

TEST TWO Thursday Evening April 1 7:30- 9:30 pm. The Friday class immediately following is canceled because of the evening exam.

What We Expect From You On The Exam

1. An understanding of the properties of conductors.
2. An understanding of capacitors. You will not need to know about the effects of dielectrics.
3. An understanding of current flow in a resistive material, e.g. how J is related to I , how E is related to J , how resistance is related to resistivity, and how to calculate it.
4. An understanding of simple circuits. For example, you should be able to set up the equations for multi-loop circuits, using Kirchoff's Laws. You should be able to derive and guess the solution to differential equations for RC circuits, and should understand the meaning of time constants ($\tau = RC$).
5. A conceptual understanding of how to determine the direction of magnetic fields of moving charges or current elements using the Biot-Savart law, e.g.

$$\vec{B} = \frac{\mu_0 q \vec{v} \times \hat{r}}{4\pi r^2} \quad d\vec{B} = \frac{\mu_0 I d\vec{s} \times \hat{r}}{4\pi r^2}.$$

You will not need to calculate any integrals using Biot-Savart Law.

6. An understanding of how to calculate the force on a current element in an external magnetic field or on a charged particle moving in an external magnetic field or in both magnetic and electric fields, including the characteristics of cyclotron motion. That is, to understand and be able to apply the equations

$$q\vec{v} \times \vec{B} = m\vec{a} \quad q(\vec{E} + \vec{v} \times \vec{B}) = m\vec{a} \quad d\vec{F} = Id\vec{s} \times \vec{B}$$

7. An understanding of the magnetic moment vector of a current loop. A conceptual understanding of how the torque exerted on a magnetic dipole in an external field arises, ($\vec{\tau} = \vec{\mu} \times \vec{B}$).

To study for this exam we suggest that you review your problem sets, in-class problems, Friday problem solving sessions, PRS in-class questions, and relevant parts of the study guide and class notes.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02

Spring 2010

Exam Two Equation Sheet

Force Law: $\vec{F} = q(\vec{E}_{ext} + \vec{v} \times \vec{B}_{ext})$

Source Equations:

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \hat{r} = \frac{dq}{4\pi\epsilon_0 r^3} \vec{r}$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{s} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \frac{Id\vec{s} \times \vec{r}}{r^3}$$

$\hat{r} = \frac{\vec{r}}{r}$ points from source to field point

Current Density and Current:

$$I = \iint_{\text{open surface}} \vec{J} \cdot d\vec{a}$$

Gauss's Law:

$$\iint_{\text{closed surface}} \vec{E} \cdot d\vec{A} = Q_{\text{inside}} / \epsilon_0$$

Gauss's Law for Magnetism:

$$\iint_{\text{closed surface}} \vec{B} \cdot d\vec{A} = 0$$

Electric Potential Difference:

$$\Delta V = V_b - V_a \equiv -\int_a^b \vec{E} \cdot d\vec{s}$$

$$\vec{E} = -\vec{\nabla}V$$

Potential Energy:

$$\Delta U = q\Delta V$$

Capacitance:

$$C = \frac{Q}{\Delta V}$$

$$U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} C \Delta V^2$$

Capacitors in Parallel: $C_{eq} = C_1 + C_2 + \dots$

Capacitors in Series: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$

Ohm's Law: $\Delta V = I R$

$\vec{J} = \sigma_c \vec{E}$ where σ_c is the conductivity

$\vec{E} = \rho_r \vec{J}$ where ρ_r is the resistivity

Resistors in Parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$

Resistors in Series: $R_{eq} = R_1 + R_2 + \dots$

Power Loss in Resistor:

$$P_{\text{Joule}} = I\Delta V = I^2 R = \Delta V^2 / R$$

Magnetic Dipole: $\vec{\mu} = I A \hat{n}$

Torque on Magnetic Dipole: $\vec{\tau} = \vec{\mu} \times \vec{B}$

Constants:

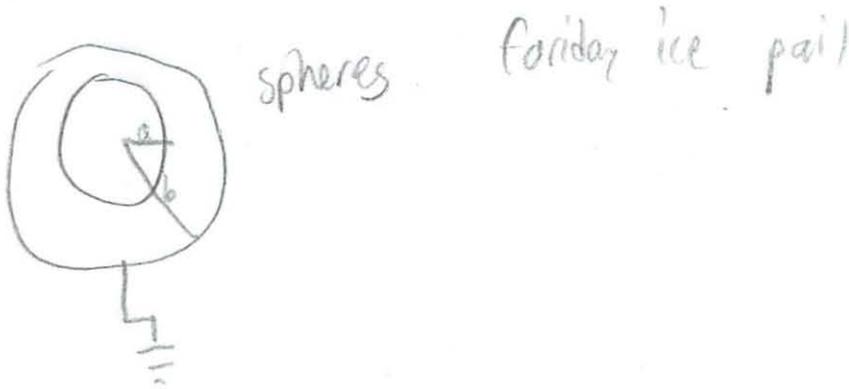
$$k_e = 1/4\pi\epsilon_0 = 9 \times 10^9 \text{ N} \cdot \text{m}^2 \cdot \text{C}^{-2}$$

$$\mu_0 / 4\pi = 10^{-7} \text{ T} \cdot \text{m} \cdot \text{A}^{-1}$$

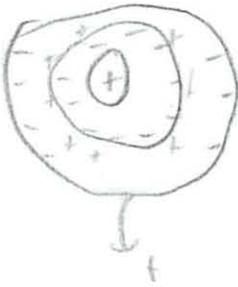
Exam Review

3/28

Conductors + Capacitors



1. Put \oplus at center



2. Touch ~~to~~ together

- the \oplus on outer ring go to ground

- the \oplus go to inner ring

3. Remove touch

4. Remove charge inside

- so just \ominus charges in inner ring
evenly distributed

Find $Q, E, V + C$

2

Remember

$$\oint E \cdot dA = \frac{q_{enc}}{\epsilon_0}$$

What surface are we doing this for



$$E = \frac{-kQ}{r^2} \hat{r}$$

* Review Gauss' Law from last test

~~$$V = \frac{kQ}{r}$$~~

$$Q = -q$$

$$E(4\pi r^2) = \frac{-Q}{\epsilon_0}$$

$$E = \frac{-Q}{4\pi\epsilon_0 r^2} = \frac{-kQ}{r^2}$$

* Total charge enclosed by any ^{closed} conductor = 0

$\oint E \cdot dA = 0 = \frac{Q_{enc}}{\epsilon_0}$ shielding inside of it

(review a little bit)

total charge must be 0

(3)

$V \rightarrow$ must integrate from B to A

$$(3) \Delta V = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right)$$

$$(1) \Delta V = -\int E dr$$

$$(2) \Delta V = -\int_a^b \frac{Q}{4\pi\epsilon_0} dr$$

* perfect exam problem

Dipoles + Torque



$$\vec{M} = I\vec{A} \quad \text{dipole moment}$$

Put dipole in field

- Feels a torque

$$\vec{T} = -\vec{M} \times \vec{B}$$

direction for torque

- screwdriver

- thumb towards torque

- fingers how to point

Be able to do right rule

$$A \times B = C$$

$$F = V \times B$$

Dir created field (B)
infinite wire

thumb towards current

here dir
into board
rotates
↻

$$\text{Torque} = \vec{M} \times \vec{B}$$

↑ dir of vector

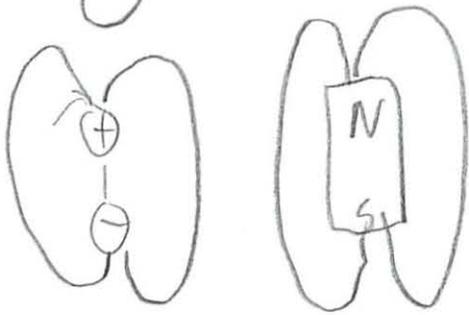
Dir \vec{M}

M is like P
"moment"

charge or mass distributed in a certain way/shape

(a) no dipole moment

now has shape \rightarrow dipole moment



how big dipole moment =
how big of a dipole moment

Everything is a dipole

μ is how strong is it / how much does it look like

bar magnet

Dipole moments want to line up w/ field

- wants to lower energy

$$U = -\mu \cdot B$$

wants as big a θ as possible

$$-\mu B \cos \theta$$

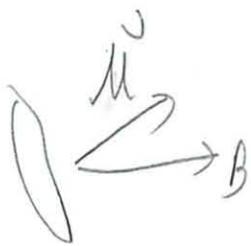
if aligned $\cos \theta = 1$

then will go to biggest B field



want to stick together
- field strongest

\otimes \otimes replace dipole w/ bar magnet and think of that



wants to line up w/ field

wire loop = dipole

~~*~~ ^{magnetic} fields never accelerate charges
- not true for currents

$$I = \frac{\mathcal{E}}{2R} \leftarrow V = IR$$

close B

now differential time varying
have parallel aspect

(I should review this)

"instant" - treat capacitor as wire
online

$$I = \frac{2\mathcal{E}}{3R}$$

* Avoid kirkoff at all costs

Review Dormskin

3/30

9-10:30 PM

know what is on test

Properties of Conductors

Capacitors + Capacitance

- no dielectrics

Current, Resistance, Circuits

~~do~~ - no solving differential circuit 😊

- know graphically

Magnetism

- forces, torques current loops

- no Ampere's law

- B-S only conceptual

Old tests cover different things

Capitance

$$Q = CV$$

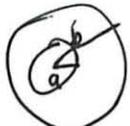
↑ charge appears when voltage diff applied

Calculate $C = \frac{Q}{|V|}$

1. How $|V|$? → $\int \vec{E} \cdot d\vec{s}$ (Gauss' law) to find E

2

3 types of capacitors

- shells  (conducting)

- parallel plates  Area A

- concentric cylinders



→ infinite length
or neglect edge effects

length l

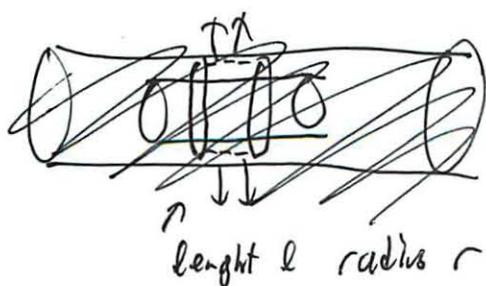
charges flow away from 1 plate to other
when voltage diff applied

Q is where (+) charge is

$$C = \frac{Q}{|\Delta V|} = \frac{Q}{|\int \vec{E} \cdot d\vec{s}|}$$

ex coaxial conducting cylinders

need to assume they are charged up
- will cancel out later



$\oint \vec{E} \cdot d\vec{a}$	$\frac{q_{enc}}{\epsilon_0}$
$E \cdot \text{area}$	$\frac{\lambda l}{\epsilon_0}$
$E \cdot 2\pi r l$	$\frac{\lambda l}{\epsilon_0}$

* Conductor so all charge on surface

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0} \frac{1}{r} \hat{r}$$

$a < r < b$

← need to be able to do (from last test) → I need to look over + review

* Also ~~know~~ remember what pitfalls are

$$C = \frac{Q}{\int \vec{E} \cdot d\vec{s}}$$

$\int_{r=a}^{r=b} \vec{E} \cdot d\vec{s}$ potential diff decreasing from inner to outer so = but abs value so does not matter

$$= - \int_a^b \frac{\lambda}{2\pi\epsilon_0} \frac{1}{r} dr$$

$$= - \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{b}{a}\right)$$

← know the rule

$$C = \frac{Q}{\frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{b}{a}\right)}$$

but what is Q ?

↳ should cancel

$$\hookrightarrow d = \frac{Q}{L} \quad \leftarrow \text{known}$$

$$C = \frac{dL}{\frac{1}{2\pi\epsilon_0} d \ln(b/a)} = \frac{L}{\frac{\ln(b/a)}{2\pi\epsilon_0}} = \boxed{\frac{2\pi\epsilon_0 L}{\ln(b/a)}}$$

Energy Stored in a Capacitor

$$U_C = \frac{Q^2}{2C} \quad \leftarrow \text{memorize}$$

$$= \frac{1}{2} C |\Delta V|^2$$

$$= \frac{Q^2}{2}$$

$$2 \left(\frac{2\pi\epsilon_0 L}{\ln(b/a)} \right) \text{ in this example}$$

$$Q = C |\Delta V|$$

OR if have voltage

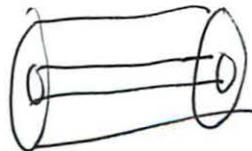
$$= \frac{1}{2} \left(\frac{2\pi\epsilon_0 L}{\ln(b/a)} \right) \Delta V^2$$

also

$$= \frac{1}{2} \epsilon_0 \iiint E^2 dV$$

$$= \frac{1}{2} \epsilon_0 \int_a^b \left(\frac{Q}{2\pi\epsilon_0 r} \right)^2$$

will get same result



$$dV = L 2\pi r dr$$

cylindrical shell

5

Try for all 3 cases (sphere, cylinder, plates)

know \rightarrow if E non constant, must do integral

What are the electric fields

? must know
figured out for last
test

Circuit Theory

Current + Resistance

$$I = \iint \vec{J} \cdot d\vec{a} \quad \text{how much } J \text{ through an area}$$

Ohm's Law

$$\Delta V_{\text{element}} = RI$$

$$R = \frac{|\Delta V_{\text{element}}|}{|I|} = \frac{|\int \vec{E} \cdot d\vec{s}|}{|\iint \vec{J} \cdot d\vec{a}|}$$

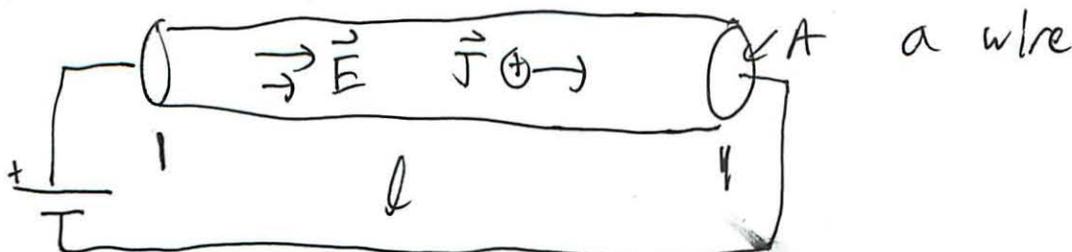
Resistivity (ρ_r)

$$\vec{E} = \rho_r \vec{J}$$

$$\vec{J} = \sigma_c \vec{E}$$

$$\sigma_c = \frac{1}{\rho_r} = \text{conductivity}$$

(*) know



$$R = \frac{|\Delta V|}{|I|} = \frac{|\int \vec{E} \cdot d\vec{s}|}{|\iint \vec{J} \cdot d\vec{a}|}$$

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$$E = \frac{\rho_c \cancel{J} l}{J A} = \frac{\rho_c L}{A}$$

← do not know at all, study

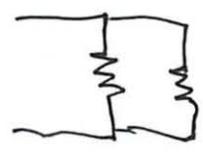
Circuits

Resistors

~~Parallel~~
~~Series~~

Series

Parallel



- Same current through each
 - Voltage drops add

- Replace w/ Req

- Same voltage across them
 - current added

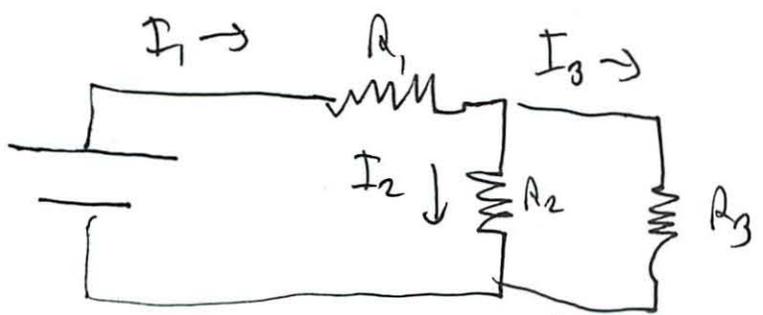
- more doors to leave theater

Can make things far simpler

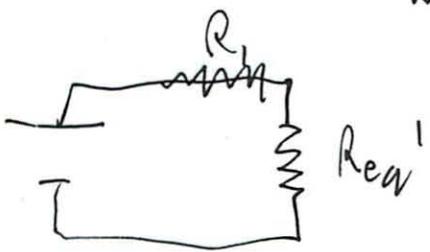
$$\Delta V = IR_1$$

$$\Delta V = IR_2$$

$$\Delta V / R_{eq} = \Delta V / R_1 + \Delta V / R_2$$



2 in parallel which together are in series is 1



$$\frac{1}{R_{eq}} = \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_{eq} = \frac{R_2 R_3}{R_2 + R_3} \quad \leftarrow \text{know}$$

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$$R_{eq} = R_1 + R_{eq}$$

$$I_1 = \frac{\Delta V}{R_{eq}}$$

to find I_2 or I_3

$$I_1 = I_2 + I_3$$

↑ just found

know ΔV_2 and $3 =$

$$\text{so } I_2 R_2 = I_3 R_3$$

$$I_2 = \frac{I_3 R_3}{R_2}$$

$$I_1 = \frac{I_3 R_3}{R_2} + I_3 = \frac{\Delta V}{R_{eq}}$$

$$I_1 = I_3 \left(\frac{R_3}{R_2} + 1 \right)$$

$$I_2 = I_3 \frac{R_3}{R_2}$$

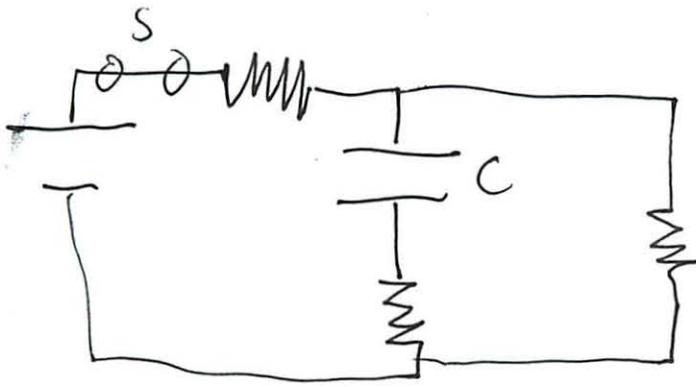
far better ✓
algebra now
thanks to physics
+ problem solving

"solve a circuit" → find current in every branch

So what happens when you add a capacitor



8



time dependent circuit
 need loop laws + differentiated
 eqs to get full
 time dependence

but they just ask - switch just closed
 - closed for really long time
 - etc

$t=0$, close $S \rightarrow$ Capacitor uncharged (like a wire)

$$R_{eq}' = \frac{R}{2}$$

$$I = \frac{\Delta V}{\frac{3R}{2}}$$

$$R_{eq} = \frac{3R}{2}$$

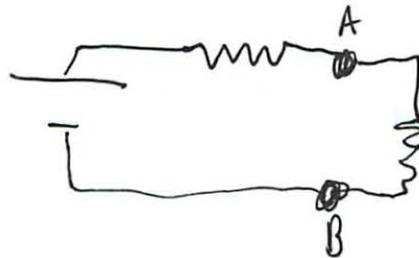
find I_2, I_3

$t =$ long time after S closed

Capacitor fully charged

$$I_2 = \text{zero}$$

$$V_C = ?$$



$$= I_{\text{final}} \cdot R$$

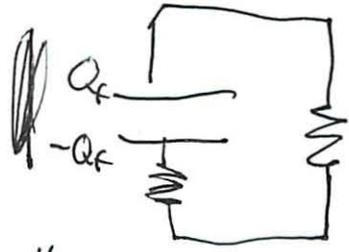
$$= \frac{\Delta V R}{2R} = \frac{\Delta V}{2}$$

half the voltage of the battery

4

New open switch ~~is~~

- discharge capacitor

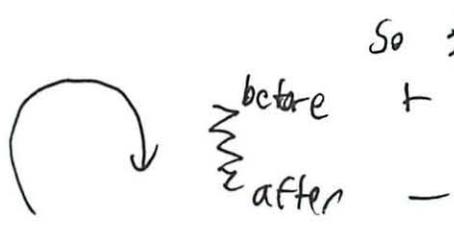


if solving differential equation

$$\Delta V = V_{\text{after}} - V_{\text{before}}$$

choice of current dir \neq choice circulation dir
 ↑ what after + before means

if going



So ~~the~~ voltages are

$$\Delta V = IR$$

$$0 = -I_2 R - I_1 R + \frac{Q}{C}$$

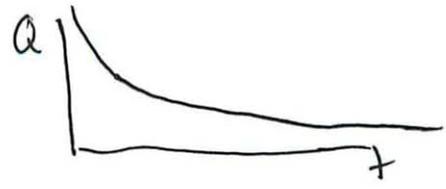
↑ since going from $\ominus \rightarrow \oplus$ plate

$$\frac{Q}{C} = 2I_2 R$$

$$I_2 = -\frac{dQ}{dt} \text{ discharging}$$

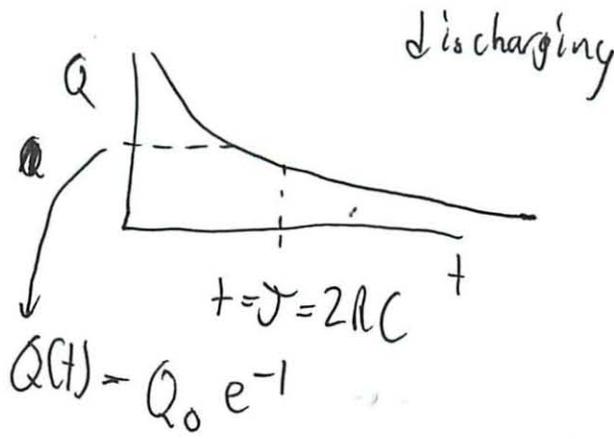
$$\frac{Q}{C} = -2 \frac{dQ}{dt} R$$

$$\frac{Q}{-2RC} = \frac{dQ}{dt}$$



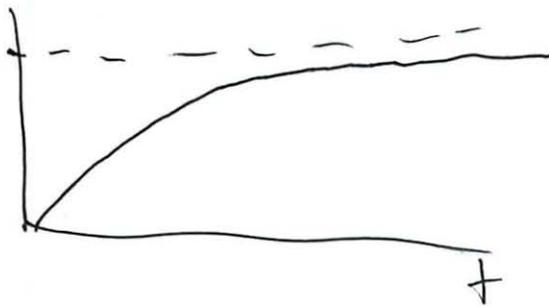
(10)

$$\frac{dQ}{Q} = \frac{dt}{2RC}$$



charging

- bit more complicated



didn't do capacitors in parallel

- do a bit more w/ graphs - like that lab

Magnetism

- currents or moving charges produce magnetic fields

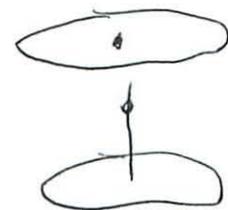
$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I \, d\vec{s} \times \hat{r}}{r^2}$$

- come from sources

- 2 special cases

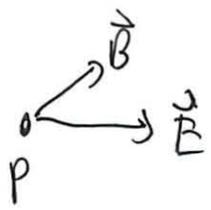
- ring w/ field at center

- ring w/ field some height z



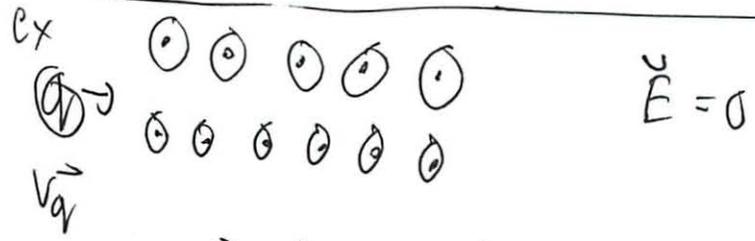
- otherwise won't use B-S much

① Given a \vec{B} field + \vec{E} field
 ↑ moving charges ↑ charges



$$F = q (\vec{E}_{ext} + \vec{v}_q \times \vec{B}_{ext})$$

Plane q at p



$$\vec{F} = m_q \vec{a}_q$$

$$q (\vec{v}_q \times \vec{B})$$

↑
 Cross product
 perpendicular
 to either
 vector

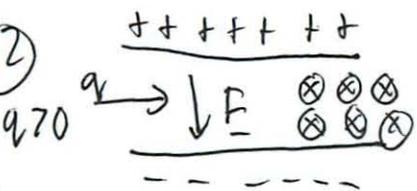
- velocity only
- change dir
- "do no work"

- If \vec{B} is uniform
- uniform force
 - will get circular motion
 - of radius r

$$v_q B = \frac{mv^2}{R}$$

← memorize
 "cyclotron"

$$T = \frac{2\pi R}{v_q}$$



$$F = m\vec{a}$$

$$q(\vec{E} + \vec{v} \times \vec{B}) = m\vec{a}$$

where it goes is up to the relative strengths

straight \rightarrow they cancel

if its not straight now force points a diff direction
so the problem has gotten a lot harder

Current loops in magnetic fields

$$\vec{F}_{\text{wire}} = \int_{\text{wire}} I d\vec{s} \times \vec{B}$$



$$dF = dq\vec{v} \times \vec{B}$$

add that up over the wire \rightarrow integrate

If \vec{B} is uniform, ~~then~~ then ~~the force on the wire~~ \rightarrow can pull out B

$$\vec{F}_{\text{wire}} = (I \int ds) \times \vec{B}$$

If wire is straight just use length

$$\vec{F}_{\text{wire}} = I \left(\int ds \right) \times \vec{B} = \cancel{I L} I \vec{L} \times \vec{B}$$

If wire is a loop (closed path) in uniform \vec{B} field

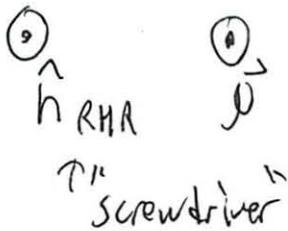
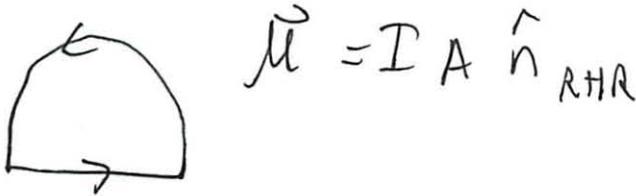
$$\vec{F}_{\text{wire}} = 0 = \left(\int ds \right) \times \vec{B}$$

13

~~Errata~~

Torque is easy to calculate

- Use concept of magnetic moment



$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Write some unit vectors



$$\vec{\tau} = I A \hat{k} \times B \hat{j}$$

$$\vec{\tau} = I A B (-\hat{i})$$

- causes object to rotate
- points along ~~the~~ axis of rotation



at this instant
- magnetic moment changing direction

(14)

Another problem "current in adj objects"

- 2 wires
 - parallel currents attract
 - antiparallel " " repel
- current loops
 - same

Did not do much conductors

Think this test far better off

- then review session 1 where I learned a lot
- only real ~~the~~ thing I did not remember was Gauss' Law
- and some formulas to memorize (but know how to use)

8.02 Test 2 Review

4/1

2:30 - exhausted - not tired but exhausted
- like long day of traveling
from math test before and long race all weekend
- need some fun
- bubbles today (first time having fun in a while)

Well test at 7 first

Lets start w/ reviewing last test + then Gauss Law

* remember opposites attract

flux is dependent on charge inc

- if it is net 0, no net flux

the systems always want to \downarrow PE

$$E = -\nabla V$$

Remember V comes from outside in
add each region

\longrightarrow $\left[\begin{array}{ccc} \longrightarrow & \longrightarrow & \longrightarrow \end{array} \right]$ Separately

$$\Delta V = -\int E \cdot d\mathbf{s} = V(P) - V(Q)$$

\int_P^Q scalar

Pill box of what is enclosed

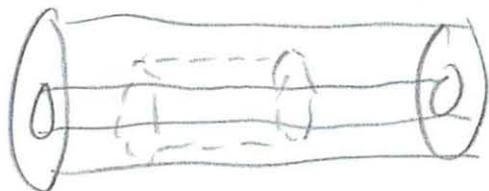
Review Gaussian shapes

Remember cylinders only outwards



② Look at Dormastin's Review

- shells
- parallel plates
- concentric cylinders of length



If did outside it would be 0, I am pretty sure of
For capacitor define what is $\pm \lambda$

Then $\int \vec{E} \cdot d\vec{a} = \frac{Q_{inc}}{\epsilon_0}$

$$E \cdot 2\pi r l$$

\int
? integrate if non uniform

did \pm have any problems like that??

$$= \frac{\lambda l}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0} \frac{1}{r}$$

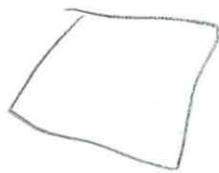
③ If other shape

Rod - kinda the same - but no radius on
the rod

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \quad \leftarrow \text{same ans actually!}$$



One ∞ large plane



σ = charge density

pill box over the shape



- only escapes on top & bottom
- Sides don't matter

$$\Phi = \iint E \cdot dA$$

$$E_1 A_1 + E_2 A_2$$

$$= (E_1 + E_2) A$$

$\uparrow \quad \uparrow$
one + one -, right

$$= 2EA$$

(4)

Now typical

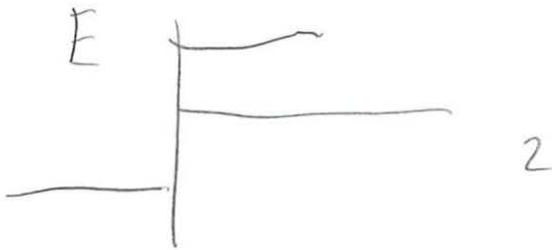
$$ZEA = \frac{\Delta Q_{inc}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma A}{\epsilon_0 Z A} = \frac{\sigma}{\epsilon_0 Z}$$

then have to split

$$\frac{\sigma}{2\epsilon_0} \quad Z > 0$$

$$-\frac{\sigma}{2\epsilon_0} \quad Z < 0$$



I pretty much remember this

I think I am pretty good (better than math) —
but I feel that I have to use time
studying

$$r \geq a$$

all 0 since charges on shell

$$r \geq a \text{ (beyond shell)}$$

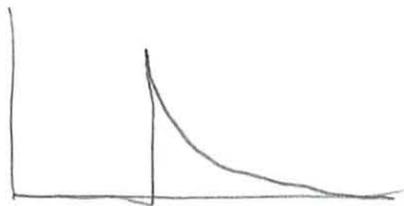
$$Q_{\text{enc}} = Q$$

$$E \cdot 4\pi r^2$$

$$\vec{E} = \frac{Q}{4\epsilon_0 r^2} = \frac{k_e Q}{r^2}$$

oh that is where that comes from

\vec{E} or electric field



Non conduction solid shell

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi a^3}$$

$$r \leq a$$

$$\Phi = E \cdot 4\pi r^2$$

$$Q_{\text{enc}} = \int_V \rho \, dV = \rho V = \rho \cdot \frac{4}{3}\pi r^3 = Q \frac{r^3}{a^3}$$

$$\textcircled{6} \quad E(4\pi r^2) = \frac{\rho}{\epsilon_0} \left(\frac{4}{3} \pi r^3 \right)$$

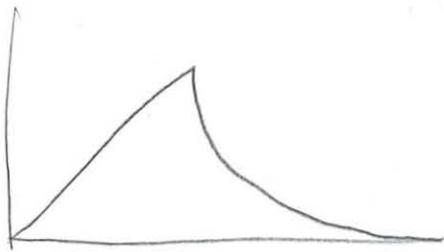
$E =$ can solve

When $r \geq a$

$k \frac{Q}{r^2}$ ~~is~~ just looking from outside

Now where is it that you have to \int integrate

(perhaps should take up before test)



Conductors

- net charge ~~is~~ inside

\vec{E} field 0 inside conductor
all on surface

$$C = \frac{Q}{\int \vec{E} \cdot d\vec{s}}$$

~~study~~

~~study~~ know the formulas now since had to learn them

⑦ Here is where we have to actually integrate

$$dV = -\int E \cdot ds$$

$$\int_a^b \frac{\lambda}{2\pi\epsilon_0} \frac{1}{r}$$

know $\int_a^b \frac{1}{r} = \ln(r) = \left(\frac{1}{b} - \frac{1}{a} \right)$

$$\ln\left(\frac{b}{a}\right)$$

↑ know that - was on last test's formula sheet

Here was another special one

or it was written on board - not on this formula sheet

$$\text{Oh } \int_a^b \frac{1}{r^2} = \left(\frac{1}{a} - \frac{1}{b} \right)$$

↑ note order - not what I thought

know that $\lambda = \frac{Q}{L}$

so it will cancel

$\lambda =$ charge density

$$\lambda = \frac{Q}{A} \quad \text{I think}$$

8 So E stored in a capacitor

$$U = \frac{Q^2}{2C} \quad \leftarrow \text{know}$$

Can plug in now for C

then can just leave

So think I can figure out sphere + cylinder

If just remember sphere $A = 4\pi r^2$

$$V = \frac{4}{3}\pi r^3$$

$$I = \iint \vec{J} \cdot d\vec{a}$$

$$V = RI = IA$$

$$R = \frac{|V|}{|I|} = \frac{\int E \cdot ds}{\int \vec{J} \cdot da} \quad \leftarrow \text{never saw it written like that}$$

resistivity all on here

Should prob switch to practice test

Well first ring

created magnetic field from current loop

$$P \cdot \vec{B}(P) = ?$$



- but this is B-S law which
is not on here - ~~is~~ except perhaps
this basic case

20 Ok Sample Test 1 Spring 08

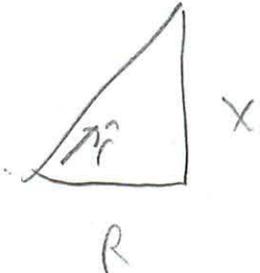
- I so don't feel like this
- Half hrs to go till German
- wasted 1/2 hr too
- so only 1 hr of work \rightarrow 8 pages

Perhaps I will think through - not write
So burnt out for all rals

- need to recover on test

Pratice test making me feel less good - but not concentrating

9



$$r = \sqrt{R^2 + x^2}$$

$$d\vec{s} \times \hat{r}$$

$$ds = R d\theta$$

$$= |d\vec{s}| \quad \text{don't really get}$$

$$\int \frac{\mu_0}{4\pi} \left(-R d\theta \hat{r} \times \frac{R(\hat{i}) \times x \hat{j}}{\sqrt{R^2 + x^2}} \right)$$

- all the horiz cancel and the vert add up

$$\frac{\int R d\theta (-\hat{k}) \times (\hat{r}_{\text{horiz}} + \hat{r}_{\text{vertical}})}{R^2 + x^2}$$

$$d\vec{s} = R d\theta (-\hat{k})$$

$$r_{\text{horiz}} = r \cos \theta$$

$$= \hat{r} \frac{R}{(R^2 + x^2)^{1/2}}$$

$$\times \text{dir} (-\hat{k} \times \hat{r}_{\text{horiz}}) = \text{dir } \hat{j}$$

$$\hat{r}_{\text{vert}} \otimes d\vec{s}$$

Skip we don't have to know this complete ...