12/6

MASSACHUSETTS INSTITUTE OF TECHNOLOGY Just to make sure

Physics Department

8.01 Physics I Fall Term 2009

Review Module on Solving N equations in N unknowns

Most students' first exposure to solving N linear equations in N unknowns occurred in high school algebra. At first you dealt with simple dimensionless numerical equations such as

$$\begin{aligned} 4x + 2y &= -2 \\ 9x + 3y &= 6 \end{aligned}$$

One approach is to solve the first equation for y as a function of x, use that to eliminate y from the second equation, then solve that for x. Once one has x one can substitute it back into the first equation to find y.

$$\begin{aligned} 4x + 2y &= -2 &\rightarrow\quad & y = -2x - 1 & & \\ 9x + 3y &= 6 &\rightarrow\quad & 9x + 3(-2x - 1) = 6 &\rightarrow\quad & \underline{x = 3} \\ 4x + 2y &= -2 &\rightarrow\quad & 12 + 2y = -2 &\rightarrow\quad & \underline{y = -7} \end{aligned}$$

Another approach is to multiply each equation by a different number to arrive at identical coefficients of one of the variables, then subtract one of the two resulting equations from the other to find an expression which contains only the other variable.

$$\begin{aligned} 3(4x + 2y) &= 3(-2) &\rightarrow\quad & 12x + 6y = -6 & & \\ 2(9x + 3y) &= 2(6) &\rightarrow\quad & 18x + 6y = 12 & & \\ 6x &= 18 &\rightarrow\quad & 18x + 6y = 12 &\rightarrow\quad & \underline{x = 3} \\ 4x + 2y &= -2 &\rightarrow\quad & 12 + 2y = -2 &\rightarrow\quad & \underline{y = -7} \end{aligned}$$

✓

Three linear equations in three unknowns can be handled in the same way, except with more iterations. Eliminate one variable to obtain two equations in two unknowns, then proceed as above. ✓

Next, you had to deal with the dreaded "word problems" where the dimensionless variables were replaced with physical ones: let f be the age of a father and s be the age of his son; or let n be the number of nickels, d the number of dimes and q the number of quarters; or let r_1 be the flow in gallons per minute from the first pipe and r_2 be the flow from the second pipe. The difficulty was not solving the equations (it was assumed you could do that), but finding the correct equations to solve. You have seen this type of problem most recently on the Math Diagnostic Exam for Physics. Of course there is an advantage here in that you can check whether your answer is reasonable: the son can't be older than his father, the numbers of each coin must be integers, and the flows from the pipes can't be negative (unless they are drains).

$4x+2y=-2$ $\leftarrow$ $2y=-2-4x$

$4x+3y=6$ $y=-1-2x$

$3x+y=2$

$3x+(-1-2x)=2$

$3x-1-2x=2$

$x-1=2$

$x=3$

$4(3)+2y=-2$

12

$2y=-14$

$y=-7$

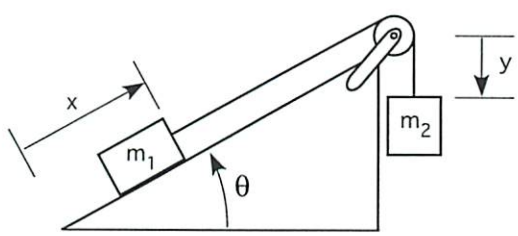
Physics, and the other sciences as well as engineering, generate many "word problems" you must solve. They differ from the high school algebra problems in that the coefficients in front of the variables are usually not pure numbers. Rather, they are expressions involving the important parameters in the problem. The down side is that it becomes more difficult to carry out what are otherwise simple algebraic manipulations. You must learn to be very careful with you math. There is one advantage though. You can check to see that your answer has the correct dimensions. If it does not, you know you have made a mistake.

Variables

8.01 focuses on teaching you how to find the appropriate equations necessary to solve a problem. Here we will give you these equations, simply telling you the physical principle upon which they are based. This review module is designed to give you practice solving the equations once they have been found.

Worked Examples

Example 1



Two blocks with masses m_1 and m_2 are connected by a massless rope of fixed length. Block 1 slides without friction on a ramp which makes an angle θ with the horizontal. The rope passes over a massless, frictionless pulley from which block 2 is freely suspended. The displacement of block 1 upward along the ramp is designated as x. The vertical displacement of block 2 below the center of the pulley is designated as y. There will be a tension T in the rope. The system is released from rest. Find the acceleration of block 2, $\ddot{y}$, in terms of the given parameters and the acceleration of gravity g.

✓ acc

The physics gives three equations relating the three unknowns: $\ddot{x}$, $\ddot{y}$, and T

$F = ma$ on block 1 gives   free body diagram, sum of forces

$$T - m_1 g \sin\theta = m_1 \ddot{x} \tag{1}$$

$F = ma$ on block 2 gives

$$m_2 g - T = m_2 \ddot{y} \tag{2}$$

The fact that the rope has a fixed length requires

don't forget the constant!

$$\boxed{\ddot{x} = \ddot{y}} \tag{3}$$

Find $\ddot{y}$

$T - m_1 g \sin\theta = m_1 \ddot{x}$

$m_2 g - T = m_2 \ddot{y}$

$\ddot{x} = \ddot{y}$

$T =$

what now $T =$

-solve for T

$T = m_1 \ddot{y} + m_1 g \sin\theta$

$T = m_2 g - m_2 \ddot{y}$

$m_1 \ddot{y} + m_1 g \sin\theta = m_2 g - m_2 \ddot{y}$

$-m_1 \ddot{y} \quad -m_2 g \quad -m_2 g \quad -m_1 \ddot{y}$

$m_1 g \sin\theta - m_2 g = -m_2 \ddot{y} - m_1 \ddot{y}$

$= -(m_1 + m_2) \ddot{y}$

$\ddot{y} = -\dfrac{m_1 g \sin\theta - m_2 g}{m_1 + m_2}$ → they reduce 1/2 more

$\dfrac{m_2 g - m_1 g \sin\theta}{m_1 + m_2}$

$\dfrac{(m_2 - m_1 \sin\theta)}{m_1 + m_2} g$ ✓

Here is one way to solve the equations. Let's solve for $\ddot{y}$. First eliminate T by adding (1) and (2).

$$m_2 g - m_1 g \sin\theta = m_1 \ddot{x} + m_2 \ddot{y} \tag{4}$$

Next use (3) to eliminate $\ddot{x}$ from (4).

$$m_2 g - m_1 g \sin\theta = m_1 \ddot{y} + m_2 \ddot{y} \tag{5}$$

Finally solve (5) for $\ddot{y}$.

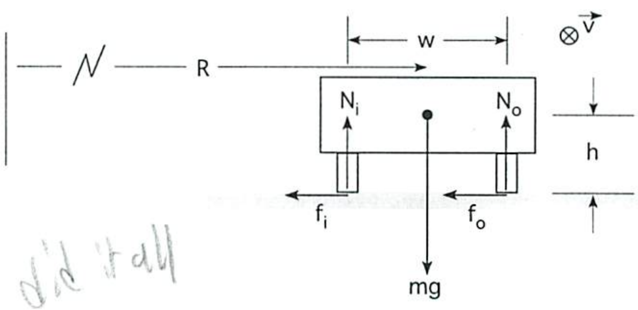
$$\ddot{y} = \frac{(m_2 - m_1 \sin\theta)}{m_1 + m_2}\, g \tag{6}$$

Now inspect the answer. The dimensions are correct. We are looking for an acceleration. The answer is in the form of a dimensionless fraction times the acceleration of gravity. The result is physically reasonable. If m_2 is sufficiently larger than m_1, block 2 accelerates downward. If m_2 is sufficiently smaller than m_1, block 2 accelerates upward. If $\theta = 90^o$ the system is balanced and does not move when $m_1 = m_2$.

A really good check

Example 2

Should also pratice setting up problem for the final now that I did it all



A car moves at constant speed v around a curved section of highway with radius R. The car has mass m. Its center of mass is a height h above the road. The span between the tires on the inside and outside of the turn is w. What is the maximum speed the car can maintain without rolling over?

Let N_i and N_o be the vertical components of the force the road exerts on the inner and outer tires respectively. Similarly, let f_i and f_o be the inwardly directed horizontal components of the friction force the road exerts on the inner and outer tires. These are the four unknowns in the problem. The car will begin to roll over when N_i goes to zero. [By Newton's third law, that is the point at which the inner tires no longer press down on the road.]

$F = ma$ on on the car in the vertical direction gives

$$N_i + N_o - mg = 0 \tag{1}$$

$F = ma$ on on the car in the horizontal direction gives

$$f_i + f_o = mv^2/R \tag{2}$$

idk how to set up

· draw pics

centrepicial

$\tau = \curvearrowright$ F $\circlearrowright$ → into ground

would roll out

-what we just did

but how does car's speed factor into it

$W = vr$

$v = \frac{W}{R}$

$\tau = I\omega$

← moment of inertia

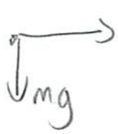
-but what is θ so it won't fall over?

N_o - just as outer wheel leaves ground

-or is it the inner wheel

← depends on which way it is turning

So inner wheel $N_i = 0$ ✓ took me a lot longer



← what force is causing it to lift?

*look at X Y not each tire

$F_x = ma = N_i + N_o - mg$

$F_y = f_i + f_o = mv^2/R$

- outer tire pushing twice as hard?

Guess no torque in this one $=0$ not rolling

- So what now set $N_o = 0$
  And do what?

write that $=0$

Write expression w/ torque

$$(f_i + f_o)h + N_i w/2 - N_o w/2 = 0$$

3 linear equations 4 unknowns

- but combine $f_i + f_o$

$$fh + N_i w/2 - N_o w/2 = 0$$

$$N_i + 0 - mg = 0$$

$$f = mv^2/R$$

← ~~#~~ realize that have 3 equations w/ 3 unknowns
~~#~~ (never thought of it this way before)

find v

$$N_i = mg$$

$$\frac{mv^2}{R} h + \frac{mgw}{2} = 0$$

$$\frac{mv^2 h}{R} = -\frac{mgw}{2}$$

$$\cancel{m} v^2 h = \frac{R \cancel{m} g w}{2}$$

$$v = \sqrt{\frac{Rgw}{2h}} \quad ✓$$

Solved it

- did the math :)
  and half the physics
- missed 1 equation

Torque = 0 about the center of mass of the car (when it has not yet begun to roll) gives

$$(f_i + f_o)h + N_i w/2 - N_o w/2 = 0 \tag{3}$$

Note that we only have 3 linear equations for 4 unknowns. We will not be able to determine them all. However, since only the sum of the two friction components appears, one could consider that sum to be a single variable. For this problem, we do not need to know f_i and f_o separately.

Let's solve for N_i. Multiply (3) by $2/W$ and move the friction term to the right hand side.

$$N_i - N_o = -2(h/w)(f_i + f_o) \tag{4}$$

Use (2) to eliminate $(f_i + f_o)$ from (4).

$$N_i - N_o = -2(h/w)mv^2/R \tag{5}$$

Add (1) and (5), then isolate N_i.

$$\underline{N_i = m[g/2 - (h/w)v^2/R]} \tag{6}$$

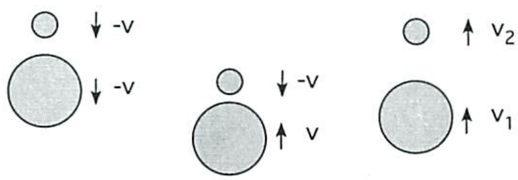
The dimensions are correct. Force has the units of mass times acceleration. Inside the [ ] we have the acceleration of gravity and a dimensionless fraction times v^2/R which also has the units of acceleration. The critical velocity is obtained by setting $N_i = 0$.

$$v_{critical} = \sqrt{gRw/2h} \tag{7}$$

$v_{critical}$ has the expected behavior. It increases with the radius of the turn and the span of the tires. It decreases as the center of gravity rises higher above the road.

Example 3

I remember this problem



Two balls are dropped at virtually the same instant. The lower ball, 1, of mass M has a vertical velocity $-v$ when it strikes the ground. The collision with the ground is elastic and it rebounds with an upward velocity v. It then strikes the upper ball, 2, of mass m and velocity $-v$ in an elastic collision. What is the subsequent velocity of the upper ball?

elastic collission
-momentum + e conserved
p=mv $\frac{1}{2}mv^2$

Conservation of momentum in the upward direction before and after the collision gives

$$Mv - mv = Mv_1 + mv_2 \tag{1}$$

Conservation of energy gives

$$(1/2)Mv^2 + (1/2)mv^2 = (1/2)Mv_1^2 + (1/2)mv_2^2 \tag{2}$$

We have two equations in the two unknowns v_1 and v_2, but one of those equations is quadratic in the variables. This means that there will be two possible solutions. We may have to solve a quadratic equation. This is going to be messy, so it is wise to clean up the equations as much as possible before looking for the solution. Rather than carrying both masses along in the algebra, we will introduce the mass ratio $r \equiv m/M$. Dividing (1) by M and (2) by $(1/2)M$ reduces the equations to

$$(1 - r)v = v_1 + rv_2 \tag{3}$$

and

$$(1 + r)v^2 = v_1^2 + rv_2^2 \tag{4}$$

Solve (3) for v_1 as a function of v_2

$$v_1 = (1 - r)v - rv_2 \tag{5}$$

Substituting this into (4) gives

$$(1 + r)v^2 = (1 - r)^2v^2 - 2r(1 - r)vv_2 + r^2v_2^2 + rv_2^2 \tag{6}$$

Collecting terms and then dividing by r leaves the quadratic equation for v_2 in terms of r and v.

$$(1 + r)v_2^2 - 2(1 - r)vv_2 - (3 - r)v^2 = 0 \tag{7}$$

Common factors 3 steps

One can use the quadratic formula to find the two solutions.

$$\underline{v_2 = \frac{3 - r}{1 + r}v \quad \text{or} \quad v_2 = -v} \tag{8}$$

The first of the solutions in (8) is the one we are looking for. Note that if $r \ll 1$ $v_2 \approx 3v$. Since the kinetic energy goes as the square of the velocity and the maximum height of the ball in the gravitational field is proportional to its energy, a much lighter upper ball will rebound to 9 times its initial release height.

The second solution in (8) is interesting. It corresponds to the initial condition <u>before</u> the collision. In elastic collision problems of this sort one of the two solutions must always correspond to the initial conditions. The wise student will remember this. Knowing one root of the quadratic equation allows one to factor the equation and find the other root without using the quadratic formula.

Ex 3

① $\cancel{m_1 V = m_1 V_1 + m_2} V_2$ ↙ use too

I think this is e

② $\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$? find

or is this 2nd equation ✱ use both ✱

small ball has initial v too!

$\cancel{v_{2f} = \sqrt{\frac{m_1 v_1^2 - m_1 v_{1f}^2}{m_2}}}$

$\cancel{\sqrt{\frac{m_1 (v_{1i}^2 - v_{1f}^2)}{m_2}}}$

2 equations 2 unknowns - one quadratic

- so messy

- introduce mass ratio $r = \frac{m}{M}$

eq $\frac{①}{M} \rightarrow (1-r)v = v_1 + r v_2$

$\frac{②}{\frac{1}{2}M} \rightarrow (1+r)v^2 = v_1^2 + r v_2^2$ ③

Solve for v_1 as function v_2

- ? how

- solve for r and plug in?

or ~~square it~~

$v_1 = (1-r)v - r v_2$ - sub into e2 - ✱ want v_{2f}

$$(1+r)v^2 = \left((1-r)v - rv_2\right)^2 + rv^2$$

now solve for v2

- expand 1st

$$(1+r)v^2 = (1-r)^2v^2 - 2rv_2(1-r)v + rv_2^2$$

everything has v_2

// Collect terms and divide by r
leaves quadratic eq in terms
of r and v

$$\cancel{(1+r)v^2} + 2rv_2(1-r)v - rv_2^2 = (1-r)^2v^2 - (1+r)v^2$$

$$v_2\left(2r(1-r)v - rv_2\right) = ?$$

$$V_2 = \frac{(1-r)^2v^2 - (1+r)v^2}{2r(1-r)v - rv_2}$$

Wrong algebra

on live terms

$$\frac{(1+r)v^2}{r} = \frac{(1-r)^2v^2}{r} - 2vv_2(1-r) + v_2^2$$

$$(1+r)v_2^2 - 2(1-r)vv_2 - (3-r)v^2 = 0$$

where that from ?

or quadratic formula
? - with variables

$$V_2 = \frac{3-r}{1+r} \qquad v_2 = -v$$

would not have got / still don't get

$$(1+r)v^2 = (1-r)^2 v^2 - 2r(1-r)vV_2 + r^2V_2^2 + rV_2^2$$

$$(1+r)v^2 = (1-r)v\left[(1-r)v - 2rV_2\right] + rV_2^2(1+r)$$

$$(1+r)(v^2 - rV_2^2) = (1-r)v\left[(1-r)v - 2rV_2\right]$$

$$(1+r)(1+r)(v^2 - rV_2^2) = (1-r)v\left[v - rv - 2rV_2\right]$$

$$\cancel{v^2} + \cancel{r}v^2 = \cancel{v^2} - r^2v^2 - 2\cancel{r}vV_2 + 2r^2vV_2 + r^2V_2^2 + \cancel{r}V_2^2$$

$$v^2 = -rv^2 - 2vV_2 + 2rvV_2 + rV_2^2 + V_2^2$$

$$v^2 + rv^2 + 2vV_2 - 2r\,vV_2 + rV_2^2 + V_2^2 = 0$$

copy error

$$(1+r)V_2^2 - 2vV_2(-1+r) + v^2(1+r) = 0$$

$$(1+r)(V_2^2 + v^2) + 2vV_2(r-1) = 0$$

$$ax^2 + bx + c = 0$$

Then do this

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

don't know how lines up w/ what given (a, b, c)

see back

$$V^2 + rV^2 = (1-2r+r^2)V^2 \mp 2rVV_2 + 2r^2VV_2 + r^2V_2^2 + rV_2^2$$

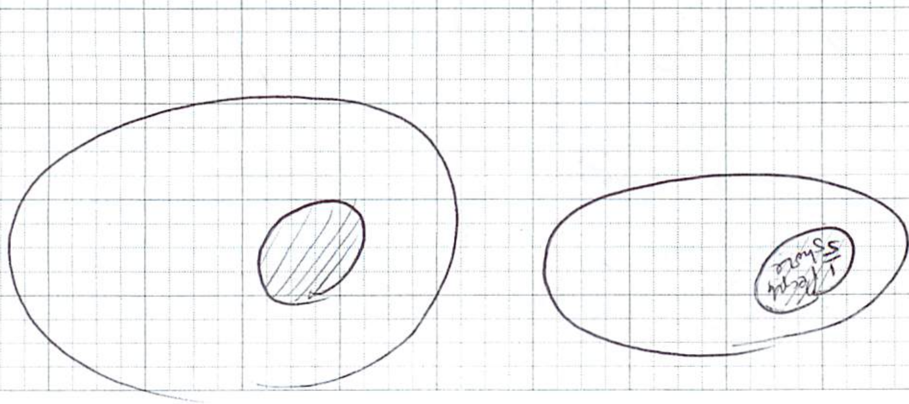
$$\cancel{V^2} + rV^2 = \cancel{V^2} - 2rV^2 + r^2V^2 - 2rVV_2 + 2r^2VV_2 + r^2V_2^2 + rV_2^2$$

$$V^2 = -2V^2 + rV^2 - 2VV_2 + 2rVV_2 + rV_2^2 + V_2^2$$

$$V^2 + 2V^2 - rV^2 + 2VV_2 - 2rVV_2 \mp rV_2^2 - V_2^2 = 0$$

$$3V^2 - rV^2 + 2VV_2 - 2rVV_2 \mp rV_2^2 - V_2^2 = 0$$

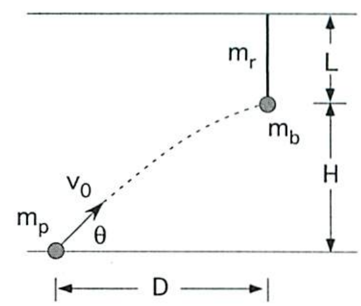
$$V^2(3-r) + 2VV_2(1-r) \mp V_2^2(r+1) = 0$$



Practice Problems

Problem 1

A cannon has muzzle velocity v_0 and is aimed at angle θ above the horizontal. Its projectile of mass m_p strikes and sticks to a pendulum hanging from the ceiling. The pendulum bob has mass m_b and is suspended by a rigid uniform rod of length L and mass m_r. The bob is a height H above, and a horizontal distance D away from, the cannon. For what value of L will the pendulum just touch the ceiling on its first swing?



This problem is subtle because the collision between the projectile and the bob is not elastic (energy is not conserved) and conservation of momentum is difficult to use since one does not know the force exerted on the rod by the ceiling. One must resort to conservation of angular momentum. Let ω be the rotation rate (counter-clockwise) of the pendulum just after the collision.

The moment of inertia of pendulum about its pivot point at the ceiling (after the projectile has become attached) is given by

$$I = (m_p + m_b)L^2 + m_r L^2/3 \tag{1}$$

Conservation of angular momentum about the pivot point during the collision gives

$$m_p L v_0 \cos\theta = I\omega \tag{2}$$

Conservation of energy between the moment after the collision and the moment the pendulum comes to rest in the horizontal position gives

$$(1/2)I\omega^2 = (m_p + m_b)Lg + m_r(L/2)g \tag{3}$$

We must solve 3 equations in three unknowns I, ω, and L to find the critical value of L.

elastic collission

p conserved

e not

but e_{after} collission = ~~pE~~ e top position

$$E_{after} = m_p + m_g \, g \, L$$

just touch

$k = 0$

but how does having e after help? $= \frac{1}{2}(m_p + m_g) v_f^2$

Solve for v_f

$$\sqrt{\frac{2\cancel{(m_p+m_g)}\, gL}{\cancel{m_p+m_g}}}$$

$$\sqrt{2gL}$$

$$m_p V_0 + 0 = (m_p + m_b) V_f$$

$$m_p V_0 = m_p + m_b \sqrt{2gL}$$

$$V_0 = \frac{(m_p + m_b)\sqrt{2gL}}{m_p}$$

they used inertia

and also did not read problem closly enough

was supposed to find L

$$I = \underbrace{(m_p + m_b) L^2}_{\text{objects}} + \underbrace{m_r \frac{L^2}{3}}_{\text{bar}}$$

$\boxed{\text{* bar has mass too *}}$

- these problems may not be set up best

Coservation of angular momentum L

$m_p L v_0 \cos\theta = I\omega$

$F \times \vec{r}$ ←?

now energy

$\frac{1}{2} I\omega^2 = (m_p + m_b) L g + m_r \frac{L}{2} g$

↑mgh ↑stick -com

Solve the 3 equations in 3 unknowns - messy

- 3 unknows I, ω, L

find when $L = 0$

$m_p L v_0 \cos\theta = (m_p + m_b) L^2 + m_r L^2/3\, \omega$

↳ in all terms

$\cancel{L = \frac{(m_p + m_b) L^2 + m_r L^2/3\,\omega}{m_p}}$

$\frac{L (m_p v_0 \cos\theta)}{L^2} = \frac{L^2 \left([m_p + m_b] + \frac{m_r \omega}{3} \right)}{L^2}$

$\frac{1}{L} m_p v_0 \cos\theta = (m_p + m_b) + \frac{m_r \omega}{3}$

$\frac{1}{L} = \frac{m_p + m_b + \frac{m_r \omega}{3}}{m_p v_0 \cos\theta}$ →

$L = \frac{m_p v_0 \cos\theta}{(m_p + m_b) + \frac{m_r \omega}{3}}$

not close to the ans they got

They eliminated w by solving 2 for w + sub into 3

$$\frac{m_p L v_0 \cos\theta}{\cancel{\ } I} = \omega$$

Sub into 3

$$\frac{1}{2} I \left(\frac{m_p L v_0 \cos\theta}{I} \right)^2 = (m_p + m_b) L g + m_r \frac{L}{2} g$$

and they factor + distribute

$$\frac{L^2 m_p^2 v_0^2 \cos\theta^2}{2I} = \cancel{\ }(m_b + m_p + \tfrac{1}{2} m_r) \cancel{L} g$$

$\cdot \frac{2I}{L}$ $\cdot \frac{2I}{L}$

$$L m_p^2 v_0^2 \cos^2\theta = 2(m_p + m_p + \tfrac{1}{2} m_r) I g$$

Sub I in from ①

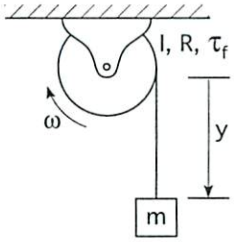
$$L m_p v_0^2 \cos 2\theta = 2(m_p + m_b + \tfrac{1}{2} m_r)(m_b + m_p + \tfrac{1}{3} m_r) L^2 g$$

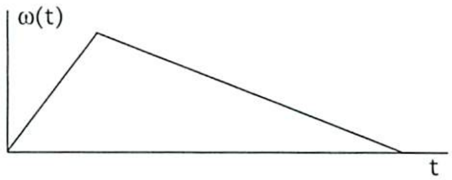
divide by L + Solve for L

- realy comblex

inlab was simplar
but far less info

Problem 2





A measurement of the moment of inertia I of a disk is carried out as shown above. The disk is mounted on a low friction bearing. One end of a string is threaded into a notch on the periphery of the disk. The string is wound around the disk several times and a weight of mass m is hung vertically from its other end. The system is released from rest. As the weight falls, the angular velocity of the disk increases at a uniform rate $\dot{\omega}_1$. After the string slips out of the notch, the angular velocity of the disk decreases at a uniform rate $\dot{\omega}_2$ (a negative quantity) due to a constant frictional torque τ_f. The disk has a radius R and the tension in the string is denoted by T. The problem is to use the measured values of $\dot{\omega}_1$ and $\dot{\omega}_2$, together with the known parameters to determine I.

While the weight is attached

$F = ma$ on the weight gives

$$mg - T = m\ddot{y} \tag{1}$$

$\tau = dL/dt$ applied to the disk gives

$$RT - \tau_f = I\dot{\omega}_1 \tag{2}$$

Equating the velocity of the string and the velocity of the point of contact on the disk gives

$$\ddot{y} = R\dot{\omega}_1 \tag{3}$$

After the weight has fallen off

$\tau = dL/dt$ applied to the disk gives

$$-\tau_f = I\dot{\omega}_2 \tag{4}$$

We are now faced with 4 equations in the 4 unknowns $\ddot{y}$, T, I and τ_f.

2 More inertia

$\alpha_1 = \dot{\omega}_1$ (+)

$\alpha_2 = \dot{\omega}_2$ (−)

define I

- do piecewise ?

N = torque from friction

~~force~~ does play a role

$\Sigma F = mg - N$

$=$ torque of whole thing

~~Σmg~~

don't mix

tension ≠ sum of force

$\cancel{T = mg - \tau_f}$

$\Rightarrow$ $mg - T = m\ddot{y}$ ← acc of y

← duh mg

$\tau = \dfrac{dL}{dt}$

$RT - \tau_f = I\dot{\omega}_1$

↑ here is where they mix but add radius

$\boxed{\ddot{y} = R\dot{\omega}_1}$ ← here equating again

after weight fallen off

$-\tau_f = I\dot{\omega}_2$

← really need to identify better

4 equations 4 unknowns

$\ddot{y}, T, I, \tau_f$

why can't I find this stuff

have the right ideas kinda

Or perhaps I am confused by sol that do it diff ways ????

goal to find I

now just algebra

$$RT + I\omega_2 = I\omega_1$$

$$mg - T = mR\omega_1$$

$$\omega_1 = \frac{mg - T}{mR}$$

$$RT + I\omega_2 = I\left(\frac{mg - T}{mR}\right)$$

$$\cancel{mR^2 T + mR I\omega_2 = I}$$

$$mR^2 T + mRI\omega_2 = Img - IT$$

$$mR^2 = I(mg - T - mR\omega_2)$$

$$I = \frac{mR^2}{mg - T - mR\omega_2}$$

not what I got

- it really should come out =

- They solved for T ← think need to keep that kinda stuff in mind elim T between 1 + 2 multipy 1 • R

$Rmg - RT = Rm\ddot{y}$ $\quad RT = I\omega_1 + T_f$

$Rmg - I\omega_1 - T_f = Rm\ddot{y}$

$Rmg - T_f = Rm\ddot{y} + I\omega_1$

$Rmg - T_f = RmR\omega_1 + I\omega_1$

$Rmg - T_f = R^2 m\omega_1 + I\omega_1$

use 4 to elim T_f ← did

$MRg + I\omega_2 = R^2 m\omega_1 + I\omega_1$

~~$I = MRg$~~

$MRg - R^2 m\omega_1 = I\omega_1 - I\omega_2$

$= I(\omega_1 - \omega_2)$

$$\boxed{I = \frac{MRg - R^2 m\omega_1}{\omega_1 - \omega_2}}$$

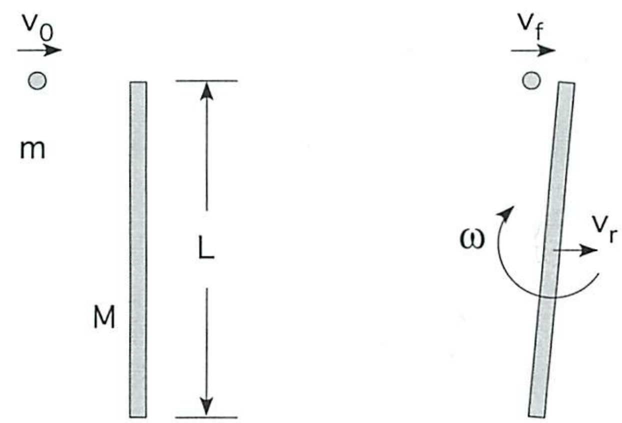
5/10 so mistakes

main * not seeing that problem wants me to get rid of T

minor and multiplying everything by R so it will =

* and alway check if makes sense and what would happen if key value = 0

Problem 3



A uniform rod of length L and mass M, initially at rest, is struck at one end by ball of mass m moving perpendicular to it at a speed v_0. The collision is completely elastic. Find the final speed of the ball, v_f, the velocity of the center of mass of the rod, v_r, and the rate at which the rod is rotating, ω. The moment of inertia of the rod about its center of mass is $I_{cm} = ML^2/12$.

Conservation of momentum gives

$$mv_0 = mv_f + Mv_r \tag{1}$$

Conservation of angular momentum about a point on the trajectory of the ball gives

$$0 = I_{cm}\omega - Mv_r L/2 \tag{2}$$

$r \times MV$

Conservation of energy gives

$$(1/2)mv_0^2 = (1/2)mv_f^2 + (1/2)Mv_r^2 + (1/2)I_{cm}\omega^2 \tag{3}$$

We must solve 3 equations in the 3 unknowns v_f, v_r and ω.

point calc-ing
the torque about

Problem 3

$$I_{cm} = \frac{ML^2}{12}$$

Uniform rod again

hit elastic (E, p conserved)

Find final speed of ball and v_{rod} of com and ω around com

$p = mv$

do seperatly

1 $m_b V_0 = m_b V_f + m_r V_r + \cancel{m_r \omega}$

3 $\frac{1}{2} m_b V_0^2 = \frac{1}{2} m_b V_f^2 + \frac{1}{2} m_r V_r^2 + \frac{1}{2} m_r \omega^2$

$L = I\omega$

inertia - resistance to change

$\int r^2 dm$

- given or table

2 $0 = I_{cm}\omega - M v_r \frac{L}{2}$ ← know this L/2 stuff

I_{cm} of com, around cm

- because its R

3 equations 3 unknowns

closer than before

angular momentum

$L = r \times p = I_{obj}\omega$

$\tau = \frac{dL}{dt}$

* Must do $\frac{L}{2}$ ← to get R

→ see back

the point it is ~~is rotating~~ ~~or is it~~ around distance from where ball hits to center from ~~the~~ Carse Notes 15

divid extended body up into N elements
having mass m_i and position $\vec{r}_{s,i}$
Inertia I_s about some poit S on fixed axis

$$L_s = \sum L_s = \sum \vec{r}_{si} \times p$$

total $= I\omega$

point object

or extended about center of mass

$$L_{total} = L_{orbital} + L_{spin}$$

$$(r_{scm} \times p_{total}) + \sum r \times m V_i$$

traditional displaceme com
orbital

$\sum mr \times v$

sum each point

abaut com of ith particle

spin

-depends only on location of com not location ith particle

i may be confused on some of this stuff

Now solving equations
- for what?
- in terms of what?

Unknowns V_f V_r w

find V_f, V_r, w

so find all - but interms of what?

Oh wow this is really messy

take 1 and divide by m

4 $$rV_0 = rV_f + V_f$$

take (2) divide by $\frac{mL}{2}$ ← how do you know to do this??

5 $$0 = \frac{wL}{6} - V_r$$

tak (3) divide by $\frac{m}{2}$

6 $$rV_0^2 = rV_f^2 + V_r^2 + \frac{(Lw)^2}{12}$$

use (5) to elim w from (6)

7 $$rV_0^2 = rV_f^2 + 4V_r^2$$

use (4) to elim V_f from (7) collect terms

$$0 = \left(4 + \frac{1}{r}\right)V_r^2 - 2V_0V_r$$

Quadratic Equation for V_r

- don't divide by V_r
- will miss that $V_r = 0$ is a solution
  ? but is initial condition
- factor out
- get linear equation for the other root
- solve for V_r

$$V_r = \frac{2m}{4m + M} V_0$$

- find other things

$$W = \frac{6}{L} \frac{2m}{4m + M} V_0$$

$$V_f = \frac{4m - M}{4m + M} V_0$$

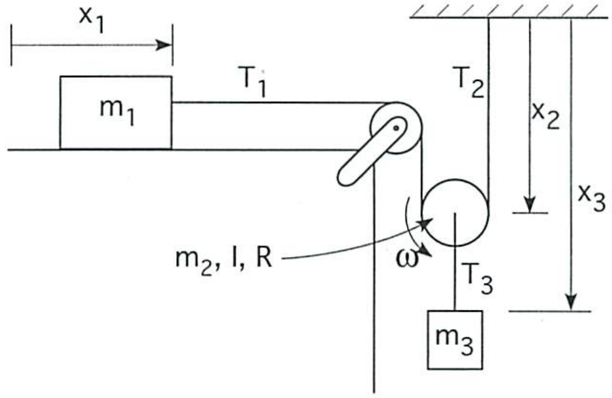
Center of mass always moves to right

rod rotates clockwise

~~M~~ Ball might change direction

$M = 4m$ rest

Problem 4



Two blocks and a massive disk are connected by ropes. The long rope goes over a massless frictionless pulley but is wrapped tightly around the disk such that the disk must rotate as it falls. This problem has 7 variables: 3 accelerations $\ddot{x}_1$, $\ddot{x}_2$ and $\ddot{x}_3$; 3 tensions T_1, T_2, and T_3; and the angular acceleration of the disk $\dot{\omega}$.

$F = ma$ on the block moving horizontally gives

$$T_1 = m_1\ddot{x}_1 \tag{1}$$

$F = ma$ on the block moving vertically gives

$$m_3 g - T_3 = m_3\ddot{x}_3 \tag{2}$$

$F = ma$ on the disk gives

$$m_2 g + T_3 - T_2 - T_1 = m_2\ddot{x}_2 \tag{3}$$

Torque equals the rate of change of the angular momentum applied to the disk gives

$$(T_2 - T_1)R = I\dot{\omega} \tag{4}$$

The disk rotates as it descends, so

$$\ddot{x}_2 = \dot{\omega}R \tag{5}$$

The lengths of the ropes are fixed, which require that

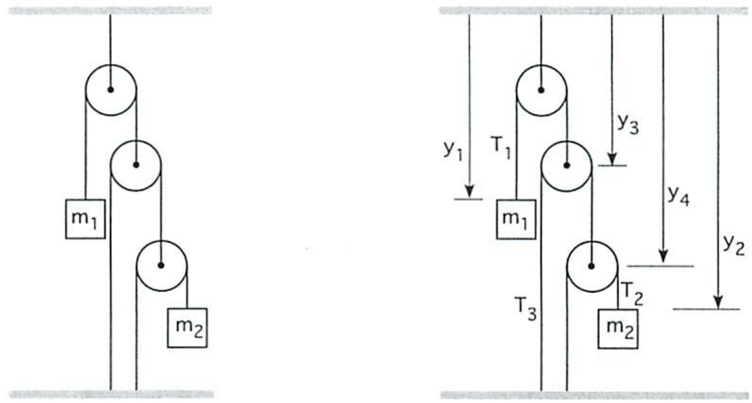
$$\ddot{x}_3 = \ddot{x}_2 \tag{6}$$

and

$$\ddot{x}_1 = 2\ddot{x}_2 \tag{7}$$

Find $\ddot{x}_1$ in terms of the three masses, the moment of inertia I and radius R of the disk, and the acceleration of gravity.

Problem 5



The system shown on the left above is made up of two massive blocks, three massless, frictionless pulleys and 3 ropes of fixed length. Students are asked to find the downward acceleration of block 2 after the system is released from rest. The figure at the right defines 4 displacements and the tensions in each of the 3 ropes that are useful in solving the problem.

$F = ma$ on block 1 gives

$$m_1 g - T_1 = m_1 \ddot{y}_1 \tag{1}$$

$F = ma$ on block 2 gives

$$m_2 g - T_2 = m_2 \ddot{y}_2 \tag{2}$$

The middle pulley will accelerate at some finite rate. However since it has no mass, unless the sum of the forces on it is zero, it would accelerate at an infinite rate. Thus

$$T_1 = 2T_3 \tag{3}$$

Similarly, the sum of the forces on the lower pulley must be zero.

$$T_3 = 2T_2 \tag{4}$$

The fact that the length of the rope with tension T_1 is fixed requires that

$$\ddot{y}_1 = -\ddot{y}_3 \tag{5}$$

The fact that the length of the rope with tension T_3 is fixed requires that

$$\ddot{y}_4 = 2\ddot{y}_3 \tag{6}$$

The fact that the length of the rope with tension T_2 is fixed requires that

$$\ddot{y}_2 = 2\ddot{y}_4 \tag{7}$$

Incidentally, finding these last 3 relations is probably the hardest part of the problem. We are now faced with solving 7 equations in seven unknowns: 4 accelerations and 3 tensions.

Solutions to Practice Problems

In all of the following solutions, the equation numbers refer back to the statement of that particular problem.

Solution, Problem 1

Eliminate ω by solving (2) for ω and substituting it into (3)

$$L^2 m_p^2 v_0^2 \cos^2\theta / 2I = (m_b + m_p + (1/2)m_r)Lg \tag{4}$$

Multiply (4) by $2I/L$

$$L m_p^2 v_0^2 \cos^2\theta = 2(m_b + m_p + (1/2)m_r)Ig \tag{5}$$

Now substitute I from (1) into (5)

$$L m_p^2 v_0^2 \cos^2\theta = 2(m_b + m_p + (1/2)m_r)(m_b + m_p + (1/3)m_r)L^2 g \tag{6}$$

Finally divide (6) by L and solve for L

$$\underline{L = \frac{m_p^2}{2(m_b + m_p + (1/2)m_r)(m_b + m_p + (1/3)m_r)} \frac{v_0^2 \cos^2\theta}{g}} \tag{7}$$

This problem illustrates that even a few simple mathematical operations can lead to "messy" answers. However we have grouped the terms to make it easy to check the units. The expression for L begins with a dimensionless ratio of masses. The final term has the units of velocity squared over an acceleration, which indeed reduces to a length. If the mass of the projectile goes to zero, so does L. If the mass of the bob or the mass of the rod is very large, L becomes very small. Finally, the necessary length of the rod grows as the initial velocity of the projectile is increased.

Solution, Problem 2

First eliminate T between (1) and (2). Multiply (1) through by R

$$MRg - TR = MR\ddot{y} \tag{5}$$

Add (5) and (2) and use (3) to eliminate $\ddot{y}$

$$MRg - \tau_f = I\dot{\omega}_1 + mR\ddot{y} = I\dot{\omega}_1 + mR^2\dot{\omega}_1 \tag{6}$$

Use (4) to eliminate τ_f

$$MRg + I\dot{\omega}_2 = I\dot{\omega}_1 + mR^2\dot{\omega}_1 \tag{7}$$

All that is left is to solve for I

$$\underline{I = \frac{mgR - mR^2\dot{\omega}_1}{\dot{\omega}_1 - \dot{\omega}_2}} \tag{8}$$

The last term in the numerator together with the denominator show the correct units for a moment of inertia: mass times distance squared. The first term in the numerator is consistent with the second since g and $R\dot{\omega}$ have the same units. There is no chance that the denominator might go to zero since we noted earlier that $\dot{\omega}_2$ was negative.

Solution, Problem 3

"Clear the decks" of extraneous material before proceeding. Define $r \equiv m/M$. Note that we are expecting a quadratic equation.

Dividing (1) by M gives

$$rv_0 = rv_f + v_r \tag{4}$$

Dividing (2) by $ML/2$ and using the given expression for I_{cm} gives

$$0 = \omega L/6 - v_r \tag{5}$$

Dividing (3) by $M/2$ and again using the expression for I_{cm} gives

$$rv_0^2 = rv_f^2 + v_r^2 + (L\omega)^2/12 \tag{6}$$

Use (5) to eliminate ω from (6) and collect terms

$$rv_0^2 = rv_f^2 + 4v_r^2 \tag{7}$$

Use (4) to eliminate v_f from (7) and collect terms

$$0 = (4 + 1/r)v_r^2 - 2v_0 v_r \tag{8}$$

This is a quadratic equation for v_r. Don't be too quick to simplify it by dividing by v_r. In doing so you might miss the fact that $v_r = 0$ is in fact a valid solution of the problem, just not the one we are looking for. $v_r = 0$ corresponds to the initial condition before the collision takes place, one that obviously must satisfy all the conservation laws we have used. Factoring out this root of the quadratic equation leaves a linear equation for the other root, the one we are looking for. Solving for v_r, then using this result to find the other two unknowns gives

$$\underline{v_r = \frac{2m}{4m+M}\, v_0} \qquad \underline{\omega = \frac{6}{L}\,\frac{2m}{4m+M}\, v_0} \qquad \underline{v_f = \frac{4m-M}{4m+M}\, v_0} \tag{9}$$

The center of mass of the rod always moves to the right and the rod always rotates clockwise. However, the ball may or may not change direction. If $M \gg m$ v_r and ω approach zero and the ball simply changes its direction with no change in speed. If $M \ll m$ v_r is half of v_0, and the ball continues along its original path with little change in speed. If $M = 4m$, the ball comes to rest after the collision.

Solution, Problem 4

Equation (3) contains most of the variables. Let's use that as a starting point. First eliminate the tensions. T_1 is given directly by (1). T_2 can be found by rearranging (4) and using the result for T_1 from (1)

$$T_2 = (I/R)\dot{\omega} + T_1 = (I/R)\dot{\omega} + m_1\ddot{x}_1 \tag{8}$$

T_3 is found by rearranging (2)

$$T_3 = m_3 g - m_3\ddot{x}_3 \tag{9}$$

Substituting these expressions for the tensions into (3) gives

$$m_2 g + m_3 g - m_3\ddot{x}_3 - (I/R)\dot{\omega} - m_1\ddot{x}_1 - m_1\ddot{x}_1 = m_2\ddot{x}_2 \tag{10}$$

Use (5) to eliminate $\dot{\omega}$ and collect terms

$$(m_2 + I/R^2)\ddot{x}_2 = (m_2 + m_3)g - 2m_1\ddot{x}_1 - m_3\ddot{x}_3 \tag{11}$$

Use (6) and (7) to eliminate $\ddot{x}_2$ and $\ddot{x}_3$

$$(1/2)(m_2 + I/R^2)\ddot{x}_1 = (m_2 + m_3)g - 2m_1\ddot{x}_1 - (1/2)m_3\ddot{x}_1 \tag{12}$$

Multiply by 2, collect terms and solve for $\ddot{x}_1$

$$\underline{\ddot{x}_1 = \frac{2(m_2 + m_3)}{4m_1 + m_2 + m_3 + I/R^2}\, g} \tag{13}$$

The dimensions are correct. We are looking for an acceleration and we have a dimensionless ratio times the acceleration of gravity. The numerator of the fraction has the masses that drive the motion, those that gravity moves directly. An increase in any of these contributes to an increase in the acceleration. In the denominator we have all the masses and moments that contribute to the inertia of the system. An increase in any of these tends to slow the acceleration. Setting $m_2 = 0$ and $I = 0$ is equivalent to replacing the disk by a massless, frictionless pulley and the problem is reduced to one often used as an example in class. If m_1 and m_3 were zero, this would be equivalent to a falling yo-yo, a problem also used as an example in class.

Solution to Problem 5

If we can find an expression for T_2 in term of T_1 we can use it in (2) then eliminate it between (1) and (2). (3) and (4) taken together give

$$T_2 = (1/4)T_1 \tag{8}$$

Substituting that into (2) and multiplying through by 4 gives

$$4m_2\ddot{y}_2 = 4m_2 g - T_1 \tag{9}$$

Subtracting (1) from (9) gives

$$4m_2\ddot{y}_2 - m_1\ddot{y}_1 = 4m_2 g - m_1 g \tag{10}$$

Using (7), (6) and (5) in succession gives

$$\ddot{y}_1 = -(1/4)\ddot{y}_2 \tag{11}$$

Substituting (11) into 10 gives

$$4m_2\ddot{y}_2 + (1/4)m_1\ddot{y}_2 = (4m_2 - m_1)g \tag{12}$$

Solving for $\ddot{y}_2$ gives the final answer

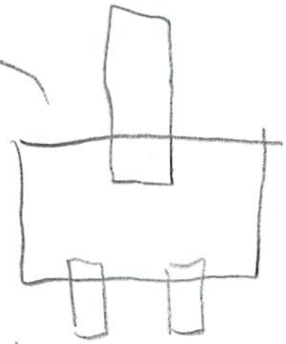
$$\underline{\ddot{y}_2 = \frac{m_2 - (1/4)m_1}{m_2 + (1/16)m_1}\, g} \tag{13}$$

The units are correct. We are looking for an acceleration and we have a dimensionless fraction times the acceleration of gravity. If $m_1 = 0$ body 2 is simply in free fall with acceleration g. In the limit $m_1 \gg m_2$ body 2 accelerates upward at 4 times the rate at which body 1 falls. These are the results we would expect on simple physical grounds.

4/2 | $\Omega I\omega = \omega$

Jared office hrs 12/8

Pset 11 #6



⊗ in

⊙ out

* do later ones - which are harder now

Car wants ↷ ⊗ in

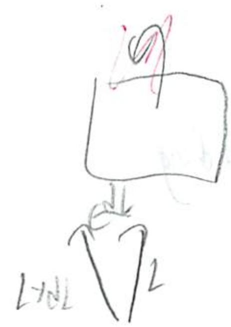
So we want wheel to go ↺ ⊙

— as a whole no external torque

So torque$_{car}$ = $-\frac{dL}{dt}$ cylinder = $-\tau$ cylinder ⊗

⊙ car turning ↺ $\quad$ τ cylinder ⊗

From top



top

dL

L

From car turning no cylinder

as turning

top view

straight

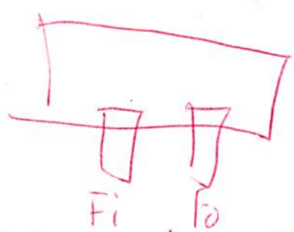
turning

So $\underrightarrow{I\omega}$

? So that is what cyliner must be moving so car goes ✗

Car turning - exerts torqu on car

TA really confused on this → meaning I am too!



b) Find ws so that $F_i = F_o$ loaded

so $F_i = F_o$

top  $dL \quad = I_w \Omega$

$= I_w \frac{v}{r}$

$\boxed{W = \frac{2 m_T L v}{m_w R^2}}$

$\cancel{F_i + F_o = m_T g}$

$\tau = I \omega \frac{v}{r}$ road exerts on car $= \frac{\frac{1}{2} m_w R^2 v}{r}$

want L spin about com

$= L(F_i + F_i)$

$\Omega = v/R$

∗ he means hori-z loading
- gave height

$= -\tau$ car exerts on road

$= -\tau$ cylinder on car

$F_i + F_o = \frac{m_T v^2}{R}$

Now solve for where =
F_i, W unknowns
solve for

$F_i = \frac{m_T v^2}{R}$

$\frac{m_w R^2 v}{2r} \omega = \frac{L m_T v^2}{R}$

$\boxed{\omega = \frac{2 r L m_T v}{m_w R^2}}$

Review ~~Review rolling w/o slipping~~ also

rolling w/o slipping

bolling ball

12/8/2009

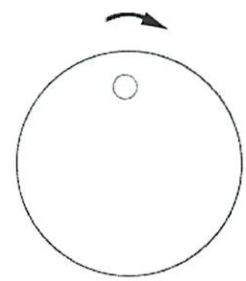
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

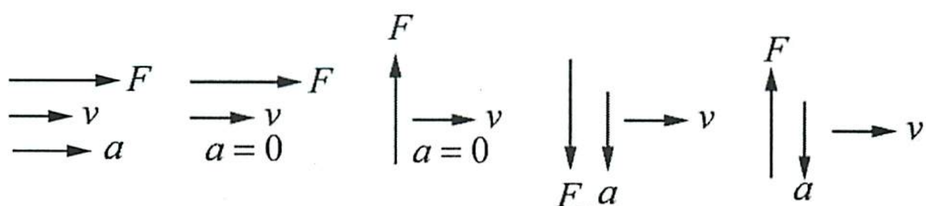
Physics 8.01 **Fall Term 2007**

Practice Problems Final Exam

Part One: Concept Questions

Problem 1: A small cylinder rests on a circular turntable, rotating at a constant speed as illustrated in the diagram below. Which of the vectors 1-5 below best describes the velocity, acceleration and net force acting on the cylinder at the point indicated in the diagram?



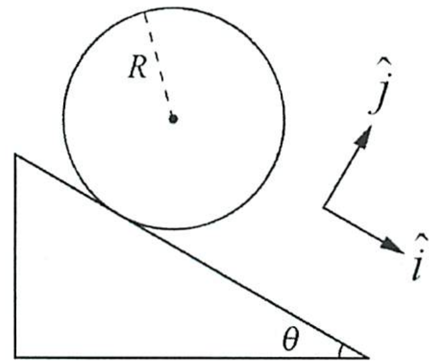


1 2 3 4 5

✓ correct

Review roll w/o slipping

Problem 2: A hollow cylinder starts from rest and rolls without slipping down an incline.



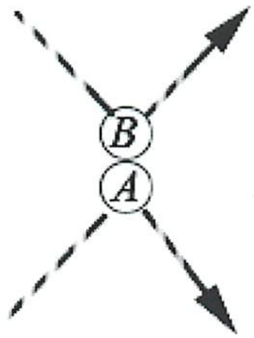
Which of the following best describes the force of friction?

1. The force of friction is kinetic friction, with $f = \mu_k N$.
2. The force of friction is static friction, with $f = \mu_s N$.
3. The force of friction is static friction, with f equal to the force necessary to prevent slipping, up to a maximum of $f_{\max} = \mu_s N$. ✓ Correct
4. The friction is zero because the cylinder rolls without slipping.

wish I could go back to AP Physics test - prob would do a lot better

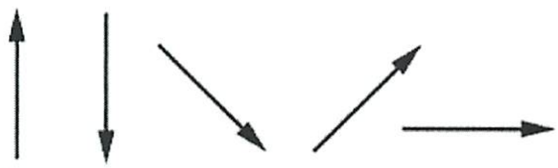
interesting to see how much better

Problem 3: The figure below depicts the path of two colliding steel balls, *A* and *B*.



Which of the arrows 1-5 best represents the impulse applied to ball *B* by ball *A* during the collision?

Ball B change in Y momentum no change in x

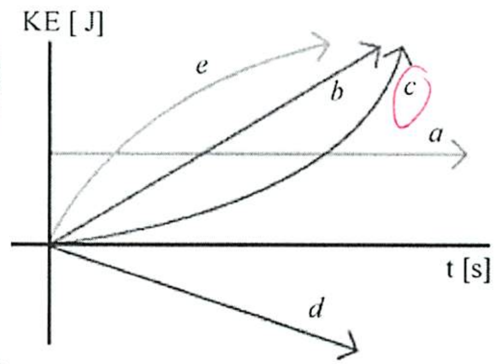


1 2 3 ~~4~~ 5

what I thought - should stick w/ intuition

Problem 4: An object is dropped to the surface of the earth from a height of 10m. Which of the following sketches best represents the kinetic energy of the object as a function of time as it approaches the earth if friction can be neglected?

*h ↓ quadratically



− mgh (constant)

1. a
2. b
3. c
4. d
5. e

↑ increasing $\frac{1}{2}mv^2$ ↑ true

top KE = 0 PE = 1

bottom KE = 1 PE = 0

but is transfer linear

thought of that but thought it was = to $\frac{1}{2}mv^2 + mgh = 0$ or is it 1

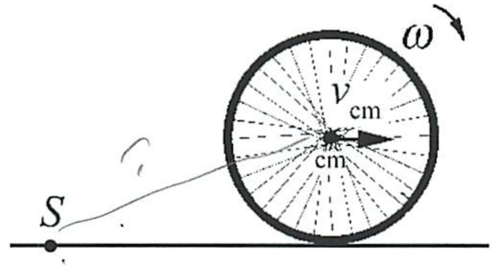
* yeah its 1 - depends where define

$$-mgh = \frac{1}{2}mv^2$$

don't cram
- learn it for real

Problem 5: A bicycle wheel is initially spinning with non-zero angular speed about the center of mass. The wheel is lowered to the ground without bouncing. As soon as the wheel touches the level ground, the wheel starts to accelerate forward until it begins to roll without slipping. S denotes a point on the ground along the line of contact between the wheel and the surface.

never really got this back →



bolling ball

force due weight + normal force) = out

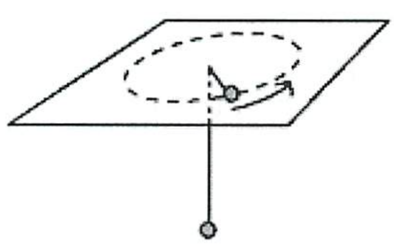
From the moment the wheel touches the ground until it just begins to roll without slipping, the angular momentum is - ? constant - don't think so

- friction force at contact pt ↑ does apply a torque only around center

What is important here →

1. constant about the wheel's center of mass.
2. constant about the point S. $r \times mv$ about this point
3. constant about both the wheel's center of mass and the point S.
4. changing about both the wheel's center of mass and the point S. about Com of Com

Problem 6: A puck of mass M is moving in a circle at uniform speed on a frictionless table as shown below. The puck is attached to a massless, frictionless string that passes through a hole in the table and which is in turn attached to a suspended bob, also of mass M, at rest below the table. What is the magnitude of the centripetal acceleration of the moving puck?



1. Less than g.
2. Equal to g.
3. Greater than g.
4. Zero
5. Insufficient information

What is happening to string
Never good at these problems

tension = weight
mg

$mg = ma$
$a = g$

Markowitz

Office hrs

measure w/ respect to this

$\tau_f = 0$

at $\underline{\text{That pt}}$

$\cancel{L = r \times m \vec{v}_{cm}}$

$\cancel{Rmv}$

$\tau_f = 0$ ← since perp

$$\tau_g = (d\hat{x} + R\hat{y}) \overset{r}{} \overset{\times}{\times} \overset{F}{(-mg\,y)} = -mgd\,\hat{z}$$

$-mgd\,\hat{z}$ ← vector cross stuff

$$N = (d\,x + R\,y) \times (mg\,y) = mg\,d\,\hat{z}$$

cancle

so no torque

torque $= 0$ $\quad L =$ constant

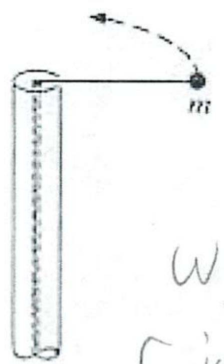
---

about com

- but then related to point $\underline{\text{moving}}$

this would not apply

Problem 7: A tetherball is attached to a post by a string. The string passes through a hole in the center of the post at the top. The string is gradually shortened by drawing it through the hole. Ignore gravity. Until the ball hits the post,



ω is changing

r is changing

1. the energy and angular momentum about the center of the post are constant.

   $I\omega$

2. the energy of the ball is constant but the angular momentum about the center of the post is changing. what is Changing here?

3. both the energy and the angular momentum about the center of the post are changing.

4. the energy of the ball is changing but the angular momentum about the center of the post is constant.

passes through center

no net torque → L constant

but displacement of ball inward

- work

- so E changes

force radially

$W = \Delta E = \int \vec{F} \cdot d\vec{r}$

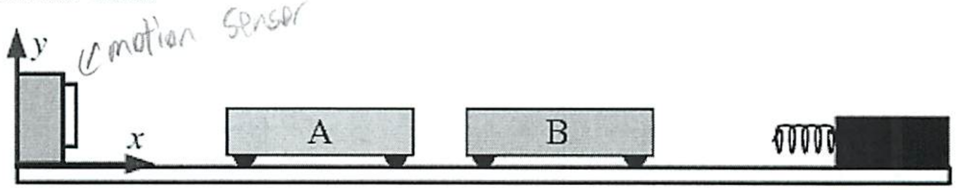
radially inward

not strictly perpendicula

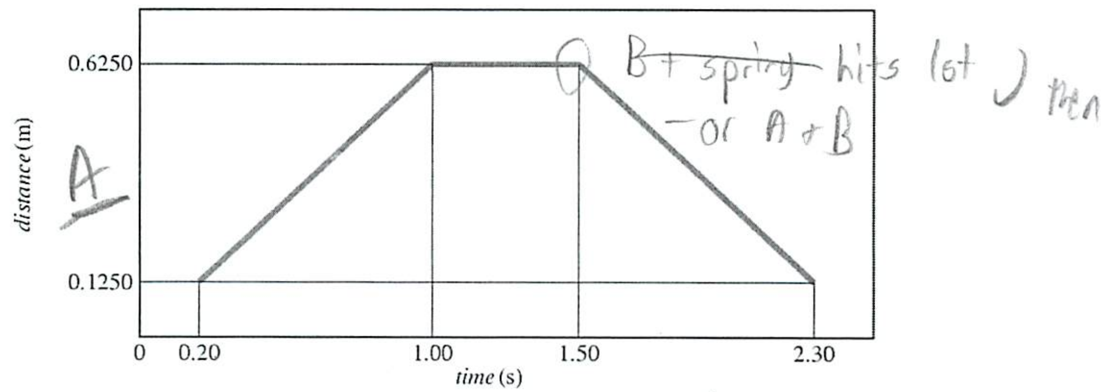
thus is net work

I think the more physics I know the more likely I am to get this wrong than thinking intuitively

Problem 8: The figure below shows the experimental setup to study the collision between two carts.



In the experiment cart A rolls to the right on the level track, away from the motion sensor at the left end of the track. The graph below shows the distance from the motion sensor to cart A as a function of time.

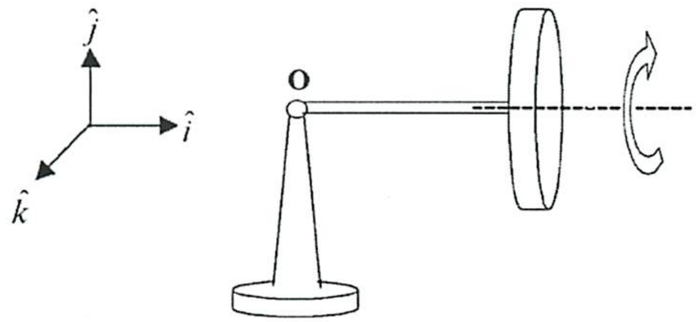


Since graph is d to A

What objects collide when $t = 1.5$ s?

1. Cart B and the spring.
2. Cart B and the motion sensor.
3. Carts A and B.
4. Cart A and the spring.
5. Cart A and the motion sensor.

Problem 9: A gyroscope has a wheel at one end of an axle, which is pivoted at point **O** as shown in the figure. The wheel spins about the axle in the direction shown by the arrow in the figure. At the moment shown in the figure, the axle is horizontal and in the plane of the page. Let $\vec{\mathbf{L}}$ be the angular momentum of the gyroscope about the center of mass of the gyroscope. You may ignore the mass of the axle and assume the spin angular velocity is much greater than the precessional angular velocity.



The direction of the vector $d\vec{\mathbf{L}}/dt$ of the gyroscope at the moment shown in the figure is:

1. $+\hat{i}$ direction.
2. $-\hat{i}$ direction.
3. $-\hat{j}$ direction.
4. $+\hat{k}$ direction.
5. $-\hat{k}$ direction.

ΔL ?? L $\Delta L + \Delta L$

↑ into page $R \times mg$

what is direction ?

↑ right what I thought ✓

↑ - shall know more about

oh duh thought backwards too but circled wrong thing since did not look at scale

but still unsure about qu like this

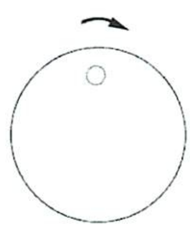
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

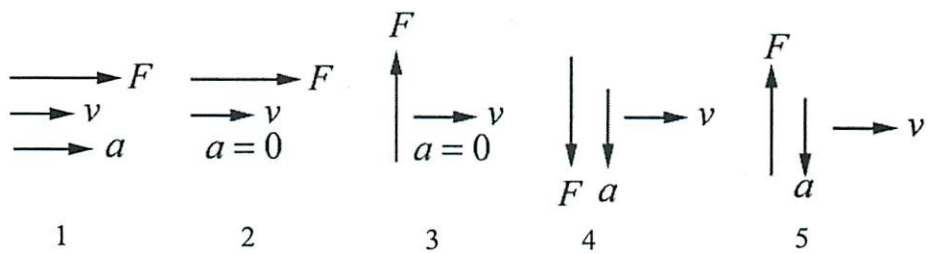
Physics 8.01 **Fall Term 2008**

Practice Problems Final Exam: Solutions

Part One: Concept Questions

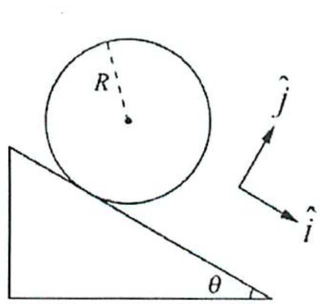
Problem 1: A small cylinder rests on a circular turntable, rotating at a constant speed as illustrated in the diagram below. Which of the vectors 1-5 below best describes the velocity, acceleration and net force acting on the cylinder at the point indicated in the diagram?





Answer: 4: (Turntables went out when Jerry Garcia did, but that's not part of the problem.) The velocity is to the right in the figure; the acceleration and the force are inward, down in the figure.

Problem 2: A hollow cylinder starts from rest and rolls without slipping down an incline.

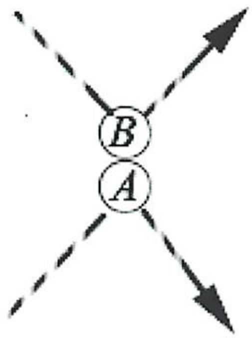


Which of the following best describes the force of friction? The magnitude of the normal force is N.

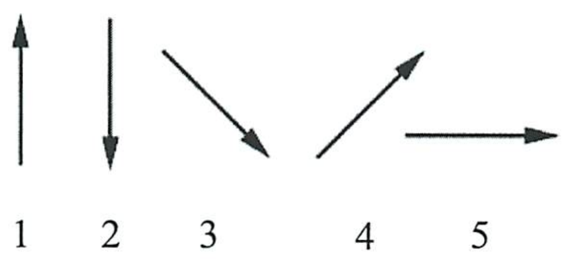
1. The force of friction is kinetic friction, with $f = \mu_k N$.
2. The force of friction is static friction, with $f = \mu_s N$.
3. The force of friction is static friction, with f equal to the force necessary to prevent slipping, up to a maximum of $f_{max} = \mu_s N$.
4. The friction is zero because the cylinder rolls without slipping.

Answer: 3. For rolling without slipping, the friction force cannot be kinetic. The static friction force must be less than the product of the coefficient of static friction μ_s and the magnitude N of the normal force.

Problem 3: The figure below depicts the paths of two colliding steel balls, A and B.

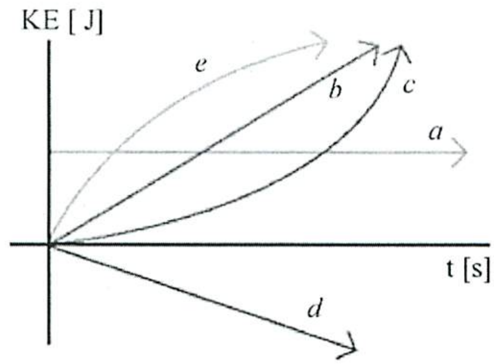


Which of the arrows 1-5 best represents the impulse applied to ball B by ball A during the collision?



Answer: 1; Ball B has changed its momentum in the upward direction in the figure, and as far as the figure can show, there is no change in its horizontal (rightward) velocity.

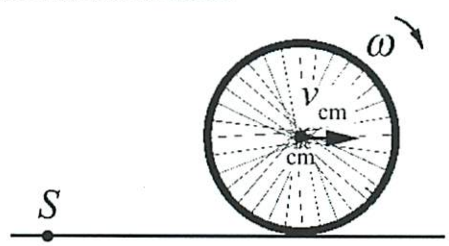
Problem 4: An object is dropped to the surface of the earth from a height of 10m. Which of the following sketches best represents the kinetic energy of the object as a function of time as it approaches the earth if friction can be neglected? Take $t = 0$ as the time when the object is dropped.



1. a 2. b 3. c 4. d 5. e

Answer: 3 c. The object manifestly has no kinetic energy at $t = 0$, and increases at a rate proportional to the square of the time t.

Problem 5: A bicycle wheel is initially spinning with non-zero angular speed about the center of mass. The wheel is lowered to the ground without bouncing. As soon as the wheel touches the level ground, the wheel starts to accelerate forward until it begins to roll without slipping. In the figure below, S denotes a point on the ground along the line of contact between the wheel and the surface.

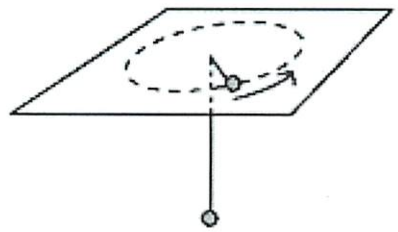


From the moment the wheel touches the ground until it just begins to roll without slipping, the angular momentum is

1. constant about the wheel's center of mass.
2. constant about the point S.
3. constant about both the wheel's center of mass and the point S.
4. changing about both the wheel's center of mass and the point S.

Answer: 2. The forces on the wheel are its weight and the normal force, and the friction force at the contact point. The weight and the normal force are equal in magnitude and opposite in direction, and hence exert no net torque about any point. The friction force, directed horizontally to the left in the figure, exerts no torque about the point S, but does exert a torque about the wheel's center.

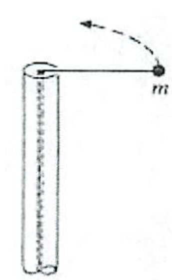
Problem 6: A puck of mass M is moving in a circle at uniform speed on a frictionless table as shown below. The puck is attached to a massless, frictionless string that passes through a hole in the table and which is in turn attached to a suspended bob, also of mass M, at rest below the table. What is the magnitude of the centripetal acceleration of the moving puck?



1. Less than g.
2. Equal to g.
3. Greater than g.
4. Zero.
5. Insufficient information.

Answer: 2 The puck is given as moving in a circle, and hence the suspended bob is not moving, and hence the tension in the string is the weight Mg of the bob. It then follows that the magnitude of the acceleration of the puck is $Mg/M = g$.

Problem 7: A tetherball is attached to a post by a string. The string passes through a hole in the center of the post at the top. The string is gradually shortened by drawing it through the hole. Ignore gravity. Until the ball hits the post,



1. the energy and angular momentum about the center of the post are constant.

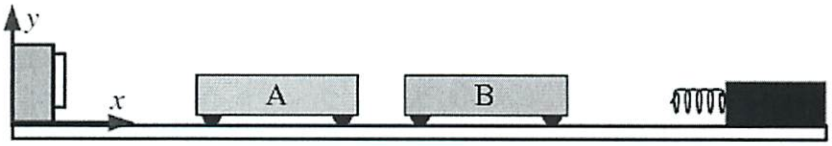
2. the energy of the ball is constant but the angular momentum about the center of the post is changing.

3. both the energy and the angular momentum about the center of the post are changing.

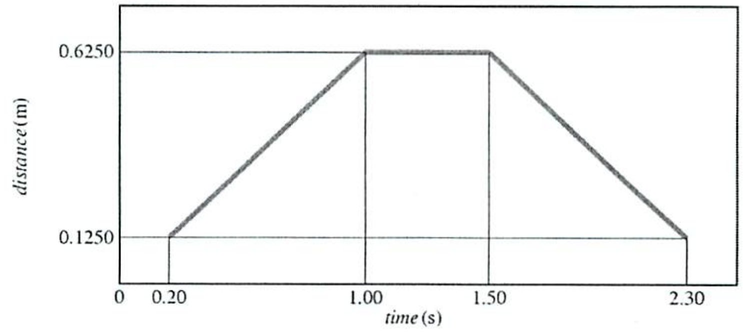
4. the energy of the ball is changing but the angular momentum about the center of the post is constant.

Answer: 4. The crucial point in this problem is that the string passes through the center of the post, and hence there is no net torque (in the absence of gravity) and hence the angular momentum about the center of the post is constant. The displacement of the ball will have an inward component, parallel to the string, and hence the string does work and the energy changes.

Problem 8: The figure below shows the experimental setup to study the collision between two carts.



In the experiment cart A rolls to the right on the level track, away from the motion sensor at the left end of the track. The graph below shows the distance from the motion sensor to cart A as a function of time.



What objects collide when $t = 1.5$ s?

1. Cart B and the spring.

2. Cart B and the motion sensor.

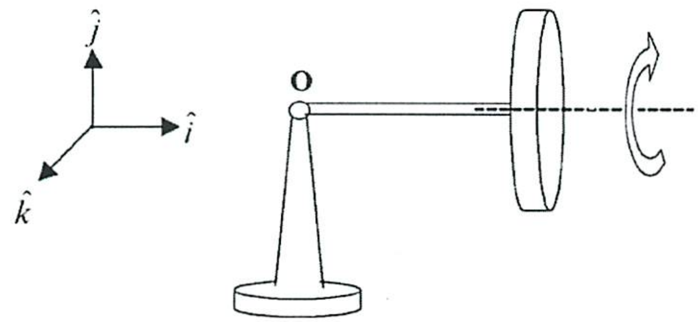
3. Carts A and B.

4. Cart A and the spring.

5. Cart A and the motion sensor.

Answer: 3. During the time interval $1.0\text{ s} < t < 1.5\text{ s}$, cart A is not moving, and only begins moving to the left (indicated by the negative slope of the graph in the figure) only after colliding with cart B, which has rebounded from the spring.

Problem 9: A gyroscope has a wheel at one end of an axle, which is pivoted at point **O** as shown in the figure. The wheel spins about the axle in the direction shown by the arrow in the figure. At the moment shown in the figure, the axle is horizontal and in the plane of the page. Let $\vec{\mathbf{L}}$ be the angular momentum of the gyroscope about the center of mass of the gyroscope. You may ignore the mass of the axle and assume the spin angular velocity is much greater than the precessional angular velocity.



The direction of the vector $d\vec{\mathbf{L}}/dt$ of the gyroscope at the moment shown in the figure is:

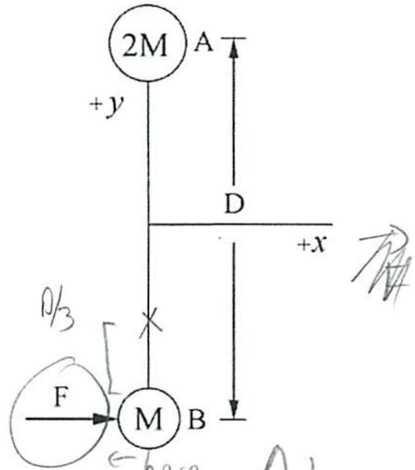
1. $+\hat{i}$. 2. $-\hat{i}$. 3. $+\hat{j}$. 4. $+\hat{k}$. 5. $-\hat{k}$.

Answer: 5, the $-\hat{\mathbf{k}}$ direction (or, $-\hat{k}$ in the notation of the figure). By the right-hand rule, the angular momentum is radially inward, the $-\hat{\mathbf{i}}$ direction in the figure. Taking the torque about $\mathbf{O}$, the net torque is due to the weight of the gyroscope, and if $\vec{\mathbf{R}}$ is the position vector from point $\mathbf{O}$ to the center of the wheel, $\vec{\mathbf{R}} \times m\vec{\mathbf{g}}$ is into the page of the figure, the $-\hat{\mathbf{k}}$ direction. Although not part of this problem, the gyroscope will precess in such a way as to move out of the page of the figure.

Part Two: Analytic problems.

Problem 1

Two point-like objects are located at the points A, and B, of respective masses $M_A = 2M$, and $M_B = M$, as shown in the figure below. The two objects are initially oriented along the y-axis and connected by a rod of negligible mass of length D, forming a rigid body. A force of magnitude $F = |\vec{\mathbf{F}}|$ along the x direction is applied to the object at B at $t = 0$ for a short time interval Δt. Neglect gravity. Give all your answers in terms of M and D as needed.



* seperate momentum + angular momentum

— just like other practice problem

a. Describe qualitatively in words how the system moves after the force is applied: direction, translation and rotation.
   does both ↻ rotates → translates ✓

see how much of this I can remember

b. How far is the center of mass of the system from the object at point B?
   $D/2$ - masses not even $\boxed{D/3}$

c. What is the direction and magnitude of the linear velocity of the center-of-mass after the collision?
   → $\frac{mV_i + Fdt}{m} = V_f$ → $\frac{0 + F\Delta t}{3M}$ 1 more step

d. What is the magnitude of the angular velocity of the system after the collision?
   see back

e. Is it possible to apply another force of magnitude F along the positive x direction to prevent the system from rotating? Does it matter where the force is applied?

f. Is it possible to apply another force of magnitude F in some direction to prevent the center of mass from translating? Does it matter where the force is applied?

momentum $p = Mv$ $\quad F = ma$

force and momentum

$W = Vr$? or similar

$mV_i + Fdt = mV_f$ $\quad$ force · time = momentum ✓ looked up

$\Delta p = Fdt$

Angular momentum

$L = I\omega$ $\quad \tau = \frac{dL}{dt}$ $\quad \tau = \frac{dI\omega}{dt}$ ↓

$I\omega_i + \tau = I\omega_f$

can you solve for torque

Consider collission in COM frame

$\tau = F\Delta t \; \frac{2}{3}D$

Remembor τ where it hits

$\omega_f = \frac{I\omega_i + \tau}{\omega_f}$

$I_{cm} = 2M \frac{D}{3}^2 + M \frac{2D}{3}^2$ ← know how to manually calc I

$\frac{2}{3}MD^2$

$\frac{0+\Delta L}{I} = \omega_f = \frac{F\Delta t \; \frac{2}{3}D}{\frac{2}{3}MD^2} = \frac{F\Delta t}{MD}$

*but don't forget where it hits

- getting closer

e) ~~Yes~~ ← in that direction

But where matters - masses ≠

- what is it MR^2

or what matters?

Guess would want it $\frac{1}{3}$ up from COM vs directly at

$\frac{2}{3}D$ from COM

but this is not on system

- isn't it top ball?

oh

I drew COM wrong

- makes more sense

f.) To stop from translating

- ~~apply anywhere~~

- right there at pt B ←

no I was originally right

can apply anywhere

duh $F = \frac{mv^2}{R}$

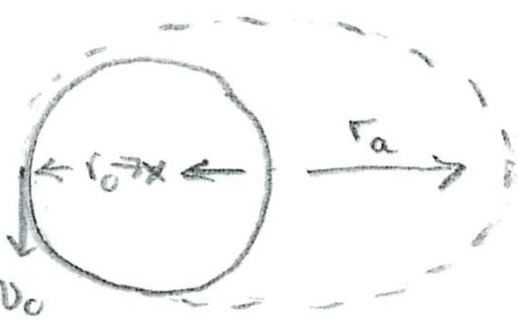
- got formula sheet
- but need to know they exist

memorize

$$\frac{Gm_1m_2}{r^2} = \frac{mV^2}{r}$$

Problem 2

A satellite of mass m_s is initially in a circular orbit of radius r_0 around the earth. The earth has mass $m_e >> m_s$ and radius R_e. Let G denote the universal gravitational constant. Express all your answers in terms R_e, m_e, m_s, G, and r_0 as needed.



a) Find an expression for the speed v_0 of the satellite when it is in the circular orbit.

b) Find an expression for the mechanical energy E_0 of the satellite when it is in the circular orbit.

= total energy

As a result of an orbital maneuver the satellite trajectory is changed to an elliptical orbit. This is accomplished by firing a rocket for a short time interval thus increasing the tangential speed of the satellite. The apogee (farthest distance from earth) of the elliptical orbit is three times the closest approach (perigee),

oh jeeze did not study these terms

$$r_a = 3r_p = 3r_0$$

look at ~~graph~~/pic/diagram

think am pulling my self to understanding - just do problems

c) Use conservation of energy and angular momentum for the elliptic orbit to find an expression for the speed of the satellite, v_p, immediately after the rocket has finished firing.

a) $\frac{Gm_em_s}{r^2} = \frac{m_sV^2}{r}$

$\cdot r \qquad \cdot r$

$\frac{Gm_em_s}{r} = m_sV^2$

$\sqrt{\frac{Gm_em_s}{rm_s}}$

$V = \sqrt{\frac{Gm_e}{r}}$

b) $\frac{1}{2}m_sV_s^2 - G\frac{m_sm_e}{r}$

$\frac{1}{2}m_s\sqrt{\frac{Gm_e}{r}}^2 - \frac{Gm_sm_e}{r}$

$E = -\frac{1}{2}m_s\frac{Gm_e}{R}$

~~ended up same thing~~ coincidence but minus

c) No clue
- skipping ellipse problems
- and prof said only ones would be conservation
- does this qualify? yeah

$r_p = r_0$
$r_a = 3r_0$

$r_p v_p = r_a v_a$

$r_0 v_p = 3r_0 v_a$

$v_a = \frac{v_p}{3}$

$$\frac{1}{2} m_s v_p^2 - G\frac{m_s m_e}{r_0} = \frac{1}{2} m_s v_a^2 - \frac{G m_s m_e}{r_a}$$

$$= \frac{1}{2} m_s \left(\frac{v_p}{3}\right)^2 - \frac{G m_s m_e}{3r_0}$$

$$\frac{4}{9} v_p^2 = \frac{2}{3} G \frac{m_e}{r_0}$$

$$v_p = \sqrt{\frac{3}{2} \frac{G m_e}{r_0}}$$

Check $v_p > v_0$

$E_f = -G m_s m_e / 4r_0$

$-G m_s m_e / A$

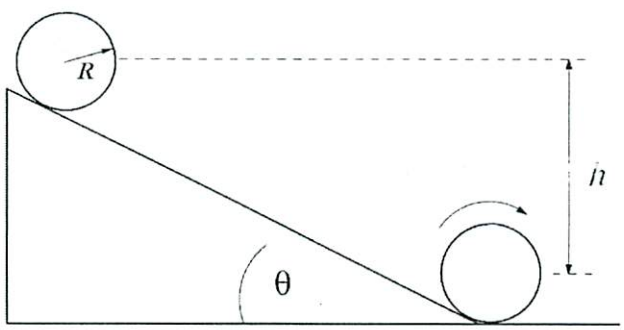
$A = r_0 + 3r_0$

Should have done this
as course was going

Problem 3

A hollow cylinder of outer radius R and mass M with moment of inertia about the center of mass $I_{cm} = MR^2$ starts from rest and moves down an incline tilted at an angle θ from the horizontal. The center of mass of the cylinder has dropped a vertical distance h when it reaches the bottom of the incline. Let g denote the gravitational constant. The coefficient of static friction between the cylinder and the surface is μ_s. The cylinder rolls without slipping down the incline. The goal of this problem is to find an expression for the smallest possible value of μ_s such that the cylinder rolls without slipping down the incline plane.

- energy?
- no friction



a) Draw a free body force diagram showing all the forces acting on the cylinder.

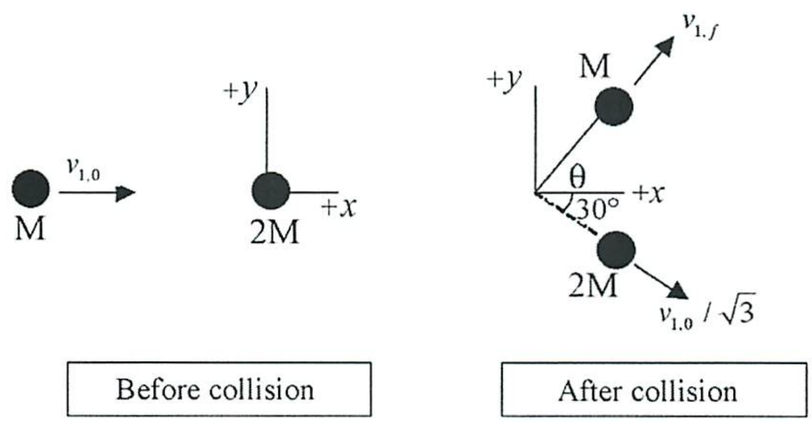
b) Find an expression for both the angular and linear acceleration of the cylinder in terms of M, R, g, θ and h as needed.

c) What is the minimum value for the coefficient of static friction μ_s such that the cylinder rolls without slipping down the incline plane? Express your answer in terms of M, R, g, θ and h as needed.

e) What is the magnitude of the velocity of the center of mass of the cylinder when it reaches the bottom of the incline? Express your answer in terms of M, R, g, θ and h as needed.

Problem 4

Particle 1 of mass M collides with particle 2 of mass $2M$. Before the collision particle 1 is moving along the x-axis with a speed $v_{1,0}$ and particle 2 is at rest. After the collision, particle 2 is moving in a direction 30^o below the x-axis with a speed $v_{1,0}/\sqrt{3}$. (Note: $\sin 30^o = 1/2$.) Particle 1 is moving upward at an angle θ to the x-axis and has a speed $v_{1,f}$.



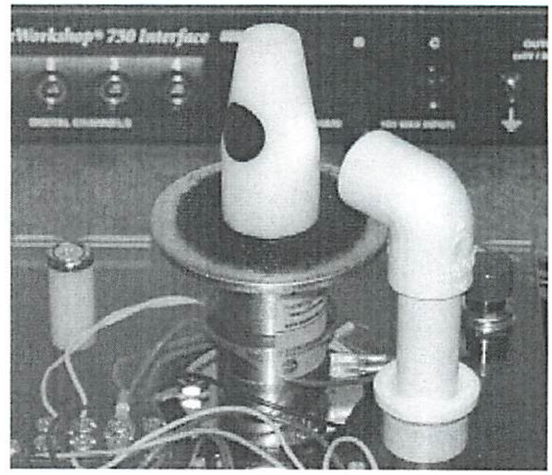
a. What is the magnitude of the velocity $v_{1,f}$ of particle 1 after the collision?

b. What is the angle θ that particle 1 makes with the x-axis after the collision?

c. Is the collision elastic or inelastic? Justify your answer.

Problem 5

In the angular momentum experiment, shown to the right, a washer is dropped smooth side down onto the spinning rotor.

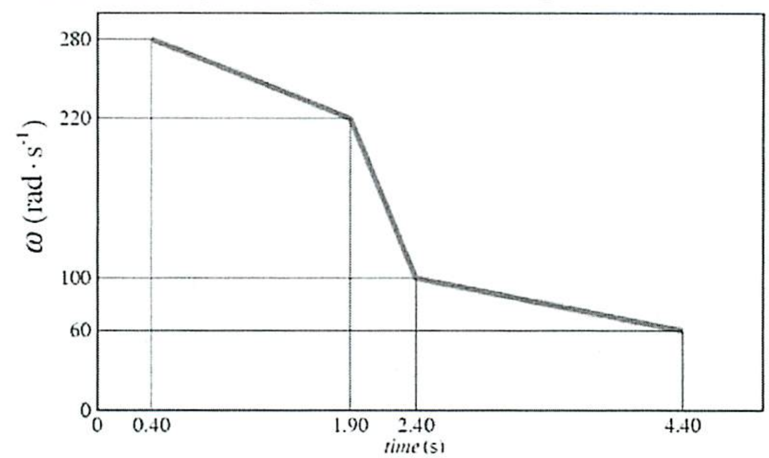


The graph below shows the rotor angular velocity ω (rad $\cdot$ s^{-1}) as a function of time.

Assume the following:

- The rotor and washer have the same moment of inertia I.
- The friction torque $\vec{\tau}_f$ on the rotor is constant during the measurement.

Note: express all of your answers in terms of I and numbers you obtain from the graph. Be sure to give an analytic expression prior to substituting the numbers from the graph.



a) Find an expression for the magnitude $\left|\vec{\tau}_f\right|$ in terms of I and numbers you obtain from the graph.

b) How much mechanical energy is lost to bearing friction during the collision (between $t = 1.90$ s and $t = 2.40$ s)?

c) How much mechanical energy is lost to friction <u>between the rotor and the washer</u> during the collision (between $t = 1.90$ s and $t = 2.40$ s)?

Problem 6

A demonstration gyroscope wheel is constructed from a bicycle wheel by removing the tire, wrapping lead wire around the rim, and taping it into place. The wheel has a radius R and the mass is m. You may assume that the entire mass is concentrated on the rim. A shaft is connected to the axle and projects a distance d at each side of the center of the wheel. A person holds the ends of the shaft in two hands. The shaft is horizontal and the wheel is spinning about the shaft with angular velocity ω_s.

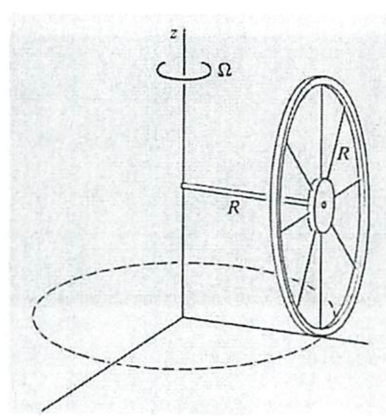
a) Find the magnitude and direction of the force each hand exerts on the shaft when the shaft is at rest.

b) Find the magnitude and direction of the force each hand exerts on the shaft when the shaft is rotating in a horizontal plane about its center with angular velocity Ω.

c) At what rate must the shaft rotate in order that it may be suspended at one end only? Draw a diagram showing which the relationship between which side the shaft is supported and which way will it rotate in the horizontal plane.

Problem 7

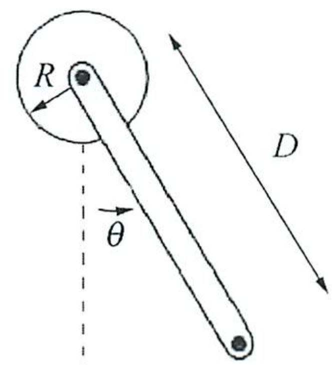
A thin hoop of mass m and radius R rolls without slipping about the z axis. It is supported by an axle of length R through its center. The hoop circles around the z axis with angular speed Ω. (Note: the moment of inertia of a hoop for an axis along its diameter is $(1/2)mR^2$.)



a) What is the instantaneous angular velocity $\vec{\omega}$ of the hoop? Specify the direction and magnitude.
b) What is the angular momentum $\vec{\mathbf{L}}$ of the hoop about a point where the axle meets the z axis? Is $\vec{\mathbf{L}}$ parallel to $\vec{\omega}$?

Problem 8:

A rigid body is composed of a uniform disk (mass m, radius R) and a uniform rod (mass m, length D) which is rigidly fixed to the center of the disk. This body is pivoted about the center of the disk around a horizontal axis which is perpendicular to the plane of the page. Assume the pivot is frictionless and the acceleration due to gravity is g .



a) Find the moment of inertia I_P about the pivot point.

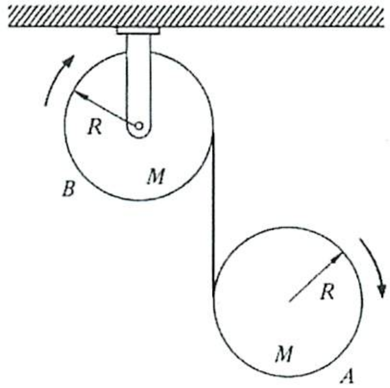
b) Suppose the pendulum is swinging freely back and forth. Write down an expression for the angular acceleration about the pivot point. You may leave your answer is terms of m, g, R, I_P, D and the angle θ as needed .

c) Suppose the angle θ is small throughout the motion. That is, you may assume $\sin\theta \simeq \theta$ and $\cos\theta \simeq 1$. What is the period for this pendulum? Express your answer in terms of m, R, D, g and I_P.

d) Now suppose there is no restriction on the value of θ (it can be large). What is the minimum angular velocity $\omega_{\min}$ which the pendulum should have at the bottom of its swing so that the pendulum can revolve completely around the pivot point?

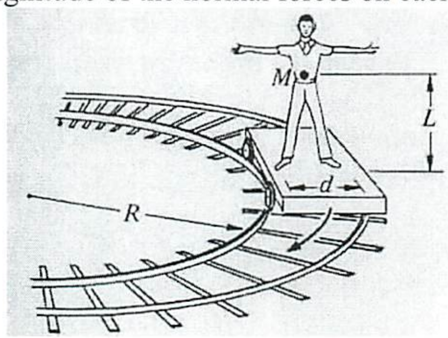
Problem 9

A drum A of mass m and radius R is suspended from a drum B also of mass m and radius R, which is free to rotate about its axis. The suspension is in the form of a massless metal tape wound around the outside of each drum, and free to unwind. Gravity is directed downwards. Both drums are initially at rest. Find the initial acceleration of drum A, assuming that it moves straight down.



Problem 10

A person of mass m is standing on a railroad car which is rounding an unbanked turn of radius R at a speed v. His center of mass is at a height of L above the car midway between his feet which are separated by a distance of d. The man is facing the direction of motion. What is the magnitude of the normal forces on each foot?



Problem 11

A proton makes a head-on collision with an unknown particle at rest. The proton rebounds straight back with $4/9$ of its initial kinetic energy. Find the ratio of the mass of the unknown particle to the mass of the proton, assuming that the collision is elastic.

Problem 12

A particle of mass m moves under an attractive central force of magnitude $F = br^3$. The angular momentum is equal to L.

a) Find the effective potential energy and make sketch of effective potential energy as a function of r.

b) Indicate on a sketch of the effective potential the total energy for circular motion.

c) The radius of the particle's orbit varies between r_0 and $2r_0$. Find r_0.

Problem 13

A wrench of mass m is pivoted a distance l_{cm} from its center of mass and allowed to swing as a physical pendulum. The period for small-angle-oscillations is T.

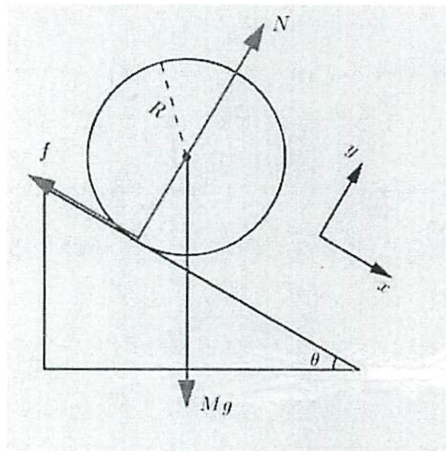
a) What is the moment of inertia of the wrench about an axis through the pivot?

b) If the wrench is initially displaced by an angle θ_0 from its equilibrium position, what is the angular speed of the wrench as it passes through the equilibrium position?

(3)

Solutions:

a) The forces are the weight, the normal force and the contact force.



b) With the coordinates system shown, Newton's Second Law, applied in the x- and y-directions in turn, yields

$$Mg\sin\theta - f = Ma$$
$$N - Mg\cos\theta = 0.$$

The equations above represent two equations in three unknowns, and so we need one more relation; this will come from torque considerations.

Of course, any point could be used for the origin in computing torques, but the "obvious" choice of the center of the cylinder turns out to make things easiest (judgment call, of course). Then, the only force exerting a torque is the friction force, and so we have

$$f\,R = I_{\text{cm}}\alpha = M\,R^2\left(a/R\right) = M\,R\,a$$

where $I_{\text{cm}} = M\,R^2$ and the kinematic constraint for the no-slipping condition $\alpha = a/R$ have been used. This leads to $f = M\,a$, and inserting this into the force equation gives the two relations

$$f = \frac{1}{2}Mg\sin\theta$$
$$a = \frac{1}{2}g\sin\theta.$$

3 more

c) For rolling without slipping, we need $f < \mu_s N$, so we need, using the second force equation above,

$$\mu_s > \frac{1}{2}\tan\theta .$$

d) The cylinder rolls a distance $L = h/\sin\theta$ down the incline, and the speed v_f at the bottom is related to the acceleration found in part (b) by

$$\begin{aligned} v_f^2 &= 2aL = 2\left(\frac{1}{2}g\sin\theta\right)(h/\sin\theta) \\ &= gh. \end{aligned}$$

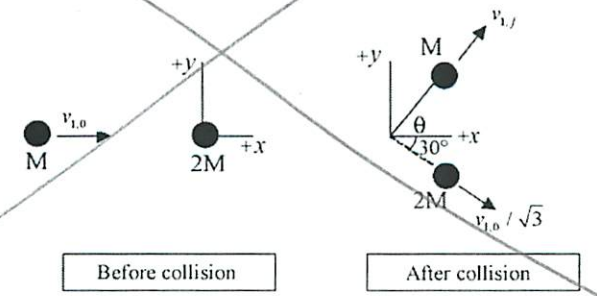
This result can and should be checked by energy conservation (for rolling without slipping, the friction force does no mechanical work). For the given moment of inertia, the final kinetic energy is

$$\begin{aligned} K_f &= \frac{1}{2}M v_f^2 + \frac{1}{2}I_{cm}\omega_f^2 \\ &= \frac{1}{2}M v_f^2 + \frac{1}{2}MR^2(v_f/R)^2 \\ &= M v_f^2, \end{aligned}$$

and setting the final kinetic energy equal to the loss of gravitational potential energy leads to the same result for the final speed.

Problem 4

Particle 1 of mass M collides with particle 2 of mass 2M. Before the collision particle 1 is moving along the x-axis with a speed $v_{1,0}$ and particle 2 is at rest. After the collision, particle 2 is moving in a direction 30° below the x-axis with a speed $v_{1,0}/\sqrt{3}$ and particle 1 is moving upward at an angle θ to the from the x-axis with speed $v_{1,f}$. (Note: $\sin 30° = 1/2$, $\cos 30° = \sqrt{3}/2$.)



4

Problem

a. What is the speed $v_{1,f}$ of particle 1 after the collision?

b. What is the angle θ that particle 1 makes with the x-axis after the collision?

c. Is the collision elastic or inelastic? Justify your answer.

Solutions:

We are not given that the collision is elastic, but in the absence of external forces that would change the momentum, the vector momentum is the same before and after the collision. Using the coordinate directions as given in the figure, the x- and y-components of momentum before and after the collision are:

$$\begin{aligned}
p_{x,0} &= M\,v_{1,0} \\
p_{x,f} &= M\,v_{1,f}\cos\theta + 2M\left(v_{1,0}/\sqrt{3}\right)\cos 30^\circ \\
&= M\,v_{1,f}\cos\theta + M\,v_{1,0} \\
p_{y,0} &= 0 \\
p_{y,f} &= M\,v_{1,f}\sin\theta - 2M\left(v_{1,0}/\sqrt{3}\right)\sin 30^\circ \\
&= M\,v_{1,f}\sin\theta - M\left(v_{1,0}/\sqrt{3}\right)
\end{aligned}$$

where $\sin 30^\circ = 1/2$, $\cos 30^\circ = \sqrt{3}/2$ have been used. Setting initial and final x-components of momentum equal and canceling the common factor of M,

$$\begin{aligned}
v_{1,0} &= v_{1,f}\cos\theta + v_{1,0} \\
0 &= v_{1,f}\cos\theta.
\end{aligned}$$

Setting initial and final y- components of momentum equal and canceling the common factor of M,

$$\begin{aligned}
0 &= v_{1,f}\sin\theta - \frac{v_{1,0}}{\sqrt{3}} \\
\frac{v_{1,0}}{\sqrt{3}} &= v_{1,f}\sin\theta.
\end{aligned}$$

All parts of the problem involve simultaneous solution of the second equations in each of the two sets, one from each momentum component found above. The method presented here follows the ordering of the parts of the problem.

a) Squaring both equations and adding, using $\cos^2\theta + \sin^2\theta = 1$, yields

$$v_{1,f}^2 = \frac{v_{1,0}^2}{3}$$

$$v_{1,f} = \frac{v_{1,0}}{\sqrt{3}}.$$

b) The second equation in found from considering the x-component of momentum gives $\cos\theta = 0$, $\theta = 90°$ immediately.

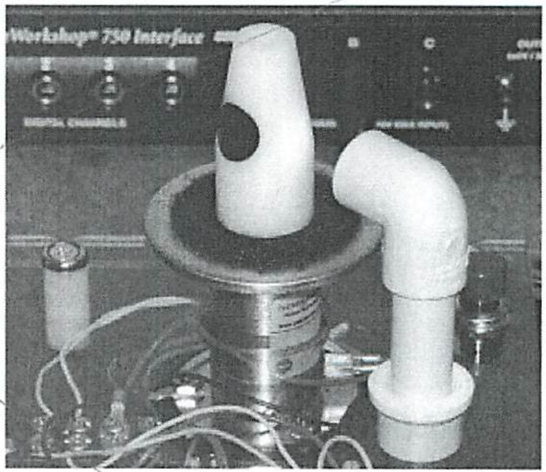
It should be noted that the angle θ, the result of part b), can be found immediately by noting, as found above, that the incident particle has no x- component of momentum after the collision, and hence must be moving perpendicular to the original direction of motion. Then, using $\sin\theta = 1$ in the above gives $v_{1,f}$ quite readily.

c) The initial kinetic energy is $K_i = (1/2)Mv_{1,0}^2$ and the final kinetic energy is

$$\begin{aligned} K_f &= \frac{1}{2}Mv_{1,f}^2 + \frac{1}{2}(2M)v_{0,f}^2 \\ &= \frac{1}{2}M\left(\frac{v_{1,0}}{\sqrt{3}}\right)^2 + \frac{1}{2}(2M)\left(\frac{v_{1,0}}{\sqrt{3}}\right)^2 \\ &= \frac{1}{2}Mv_{1,0}^2; \end{aligned}$$

the collision is elastic.
The fact that the particles have the same final speed is mere coincidence.

Problem 5



In the angular momentum experiment, shown to the right, a washer is dropped smooth side down onto the spinning rotor.

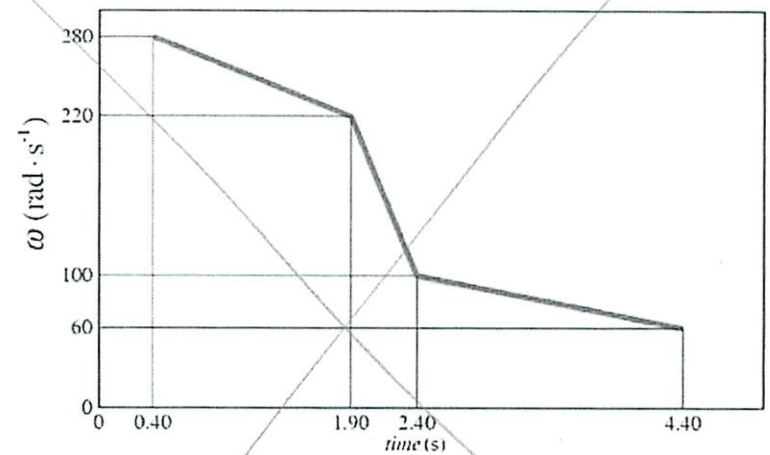
Problem

The graph below shows the rotor angular velocity $\omega\left(\mathrm{rad}\cdot\mathrm{s}^{-1}\right)$ as a function of time.

Assume the following:

- The rotor and washer have the same moment of inertia I.
- The friction torque $\vec{\tau}_f$ on the rotor is constant during the measurement.

Note: express all of your answers in terms of I and numbers you obtain from the graph. Be sure to give an analytic expression prior to substituting the numbers from the graph.



Problem

a) Find an expression for the magnitude $|\vec{\tau}_f|$ in terms of I and numbers you obtain from the graph.

b) How much mechanical energy is lost to bearing friction during the collision (between $t = 1.90\ \mathrm{s}$ and $t = 2.40\ \mathrm{s}$)?

c) How much mechanical energy is lost to friction *between the rotor and the washer* during the collision (between $t = 1.90\ \mathrm{s}$ and $t = 2.40\ \mathrm{s}$)?

5

Solutions:

a) First, make sure that the problem makes sense. Between times $t = 0.40\mathrm{s}$ and $t = 1.90\mathrm{s}$, the magnitude of the angular acceleration is $\Delta\omega / \Delta t = 40\,\mathrm{rad \cdot s^{-2}}$ and between times $t = 2.40\mathrm{s}$ and $t = 4.40\mathrm{s}$ the magnitude of the angular acceleration is $\Delta\omega / \Delta t = 20\,\mathrm{rad \cdot s^{-2}}$. During these two time intervals, the only torque is the friction torque, assumed constant, and doubling the net moment of inertia halves the angular acceleration.

We then have $|\vec{\tau}_f| = I\left(40\,\mathrm{rad \cdot s^{-2}}\right)$. This is also $|\vec{\tau}_f| = 2I\left(20\,\mathrm{rad \cdot s^{-2}}\right)$, but that's not part of this problem, just a consistency check.

For parts (b) and (c), denote $\omega_{\mathrm{initial}} = 220\,\mathrm{rad \cdot s^{-1}}$, $\omega_{\mathrm{final}} = 100\,\mathrm{rad \cdot s^{-1}}$, so that

$$K_{\text{initial}} = \frac{1}{2} I \omega_{\text{initial}}^2 = I\left(24,200\,\text{rad}^2 \cdot \text{s}^{-2}\right)$$

$$K_{\text{final}} = \frac{1}{2}(2I)\,\omega_{\text{final}}^2 = I\left(10,000\,\text{rad}^2 \cdot \text{s}^{-2}\right).$$

b) The mechanical energy lost due to the bearing friction is the product of the magnitude of the frictional torque and the total angle $\Delta\theta$ through which the bearing has turned during the collision. A quick way to calculate $\Delta\theta$ is to use

$$\Delta\theta = \omega_{\text{ave}} \Delta t = \left(160\,\text{rad} \cdot \text{s}^{-1}\right)\left(0.50\,\text{s}\right) = 80\,\text{rad}\,,$$

so $-\Delta E_{\text{bearing}} = \left|\vec{\tau}_f\right| \Delta\theta = I\left(3200\,\text{rad}^2\right)$.

c) The energy lost due to friction between the rotor and the washer is then

$$-\Delta K + -\Delta E_{\text{bearing}} = K_{\text{initial}} - K_{\text{final}} - I\left(3200\,\text{rad}^2\right) = I\left(11,000\,\text{rad}^2\right).$$

problem

Problem 6

A demonstration gyroscope wheel is constructed from a bicycle wheel by removing the tire, wrapping lead wire around the rim, and taping it into place. The wheel has a radius R and the mass is m. You may assume that the entire mass is concentrated on the rim. A shaft is connected to the axle and projects a distance d at each side of the center of the wheel. A person holds the ends of the shaft in two hands. The shaft is horizontal and the wheel is spinning about the shaft with angular velocity ω_{spin}.

a) Find the magnitude and direction of the force each hand exerts on the shaft when the shaft is at rest.

b) Find the magnitude and direction of the force each hand exerts on the shaft when the shaft is rotating in a horizontal plane about its center with angular velocity Ω.

c) At what rate must the shaft rotate in order that it may be suspended at one end only? Draw a diagram showing which the relationship between which side the shaft is supported and which way will it rotate in the horizontal plane.

6

Solutions:

a) If the shaft is at rest, there is no net torque on the gyroscope, and each hand exerts an upward force with magnitude equal to half the weight mg.

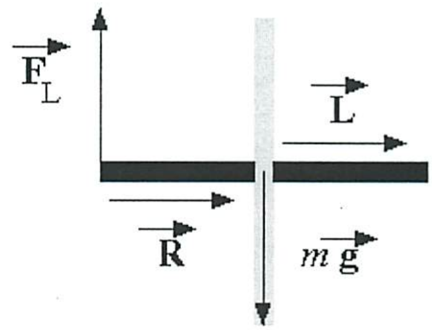
b) The net torque about the center will be the product of the precession frequency Ω and the horizontal component of the wheel's angular momentum, $L_{\text{horiz}} = mR^2\omega_{\text{spin}}$. Thus the difference between the magnitudes of the applied forces, multiplied by the distance d, is the product $\Omega mR^2\omega_{\text{spin}}$. Denoting the two forces as F_{L} and F_{R} (for "Left" and "Right"), we have the two equations

$$\begin{aligned}(F_{\text{L}} - F_{\text{R}})d &= \Omega mR^2\omega_{\text{spin}} \\ F_{\text{L}} + F_{\text{R}} &= mg.\end{aligned}$$

Dividing the first by d and adding to the second, and then subtracting as well, yields

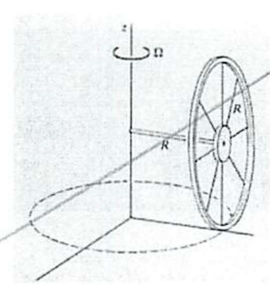
$$\begin{aligned}F_{\text{L}} &= \frac{m}{2}\left(g + \frac{\Omega R^2\omega_{\text{spin}}}{d}\right) \\ F_{\text{R}} &= \frac{m}{2}\left(g - \frac{\Omega R^2\omega_{\text{spin}}}{d}\right).\end{aligned}$$

c) From the above, when $\Omega = gd / R^2\omega_{\text{spin}}$, the force that one hand exerts goes to zero. In the diagram below, the wheel is shown from the side, the person holding the wheel is facing the wheel (and into the plane of the figure), F_R has vanished and the angular momentum of the wheel is as shown. Taking torques about the support point (the left hand), $\vec{\mathbf{R}}$ is directed to the right and the torque is into the page; the wheel will precess away from the holder. Taking torques about the center of the wheel yields the same result.



Problem 7

A thin hoop of mass m and radius R rolls without slipping about the z axis. It is supported by an axle of length R through its center. The hoop circles around the z axis with angular speed Ω. (Note: the moment of inertia of a hoop for an axis along a diameter is $(1/2)mR^2$.)



a) What is the instantaneous angular velocity $\vec{\omega}$ of the hoop? Specify the direction and magnitude.
b) What is the angular momentum $\vec{\mathbf{L}}$ of the hoop about a point where the axle meets the z axis? Is $\vec{\mathbf{L}}$ parallel to $\vec{\omega}$?

5

Solutions:

a) Because the radius of the hoop and the length of the axle are the same, when the hoop completes one circuit around the circle it also completes one complete revolution about the axle. The result is that the spin angular velocity has the same magnitude as the orbital angular speed, $\omega_{\text{spin}} = \Omega$. Due to this restriction, we cannot neglect the vertical component of angular velocity or angular momentum. The angular velocity of the hoop about its center is $\vec{\boldsymbol{\omega}} = \Omega\left(\hat{\mathbf{k}} - \hat{\mathbf{r}}\right)$ (note that the horizontal component is directed radially inward in the above figure.

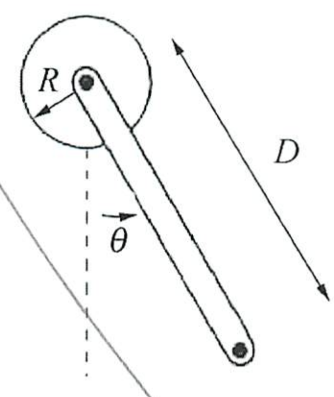
b) About the specified point, there are three contributions to the angular momentum: the horizontal component (often known as the "spin" angular momentum), the motion of the center of the wheel about the central shaft (often known as the "orbital" angular momentum) and the fact that the wheel is also rotating about a vertical axis. The angular momentum is then given by

$$\vec{\mathbf{L}} = \omega_{\text{spin}} mR^2\left(-\hat{\mathbf{r}}\right) + \Omega mR^2\left(\hat{\mathbf{k}}\right) + \Omega\frac{1}{2}mR^2\left(\hat{\mathbf{k}}\right) = \Omega mR^2\left(\frac{3}{2}\hat{\mathbf{k}} - \hat{\mathbf{r}}\right);$$

the angular momentum is not parallel to the angular velocity.

Problem 8:

A rigid body is composed of a uniform disk (mass m, radius R) and a uniform rod (mass m, length D) that is rigidly fixed to the center of the disk. This body is pivoted about the center of the disk around a horizontal axis that is perpendicular to the plane of the page. Assume the pivot is frictionless and the acceleration due to gravity is g.



a) Find the moment of inertia I_{pivot} about the pivot point.

b) Suppose the pendulum is swinging freely back and forth. Write down an expression for the angular acceleration about the pivot point. You may leave your answer in terms of m, g, R, I_{pivot}, D and the angle θ as needed .

c) Suppose the angle θ is small throughout the motion. That is, you may assume $\sin\theta \approx \theta$ and $\cos\theta \approx 1$. What is the period for this pendulum? Express your answer in terms of m, R, D, g and I_{pivot}.

d) Now suppose there is no restriction on the value of θ (it can be large). What is the minimum angular speed ω_{min} that the pendulum should have at the bottom of its swing so that the pendulum can revolve completely around the pivot point?

8

Solutions:

Note: The answers given here will use the result of part (a) for the moment of inertia of the pendulum about the pivot point.

a) From the parallel axis theorem, or a handy formula sheet, the moment of inertia of the rod about the pivot point is $I_{\text{rod,pivot}} = mD^2/3$. The pivot is the center of the disc, so $I_{\text{disc, pivot}} = mR^2/2$ and the total moment of inertial about the pivot is $I_{\text{pivot}} = m\left(R^2/2 + D^2/3\right)$.

b) The weight of the disc (and any contact force between the disc bearings and the pendulum) exert no torque, and the torque exerted by the weight of the rod, directed into the page in the figure, is $\tau = mg(D/2)\sin\theta$. The angular acceleration is then

$$\alpha = -\frac{\tau}{I_{\text{pivot}}} = -\frac{mg(D/2)\sin\theta}{m\left(R^2/2 + D^2/3\right)} = -\frac{g\sin\theta}{R^2/D + 2D/3},$$

with the negative signs indicating a restoring torque.

c) The square of the frequency of small oscillations is given by the negative of the term multiplying $\sin\theta$ in part (b), and so the period of small oscillations is

$$T = 2\pi\sqrt{\frac{R^2/D + 2D/3}{g}}.$$

d) Without the small angle approximation, this part of the problem cannot be solved directly by using torques; energy considerations must be use. At the bottom of the swing, the kinetic energy is $(1/2)I_{\text{pivot}}\omega_{\text{min}}^2$ and to just make it around the pivot point, the kinetic energy at the top should be taken to be zero. Note that the center of mass of the disc does not move, so in going from the bottom to the top, the change in gravitational potential energy is due only to the change in height of the center of mass of the rod and hence increased by $\Delta U = mgD$. Therefore setting the change in kinetic energy equal to the negative of the change in potential energy,

$$\frac{1}{2} I_{\text{pivot}} \omega_{\min}^2 = mgD$$

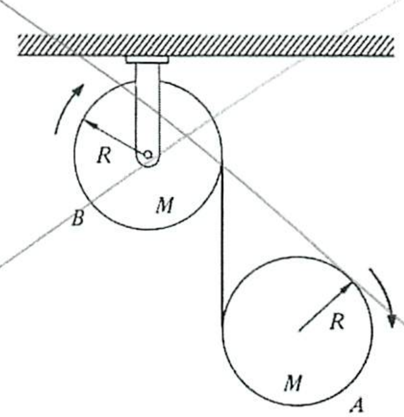
$$\omega_{\min}^2 = \frac{mgD}{I_{\text{pivot}}} = \frac{2gD}{R^2/2 + D^2/3}$$

Therefore the angular speed is

$$\omega_{\min} = \sqrt{\frac{2gD}{R^2/2 + D^2/3}}$$

Problem 9

A drum A of mass m and radius R is suspended from a drum B also of mass m and radius R, which is free to rotate about its axis. The suspension is in the form of a massless metal tape wound around the outside of each drum, and free to unwind. Gravity is directed downwards. Both drums are initially at rest. Find the initial acceleration of drum A, assuming that it moves straight down.



9

Solution:

The key to solving this problem is to determine the relation between the three kinematic quantities α_A, α_B and a_A, the angular accelerations of the two drums and the linear acceleration of drum A. One way to do this is to introduce the auxiliary variable z for the length of the tape that is unwound from the upper drum. Then, $\alpha_B R = \frac{d^2 z}{dt^2}$. The linear velocity v_A may then be expressed as the sum of two terms, the rate $\frac{dz}{dt}$ at which the tape is unwinding from the upper drum and the rate $\omega_A R$ at which the falling drum is moving relative to the lower end of the tape. Taking derivatives, we obtain

$$a_A = \frac{d^2 z}{dt^2} + \alpha_A R = \alpha_B R + \alpha_A R .$$

Denote the tension in the tape as (what else) T. The net torque on the upper drum about its center is then $\tau_B = TR$, directed clockwise in the figure, and the net torque on the falling drum about its center is also $\tau_A = TR$, also directed clockwise. Thus, $\alpha_B = TR / I = 2T / MR$, $\alpha_A = TR / I = 2T / MR$. Where we have assumed that the moment of inertia of the drum and unwinding tape is $I = (1/2)MR^2$. Newton's Second Law, applied to the falling drum, with the positive direction downward, is $Mg - T = Ma_A$. We now have five equations,

$$\alpha_B R = \frac{d^2 z}{dt^2}, \quad a_A = \frac{d^2 z}{dt^2} + \alpha_A R, \quad \alpha_B = \frac{2T}{MR}, \quad \alpha_A = \frac{2T}{MR}, \quad Mg - T = Ma_A,$$

in the five unknowns α_A, α_B, a_A, $\frac{d^2 z}{dt^2}$ and T.

It's easy to see that

$$\alpha_A = \alpha_B .$$

Therefore

$$a_A = \alpha_B R + \alpha_A R = 2\alpha_A R .$$

The tension in the tape is then

$$T = \frac{\alpha_A MR}{2} = \frac{a_A MR}{4R} = \frac{Ma_A}{4}$$

Newton's Second Law then becomes

$$Mg - \frac{Ma_A}{4} = Ma_A .$$

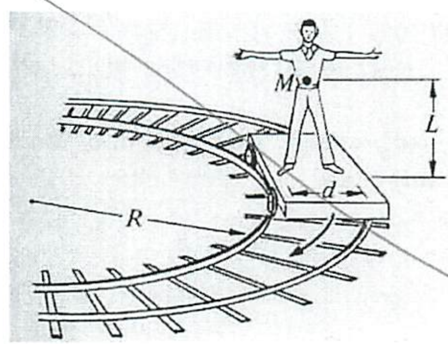
Therefore solving for the acceleration yields

$$a_A = \frac{4}{5} g$$

This result is certainly plausible. We expect $a_A < g$, and we also expect that with both drums free to rotate, the acceleration will be almost but not quite g.

Problem 10

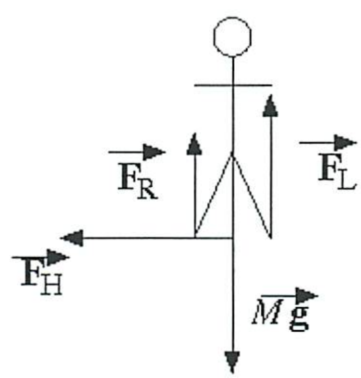
A person of mass M is standing on a railroad car, which is rounding an unbanked turn of radius R at a speed v. His center of mass is at a height of L above the car midway between his feet, which are separated by a distance of d. The man is facing the direction of motion. What is the magnitude of the normal force on each foot?



10

Solution:

A free-body diagram, done by an unskilled artist, is shown below. The distances d and L are not shown to avoid clutter, and one would have to guess at the position of the center of mass of the person.



We expect that there will be some friction force, or other horizontal contact force, between the feet and the car, but we aren't given anything about the nature of these forces. Using some foresight, we notice that there must be a net horizontal inward force, shown as $\vec{\mathbf{F}}_{\mathrm{H}}$ in the figure, of magnitude Mv^2/R applied to the feet.

If we take torques about the center of mass of the person, and denote the normal forces on the feet as F_{R} and F_{L} for "Right" and Left", the clockwise torque is the sum $\left(Mv^2/R\right)L + F_{\mathrm{R}}d/2$ and the counterclockwise torque is $F_{\mathrm{L}}d/2$ (note that F_{R} and F_{L} are the person's right and left feet, not right and left in the diagram above). Equating these torque magnitudes and using $F_{\mathrm{R}} + F_{\mathrm{L}} = Mg$ leads, after some basic algebra, to

$$F_{\mathrm{L}} = M\left(g/2 + v^2L/Rd\right), \quad F_{\mathrm{R}} = M\left(g/2 - v^2L/Rd\right).$$

This makes sense; the larger force is on the outer foot, and if the car is moving fast enough at some speed the force on the right foot will vanish, and the person will fall over.

There are other ways to do this problem that would not involve introduction of $\vec{\mathbf{F}}_{\mathrm{H}}$ at all, and it's reasonable to hope that there might be some simplification. For instance, choose the point above the center of the circle as the origin (at the horizontal level of the person's feet). Then, the angular momentum has two components, a constant vertical component with $L_{\mathrm{vert}} = MvR$ and a horizontal component with constant magnitude $L_{\mathrm{horiz}} = MvL$ (if you use this method, note the danger of confusing the distance "L" in the problem with any angular momentum). The horizontal component changes direction, and the magnitude of the rate of change is

$$\left|\frac{d}{dt}\vec{\mathbf{L}}\right| = \Omega L_{\mathrm{horiz}} = \frac{v}{R}MvL = \frac{Mv^2L}{R}.$$

The torque about this origin has no contribution from the horizontal forces on the feet; this torque is horizontal and has magnitude

$$|\tau| = F_{\mathrm{L}}(R+d) + F_{\mathrm{R}}(R-d) - MgR.$$

Setting this equal to the magnitude of the rate of change of angular momentum and substituting first $F_{\mathrm{L}} = Mg - F_{\mathrm{R}}$ and solving for F_{L} and then doing an almost identical calculation gives the result found above.

Problem 11

A proton makes a head-on collision with an unknown particle at rest. The proton rebounds straight back with $4/9$ of its initial kinetic energy. Find the ratio of the mass of the unknown particle to the mass of the proton, assuming that the collision is elastic.

11

Solution:

Two methods will be presented here, one "standard" and one "almost too slick."

Standard: For the head-on collision, given that the incident proton recoils with $4/9$ of its initial kinetic energy, it must recoil with $2/3$ its initial speed. Taking the initial direction of the proton to be the positive direction, and using m_{p} for the mass of the proton, M_{X}

for the unknown mass, v_0 for the initial speed of the proton and V_X for the final speed of the unknown particle, we have from conservation of linear momentum

$$m_p v_0 = -\frac{2}{3} m_p v_0 + M_X V_X$$
$$\frac{5}{3} m_p v_0 = M_X V_X.$$

Equating initial and final kinetic energies and employing minimal algebra gives

$$\frac{1}{2} m_p v_0^2 = \frac{1}{2}\left(\frac{4}{9} m_p v_0^2\right) + \frac{1}{2} M_X V_X^2$$
$$\frac{5}{9} m_p v_0^2 = M_X V_X^2.$$

Squaring the result of the momentum equation gives $(25/9) m_p^2 v_0^2 = M_X^2 V_X^2$; dividing by the simplified kinetic energy equation, the masses cancel and $M_X = 5m_p$.

Slick: In order for a rebound velocity of $(-2/3)v_0$ in a completely elastic collision, the center of mass of the system must be moving with speed $(1/2)(v_0 + (-2/3)v_0) = (1/6)v_0$. This speed is $v_{cm} = v_0 m_p / (m_p + M_X)$, leading to $M_X = 5m_p$. If you've memorized, or can rederive the expression $v_p' = (m_p - M_X)/(m_p + M_X) v_0$, the result is the same.
(Note: There are no known stable nuclei with mass equal to five time the proton mass.)

Problem 12

A particle of mass m moves under an attractive central force of magnitude $F = br^3$. The angular momentum is equal to L.

a) Find the effective potential energy and make sketch of effective potential energy as a function of r.

b) Indicate on a sketch of the effective potential the total energy for circular motion.

c) The radius of the particle's orbit varies between r_0 and $2r_0$. Find r_0.

12

Solutions:

a) The potential energy is, taking the zero of potential energy to be at $r = 0$, is

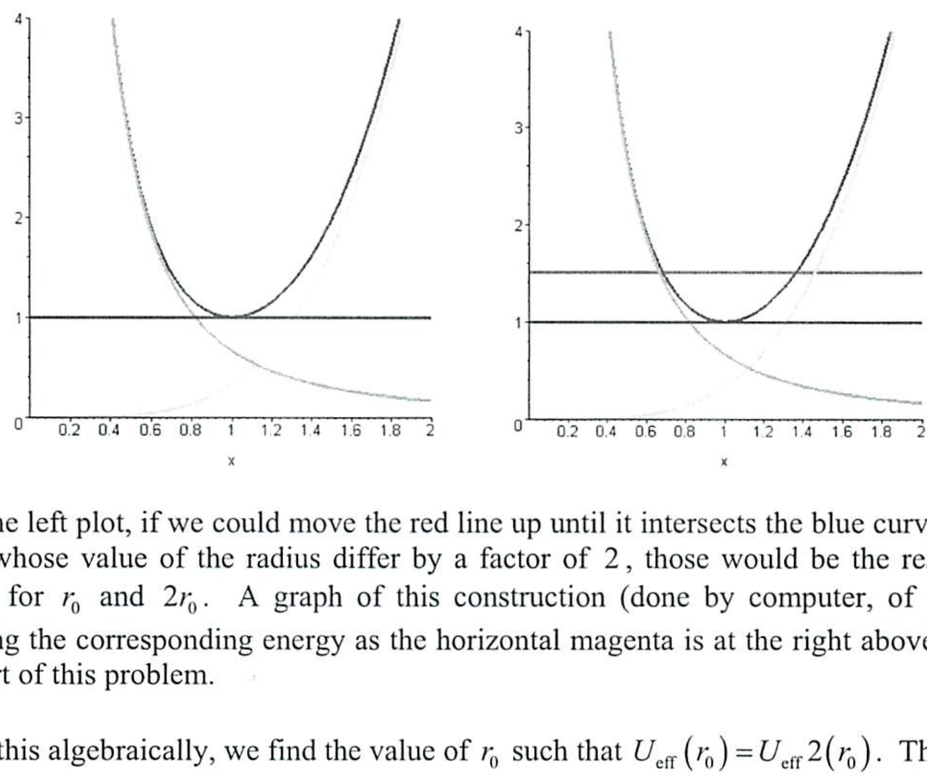
$$U(r) = -\int_0^r \left(-br'^3\right) dr' = \frac{b}{4} r^4$$

and the effective potential is

$$U_{\text{eff}}(r) = \frac{L^2}{2mr^2} + U(r) = \frac{L^2}{2mr^2} + \frac{b}{4} r^4 .$$

A plot is shown below, including the potential (yellow if seen in color), the term $L^2/2m$ (green) and the effective potential (blue). The minimum effective potential energy is the horizontal line (red). The horizontal scale is in units of the radius of the circular orbit and the vertical scale is in units of the minimum effective potential.

b) See the solution to part (a) above and the plot to the left below.



c) In the left plot, if we could move the red line up until it intersects the blue curve at two point whose value of the radius differ by a factor of 2, those would be the respective values for r_0 and $2r_0$. A graph of this construction (done by computer, of course), showing the corresponding energy as the horizontal magenta is at the right above, and is not part of this problem.

To do this algebraically, we find the value of r_0 such that $U_{\text{eff}}(r_0) = U_{\text{eff}} 2(r_0)$. This is

$$\frac{L^2}{mr_0^2} + \frac{b}{4} r_0^4 = \frac{L^2}{m(2r_0)^2} + \frac{b}{4}(2r_0)^4 .$$

Rearranging and combining terms, and then solving for r_0,

$$\frac{3}{8}\frac{L^2}{m}\frac{1}{r_0^2} = \frac{15}{4} b r_0^4$$

$$r_0^6 = \frac{1}{10}\frac{L^2}{mb}.$$

Thus, $r_0 = \left(1/\sqrt{10}\right) r_{\text{circular}}$ (not part of the problem), consistent with the auxiliary figure on the right above.

Problem 13

A wrench of mass m is pivoted a distance l_{cm} from its center of mass and allowed to swing as a physical pendulum. The period for small-angle-oscillations is T.

a) What is the moment of inertia of the wrench about an axis through the pivot?

b) If the wrench is initially displaced by an angle θ_0 from its equilibrium position, what is the angular speed of the wrench as it passes through the equilibrium position?

13

Solutions:

a) The period of the physical pendulum for small angles is $T = 2\pi\sqrt{I_{\text{P}} / ml_{\text{cm}}g}$; solving for the moment of inertia,

$$I_{\text{P}} = \frac{T^2 ml_{\text{cm}}g}{4\pi^2}.$$

b) For this part, we are not given a small-angle approximation, and should not assume that θ_0 is a small angle. We will need to use energy considerations, and assume that the pendulum is released from rest.

Taking the zero of potential energy to be at the bottom of the pendulum's swing, the initial potential energy is $U_{\text{initial}} = mgl_{\text{cm}}(1-\cos\theta_0)$ and the final kinetic energy at the bottom of the swing is $U_{\text{final}} = 0$. The initial kinetic energy is $K_{\text{initial}} = 0$ and the final kinetic energy is related to the angular speed ω_{final} at the bottom of the swing by $K_{\text{final}} = (1/2) I_{\text{P}}\omega_{\text{final}}^2$. Equating initial potential energy to final kinetic energy yields

$$\omega_{\text{final}}^2 = \frac{2mgl_{\text{cm}}(1-\cos\theta_0)}{I_{\text{P}}} = \frac{8\pi^2}{T^2}(1-\cos\theta_0).$$

1

Solutions:

a) The center of mass will move to the right in the figure, and the two masses will rotate about the center of mass, counterclockwise in the figure.

b) The distance from the object originally at point B is $M_A D/(M_A + M_B) = (2/3)D$, at a position $y_{cm} = D/3$ in the figure.

c) The magnitude of the linear momentum will be the magnitude $F\Delta t$ and, as in part (a), the direction will be the right. The magnitude of the velocity is then $(F\Delta t)/(3M)$

d) The quickest way to find the angular velocity is to consider the collision in the center of mass frame. In this frame the angular impulse, and hence the magnitude of the angular momentum, is $(F\Delta t)(2/3)D$. The momentum of inertia about the center of mass is

$$I_{cm} = (2M)(D/3)^2 + (M)(2D/3)^2 = (2/3)MD^2$$

and the magnitude ω_f of the final angular momentum is

$$\omega_f = \frac{(F\Delta t)(2/3)D}{(2/3)MD^2} = \frac{F\Delta t}{MD}.$$

e) No. The force additional force would have to be applied at a distance $2D/3$ above the center of mass, which is not a physical point of the system.

f) An additional force of the same magnitude, in the negative x direction, would result in no net force and hence no acceleration of the center of mass.

(2)

Solutions:

a) This preliminary part should be found directly from Newton's Second Law and the Universal Law of Gravitation. The magnitude of the acceleration for the circular orbit is v_0^2 / r_0, and so

$$m_s \frac{v_0^2}{r_0} = G\frac{m_s m_e}{r_0^2}$$
$$v_0 = \sqrt{Gm_e / r_0}.$$

b) The total mechanical energy is the sum of the kinetic energy and the gravitational potential energy,

$$E_0 = -G\frac{m_s m_e}{r_0} + \frac{1}{2}m_s v_0^2$$
$$= -\frac{1}{2}G\frac{m_s m_e}{r_0}.$$

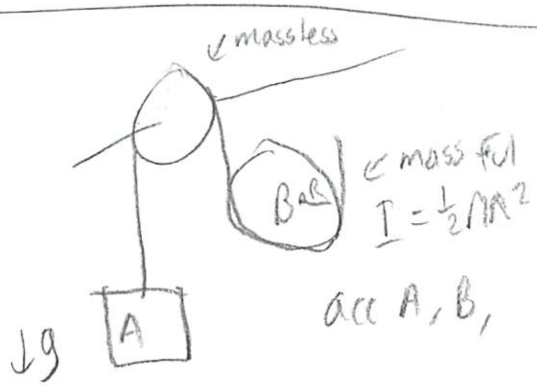
c) Since $r_p = r_0$, and $r_a = 3r_0$, the condition that angular momentum is constant $r_p v_p = r_a v_a$ becomes $r_0 v_p = (3r_0) v_a$, so $v_a = v_p / 3$. The condition that the mechanical energy is constant then becomes,

$$\frac{1}{2}m_s v_p^2 - G\frac{m_s m_e}{r_0} = \frac{1}{2}m_s v_a^2 - G\frac{m_s m_e}{r_a}$$
$$= \frac{1}{2}m_s\left(\frac{v_p}{3}\right)^2 - G\frac{m_s m_e}{3r_0}$$
$$\frac{4}{9}v_p^2 = \frac{2}{3}G\frac{m_e}{r_0}$$
$$v_p = \sqrt{(3/2)Gm_e / r_0}.$$

As a simple check, note that $v_p > v_0$. As a further check, some minor algebra shows that after the rocket burn, the final mechanical energy is $E_f = -Gm_s m_e / (4r_0) = -Gm_s m_e / A$, where $A = r_0 + 3r_0$ is the major axis of the ellipse.

① 8.01 Final Review Dumaskin 12/12

- think about getting started
- need idea to ~~start~~



$M_A = M_B$

acc A, B, α A

How get started
fixed axis rotation
com B translating

Think about it this way

1. Ideas: $\tau_{cm,B} = I_{cm}\vec{\alpha}$ for B

$F_a = m\vec{a}_a$

$F_b = m\vec{a}_b$

same mass

Not gyro - just rolling

Constraint $\vec{a}_a, \vec{a}_b, \vec{\alpha}_B$

don't just use formula

2. Tools will use

$F = ma$

(well drawn free body diagram

$\tau = I\alpha$

(torque diagram

(diagram for constraint

will be 4 equations
- force
- force
- constraint
- torque

unknowns: T, a_a, a_b, α_b

(2)

~~$T_A \approx T_B$~~ not yet

← torque diagram

apply forces where They are happening

* pick coord system

$J_A \downarrow \oplus \quad J_B \downarrow \oplus$

* be careful sign of α

so τ + is in board

$$\alpha = \frac{d^2\theta}{dt^2}$$

so + α is same way

Now can write equations

	$\vec{F}_a$	$= m_a \vec{a}_a$	$\vec{F}_b$	$= m_b a_b$	
J_a	$m_a g - T$	$m_a a_a$ (1)	$m_b g - T$	$= m_b a_b$	(2)

stop and check
- have 2 equations
- signs w/ diagram

- on this problem both objects could fall down
- depends on mass ratio

- pully has no effect

- massless

$T_1 R - T_2 R = \cancel{I_p \alpha_p}$ 0 so no torque $T_1 - T_2 = 0$

③ * $T \neq mg$ when $a \neq 0$

↑ can't assume

Now Torque diagram

about com

↑ for pullies.

$$\tau = \frac{dL}{dt}$$

$\vec{r}_{cm} \times \vec{T}$	$I_{cm}\,\alpha$ ⊗

↓ from com to where applies

$L_{cm} = I_{cm}\vec{\omega}$

↑ direction not changing (only gyros) magnitude

cross product start from one to other ⊗

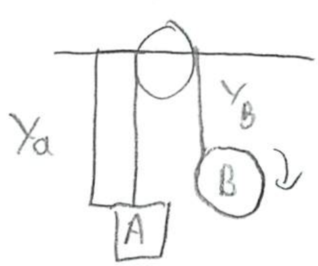
↑ defined as (+)

$$\boxed{RT = I_{cm}\,\alpha} \quad ③$$

If not sure how to get started
Do all this

- Now constraint
  - difficult

$$\ell(T) = y_a + y_B$$



$\alpha, \omega, \theta(t)$

$\theta(t) - \theta_0$ = amt angle B has changed as tape unwound

arclength →
$R(\theta(t) - \theta_0) = s$ = amt tape unwound

I think I kinda know this stuff - need to apply now when running from scratch - Need to do process more

(4)

$$\ell(t) = \ell_0 + R(\theta(t) - \theta_0)$$

$Y_a + Y_B = \ell_0 + R(\theta(t) - \theta_0)$ ← lenght of tape getting longer as this unwinds

take 2 derivs to get to α ← how in all world suppose to know that - well get it in terms of a given

(4) $\boxed{a_A + a_B = R\alpha}$ super-hard constraint

- if held A - B would unwind - rolling w/o slipping

---

① $TR = I_{cm}\alpha$

② $m_a g - T = m_a a_a$

③ $m_b g - T = m_B a_B$

④ $a_A + A_B = R\alpha$

now algebra ← but this is heart of what to do

- had gotten a lot better at solving systems from the review

① Torque
② Force
③ Constraint

Can Review constraints in different situations

do some in class problems, quizzes

~~- really, compl~~

I need to do the past quiz ones

Memorize Concepts + tools

It's just knowing how to do it + doing it

MIT throws hard stuff at you (figure out the easy stuff as you go)

- I think good

Look at how you think

(5) New problem

m moves in central force $F = -br^3\hat{r}$

The particle moves in circular orbit w/ fixed angular momentum L.

a) Find PE as function of R w/ $U(r=0)=0$

* not gravitation $\cancel{\frac{GMm}{r^2}}$ can't use

- integrate force to get PE

F = ma circular

- get relationship r_0 v_0

and have 1 eq and 2 unknowns

~~r=mv~~ $\vec{L}_0 = \vec{r}_0 \times m\vec{v}_0$

$U(B) - U(A) = -\int_A^B \vec{F}, dr$ ← must calculate PE w/ integration

↑ def potential Energy

must integrate

where is my 0?

$E_0 = k_0 + U_0$

* keys: Angular momentum and energy

- what are their energies

PE from integrating force

Eaiser w/ circular, eliptical = hard

F=ma

(6)

$\circlearrowleft$ + so $\odot$

$\to \hat{r}$

F	$= ma$
$-br_0^3$	$= -\frac{mv_0^2}{r_0}$
	↑ inward

forgot $a = \frac{v^2}{r}$ uniform circular motion since $= v\omega$, $a = v\frac{d\theta}{dt}$

← what is best way to write a_{circ}

$\boxed{\begin{matrix} v = \omega r \\ \omega = \frac{v}{r} \end{matrix}}$

$$r_0^4 = \frac{mv_0^2}{b}$$

$$r_0 = \left(\frac{mv_0^2}{b}\right)^{1/4}$$

check after completing idea

$$\vec{L}_0 = \vec{r}_0 \times m\vec{v}_0$$

← from COM to where is

$\odot$

$$L_0 = m r_0 v_0$$

Do more like this

Solve for v_0, r_0 in terms of b, L, m

$$U(B) - U(A) = -\int_A^B \vec{F} \cdot d\vec{r}$$

← minus work done by force

↑ endpoints really matter

$$E_0 = \frac{1}{2}mv_0^2 + U$$

thinking pt: A is the reference potential 0

B is variable pt r

and can look at at any moment

think for easier to write on paper, no distractions, can memorize based on pg position

(7) $U(r) - \underset{? 0}{U(0)} = -\int_0^r \; ?$

? must get into 1 dimesion
if don't know vector calculus
-don't turn away

$$\vec{F} \cdot d\vec{r} \qquad \vec{F} = \overbrace{-br^3}^{F_r}\hat{r}$$

$$\| \qquad d\vec{r} = dr\,\hat{r}$$

$$\underbrace{-br^3}_{F_r}dr \qquad F_x\,dx$$

did not get

I think just squashed it in - nothing changed

$$U(r) = -\int_0^r (-br^3)\,dr = \frac{br^4}{4}\Big|_0^4$$

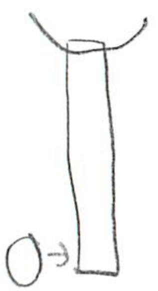
$$E_0 = \frac{1}{2}mv_0^2 + \frac{br_0^4}{4}$$

For this type of problems
2 unknowns
radius + speed
If know energy can solve for
Lots of alternate problems

* key: lots of ellipitical orbits

New problem

8 Moving towards idea of rotation + translation



$I = \frac{1}{12} m l^2$

Gyro - hard since just about COM
- several types of rotation at once

pure rotation about pivot

* translation since COM moving *

a) $K_p = \frac{1}{2} I_p w^2$ ← angular KE

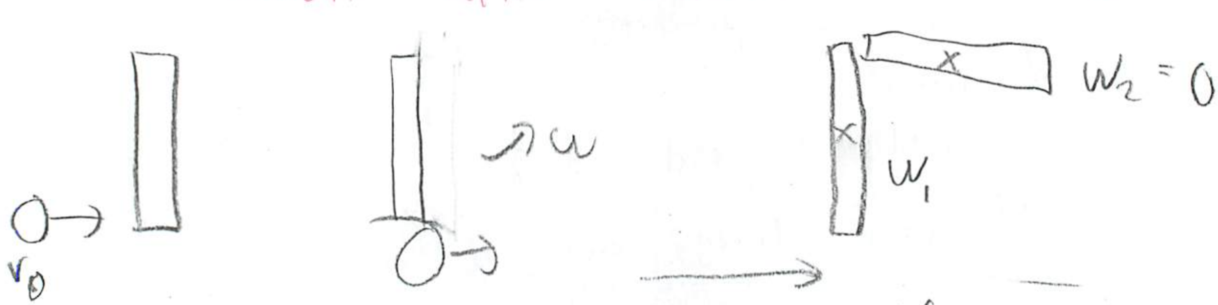
b) $L_p = I_p w$ ← fixed axis, no gyro

c) are pivot forces, include $F = ma$

← dumaskin + graytek do this differently
dumaskin more general
- short term that confuses

* 2 stage process
- collission
- moment after collision

Not ~~$\frac{1}{2} m_b v_b^2 = (m_s + m_b) g h$~~



$\overrightarrow{\text{Collission}}$ A

$\frac{V_0}{2}$ ← ball still moving like that

after collision B

* Each stage has a different problem

(9) Stage A

When objects collide - are there nonconserivive values?

elastic = E constant

external forces = <u>not</u> constant momentum

Conservation Laws

1. Energy
2. Momentum
3. Angular momentum

know when fixed
when changing

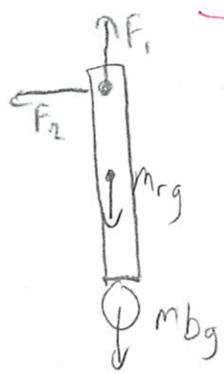
know the rules explicitly

* Note the COM changes

- momentum not constant 60-70% scewed this up
- external force → <u>pivot</u> forces

Ball hits rod
rod counters Ball) = and opposite so $\tau_p = 0$

* Only angular momentum constant



$\tau_p = \vec{r}_p \times \vec{F}_p = 0$ from pivot to where force acting

$\tau_p = \vec{r}_p \times m_r g = 0$

$\tau_p = \vec{r}_p \times \vec{r}_b g = 0$

5 forces all $\tau = 0$

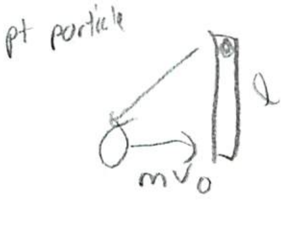
- What quantities are conserved?

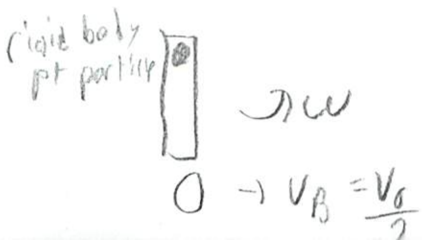
$\vec{L}_p$ = constant for system

$L_p = L_p$

$\vec{L}_p = \vec{r}_{pcm} \times m\vec{v}_{cm}$ point particle

$= I_p \vec{\omega}$ rigid body





$\vec{L}$ will be $\underline{\otimes}$ fixed axis

10 $\vec{L}_{p_0} = m l v \odot$ $\qquad$ $\vec{L}_{p_1} = m l \frac{v_0}{2} \odot + I_p \omega \odot$

$$m l v = m l \frac{v_0}{2} + I_p \omega$$

Solve for ω in terms of I_p, v_0, l, m

- will give I_{com} and parallel axis teorm
  need to find I_p

Stage B

Energy + L conserved

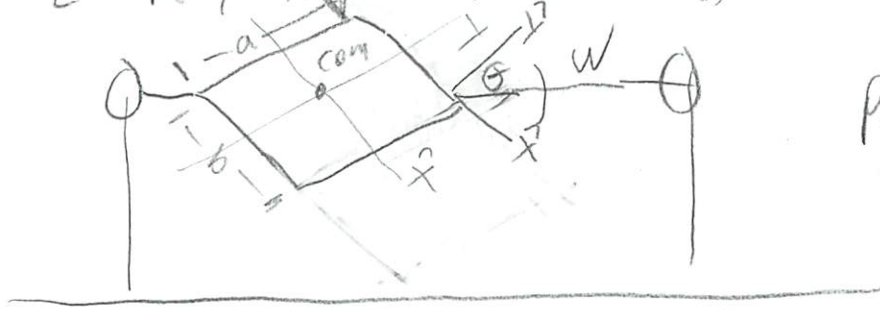
Remember its the distance COM rises

$$E_1 = \frac{1}{2} I_p \omega^2 = E_2 = m_r g \frac{l}{2}$$

Need to know when + what stuff is conserved

① Gyroscope Problems

- 2 very important differences



Parallel axis theorm

Idea $\vec{\omega} = \vec{w}_x + \vec{w}_y$

Need to get direction of ω
decompose vectors

$$= w \sin\theta \,\hat{\imath} + w\cos\theta \,\hat{J}$$

$$\vec{L}_{cm} = I_x \vec{w}_x + I_y \vec{w}_y$$
parallel

$$I_y = \frac{1}{12} m b^2$$
$$I_x = \frac{1}{12} m a^2$$

$$L_{cm} = \frac{1}{12} m a^2 \omega \sin\theta \,\hat{\imath} + \frac{1}{12} m b^2 \omega \cos\theta \,\hat{J}$$

Tilted wheels = very hard

$$k_{cm} = \frac{1}{2} I_x \omega^2 + \frac{1}{2} I_y w_y^2$$

* Look at the 2 types of rotation

~~Th~~

(12)

only 1 spin direction

Write down L

key then How does L change

1 # 8.01 Quiz Redos

12/12

6 quizzes
2 exams

Quiz 1 (69)

~~Geraldine~~ Learning to drive
start at rest $t=0$
lurches forward
stomps on break vel $\to 0$
then at rest

$$\ddot{x}(t) = \alpha t - \beta t^3$$

a) units of α and B

$$\alpha = \frac{\text{meters}}{\text{sec}^{3}} (\circ \text{ sec})$$

$$B \quad \frac{\text{meters}}{\text{sec}^{5}} (\text{sec}^3)$$

Since $\ddot{x} = \frac{\text{meters}}{\text{sec}^2}$
note not x
got tricked again
got it right originally

b) How long was car in motion?

$\longrightarrow$

(2) How long was car in motion?
- all variables
- have to stop + think even now

$$acc = \frac{\Delta vel}{time} = \frac{d^2x}{dt^2}$$

how far it traveled

~~Acc~~

when $\ddot{x}(t) = 0$ for second time
then it is at rest

$$\ddot{x}(t) = \ddot{x}(t_0) = 0 = \alpha t - \beta t^3$$ ← was thrown off last time non standard

but not the ans

they integrate and set $\dot{x}(t) = 0$
- Yeah when not acc does not mean at rest duh

So integrate

$$\int \alpha t - \beta t^3$$

$$\alpha \frac{t^2}{2} - \beta \frac{t^4}{4}$$ ← and now I know how to integrate

did better this time, but not perfect

$$0 = \alpha \frac{t^2}{2} - \beta \frac{t^4}{4}$$ Solve for t

$$\beta \frac{t^4}{4} = \alpha \frac{t^2}{2}$$

$$\cdot 4 \qquad \cdot 4$$

$$\frac{\beta t^4}{t^4} = \frac{2 \alpha t^2}{2\alpha} \qquad \frac{\beta}{2\alpha} = \frac{t^2}{t^4}$$

3

$$\frac{B}{2\alpha} = \frac{1}{t^2}$$

$$\frac{2\alpha}{\beta} = t^2$$

$$t = \sqrt{\frac{2\alpha}{\beta}}$$ ✓ got the solving down

c) what is max velocity ?

kinda sam ans for
when $a = 0$ first time

$$0 = \alpha t - \beta t^3$$

Solve for t

$$\frac{\alpha t}{t} = \frac{\beta t^3}{\beta}$$

$$\frac{\alpha}{\beta} = \frac{t^3}{t} = t^2$$

$$t = \sqrt{\frac{\alpha}{\beta}}$$ ← before was so confused on how got this From doing solving p-set
Should have done all p-sets
but no time!

now need to plug this in ! (don't forgit!)

$$\cancel{\frac{1}{2}\alpha}\ \dot{x}\left(\sqrt{\frac{\alpha}{B}}\right) = \frac{\alpha\left(\frac{\alpha}{B}\right)}{2} - \frac{B\left(\frac{\alpha^2}{B^2}\right)}{4}$$

$$V = \frac{\alpha^2}{2B} - \frac{\alpha^2}{4B}$$

can simp $\frac{1}{2}\frac{\alpha^2}{B} - \frac{1}{4}\frac{\alpha^2}{4} = \frac{1}{4}\alpha^2/B$ ✓

(4) How far does car travel

$x(t_{final})$?

$t = \sqrt{\frac{2\alpha}{\beta}}$

need to integrate $\dot{x}$

$$\int \frac{\alpha t^2}{2} - \frac{\beta t^4}{4}$$

$$\alpha \frac{t^3}{6} - \frac{\beta t^5}{20}$$

$$\frac{\alpha \sqrt{\frac{2\alpha}{\beta}}^3}{6} - \frac{\beta \sqrt{\frac{2\alpha}{\beta}}^5}{20}$$

can simplify further

$$\frac{1}{15} \alpha \left(\frac{2\alpha}{\beta}\right)^{3/2}$$

$2^{3/2}$

$\sqrt{2 \cdot 2 \cdot 2}$

$\sqrt{8}$

oh and subtract

I suppose factor + subtract

where does β go?

~~en~~

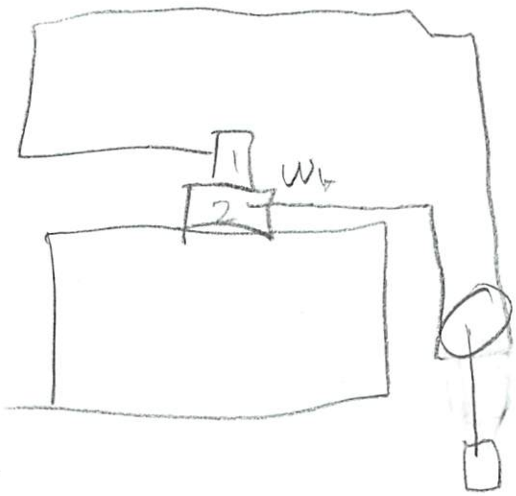
make sketches

- was always bad at
- just think about carefully

(5) Quiz #2

- Don't have solution files

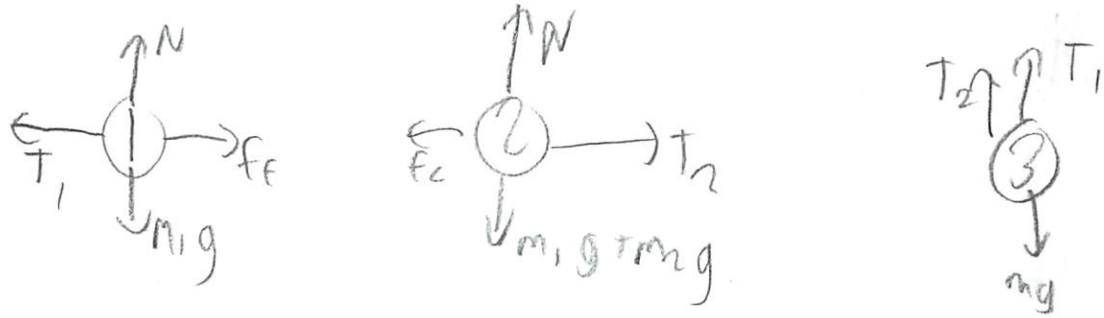
Pully problems - how did I get a 79%?!



define coords

↑ (+) →

a) Free Body diagram it



given $a \neq 0$

* dont forget the constraints $T_1 = T_1$

$T_2 = T_2$

$F = ma$ $\quad$ $F = ma$ $\quad$ $F = ma$

$F_x = f_f - T_1 = m_1 a_1$ $\quad$ $F_x\ T_2 - f_f = m_2 a_2$ $\quad$ $F_x = 0$

$\sum F_y = 0$ $\quad$ $F_y = 0$ $\quad$ $F_y = T_1 + T_2 - m_3 g = m_3 a_3$

can ignore I belive

6

b) Friction force

$f_f = \mu N$

$\mu + m_1 g$ ← just the top one

$+ \mu m_1 g$ ✓ normal force ↑ ~~stad~~ since ↑ defined as ⊕ pay attention, don't just go through motions

c) Ok the long part
find N equations that must be solved w/ N unknowns

F_1
F_2) and if applicable torques,
F_3
constraint

kinda did already on reverse

~~FA~~

$f_f - T_1 = m_1 a_1$

$T_2 - f_f = m_2 a_2$

$T_1 + T_2 - m_3 g = m_3 a_{3y}$

String is the constraint - missed concept 1st time

So $-\Delta y_1 = \frac{1}{2} \Delta x_3$) how to write?

$\Delta y_2 = \frac{1}{2} \Delta x_3$ how to integrate

ch friction $f_f = \mu m_1 g$ from above

don't have solution for so can't really do

(7) Quiz 3 10/28/09 -oh wow got a 70

a) Which gie rise to SHM Give w

really forget and really doubt SHM ~~the~~ is on test

think in terms of amp/phase representation

$x(t) = A\sin(wt + \phi)$

(d) $\frac{d^2x}{dt^2} = -\beta x$

Oh take deriv or 2nd deriv and see if it fits in

$w = \sqrt{\beta}$

really tricky
would def. get that wrong

b) Ok here is the real hard problem

min speed v_0 so starts from A and just gets to B -no slack

Where to start?
think these have gotten harder since not specialized in it
But prof said these are not on the test

(8)

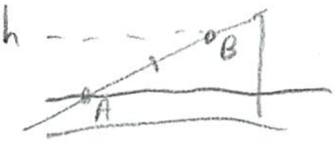
So if even can just spin it at ω

$V = \omega r$

But it loses <u>Energy</u> as goes up

∴ do w Energy

mgh



$A \rightarrow U = 0 = mgh$

$KE = \frac{1}{2} I \omega_0^2 = \frac{1}{2} I \left(\frac{v}{r}\right)^2$

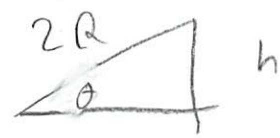
$B \rightarrow U = mgh$

$K = 0$ ~~= ...~~

notice 2R! :)

what is this?

$\frac{1}{2} I \left(\frac{V}{R}\right)^2 = mgh$



$\frac{1}{2}MR^2 \quad = mg\, 2R\sin\theta$

$\sin\theta = \frac{h}{2R}$

$h = 2R\sin\theta$

$\frac{1}{4} M \cancel{R^2} \frac{V^2}{\cancel{R^2}} = mg 2R\sin\theta$ - friction

Look at quantities you have

$\frac{1}{4} V^2 = 2gR\sin\theta$

Solve for V

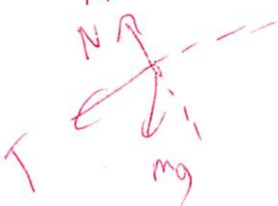
$\cdot 4 \qquad \cdot 4$

$V = \sqrt{8gR\sin\theta}$

← much better - glad I can now do these problems fairly well

(9)

Supposed to have found where T = 0



- subing 1 error for another

$$\Sigma F = ma$$

$$T + mg \sin\theta = \frac{mv^2}{R}$$

↑ slight

↑ set to 0

$$v = \sqrt{Rg \sin\theta}$$

↑ the vel at end kinda what I got

- oh yeah I kinda did
  - neglected y component of g however ??

- well just assumed T = 0 at top
  ↑ if V = 0 but v ≠ 0 at top

~~and~~

And fogot friction - part of E final

$$\int F_{friction} \cdot dr$$

↑ E = Δwork

$$\int \mu N \, dr$$

$$\int \mu mg \cos\theta \, dr$$

$$\mu mg \cos\theta \, \pi R$$

↑ direction R is acting in - where this from?

Oh the swing arc lenght

oh this was the Dumaskin review - no wonder I got it

=

(10)

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_t^2 + mg(2R\sin\theta) + (\mu mg\cos\theta)(\pi R)$$

? Oh and speed is not 0 at top
(would have gone slack before this)

tablet
- Pro: organization slides
- Con: writing quickly, ref quickly, memorizing, concentrating

$$V_0^2 = V_t^2 + 4gR\sin\theta + 2\pi\mu mg\cos\theta$$

↓ where does this go and why? $V_t^2 \to gR\sin\theta$??

$$V_0 = \sqrt{5gR\sin\theta + 2\pi\mu mg\cos\theta}$$

mistakes

(2) forgetting friction

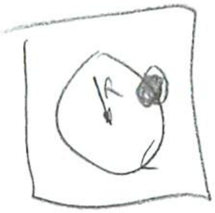
(1) assuming $V=0$ at top
- and thus not calculating friction

(11) Quiz 4 ~~No solutions either~~ Re washing machine !! :)

Lets see how much I learned!

horiz axis

Really hard I thought when I took it
But now I see it was not that difficult



side

$R'_c = R\sin\omega t\,\hat{i} + R\cos\omega t\,\hat{j}$

means not much to problem

relative to washing machine

abs location $\vec{r}_c = r_w + r'_c$

location washer

a) Put washer on frictionless surface

horiz displacement ⟷

as function of time

I don't really get this easily

If spinning ⟲ I guess will migrate →

But how - through a torque

- $\tau = \Delta L$ ← are there any changes in angular momentum?
  - I know there are changes
  - τ is like a circular force

$\tau = I\alpha$

$d I\omega$

~~Oh no change τ~~
~~but $\tau \neq 0$~~

$L = I\omega$

ball - not given so don't this plays a role

(12)

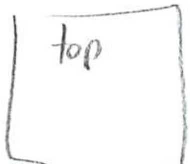
So what is it then

Something related to equation

Study best when lose track of hours

Mix it up

Free body washing machine



what is causing it to move
- clothes
- but how write this

Ans No external forces in horiz direction

So COM fixed

X component COM = 0

So write fixed equation

$$0 = X_{cm} = \frac{m_w x_w + m_c x_c}{m_w + m_c}$$

this is like COM equation
- had not thought of anything like this
- all about knowing how to start

now fill in for x_c

$$\frac{m_w x_w + m_c (\underset{\text{loc washer}}{x_w} + \underset{\text{relative}}{R \sin \omega t})}{m_w + m_c}$$

Solve for x_w which displacement of washer

$X_{cm}(m_w + m_c) = m_w x_w + m_c x_w + m_c R \sin \omega t$

$x_w (m_w + m_c)$

$X_{cm}(m_w + m_c) - m_c R \sin \omega t = x_w (m_w + m_c)$

(13) $X_w = X_{cm} - \dfrac{m_c R \sin \omega t}{m_c + m_w}$ ← they call this D (displacement from to

center of mass of both

← understanding this problem for 1st time

b) If left side of machine attached to wall (r_w fixed)

← 1st time I realized it said left

← washer fixed down

Find the force

$X_{cm} = D = 0 = \dfrac{m_w X_w + m_c (X_w + R \sin \omega t)}{m_c + m_w}$ ← 0 fixed

↑ think wrong track

$F_x(t)$ Only momentum due to clothes

Ok perhaps go back to what I wrote before. But now I am at what I struggled at point 1. What are the clothes actually doing?

$F_x(t) = \dfrac{dp_x}{dt} = \dfrac{d}{dt}(m_c \dot{x}_c)$ ← change in momentum horiz - not circular

So yeah horiz in x direction - plug in equation for horiz as given

$= \dfrac{d}{dt}(m_c \omega R \cos \omega t)$ ← now fill in w/ deriv of $x = \dot{x}$

(14) Does not seem so hard now

Why Greytak ~~too~~ supprised we did bad

Don't forget to deriv the entire thing

with respect to what?

m_c = constant

ω is the thing

$$\frac{d}{dx} \underbrace{m_c \omega R}_{\text{constant I guess - why?}} \cos \omega t$$

~~product rule~~

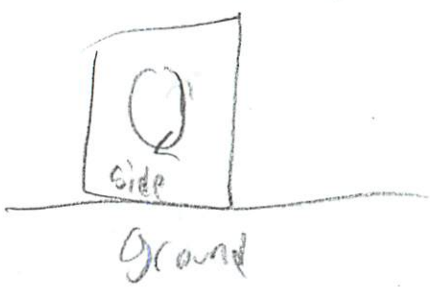
Chair rule

$$\cancel{m_c R \left(\omega \sin \omega t \cdot d + \cos \omega t\right.}\left. d\right)$$

would have gotten this wrong too

$$F_{(x)} t = \boxed{-m_c \omega^2 R \sin \omega t}$$

c) If machie is free standing - not attached. At what ω value jump off floor



Ok this is just like B except F_y and when it $= -(m_w + m_c) g$

$$F_y = (m_w + m_c) g = \frac{dP_y}{dx} = d(m_c \cancel{x}_c)$$

when normal force = 0

$$= \frac{d}{dt}(m_c \omega R \sin \omega t)$$

$$= -m_c \omega^2 R \cos \omega t$$

← they use N

$$-(m_w + m_c) g = -m_c \omega^2 R \cos \omega t$$

(15) Now solve for w

whe

$$w = \sqrt{\frac{-m_w g - m_c g}{-m_c R \cos wt}}$$

?but w is here

go back a step or 2

$$w^2 \cos wt = \frac{-m_w g - m_c g}{-m_c R}$$

oh same as here

$$w = \sqrt{\left(\frac{m_w + m_c}{m_c}\right)\frac{g}{R}}$$

but where does cos wt go → they just turn it to 1

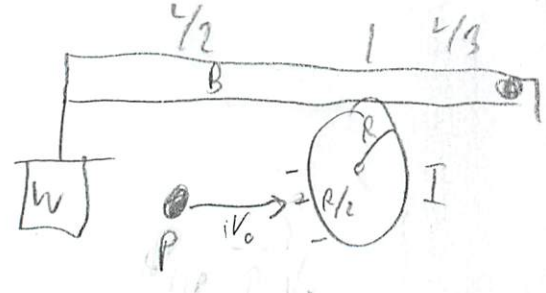
why??

(16) Quiz 5

I recognize this one fairly recent

And very similar to what is going to be on final

Think I've learned more than last time to - torques



break bar

a) Find ω disk after collission

τ yes - outside force

L ~~not~~ is conserved w/ both projectile + disk

$L_i = \cancel{I\omega_0}$ at rest initally

keep as

Collision inelastic

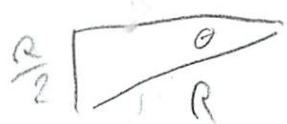
P conserved

E not

L ~~will change~~

Conserved (disk + projectile)

$L_f = I\omega_0 + \ldots$



$\vec{\tau} = \vec{R} \times \vec{F}$ $\frac{1}{\sqrt{2}}$

$F R \frac{1}{\sqrt{2}} = dL$

- break bar plays no role

L will be conserved right before hits infitesmly small gap

$L_i = \frac{R}{2} M_p V_0$ $L = r \times p$

well of disk

(17)

$$I\omega + M_p R^2 \omega$$

$$L_f = \left(I + M_p R^2\right)\omega_0$$

spin + object

Ro Mov

$\tau v = R\omega$ ← correct to

would not have done - guess would be = just longer

τ counterclockwise ⟲

Now find w_f ← which is w_0 in this case

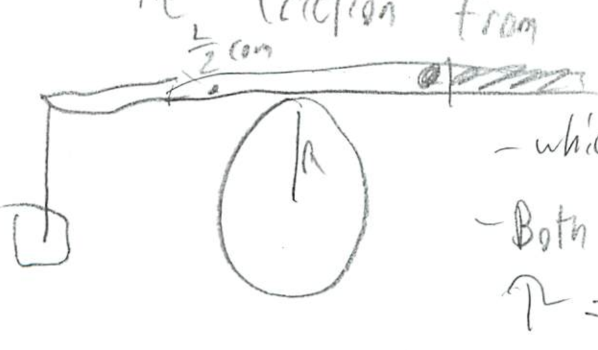
$$\frac{R}{2} M_p V_0 = \left(I + M_p R^2\right) w_0$$

$$\cancel{w_0 = I + M_p R^2}$$

$$w_0 = \frac{\frac{R}{2} M_p V_0}{I + M_p R^2} = \frac{R M_p V_0}{2\left(I + M_p R^2\right)}$$ ✓

b) How long does it take for the disk to come to rest after a collision?

- The friction from break bar



- which is a torque
- Both the bar

$$\tau = \vec{r} \times \vec{F}$$

but is it from com to pivot $\frac{L}{2}$

Or from com to object

⟶ Or from object to pivot $\frac{L}{3}$ ←

(18) Friction is also a torque on wheel

* define what is +

~~$\tau \rightarrow \oplus$~~

$\ominus$ ← I thought it was trust intuition since opposes motion

$\tau_f = R F_f$
wheel

$F_f = \mu N$

$\tau_f = R\mu N$

→ Later

Torques around pivot pt

Do 1st

$\tau = r \times F$

$L \; m_w g$

$\tau = N \frac{L}{3}$

? what is normal force

oh forgot gravity on bar's weight

$\tau = \frac{L}{2} \cdot mg$

well bar does not move torques must be =

$\frac{L}{2} mg + L m_w g = \frac{L}{3} N$

$\div \frac{L}{3}$ or $\cdot \frac{3}{L}$ $\quad \div \frac{L}{3}$ or $\cdot \frac{3}{L}$

← and math error

~~$\frac{2L}{3} m_w g = N$~~ ← L's will cancle

$3\left(M_w + \frac{M_B}{2}\right) g = N$

(19)

$\tau_{f\,wheel} = -R\,\mu\,\cancel{\frac{2L\,m_w g}{3}}$ wrong from before $= -3\mu_k\left(M_w + \frac{M_B}{2}\right)gR$

minus

don't forget mu

this $\tau = \frac{dL}{dt} = (I + M_p R^2)\dot{\omega}$ ← why is it α deriv

So goal is t for stop

~~what~~ α

$\cancel{L_i - \tau_{friction} = 0}$ ← wrong form

$\cancel{(I + M_p R^2)\omega_0}$ z

$L_i \to 0$

through τ_f

$\tau = \frac{dL}{dt}$ ← but how to get how long it applies

duh $\tau = L_f - 0$

← $\dot{\omega} = \alpha$

$\tau = L_f$ just what I thought

but how long?

now solve for $\dot{\omega} = \alpha$ $\qquad \tau = \frac{dL}{dt}$ ← simple + beautiful

$$\dot{\omega} = \alpha = \frac{-3\mu_k\left(M_w + \frac{M_B}{2}\right)gR}{(I + M_p R^2)}$$

(20) $\dot{\omega}$ is constant

integrate

$$\omega(t) = \omega_0 + \dot{\omega} t$$

Comes to rest $t = \frac{\omega_0}{-\dot{\omega}}$ chow in all world

$\cancel{\omega(t)} = \omega_0 + \omega t$

$\cancel{\omega(t)} - \omega t = \omega_0$

$\cancel{\omega(t)}\ t = \frac{\omega_0}{\omega}$

? just takes 1 side of expression. Is that even legal? would not have gotten.

$$t_{stop} = \frac{M_p R V_0}{2(I + M_p R^2)} \cdot \frac{(I + M_p R^2)}{3\mu_k \left(M_w + \frac{M_B}{2}\right) g a}$$

$$= \frac{M_p V_0}{6\mu_k \left(M_w + \frac{M_B}{2}\right) g}$$

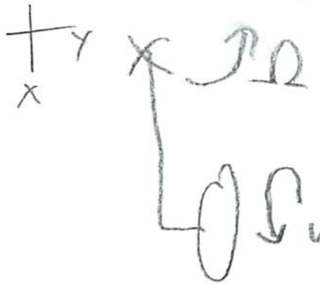
really really relly complex

- no way
- I had some basic concepts

can check units to see if right

- More than before. I think

(21) Quiz 6 - last one (well exams)
aircraft landing gear
gyro problem
- need to review gyros too



Wheel spins as landing gear retracts

a) When at 45° what is L?
x, y, z

$L = I\omega$

break into x, y

- but what ω just spins around 2 points → +y

but then have it moving up. how is it affected
- the gyro problem

$I_{origin} = I_d + MD^2$
← well parallel axis theorem

$\vec{L} = L_{spin} + L_{CM\ motion}$

$|L_{spin}| \left(\frac{\hat{\imath} + \hat{\jmath}}{\sqrt{2}}\right)$ ← the 45° thing $+ I_{origin} \Omega \hat{k}$

$\frac{I_0 \omega}{\sqrt{2}} \hat{\imath} + \frac{I_0 \omega}{\sqrt{2}} \hat{\jmath} + (I_d + MD^2) \Omega \hat{k}$

→ remember the axis don't turn

↑ just add

22 b) What components are chaning w/ time

- well the angle
  - won't be $\sqrt{2}$ anymore

Yeah of $\vec{L}_{spin}$

magnitude constant

rotating around $\vec{z}$ axis at Ω

c) What torqe T_b must be aplied by bearing at 45°

Give in $\hat{i}, \hat{j}, \hat{k}$

$$\vec{\tau} = \frac{dL}{dt} = L_{spin}\,\Omega\,\frac{-i+j}{\sqrt{2}}$$

$$I\omega\,\Omega\,\frac{-i+j}{\sqrt{2}}$$

$$= \vec{\tau}_{bearing} + \vec{\tau}_{gravity}$$

$$\vec{\tau}_{bearing} - \frac{MgD}{\sqrt{2}}\hat{k}$$

$$\vec{\tau}_{Bearing} = I_0\omega\,\Omega\,\frac{-\hat{i}+\hat{j}}{\sqrt{2}} + \frac{MgD}{\sqrt{2}}\hat{k}$$

need to study this type more

spent 5 hrs on these 22 pgs

While Studying

12/12

When you see it - it seems so easy,

The trik is to to <u>be able to do it</u>

Never really understand the $\vec{r}_{cm}$ to where Force acting $\times F$

$0° = 0$

$90° = 1$

Review Central Force concepts
from text book or notes

Review notes from today pg 5 - that problem
and the rolling bolling pall

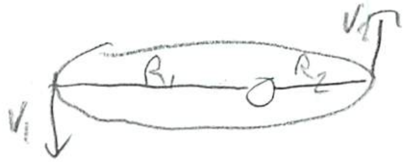
Really need
to study + redo past quizzes

Review Gyro Problems
- starting pts
- formulas

8.01 Textbook Reread 12/13

Central Force

- only need to know angular momentum constant at endpoints



$L = R \times p$

$Rmv =$ constant

$R_1 v_1 = R_2 v_2$

and energy

$$U = -\frac{GMm}{R}$$

$$\text{Force} = \frac{GMm}{R^2} = \frac{mv^2}{R}$$

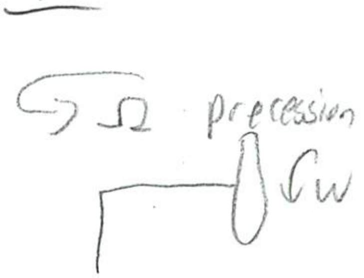
$$= -\frac{d}{dx}$$

So $\frac{1}{2}mv^2 - \frac{GMm}{R}$

$$U = -\int_A^B F\,dr$$

definition of potential energy

Gyro



$$\tau = \frac{dL}{dt}$$

→ ω along x axis

↓ weight is torque down causes it to fall

$$dL = \tau\,dt$$

(2)

Pset 8, 9, 10, 11

When spinning $\vec{L} \neq 0$

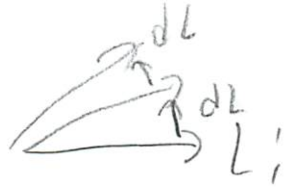
τ perpendicular $= \vec{r} \times \vec{w}$
- of it falling

So points into page $\odot$

$\longrightarrow \vec{r}$ thumb points into page

Change dL are in horizontal plane

top



τ change which is perpendicular will change direction not magnitude

$$\Omega = \frac{d\phi}{dt} = \frac{|dL| / |L|}{dt} = \frac{\tau}{L} = \frac{wr}{I\omega}$$

Precession speed inversly proportional to angular speed

↑ don't just memorize formula

Ok do some problems like car

* in falling flywheel rotates around pivot and gets angular momentum L

- direction L stays constant

① ~~Pset #11 #6 again~~ later

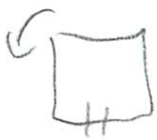
8.01 More Problems

12/13

Now lets do on in class examples

which way should gyro point

car will want to turn



want gyro to have force like this

so it turns

⊗ velocity

into page

get notation right!

So to right in this problem

realize

otherwise right

~~change~~ now suitcase which way

turning

moves out

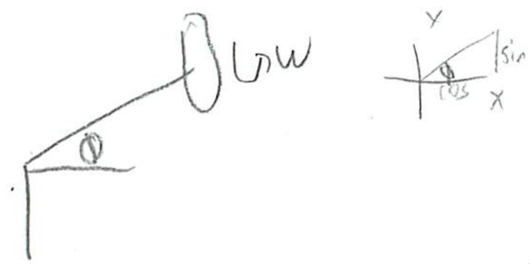
So ω is out

in this case up + left ✓

(21)

W13D2-1 Tilted Gyro

Wheel at one end of axle ℓ



note the axis

a) magnitudes + direction of torques around pivot

So this is going to fall right

- what text book said
- unless going exact right speed

- unless it's like the aircraft landing gear Quiz

$\tau = mg\,\ell\cos\phi$

so $\vec{r} \times \vec{F}$

why cos

oh $\times$ product always cos

textbook a1

weight τ is back $-\hat{k}$ direction

but how magnitude? - write it out

x and ~~x~~ are like the ω_2

~~$\sin\phi\,\hat{x} + \cos\phi\,\hat{y} + \Omega\,\hat{z}$~~

these are not torques that is

L - the angular momentum

τ is Δ in angular momentum

- pay attention, know the difference

b)

(4) Def study this more

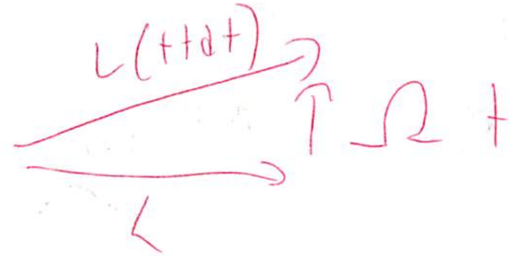
~~The vertical com~~ Vertical component of spin angular momentum does not change

$L_{cm} = I_{cm}\,\omega_s \cos\phi \,\hat{\imath}$ changes into page

yeah what I said about it falling

So take derivitive

$\tau = I_{cm}\,\omega_s \cos\phi \,\Omega\,\hat{\jmath}$



this is whats happening when take derivitive

So now set =

$mgl\cos\phi\,\hat{\jmath} = I_{cm}\,\omega\cos\phi\,\Omega\,\hat{\jmath}$

torque due to gravity from #1

this deriv of changing L which gets it to precess

* it will stay at ϕ because going at right speed - won't not know how to solve for - should be able to figure out

now solve for Ω of precession

$$\Omega = \frac{mgl\cancel{\cos\phi}}{I_{cm}\,\omega\cancel{\cos\phi}} = \frac{mgl}{I_{cm}\,\omega}$$

- independent of angle ϕ

③ b) The wheel has horiz and vert component L

- here can use L

$$L = \underline{I_0}\, \omega \sin\theta\, \vec{x} + I_0\, \omega \cos\theta\, \vec{y} +$$

the other extended

$$(I_0 + M\ell^2)\, \Omega\, \vec{z}$$

horiz components = spin

vert components = precession

in this case want

$$L_p = \vec{r}_{cm \to p} \times m\vec{v} + \vec{L}_{cm}$$

of COM — around com - spinning or not vertical

this is $\vec{L}$ around the pivot

rot mass less is

← parallel axis theorem

$I = M\ell^2 + I$

$$\ell v m\, (\cos\phi\, \hat{k} - \sin\phi\, \hat{\imath}) + \cancel{L_{cm}}$$

speed COM = $V_{cm} = \ell \cos\phi\, \Omega$

ask yourself what is COM

- on pole

total = pivot + spin vector sum

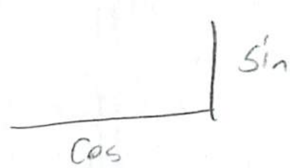
Spin

$\vec{r}$ its working in

$$L_{cm} = I_{cm}\omega(\cos\phi\, \hat{\imath} + \sin\phi\, \hat{k})$$

^ so what I wrote above except cos and sin flipped

I had done L_{cm} - but did not realize it since copied the model



(5) c) For the special case of $\phi = 0$ ~~pi~~

-horizontal

write total L pivot pt

$\tilde{I}_{cm}$ is parallel to face - how is that helpful?

$$\vec{L} = L_{spin} + L_{pivot}$$

$$L = I_{cm}\,w(\cos\phi\,\hat{\imath} + \sin\phi\,\hat{k}) + l v m\,(\cos\theta - \sin\phi) + \cancel{l v m}$$

function of time

$$I_{cm}\,w\,\hat{\imath} + \underline{lvm} + L_{cm}$$

↑don't want to apply here $mr^2\Omega + \frac{I_{cm}}{2}$

$\cancel{L_{pcm} \times m v + L_{cm}}$

L pivot

$\cancel{m l \Omega^2\,\hat{k} + I_{cm} w (\cos \Omega\,\hat{\imath}}$

↑perpendicular through diameter

parallel axis back

2 different moments of inertia

I_{cm} an axis passing through COM perp disk

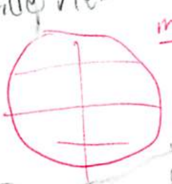
$\tilde{I}_{cm}$ parallel to face

$\tilde{I}_{cm} = \frac{1}{2} I_{cm}$ ← how do you find?

side view disk

$\frac{mr^2}{2}$

side view



$\frac{mr^2}{4}$

↑integrating from top

$+ L_{spin}$

$I\,\omega\,\hat{r}$ direction of Radius

↑ $\cos \Omega t + \sin \Omega t$

top view

 Ω

6

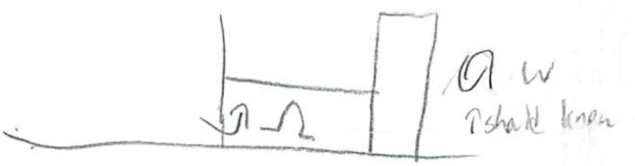
Parallel axis theorm

- moment of inertia about any pt
  - if have I on parallel axis through COM
  - and perp distance to pt

$$I_2 = I_{cm} + md^2$$

Ok let me do ~~that~~ 2 ~~pt~~ similar problem Grain Mill

R, b, M, Ω, g



~~Ω~~ contact force = 2 x weight

a) How is ω related to Ω
- ok more confused about gyros now

$\overleftarrow{\omega}$ $\tau_g = \vec{r} \times m\vec{g}$

$\rightarrow \downarrow$ $\tau_g = \otimes$ page

So precesses right way

but w/ magnitudes

$V = \Omega R$ $V = \omega b$ ← rolling w/out slipping equation

← know of whole thing

$\Omega R = \omega b$

$\omega = \frac{\Omega R}{b}$ ✓

Study his rolling w/o slipping
- like w/ rolling ball

(7) Rolling w/o slipping

Velocity of COM = cross product of angular velocity from pt of contact to com

$V = R\omega$

- but it somehow applies for both

b) What is the horiz component of angular momentum about point P (not drawn, guessing its pivot)

~~Can compute~~ ignore verticle axle

So gravity $\tau = 0$

$L = L_{spin} + L_{pivot}$

? think its asking for both

vertical

$I = \frac{1}{2}Mb^2$

$L_p = I_{cm}\omega + L_{of\ com} + L_{around\ com}$

? what was this one again?

$\vec{r} \times m\vec{v} + L_{around\ com}$ through diameter around

$RM\frac{\omega}{R}$ ← found in part A

$V = \omega R$

$\frac{1}{4}MR^2\omega$

just wants L_{spin}

$L = I_{cm}\omega$

$\frac{1}{2}Mb^2 \frac{\Omega R}{b}$

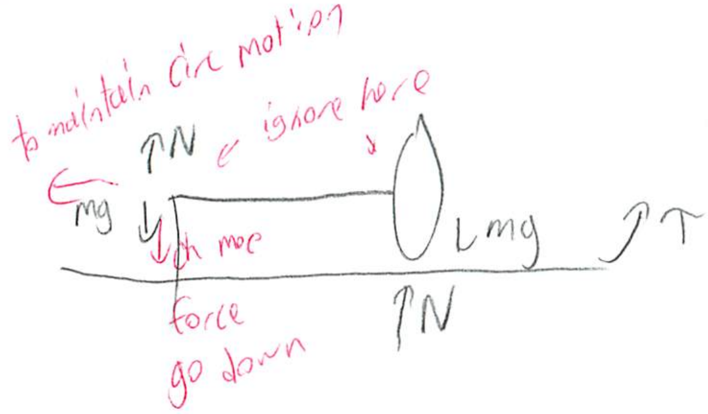
$L = \frac{1}{2}Mb\Omega R$ ✓

Since horizontal

read problem carefully

~~perhaps~~

(8) c) Free body diagram of forces acting on this



d) What is torque about the joint?

$$\tau = r \times F$$

$$\tau_g = mgr \quad \otimes$$

$$\tau_N = -2mgr \quad \odot$$

problem says this is dable
remember to use problem info

- no more complex - nope those are only 2 forces - add

$$\tau = MgR - 2MgR = -MgR$$

e) knew that :)

Now set = to

$$-MgR = \frac{dL}{dt} = d\left(\tfrac{1}{2} Mb\,\Omega R\right)$$

$$-MgR = \tfrac{1}{2} Mb\,\Omega^2$$

note $\Omega = \frac{mgr}{I\omega}$
- all problem about deriving this

but they want angular speed about vert axis

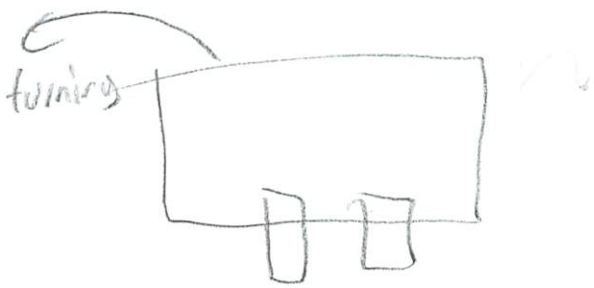
solve for $\Omega = \sqrt{\frac{2g}{b}}$

- prob did not say anything about this - oh this is E
- starting to get

Should now do that Pset problem of gyro in car

P set #11 #6

Car is rounding a curve at high speed



- when inside wheel load → 0


will swing at ⊗


need force ⊙


so gyro this way ⊗

a) Sense of rotation

↷ ⊗

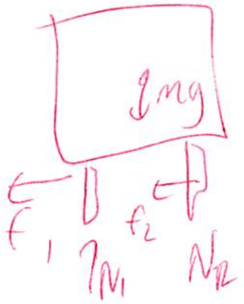
b) Show that for flywheel of m/R find w

$$w = 2v \frac{m_T L}{m_w R^2}$$ for = loading

m_T = total mass of car + flywheel

L = height com above road

(10) Just going to copy this one



when $N_1 = N_2$ = equal loading

$\tau = r \times F$

$\tau_1 = r \times \frac{1}{2}\cancel{mg} + r \times \frac{1}{2}mg$

include both F and N

$r_1(N_1 + f_1) + r_2(N_2 + f_2)$

all into page

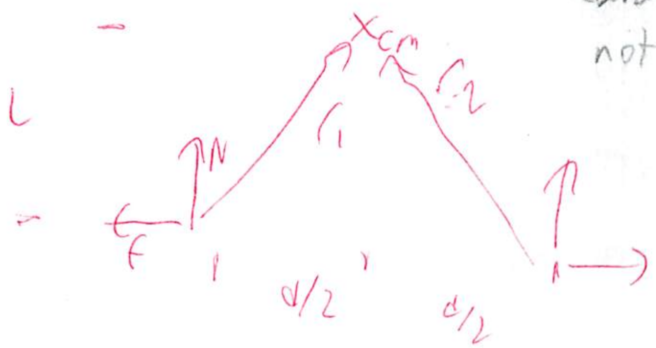
except N_2

oh draw like this

-what I thought

think about physics

not how solved problem before



rotate to R Right hand rule

(11)

top

$L(t+dt)$ $\uparrow dL$ see how the thing is changing

L

don't forget about this

- yeah increases load turning left
- if it tries to turn right, opposes turn

$$L = I\omega$$

derive

$$\frac{dL}{dt} = \tau = I\omega\Omega$$

so set $\tau =$

$$I\omega\Omega = r_1(N_1 + f_1) + r_2(N_2 + f_2)$$

$\frac{1}{2}MR^2$ $\quad \frac{v}{R}$

don't forget these conversion steps

$$\tau = \frac{-mR^2\omega v}{2r}$$

Because car is ~~not~~ stable $\tau = 0$

typo

yeah set = to

then this problem assumes $N_1 = N_2$

$$0 = L(f_1 + f_2) - \frac{mR^2\omega v}{2r}$$

com going to center

(13) Newton's 2nd Law

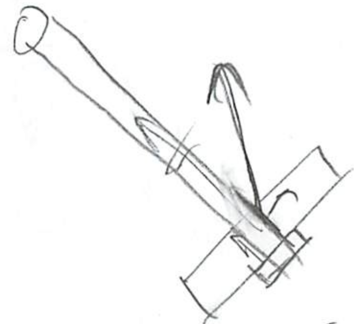
$$F = ma$$

$$f_1 + f_2 = \frac{mv^2}{R}$$

Sub in

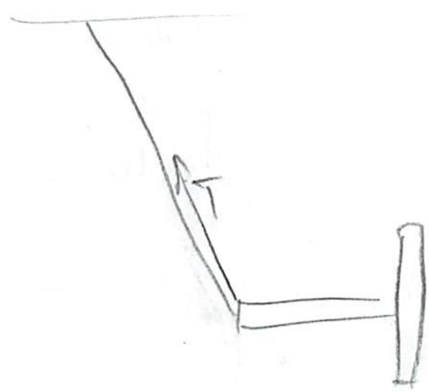
$$\frac{L m_T V^2}{r} = \frac{m_w R^2 \omega_s V_{cm}}{2r}$$

$$\omega = \frac{2 L m_T V_{cm}}{m_w R^2}$$



$$F_x = m \Omega^2 r \cos\theta$$

$$\boxed{F_y = mg}$$



(13) Should be able to do Quiz 6 better now

the aircraft landing gear gyro problem

I_o center

I_d diameter know I see why need it

- the more I know the less likely I am to read always read problem

a) What is L of wheel about origin when at 45° know a lot better how arrived at answer

$L_{total} = L_{spin} + L'_{pivot}$ ← have already

$+ L$ of com $+$ ~~L around com~~

- Some where parallel axis theorm ? just one of com

$I_o \omega \hat{r}$ $\frac{1}{\sqrt{2}}\hat{x}$ $\frac{1}{\sqrt{2}}\hat{y}$ $+ \cancel{(I_o + Ml^2)\omega \hat{r}} +$ $\hat{x}, \hat{y}$ and decompose again

this is vertical $+ I_o \Omega \hat{z}$

sub for this see back

$\cancel{\frac{I_o\omega}{\sqrt{2}}\hat{x} + \frac{I_o\omega}{\sqrt{2}}\hat{y} + \frac{I_o\omega}{\sqrt{2}}\hat{x} + \frac{I_o\omega}{\sqrt{2}}\hat{y} + \frac{Ml^2\omega}{\sqrt{2}}\hat{x}} +$

(14)

$\cancel{\frac{M\ell^2 \omega}{\sqrt{2}} \hat{y}} + I_d \Omega \hat{z}$

Well $I_0 = I_d + MD^2$
parallel axis theorm

$2\frac{I_0 \omega}{\sqrt{2}} \hat{x} + 2\frac{I_0 \omega}{\sqrt{2}} \hat{y} + \cancel{\frac{M\ell^2 \omega}{\sqrt{2}}} \hat{x} + \cancel{\frac{M\ell^2 \omega}{\sqrt{2}}} \hat{y} + I_0 \Omega \hat{z}$

not d

~~$2 I_0 \omega$~~

$\cancel{\frac{\omega(2I_0 + M\ell^2)}{\sqrt{2}} \hat{x} + \frac{\omega(2I_0 + M\ell^2)}{\sqrt{2}} \hat{y}} + I_d \Omega \hat{z}$

So I think that's it unless simp more

got too complex

think Jared tood me too many of those things - don't need all of them

Yeah ~~only~~ I had around com twice

$L_{around\ com} = L_{spin}$

Jared did not find + fix

~~and then use parallel axis theorm~~

Oh wheel about origin ~~don't count~~ Ω

I_d - instead just he origin - which is parallel axis theorm

- get it better now

(15) b) Which components are changing w/ time

the angle - no longer $\sqrt{2}$

~~also~~ Ω is constant

L constant

L_{spin} not constant

Since direction changing (mini-trick)

$|$magnitude$|$ = same

c) What is the τ_b that is applied to landing gear

$\tau = \frac{dL}{dt}$ ← remember

possible part d

So find τ

then derive L ← yeah but L_{spin} only

Set = ~~th~~

find something

and do so in this problem

$\tau_w = r \times mg$ + $\tau_{bearing} = \tau_{total} = \frac{dL}{dt}$

rmg

τ to pull it up

- perhaps just differentiate L_{spin} and set = to

$$\tau \quad \frac{I_0 \omega \Omega}{\sqrt{2}} \hat{\imath} + \frac{I_0 \omega \Omega}{\sqrt{2}} \hat{\jmath} + (I_\perp + MD^2)\Omega^2 \hat{k}$$

- what if $\omega \to 0$

- don't think this is right

(16) Yeah wanted you to differentiate L_{spin} ← why L_{spin} only? - because its the only thing changing → (direction that is)

↓ this is what adding

$$\tau = \frac{dL}{dt} = |L_{spin}| \, \Omega \left(\frac{-\hat{i} + \hat{j}}{\sqrt{2}} \right)$$

$$I_0 \, w \, \Omega \left(\frac{-\hat{i} + \hat{j}}{\sqrt{2}} \right)$$

and set it = to

$$= \tau_{bearing} - \tau_{gravity}$$

$$\tau_{bearing} - \frac{Mg D}{\sqrt{2}} \hat{k}$$ ← note the cos 45

and solve for $\tau_{bearing}$

↑ should have read qu closer

what is it asking for

$$I_0 w \Omega \left(\frac{-\hat{i} + \hat{j}}{\sqrt{2}} \right) = \tau_{bearing} - \frac{Mg D}{\sqrt{2}} \hat{k}$$

$\cdot\sqrt{2}$ $\cdot\sqrt{2}$ $\cdot\sqrt{2}$

$$I_0 W \Omega (-\hat{i} + \hat{j}) = \sqrt{2}\,\tau_{bearing}$$

$+ Mgd\,\hat{k}$

$$\tau_{bearing} = \frac{I_0 w \Omega (-\hat{i} + \hat{j}) + Mgd\,\hat{k}}{\sqrt{2}}$$

- don't forget the gravity

there is no way I will get a 100 on this

- but should be good for partial credit

- d/c moving of gyro

17 Rolling w/o slipping Rolling Ball

When does the ball start to roll?

ball mass M radius R

thrown down ally

Slides but friction gets it to start to roll

$I_{cm} = \frac{2}{5} mR^2$

V_f? just as starts to roll w/o slipping

friction is what does it

~~static~~ kinetic → static

Can solve w/ Torque + angular momentum

1. 2nd law for translational

2. angular momentum constant before and after

I would prob use some combo of the 2 methods w/o realizing it

ω ⊗

) cancles ✓

$\tau_f = r \times F_f$ ← remember

$-R \mu mg = I \alpha$

(18) In this case α is (+)

- look at chart

* Also look at kinematics

$$a_{cm} = \frac{-f_{friction}}{mass}$$

don't forget normal kinematics

a will be $\ominus$ as expected

$$\omega(t) = \alpha t = \frac{Rf}{I} t$$

can't depend on rolling since skidding does both

$$v(t) = V_0 - \frac{f_k}{m} t$$

* As soon as ball stops slipping kinetic friction no longer acts t_f

- constant angular + linear velocity

$$v_f = R\omega_f$$

$$v_f = R^2 \frac{f_x}{I_{cm}} t_f$$

yeah its all simple kinematics can I do it?

$$v_f = V_0 - \frac{f_k}{m} t_f$$

Solve for t_f and sub in

circular reference!!

$$t_f = \frac{I_{cm}}{f_k R^2} v_f$$

$$v_f = V_0 - \frac{f_k}{m} \frac{I_{cm}}{f_k R^2} v_f$$

$$v_f = V_0 - \frac{I_{cm}}{m R^2} v_f$$

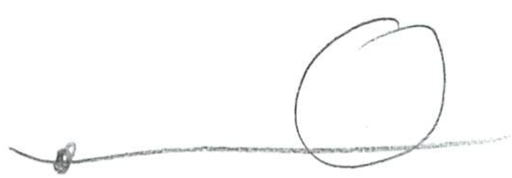
(19) Solve for v_f

* note $I_{cm} = \frac{2}{5} m R^2$

$$v_f = \frac{v_0}{1 + \frac{I_{cm}}{mR^2}} = \frac{5}{7} v_0$$

* shows ~~chan~~ that μ does not affects speed it starts rolling at but μ will affect time

2. Since no friction forces in final result should pick a pt to find torque + angular momentum which does not involve μ



↑ good point

friction cancles to 0

pt of friction moves down line but always parallel to 0

gravity + normal cancle

no torques → L conserved

* inital L only due to translation

$$L_0 = Rp \hat{k} = Rmv_0 \hat{k}$$

final L has rotation + translation

$$(Rp + I\omega) \hat{k}$$

(20) Convert $\qquad V = \omega R$

$$\left(Rmv + \frac{2}{5}MR^2 \frac{v}{\cancel{R}}\right)\hat{k}$$

$$mRv\left(1 + \frac{2}{5}\right)$$

$$\frac{7}{5} mRv \hat{k}$$

* set inital final =

$$\cancel{R}mv_0 = \frac{7}{5} \cancel{R}\cancel{M}v_f$$

$$v_0 = \frac{7}{5} v_f$$

$$\cdot\frac{5}{7} \quad \cdot\frac{5}{7}$$

$$v_f = \frac{5}{7} v_f$$

Same result (V)

2nd way much better

- and I know all the steps

Now just need to think to do them

know what its = to

(21) In depth on part

$$V_f = V_0 - \frac{I_{cm}}{mR^2} V_f$$

$$\cancel{V_f^2 = V_0 - \frac{I_{cm}}{MR^2}}$$ illegal

$$V_f^2 + \frac{I_{cm}}{MR^2} = V_0$$

$$V_f^2 \cdot MR^2 + I_{cm} = MR^2 V_0$$

$$\cancel{V_f^2 = V_0 V_f - \frac{I_{cm}}{MR^2}}$$

* perfectly legal to have V_f on both sides

its like

$$3x^2 + 7 = x^2$$

and w/ coefficents in front - its not circular

find V_f

$$7 = -2x^2$$

$$\frac{7}{-2} = x^2$$

$$x = \sqrt{\frac{7}{-2}}$$

know the algebra

$$V_f + \frac{I_{cm}}{mR^2} V_f = V_0$$

$$V_f \left(1 + \frac{I_{cm}}{MR^2}\right) = V_0$$

$$V_f = \frac{V_0}{1 + \frac{I_{cm}}{MR^2}}$$

Oh yeah that was easy

(22) Ok did rolling w/o slipping
- what else 1hr left
- central force
  - did some earlier
  - remember get formula sheet
- know the R x F much better now
- did everything else on check list

<u>on test</u>

angular collission
- the bullet hitting block
- L conserved very similar to rolling w/o slipping method 2

- central force
- gyros
  - did 3 hrs of that
- turning car
  - kinda did that
  - could do again
- mass flow
  - ~~[illegible]~~ prob not on there would be very bad it was
- otherwise studied everything else a lot

23 Turning car

- but not PSet 11 #6
- where was this problem?

Or a yo-yo problem

Principal axis therm

- think just in gyros

Newton's 2nd law $F=ma$

in Central Force $\leftarrow F=\frac{GMm}{R^2}=\frac{mv^2}{R}$

Circ motion

- constant speed towards center $\frac{v^2}{R}$
- tangental component w/ speed changing $\neq 0$
- net force towards center

3rd law

= oppesit

2nd Liear + circular

$W=\int F\,dr=\Delta E$

$PE=mgh$

$\frac{1}{2}kx^2$

$\frac{GMm}{R}$

Conservitive + Non conservitive forces

~~Work~~ Δ

(24) Momentum $= p = mv$

Impulse $= \Delta p$

$F = ma = \frac{d}{dt} mv$

~~p~~

internal + external

↑ cancles ↑ what changes a

1 and 2 d collissions

Sounds like I know all this we will see

Conservation Laws

~~m~~, E, p, L

SHO

$F = -kx$

position $= A\cos(\omega t + \varphi)$

Velocity $= -A\omega\sin(\omega t + \delta)$

am not good with

acc $= -A\omega^2\cos(\omega t + \delta)$

periodic = time

harmonic = space

period $= 2\pi\sqrt{\frac{m}{k}}$ $\omega = \sqrt{\frac{k}{m}}$

$f = \frac{\omega}{2\pi}$

26

$$\tau = -b\hat{r} \times -T\hat{j} = bT\hat{k}$$

notice what is going on
right hand rule
never saw it written like that

feel kinda
well prepared
but test will
prob be hard
guess 70%
pass class

$$mg - T = ma$$

really simple, just remember ΣF

$$bT = I\alpha$$

and equivilint $a = \alpha b$

$$bT = I\frac{a}{b}\alpha$$

$\cdot b \qquad \cdot b$

$$\cancel{b^2 T = I}$$

solve for α

$$mg - T = mb\alpha = mb\left(\frac{bT}{I}\right)$$

now solve for T

$$mg - T = \frac{mb^2 T}{I}$$

$$Img - TI = mb^2 T$$

$$Img = mb^2 T + TI$$

$$Img = T(mb^2 + I)$$

$$T = \frac{Img}{mb^2 + I} = \frac{mg}{1 + \frac{2b^2}{R^2}}$$

eairlier

they expand
I

- all about what you want to solve for

25

~~Newton~~

Equations of motion SHO using Energy, Force, torque

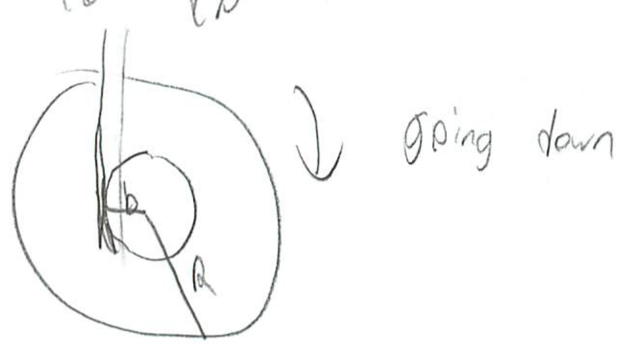
equations of motion, angular freq of spring, pendulum
physical pendulum

Really don't get SHO

Yo-Yo problem

Central force problem

Yo-Yo



a) tension as assends + decends ?

τ is in ⊛ direction

so do both $\sum F = ma$

and τ which uses position

$r \times F$

(27) Now sole for α

$$\alpha = \frac{bT}{I} = \frac{2bg}{(R^2 + 2b^2)}$$

Can find a from $a = R\alpha$

$$a = \frac{2b^2 g}{(R^2 + 2b^2)} = \frac{g}{1 + \frac{R^2}{2b^2}}$$

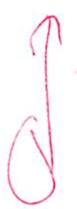
acc less than free fall

now as yo-yo starts to move $\uparrow$

τ causes ω to $\downarrow$

ΣF cause com $v \downarrow$

still $mg - T = ma$



this side now

$\odot$ torque out

$$-b\,T_{up} = I a$$

$$\alpha = \frac{-b\,T_{up}}{I}$$

Tangular speed slowing

$$a = -b\,\alpha$$

(28) Sub in ~~$a = b\alpha$~~

$$a = \frac{b^2 T_{up}}{I}$$

← set = to

$$mg - T_{up} = \frac{m b^2 T_{up}}{I}$$

and solve for what you want T_{up}

$$T_{up} = \frac{mg}{1 + \frac{mb^2}{I}} = \frac{mg}{1 + \frac{2b^2}{R^2}}$$

b) use conservation of Energy to find ω at bottom of string

- set $U = 0$ at bottom

$$E_i = -mg - l = mgl$$

$$E_f = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$ ← not stopped at bottom

$$E_i = E_f$$

* constraint $V_f = b\omega_f$ ← typical

$$I = \frac{1}{2}mv^2$$

$$\omega_f = \sqrt{\frac{4gl}{(2b^2 + R^2)}}$$

(29) Could ~~a~~ use kinematics to determine final angular velocity

$$\Delta t = \sqrt{\frac{2l}{a}} = \sqrt{\frac{l(R^2+2b^2)}{b^2 g}}$$

where in all world do they get that?

oh something w/

$v = at$

$x = vt$

$d = \frac{1}{2}at^2$

$$\boxed{v_f^2 = 2ad}$$

basic kinematics

$$\omega_f = \alpha \Delta t = \sqrt{\frac{4gl}{(R^2+2b^2)}}$$

↑ oh same as above

midnight - can't really concentrate

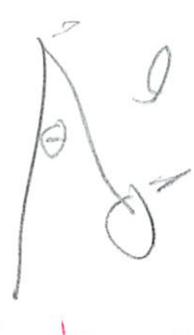
SHM ~

- what do they give us
- nothing

Rotational
- sum torques = 0

Translation
Sum forces = 0

Ok 1 pendulum problem



$\theta = 1$ rad

$\sin\theta \cong \theta$

circular arc
- centered at pivot point

polar coords

(30)

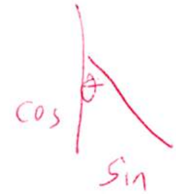
← split mg vector

$$\vec{mg} = mg(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta})$$

want tangental component of gravitational force

$$F_\theta = -mg\sin\theta$$

↑ why is it sin



→ tangental

Oh

still should be cos ↑

restores pendilum to original value

$\theta = 0$

$\theta > 0 \quad F_\theta > 0$ $\qquad -\pi < \theta < \pi$

$\theta < 0 \quad F_\theta > 0$

$$a_\theta = l\alpha = l\frac{d^2\theta}{dt}$$

↑ r ↑ angular ↑ 2nd deriv

as well as $F = ma$

$$-mgl\sin\theta = ml2\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt} + \frac{g}{l}\sin\theta = 0$$

no based off this - set = to

α + ↑ acc due to gravity ???

(31)

Solve

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta$$

Notice similar to SHM

$$\frac{d^2x}{dt^2} = \frac{-k}{m}x$$

W of angular freq of oscillation $= \sqrt{\frac{k}{m}}$

W of pendilum $W_0 \cong \sqrt{\frac{g}{l}}$

w/ period $T = \frac{2\pi}{W_0} \cong 2\pi\sqrt{\frac{l}{g}}$

Now use SHM

$\theta(t) = \theta_A \cos(\omega t)$

fill in

$\theta \cos\left(\frac{2\pi}{T}t\right)$

$\theta \cos\left(\sqrt{\frac{g}{l}}\,t\right)$

? velocity - differentiate

$\frac{d}{dx}\,\theta \cos\sqrt{\frac{g}{l}}\,t$

$-\sqrt{\frac{g}{l}}\,\theta \sin\sqrt{\frac{g}{l}}\,t$

(32) keep in mind $w = \frac{d\theta}{dt}$ is a kinematic variable that changes w/ time in oscillatory manner / w_0 describes that system

$w \rightarrow$ time dependent, depends on θ_0

w_0 not dependent on θ_0

Can find $t_{\text{hits bottom}}$ by setting $\theta = 0$

$$0 = \theta \cos\left(\sqrt{\frac{g}{l}}\, t_1\right)$$

So when $\sqrt{\frac{g}{l}}\, t = \frac{\pi}{2}$

So when $t = -\sqrt{\frac{g}{l}}\, \theta$

$$\frac{d\theta}{dt}(t_1) = -\sqrt{\frac{g}{l}}\, \theta \sin \sqrt{\frac{g}{l}}\, t_1$$

$$-\sqrt{\frac{g}{l}}\, \theta \sin\left(\frac{\pi}{2}\right)$$

$$-\sqrt{\frac{g}{l}}\, \theta$$

$\ominus$ means θ moving in negitive $\hat{\theta}$ direction at bottom of arc 1st time