, 3)`

Use `sm.Repeat` and a very simple state machine that you define to create a new state machine `MyNewSM`, such that `MyNewSM` is equivalent to an instance of `MySM`.

∴ don't understand what they want

```

class MyNewSM Sum SM(sm):
  def start state = 0 getNextValue(self, state, inp):
    y = state
    y = y + inp
    if y < 100:
      return (y, y)
    if y ≥ 100:
      return (y, y)
      sm.done(y)
  def done(self, state):
    return state ≥ 100

```

How exactly does done operate again?

`MyNewSM = sm.Repeat(SM(), 3)`

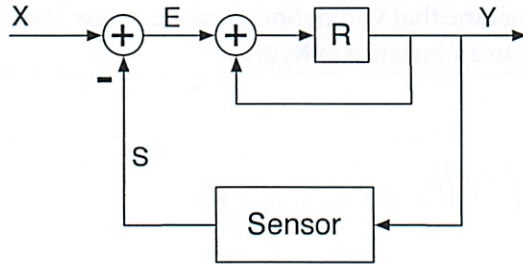
? need to include as well



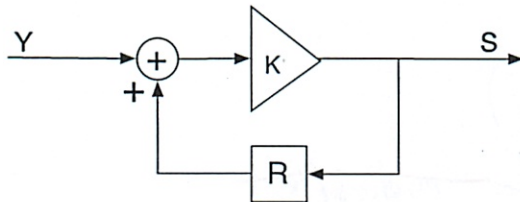
*How much do I remember?*

### 3 Linear Systems (20 points)

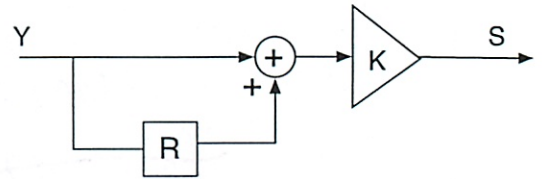
In the following system, we will consider some different possible sensors.



We can buy sensor A or sensor B as shown in the diagrams below. For each of them we can also specify the gain  $k$  at the time we order it.



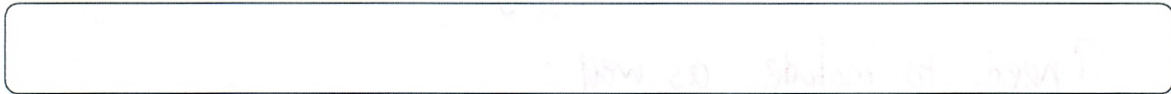
Sensor A



Sensor B

- A. Just considering the sensor, not yet connected into the system, if we feed a **step function** (going from 0 to 1) into a sensor **A** with  $k = 0.5$ , what will the output value be (approximately) after 100 steps? Assume the system starts at rest.

*how to approach again - system function*



~~A)  $S[t+1] = k Y[t] + S[t-1]$~~

~~$S[t] = k Y[t-1] + S[t-1]$~~

~~$S - SR^2 = .5 YR$~~

~~$S(1-R^2) = .5 YR$~~

$\frac{S}{Y} = \frac{.5R}{1-R^2}$

*still forget how to ans*

See review session  $\sim 1$  ✓

Let me retry w/ Blacks



$H_1$

$$Y[n] = X[n-1] + Y[n-1]$$

$$Y = XR + YR$$

$$Y - YR = XR$$

$$Y(1-R) = XR$$

$$\frac{Y}{X} = \frac{R}{1-R}$$

$H_2$  sensor A

$$S[n] = k Y[n] + k S[n-1]$$

$$S = kY + kSR$$

$$S - kSR = kY$$

$$S(1-kR) = kY$$

$$\frac{S}{Y} = \frac{k}{1-kR}$$

Blacks

$$\frac{\frac{R}{1-R}}{1 + \frac{R}{1-R} \cdot \frac{1}{1-kR}} = \frac{\frac{R}{1-R}}{1 + \frac{R}{1-R-kR+kR^2}}$$

$$\frac{R}{1-R} \cdot \left( \frac{1}{1} + \frac{1-R-kR+kR^2}{R} \right)$$

been awhile since I  
did this

$$\frac{R}{1-R} + \frac{R}{1-R} \cdot \frac{1-R-kR+kR^2}{R}$$

$$\frac{R}{1-R} + \frac{1-R-kR+kR^2}{1-R}$$

common denom, add

$$\frac{1-kR+kR^2}{1-R} \quad \text{why is mine flipped?}$$

Go back + try + find when did in class

Need to be able to do this

✓ did on other paper

try again

$$H_{1/} = Y[n] = x[n-1] + y[n-1]$$

$$Y = xR + yR$$

$$Y - yR = xR$$

$$Y(1-R) = xR$$

$$\frac{Y}{x} = \frac{R}{1-R}$$

H<sub>2</sub>

$$S[n] = k y[n] + k s[n-1]$$

$$S = k y + k s R$$

$$S - k s R = k y$$

$$S(1 - kR) = k y$$

$$\frac{S}{Y} = \frac{k}{1 - kR}$$

H<sub>1</sub>

$$\frac{H_1}{1 + H_1 H_2}$$

← mistake in memorizing formula

$\frac{Y}{X}$

$$\frac{\frac{R}{1-R}}{1 + \left(\frac{R}{1-R}\right) \left(\frac{k}{1-kR}\right)}$$

now they multiplied both ~~parts~~ halves  
by  $1-R$

$$\frac{R}{1-R + \left(\frac{Rk}{1-kR}\right)}$$

multiply both sides by  $1-kR$   
again - try

$$\frac{R(1-kR)}{(1-R)(1-kR) + Rk}$$

$$\frac{R(1-kR)}{1-R-kR+kR^2+kR}$$

$$\frac{R(1-kR)}{1-R+kR^2}$$

✓ now correct

- just do it that way in future

- Blacks

(w/ a +)

- and get rid by multiplying ~~both sides~~ by  
denom thing

Why does my flippy thing not work?

B. Just considering the sensor, not yet connected into the system, if we feed a **step function** (going from 0 to 1) into a sensor B with  $k = 0.8$ , what will the output value be (approximately) after 100 steps? Assume the system starts at rest.

See review session

$$S[n] = kY[n] + kY[n-1]$$

$$= .8$$

$$= .16 \dots$$

1.6 ✓

C. If you plug sensor A into the overall system, what is the system function relating  $Y$  to  $X$ ? (Leave  $k$  as a variable)

Did not do in review

$$Y[n] = X[n-1] + Y[n-1] + \text{sensor}$$

$$S[n] = kY[n] + kS[n-1]$$

$$Y[n] = X[n-1] + Y[n-1] - kY[n] + kX[n-1]$$

I am I converting this right

System function opp eq

$$Y = XR + YR - kY + kXR$$

$$Y - YR + kY = XR + kXR$$

$$Y(1 - R + k) = X(R + kR)$$

perfect spacing

$$\frac{Y}{X} = \frac{R + kR}{1 - R + k}$$

$$\frac{R(1 + k)}{1 - R + kR}$$

new  $q_v$

D. If we put a sensor of type B into the system, then the overall system function is

$$\frac{R}{kR^2 - R + kR + 1}$$

1. For  $k = 0.25$ , what are the poles of the system?

$$\frac{R}{1.25R^2 + 0.75R - R + 1} = \frac{1}{z} = R$$

replace  $R \Rightarrow \frac{1}{z}$   
 multiply by  $z^2$   
~~as so~~

$$\frac{1.25}{z^2} - \frac{0.75}{z} + 1$$

then what missing something

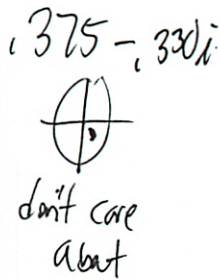
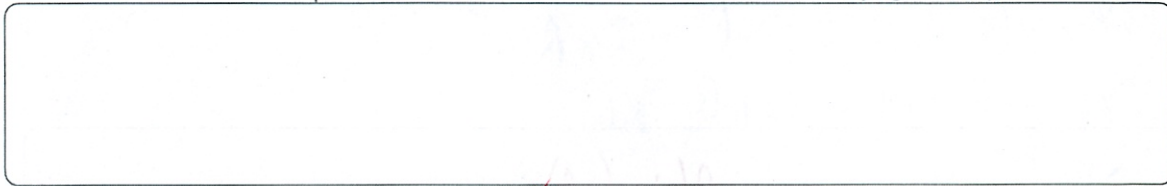
$$1.25z^2 - 0.75z + z^2 \leftarrow \text{you end up w/ this}$$

$-b \pm \sqrt{b^2 - 4ac}$   
 $2a$   
 remember the formula

$$1.75 \pm \sqrt{1.75^2 - 4 \cdot 1 \cdot 0.25} = 1.375 \pm 0.330j$$

Need to convert  $\sqrt{0.375^2 + 0.330^2} = 0.5$

look up + fix - almost have it and other one



Is it stable?

Yes converges to 0 ✓

Does it oscillate?

Yes ✓

2. for  $k = 0.1$ , the poles of the system are at 0.77 and 0.13.

Is it stable?

Yes ✓

Does it oscillate?

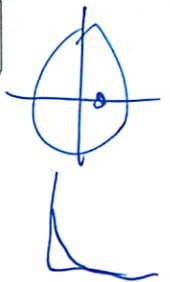
No ✓

increase |decrease  
 monotonically

Ls real

persistent mag dom pole = 1  
 ∴ not oscillate (if not in notes)

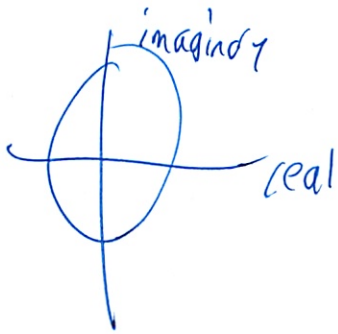
What does stable mean?



Can look on exam 2

Now review finding poles  
✓ did on other paper

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



$$\text{mag} = \sqrt{\text{real}^2 + \text{imag}^2}$$

Then look at lecture notes  
or midterm 2



3. If you were buying a sensor of type B, which of these gains would give you faster response? Explain.

**Scratch Paper**

System controlled by mag dom pole

Want want gain 125 or 11

Higher gain = faster, but more oscillation

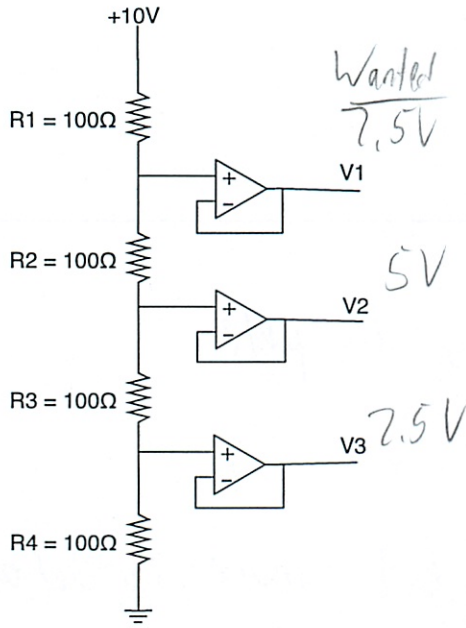
—always



Lets' see 'if remember better

### 4 Circuits (20 points)

A. Dana, Emerson, and Flann have a supply of +10 V and want to make buffered supplies of 2.5 V, 5.0 V and 7.5 V for use in their circuit, but they are having a disagreement about how to do it. Here is the circuit Dana suggests:



Oh - dont remember well  
Review by doing ans  
- even after review session

Wanted  
7.5V

5V

2.5V

was right  
- now confirm in my

1. In Dana's circuit, what are the actual values of  $V_1$ ,  $V_2$ , and  $V_3$ ?

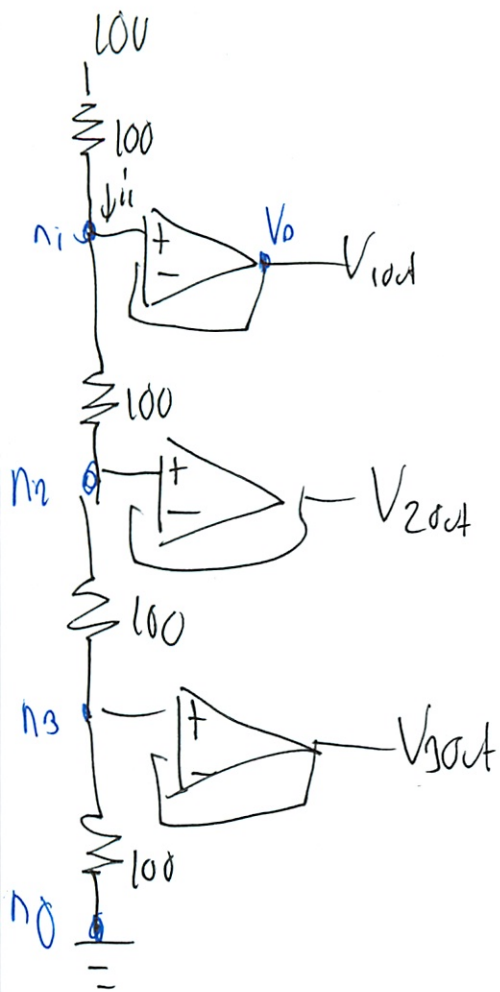
$V_1 =$    $V_2 =$    $V_3 =$

notes  
+ became

If they are incorrect, can you change the resistor values to make it work? If so, how?

Correct

requires



$$V = IR$$

$$V_+ = V_-$$

$$10V - V_1 = i_{i1} \cdot 100$$

(I don't think this is fastest way)

we'll have to set = to

~~$$V_1 = V_+$$~~

$$V_1 = V_+ = V_-$$

$V_{1out} = \text{read notes}$

the normal op amp

$$V_{1out} = V_+ = V_- = V_1$$

Now  $V_1$  is this voltage divider

shortcut thing

- let me read up on

If I remember this one correctly you start at bottom  
Midterm 2 was like this #3

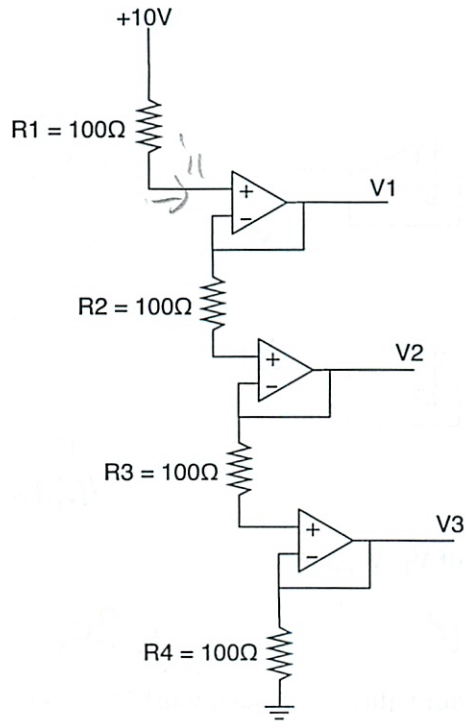
$$V_{end} = \frac{\text{bottom } \Omega}{\text{whole } \Omega} \cdot \text{start } V$$

$$\frac{100 \Omega}{400 \Omega} \cdot 10V = 2.5V \text{ @}$$

$$\frac{200 \Omega}{400} \cdot 10 = 5V$$

? So combine ones below — and op amp provides separation/buffering

Here is the circuit that Emerson suggests:



*Seperate paper*

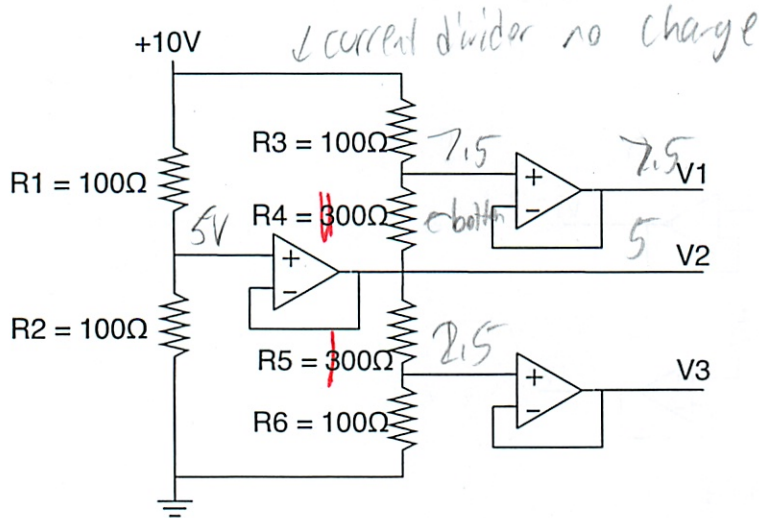
2. In Emerson's circuit, what are the actual values of  $V_1$ ,  $V_2$ , and  $V_3$ ?

$V_1 =$    $V_2 =$    $V_3 =$

If they are incorrect, can you change the resistor values to make it work? If so, how?

*\* Voltage is the same, but not current*

Here is the circuit that Flann suggests:



Seems right  
but is there  
a trick

Again - don't get  
what they did  
here  
- Ohm's law

3. In Flann's circuit, what are the actual values of  $V_1$ ,  $V_2$ , and  $V_3$ ?

$V_1 = 8.75$     $V_2 = 5V$     $V_3 = 6.25V$

If they are incorrect, can you change the resistor values to make it work? If so, how?

$R_4, R_5 = 100\Omega$

Wrong - but what is it

Check op amp sheet

Non inverting amplifier - w/ 0  $R_2$

But how figure out what each is

(I like building my own circuits - not debugging others)

Well  $V_+ = V_- = V_{out}$  w/ these

So maybe it is the same

- yeah it is!

⊗ 10V 10V 10V

Can't fix

Somehow always bumped to 10V

My first intuition was correct

But should be able to do MVCC

(I wish they showed solution on this)

~~10V~~

$$10V - V_{it} = i_1 \cdot 100$$

$$\frac{10V - V_{it}}{100} = i_1$$

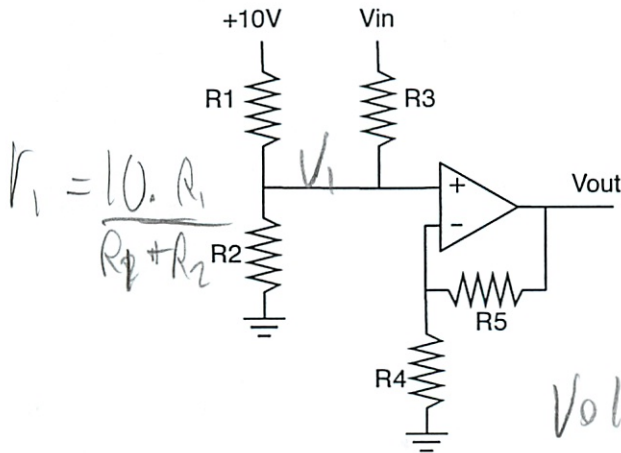
$$V_{it} = V_{i-}$$

$$V_{it} = V_{i-} = V_{out}$$

no confusion  
- skip for now

Did in review  
- can I remember

B. Consider the following circuit



$$V_1 = 10 \cdot \frac{R_2}{R_1 + R_2}$$

Voltage adder roughly

1. Write a formula for  $V_+$  (the voltage on the positive input to the op-amp) in terms of  $V_{in}$  and the resistor values for this circuit:

- Try Nodal from lecture

$$\frac{V_{out} - V_-}{R_5} = \frac{V_- - 0}{R_4}$$

$$\frac{V_1 - V_+}{R_1 + R_2} = \frac{V_+ - V_{in}}{R_3}$$

divider

$$V_+ = V_-$$

first solve each for  $V_-$ ,  $V_+$  respectively

$$R_4(V_{out} - V_-) = R_5 V_-$$

$$V_{out} R_4 - R_4 V_- = R_5 V_-$$

$$V_{out} R_4 = (R_4 + R_5) V_-$$

$$V_- = \frac{V_{out} R_4}{R_4 + R_5}$$

$$R_3 V_1 - R_3 V_+ = (R_1 + R_2) V_+ - (R_1 + R_2) V_{in}$$

$$R_3 V_1 + (R_1 + R_2) V_{in} = (R_1 + R_2) V_+ + R_3 V_+$$

$$V_+ = \frac{(R_1 + R_2) V_{in} + R_3 V_1}{R_1 + R_2 + R_3}$$

$V_+ =$

~~$$\frac{(R_1 + R_2) V_{in} + R_3 V_1}{R_1 + R_2 + R_3} = \frac{V_{out} R_4}{R_4 + R_5}$$~~

again right in space  
but need right  $V_+$

~~$$R_3(10R_2 + R_1 V_{in})$$~~

~~$$R_1 R_3 + R_1 R_2 + R_2 R_3$$~~

I don't think I had anything else

2. Write a formula for  $V_{\text{out}}$  in terms of  $V_+$  and the resistor values for this circuit:

$V_{\text{out}} =$

3. For each of these relationships, state whether it is possible to choose resistor values that make it hold in the circuit above. Write **Yes** or **No**; it is **not necessary** to provide the resistor values.

a.  $V_{\text{out}} = 2.5 - \frac{3}{16} V_{\text{in}}$

b.  $V_{\text{out}} = 2.5 + \frac{3}{16} V_{\text{in}}$

c.  $V_{\text{out}} = -2.5 + \frac{3}{16} V_{\text{in}}$



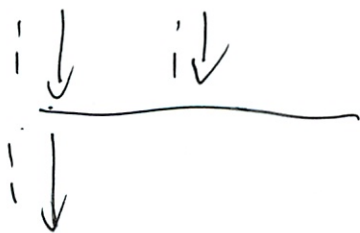
Let me review w/ review session

$$i_2 = i_1 + i_3$$

$$\frac{V_+ - 0}{R_2} = \frac{10 - V_+}{R_1} + \frac{V_{in} - V_+}{R_3}$$

Much better

Not trying to force it



$$R_1 R_3 V_+ = R_2 R_3 (10 - V_+) + R_1 R_2 (V_{in} - V_+)$$

Now try ~~on~~ own

$$V_+ = \frac{R_2 R_3 (10 - V_+)}{R_1 R_3} + \frac{R_1 R_2 (V_{in} - V_+)}{R_1 R_3}$$

$$= \frac{R_2}{R_1} (10 - V_+) + \frac{R_2}{R_3} (V_{in} - V_+)$$

No! How do you get to

$$\frac{R_3 (10R_2 + R_1 V_{in})}{R_1 R_3 + R_1 R_2 + R_2 R_3}$$

Why am I so  
bad at circuits  
- can't even research ans!

Let me review ans b

$$i_4 = i_5$$

$$\frac{V_+ - 0}{R_4} = \frac{V_{out} - 0}{R_4 + R_5}$$

req resistance

$$V_{out} = \frac{R_4 + R_5}{R_4} V_+$$

Now why can't I do this  
Review circuit problems from past exams  
+ redo the Thevenin + Norton

### 5 State estimation (10 points)

We are interested in deciding whether the earth is in one of three states, in any given century: *warming* ( $w$ ), *stable* ( $s$ ), *cooling* ( $c$ ).

A. We can do an experiment on an ice sample which gives one of two observations: melting ( $m$ ), not melting ( $n$ ).

Here are the observation probabilities for the experiment:

$$\Pr(O = m \mid S = w) = 0.9$$

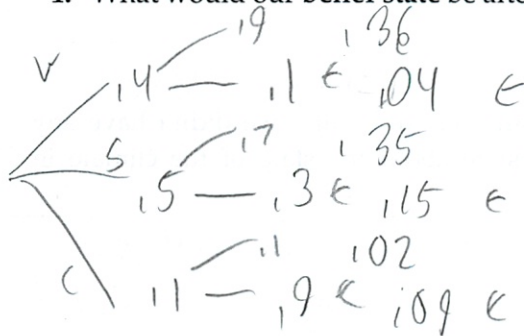
$$\Pr(O = m \mid S = s) = 0.7$$

$$\Pr(O = m \mid S = c) = 0.1$$

$$P_n = \begin{pmatrix} .1 \\ .3 \\ .9 \end{pmatrix}$$

Assume our initial belief state is  $\Pr(S = w) = 0.4$ ,  $\Pr(S = s) = 0.5$ ,  $\Pr(S = c) = 0.1$ .

1. What would our **belief state** be after doing the experiment and observing  $n$ ?



but need to normalize?

$$.04 + .15 + .09$$

$$.28$$

$w/$	$s/$	$c/$	
$\frac{.04}{.28}$	$\frac{.15}{.28}$	$\frac{.09}{.28}$	
.14	.535	.32	= 1

dist.  $P(\text{Dist}(\{w\} = .14, \{s\} = .535, \{c\} = .32))$

think figured it out

Markov chain

B. Now, let's assume that the state of the planet is a Markov process whose transitions can be described, on the scale of centuries, as follows (of course, this is completely climatologically bogus):

weird way of writing it

old

		new S <sub>t+1</sub>		
		w	s	c
S <sub>t</sub>	w	0.7	0.3	0.0
	s	0.4	0.2	0.4
	c	0.0	0.3	0.7

1. Circle the following sequences of states that are possible.

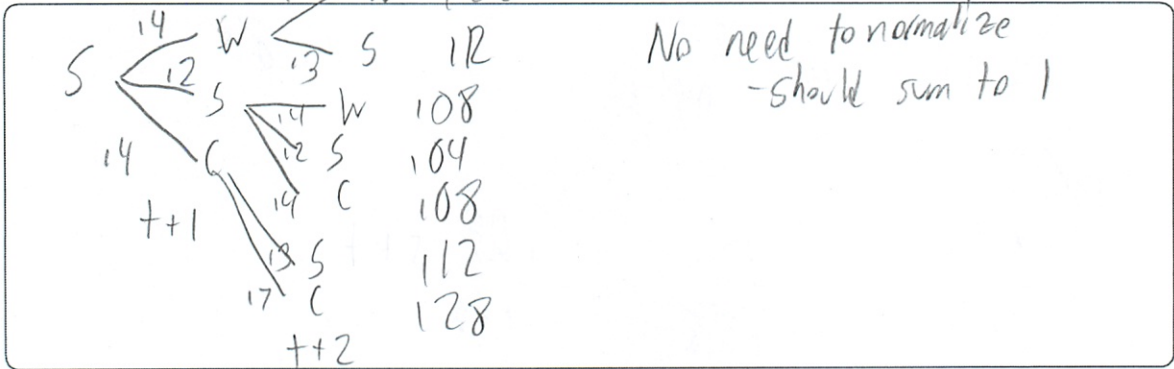
a. ~~w, s, w, w, c, c, s, w~~

b. c, c, c, s, s, c, s, w ✓

c. ~~w, s, c, w, s, c, s, w~~

2. If we were certain that climate was stable in some century t, and we didn't have any experimental evidence, what would our belief state about the state of the climate in century t + 2 be?

s - not markov stable



No need to normalize  
- should sum to 1

$$W = 128 + 108 = 236$$

$$S = 12 + 104 + 112 = 228$$

$$C = 108 + 128 = 236$$

did in review session

### 6 Search (20 points)

A. We want to improve the search performance in the wolf-goat-cabbage problem (summarized below; it is the same as in the tutor problem). *↓ forget but should be able to re-gen what was the solution*

- The farmer has a goat, a wolf and a head of cabbage. They come to a river (they're on the left bank) and need to get everything and everyone to the other side (the right bank).
- There's a boat there that fits at most two of them; the farmer must always be one of the two in the boat.
- If the farmer leaves the goat alone with the cabbage, the goat will eat the cabbage (so that's not a legal state). Similarly, if the farmer leaves the goat alone with the wolf... (so that's not a legal state).

Let  $n(s)$  be the number of objects (wolf, goat, cabbage) that are on the incorrect side of the river in state  $s$ .

1. Andrea suggests that a good heuristic would be  $n(s) - 1$ . Is it admissible? Why or why not?

2. Bobbie suggests that a good heuristic would be  $2n(s) - 1$ . Is it admissible? Why or why not?

3. Casey suggests that a good heuristic would be  $3n(s) - 1$ . Is it admissible? Why or why not?

4. Which heuristic would be likely to reduce the search the most, while retaining optimality of the answer?

# of steps is # of moves  
(need to understand what the heuristic is of)

? have no real clue which heuristic would be work

B. We need to travel over a network of roads through the mountains in the snow. Each road has a current condition: *clear*, *slippery*, or *buried*. There are two possible vehicles you can use: a sports car, which can only traverse clear roads or an SUV, which can traverse any road.

You start in the sports car (in location S), but if you are driving one vehicle, and you're in the same location as another vehicle, you can trade vehicles; if you drive your sports car to the location of the SUV (which starts in location A), and trade, then when you move, you will move with the SUV and the sports car will be left at that location.

We will specify the map using the data structure below, which characterizes, for each location, the roads leading out of it. Each road is described by a triple indicating the next location, the length of the road, and the condition of the road.

```
map1dist = {'S' : [('A', 2, 'clear'), ('B', 1, 'slippery')],
            'A' : [('S', 2, 'clear'), ('C', 3, 'clear'), ('D', 10, 'slippery')],
            'B' : [('S', 1, 'slippery'), ('D', 2, 'slippery')],
            'C' : [('A', 3, 'clear')],
            'D' : [('A', 10, 'slippery'), ('B', 2, 'slippery')]}
```

↓ take next lowest cost

We are going to formulate this as a search problem with costs, to be solved using UC search. Let the cost to traverse a road just be the the length of the road times a multiplier: the multiplier is 1 for the sports car and 2 for the SUV. There is a cost of 1 for the action of swapping cars.

The possible actions are to drive on one of the roads emanating from a current location or to swap cars.

1. What information do you need to keep in each state? How will you represent it in Python?

~~- current location you~~ ✓  
~~- current car~~ ✓  
~~- current loc sports~~

~~- current loc SUV~~  
~~- cost so far~~  
 ← don't need all of them, but will do

don't need to solve yet

2. How would you represent the starting state (as a Python expression)? [loc, car, sportsLoc, SUVLoc, cost]

[ 'S', 'sports', 'S', 'A', 0 ]

['S', 'car']

only 1 place to switch

Back from review session - where did I try again

print documentation ✓

3. What would you pass in as the second argument to `ucSearch.search`, the goal test, if the goal is to end in location 'D'?

↑ goalTest

Write Python expression(s)

```
def goalTest(self, state):
    return state[0] == 'D' ✓
```

or w/ lambda

4. Let the actions be described by (action, roadNum), where action is one of 'drive' or 'swapCars', and roadNum is an integer that means which road to drive on out of an intersection. The roadnum can be used as an index into the list of results in `map1dist`. When action is 'swapCars', then the roadNum is ignored.

just array order

If `drivingDynamics` is an instance of `sm.SM` that describes this planning domain, using your state representation, what would the output of this expression be:

```
>>> drivingDynamics.transduce([( 'drive', 0), ('swapCars', 0), ('drive', 1)])
```

Write a list of states.

here they are using code

- what would this do

```
(S, sports) ← don't forget lst!
(A, sports)
(A, SUV)
(D, SUV)
```

Using review session convention

are we using UC

- draw pic
- see (D, SUV) too expensive
- would not be the next one

See pick from review session

5. From that same start state, what path through state space would be found by breadth-first search, when the goal is to be in location 'D'? Provide a list of states. no cost  
end path

S, sports  
A, sports  
A, SUV  
D, SUV

breath first - queue  
added in order that is specified

✓

6. From that same start state, what path through state space would be found by uniform-cost search? Provide a list of states.

S, sports  
A, sports  
A SUV  
S SUV  
B SUV  
D SUV

✓

What is its cost? 13 ✓



# 6.01 Final Exam: Fall 2009

Name: ANSWERS

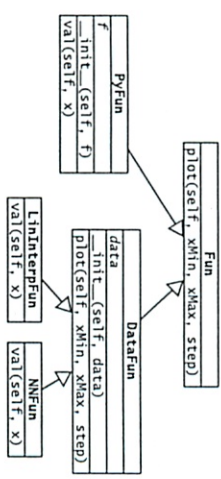
Enter all answers in the boxes provided.  
You may use any paper materials you have brought.  
You may use a calculator.  
No computers, cell phones, or music players.

For staff use:

1.	/20
2.	/10
3.	/20
4.	/20
5.	/10
6.	/20
total:	/100

## 1 Object-Oriented Programming (20 points)

We will build a set of classes for representing functions (that is, mathematical functions, mapping real numbers to real numbers) in Python. Here is a diagram of the class hierarchy. Each box corresponds to a class, with its name shown in bold. Class attributes are shown in italics and methods are shown in regular text with their arguments. Arrows point upward to the superclass of a class. Fun has no superclass.



Every instance of a subclass of Fun that we create will have two methods: *val*, which takes an *x* value and returns the associated *y* value; and *plot*, which takes a description of the minimum and maximum *x* values for plotting, and the spacing between plotted points, and makes a plot. Different functions will be represented internally in different ways. In this problem, we will implement the Fun, PyFun, DataFun, and LinInterpFun classes; the NNFun class is in the diagram to illustrate where we might put a subclass of DataFun that uses a different interpolation strategy.

A. The class `Fun` is an abstract superclass. It won't be useful to make an instance of it, but we can put the `plot` method, which is shared among its subclasses, in it. The `plot` method should create two lists:

- `xVals` is a list whose first value is `xMin`, and whose subsequent values are `xMin + step`, `xMin + step + step`, etc., stopping so that the last value is as large as it can be, but less than `xMax`. `step` will typically be a float.

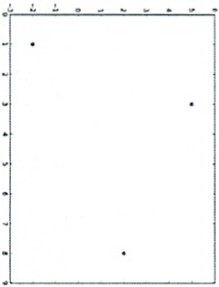
— `yVals` is a list of the y values of the function, corresponding to the x values in `xVals`. You can use the `val(self, x)` method, which is required to be defined for any actual instance of a subclass of `Fun`.

Then, it can call the global function `makePlot` on those lists. Assume `makePlot` is already defined.

So, for example,

```
makePlot([1, 3, 8], [-2, 5, 2])
```

would make the plot on the right.

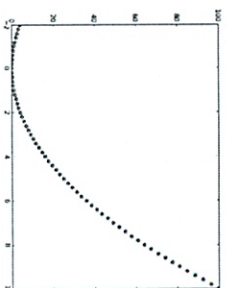


Implement the `plot` method. Recall that Python's `range` only works with integer step size.

```
class Fun:
    def plot(self, xMin, xMax, step):
        xVals = []
        yVals = []
        x = xMin
        while x <= xMax:
            xVals.append(x)
            yVals.append(self.val(x))
            x += step
        makePlot(xVals, yVals)
```

B. The `PyFun` class is a subclass of `Fun`. It represents a function just using a Python procedure of a single argument. It should operate as follows:

```
>>> t1 = PyFun(lambda x: x**2)
>>> t1.val(2)
4
>>> t1.val(-3)
9
>>> t1.plot(-2, 10, 0.2)
```



Implement the `PyFun` class.

```
class PyFun(Fun):
    def __init__(self, f):
        self.f = f
    def val(self, x):
        return self.f(x)
```

- C. The DataFun class is a subclass of Fun, representing functions using a list of  $(x, y)$  data points, and using an interpolation rule to evaluate the function at values of  $x$  that don't appear in the data set. The data points are not sorted in any particular way. Different subclasses of DataFun will provide different interpolation strategies by implementing different `val` methods. DataFun is an abstract class, because it does not itself provide a `val` method. It will provide useful `__init__` and `plot` methods, however.
- The `__init__` method should take a list of  $(x, y)$  data pairs as input and store it in an attribute called `data`.
  - The `plot` method should first plot the function with regularly-spaced  $x$  values, using the `plot` method from the parent class, Fun; then it should plot the actual data points stored in the `data` attribute.

Implement the DataFun class.

```
class DataFun(Fun):
    def __init__(self, data):
        self.data = data
    def plot(self, xmin, xmax, step):
        Fun.plot(self, xmin, xmax, step)
        makePlot([X for (X,Y) in self.data],
                 [Y for (X,Y) in self.data])
```

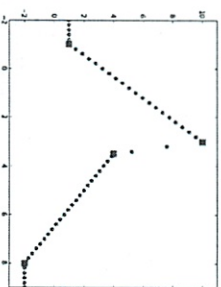
- This part of the problem is worth 5 points; we suggest that you do it only after you have finished the rest of the exam.
- D. The `LinInterpFun` class is a subclass of `DataFun`. Its only method is `val`, which performs linear interpolation. That is, given an  $x_i$  value as input, it should look through the data to find the  $x_{i0}$  value that is, of all  $x$  values less than or equal to  $x_i$ , closest to  $x_i$ , and the  $x_{i1}$  value that is, of all  $x$  values greater than or equal to  $x_i$ , closest to  $x_i$ . Let  $y_{i0}$  be the  $y$  value associated with  $x_{i0}$  and  $y_{i1}$  be the  $y$  value associated with  $x_{i1}$ . The method should return the linear interpolation value of  $y$  for input  $x_i$ , which is:

$$y = y_{i0} + (x_i - x_{i0}) \frac{(y_{i1} - y_{i0})}{(x_{i1} - x_{i0})}$$

If the query  $x_i$  is lower than any  $x$  value in the data, then the method should return the  $y$  value associated with the smallest  $x$  value. Values of  $x_i$  that are higher than any  $x$  value in the data should be handled analogously. You can assume that all numbers are floats; do not assume that the data are sorted in any way.

Here is an example plot made from an `LinInterpFun` instance. The large squares are the actual stored data points.

```
t3 = LinInterpFun([(3, 10), (-1, 1), (8, -2), (3.5, 4)])
t3.plot(-2, 10, 0.2)
```



Write your answer in the box on the next page.

Implement the `LinInterpFun` class.

```
class LinInterpFun(Fun):
    def eval(self, X):
        xvals = [dx for (dx,dy) in self.data]
        xlo = min(xvals)
        xhi = max(xvals)
        if X <= xlo: return xlo
        if X >= xhi: return xhi
        for (dx, dy) in data:
            if dx < X and dx >= xlo: (xlo, ylo) = (dx, dy)
            if dx > X and dx <= xhi: (xhi, yhi) = (dx, dy)
        return ylo + (X - xlo) * (yhi - ylo) / (xhi - xlo)
```

## 2 State machines (10 points)

Consider the following program

```
def thing(inputList):
    output = []
    i = 0
    for x in range(3):
        y = 0
        while y < 100 and i < len(inputList):
            y = y + inputList[i]
            output.append(y)
            i = i + 1
    return output
```

A. What is the value of

```
thing([1, 2, 3, 100, 4, 9, 500, 51, -2, 57, 103, 1, 1, 1, 1, -10, 207, 3, 1])
```

```
[1, 3, 6, 106, 4, 13, 613, 51, 49, 106]
```

B. Write a single state machine class `MySM` such that `MySM().transduce(inputList)` gives the same result as `thing(inputList)`, if `inputList` is a list of numbers. Remember to include a `done` method, that will cause it to terminate at the same time as `thing`.

```
class MySM(sm.SM):
    startState = (0, 0)
    def getNextValues(self, state, inp):
        (x, y) = state
        y += inp
        if y >= 100:
            return ((x + 1, 0), y)
        return ((x, y), y)
    def done(self, state):
        (x, y) = state
        return x >= 3
```

B. Just considering the sensor, not yet connected into the system, if we feed a step function (going from 0 to 1) into a sensor B with  $k = 0.8$ , what will the output value be (approximately) after 100 steps? Assume the system starts at rest.

1.6

C. If you plug sensor A into the overall system, what is the system function relating Y to X? (Leave k as a variable)

$$\frac{Y}{X} = \frac{R(1 - kR)}{1 - R + kR^2}$$

D. If we put a sensor of type B into the system, then the overall system function is

$$\frac{R}{kR^2 - R + kR + 1}$$

1. For  $k = 0.25$ , what are the poles of the system?

$$\frac{1}{8}(13 \pm \sqrt{77})$$

Is it stable?

Yes

Does it oscillate?

Yes

2. for  $k = 0.1$ , the poles of the system are at 0.77 and 0.13.

Is it stable?

Yes

Does it oscillate?

No

C. Recall the definition of `sm.Repeat(m, n)`: Given a terminating state machine `m`, it returns a new terminating state machine that will execute the machine `m` to completion `n` times, and then terminate.

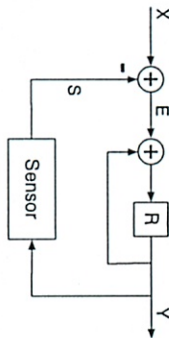
Use `sm.Repeat` and a very simple state machine that you define to create a new state machine `MyNewSM`, such that `MyNewSM` is equivalent to an instance of `MySM`.

```
class Sum(sm.SM):
    startState = 0
    def getNextValues(self, state, inp):
        return (state + inp, state + inp)
    def done(self, state):
        return state > 100

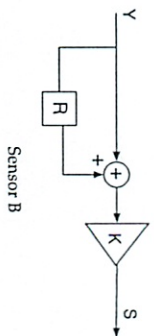
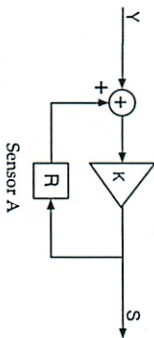
myNewSM = sm.Repeat(Sum(), 3)
```

### 3 Linear Systems (20 points)

In the following system, we will consider some different possible sensors.



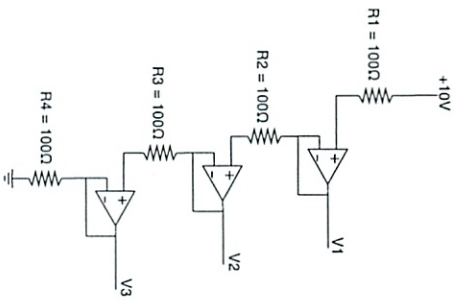
We can buy sensor A or sensor B as shown in the diagrams below. For each of them we can also specify the gain `k` at the time we order it.



A. Just considering the sensor, not yet connected into the system, if we feed a step function (going from 0 to 1) into a sensor A with `k = 0.5`, what will the output value be (approximately) after 100 steps? Assume the system starts at rest.

3/4, 7/8, 15/16, ..., 1

Here is the circuit that Emerson suggests:



2. In Emerson's circuit, what are the actual values of  $V_1$ ,  $V_2$ , and  $V_3$ ?

$V_1 =$

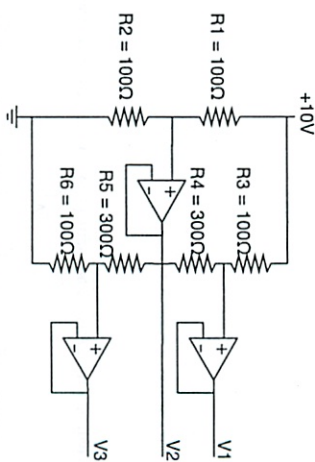
$V_2 =$

$V_3 =$

If they are incorrect, can you change the resistor values to make it work? If so, how?

No.

Here is the circuit that Flann suggests:



3. In Flann's circuit, what are the actual values of  $V_1$ ,  $V_2$ , and  $V_3$ ?

$V_1 =$

$V_2 =$

$V_3 =$

If they are incorrect, can you change the resistor values to make it work? If so, how?

Set  $R_4 = 100\Omega$  and  $R_5 = 100\Omega$

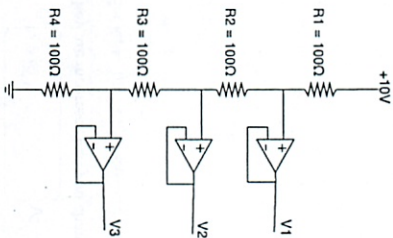
3. If you were buying a sensor of type B, which of these gains would give you faster response? Explain.

Pick  $k = 0.25$  because it converges faster.

Scratch Paper

#### 4 Circuits (20 points)

- A. Dana, Emerson, and Flann have a supply of +10 V and want to make buffered supplies of 2.5 V, 5.0 V and 7.5 V for use in their circuit, but they are having a disagreement about how to do it. Here is the circuit Dana suggests:



1. In Dana's circuit, what are the actual values of  $V_1$ ,  $V_2$ , and  $V_3$ ?

$V_1 =$

$V_2 =$

$V_3 =$

If they are incorrect, can you change the resistor values to make it work? If so, how?

CORRECT



## 5 State estimation (10 points)

We are interested in deciding whether the earth is in one of three states, in any given century: warming ( $w$ ), stable ( $s$ ), cooling ( $c$ ).

A. We can do an experiment on an ice sample which gives one of two observations: melting ( $m$ ), not melting ( $n$ ).

Here are the observation probabilities for the experiment:

$$\Pr(O = m \mid S = w) = 0.9$$

$$\Pr(O = m \mid S = s) = 0.7$$

$$\Pr(O = m \mid S = c) = 0.1$$

Assume our initial belief state is  $\Pr(S = w) = 0.4$ ,  $\Pr(S = s) = 0.5$ ,  $\Pr(S = c) = 0.1$ .

1. What would our belief state be after doing the experiment and observing  $n$ ?

$$\left( \frac{1}{7}, \frac{15}{28}, \frac{9}{28} \right)$$

B. Now, let's assume that the state of the planet is a Markov process whose transitions can be described, on the scale of centuries, as follows (of course, this is completely climatologically bogus):

	$S_{t+1}$		
	$w$	$s$	$c$
$w$	0.7	0.3	0.0
$s$	0.4	0.2	0.4
$c$	0.0	0.3	0.7

1. Circle the following sequences of states that are possible.

a.  $w, s, w, w, c, c, s, w$  Not possible

b.  $c, c, c, s, s, c, s, w$  Possible

c.  $w, s, c, w, s, c, s, w$  Not possible

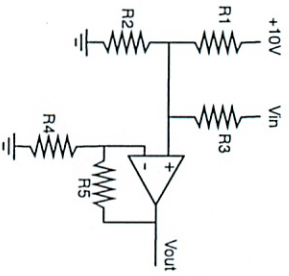
2. If we were certain that climate was stable in some century  $t$ , and we didn't have any experimental evidence, what would our belief state about the state of the climate in century  $t + 2$  be?

$$t = 0: (0.1, 0)$$

$$t = 1: (0.4, 0.2, 0.4)$$

$$t = 2: (0.36, 0.28, 0.36)$$

B. Consider the following circuit:



1. Write a formula for  $V_+$  (the voltage on the positive input to the op-amp) in terms of  $V_{in}$  and the resistor values for this circuit:

$$V_+ = \frac{R_3(10R_2 + R_1V_{in})}{R_1R_2 + R_1R_3 + R_2R_3}$$

2. Write a formula for  $V_{out}$  in terms of  $V_+$  and the resistor values for this circuit:

$$V_{out} = \left( \frac{R_4 + R_5}{R_4} \right) V_+$$

3. For each of these relationships, state whether it is possible to choose resistor values that make it hold in the circuit above. Write Yes or No; it is not necessary to provide the resistor values.

a.  $V_{out} = 2.5 - \frac{3}{16} V_{in}$

No

b.  $V_{out} = 2.5 + \frac{3}{16} V_{in}$

Yes

c.  $V_{out} = -2.5 + \frac{3}{16} V_{in}$

No

3. What would you pass in as the second argument to `ucSearch.search`, the goal test, if the goal is to end in location 'D'?

Write Python expression(s)

```
lambda s: s[0] == 'D'
```

4. Let the actions be described by (`action`, `roadnum`), where `action` is one of 'drive' or 'swapCars', and `roadnum` is an integer that means which road to drive on out of an intersection. The roadnum can be used as an index into the list of results in `mapdist`. When `action` is 'swapCars', then the `roadnum` is ignored.

If `drivingDynamics` is an instance of `sm.SM` that describes this planning domain, using your state representation, what would the output of this expression be:

```
>>> drivingDynamics.transduce([('drive', 0), ('swapCars', 0), ('drive', 1)])
```

Write a list of states.

```
{('S', 'car'), ('A', 'car'), ('A', 'bus'), ('C', 'bus')}
```

5. From that same start state, what path through state space would be found by breadth-first search, when the goal is to be in location 'D'? Provide a list of states.

```
[('S', 'car'), ('A', 'car'), ('A', 'bus'), ('D', 'bus')]
```

6. From that same start state, what path through state space would be found by uniform-cost search? Provide a list of states.

```
[('S', 'car'), ('A', 'car'), ('A', 'bus'), ('S', 'bus'), ('B', 'bus'), ('D', 'bus')]
```

What is its cost?

13

## 6 Search (20 points)

A. We want to improve the search performance in the wolf-goat-cabbage problem (summarized below; it is the same as in the tutor problem).

- The farmer has a goat, a wolf and a head of cabbage. They come to a river (they're on the left bank) and need to get everything and everyone to the other side (the right bank).
- There's a boat there that fits at most two of them; the farmer must always be one of the two in the boat.
- If the farmer leaves the goat alone with the cabbage, the goat will eat the cabbage (so that's not a legal state). Similarly, if the farmer leaves the goat alone with the wolf... (so that's not a legal state).

Let  $n(s)$  be the number of objects (wolf, goat, cabbage) that are on the incorrect side of the river in state  $s$ .

1. Andrea suggests that a good heuristic would be  $n(s) - 1$ . Is it admissible? Why or why not?

Yes, always less than the number of steps to go.

2. Bobbie suggests that a good heuristic would be  $2n(s) - 1$ . Is it admissible? Why or why not?

Yes, always less than or equal the number of steps to go.

3. Casey suggests that a good heuristic would be  $3n(s) - 1$ . Is it admissible? Why or why not?

No, on the first step,  $3n(s) - 1 = 2$  but there are 7 steps to go.

4. Which heuristic would be likely to reduce the search the most, while retaining optimality of the answer?

Use Bobbie's,  $2n(s) - 1$ , it's the largest admissible heuristic.

B. We need to travel over a network of roads through the mountains in the snow. Each road has a current condition: *clear*, *slippery*, or *buried*. There are two possible vehicles you can use: a sports car, which can only traverse clear roads or an SUV, which can traverse any road.

You start in the sports car (in location S), but if you are driving one vehicle and you're in the same location as another vehicle, you can trade vehicles; if you drive your sports car to the location of the SUV (which starts in location A), and trade, then when you move, you will move with the SUV and the sports car will be left at that location.

We will specify the map using the data structure below, which characterizes, for each location, the roads leading out of it. Each road is described by a triple indicating the next location, the length of the road, and the condition of the road.

```
mapdict = {'S': [(('A', 2, 'clear'), ('B', 1, 'slippery'))],
           'A': [(('S', 2, 'clear'), ('C', 3, 'clear'), ('D', 10, 'slippery'))],
           'B': [(('S', 1, 'slippery'), ('D', 2, 'slippery'))],
           'C': [(('A', 3, 'clear'))],
           'D': [(('A', 10, 'slippery'), ('B', 2, 'slippery'))]}
```

We are going to formulate this as a search problem with costs, to be solved using UC search. Let the cost to traverse a road just be the length of the road times a multiplier: the multiplier is 1 for the sports car and 2 for the SUV. There is a cost of 1 for the action of swapping cars.

The possible actions are to drive on one of the roads emanating from a current location or to swap cars.

1. What information do you need to keep in each state? How will you represent it in Python?

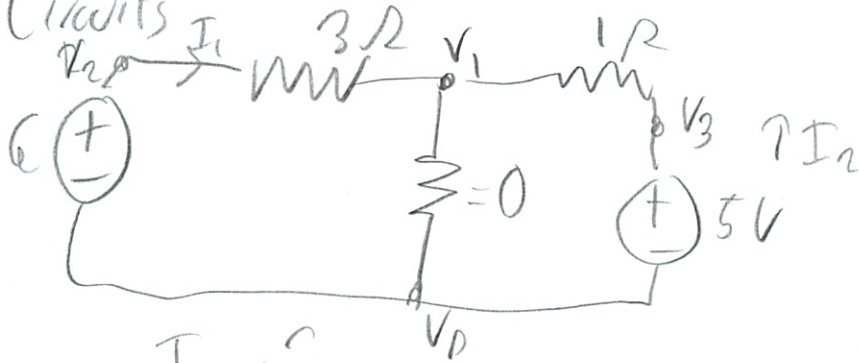
Our location and which vehicle we are using (Loc, car)  
We don't need to keep in the state where the other vehicle is. Once we change vehicles, we don't ever change back.

2. How would you represent the starting state (as a Python expression)?

```
{'S', 'car'}
```

# Ge01 Midterm 2 Spring 09

1. Circuits



$$I_1 = ?$$

do full formal way

$$V_2 - V_0 = 6V$$

$$V_2 - V_1 = I_1 \cdot 3\Omega$$

$$\frac{V_2 - V_1}{3} = I_1$$

$$V_3 - V_0 = 5V$$

$$V_3 - V_1 = I_2 \cdot 1\Omega$$

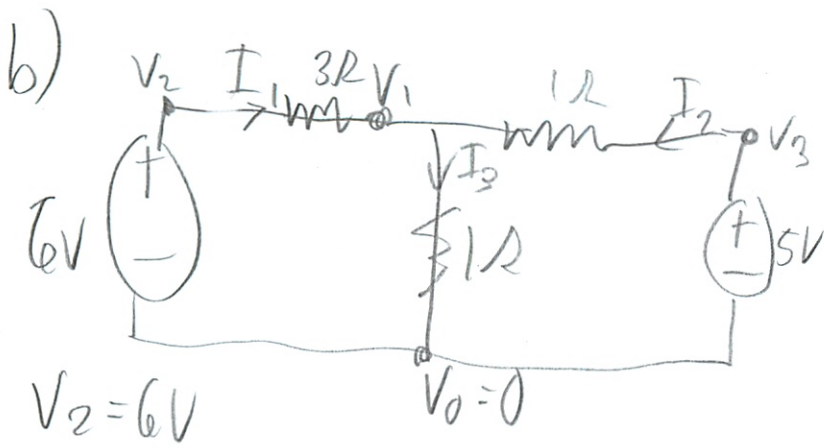
$$\frac{V_3 - V_1}{1} = I_2$$

$$V_1 = V_0 \text{ here } *$$

$$6V = I_1 \cdot 3\Omega$$

$$I_1 = 2A \text{ (✓)}$$

a bit long - but right



oh yeah drh

$$V_3 = 5V$$

$$V_3 - V_0 = V_3 - 0 = V_3$$

↙ i may not be 2

$$V_2 - V_1 = I_1 \cdot 3\Omega$$

$$\frac{V_2 - V_1}{3} = I_1 \quad \leftarrow \text{most write like th's}$$

oh yeah  $I_1 + I_2 = I_3$

$$\frac{V_3 - V_1}{1} = I_2$$

$$\frac{V_1 - V_0}{1} = I_3$$

$$\frac{V_2 - V_1}{3} + \frac{V_3 - V_1}{1} = \frac{V_1 - 0}{1}$$

$$\frac{6V - V_1}{3} + 5V - V_1 = V_1$$

$$2V + 5V = V_1 + V_1 + \frac{V_1}{3}$$

$$7V = \frac{7}{3}V_1$$

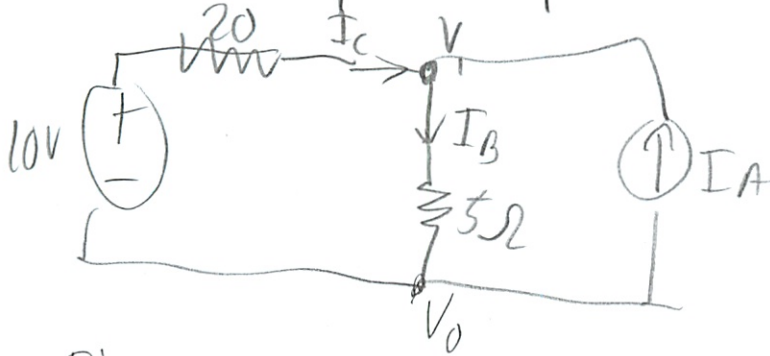
· 3/7

$$V_1 = 7 \cdot \frac{3}{7} = \textcircled{3V} \quad \textcircled{\checkmark}$$

See I can do it - all by myself  
 but what about an op-amp one

Q101 Final  
Fall 08 #6

- key final practice problem!



Find  $I_c$  if  $I_A = 0$

- think I know it better now

realized  
just plug  
it in

$$\frac{10V - V_1}{20} = I_c$$

$$\frac{V_1 - 0}{5} = I_B$$

$$\rightarrow I_A + I_c = I_B$$

$$\text{if } I_A = 0 \quad \underbrace{\frac{10V - V_1}{20}}_{\text{find}} = \frac{V_1 - 0}{5}$$

first need to find  $V_1$

$$50V - 5V_1 = 20V_1$$

$$50 = 25V_1$$

$$2 = V_1$$

$$I_c = \frac{10 - 2}{20} = \frac{8}{20} = \frac{4}{10} = \left(\frac{2}{5}\right) A$$

really long -  
but that's Ok

✓ did not get  
before

b) Find  $i_B$  if  $I_A = 5A$

$$5A + \frac{10 - V_1}{20} = \frac{V_1 - 0}{5} \quad \leftarrow \text{may have changed}$$

$$25 + 50 - 5V_1 = 20V_1$$

$$75 = 25V_1$$

$$V_1 = 3$$

ewolfram alpha says  $V = 22$

so this was  
problem  
- bad algebra

$$I_B = \frac{3}{5} A \quad \otimes 4.4 = \frac{22}{5}$$

$\leftarrow$  seems funny  $5 - 1.6 = 4.4$

What about other way

$$5 + \frac{10 - 3}{20}$$

$$5.35$$

ii

c) Find  $I_A$  so  $I_C = 0$

$$I_A + 0 = I_B = \frac{V_1 - 0}{5}$$

$$0 = \frac{10V - V_1}{20}$$

$$0 = 10V - V_1$$

$$V_1 = 10V$$



$$I_B = \frac{10}{5} = 2$$

$$I_A = 2 \text{ A} \quad \text{D}$$

much less of a disaster than last time

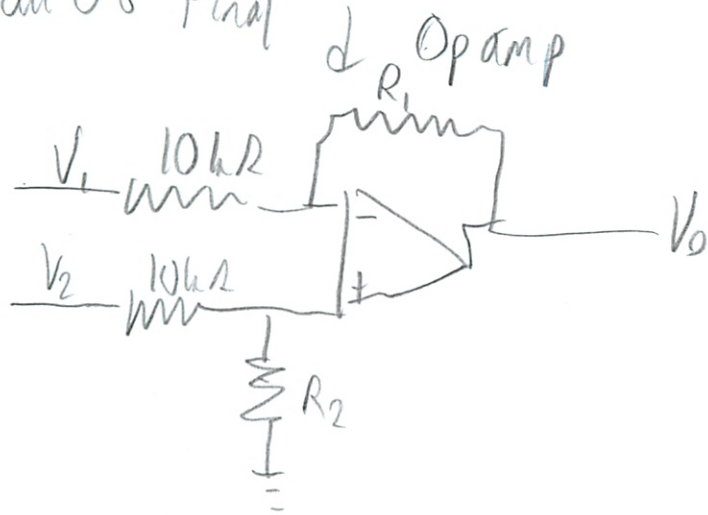
but why part B wrong?

Oh I also did something w/ current dividers

On separate page

- I like my ans here better

Fall 08 Final



Fill in table

- First check notes for which  $\rightarrow$  subtractor
- Find  $R_1, R_2$  to satisfy

First let me try to find eq on my own

$$\frac{V_1 - V_-}{10k\Omega} = \frac{V_- - V_o}{R_1}$$

Ⓟ be sure to go  
~~right~~ right direction

$$\frac{V_2 - V_+}{10k\Omega} = \frac{V_+ - 0}{R_2}$$

~~Then solve each for  $V_-, V_+$  respectively~~ Then solve each for  $V_-, V_+$  respectively

$$V_1 R_1 - V_- R_1 = 10k V_- - 10k V_o$$

$$10k V_- + V_- R_1 = V_1 R_1 + 10k V_o$$

$$V_- (10k + R_1) = V_1 R_1 + 10k V_o$$

$$V_- = \frac{V_1 R_1 + 10k V_o}{10k + R_1}$$

$$V_+ = R_2 V_2 - R_2 V_+ = 10k V_+$$

$$10k V_+ + R_2 V_+ = R_2 V_2$$

$$V_+ = \frac{R_2 V_2}{(10k + R_2)}$$

$$V_- = V_+$$

$$\frac{V_1 R_1 + 10k V_0}{10k + R_1} = \frac{R_2 V_2}{(10k + R_2)}$$

? now what

$V_0$  - in terms of  $V_1, V_2, R_1, R_2$

$$\frac{10k V_0}{10k + R_1} = \frac{R_2 V_2}{10k + R_2} - \frac{V_1 R_1}{10k + R_1}$$

$$10k V_0 = \frac{R_2 V_2 (10k + R_1)}{10k + R_2} - \frac{V_1 R_1 (10k + R_1)}{10k + R_1}$$

$\overline{10k}$

$$V_0 = \frac{R_2 V_2 (10k + R_1)}{(10k + R_2) 10k} - \frac{V_1 R_1}{10k}$$

- If all  $10k$  then should

$$= V_2 - V_1$$

Test by setting  $R_1 = R_2 = 10k$

$$\frac{10k V_2 (\cancel{10k + 10k})}{(\cancel{10k + 10k}) \cancel{10k}} - \frac{V_1 \cancel{10k}}{\cancel{10k}} \quad \text{✓ checks out}$$

Ok now back to our regularly scheduled problem

$$V_0 = 2V_2 - 2V_1$$

So want

$$\frac{R_1}{10k} = 2$$

$$\frac{R_2 (10k + R_1)}{(10k + R_2) 10k} = 2$$

$$= 2$$

to  
(connected)

~~to solve~~

either start here

$$R_1 = 20k$$

now back

$$\frac{R_2 (10k + 20k)}{(10k + R_2) 10k} = 2$$

$$= 2$$

$$\frac{30k R_2}{10k (10k + R_2)} = 2$$

$$= 2$$

$$\frac{3R_2}{(10k + R_2)} = 2$$

wolfram solve

$$x = 20,000 = 20k$$

could have  $g+V$

⊙ Took me 4 pages but is correct

$$V_0 = 2V_2 - V_1$$

$$\frac{R_1}{10k} = 1$$

$$R_1 = 10k$$

$$\frac{R_2 (10k + \overset{\downarrow R_1}{10k})}{(10k + R_2) 10k} = 2$$

$$\frac{2R_2}{(10k + R_2)} = 2$$

$$2R_2 = 20k + 2R_2 \quad \text{ouch here how to solve by hand}$$

$$0R_2 = 20k$$

$$R_2 = \infty - \text{anything} \quad \checkmark$$

$$V_0 = V_2 - 2V_1$$

$$\frac{R_1}{10k} = 2$$

$$R_1 = 20k$$

$$\frac{R_2 (10k + 20k)}{(10k + R_2) 10k} = 1$$

$$\frac{3R_2}{(10k + R_2)} = 1$$

$$3R_2 = 10k + R_2$$

$$2R_2 = 10k$$

$$R_2 = 5k \quad \checkmark$$

---

$$V_o = 4V_2 - 2V_1$$

$$\frac{R_1}{10k} = 2$$

$$R_1 = 20k$$

$$\frac{R_2 (10k + 20k)}{(10k + R_2) / 10k} = 4$$

$$\frac{3R_2}{(10k + R_2)} = 4$$

$$3R_2 = 40k + 4R_2$$
$$-4R_2 \quad \quad -4R_2$$

$$-R_1 = 40k$$

$$R_1 = -40k$$

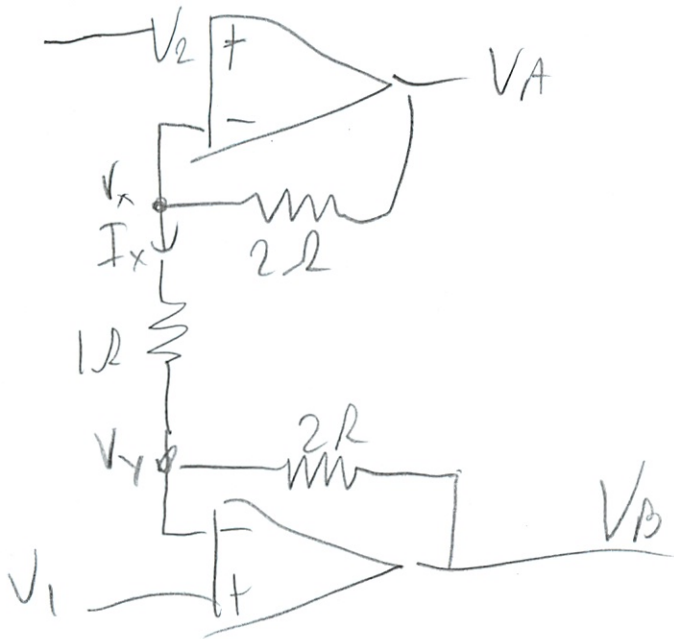
impossible



Good - getting the hang of circuits

# Nano Quiz 10

- may be a problem - can I remember  $V_+ = V_-$



$$V_+ = V_-$$

a) Determine  $I_x$  when  $V_1 = 1V$   $V_2 = 2V$

~~$$V_2 = V_+ = V_-$$~~

~~$$\frac{V_A - V_-}{2} = I_x$$~~

~~$$V_2 = V_x$$~~

$$\frac{V_A - V_x}{2} = I_x$$

$$\frac{V_x - V_y}{1} = I_x$$

direction is critical!

$$\frac{V_Y - V_B}{2} = I_x$$

Which dir. is current flowing in?

$$V_1 = V_Y$$

$$\frac{V_A - V_x}{2} = \frac{V_x - V_Y}{1} = \frac{V_Y - V_B}{2}$$

$$V_x = V_2$$

$$V_Y = V_1$$

$$\frac{V_A - V_2}{2} = V_2 - V_1 = \frac{V_1 - V_B}{2} = I_x$$

$$\frac{V_A - 2}{2} = 2 - 1 = \frac{1 - V_B}{2} = I_x$$

$$1 = I_x \quad \text{✓}$$

I think I did way too much there



b) Determine  $V_A$  when  $V_1 = 1$   $V_2 = 2$

So use same formula

— nothing's changed — can use old #

$$\frac{V_A - 2}{2} = 1$$

$$V_A - 2 = 2$$

$$V_A = 4 \quad \text{D}$$

It seems like each time around I solve it differently — the, used too many diff methods

c) Determine an expression for  $V_A$  in terms  $V_1, V_2$

$$\frac{V_A - V_2}{2} = V_2 - V_1$$

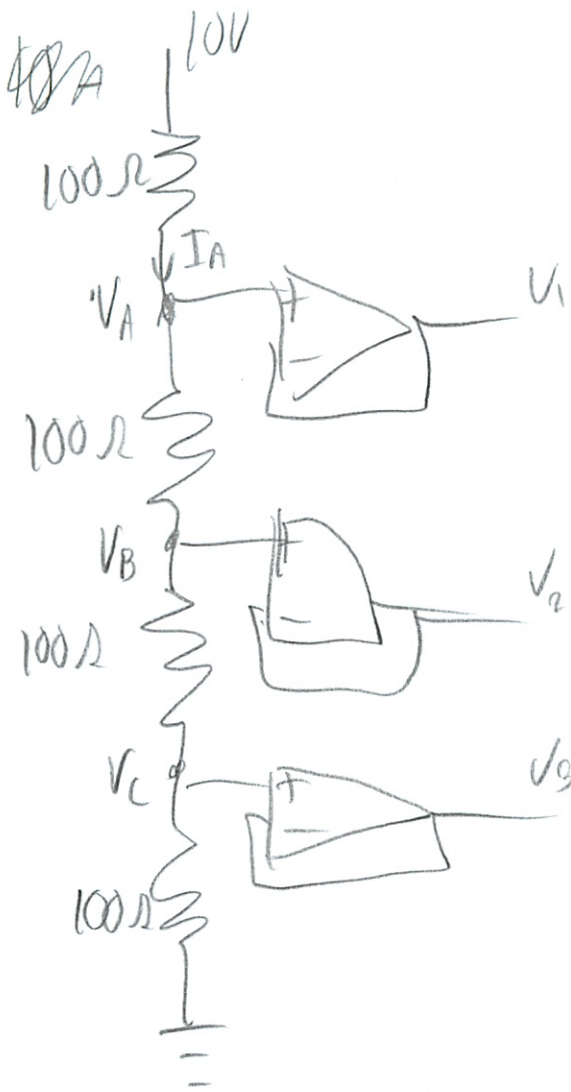
$$V_A - V_2 = 2(V_2 - V_1)$$

$$V_A = 2(V_2 - V_1) + V_2$$

$$V_A = 2V_2 - 2V_1 + V_2$$

$$V_A = 3V_2 - 2V_1 \quad \text{D}$$

I seem to be getting this circuit thing!



$$\frac{10V - V_A}{100} = I_A$$

$$V_1 = V_A \text{ buffered}$$

$$\frac{V_A - V_B}{100} = I_B$$

$$I_A = I_B \text{ same current!}$$

$$\frac{V_B - V_C}{100}$$

$$V_2 = V_B$$

$$\frac{V_C - 0}{100}$$

$$V_3 = V_C$$

$$\frac{10V - V_A}{100} = \frac{V_A - V_B}{100} = \frac{V_B - V_C}{100} = \frac{V_C}{100}$$

Right???

- no where to confirm

← Voltage divider thing

$$10 \cdot \frac{100}{400} = V_C = 2.5$$

etc

- try others

Emerson

oh here  $I_s$  not the same

↳

What is it  $\forall$  currents + op amps again?

~~VA~~  $I$  at  $V_+, V_- = 0$  (draws no current)

$I$  generated needed at put current

$$\frac{10V - V_A}{100} = I_1 \quad \text{but } I_1 = 0$$

$$V_A = V_+ = V_- = V_1$$

$$\frac{V_A - V_B}{100} = I_B \quad \text{from op amp}$$

So what ever so  $V_A = V_B$

~~10/100~~

...

Somehow all are 10V

No way to fix

---

Well  $V_1$  always 10V - that's what's coming in  
-  $\therefore$  no voltage drop over resistor.

$$V = IR$$

$$10 = \frac{0}{100} \cdot 100$$

$\uparrow$  since no current

or whatever current so 10V

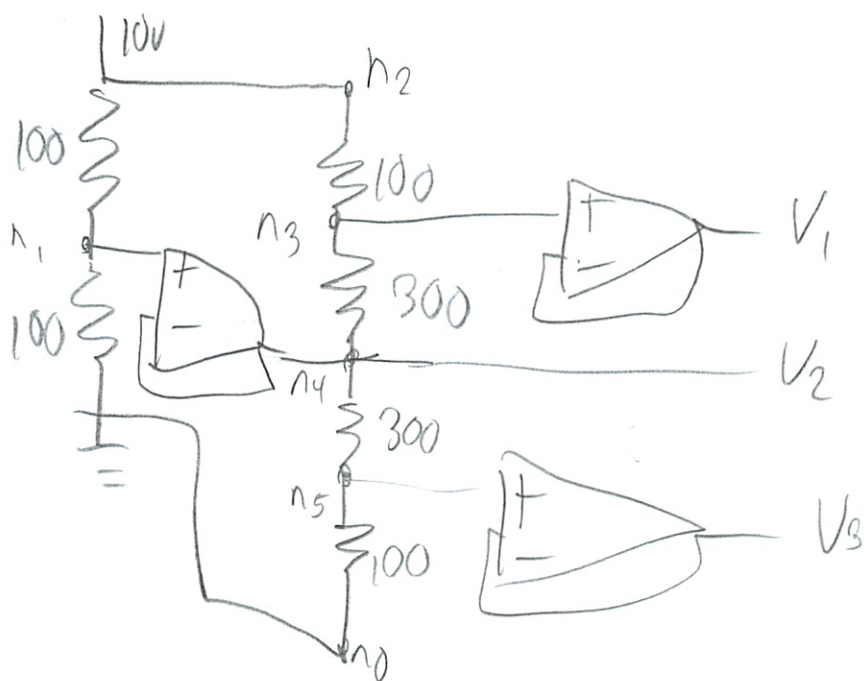
Plan next



Same as above

(still don't really get - but ship)

Flann



$$\frac{10 - V_1}{100} = \frac{V_1 - 0}{100}$$

$$V_1 = 5$$

$$\frac{n_2 - n_3}{100} = \frac{n_3 - n_4}{300}$$

$$n_4 = 5V$$

from opamp

$$n_2 = 10V \text{ (source)}$$

↓ opamp draws no voltage

$$\frac{10V - n_3}{100} = \frac{n_3 - 5V}{300}$$

find  $n_3$

← Volt from algebra =  $\frac{35}{4} = 8.75$

~~$$300 - 300n_3 = 100n_3 - 500$$~~

So problem is in solving

~~$$800 = 400n_3$$~~

~~$$2 = n_3 \text{ should be } 8.75$$~~

but got circuits

Does current go out w/  $V_2$   
 — no — nothing is hooked up

fix this now

what am I doing wrong?

can't do that distribution  
 let me try something else

division of multiplication

$$\frac{10 - n_3}{100} = \frac{n_3 - 5}{300}$$

$$\cdot 300 \qquad \qquad \cdot 300$$

$$30 - 3n_3 = n_3 - 5$$

$$\qquad + 3n_3 \qquad + 5$$

$$35 = 4n_3$$

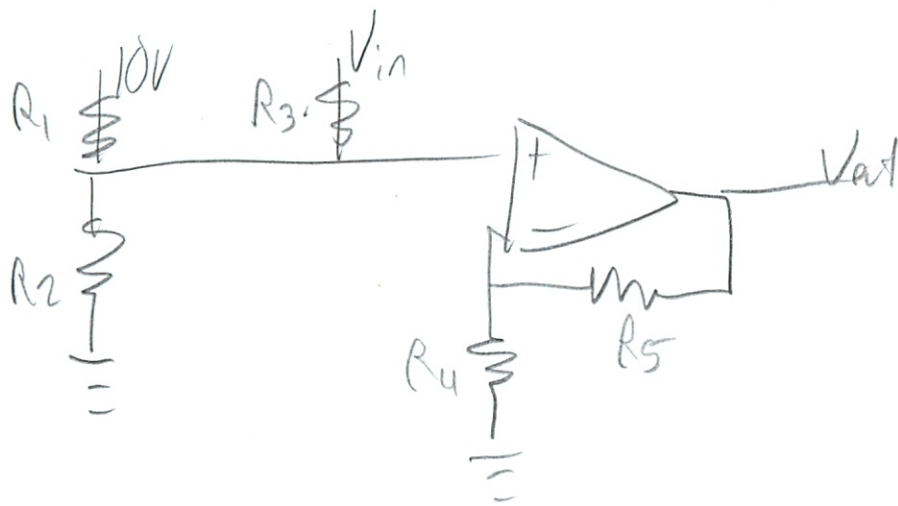
$$\frac{35}{4} \text{ (✓)}$$

why can't I get to simple

— also in black's formula  
 — end in 14.01

algebra down

B 3rd time



Formula for  $V_t$

Which way current  $\downarrow \downarrow$

$$\frac{10V - V_t}{R_1} + \frac{V_{in} - V_t}{R_3} = \frac{V_t - 0}{R_2} \quad (1)$$

Solve for  $V_t$

Again w/ denominators

oh right - get common denom *get a common denom!* *get the algebra down!*

$$\frac{R_3(10V - V_t)}{R_1 R_3} + \frac{R_1(V_{in} - V_t)}{R_3 R_1} = \frac{V_t}{R_2}$$

$$\frac{10V R_3 - V_t R_3 + R_1 V_{in} - V_t R_1}{R_1 R_3} = \frac{V_t}{R_2}$$

Well - what do we want - all  $V_t$  on 1 side of page  
 now can cross multiply?

$$\frac{V_+}{R_2} + \frac{V_+ R_3}{R_1 R_3} + \frac{V_+ R_1}{R_1 R_3} = \frac{10V R_3 + R_1 V_{in}}{R_1 R_3}$$

Now need common denom

$$\frac{V_+ \cdot R_1 \cdot R_3}{R_2 \cdot R_1 \cdot R_3} + \frac{V_+ R_2 R_3}{\dots} + \frac{V_+ R_1 R_2}{+ \dots} = \frac{10V R_3 + R_1 V_{in}}{R_1 R_3}$$

$\cdot R_1 R_2 R_3$

$$V_+ (\dots) = \frac{(10V R_3 + R_1 V_{in}) R_1 R_2 R_3}{R_1 R_3}$$

$$V_+ (\dots) = 10V R_2 R_3 + R_1 R_2 V_{in}$$

$$V_+ = \frac{10V R_2 \cdot R_3 + R_1 R_2 V_{in}}{R_1 R_3 + R_2 R_3 + R_1 R_2}$$

$$= \frac{R_3 (10V R_2 + V_{in} R_1)}{R_1 R_3 + R_2 R_3 + R_1 R_2}$$

$$\Rightarrow R_1 R_3 + R_2 R_3 + R_1 R_2$$

how did I switch those two?

- but much nicer!

Now  $V_{out}$  in terms  $V_+$

$$V_+ = V_-$$

$$\frac{V_- - 0}{R_4} = \frac{V_- - V_{out}}{R_5}$$

↓  $\frac{V_{out} - 0}{R_4 + R_5}$

~~=  $\frac{V_- - V_{out}}{R_5}$~~  I must be some op amp process

$$V_{out} = \frac{R_4 + R_5}{R_4} V_+$$

(I don't care about this anymore)