6.02 Quiz 2

SNR/BER

Mean $\mu_x = \frac{1}{N}\sum_{n=1}^{N} x[n]$

Power $P_x = \frac{1}{N}\sum_{n=1}^{N} x[n]^2$

often factor mean out $\hat{P}_x = \frac{1}{N}\sum_{n=1}^{N}(x[n]-\mu_x)^2 = \sigma^2 =$ var

Energy $E_x = \sum_{n=1}^{N} x[n]^2$

$\hat{E}_x = \sum_{n=1}^{N}(x[n]-\mu_x]^2$

SNR $= \frac{\tilde{P}_{signal}}{\hat{P}_{noise}}$ in db $= 10\log\left(\frac{\tilde{P}_{signal}}{\tilde{P}_{noise}}\right)$

Stationary - not affected by shifts

ergodic - " " " time

Find P_x, then P_{noise}

No noise $P_N = 0$ · SNR $= \infty$ BER $= 0$

Noise $\sim \sigma N + \mu$

$P(\text{Noise} < t) = P(\sigma N + \mu < t)$

$= \Phi\left(\frac{t-\mu}{\sigma}\right)$

Sometimes $\mu_x = \frac{1}{2}0 + \frac{1}{2}\cdot 1 = \frac{1}{2}$

$\sigma_x^2 = \frac{1}{2}\cdot 0^2 + \frac{1}{2}\cdot 1^2 - \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

BER $= P(\text{noise} > .5 \mid 0)\cdot P(0) + P(< .5 \mid 1)P(1)$

$= \Phi\left(-\frac{1}{2\sigma}\right)\cdot\frac{1}{2} + \frac{1}{2}\Phi\left(-\frac{1}{2\sigma}\right)$

$= \Phi\left(-\sqrt{SNR}\right)$

SNR(db) $= 10\log\left(\frac{P_{signal}}{\sigma^2}\right)$

- So for BER, Get σ^2 from here, plug from SNR

Intersymbol Interference ISI

- eye diagram
  - all possible cases on top of each other
- for BER $P(11)P(\text{error}\mid 11) + P(10)P(\text{error}\mid 10)$ + etc
  $\uparrow \Phi\left(\frac{\text{midline - voltage it will be}}{\sigma}\right)$
- V_{TH} to minimize error rate

Ways to avoid errors

Packet = {#, msg, Chk}

Hash to detect error

Checksum - Just add up bits

- easy for one error to offset another

$P(>1 \text{ error}) = 1 - P(\text{no error}) = 1-(1-BER)^k$

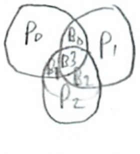
$P(>2 \text{ errors}) = 1 - P(\text{no error}) - P(1 \text{ error})$

$= 1(1-BER)^k - k\cdot BER\cdot(1-BER)^{k-1}$

Hamming Distance = # digit positions wrong

D = min hamming dist - can correct $\left\lfloor\frac{D-1}{2}\right\rfloor$

Single error codes



$E_0 = B_0 \oplus B_1 \oplus B_3 \oplus P_0$

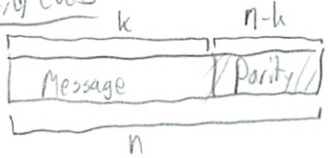
$E_1 = B_0 \oplus B_2 \oplus B_3 \oplus P_1$

$E_2 = B_1 \oplus B_2 \oplus B_3 \oplus P_2$

↑ should all be 0s

↖ (n+1, n, 2)

(n,k,d) Codes



d = Min Hamming distance can correct

k/n = code rate, ≤ 1, larger better

Rectangular

Need $n \leq 2^{n-k} - 1$

B_0	B_1	P_0
B_2	B_3	P_1
P_2	P_3	

← (8,4,3)

Replicating (n, 1, n) Good for code w/ Hamming = 2 = d

SECC (n, n-p, 3) $n = 2^p - 1$ for $p > 1$

Interleaving for burst errors

- Needs framing to know where to start
- 8b10 works

Reed-Solomon

- the matrix, Galois field thing
- common (255, 223)
- (n, k) solves up to (n-k)/2 errors

Convolutional Codes

4/10

- always a fn of what came before

$P_i[n] = \left(\sum_{j=0}^{k-1} g_i[j]\,x[n-j]\right) \bmod 2$

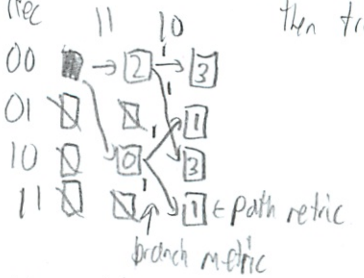
g_i = k-element generator polynomial for parity bit P_i

- either 1 or 0
- given
- interleave data parity for rate = 1/2

$= P_0[0]P_1[0]P_0[1]P_1[1]\ldots$

message never transmitted

- like a SM or Trellise (over time)
- look for min hamming for most likely then trace backward



Hard decision - digitized 1 or 0

Soft decision

for each pt - like 00 $(V_{P0}-0)^2 + (V_{P1}-0)^2$ no √!

k = # places using a bit - higher better

Protocols for Multi User

Goals

- high utilization $U = \frac{\text{throughput all nodes}}{\text{max data rate of channel}}$
- fairness $F = \frac{\left(\sum_{i=1}^{N} x_i\right)^2}{N\sum_{i=1}^{N} x_i^2}$ $\frac{1}{N} \leq F \leq 1$
- bounded wait
- Scalability

TDMA - just give each person a slot

Aloha - p send packet

$U_{slotted} = N\underbrace{p(1-p)^{N-1}}_{P(\text{only 1 sender})}$

$p_{best} = \frac{1}{N}$ leads to $U = \frac{1}{e} \approx 37\%$

Can ↓ p if collision

- if half $= 2^{-k}$ = binary exp back off

but bound $p = \max(p_{min}, p/2)$

$p_{min} \ll 1/\max(N)$

$U_{unslotted} \approx \frac{Np(1-p)^{(2T-1)N-1}}{1/t} = \frac{TNp(1-p)}{\exp(2T-1)N-1}$

with large N

$$U_{max} \approx \left(\frac{T}{2T-1}\right)\frac{1}{e}$$

w/ large N.T

$$U_{max} \approx \frac{1}{2e}$$

$$V_{max} = \frac{T}{2T-1}\left(1 - \frac{1}{(2T-1)N}\right)^{(2T-1)(N-1)}$$

Unslotted

Carrier sense - even better

- need random wait time after backoff
- * inside a contention window

CDMA - using orthoginal vectors

Freq Division Multiplexing

- seq must be periodic (repeat)

$$x[n] = x[n+N]$$

- in N samples - fun freq $= \frac{2\pi}{N}$
- harmonics possible $k \cdot \frac{2\pi}{N}$
- negitive means go other direction
- complex exponential

$$e^{j\varphi} = \cos(\varphi) + j\sin(\varphi)$$

$$\cos(\varphi) = \frac{1}{2}e^{j\varphi} + \frac{1}{2}e^{-j\varphi}$$

$$\sin(\varphi) = \frac{1}{2}e^{j\varphi} - \frac{1}{2}e^{-j\varphi}$$

When $\varphi = 0$

$$e^{j0} = \cos(0) + j\sin(0) = 1 + 0j$$

$\varphi = \pm\pi$

$$e^{j\pi} = e^{-j\pi} = -1$$

$$e^{j\pi n} = e^{-j\pi n} = (-1)^n$$

Summing

$$\sum_{n=\langle n \rangle} e^{jk\frac{2\pi}{N}n} = \begin{cases} N & k = 0, \pm N, \pm 2N, \ldots \\ 0 & \text{otherwise} \end{cases}$$

Discrete Time Fourier Seq

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\left(\frac{2\pi}{N}\right)n}$$

Synthesis freq → time

k is over range of ints

- 0 for $0 \leq$ freq $< 2\pi$
- $-N/2$ for $-\pi \leq$ freq $\leq \pi$

a_k = spectrial coefficent (complex)

$$a_k = \frac{1}{N}\sum_{n=\langle N \rangle} x[n] e^{-jk\left(\frac{2\pi}{N}\right)n}$$

Analysis time to freq

= k th spectrial coefficent

↑ = index = # cycles in N samples

Need to truncate/limit to a freq

- loses fine detail, like an avg

$$H(e^{j\Omega}) = \sum_m h[m] e^{-j\Omega m}$$

↑ calc for a certain Ω

channel response

$$\delta[n] \rightarrow \boxed{H(e^{j\Omega})} \rightarrow h(n)$$

↑ unit sample, periodic N timesteps

↑ unit sample response

$$a_k = \frac{1}{N}x[0]e^{-jk\frac{2\pi}{N}0} = \frac{1}{N}$$

$$h[n] = \sum_{k=\langle N \rangle} H\left(e^{jk\frac{2\pi}{N}}\right)e^{jk\frac{2\pi}{N}n}$$

Finding the 0s at $\pm\varphi$

$$H(e^{j\Omega}) = (e^{j\Omega})^2 - 2\cos(\varphi)(e^{j\Omega}) + 1 = \boxed{0}$$

$h[0] = 1$ $h[1] = -2\cos(\varphi)$ $h[2] = 1$ for these, 0 elsewhere

Convolution in time domain

multiplication in freq domain

poor mans low pass = convole a bunch of 0s together

$$x\alpha \rightarrow \square \rightarrow y\alpha$$

Random

$$a_k e^{j\Omega kn} \rightarrow \square \rightarrow a_k H(e^{j\Omega k}) \cdot e^{j\Omega kn}$$

Ω is the band you are assigned

all on 1 channel

have 0s at start + end to bound

$$\Omega = 2\pi\frac{f}{f_s} = 2\pi\frac{k}{N}$$

signal freq cycles/sec

↑ sample freq samples/sec

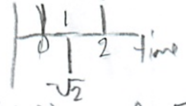
k ← spectral coef

Ω ← angular freq

Filter to only certain freq

- as an LTI channel - choose h()
  - want 0 at many pts
  - convolve many pts together

$$h[n] = \delta[n] - \sqrt{2}\,\delta[n-1] + \delta[n-2]$$



$$H(e^{j\Omega}) = e^0 - \sqrt{2}e^{-j\Omega} + e^{-2j\Omega}$$

$$= 1 - \sqrt{2}\cos{-\varphi} - \sqrt{2}\sin{-\varphi}\,i + \boxed{\cos{-2\varphi} + \sin{-2\varphi}\,i}$$

$$= 1 - \sqrt{2}x + x^2$$

if find 0, set = to 0, find x

$x = \cos\varphi + \sin\varphi$, find φ

$$C(\sqrt{2}\text{ here}) = \frac{1 + e^{-2\Omega j}}{-e^{-\Omega j}}$$

Cos

$$a_p = e^{\varphi} \cdot \frac{1}{2} + \frac{1}{2}e^{-\varphi} \text{ for } 0 \leq p \leq N-1$$

$$\text{or } \left[-\frac{N}{2}\right] \leq p \leq \left[\frac{N}{2}\right]$$

$$\cos\left(p\left(\frac{2\pi}{11}\right)n\right) \text{ for } -5 \leq p \leq 5$$

works out to $a_k = \begin{cases} 1/2 & k = \pm r \\ 0 & \text{otherwise} \end{cases}$

Sin

$$a_p = \frac{j}{2} + e^{\varphi} = \frac{j}{2} + \cos\varphi + \sin\varphi\, i$$

$$a_{-p} = -\frac{j}{2} + e^{\varphi} = -\frac{j}{2} + \cos\varphi + \sin\varphi\, i$$

$$\text{so } x[n] = 1 + 2\cos\left(3\frac{2\pi}{11}n\right) - 3\sin\left(5\frac{2\pi}{11n}\right)$$

$$a_{\pm 3} = 2 \cdot \frac{1}{2} = 1 \quad (\cos)$$

$$a_{-5} = -3\frac{j}{2} = -1.5j$$

$$a_5 = -3 \cdot -\frac{j}{2} = 1.5j$$

$a_k = 0$ otherwise

---

states on state diagram $= 2^{k-1}$

$$P(\text{noise} > .5) = P(\sigma N > .5) = P\left(N > \frac{.5}{\sigma}\right)$$

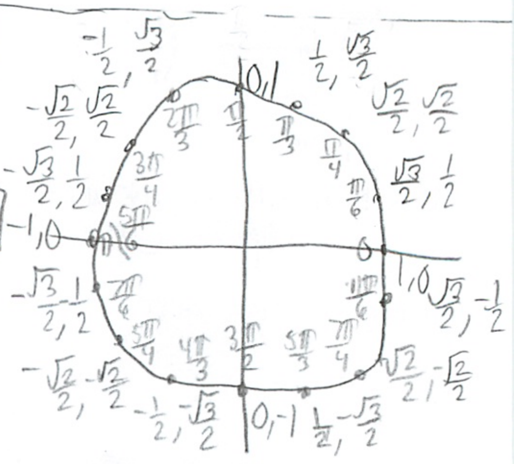
$$= P\left(N < \frac{-.5}{\sigma}\right) = \Phi\left(-\frac{.5}{\sigma}\right) = 1 - \Phi\left(\frac{.5}{\sigma}\right)$$

(n,k) has at most 2^{n-k} patterns

With min $d = 2t+1$ can corr t errors

---

k = Constraint lenght



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6.02 Tutorial Problems: Noise & Bit Errors

Problem 1.

Suppose the bit detection sample at the receiver is `V + noise` volts when the sample corresponds to a transmitted '1', and `0.0 + noise` volts when the sample corresponds to a transmitted '0', where `noise` is a zero-mean Normal(Gaussian) random variable with standard deviation σ_{NOISE}.

A. If the transmitter is equally likely to send '0"s or '1"s, and `V/2` volts is used as the threshold for deciding whether the received bit is a '0' or a '1', give an expression for the bit-error rate (BER) in terms of the zero-mean unit standard deviation Normal cumulative distribution function, Φ, and σ_{NOISE}.

Show Answer

B. Suppose the transmitter is equally likely to send zeros or ones and uses zero volt samples to represent a '0' and one volt samples to represent a '1'. If the receiver uses 0.5 volts as the threshold for deciding bit value, for what value of σ_{NOISE} is the probability of a bit error approximately equal to 1/5? Note that $\Phi(0.85) \approx 4/5$.

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C. Will your answer for σ_{NOISE} in part (B) change if the threshold used by the receiver is shifted to 0.6 volts? Do not try to determine σ_{NOISE}, but justify your answer.

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D. Will your answer for σ_{NOISE} in part (B) change if the transmitter is twice as likely to send ones as zeros, but the receiver still uses a threshold of 0.5 volts? Do not try to determine σ_{NOISE}, but justify your answer.

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Problem 2.

Ben Bitdiddle is doing a 6.02 lab on understanding the effect of noise on data receptions, and is confused about the following questions. Please help him by answering them.

In these questions, assume that:

1. The sender sends 0 Volts for a "0" bit and 1 Volt for a "1" bit
2. P_ij = Probability that a bit transmitted as "i" was received as a "j" bit (for all four combinations of i and j, 00, 01, 10, 11)
3. alpha = Probability that the sender sent bit 0
4. beta = Probability that the sender sent bit 1
5. and, obviously, alpha + beta = 1

The channel has non-zero random noise, but unless stated otherwise, assume that the noise has 0 mean and that it is a Gaussian with finite variance. The noise affects the received samples in an additive manner, as in the labs you've done.

1. Which of these properties does the bit error rate of this channel depend on?
   a. The voltage levels used by the transmitter to send "0" and "1"
   b. The variance of the noise distribution
   c. The voltage threshold used to determine if a sample is a "0" or a "1"
   d. The number of samples per bit used by the sender and receiver

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2. Suppose Ben picks a voltage threshold that minimizes the bit error rate. For each choice below, determine whether it's true or false.
   a. P_01 + P_10 is minimized for all alpha and beta
   b. alpha * P_01 + beta * P_10 is minimized
   c. P_01 = P_10 for all alpha and beta
   d. if alpha > beta then P_10 > P_01
   e. The voltage threshold that minimizes BER depends on the noise variance if alpha = beta

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3. Suppose alpha = beta. If the noise variance doubles, what happens to the bit error rate?

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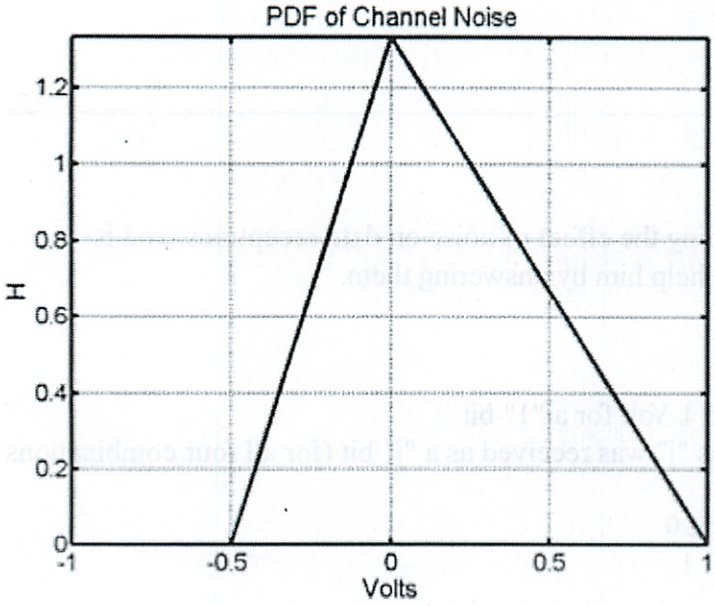
Problem 3.

Messages are transmitted along a noisy channel using the following protocol: a "0" bit is transmitted as -0.5 Volt and a "1" bit as 0.5 Volt. The PDF of the total noise added by the channel, H, is shown below.

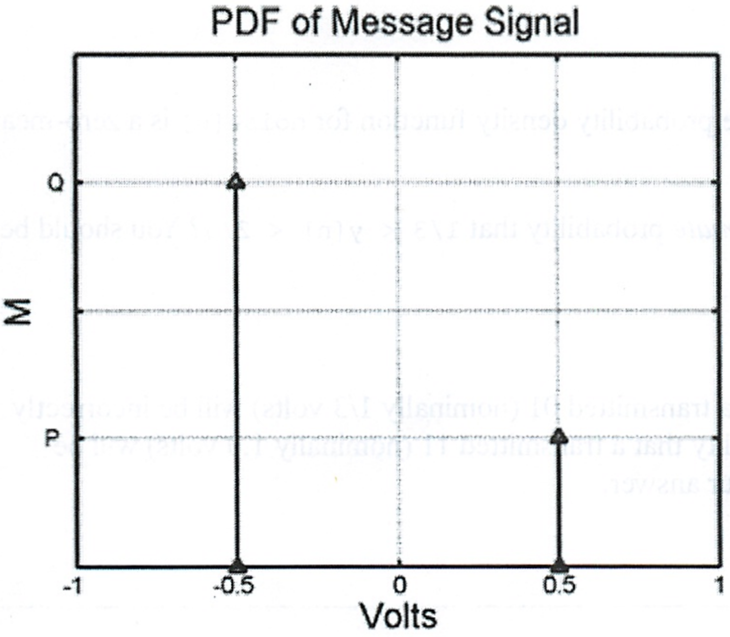


Not normal Anow
- pyrmidd

a. Compute H(0), the maximum value of H.

Show Answer

b. It is known that a "0" bits 3 times as likely to be transmitted as a "1" bit. The PDF of the message signal, M, is shown below. Fill in the values P and Q.



Show Answer

c. If the digitization threshold voltage is 0V, what is the bit error rate?

Show Answer

d. What digitization threshold voltage would minimize the bit error rate?

Show Answer

Problem 4.

Consider a transmitter that encodes pairs of bits using four voltage values. Specifically:

00 is encoded as zero volts,
01 is encoded as `(1/3)Vhigh` volts,
10 is encoded as `(2/3)Vhigh` volts and
11 is encoded as `Vhigh` volts.

For this problem we will assume a wire that only adds noise. That is,

```
y[n] = x[n] + noise[n]
```

where `y[n]` is the received sample, `x[n]` the transmitted sample whose value is one of the above four voltages, and `noise[n]` is a random variable.

Please assume all bit patterns are *equally likely* to be transmitted.

Suppose the probability density function for `noise[n]` is a constant, `K`, from -0.05 volts to 0.05 volts and zero elsewhere.

A. What is the value of `K`?

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Suppose now `Vhigh= 1.0` volts and the probability density function for `noise[n]` is a zero-mean Normal with standard deviation σ.

C. If $\sigma = 0.001$, what is the *approximate* probability that `1/3 < y[n] < 2/3`? You should be able to give a numerical answer.

Show Answer

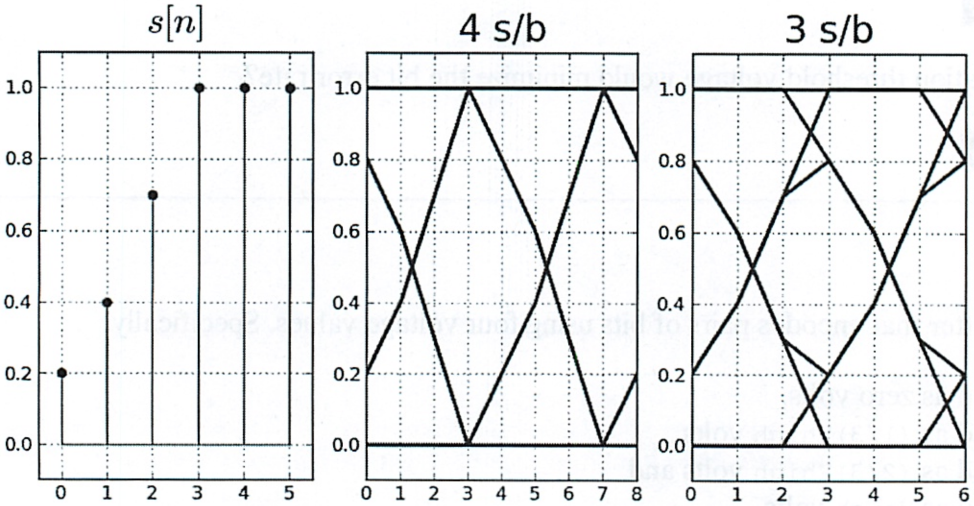
D. If $\sigma = 0.1$, is the probability that a transmitted 01 (nominally 1/3 volts) will be incorrectly received the same as the probability that a transmitted 11 (nominally 1.0 volts) will be incorrectly received? Explain your answer.

Show Answer

Problem 5.

Consider the figure below, which shows the step response for a particular transmission channel along with the eye diagram for channel response when transmitting 4 samples/bit and 3 samples/bit.

Char of channel



A. If the transmitter uses 3 samples/bit, under what conditions will it be possible to reliably (i.e., correctly) receive any sequence of transmitted bits?

Show Answer

Suppose now that there is additive noise on this channel so that sometimes a transmitted bit is misidentified at the receiver. Let's investigate how the rate of bit errors is affected by changes in noise probability density functions and number of samples per bit. In answering the questions below, please assume that the receiver uses the optimal detection sample for each bit (corresponding to the "center" of the eye) and uses a detection threshold of 0.5V.

B. If we send 4 samples/bit down the noisy channel, the received voltage will be `1.0 + noise` when receiving a transmitted '1' bit, and `0.0 + noise` volts when receiving a transmitted '0' bit. If the noise is zero-mean Gaussian with standard deviation $\sigma=0.25$, what is the bit error rate? Assume that '0' and '1' bits are transmitted with a probability of 0.5, and that the noise is independent of the bit being transmitted.

Show Answer

C. If 3 samples/bit are used by the transmitter, the received voltage will be

`1.0+noise` or `0.8+noise` when receiving a transmitted '1' bit, and

`0.2+noise` or `0.0+noise` volts when receiving a transmitted '0' bit.

If the noise is **uniformly** distributed between the voltage values -1 and 1 volts, what is the bit error rate? Hint: are all four cases of received voltages equally likely?

Show Answer

Problem 6.

Suppose a channel has both noise and intersymbol interference, and further suppose the voltage at the receiver is:

`8.0+noise` volts when the transmitter sends a '1' bit preceded by a '1' bit
`6.0+noise` volts when the transmitter sends a '1' bit preceded by a '0' bit
`2.0+noise` volts when the transmitter sends a '0' bit preceded by a '1' bit
`0.0+noise` volts when the transmitter sends a '0' bit preceded by a '0' bit.

In answering the following parts, please assume the receiver uses 4.0 volts as the threshold for deciding the bit value.

A. Suppose noise is Gaussian with standard deviation $\sigma=1$ and all bit patterns are equally likely. Please determine the probability of a bit error.

Show Answer

B. Again suppose noise is Gaussian with standard deviation $\sigma= 1$, and suppose that for a particular set of transmitted data, which we will refer to as checkerboard data, there is an increased probability of unequal contiguous bits. That is, for checkerboard data

$p(00) = p(11) = 1/6$
$p(01) = p(10) = 1/3$

What is the probability of bit error for the checkerboard case?

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6.02 Tutorial Problems: Error Correcting Codes

Problem 1. For each of the following sets of codewords, please give the appropriate (n,k,d) designation where n is number of bits in each codeword, k is the number of message bits transmitted by each code word and d is the minimum Hamming distance between codewords. Also give the code rate.

A. {111, 100, 001, 010}

Show Answer

B. {00000, 01111, 10100, 11011}

Show Answer

C. {00000}

Show Answer

Problem 2. Suppose management has decided to use 20-bit data blocks in the company's new (n,20,3) error correcting code. What's the minimum value of n that will permit the code to be used for single bit error correction, i.e., that will achieve a minimum Hamming distance of 3 between code words?

Show Answer

Problem 3. The Registrar has asked for an encoding of class year ("Freshman", "Sophomore", "Junior", "Senior") that will allow single error correction. Please give an appropriate 5-bit binary encoding for each of the four years.

Show Answer

Problem 4. For any block code with minimum Hamming distance at least 2t + 1 between code words, show that:

$$2^{n-k} \geq 1 + \binom{n}{1} + \binom{n}{2} + \ldots \binom{n}{t}.$$

Show Answer

Problem 5. Pairwise Communications has developed a block code with three data (D1, D2, D3) and three parity bits (P1, P2, P3):

```
P1 = D1 + D2
P2 = D2 + D3
P3 = D3 + D1
```

A. What is the (n,k,d) designation for this code.

Show Answer

B. The receiver computes three syndrome bits from the (possibly corrupted) received data and parity bits:

```
E1 = D1 + D2 + P1
E2 = D2 + D3 + P2
E3 = D3 + D1 + P3.
```

The receiver performs maximum likelihood decoding using the syndrome bits. For the combinations of syndrome bits listed below, state what the maximum-likelihood decoder believes has occured: no errors, a single error in a speciï¬c bit (state which one), or multiple errors.

```
E1 E2 E3 = 000
E1 E2 E3 = 010
E1 E2 E3 = 101
E1 E2 E3 = 111
```

Show Answer

Problem 6. Dos Equis Encodings, Inc. specializes in codes that use 20-bit transmit blocks. They are trying to design a (20, 16) linear block code for single error correction. Explain whether they are likely to succeed or not.

Show Answer

Problem 7. Consider the following (n,k,d) block code:

```
D0  D1  D2  D3  D4   | P0
D5  D6  D7  D8  D9   | P1
D10 D11 D12 D13 D14  | P2
-------------------------
P3  P4  P5  P6  P7   |
```

where D0-D14 are data bits, P0-P2 are row parity bits and P3-P7 are column parity bits. The transmitted code word will be:

D0 D1 D2 ... D13 D14 P0 P1 ... P6 P7

A. Please give the values for n, k, d for the code above.

Show Answer

B. If `D0 D1 D2 ... D13 D14 = 0 1 0 1 0, 0 1 0 0 1, 1 0 0 0 1`, please compute P0 through P7.

Show Answer

C. Now we receive the four following code words:

```
M1: 0 1 0 1 0, 0 1 0 0 1, 1 0 0 0 1, 0 0 0 1 1 0 1 0
M2: 0 1 0 1 0, 0 1 0 0 1, 1 0 0 0 1, 0 0 1 1 1 0 1 0
M3: 0 1 0 1 0, 0 1 0 0 1, 1 0 0 0 1, 1 1 0 1 1 0 1 0
M4: 0 1 0 1 0, 0 1 0 0 1, 1 0 0 0 1, 1 0 0 1 1 0 1 0
```

For each of received code words, indicate the number of errors. If there are errors, indicate if they are correctable, and if they are, what the correction should be.

Show Answer

Problem 8. The following matrix shows a rectangular single error correcting code consisting of 9 data bits, 3 row parity bits and 3 column parity bits. For each of the examples that follow, please indicate the correction the receiver must perform: give the position of the bit that needs correcting (e.g., D7, R1), or "no" if there are no errors, or "M" if there is a multi-bit uncorrectable error.

D1	D2	D3	R1
D4	D5	D6	R2
D7	D8	D9	R3
C1	C2	C3	—

None

1	1	1	1
1	1	0	0
0	1	1	0
0	1	1	–

Wrong

Opps did not see

0	0	1	0
0	1	1	1
0	1	1	0
0	0	1	–

M ✓

1	1	0	0
1	0	0	1
0	0	1	0
0	0	1	–

✓

1	0	1	0
1	0	0	1
1	1	0	0
1	1	1	–

None ✓

1	1	1	0
1	1	1	1
1	0	0	0
0	0	1	–

M ✓

Show Answer

Problem 9. Consider two convolutional coding schemes - I and II. The generator polynomials for the two schemes are

Scheme I: G0 = 1101, G1 = 1110
Scheme II: G0 = 110101, G1 = 111011

Notation is follows: if the generator polynomial is, say, 1101, then the corresponding parity bit for message bit `n` is

(x[n] + x[n-1] + x[n-3]) mod 2

where x[n] is the message sequence.

A. Indicate TRUE or FALSE
  a. Code rate of Scheme I is 1/4.
  b. Constraint length of Scheme II is 4.
  c. Code rate of Scheme II is equal to code rate of Scheme I.
  d. Constraint length of Scheme I is 4.

Show Answer

B. How many states will there be in the state diagram for Scheme I? For Scheme II?

Show Answer

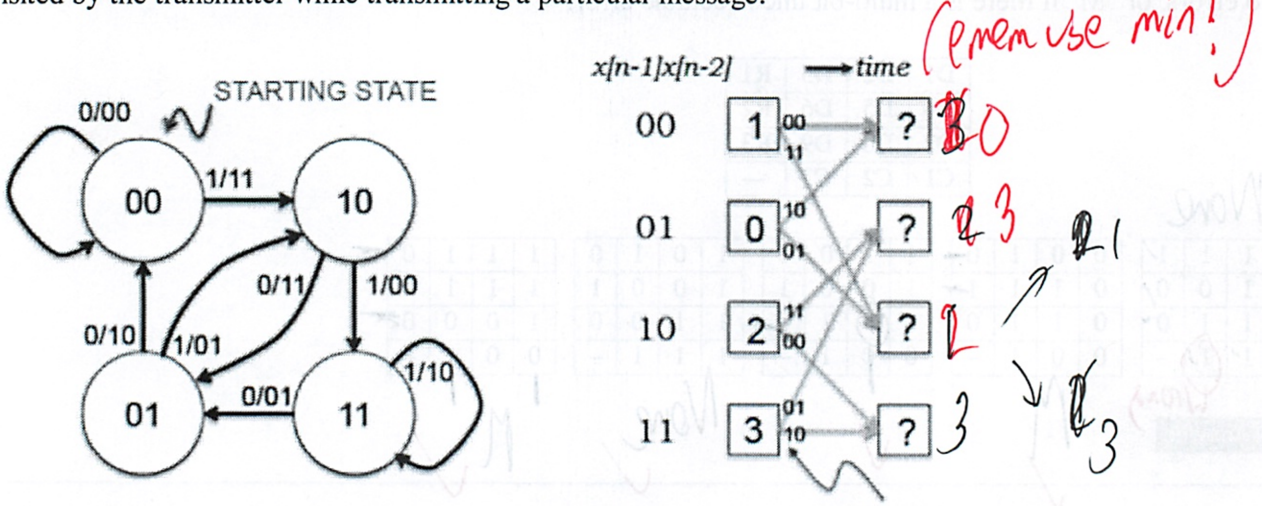
C. Which code will lead to a lower bit error rate? Why?

Show Answer

D. Alyssa P. Hacker suggests a modification to Scheme I which involves adding a third generator polynomial G2 = 1001. What is the code rate `r` of Alyssa's coding scheme? What about constraint length `k`? Alyssa claims that her scheme is stronger than Scheme I. Based on your computations for `r` and `k`, is her statement true?

Show Answer

Problem 10. Consider a convolution code that uses two generator polynomials: G0 = 111 and G1 = 110. You are given a particular snapshot of the decoding trellis used to determine the most likely sequence of states visited by the transmitter while transmitting a particular message:



Received parity bits = 0.6, 0.4

Top arrow for x[n] = 0, else bottom for 1

A. Complete the Viterbi step, i.e., fill in the question marks in the matrix, assuming a hard branch metric based on the Hamming distance between expected an received parity where the received voltages are digitized using a 0.5V threshold.

Show Answer

B. Complete the Viterbi step, i.e., fill in the question marks in the matrix, assuming a soft branch metric based on the square of the Euclidean distance between expected an received parity voltages. Note that your branch and path metrics will not necessarily be integers.

Show Answer

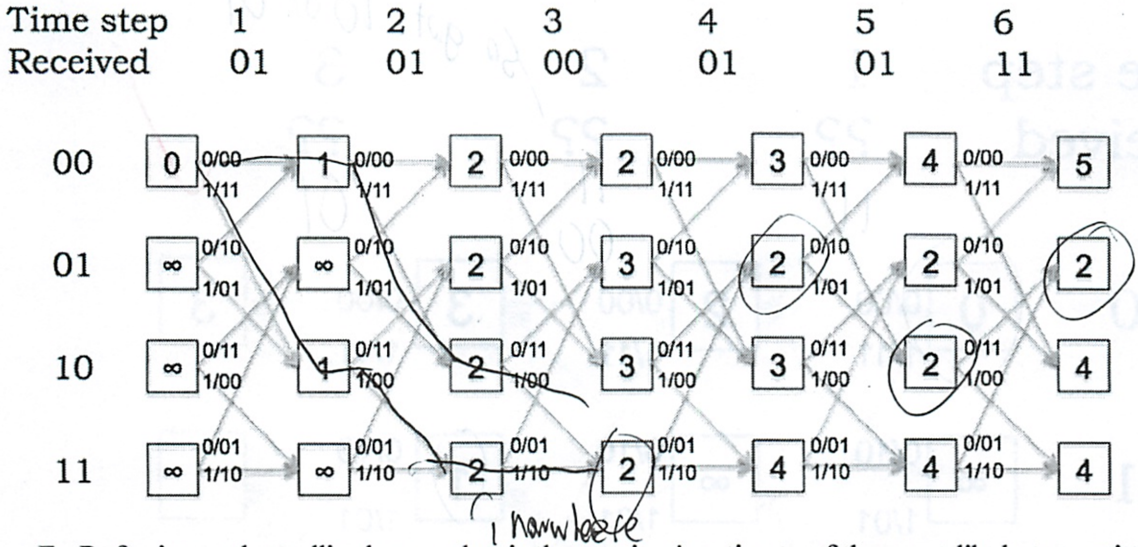
C. Does the soft metric give a different answer than the hard metric? Base your response in terms of the relative ordering of the states in the second column and the survivor paths.

Show Answer

D. If the transmitted message starts with the bits "01011", what is the sequence of bits produced by the convolutional encoder?

Show Answer

The receiver determines the most-likely transmitted message by using the Viterbi algorithm to process the (possibly corrupted) received parity bits. The path metric trellis generated from a particular set of received parity bits is shown below. The boxes in the trellis contain the minimum path metric as computed by the Viterbi algorithm.



E. Referring to the trellis above, what is the receiver's estimate of the most-likely transmitter state after processing the bits received at time step 6? —arbitary

Show Answer

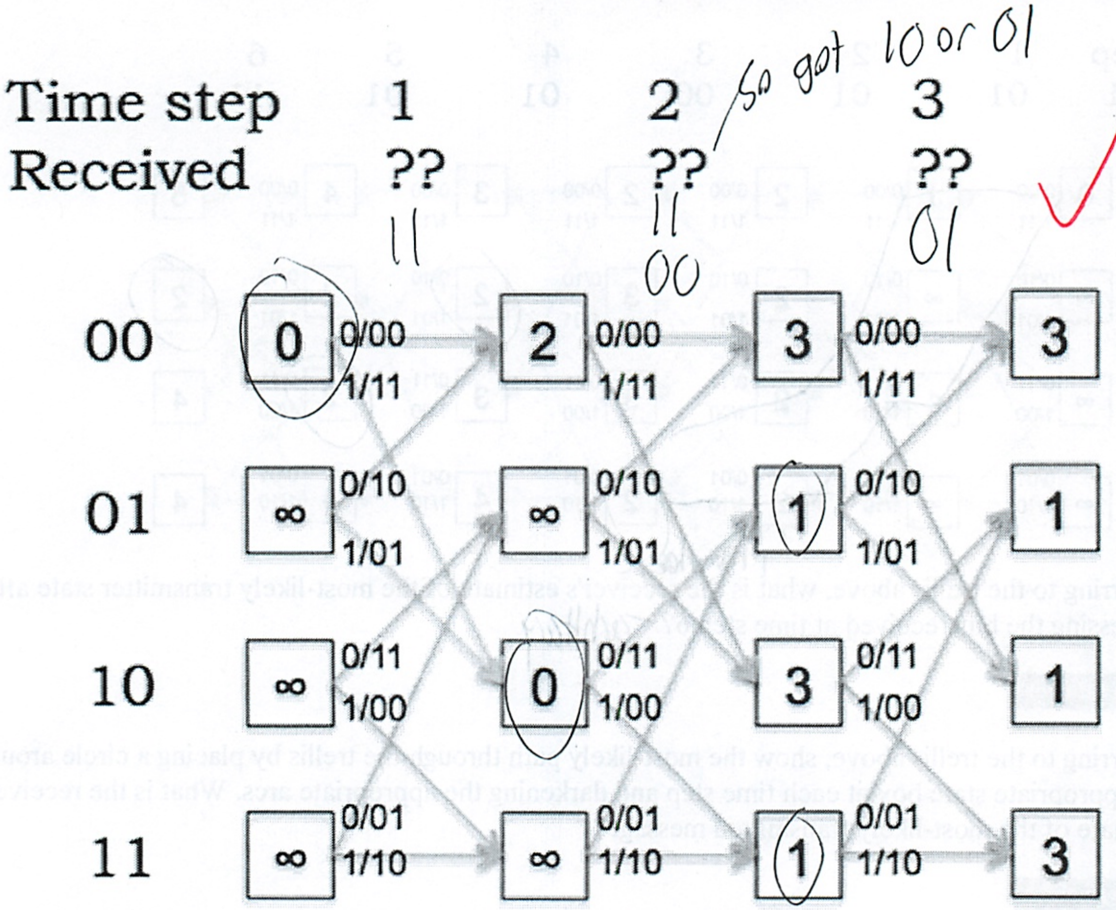
F. Referring to the trellis above, show the most-likely path through the trellis by placing a circle around the appropriate state box at each time step and darkening the appropriate arcs. What is the receiver's estimate of the most-likely transmitted message?

Show Answer

G. Referring to the trellis above, and given the receiver's estimate of the most-likely transmitted message, at what time step(s) were errors detected by the receiver? Briefly explain your reason- ing.

Show Answer

H. Now consider the path metric trellis generated from a *different* set of received parity bits.



Referring to the trellis above, determine which pair(s) of parity bits could have been been received at time steps 1, 2 and 3. Briefly explain your reasoning.

Show Answer

Problem 11. Consider a binary convolutional code specified by the generators (1011, 1101, 1111).

A. What are the values of
   a. constraint length of the code
   b. rate of the code
   c. number of states at each time step of the trellis
   d. number of branches transitioning into each state
   e. number of branches transitioning out of each state
   f. number of expected parity bits on each branch

Show Answer

A 10000-bit message is encoded with the above code and transmitted over a noisy channel. During Viterbi decoding at the receiver, the state 010 had the lowest path metric (a value of 621) in the final time step, and the survivor path from that state was traced back to recover the original message.

B. What is the likely number of bit errors that are corrected by the decoder? How many errors are likely left uncorrected in the decoded message?

Show Answer

C. If you are told that the decoded message had no uncorrected errors, can you guess the approximate number of bit errors that would have occured had the 10000 bit message been transmitted without any coding on the same channel?

Show Answer

D. From knowing the final state of the trellis (010, as given above), can you infer what the last bit of the original message was? What about the last-but-one bit? The last 4 bits?

Show Answer

Consider a transition branch between two states on the trellis that has 000 as the expected set of parity bits. Assume that 0V and 1V are used as the signaling voltages to transmit a 0 and 1 respectively, and 0.5V is used as the digitization threshold.

E. Assuming hard decision decoding, which of the two set of received voltages will be considered more likely to correspond to the expected parity bits on the transition: (0V, 0.501V, 0.501V) or (0V, 0V, 0.9V)? What if one is using soft decision decoding?

Show Answer

Problem 12. Indicate whether each of the statements below is true or false, and a brief reason why you think so.

A. If the number states in the trellis of a convolutional code is S, then the number of survivor paths at any point of time is S. Remember that if there is "tie" between to incoming branches (i.e., they both result in the same path metric), we arbitrarilly choose only one as the predecessor.

Show Answer

B. The path metric of a state s1 in the trellis indicates the number of residual uncorrected errors left along the trellis path from the start state to s1.

Show Answer

C. Among the survivor paths left at any point during the decoding, no two can be leaving the same state at any stage of the trellis.

Show Answer

D. Among the survivor paths left at any point during the decoding, no two can be entering the same state at any stage of the trellis. Remember that if there is "tie" between to incoming branches (i.e., they both result in the same path metric), we arbitrarilly choose only one as the predecessor.

Show Answer

E. For a given state machine of a convolutional code, a particular input message bit stream always produces the same output parity bits.

Show Answer

Problem 13. Consider a convolution code with two generator polynomials: G0=101 and G1=110.

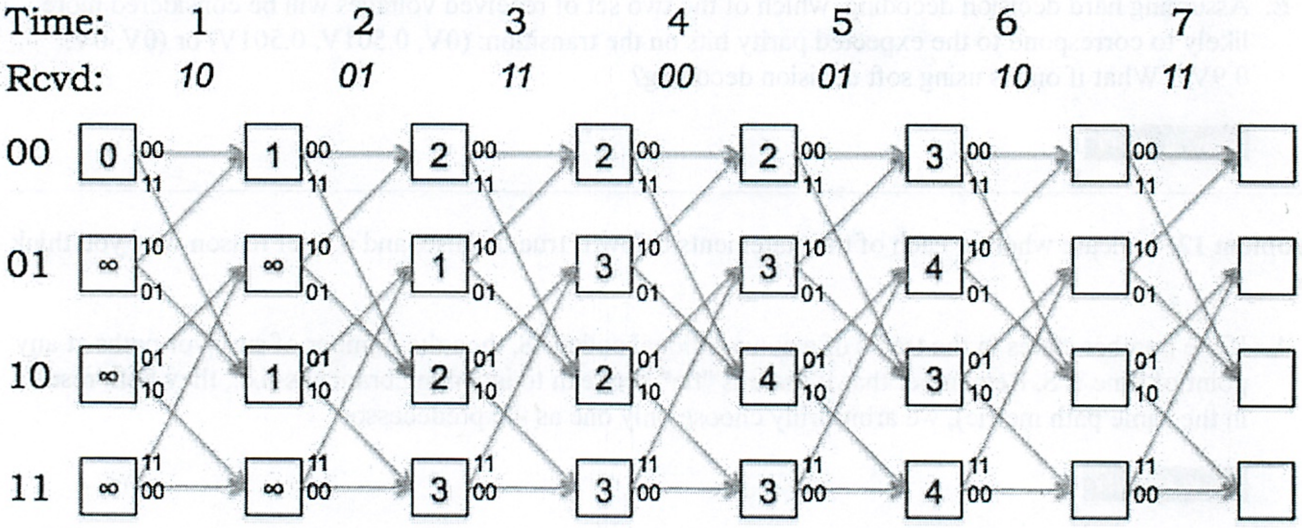
A. What is code rate `r` and constraint length `k` for this code?

Show Answer

B. Draw the state transition diagram for a transmitter that uses this convolutional code. The states should be labeled with the binary string $x_{n-1}\ldots x_{n-k+1}$ and the arcs labeled with x_n/p_0p_1 where `x[n]` is the next message bit and p_0 and p_1 are the two parity bits computed from G0 and G1 respectively.

Show Answer

The figure below is a snapshot of the decoding trellis showing a particular state of a maximum likelihood decoder implemented using the Viterbi algorithm. The labels in the boxes show the path metrics computed for each state after receiving the incoming parity bits at time `t`. The labels on the arcs show the expected parity bits for each transition; the actual received bits at each time are shown above the trellis.



C. Fill in the path metrics in the empty boxes in the diagram above (corresponding to the Viterbi calculations for times 6 and 7).

Show Answer

D. Based on the updated trellis, what is the most-likely final state of the transmitter? How many errors were detected along the most-likely path to the most-likely final state?

Show Answer

E. What's the most-likely path through the trellis (i.e., what's the most-likely sequence of states for the transmitter)? What's the decoded message?

Show Answer

F. Based on your choice of the most-likely path through the trellis, at what times did the errors occur?

Show Answer

Show All Answers Hide All Answers

6.02 Tutorial Problems: MAC protocols

Problem 1.

Which of these statements are true for correctly implemented versions of stabilized unslotted Aloha, stabilized slotted Aloha, and Time Division Multiple Access (TDMA)? Assume that the slotted and unslotted versions of Aloha use the same stabilization method and parameters.

A. When the number of nodes is large, unslotted Aloha has a lower maximum throughput than slotted Aloha.

Show Answer

B. When the number of nodes is large and nodes transmit data according to a Poisson process, there exists *some* offered load for which the throughput of unslotted Aloha is higher than the throughput of slotted Aloha.

Show Answer

C. TDMA has no packet collisions.

Show Answer

D. There exists some offered load pattern for which TDMA has lower throughput than slotted Aloha.

Show Answer

Problem 2.

Binary exponential backoff is a mechanism used in some MAC protocols. Which of the following statements is correct?

A. It ensures that two nodes that experience a collision in a time slot will *never* collide with each other when they each retry that packet.

B. It ensures that two or more nodes that experience a collision in a time slot will experience a lower probability of colliding with each ther when they each retry that packet.

C. It can be used with slotted Aloha but not with carrier sense multiple access.

D. Over short time scales, it improves the fairness of the throughput achieved by different nodes compared to not using the mechanism.

Show Answer

Problem 3.

In the Aloha stabilization protocols we studied, when a node experiences a collision, it decreases its transmission probability, but sets a lower bound, p_min. When it transmits successfully, it increases its transmission probability, but sets an upper bound, p_max.

A. Why would we set a lower bound on p_min that is not too close to 0?

Show Answer

B. Why would we set p_max to be significantly smaller than 1?

Show Answer

C. Let N be the average number of backlogged nodes. What happens if we set p_min >> 1/N?

Show Answer

Problem 4.

Consider a shared medium with N nodes running the slotted Aloha MAC protocol without any backoffs. A "wasted slot" is one in which no node sends data. The fraction of time during which no node uses the medium is the "waste" of the protocol.

A. If the sending probability is p, what is the waste? What are the smallest and largest possible values of the waste?

Show Answer

B. If the Aloha sending probability, p, for each node is picked so as to maximize the utilization, what is the corresponding waste?

Show Answer

Problem 5.

True or false?

Assume that the shared medium has N nodes and they are always backlogged.

A. In a slotted Aloha MAC protocol using binary exponential backoff, the probability of transmission will always eventually converge to some value p, and all nodes will eventually transmit with probability p.

Show Answer

B. Using carrier sense multiple access (CSMA), suppose that a node "hears" that the channel is busy at time slot t. To maximize utilization, the node should not transmit in slot t and instead transmit the packet in the next time slot with probability 1.

Show Answer

C. There is some workload for which an unslotted Aloha with perfect CSMA will not achieve 100% utilization.

Show Answer

Problem 6.

Eight Cell Processor cores are connected together with a shared bus. To simplify bus arbitration, Ben Bittdidle, young IBM engineer, suggested time-domain multiplexing (TDM) as an arbitration mechanism. Using TDM each of the processors is allocated a equal-sized time slots on the common bus in a round-robin fashion. He's been asked to evaluate the proposed scheme on two types of applications: 1) core-to-core streaming, 2) random loads.

1) Core-to-core streaming setup: Assume each core has the same stream bandwidth requirement.

2) Random loads setup: core 1 load = 20%, core 2 load = 30%, core 3 load = 10%, core 4 = 5%, core 5 = 1%, core 6 = 3%, core 7 = 1%, core 8 load = 30%

Help Ben out by evaluating the effectiveness (bus utilization) of TDM under these two traffic scenarios.

Show Answer

Problem 7.

Randomized exponential backoff is a mechanism used to stabilize contention MAC protocols. Which of the following statements is correct?

A. It ensures that two nodes that experience a collision in a time-slot will *never* collide with each other when they each retry that packet.

Show Answer

B. It ensures that two or more nodes that experience a collision n a time-slot will experience a lower probability of colliding with ach other when they each retry that packet.

Show Answer

C. It can be used with slotted Aloha but not with CSMA.

Show Answer

Problem 8.

Three users X, Y and Z use a shared link to connect to the Internet. Only one of X, Y or Z can use the link at a given time. The link has a capacity of 1Mbps. There are two possible strategies for accessing the shared link:

- TDMA: equal slots of 0.1 seconds.
- "Taking turns": adds a latency of 0.05 seconds before taking the turn. The user can then use the link for as long as it has data to send. A user requests the link only when it has data to send.

In each of the following two cases, which strategy would you pick and why?

A. X, Y and Z send a 40KBytes file every 1sec.

Show Answer

B. X sends 80KBytes files every 1sec, while Y and Z send 10KBytes files every 1sec.

Show Answer

Problem 9.

Alyssa P. Hacker is setting up an 8-node broadcast network in her apartment building in which all nodes can hear each other. Nodes send packets of the same size. If packet collisions occur, both packets are corrupted and lost; no other packet losses occur. All nodes generate equal load on average.

Alyssa observes a utilization of 0.5. Which of the following are consistent with the observed utilization?

A. True/False: Four nodes are backlogged on average, and the network is using Slotted Aloha with stabilization, and the fairness is close to 1.

Show Answer

B. True/False: Four nodes are backlogged on average, and the network is using TDMA, and the fairness is close to 1.

Show Answer

Now suppose Alyssa's 8-node network runs the Carrier Sense Multiple Access (CSMA) MAC protocol. The maximum data rate of the network is 10 Megabits/s. Including retries, each node sends traffic according to some unknown random process at an average rate of 1 Megabit/s per node. Alyssa measures the network's utilization and finds that it is 0.75. No packets get dropped in the network except due to collisions.

C. What fraction of packets sent by the nodes (including retries) experience a collision?

Show Answer

Problem 10.

Note: this problem is useful to review how to set up and solve problems related to Aloha-like access *protocols, but the calculations shown in the answer are more complex than we would ask on a quiz.*

Consider a network with four nodes, where each node has a dedicated channel to each other node. The probability that any node transmits is p. Each node can only send OR receive one packet at a time. What is the utilization of the network? Assume each packet takes 1 time slot. Assume each node has 3 queues -- one for each other node -- and that each is backlogged.

Show Answer

Problem 11.

Suppose that there are three nodes seeking access to a shared medium using slotted Aloha, where each packet takes one slot to transmit. Assume that the nodes are always backlogged, and that each has probability p_i of sending a packet in each slot, where i = 1, 2 and 3 indexes the node. Suppose that we assign more the sending probabilities so that

p_1 = 2(p_2) and p_2 = p_3

A. What is the utilization of the shared medium?

Show Answer

B. What are the probabilities that maximize the utilization and the corresponding utilization?

Show Answer

Problem 12.

Suppose that two nodes are seeking access to a shared medium using slotted Aloha with binary exponential backoff subject to maximum and minimum limits of the probability pmax = 0.8 and pmin = 0.1. Suppose that both nodes are backlogged, and at slot n, the probabilities the two nodes transmit packets are p_1 = 0.5 and p_2 = 0.3.

A. What are the possible values of p_1 at slot n+1? What are the probabilities assocated with each possible value?

Show Answer

B. What are the possible values of p_2 at slot n+1? What are the probabilities associated with each possible value?

Show Answer

Problem 13. Bluetooth is a wireless technology found on many mobile devices, including laptops, mobile phones, GPS navigation devices, headsets, and so on. It uses a MAC protocol called Time Division Duplex (TDD). In TDD, the shared medium network has 1 master and N slaves. You may assume that the network has already been configured with one device as the master and the others as slaves. Each slave has a unique identifier (ID) that serves as its address, an integer between 1 and N . Assume that no devices ever turn off during the operation of the protocol. Unless otherwise mentioned, assume that no packets are lost.

The MAC protocol works as follows. Time is slotted and each packet is one time slot long.

In every *odd* time slot (1, 3, 5, …, 2t–1, …), the master sends a packet addressed to some slave for which it has packets backlogged, in round-robin order (i.e., cycling through the slaves in numeric order).

In every *even* time slot (2, 4, 6, …, 2t, …), the slave that received a packet from the master in the immediately preceding time slot gets to send a packet to the master, if it has a packet to send. If it has no packet to send, then that time slot is left unused, and the slot is wasted.

A. Alyssa P. Hacker finds a problem with the TDD protocol described above, and implements the following rule in addition:

> From time to time, in an odd time slot, the master sends a "dummy" packet addressed to a slave even if it has no other data packets to send to the slave (and even if it has packets for other slaves).

Why does Alyssa's rule improve the TDD protocol?

Show Answer

Henceforth, the term "TDD" will refer to the protocol described above, augmented with Alyssa's rule. Moreover, whenever a "dummy" packet is sent, that time slot will be considered a wasted slot.

B. Alyssa's goal is to emulate a round-robin TDMA scheme amongst the N slaves. Propose a way to achieve this goal by specifying the ID of the slave that the master should send a data or dummy packet to, in time slot 2t–1 (note that $1 \le t \le \infty$).

Show Answer

Henceforth, assume that the TDD scheme implements round-robin TDMA amongst the slaves. Suppose the master always has data packets to send only to an arbitrary (but fixed) subset of the N slaves. In addition, a (possibly different) subset of the slaves always has packets to send to the master. Each subset is of size r, a fixed value. Answer the questions below (you may find it helpful to think about different subsets of slaves).

C. What is the *maximum possible* utilization of such a configuration?

Show Answer

D. What is the *minimum possible* utilization (for a given value of r) of such a configuration? Assume that r > N/2. Note that if the master does not have a data packet to send to a slave in a round, it sends a "dummy" packet to that slave instead. A dummy packet does not count toward the utilization of the medium.

Show Answer

Problem 14.

Recall the MAC protocol with contention windows. Here, each node maintains a contention window, W, and sends a packet t idle time slots after the current slot, where t is an integer picked uniformly in [1, W]. Assume that each packet is 1 slot long.

Suppose there are two backlogged nodes in the network with contention windows W1 and W2, respectively. Assume that both nodes pick randomly from [1, W1] and [1, W2] *at the current time* and that W1 ≥ W2. What is the probability that the two nodes will collide the next time they each transmit?

Show Answer

Problem 15.

Eager B. Eaver gets a new computer with two radios. There are N other devices on the shared medium network to which he connects, but each of the other devices has only one radio. The MAC protocol is slotted Aloha with a packet size equal to 1 time slot. Each device uses a fixed transmission probability, and only one packet can be sent successfully in any time slot. All devices are backlogged.

Eager persuades you that because he has paid for two radios, his computer has a moral right to get twice the throughput of any other device in the network. You begrudgingly agree. Eager develops two protocols:

Protocol A: Each radio on Eager's computer runs its MAC protocol independently. That is, each radio sends a packet with fixed probability p. Each other device on the network sends a packet with probability p as well.

Protocol B: Eager's computer runs a single MAC protocol across its two radios, sending packets with probability 2p, and alternating transmissions between the two radios. Each other device on the network sends a packet with probability p.

A. With which protocol, A or B, will Eager achieve higher throughput?

   Show Answer

B. Which of the two protocols would you allow Eager to use on the network so that his expected throughput is double any other device's?

   Show Answer

Show All Answers Hide All Answers

6.02 Tutorial Problems: Frequency Domain & Filters

Problem 1.

Give an expression for the magnitude of a complex exponential with frequency φ, i.e., $|e^{j\varphi}|$. Hint: it's a numeric value independent of φ.

Show Answer

Problem 2.

A. Prove the validity of the following formula, often referred to as the *finite sum formala*:

$$\sum_{n=0}^{N-1} \alpha^n = \begin{cases} N & \alpha = 1 \\ \dfrac{1-\alpha^N}{1-\alpha} & \text{for any complex number } \alpha \neq 1 \end{cases}$$

Show Answer

B. Use the finite sum formula to show:

$$\sum_{n=\langle N \rangle} e^{jk\frac{2\pi}{N}n} = \begin{cases} N & k = 0, \pm N, \pm 2N, \ldots \\ 0 & \text{otherwise} \end{cases}$$

Show Answer

Problem 3.

Consider the following signals, which are periodic with period `N`. For each signal compute the spectral coefficients a_k. In 6.02, we usually choose `N` consecutive `k`'s starting with `-N/2`.

A. `x[n] = 1 + sin((2π/N)*n) + 3*cos((2π/N)*n) + cos(2*(2π/N)*n + π/2).`

Show Answer

B. `x[n] = 5*cos(6πn + π) + 7*cos(3πn)`

Show Answer

C. `x[n]` is a square wave with period N=4 with the following values: `x[0]=1, x[1]=1, x[2]=0, x[3]=0.`

Show Answer

D. $x[n] = \cos(2\pi n/3)*\sin(2\pi n/9)$

Show Answer

E. $x[n]$ has exactly one non-zero value per period, i.e., $x[m] \neq 0$ for some m and 0 otherwise. Compute the *magnitude* of a_k.

Show Answer

Problem 4.

If $x[n]$ is real, *even* (i.e., $x[n] = x[-n]$) and periodic with period N, show that all the a_k are real.

Show Answer

Problem 5.

Suppose you're given the spectral coefficients a_k for a particular periodic sequence $x[n]$. Compute the spectral coefficients b_k for $w[n] = x[n-\alpha]$, i.e., x time shifted by α samples, in terms of the a_k.

Show Answer

Problem 6.

Consider an LTI system characterized by the unit-sample response $h[n]$.

A. Give an expression for the frequency response of the system $H(e^{j\Omega})$ in terms of $h[n]$.

Show Answer

B. If $h[0]=1$, $h[1]=0$, $h[2]=1$, and $h[n]=0$ for all other n, what is $H(e^{j\Omega})$?

Show Answer

C. Let $h[n]$ be defined as in part B and $x[n] = \cos(\varphi n)$. Is there a value of φ such that $y[n]=0$ for all n and $0 \leq \varphi \leq \pi$?

Show Answer

D. Let $h[n]$ be defined as in part B. Find the maximum magnitude of $y[n]$ if $x[n] = \cos(\pi n/4)$.

Show Answer

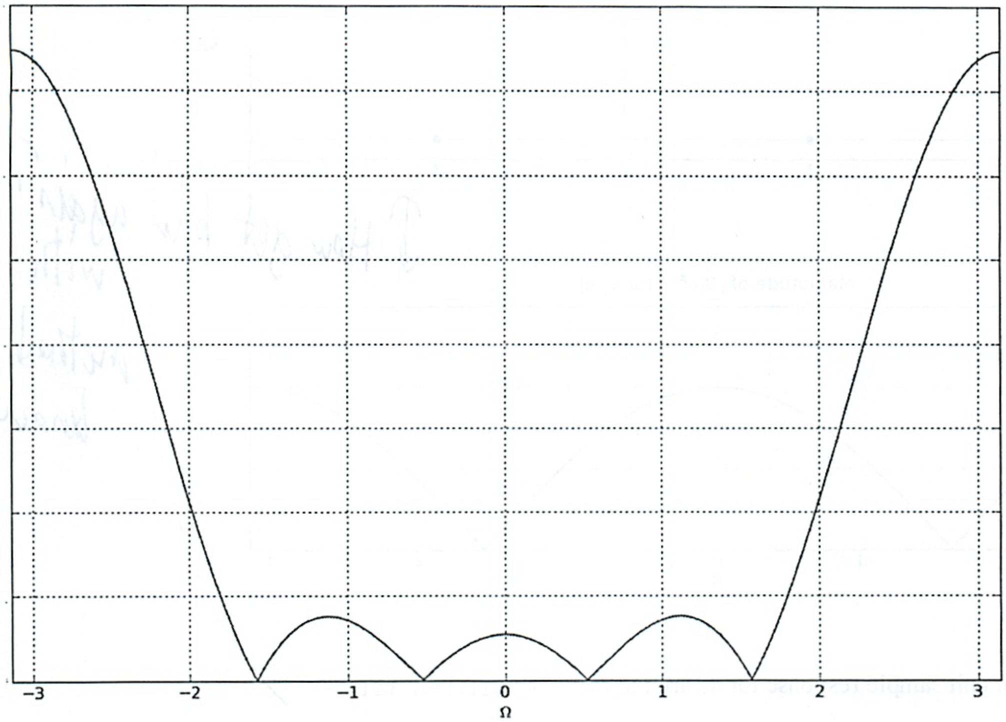
E. Let $h[n]$ be defined as in part B. Find the maximum magnitude of $y[n]$ if $x[n] = \cos(-(\pi/2)n)$.

Show Answer

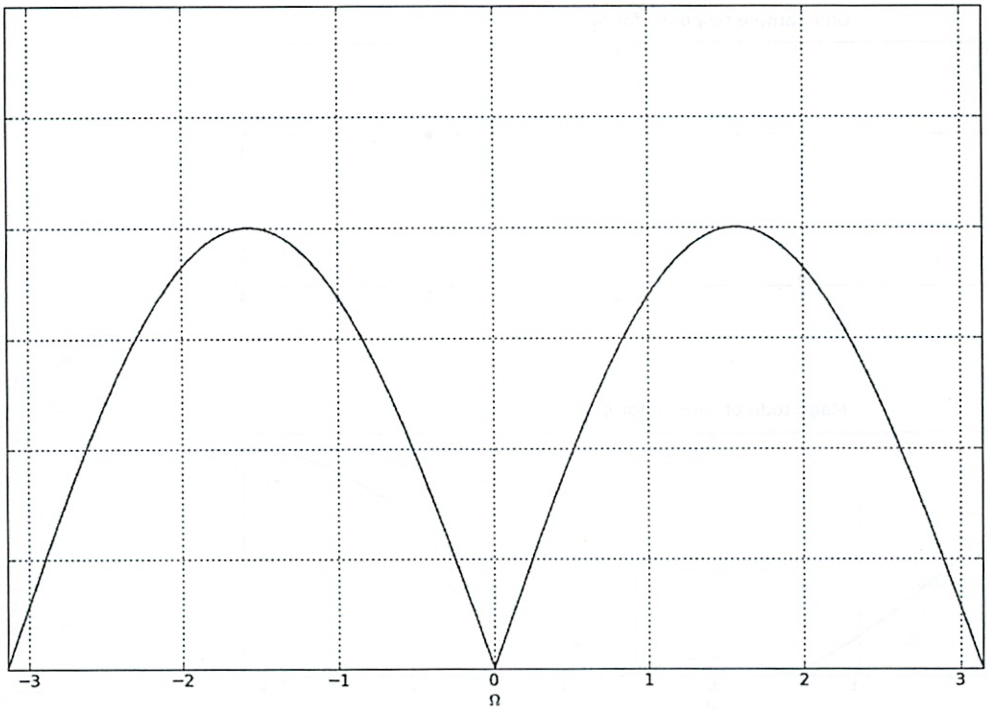
Problem 7.

In answering the questions below, please consider the unit sample response and frequency response of two filters, H_1 and H_2, In answering the several parts of this review question consider four linear time-invariant systems, denoted A, B, C, and D, each characterized by the magnitude of its frequency response, $|H_A(e^{j\Omega})|$, $|H_B(e^{j\Omega})|$, $|H_C(e^{j\Omega})|$, and $|H_D(e^{j\Omega})|$ respectively, as given in the plots below. This is a review problem, not an actual exam question, so similar concepts are tested multiple times to give you practice

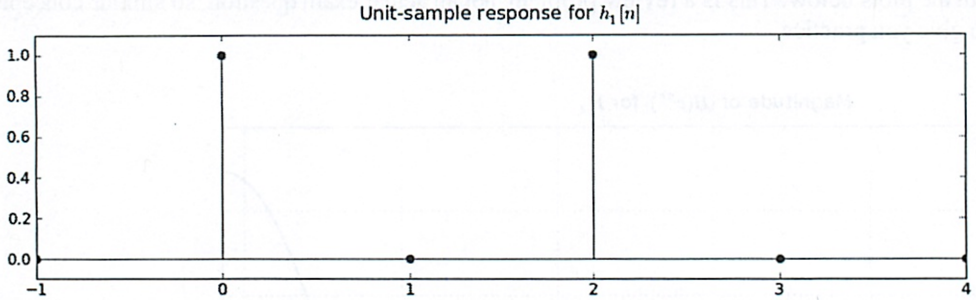
Magnitude of $|H(e^{j\Omega})|$ for H_A

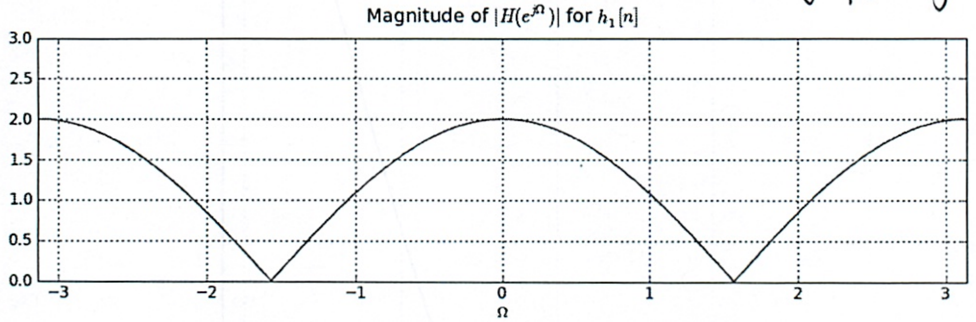


Magnitude of $|H(e^{j\Omega})|$ for H_B



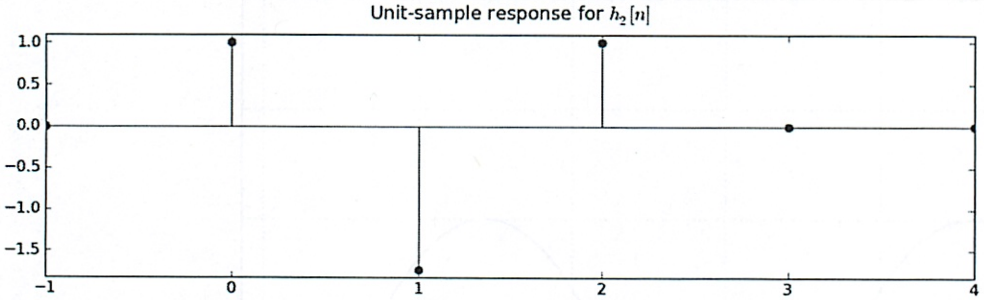
plotted below.

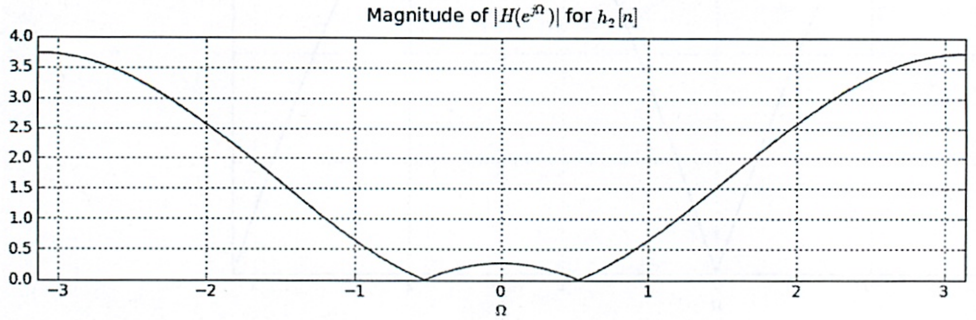




How get b/w again? -with the method I know

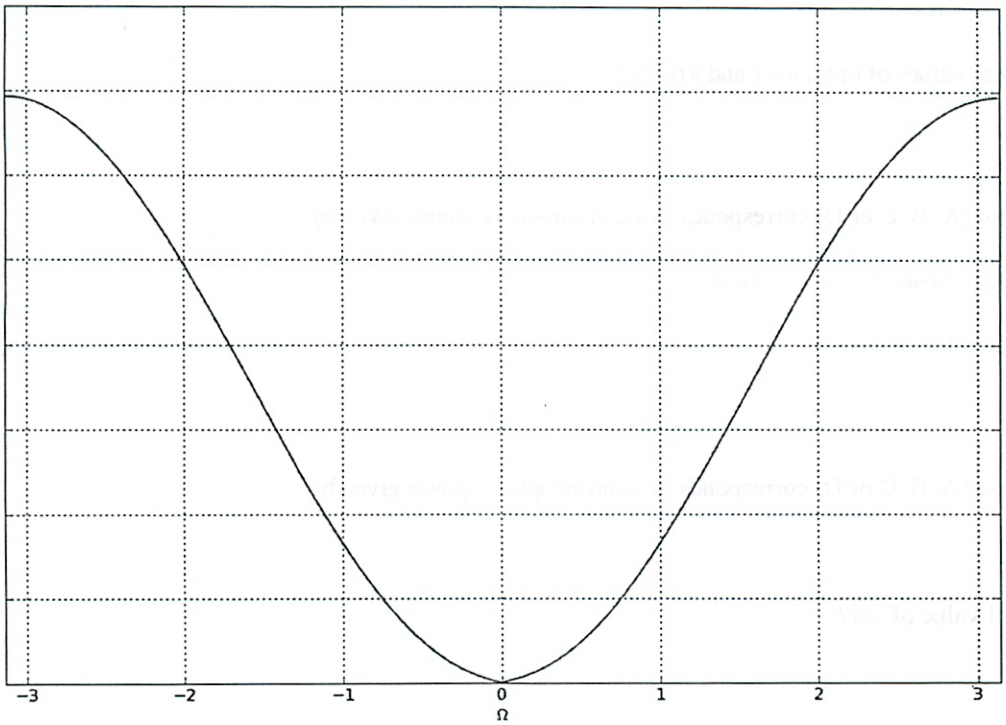
Note: the only nonzero values of unit sample response for `H1` are : `h1[0] = 1, h1[1]=0, h1[2]=1.`



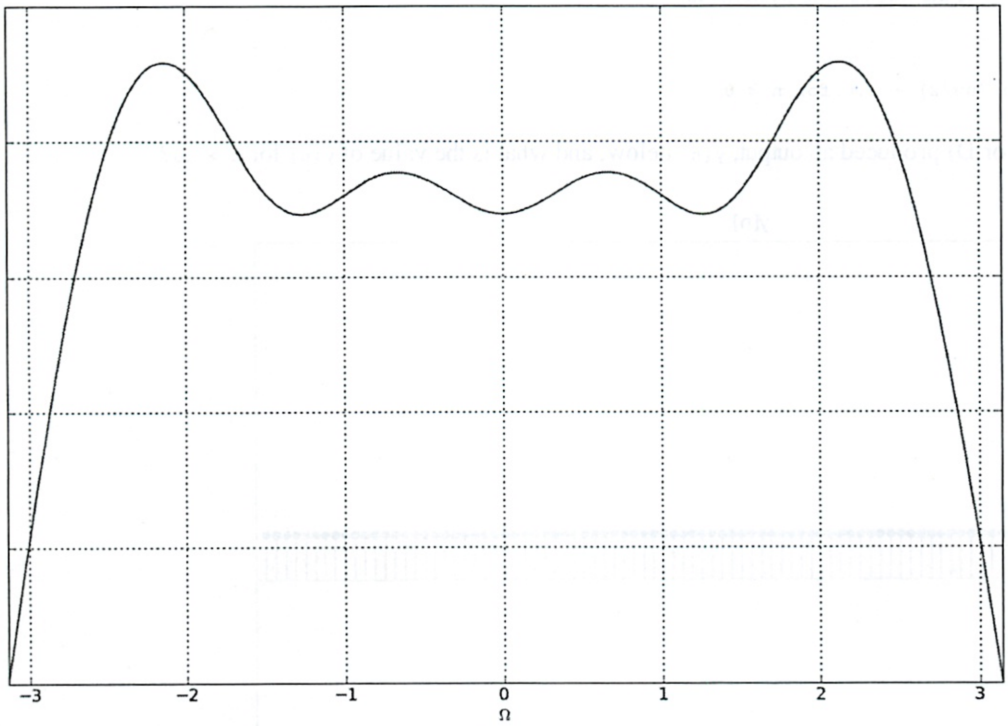


Note, the only nonzero values of unit sample response for `H2` are : `h2[0] = 1, h2[1]=-sqrt(3), h2[2]=1.`

Magnitude of $|H(e^{j\Omega})|$ for H_C



Magnitude of $|H(e^{j\Omega})|$ for H_D



A. Which frequency response (A, B, C or D) corresponds to a unit sample response given by

$h[n] = \alpha\ \delta[n] - h_1[n]$

and what is the numerical value of $|\alpha|$?

Show Answer

B. Which frequency response (A, B, C or D) corresponds to a unit sample response given by

$h[n] = \Sigma_m h_1[m] h_2[n-m]$ for $m = 0$ to n

and what are the numerical values of $h[2]$, $h[3]$ and $H(e^{j0})$?

Show Answer

C. Which frequency response (A, B, C or D) corresponds to a unit sample response given by

$h[n] = \alpha\ \delta[n] - \Sigma_m h_1[m] h_2[n-m]$ for $m = 0$ to n

and what is the numerical value of $|\alpha|$?

Show Answer

D. Which frequency response (A, B, C or D) corresponds to a unit sample response given by

$h[n] = \alpha\ \delta[n] - h_2[n]$

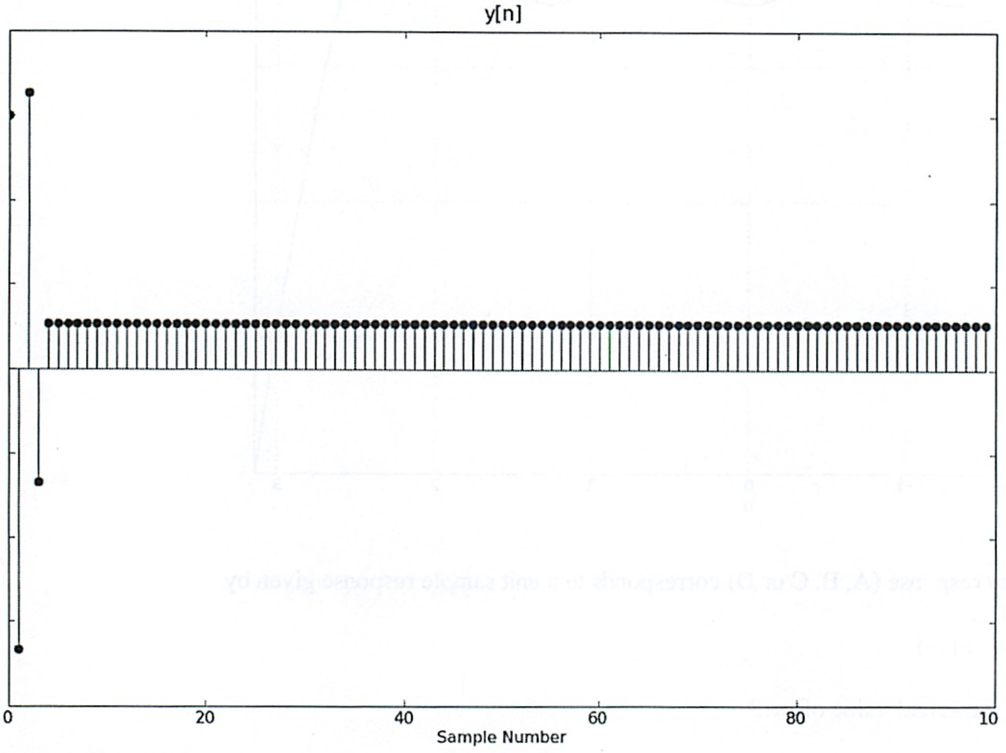
and what is the numerical value of $|\alpha|$?

Show Answer

E. Suppose the input to each of the above four systems is

$x[n]=0$ for $n < 0$ and
$x[n] = \cos(n\pi/6) + \cos(n\pi/2) + 1.0$ for $n \geq 0$

Which system (A, B, C or D) produced an output, $y[n]$ below, and what is the value of $y[n]$ for $n > 10$?

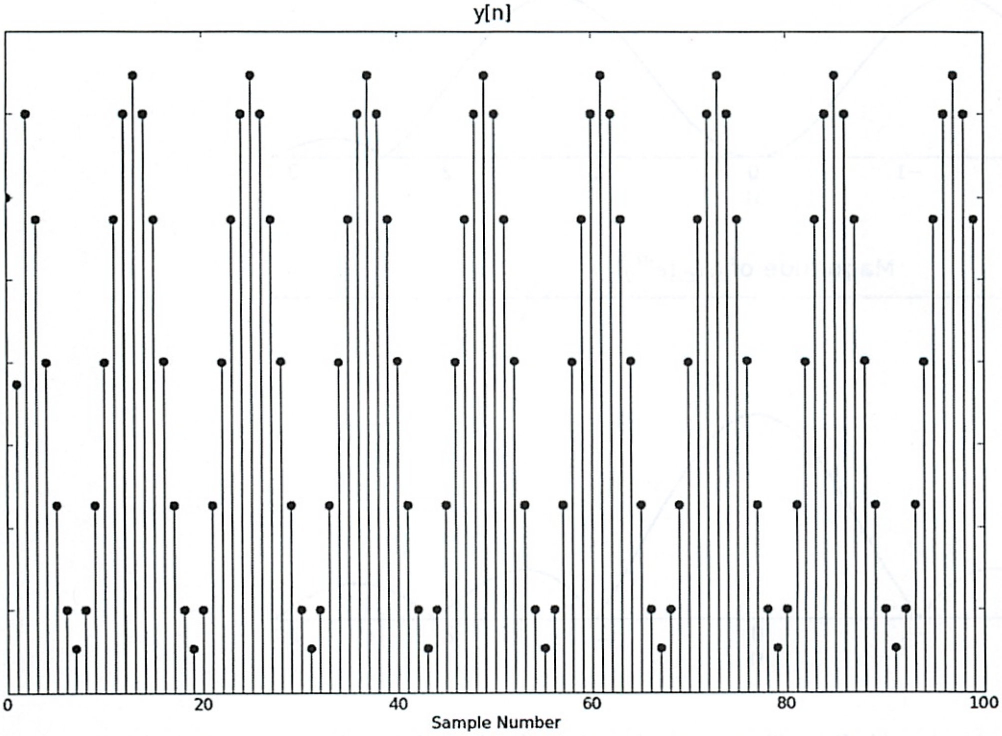


Show Answer

F. Suppose the input to each of the above four systems is

`x[n]=0 for n < 0` and
`x[n] = cos(nπ/6) + cos(nπ/2) + 1.0 for n ≥ 0`

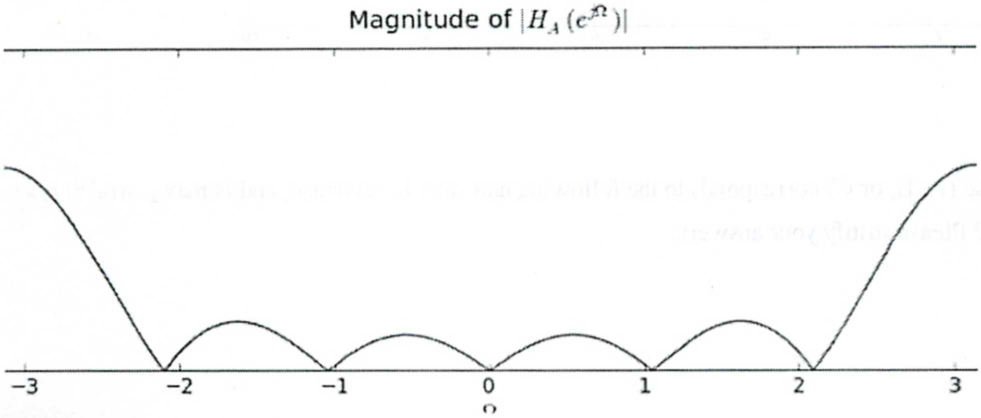
Which system (`H1` or `H2`) produced an output, `y[n]` below, and what is the value of `y[22]`?

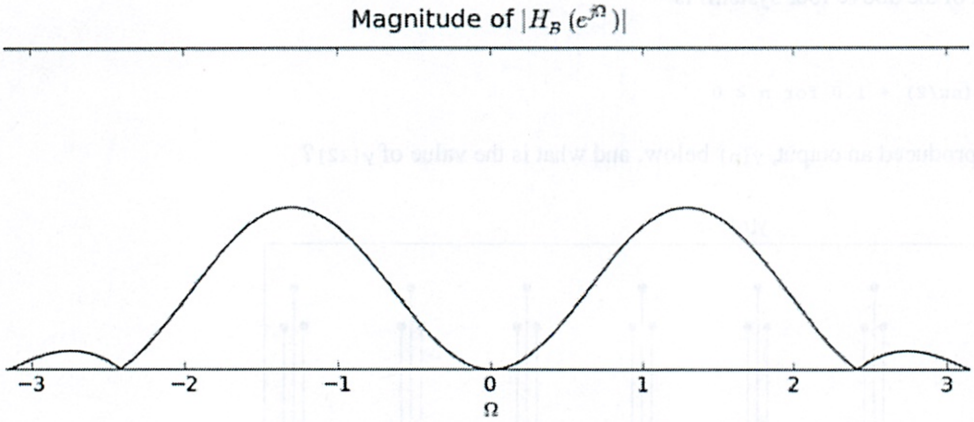


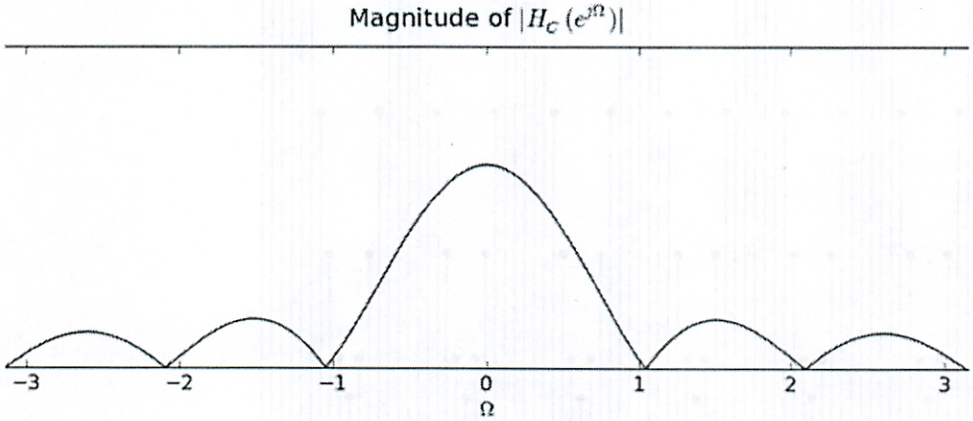
Show Answer

Problem 8.

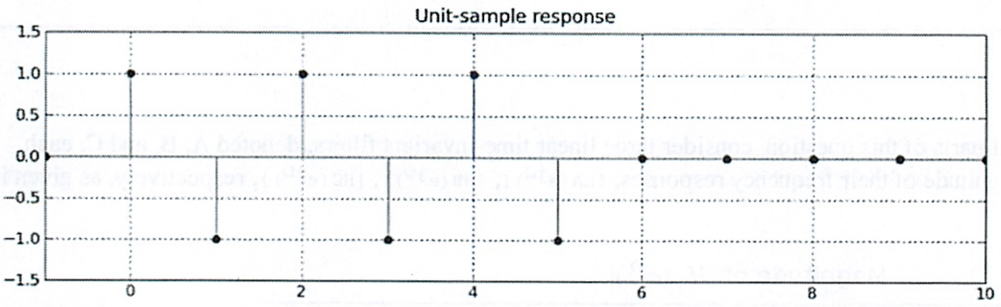
In answering the several parts of this question, consider three linear time-invariant filters, denoted A, B, and C, each characterized by the magnitude of their frequency responses, `|HA(ejΩ)|`, `|HB(ejΩ)|`, `|HC(ejΩ)|`, respectively, as given in the plots below.





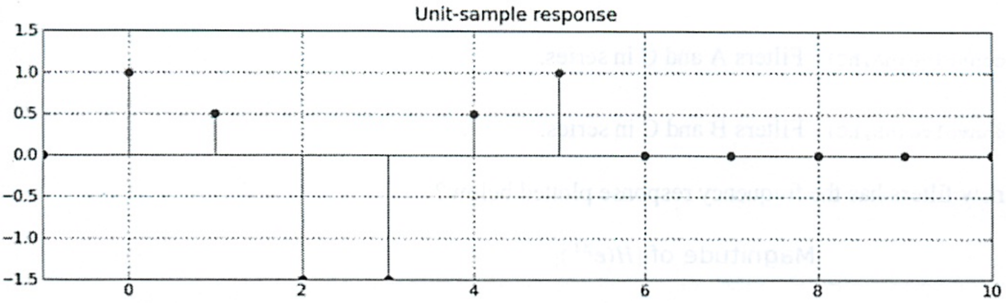


A. Which frequency response (A, B, or C) corresponds to the following unit sample response, and what is $\max_\Omega$ $|H(e^{j\Omega})|$ for your selected filter? Please justify your selection.



Show Answer

B. Which frequency response (A, B, or C) corresponds to the following unit sample response, and is $\max_\Omega$ $|H(e^{j\Omega})| > 6$ for your selected filter? Please justify your answers.

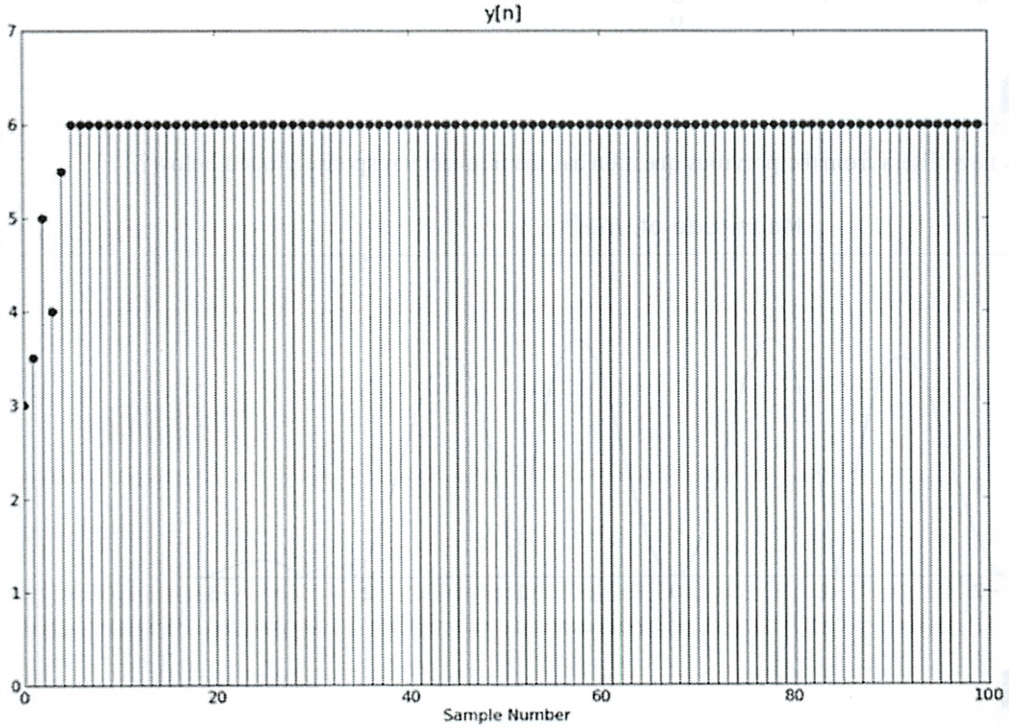


Show Answer

C. Suppose the input to each of the above three filters is `x[n] = 0` for `n < 0` and for `n ≥ 0` is

`x[n] = cos((π/3)n) + cos(πn) + 1.0`

Which filter (A, B,or C) produced the output, `y[n]` below, and what is $\max_\Omega |H(e^{j\Omega})|$ for your selected system?



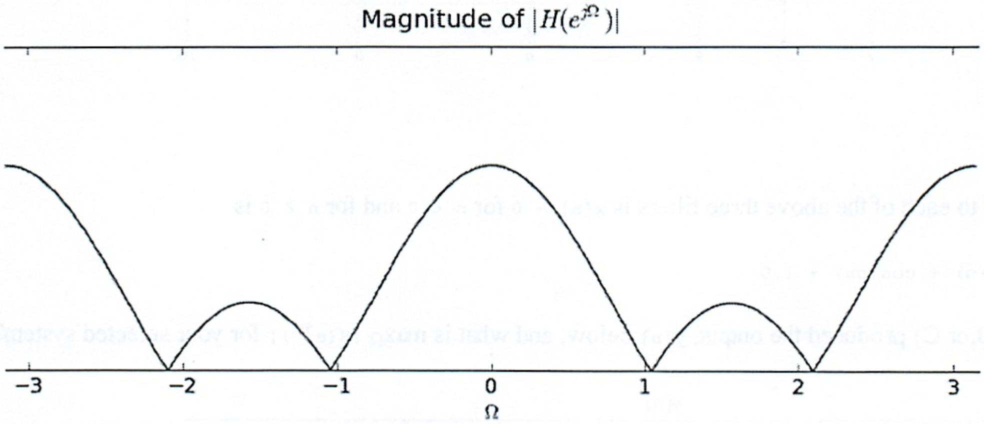
Show Answer

D. Six new filters were generated using the unit sample responses of filters A, B and C, denoted `hA[n]`, `hB[n]`, and `hC[n]` respectively. The unit sample responses of the new filters were gener- ated in the following way:

- ○ `h1[n] = hA[n] + hB[n]`. Filters A and B in parallel.
- ○ `h2[n] = hA[n] + hC[n]`. Filters A and C in parallel.
- ○ `h3[n] = hB[n] + hD[n]`. Filters B and C in parallel.
- ○ `h4[n] = convolve(hA,hB)`. Filters A and B in series.

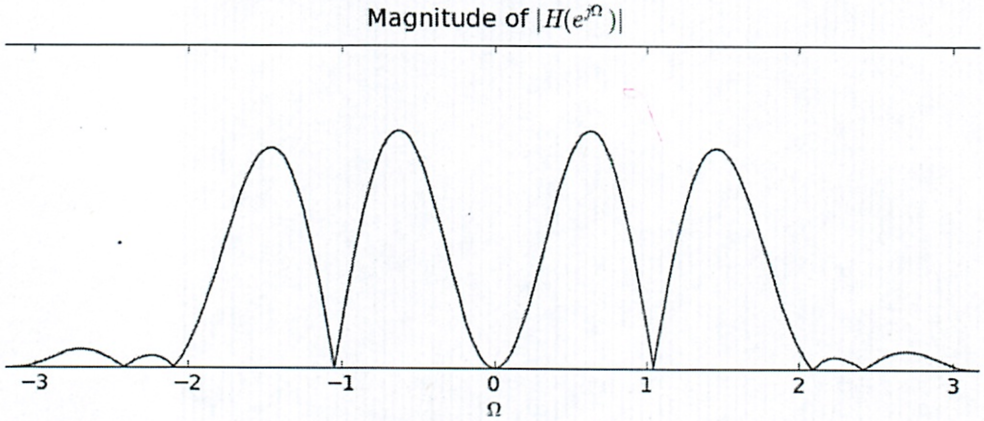
- ○ `h5[n] = convolve(hA,hC)`. Filters A and C in series.
- ○ `h6[n] = convolve(hB,hC)`. Filters B and C in series.

Which of the six new filters has the frequency response plotted below?



Show Answer

Which of the six new filters from the previous question has the frequency response plotted below?



Show Answer

6.02 Quiz 2 Review

4/11

Noise + Bit errors

1.a. V + noise $\qquad$ noise = $N(0, \sigma_{Noise})$

0 + noise

Give BER expression

$$BER = P(\text{noise} > V/2 \mid 0)\, P(0) + P(\text{noise} < \tfrac{V}{2} \mid 1)\, P(1)$$

$$= \phi\left(-\frac{1}{2\sigma}\right) \cdot \frac{1}{2} + \frac{1}{2}\left(\phi\left(-\frac{1}{2\sigma}\right)\right)$$

$$= \phi\left(-\frac{1}{2\sigma}\right)$$

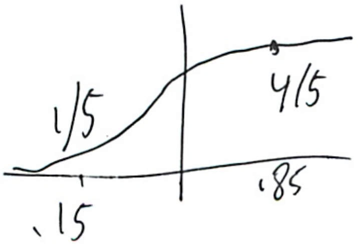
b. $1/5 = \phi\left(-\frac{V_{th}}{2\sigma}\right)$ ← if not at 1

? how get by hand - solve for σ

What is BER? $\frac{\#\text{errors}}{\#\text{bits}}$?

But where does $\phi(.85) = 4/5$ come in

Unless its a match



$$.15 = -\frac{1}{2\sigma}$$

②

$-.3 = \sigma$

↑ but cant be −

$BER = 1 - \Phi\left(\frac{.5}{\sigma}\right)$

if want $BER = .2$

$\frac{1}{5} = 1 - \Phi\left(\frac{.5}{\sigma_{noise}}\right)$ ← I forgot the "1−" this step]

$\Phi\left(\frac{1-.5}{\sigma}\right) = 1 - \Phi\left(\frac{.5}{\sigma}\right)$

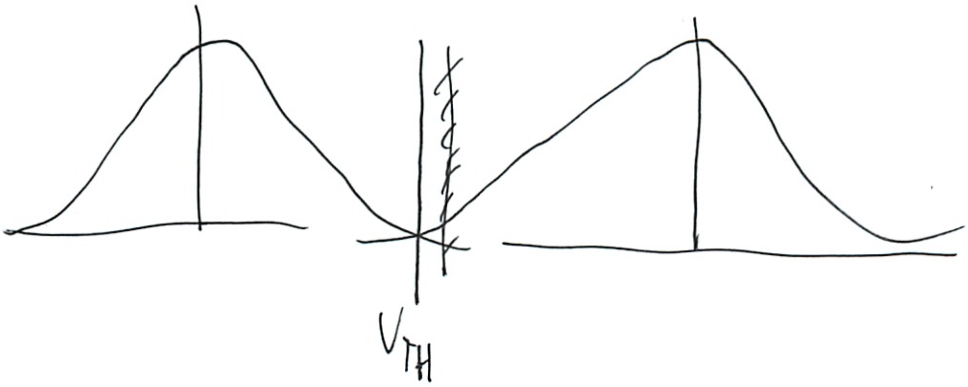
b)

So $\Phi\left(\frac{.5}{\sigma}\right) = 4/5$

So $.85 = \frac{.5}{\sigma}$

So $\sigma = .588$

c) ~~No~~ − Yes



Shifting ↑ prob of bit error at σ

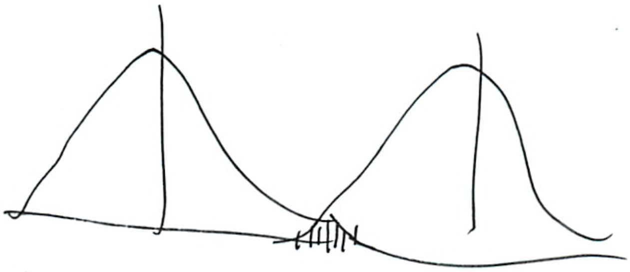
So if prob bit error fixed, σ will ↓ ✓

d) ~~Yes~~ Formula $P(error|0)\,P(0) + P(error|1)\,P(1)$

↑ same .33 ↑ same .66

actually still adds to same ✓ −no

4a) Isn't that fixed they said?
I guess that varies while everything else is fixed



noise + ISI

a yes move closer
b yes wider ✓
c yes where line drawn
d no - this is just on samples level
- later it will mater

b.) a) True? only if $\alpha = \beta$
b) α & β fixed, so ~~does~~ same as a ✓
c) No
d) No True review
e) Yes No

c) Increases - but by how much?

④

3.a : 1.25 : t

I guess they want you to do geometry

$$\frac{1}{2} \cdot .5 \cdot h + \frac{1}{2} \cdot 1 \cdot h = 1$$

$$\frac{3}{4}h = 1$$

$$\cdot \frac{4}{3}$$

$$h = \frac{4}{3} = 1.33$$ ✓

b) Same

0 3x likly

$a =$ ~~0 trans~~ $P(0\ \text{trans})$
$b =$ ~~1 trans~~ $P(1\ \text{trans})$

$$a = 3b$$

$$a + b = 1$$

$$a = 3(1-a)$$

$$a = 3 - 3a$$ ← the fancy algebra way!
- after 115 would not have been able to do

$$4a = 3$$

$$a = \frac{3}{4}$$ ✓

$$b = \frac{1}{4}$$ ✓

⑤

c) Signal + noise?

$P[>0 \mid 0 \text{ trans}]P(0) + P[<0 \mid 1 \text{ trans}]P(1)$

So to 0 look at PDF

$\frac{1}{2} \cdot .5 \cdot \_\_\_$

So need the blank

$\frac{1.33}{1} = \frac{4/6}{.5}$ $= 1/3$

$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{\cancel{3}\,2} = \cancel{\frac{1}{12}}$ $\boxed{\frac{1}{8}}$

Look at 1 - assume noise just translated

- to error prob here

So all together

$\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$ = BER

They did something weird - adjusted it down

⑥

d) What would minimize error rate

- think about - don't try to "solve"

Somewhere b/n 0, .5

Perhaps .25

That would be

$$\frac{1.33}{1} = \frac{4/3/4}{.25} \rightarrow \frac{4}{3}\cdot\frac{1}{4} = \frac{4}{12} = \frac{1}{3}$$

$$\frac{1.33}{\cancel{33}\ .5} = \frac{4/3/2}{.25} \rightarrow \frac{4}{3}\cdot\frac{1}{2} = \frac{4}{6} = \frac{2}{3}$$

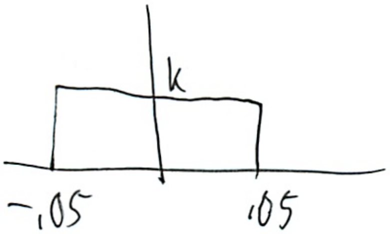
$$\frac{3}{4}\cdot\frac{1}{3} + \frac{1}{4}\cdot\frac{2}{3} = \frac{1}{4} + \frac{1}{6} = \frac{10}{24} = \frac{5}{12} = BER$$

Guess could generalize + "solve"

$x = .3V$

⑦

4. PDF noise is constant



a) $k = ?$

10 ✓

b) Now $V_{high} = 1$ Noise Normal$(0, \sigma)$

If $\sigma = .001$ what is prop $\frac{1}{3} \leq y < \frac{2}{3}$

~~Almost 0~~

No 01 is $\frac{1}{3}$

So its (half of the normal + half normal) $\frac{1}{2}$

$$\frac{1}{2}\phi\left(\frac{1/2}{.001}\right)$$

$\frac{1}{4}$ I don't think this is right

⑧

d) If $\sigma = .1$

$N_0 - V_{Th} = .5$ – best place

So 01 more likly to be an error ✓

---

5. a) colors would be nice

$V_{TH} = \frac{1}{2}$

= prob

Yes eye still wide enough

if noise < .3 V

– just look at diagram

b) So now noise added

How is BER change?

– problem same as before or very similar

– ~~Skipping2 sect2~~ – ~~do~~ how does it change w/ multiple?

= oh C is where this is asked

– just add up ϕ – check the liklihoods

01	10	← think its only 2
11	00	

So then is =ly likly

But is it 010 ?

etc

⑩ ECCs

1. Give (n, k, d), rate
- But what exactly is transmitted here
  - These are codewords
    - but for what

n = # bits in codeword

k = # message bits trans. by each codeword

d = min hamming dist

a) {111, 100, 001, 010}

So n = 3 since len 3 ✓

k = ~~4~~ 2 since 4 of them ~~…~~ $2^2=4$

d = 2 hamming d of them

$r = \cancel{\frac{1}{3}}$ but $\frac{k}{n} = \cancel{\frac{4}{3}}\ \frac{2}{3}$

what is right?

- Just code word sent ↑ just look at

b) {00000, 01111, 10100, 11011}

- don't really get what its asking

n = 5

k = ~~4~~ 2 since 4 codewords

~~$r = \frac{1}{5}$ but → $\frac{k}{n} = \frac{4}{5}$~~ $\frac{2}{5}$

d = 2

⑪

All ( ) and what in all world is this

trick

$n = 5$

$k = 0$ rate $= 0$

$d = $ und.

no useful info transmitted

This was a stupid qu

2. [illegible] 20 bit data blocks in $(n, 20, 3)$
what is min n so $d = 3$

$n \leq 2^{n-k} - 1$ is it? ✓

min so

$n = 2^{n-20} - 1$

$n + 1 = 2^{n-20}$

$n = 22$
$23 = 2^3$ ⊗

$n = 23$
$24 = 2^4$ x

$n = 25$
$26 = 2^5$ ⊗
32

← should work

guess math error

but no d here!

n does not matter

oh the = does not apply
- its just when it goes over the threshhold

⑫ Still a bit shaky on n,k,d thing

3. Registrar wants to encode class year w/ SEC
So hamming distance of 2 or 3
-well as much as possible

00000 ✓

~~11111~~ don't do this

00111 ✓ $d = \cancel{2}\ 3$ ← can get 3

-but how do we know this must be 3?

11000

11011

11100

4. For any block code w/ $d \leq 2t+1$ show

$$2^{n-k} \geq 1 + \binom{n}{1} + \binom{n}{2} + \dots \binom{n}{t}$$

? How would you show this?
We never did this

(n,k) has at most 2^{n-k} patterns - must cover all error cases we wish to cover as well as no errors. With min $d = 2t+1$ can correct up to t errors. So can exp $0, 1, 2, \dots, t$ errors as $1 + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{t}$ ← can't $> 2^{n-k}$

5. Pair wise

$P_1 = D_1 + D_2$

$P_2 = D_2 + D_3$

$P_3 = P_3 + D_1$

a) n = But what is sent?

$D_1 \ D_2 \ D_3 \ P_1 \ P_2 \ P_3$

k = 3 ✓

n = 6 ✓

d = ← ? How to find again? list 8 possible code words + look

?? 2 b/c 1 ~~wrong correctable~~

2 " not "

3 ← Actually look

b) Reciever

$E_1 = D_1 + D_2 + P_1$

$E_2 = D_2 + D_3 + P_2$

$E_3 = D_3 + D_1 + P_3$

What is max liklyhood

a) No error ✓

b) E_2 is wrong

D_2 D_3 P_2

↑ likly ✓

c) E1 E2 E3 = 1 01

↑ D1 D2 P1 ↑ D3 D1 P3

↑ likly ✓

d) E1 E2 E3 = 111

Multipl errors ✓

6, Want a (20,16) linear block code

What is linear again?

-not on my cheatsheet lots of codes are linear?

20 = n 16 = k

What does it mean to be single error correctable?

↑ must be able to uniqely identify n+1 patterns

-n error patterns + 1 w/o errors.

So $2^{n-k} \geq n+1$. Not true w/ n = 20 k=16 ← do the check 1st

⑮

8) So $1+1=0$

$1+1+1=1$

$1=1$

$0=0$

On paper ✓

9) Conv scheme

~~[illegible]~~ Scheme 1 ; $G0 = 1101$ $G1 = 1110$

2 $G0 = 110101$ $G1 = 111011$

So notation 1101 means parity for n is

$(x[n] + x[n-1] + 0 + x[n-3])$ mod 2

↑since bit

a) ~~True~~ False ✓

- 2 bits set for every message bit $\frac{1}{2}$
- message bit never sent

I $r = 1/2$ $k = 4$

↑code lenght = Constraint lenght

II $r = 1/2$ $k = 6$

⑯

b) Constraint lenght 2 is 4

- what is constraint lenght ?

I think just k - lenght of history generator looks at

False is 6

c) True

d) ?True ✓

b) How many states ?

- Oh I did not study state diagram

$\boxed{2^{k-1}}$

I) 8 II 32

c) 2 - since more history ✓

d) $r = \frac{1}{3}$ lower code rate

$k = 4$ - more redundancy

So scheme stronger

⑰

10. The Virterbi
- will be last I do in this section

a. G0 = 111
G1 = 110

Rec .6, .4
1, 0

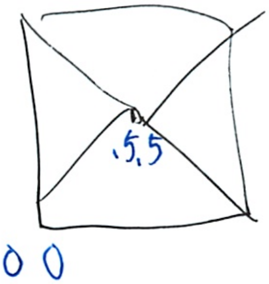
Start 00

? So apply gereator - well see
Go to 10

Why did I get wrong?

I know - I only did for 1st block
- This is not 1st time step!
- must look at other things

b) Grr - what is formulam again



√~~thing~~

No √

$(.5-0)^2 + (.5-0)^2$

⑱

C) Did not fully do   path firmed up

0 1 0 1 1

d) 00 11 11 01 00 ✓

~~d)~~ e) Min path used   state = 01

?oh at 6

e) What is next step?

~~What~~ It does not recieve anything ...

00 ← 10 ← 11 ← 01 ← 10 ← 01

f) Isn't there 2 paths?   So 011010

Yes

H) See paper ✓

⑲

MAC

1.a) Which are true?

Slotted [illegible] $U = \frac{1}{e}$ $U = \frac{1}{2e}$

What is larger

$e \approx 3$ say

$\frac{1}{3}$ is larger

Slotted larger

True ✓

b) Is there some load – ~~yeah when~~

yeah when only 1 transmitter backlogged/has something to transmit

~~True~~ False ~~Wha~~ slotted is always better than unslotted

c) True if done right ✓ – actually makes sense – less time to conflict

⑳

D) Oh yeah ✓

---

2. Binary exponential backoff. What is right

a) Not never. False

B) True ✓
-prob is lower

but lower than what?

C) False

D) Does nothing for fairness I think

---

3. a) So that one does not get so low it won't transmit again ✓

"starving"

b) While don't want one to dominate - by always sending "capture effect" ✓

c) Min to high - lots of ✓ collisons

V ~ 0

4.a) Slotted aloha

~~M~~ If p, what is waste?

$U = N_p(1-p)^{N-1}$

$(1-p)^N$

So waste = 1 − U

? oh something about them all togeter

b) p max = $\frac{1}{N}$ ≈ 37%

min ≃ 0 max ≃ 1

waste = 1 − 37% = 63%

$\frac{1}{e}$

~~but~~ isn't U+waste = 1?

Or are they not counting collissons as waste?

5. True or false?

Nodes backlogged

a) Will converge to p w/ all nodes w/ binary exponential backoff

No – will never converge ✓

b) Carrier sense - Transmit in next slot

No - wait random time ✓

(I feel like I get this section)

㉒

C) Some workload where not 100% util

- are blank spots

So true ✓

---

6. Processor cores

1. Streaming - very good 100% ✓
2. Random - some empty time

Do we need to calulate explictly?

if any has load < $\frac{100}{8}\%$ → waste

load > $\frac{100}{8}\%$ → backed up

add $\sum \max(12.5, \text{actual})$

{ could have easily have added ✓

㉓

7. Skip, similar
8. " "
9. " "
10 " complex
11 " similar
12 " "

13.a) I don't know why sending dummy packet helps
Unless there is something similar about it
Prevents slave from being starved - w/o slave w/ packets will never send packets

Oh I get it now

b) Yeah they said dummy = waste
So it sends dummy packet ~~w/ [illegible]~~ when nothing to send?

did not read close enough to see

㉔

Freq Domain + Filters

This section I should know

1. = $e^{j\varphi}$ = $\cos(\varphi) + j\sin(\varphi)$

add → $\sqrt{\cos^2\varphi + \sin^2\varphi}$

always = 1 I think ✓

2. a) Prove

$$\sum_{n=0}^{N-1} \alpha^n = \begin{cases} N & \alpha = 1 \\ \frac{1-\alpha^n}{1-\alpha} & \text{for complex \# } \alpha \neq 1 \end{cases}$$

Case1 So when $\alpha = 1$

$$\sum_{n=0}^{N-1} 1^n = 1^0 + 1^1 + 1^2 + \dots 1^{N-1}$$

$$= N \cdot 1$$

$$= N$$

Case 2 $\alpha^0 + \alpha^1 + \alpha^2 + \dots + \alpha^{N+1}$

- same as 6.042 today

$\alpha^0 = 1$

㉕

I don't get the rest of it

-Not in 6.042 either

For $\alpha \neq 1$

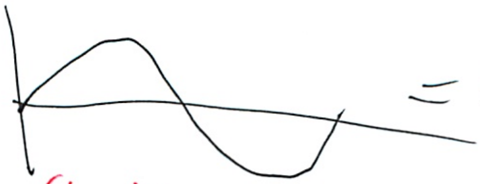
1. expand the sum - What would this be $1+\alpha+\alpha^2+\dots+\alpha^{N+1}$
2. Multiply by $\frac{1-\alpha}{1-\alpha}$ ← So its always this
3. Simplify the numerator

b) Use finite formula to show

$$\sum_{n=\langle N \rangle} e^{jk\frac{2\pi}{N}n} = \begin{cases} N & k=0, \pm N, \pm 2N \dots \\ 0 & \text{otherwise} \end{cases}$$

So it is 0 elsewhere

Adds up to ballance at



$= 0$

Simplify, use finite sum formula and $e^{j\pi}=-1$

↑ oh tricky

? otherwise cancles out?

? So when it is 1?

- when it cancles N?

(26 pages in - so sick of this)

3a) Compute a_k

$$x[n] = 1 + \sin \frac{2\pi}{N} \cdot n + 3\cos\left(\frac{2\pi}{N} n\right) + \cos\left(2 \frac{2\pi}{N} \cdot n + \frac{\pi}{2}\right)$$

? What does this do?

I'm guessing offsets it so is like if sin did it

$a_{+1} = \frac{-j}{2}$

$a_{-1} = \frac{j}{2}$

what do you use

gather terms

add

$a_1 = 3 \cdot \frac{1}{2}$

$a_{-1} = 3 \cdot \frac{1}{2}$

$a_{\pm 2} = 2 \cdot \frac{1}{2}$

a_k are periodic w/ period N

-oh just means it repeats

$a_{-2} = -\frac{1}{2} j$

$a_{-1} = \frac{3}{2} - \frac{1}{2} j = \frac{3}{2} + \frac{1}{2} j$

$a_0 = 1$ ← don't forget

$a_1 = \frac{3}{2} + \frac{1}{2} j = \frac{3}{2} - \frac{1}{2} j$

$a_2 = \frac{1}{2} j$

I see now

(27)

b) $x[n] = 5\cos(6\pi n + \pi) + 7\cos(3\pi n)$

find smallest N where periodic
rewrite for that
N = 2 here

$a_{\pm 6} = 5 \cdot \frac{1}{2}$

$a_{\pm 3} = 7 \cdot \frac{1}{2}$

$= 5\cos(\pi) + 7\cos(1 \cdot \frac{2\pi}{2} \cdot n)$

$a_0 = -5$

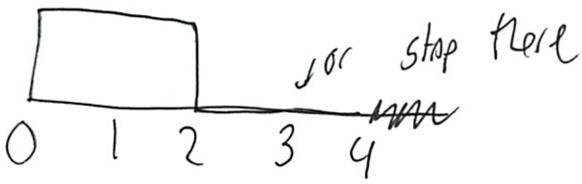
$a_1 = \frac{7}{2}$

↑ cos(π) is just 1

period $= \frac{2\pi}{\Omega} = \frac{2\pi}{6\pi} = \frac{1}{3}$???

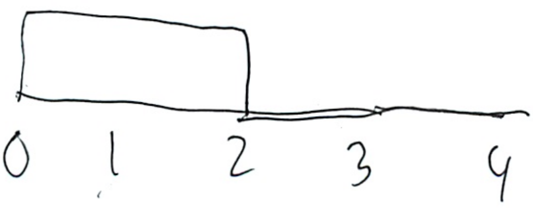
c) x[n] is square wave w/ period N=4

$x[0] = 1$ $x[1] = 1$ $x[2] = 0$ $x[3] = 0$



$a_k = \frac{1}{N} \sum_{n\langle n\rangle} x[n] e^{-jk\frac{2\pi}{N}n}$

↑ do it manually if need to



$a_{-2} = 0 = \frac{1 + -1}{4}$

$a_{-1} = \frac{1+j}{4}$

↑ no think go toy $a_0 = 1$

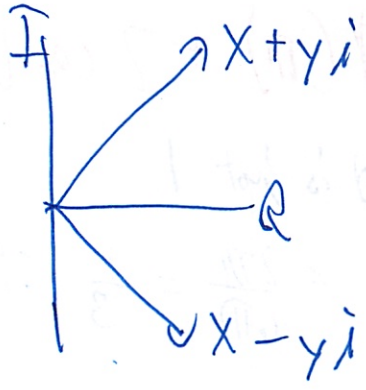
but I don't know how to get a_k $a_1 = \frac{1-j}{4}$

remember a_k is complex conj of a_{-k} when x[n] is real

Complex Conj = pair of complex #

same real part

flip signs on imag part



㉘

d) $x[n] = \cos\left(\frac{2\pi n}{3}\right) \circ \sin\left(\frac{2\pi n}{9}\right)$

Just look at ans

Smallest N where period is is 9

? where/how?

$\frac{2\pi}{\frac{2}{3}} = \frac{2\pi \cdot 3}{2} = \frac{6\pi}{2} = 3\pi$

$a_{-4} = \frac{j}{4}$ $\quad a_k = 0$

$\frac{2\pi}{\frac{2}{9}} = \frac{18\pi}{2} = 9\pi$

$a_{-2} = -\frac{j}{4}$

thank $\frac{2\pi}{N} \cdot k$ $\quad$ I think I will skip this

$a_0 = 0$

so $N=2$, $k=9$ / but not what got before

$a_2 = \frac{j}{4}$

$a_4 = -\frac{j}{4}$

I'm just not seeming to learn this part

e) x[n] has 1 non-zero value per period

for $x[m] \neq 0$ for m and 0 otherwise

Mag of ak

(which ak?

$$|a_k| = \left|\frac{1}{N}\sum_{\langle N\rangle} x[n] e^{-jk\left(\frac{2\pi}{N}\right)n}\right|$$

$$= \frac{1}{N}\left|x[m]\right|\left|e^{-jk\frac{2\pi}{N}n}\right|$$

$$= \cancel{\left|\frac{x[m]}{N}\right|} \quad \boxed{\frac{|x[m]|}{N}}$$

Since $|e^{j\varphi}|$ is 1 for any φ

(29)

4. If $x[n)$ is real, even ~~Both~~ periodic w/ period N
Show that all a_k are real

$$a_k = \frac{1}{N}\sum_{\langle N\rangle} x[n]\, e^{-jk\frac{2\pi}{N}n}$$

Noting that $x[1] = x[-1]$, $x[2] = x[-2]$, ...
if N is odd, can rewrite

$$a_k = \frac{1}{N}\Big[x[0] + x[1]\big(e^{jk\frac{2\pi}{N}} + e^{-jk\frac{2\pi}{N}}\big) + x[2]\big(e^{jk\frac{2\pi}{N}2} + e^{-jk(\frac{2\pi}{N})2}\big) + \ldots\Big]$$

$$= \frac{1}{N}\Big[x[0] \overset{x[1]}{\cdot} 2\cos\big(\frac{2\pi k}{N}\big) + x[2]\cdot 2\cos\big(\frac{4\pi k}{N}\big) + \ldots$$

Which is a sum of real #s

If N is even there will be ~~[illegible]~~ 1 - index in eq $n = -\frac{N}{2}$ that is not matched w/ (+) counterpart

But $x[-N/2]\, e^{-jk(\frac{2\pi}{N})(-\frac{N}{2}}$.

$$= x[-\tfrac{N}{2}]\, e^{jk\pi}$$

$$= x[-\tfrac{N}{2}]\cdot(-1)^k$$

which is real

did not follow!

㉚

5. Given a_k for a seq $x[n]$ ~~for~~
Compute b_k for $w[n] = x[n-\alpha]$

? Just shift?

$$b_k = \frac{1}{N} \sum_{n=\langle N \rangle} w[n] e^{-jk\left(\frac{2\pi}{N}\right)n}$$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} x[n-\alpha] e^{-jk\left(\frac{2\pi}{N}\right)n}$$

This time run indices of summation from α to $\alpha+(N-1)$

$$b_k = \frac{1}{N} \sum_{\alpha \leq n \leq \alpha+(N-1)} x[n-\alpha] e^{-jk\left(\frac{2\pi}{N}\right)n}$$

$$= \quad " \quad " \quad x[n] e^{-jk\left(\frac{2\pi}{N}\right)n+\alpha}$$

$$= \quad " \quad " \quad x[n] e^{-jk\left(\frac{2\pi}{N}\right)n} + e^{-jk\frac{2\pi}{N}\alpha}$$

$$= e^{-jk\frac{2\pi}{N}\alpha} \cdot \frac{1}{N} \sum_{0 \leq n \leq (N-1)} x[n] e^{-jk\frac{2\pi}{N}n}$$

$$= a_k e^{-jk\left(\frac{2\pi}{N}\right)\alpha}$$

㉛ | I know how to do certain things + useless at other stuff |

6. Consider an LTI system w/ $h(n)$

a) Give $H(e^{j\Omega})$ in terms of $h[n]$

$$= \sum_{m} h[m] e^{-j\Omega m}$$ ✓

b) If $h[0]=1$ $h[1]=0$ $h[2]=1$ $h[n]=0$ for other n

What is $H(e^{j\Omega})$

(neg since back in time

$$= e^{j\Omega \cdot 0} + 0 e^{j\Omega \cdot 1} + 1 \cdot e^{j\Omega \cdot -2}$$

$$= 1 + 0 + e^{-j\Omega 2}$$ ✓

$$= 1 + \cos(-2\Omega) + \sin(-2\Omega) i$$

I don't know how much this is helping — I'll look at ans

㉜

c) Let $h[n]$ be b

$x[n] = \cos \varphi n$

Is there a φ so $y[n] = 0$ for all n $\quad 0 \leq \varphi \leq \pi$

So

~~$0 = 1 + \cos 2(2\varphi$~~

$0 = 1 + 2x$

$-1 = 2x$

$x = -\frac{1}{2}$

$-\frac{1}{2} = \cos(\varphi)$ ↑ imag is 0

$\frac{2\pi}{3}$ $\quad \frac{\pi}{2}$ should be

↑ only top half of circle

d) Find max mag of $y(n)$ if $x[n] = \cos\left(\pi \frac{n}{4}\right)$

What does this mean again?

$\varphi = \frac{\pi}{4}$

$= 1 + \cos\left(2 \frac{\pi}{4}\right) + \sin\left(2 \frac{\pi}{4}\right) i$

$= 1 + 0 \cancel{+} 1i$

$= 1 \cancel{+} i$

- what does max mag mean?

$= \sqrt{2}$

(33)

e) h[n] as before. Find max mag of y[n] if $x[n] = \cos\left(\frac{-\pi}{2} n\right)$

So same, but $\varphi = -\frac{\pi}{2}$

$$= 1 + \cos\left(2 \cdot -\frac{\pi}{2}\right) + \sin\left(2 \cdot -\frac{\pi}{2}\right) i$$

$$= 1 + \cos(-\pi) + \sin(-\pi) i$$

$$= 1 + \cos(\pi) + \sin(\pi) i$$

$$= 1 + -1 + 0$$

$= 0$ ✓

㉞

7. 2 filters

~~D~~ 4 systems

a) what is $h[n] = \alpha\, \delta[n] - h_1[n]$

and what is α

- no idea

~~A~~ δ is like 1 ?

C ?

D - I'm just pattern matching

B has only freq response w/ same values at 0, $\pm\pi$, $\pm 2\pi$

$\alpha = 2$ as $H_1(e^{j0}) = 2$ But $H_B(e^{j0}) = 0$

B) $h[n] = \sum_m h_1[m] h[n-m]$ for m = 0 to n

H_A since $H_1(e^{j\Omega}) H_2(e^{j\Omega}) = H(e^{j\Omega})$

so $|H(e^{j\Omega})| = 0$ when $H_1(e^{j\Omega}) = 0$ or $H_2(e^{j\Omega}) = 0$

8. Last problem!

3 filters as shown

a) What freq response is this?

What is max Ω