Physics 8.02 **Equation Sheet Exam One**

Please Remove this Tear Sheet from Your Exam

$$\vec{\mathbf{E}} = \frac{q}{4\pi\varepsilon_o r^2}\hat{\mathbf{r}} = \frac{q}{4\pi\varepsilon_o r^3}\vec{\mathbf{r}}$$

$\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}}}{r}$ *points* **from** *source q* **to** *observer*

$$\vec{\mathbf{E}}_{\text{many point charges}} = \sum_{i=1}^{N} \frac{q_i}{4\pi\varepsilon_o \left|\vec{\mathbf{r}} - \vec{\mathbf{r}}_i\right|^3}(\vec{\mathbf{r}} - \vec{\mathbf{r}}_i)$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\int_{source} \frac{dq}{\left|\vec{\mathbf{r}} - \vec{\mathbf{r}}'\right|^2}\hat{\mathbf{r}}$$

$$\vec{\mathbf{F}}_q = q\vec{\mathbf{E}}_{source}$$

$$\oiint_{\substack{closed\\surface}} \vec{\mathbf{E}} \cdot \mathbf{d}\vec{\mathbf{A}} = \frac{Q_{\text{enc}}}{\varepsilon_o}$$

$\mathbf{d}\vec{\mathbf{A}}$ points from inside to outside

$$\oint_{\substack{closed\\path}} \vec{\mathbf{E}} \cdot \mathbf{d}\vec{\mathbf{s}} = 0$$

$$\Delta V_{moving\ from\ a\ to\ b} = V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot \mathbf{d}\vec{\mathbf{s}}$$

$$\Delta U = q\Delta V$$

$$V_{\text{point charge}} = \frac{q}{4\pi\varepsilon_o r}; V(\infty) = 0$$

$$V_{\text{many point charges}} = \sum_{i=1}^{N} \frac{q_i}{4\pi\varepsilon_o \left|\vec{\mathbf{r}} - \vec{\mathbf{r}}_i\right|}; V(\infty) = 0$$

$$V(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\int_{source} \frac{dq}{\left|\vec{\mathbf{r}} - \vec{\mathbf{r}}'\right|}; V(\infty) = 0$$

$$U = \sum_{all\ pairs} \frac{q_i q_j}{4\pi\varepsilon_o \left|\vec{\mathbf{r}}_i - \vec{\mathbf{r}}_j\right|}; U(\infty) = 0$$

$$U = \frac{1}{2}\varepsilon_o \iiint_{all\ space} E^2 dV_{vol}$$

$E_r = -\frac{\partial V}{\partial r}$ for spherical symmetry,

$$\vec{\mathbf{E}} = -\vec{\nabla} V$$

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

$$C = \frac{|Q|}{|\Delta V|} \qquad U = \frac{1}{2}C\Delta V^2 = \frac{Q^2}{2C}$$

Circumferences, Areas, Volumes:

1) The area of a circle of radius r is πr^2
   Its circumference is $2\pi r$

2) The surface area of a sphere of radius r is $4\pi r^2$. Its volume is $(4/3)\pi r^3$

3) The area of the sides of a cylinder of radius r and height h is $2\pi rh$. Its volume is $\pi r^2 h$

Integrals that may be useful

$$\int_a^b dr = b - a$$

$$\int_a^b \frac{dr}{r} = \ln(b/a)$$

$$\int_a^b \frac{1}{r^2}dr = \left(\frac{1}{a} - \frac{1}{b}\right)$$

Some potentially useful numbers

$$k_e = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{\text{N m}^2}{\text{C}^2}$$

8.02 Exam One Spring 2010

P	L	A	S	M	E	I	E	R							

FAMILY (last) NAME

M	I	C	H	A	E	L									

GIVEN (first) NAME

9	2	1		6	4		5	2	6	1

Student ID Number

Your Section:

X **L01 MW 9 am** ____**L02 MW 11 am** ____**L03 MW 1 pm** ____**L04 MW 3 pm**
____**L05 TR 9 am** ____**L06 TR 11 am** ____**L07 TR 1 pm** ____**L08 TR 3 pm**

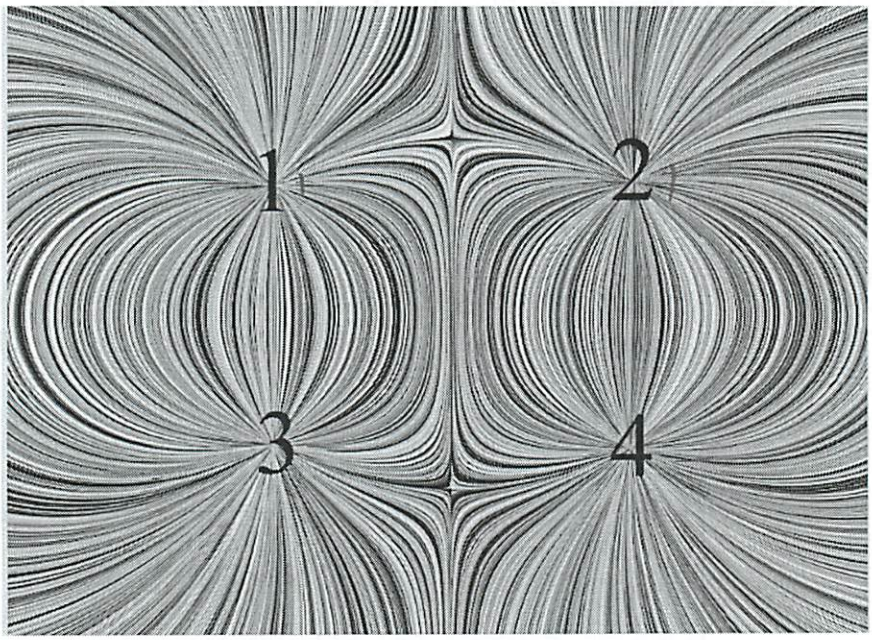
Your Table and Group (e.g. 10A): 11C

	Score	**Grader**
Problem 1 (25 points)	25	PHF
Problem 2 (25 points)	17	EJ
Problem 3 (25 points)	14	AES
Problem 4 (25 points)	15	EF
TOTAL	71	

Problem 1 (25 points)

In this problem you are asked to answer 5 questions, each worth 5 points. You do not have to show your work; in most cases you may simply circle the chosen answer.

Question 1 (5 points)



1. Above we show the grass seeds representation of the field of four point charges, located at the positions indicated by the numbers. Which statement is true about the signs of these charges:

a) All four charges have the same sign.

b) Charges 1 and 2 have the same sign, and that sign is opposite the sign of 3 and 4.

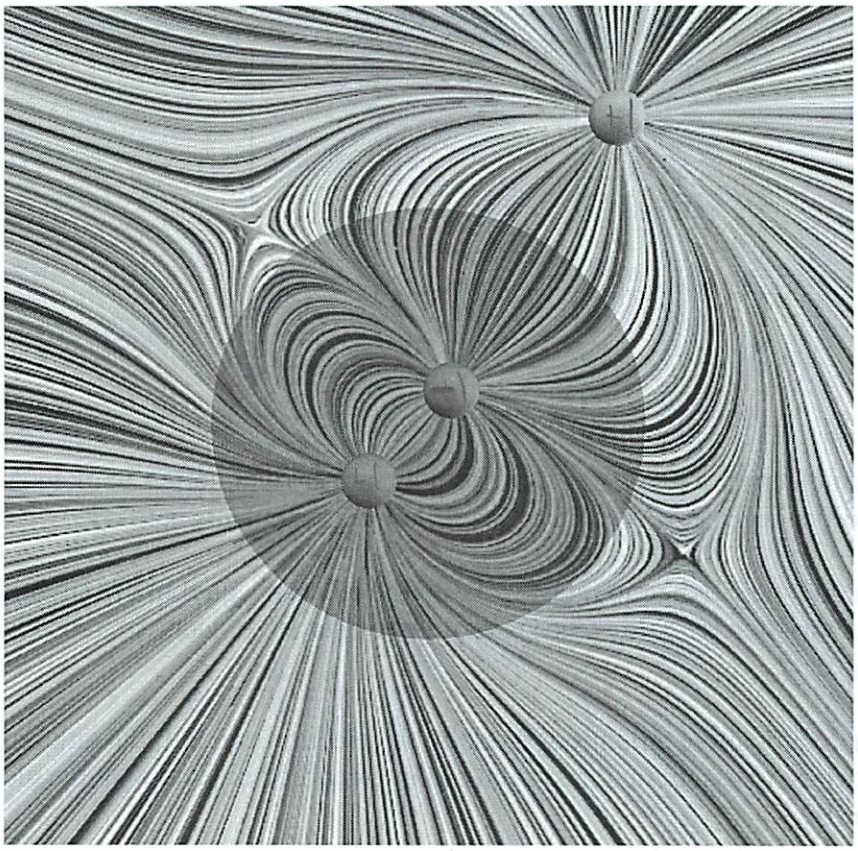
c) Charges 1 and 3 have the same sign, and that sign is opposite the sign of 2 and 4.

d) Charges 1 and 4 have the same sign, and that sign is opposite the sign of 2 and 3.

e) None of the above.

Question 2 (5 points)

The grass seeds figure below shows the electric field of three charges with charges +1, +1, and -1, The Gaussian surface in the figure is a sphere containing two of the charges.



The total electric flux through the spherical Gaussian surface is

a) Positive

b) Negative

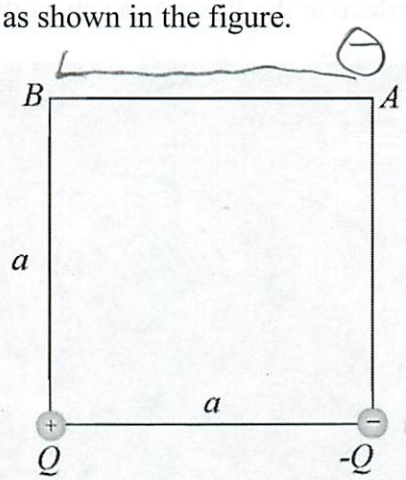
Net $Q_{enc} = 0$

c) Zero

d) Impossible to determine without more information

Question 3 (5 points)

Two point-like charged objects with charges $+Q$ and $-Q$ are placed on the bottom corners of a square of side a, as shown in the figure.



easy to do

you − work

You move an electron with charge $-e$ from the upper right corner marked A to the upper left corner marked B. Which of the following statements is true?

a) You do a negative amount of work on the electron equal to the amount of energy necessary to assemble the system of three charged objects with the electron at point B.

2 diff things

b) You do a positive amount of work on the electron equal to the amount of energy necessary to assemble the system of three charged objects with the electron at point B.

c) You do a positive amount of work on the electron and the potential energy of the system of three charged objects increases.

d) You do a negative amount of work on the electron and the potential energy of the system of three charged objects decreases. always moves naturally to ↓ PE naturally

e) You do a negative amount of work on the electron and the potential energy of the system of three charged objects increases.

f) You do a positive amount of work on the electron and the potential energy of the system of three charged objects decreases.

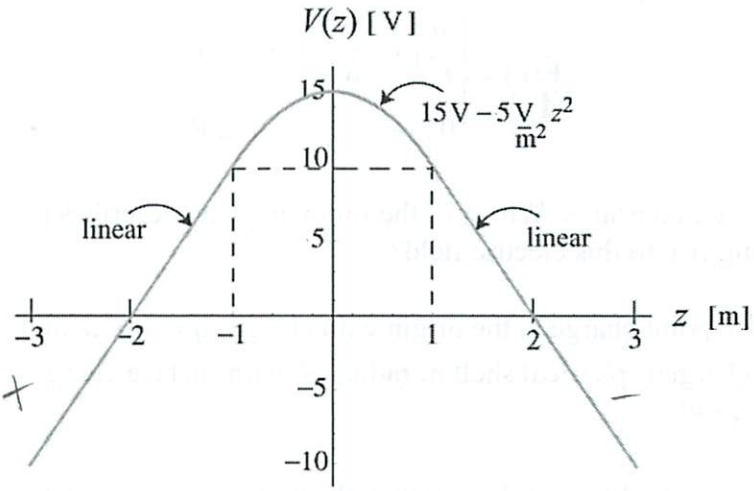
$W = \Delta U$

you − work

system ↓ energy

Question 4 (5 points)

A graph of the electric potential $V(z)$ vs. z is shown in the figure below.



Gradient

Which of the following statements about the z-component of the electric field E_z is true?

Slope

a) $E_z < 0$ for $-3\,\text{m} < z < 0$ and $E_z < 0$ for $0 < z < 3\,\text{m}$.

b) $E_z < 0$ for $-3\,\text{m} < z < 0$ and $E_z > 0$ for $0 < z < 3\,\text{m}$.

c) $E_z > 0$ for $-3\,\text{m} < z < 0$ and $E_z < 0$ for $0 < z < 3\,\text{m}$.

d) $E_z > 0$ for $-3\,\text{m} < z < 0$ and $E_z > 0$ for $0 < z < 3\,\text{m}$.

e) None of the above because E_z cannot be determined from information in the graph for the regions $-3\,\text{m} < z < 0$ and $0 < z < 3\,\text{m}$.

– no you can by looking around

$E = -\nabla$

⊕

Question 5 (5 points)

1, 5

2 -3

Careful measurements reveal an electric field

$$\vec{\mathbf{E}}(r) = \begin{cases} \dfrac{a}{r^2}\left(1 - \dfrac{r^3}{R^3}\right)\hat{\mathbf{r}}\,; & r \le R \\ \vec{0}\,; & r \ge R \end{cases}$$

where a and R are constants. Which of the following best describes the charge distribution giving rise to this electric field?

a) A negative point charge at the origin with charge $q = 4\pi\varepsilon_0 a$ and a uniformly positive charged spherical shell of radius R with surface charge density $\sigma = -q/4\pi R^2$.

b) A positive point charge at the origin with charge $q = 4\pi\varepsilon_0 a$ and a uniformly negative charged spherical shell of radius R with surface charge density $\sigma = -q/4\pi R^2$.

is it a sphere or shell?

c) A positive point charge at the origin with charge $q = 4\pi\varepsilon_0 a$ and a uniformly negative charged sphere of radius R with charge density $\rho = -q/(4\pi R^3/3)$.

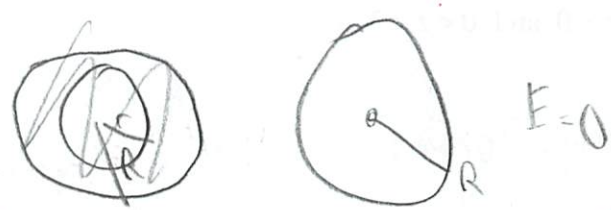
d) A negative point charge at the origin with charge $-q = -4\pi\varepsilon_0 a$ and a uniformly positive charged sphere of radius R with charge density $\rho = q/(4\pi R^3/3)$.

also credit – but does not really work

e) Impossible to determine from the given information.

could it be – don't know which +/–

a is ⊕ did not say so d or e also correct ambigeous



from source to field pt

this is confusing

a is ⊕ so

⊕ in middle

✓ great performance on part 1

$\sqrt{a^2+a^2} = \sqrt{2a^2} = \sqrt{2}a$

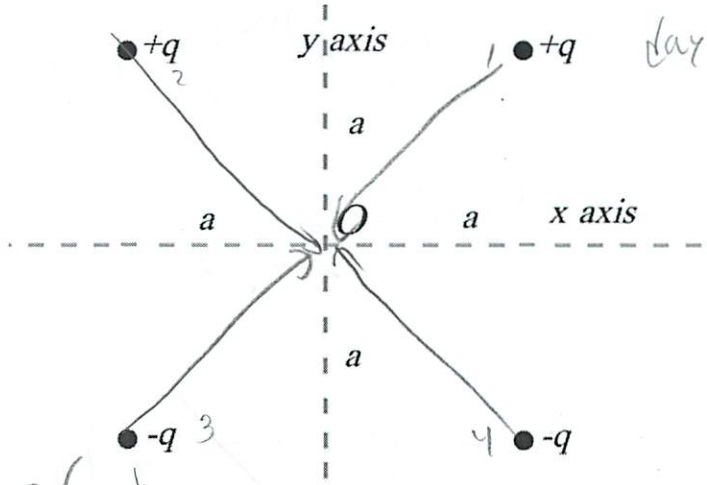
Problem 2 (25 points)

NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!) .

Coulomb's law

day 2

Four charged point-like objects, two of charge $+q$ and two of charge $-q$, are arranged on the vertices of a square with sides of length $2a$, as shown in the sketch.



a) What is the electric field at point O, which is at the center of the square? Indicate the direction and the magnitude.

$$\left[kq\left(\frac{a}{\sqrt{a^2+a^2}}\hat{i} - \frac{a}{\sqrt{a^2+a^2}}\hat{j}\right) + kq\left(-\frac{a}{\sqrt{a^2+a^2}}\hat{i} - \frac{a}{\sqrt{a^2+a^2}}\hat{j}\right) \right.$$

$$\left. + kq\left(\frac{a}{\sqrt{a^2+a^2}}\hat{i} + \frac{a}{\sqrt{a^2+a^2}}\hat{j}\right) + kq\left(-\frac{a}{\sqrt{a^2+a^2}}\hat{i} + \frac{a}{\sqrt{a^2+a^2}}\hat{j}\right) \right]$$

top 2 cancel horiz vert ↓

bottom 2 horiz cancel vert ↓

$$\vec{E} = -2\frac{kqa}{\sqrt{2}a\sqrt{2}a\sqrt{2}a}\hat{i} + \frac{2kqa}{\sqrt{2}a\sqrt{2}a\sqrt{2}a}\hat{j}$$

↑ cancles

$$= \frac{2kq}{2\sqrt{2}a^2}\hat{i} - \frac{2kq}{2\sqrt{2}a^2}\hat{j}$$

$$\vec{E} = \left(\frac{kq}{\sqrt{2}a^2}\right)\overbrace{(\hat{i} - \hat{j})}^{\geq 1}\frac{1}{\sqrt{2}}$$

e direction? since it is 45°

why? - that is the ↓ component

4 · the $\frac{1}{\sqrt{2}}$ direction

$$|\vec{E}| = \sqrt{\left(\frac{kq}{\sqrt{2}a^2}\right)^2 + \left(\frac{kq}{\sqrt{2}a^2}\right)^2}$$

$$= \sqrt{\frac{k^2q^2}{2a^4} + \frac{k^2q^2}{2a^4}}$$

$$= \sqrt{\frac{2k^2q^2}{2a^4}}$$

$$= -\frac{kq}{a^2}$$

e why no $\sqrt{2}$

$$E = 4|\vec{E}|\sin\theta \; -\hat{j}$$

$$4k\frac{q}{2a}\frac{1}{\sqrt{2}}\hat{j}$$

so have 4 times pointing ↓

-3

b) What is the electric potential V at point O, the center of the square, taking the potential at infinity to be zero?

$$V(P) - V(\infty) = V(P) - 0 = V(P) = -\int E \cdot ds$$

$$-\int \frac{kq}{\sqrt{2}a^2}\uparrow - \frac{kq}{\sqrt{2}a^2}\uparrow \cdot ds$$

$$-\frac{kq}{\sqrt{2}}\left(\int \frac{1}{a^2}\uparrow - \int \frac{1}{a^2}\uparrow\right)$$

$$-\frac{kq}{\sqrt{2}}\left(-\frac{1}{a}\uparrow - -\frac{1}{a}\uparrow\right)$$

$\frac{1}{a^2}$

a^{-2}

$\frac{a^{-1}}{-1}$

$= -\frac{1}{a}$

$$V(P) = \frac{kq}{\sqrt{2}a}\uparrow - \frac{kq}{\sqrt{2}a}\uparrow$$

← no direction! (scalar)
So I almost had it
grr

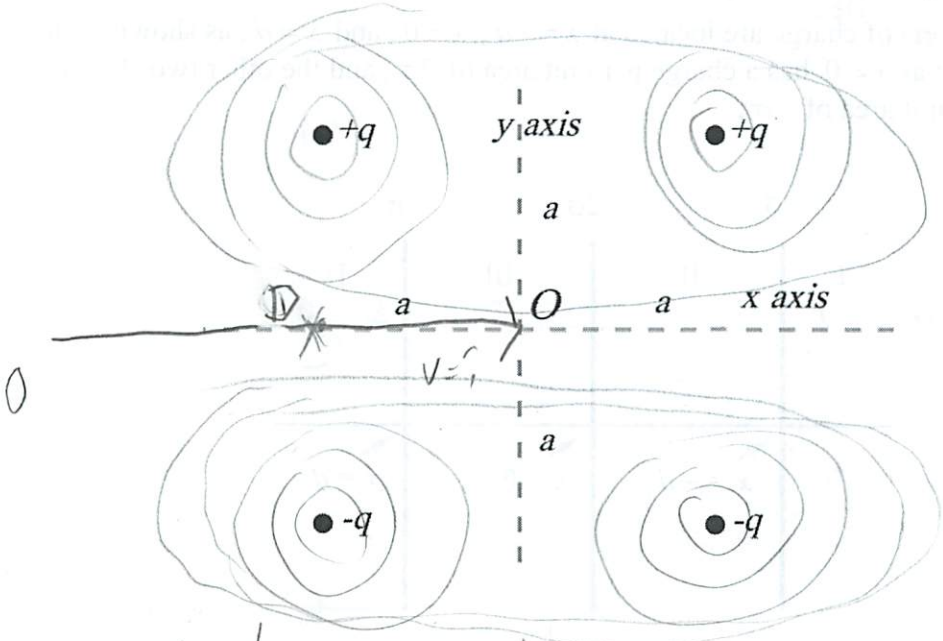
looks right

$$V(P) = 0$$

$$\frac{kq}{\sqrt{2}a} + \frac{kq}{\sqrt{2}a} + \frac{k\,-q}{\sqrt{2}a} + \frac{k\,-q}{\sqrt{2}a} = 0$$

So write it out full
(like on pratice test)
and use that

c) Sketch on the figure below one path leading from infinity to the origin at O where the integral $\int_{\infty}^{O} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$ is trivial to do by inspection. Does your answer here agree with your result in b)? like experiment



equipotential curve

$$V = \frac{kq}{\sqrt{2}a}\uparrow - \frac{kq}{\sqrt{2}a}\uparrow$$

sums to 0

Voltage at top near ⊕ = total voltage

bottom near ⊖ = 0

Voltage at pt D is $\frac{1}{2}$ total voltage

$$= \int_{\infty}^{O} \frac{kq}{\sqrt{2}a}\uparrow - \frac{kq}{\sqrt{2}a}\uparrow$$

Should have better studied the plane ones - not just sphere

- the practice test I did was conductors - crap - why did they have to do that - this is like P-set one

Problem 3 (25 points)

NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!)

if line would be λ not σ

slabs that are thin

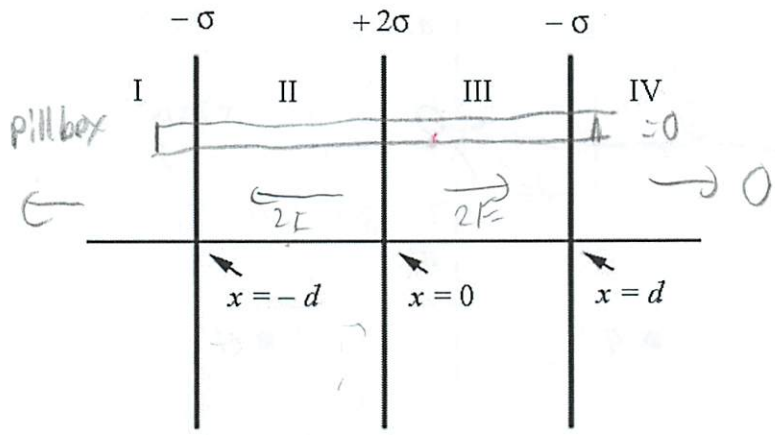
Three infinite sheets of charge are located at $x = -d$, $x = 0$, and $x = d$, as shown in the sketch. The sheet at $x = 0$ has a charge per unit area of 2σ, and the other two sheets have charge per unit area of $-\sigma$.

constant

pt $\frac{1}{r^3}$

line $\frac{1}{r^2}$

plane slap $\frac{1}{r}$



E

remember linear

+2/2

a) What is the electric field in each of the four regions I-IV labeled in the sketch? Clearly present your reasoning, relevant figures, and any accompanying calculations. Plot the x component of the electric field, E_x, on the graph on the bottom of the next page. Clearly indicate on the vertical axis the values of E_x for the different regions.

E in 1 and 4 is 0 since the charges in the

+3/3

pill box balance out (no net charge). Also with a slab the charge beyond it is constant.

E in regions 2 and 3

On both the right and the left will be positive

$$EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma A}{\epsilon_0 A} = \frac{\sigma}{\epsilon_0} \quad \cancel{\epsilon_0 \sigma}$$

2/4

$-\frac{\sigma}{\epsilon_0}$ ↑ $-d<x<0$

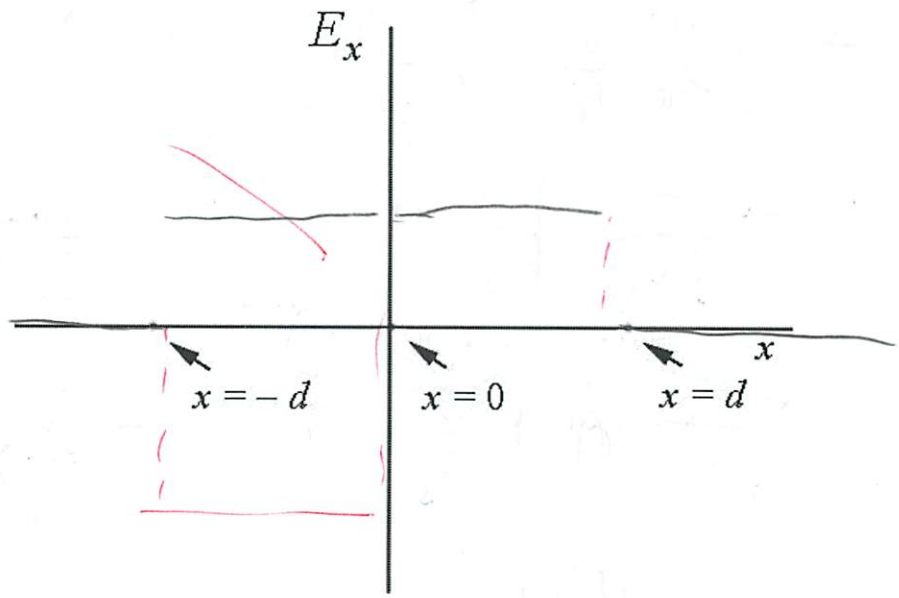
$\frac{\sigma}{\epsilon_0}$ ↑ $0<x<d$

so I only made a sign mistake. Thats not -3

Also not 20

$$2EA = \frac{2\sigma A}{\varepsilon_0}$$

should have thought more about not just wrote as sidebar



b) Find the electric potential in each of the four regions I-IV labeled above, with the choice that the potential is zero at $x = +\infty$ i.e. $V(+\infty) = 0$. Show your calculations. Plot the electric potential as a function of x on the graph on the bottom of the next page. Indicate units on the vertical axis.

$V(P) = V(P) - V(\infty) = V(P) - 0 = -\int E \cdot ds$

1 $\rightarrow = -\int_{-\infty}^{-d} 0 \, ds$ 4 $\rightarrow -\int_{d}^{\infty} 0 \, ds$

$= -r \Big|_{-\infty}^{-d}$ $= -r \Big|_{d}^{\infty}$

$-(-d) - (-\infty)$ $-\infty - d$

$\cancel{d}$ 0 $\cancel{-d}$ 0

$50 = 0$

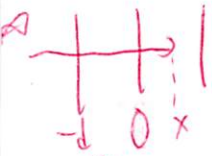
grrr math error

2 $\rightarrow -\int_{-d}^{0} \frac{\sigma}{\epsilon_0} \, ds - \int_{0}^{x} \frac{\sigma}{\epsilon_0} \, ds$ 3 $\rightarrow -\int_{d}^{x} \frac{\sigma}{\epsilon_0} \, ds - \int_{-\infty}^{d} \frac{\sigma}{\epsilon_0}$

$\cancel{-\frac{\sigma}{\epsilon_0} r \Big|_{-d}^{0}}$ $\cancel{-\frac{\sigma}{\epsilon_0} r \Big|_{0}^{d}}$

$-\frac{\sigma}{\epsilon_0} \cdot d + \frac{\sigma}{\epsilon_0} x$ $-\frac{\sigma}{\epsilon_0} x - \frac{\sigma}{\epsilon_0} d$

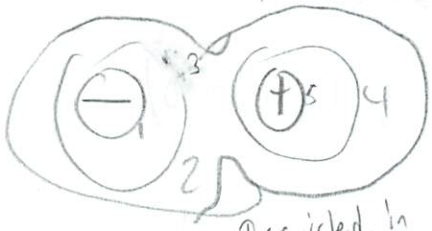
$\cancel{x} \rightarrow \cancel{\frac{\sigma}{\epsilon_0} d}$ $\cancel{-\frac{\sigma}{\epsilon_0} d}$



know what going from to to and what you have to go through to get there

3/6

think conceptually from experiment

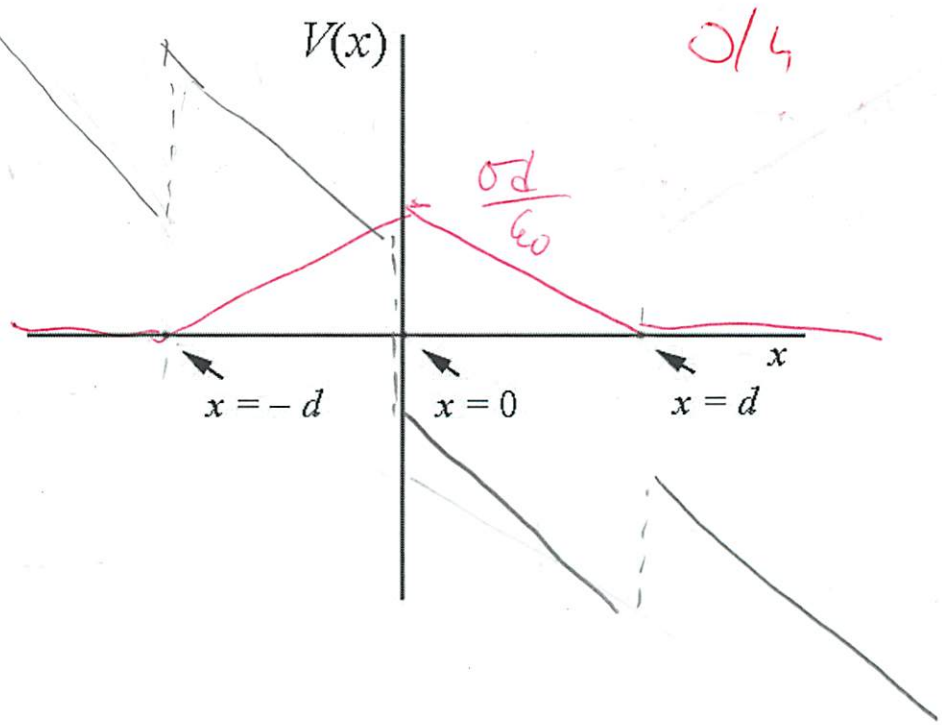


↑ squished in middle b/w largest voltage change

0	1
$\frac{\sigma}{\epsilon_0} d + \frac{\sigma}{\epsilon_0} x$	2
$\frac{\sigma}{\epsilon_0} d - \frac{\sigma}{\epsilon_0} x$	3
0	4

$\frac{\sigma}{\epsilon_0} d - \frac{\sigma}{\epsilon_0} |x|$

0/4



they are not slopes - remember how to graph

note page is not slopes

c) How much work must you do to bring a point like object with charge $+Q$ in from infinity to the origin $x = 0$?

$W = \cancel{qE} = \Delta U = qV = q(V(P) - 0)$

$W = +Q\,V(x=0)$

or

~~The sheet has charge (+) so how can you bring +Q into it - it will repel there is no way you can get it to touch?~~

← my mistake from before - I just jammed it in here w/o thinking

$W = q\frac{\sigma}{\varepsilon_0}d$

$W = \frac{q\sigma d}{\varepsilon_0}$

3/5

↑ Should be worth more

$\frac{Q\sigma}{\varepsilon_0}d$

did not study cylinders too much

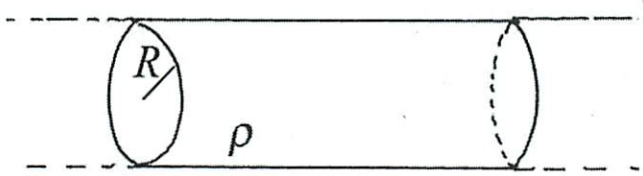
Problem 4 (25 points)

NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!). You may find the following integrals helpful in this answering this question.

$$\int_{r_a}^{r_b}\frac{dr}{r^2}=-\left(\frac{1}{r_b}-\frac{1}{r_a}\right),\ \int_{r_a}^{r_b}\frac{dr}{r}=\ln(r_b/r_a),\ \int_{r_a}^{r_b}dr=r_b-r_a,\ \int_{r_a}^{r_b}rdr=\frac{1}{2}\left(r_b^2-r_a^2\right).$$

$$\int_{r_a}^{r_b} r^n\,dr=\frac{1}{n+1}\left(r_b^{n+1}-r_a^{n+1}\right)$$

Consider a charged infinite cylinder of radius R.



The charge density is non-uniform and given by

$$\rho(r)=br\,;\ r<R,$$

where r is the distance from the central axis and b is a constant.

a) Find an expression for the direction and magnitude of the electric field everywhere i.e. inside and outside the cylinder. Clearly present your reasoning, relevant figures, and any accompanying calculations.

gaussian surface = larger cylinders + smaller

inside

- it is leaking charge on both sides and ends

$$EA=\frac{\rho V}{\varepsilon_0}$$

$$E(\cancel{\pi r^2}+2\pi r h)=\int\frac{br\,2\pi r h}{\varepsilon_0}\,dr$$

Since its ∞ long no endcaps! −2

$$\frac{1}{\varepsilon_0}\int_0^{r'} br^2\,2\pi h\,dr$$

$$\frac{2}{\varepsilon_0}b\pi h\int_0^{r'} r^2\,dr$$

prob missing some small think

$$\frac{1}{\epsilon_0} b \pi h \frac{r^3}{3} \Big|_0^{r'}$$

$$E = \frac{2 b \pi h r^3}{\epsilon_0 3 (\cancel{2\pi r^2} + 2\pi r h)}$$

$$= \frac{b \cancel{\pi} h r^{\cancel{3}2}}{\epsilon_0 8 \cancel{\pi} r^2 + \epsilon_0 8 \cancel{\pi} r h}$$

$$= \frac{b h r^2}{\epsilon_0 8 (1 + h)} \; ?$$

$$\frac{b r^2}{3 \epsilon_0} \hat{r} \qquad r < R$$

outside

$$E A = \frac{\rho V}{\epsilon_0}$$

$$E (\cancel{2\pi r^2} + 2\pi r h) = \frac{\int_0^R \rho \, 2\pi r \, h \, dr}{\epsilon_0}$$

-2

$$\frac{b R^2}{3 \epsilon_0} \frac{1}{r} \qquad r > R$$

$$\int_0^R b r \cdot \pi r h \, dr$$

$$\pi h \cdot \frac{b r^2}{2} \Big|_0^R = \frac{b R^2 \pi h}{2}$$

$$E = \frac{b R^2 \pi h}{\epsilon_0 2 (2\pi R^2 + 2\pi R h)} = \frac{b R^{\cancel{2}} \cancel{\pi} h}{\epsilon_0 2 \cancel{\pi} \cancel{R} (2R + 2h)} \quad -2$$

endcaps still don't matter

$$E = \frac{b R h}{\epsilon_0 4 (R + h)}$$

$r \neq R$!

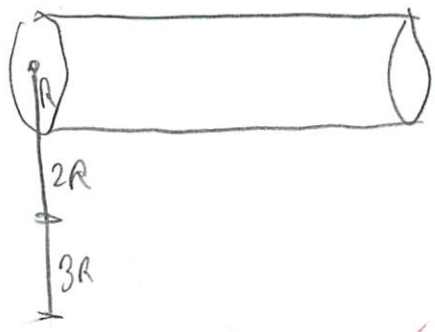
don't screw this up - its sometimes true sometimes not be disciplined

I had it and erased it so could simplify more

Energy approch

did not look too long at since it was last qu

b) A point-like object with charge $+q$ and mass m is released from rest at the point a distance $2R$ from the central axis of the cylinder. Find the speed of the object when it reaches a distance $3R$ from the central axis of the cylinder



$0 = U + K$

$U = q\Delta V$

↑ do I have to find ... grrr

$K = \frac{1}{2}mv^2$

$q\Delta V = \frac{1}{2}mv^2$

$\Delta V_{\text{pot diff}} = -\int_{2R}^{3R} E \cdot ds$

$\Delta V = -\int \frac{bR^{-h}}{\varepsilon_0 4(R+h)}\, dr$

$\frac{1}{-R}$

R^{-1}

$\ln R$

$\Delta V = -\frac{bh \ln(3R/2R)}{\varepsilon_0 4(1+h)(2)}\left((3R)^2 - (2R)^2\right)$

$\Delta V = -\frac{bh \ln(3R/2R)(9R^2 - 4R^2)}{\varepsilon_0 8(1+h)}$

velocity $= \sqrt{\frac{2q\Delta V}{m}}$ plug in

velocity $= \sqrt{\frac{2q\left(\frac{-bh \ln(3R/2R)(9R^2-4R^2)}{\varepsilon_0 8(1+h)}\right)}{m}}$

vel $= \sqrt{\frac{2q\Delta V}{m}}$

??

−4

no need to integrate just subtract

$K(3R) - K(2R)$

$= -(U(3R) - U(2R))$

$= -q(V(3R) - V(2R))$

$K(2R) = 0$

$K(3R) = \frac{1}{2}mv_f^2$

ok

$v_f = \sqrt{\frac{2qbR^3}{3m\varepsilon_0}\ln(3/2)}$

Oh they just write V

still no end caps

think I major screwed up - sure side and ends?
- but have been messy problems befor

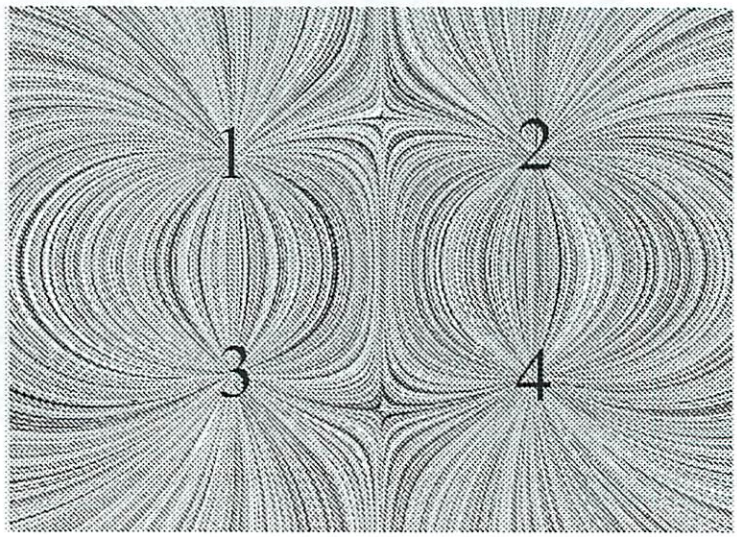
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2010**

8.02 Exam One Solutions Spring 2010

Problem 1 (25 points)

Question 1 (5 points)



1. Above we show the grass seeds representation of the field of four point charges, located at the positions indicated by the numbers. Which statement is true about the signs of these charges:

a) All four charges have the same sign.

b) Charges 1 and 2 have the same sign, and that sign is opposite the sign of 3 and 4.

c) Charges 1 and 3 have the same sign, and that sign is opposite the sign of 2 and 4.

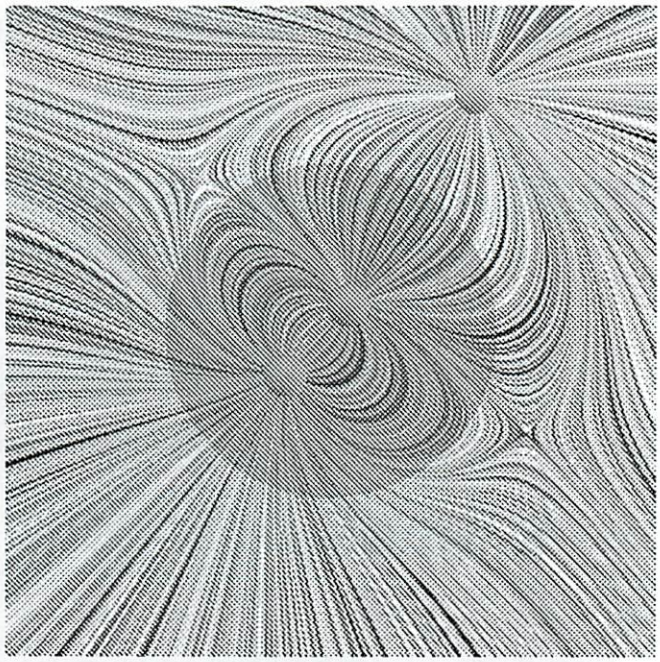
d) Charges 1 and 4 have the same sign, and that sign is opposite the sign of 2 and 3.

e) None of the above.

Solution b. Field lines continuously connect charges 1 and 3, and 2 and 4 respectively, indicating that the charge of those pairs are opposite in sign. The field is a zero between charges 1 and 2 indicating that they repel and hence are of the same sign. A smilar argument holds for charges 3 and 4.

Question 2 (5 points)

The grass seeds figure below shows the electric field of three charges with charges +1, +1, and -1, The Gaussian surface in the figure is a sphere containing two of the charges.



The total electric flux through the spherical Gaussian surface is

a) Positive

b) Negative

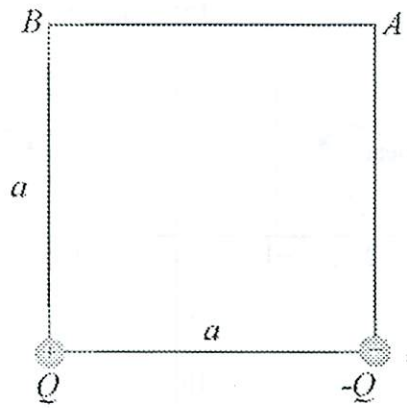
c) Zero

d) Impossible to determine without more information

Solution c. Because the field lines connect the two charges within the Gaussian surface they must have opposite sign. Therefore the charge enclosed in the Gaussian surface is zero. Hence the electric flux through the surface of the Gaussian surface is also zero.

Question 3 (5 points)

Two point-like charged objects with charges $+Q$ and $-Q$ are placed on the bottom corners of a square of side a, as shown in the figure.



You move an electron with charge $-e$ from the upper right corner marked A to the upper left corner marked B. Which of the following statements is true?

a) You do a negative amount of work on the electron equal to the amount of energy necessary to assemble the system of three charged objects with the electron at point B.

b) You do a positive amount of work on the electron equal to the amount of energy necessary to assemble the system of three charged objects with the electron at point B.

c) You do a positive amount of work on the electron and the potential energy of the system of three charged objects increases.

d) You do a negative amount of work on the electron and the potential energy of the system of three charged objects decreases.

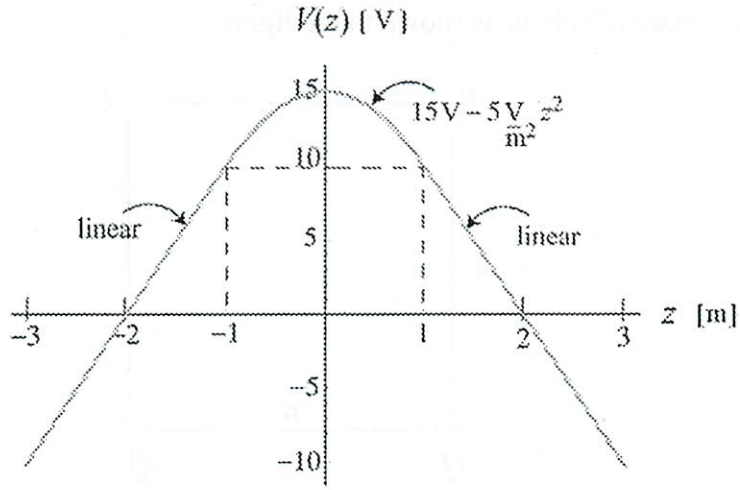
e) You do a negative amount of work on the electron and the potential energy of the system of three charged objects increases.

f) You do a positive amount of work on the electron and the potential energy of the system of three charged objects decreases.

Solution d. Because point B is closer to the positive charge than the point A, the electric potential difference $V(B)-V(A)>0$. When you move an electron with charge $-e$ from the upper right corner marked A to the upper left corner marked B, the potential energy difference is $U(B)-U(A)=-e(V(B)-V(A))<0$. This means that you do a negative amount of work and the potential energy of the system decreases.

Question 4 (5 points)

A graph of the electric potential $V(z)$ vs. z is shown in the figure below.



Which of the following statements about the z-component of the electric field E_z is true?

a) $E_z < 0$ for $-3\,\text{m} < z < 0$ and $E_z < 0$ for $0 < z < 3\,\text{m}$.

b) $E_z < 0$ for $-3\,\text{m} < z < 0$ and $E_z > 0$ for $0 < z < 3\,\text{m}$.

c) $E_z > 0$ for $-3\,\text{m} < z < 0$ and $E_z < 0$ for $0 < z < 3\,\text{m}$.

d) $E_z > 0$ for $-3\,\text{m} < z < 0$ and $E_z > 0$ for $0 < z < 3\,\text{m}$.

e) None of the above because E_z cannot be determined from information in the graph for the regions $-3\,\text{m} < z < 0$ and $0 < z < 3\,\text{m}$.

Solution b. For values of $-3\,\text{m} < z < 0$, the derivative $dV(z)/dz > 0$, and $E_z = -dV(z)/dz < 0$. For values of $0 < z < 3\,\text{m}$, the derivative $dV(z)/dz < 0$, and $E_z = -dV(z)/dz > 0$.

Question 5 (5 points)

Careful measurements reveal an electric field

$$\vec{\mathbf{E}}(r) = \begin{cases} \dfrac{a}{r^2}\left(1 - \dfrac{r^3}{R^3}\right)\hat{\mathbf{r}}\,; & r \le R \\ \vec{0}\,; & r \ge R \end{cases}$$

where a and R are constants. Which of the following best describes the charge distribution giving rise to this electric field?

a) A negative point charge at the origin with charge $q = 4\pi\varepsilon_0 a$ and a uniformly positive charged spherical shell of radius R with surface charge density $\sigma = -q/4\pi R^2$.

b) A positive point charge at the origin with charge $q = 4\pi\varepsilon_0 a$ and a uniformly negative charged spherical shell of radius R with surface charge density $\sigma = -q/4\pi R^2$.

c) A positive point charge at the origin with charge $q = 4\pi\varepsilon_0 a$ and a uniformly negative charged sphere of radius R with charge density $\rho = -q/(4\pi R^3/3)$.

d) A negative point charge at the origin with charge $-q = -4\pi\varepsilon_0 a$ and a uniformly positive charged sphere of radius R with charge density $\rho = q/(4\pi R^3/3)$.

e) Impossible to determine from the given information.

Was Confused on

Solution c. As you shall see below the answer should be c. because the problem does not specify the sign of the constant a. However both description c. and d. do seem plausible so we shall accept answers c., d., and e.

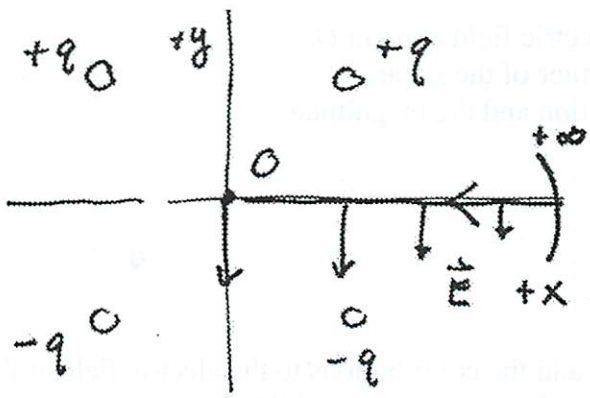
Assume $a > 0$. Then the electric field can be thought of as the superposition of two fields, $\vec{\mathbf{E}}_+(r) = \dfrac{a}{r^2}\hat{\mathbf{r}}$ and $\vec{\mathbf{E}}_-(r) = -\dfrac{ar}{R^3}\hat{\mathbf{r}}$. $\vec{\mathbf{E}}_+(r)$ is the electric field of a positive point charge at the origin with $q = 4\pi\varepsilon_0 a$. $\vec{\mathbf{E}}_-(r)$ is the electric field of a uniformly negative charged sphere of radius R. Because the electric field for radius $r > R$ is zero, the sum of the two charges distributions must be zero. Therefore the charge density must satisfy $\rho = -q/(4\pi R^3/3) = -4\pi\varepsilon_0 a/(4\pi R^3/3) = -3\varepsilon_0 a/R^3$.

Now assume $a < 0$. Suppose the electric field can now be thought of as the superposition of two fields, $\vec{\mathbf{E}}_-(r) = \frac{a}{r^2}\hat{\mathbf{r}}$ and $\vec{\mathbf{E}}_+(r) = -\frac{ar}{R^3}\hat{\mathbf{r}}$. $\vec{\mathbf{E}}_-(r)$ is the electric field of a negative point charge at the origin with $-q = 4\pi\varepsilon_0 a > 0$, hence $q < 0$. $\vec{\mathbf{E}}_+(r)$ is the electric field of a uniformly positively charged sphere of radius R. Because the electric field for radius $r > R$ is zero, the sum of the two charges distributions must be zero. Therefore the charge density must satisfy $\rho = q/(4\pi R^3/3) < 0$. Therefore when $a < 0$ the only possible answer d. cannot be correct.

sum

$$V(O)-V(\infty)=V(O)=k\frac{q}{(2a^2)^{1/2}}+k\frac{q}{(2a^2)^{1/2}}+k\frac{(-q)}{(2a^2)^{1/2}}+k\frac{(-q)}{(2a^2)^{1/2}}=0.$$

c) Sketch on the figure below one path leading from infinity to the origin at O where the integral $\int_{\infty}^{O}\vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}$ is trivial to do by inspection. Does your answer here agree with your result in b)?

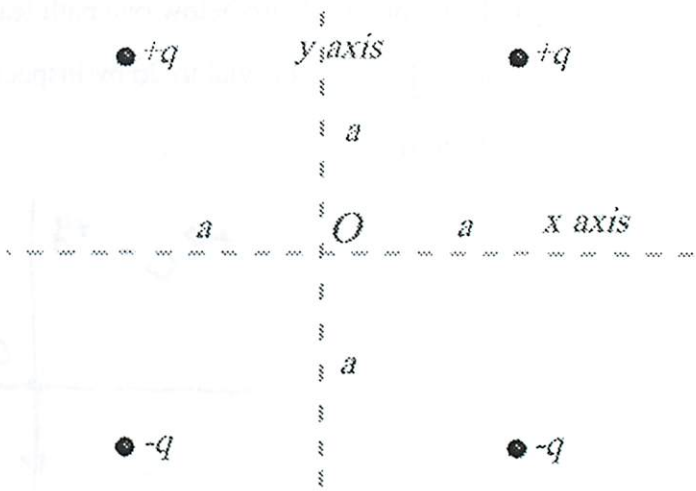


Solution: The electric field at any point along the x-axis is points in the –y-direction. Therefore for a path from infinity to the origin at O along the x-axis, the dot product $\vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}=0$ and hence the integral $\int_{\infty}^{O}\vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}=0$. Because by definition $\int_{\infty}^{O}\vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}=-(V(O)-V(\infty))=0$, and the integral is path independent, our answer for the above path along the x-axis sagrees with our result in part b.

Problem 2 (25 points)

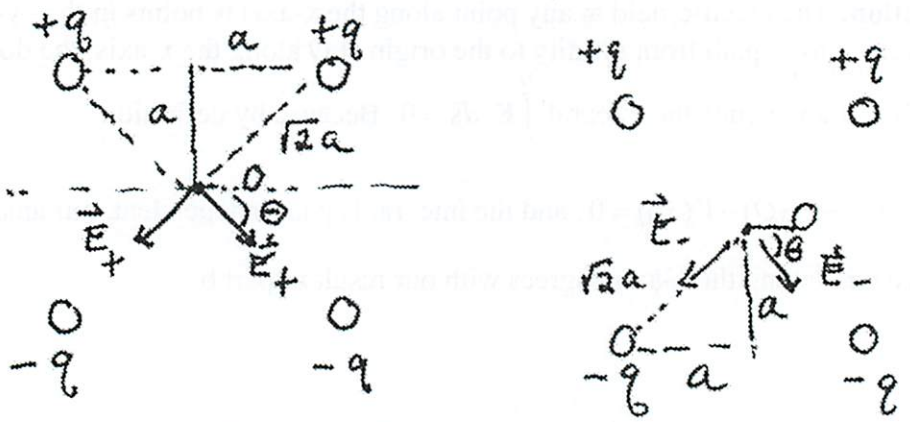
NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!) .

Four charged point-like objects, two of charge $+q$ and two of charge $-q$, are arranged on the vertices of a square with sides of length $2a$, as shown in the sketch.



a) What is the electric field at point O, which is at the center of the square? Indicate the direction and the magnitude.

Solution: When I add the contributions to the electric field at the origin from the two positive charges on the upper corners of the square, the horizontal component cancels and the vertical component points down.



A similar argument holds for the contributions to the electric field at the origin from the two negative charges on the lower corners of the square. Therefore the electric field at the origin is

$$\vec{\mathbf{E}}_O = 4\left|\vec{\mathbf{E}}_{+q}\right|\sin\theta(-\hat{\mathbf{j}}) = 4k\frac{q}{2a^2}\left(\frac{1}{\sqrt{2}}\right)(-\hat{\mathbf{j}}) = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}q}{a^2}(-\hat{\mathbf{j}})$$

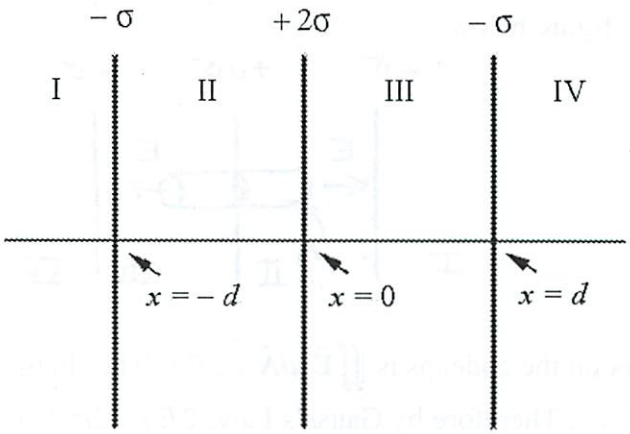
b) What is the electric potential V at point O, the center of the square, taking the potential at infinity to be zero?

Solution zero. The electric potential difference between infinity and the origin is just the

Problem 3 (25 points)

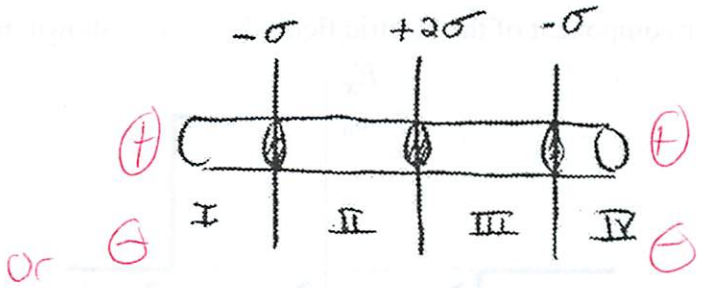
NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!)

Three infinite sheets of charge are located at $x = -d$, $x = 0$, and $x = d$, as shown in the sketch. The sheet at $x = 0$ has a charge per unit area of 2σ, and the other two sheets have charge per unit area of $-\sigma$.



a) What is the electric field in each of the four regions I-IV labeled in the sketch? Clearly present your reasoning, relevant figures, and any accompanying calculations. Plot the x component of the electric field, E_x, on the graph on the bottom of the next page. Clearly indicate on the vertical axis the values of E_x for the different regions.

Solution: We begin by choosing a Gaussian cylinder with end caps in regions I and IV as shown in the figure below. The total charge enclosed is zero and hence the electric flux on the endcaps must be zero. Thus the electric field must be zero in regions I and IV.



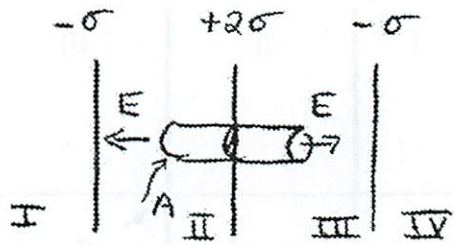
This turns out to be correct but the conclusion depends on an additional argument based on symmetry. If the electric field is non-zero on the endcaps it must point either in the +x-direction in both regions I and IV or in the –x-direction in both regions I and IV. Neither is possible due to the symmetry of the charge distribution. For example, if the electric field pointed in the +x-direction in both regions I and IV. Then if we looked at

the charge distribution from the other side of the plane of the paper, the field should point in the –x-direction. However the charge distribution is identical when looking from the other side of the paper. Therefore the field must point in the +x-direction according to our original assertion. Therefore by symmetry the only possibility is for the fields in regions I and IV to point toward $x = 0$ or away from $x = 0$. In the first case the flux would be non-zero on our Gaussian surface but it must be zero because the charge enclosed is zero. Hence the electric field in regions I and IV is zero. (A similar argument holds if we assume that the field points in the –x-direction in both regions I and IV.)

???

Don't get at ALL

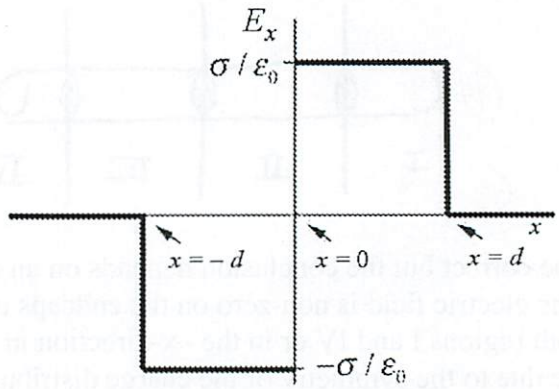
For regions II and III, we choose a Gaussian cylinder with end caps in regions II and III as shown in the figure below.



The electric flux on the endcaps is $\iint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 2EA$. The charge enclosed divided by ε_0 is $Q_{enc} / \varepsilon_0 = 2\sigma A / \varepsilon_0$. Therefore by Gauss's Law, $2EA = 2\sigma A / \varepsilon_0$ which implies that the magnitude of the electric field is $E = \sigma / \varepsilon_0$. Thus the electric field is given by

$$\vec{\mathbf{E}} = \begin{cases} \vec{\mathbf{0}} \; ; & x < -d \\ -\dfrac{\sigma}{\varepsilon_0}\hat{\mathbf{i}} \; ; & -d < x < 0 \\ \dfrac{\sigma}{\varepsilon_0}\hat{\mathbf{i}} \; ; & 0 < x < +d \\ \vec{\mathbf{0}} \; ; & d < x \end{cases}$$

The graph of the x component of the electric field, E_x vs x is shown on the graph below.



b) Find the electric potential in each of the four regions I-IV labeled above, with the choice that the potential is zero at $x = +\infty$ i.e. $V(+\infty) = 0$. Show your calculations. Plot the electric potential as a function of x on the graph on the bottom of the next page. Indicate units on the vertical axis.

Solution: The electric potential difference between infinity and a point P located at x, is given by

$$V(x) - V(\infty) = -\int_{\infty}^{P} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} .$$

We shall evaluate this integral for points in each region. We start with P anywhere in region IV, $d < x$. Because the electric field in region IV is zero, the integral is zero,

$$V(x) - V(\infty) = -\int_{\infty}^{x} \vec{\mathbf{E}}_{IV} \cdot d\vec{\mathbf{s}} = 0 .$$

If P is anywhere in region III, $0 < x < +d$ then

$$\begin{aligned} V(x) - V(\infty) &= -\int_{\infty}^{d} \vec{\mathbf{E}}_{IV} \cdot d\vec{\mathbf{s}} - \int_{d}^{x} \vec{\mathbf{E}}_{III} \cdot d\vec{\mathbf{s}} \\ &= 0 - \int_{d}^{x} E_x dx = -\int_{d}^{x} \frac{\sigma}{\varepsilon_0} dx = -\frac{\sigma}{\varepsilon_0}(x-d) = \frac{\sigma}{\varepsilon_0} d - \frac{\sigma}{\varepsilon_0} x \end{aligned}$$

If P is anywhere in region II, $-d < x < 0$ then

$$\begin{aligned} V(x) - V(\infty) &= -\int_{\infty}^{d} \vec{\mathbf{E}}_{IV} \cdot d\vec{\mathbf{s}} - \int_{d}^{0} \vec{\mathbf{E}}_{III} \cdot d\vec{\mathbf{s}} - \int_{0}^{x} \vec{\mathbf{E}}_{II} \cdot d\vec{\mathbf{s}} \\ &= 0 - \int_{d}^{0} \frac{\sigma}{\varepsilon_0} dx - \int_{0}^{x} -\frac{\sigma}{\varepsilon_0} dx = \frac{\sigma}{\varepsilon_0} d + \frac{\sigma}{\varepsilon_0} x \end{aligned}$$

If P is anywhere in region I, $x < -d$ then

$$\begin{aligned} V(x) - V(\infty) &= -\int_{\infty}^{d} \vec{\mathbf{E}}_{IV} \cdot d\vec{\mathbf{s}} - \int_{d}^{0} \vec{\mathbf{E}}_{III} \cdot d\vec{\mathbf{s}} - \int_{0}^{-d} \vec{\mathbf{E}}_{II} \cdot d\vec{\mathbf{s}} - \int_{-d}^{x} \vec{\mathbf{E}}_{I} \cdot d\vec{\mathbf{s}} \\ &= 0 - \int_{d}^{0} \frac{\sigma}{\varepsilon_0} dx - \int_{0}^{-d} -\frac{\sigma}{\varepsilon_0} dx - 0 = \frac{\sigma}{\varepsilon_0} d - \frac{\sigma}{\varepsilon_0} d = 0 \end{aligned}$$

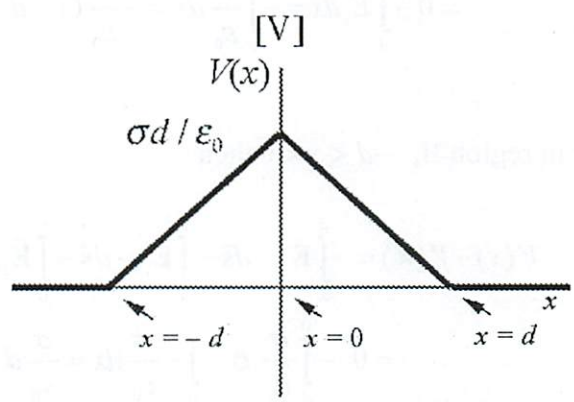
Because the electric field is continuous we can write our result as

$$V(x)-V(\infty)=\begin{cases} 0 \;; & x\le -d \\ \dfrac{\sigma}{\varepsilon_0}d+\dfrac{\sigma}{\varepsilon_0}x\;; & -d\le x\le 0 \\ \dfrac{\sigma}{\varepsilon_0}d-\dfrac{\sigma}{\varepsilon_0}x\;; & 0\le x\le +d \\ 0 \;; & d\le x \end{cases}.$$

Note this can be written as

$$V(x)-V(\infty)=\begin{cases} 0 \;; & x\le -d \\ \dfrac{\sigma}{\varepsilon_0}d-\dfrac{\sigma}{\varepsilon_0}|x|\;; & -d\le x\le d \\ 0 \;; & d\le x \end{cases}.$$

This result looks good because the area under the graph of the x component of the electric field, E_x vs x for the region $-d<x<d$ is zero. The plot of the electric potential as a function of x on the graph is shown below with units of [V] on the vertical axis.



c) How much work must you do to bring a point-like object with charge $+Q$ in from infinity to the origin $x=0$?

Solution. The work you must do is equal to the change in potential energy (assuming the point-like object begins and ends at rest). Therefore

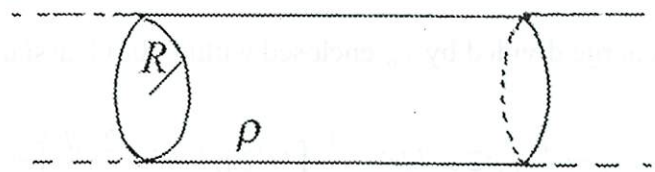
$$W=U(0)-U(\infty))=+Q(V(0)-V(\infty))=+\frac{Q\sigma}{\varepsilon_0}d\,.$$

Problem 4 (25 points)

NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!). You may find the following integrals helpful in this answering this question.

$$\int_{r_a}^{r_b} r^n dr = \frac{1}{n+1}\left(r_b^{\,n+1} - r_a^{\,n+1}\right); n \neq 1 , \qquad \int_{r_a}^{r_b} \frac{dr}{r} = \ln(r_b / r_a) .$$

Consider a charged infinite cylinder of radius R.



The charge density is non-uniform and given by

$$\rho(r) = br ; \; r < R ,$$

where r is the distance from the central axis and b is a constant.

a) Find an expression for the direction and magnitude of the electric field everywhere i.e. inside and outside the cylinder. Clearly present your reasoning, relevant figures, and any accompanying calculations.

Solution. Because the charge distribution defines two distinct regions of space, region I defined by $r < R$ and region II defined by $r > R$, we must apply Gauss's Law twice to find the electric field everywhere.

In region I, where $r < R$, we choose a Gaussian cylinder of radius r and length l.

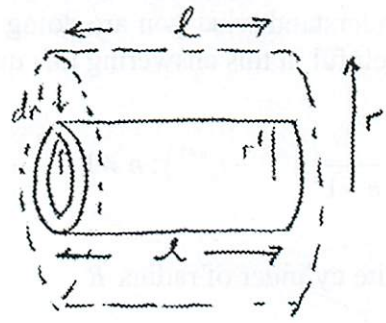
* infinitly long
- So no endcaps

grr

Because the electric field points away from the central axis, the electric flux on our Gaussian surface is

$$\oiint \vec{\mathbf{E}}_I \cdot d\vec{\mathbf{A}} = E_I 2\pi r l .$$

Because the charge density is non-uniform, we must integrate the charge density. We choose as our integration volume a cylindrical shell of radius r', length l and thickness dr'. The integration volume is then $dV' = 2\pi r' l dr'$.



Therefore the charge divided by ε_0 enclosed within our Gaussian surface is

$$Q_{enc} / \varepsilon_0 = \frac{1}{\varepsilon_0}\int_0^r \rho 2\pi r' l dr' = \frac{1}{\varepsilon_0}\int_0^r b r' 2\pi r' l dr' = \frac{2\pi l b}{\varepsilon_0}\int_0^r r'^2 dr' = \frac{2\pi l b r^3}{3\varepsilon_0}.$$

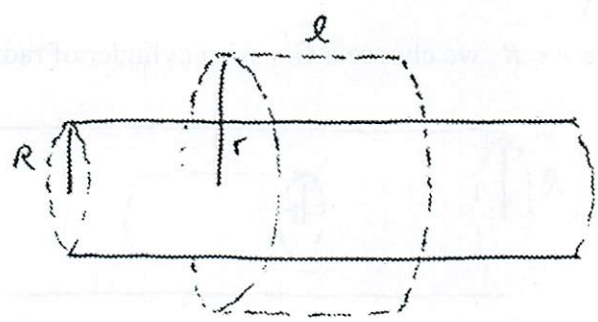
Therefore Gauss's Law becomes

$$E_I 2\pi r l = 2\pi l b r^3 / 3.$$

We can now solve for the direction and magnitude of the electric field when $r < R$,

$$\vec{\mathbf{E}}_I = \frac{b r^2}{3\varepsilon_0}\hat{\mathbf{r}}.$$

In region II where $r > R$, we choose a Gaussian cylinder of radius r and length l.



Because the electric field points away from the central axis, the electric flux on our Gaussian surface is

$$\iint \vec{\mathbf{E}}_{II} \cdot d\vec{\mathbf{A}} = E_{II} 2\pi r l.$$

We again must integrate the charge density but this time taking our endpoints as $r = 0$ and $r = R$. Therefore the charge divided by ε_0 enclosed within our Gaussian surface is

$$Q_{enc} / \varepsilon_0 = \frac{1}{\varepsilon_0}\int_0^r \rho 2\pi r' l dr' = \frac{1}{\varepsilon_0}\int_0^R b r' 2\pi r' l dr' = \frac{2\pi l b}{\varepsilon_0}\int_0^R r'^2 dr' = \frac{2\pi l b R^3}{3\varepsilon_0}.$$

Therefore Gauss's Law becomes

$$E_{II} 2\pi r l = 2\pi l b R^3 / 3.$$

We can now solve for the direction and magnitude of the electric field when $r > R$,

$$\vec{\mathbf{E}}_{II} = \frac{bR^3}{3\varepsilon_0}\frac{1}{r}\hat{\mathbf{r}}.$$

Collected our results we have that

$$\vec{\mathbf{E}} = \begin{cases} \dfrac{br^2}{3\varepsilon_0}\hat{\mathbf{r}}; & r < R \\ \dfrac{bR^3}{3\varepsilon_0}\dfrac{1}{r}\hat{\mathbf{r}}; & r > R \end{cases}$$

b) A point-like object with charge $+q$ and mass m is released from rest at the point a distance $2R$ from the central axis of the cylinder. Find the speed of the object when it reaches a distance $3R$ from the central axis of the cylinder.

Solution: The change in kinetic energy when the object moves from a distance $2R$ from the central axis of the cylinder to a distance $3R$ is given by

$$K(3R) - K(2R) = -(U(3R) - U(2R)) = -q(V(3R) - V(2R)).$$

Because the particle was released at rest, $K(2R) = 0$, and $K(3R) = (1/2)mv_f^2$, the final speed of the object is

$$v_f = \sqrt{-\frac{2q}{m}(V(3R) - V(2R))}.$$

The electric potential difference between two points in region II is given by

$$V(3R)-V(2R)=-\int_{2R}^{3R}\vec{\mathbf{E}}_{II}\cdot d\vec{\mathbf{s}}=-\int_{2R}^{3R}\frac{bR^3}{3\varepsilon_0}\frac{1}{r}\hat{\mathbf{r}}\cdot d\vec{\mathbf{s}}$$

$$=-\int_{2R}^{3R}\frac{bR^3}{3\varepsilon_0}\frac{1}{r}dr=-\frac{bR^3}{3\varepsilon_0}\ln\frac{3R}{2R}=-\frac{bR^3}{3\varepsilon_0}\ln(3/2)$$

Therefore the speed of the object when it reaches a distance $3R$ from the central axis of the cylinder is

$$v_f=\sqrt{\frac{2qbR^3}{3m\varepsilon_0}\ln(3/2)}\,.$$