Physics Review Exam

2/23

Colomb's Law

- electrostatic interaction b/w charged particles

$$F = k_e \frac{q_1 q_2}{r^2}$$

vector r_{21} from $2 \rightarrow 1$

Why is this seeming easy to me

Reviewed on weekend

Like math

- I think I know it, but can't do it or run into trouble w/ subtilies

What should I review

- do pratice problems

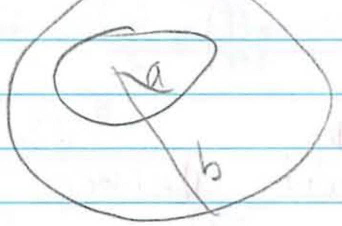
This semester its not just 1 class per day

- lots of work on weekend
- small work weekdays

After fixing p-set seems really hard

Rings of charge

Dumaslin 2/23



~~ring~~ $\frac{kq}{r}$ ~~at~~ pt charge at 0

$dq \quad dr$) sum rings

each ring is r distance away from center so sum

$$\int_a^b \frac{k\,dq}{r}$$

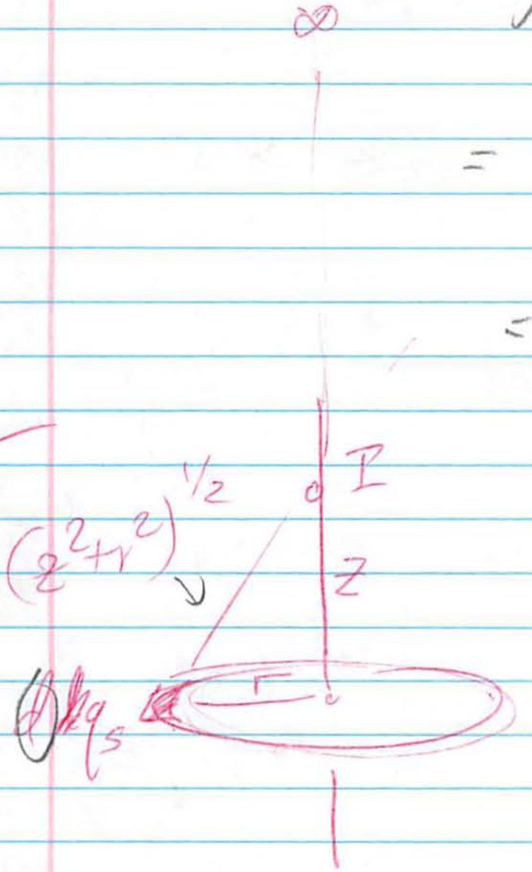
$q = \sigma A$

$$\int_a^b \frac{k\sigma 2\pi r\,dr}{r}$$

$$= \frac{1}{4\pi\epsilon_0}\sigma 2\pi (b-a)$$

$$= \frac{\sigma}{2\epsilon_0}(b-a)$$

for 1 ring



$$= -\int_\infty^P \frac{kq_s\,dr}{r^2}$$

$$\phi(P) - \phi(\infty) = \frac{kq_s}{r_{sp}}$$

add charges around in a circle

$$= \int_{ring} \frac{k\,dq_s}{(r^2+z^2)^{1/2}}$$

$$= \frac{k}{(r^2+z^2)^{1/2}} \int_{ring} dq_s = \frac{kq_{ring}}{(r^2+z^2)^{1/2}}$$

then add rings up

r becomes variable
-each a different distance
from center

*

Dumaskin Review Session

2/23

1. Discrete charges ≡ Sources

Classic qu

Source distribution

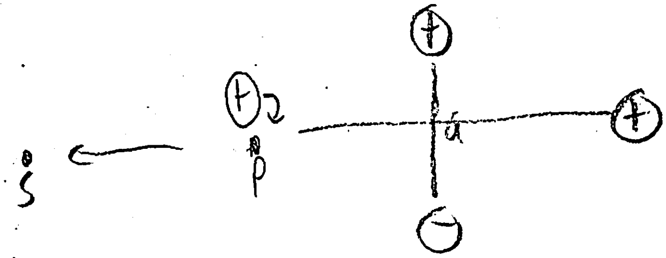
- electric field
- potential difference

$\vec{V}(F) - \vec{V}(P)$

reference pt

- how much energy does it take to assemble source

If place additional charge near these sources



What is force on q at pt p

$F_q = Q\,\vec{E_s}(P)$

Move q from P to S

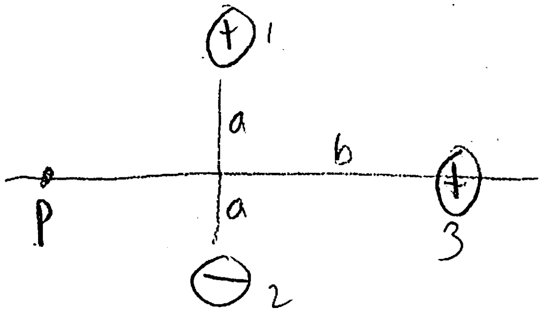
$\Delta U = U(S) - U(P) = Q\,\Delta V_s$

potential

If release Q from rest, at P, what is its Speed at S

$\Delta k + \Delta U = 0$

(2)

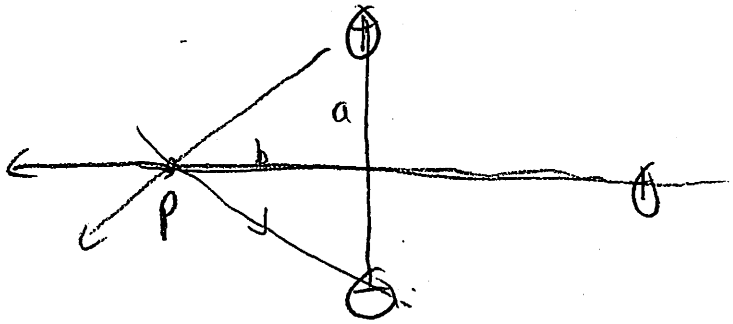


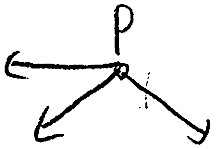
E from + to −

- superposition + vector addition

$$\vec{E}_p = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$

- can draw vectors knowing + −
- or use $\hat{r}$ q formula approch

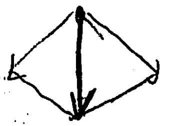




← all same lenght

$$E_3 = -\hat{\imath} \rightarrow \frac{kq}{d^2} = \frac{kq}{(2b)^2}\hat{\imath}$$

$E_1 + E_2$



$-\hat{\jmath} \rightarrow \left(2|\vec{E}_1|\cos\theta \; -\hat{\jmath}\right)$

$$|E_1| = \frac{kq}{d^2} = \frac{kq}{\sqrt{a^2+b^2}^{\,2}} = \frac{kq}{a^2+b^2}$$

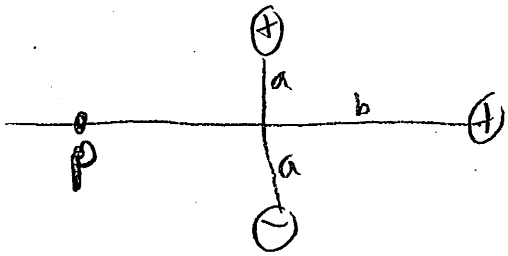
$\cos\theta =$  $= \frac{a}{\sqrt{a^2+b^2}}$

(3)

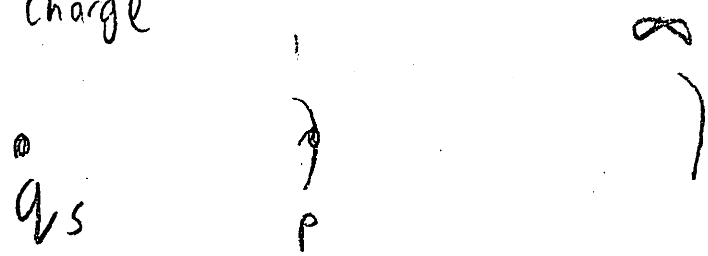
$$\vec{E}(P) = \frac{2kq}{(a^2+b^2)}\frac{a}{a^2+b^2}(-\hat{j}) - \frac{kq}{4b^2}\hat{i}$$

* now about adding vectors
* de nom always same
* Could decompose into $\hat{i}$ $\hat{j}$
* So I was <u>wrong</u> w/ writing d in numerator
- or it only worked here

Scalar Potential



single charge



$$V(P) - V(\infty) = \frac{kq_s}{r_{sp}}$$

↑ integral of electric field

$$V(P) - V(\infty) = -\int_{\infty}^{P} \vec{E}\cdot d\vec{s} = -\int \frac{kq_s}{r^2}\,dr$$

on straight line only ....

dir of path by dir of points

$$= \frac{kq_s}{r}\Big|_{\infty}^{r_{sp}}$$

5

Difference   q factor out

$$V(S) - V(P) = \frac{kq}{b}\left(\frac{1}{3} - \frac{1}{2}\right) = \frac{kq}{6b}$$

$$\Delta U = Q\,\Delta V_s = -\frac{Q\,kq}{6b}$$

$$\Delta k + \Delta U = 0$$

$$\Delta k = -\Delta U = \frac{Q\,kq}{6b}$$

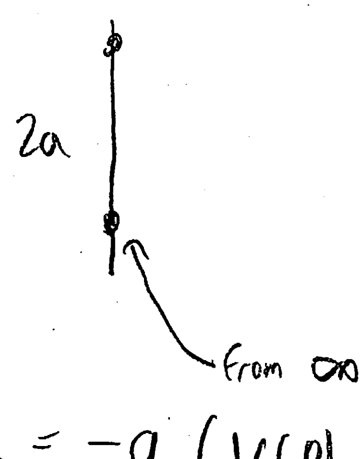
$$\frac{1}{2} m v_f^2 - 0 = \frac{Q\,kq}{6b}$$

$$v_f = \sqrt{\frac{2Q\,kq}{6mb}}$$

How much energy to assemble these charges

First is free

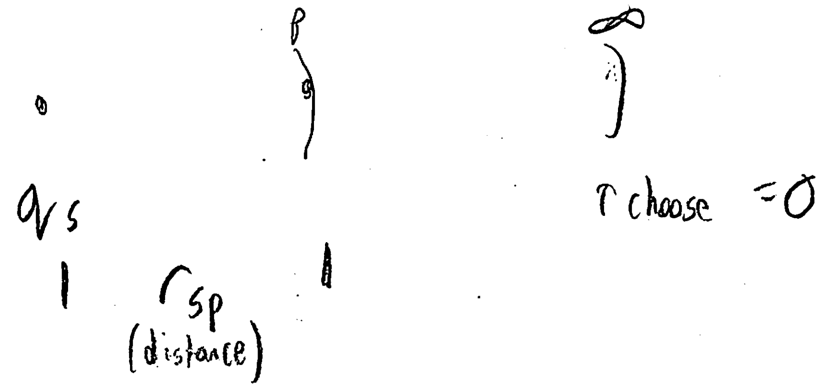
2nd



$$\Delta U_2 = -q\left(V(P) - V(\infty)\right)$$

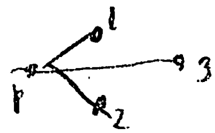
$$= -q\,\frac{kq}{2a}$$

(4)



Can do superposition + add them

$$V(\infty) = 0$$



$$V(P) = V_1(P) + V_2(P) + V_3(P)$$ - scalar, no vectors

$$\frac{kq}{(a^2+b^2)^{1/2}} - \frac{kq}{(a^2+b^2)^{1/2}} + \frac{kq}{2b} \leftarrow \frac{kq}{r} \text{ not } \frac{kq}{r^2}$$

$$= \frac{kq}{2b}$$

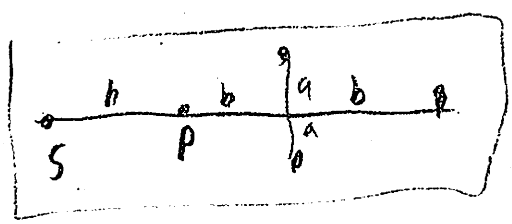
If $V(\infty) \neq 0$

then have to add it for each charge

$$V(\infty) + \frac{kq}{r}$$

If move $P \to S$

Remember P_1 P_2 cancle



$$V(S) = \frac{kq}{3b}$$ by itself

6

Bring 3rd in

- ~~Non~~ Must sum energy w/ 1 and energy w/ 2

$$\Delta U = \Delta U_{12} + \Delta U_{13} + \Delta U_{23}$$

$$= (-q)\frac{kq}{2a} + q\frac{kq}{\sqrt{a^2+b^2}} + \frac{(q)(k)(-q)}{\sqrt{a^2+b^2}}$$

$$= -\frac{kq^2}{2a}$$

↑ $\ominus$ sign means does it on own

energy stored in config

How to use Gauss' Law

3 types of problems

Spheres

Cylinders

planes

- or combos, or concentrics

Slabs, planes

need to know how to choose right surface

! - draw pic

- where is charge inclosed

Straighten out ρ, λ, σ

is it just σV or do I need to integrate

↑ σ constant ↑ σ varing

↑ do example of this

8) $0<r<a$

← - - - = guassian

$\oint E\cdot d\vec{a}$	$\frac{q_{inc}}{\epsilon_0}$
$E_1\cdot 4\pi r^2$	$\frac{1}{\epsilon_0}\int \rho\, dv$

how much is pointing out - just surface area here

what is the dV

$$dV = \underbrace{4\pi r'^2}_{\text{area}}\ \underbrace{dr}_{\text{thickness}} \text{ of shell}$$

$$\rho\, dV = hr'\,4\pi r'^2\, dr$$

radius guassian surface you picked

$$\int_0^r \rho\, dV = \int_0^r hr'\,4\pi r'^2\, dr'$$

1 D integral

$$q_{inc} = \int_0^r hr'\,4\pi r'^2\, dr$$

$$= h4\pi \int_0^r r'^3\, dr'$$

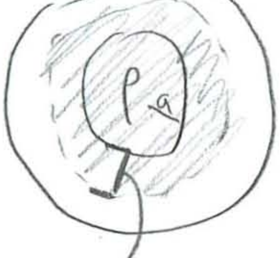
$$= \frac{h4\pi r^4}{4}$$

$$= hr^4$$

$E_1 4\pi r^2$	$\frac{h\pi r^4}{\epsilon_0}$

$$\boxed{E_1 = \frac{h}{\epsilon_0} r^2 \hat{r} \quad 0<r<a}$$

(9) $a < r < b$

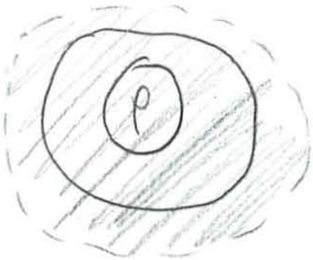


no charge here nothing to integrate here

$\oint E \cdot da$	$\frac{q_{inc}}{\varepsilon_0}$
$E_2 4\pi r^2$	$\frac{1}{\varepsilon_0}\int_0^a h r' \, 4\pi r'^2 \, dr'$
$E_2 =$	$\frac{1}{4\pi r^2 \varepsilon_0} \, h 4\pi \frac{a^4}{4}$ ← may be wrong
$E_2 =$	$\frac{h a^4}{4\varepsilon_0} \frac{1}{r^2}\hat{r} \quad a < r < b$ ← right

$\underbrace{\quad\quad\quad}_{\text{inverse square}}$

$r < b$



$E \, da$	$\frac{q_{inc}}{\varepsilon_0}$
~~$E_3 4\pi r^2$~~	~~$\frac{1}{\varepsilon_0}\int_0^b h r' \, 4\pi r'^2 \, dr'$~~
~~$E_3 =$~~	~~$\frac{1}{4\pi r^2 \varepsilon_0} \frac{h 4\pi b^4}{4}$~~
~~$E_3 =$~~	~~$\frac{h b^4}{4\varepsilon_0} \frac{1}{r^2}\hat{r}$~~

$$\boxed{E_3 = \frac{h a^4}{4\varepsilon_0}\frac{1}{r^2}\hat{r} + \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}\hat{r}}$$ superposition argument

or traditional

$$E_3 4\pi r^2 = \frac{1}{\varepsilon_0}\left(\int_0^a h r' \, 4\pi r'^2 \, dr' + Q\right)$$

$$E_3 = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\left(\frac{h 4\pi a^2}{4} + Q\right)$$

not finding gradients

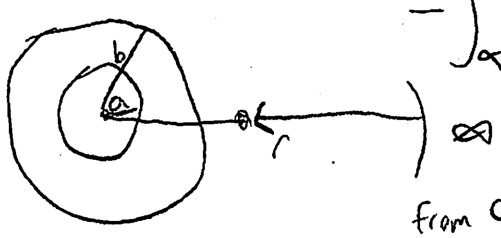
(10) If need $V(R)$ everywhere in space
- choose where to have it 0
- this is the difficult part

can do ∞ or 0
- does not matter
- will choose $V(\infty) = 0$

$$V(r) - \underbrace{V(\infty)}_{0} = \begin{cases} \_\_\_ & r > b \\ \_\_\_ & a < r < b \\ \_\_\_ & r < a \end{cases}$$

$\leftarrow$ hardest

<u>$r > b$</u>



from $\infty \to r$

$$-\int_{\infty}^{r} \vec{E}_3 \cdot ds$$

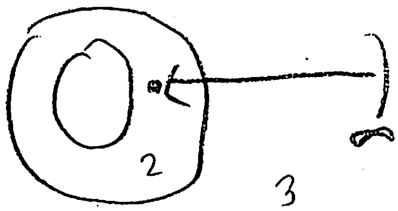
$$= -\int_{\infty}^{r} \frac{Q_{inc}}{4\pi\epsilon_0} \frac{dr'}{r'^2}$$

$$\boxed{\int^{r} -\frac{dr}{r^2} = \frac{1}{r}}$$

looks just like formula for pt charge

$$= \frac{Q_{inc}}{4\pi\epsilon_0} \frac{1}{r}$$

(11) $a < r < b$



going through regions 3 and 2

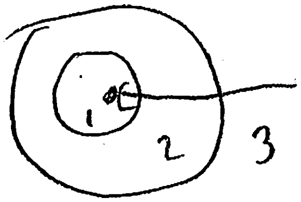
$$V(r) - V(\infty) = -\int_{\infty}^{b} \vec{E_3} \cdot ds - \int_{b}^{r} \vec{E_2} \cdot \vec{ds}$$

$$= -\int_{\infty}^{b} \frac{Q_{inc}}{4\pi\epsilon_0} \frac{\pi}{r^2}\, dr - \int_{b}^{r} \frac{h a^4}{4\epsilon_0} \frac{1}{r^2}\, dr$$

$$= \frac{Q_{enc}}{4\pi\epsilon_0} \frac{1}{b} + \frac{h a^4}{4\epsilon_0} \left(\frac{1}{r} - \frac{1}{b}\right)$$

$Q_{inc} = h\pi a^4 + Q$ ← is a shortcut so don't have to rewrite

$r < a$



$$V(r) - V(\infty) = -\int_{\infty}^{b} \vec{E_3} \cdot ds - \int_{b}^{a} \vec{E_2} \cdot ds - \int_{a}^{r} \vec{E_1} \cdot ds$$

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the guassian surface is the variable

get E field for each part (are vectors)

The potential difference transverses a path

- need E field for each region

(12) $V(0) = V(\infty) + \Delta V_{0\infty}$

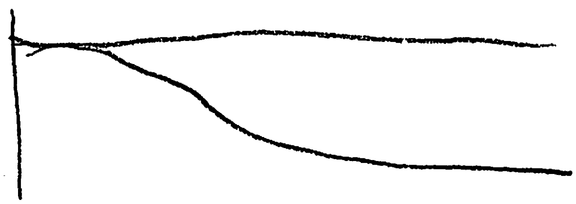
↑ you calculated

$r(\infty) = 0$

r $\infty \rightarrow$

Just where you start from

$r(0) = 0$



$\infty \rightarrow$

r

Be able to do this for planes + cylinders

↑ easy

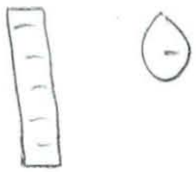
↑ bit tricker

Not doing find E from V

Day 10

2/24

How to approch problems

1. 

Use Guasses Law
- but not inclosed

- do as much goes tangelt through surface

$\Phi = 0$; not enclosed

- but its flux - still some

One of the 6 sides of a cube

$\frac{-q}{6\varepsilon_0}$ ⊖ charge, $\frac{1}{6}$ of cube

2. Diapoles are just charges
   - subject to Colomb's law

On the left · will
left + clockwise (D)

4. Equipotential lines — topo map

~~opposite~~ + smaller

Same sign

* don't screw up → same sige = opposit charge

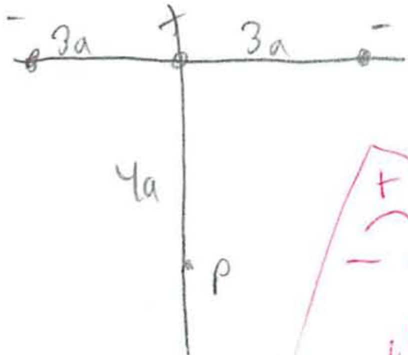
or

$-$
$+$
$+$

-2 ← see that larger
$+$
$+$

3 charges
equidistant

I did not read all of
the answers
to see — missed



always add does not matter

$$\vec{E} = \frac{kqq}{r^2} = \frac{k\,{-q}\,3a}{(3a^2+4a^2)}\hat{\imath} + \frac{k\,{-q}\,3a}{(3a^2+4a^2)}\hat{\imath} + kq$$

always charge to point!

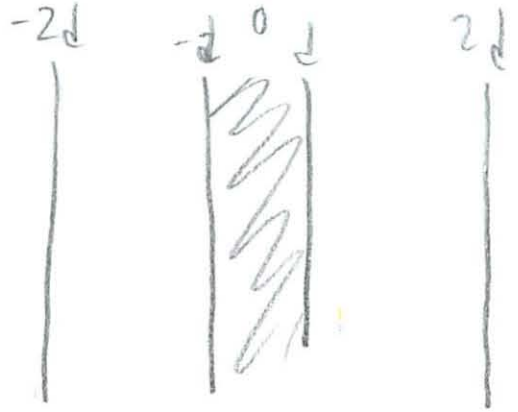
$$\frac{1}{r^2}\ \frac{\lambda R}{2\epsilon_0}\ \frac{2}{\sqrt{R^2+z^2}^{\,3}}$$

All charge same distance
almost like pt charge
only vertical component that
survives

what happens when $z \to 0$

$$\boxed{\vec{F} = \frac{kq}{r^2}}\quad \frac{l}{\ }$$

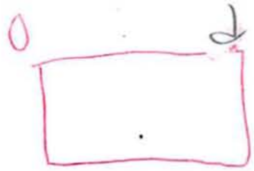
(3)



not drawn to scale

pill box

take advantage of symmetry



$V(-d) = 0$

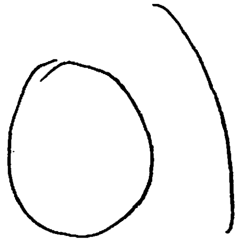
Straight forward - read it later

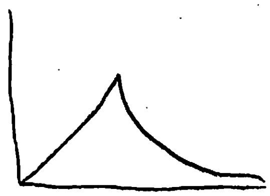
* think about what you need to solve problem

①

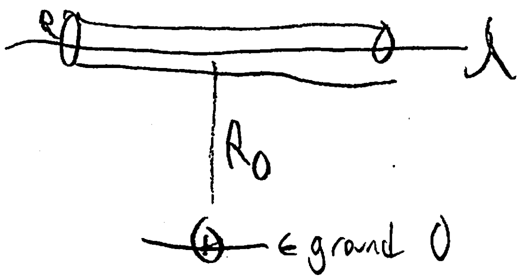
Office Hrs

2/24

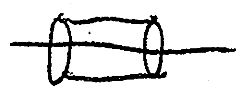




Problem 4 Pset 3 Power Lines



$E = \frac{\lambda}{2\pi\epsilon_0 r}$ from doing cylinder



$V = \frac{-\lambda}{2\pi\epsilon_0} \ln(r) + C$

$\hookrightarrow 0 = \frac{-\lambda}{2\pi\epsilon_0} \ln R_0 + C$

has to sum to 0

$C = \frac{\lambda}{2\pi\epsilon_0} \ln(R_0)$

$V = \frac{-\lambda}{2\pi\epsilon_0} \ln\left(\frac{R}{r_0}\right)$

put together

+ flip

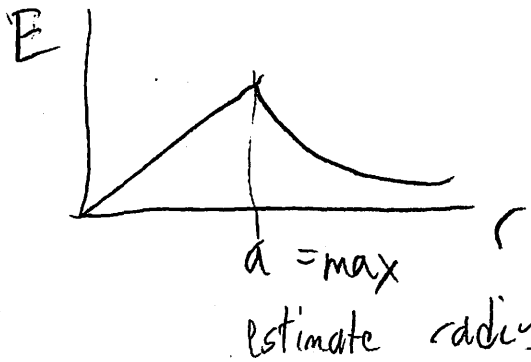
$V = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_0}{R}\right)$

Remember

$\ln r - \ln(R_0) = \ln \frac{r}{R_0}$

(2) Estimate radius of line

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$



$$\boxed{\frac{\lambda}{2\pi\epsilon_0 a} = 10^6\ V/m}$$

$$V = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{R_0}{a}$$

$$\lambda = rE$$

$$V = rE \ln \frac{R_0}{r}$$

potential function of E

0 at ground

$$\boxed{V = 1cm \cdot (10^6\ V/m) \ln\left(\frac{10m}{1cm}\right)}$$

You learn more from doing 1 problem slowly than lots of problems fast

(3)

E field points from (+) to (−)

Voltage (+) charge goes to lower potential

(−) charge goes to higher potential

~~When summing config energy~~

~~Summing config energy~~ from charge to point measuring

Calcing E field at p

---

equipotential $\perp$ field lines

againts E field potential ↑

↑ E must be (−)

Potential = work to do to move (+) charge

potential ↑ have to do work

Config energy ≠ calc field at P

#5 from class tody

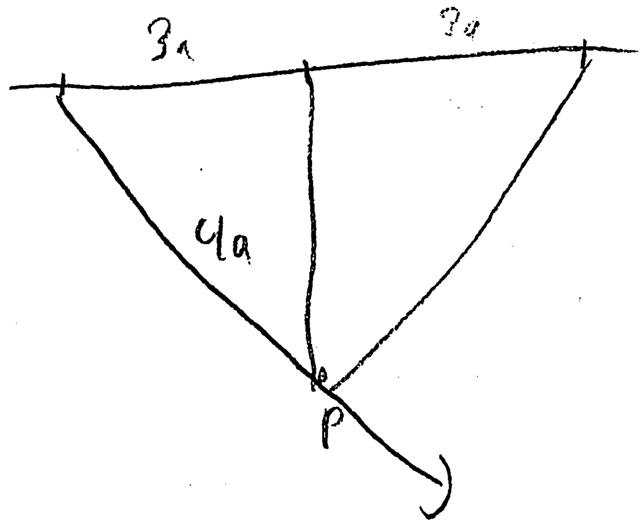
$$E = \frac{kq(\vec{r}-\vec{r}')}{|r-r'|^3}$$

r = where ~~at~~ measuring P

r' = Charge where

apply 3 times

(4.)



from charge → measuring

$\vec{r} = -4a\,\hat{j}$ ← measuring

$\vec{r}\,' = -3a\,\hat{i}$ ← charge

Using origin to measure from

$\vec{r} - \vec{r}\,' = -4a\,\hat{j} - -3a\,\hat{i}$

$= -4a\,\hat{j} + 3a\,\hat{i}$

$|\vec{r} - \vec{r}\,'| = \sqrt{16a^2 + 9a^2}$

$5a$

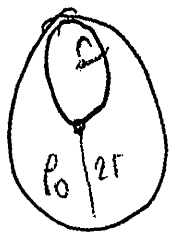
$\frac{1}{r^3} = \frac{\hat{r}}{r^2} = \frac{\vec{r}}{r^3}$

Since $\frac{r}{r^3} = \frac{1}{r^2}$ to get units to work out

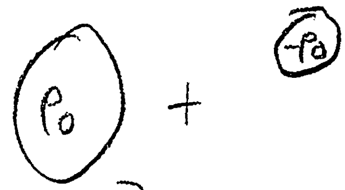
$$\frac{k(-1)(-4a\,\hat{j} + 3a\,\hat{i})}{(5a)^3}$$

now do this for other 2 vectors

5) Cavity Problem Pset 2



Super position fully charged + empty



$$\vec{E} \text{ field} = \vec{E}_{large} + \vec{E}_{small}$$

$$E = \frac{\rho}{3\epsilon_0} \vec{r}_{center_1} + \frac{-\rho_0 \vec{r}_{center_2}}{3\epsilon_0}$$

$$\frac{\rho \, r_{center}}{3\epsilon_0} +$$

Vector $\rightarrow$ $\vec{r}_{center2} = \vec{r}_{center_1} + \underset{\text{the radius}}{R}$

have = + opposite charge
So <u>not</u> like comparing volumes

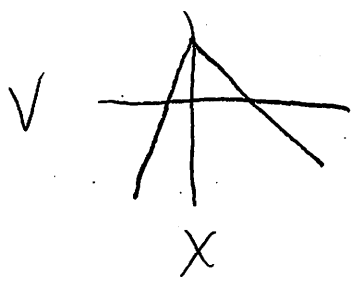
E from V/Gradiant

2/24

Oumashkin did not do, so I will

$E = -\nabla V$

↑ differenltate each part



$x > 0$

magnitude of E smaller

(since not as steep)

as $x < 0$

* Don't get tricked by concept qu where the ans is IDK b/c I have to look around

* And it is the − gradient

---

* units $E = \frac{V}{m}$

Other Review

Collecting E from charges $\frac{1}{r^3}$

$F = \frac{kqQ}{r^2}$

$E = \frac{kQ}{r^2}$ $\hat{r}$ from charge to observer $= \frac{kQ\vec{r}}{r^3}$

② $F = qE$

$p = qd$) don't think used much

$\tau = \vec{p} \times \vec{E}$

$E = k \sum \frac{q}{r^2}$

Config E - moving pt in

- work
- $\frac{kq}{r}$ + summing

$W = qV$

Not finding E at a pt

$\Delta U = qE$

E falls off

Diapole $\frac{1}{r^3}$
Pt $\frac{1}{r^2}$
Line $\frac{1}{r}$
Plane 1 - constant

Physics 8.02 **Equation Sheet Exam One**

Please Remove this Tear Sheet from Your Exam

$$\vec{\mathbf{E}} = \frac{q}{4\pi\varepsilon_o r^2}\hat{\mathbf{r}} = \frac{q}{4\pi\varepsilon_o r^3}\vec{\mathbf{r}}$$

$\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}}}{r}$ *points* **from** *source q* **to** *observer*

$$\vec{\mathbf{E}}_{\text{many point charges}} = \sum_{i=1}^{N} \frac{q_i}{4\pi\varepsilon_o |\vec{\mathbf{r}} - \vec{\mathbf{r}}_i|^3}(\vec{\mathbf{r}} - \vec{\mathbf{r}}_i)$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\int_{source} \frac{dq}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}'|^2}\hat{\mathbf{r}}$$

$$\vec{\mathbf{F}}_q = q\vec{\mathbf{E}}_{source}$$

$$\oiint_{\substack{closed\\surface}} \vec{\mathbf{E}} \cdot \mathbf{d}\vec{\mathbf{A}} = \frac{Q_{\text{enc}}}{\varepsilon_o}$$

$\mathbf{d}\vec{\mathbf{A}}$ points from inside to outside

$$\oint_{\substack{closed\\path}} \vec{\mathbf{E}} \cdot \mathbf{d}\vec{\mathbf{s}} = 0$$

$$\Delta V_{moving\ from\ a\ to\ b} = V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot \mathbf{d}\vec{\mathbf{s}}$$

$$\Delta U = q\Delta V$$

$$V_{\text{point charge}} = \frac{q}{4\pi\varepsilon_o r}; V(\infty) = 0$$

$$V_{\text{many point charges}} = \sum_{i=1}^{N} \frac{q_i}{4\pi\varepsilon_o |\vec{\mathbf{r}} - \vec{\mathbf{r}}_i|}; V(\infty) = 0$$

$$V(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\int_{source} \frac{dq}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}'|}; V(\infty) = 0$$

$$U = \sum_{all\ pairs} \frac{q_i q_j}{4\pi\varepsilon_o |\vec{\mathbf{r}}_i - \vec{\mathbf{r}}_j|}; U(\infty) = 0$$

$$U = \frac{1}{2}\varepsilon_o \iiint_{all\ space} E^2 dV_{vol}$$

$E_r = -\frac{\partial V}{\partial r}$ for spherical symmetry,

$$\vec{\mathbf{E}} = -\vec{\nabla} V$$

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

$$C = \frac{|Q|}{|\Delta V|} \qquad U = \frac{1}{2}C\Delta V^2 = \frac{Q^2}{2C}$$

Circumferences, Areas, Volumes:

1) The area of a circle of radius r is πr^2
Its circumference is $2\pi r$

2) The surface area of a sphere of radius r is $4\pi r^2$. Its volume is $(4/3)\pi r^3$

3) The area of the sides of a cylinder of radius r and height h is $2\pi rh$. Its volume is $\pi r^2 h$

Integrals that may be useful

$$\int_a^b dr = b - a$$

$$\int_a^b \frac{dr}{r} = \ln(b/a)$$

$$\int_a^b \frac{1}{r^2}dr = \left(\frac{1}{a} - \frac{1}{b}\right)$$

Some potentially useful numbers

$$k_e = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{\text{N m}^2}{\text{C}^2}$$

2/24/10

8.02 Exam One Spring 2008

FAMILY (last) NAME

GIVEN (first) NAME

Student ID Number

Your Section:

____L01 MW 9 am ____L02 MW 11 am ____L03 MW 1 pm ____L04 MW 3 pm
____L05 TR 9 am ____L06 TR 11 am ____L07 TR 1 pm ____L08 TR 3 pm

Your Group (e.g. 10A): ____________

	Score	**Grader**
Section I (25 points)		
Problem 1 (25 points)		
Problem 2 (25 points)		
Problem 3 (25 points)		
TOTAL		

SECTION I

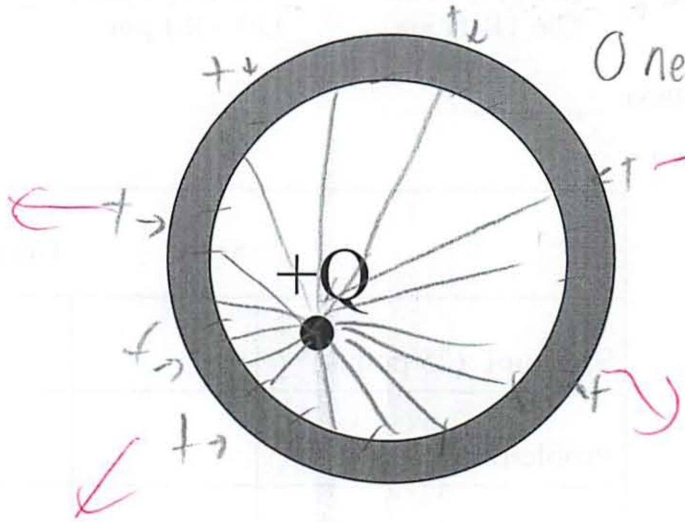
In this section you are asked to answer 5 questions, each worth 5 points. You do not have to show your work; in most cases you may simply circle the chosen answer.

Question 1 (5 points):

Consider the hollow conductor shown in the figure which has zero net charge. As shown, an object with charge $+Q$ is placed inside the neutral, hollow, spherical conductor. Indicate the direction of the electric field that arises after static equilibrium has been achieved by sketching several representative field lines everywhere in space as needed.

no Conductors On real test

O net charge



draw lines outside – I was thinking I should but decided againts it

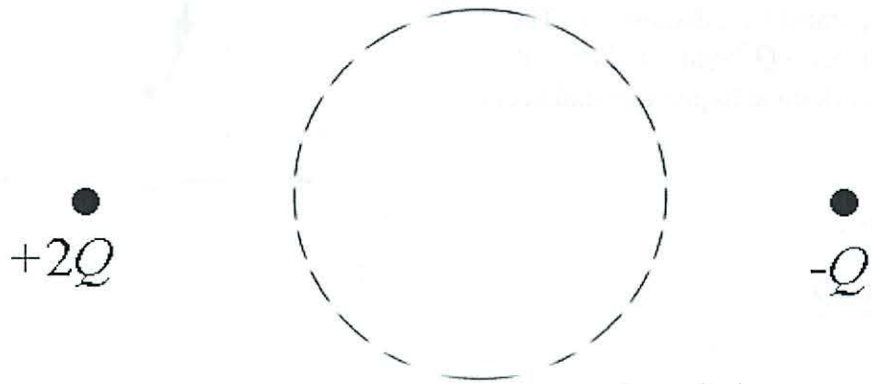
– goes to inner ring

\+ goes to outer ring

– no field lines

Question 2 (5 points):

Consider the two charges $+2Q$ and $-Q$, and a mathematical spherical surface (it does not physically exist) as shown in the figure below.



The electric flux on the spherical surface between the charges is…

a) positive.

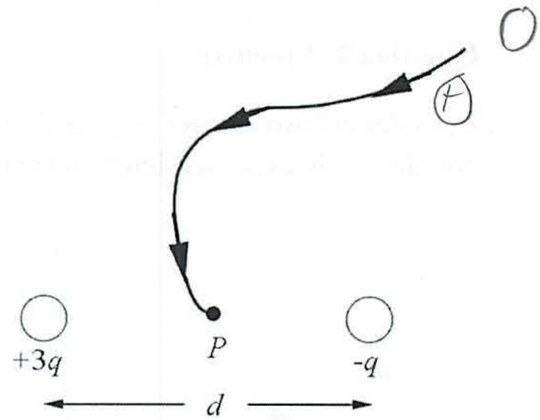
b) negative. net

c) zero. nothing inside

d) undetermined by the information given.

Question 3 (5 points):

A positive charge $+Q$ is moved by an external agent from infinity to a point midway between two charges $+3q$ and $-q$ that are separated by a distance d. The positive charge $+Q$ begins and ends at rest. The work done by the external agent is



a) zero.

b) $+\frac{k_e qQ}{d}$.

c) $-\frac{k_e qQ}{d}$.

d) $+\frac{2k_e qQ}{d}$.

e) $-\frac{2k_e qQ}{d}$.

f) $+\frac{3k_e qQ}{d}$.

g) $-\frac{3k_e qQ}{d}$.

h) $+\frac{4k_e qQ}{d}$.

i) $-\frac{4k_e qQ}{d}$.

j) none of the above.

Should know

have to do work → ans +

"Config energy"

$q_{net} = 2$ so d

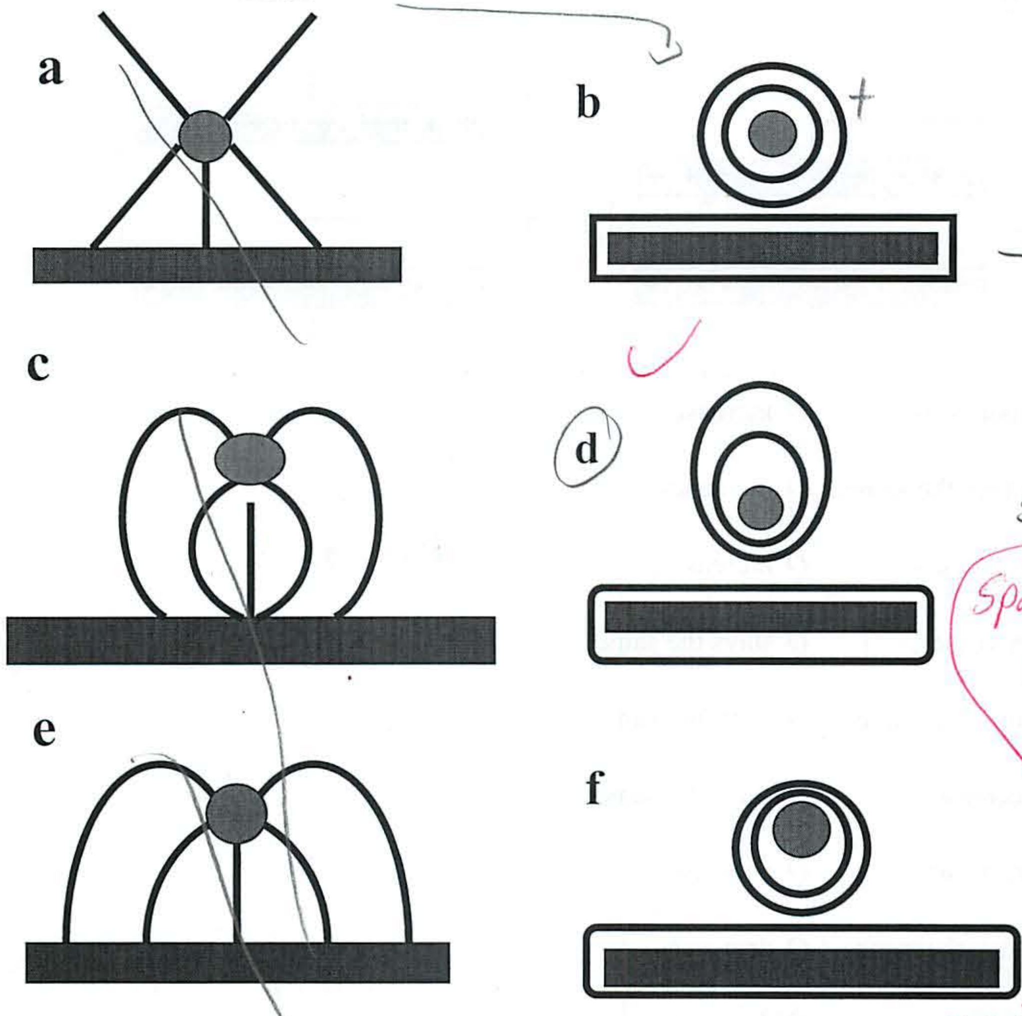
$$Q\left(\frac{3kq}{\frac{d}{2}} - \frac{kq}{d/2}\right) = \frac{4kqQ}{d}$$

oh b/c $\frac{d}{2}$

tricky!

Question 4 (5 points):

The circle is at +5 V relative to the plate. Which of the below are the most accurate contours for the equipotentials (**equipotential map**)?



So

Spaced closet to the 2 Conductors

not ⊕/⊖

~~Close to ⊕~~ equi spaced

? So point towards or away

⊖ low # black

⊕ high # red

further to fall

think ~~experiment~~

+

−

E from + → −

was it tight around

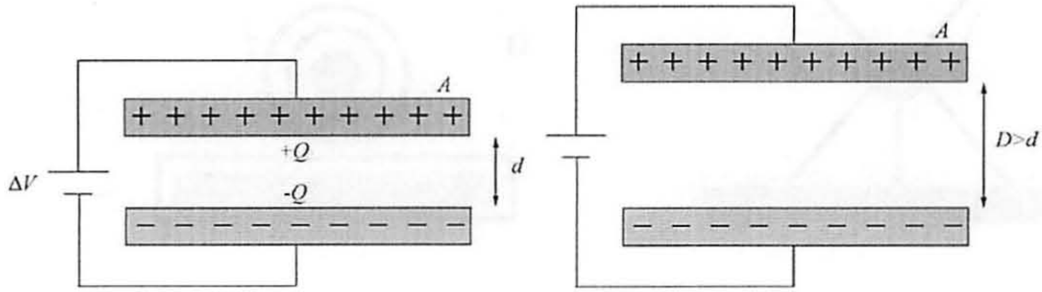
+ ~~or~~ −

~~−~~ since

~~charge flows from the~~

Question 5 (5 points):

A parallel plate capacitor has plates with equal and opposite charges $\pm Q$, separated by a distance d, **and is connected to a battery**. The plates are pulled apart to a distance $D > d$. What happens to the magnitudes of the electric field E in the gap and the charge Q?



(a) E increases, Q increases.

(b) E stays the same, Q increases.

(c) E decreases, Q increases.

(d) E increases, Q stays the same.

(e) E stays the same, Q stays the same.

(f) E decreases, Q stays the same.

(g) E increases, Q decreases.

(h) E stays the same, Q decreases.

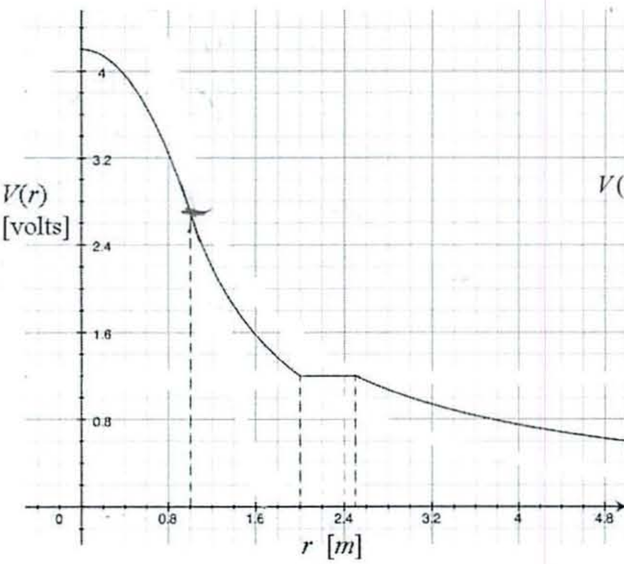
(i) E decreases, Q decreases.

Skip
not on test

SECTION II In this section you are asked to solve three problems. **Please be careful to show your work, and remember to include units where appropriate.**

Problem 1 (25 points)

The graph below shows the variation of an electric potential $V(r)$ with radial distance r with choice $V(\infty)=0$. The potential is described by the function



$$V(r)=\begin{cases}(3\text{ V})\left(\dfrac{7}{5}-\dfrac{r^2}{2\text{ m}^2}\right);\ 0\text{ m}\le r<1.0\text{ m}\\[2mm] (3\text{ V}\cdot\text{m})\left(\dfrac{1}{\text{r}}-\dfrac{1}{10\text{ m}}\right);\ 1.0\text{ m}<r<2.0\text{ m}\\[2mm] \left(\dfrac{6}{5}\text{V}\right);\ 2.0\text{ m}<r<2.5\text{ m}\\[2mm] (3\text{ V}\cdot\text{m})\dfrac{1}{r};\ r>2.5\text{ m}\end{cases}$$

a) Give the electric field vector $\vec{\mathbf{E}}$ for each of the four regions in (i) to (iv) below?

(i) $0\text{ m}\le r<1.0\text{ m}$

Gradiant - look up

$E = -\frac{dV}{dx} = -\frac{4.2-2.6}{1-0} = \frac{-6.8}{1} = -6.8$

(ii) $1.0\text{ m}<r<2.0\text{ m}$ Oh they give formula - can differentiate literal
- lets see if I remember

$-\frac{d}{dx}3\left(\frac{1}{r}-\frac{1}{10}\right)$

$3\left(\ln r-\frac{1}{10}\right)$

$3\ln r - \frac{3}{10}$

$3V\cdot m\cdot\frac{1}{r^2}$

try again

$3\left(\frac{d}{dr}\frac{1}{r}-\frac{d}{dr}\frac{1}{10}\right)$

$3\cdot\cancel{\ln r}\ \frac{d}{dr}\frac{1}{r}\neq\ln$

$3\cdot\frac{-1}{r^2}$

and do units

$-3\left(\frac{7}{5}-\frac{2r}{2m}\right)$

$-3\left(\frac{7}{5}-r\right)$

Simplify

$\boxed{3\frac{V}{m^2}r\hat{r}}$

test $1m = 3\frac{V}{m}$

right units

So why is this so wrong
-b/c I can't differentiate

This is just a math q!

1. factor out constants

$-3\left(\frac{d}{dr}\left(\frac{7}{5}-\frac{r^2}{2}\right)\right)$

2. Differentiate sum term by term

$-3\left(\frac{d}{dr}\left(\frac{7}{5}\right)-\frac{1}{2}\frac{d}{dr}(r^2)\right)$

* 3. differentiate $\frac{7}{5}=0$

$\frac{3}{2}\frac{d}{dr}(r^2)$

$=3r$

So need to know what factors out + what goes to 0

(iii) 2.0 m < r < 2.5 m

Lets see if this goes better

$\frac{d}{dx}\frac{6}{5}V = \boxed{0\,\frac{V}{m}}$

↑ flat line

(iv) r > 2.5 m

$\frac{d}{dx}\,3\,V\cdot m\,\frac{1}{r}$

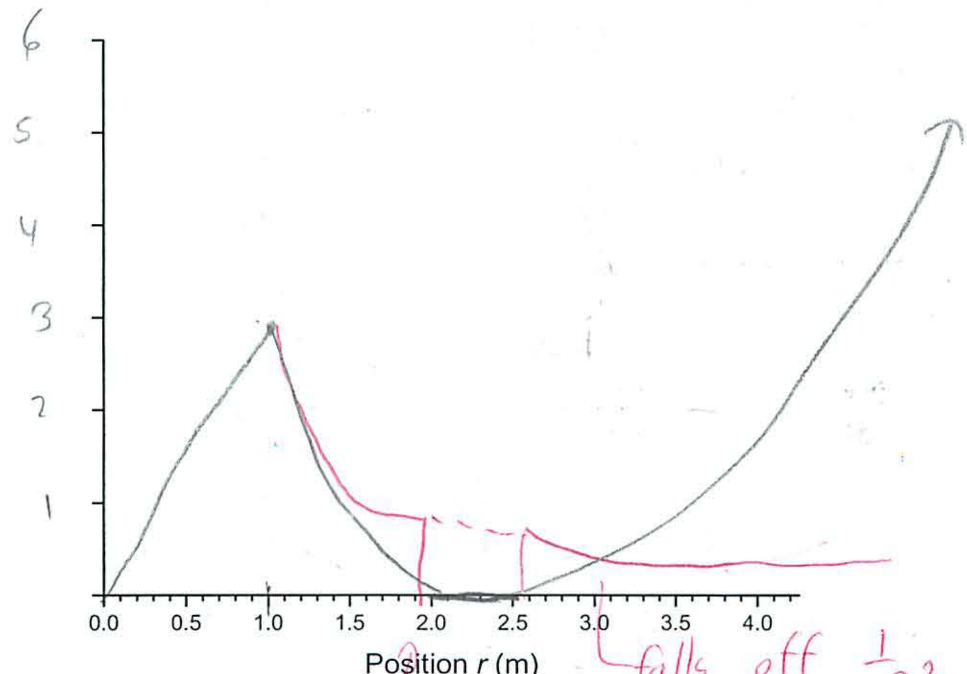
$-3\frac{d}{dx}\frac{1}{r}$

$-3\cdot-\frac{1}{r^2}$

$\boxed{+\frac{3}{r^2}\,\frac{V}{m}}$ ✓

but units $3\,V\cdot m\cdot\frac{1}{r^2}$

b) On the axes below, sketch the magnitude of the radial component of the electric field, E_r, as a function of r. Make sure you label the axes to indicate the numeric magnitude of the field.

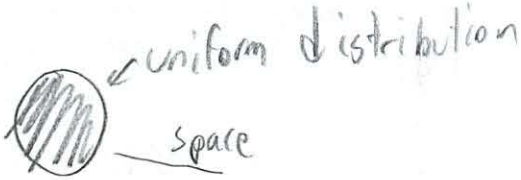


falls off $\frac{1}{r^2}$

try plugging # in if doing for real

c) Qualitatively describe the distribution of charges that gives rise to this potential landscape and hence the electric fields you calculated. That is, where are the charges, what sign are they, what shape are they (solid, shell…)?

$\frac{1}{r^2}$ = sphere?



I don't know what the gap is that does not effect anything

Oh E=0 gap = conductor

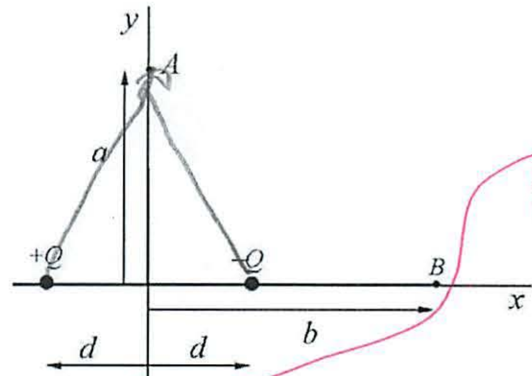
R

but ~~At~~ the whole shell is evenly distributed

~~this is Matt~~

Problem 2 (25 points)

Two charges $+Q$ and $-Q$ lie along the x-axis and are separated by a distance $2d$.



W = qV
which is about work
this is not Config E like

day 2 Coulomb dipoles

for U

(a) Calculate the total electric field $\vec{\mathbf{E}}$ at position A, a distance a from the x-axis. Indicate its direction on the sketch (draw an arrow)

~~$\vec{E}$ = super position~~ or vector approach

~~1st free~~
~~2nd =~~ ~~don't remember~~
would not work for this

Coulomb
$\frac{\Delta x}{r^3}\hat{i} + \frac{\Delta x}{r^3}\hat{j}$

From charge to pt

$\hat{j}$ cancles

$\hat{j}$ does not matter here why?

$\cancel{+\frac{kq\,a}{\sqrt{d^2+a^2}} + \frac{k+q\,a}{\sqrt{d^2+a^2}}\hat{j}}$? is that right?

* Also vectors +charges ~~cancle~~ contribute *

$\hat{i}\,(+q\cdot -d) + (-q\cdot +d) = -2qd$ ← does not cancle

$\hat{j}\,(+q\cdot +a) + (-q\cdot +a) = 0$ ← this cancles!

(b) Calculate the total electric field $\vec{\mathbf{E}}$ at position B, a distance b from the y-axis. Indicate its direction on the sketch (draw an arrow)

$$k\frac{Q}{a^2+d^2}\left(\frac{d}{\sqrt{a^2+d^2}}\hat{i} + \frac{a}{\sqrt{a^2+d^2}}\hat{j}\right)$$

$\hat{i}$ just over r

$$k\frac{2Qd}{(a^2+d^2)^{3/2}}\hat{i}$$

This time $\hat{i}$ cancles

$(q\cdot d+b) + (-q\cdot b-d)$
↑ Clever makes you think

$$\frac{kQ\,\cancel{(d+b)}}{(d+b)^{\cancel{3}\,2}} + \frac{k-Q\,\cancel{(b-d)}}{(b-d)^{\cancel{3}\,2}}\hat{j}$$

← just 2

Oh I see don't need seperately in numerator

this is making me more confused!
- well I never knew diff and now I am finding it
- so good confused now where can spend hr to fix

Now, assuming that the electric potential is defined to be zero at infinity,

Look over Dimasti's review

(c) Find the electric potential V at position A.

$V = -\int E ds$

$V = -\int k \frac{2Qd}{(a^2+d^2)^{3/2}} ds$

?? how integrate w/ respect to d ?? yeah

$V = 0$

$V = k \frac{Q}{(a^2+d^2)^{1/2}}$ ← droped d

↑ went from $\frac{3}{2}$ → $\frac{1}{2}$

(d) Find the electric potential V at position B.

$\frac{1}{4\pi\epsilon_0}\left(-\frac{Q}{b-d} + \frac{Q}{b+d}\right)$ ← dropped 2

- so I guess really simple integration

(e) A negatively charged dust particle with mass m and charge $-q$ is released from rest at point B. In what direction will it accelerate (circle the correct answer)?

~~up hill to higher potential~~

LEFT RIGHT UP DOWN It Won't Accelerate

~~towards the (+) ∴ ← higher #s~~

✓ Oh I was thinking ← left wrong - but happened to be right

just the ⊖ will repell the ⊖ →

but in another concept Q it was the other way - you have to be past a pt - but how will you know that?

(f) Again, assuming the negatively charged dust particle was released from rest, what will its speed be after it has traveled a distance s from its original position at point B? HINT: Don't try to simplify any fractions

$F = ma = qE$

$a = \frac{qE}{m}$

$a = q\,k\left(-\frac{Q}{(b-d)^2} + \frac{Q}{(b+d)^2}\right)$

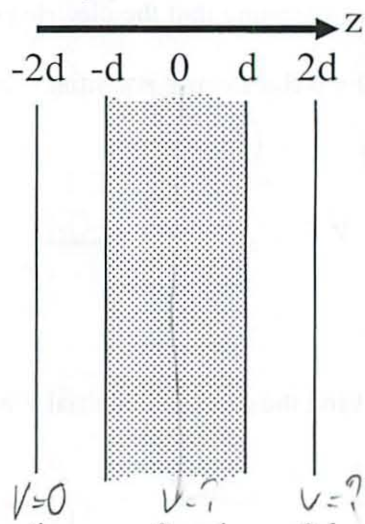
↑ but need velocity $\int$ integrate

they do it w/ energy

See sheet at end

Here's the Gauss' Law problem did in Class 10

Problem 3 (25 points):



Consider a semi-infinite **non-conducting** charged slab of thickness $2d$ centered on the xy-plane (filling all space between $z=-d$ and $z=d$). It has a positive, linear charge distribution

← non constant

$$\rho(z)=\rho_0|z|/d.$$

Sandwiching this charged slab are two semi-infinite neutral conductors, each of thickness d, located between $z=-2d$ and $z=-d$ and between $z=d$ and $z=2d$. The left hand side of the sandwich (at $z=-2d$) is held at $V=0$. Given this arrangement, calculate the electric potential V at two locations: $z=0$ and $z=+2d$.

NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!) To make our lives easier (and your grade higher) you should outline how you plan to approach the problem

.

Conductors $\vec{E}=0$

would not be on test

So need to find $\vec{E}$ for each section

- bars = 0 slabs = ?

Integrate each section

← Round ends or ends should fall off – yeah it falls off

pill box

$$2EA=\frac{Q_{inc}}{\epsilon_0}=\frac{\rho V}{\epsilon_0}$$

$h=2d$

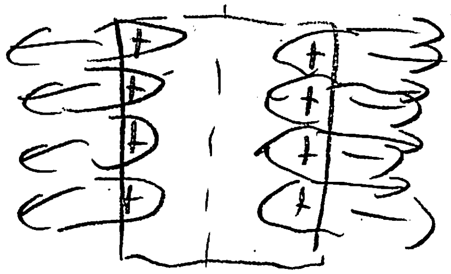
$$2E\pi r^2=\frac{\rho_0|z|}{d}\cdot\pi r^2 h$$

$$E=\frac{\rho_0|z|\cancel{\pi r^2}\,\cancel{2}d}{\cancel{2}\cancel{\pi r^2}}$$

$$E=\rho_0|z|\,d$$

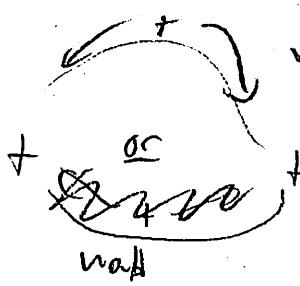
3

So why is E=0 in middle
B/c charge repels to outside



? ~~so what is inside~~
ok so yeah E=0
and why is V max there?

-b/c the | barrier to slab is a conductor
So most potential in middle
(i so if drop a + there



would get pushed
like field lines
-wish I could see them

try to see animation online
(think wrapped up a lot of confusion tonight)

nergy method 2f

$$\Delta U = U(\text{final}) - U(B) = -q\left(V(\text{final}) - V(B)\right)$$

Change in potential E

$$U(\text{Final}) = k_e\left(-\frac{Q}{b+s-d} + \frac{Q}{b+s+d}\right)$$

$$\Delta U = -q\left(k_e\left(-\frac{Q}{b+s-d} + \frac{Q}{b+s+d}\right) - k_e\left(-\frac{Q}{b-d} + \frac{Q}{b+d}\right)\right)$$

$$\Delta k + \Delta U = 0$$

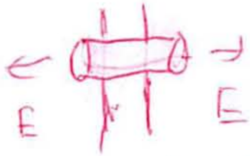
$k(B) = 0$ so goes to $k(\text{Final}) = \frac{1}{2} m v_f^2$

$$\frac{1}{2} m v_f^2 = -\Delta U = q\left(k_e\left(\frac{-Q}{b+s-d} + \frac{Q}{b+s+d}\right) - k_e\left(-\frac{Q}{b-d} + \frac{Q}{b+d}\right)\right)$$

$$v_f = \sqrt{\frac{2k_e q}{m}\left(\left(-\frac{Q}{b+s-d} + \frac{Q}{b+s+d}\right) - \left(-\frac{Q}{b-d} + \frac{Q}{b+d}\right)\right)}$$

Would never have gotten that!

Calc electric potentiall



E=0

So it is 0 at middle

* like ~~charges~~ repel to the outside *

↑ If I forget this 1 more time!!!

+V peak at center

Or they say (+) potential peaked at center and falls off to outside (what I did)

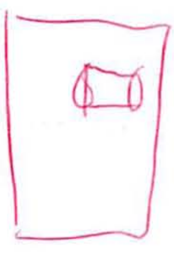
so why is what I assumed more right than above?

Potential also 0 at $z=d$

↑ that's that conductor stuff we don't have to do

Only need to calculate in slab

$E=0$ in middle



$$EA = \frac{1}{\varepsilon_0}\int_0^z \rho \, dv$$

↑ distance

$\boxed{\text{* Need } \int \text{ if } \rho \text{ not constant}}$

$$\frac{1}{\varepsilon_0}\int \rho_0 \frac{z'}{d} A \, dz'$$

$$\frac{1}{\varepsilon_0} \rho_0 \frac{z^2}{2d} A$$

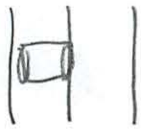
$$\boxed{E = \frac{\rho_0}{\varepsilon_0}\frac{z^2}{2d}} \leftarrow (-)\ (+) \rightarrow$$

$$V = -\int_{-d}^{0} E(z)\, dz = -\int_{-d}^{0} -\frac{\rho_0}{\varepsilon_0}\frac{z^2}{2d}\, dz = \frac{\rho_0}{\varepsilon_0}\frac{z^3}{6d}\Big|_{-d}^{0} = \boxed{\frac{\rho_0 d^2}{\varepsilon_0 6}}$$

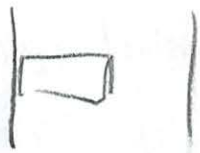
→ think on next page (after 2f)

$\tilde{E}$ in the space inbetween the fields

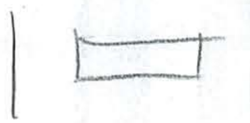
- constant remember the chart
- ~~But~~ - or thats what I found
- need to find inside



~~$2EA =$~~ ~~one of the~~ ? is one half 0 - no b/c its all +



So do from 1side



or middle

$$2EA = \frac{\rho V}{\epsilon_0}$$

~~2EA~~ ~~$E = \rho_0 |z| 2d$~~

$$E = \frac{\rho_0 |z| \cancel{\pi r^2} 2dd}{2\cancel{\pi r^2}}$$

this is confusing - how to make integral

$E = \rho_0 |z| dd$ ← d we are using

? yeah ∫ the damn thing! why did I not think of this!

$$V = \int_{-2d}^{-d} \rho_0 |z| d \, ds + \int_{-d}^{0} \rho_0 |z| dd \, ds$$

$$V = \int_{-2d}^{d} \rho_0 |z| dd s + Q + \int_{d}^{2d} \rho_0 |z| d \, ds$$

(This page intentionally blank for extra room to work)

Copy ans on back ⟶

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 Spring 2008

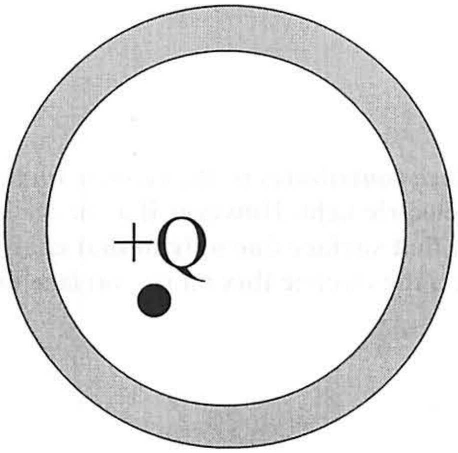
8.02 Exam One Solutions Spring 2008

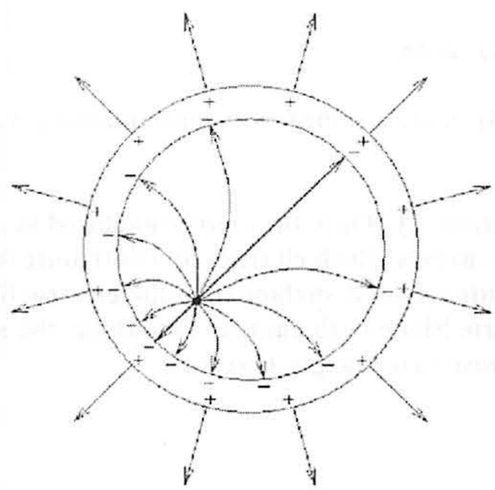
SECTION I

In this section you are asked to answer 5 questions, each worth 5 points. You do not have to show your work; in most cases you may simply circle the chosen answer.

Question 1 (5 points):

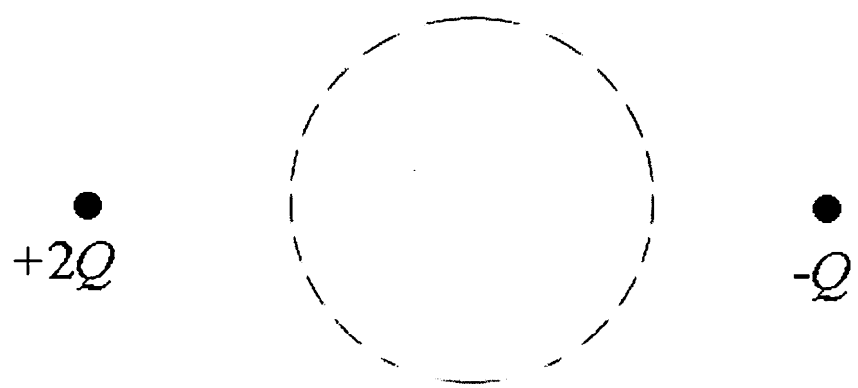
Consider the hollow conductor shown in the figure which has zero net charge. As shown, an object with charge $+Q$ is placed inside the neutral, hollow, spherical conductor. Indicate the direction of the electric field that arises after static equilibrium has been achieved by sketching several representative field lines everywhere in space as needed.





Question 2 (5 points):

Consider the two charges $+2Q$ and $-Q$, and a mathematical spherical surface (it does not physically exist) as shown in the figure below.



The electric flux on the spherical surface between the charges is…

a) positive.

b) negative.

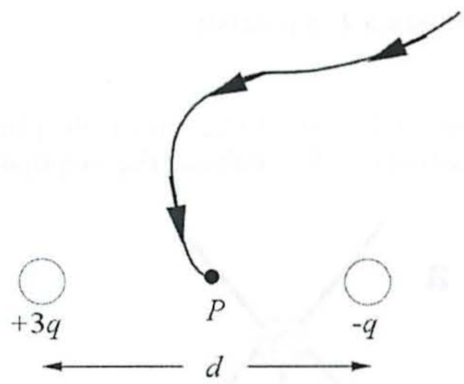
c) zero.

d) undetermined by the information given.

Solution: c). Only the charge enclosed in a surface contributes to the electric flux on that surface. Both charges do contribute to the electric field. However if a charge is outside a closed surface then the electric flux on that surface due only to that charge is zero. Since both charges lie outside the surface, the electric flux on the surface due to those two charges is zero.

Question 3 (5 points):

A positive charge $+Q$ is moved by an external agent from infinity to a point midway between two charges $+3q$ and $-q$ that are separated by a distance d. The positive charge $+Q$ begins and ends at rest. The work done by the external agent is



a) zero.

b) $+\frac{k_e qQ}{d}$.

c) $-\frac{k_e qQ}{d}$.

d) $+\frac{2k_e qQ}{d}$.

e) $-\frac{2k_e qQ}{d}$.

f) $+\frac{3k_e qQ}{d}$.

g) $-\frac{3k_e qQ}{d}$.

h) $+\frac{4k_e qQ}{d}$.

i) $-\frac{4k_e qQ}{d}$.

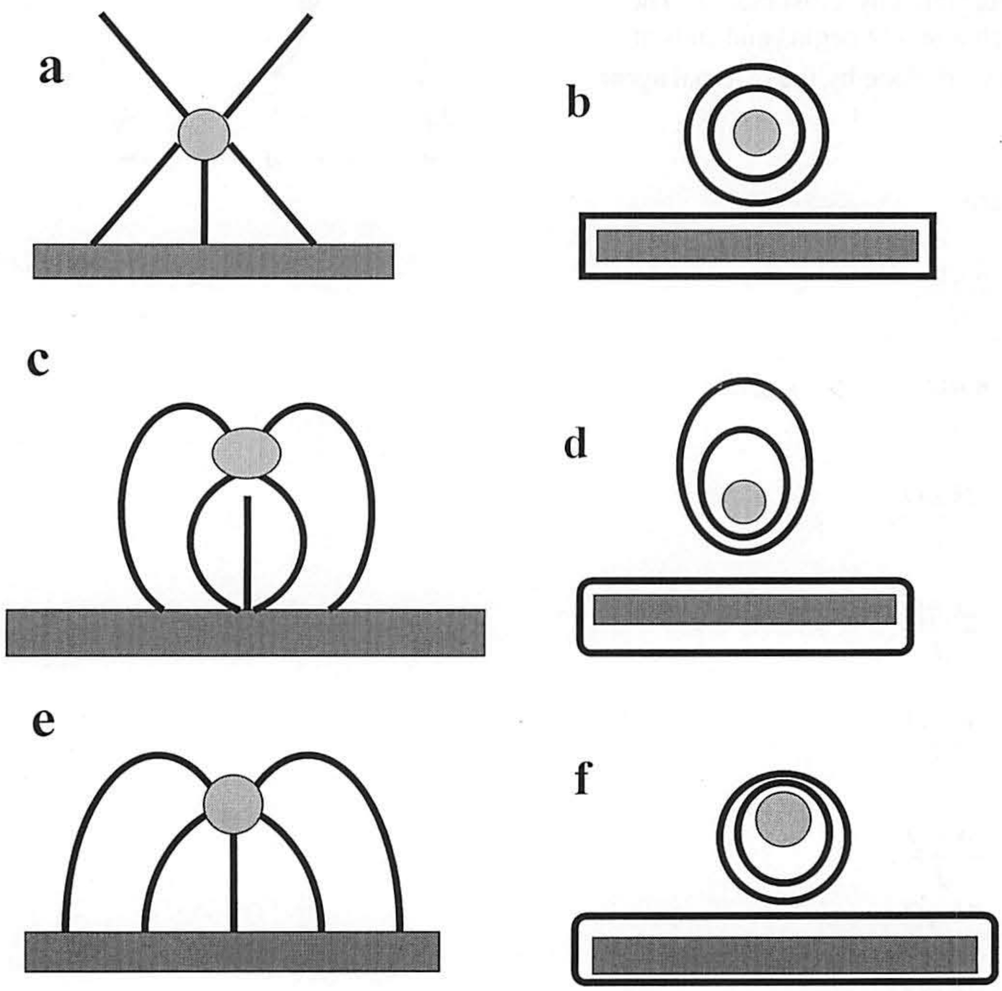
j) none of the above.

Solution: h). The work done by the external agent is equal to the change in electric potential energy which is given by the charge $+Q$ times the electric potential difference between infinity and the point between the charges:

$$W_{ext} = +Q(V(P) - V(\infty)) = +Q\left(\frac{3k_e q}{d/2} - \frac{k_e q}{d/2}\right) = +\frac{4k_e qQ}{d}$$

Question 4 (5 points):

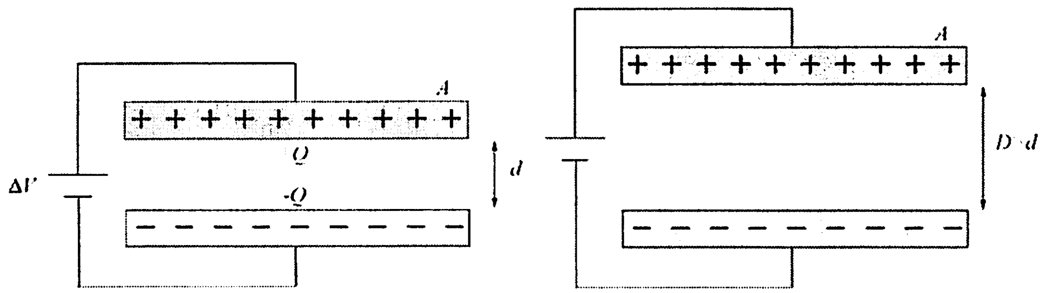
The circle is at +5 V relative to the plate. Which of the below are the most accurate contours for the equipotentials (**equipotential map**)?



Solution d). The strongest electric field is in the region between the circle and the plate and so the gradient of the electric potential is largest there. Thus we should have more contours in that region. Figure e) is the correct figure for the electric field lines. We can rule out answer b), because equal spaced contours above and below the circle at 5V implies that the electric field is radially symmetric but it must curve down to the plate as shown in e) in order to be perpendicular to the conductor. The contours in f) imply the electric field is strongest above the circle which is not correct.

Question 5 (5 points):

A parallel plate capacitor has plates with equal and opposite charges $\pm Q$, separated by a distance *d*, **and is connected to a battery**. The plates are pulled apart to a distance $D > d$. What happens to the magnitudes of the electric field E in the gap and the charge Q?



(a) E increases, Q increases.

(b) E stays the same, Q increases.

(c) E decreases, Q increases.

(d) E increases, Q stays the same.

(e) E stays the same, Q stays the same.

(f) E decreases, Q stays the same.

(g) E increases, Q decreases.

(h) E stays the same, Q decreases.

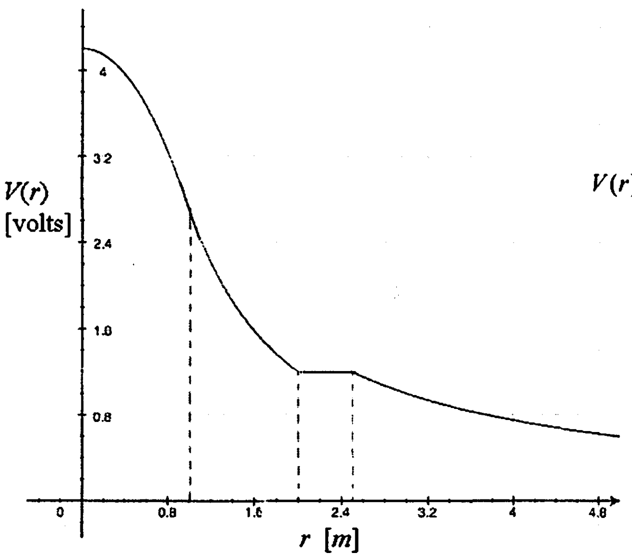
(i) E decreases, Q decreases.

Solution (i): Since the plates are connected to the battery, the potential difference between the plates ΔV does not change when the plates are moved. Before the plates were moved, $|\Delta V| = |\vec{E}|d = Ed$. After the plates were moved, $|\Delta V| = E_{new}D$. Since $D > d$, $E_{new} < E$. Since the electric field has decreased in magnitude, the magnitude of the charge on either plate must also decrease.

SECTION II In this section you are asked to solve three problems. **Please be careful to show your work, and remember to include units where appropriate.**

Problem 1 (25 points)

The graph below shows the variation of an electric potential $V(r)$ with radial distance r with choice $V(\infty)=0$. The potential is described by the function



$$V(r)=\begin{cases}(3\ \text{V})\left(\dfrac{7}{5}-\dfrac{r^2}{2\ \text{m}^2}\right);\ 0\ \text{m}\le r<1.0\ \text{m}\\ (3\ \text{V}\cdot\text{m})\left(\dfrac{1}{\text{r}}-\dfrac{1}{10\ \text{m}}\right);\ 1.0\ \text{m}<r<2.0\ \text{m}\\ \left(\dfrac{6}{5}\ \text{V}\right);\ 2.0\ \text{m}<r<2.5\ \text{m}\\ (3\ \text{V}\cdot\text{m})\dfrac{1}{r};\ r>2.5\ \text{m}\end{cases}$$

a) Give the electric field vector $\vec{\mathbf{E}}$ for each of the four regions in (i) to (iv) below?

Solution: We shall the fact that $\vec{\mathbf{E}}=-\vec{\nabla}V$. Since the electric potential only depends on the radial distance r, we have that the radial component of the electric field is given by

$$E_r=-\frac{dV}{dr}.$$

The electric field vector is then given by

$$\vec{\mathbf{E}}=E_r\hat{\mathbf{r}}=-\frac{dV}{dr}\hat{\mathbf{r}}$$

(i) $0\text{ m} \le r < 1.0\text{ m}$

$$\vec{\mathbf{E}} = -\frac{dV}{dr}\hat{\mathbf{r}} = -\frac{d}{dr}\left((3\text{ V})\left(\frac{7}{5} - \frac{r^2}{2\text{ m}^2}\right)\right)\hat{\mathbf{r}} = \left(3\frac{\text{V}}{\text{m}^2}\right) r\,\hat{\mathbf{r}}.$$

Note that the radial variable has units of meters, so the value of the electric field at a point just inside $r = 1.0\text{ m}$ is given by

$$\vec{\mathbf{E}}^- = \left(3\frac{\text{V}}{\text{m}^2}\right)(1.0\text{ m})\,\hat{\mathbf{r}} = 3\frac{\text{V}}{\text{m}}\hat{\mathbf{r}}$$

Note that the component of the electric field, $3\dfrac{\text{V}}{\text{m}}$, has the correct units .

(ii) $1.0\text{ m} < r < 2.0\text{ m}$

$$\vec{\mathbf{E}} = -\frac{dV}{dr}\hat{\mathbf{r}} = -\frac{d}{dr}\left((3\text{ V}\cdot\text{m})\left(\frac{1}{\text{r}} - \frac{1}{10\text{ m}}\right)\right)\hat{\mathbf{r}} = (3\text{ V}\cdot\text{m})\frac{1}{\text{r}^2}\hat{\mathbf{r}}$$

At r^- just less than 2.0 m, the component of the electric field is given by $\dfrac{3}{4}\dfrac{\text{V}}{\text{m}}$ and has the correct units.

(iii) $2.0\text{ m} < r < 2.5\text{ m}$

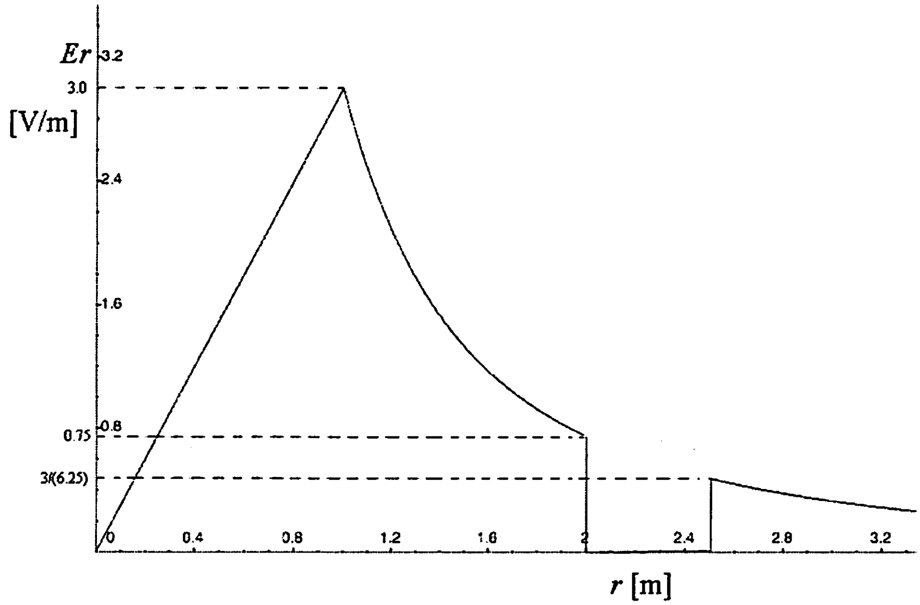
$$\vec{\mathbf{E}} = -\frac{dV}{dr}\hat{\mathbf{r}} = -\frac{d}{dr}\left(\left(\frac{6}{5}\text{ V}\right)\right)\hat{\mathbf{r}} = \vec{\mathbf{0}}$$

(iv) $r > 2.5\text{ m}$

$$\vec{\mathbf{E}} = -\frac{dV}{dr}\hat{\mathbf{r}} = -\frac{d}{dr}\left((3\text{ V}\cdot\text{m})\frac{1}{r}\right)\hat{\mathbf{r}} = (3\text{ V}\cdot\text{m})\frac{1}{\text{r}^2}\hat{\mathbf{r}}$$

At r^- just greater than 2.5 m, the component of the electric field is given by $\dfrac{3}{(2.5)^2}\dfrac{\text{V}}{\text{m}}$ and has the correct units.

b) On the axes below, sketch the magnitude of the radial component of the electric field, E_r, as a function of r. Make sure you label the axes to indicate the numeric magnitude of the field.



c) Qualitatively describe the distribution of charges that gives rise to this potential landscape and hence the electric fields you calculated. That is, where are the charges, what sign are they, what shape are they (solid, shell…)?

Solution: The symmetry is radial so the charges are spherical symmetric distributions. For the regions:

(i) $0\ \text{m} \le r < 1.0\ \text{m}$ **: The electric field grows linear with radial distance so this corresponds to a sphere of uniform charge density** ρ**.**

(ii) $1.0\ \text{m} < r < 2.0\ \text{m}$ **: The electric field falls off like** $1/r^2$**, so there is no charge in this region.**

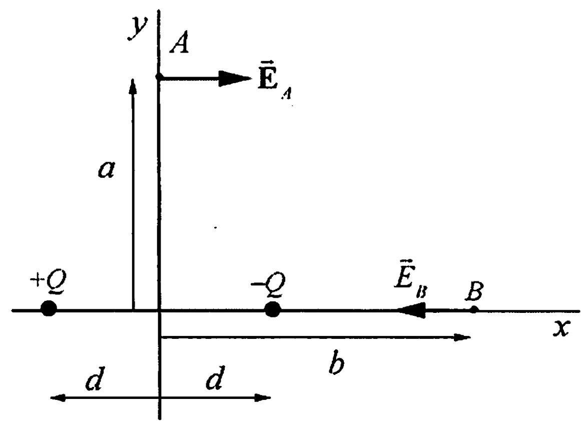
At $r = 2.0\ \text{m}$**, there is a discontinuity in the electric field corresponding to a very thin spherical shell of negative charge and at** $r = 2.5\ \text{m}$**, there is a discontinuity in the electric field corresponding to a very thin spherical shell of positive charge**

(iii) $2.0\ \text{m} < r < 2.5\ \text{m}$ **: Since the electric field is zero, and this region is bounded by two shells of opposite surface charge (negative inside, positive outside), this is a conductor. It could also just be open space with shells on either side.**

(iv) $r > 2.5\ \text{m}$ **: The electric field falls off like** $1/r^2$**, so there is no charge in this region.**

Problem 2 (25 points)

Two charges $+Q$ and $-Q$ lie along the x-axis and are separated by a distance $2d$.



(a) Calculate the total electric field $\vec{\mathbf{E}}$ at position A, a distance a from the x-axis. Indicate its direction on the sketch (draw an arrow)

$$\vec{\mathbf{E}}_A = k\frac{Q}{a^2+d^2}\left(\frac{d}{(a^2+d^2)^{1/2}}\hat{i}+\frac{a}{(a^2+d^2)^{1/2}}\hat{j}\right)+k\frac{Q}{a^2+d^2}\left(\frac{d}{(a^2+d^2)^{1/2}}\hat{i}-\frac{a}{(a^2+d^2)^{1/2}}\hat{j}\right)$$

$$\vec{\mathbf{E}}_A = k_e\frac{2Qd}{(a^2+d^2)^{3/2}}\hat{\mathbf{i}} = \frac{1}{4\pi\varepsilon_0}\frac{2Qd}{(a^2+d^2)^{3/2}}\hat{\mathbf{i}}$$

See sketch for arrow at point A.

(b) Calculate the total electric field $\vec{\mathbf{E}}$ at position B, a distance b from the y-axis. Indicate its direction on the sketch (draw an arrow)

$$\vec{\mathbf{E}}_B = k_e\left(-\frac{Q}{(b-d)^2}+\frac{Q}{(b+d)^2}\right)\hat{\mathbf{i}} = \frac{1}{4\pi\varepsilon_0}\left(-\frac{Q}{(b-d)^2}+\frac{Q}{(b+d)^2}\right)\hat{\mathbf{i}}$$

See sketch for arrow at point B.

Now, assuming that the electric potential is defined to be zero at infinity,

(c) Find the electric potential V at position A.

$$V(A)-V(\infty)=V(A)=-k_e\frac{Q}{(a^2+d^2)^{1/2}}+k_e\frac{Q}{(a^2+d^2)^{1/2}}=0$$

(d) Find the electric potential V at position B.

$$V(B)-V(\infty)=V(B)=k_e\left(-\frac{Q}{b-d}+\frac{Q}{b+d}\right)=\frac{1}{4\pi\varepsilon_0}\left(-\frac{Q}{b-d}+\frac{Q}{b+d}\right)$$

(e) A negatively charged dust particle with mass m and charge $-q$ is released from rest at point B. In what direction will it accelerate (circle the correct answer)?

LEFT RIGHT UP DOWN It Won't Accelerate

Solution: The negatively charged dust particle will accelerate to the right because the charge $-Q$ is closer to the dust particle and hence exerts a greater repulsive force than the positive charge $+Q$ which exerts a smaller attractive force because it is further away from the dust particle.

(f) Again, assuming the negatively charged dust particle was released from rest, what will its speed be after it has traveled a distance s from its original position at point B? HINT: Don't try to simplify any fractions

Solution: Assuming that there are no other forces acting on the dust particle, the change in potential energy of the dust particle as it moves from point B to a point a distance s from B to the right along the x-axis, is given by

$$\Delta U \equiv U(final)-U(B)=-q(V(final)-V(B))$$

The final electric potential is given by

$$V(final)=k_e\left(-\frac{Q}{b+s-d}+\frac{Q}{b+s+d}\right).$$

So the change in potential energy is

$$\Delta U=-q\left(k_e\left(-\frac{Q}{b+s-d}+\frac{Q}{b+s+d}\right)-k_e\left(-\frac{Q}{b-d}+\frac{Q}{b+d}\right)\right)$$

From conservation of energy

$$\Delta K+\Delta U=0.$$

Since it was released from rest at point B, $K(B)=0$, so the change in kinetic energy is

$$\Delta K \equiv K(final)=\frac{1}{2}m{v_f}^2$$

From conservation of energy $\Delta K + \Delta U = 0$ implies that $\Delta K = -\Delta U$, so

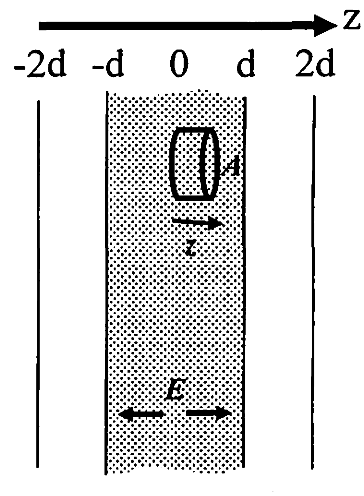
$$\frac{1}{2}mv_f^{\,2} = -\Delta U = q\left(k_e\left(-\frac{Q}{b+s-d}+\frac{Q}{b+s+d}\right)-k_e\left(-\frac{Q}{b-d}+\frac{Q}{b+d}\right)\right)$$

The final speed is thus

$$v_f = \sqrt{\frac{2qk_e}{m}\left(\left(-\frac{Q}{b+s-d}+\frac{Q}{b+s+d}\right)-\left(-\frac{Q}{b-d}+\frac{Q}{b+d}\right)\right)}$$

Problem 3 (25 points):
Consider a semi-infinite **non-conducting** charged slab of thickness $2d$ centered on the xy-plane (filling all space between $z = -d$ and $z = d$). It has a positive, linear charge distribution

$$\rho(z) = \rho_0 |z|/d .$$



Sandwiching this charged slab are two semi-infinite neutral conductors, each of thickness d, located between $z = -2d$ and $z = -d$ and between $z = d$ and $z = 2d$. The left hand side of the sandwich (at $z = -2d$) is held at $V = 0$. Given this arrangement, calculate the electric potential V at two locations: $z = 0$ and $z = +2d$.

NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!) To make our lives easier (and your grade higher) you should outline how you plan to approach the problem.

Solution: We need to calculate the electric potential. We will do this by calculating the electric field using Gauss's Law and then integrating to find the potential difference. NOTE: Some tried to treat the system as a collection of point charges and integrate the potential from each of them. This is incredibly difficult and definitely not the way to proceed.

Before launching into detailed calculations it helps to get a conceptual picture of the situation. The charged slab is positive and symmetric. So it is going to make an electric field that points outward (to the left on the left, to the right on the right). By symmetry, the E field will be zero at the center. This corresponds to a positive potential that is peaked at the center (z=0) and falls as you move outward. What do the conductors do? They set the electric field to zero and hence keep the potential constant.

<u>Calculate the Electric Field</u>

We know the potential is zero at $z = -2d$. That means that the potential is also zero at $z = -d$ (a conductor is an equipotential surface). Since we want the potential at $z = 0$ and at $z = 2d$ (which is the same as at $z = d$) we need to know the field everywhere in the slab. We do NOT need to calculate it anywhere else. So, we use the Gaussian Pillbox pictured above, with one surface at z=0 (where the field is zero) and the other surface at z, where we want to calculate the E field. The pillbox has a cross sectional area of A (note that there is NO reason to give the radius of the pillbox, we just need its area).

By Gauss's Law:

$$\oiint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E(z)A = \frac{Q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\iiint \rho dV = \frac{1}{\varepsilon_0}\int_{z'=0}^{z} \rho_0 \frac{z'}{d} A dz' = \frac{1}{\varepsilon_0}\rho_0 \frac{z^2}{2d} A$$

$$\boxed{E(z) = \frac{\rho_0}{\varepsilon_0}\frac{z^2}{2d}}$$

Note that we already determined the direction above (to the right on the right, to the left on the left), so although there is no reason to include any sign here when we integrate to find the potential we have to be careful to have E negative on the left and positive on the right.

Calculate the Potential

To find the potential we need to integrate the electric field from where we know the potential to where we want to know it. It was defined at $z = -2d$, then the equipotential conductor lets us work from $z = -d$:

$$\Delta V = V(0) - \underbrace{V(-d)}_{0} = -\int_{z=-d}^{0} E(z)dz = -\int_{z=-d}^{0} -\frac{\rho_0}{\varepsilon_0}\frac{z^2}{2d}dz = \frac{\rho_0}{\varepsilon_0}\frac{z^3}{6d}\bigg|_{-d}^{0} = \boxed{\frac{\rho_0}{\varepsilon_0}\frac{d^2}{6} = V(0)}$$

To find V($z = 2d$) we could integrate from 0 to d, but the easier way to determine it is by symmetry. Since the field down on the right is the same as the field down on the left the potential difference must be the same, hence $\boxed{V(z = 2d) = 0}$.

Common Mistakes

People made a wide array of mistakes on this problem. Conceptually many people didn't realize that the conductors had zero field in them and hence were equipotential objects. Also conceptually, people didn't realize that you need E(z) in order to integrate and find the potential. Instead they found the electric field at a specific point, for example E(0) or E(d) and then "integrated" that formula. This is a major misunderstanding – to get from electric field to potential or vice versa you need to know how they behave as a function of position. In particular, a number of people said that because E(0) = 0 that V(0) must also equal zero. This is nonsense. Just because a surface is locally flat (E=0) doesn't mean that it has to be at zero height (V=0). The floor, a table and the ceiling all are flat but have different heights. its flat, but you don't know where

Calculating the E field with Gauss's Law was very difficult for people even if they knew they wanted it as a function of position. Don't forget to put the endcaps of the pillbox where you want to calculate the field. So if you want it in the slab, don't draw the pillbox with the endcaps outside of the slab. Some people tried to put one of the endcaps in the conductor, not a bad idea since E = 0 there. Unfortunately, there is a surface charge distribution at the conductor/slab interface, so this requires an extra step to calculate if you want to go this way. Much easier just to work from the slab center as we did above. Finally, when determining the charge enclosed you can't just multiply the formula for the charge density by the volume enclosed, you must integrate since it is non-uniform.

Please Remove this Tear Sheet from Your Exam

$$\vec{\mathbf{E}} = \frac{q}{4\pi\varepsilon_o r^2}\hat{\mathbf{r}} = \frac{q}{4\pi\varepsilon_o r^3}\vec{\mathbf{r}}$$

$\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}}}{r}$ *points* **from** *source q* **to** *observer*

$$\oiint_{closed\ surface} \kappa \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{Q_{\text{inside-free}}}{\varepsilon_o}$$

$d\vec{\mathbf{A}}$ points from inside to outside

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

$$\Delta V_{moving\ from\ a\ to\ b} = V_b - V_a = -\int_a^b \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}$$

$$\oint_{closed\ path} \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = 0$$

$$W = \Delta U = q\Delta V$$

$$V_{\text{point charge}} = \frac{q}{4\pi\varepsilon_o r}$$

$$U = \sum_{all\ pairs} \frac{q_i q_j}{4\pi\varepsilon_o |\vec{\mathbf{r}}_i - \vec{\mathbf{r}}_j|}$$

$$U = \oiiint \left[\frac{1}{2}\varepsilon_o E^2\right] dV_{vol}$$

$$E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}$$

$E_r = -\frac{\partial V}{\partial r}$ for spherical symmetry

$$C = \frac{|Q|}{|\Delta V|} \qquad U = \frac{1}{2}C\Delta V^2 = \frac{Q^2}{2C}$$

$$C_{parallel} = C_1 + C_2 \qquad C_{series} = \frac{C_1 C_2}{C_1 + C_2}$$

Circumferences, Areas, Volumes:

Circle of radius r:
- Area = πr^2
- Circumference = $2\pi r$

Sphere of radius r
- Surface Area = $4\pi r^2$
- Volume = $\frac{4}{3}\pi r^3$

Cylinder of radius r and height h
- Side surface area = $2\pi r h$
- End cap surface area = $2\pi r^2$
- Volume = $\pi r^2 h$

Definition of trig functions

sin is opposite/hypotenuse;
cos is adjacent/hypotenuse;
tangent is opposite over adjacent;

Properties of 30, 45, and 60 degrees
($\pi/6$, $\pi/4$, and $\pi/3$ radians):
$\sin(\pi/6) = \cos(\pi/3) = 1/2$,
$\sin(\pi/3) = \cos(\pi/6) = \sqrt{3}/2$;
$\sin(\pi/4) = \cos(\pi/4) = 1/\sqrt{2}$;

Integrals that may be useful

$$\int_a^b dr = b - a$$

$$\int_a^b \frac{1}{r}dr = \ln(b/a)$$

$$\int_a^b \frac{1}{r^2}dr = \left(\frac{1}{a} - \frac{1}{b}\right)$$

Some potentially useful numbers

$$k_e = \frac{1}{4\pi\varepsilon_0} = 9\times 10^9 \frac{\text{N m}^2}{\text{C}^2}$$

Breakdown of air	$E \sim 3 \times 10^6$ V/m
Speed of light	$c = 3 \times 10^8$ m/s
Electron charge	$e = 1.6 \times 10^{-19}$ C
Avogadro's number	$N_A = 6.02 \times 10^{23}$ mol^{-1}

Guass's law tips

1. put Guassian surface where want to know or know field

2. integrate non uniform ρ λ σ

3. $\vec{E}$ field is a vector write w/ $d\vec{r}$

2/25

MIT PHYSICS DEPARTMENT

I don't like to see dates passing - but really making the most of it

8.02 Exam One Spring 2009

FAMILY (last) NAME

GIVEN (first) NAME

Student ID Number

Your Section:

___L01 MW 9 am ____L02 MW 11 am ____L03 MW 1 pm ____L04 MW 3 pm

___L05 TR 9 am ____L06 TR 11 am ____L07 TR 1 pm ____L08 TR 3 pm

Your Group (e.g. 10A): ______________

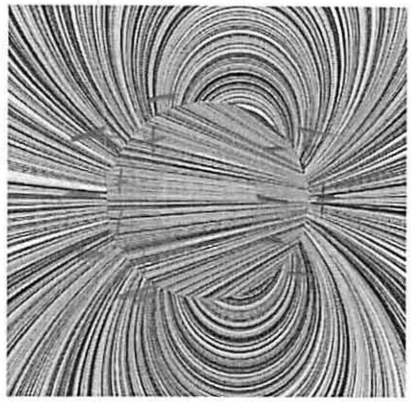
Problem	**Score**	**Grader**
1 (15 points)		
2 (10 points)		
3 (25 points)		
4 (25 points)		
5 (25 points)		
TOTAL		

hudson, this not a useful sample exam

Cleared a lot of things up + learned a lot while studying

Problem 1: Five Short Questions. Circle your choice for the correct answer.

Question A (3 points out of 15 points):



A positive charge is brought near a neutral, spherical, conducting shell, causing the shell to generate an electric field with the grass seed representation at right. The combination of this induced electric field and the field of the charge itself combine together to make a **potential landscape** best represented in which of the figures below (with streaks parallel to equipotentials)?

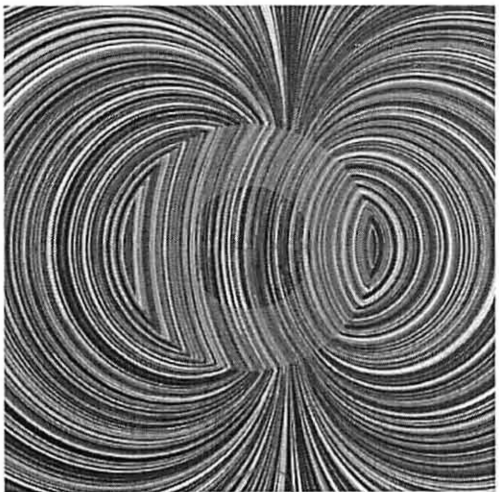
So this goes from

High # near red ⊕

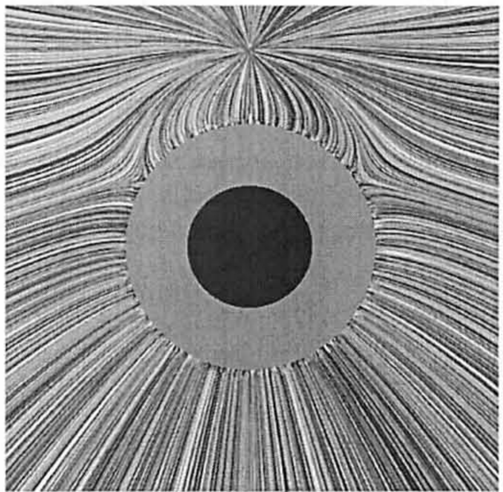
low # near the ⊖

circle around charge

I

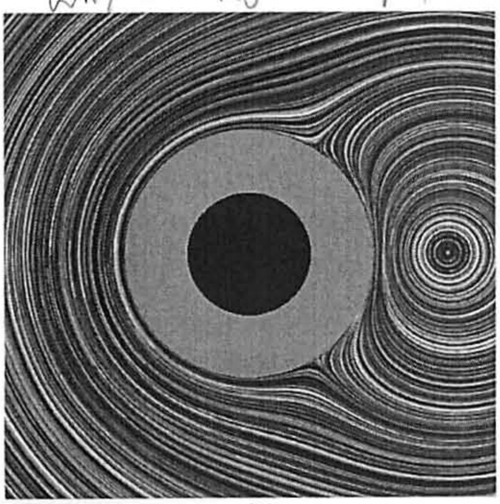


(1)

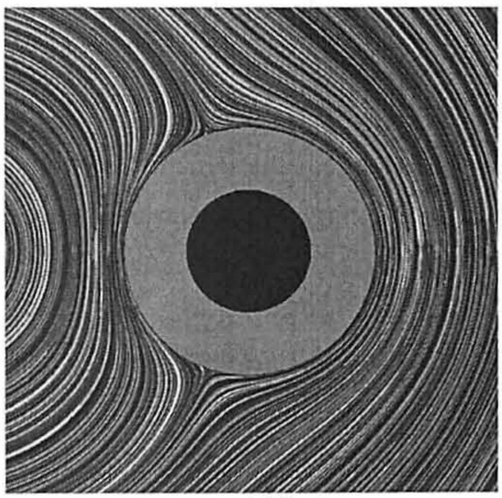


~~(2)~~

why is it lighter but



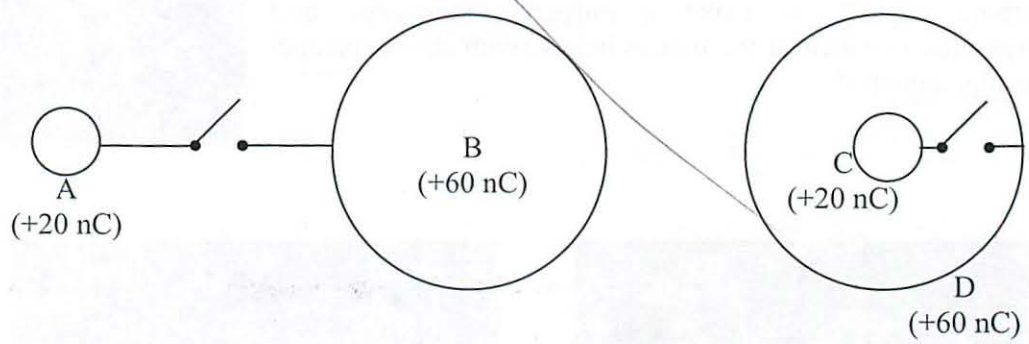
(3)



(4)

Question B (3 points out of 15 points):

Two pairs of hollow, spherical conducting shells are connected with wires and switches. The system AB (where A and B are far apart) is extremely far from CD. In both systems the large shells have four times the radius of the small shells. ***Before the switches are closed*** each pair has a charge of +20 nC on the small shell (A,C) and +60 nC on the large shell (B,D)

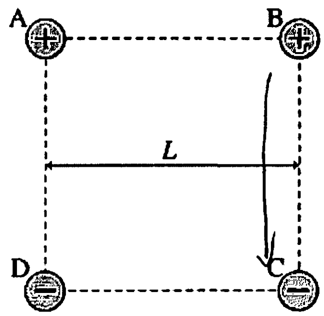


When the switches are closed, charge is free to flow along the conducting wires connecting the spherical shells. What is the rank of the magnitude of the charge on each of the shells after the switches have been closed?

1. $Q_A = Q_B < Q_C = Q_D$
2. $Q_C = Q_D < Q_A = Q_B$
3. $Q_A = Q_C < Q_B = Q_D$
4. $Q_B = Q_D < Q_A = Q_C$
5. $Q_A < Q_C < Q_D < Q_B$
6. $Q_B < Q_D < Q_C < Q_A$
7. $Q_C < Q_A < Q_B < Q_D$
8. $Q_C < Q_B < Q_A < Q_D$
9. The charge is the same for all four shells
10. The ranking of the electric charge cannot be determined or is none of the above

Skip

Question D (3 points out of 15 points):



Four charges of equal magnitude (two positive and two negative) are placed on the corners of a square as pictured at left (positive charges at positions A & B, negative at positions C & D).

You decide that you want to swap charges B & C so that the charges of like sign will be on the same diagonal rather than on the same side of the square.

In order to do this, you must do…

1. … positive work
2. … negative work
3. … no net work
4. … an indeterminate amount of work, as it depends on exactly how you choose to move the charges.

b will be attracted
C " " "

So — work

happens automaticly

Question E (3 points out of 15 points):
Two uncharged conductors, A and B, are of different sizes. They are charged as follows:

1. A is charged from an electrostatic generator to charge q.
2. A is briefly touched to B.
3. Steps 1 and 2 are repeated until the charge on B reaches a maximum value.

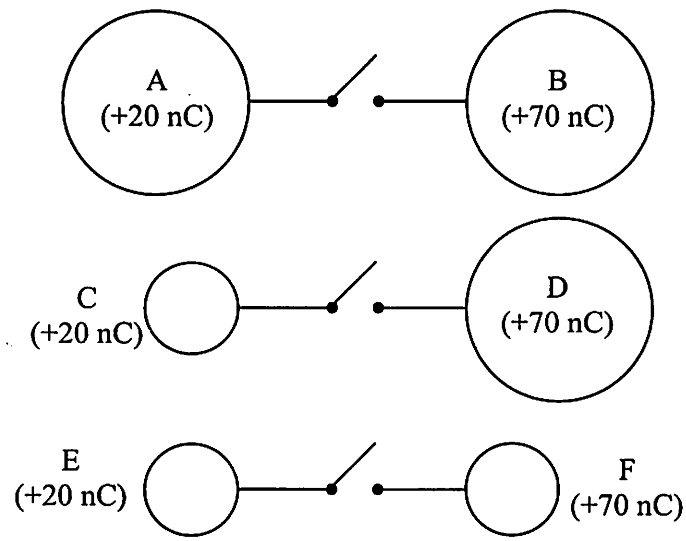
If the **final** charge on **B** is $3q$, what was the charge on **A** after the **first** time it touched B?

1. $5q/6$
2. $3q/4$
3. $2q/3$
4. $q/2$
5. $q/3$
6. $q/4$

Skip

Question C (3 points out of 15 points):

Three pairs of charged conducting spheres are connected with wires and switches. The spheres are all very far apart. The large spheres have twice the radius of the small spheres. Each sphere on the left has a charge of +20 nC and each sphere on the right has a charge of +70 nC before the switches are closed.



All of the switches are closed and then, after a long time, opened again. Which of the below rankings of the electric potentials on four of these spheres (A, C, D, E) is correct?

1. $V_A < V_C = V_D < V_E$
2. $V_E < V_C = V_D < V_A$
3. $V_A = V_D < V_C = V_E$
4. $V_C = V_E < V_A = V_D$
5. $V_A < V_D < V_C < V_E$
6. $V_E < V_C < V_D < V_A$
7. The electric potential is the same for all four spheres (A, C, D & E)
8. The ranking of the electric potential cannot be determined or is none of the above

skip

Problem 2: Back of the Envelope Calculation (10 Pts)

Don't let the sparks fly!

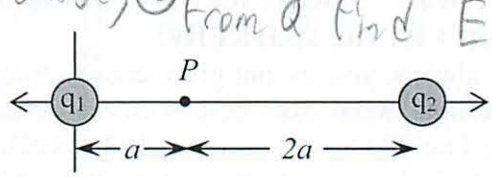
As always, you are not given enough information to exactly determine the answer to this question. Make your best estimates for unknowns, clearly indicating what your estimates are (e.g. Radius R ~ ....) NO CREDIT will be given for simply guessing a final numerical answer from scratch. It must be properly motivated (i.e. write equations, use words!)

Rub a balloon on your head and you can stick it to the ceiling. Using electrostatic arguments (i.e. not just experience), what is the largest mass you could support from the balloon?

no back of envlope
calcs

Problem 3: Charges (25 points) ① pt charge, ② from Q find E then V

Two point-like particles with charges q_1 and q_2 sit fixed on the x-axis at the origin and $x = 3a$ respectively. A point P lies on the x-axis between the two charges at $x = a$ (as pictured at right).



③ ~~Field~~ from V find E then Q

(a) What is the electric field $\vec{\mathbf{E}}$ at point P due to these two charges?

E = superposition colombs law $\frac{r}{r^3} = \frac{1}{r^2}$

- only in $\hat{i}$ direction

$\vec{E}$

$$\frac{kq_1}{(a)^2} - \frac{kq_2}{(2a)^2}$$ ✓ = good

Can simplify

$$k\left(\frac{q_1}{a^2} - \frac{q_2}{4a^2}\right)$$

one more

simplify

$$\frac{k}{a^2}\left(q_1 - \frac{q_2}{4}\right)$$

(b) What is the electric potential V at point P due to these two charges, assuming that potential is defined such that $V = +10$ V at infinity?

$$V = V(P) - V(\infty) = -\int E\,ds - 10$$

remember - visualize

$$-\int \frac{k}{a^2}\left(q_1 - \frac{q_2}{4}\right)da + 10$$

they used un simplified - better

$$-k \int \frac{1}{a^2} \cdot \cancel{\left(q_1 - \frac{q_2}{4}\right)}$$

~~chain rule~~

a^{-2}

$-2a^{-1}$

$-\frac{2}{a}$

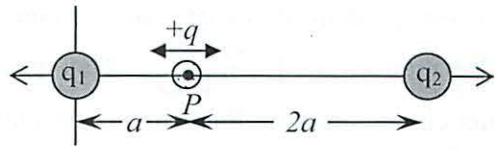
$$+\frac{2k}{a}\left(q_1 - \frac{q_2}{4}\right) - 10$$

$$\boxed{\frac{kq_1}{a} + \frac{kq_2}{2a} + 10}$$

Problem 3 *continued*: Charges

A third particle of mass m and known positive charge $+q$ is now brought in from infinity to point P. Once there it is confined to move on the x-axis, and it is found that if it is slightly displaced from point P that it will undergo simple harmonic motion ("small oscillations") about point P due to the electric forces from the first two charges (i.e. ignore gravity).

work done



I am kinda getting them right

– but centered at pt P?

(c) From the above information, what can you determine about the signs and magnitudes of the two charges q_1 and q_2? For ease of notation, assume that the magnitude of q_1 is Q, where Q is a positive number (so if q_1 were negative you would write $q_1 = -Q$). **Explain** how you know this!

$q_1 = -2Q$ ← but this is based of t

$q = Q$

$\frac{1}{r^3}$ ← For pt charge

So not that simple

but what is full eq for pt charge feel $F = qE$

So what plug in

$F_2 = q\,k\left(\frac{q_1}{a^2} - \frac{q_2}{4a^2}\right)$

↑ but how know this?

Field in middle must $= 0$ ← duh should have thought of

and one must be ⊕ other ⊕ to be in middle (again duh)

(d) How much energy did it take to move the charge $+q$ from infinity in to P? Briefly explain the meaning of the sign of this energy (whether positive, negative or zero).

define $q_1 = Q$ $q_2 \Rightarrow q_1 - \frac{q_2}{4} = 0$ $q_2 = 4Q$

Look at eqn sheet

$W = \Delta U = q\Delta V$

$\Delta U = q\left(\frac{kq_1}{a} + \frac{kq_2}{2a} + \cancel{k0}\right)$

difference

$= q\left(\frac{kQ}{a} + \frac{k4q}{2a} + \cancel{k0}\right)$

Simplify

$\boxed{W = \frac{3kqQ}{a}}$

oh the long problem

Problem 4: A Sphere's Potential (25 points)

A spherical insulator of radius a contains a non-uniform spherically symmetric charge distribution $\rho(r) = Q/4\pi a r^2$. This insulating sphere is wrapped with a conductor with no net charge, inner radius a and outer radius $3a$, as pictured below. The zero of potential is defined on the midline of the conductor. That is, $V(r = 2a) = 0$. This is simply a definition – the conductor is NOT grounded. Although these two objects are in contact, the charge on the insulator is firmly attached and none will "leak into" the conductor.

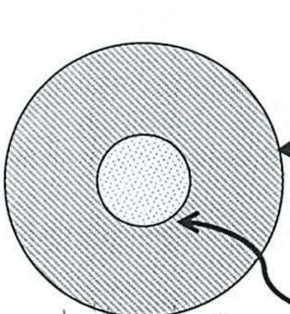
non uniform

$E = 0$

Skip conductor

NOTE: This problem will be graded on your **displayed understanding** of what you are doing. SHOW WORK. Answers without explanations (in words, clear derivations and pictures) will receive no credit. If you can intuit a part of the answer, for example using symmetry, it is fine to state the result without mathematical derivation but **justify it with a brief statement**.

(a) Calculate the electric potential V throughout space due to this charge distribution
Make sure that your final answer is clearly identified (boxed)



1. Find E
2. Find V

- will be harder since stuff have not learned

will just do ports

E inside →

$$EA = \int \frac{\rho V}{\varepsilon_0}$$

"what to integrate — need to practice

$$E\, 4\pi r^2 = \int \frac{Q\, 4\pi r^3}{4\pi a r^2\, 3}$$

$$E = \int \frac{Q\, \cancel{4\pi r^3}}{4\pi a r^2\, 3\, \cancel{4\pi r^2}}$$

Problem 4 ***continued…*****: A Sphere's Potential**

port 2 - find V (after p 15 + 16)

$V = V(P) - V(2a) = 0$ also I am just doing normal shell

$V = -\int E\,ds = -\int \frac{kq}{ar}\,dr$

$-\frac{kq}{a} \ln\left(\frac{r}{a}\right)\Big|_0^r$

? why a ? - Dumashin review did not do

oh

$\int_a^b \frac{1}{r}\,dr = \ln\left(\frac{b}{a}\right)$ ← know

also know $\int_a^b \frac{1}{r^2}\,dr = \left(\frac{1}{a} - \frac{1}{b}\right)$

(b) A negligibly small hole is drilled in the above conducting sphere down to a radius of $r = 2a$, so that a conducting wire may be attached to the conductor at that radius (where $V = 0$). The other end of the wire is then attached to the outer surface of a solid conducting sphere of radius $2a$, and with no net charge, a very large distance away. Is any charge transferred through this wire? If yes, what sign of charge travels to the smaller conducting sphere and why? If no, why not?

Problem 4 ***continued…*****: A Sphere's Potential**
(Blank page in case space is needed. Problem continues on next page)

$$\oiint E \cdot d\vec{A} = E 4\pi r^2 = \frac{Q_{enc}}{\varepsilon_0} = \underbrace{\frac{1}{\varepsilon_0} \iiint \rho(r') \, dV}_{\text{what I forgot - is how to integrate}}$$

So are $\int$ w/ respect to r' which is what we are adding up
~~to~~ each bigger + bigger sphere - w/ diff ρ

$$= \frac{1}{\varepsilon_0} \int_{r'=0}^{r} \frac{Q}{4\pi a r'^2} 4\pi r'^2 \, dr'$$

$$= \frac{Q}{\varepsilon_0 a} r$$

1. Factor out $\frac{1}{\varepsilon_0}$

2. Integrate ρ as a function of r' w/ respect to volume ← why w/ respect to V?
 - has R as a component

$$E = \frac{kQ}{a r} \hat{r}$$

3. So write it out

$$\int_{r'=0}^{r} \underbrace{\frac{Q}{4\pi a r'^2}}_{\rho(r')} \cdot \underbrace{4\pi r'^2}_{A\,??} \, dr'$$

why this here - not volume? Or is it volume

$$\int \frac{Q \, \cancel{4\pi r'^2}}{\cancel{4\pi a r'^2}} = \frac{Q}{\varepsilon_0 a} r \leftarrow \text{since } \int 1 \, dr = r$$

ε_0 ← add back

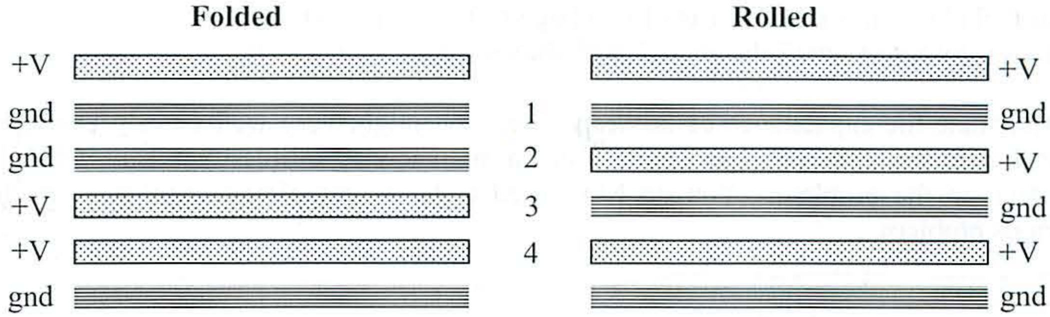
4. From r to 0
top - bottom then divide area

- check Dmaskin reviews

- contined p 15

Note later

Problem 5 ***continued…*****: Parallel Plate Capacitor**
If we zoom in on some arbitrary section of the folded and rolled capacitors we will find:



I have shaded and labeled the subplates to indicate which ones are attached together, but don't show those attachments at the edges (since I'm only looking near the centers of the subplates). I have also numbered the central subplates 1 through 4.

Your job in this problem is to determine which method, if either, makes a better capacitor. You begin by attaching a battery to each capacitor and charging one plate (now folded or rolled) to a potential $+V$ relative to the other plate (denoted "ground" or "gnd").

(b) In terms of this potential $+V$ and the dimensions of the subplates (length ℓ, width w separated by distance d), what is the electric field everywhere between subplates 1 and 4?

Skip

#4 E continued

Let me try integral again

$EA = \int \frac{\rho V}{\epsilon_0}$

area ~~not~~ volume

$E 4\pi r^2 = \frac{1}{\epsilon_0}\int_0^r \frac{Q}{4\pi a r^2} \cdot \cancel{4}\pi r'^2 \underbrace{dr'}_{\text{thickness}}$

$E 4\pi r^2 = \frac{1}{\epsilon_0} \frac{Q \cdot \cancel{4\pi}}{4\pi a \cancel{4}} \cdot \int_0^r \frac{1 r'^2}{r'^2} dr' = r$

$E 4\pi r^2 = \frac{1Qr}{\epsilon_0 a \cancel{4}}$

$E = \frac{Qr}{\epsilon_0 a \cancel{4} \cdot 4\pi r^2} = \frac{kQ}{\cancel{4} r a}$ Close!

Problem 5: Parallel Plate Capacitor (25 pts)

A parallel plate capacitor consists of two large rectangular conductors (length L, width w) held apart by a very small distance d (with dimensions $L >> w >> d$).

(a) Calculate the capacitance of this capacitor. You might very well already know the answer to this – feel free to just write it down, but showing a little work here will help you later in the problem. You do NOT need to be as complete as you were in the previous problem. Skip

#4 E continued - Dumashkin's review session

$$E \cdot 4\pi r^2 = \frac{1}{\epsilon_0}\int \rho \, dV$$

$$\hookrightarrow dV = \underbrace{4\pi r'^2}_{\text{area}} \underbrace{dr}_{\text{thickness of shell}}$$

$$\frac{1}{\epsilon_0}\int_0^r \underbrace{h r'}_{\rho} 4\pi r'^2 \, dr$$

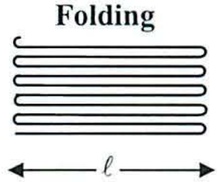
↓ 1 Dimension integral
factor at constants

$$\frac{1}{\epsilon_0} h \, 4\pi \int_0^r r'^3 \, dr$$

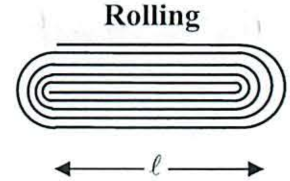
$$\frac{h \, 4\pi r^4}{4} = h r^4$$

This capacitor is too big (spatially) to be of much use for us, so you need to pack it up into a more compact object. Here are two suggested methods for your consideration:

Folding

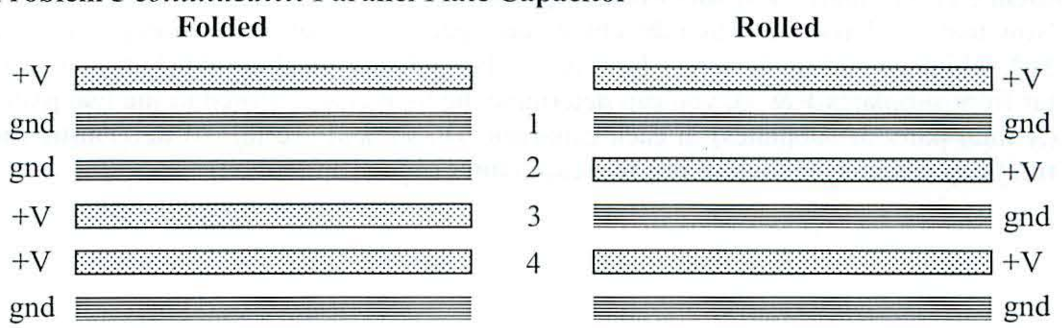


Rolling



Note that these diagrams, in which the pair of plates is drawn as a single line, are NOT to scale and hence are slightly misleading. You will use the same spacing d between neighboring plates as between the plates. Also, because the length of each of the N pairs of "subplates" $\ell \equiv L/N$ is much larger than the spacing d, the rounded edge sections are not nearly as significant as they appear here. **You can ignore the edges.**

Problem 5 *continued…*: Parallel Plate Capacitor



(c) For each of the two capacitors, calculate the charge on each of the four central conducting subplates pictured above (subplates 1-4). Where is that charge located on each of the subplates?

Skip

Problem 5 ***continued…*****: Parallel Plate Capacitor**

(d) Now that you have calculated the charge on a pair of subplates (for example plates 2 & 3, which were directly across from each other in the original unfolded capacitor but far from subplates 1 & 4), you can determine the total charge stored in the two plates (N total pairs of subplates) in each capacitor. Do so, and use this to **determine the total capacitance of each of the two capacitors** (folded and rolled)

skip

(e) Which, if any, of the capacitors (folded, rolled or the original flat one) is best? Why?

skip

Please Remove this Tear Sheet from Your Exam

$$\vec{\mathbf{E}} = \frac{q}{4\pi\varepsilon_o r^2}\hat{\mathbf{r}} = \frac{q}{4\pi\varepsilon_o r^3}\vec{\mathbf{r}}$$

$\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}}}{r}$ *points* **from** *source q* **to** *observer*

$$\oiint_{closed\ surface} \kappa\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{Q_{\text{inside-free}}}{\varepsilon_o}$$

$d\vec{\mathbf{A}}$ points from inside to outside

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

$$\Delta V_{moving\ from\ a\ to\ b} = V_b - V_a = -\int_a^b \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}$$

$$\oint_{closed\ path} \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = 0$$

$$W = \Delta U = q\Delta V$$

$$V_{\text{point charge}} = \frac{q}{4\pi\varepsilon_o r}$$

$$U = \sum_{all\ pairs} \frac{q_i q_j}{4\pi\varepsilon_o\left|\vec{\mathbf{r}}_i - \vec{\mathbf{r}}_j\right|}$$

$$U = \oiiint\left[\frac{1}{2}\varepsilon_o E^2\right]dV_{vol}$$

$$E_x = -\frac{\partial V}{\partial x},\quad E_y = -\frac{\partial V}{\partial y},\quad E_z = -\frac{\partial V}{\partial z}$$

$E_r = -\frac{\partial V}{\partial r}$ for spherical symmetry

$$C = \frac{|Q|}{|\Delta V|} \qquad U = \frac{1}{2}C\,\Delta V^2 = \frac{Q^2}{2C}$$

$$C_{parallel} = C_1 + C_2 \qquad C_{series} = \frac{C_1 C_2}{C_1 + C_2}$$

Circumferences, Areas, Volumes:

Circle of radius r:

Area = πr^2

Circumference = $2\pi r$

Sphere of radius r

Surface Area = $4\pi r^2$

Volume = $\frac{4}{3}\pi r^3$

Cylinder of radius r and height h

Side surface area = $2\pi r h$

End cap surface area = $2\pi r^2$

Volume = $\pi r^2 h$

Definition of trig functions

sin is opposite/hypotenuse;

cos is adjacent/hypotenuse;

tangent is opposite over adjacent;

Properties of 30, 45, and 60 degrees ($\pi/6$, $\pi/4$, and $\pi/3$ radians):

$\sin(\pi/6) = \cos(\pi/3) = 1/2$,

$\sin(\pi/3) = \cos(\pi/6) = \sqrt{3}/2$;

$\sin(\pi/4) = \cos(\pi/4) = 1/\sqrt{2}$;

Integrals that may be useful

$$\int_a^b dr = b - a$$

$$\int_a^b \frac{1}{r}dr = \ln(b/a)$$

$$\int_a^b \frac{1}{r^2}dr = \left(\frac{1}{a} - \frac{1}{b}\right)$$

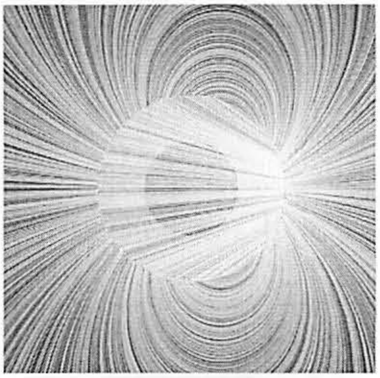
Some potentially useful numbers

$$k_e = \frac{1}{4\pi\varepsilon_0} = 9\times10^9\,\frac{\text{N m}^2}{\text{C}^2}$$

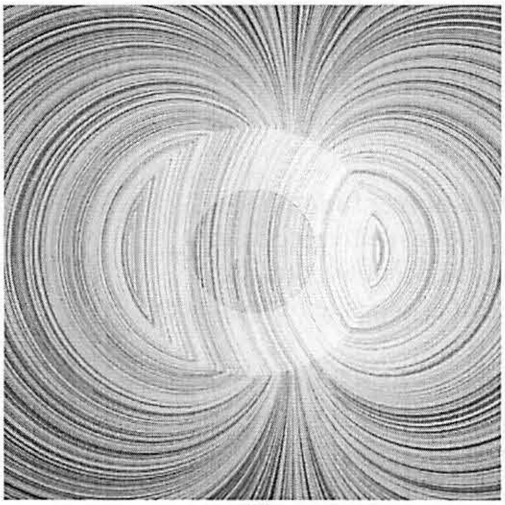
Breakdown of air	$E \sim 3 \times 10^6$ V/m
Speed of light	$c = 3 \times 10^8$ m/s
Electron charge	$e = 1.6 \times 10^{-19}$ C
Avogadro's number	$N_A = 6.02 \times 10^{23}\ \text{mol}^{-1}$

Problem 1: Five Short Questions. Circle your choice for the correct answer.

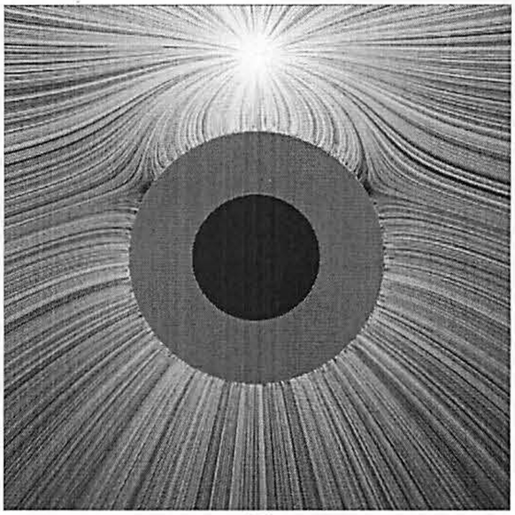
Question A (3 points out of 15 points):



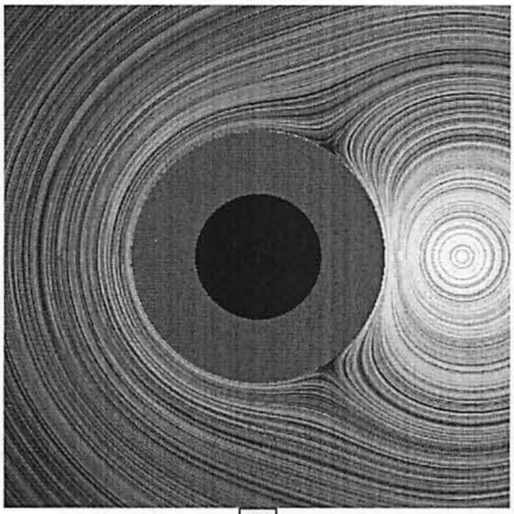
A positive charge is brought near a neutral, spherical, conducting shell, causing the shell to generate an electric field with the grass seed representation at right. The combination of this induced electric field and the field of the charge itself combine together to make a **potential landscape** best represented in which of the figures below (with streaks parallel to equipotentials)?



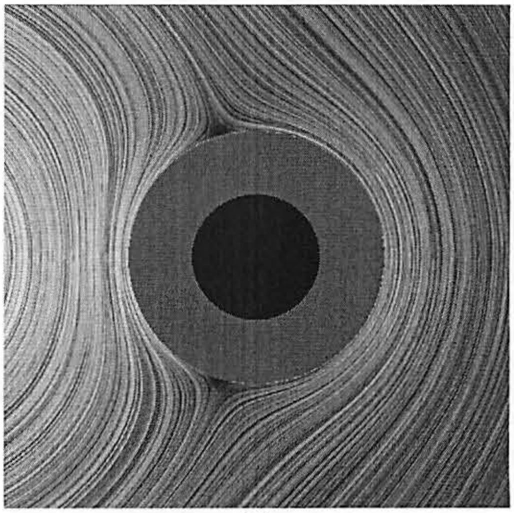
(1)



(2)



$\boxed{3}$

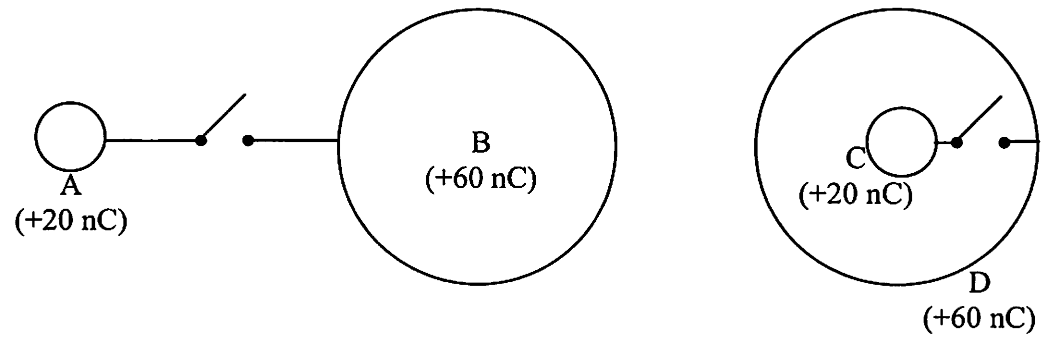


(4)

The positive charge is to the right of the shell (field is stronger there and diverges from there). So equipotentials surround it, as pictured.

Question B (3 points out of 15 points):

Two pairs of hollow, spherical conducting shells are connected with wires and switches. The system AB (where A and B are far apart) is extremely far from CD. In both systems the large shells have four times the radius of the small shells. ***Before the switches are closed*** each pair has a charge of +20 nC on the small shell (A,C) and +60 nC on the large shell (B,D)



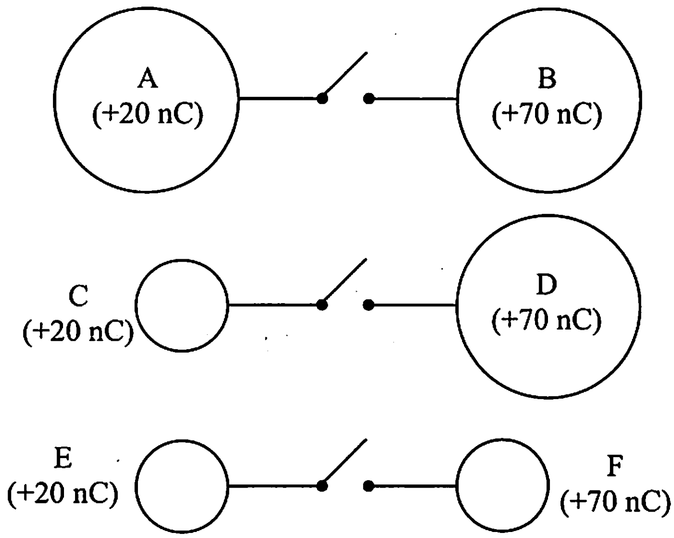
When the switches are closed, charge is free to flow along the conducting wires connecting the spherical shells. What is the rank of the magnitude of the charge on each of the shells after the switches have been closed?

1. $Q_A = Q_B < Q_C = Q_D$
2. $Q_C = Q_D < Q_A = Q_B$
3. $Q_A = Q_C < Q_B = Q_D$
4. $Q_B = Q_D < Q_A = Q_C$
5. $Q_A < Q_C < Q_D < Q_B$
6. $Q_B < Q_D < Q_C < Q_A$
7. $\boxed{Q_C < Q_A < Q_B < Q_D}$
8. $Q_C < Q_B < Q_A < Q_D$
9. The charge is the same for all four shells
10. The ranking of the electric charge cannot be determined or is none of the above

When connected, the shells will come to the same potential. In the CD system all charge will leave the C conductor and flow to D (that is how it gets far away from itself). In the AB system more charge will be on B than on A because kQ/r must be the same for the two.

Question C (3 points out of 15 points):

Three pairs of charged conducting spheres are connected with wires and switches. The spheres are all very far apart. The large spheres have twice the radius of the small spheres. Each sphere on the left has a charge of +20 nC and each sphere on the right has a charge of +70 nC before the switches are closed.

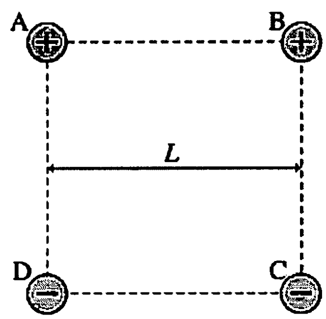


All of the switches are closed and then, after a long time, opened again. Which of the below rankings of the electric potentials on four of these spheres (A, C, D, E) is correct?

1. $V_A < V_C = V_D < V_E$
2. $V_E < V_C = V_D < V_A$
3. $V_A = V_D < V_C = V_E$
4. $V_C = V_E < V_A = V_D$
5. $V_A < V_D < V_C < V_E$
6. $V_E < V_C < V_D < V_A$
7. The electric potential is the same for all four spheres (A, C, D & E)
8. The ranking of the electric potential cannot be determined or is none of the above

Again, when connected the spheres will come to the same potential. For AB and EF this means the same charge (45 nC) will be on each sphere. For CD less charge will be on C than on D (you can calculate it will be half as much, 30 nC on C, 60 nC on D, but that isn't necessary to answer the question).

Question D (3 points out of 15 points):



Four charges of equal magnitude (two positive and two negative) are placed on the corners of a square as pictured at left (positive charges at positions A & B, negative at positions C & D).

You decide that you want to swap charges B & C so that the charges of like sign will be on the same diagonal rather than on the same side of the square.

In order to do this, you must do…

1. … positive work
2. … negative work
3. … no net work
4. … an indeterminate amount of work, as it depends on exactly how you choose to move the charges.

By moving like sign charges farther apart and opposite sign charges closer together you reduce the energy of the system

Question E (3 points out of 15 points):
Two uncharged conductors, A and B, are of different sizes. They are charged as follows:

1. A is charged from an electrostatic generator to charge q.
2. A is briefly touched to B.
3. Steps 1 and 2 are repeated until the charge on B reaches a maximum value.

If the **final** charge on **B** is $3q$, what was the charge on **A** after the **first** time it touched B?

1. $5q/6$
2. $3q/4$
3. $2q/3$
4. $q/2$
5. $q/3$
6. $q/4$

Charge is proportional to potential. For simplicity write $V_A = a\,q_A; V_B = b\,q_B$
In the end, the charge on A is $q_A = q$.
So $a/b = q_B/q_A = 3$.
So the first time, for their potentials to be equal, $a\,q_A = b\,(q-q_A)$

Problem 2: Back of the Envelope Calculation (10 Pts)

Don't let the sparks fly!

As always, you are not given enough information to exactly determine the answer to this question. Make your best estimates for unknowns, clearly indicating what your estimates are (e.g. Radius R ~ ....) NO CREDIT will be given for simply guessing a final numerical answer from scratch. It must be properly motivated (i.e. write equations, use words!)

Rub a balloon on your head and you can stick it to the ceiling. Using electrostatic arguments (i.e. not just experience), what is the largest mass you could support from the balloon?

As always for these types of problems, there are many ways to address this. Conceptually, rubbing the balloon puts some charge on it. When brought near the ceiling there will be an image charge on the ceiling and thus they attract. The problem is that we need some numbers.

We do know the biggest possible field between the balloon and the wall – the breakdown field of air, 3 x 10^6 V/m. If we pretend that the balloon is one plate of a parallel plate capacitor (not a terrible abstraction given that the balloon is very close to the ceiling) then we can write:

$$F \sim qE = \sigma AE \sim \varepsilon_0 EAE = \varepsilon_0 AE^2$$

This isn't quite right: in a parallel plate capacitor the force on one plate is half what I have written here since half the electric field comes from each plate, and a plate can't exert a net force on itself. But factors of two aren't relevant for these kinds of problems. So now we just plug in some numbers. I'll rewrite in terms of k, since I know that:

$$F \sim \frac{(4\pi\varepsilon_0)AE^2}{4\pi} \sim \frac{\left(9\times10^9\ \mathrm{Nm^2C^{-2}}\right)^{-1}\left(10^{-2}\mathrm{m^2}\right)\left(3\times10^6\mathrm{NC^{-1}}\right)^2}{10} = 1\,\mathrm{N}$$

Here I've used an area on 10 cm square. This is probably a little on the big side, but we want a maximum mass, so that's ok. Now I just need to convert this to mass using gravity:

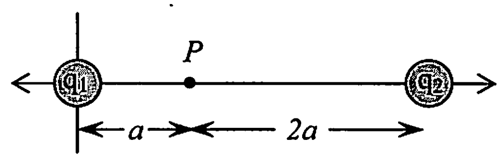
$$m = \frac{F}{g} \sim \frac{1\,\mathrm{N}}{10\,\mathrm{ms^{-2}}} = \boxed{0.1\ \mathrm{kg}}$$

Reality check: 100 g? Seems reasonable!

NOTE: I was stunned by how many students simply guessed at how much charge was on the balloon. The guesses ranged from 1 e^- to 10^5 C. 100 electrons seemed popular for some reason. The problem is, these are simply guesses and these problems do NOT involve guessing. They involve estimation based on knowledge. You know the breakdown E field of air and probably know that the air often crackles as you rub balloons on your head (or could surmise it from the title of the problem). Also, CHECK your answer (they ranged between CLEARLY WRONG extremes 10^{-100} g to 10^{29} kg).

Problem 3: Charges (25 points)

Two point-like particles with charges q_1 and q_2 sit fixed on the x-axis at the origin and $x = 3a$ respectively. A point P lies on the x-axis between the two charges at $x = a$ (as pictured at right).



(a) What is the electric field $\vec{\mathbf{E}}$ at point P due to these two charges?

$$\vec{\mathbf{E}} = \frac{kq_1}{a^2}\hat{\mathbf{i}} - \frac{kq_2}{4a^2}\hat{\mathbf{i}} = \frac{k}{a^2}\hat{\mathbf{i}}\left(q_1 - \frac{q_2}{4}\right)$$

(b) What is the electric potential V at point P due to these two charges, assuming that potential is defined such that $V = +10$ V at infinity?

We start with just writing the potential difference relative to infinity and using superposition, the way we always do:

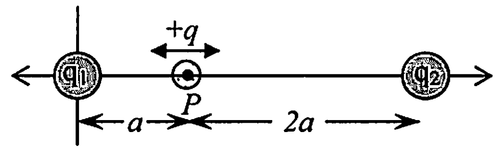
$$\Delta V = \frac{kq_1}{a} + \frac{kq_2}{2a}$$

But we asked for potential with infinity at +10 V so

$$\boxed{V = \frac{kq_1}{a} + \frac{kq_2}{2a} + 10\text{V}}$$

Problem 3 *continued*: Charges

A third particle of mass m and known positive charge $+q$ is now brought in from infinity to point P. Once there it is confined to move on the x-axis, and it is found that if it is slightly displaced from point P that it will undergo simple harmonic motion ("small oscillations") about point P due to the electric forces from the first two charges (i.e. ignore gravity).



(c) From the above information, what can you determine about the signs and magnitudes of the two charges q_1 and q_2? For ease of notation, assume that the magnitude of q_1 is Q, where Q is a positive number (so if q_1 were negative you would write $q_1 = -Q$). **Explain** how you know this!

If it is undergoing simple harmonic motion, the field at point P must be 0 and the charges must both be positive (if either or both were negative, when it got closer to the negative one it would just snap to it).

So, $\boxed{q_1 = Q}$

And from (a):

$$\left(q_1 - \frac{q_2}{4}\right) = 0 \Rightarrow \boxed{q_2 = 4Q}$$

(d) How much energy did it take to move the charge $+q$ from infinity in to P? Briefly explain the meaning of the sign of this energy (whether positive, negative or zero).

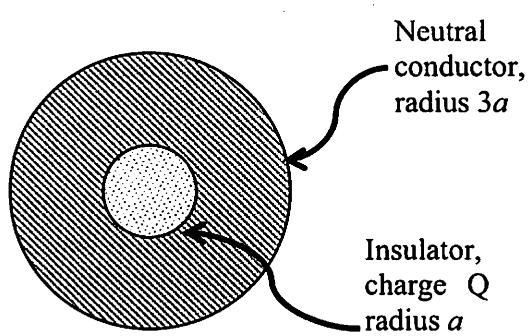
We get energy (word) required by the potential difference of (b):

$$W = q\Delta V = q\left(\frac{kq_1}{a} + \frac{kq_2}{2a}\right) = q\left(\frac{kQ}{a} + \frac{k4Q}{2a}\right) = \boxed{\frac{3kqQ}{a}}$$

This is a positive energy because we needed to push the positive charge close to the other two positive charges (it doesn't naturally want to be there).

Problem 4: A Sphere's Potential (25 points)
A spherical insulator of radius a contains a non-uniform spherically symmetric charge distribution $\rho(r) = Q/4\pi a r^2$. This insulating sphere is wrapped with a conductor with no net charge, inner radius a and outer radius $3a$, as pictured below. The zero of potential is defined on the midline of the conductor. That is, $V(r = 2a) = 0$. This is simply a definition – the conductor is NOT grounded. Although these two objects are in contact, the charge on the insulator is firmly attached and none will "leak into" the conductor.
NOTE: This problem will be graded on your **displayed understanding** of what you are doing. SHOW WORK. Answers without explanations (in words, clear derivations and pictures) will receive no credit. If you can intuit a part of the answer, for example using symmetry, it is fine to state the result without mathematical derivation but **justify it with a brief statement**.

(a) Calculate the electric potential V throughout space due to this charge distribution
Make sure that your final answer is clearly identified (boxed)



<u>**Here is an outline of our procedure:**</u>

The conductor is an equipotential surface, so we know that the potential is zero throughout it. To calculate the potential elsewhere we need to integrate the electric field, which we can calculate using Gauss's law with spherical symmetry:

<u>**Inside the insulator ($r < a$):**</u>

$$\oint\!\!\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E 4\pi r^2 = \frac{Q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\iiint \rho(r')\,dV = \frac{1}{\varepsilon_0}\int_{r'=0}^{r} \frac{Q}{4\pi a r'^2} 4\pi r'^2\,dr' = \frac{Q}{\varepsilon_0 a} r \Rightarrow \vec{\mathbf{E}} = \frac{kQ}{ar}\hat{\mathbf{r}}$$

We obtain the potential by integrating:

$$\Delta V = V(r) - \underbrace{V(a)}_{0} = -\int_a^r \frac{kQ}{ar'}dr' = -\frac{kQ}{a}\ln\left(\frac{r}{a}\right) \Rightarrow \boxed{V(r) = \frac{kQ}{a}\ln\left(\frac{a}{r}\right)}$$

Note that this is positive, as it should be since we are moving into positive charge.

<u>**Outside the insulator ($r > 3a$):**</u>
Here we don't even need to use Gauss's Law. We are outside of a spherical object with net charge $+Q$ (plug $r = a$ into Q_{enc}), so $\vec{\mathbf{E}} = \frac{kQ}{r^2}\hat{\mathbf{r}}$ The integral of that:

$$\Delta V = V(r) - \underbrace{V(3a)}_{0} = -\int_{3a}^{r} \frac{kQ}{r'^2}dr' = kQ\left[\frac{1}{r} - \frac{1}{3a}\right] \Rightarrow \boxed{V(r) = -kQ\left[\frac{1}{3a} - \frac{1}{r}\right]}$$

I rewrite it like that to emphasize that the potential is negative, as $r > 3a$. This makes sense, we had to walk uphill from infinity to get to the positively charged outer surface of the shell.

Problem 4 *continued…*: A Sphere's Potential

(b) A negligibly small hole is drilled in the above conducting sphere down to a radius of $r = 2a$, so that a conducting wire may be attached to the conductor at that radius (where $V = 0$). The other end of the wire is then attached to the outer surface of a solid conducting sphere of radius $2a$, and with no net charge, a very large distance away. Is any charge transferred through this wire? If yes, what sign of charge travels to the smaller conducting sphere and why? If no, why not?

To answer this you need to realize two things:

1) Where we choose to connect the wire to the conductor, an equipotential body, is irrelevant. Even though there is no *net* charge in the center of the conductor, if this wire is connected to an object with a different potential then charge will separate and flow.
2) The distant uncharged conductor will be at the same potential as infinity $V(\infty) = -\frac{kQ}{3a}$. Since this is ***LOWER*** than the potential of the radius 3a conductor, ***POSITIVE*** charges will flow to the second conductor. The negative charges which are left behind when the positive charges are stripped out of the conductor will flow to the surface and begin to neutralize it.

Although you weren't asked how much charge would flow, we can easily calculate this recalling that the charge will flow to equalize potentials. If we redefine our zero of potential to be zero at infinity (this doesn't physically change the system, just makes calculations easier) we have:

$$\frac{kQ_{BigSphere}}{3a} = \frac{kQ_{SmallSphere}}{2a} \Rightarrow Q_{BigSphere} = \tfrac{3}{2} Q_{SmallSphere}$$

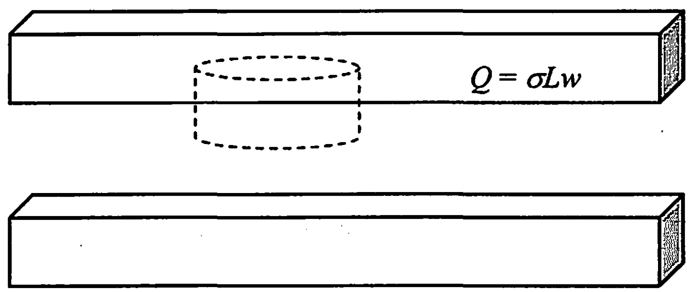
$$Q_{BigSphere} + Q_{SmallSphere} = Q \Rightarrow \boxed{Q_{SmallSphere} = \frac{2Q}{5}}$$

Note also that this idea is very similar to problem 1E

Problem 5: Parallel Plate Capacitor (25 pts)

A parallel plate capacitor consists of two large rectangular conductors (length L, width w) held apart by a very small distance d (with dimensions $L >> w >> d$).

(a) Calculate the capacitance of this capacitor. You might very well already know the answer to this – feel free to just write it down, but showing a little work here will help you later in the problem. You do NOT need to be as complete as you were in the previous problem.



We get E from Gauss's law with a pillbox of area A:

$$\oiint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EA = \frac{Q_{enc}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 L w}$$

We integrate between the two plates to get the potential difference:

$$\Delta V = -\int E \cdot ds = Ed = \frac{Qd}{\varepsilon_0 L w}$$

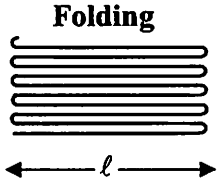
Then the capacitance is just:

$$\boxed{C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 L w}{d}}$$

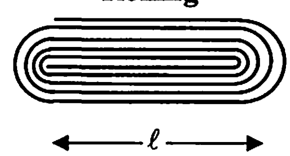
(NOTE: It was fine to just write this down, but of course if you wrote it down wrong there could be no partial credit)

This capacitor is too big (spatially) to be of much use for us, so you need to pack it up into a more compact object. Here are two suggested methods for your consideration:

Folding



Rolling

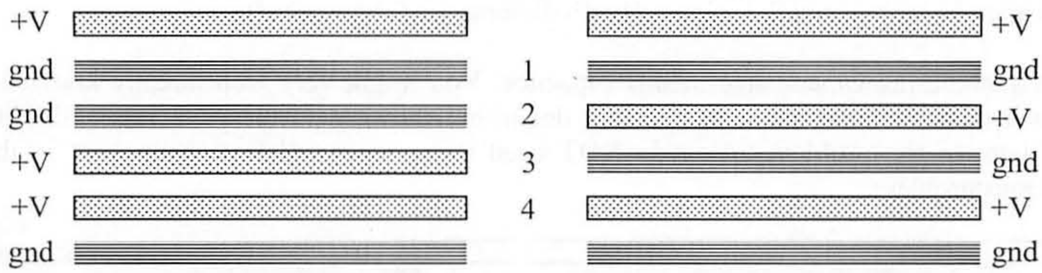


Note that these diagrams, in which the pair of plates is drawn as a single line, are NOT to scale and hence are slightly misleading. You will use the same spacing d between neighboring plates as between the plates. Also, because the length of each of the N "subplates" $\ell \equiv L/N$ is much larger than the spacing d, the rounded edge sections are not nearly as significant as they appear here. **You can ignore the edges.**

Problem 5 *continued…*: Parallel Plate Capacitor

If we zoom in on some arbitrary section of the folded and rolled capacitors we will find:

Folded **Rolled**



I have colored and labeled the plates to indicate which ones are attached together, but don't show those attachments (since I'm only looking near the centers of the plates). I have also numbered the central subplates 1 through 4.

Your job in this problem is to determine which method, if either, makes a better capacitor. You begin by attaching a battery to each capacitor and charging one plate (now folded or rolled) to a potential $+V$ relative to the other plate (denoted "ground" or "gnd").

(b) In terms of this potential $+V$ and the dimensions of the subplates (length ℓ, width w separated by distance d), what is the electric field everywhere between subplates 1 and 4?

In all cases we are in a planar geometry so the electric field is simply proportional to the potential: $E = V / d$ between the subplates (pointing from high to low potential) and zero in the subplates.

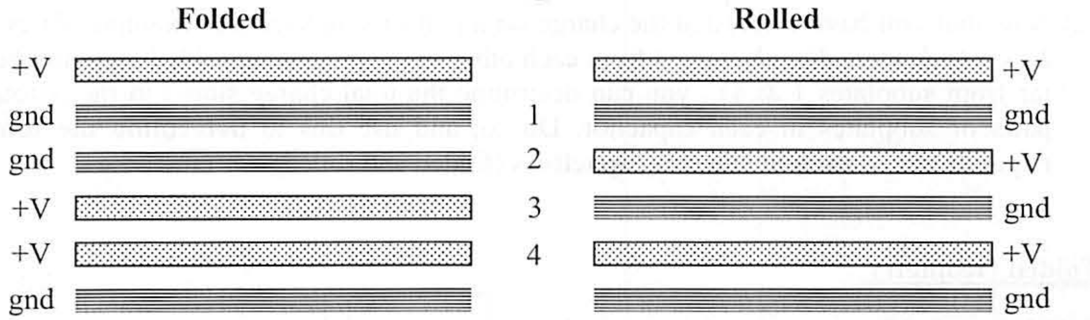
<u>Folded Geometry:</u>

Here the neighboring subplates occasionally have the same potential, so the E field is zero between these plates. So between 1 & 4 the only place with a non-zero E field is between 2 and 3. There the electric field is V/d upwards (from high to low potential)..

<u>Rolled Geometry:</u>

Here the neighboring subplates always alternate potential, so the E field is never zero between the subplates. It alternates between pointing up (between 1&2 and 3&4) and pointing down (between 2&3).

Problem 5 ***continued…*****: Parallel Plate Capacitor**

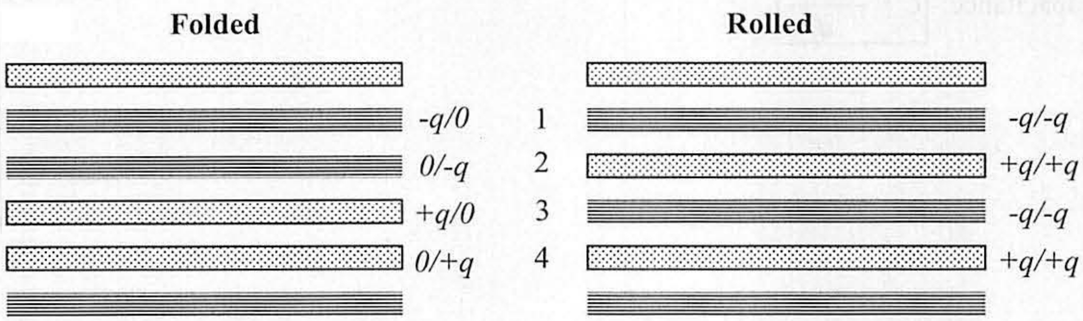


(c) For each of the two capacitors, calculate the charge on each of the four central conducting subplates pictured above (subplates 1-4). Where is that charge located on each of the subplates?

Charge is always on the surface of conductors. Wherever there is an electric field E between the subplates there will be a charge on the surface of the subplates such that $E = \sigma/\varepsilon_0$. So, using what we know:

$$\boxed{q = \sigma A_{\text{subplate}} = \varepsilon_0 E \ell w = \varepsilon_0 \ell w \frac{V}{d}}$$

That charge will be positive on high potential surfaces and negative on the low potential (gnd) surfaces. So:



where I have used the notation q1/q2 to indicate the charge on the top/bottom surfaces of each subplate

Thus, in the folded geometry each 'gnd' subplate has charge $-q$, each $+V$ subplate has charge $+q$. In the rolled geometry, on the other hand, each 'gnd' subplate has charge $-2q$, each $+V$ subplate has charge $+2q$.

Problem 5 *continued…*: Parallel Plate Capacitor

(d) Now that you have calculated the charge on a pair of subplates (for example plates 2 & 3, which were directly across from each other in the original unfolded capacitor but far from subplates 1 & 4) , you can determine the total charge stored in the N total pairs of subplates in each capacitor. Do so, and use this to **determine the total capacitance of each of the two capacitors** (folded and rolled)

Folded Geometry

Subplates 2 & 3 have equal and opposite charge $\boxed{q = \pm\varepsilon_0 \ell w \frac{V}{d}}$

So the total system has charge $\boxed{Q = Nq = \varepsilon_0 N \ell w \frac{V}{d} = \varepsilon_0 L w \frac{V}{d}}$

And the capacitance: $\boxed{C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 L w}{d}}$

Rolled Geometry

We had twice as much charge for the same potential, so we have twice as much capacitance: $\boxed{C = \frac{2\varepsilon_0 L w}{d}}$

(e) Which, if any, of the capacitors (folded, rolled or the original flat one) is best? Why?

The folded and original unfolded capacitors have the same capacitance. **The rolled capacitor is better** because it has a bigger capacitance, meaning it can store more charge (twice as much!) for a given potential difference (and given internal electric field).