Current year

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2010**

Final Exam Equation Sheet

Force Law: $\vec{\mathbf{F}}_q = q(\vec{\mathbf{E}}_{ext} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}_{ext})$ $F = \frac{kqQ}{R^2}$

$F_B = I\vec{w} \times \vec{B}$
lenght

Force on Wire: $\vec{\mathbf{F}}_q = \int_{wire} Id\vec{\mathbf{s}} \times \vec{\mathbf{B}}$

Faraday's law
$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt} \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{a}}$$ $\mathcal{E} = \frac{-d}{dt} \Phi$

Gauss's Law:
$$\underset{\text{closed surface}}{\oiint} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = Q_{\text{inside}} / \varepsilon_o$$

Gauss's Law for Magnetism:
$$\underset{\text{closed surface}}{\oiint} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

Ampere's Law
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_o \iint \vec{\mathbf{J}} \cdot d\vec{\mathbf{a}} + \mu_o \varepsilon_0 \frac{d}{dt} \iint \vec{\mathbf{E}} \cdot d\vec{\mathbf{a}}$$

Electric Potential Difference:
$$\Delta V = V_b - V_a \equiv -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$
$$\vec{\mathbf{E}} = -\vec{\nabla} V$$

Potential Energy:
$\Delta U = q\Delta V$
$U = qV$

Capacitance:
$$C = \frac{Q}{\Delta V}$$
$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} C \Delta V^2$$

Inductance: $L = N\Phi_B / I$

Energy

$\mathcal{E}_{\text{back}} = -LdI/dt \qquad U_L = \frac{1}{2} LI^2$

Energy Density Stored in Fields:
$u_E = \frac{1}{2}\varepsilon_0 E^2 \;;\; u_B = \frac{1}{2} B^2 / \mu_0$

Current Density and Current:
$$I = \underset{\text{open surface}}{\iint} \vec{\mathbf{J}} \cdot d\vec{\mathbf{a}}$$

Ohm's Law: $\Delta V = I\,R$

$\vec{\mathbf{J}} = \sigma_c \vec{\mathbf{E}}$ where σ_c is the conductivity

$\vec{\mathbf{E}} = \rho_r \vec{\mathbf{J}}$ where ρ_r is the resistivity

Power Dissipated in Resistor:
$P_{\text{Joule}} = I\Delta V = I^2 R = \Delta V^2 / R$

magnetic dipole moment

$\tau = \mu \times B$
$= I A \hat{n} \times \vec{B}$
area normal

Constants:
$k_e = 1/4\pi\varepsilon_0 = 9 \times 10^9 \ \mathrm{N \cdot m^2 \cdot C^{-2}}$

$\mu_0 / 4\pi = 10^{-7}\ \mathrm{T \cdot m \cdot A^{-1}}$

Differential Equations and Solutions:

$$\mathcal{E} - IR - L\frac{dI}{dt} = 0 \qquad I(t) = \frac{\mathcal{E}}{R}(1 - e^{-tR/L})$$

$$IR + L\frac{dI}{dt} = 0 \qquad I(t) = I_0 e^{-tR/L}$$

$V = Ed$

AC Circuits: $\omega_0 = 1/\sqrt{LC}$;
$X_L = \omega L$; $X_C = 1/\omega C$; $X_R = R$

Series RLC**:**
$$Z = \sqrt{R^2 + X^2} = \sqrt{R^2 + (X_L - X_c)^2} \;;$$

$\Phi_E = \int E\,dA$

$\Phi_B = \iint_S B\,dS = BA\cos\theta$

B-S Law

differential current element

source to point vector line

$\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \vec{r}}{r^2}$

$\tan\phi = (X_L - X_C)/R$; $V_0 = I_0 Z$

$\tan(\pm\pi/4) = \pm 1$; $\sin(\pm\pi/4) = \pm\sqrt{2}/2$;

$\cos(\pm\pi/4) = \sqrt{2}/2$

Waves: $c = 1/\sqrt{\mu_0 \varepsilon_0}$

$f = 1/T$ $\omega = 2\pi f$ $k = 2\pi/\lambda$

$c = \lambda/T = \lambda f = \omega/k$

Double Slit Interference:

Constructive:
$d\sin\theta = m\lambda$; $m = 0, \pm 1, \pm 2, \cdots$

Destructive:
$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$; $m = 0, \pm 1, \pm 2, \cdots$

Single Slit Diffraction: Destructive:
$a\sin\theta = n\lambda$; $n = \pm 1, \pm 2, \cdots$

Colombs Law review

5/13

electric charge units

Colombs

$$F = k_e \frac{q_1 q_2}{r^2} \hat{r} \qquad \hat{r} = \frac{\vec{r}}{r}$$

interaction b/w charges

Superposition of charges

likely <u>not</u> to be on exam

$$k = \frac{1}{4\pi\epsilon_0}$$

e add

electric field lines

$$\vec{F} = q\vec{E}$$

↑ $\vec{E}$ field

$$a = \frac{F_e}{m}$$ ← for those weird hybrid acc problems

diapole = 2 opposet, but = Charges

$\tau = \vec{p} \times \vec{E}$

(never quite got this)

rotate diapole clockwise so ~~all~~ dipole moment $\vec{p}$
aligns w/ Field $\vec{E}$

Work and PE

(I always forget!)

$W = pE(\cos\theta - \cos\theta_0)$

$\Delta U = -W = -pE(\cos\theta - \cos\theta_0)$

$U = -pE\cos\theta$

Charge density ρ

$\rho = \frac{dq}{dV}$

$Q = \sum q = \int \rho \, dV$ - yeah add up all of the charge densities

surface charge density σ

$\sigma = dq/dA$

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$$E = \frac{1}{4\pi\epsilon} \int_V \frac{dq}{r^2} \hat{r}$$

I understand all tese 5 things now!

Review E field on ring solution

Gauss' Law

~~El~~ (know this stuff a lot better)

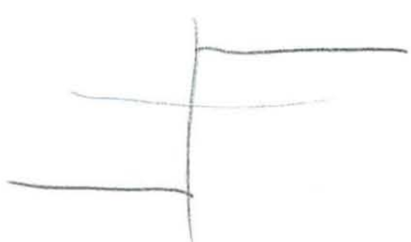
$\phi = \vec{E} \cdot \vec{A}$

- used later 8.02
- did today 18.02

$\int$ over te area

different types of shapes
- know all te shapes
- don't be suppirized

plane



two planes → last test

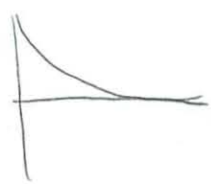
4

Where is the field in the first place?

- Conductor → 0 inside
- coaxal

What is your Gaussian surface?

- pill box?



transfer charges from one to other

ring on top of

perhaps do last p-set again

8.02

5/15

Test monday

Plan Do a past year's Final

Then study

Physics 8.02 **Equation Sheet Final**
Please Remove this Tear Sheet from Your Exam

$U = qV$

Force Laws: $\vec{\mathbf{F}} = q(\vec{\mathbf{E}}_{ext} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}_{ext})$

for current $\vec{\mathbf{F}}_B = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$

Source Equations:

Coulomb Law: $d\vec{\mathbf{E}} = \dfrac{1}{4\pi\varepsilon_o}\dfrac{dq}{r^2}\hat{\mathbf{r}}$

$d\vec{\mathbf{B}} = \dfrac{\mu_o I}{4\pi}\dfrac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$

$\hat{\mathbf{r}} = \dfrac{\vec{\mathbf{r}}}{r}$ points from source to field point

Current Density and Current:

$$I = \iint_{\text{open surface}} \vec{\mathbf{J}} \cdot d\vec{\mathbf{a}} = \frac{dq}{dt}$$

Gauss's Law:

$$\oiint_{\text{closed surface}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{inside}}}{\varepsilon_o}$$

Ampere's Law:

$$\oint_{\text{closed path}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \iint_{\text{open surface}} \vec{\mathbf{J}} \cdot d\vec{\mathbf{a}} = \mu_0 I_{enc}$$

plus Displacement Current

$$I_d = \varepsilon_0 \frac{d}{dt}\Phi_{Electric}$$

$$\oint_{\text{closed path}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \left(I_{enc} + I_d\right)$$

Gauss's Law for Magnetism:

$$\oiint_{\substack{closed\\surface}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

Faraday's Law:

$$\mathcal{E} = -\frac{d}{dt}\Phi_{magnetic}$$

Current Loop flux:

$$\Phi_{magnetic}^{total} = N\,\Phi_{magnetic}^{single\ turn}$$

Electric Potential Difference:

$$\Delta V = V_b - V_a \equiv -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

Resistance: $R = \dfrac{\rho L}{A}$

Capacitance: $C = \dfrac{Q}{\Delta V}$

$U = \dfrac{1}{2}\dfrac{Q^2}{C} = \dfrac{1}{2}C\Delta V^2$

Capacitors in Parallel: $C_{eq} = C_1 + C_2 + \cdots$

Capacitors in Series: $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots$

$\tau_{RC} = RC$

Ohm's Law: $\Delta V = I\,R$

$\vec{\mathbf{J}} = \sigma_c \vec{\mathbf{E}}$ where σ_c is the conductivity

$\vec{\mathbf{E}} = \rho_r \vec{\mathbf{J}}$ where ρ_r is the resistivity

Resistors in Parallel: $\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots$

Resistors in Series: $R_{eq} = R_1 + R_2 + \cdots$

Joule Heating: $P_{\text{Joule}} = I\Delta V = I^2 R = \dfrac{\Delta V^2}{R}$

Magnetic Dipoles: $\vec{\boldsymbol{\mu}} = I\,A\,\hat{\mathbf{n}}$

Torque: $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$

Force: $F_z = \mu_z \dfrac{dB_z}{dz}$

Inductance:

$L = \dfrac{N\Phi_{\text{B,self,sgl coil}}}{I}$

$\mathcal{E}_{\text{back}} = -L\dfrac{dI}{dt}$

$U_L = \dfrac{1}{2}LI^2$

$\tau_{RL} = L/R$

Kirkhoff's Laws

$\sum_i I_i^{in} = \sum_j I_j^{out}$ for any junction

$$\sum_i \Delta V_i - \sum_j L_j \frac{dI_j}{dt} = 0$$

AC Circuits:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$X_L = \omega L$, I lags V $X_C = \frac{1}{\omega C}$, I Leads V,

$X_R = R$, in phase

Series RLC **:** $I(t) = I_0 \sin(\omega t)$

$$Z = \sqrt{R^2 + X^2} = \sqrt{R^2 + (X_L - X_c)^2}$$

$\tan\phi = (X_L - X_C)/R$; $V_0 = I_0 Z$

Energy stored in fields:

$$u_E = \frac{1}{2}\varepsilon_0 E^2 \; ; \; u_B = \frac{B^2}{2\mu_0}$$

Energy Flow:

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}} \quad P_{power} = \iint_{surface} \vec{\mathbf{S}} \cdot d\vec{\mathbf{a}}$$

$\vec{\mathbf{p}} = \frac{U}{c} \quad P = \frac{|\vec{\mathbf{S}}|}{c}$ absorbed waves

$\vec{\mathbf{p}} = \frac{2U}{c} \quad P = \frac{2|\vec{\mathbf{S}}|}{c}$ reflected waves

Intensity $I = \langle |\vec{\mathbf{S}}| \rangle$

Waves:

$$c = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} = \omega/k = f\lambda$$

$$f = 1/T = \omega/2\pi \quad k = 2\pi/\lambda$$

$$c = \lambda/T = \lambda f = \omega/k$$

Interference:

Phase difference $\delta = \begin{matrix} m\lambda \text{ Constructive} \\ (m+\frac{1}{2})\lambda \text{ Destructive} \end{matrix}$

Far field: $\sin\theta = y/L$

2 Slit interference, spacing d: $\delta = d\sin\theta$

1 Slit diffraction Slit width a: $\delta = \frac{a}{2}\sin\theta$

Cross-products of unit vectors:

$$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$$

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \quad \hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}} \quad \hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}}$$

Kinematics:

$$|\vec{\mathbf{a}}_{cent}| = v^2/r$$

Circumferences, Areas, Volumes:

1) The area of a circle of radius r is πr^2, the circumference is $2\pi r$
2) The surface area of a sphere of radius r is $4\pi r^2$, the volume $(4/3)\pi r^3$
3) The area of the sides of a cylinder of radius r and length l is $2\pi r l$. Its volume is $\pi r^2 l$

Integrals that may be useful

$$\int_a^b dr = b - a$$

$$\int_a^b \frac{dr}{r} = \ln(b/a)$$

$$\int_a^b \frac{1}{r^2} dr = \left(\frac{1}{a} - \frac{1}{b}\right)$$

Some potentially useful numbers

$$k_e = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \ \mathrm{N \cdot m^2 \cdot C^{-2}}$$

$$\frac{\mu_0}{4\pi} = 10^{-7} \ \mathrm{T \cdot m \cdot A^{-1}}$$

try to do as much w/o looking — not check every qv

5/15

8.02 Final Exam Fall 2008

FAMILY (last) NAME

GIVEN (first) NAME

Student ID Number

Your Section: ____L01 MW 10 am ____L02 MW 12 pm
Your Group (e.g. 10A): ______________

No notes or calculators allowed. Please show all your work on the analytical questions. Answers quoted without any work will receive no credit, regardless of whether or not they are actually correct.

	Score	Grader
Problem 1 (40 points)		
Problem 2 (20 points)		
Problem 3 (30 points)		
Problem 4 (30 points)		
Problem 5 (20 points)		
Problem 6 (40 points)		
TOTAL (180 points)		

Problem 1 10 Concept questions, 4 points each, 40 points total

1.A (4 pts) If I have a uniformly charged insulating sphere of radius R with total charge Q, I find that outside the sphere at some arbitrary distance r away, I have an electric field of $\vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}$. Which of the following actions would change this $\vec{\mathbf{E}}$ field?

r R not in form
x

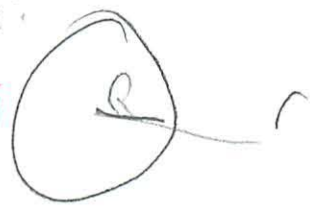
a) Reducing R by ½.
b) Making the sphere conducting rather than insulating
c) Breaking the spherical symmetry, such as deforming the sphere into the shape of an egg
d) None of the above
e) I do not know (1 pointt)

more outside

r :)

Some charge will be closer/furter away breaking field

all above?

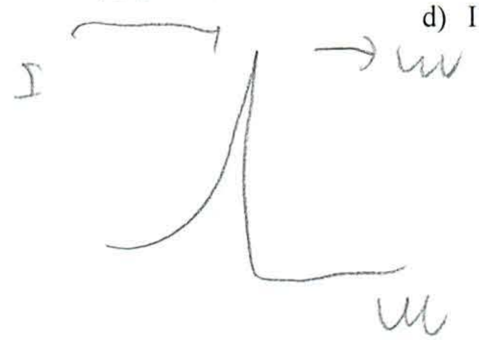


$E = \frac{Q}{4\pi\varepsilon_0 r^2}$

1.B (4 pts) I have an AC power supply and two *different* circuit elements in series, R, L, or C. If I arbitrarily increase the driving frequency of my power supply, I find the amplitude of the current measured always increases. Which two elements are in my circuit?

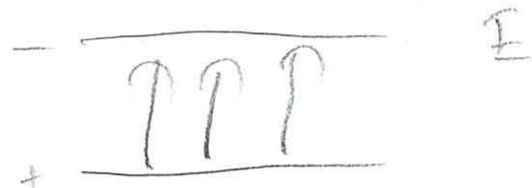
a) R and C
b) R and L
c) L and C
d) I do not know (1 pt)

must be here



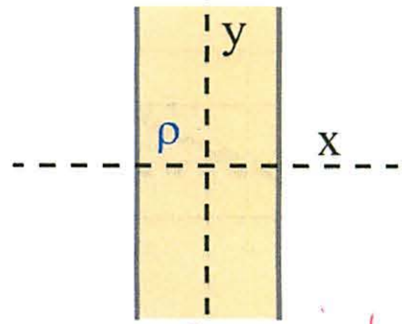
1.C (4 pts) If you place a negatively charged particle in an electric field, the charge will move

a) from higher to lower electric potential and from lower to higher potential energy. (+)
b) from higher to lower electric potential and from higher to lower potential energy.
c) from lower to higher electric potential and from lower to higher potential energy.
d) from lower to higher electric potential and from higher to lower potential energy. (−)
e) I don't know (1 point)



(+) charge goes down hill potential

shells



1.D (4 pts) If I have a uniform charge distribution ρ over some semi-infinite volume as pictured at right, ***inside the volume*** the electric field dependence on distance x from the center is

a) $|\vec{\mathbf{E}}| \propto \frac{1}{x^2}$

b) $|\vec{\mathbf{E}}| \propto \frac{1}{x}$

c) $|\vec{\mathbf{E}}| \propto \ln(x)$

d) Does not depend on x ; Solenoid

e) $|\vec{\mathbf{E}}| \propto x$

f) $|\vec{\mathbf{E}}| \propto x^2$

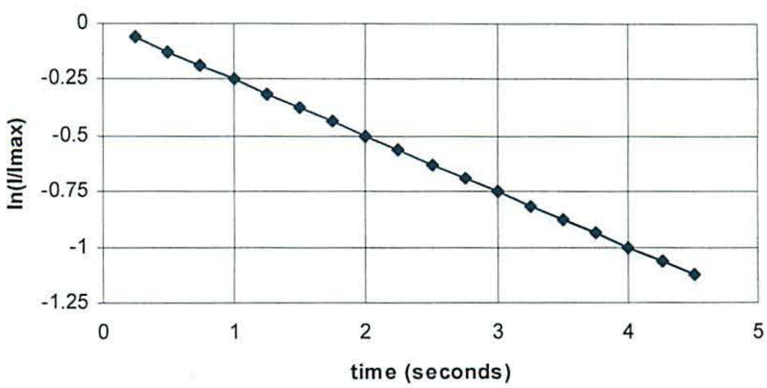
g) I don't know (1 pt)

$q = \rho A x$

flux $= EA$

just know examples

1.E (4 pts) I In experiment 6 you measure the current on the right for your RC circuit, and plot $\ln\left(\frac{I(t)}{I_{max}}\right)$ vs. t What is the time constant?



a. 2 seconds
b. 4 seconds
c. 0.5 seconds
d. 0.25 seconds
e. I do not know (worth 1 point)

$\tau = ?$

fudge - not good at

RC → current will start at max + decay to 0

$I(t) = I_{max}\, e^{-1/\tau}$

Since log scale → $\ln\left(\frac{I(t)}{I_{max}}\right) = -\left(\frac{1}{\tau}\right) t$

did not know converted to this

well do calculations + work backwards

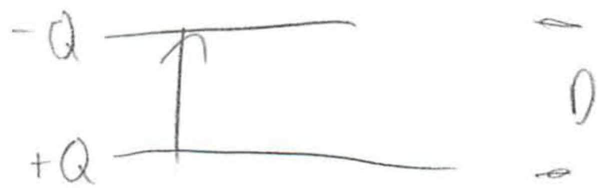
$\tau = \frac{.25}{1} = 4$

If charge does not change → neither does E field
But $U \propto (E^2, vol)$ E so U ↓
$V = E \cdot d$ so V ↓
voltage

1.F (4 pts) A parallel plate capacitor is charged up to charge Q with the plates separated by a distance D. The battery is then disconnected, then the plates are moved closer, to a separation of $d<D$. What happens to the Stored Energy in the capacitor U and the Voltage V?

a) U increases, V increases
b) U increases, V stays the same
c) U increases, V decreases
d) U stays the same, V increases
e) U stays the same, V stays the same
f) U stays the same, V decreases
g) U decreases, V increases
h) U decreases, V stays the same
i) U decreases, V decreases
j) I do not know (1 point)

forget this

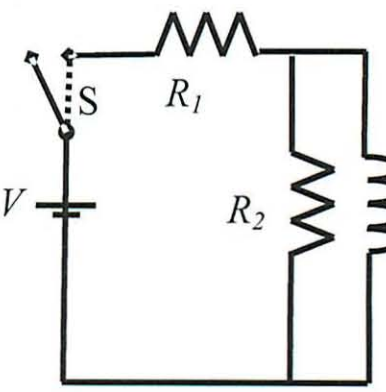


$V = \frac{Q}{C}$ C ↑ changes

1.G (4 pts) A circuit consisting of an inductor of inductance L,, a battery providing voltage V, a switch S, and two resistors of resistance R_1 and R_2 is shown on the right. At time t=0, the switch is closed. ***After a long time***, what is the current through resistor R_2?

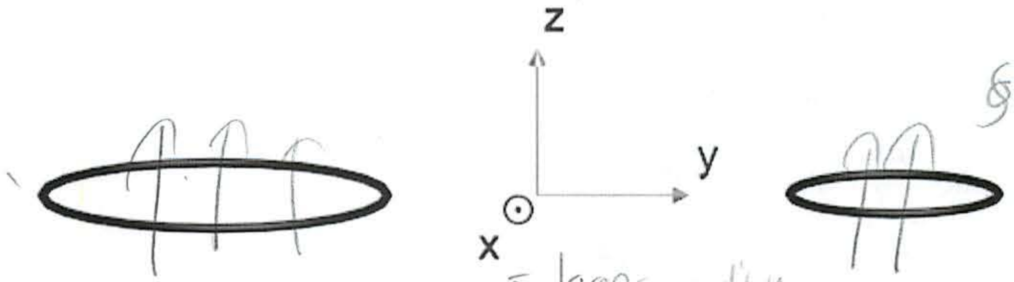
a) 0
b) V / R_2
c) $V / (R_1 + R_2)$
d) $V / (R_2 + L)$
e) $V \times t / L$
f) I don't know (1 point)

all goes inductor ✓



parallel!

1.H (4 pts) Two charged particles of identical mass and charge move in circular orbits in the same constant magnetic field $\mathbf{B} = B_o\hat{\mathbf{z}}$. The two particles have different speeds.



∮ B·ds

= larger radius

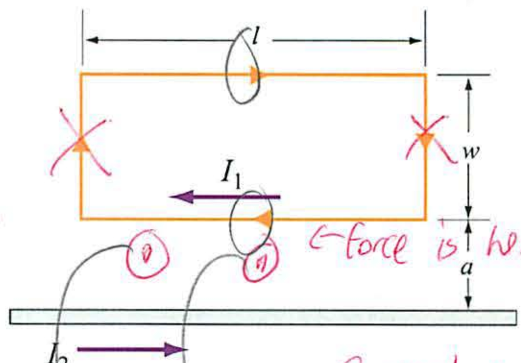
The orbit of the particle with the ***larger speed*** will
a) have a larger radius and a longer period as compared to that of the slower particle
b) have a larger radius and the same period as compared to that of the slower particle
c) have a larger radius and a shorter period as compared to that of the slower particle ; formula
d) have a smaller radius and a longer period as compared to that of the slower particle
different speed
e) have a smaller radius and the same period as compared to that of the slower particle
f) have a smaller radius and a shorter period as compared to that of the slower particle
g) I do not know (1 point)

So ~~same~~ diff speed

✓ ; f

1.I (4 pts) A wire loop carries current I_1, and is located near an infinite wire carrying current I_2. The currents flow in the directions shown. The net force on the wire loop due to the presence of the infinite wire is
a) upwards
b) downwards
c) to the left
d) to the right
e) I don't know (1 point)

✓ decreases ↑ w/ ↑ distance



← force is here

; repel so up ;

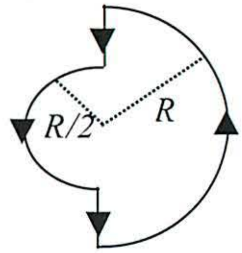
$F = I(\vec{v} \times B)$
$t \times ⊙ = \uparrow$

; right

glad this is just pratice,

but will prob end up w 75% - which would cement my final grade

1.J (4 pts) A current of value I goes in a semicircle of radius R, then radially in a distance of $R/2$, then in another semicircle, and rejoins with the first semicircle, as depicted. What is the magnetic field in the center of the circuit?



a) $\frac{3\mu_0 I}{4R}$, into the page

b) $\frac{3\mu_0 I}{4R}$, out of the page ✓

c) $\frac{3\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi R}$, into the page

d) $\frac{3\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi R}$, out of the page

e) I don't know (1 point)

Oh I rember studying this

$\oint B \cdot ds = \mu_0 I_{enc}$

$2\pi R \quad I \pi R^2$

$B = \mu_0 \frac{I \cancel{\pi} R^{\cancel{2}}}{2 \cancel{\pi} \cancel{R}} = \frac{\mu_0 I R}{2}$

$\downarrow +$

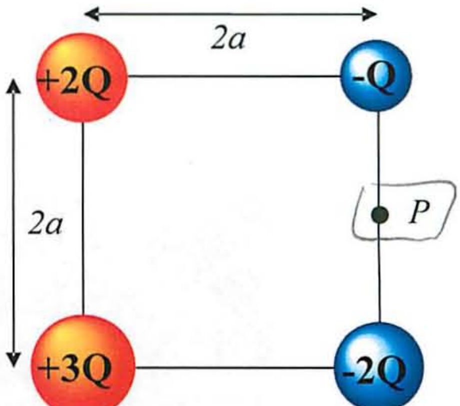
$\frac{\mu_0 I \cancel{\pi} \frac{R}{2}}{2} = \frac{\mu_0 I \cancel{\pi} R}{4}$

Out of pg

$\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

Remember to use formula sheet

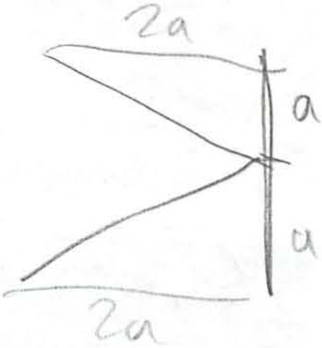
Problem 2 Charge Configuration (20 points)
4 different charges lie on a square of side length $2a$ as depicted at right. The magnitude of charge starting from top left and moving clockwise is $+2Q$, $-Q$, $-2Q$, $+3Q$. We are interested in the point P, which is on the right side of the square, halfway between the corners



a) Use Coulomb's Law and superpostition to find the Electric field $\mathbf{E}$ at point P.

grr

kinda remember
was only on last years
formula sheet



$E = k\dfrac{dq}{r^2}$

$\dfrac{-q}{a^2} + \dfrac{-2q}{a^2} + \dfrac{+2q}{5a^2} + \dfrac{3q}{5a^2}$

$r_1 = \sqrt{a^2+(2a)^2} = \sqrt{a^2+4a^2} = \sqrt{5a^2}$

more complex then what I wrote
have to break up into coordinate,
difficult ↑ + ↗

$\dfrac{5q}{5a^2} - \dfrac{3q}{a^2}$

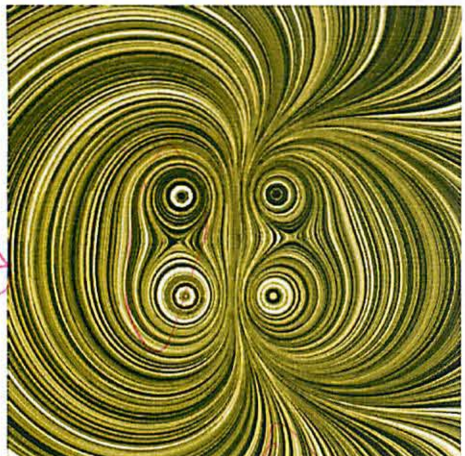
$\dfrac{q-3q}{a^2} = \boxed{-\dfrac{2q}{a^2}}$

b) Which of the following "grass seed" representations (A-F) most accurately describes the

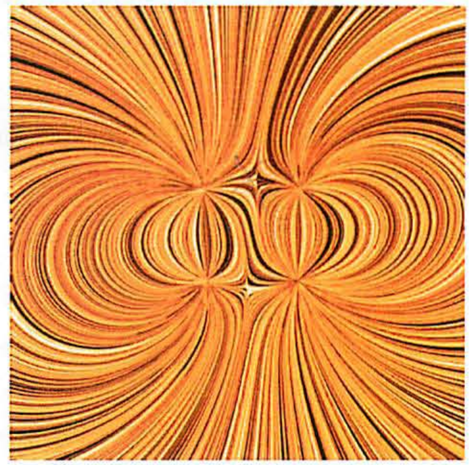
i) field lines?__________ yellow C

ii) equipotential lines? __________ green ~~C~~ A

like charges loop

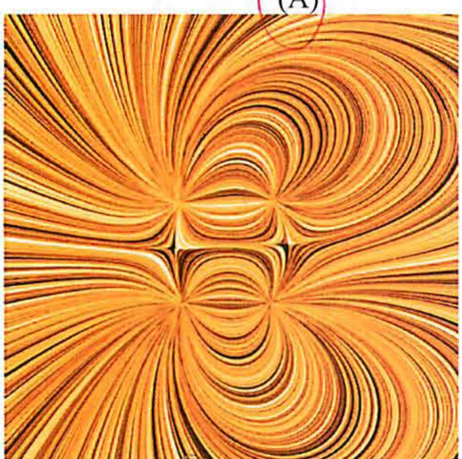


(A)



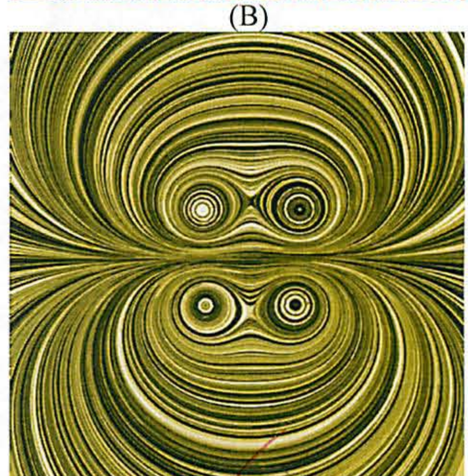
(B)

opposet



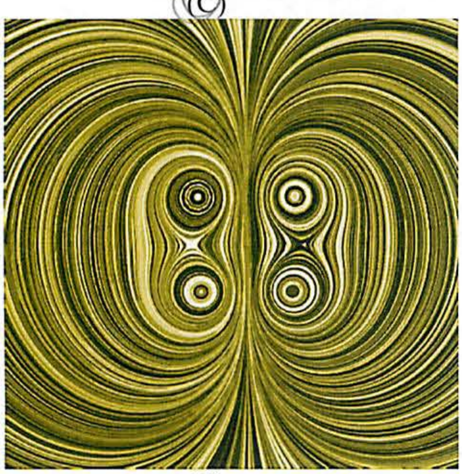
(C)

appears different levels

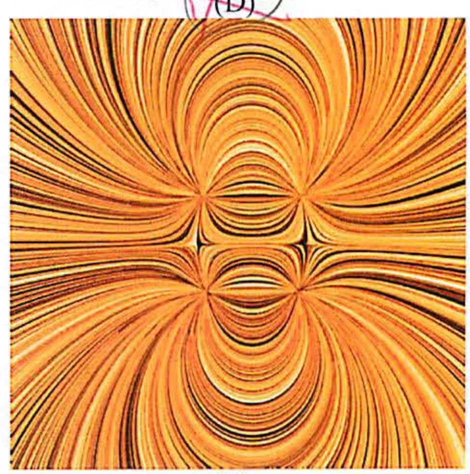


(D)

opposet don't !



(E)



(F)

c) How much energy would it take to move a test charge $+q$ ($q>0$) to point P assuming it started infinitely far away, where the potential is zero?

Familiar qu but forgot

U = what again qV

add up Voltage of each but not vector $\boxed{\Sigma V = \frac{kq}{r}}$

$E = \frac{kq}{r^2}$

d) You are given a single point charge whose position and magnitude you choose, with the goal of making the $\vec{\mathbf{E}}$ field zero at point P. Where do you put the charge and what magnitude do you use? **[Only f you did not get an answer to part a):** assume the field at P has the form $\vec{\mathbf{E}} = E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}}$ and use that to solve the problem symbolically].

So go $-2q/a^2$

Place $+2q$

distance a away

like at center

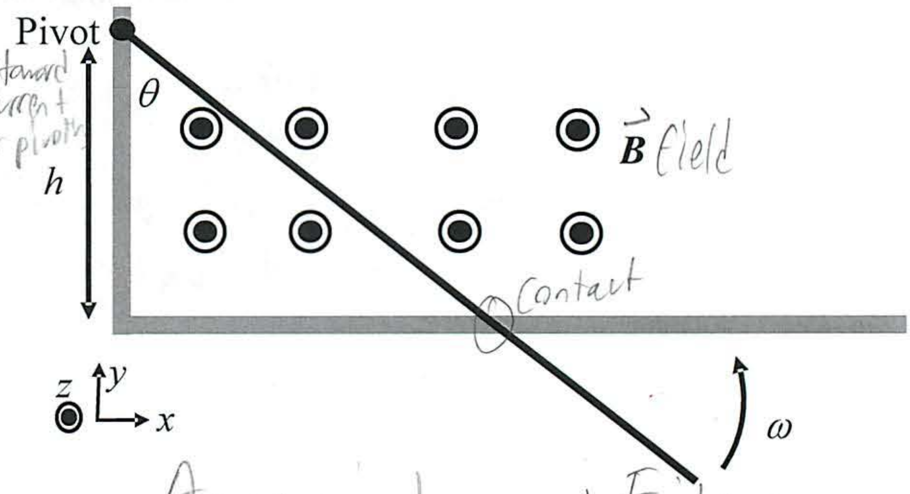
in order to get 0

$$\left[a + \frac{2}{\sqrt{5}}a,\ -a\left(1 - \frac{1}{5\sqrt{5}}\right)\right]$$

then find magnitude

Problem 3   Resistive Bar on a pivot (30 points)

A very long resistive bar with **resistance per unit length** r is attached to a conducting right angle frame at one end by a pivot a distance h from the right angle, with the other end resting on the bar, making a triangular circuit. To simplify things assume the frame has negligible resistance. A constant magnetic field with magnitude B perpendicular to the apparatus permeates the area, coming out of the page as shown.



Ampere's Law + Faradays

The bar starts at $t=0$ along the y axis and is rotated counterclockwise about the pivot with constant angular velocity ω, such that the pivot angle $\theta = \omega t$. Assume the bar is long enough such that it never loses contact with the horizontal side of the frame.

$\cos\theta = \frac{h}{r}$

$r\cos\theta = h$

$r = \frac{h}{\cos\theta}$

$\tan\theta = \frac{r}{h}$

$r = h\tan\theta$

a) What is the magnetic flux penetrating the circuit as a function of (t, ω, B, h, r)?

$\oint B \cdot ds = \mu_0 I_{enc}$

$B \frac{1}{2} h^2 \tan(\omega t)$ ✓ was right

$\vec{B} = h + h\tan\theta + \frac{h}{\cos\theta} = \mu_0 \frac{1}{2}(h)\cdot(h\tan\theta) I$

$\theta = \omega t$ ✓

$\phi = \int B \cdot dA$ ← right? not on sheet

$\frac{\mu_0 \frac{1}{2} h^2 \tan\theta}{\cancel{h(1+\tan\theta+\frac{1}{\cos\theta})}} \cdot \cancel{h + h\tan\theta} + \frac{h}{\cos\theta} = \boxed{\frac{\mu_0 h^2 \tan\theta I}{2}}$

They don't define B

b) What is the resistance of the circuit as a function of (t, ω, B, h, r)?

$V = IR$

$R = rl = \frac{rh}{\cos(\omega t)}$

$\varepsilon = -\frac{d}{dt}\phi = \cancel{\varepsilon_0 \frac{d}{dt}\phi R}$

confusing $\vec{E}$ + $\vec{B}$ flux

duh simple explination

= circular?

$\frac{-\frac{d\phi}{dt}}{\frac{1}{2}h^2\tan\theta I}$ ← whats changing θ − deriv of $\tan\theta = ??$

$\frac{\mu_0 h^2 I \frac{d}{dt}\tan\theta}{h^2 \tan\theta I} = \frac{\mu_0 \frac{d}{dt}\tan\theta}{\tan\theta}$

seems wrong

c) Using Faraday's law, find the induced emf for our circuit and the resulting current, again in terms of t, ω, B, h, and r. Indicate which way the current will flow on the diagram. It may help to know that $\frac{d}{d\theta}(\tan\theta) = \frac{1}{\cos^2\theta}$. dh think I jumped ahead

$$\mathcal{E} = -\frac{d}{dt}\Phi = -\frac{\mu_0 h^2 I}{2\cos^2\theta} \quad I = \frac{Bh\omega}{2r\cos(\omega t)}$$

Clockwise

need to define

d) What is the force on the resistive bar in terms of t, ω, B, h, and r? Be sure to indicate magnitude and direction. Does it get easier or harder as time goes on to keep it rotating at constant ω?

Resistive Force

~~R~~ was that the R looking force

what is diff resistance + resistive force?

property of wire / force puts on

$$F = q(\vec{v} \times \vec{B}) = I(\vec{L} \times \vec{B}) = ILB$$

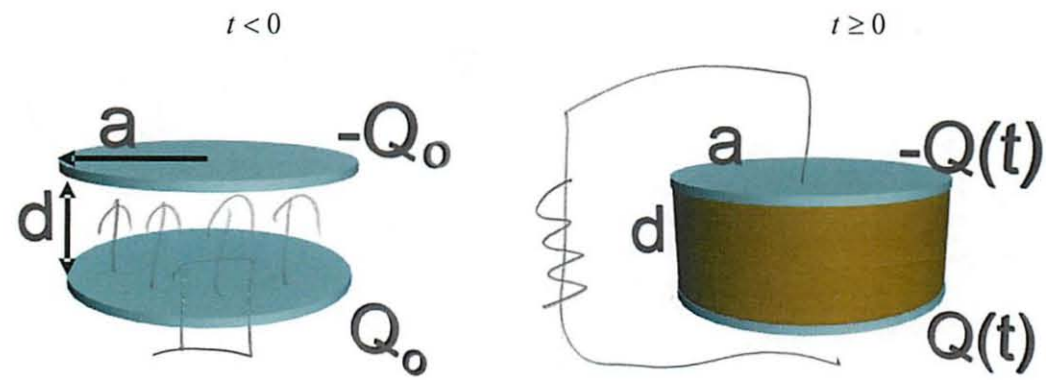
whats L again?

lenght

$$= \frac{Bh\omega}{2r\cos(\omega t)} \frac{h}{\cos(\omega t)} B = \frac{B^2h^2\omega}{2r\cos(\omega t)}$$

Vector so need to paramitrize bar

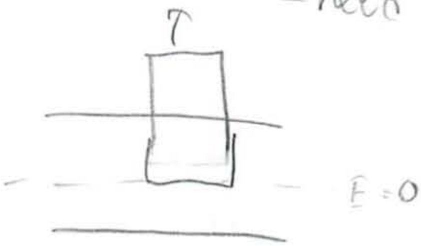
Problem 4 Discharging Capacitor through a resistor (30 points)
A capacitor consists of two parallel circular plates of radius a separated by a distance d (assume $a >> d$). The capacitor is initially charged to a charge Q_o. At $t = 0$, this capacitor begins to discharge because **we insert a circular resistor** of radius a and height d between the plates, such that the ends of the resistor make good electrical contact with the plates of the capacitor.. The capacitor then discharges through this resistor for $t \geq 0$, so the charge on the capacitor becomes a function of time $Q(t)$. *Throughout this problem you may ignore edge effects.*



a) Use Gauss's Law to find the electric field between the plates in terms of $Q(t)$. Is this electric field upward or downward?

pill box

-need to review to lock down this answer



$$\oint E \cdot dA = \frac{Q}{\epsilon_0}$$

$$E \cdot \pi a^2 = \frac{Q_0}{\epsilon_0} \cdot \text{area}$$

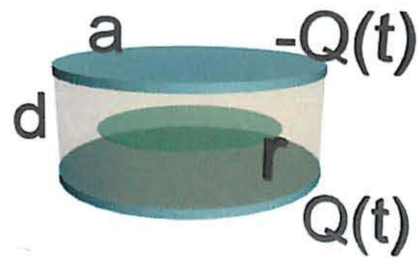
$$E = \frac{Q_0}{\epsilon_0 \cancel{\pi a^2}} \quad \boxed{UP}$$

✓

this problem → need to memorize

b) For $t \geq 0$, consider an imaginary open surface of radius $r < a$ inside the capacitor with its normal $\mathbf{d\vec{A}}$ upward (see figure)

think doing ok on this



displacement current stuff

For $t \geq 0$, what is the conduction current flowing through this open surface in terms of $Q(t)$ or $\frac{dQ(t)}{dt}$ and the parameters given. Define the direction of positive current to be upward, and be careful about signs, in particular because $\frac{dQ}{dt} < 0$.

I

$J = \frac{-dQ}{dt} \frac{1}{\pi a^2}$

$I_d = \epsilon_0 \frac{d}{dt} \Phi$ Electric

how are we supposed to know?

$\epsilon \pi r^2$ Look at ans

$= J \pi r^2 = \frac{-dQ}{dt} \frac{r^2}{a^2}$ or $-I \frac{\pi r^2}{\pi a^2}$

portion of I that goes trough

c) For this same imaginary open surface, what is the **time rate of change** of the electric flux though the surface, in terms of $Q(t)$ or $\frac{dQ(t)}{dt}$ and the parameters given (hint: use your answer above for **E**).

forget this one too

$\Phi(t) = \epsilon \pi r^2 \ldots$

$\vec{E} = \frac{Q}{\pi a^2 \epsilon}$ $\Phi = \frac{Q \pi r^2}{\pi a^2 \epsilon_0} = \frac{Q}{\epsilon_0}\left(\frac{r^2}{a^2}\right)$

$\frac{d\Phi_E}{dt} = \frac{1}{\epsilon_0}\left(\frac{r^2}{a^2}\right)\frac{dQ}{dt}$ ← yeah change $Q \to \frac{dQ}{dt}$ + leave

d) What is the integral of the magnetic field around the contour bounding this open circle, using the Ampere-Maxwell Law? Be careful of signs.

$\int B \cdot ds = \mu_0 I_{enc}$

add up currents + apply Ampere's law

$\oint B \cdot dl = \mu_0 \left(\frac{-dQ}{dt} \frac{r^2}{a^2} + \epsilon_0 \left[\frac{1}{\epsilon_0} \frac{r^2}{a^2} \frac{dQ}{dt} \right] \right)$

$= 0$

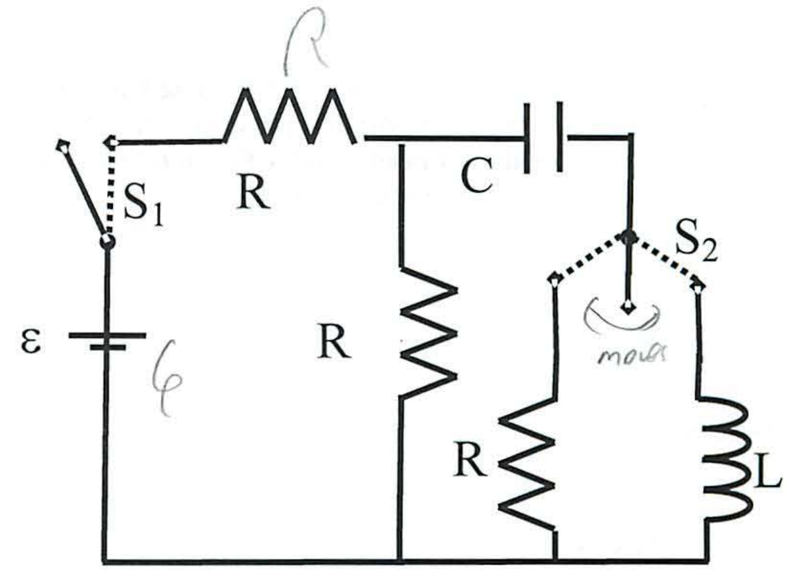
e) Does your answer in (d) make sense in terms of the energy dissipation and energy flow in this problem? You must explain your answer clearly and logically to get credit.

Yeah really need to review problem

Yes b/c energy flowing out of capicator immediaty disipated by resistor

Problem 3 (25 pts): RC circuit

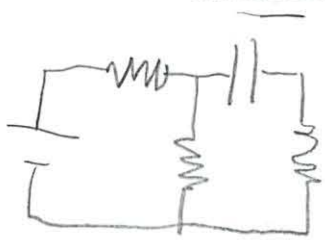
The circuit at right contains a battery of emf ε, 3 resistors each of resistance R, a capacitor with capacitance C, an inductor with inductance L, and **two switches** – switch S_1 switches the battery into the circuit, and S_2 switches *either* the resistor or the inductor into the circuit (the dotted lines in the figure indicate possible connections for the switches).



At first, S_2 is set to connect to the resistor into the circuit.

remember Capicator like a Short at first!!!

a. At $t=0^+$, S_1 is closed, putting the battery into the circuit. What is the instantaneous current into the capacitorat time t=0+?



$I = 0$ into capicator

takes time to charge up

(or am I flipping this)

$I = \frac{\varepsilon}{3R}$

b. After the capacitor is fully charged (at t = T), what is the total charge on the capacitor?

$Q = CV = \frac{C\varepsilon}{2}$

what is this?

do I have to do full loop law?

no current flowing

$I_{tot} = \frac{\varepsilon}{2R}$ - Resistors in parallel

c. Now, you open the switch S_1. Use Kirchoff's laws to write a differential equation which governs the discharging of the capacitor. (S_2 still connects to the resistor). **You do not need to solve the equation.** What is the time constant for discharging the capacitor?

$$\frac{Q}{C} - IR - IR = 0 \qquad \frac{Q}{C} - I2R = 0$$

$$I(t)\ \frac{Q}{2RC}$$ ✓ $\tau = 2RC$

, more

<u>Once the capacitor is fully discharged, S_2 is set to connect to the inductor into the circuit.</u>

d. Let's reset the time to $t=0$ again and close switch S_1. Now what is the instantaneous current in the capacitor? Explain your answer.

$I=0$ again since ~~capacitor~~ inductor takes time to charge ✓

e. After a while, after the capacitor becomes fully charged, you open switch S_1 again. Now you have a damped undriven oscillator. Use Kirchoff's laws to write down a differential equation describing the circuit behavior. **You do not need to solve the equation.** If one ignores the small effect of the resistance, what is the fundamental frequency of this oscillator ω_0?

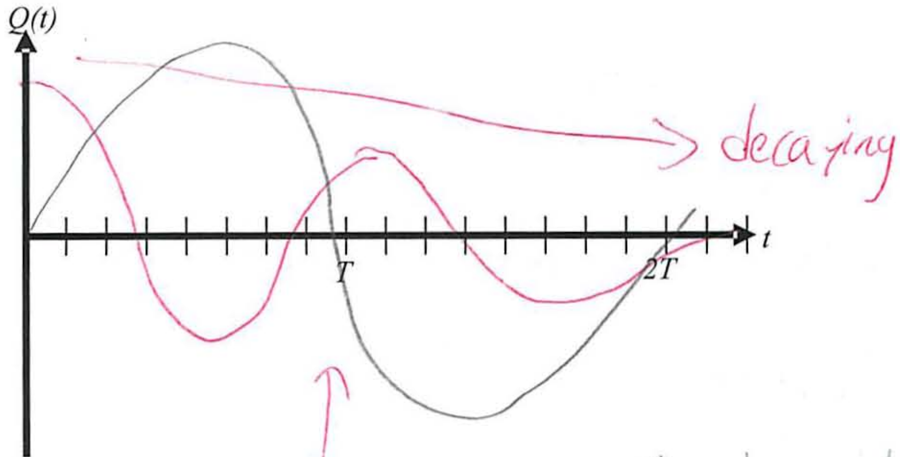
$$\frac{dI}{dt} = \frac{d^2Q}{dt}$$

$$\frac{Q}{C} - L\frac{dI}{dt} - IR = 0$$

$$I(t) = I_0 \sin(\omega t)$$

$$\omega = \frac{1}{\sqrt{LC}}$$ ✓

f. If the decay time τ for this circuit is $2T$, where $T = \frac{2\pi}{\omega}$ is the period of oscillation, sketch the **charge** on the capacitor as a function of time measured in units of the period T on the graph below (be sure to label the y axis scale using the amount of charge at $t=0$, when you first open the switch)



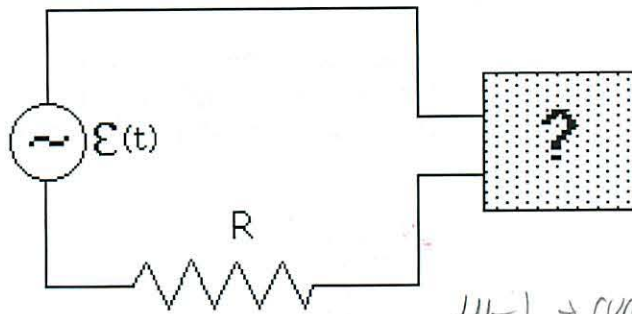
one period

~~duh~~ think about

(had an inkling)

Guessing w/o much thought

Problem 4 (26 pts): What's in the box? The circuit shown contains an AC generator which provides a source of sinusoidally varying emf $\mathcal{E}(t) = \mathcal{E}_m \sin(\omega t)$, a resistor with resistance $R = 2$ ohm, and a "black box", which contains *either* an inductor *or* a capacitor, *or both*.



The amplitude of the driving emf, $\mathcal{E}_m$, is 1 Volt. We first measure the current in the circuit at an angular frequency ω =1 radians/sec and find that it is leads the driving emf by exactly $\pi/3$ radians. We then measure the current in the circuit at an angular frequency ω =2 radians/sec and find that it lags the driving emf by exactly $\pi/3$ radians. [Note: $\pi/3$ radians = 60 degrees, $\tan(\pi/3) = \sqrt{3}$].

$\omega=1 \rightarrow$ current leads voltage $\frac{\pi}{3}$

$\omega=2 \rightarrow$ current lags voltage $\frac{\pi}{3}$

a. What does the black box contain--an inductor or a capacitor, or both? Explain your reasoning.

both

changing ω ✓

b. What is the numerical value of the current in the circuit at ω =1 radians/sec? At ω =2 radians/sec? You ***do not have to determine*** the unknown L and/or C to answer this question, and the variables L and C should not appear in your answer. Your answer can involve square roots.

$I = I_0 \sin(\omega t)$

anything else?

it varies w/ time

they want I_0

$$I = \frac{V}{Z} = \frac{1V}{\sqrt{R^2+(X_L-X_C)^2}} = \frac{1V}{R\sqrt{1+\left(\frac{X_L-X_C}{R}\right)^2}}$$

$$= \frac{1V}{2\Omega\sqrt{1+\underbrace{\tan^2\phi}_{=\sqrt{3}}}} = \boxed{\frac{1}{4}A}$$

Use information from each freq seperatly. At each freq the impedance is 2R so get 2 eq. to solve for L + C

c. What is the numerical value of the capacitance *C or* of the inductance *L, or of both*, as the case may be? Indicate units. Your answer(s) will involve simple fractions and square roots. You will not need a calculator to find the value(s).

What is the shortest way to find this again?

$\boxed{\omega L - \frac{1}{\omega C} = \tan\phi \, R}$

Find $\omega = 1.5$

$L - \frac{1}{1C} = -\sqrt{3}(2)$ $\quad$ $1.5 = \frac{1}{\sqrt{LC}}$ $\quad$ don't think that is right esp since bottom uses

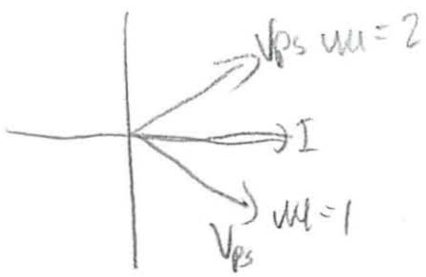
$L - \frac{1}{2C} = \sqrt{3}(2)$ .... $\quad$ $L = 2\sqrt{3}\,H$

$C = \frac{1}{4\sqrt{3}}\,F$

d. Doing no math, tell us in which frequency range you would drive this circuit to produce the maximum current (circle one)?

< 1 rad/sec $\qquad$ between 1 rad/sec and 2 rad/sec ✓ $\qquad$ > 2 rad/sec

Briefly explain why you choose the range you did (a plot might help).



must be in middle

e. Now calculate the numerical value of the angular frequency for which the current in the circuit will be a maximum. If you are uncertain in your answer in (c), make sure you give you answer in terms of variables first.

~~ω 1.5 in the middle~~ no use ft

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\sqrt{3} \cdot \frac{1}{4\sqrt{3}}}} = \sqrt{2} \text{ rad/sec}$$

If your circuit contains an inductor, give an expression for the voltage drop across the inductor as a function of time when the current is at a maximum. If you circuit does not contain an inductor, give an expression for the voltage drop across the capacitor as a

Think not too burnt out to continue working function of time when the current is at a maximum. You need only give one expression. If you cannot decide which it was, choose one, but in all cases clearly indicate whether your expression is for an inductor or for a capacitor. The amplitude of your time function should be a numerical value in Volts.

$$V_C = \frac{Q}{C}$$

$$V_I = L\frac{dI}{dt} = I X_L = I\omega L$$

Shald use it oh well
will try harder to at least right something on final

$$I = \frac{V}{R} = \frac{1}{2} A$$

but inductor leads current by $90°$

$$\sin(\omega t + \frac{\pi}{2}) = \cos(\omega_0 t)$$

$$V_L = \frac{1}{2} A \left(\sqrt{2}\ \frac{rad}{sec}\right) 2\sqrt{3}\ H \cos(\sqrt{2}t)$$

$$= \sqrt{6} \cos(\sqrt{2}t) A$$

This is like way too hard

Solutions

8.02 Final Exam Fall 2008

FAMILY (last) NAME

GIVEN (first) NAME

Student ID Number

Your Section: ____L01 MW 10 am   ____L02 MW 12 pm
Your Group (e.g. 10A): ______________

No notes or calculators allowed. Please show all your work on the analytical questions. Answers quoted without any work will receive no credit, regardless of whether or not they are actually correct.

	Score	**Grader**
Problem 1 (40 points)		
Problem 2 (20 points)		
Problem 3 (30 points)		
Problem 4 (30 points)		
Problem 5 (20 points)		
Problem 6 (40 points)		
TOTAL (180 points)		

Problem 1 10 Concept questions, 4 points each, 40 points total

1.A (4 pts) If I have a uniformly charged insulating sphere of radius R with total charge Q, I find that outside the sphere at some arbitrary distance r away, I have an electric field of $\vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}$. Which of the following actions would change this $\vec{\mathbf{E}}$ field?

a) Reducing R by ½.
b) Making the sphere conducting rather than insulating
c) Breaking the spherical symmetry, such as deforming the sphere into the shape of an egg
d) None of the above
e) I do not know (1 pointt)

None of the options affect the enclosed charge, while for c) some charge will be closer and other will be further away, thus changing the field.

1.B (4 pts) I have an AC power supply and two *different* circuit elements in series, R, L, or C. If I arbitrarily increase the driving frequency of my power supply, I find the amplitude of the current measured always increases. Which two elements are in my circuit?

a) R and C
b) R and L
c) L and C
d) I do not know (1 pt)

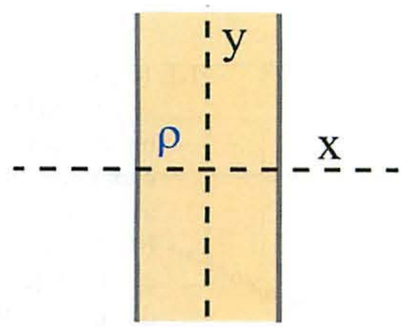
In this case as you increase the frequency the current goes up, so the impedance goes down, so there cannot be an inductor in the circuit. There is a resistor and a capacitor, and because the capacitor impedance decreases with increasing frequency, you always get higher current for higher frequency.

1.C (4 pts) If you place a negatively charged particle in an electric field, the charge will move

a) from higher to lower electric potential and from lower to higher potential energy.
b) from higher to lower electric potential and from higher to lower potential energy.
c) from lower to higher electric potential and from lower to higher potential energy.
d) from lower to higher electric potential and from higher to lower potential energy.
e) I don't know (1 point)

Particles always move towards lower potential energy (remember $\vec{\mathbf{F}} = -\nabla U$) but for a negative charge that means moving towards higher electric potential

1.D (4 pts) If I have a uniform charge distribution ρ over some semi-infinite volume as pictured at right, ***inside the volume*** the electric field dependence on distance x from the center is

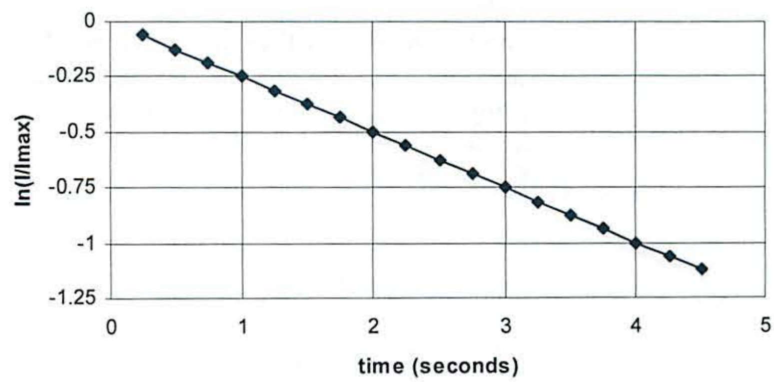


a) $\left|\vec{\mathbf{E}}\right| \propto \frac{1}{x^2}$
b) $\left|\vec{\mathbf{E}}\right| \propto \frac{1}{x}$
c) $\left|\vec{\mathbf{E}}\right| \propto \ln(x)$
d) Does not depend on x
e) $\left|\vec{\mathbf{E}}\right| \propto x$
f) $\left|\vec{\mathbf{E}}\right| \propto x^2$
g) I don't know (1 pt)

Inside a volume, the total charge enclosed will be $\rho A x$ where x is the distance from the symmetry axis, and the flux will be EA, so the E field will be proportional to x.

1 A. (4 pts) In experiment 6 you measure the current on the right for your RC circuit, and plot $\ln\left(I(t) / I_{\max} \right)$ vs. t What is the time constant?

a. 2 seconds
b. 4 seconds
c. 0.5 seconds
d. 0.25 seconds
e. I do not know (worth 1 point)



In an RC circuit, the current will start at a maximum and then decay to zero as the charge builds up, according to the expression $I(t) = I_{\max} e^{-t/\tau}$ *so* $\ln\left(I(t) / I_{\max} \right) = -\left(\frac{1}{\tau}\right) t$.

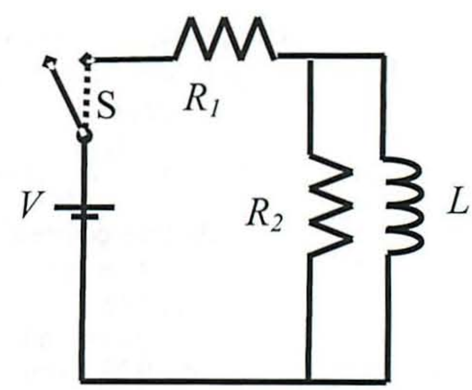
This means you get the time constant from the slope of the line, $\tau = \left(0.25 / 1s\right)^{-1} = 4\,s$

1.E (4 pts) A parallel plate capacitor is charged up to charge Q with the plates separated by a distance *D*. The battery is then disconnected, then the plates are moved closer, to a separation of $d<D$. What happens to the Stored Energy in the capacitor *U* and the Voltage *V*?

a) *U* increases, *V* increases
b) *U* increases, *V* stays the same
c) *U* increases, *V* decreases
d) *U* stays the same, *V* increases
e) *U* stays the same, *V* stays the same
f) *U* stays the same, *V* decreases
g) *U* decreases, *V* increases
h) *U* decreases, *V* stays the same
i) *U* decreases, *V* decreases
j) I do not know (1 point)

If the charge doesn't change, neither does the E field, but the total energy is proportional to the E^2 times the volume, and the volume decreases so the Energy decreases, and the Voltage is E time the distance, which also decreases, so the Voltage decreases.

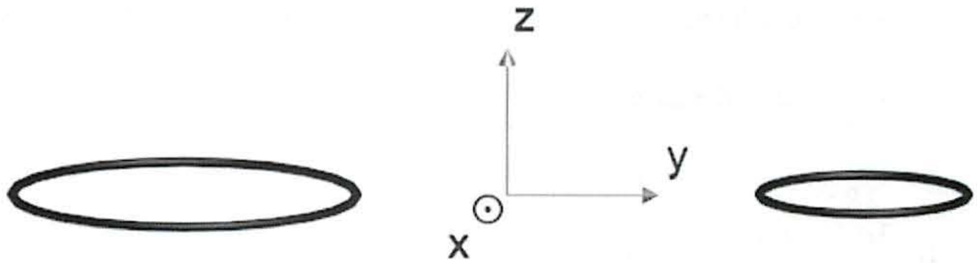
1.F (4 pts) A circuit consisting of an inductor of inductance *L*,, a battery providing voltage *V*, a switch *S*, and two resistors of resistance R_1 and R_2 is shown on the right. At time t=0, the switch is closed. ***After a long time***, what is the current through resistor R_2?



a) 0
b) V / R_2
c) $V / (R_1 + R_2)$
d) $V / (R_2 + L)$
e) $V \times t / L$
f) I don't know (1 point)

After a long time, the inductor acts like a short. The current will take the path of least resistance, and you can't get less than zero! So all the current will run down the inductor, none through the resistor.

1.G (4 pts) Two charged particles of identical mass and charge move in circular orbits in the same constant magnetic field $\mathbf{B} = B_o\hat{\mathbf{z}}$. The two particles have different speeds.

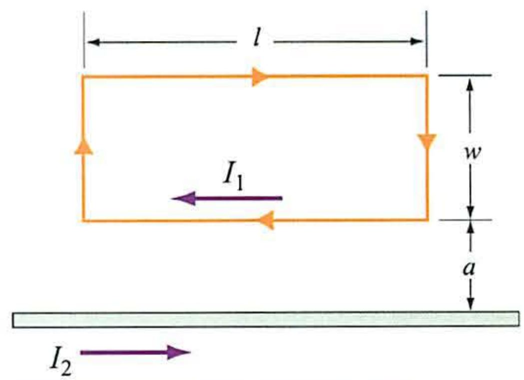


The orbit of the particle with the ***larger speed*** will

a) have a larger radius and a longer period as compared to that of the slower particle
b) have a larger radius and the same period as compared to that of the slower particle
c) have a larger radius and a shorter period as compared to that of the slower particle
d) have a smaller radius and a longer period as compared to that of the slower particle
e) have a smaller radius and the same period as compared to that of the slower particle
f) have a smaller radius and a shorter period as compared to that of the slower particle
g) I do not know (1 point)

Here we combine the Magnetic Lorentz force with circular motion to find $\frac{mv^2}{r} = qvB \rightarrow r = \left(\frac{m}{qB}\right)v$ so the larger speed will have the larger radius. The period is given by $T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$ so that is the same for both particles.

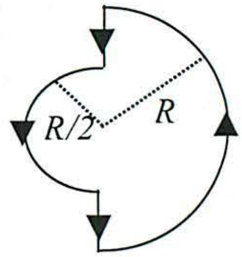
1.H (4 pts) A wire loop carries current I_1, and is located near an infinite wire carrying current I_2. The currents flow in the directions shown. The net force on the wire loop due to the presence of the infinite wire is



a) upwards
b) downwards
c) to the left
d) to the right
e) I don't know (1 point)

The current I_2 generates a magnetic field out of the page, which decreases with increasing distance. For the two vertical legs the forces are equal and opposite, so they cancel, while for the horizontal legs the lower one is in the area of higher field, so it is not completely canceled by the upper leg's contribution. From the right hand rule, the force from the lower leg and therefore the net force is up.

1.I (4 pts) A current of value I goes in a semicircle of radius R, then radially in a distance of $R/2$, then in another semicircle, and rejoins with the first semicircle, as depicted. What is the magnetic field in the center of the circuit?



a) $\frac{3\mu_0 I}{4R}$, into the page

b) $\frac{3\mu_0 I}{4R}$, out of the page

c) $\frac{3\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi R}$, into the page

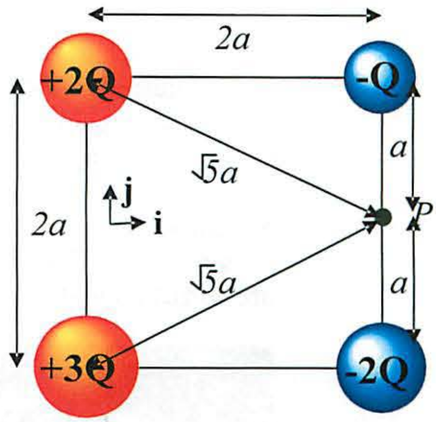
d) $\frac{3\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi R}$, out of the page

e) I don't know (1 point)

This is an application of the Biot-Savart law. The two semicircles by the right hand rule give a contribution out of the page. For a semicircle, we know $\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\frac{d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2} \Rightarrow \left|\vec{\mathbf{B}}\right| = \frac{\mu_0 I}{4\pi}\frac{\int R\,d\theta}{R^2} = \frac{\mu_0 I}{4R}$, so we have to add the two together to find $\left|\vec{\mathbf{B}}\right| = \frac{\mu_0 I}{4R} + \frac{\mu_0 I}{4\frac{R}{2}} = \frac{3\mu_0 I}{4R}$. The radial legs do not contribute at all, since the path is parallel to the displacement, so the cross product is zero.

Problem 2 Charge Configuration (20 points)

4 different charges lie on a square of side length $2a$ as depicted at right. The magnitude of charge starting from top left and moving clockwise is $+2Q$, $-Q$, $-2Q$, $+3Q$. We are interested in the point P, which is on the right side of the square, halfway between the corners



a) Use Coulomb's Law and superpostition to find the Electric field $\vec{\mathbf{E}}$ at point P.

Here we need to figure out the contribution from each charge, and then add them all up. The positive charges are a distance $\sqrt{a^2+(2a)^2}=\sqrt{5}a$, and the negative ones are a distance a away. It is easiest to use the r^3 version of Coulomb's law: $\vec{\mathbf{E}}=\frac{q}{4\pi\varepsilon_0 r^3}\boxed{\vec{\mathbf{r}}}$. Breaking up into coordinates, we have:

$$\vec{\mathbf{E}}_1=\frac{2Q}{4\pi\varepsilon_0\left(\sqrt{5}a\right)^3}\left[2a\hat{\mathbf{i}}-a\hat{\mathbf{j}}\right]=\frac{Q}{4\pi\varepsilon_0 a^2}\left[\frac{4}{5\sqrt{5}}\hat{\mathbf{i}}-\frac{2}{5\sqrt{5}}\hat{\mathbf{j}}\right]$$

directional aspect

$$\vec{\mathbf{E}}_2=\frac{3Q}{4\pi\varepsilon_0\left(\sqrt{5}a\right)^3}\left[2a\hat{\mathbf{i}}+a\hat{\mathbf{j}}\right]=\frac{Q}{4\pi\varepsilon_0 a^2}\left[\frac{6}{5\sqrt{5}}\hat{\mathbf{i}}+\frac{3}{5\sqrt{5}}\hat{\mathbf{j}}\right]$$

$$\vec{\mathbf{E}}_3=\frac{-2Q}{4\pi\varepsilon_0(a)^3}\left[a\hat{\mathbf{j}}\right]=\frac{Q}{4\pi\varepsilon_0 a^2}\left[-2\hat{\mathbf{j}}\right]$$

$$\vec{\mathbf{E}}_4=\frac{-Q}{4\pi\varepsilon_0(a)^3}\left[-a\hat{\mathbf{j}}\right]=\frac{Q}{4\pi\varepsilon_0 a^2}\left[\hat{\mathbf{j}}\right]$$

$$\vec{\mathbf{E}}_{\text{total}}=\sum_{i=1}^{4}\vec{\mathbf{E}}_i=\frac{Q}{4\pi\varepsilon_0 a^2}\left[\left(\frac{6}{5\sqrt{5}}+\frac{4}{5\sqrt{5}}\right)\hat{\mathbf{i}}+\left(\frac{3}{5\sqrt{5}}-\frac{2}{5\sqrt{5}}+1-2\right)\hat{\mathbf{j}}\right]$$

$$\boxed{\vec{\mathbf{E}}_{\text{total}}=\frac{Q}{4\pi\varepsilon_0 a^2}\left[\frac{2}{\sqrt{5}}\hat{\mathbf{i}}-\left(1-\frac{1}{5\sqrt{5}}\right)\hat{\mathbf{j}}\right]}$$

add $\hat{i}$ and $\hat{j}$ seperatly

vector answer!

(perhaps try eaiser problem)

$k\frac{Q}{r^3}[\vec{r}]$

vector component of r

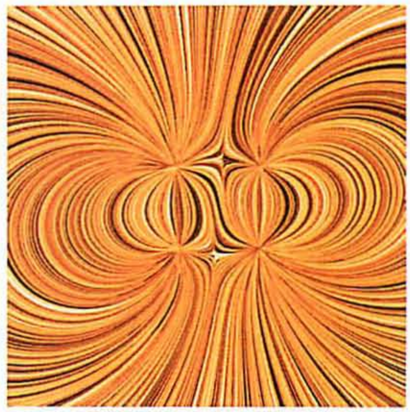
add up ~~..~~ $\hat{i}$ + $\hat{j}$ seperatly

b) Which of the following "grass seed" representations (A-F) most accurately describes the
   i) field lines? ___C___
   ii) equipotential lines? ____A____

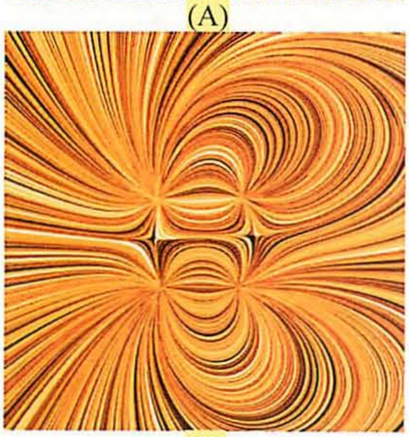
For the field lines, the zeros should be aligned along a horizontal line between the top and bottom, but the lines should not be left right symmetric. Similarly, the equipotential lines are vertical but also are not left-right symmetric.

repell

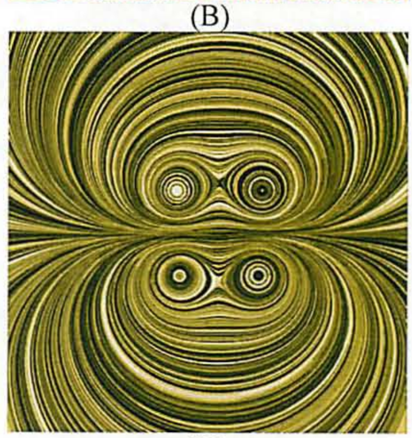
(A)



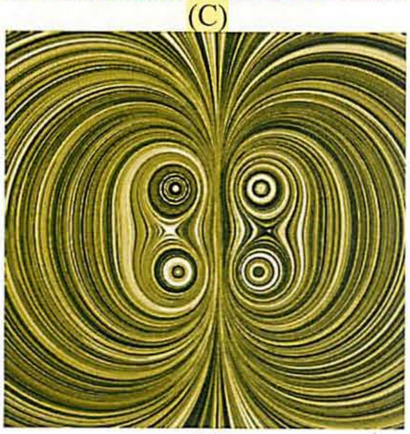
(B)



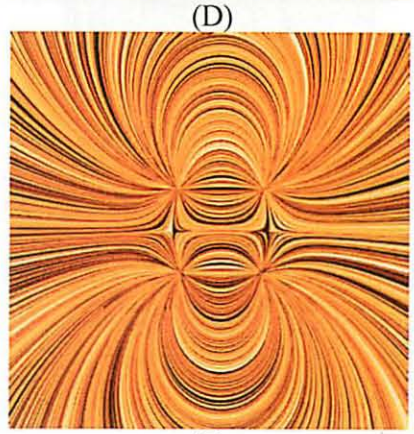
(C)



(D)



(E)



(F)

attract as many a

repell

c) How much energy would it take to move a test charge $+q$ ($q>0$) to point P assuming it started infinitely far away, where the potential is zero?

Now we have to calculate the voltage, which we do in the same way, although it is a little easier without the vector stuff:

$$V_1 = \frac{2Q}{4\pi\varepsilon_0\left(\sqrt{5}a\right)}$$

$$V_2 = \frac{3Q}{4\pi\varepsilon_0\left(\sqrt{5}a\right)}$$

$$V_3 = \frac{-2Q}{4\pi\varepsilon_0 a}$$

$$V_4 = \frac{-Q}{4\pi\varepsilon_0 a}$$

$$V_{\text{total}} = \sum_{i=1}^{4} V_i = \frac{Q}{4\pi\varepsilon_0 a}\left[\frac{2+3}{\sqrt{5}} - 3\right] = \frac{Q}{4\pi\varepsilon_0 a}\left[\sqrt{5}-3\right]$$

Now one just uses $U=qV$ and finds $\boxed{U = qV_{\text{total}} = \frac{qQ}{4\pi\varepsilon_0 a}\left[\sqrt{5}-3\right]}$

d) You are given a single point charge whose position and magnitude you choose, with the goal of making the $\vec{\mathbf{E}}$ field zero at point P. Where do you put the charge and what magnitude do you use? **[Only if you did not get an answer to part a):** assume the field at P has the form $\vec{\mathbf{E}} = E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}}$ and use that to solve the problem symbolically].

With just one charge, one has to place the charge in the line defined by the vector to be able to cancel out both components, and then adjust the charge appropriately, depending on how far away along that line you put it. Since P is at $(a,0)$, I can put the charge q at $\boxed{\left(a+\frac{2}{\sqrt{5}}a, -a\left(1-\frac{1}{5\sqrt{5}}\right)\right)}$, it gives a contribution at P of:

$$\vec{\mathbf{E}}_{\text{new}} = \frac{q}{4\pi\varepsilon_0 a^2\left(\left(\frac{2}{\sqrt{5}}\right)^2 + \left(1-\frac{1}{5\sqrt{5}}\right)^2\right)^{\frac{3}{2}}}\left[-\frac{2}{\sqrt{5}}\hat{\mathbf{i}} + \left(1-\frac{1}{5\sqrt{5}}\right)\hat{\mathbf{j}}\right]$$

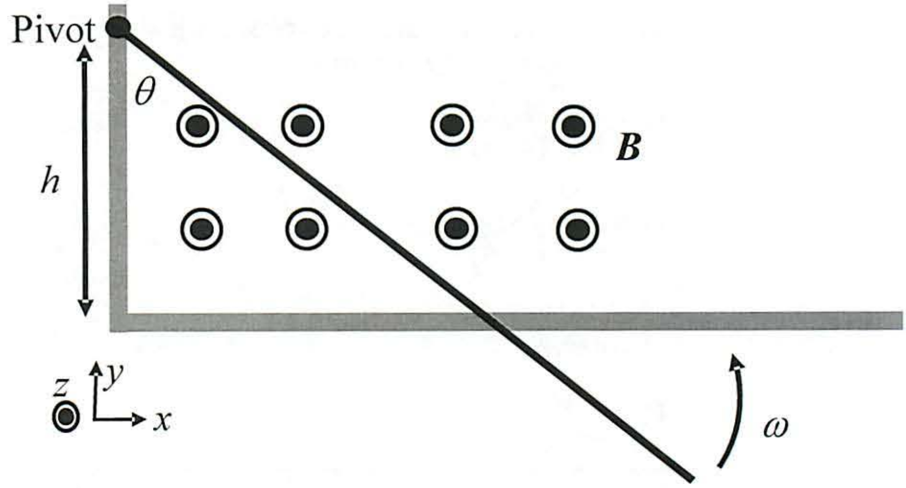
Adding this to our previous field, we find

$$\vec{\mathbf{E}}_{\text{total}} = \frac{1}{4\pi\varepsilon_0 a^2}\left[\frac{2}{\sqrt{5}}\hat{\mathbf{i}} - \left(1-\frac{1}{5\sqrt{5}}\right)\hat{\mathbf{j}}\right]\left(Q - \frac{q}{\left(\left(\frac{2}{\sqrt{5}}\right)^2 + \left(1-\frac{1}{5\sqrt{5}}\right)^2\right)^{\frac{3}{2}}}\right)$$

so from here, to get zero field we just have to specify the magnitude $\boxed{q = Q\left(\left(\frac{2}{\sqrt{5}}\right)^2 + \left(1-\frac{1}{5\sqrt{5}}\right)^2\right)^{\frac{3}{2}}}$

Problem 3 Resistive Bar on a pivot (30 points)

A very long resistive bar with **resistance per unit length** r is attached to a conducting right angle frame at one end by a pivot a distance h from the right angle, with the other end resting on the bar, making a triangular circuit. To simplify things assume the frame has negligible resistance. A constant magnetic field with magnitude B perpendicular to the apparatus permeates the area, coming out of the page as shown.



The bar starts at $t=0$ along the y axis and is rotated counterclockwise about the pivot with constant angular velocity ω, such that the pivot angle $\theta = \omega t$. Assume the bar is long enough such that it never loses contact with the horizontal side of the frame.

a) What is the magnetic flux penetrating the circuit as a function of $(t,\ \omega,\ B,\ h\ ,r)$?

The magnetic flux is just the triangular area times the **B** field. The area of the triangle is just ½ base time height, but the base length is changing with time as $b = h\tan(\omega t)$, so we have $A = \frac{1}{2}h^2\tan(\omega t) \rightarrow \boxed{\Phi_M = \frac{1}{2}Bh^2\tan(\omega t)}$

b) What is the resistance of the circuit as a function of $(t,\ \omega,\ B,\ h,\ r)$?

The resistance comes from the length of the rod in the circuit, which is also growing since the length of the hypotenuse grows with time. The length of the hypotenuse is

$$l = \frac{h}{\cos(\omega t)} \Rightarrow \boxed{R = rl = \frac{rh}{\cos(\omega t)}}$$

c) Using Faraday's law, find the induced emf for our circuit and the resulting current, again in terms of t, ω, B,h, and r. Indicate which way the current will flow on the diagram. It may help to know that $\frac{d}{d\theta}(\tan\theta)=\frac{1}{\cos^2\theta}$.

Here we have to use Faraday's law, to find the emf, and then divide by the Resistance for the current:

$$\varepsilon=-\frac{d\Phi_M}{dt}=-\frac{1}{2}Bh^2\frac{d}{dt}\left(\tan(\omega t)\right)=-\frac{1}{2}Bh^2\omega\left(\frac{1}{\cos^2(\omega t)}\right)$$

$$I=\frac{|\varepsilon|}{R}=\frac{1}{2}Bh^2\omega\left(\frac{1}{\cos^2(\omega t)}\right)\times\left(\frac{\cos(\omega t)}{rh}\right)$$

$$\boxed{I=\frac{Bh\omega}{2r\cos(\omega t)}}$$

For direction, as the bar rotates into the frame, the flux outward is increasing, so by Lenz's law the flux will be $\boxed{\text{clockwise}}$

d) What is the force on the resistive bar in terms of t, ω, B,h, and r? Be sure to indicate magnitude and direction. Does it get easier or harder as time goes on to keep it rotating at constant ω?

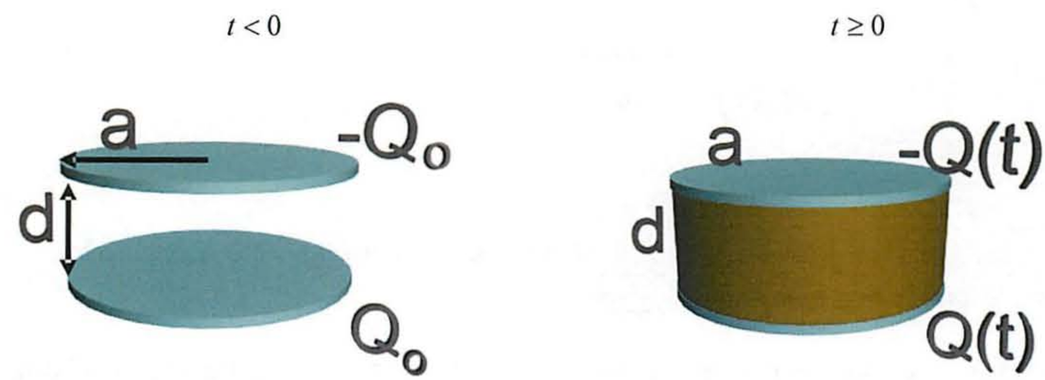
Note that the force will always be perpendicular to the bar and to the B field, so it always points opposite the direction of rotation, which is all you really need to say for direction. For magnitude, since the bar and the B field are always at right angles, the magnitude will be $|\vec{\mathbf{F}}|=ILB=\frac{Bh\omega}{2r\cos(\omega t)}\frac{h}{\cos(\omega t)}B=\boxed{\frac{B^2h^2\omega}{2r\cos^2(\omega t)}}$. To do it mathematically, one can parameterize the bar as $\vec{\mathbf{L}}=\frac{h}{\cos(\omega t)}\left[\sin(\omega t)\hat{\mathbf{i}}-\cos(\omega t)\hat{\mathbf{j}}\right]$ and just do the cross product:

$$\vec{\mathbf{F}}=\frac{Bh\omega}{2r\cos(\omega t)}\left[\frac{h}{\cos(\omega t)}\left[\sin(\omega t)\hat{\mathbf{i}}-\cos(\omega t)\hat{\mathbf{j}}\right]\times B\hat{\mathbf{k}}\right]$$

$$\boxed{\vec{\mathbf{F}}=\frac{Bh^2\omega}{2r\cos^2(\omega t)}\left[-\cos(\omega t)\hat{\mathbf{i}}-\sin(\omega t)\hat{\mathbf{j}}\right]}$$

Problem 4 Discharging Capacitor through a resistor (30 points)

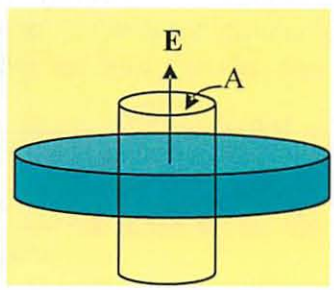
A capacitor consists of two parallel circular plates of radius a separated by a distance d (assume $a >> d$). The capacitor is initially charged to a charge Q_o. At $t = 0$, this capacitor begins to discharge because **we insert a circular resistor** of radius a and height d between the plates, such that the ends of the resistor make good electrical contact with the plates of the capacitor.. The capacitor then discharges through this resistor for $t \geq 0$, so the charge on the capacitor becomes a function of time $Q(t)$. *Throughout this problem you may ignore edge effects.*



a) Use Gauss's Law to find the electric field between the plates in terms of $Q(t)$. Is this electric field upward or downward?

To use Gauss's law, we must construct a volume in which to enclose charge – let's use a pillbox around the lower disk of the capacitor, which has surface charge density of $\sigma = \dfrac{Q}{\pi a^2}$, applying Gauss's law we have:

$$\oiint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EA = \frac{1}{\varepsilon_0}(\sigma A) \Rightarrow \boxed{\vec{\mathbf{E}} = \frac{Q}{\pi a^2 \varepsilon_0}\hat{\mathbf{k}} \text{ or "up"}}$$



b) For $t \geq 0$, consider an imaginary open surface of radius $r < a$ inside the capacitor with its normal $d\vec{\mathbf{A}}$ upward (see figure)

For $t \geq 0$, what is the conduction current flowing through this open surface in terms of $Q(t)$ or $\frac{dQ(t)}{dt}$ and the parameters given. Define the direction of positive current to be upward, and be careful about signs, in particular because $\frac{dQ}{dt} < 0$.

Here we want the conduction current flowing through the resistor in the area defined by r. The current density is $\vec{\mathbf{J}} = \left(-\frac{dQ}{dt}\right)\frac{1}{\pi a^2}\hat{\mathbf{k}}$ where the minus sign is to make sure positive current goes up, in the $\hat{\mathbf{k}}$ direction. So, the current inside a loop of radius r is

$$\boxed{I = \left|\vec{\mathbf{J}}\right|\pi r^2 = -\frac{dQ}{dt}\frac{r^2}{a^2}}$$

c) For this same imaginary open surface, what is the **time rate of change** of the electric flux though the surface, in terms of $Q(t)$ or $\frac{dQ(t)}{dt}$ and the parameters given (hint: use your answer above for **E**).

Before we found $\vec{\mathbf{E}} = \frac{Q}{\pi a^2 \varepsilon_0}\hat{\mathbf{k}}$ so we have $\Phi_E = \frac{Q}{\pi a^2 \varepsilon_0}\left(\pi r^2\right) = \frac{Q}{\varepsilon_0}\left(\frac{r^2}{a^2}\right)$, and if we take a time derivative we get: $\boxed{\frac{d\Phi_E}{dt} = \frac{1}{\varepsilon_0}\left(\frac{r^2}{a^2}\right)\frac{dQ}{dt}}$

d) What is the integral of the magnetic field around the contour bounding this open circle, using the Ampere-Maxwell Law? Be careful of signs.

Here you just add up the currents and apply Ampere's law, and find

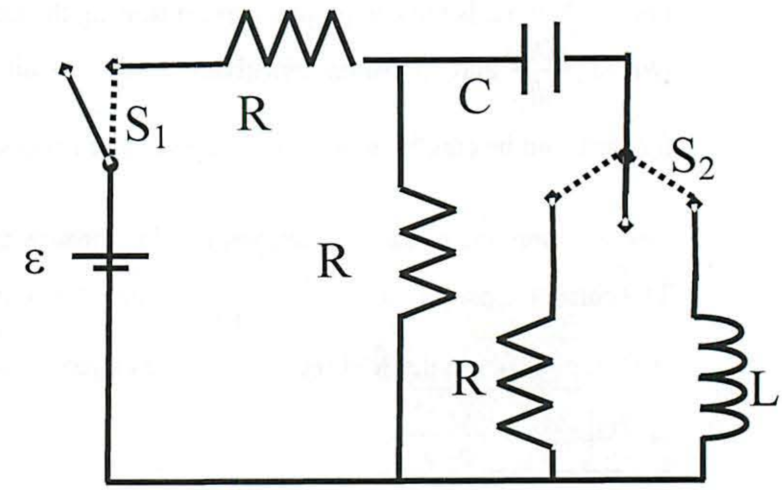
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{l}} = \mu_0\left(-\frac{dQ}{dt}\frac{r^2}{a^2} + \cancel{\varepsilon_0}\left[\frac{1}{\cancel{\varepsilon_0}}\frac{r^2}{a^2}\frac{dQ}{dt}\right]\right)\boxed{= 0}$$

that the integral vanishes

e) Does your answer in (d) make sense in terms of the energy dissipation and energy flow in this problem? You must explain your answer clearly and logically to get credit.

Yes, it does, because the energy that is flowing out of the capacitor is being immediately dissipated in the form of Joule Heating by the resistor.

Problem 3 (25 pts): RC circuit



The circuit at right contains a battery of emf ε, 3 resistors each of resistance R, a capacitor with capacitance C, an inductor with inductance L, and **two switches** – switch S_1 switches the battery into the circuit, and S_2 switches *either* the resistor or the inductor into the circuit (the dotted lines at right indicate possible connections for the switches).

<u>At first, S_2 is set to connect to the resistor into the circuit.</u>

a. At $t=0^+$, S_1 is closed, putting the battery into the circuit. What is the instantaneous current into the capacitor?

The capacitor acts like a short, so the circuit looks like a resistor in series with two resistors which are in parallel. The total resistance is $R_{tot} = R + \left(\frac{1}{R}+\frac{1}{R}\right)^{-1} = \frac{3}{2}R$, *so the total current will be* $I_{tot} = \frac{\varepsilon}{\frac{3}{2}R} = \frac{2\varepsilon}{3R}$ *but only ~~half~~ goes through the capacitor:* $\boxed{I_c = \frac{\varepsilon}{3R}}$

half

b. After the capacitor is fully charged (at t = T), what is the total charge on the capacitor?

Now no current flows in this leg, so the total current has gone down to $I_{tot} = \frac{\varepsilon}{2R}$, *and the voltage across the capacitor is just the voltage drop on the resistor in parallel:*

$$V_C = V_R = RI_{tot} = \frac{\varepsilon}{2R}R \Rightarrow V_C = \frac{\varepsilon}{2} \Rightarrow \boxed{Q = CV = \frac{C\varepsilon}{2}}$$

c. Now, you open the switch S_1. Use Kirchoff's laws to write a differential equation which governs the discharging of the capacitor. (S_2 still connects to the resistor). **You do not need to solve the equation** What is the time constant for discharging the capacitor?

The capacitor discharges through the two resistors in the loop, so we have

$$\frac{Q}{C} - IR - IR = 0 \quad I = -\frac{dQ}{dt} \Rightarrow \boxed{\frac{dQ}{dt} = -\frac{1}{2RC}Q \quad \tau = 2RC}$$ *which makes sense because with two resistors it will take longer to discharge the capacitor.*

<u>Once the capacitor is fully discharged, S_2 is set to connect to the inductor into the circuit.</u>

d. Let's reset the time to $t=0$ again and close switch S_1. Now what is the instantaneous current in the capacitor? Explain your answer.

Instantaneously the current will be zero still, because the inductor acts like an open and doesn't allow current to flow instantaneously.

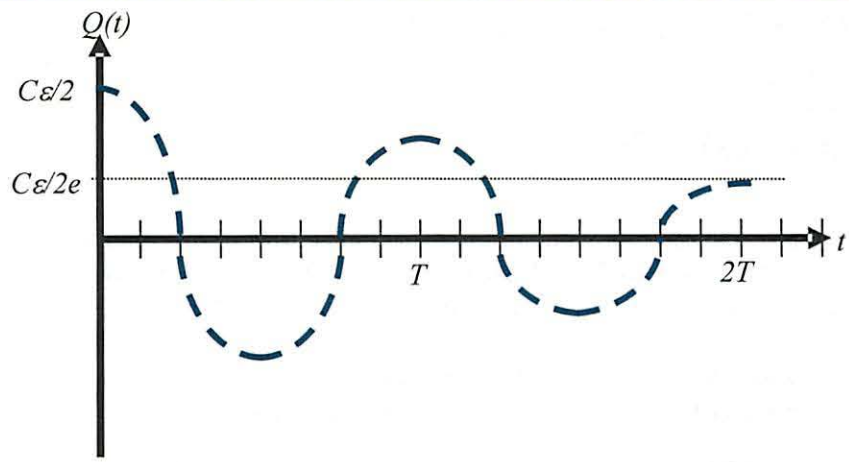
e. After a while, after the capacitor becomes fully charged, you open switch S_1 again. Now you have a damped undriven oscillator. Use Kirchoff's laws to write down a differential equation describing the circuit behavior. **You do not need to solve the equation**. If one ignores the small effect of the resistance, what is the fundamental frequency of this oscillator ω?

Now it's a loop with Capacitor, Resistor, and Inductor, so from Kirchoff's Law #2 with discharging capacitor: $\frac{Q}{C} - IR - L\frac{dI}{dt} = 0 \quad I = \frac{-dQ}{dt} \Rightarrow \boxed{\frac{d^2Q}{dt^2} + \frac{1}{LC}Q = -\frac{R}{L}\frac{dQ}{dt}}$.

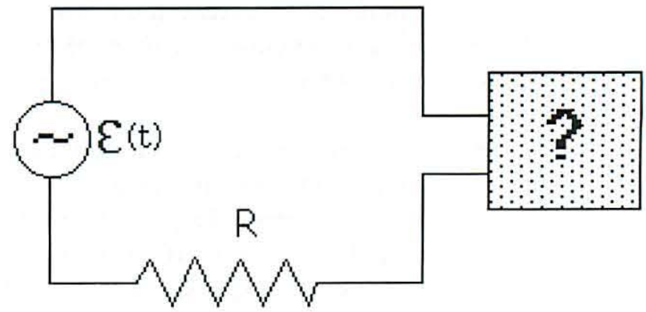
Ignoring the resistance I get the equations for a simple harmonic oscillator, with the fundamental frequency being $\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$

f. If the decay time τ for this circuit is $2T$, where $T = \frac{2\pi}{\omega}$ is the period of oscillation, sketch the **charge** on the capacitor as a function of time measured in units of the period T on the graph below (be sure to label the y axis scale using the amount of charge at $t=0$, when you first open the switch)

Before we open the switch, the capacitor will charged up and there will be no current flowing through it or the inductor, so the voltage drop will be the same as across the resistor just like in part b), $V_C = \frac{\varepsilon}{2} \Rightarrow \boxed{Q(0) = \frac{C\varepsilon}{2}}$ *Then the charge will oscillate and decay, with its maximum at time $t=2T$ down one factor of e compared to at $t=0$*



Problem 4 (26 pts): What's in the box?



The circuit shown contains an AC generator which provides a source of sinusoidally varying emf $\mathcal{E}(t) = \mathcal{E}_m \sin(\omega t)$, a resistor with resistance $R = 2$ ohm, and a "black box", which contains *either* an inductor *or* a capacitor, *or both.*

The amplitude of the driving emf, $\mathcal{E}_m$, is 1 Volt. We first measure the current in the circuit at an angular frequency ω =1 radians/sec and find that it is leads the driving emf by exactly $\pi/3$ radians. We then measure the current in the circuit at an angular frequency ω =2 radians/sec and find that it lags the driving emf by exactly $\pi/3$ radians. [Note: $\pi/3$ radians = 60 degrees, $\tan(\pi/3) = \sqrt{3}$].

a. What does the black box contain--an inductor or a capacitor, or both? Explain your reasoning.

For a circuit to be "capacitor like" (Current leading voltage) at one frequency and "inductor-like" (Voltage leading current) at a higher frequency, you need to have ***both.***

b. What is the numerical value of the current in the circuit at ω =1 radians/sec? At ω =2 radians/sec? You ***do not have to determine*** the unknown L and/or C to answer this question, and the variables L and C should not appear in your answer. Your answer can involve square roots.

We use the phase information to find out: $\tan(\phi) = \dfrac{X_L - X_C}{R}$ *and* $I = \dfrac{V}{Z}$. *Combining,*

$I = \dfrac{1\text{ V}}{\sqrt{R^2 + (X_L - X_C)^2}} = \dfrac{1\text{ V}}{R\sqrt{1 + \left(\frac{X_L - X_C}{R}\right)^2}} = \dfrac{1\text{ V}}{2\Omega\sqrt{1 + \tan^2\phi}}$ *so for both frequencies we have* $I = \dfrac{1\text{ V}}{2\Omega\sqrt{1 + \left(\sqrt{3}\right)^2}} = \boxed{\dfrac{1}{4}\text{ A}}$

c. What is the numerical value of the capacitance C *or* of the inductance L, *or of both,* as the case may be? Indicate units. Your answer(s) will involve simple fractions only. You will not need a calculator to find the value(s).

Now we have to use the information at each frequency separately. At each frequency, the impedance from the capacitor/inductor combination is twice the resistance, which gives us two equations to solve for L and C. The general equation is $\omega L - \dfrac{1}{\omega C} = \tan(\phi) R$.

Plugging in for our two values we have

$$(1)L-\frac{1}{(1)C}=-\sqrt{3}(2\Omega)\rightarrow\frac{1}{C}=2\sqrt{3}+L$$

$$(2)L-\frac{1}{(2)C}=\sqrt{3}(2\Omega)\rightarrow 2L-\tfrac{1}{2}\left(2\sqrt{3}+L\right)=2\sqrt{3}\Rightarrow\frac{3}{2}L=3\sqrt{3}\Rightarrow\boxed{L=2\sqrt{3}\,\mathrm{H}}$$

$$\frac{1}{C}=2\sqrt{3}+2\sqrt{3}=4\sqrt{3}\Rightarrow\boxed{C=\frac{1}{4\sqrt{3}}\mathrm{F}}$$

d. Doing no math, tell us in which frequency range you would drive this circuit to produce the maximum current (circle one)?

< 1 rad/sec   between 1 rad/sec and 2 rad/sec   > 2 rad/sec

Briefly explain why you choose the range you did (a plot might help).

If we are capacitor-like at 1 rad/s, which is below resonance, and inductor-like at 2 rad/s, i.e. above resonance, then somewhere between those two is resonance, which gives the minimum impedance of R= 2 Ohms and therefore the maximum current.

e. Now calculate the numerical value of the angular frequency for which the current in the circuit will be a maximum. If you are uncertain in your answer in (c), make sure you give you answer in terms of variables first.

At resonance, the capacitor and inductor impedance cancel, so $X_L = X_C$.

$$\omega_0 L=\frac{1}{\omega_0 C}\Rightarrow\omega_0=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{2\sqrt{3}\times\frac{1}{4\sqrt{3}}}}=\boxed{\sqrt{2}\,\frac{\mathrm{rads}}{s}}$$

f. If your circuit contains an inductor, give an expression for the voltage drop across the inductor as a function of time when the current is at a maximum. If you circuit does not contain an inductor, give an expression for the voltage drop across the capacitor as a function of time when the current is at a maximum. You need only give one expression. If you do not know, choose one, but in all cases clearly indicate whether your expression is for an inductor or for a capacitor. The amplitude of your time function should be a numerical value in Volts.

We have an inductor, for which we know the Voltage is $V_L = IX_L = I\omega L$. *At resonance, the current is determined just by the resistor:* $I=\frac{V}{R}=\frac{1\mathrm{V}}{2\Omega}=\frac{1}{2}\mathrm{A}$ *but the inductor voltage leads the current by 90 degrees, so it will go like* $\sin\left(\omega_0 t+\frac{\pi}{2}\right)=\cos\left(\omega_0 t\right)$. *Putting all this together, we have* $V_L=\frac{1}{2}A\left(\sqrt{2}\frac{\mathrm{rad}}{s}\right)\left(2\sqrt{3}\,\mathrm{H}\right)\cos\left(\sqrt{2}t\right)=\boxed{\sqrt{6}\cos\left(\sqrt{2}t\right)\mathrm{A}}$.

E field $= -\nabla V$

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Please Remove this Tear Sheet from Your Exam

Some (possibly useful) Relations:

$$d\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_o}\frac{dq}{r^2}\hat{\mathbf{r}} \; ; \; \hat{\mathbf{r}} = \frac{\vec{\mathbf{r}}}{r} \text{ from } dq \text{ to } obs.$$

$$\oiint_{\text{closed surface}} \kappa \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{free, inside}}}{\varepsilon_o}$$

$d\vec{\mathbf{A}}$ points from inside to outside

$$\Delta V_{\text{moving from } a \text{ to } b} = V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$W = \Delta U = q\Delta V$$

$$E_x = -\frac{\partial V}{\partial x}, \; E_y = -\frac{\partial V}{\partial y}, \; E_z = -\frac{\partial V}{\partial z}$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} \qquad \varepsilon = -N\frac{d\Phi_{\text{sgl loop}}}{dt}$$

$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{q\,\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2} \quad |\vec{\mathbf{v}}| << c \quad d\vec{\mathbf{B}} = \frac{\mu_o I}{4\pi}\frac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

where $\hat{\mathbf{r}}$ points **from** source **to** observer

$$\oiint_{\substack{closed\\surface}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

$$\oint_{\text{contour}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_o\left(I_{\text{through}} + \varepsilon_0 \frac{d\Phi_E}{dt}\right)$$

where I_{through} is the current flowing through any open surface bounded by the contour:

$$I_{\text{through}} = \int_{\text{open surface}} \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}$$

ds is right-handed with respect to d**A**

$$u_E = \frac{1}{2}\varepsilon_0 E^2 \qquad u_B = \frac{B^2}{2\mu_o}$$

$$\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}_{ext}\right) \qquad d\vec{\mathbf{F}} = I\, d\vec{\mathbf{s}} \times \vec{\mathbf{B}}_{ext}$$

$$F_{\text{cent.}} = mv^2/r$$

$$\vec{\boldsymbol{\mu}} = IA\,\hat{\mathbf{n}}$$

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}} \qquad F_z = \mu_z \frac{dB_z}{dz}$$

$$\Delta V = IR$$

$E = \frac{V}{d}$

$$P_{\text{ohmic heating}} = I\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

e field constant inside capacitor

$$C = \frac{Q}{\Delta V} \qquad U = \frac{1}{2}C\Delta V^2 = Q^2/2C$$

$$\tau = RC \qquad X_C = 1/\omega C$$

$$L = \frac{N\Phi_{\text{B,self,sgl coil}}}{I} \qquad \varepsilon_{\text{back}} = -L\frac{dI}{dt}$$

$$U_L = \tfrac{1}{2}LI^2$$

$$\tau = L/R \qquad X_L = \omega L$$

Series RLC: $Z = \sqrt{R^2 + X^2} = \sqrt{R^2 + (X_L - X_C)^2}$

$\tan\varphi = X/R \qquad V_0 = I_0 R$

$$\omega = 2\pi f = 2\pi/T \qquad k = 2\pi/\lambda$$

$$c = \lambda/T = \lambda f = \omega/k = (\mu_0\varepsilon_0)^{-1/2}$$

$$E_0 = v_{\text{light}} B_0 \qquad \hat{\mathbf{E}} \times \hat{\mathbf{B}} = \hat{\mathbf{p}}$$

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}} \qquad P_{\text{absorb}} = \frac{S}{c}; \; P_{\text{reflect}} = \frac{2S}{c}$$

$V = Ed$

Cross-products of unit vectors:

$$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$$

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \quad \hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}} \quad \hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}}$$

Some potentially useful numbers

$$k_e = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{\text{N m}^2}{\text{C}^2} \qquad \mu_o = 4\pi \times 10^{-7} \frac{\text{T m}}{\text{A}}$$

Breakdown of air	$E \sim 3 \times 10^6$ V/m
Earth's B Field	$B \sim 5 \times 10^{-5}$ T = 0.5 Gauss
Speed of light	$c = 3 \times 10^8$ m/s
Light (blue to red)	λ = 400 nm to 700 nm
Electron charge	$e = 1.6 \times 10^{-19}$ C
Avogadro's number	$N_A = 6.02 \times 10^{23}$ mol^{-1}

8.02 Final Exam Spring 2006

FAMILY (last) NAME

GIVEN (first) NAME

Student ID Number

___L01 MW 9 am ____L02 MW 11 am ____L03 MW 1 pm ____L04 MW 3 pm

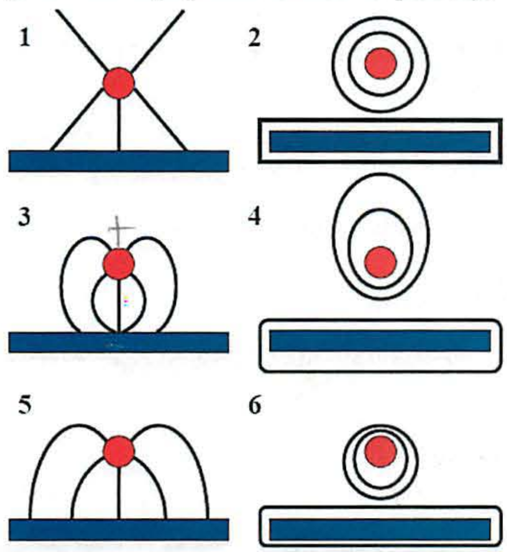
___L05 TR 9 am ____L06 TR 11 am ____L07 TR 1 pm ____L08 TR 3 pm

Your Group (e.g. 10A): ______________

Problem	**Score**	**Grader**
1 (40 points)		
2 (20 points)		
3 (10 points)		
4 (20 points)		
5 (20 points)		
6 (30 points)		
7 (60 points)		
TOTAL		

Problem 1: Eight Short Questions. Circle your choice for the correct answer
Each problem is worth 5 points for the correct answer, 2 points for a partially correct answer (at our discretion). If you don't know the answer you can earn 1 point for admitting that by leaving it unanswered and writing "I don't know" (make this clear!).

Question A (5 points out of 40 points):



In lab 1 you fixed the potential difference between two plates and measured equipotential lines from which you determined electric field lines and approximate charge distributions. You are given a layout with a conducting plate and conducting circle, as pictured at left, with the circle held at +5 V relative to the plate. Identify the most accurate representation of the equipotential lines and the electric field lines:

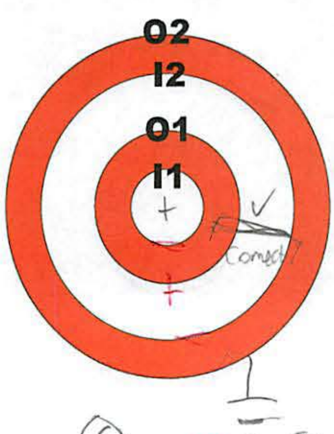
even
Equipotential Lines: # 4

odd
Electric Field Lines: # 5

Oh yeah think like that (on other test)

Yeah spread out more

Question B (5 points out of 40 points):



In the second lab you worked with a Faraday pail, two nested conducting cylinders as pictured at left (in the lab the shaded regions were thin). You held the outer cylinder at ground (i.e. at the same potential as infinity) and measured the potential of the inner cylinder relative to the outer cylinder. For one of the measurements you started from a condition where both cylinders were uncharged, introduced a positive charge producer into the central region, briefly connected the inner at outer cylinders with a conductor (your finger) and, after removing the connection, removed the positive charge producer. The positive charge producer never touched either of the cylinders during this measurement. Identify (circle) the sign of the charge (positive, negative or zero) on each surface after doing this:

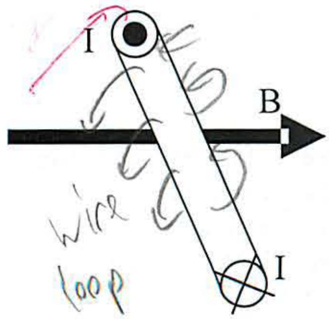
I1: >0 =0 <0 O1: >0 =0 <0

I2: >0 =0 <0 O2: >0 =0 <0

grounded

when touching the ⊖ flow in
but then it would not be touching
i don't get and I thought I knew it

Question C (5 points out of 40 points):



current flowing

In the third lab you studied the effects of magnetic fields. A current-carrying coil is placed in a uniform magnetic field pointing to the right. The current flows as shown, out of the page in the upper left and in on the lower right.

What are the direction of the force and torque on the coil (circle one direction for each)?

Force (dipole will tend to move…):

Mostly left Mostly Right Zero

no force in uniform field

Torque (dipole will tend to rotate…):

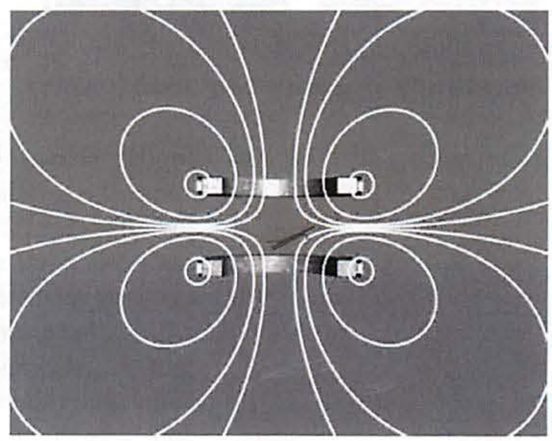
Clockwise Counterclockwise None

kinda guessing

diapolemement up + right
r redew

Question D (5 points out of 40 points):

In the fourth lab you measured the force and torque on a magnetic dipole in the field of a Helmholtz coil (which you could energize in either Helmholtz or Anti-Helmholtz mode). The picture at right shows the field configuration created by the coils after you have energized them in one of these two ways.



If, before the above field is turned on, you place a dipole so that it is very slightly below center and points very slightly away from alignment with the eventual field of the Helmholtz coil after it is energized, what force and torque will it feel when the coils are energized?

It will feel a force: up (towards center) down (away from center) no force

It will feel a torque: to align to anti-align no torque

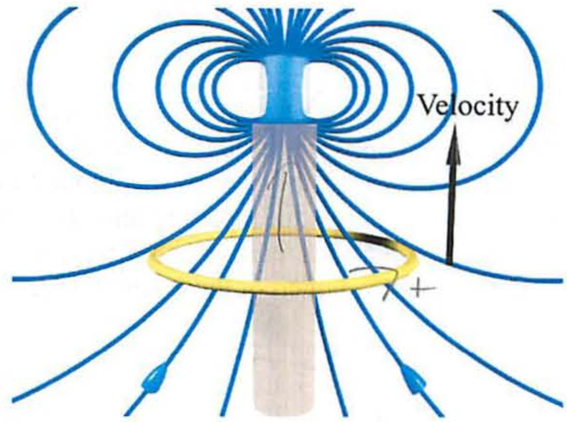
I don't really know

anti Helmholtz = 0 at center

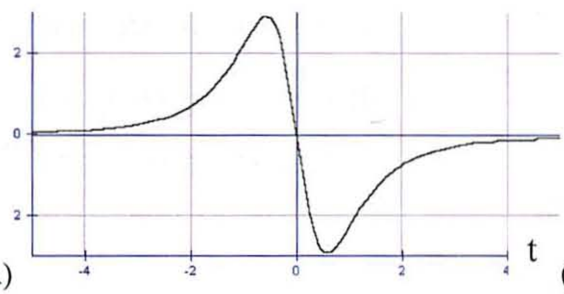
Strongest field ↓

One qv from each lab

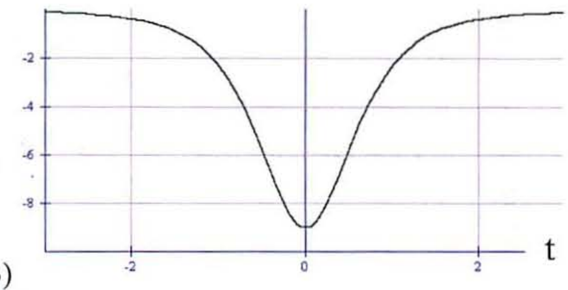
Question E (5 points out of 40 points):
In the fifth lab you measured the current and calculated flux generated in a wire coil that was moved from well above a magnet with its North pole facing upwards to well below the magnet and then back up again (see figure). We defined a counter-clockwise (as viewed from above) current as positive and defined the positive flux direction accordingly. For the portion of the motion from well below to well above the magnet, which two of the following diagrams most closely resemble what you should have measured for flux and current respectively?



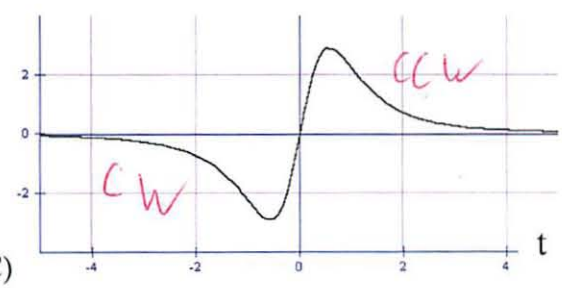
I



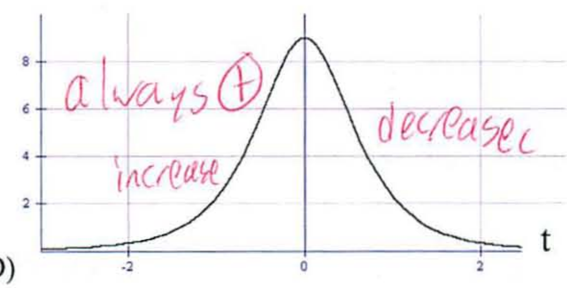
(A)



(B)



(C)



(D)

Flux: ~~C~~ D

Current: ~~B~~ C

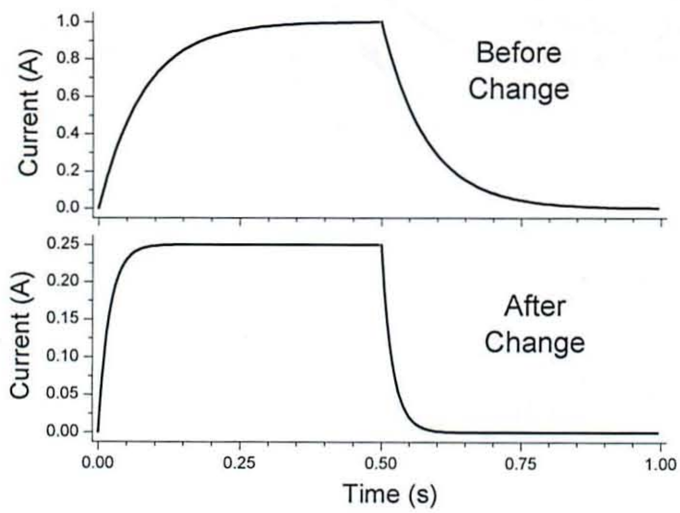
always forget this

total guess

$\Phi = BA\cos\theta$

Question F (5 points out of 40 points):

In experiment six you set up a simple series LR circuit which consisted of the 750 function generator and the coil (which as you may recall has both a resistance and an inductance). The 750 power supply was used as a "variable battery" which would periodically turn on and off, and the current through the battery was plotted vs. time. In this experiment you had the opportunity to measure the effect of inserting and removing an iron core from the coil as well as the effect of adding an additional resistor either in series or in parallel with the coil. In moving between the two plots below, which of those four things was done (circle one)?



1) Core was added

2) Core was removed

3) Resistor was added in parallel

4) Resistor was added in series

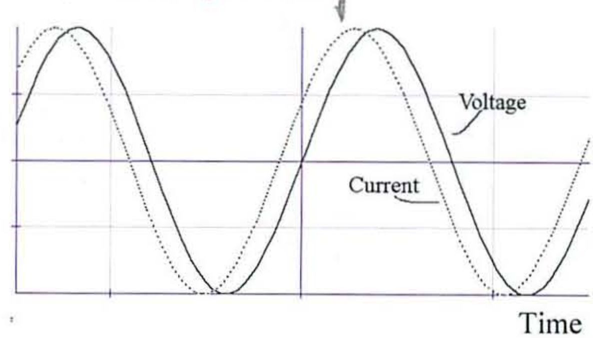
5) I don't know (1 point)

core = extra inductance ?
↑ L

current ↓ quickly (dispersed)

Question G (5 points out of 40 points):

In experiment seven you studied a driven series LRC circuit and recorded both the power supply voltage (solid curve at right) and current (dashed curve). Which leads and are we at resonance or above or below the resonance frequency ?



C
V

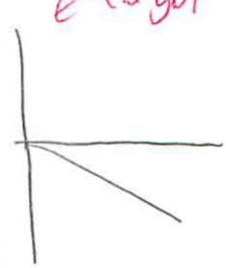
Check time

Which leads? Current Voltage neither

Above or below? Above Below On Resonance

← forgot this orientation

L



C

→

always screw up

~~inductor~~ like
Capacitor

Question H (5 points out of 40 points):

In experiment eight you measured the angular dependence of the radiation from a spark gap antenna by moving your receiver either horizontally or vertically around the transmitter.

Angular dependence - Horizontal

Angular dependence - Vertical

Which kind of motion, horizontal or vertical, shows a larger change in radiation intensity over the range of motion?

1. Horizontal
2. Vertical
3. Both show same range of change
4. I don't know (1 point)

I did alful on these
and I thought I knew it

Problem 2: Maxwell's Equations (20 Pts)
The content of this course can almost completely be summarized in Maxwell's equations. For each of Maxwell's equations please do the following:

1) State the NAME of the equation
2) Write the INTEGRAL FORM of the equation (in other words, write down the equation as you have learned it)
3) Briefly EXPLAIN THE MEANING of the equation (that is, in words, explain the IDEA behind the equation – do not simply give the meaning of the symbols).
4) For TWO OF THE FOUR equations (your choice): give a REAL WORLD EXAMPLE of how you would use the equation to make an approximation of something. You have been given lots of these on the problem sets this semester – feel free to choose one of those or make up one of your own. Give values for all quantities in your approximation. Note that you don't need to do any more work here than you would if you really were doing the approximation (i.e. don't work through the problem in gruesome detail, just show how you can make a quick approximation).

Oh no essay :(

~~Ampere~~ Gauss

$$\oint E\, ds = \frac{Q}{\epsilon_0}$$

Ampere w/ Maxwell

$$\oint B\, ds = \mu_0 \left(I_{thru} + \epsilon_0 \frac{d\Phi_E}{dt} \right)$$

Faraday

$$\oint E \cdot ds = -\frac{d}{dt} \underbrace{\iint \vec{B} \cdot d\vec{a}}_{\Phi_B}$$

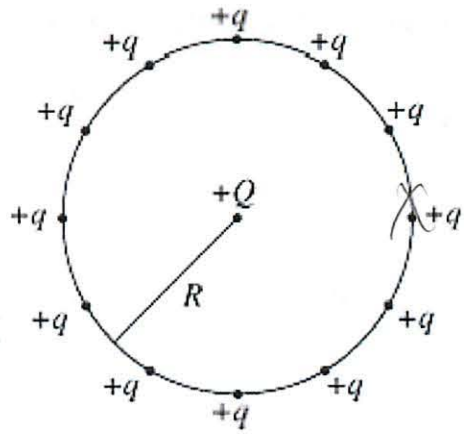
Problem 2: Maxwell's Equations continued…

What is 4th?

Magnetic guass

$\oint B \cdot dA = 0$ no magnetic monopoles

Problem 3: Charges (10 points):



Twelve equal charges $+q$ are situated in a circle with radius R, and they are equally spaced (see the figure).

Colomb's

(a) What is the net force (magnitude and direction) on a charge $+Q$ at the center of the circle.

not much space for vectors
i don't cancle out

No net force $=0$ Symmitry

We remove only the +q charge which is located at "3-o'clock."
(b) What now is the force (magnitude and direction) on the charge +Q at the center of the circle?

9 o clock unbalanced

$\vec{F} = \frac{kqQ}{R^2}$ to right

vector

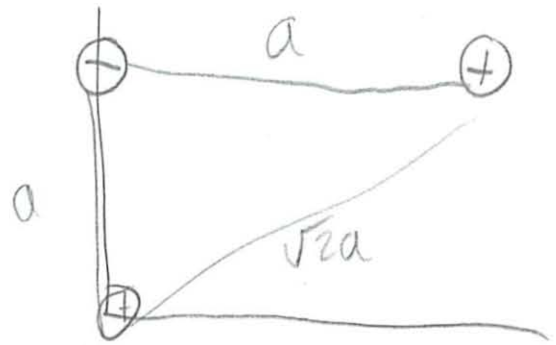
Review Colomb's

5/15

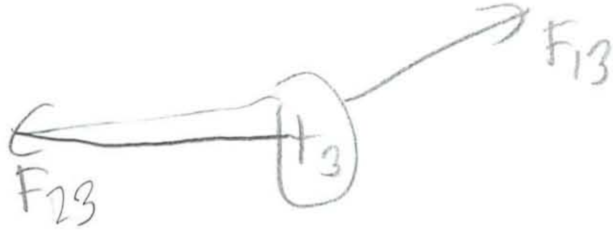
$$\vec{F} = k_e \frac{q_1 q_2}{R}$$

Super position

$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23} = \cancel{k}\, k\left(\frac{qa}{r^2} + \frac{qq}{r^2}\right)$$



force on 3 we are considering



unit vector

$$k\left(\frac{q_1 q_3}{(\sqrt{2}a)^2}\hat{r} + \frac{q_2 q_3}{a^2}\hat{r}\right)$$

$+\hat{i}$

$$\frac{-\hat{i}-\hat{j}}{\sqrt{2}\,a} \leftarrow \frac{\sqrt{2}}{2}(\hat{i}+\hat{j}) = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$

$\frac{\sqrt{2}}{2} \rightarrow$ was $\frac{1}{\sqrt{2}}$ yeah I was right

Now add up $\uparrow$ and $\rightarrow$ sepertly

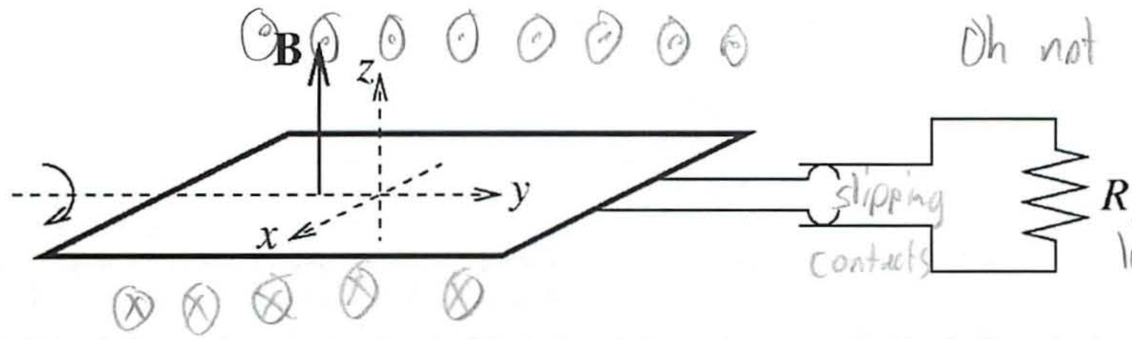
$\phi = \tan^{-1}\left(\frac{F_{3y}}{F_{3x}}\right) = \tan^{-1}\left[\frac{\sqrt{2}/4}{-1+\sqrt{2}/4}\right] = 151.3°$

not what is drawn
- pay attention to #
- limit what you ~~can~~ assume

would not know how to deal with - guess could figure out

Problem 4: Generator (20 points)

A simple electric generator (as shown below) is rotating about the y-axis with a frequency of f [Hz]. There is a uniform magnetic field B [T] in the +z direction. The rotor consists of a coil of n windings each with an area S [m^2]. The generator, through slipping contacts, is powering a light bulb whose resistance is R [Ω]. The ohmic resistance of the coil is negligibly small compared to that of the light bulb. You may also assume here, for simplicity, that the self-inductance of the coil is negligibly small.



Oh not good at this

(a) What is the maximum value (I_{max}) of the induced alternating current? Also indicate in the figure one of the two positions of the coil when this maximum current occurs.

$I_0 =$ what is principal here?

current driven by $\mathcal{E}$ from changing Φ

$$I = \frac{\mathcal{E}}{R} = \frac{1}{R}\frac{d\Phi}{dt} = \frac{1}{R}\frac{d}{dt} BnS\sin(2\pi f t)$$

freq

$$= \frac{2\pi f}{R} BnS\cos(2\pi f t)$$

(b) What is the time-averaged mechanical power (in Watts) that must be supplied to maintain the rotation (neglect friction in the bearings)?

$$I_{max} = \frac{2\pi f}{R} BnS$$

must also be able to do w/o looking

power = work / time

$$= I^2 R$$

$$= \left\langle \left(\frac{2\pi f}{R} BnS\cos(2\pi f t)\right)^2 R \right\rangle$$

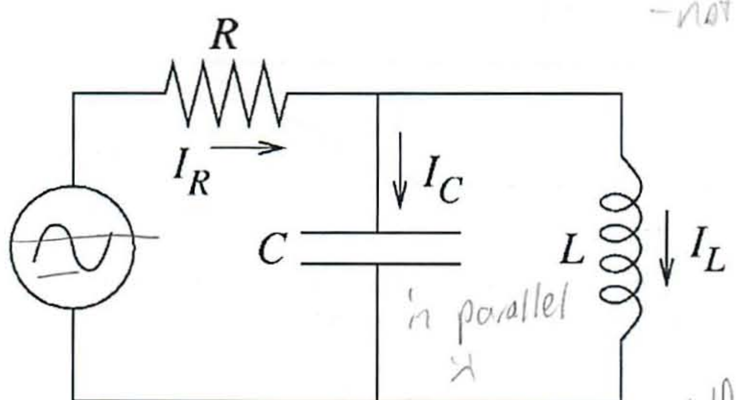
$$= \frac{1}{2}\frac{4\pi^2 f^2 B^2 n^2 S^2}{R}$$

should take some time to read through notes

Problem 5: Circuit (20 points)

The LRC circuit as shown is driven by a power supply whose EMF = $V_0\cos(\omega t)$. In steady state, the current through the ideal self-inductor is I_L, the current through the ideal capacitor is I_C and the current through the resistor is I_R. **Steady state means that you wait a long time so that all transient phenomena have died out. Don't even THINK of writing down a differential equation. This problem is designed to see whether you have an appreciation for how a capacitor and a self-inductor behave in extreme situations. No fancy math is needed.** Express all your answers in terms of L, R, C and V_0.

? good
– not like other exam



what are right eq here?

DS source

(a) What are the maximum values of I_L, I_C and I_R in case ω=0 (zero frequency means that the power supply is now a simple battery with zero internal resistance). **We are asking you for steady state solutions, NOT transient solutions.**

max   not (t)   extream circumstances

low freq C has high impediance So all Chooses inductor

$I_C = 0$

$I_R = I_L = \frac{V_0}{R}$

$\mathcal{E} - IR - \frac{Q}{C} = 0$ $\mathcal{E} = IR$ $I_R = \frac{\mathcal{E}}{R}$ $\frac{\mathcal{E} - \frac{Q}{C}}{R} = I_C$

$\mathcal{E} - IR - L\frac{dI}{dt} = 0$ $\frac{\mathcal{E} - L\frac{dI}{dt}}{R} = I_L$

? what else could it be?

(b) Answer the same question as under "(a)" for the other extreme when ω approaches a value which is infinitely high.

Very inductor like

$\mathcal{E} - IR - L\frac{dI}{dt} = 0$

$I_L = \frac{\mathcal{E} - L\frac{dI}{dt}}{R}$

$I_R = I_C = \frac{V_0}{R}$ thought something simple

$I_L = 0$

also I rarely come
across same problem
so memorizing exactly not effective

Problem 5: Circuit *continued…*

(c) Do you expect the maximum value of the current I_R to be higher or lower than the value you found under "(a)" in the case that the frequency is somewhere between the two extremes? **Give your reasons.**

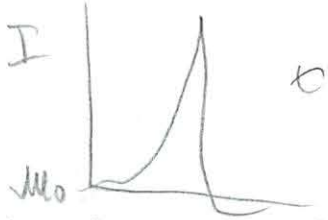
? same - guess thats wrong

in 2 extrem limits one is a short (wire)

intermediate Freq → not cases so impedeance higher
current lower

but I thought

$\boxed{\text{Parallel}}$



true
- but parallel circuit ?

(d) There is one frequency (**in steady state**) for which I_R is zero. This is not so intuitive, but given the fact that this is so, what do you think that frequency is? **Please do not try to calculate this frequency.**

? is it resonance ✓ would have guessed
- no ?? what is it ?

why ?

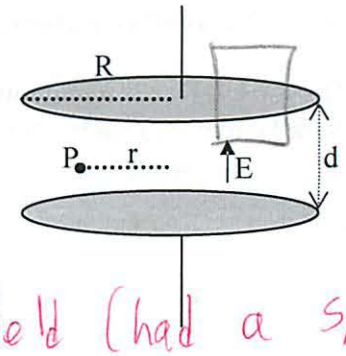
yes

$\omega = \frac{1}{\sqrt{LC}}$

here we go again

Problem 6: Capacitor (30 points)

A parallel-plate capacitor consists of two circular plates, each with radius *R*, separated by a distance *d*. The electric field ***E*** between the plates is uniform and directed upwards (see sketch).



read the qu!

(a) What is the total energy stored in the electric field of the capacitor? Assume that the electric field is uniform between the plates and zero outside of the plates (i.e., neglect fringing fields).

$\rightarrow$ not E field (had a small inkling)

$$\int E \cdot ds = \frac{Q}{\epsilon_0}$$

$$\cancel{E \cdot A = \frac{\sigma A}{\epsilon_0}}$$

$$\cancel{E = \frac{Q}{\epsilon}}$$

Q seems sparse

$u_E \cdot V$

$$\frac{1}{2}\epsilon_0 E^2 \cdot \pi R^2 d$$

$$= \frac{\epsilon_0 \pi R^2 d E^2}{2}$$

(b) Now, suppose that the electric field is *increasing* with time ($dE/dt > 0$). The point *P* is located between the plates at radius $r < R$ (see sketch). *Derive* an expression for the magnitude of the magnetic field ***B*** at point *P* and indicate its direction there on the sketch.

✓ didn't mention displacement current still must do

$$\frac{dE}{dt} > 0 \qquad \frac{dQ}{dt}$$

w/ $E\uparrow$, upward displacement current

$$\int B \cdot ds = \mu_0 I_{displacement}$$

$$B \cdot 2\pi \cancel{R}_{r?} = \mu_0 \cancel{\pi r^2} \pi r^2 \frac{dE}{dt}$$

$$B = \frac{\mu_0 \pi r^2}{2\pi R} = \frac{\mu_0 r \epsilon_0}{2} \frac{dE}{dt} \text{ CCW}$$

this problem use E as ~~total~~ variable

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt} = \frac{d(E\pi r^2)}{dt} = \epsilon_0 \pi r^2 \frac{dE}{dt}$$

Problem 6: Capacitor continued…

(c) What is the Poynting vector at point *P*? Give both direction and magnitude.

~~Inside~~ radially inward

$$\frac{\frac{\sigma}{\epsilon} \cdot \frac{\mu_0 r^2}{2R}}{\mu_0}$$

$$\frac{1}{\mu_0}\frac{E}{2}\mu_0 \epsilon_0 r \frac{dE}{dt}$$

?

they just use $\underline{E}$

$$= \frac{\sigma \mu_0 r^2}{\epsilon 2R}$$

(d) Using the Poynting vector, determine the total electromagnetic energy flowing into or out of the capacitor per unit time across $r = R$. Which is it (into or out of)? Write down an equation relating this quantity to the electric energy contained in the capacitor (see part (a)).

it always asks you to compare

$$\int_0^{2\pi}\int_0^{r}\int_0^{d} \frac{\sigma \mu_0 r^2}{\epsilon 2R}\, dz\, r\, dr\, d\theta$$

$$\int_0^{r} \frac{\sigma \mu_0 r^3}{\epsilon 2R} = \frac{\sigma \mu_0 r^4}{\epsilon 2R}$$

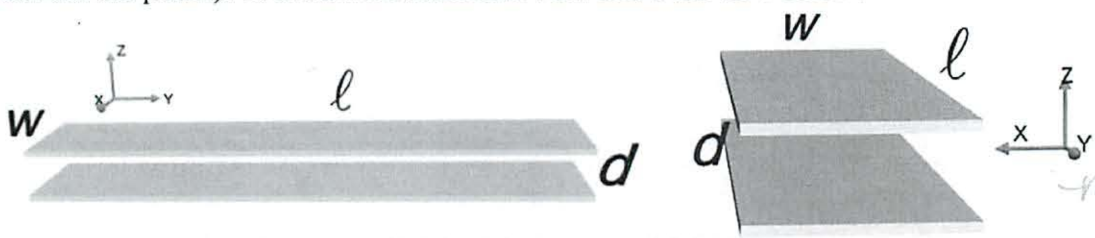
$$\frac{\sigma \mu_0 r^4 2\pi}{\epsilon 2R}$$

remember can just multiply by volume

Problem 7: Transmission Line (60 pts) this was a problem before

The rest of this exam is an extended question dealing with transmission lines. There are a variety of transmission lines used in the world. A simple example is two wires running next to each other with current flowing one direction in one and the opposite in the other. Another example that you considered in problem set 12 was the coaxial cable, where current flowed up the inside wire and back along the outer shield.

In this problem you will calculate the properties of a microstrip transmission line. It consists of two thin parallel plates of width w and length ℓ, separated by a small distance d (they are typically held apart by a dielectric, but to make your life simple let's just pretend there is air between the plates). It is shown both in side view and front view below.



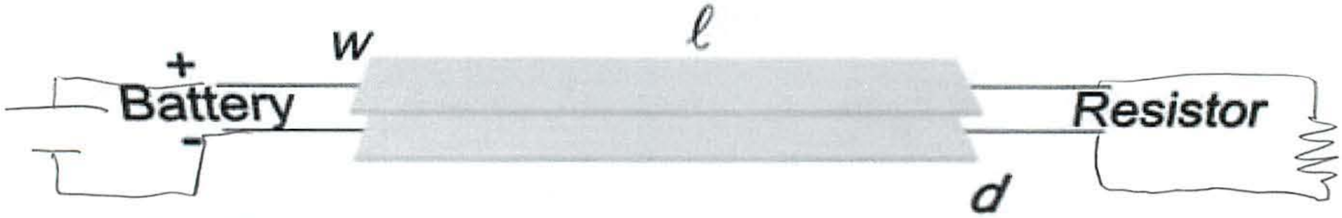
flat capacitors

-much better at round[illegible]

The dimensions are such that **you should assume that any fields created by the transmission line are confined to the region between the two plates.**

no edge effects size?

We use transmission lines to carry power from batteries or power supplies to loads (typically modeled as resistors):



In this problem you will calculate the capacitance and inductance of the microstrip transmission line and then study energy flow at DC.

NOTE: PLEASE READ THIS CAREFULLY

In several parts of this problem you will be asked to calculate something that will require the use of one of Maxwell's equations. Make sure that you state the name of the equation and the write it in the form that you plan to use it before you do that part. You do not need to describe the equation as you were asked to do earlier in this exam, but you do need to be explicit in the calculations and draw and label anything that you need to use to do the calculation. I will not provide any further drawings. Please duplicate drawings from this page (simplified to remove the perspective of course) when you think they will be useful.

Do not forget to give both magnitude and direction of vector quantities.

Feel free to tear out this page so that you do not have to continually turn back to it.

Problem 7A: Capacitance of the Microstrip Transmission Line

In the first two parts of this problem (A and B) we will consider the transmission line in isolation (no battery or load resistor).

Calculate the capacitance of the transmission line.

$C = ?$ oh did not study much property of surface

$$C = \frac{Q}{V}$$

Find $\vec{E}$ field

$$\oint E\,dA = EA = \frac{Q_{enc}}{\epsilon_0}$$

$$E = \frac{Q}{w l \epsilon_0}$$

dont forget to describe it

$$\Delta V = Ed = \frac{Qd}{w l \epsilon_0}$$

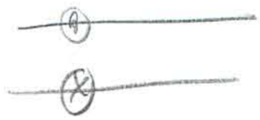
$$C = \frac{Q}{V} = \frac{Q}{\frac{Qd}{w l \epsilon_0}} = \frac{w l \epsilon_0}{d}$$

So I was close I just gave up too early

— liable to make this mistake a lot

Problem 7A: Capacitance of the Microstrip Transmission Line ***continued***

Problem 7B: Inductance of the Microstrip Transmission Line



Calculate the inductance of the transmission line.
NOTE: There are two ways to do this. If you don't recall either of them then I suggest that you at least send some current through the transmission line and calculate the magnetic energy between the plates.

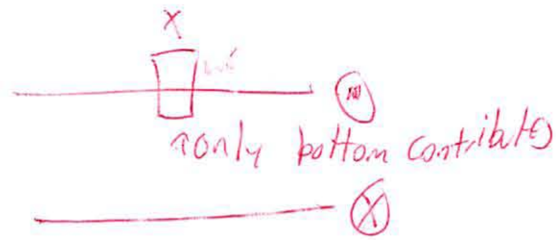
?, forget this ton

?, does it loop arand

$$L = \frac{N \Phi_{B\ \text{self}\ \text{single coil}}}{I}$$

$$\Phi_B = \int B \cdot A$$

$$\oint B \cdot ds = \mu_0 I_{enc}$$



$$B \cdot 2\pi r = \mu_0 \pi r^2$$

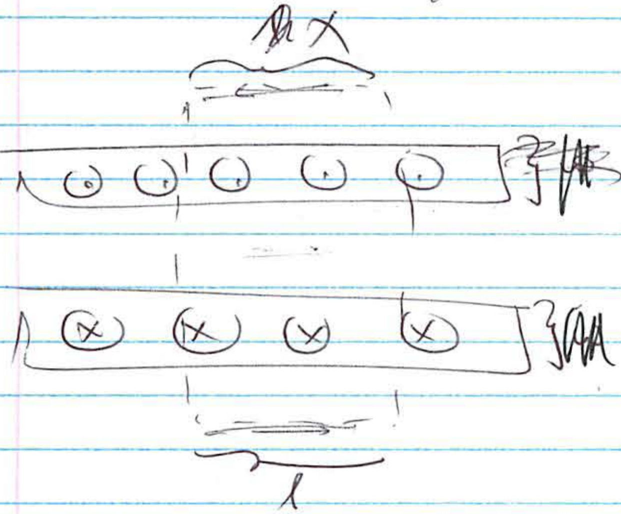
$$B x = \mu_0 \frac{x}{w} I$$

$$B = -\frac{\mu_0 I}{w}$$

~~UBT~~ Energy to calculate inductance (energy density · volume)

$$\frac{B^2}{2\mu_0} wld = \left(\frac{\mu_0 I}{w}\right)^2 \cdot \frac{wld}{2\mu_0} = \frac{1}{2}\overbrace{\boxed{\frac{\mu_0 ld}{w}}}^{L} I^2$$

Problem 7B: Inductance of the Microstrip Transmission Line ***continued***

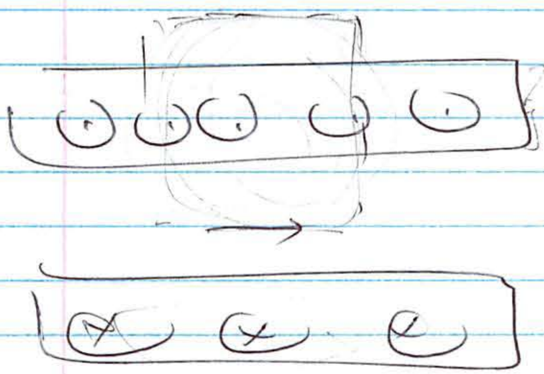


$2B\cancel{x} = \mu_0 I_{enc}$

I_{enc}

$\vec{B} = 0$

$B\cancel{l}^{x} = \mu_0 \underline{I_{enc}}$

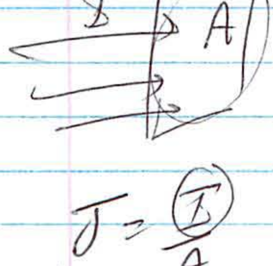


$\cancel{I_{enc} = \frac{x\,w}{d}}$

$\cancel{I_{enc}} =$

$I_{enc} = \int \vec{K} \cdot d\vec{s} =$

$B\cancel{l} = \mu_0 \frac{I}{w} \cancel{x}$



$I = \iint \vec{J} \cdot d\vec{A}$

K = current per unit lenght

$J = \frac{I}{A}$

$\int \vec{K} \cdot d\vec{s} = I$ $\quad K \int d\vec{s}$

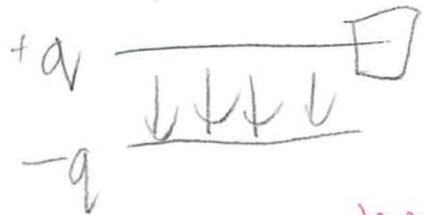
$\frac{I}{w} x$ $\quad K = \frac{I}{S}$

I is passing through with

Problem 7C: DC Power Transmission with the Microstrip Transmission Line

We now connect the transmission line to a battery (EMF ε) on the left and a resistor (resistance R) on the right, as pictured at the beginning of this problem. We are interested in what happens a long time after this connection has been made (after any transient behavior has passed).

(a) What is the electric field between the plates? HINT: This is much easier than you probably think now that the battery fixes the potential difference between the plates.



$\cancel{\oint E \cdot dA = \frac{Q}{\epsilon_0}}$

$\cancel{EA = \frac{qA}{\epsilon_0}}$

no Guass

$E = \cancel{\frac{q}{\epsilon_0}} = \sim \frac{\epsilon}{d}$

$\boxed{E = \frac{V}{d}}$

(b) What is the magnetic field between the plates? HINT: You probably already did this in 7B. Feel free to just quote your previous result.

$\oint B \cdot ds = \mu_0 I_{enc}$ ← only part that matters

$B \cdot \cancel{2\pi r}\, x = \mu_0 \cancel{\pi r^2} \frac{x}{w} I$

$B = \mu_0 \frac{\pi r^2}{2\pi r}$ ← like the only calculation I can do

not a circle

$B = \cancel{\frac{\mu_0 r}{2}}$ $\frac{\mu_0 I}{w} = \frac{\mu_0}{w} \frac{\epsilon}{R}$

CW

right?

Problem 7C: DC Power Transmission with the Microstrip Transmission Line ***continued***

(c) What is the Poynting vector between the plates?

$$\frac{\vec{E} \times \vec{B}}{\mu_0}$$ radially inward charging

$$\frac{q}{\varepsilon_0} \cdot \frac{\mu_0 r}{2 \cancel{\mu_0}} = \frac{q r}{\varepsilon_0 2}$$ did right

(d) Integrate the Poynting vector over a relevant area and show that the result simplifies to what you would expect given the meaning of the Poynting vector.

~~Volume~~ simplify

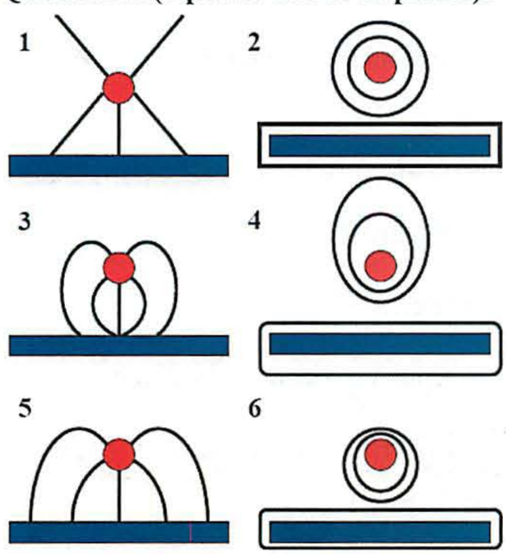
Surface area

Need to get passed doing work + study + learn

Still getting most wrong → not good!

Problem 1: Eight Short Questions. Circle your choice for the correct answer
Each problem is worth 5 points for the correct answer, 2 points for a partially correct answer (at our discretion). If you don't know the answer you can earn 1 point for admitting that by leaving it unanswered and writing "I don't know" (make this clear!).

Question A (5 points out of 40 points):



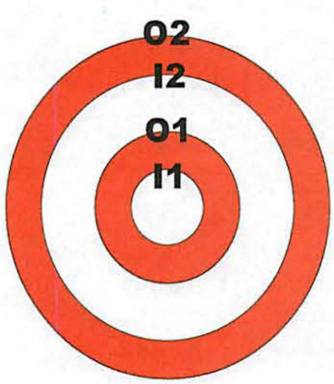
In lab 1 you fixed the potential difference between two plates and measured equipotential lines from which you determined electric field lines and approximate charge distributions. You are given a layout with a conducting plate and conducting circle, as pictured at left, with the circle held at +5 V relative to the plate. Identify the most accurate representation of the equipotential lines and the electric field lines:

Equipotential Lines: # 4

Electric Field Lines: # 5

Field lines must be perpendicular to surfaces. Equipotentials must be closest where field is strongest (between conductors)

Question B (5 points out of 40 points):

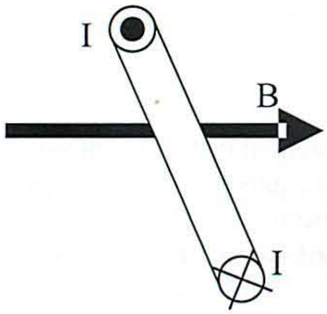


In the second lab you worked with a Faraday pail, two nested conducting cylinders as pictured at left (in the lab the shaded regions were thin). You held the outer cylinder at ground (i.e. at the same potential as infinity) and measured the potential of the inner cylinder relative to the outer cylinder. For one of the measurements you started from a condition where both cylinders were uncharged, introduced a positive charge producer into the central region, briefly connected the inner at outer cylinders with a conductor (your finger) and, after removing the connection, removed the positive charge producer. The positive charge producer never touched either of the cylinders during this measurement. Identify (circle) the sign of the charge (positive, negative or zero) on each surface after doing this:

I1:	>0	$\boxed{=0}$	<0	O1:	>0	=0	$\boxed{<0}$
I2:	$\boxed{>0}$	=0	<0	O2:	>0	$\boxed{=0}$	<0

When touching the two together negative charges flow to the inner conductor to shield the positive charge. In the end they move to O1 and positive charges shield them at I2. Nothing is at O2 because it is grounded, or at I1 because it is an interior surface.

Question C (5 points out of 40 points):



In the third lab you studied the effects of magnetic fields. A current-carrying coil is placed in a uniform magnetic field pointing to the right. The current flows as shown, out of the page in the upper left and in on the lower right.

What are the direction of the force and torque on the coil (circle one direction for each)?

Force (dipole will tend to move...):

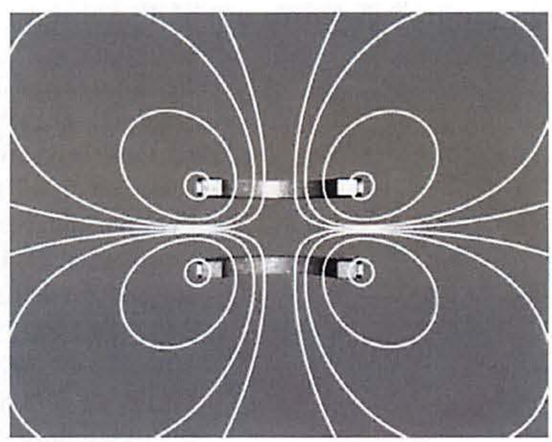
Mostly left   Mostly Right $\boxed{\text{Zero}}$

Torque (dipole will tend to rotate...):

$\boxed{\text{Clockwise}}$ Counterclockwise   None

No force in a uniform field. The dipole moment is up and to right (from right hand rule) so torque to align makes it rotate clockwise.

Question D (5 points out of 40 points):



In the fourth lab you measured the force and torque on a magnetic dipole in the field of a Helmholtz coil (which you could energize in either Helmholtz or Anti-Helmholtz mode). The picture at right shows the field configuration created by the coils after you have energized them in one of these two ways.

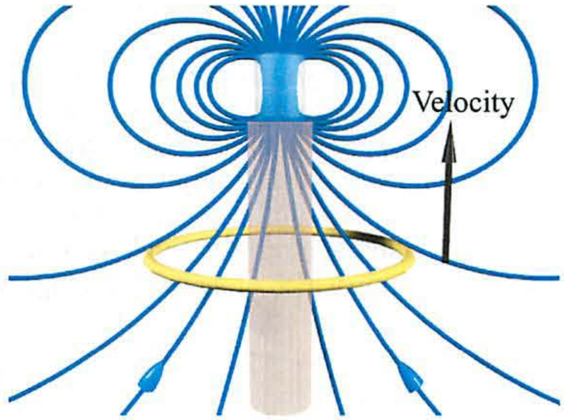
If, before the above field is turned on, you place a dipole so that it is very slightly below center and points very slightly away from alignment with the eventual field of the Helmholtz coil after it is energized, what force and torque will it feel when the coils are energized?

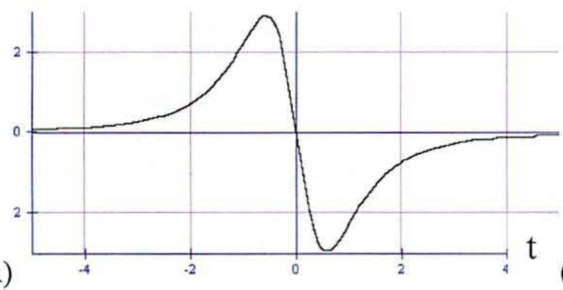
It will feel a force: up (towards center) $\boxed{\text{down (away from center)}}$ no force

It will feel a torque: $\boxed{\text{to align}}$ to anti-align   no torque

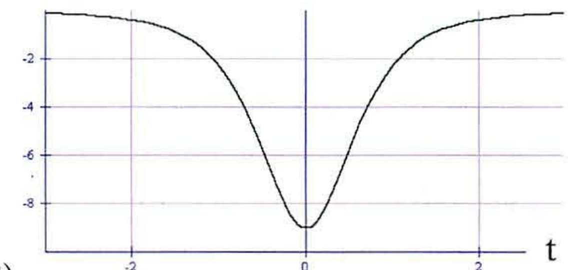
This is an anti-Helmholtz configuration (field is zero at center). We are below center so when the field is energized we will align with the field (ALWAYS!) and then move to the region of strongest field, which is downwards.

Question E (5 points out of 40 points):
In the fifth lab you measured the current and calculated flux generated in a wire coil that was moved from well above a magnet with its North pole facing upwards to well below the magnet and then back up again (see figure). We defined a counter-clockwise (as viewed from above) current as positive and defined the positive flux direction accordingly. For the portion of the motion from well below to well above the magnet, which two of the following diagrams most closely resemble what you should have measured for flux and current respectively?

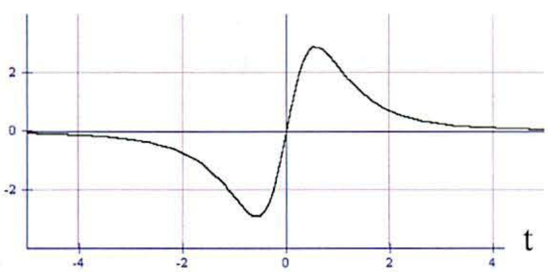




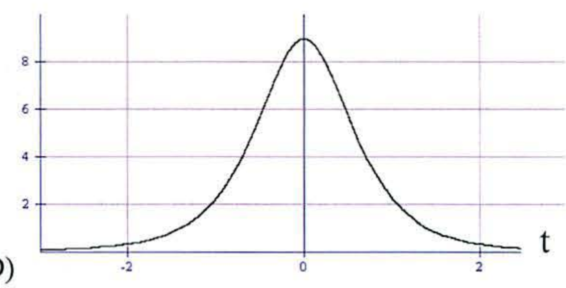
(A)



(B)



(C)



(D)

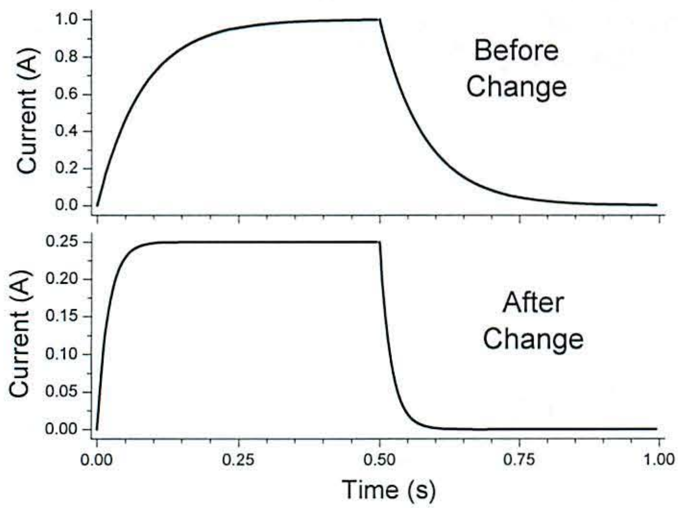
Flux: D

Current: C

Flux is always upwards (positive). The flux will increase then decrease (D). The current will fight the increase by flowing clockwise, then fight the decrease by flowing ccw (C).

Question F (5 points out of 40 points):

In experiment six you set up a simple series LR circuit which consisted of the 750 function generator and the coil (which as you may recall has both a resistance and an inductance). The 750 power supply was used as a "variable battery" which would periodically turn on and off, and the current through the battery was plotted vs. time. In this experiment you had the opportunity to measure the effect of inserting and removing an iron core from the coil as well as the effect of adding an additional resistor either in series or in parallel with the coil. In moving between the two plots below, which of those four things was done (circle one)?



1) Core was added

2) Core was removed

3) Resistor was added in parallel

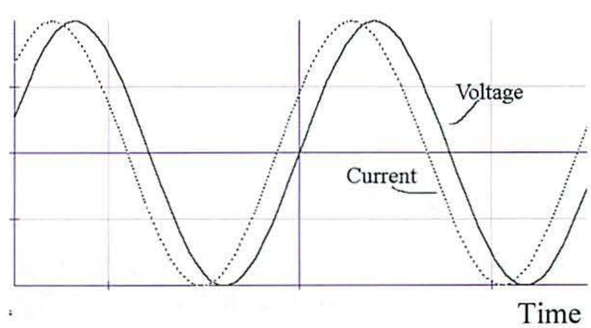
4) Resistor was added in series

5) I don't know (1 point)

The current decreased so resistance must have been added in series. In addition, the time constant decreased ($\tau = L/R$) so this also makes sense.

Question G (5 points out of 40 points):

In experiment seven you studied a driven series LRC circuit and recorded both the power supply voltage (solid curve at right) and current (dashed curve). Which leads and are we at resonance or above or below the resonance frequency ?



Which leads?	Current	Voltage	neither
Above or below?	Above	Below	On Resonance

The current peaks first so it leads. Current leading is capacitor-like, so we are below the resonance frequency, where the capacitor dominates.

Question H (5 points out of 40 points):

In experiment eight you measured the angular dependence of the radiation from a spark gap antenna by moving your receiver either horizontally or vertically around the transmitter.

Angular dependence - Horizontal

Angular dependence - Vertical

Which kind of motion, horizontal or vertical, shows a larger change in radiation intensity over the range of motion?

1. Horizontal
2. Vertical
3. Both show same range of change
4. I don't know (1 point)

The vertical motion shows no change in intensity because there is symmetry that direction. Horizontally the intensity decreases as you move away from alignment between transmitter and receiver.

Problem 2: Maxwell's Equations (20 Pts)
The content of this course can almost completely be summarized in Maxwell's equations. For each of Maxwell's equations please do the following:

1) State the NAME of the equation
2) Write the INTEGRAL FORM of the equation (in other words, write down the equation as you have learned it)
3) Briefly EXPLAIN THE MEANING of the equation (that is, in words, explain the IDEA behind the equation – do not simply give the meaning of the symbols).
4) For TWO OF THE FOUR equations (your choice): give a REAL WORLD EXAMPLE of how you would use the equation to make an approximation of something. You have been given lots of these on the problem sets this semester – feel free to choose one of those or make up one of your own. Give values for all quantities in your approximation. Note that you don't need to do any more work here than you would if you really were doing the approximation (i.e. don't work through the problem in gruesome detail, just show how you can make a quick approximation).

Gauss's Law: $\oiint_{\text{closed surface}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \dfrac{Q_{\text{free, inside}}}{\varepsilon_o}$ means that charges create diverging electric fields

How much excess charge is on your finger when you get a shock on a doorknob?

$E \approx 3\times10^6 \ \dfrac{\text{V}}{\text{m}}$; $A \approx$ sphere of radius 1 cm $= 4\pi(1 \text{ cm})^2 \approx 10^{-3}\,\text{m}^2; \varepsilon_0 \approx 9\times10^{-12} \ \dfrac{\text{F}}{\text{m}}$

So $Q \sim \varepsilon_0 EA \approx \left(9\times10^{-12} \ \dfrac{\text{F}}{\text{m}}\right)\left(3\times10^6 \ \dfrac{\text{V}}{\text{m}}\right)\left(10^{-3}\text{m}^2\right) \sim 3\times10^{-8} \text{ C}$

Ampere-Maxwell Law: $\oint_{\text{contour}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_o \left(I_{\text{through}} + \varepsilon_0 \dfrac{d\Phi_E}{dt} \right)$ means that current and changing electric fields create curling magnetic fields

How much magnetic field do you feel from a power line going into your house.

$s \approx$ circumference of 3 m circle $= 2\pi(3 \text{ m}) \approx 20 \text{ m}; \mu_o \approx 4\pi\times10^{-7} \ \dfrac{\text{T m}}{\text{A}}; I = 100 \text{ A}$

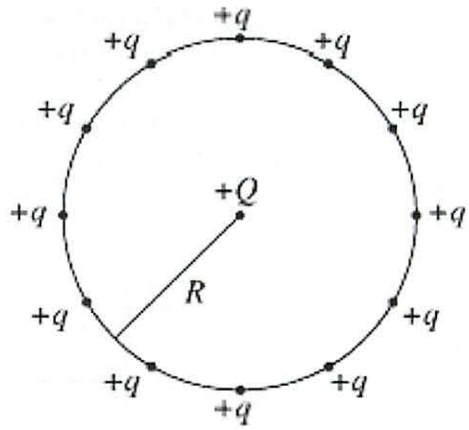
So $B \sim \dfrac{\mu_o I}{s} \approx \left(10^{-6} \ \dfrac{\text{T m}}{\text{A}}\right)(100 \text{ A})(20 \text{ m})^{-1} \sim 2\times10^{-3} \text{ T}$

(of course, there is typically another wire nearby taking current in the opposite direction that will reduce this, but as a first approximation this is fine).

Magnetic Gauss's Law: $\oiint_{\substack{\text{closed}\\ \text{surface}}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$ means no magnetic monopoles

Faraday's Law: $\oint \vec{\mathbf{E}} \cdot \mathbf{d}\vec{\mathbf{s}} = -\dfrac{d}{dt} \iint \vec{\mathbf{B}} \cdot \mathbf{d}\vec{\mathbf{A}}$ means that changing magnetic fields can induce curling electric fields

Problem 3: Charges (10 points):



Twelve equal charges $+q$ are situated in a circle with radius R, and they are equally spaced (see the figure).

(a) What is the net force (magnitude and direction) on a charge $+Q$ at the center of the circle.

4 points:

By symmetry the net force is zero.

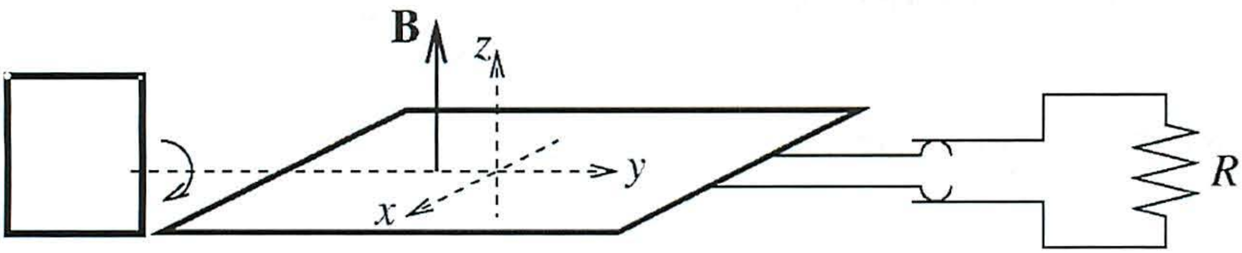
We remove only the +q charge which is located at "3-o'clock."
(b) What now is the force (magnitude and direction) on the charge +Q at the center of the circle?

6 points:

With the 3-o'clock charge removed, the 9-o'clock charge is now unbalanced, so it exerts a force:

$$\vec{\mathbf{F}} = \frac{kqQ}{R^2} \text{ to the right}$$

Problem 4: Generator (20 points)
A simple electric generator (as shown below) is rotating about the y-axis with a frequency of f [Hz]. There is a uniform magnetic field B [T] in the +z direction. The rotor consists of a coil of n windings each with an area S [m^2]. The generator, through slipping contacts, is powering a light bulb whose resistance is R [Ω]. The ohmic resistance of the coil is negligibly small compared to that of the light bulb. You may also assume here, for simplicity, that the self-inductance of the coil is negligibly small.



(a) What is the maximum value (I_{max}) of the induced alternating current? Also indicate in the figure one of the two positions of the coil when this maximum current occurs.

10 points:

The current is driven by the EMF induced by changing magnetic flux through the loop (Faraday's Law):

$$I = \frac{\varepsilon}{R} = \frac{1}{R}\frac{d\Phi}{dt} = \frac{1}{R}\frac{d}{dt} BnS\sin(2\pi ft) = \frac{2\pi f}{R} BnS\cos(2\pi ft)$$

So the maximum of the current is $I_{\text{Max}} = \dfrac{2\pi f}{R} BnS$

The current is a max when the flux is changing the most which is when the loop is 90 degrees to the way it is pictured above

(b) What is the time-averaged mechanical power (in Watts) that must be supplied to maintain the rotation (neglect friction in the bearings)?

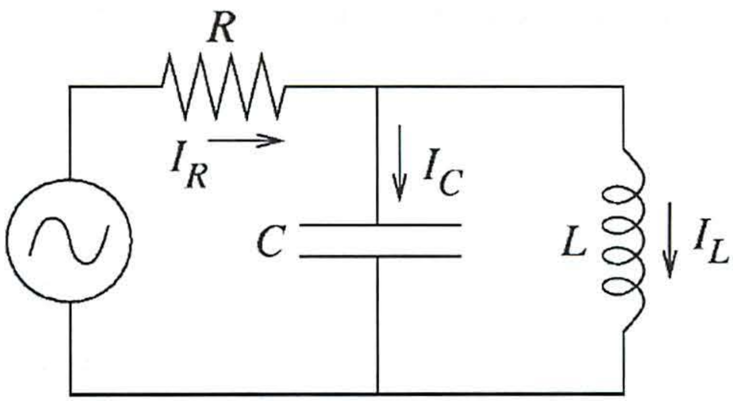
10 points:

$$\langle P\rangle = \langle I^2 R\rangle = \left\langle \left(\frac{2\pi f}{R} BnS\cos(2\pi ft)\right)^2 R\right\rangle = \frac{1}{2}\frac{4\pi^2 f^2 B^2 n^2 S^2}{R}$$

Where the ½ out front comes from the time average of cos^2.

Problem 5: Circuit (20 points)

The LRC circuit as shown is driven by a power supply whose EMF = $V_0\cos(\omega t)$. In steady state, the current through the ideal self-inductor is I_L, the current through the ideal capacitor is I_C and the current through the resistor is I_R. **Steady state means that you wait a long time so that all transient phenomena have died out. Don't even THINK of writing down a differential equation. This problem is designed to see whether you have an appreciation for how a capacitor and a self-inductor behave in extreme situations. No fancy math is needed.** Express all your answers in terms of L, R, C and V_0.



(a) What are the maximum values of I_L, I_C and I_R in case ω=0 (zero frequency means that the power supply is now a simple battery with zero internal resistance). **We are asking you for steady state solutions, NOT transient solutions.**

6 points:

At low frequency the capacitor will have a high impedance and the inductor will have a near zero impedance, so all current goes through the inductor:

$$I_R = I_L = \frac{V_0}{R};\quad I_C = 0$$

(b) Answer the same question as under "(a)" for the other extreme when ω approaches a value which is infinitely high.

6 points:

At high frequency the inductor will have a high impedance and the capacitor will have a near zero impedance, so all current goes through the capacitor:

$$I_R = I_C = \frac{V_0}{R};\quad I_L = 0$$

Problem 5: Circuit ***continued…***

(c) Do you expect the maximum value of the current I_R to be higher or lower than the value you found under "(a)" in the case that the frequency is somewhere between the two extremes? **Give your reasons.**

5 points:

In the two extreme limits one of the two parallel elements provides a short. In intermediate frequencies this will not be the case so the impedance will be HIGHER and the current will be LOWER.

$$I_0 = \sqrt{I_{R_0}^2 + (I_{C0} - I_{L0})^2}$$

(d) There is one frequency (**in steady state**) for which I_R is zero. This is not so intuitive, but given the fact that this is so, what do you think that frequency is? **Please do not try to calculate this frequency.**

3 points:

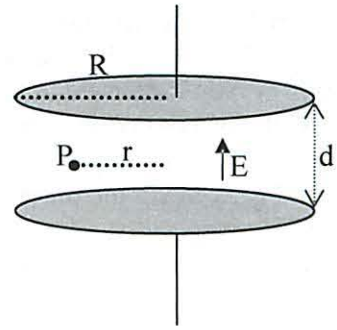
This will happen when the frequency is such that the inductor and capacitor ring:

$$\omega = \frac{1}{\sqrt{LC}}$$

It seems like we have to memorize the whole book - well be able to apply the rules

Problem 6: Capacitor (30 points)

A parallel-plate capacitor consists of two circular plates, each with radius R, separated by a distance d. The electric field $\boldsymbol{E}$ between the plates is uniform and directed upwards (see sketch).



(a) What is the total energy stored in the electric field of the capacitor? Assume that the electric field is uniform between the plates and zero outside of the plates (i.e., neglect fringing fields).

The energy stored is in the electric field. Since E is nearly constant we can just multiply the energy density by the volume inside the capacitor:

$$U_E = u_E \cdot V = \frac{\varepsilon_0 E^2}{2}\pi R^2 d = \frac{\varepsilon_0 \pi R^2 d}{2}E^2$$

(b) Now, suppose that the electric field is *increasing* with time ($dE/dt > 0$). The point P is located between the plates at radius $r < R$ (see sketch). *Derive* an expression for the magnitude of the magnetic field $\boldsymbol{B}$ at point P and indicate its direction there on the sketch.

With the electric field increasing, we have an upwards displacement current:

$$I_{displacement} = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d\left(E\pi r^2\right)}{dt} = \varepsilon_0 \pi r^2 \frac{dE}{dt}$$

$$\oint B \cdot dl = B \cdot 2\pi r = \mu_0 I_{displacement} = \mu_0 \varepsilon_0 \pi r^2 \frac{dE}{dt} \Rightarrow B = \frac{1}{2}\mu_0 \varepsilon_0 r \frac{dE}{dt} \text{ out of page at P}$$

(c) What is the Poynting vector at point P? Give both direction and magnitude.

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0} = \frac{1}{\mu_0}E\frac{1}{2}\mu_0\varepsilon_0 r\frac{dE}{dt} = \frac{1}{2}\varepsilon_0 rE\frac{dE}{dt} \text{ (to the right/inwards!)}$$

(d) Using the Poynting vector, determine the total electromagnetic energy flowing into or out of the capacitor per unit time across $r = R$. Which is it (into or out of)? Write down an equation relating this quantity to the electric energy contained in the capacitor (see part (a)).

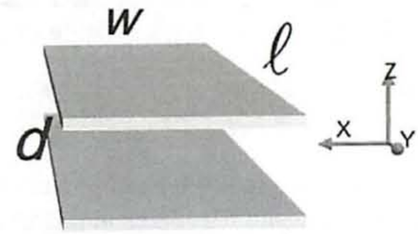
To find the total energy flowing in consider that the band at $r = R$ has an area $A = 2\pi Rd$, so

$$\frac{dU}{dt} = \vec{S}(r = R)\cdot\vec{A} = \left(\frac{1}{2}\varepsilon_0 RE\frac{dE}{dt}\right)(2\pi Rd) = \varepsilon_0 \pi R^2 dE\frac{dE}{dt}.$$

Notice that this is indeed the time derivative of U_E that we calculated in part (a).

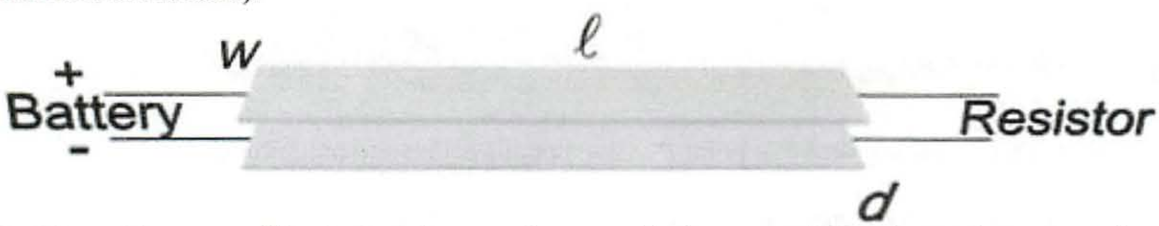
Problem 7: Transmission Line (60 pts)
The rest of this exam is an extended question dealing with transmission lines. There are a variety of transmission lines used in the world. A simple example is two wires running next to each other with current flowing one direction in one and the opposite in the other. Another example that you considered in problem set 12 was the coaxial cable, where current flowed up the inside wire and back along the outer shield.

In this problem you will calculate the properties of a microstrip transmission line. It consists of two thin parallel plates of width w and length ℓ, separated by a small distance d (they are typically held apart by a dielectric, but to make your life simple let's just pretend there is air between the plates). It is shown both in side view and front view below.



The dimensions are such that **you should assume that any fields created by the transmission line are confined to the region between the two plates.**

We use transmission lines to carry power from batteries or power supplies to loads (typically modeled as resistors):



In this problem you will calculate the capacitance and inductance of the microstrip transmission line and then study energy flow at DC.

NOTE: PLEASE READ THIS CAREFULLY

In several parts of this problem you will be asked to calculate something that will require the use of one of Maxwell's equations. Make sure that you state the name of the equation and the write it in the form that you plan to use it before you do that part. You do not need to describe the equation as you were asked to do earlier in this exam, but you do need to be explicit in the calculations and draw and label anything that you need to use to do the calculation. I will not provide any further drawings. Please duplicate drawings from this page (simplified to remove the perspective of course) when you think they will be useful.

Do not forget to give both magnitude and direction of vector quantities.

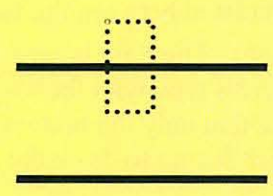
Feel free to tear out this page so that you do not have to continually turn back to it.

Problem 7A: Capacitance of the Microstrip Transmission Line

In the first two parts of this problem (A and B) we will consider the transmission line in isolation (no battery or load resistor).

Calculate the capacitance of the transmission line.

STEP 1: Place ±Q on the plates and calculate the electric field between them?



We have a charge +Q on the top plate so an electric field will be created pointing downwards. We will use Gauss's Law to calculate the electric field between the plates: $\oiint_{\text{Pillbox}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{enc}}}{\varepsilon_o}$

We use a Gaussian pillbox with end cap area A. The only surface of the pillbox we care about is the one between the plates. The field runs perpendicular to the area vector on the sides (doesn't penetrate them) and the field is zero outside because the fields from the two plates cancel there.

$$\oiint_{\text{Pillbox}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EA = \frac{Q_{\text{enc}}}{\varepsilon_o} = \frac{Q}{\varepsilon_o}\frac{A}{w\ell} \Rightarrow \boxed{\vec{\mathbf{E}} = -\frac{Q}{w\ell\varepsilon_o}\hat{\mathbf{k}}}$$

STEP 2: Calculate the voltage difference between them

$$\Delta V = Ed = \frac{Qd}{w\ell\varepsilon_o}$$

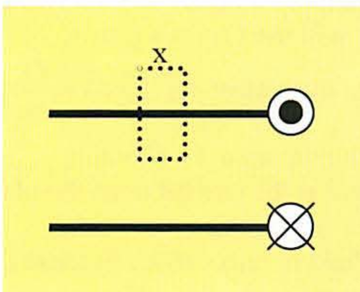
STEP 3: Calculate the capacitance

$$C = \frac{Q}{\Delta V} = \frac{w\ell\varepsilon_o}{d}$$

Problem 7B: Inductance of the Microstrip Transmission Line

Calculate the inductance of the transmission line.
NOTE: There are two ways to do this. If you don't recall either of them then I suggest that you at least send some current through the transmission line and calculate the magnetic energy between the plates.

STEP 1: Place current $\pm I$ on the plates and calculate the magnetic field between them?



We have a current I flowing out the top plate and in the bottom plate meaning that a magnetic field is created between the two plates pointing to the right ($-\hat{\mathbf{i}}$ direction). The field is zero outside by cancellation. We use Ampere's Law with the Amperian loop pictured at left and note that only the bottom leg contributes (B=0 at top and is perpendicular to ds on the sides):

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = Bx = \mu_o I_{\text{enc}} = \mu_o \frac{x}{w} I \Rightarrow \boxed{\vec{\mathbf{B}} = -\frac{\mu_o I}{w}\hat{\mathbf{i}}}$$

STEP 2: Calculate the inductance

We will use energy to calculate the inductance. The magnetic field is uniform so we can just multiply the energy density by the volume:

$$U_B = \frac{B^2}{2\mu_o} \cdot w\ell d = \left(\frac{\mu_o I}{w}\right)^2 \frac{w\ell d}{2\mu_o} = \frac{1}{2}\frac{\mu_o \ell d}{w} I^2 = \frac{1}{2} L I^2 \Rightarrow \boxed{L = \frac{\mu_o \ell d}{w}}$$

Problem 7C: DC Power Transmission with the Microstrip Transmission Line

We now connect the transmission line to a battery (EMF ε) on the left and a resistor (resistance R) on the right, as pictured at the beginning of this problem. We are interested in what happens a long time after this connection has been made (after any transient behavior has passed).

(a) What is the electric field between the plates? HINT: This is much easier than you probably think now that the battery fixes the potential difference between the plates.

$$\vec{\mathbf{E}} = -\frac{\varepsilon}{d}\hat{\mathbf{k}}$$

(b) What is the magnetic field between the plates? HINT: You probably already did this in 7B. Feel free to just quote your previous result.

$$\vec{\mathbf{B}} = -\frac{\mu_o}{w}I\hat{\mathbf{i}} = -\frac{\mu_o}{w}\frac{\varepsilon}{R}\hat{\mathbf{i}}$$

(c) What is the Poynting vector between the plates?

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}} = \frac{1}{\mu_0}\left(-\frac{\varepsilon}{d}\hat{\mathbf{k}}\right)\times\left(-\frac{\mu_o}{w}\frac{\varepsilon}{R}\hat{\mathbf{i}}\right) = \frac{\varepsilon^2}{R}\frac{1}{wd}\hat{\mathbf{j}}$$

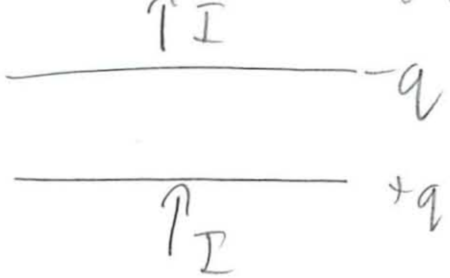
(d) Integrate the Poynting vector over a relevant area and show that the result simplifies to what you would expect given the meaning of the Poynting vector.

The relevant area is the cross-sectional area of the transmission line, *wd*. The Poynting vector is uniform so we can just multiply rather than integrate:

$$\iint \vec{\mathbf{S}}\cdot d\vec{\mathbf{A}} = SA = \frac{\varepsilon^2}{R} = \text{Power dissipated by the resistor}$$

Review P-Set 12

8. The induction current one



did Find Guass' law fine

b) E Stored in E field

$$\frac{1}{2}\epsilon_0 E^2 \cdot \text{volume}$$

$$\frac{1}{2}\cancel{\epsilon_0}\left(\frac{Q}{\pi R^2 \cancel{\epsilon_0}}\right)^2 \cdot \text{volume}$$

$$\frac{Q^2}{2\cancel{\pi^2} \cancel{R^4}\epsilon_0} \cdot \cancel{\pi}\cancel{R^2} d = \frac{Q^2 d}{2\pi R^2 \epsilon_0}$$

~~did not have to~~

must sum up Volume

$$\frac{1}{2}\epsilon_0 \int E^2 \, dV$$

well just put in volume

$$\frac{1}{2}\epsilon_0 E^2 \,\pi R^2 d$$

yeah still have to · volume

(2)

c) What is rate of change of Energy

$$dU = d\,\frac{1}{2}\,\frac{Q^2 d}{\pi R^2}$$

what changes Q

$$\frac{1}{2}\,dQ^2$$

$$Q\,\frac{d}{2\pi R^2}\,\underbrace{\frac{dQ}{dt}}_{I}$$ ← why in all world

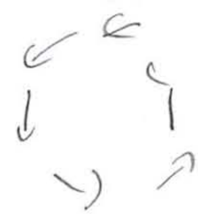
$$\frac{d}{dt}U_{elec} \quad \frac{Q\,I\,d}{\pi R^2 \epsilon_0}$$

d) B Field

$$\oint B\cdot ds = \mu_0 I_{enc}$$

$$B\cdot \underset{\text{duh}}{\overset{2\pi r}{\cancel{\pi r^2}}} = \mu_0\,\frac{\pi r^2}{\pi R^2}\,I$$

what dir is it?



← duh picture is ccw

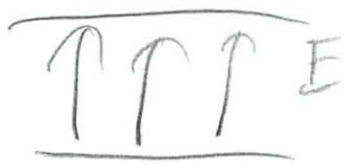
CCW

$$B = \frac{\mu_0 I \cancel{\pi} r^{\cancel{2}}}{2\cancel{\pi r}\,\pi R^2} = \frac{\mu_0 I r}{2\pi R^2}$$

$0 < r < R$

③

e) Sketch $\vec{E}$ + $\vec{B}$



side

top

f) Dir + magnitude poynting vector

$$\begin{array}{ccc} & \downarrow & \\ \rightarrow & & \leftarrow \\ & \uparrow & \end{array}$$ inside so charging

magnitude $\frac{E \times B}{\mu_0}$ $\qquad \frac{\frac{Q}{\pi R^2 \epsilon_0} \cdot \frac{\mu_0 I r}{2\pi R^2}}{\mu_0}$

$$\frac{Q I r}{2\pi^2 R^4 \epsilon_0} \quad -\hat{r} \qquad r = R$$

$$\frac{Q I}{2\pi^2 R^3 \epsilon_0}$$ ✓

actually doing Ok besides ~~first 4V~~ energy

(4)

Integrate $\vec{S}$ over surface to find power

$$\int_0^{2\pi}\int_0^{R}\int \frac{Q\,I}{\pi R^2 \epsilon_0\, 2\pi \cancel{R}}\,\cancel{R}\,dr\,d\theta\,dz$$

$$\int \frac{1}{r^2} = \int r^{-2} = \frac{r^{-1}}{-1} = -\frac{1}{r}$$

$$-\frac{Q\,I}{\pi R\,\epsilon_0\,2\pi}$$

$$-\frac{Q\,I\,\cancel{2\pi}}{\cancel{2}\pi^{\cancel{2}} R\,\epsilon_0} =$$

$$\frac{-Q\,I}{\pi R^2 \epsilon_0} \downarrow$$

well pointing does not change
area · poynting
closer still

(5)

How does answer compare to part c

Real answer: Helped me work backward to solve C

Should be =

Ok review energy argument

$U_{ele} = \frac{1}{2} \epsilon_0 E^2$

but must integrate over volume

energy density

That was my problem

And when differentiating only a part for some reason

What else from this p-set?

Other wise should do another pratice test

Hope it works better

Well finish reviewing pratice test

(6).

Electric flux

Oh yeah just $\vec{E}$ through area

$$\int E \cdot dA$$

Slits + optics - don't need to know too much

I think all rest of P-set 12 I will know how to do when I get sheet

- but did poorly on pratice test

Lets do another praticy

Physics 8.02 | Final Exam | Fall 2005

Please Remove this Tear Sheet from Your Exam

Some (possibly useful) Relations:

$$d\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_o}\frac{dq}{r^2}\hat{\mathbf{r}}$$

$$\oiint_{\text{closed surface}} \kappa \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{free, inside}}}{\varepsilon_o}$$

$d\vec{\mathbf{A}}$ points from inside to outside

$$\oint \vec{\mathbf{E}} \cdot \mathbf{d}\vec{\mathbf{s}} = -\frac{d}{dt}\iint \vec{\mathbf{B}} \cdot \mathbf{d}\vec{\mathbf{A}} \qquad \varepsilon = -N\frac{d\Phi_{\text{sgl loop}}}{dt}$$

$$\Delta V_{\text{moving from } a \text{ to } b} = V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{q\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2} \quad |\vec{\mathbf{v}}| << c \quad d\vec{\mathbf{B}} = \frac{\mu_o I}{4\pi}\frac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

where $\hat{\mathbf{r}}$ points **from** source **to** observer

$$\oiint_{\text{closed surface}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

$$\oint_{\text{contour}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_o\left(I_{\text{through}} + \varepsilon_0\frac{d\Phi_E}{dt}\right)$$

where I_{through} is the current flowing through any open surface bounded by the contour:

$$I_{\text{through}} = \int_{\text{open surface}} \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}$$

ds is right-handed with respect to dA

$$u_E = \frac{1}{2}\varepsilon_0 E^2 \qquad u_B = \frac{B^2}{2\mu_o}$$

$$\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}_{ext}\right) \qquad d\vec{\mathbf{F}} = I\, d\vec{\mathbf{s}} \times \vec{\mathbf{B}}_{ext}$$

$$F_{\text{cent.}} = mv^2/r$$

$$\vec{\boldsymbol{\mu}} = IA\,\hat{\mathbf{n}}$$

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}} \qquad F_z = \mu_z \frac{dB_z}{dz}$$

$$\Delta V = IR \qquad R = \frac{\rho L}{A}$$

$$P_{\text{ohmic heating}} = I\Delta V = I^2 R = \frac{\Delta V^2}{R} = FV$$

$$C = \frac{Q}{\Delta V}$$

$$U = \frac{1}{2}C\Delta V^2 = Q^2/2C$$

$$\tau = RC \qquad X_C = 1/\omega C$$

$$L = \frac{N\Phi_{\text{B,self,sgl coil}}}{I} \qquad \varepsilon_{\text{back}} = -L\frac{dI}{dt}$$

$$U_L = \tfrac{1}{2}LI^2$$

$$\tau = L/R \qquad X_L = \omega L$$

Series RLC: $Z = \sqrt{R^2 + X^2} = \sqrt{R^2 + (X_L - X_C)^2}$

$\tan\varphi = X/R \qquad V_0 = I_0 R$

$$\omega = 2\pi f = 2\pi/T \qquad k = 2\pi/\lambda$$

$$c = \lambda/T = \lambda f = \omega/k = (\mu_0\varepsilon_0)^{-1/2}$$

$$E_0 = v_{\text{light}} B_0 \qquad \hat{\mathbf{E}} \times \hat{\mathbf{B}} = \hat{\mathbf{p}}$$

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}} \qquad P_{\text{absorb}} = \frac{S}{c};\; P_{\text{reflect}} = \frac{2S}{c}$$

If the function $D(t)$ satisfies the equation

$$\frac{d}{dt}D(t) = -\frac{D(t)}{\tau}, \text{ then } D(t) = D_o e^{-t/\tau}$$

Cross-products of unit vectors:

$$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$$

$$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \quad \hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}} \quad \hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}}$$

Some potentially useful numbers

$$k_e = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{\text{N m}^2}{\text{C}^2} \qquad \mu_o = 4\pi \times 10^{-7}\frac{\text{T m}}{\text{A}}$$

Breakdown of air	$E \sim 3 \times 10^6$ V/m
Earth's B Field	$B \sim 5 \times 10^{-5}$ T = 0.5 Gauss
Speed of light	$c = 3 \times 10^8$ m/s
Light (blue to red)	λ = 400 nm to 700 nm
Electron charge	$e = 1.6 \times 10^{-19}$ C
Avogadro's number	$N_A = 6.02 \times 10^{23}$ mol^{-1}
Calories	1 cal = 10^{-3} Cal = 4.184 J

8.02 Final Exam Fall 2005

FAMILY (last) NAME

GIVEN (first) NAME

Student ID Number

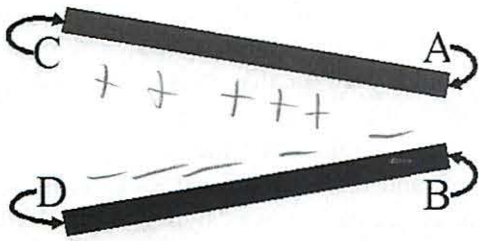
Your Section: ___L01 MW 10 am ____L02 MW 1 pm

Your Group (e.g. 10A): ______________

Problem	**Score**	**Grader**
1 (40 points)		
2 (15 points)		
3 (25 points)		
4 (45 points)		
5 (75 points)		
TOTAL		

Problem 1: Ten Short Questions. Circle your choice for the correct answer
Each problem is worth 4 points for the correct answer or 1 point for admitting that you don't know how to do it. Unanswered or incorrectly answered problems will earn a 0.

Question A (4 points out of 40 points):

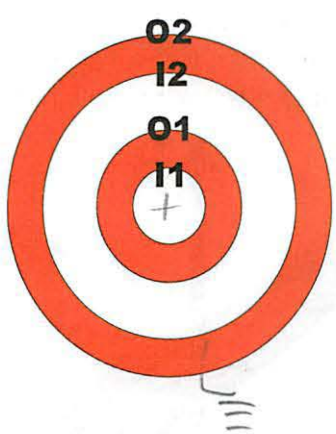


In lab 1 you fixed the potential difference between two plates and measured equipotential lines from which you determined electric field lines and approximate charge distributions. You are given the two conducting plates at left, with the top plate held at +5 V relative to the bottom plate. What can you say about the relative magnitude of the charge densities near the four locations indicated?

1. |Q(A)| ~ |Q(C)| > |Q(B)| ~ |Q(D)|
2. |Q(A)| > |Q(B)| ~ |Q(C)| > |Q(D)|
3. |Q(A)| ~ |Q(B)| > |Q(C)| ~ |Q(D)|
4. |Q(D)| ~ |Q(C)| > |Q(B)| ~ |Q(A)|
5. |Q(B)| ~ |Q(D)| > |Q(A)| ~ |Q(C)|
6. |Q(A)| > |Q(D)| ~ |Q(C)| > |Q(B)|
7. |Q(C)| > |Q(A)| ~ |Q(B)| > |Q(D)|
8. I don't know (this answer is worth 1 point)

? tricky

Question B (4 points out of 40 points):



In the second lab you worked with a Faraday pail, two nested conducting cylinders as pictured at left. You held the outer cylinder at ground (i.e. at the same potential as infinity) and measured the potential of the inner cylinder relative to the outer cylinder. For one of the measurements you started from a condition where both cylinders were uncharged, introduced a positive charge producer into the central region, briefly connected the inner at outer cylinders with a conductor (your finger) and, after removing the connection, removed the positive charge producer. The positive charge producer never touched either of the cylinders during this measurement. Which of the following statements about the surface charges at the end of this measurement is true?

1. Q(I1) = 0; Q(O1) = 0; Q(I2) = 0; Q(O2) = 0
2. Q(I1) = 0; Q(O1) < 0; Q(I2) > 0; Q(O2) = 0
3. Q(I1) = 0; Q(O1) < 0; Q(I2) > 0; Q(O2) < 0
4. Q(I1) = 0; Q(O1) > 0; Q(I2) < 0; Q(O2) = 0
5. Q(I1) = 0; Q(O1) > 0; Q(I2) < 0; Q(O2) > 0
6. Q(I1) < 0; Q(O1) > 0; Q(I2) < 0; Q(O2) > 0
7. I don't know (this answer is worth 1 point)

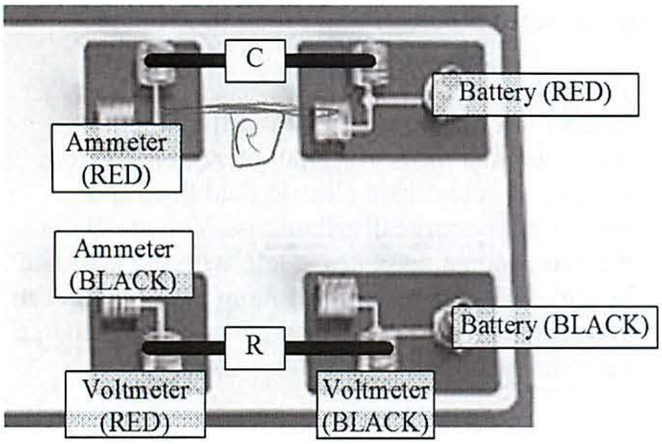
remember from last time

O2 = 0 I2 +

I1 = 0

O1 −

Question C (4 points out of 40 points):



In the third lab you constructed the circuit at left in order to study the effects of capacitors in circuits. You measured the current through and voltage across the resistor *R* using the ammeter and voltmeter as pictured at left. The battery would periodically switch on and off, allowing you to measure their "initial" values (right after the battery switched on) and their "final" values (a long time after the battery was switched on). After measuring the behavior in this circuit you had the opportunity to add a second resistor **in parallel** with the capacitor *C*.

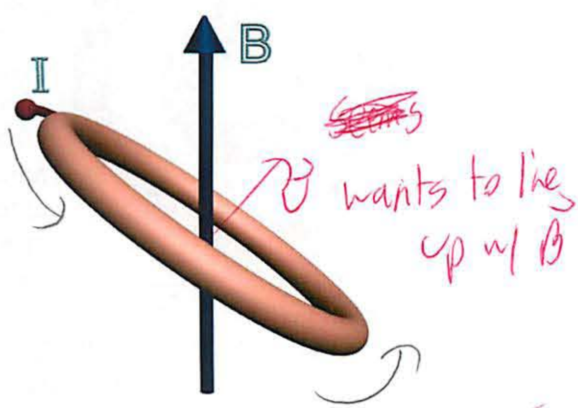
After adding the second resistor which of the following statements is true?

1. Neither the initial current nor the final voltage changed
2. The initial current was smaller but the final voltage was the same
3. The initial current was larger but the final voltage was the same
4. The initial current was the same but the final voltage was smaller
5. The initial current was the same but the final voltage was larger
6. I don't know (this answer is worth 1 point)

C = initally wire
final voltage
V before V = 0
since no current flowing
??? or V = ∞

Question D (4 points out of 40 points):

In the fourth lab you studied the effects of magnetic fields. A current-carrying coil is placed in a uniform magnetic field pointing upward. The current flows as shown, out of the page in the upper left and in on the lower right.



~~seems~~ $\vec{\mu}$ wants to line up w/ B

What are the force and torque on the coil?

1. No force or torque
2. No force, torque to rotate clockwise
3. No force, torque to rotate counterclockwise
4. Force up, no torque
5. Force up, torque to rotate clockwise
6. Force up, torque to rotate counterclockwise
7. Force down, no torque
8. Force down, torque to rotate clockwise
9. Force down, torque to rotate counterclockwise
10. I don't know (this answer is worth 1 point)

$\cancel{F = I(L \times B)}$

$\cancel{\tau = I A \hat{n} \times \vec{B}}$

helpless

Could also apply differenty

really need to learn this

See next pg

which way is L?

or think of it like past test

no force B constant

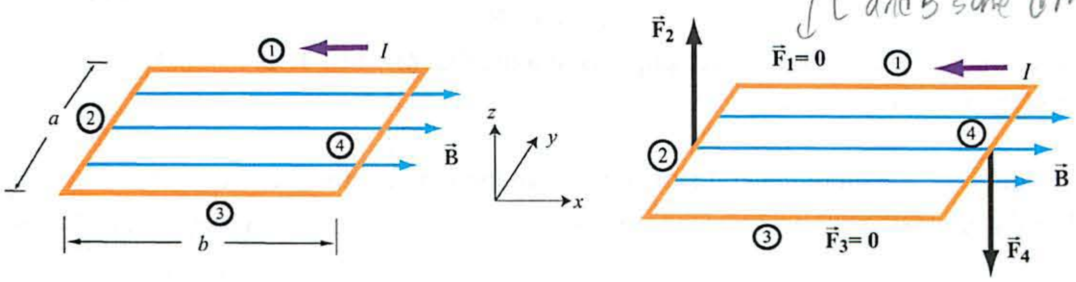
clockwise

Course Notes

8.4 Torque on a Current Loop

What happens when we place a rectangular loop carrying a current I in the xy plane and switch on a uniform magnetic field $\vec{\mathbf{B}} = B\hat{\mathbf{i}}$ which runs parallel to the plane of the loop, as shown in Figure 8.4.1(a)?

L and B same dir



$\cancel{F = I \times B}$

$I(L \times B)$

remember

Figure 8.4.1 (a) A rectangular current loop placed in a uniform magnetic field. (b) The magnetic forces acting on sides 2 and 4.

From Eq. 8.4.1, we see the magnetic forces acting on sides 1 and 3 vanish because the length vectors $\vec{\ell}_1 = -b\hat{\mathbf{i}}$ and $\vec{\ell}_3 = b\hat{\mathbf{i}}$ are parallel and anti-parallel to $\vec{\mathbf{B}}$ and their cross products vanish. On the other hand, the magnetic forces acting on segments 2 and 4 are non-vanishing:

how do they do the dir of a?

$$\begin{cases} \vec{\mathbf{F}}_2 = I(-a\,\hat{\mathbf{j}}) \times (B\hat{\mathbf{i}}) = IaB\hat{\mathbf{k}} \\ \vec{\mathbf{F}}_4 = I(a\,\hat{\mathbf{j}}) \times (B\hat{\mathbf{i}}) = -IaB\hat{\mathbf{k}} \end{cases} \tag{8.4.1}$$

from I?

with $\vec{\mathbf{F}}_2$ pointing out of the page and $\vec{\mathbf{F}}_4$ into the page. Thus, the net force on the rectangular loop is

$$\vec{\mathbf{F}}_{\text{net}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 + \vec{\mathbf{F}}_4 = \boxed{\vec{\mathbf{0}}} \tag{8.4.2}$$

as expected. Even though the net force on the loop vanishes, the forces $\vec{\mathbf{F}}_2$ and $\vec{\mathbf{F}}_4$ will produce a torque which causes the loop to rotate about the y-axis (Figure 8.4.2). The torque with respect to the center of the loop is

$\tau = \vec{\mu} \times \vec{B}$

$$\begin{aligned} \vec{\boldsymbol{\tau}} &= \left(-\frac{b}{2}\hat{\mathbf{i}}\right) \times \vec{\mathbf{F}}_2 + \left(\frac{b}{2}\hat{\mathbf{i}}\right) \times \vec{\mathbf{F}}_4 = \left(-\frac{b}{2}\hat{\mathbf{i}}\right) \times \underbrace{\left(IaB\hat{\mathbf{k}}\right)}_{F_2} + \left(\frac{b}{2}\hat{\mathbf{i}}\right) \times \underbrace{\left(-IaB\hat{\mathbf{k}}\right)}_{F_4} \\ &= \left(\frac{IabB}{2} + \frac{IabB}{2}\right)\hat{\mathbf{j}} = IabB\hat{\mathbf{j}} = \boxed{IAB\hat{\mathbf{j}}} \end{aligned} \tag{8.4.3}$$

how all calculated

why?

$\tau = I A \hat{n} \times \vec{B}$
area normal B field
area

orea

where $A = ab$ represents the area of the loop and the positive sign indicates that the rotation is clockwise about the y-axis. It is convenient to introduce the area vector $\vec{\mathbf{A}} = A\hat{\mathbf{n}}$ where $\hat{\mathbf{n}}$ is a unit vector in the direction normal to the plane of the loop. The direction of the positive sense of $\hat{\mathbf{n}}$ is set by the conventional right-hand rule. In our case, we have $\hat{\mathbf{n}} = +\hat{\mathbf{k}}$. The above expression for torque can then be rewritten as

$$\vec{\boldsymbol{\tau}} = I\vec{\mathbf{A}} \times \vec{\mathbf{B}} \tag{8.4.4}$$

Notice that the magnitude of the torque is at a maximum when $\vec{\mathbf{B}}$ is parallel to the plane of the loop (or perpendicular to $\vec{\mathbf{A}}$).

Consider now the more general situation where the loop (or the area vector $\vec{\mathbf{A}}$) makes an angle θ with respect to the magnetic field.

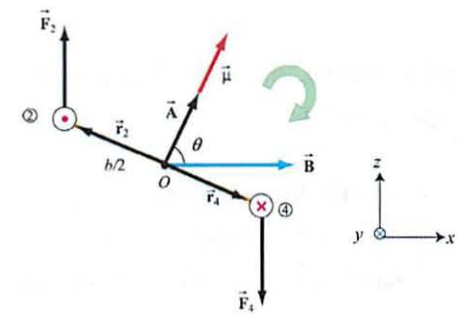


Figure 8.4.2 Rotation of a rectangular current loop

From Figure 8.4.2, the lever arms and can be expressed as:

$$\vec{\mathbf{r}}_2 = \frac{b}{2}\left(-\sin\theta\,\hat{\mathbf{i}} + \cos\theta\,\hat{\mathbf{k}}\right) = -\vec{\mathbf{r}}_4 \tag{8.4.5}$$

and the net torque becomes

$$\begin{aligned} \vec{\boldsymbol{\tau}} &= \vec{\mathbf{r}}_2 \times \vec{\mathbf{F}}_2 + \vec{\mathbf{r}}_4 \times \vec{\mathbf{F}}_4 = 2\vec{\mathbf{r}}_2 \times \vec{\mathbf{F}}_2 = 2\cdot\frac{b}{2}\left(-\sin\theta\,\hat{\mathbf{i}} + \cos\theta\,\hat{\mathbf{k}}\right)\times\left(IaB\,\hat{\mathbf{k}}\right) \\ &= IabB\sin\theta\,\hat{\mathbf{j}} = I\vec{\mathbf{A}} \times \vec{\mathbf{B}} \end{aligned} \tag{8.4.6}$$

For a loop consisting of N turns, the magnitude of the toque is

$$\tau = NIAB\sin\theta \tag{8.4.7}$$

The quantity $NI\vec{\mathbf{A}}$ is called the magnetic dipole moment $\vec{\boldsymbol{\mu}}$:

$$\boxed{\vec{\boldsymbol{\mu}} = NI\vec{\mathbf{A}}} \tag{8.4.8}$$

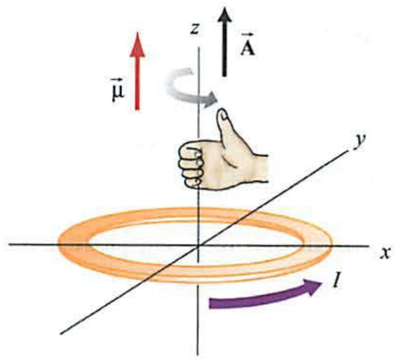


Figure 8.4.3 Right-hand rule for determining the direction of $\vec{\boldsymbol{\mu}}$

The direction of $\vec{\boldsymbol{\mu}}$ is the same as the area vector $\vec{\mathbf{A}}$ (perpendicular to the plane of the loop) and is determined by the right-hand rule (Figure 8.4.3). The SI unit for the magnetic dipole moment is ampere-meter^2 $(\mathrm{A}\cdot\mathrm{m}^2)$. Using the expression for $\vec{\boldsymbol{\mu}}$, the torque exerted on a current-carrying loop can be rewritten as

magnetic diapole moment

$$\boxed{\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}} \tag{8.4.9}$$

The above equation is analogous to $\vec{\boldsymbol{\tau}} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$ in Eq. (2.8.3), the torque exerted on an electric dipole moment $\vec{\mathbf{p}}$ in the presence of an electric field $\vec{\mathbf{E}}$. Recalling that the potential energy for an electric dipole is $U = -\vec{\mathbf{p}}\cdot\vec{\mathbf{E}}$ [see Eq. (2.8.7)], a similar form is expected for the magnetic case. The work done by an external agent to rotate the magnetic dipole from an angle θ_0 to θ is given by

$$\begin{aligned} W_{\text{ext}} &= \int_{\theta_0}^{\theta} \tau\, d\theta' = \int_{\theta_0}^{\theta} (\mu B \sin\theta')\, d\theta' = \mu B\left(\cos\theta_0 - \cos\theta\right) \\ &= \Delta U = U - U_0 \end{aligned} \tag{8.4.10}$$

Once again, $W_{\text{ext}} = -W$, where W is the work done by the magnetic field. Choosing $U_0 = 0$ at $\theta_0 = \pi/2$, the dipole in the presence of an external field then has a potential energy of

$$U = -\mu B \cos\theta = -\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}} \tag{8.4.11}$$

The configuration is at a stable equilibrium when $\vec{\boldsymbol{\mu}}$ is aligned parallel to $\vec{\mathbf{B}}$, making U a minimum with $U_{\min} = -\mu B$. On the other hand, when $\vec{\boldsymbol{\mu}}$ and $\vec{\mathbf{B}}$ are anti-parallel, $U_{\max} = +\mu B$ is a maximum and the system is unstable.

8.4.1 Magnetic force on a dipole

As we have shown above, the force experienced by a current-carrying rectangular loop (i.e., a magnetic dipole) placed in a uniform magnetic field is zero. What happens if the magnetic field is non-uniform? In this case, there will be a net force acting on the dipole.

Consider the situation where a small dipole $\vec{\boldsymbol{\mu}}$ is placed along the symmetric axis of a bar magnet, as shown in Figure 8.4.4.

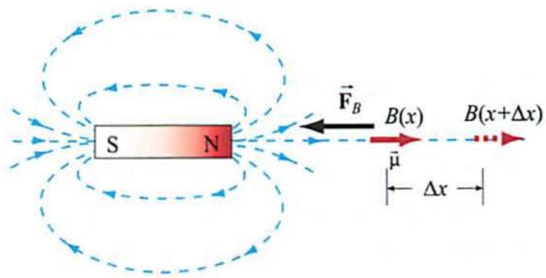


Figure 8.4.4 A magnetic dipole near a bar magnet.

The dipole experiences an attractive force by the bar magnet whose magnetic field is non-uniform in space. Thus, an external force must be applied to move the dipole to the right. The amount of force F_{ext} exerted by an external agent to move the dipole by a distance Δx is given by

$$F_{\text{ext}}\Delta x = W_{\text{ext}} = \Delta U = -\mu B(x+\Delta x) + \mu B(x) = -\mu[B(x+\Delta x) - B(x)] \qquad (8.4.12)$$

where we have used Eq. (8.4.11). For small Δx, the external force may be obtained as

$$F_{\text{ext}} = -\mu\frac{[B(x+\Delta x) - B(x)]}{\Delta x} = -\mu\frac{dB}{dx} \qquad (8.4.13)$$

which is a positive quantity since $dB/dx < 0$, i.e., the magnetic field decreases with increasing x. This is precisely the force needed to overcome the attractive force due to the bar magnet. Thus, we have

$$F_B = \mu\frac{dB}{dx} = \frac{d}{dx}(\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}) \qquad (8.4.14)$$

More generally, the magnetic force experienced by a dipole $\vec{\boldsymbol{\mu}}$ placed in a non-uniform magnetic field $\vec{\mathbf{B}}$ can be written as

$$\vec{\mathbf{F}}_B = \nabla(\vec{\boldsymbol{\mu}}\cdot\vec{\mathbf{B}}) \qquad (8.4.15)$$

where

But all this was helpless in problem

Seems ⓧ goes towards B
and only force if B is changing

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Department of Physics
8.02

Experiment 4 *Solutions*: Forces and Torques on Magnetic Dipoles

MEASUREMENTS

REQUIRED

Part 1: Dipole in Helmholtz Mode

Question 1:

Did the disk magnet rotate? (Was there a torque on the magnet?)

Yes, it rotated to align with the field

Question 2:

Did the spring stretch or compress? (Was there a force on the magnet?)

No, there is no force on the magnet (it is sitting at the field maximum already)

B did not change

Part 2: Reversing the Leads

Question 3:

What happened to the orientation of the disk magnet when you change the current direction in the coils in the Helmholtz configuration? Is this what you expect? Why?

It did different things depending on how careful we were. Most of the time it flipped over to align with the newly oriented field, but sometimes it would sit in the unstable equilibrium of pointing OPPOSITE the field, until the table was bumped and it quickly flipped over.

Part 3: Moving a Dipole Along the Axis of the Helmholtz Apparatus

Question 4:

Starting from the bottom, describe the direction of the force (up or down) and the orientation of the disk magnet, paying careful attention to locations where they change.

At the bottom the string is slightly compressed (there is an upwards force). As we raise upwards to the center that compression decreases. Above the center the spring stretches. Clearly the magnet wants to be at the center of the apparatus where the field is the strongest. The magnet never rotates (it is always aligned with the field).

Question 5:

Where does the force appear to be the largest? The smallest? How should you know this?

The force is the smallest (zero) at the center. The magnet is happy being there so we know there should be no force there. The force is the largest where the gradient is the largest, out towards the coils.

no force at center

OPTIONAL

Part 4: Dipole in Anti-Helmholtz

Question 6:

Did the disk magnet rotate? (Was there a torque on the magnet?)

Once the magnet moves it then does rotate to align with the field. It is hard to tell if it rotates or moves first, but we know that it shouldn't feel any torque when exactly at the center because the field there is zero.

Question 7:

field still 0 at center

Did the spring stretch or compress? (Was there a force on the magnet?)

The spring did compress as the magnet leapt upwards.

Part 5: Moving a Dipole Along the Axis of an Anti-Helmholtz Coil

Question 8:

Starting from the bottom, describe the direction of the force (up or down) and the orientation of the disk magnet, paying careful attention to locations where they change.

At the bottom the spring appears to be slightly compressed, like the magnet wants to go upwards. It's hard to tell here. As we pull upwards, the spring definitely stretches downwards, pulling the dipole down towards the bottom coil. This whole time the dipole is pointing down. Slightly above the center line the magnet flips over and then the spring compresses as the magnet tries to push up towards the top coil. Then as we continue to pull upwards the magnet remains oriented upwards, and the spring becomes uncompressed and then stretches, again trying to get the magnet to near the center of the coils (slightly outside of them).

Question 9:

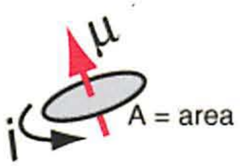
Where does the force appear to be the largest? The smallest? How should you know this?

The force seems to be the largest at the center of the coil, when the magnet gets a BIG jump when the dipole flips over. This makes sense, since the gradient of the field is largest there. The force should be zero where the field is a maximum (minimum), and it is, reaching nearly zero just above the top coil and just below the bottom coil.

force largest at center

Magnetic Dipole Moment

From the expression for the torque on a current loop, the characteristics of the current loop are summarized in its magnetic moment



$$\boxed{\mu = IA}.$$

~~is~~ ; the normal vector in dir of right hand rule

The magnetic moment can be considered to be a vector quantity with direction perpendicular to the current loop in the right-hand-rule direction. The torque is given by

normal vector

$$\tau = \mu \ x \ B$$

As seen in the geometry of a current loop, this torque tends to line up the magnetic moment with the magnetic field B, so this represents its lowest energy configuration. The potential energy associated with the magnetic moment is

$$U(\theta) = -\mu \cdot B$$

so that the difference in energy between aligned and anti-aligned is

$$\Delta U = 2\mu B$$

These relationships for a finite current loop extend to the magnetic dipoles of electron orbits and to the intrinsic magnetic moment associated with electron spin. Also important are nuclear magnetic moments.

HyperPhysics***** Electricity and Magnetism — R Nave

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Magnetic force applications

Magnetic field concepts

Go Back

Ok - does that match problem

$\vec{B}$ $\vec{U}$

U is amt that X cross product so

that much but dir

no $\vec{U}$ lines up to B

Torque on a Current Loop

The torque on a current-carrying coil, as in a DC motor, can be related to the characteristics of the coil by the "magnetic moment" or "magnetic dipole

Index

Magnetic force

moment". The torque exerted by the magnetic force (including both sides of the coil) is given by

$$\tau = BILW\sin\theta$$

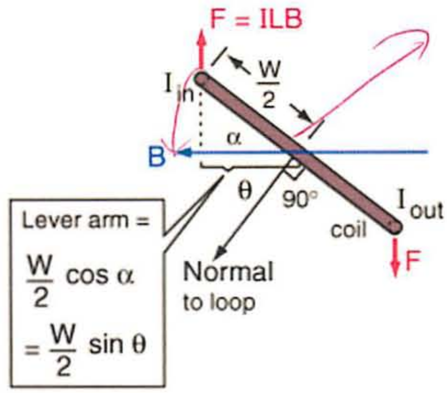
The coil characteristics can be grouped as

$$\mu = IA \quad (or\ \mu = NIA \quad for\ \ n\ \ loops)$$

called the magnetic moment of the loop, and the torque written as

$$\tau = \mu B\sin\theta$$

yeah



The direction of the magnetic moment is perpendicular to the current loop in the right-hand-rule direction, the direction of the normal to the loop in the illustration. Considering torque as a vector quantity, this can be written as the vector product

$$\tau = \mu \ x \ B$$

but they never ask amt just direction

Since this torque acts perpendicular to the magnetic moment, then it can cause the magnetic moment to process around the magnetic field at a characteristic frequency called the Larmor frequency.

If you exerted the necessary torque to overcome the magnetic torque and rotate the loop from angle zero to 180 degrees, you would do an amount of rotational work given by the integral

$$W = -\int_0^{\pi} \tau d\theta = -\int_0^{\pi} \mu B\sin\theta d\theta = -\mu B\cos\theta|_0^{\pi} = 2\mu B$$

The position where the magnetic moment is opposite to the magnetic field is said to have a higher magnetic potential energy.

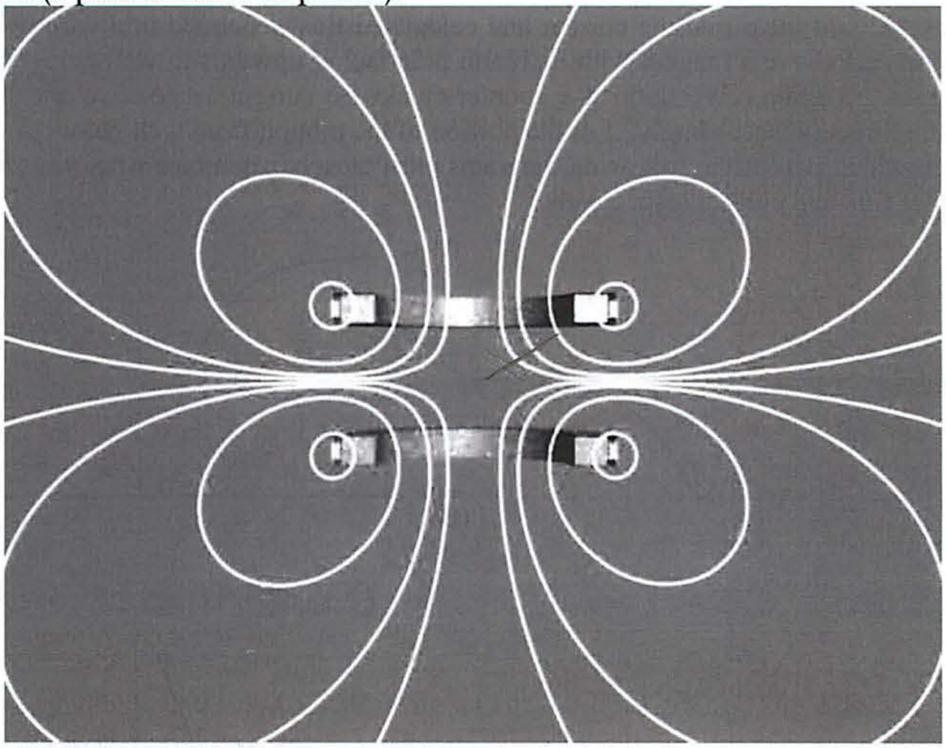
Magnetic force applications

HyperPhysics***** Electricity and Magnetism R Nave

Go Back

Question E (4 points out of 40 points):

in helmholtz
0 at center



better
get this now

In the fifth lab you measured the force and torque on a magnetic dipole in the field of a Helmholtz coil (which you could energize in either Helmholtz or Anti-Helmholtz mode). The picture above shows the field configuration of the coils after you have energized them in one of these two ways.

If, before the above field is turned on, you place a dipole so that it is very slightly above center and points very slightly away from alignment with the eventual field, what force and torque will it feel when the coils are energized?

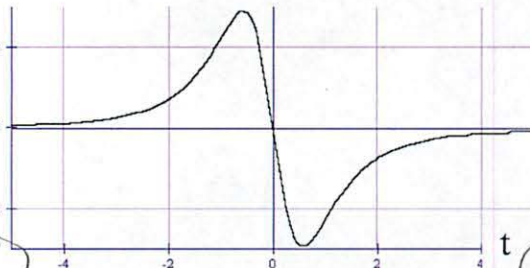
it does turn on and off

1. It will feel no force or torque
2. It will feel a force down (towards the center) but no torque
3. It will feel a force up (away from the center) but no torque
4. It will feel no force but a torque to align with the field
5. It will feel a force down (towards the center) and a torque to align with the field
6. It will feel a force up (away from the center) and a torque to align with the field
7. It will feel no force but a torque to anti-align with the field
8. It will feel a force down (towards the center) and a torque to anti-align with the field
9. It will feel a force up (away from the center) and a torque to anti-align with the field
10. I don't know (this answer is worth 1 point)

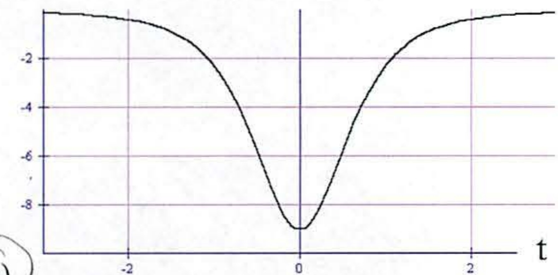
still wrong

force → is B moving no
will always align w/field
force up – yeah where field strongest
~~..~~ but is B moving ???

Question F (4 points out of 40 points):
In the sixth lab you measured the current and calculated flux generated in a wire coil that was moved from ~~well~~ above a magnet with its North pole facing upwards to well below the magnet and then back up again. We defined a counter-clockwise current as positive and defined the positive flux direction accordingly. For the portion of the motion from well below to well above the magnet, which two of the following diagrams most closely resembles what you should have measured for flux and current respectively?

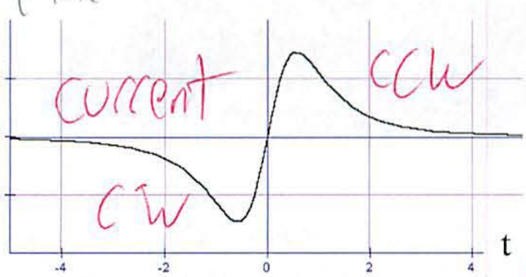


(A)



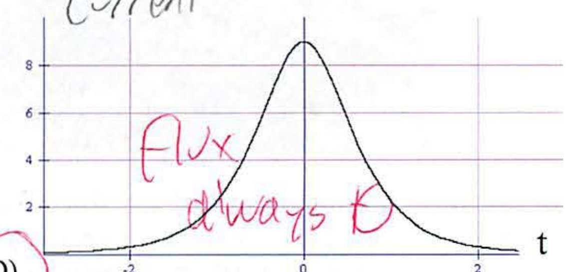
(B)

flux



(C)

current



(D)

1. A (flux) & B (current)
2. A & D
3. C & B
4. C & D
5. B & A
6. B & C
7. D & A
8. D & C
9. I don't know (this answer is worth 1 point)

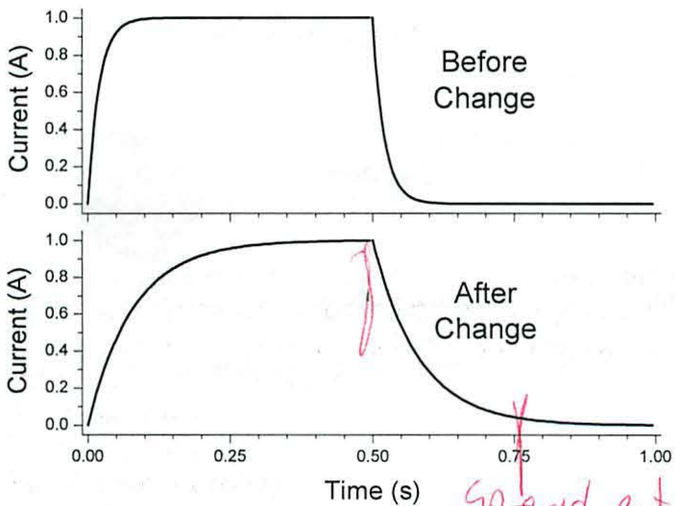
lets see how well I remember

No !

Do the numbers I guess if confused still

Question G (4 points out of 40 points):

In experiment seven you set up a simple series LR circuit which consisted of the 750 function generator and the coil (which as you may recall has both a resistance and an inductance). The 750 power supply was used as a "variable battery" which would periodically turn on and off, and the current through the battery was plotted vs. time. In this experiment you had the opportunity to measure the effect of inserting and removing an iron core from the coil as well as the effect of adding an additional resistor either in series or in parallel with the coil. In moving between the two plots below, which of those four things was done?



1) Core was added
2) Core was removed
3) Resistor was added in parallel
4) Resistor was added in series
5) I don't know (1 point)

~~Same as last the~~ different graph

spread out longer (higher L)

i could also say remved capacitor?

Question H (4 points out of 40 points):

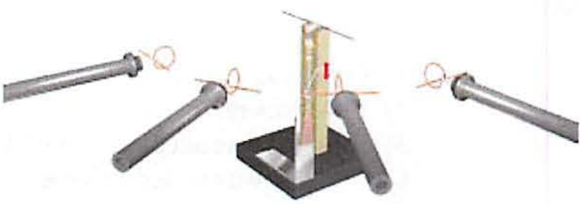
In experiment eight you studied an undriven series LRC circuit and made a plot of energy stored in the capacitor and in the inductor vs. time, which, in addition to oscillating with time, also decayed with time. The total energy (the sum of these two) also decayed in time, but not always at the same rate. When did the total energy in the system decrease most rapidly?

1. When the voltage across the capacitor was a maximum
2. When the voltage across the resistor was a maximum ← what causes decay
3. When the voltage across the inductor was a maximum
4. More than one of the above
5. All of the above
6. None of the above
7. I don't know (1 point)

✓ ?!

Question I (4 points out of 40 points):

In experiment nine you measured the angular dependence of the radiation from a spark gap antenna by moving your receiver either horizontally or vertically around the transmitter.



Angular dependence - Horizontal

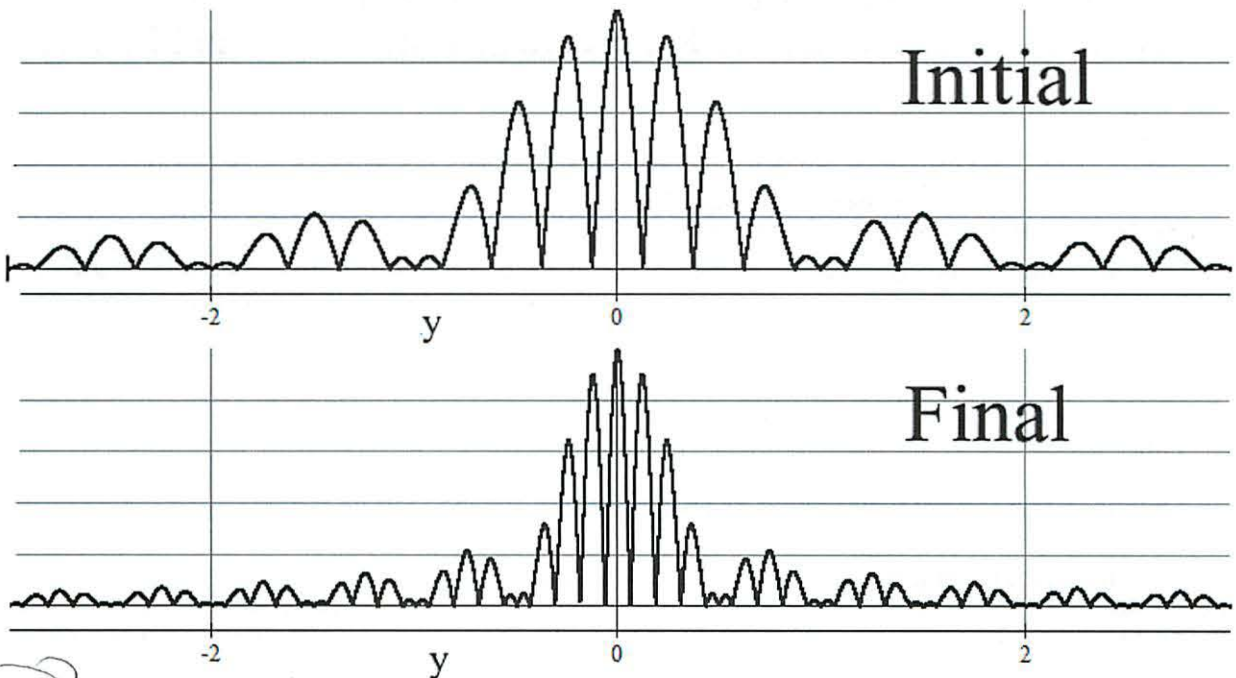
Angular dependence - Vertical

Which kind of motion, horizontal or vertical, shows a larger change in radiation intensity over the range of motion?

1. Horizontal
2. Vertical
3. Both show same range of change
4. I don't know (1 point)

Question J (4 points out of 40 points):

In experiment ten you observed an "Initial" intensity pattern for light coming from two slits and hitting a screen. If you had used a green laser rather than a red one, would you have seen a pattern similar to "Final" below?



1. Yes peaked
2. No, the distance d between the slits must have changed in going from Initial to Final
3. No, the width a of the slits must have changed in going from Initial to Final
4. No, the change depicted results from a change in wavelength the other direction (as if we had started with green light in Initial and moved to red light in Final)
5. I don't know (this answer is worth 1 point)

no clue on light

guess Green is lower wave lengh

Green → Red → Blue

Problem 2: Back of the Envelope Calculation – Numerical Estimation (15 Pts)
I hope that the back of the envelope calculations this semester have given you the confidence to make numerical estimates about things that you aren't exactly sure about (in addition to improving your Google skills). To see if this is true, please estimate the following values. NOTE: I know that you don't necessarily know the answers to these (for example, the radius of the Earth might not be stored in your brain). That is the point. You should be able to make good estimates based on what you do know. **For credit, give all answers in SI units.**

Length

Thickness of notebook paper .... ____________________

Length of the infinite corridor: .... ____________________

Radius of the Earth: .... ____________________

don't think this will be included

Time

Time for Earth to rotate once about its own axis: .... ____________________

Time for Earth to orbit the sun: .... ____________________

Velocity

Speed of a commercial airplane: .... ____________________

Speed of sound in air at atmospheric pressure: .... ____________________

Acceleration

Peak acceleration of a good car: .... ____________________

Energy

Energy stored in a cell phone battery: .... ____________________

Power

Power consumed by a light bulb .... ____________________

Power generated by a power plant: .... ____________________

Electric Field

Electric field at your face from the lamp on your desk..........____________________

Voltage

Voltage between finger and door when getting a shock ....____________________

Magnetic Field

Magnetic field generated by an MRI magnet ....____________________

And finally…

Number of times (it might be less that one) you'd need to run from the bottom to the top of the Green building to burn the calories in a candy bar

....____________________

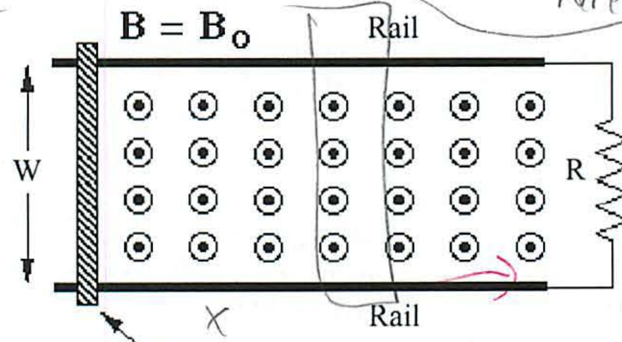
Problem 3 Sliding Bar (25 points): A conducting bar has mass M. It slides to the right along two frictionless horizontal rails separated by a distance W, as shown in the sketch. The rails are connected on the far right by a resistor of resistance R. The bar itself and the rails have zero resistance.

constant speed

At time t = 0 the bar has slid (under its own inertia – no one is pushing it anymore) to the point pictured below, where everywhere to its right there is a constant magnetic field $\mathbf{B}_0$ directed out of the page. This is the ***only*** B field that you are to think about it this problem. At this time it has a velocity $\vec{\mathbf{v}}(t=0)=V_0\hat{\mathbf{i}}$,

where $\hat{\mathbf{i}}$ points to the right in below picture.

here we go the familiar picture



Bar with mass M sliding to the right

(a) As time goes on what will happen to the velocity of the conducting bar? (Circle ans)

1. Increases without limit
2. Increases to limiting value
3. Remains constant
4. Decreases to zero
5. Decreases to zero, then reverses directions

Area shrinking! (did not think through)

unless something else occurs ??

✓ – the induction I keep forgetting + do not clearly know (Study)

(b) Briefly explain why this happens (use words, ***not*** equations). If you will need to use a Maxwell equation to determine subsequent motion of the bar then explicitly state which equation (by name), write the equation, and briefly explain what it means.

~~$\oint B \cdot ds = \mu_0 I_{enc}$~~ $d\Phi = B \cdot dA$

~~$B \cdot (2w+2x) = \mu_0$~~ $d\Phi = B_0 \cdot W \cdot dx$

$d\Phi \quad B_0 \cdot W \cdot V$

Faraday ✓

$\boxed{\text{That supposed to do here}}$

loop area ↓

nature opposes this

So force on bar to stop it

* remember *

Force always opposes change in flux

$\mathcal{E} = -\frac{d\Phi}{dt}$ changing $B \rightarrow$ generates E

So creates a current

(c) Assume that at some later time t the speed of the bar is $v(t)$. What is the current, if any, in the circuit?

$\mathcal{E} = -\frac{d\Phi}{dt}$

$\mathcal{E} = IR$

$I = \frac{\mathcal{E}}{R}$ ✓

$I = \frac{-\frac{d\Phi}{dt}}{R}$ ✓

$I = \frac{B_0 W V}{R}$ ✓ Bingo

Problem 3: Sliding Bar ***continued***

(d) What, if any, is the total magnetic force $\vec{\mathbf{F}}_B$ on the moving bar at this time?

bar height in dir current ?

? found via Lenz'

"screwdriver" right hand rule

$$F_B = \vec{v} \times \vec{B} = I\vec{w} \times \vec{B}$$

? are we supposed to find something.

Force ↓ × ⊙ = ← left

(? this would not effect bar?)

$$\frac{B_0 w v}{R} \cdot w \times B$$

$$= \frac{B^2 w^2 v(t)}{R}$$

(e) Is the kinetic energy of the bar changing? If so, where is that energy going to or coming from? Do a calculation to demonstrate that your answer to this question is true. If not, simply write down an expression for the kinetic energy of the bar.

No ? ~~since force is down?~~

~~But I don't think that is right~~

if no net force must be push in other direction

but no push

So KE is ↓ as bar is slowing

Power dissipated by resistor

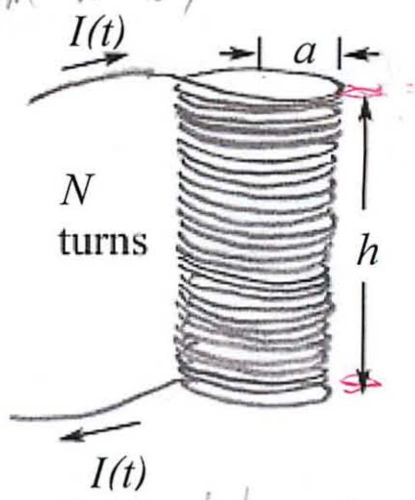
$$P = \vec{F}\vec{v} = \frac{B^2 w^2 v^2(t)}{R} = I^2 R$$

? need to really review power

Instead of just doing pratice test – read up on material then do it from memory

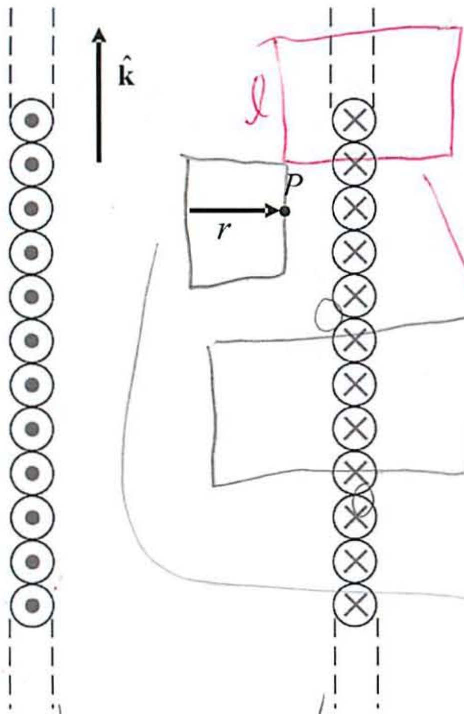
Problem 4: Solenoid (45 pts)

Consider a very long solenoid of N turns, radius a, and length h ($h >> a$) (as pictured at right). It is coaxial with the z-axis.



(a) By looking at the solenoid in cross-section (as I do below) you can calculate the magnetic field at an arbitrary point P, at a radius $r < a$. If the current through the solenoid is $I(t)$, explicitly calculate the magnetic field $\vec{\mathbf{B}}(t)$ at point P. Make sure that you state and briefly explain which Maxwell's equation you are using, and that you draw and label anything needed to do the calculation on the below image. Be completely clear about every step you take (for example, if anything in the calculation is zero, explain why).

Have to really study magnetic field stuff



$N =$ # turns

$\oint B l = \mu_0 N I$

ask your self what I is in this case 1 wire

$B = \frac{\mu_0 N I}{l}$

really know all scenarios for orienting the box

0 →

$Bl = \mu_0 I$ Some fraction thing?

?0 since no wires

their box

$Bl = \mu_0 I_{enc}$

$= \mu_0 N \frac{l}{h} I$

?fraction of current

B is constant here? really need to learn this

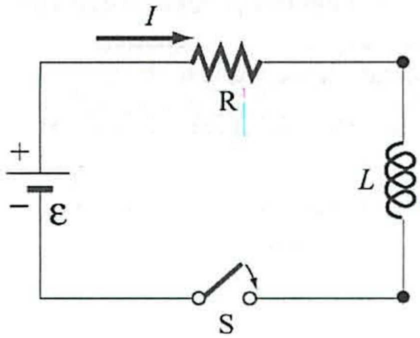
and 0 here

$B = \frac{\mu_0 N I}{h}$

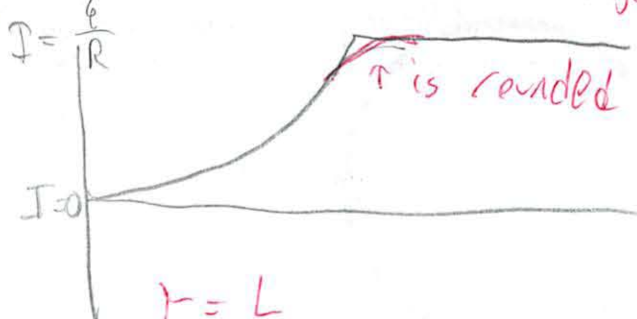
$L = \frac{N\Phi}{I}$

Problem 4: Solenoid ***continued***

This solenoid is an inductor of inductance L (which I'm sure you can calculate so I won't ask you to). We put it in a series LR circuit (pictured below) consisting of a battery with EMF ε, a resistor of resistance R and a switch S. At time $t = 0$ we close the switch in the circuit.



(b) Sketch the time dependence of the current in the circuit, and write an equation for $I(t)$. Clearly identify the initial (t=0) and final (t=∞) values of the current. Briefly explain why the current behaves the way it does. ✓ got reasoning



↑ is rounded

← really need to memorize this!

$I = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)$

$\tau = \frac{L}{R}$

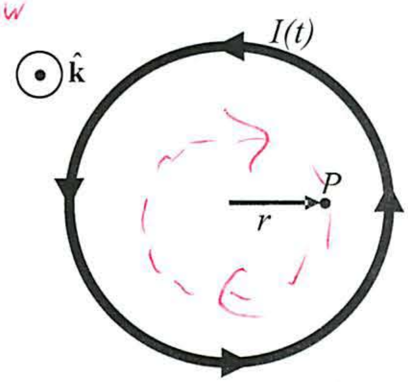
(c) From part (a) you know the magnetic field $\vec{\mathbf{B}}(t)$ as a function of the current $I(t)$, which you have just calculated in part (b). Now let's look at the electric field that is induced inside the solenoid. It is easiest to do that in a top view of the solenoid, which is provided below. Calculate the induced electric field $\vec{\mathbf{E}}(t)$ at point P. If you need to use one of Maxwell's equations then name and briefly explain it before using it. Be very explicit about how you do the calculation, drawing and labeling anything that you need on the figure below. Feel free to leave your answer in terms of $\vec{\mathbf{B}}(t)$ or $I(t)$ as you find convenient – there is no need to substitute your results from part (a) or (b).

blam

Stalk the formula sheet if don't know

induced by Lenz

Opposes changing $\vec{B}$ field tries to make it in



E Gauss

$\oint E \cdot ds = \frac{Q}{\varepsilon_0}$

Faradays

I don't remember this problem

$E \cdot 2\pi r = \frac{-d\Phi_B}{dt}$

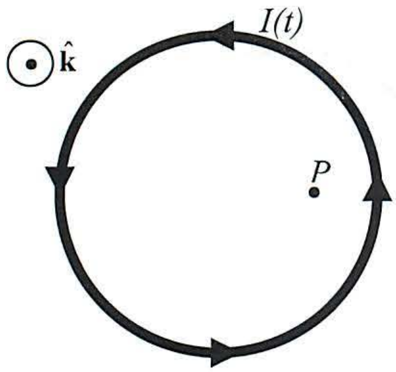
$= \frac{d(B \cdot A)}{dt}$

$= \pi r^2 \frac{dB}{dt}$

$E = \frac{r}{2}\frac{d}{dt}(\vec{B})$ down

Problem 4: Solenoid ***continued***

(d) You now have an electric field $\vec{\mathbf{E}}(t)$ and a magnetic field $\vec{\mathbf{B}}(t)$ at point *P*, meaning that there is a Poynting vector there. Briefly explain the meaning of the Poynting vector (for example, what units does it have?) and then calculate its value at point *P*. Feel free to leave the answer in terms of the field magnitudes $E(t)$ and $B(t)$ (you do *not* need to plug in your answers from previous parts) but **do explicitly state its direction**. To be clear please indicate the direction of the Poynting vector at point *P* on the diagram below. What does the direction indicate about this system?

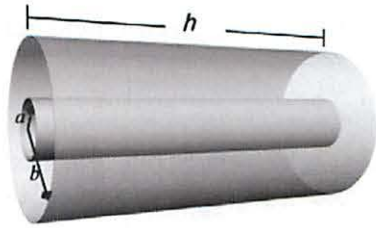


(e) To demonstrate the meaning of the Poynting vector we typically have you integrate it over some area and show that it is equal to something else. In the case of this solenoid, over what area should we integrate? Be very clear here – probably the easiest thing to do is to state an equation for the area. What should that integral be equal to (state this both in words and as an equation)? There is no need to do the actual integral or to plug in values you calculated above to demonstrate that this is indeed the case.

Problem 5: Transmission Line (75 pts) The coaxial problem

The rest of this exam is an extended question dealing with transmission lines. There are a variety of transmission lines used in the world. A simple example is two wires running next to each other with current flowing one direction in one and the opposite in the other.

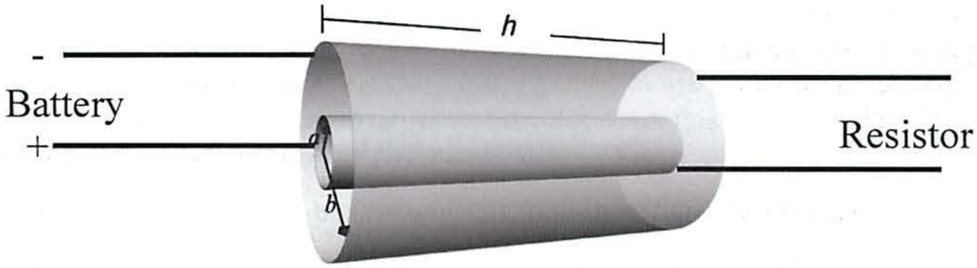
In this problem you will calculate the properties of a coaxial cable. It consists of a solid core or radius a and a thin outer "shield" conductor of radius b, both of length h. They are typically held apart by a dielectric, but to make your life simple let's just pretend there is vacuum between the conductors. It is shown in perspective at right.



at least circular better

The dimensions are such that **you should assume that any fields created by the transmission line are confined to the region between to two conductors.**

We use transmission lines to carry power from batteries or power supplies to loads (typically modeled as resistors):



In this problem you will calculate the capacitance per unit length and inductance per unit length of the microstrip transmission line and then study energy flow at DC. Finally, you will describe its behavior when driven by an AC function generator.

NOTE: PLEASE READ THIS CAREFULLY

In several parts of this problem you will be asked to calculate something that will require the use of one of Maxwell's equations. Make sure that you state the name of the equation and the write it in the form that you plan to use it before you do that part. You do not need to describe the equation as you were asked to do in earlier parts of this exam, but you do need to be explicit in the calculations and draw and label anything that you need to use to do the calculation. I will not provide any further drawings. Please duplicate drawings from this page (simplified to remove the perspective of course) when you think they will be useful.

not this exam

Do not forget to give both magnitude and direction of vector quantities.

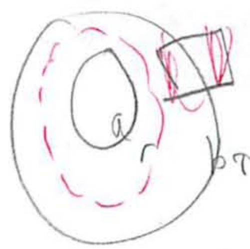
Feel free to tear out this page so that you do not have to continually turn back to it.

Lets see how much I rember

Problem 5A: Capacitance of the Coaxial Cable

In this part we will consider the transmission line in isolation (no battery or load resistor). Assume that the inner conductor has a charge $+Q$ and the outer conductor has a charge $-Q$.

(a) What is the electric field between the conductors?

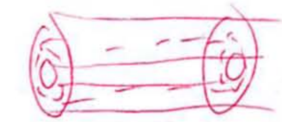


$\oint E \cdot da = \frac{Q}{\epsilon_0}$

what is surface, cylinder

duh ~~[illegible]~~ in middle

don't I remember anything?



$\oint E \cdot 2\pi r L = \frac{\cancel{\pi a^2 I}}{\epsilon_0}$ keep as Q, $\frac{L}{h}$

$E = \cancel{\frac{\pi a^2 I}{2\pi r \epsilon_0}}$ $\frac{Q \cancel{L}}{\epsilon_0 2\pi r \cancel{L} h}$

if partial only, or always?

So many different formulas
So hard to remember
esp from start of semester

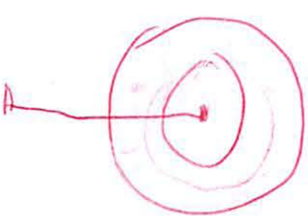
Problem 5A: Capacitance of the Coaxial Cable *continued*

(b) What is the voltage of the outer conductor relative to the inner conductor (that is, what is the voltage difference $\Delta V = V_{outer} - V_{inner}$ between them)?

$\cancel{V = \frac{Q}{C}}$

$$V = -\int_{inner}^{outer} E \cdot ds = -\int_{r=a}^{r=B} \frac{Q}{\epsilon_0 2\pi r h} dr$$

Remember



remember $\int \frac{1}{r} = \ln r = \ln\left(\frac{b}{a}\right)$

$$= \frac{Q}{\epsilon_0 2\pi h} \ln\left(\frac{b}{a}\right)$$

(c) What is the capacitance of the transmission line?

$$C = \frac{Q}{V} \quad \frac{Q}{\frac{Q}{\epsilon_0 2\pi h} \ln\left(\frac{b}{a}\right)} = \frac{\epsilon_0 2\pi h}{\ln\left(\frac{b}{a}\right)} \quad ✓$$

$\frac{Q}{1} \cdot \frac{1}{Q}$

(d) What is the capacitance per unit length of the transmission line? Note that *h* should not appear in this answer – what I mean by "per unit length" is that you need to multiply by the length *h* to get the total capacitance.

Then divide by h

total = C per unit · h

$$\frac{\epsilon_0 2\pi}{\ln\left(\frac{B}{a}\right)} \quad ✓$$

pointless

ϕB is the like ϕ thing I always want to find

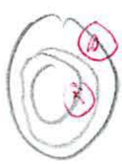
Problem 5B: Inductance of the Coaxial Cable

In this part we will assume that the transmission line has a constant current I traveling down the inner conductor (in the $+\hat{\mathbf{k}}$ direction, to the right) and back along the outer conductor (in the $-\hat{\mathbf{k}}$ direction).

(a) What is the magnetic field between the conductors?

$$L = \frac{N\phi}{I} \qquad \phi = BA$$

$$\oint B\, ds = \mu_0 I_{enc}$$



$$B \cdot 2\pi r = \mu_0 \cancel{\pi a^2} I$$

$$\cancel{B = \frac{2\pi r I}{\mu_0 \pi a^2} \cdot \pi r^2 = \frac{2\pi r^3 I}{\mu_0 a^2}}$$

just keep at I ~~unless~~ since I is defined as current over it - only put stuff in front if fraction of I

$$\cancel{\frac{N}{I}\frac{2\pi r^3 I}{\mu_0 a^2} = \frac{N 2\pi r^3}{\mu_0 a^2}}$$

$$\phi = B = \frac{\mu_0 I}{2\pi r} \cdot \cancel{A = \pi r^2}$$

✓ the just B part is right

dir? cw

$$\cancel{\frac{N}{I}\frac{\mu_0 I r}{2} = \frac{N\mu_0 r}{2}}$$

why do they use energy + get something totally different → N may not be appropriet either

Problem 5B: Inductance of the Coaxial Cable ***continued***

(b) What is the inductance of the transmission line? NOTE: There are two ways to do this. If you don't recall either of them then I suggest that you at least calculate the magnetic energy between the conductors.

Oh they just ask B there -

~~take~~ wtf?

$L = \frac{N\Phi_B}{I}$ $\boxed{\iiint u_B = \frac{1}{2}LI^2}$

See answer sheet

why does n method not work

$L = \frac{h\mu_0}{2\pi} \ln\left(\frac{b}{a}\right)$

(c) What is the inductance per unit length of the transmission line?

$\frac{L}{\text{length}} = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right)$

I'm not rembering stuff from studying - may be the to lead home son

Problem 5C: DC Power Transmission with the Coaxial Cable

We now connect the transmission line to a battery (EMF ε) on the left and a resistor (resistance R) on the right, as pictured at the beginning of this problem. We are interested in what happens a long time after this connection has been made (after any transient behavior has passed). In answering the below questions feel free to use the results from previous sections of this problem (you do not need to derive them again) but express your answers only in terms of variables given here and at the beginning of the problem (NOT in terms of Q or I from parts 5A and 5B).

(a) What is the electric field between the conductors?

Right $E = \frac{V}{d} = \frac{Q \ln\left(\frac{b}{a}\right)}{d\, \varepsilon_0 2\pi h d}$

(b) What is the magnetic field between the conductors?

$\vec{B}$ found before

$$\frac{\mu_0 I}{2\pi r}$$

Problem 5C: DC Power Transmission with the Coaxial Cable *continued*

(c) What is the Poynting vector between the conductors?

$$\frac{\dfrac{Q \ln(b/a)}{d\epsilon_0 2\pi h d} \cdot \dfrac{\mu_0 I}{2\pi r}}{\mu_0}$$

think dir $\hat{r}$

into

(d) Integrate the Poynting vector over a relevant area and show that the result simplifies to what you would expect given the meaning of the Poynting vector.

- Surface area

~~volume~~

must actually S

$$\iint S \cdot d\vec{A} = \int_a^b \frac{\mathcal{E}^2}{2\pi r^2 R(\ln(b/a))} \cdot 2\pi r\, dr$$

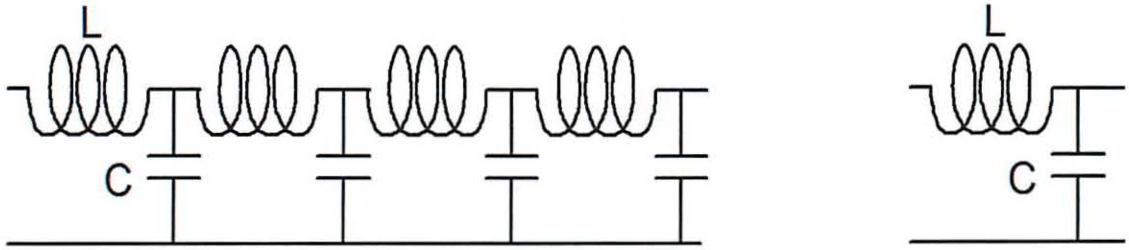
$$= \frac{\mathcal{E}^2}{R} \int_a^b \frac{dr}{\ln(b/a)} = \frac{\mathcal{E}^2}{R}$$

power dissipated by resistor

V=Ed

Problem 5D: Transients and AC Transmission in the Coaxial Cable

You calculated that the transmission line has an inductance per unit length and a capacitance per unit length. A typical way to model the behavior of the transmission line is as a collection of inductors and capacitors, as pictured below left, or even more simply as just a single inductor and capacitor, as pictured below right.



These are "lossless" models – we are ignoring the resistance of the transmission line itself.

(a) Let's first think about the transient behavior of this circuit. The instant after you attach a battery (ε) on the left and a resistive load (R) on the right, what is the current through the load? Why? Describe what the inductive and capacitive parts of the transmission are behaving like at this instant.

I is varable

Read carefully

no current in inductor

current short circuit

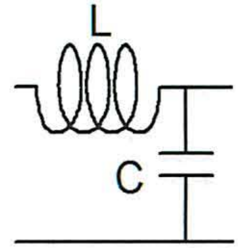
(b) A long time after the battery ε and load resistor R have been connected what is the current through R? Why? Describe what the inductive and capacitive parts of the transmission line are behaving like at this instant.

Capicator fullly charged =0

inductor open I

Problem 5D: Transients and AC Transmission ***continued***

Now instead of attaching a battery on the left, let's attach a function generator, driving a voltage $V = V_0 \sin \omega t$. On the right we still have a resistive load, but to make life simpler let's assume that it is a very large resistor R.



(c) You have already discussed the very low frequency (DC) behavior of the transmission line. As we turn up the $\boxed{\text{frequency}}$ of the power supply, **QUALITATIVELY** describe (no equations) what happens to the voltage that the load sees. Why?

Besides that it varies w/ time

$\uparrow$ not Freq Δ

$\downarrow$ $\downarrow$ reactance

well switches from capicator dominated

to inductor dominated

$\uparrow$ inductor gets $\uparrow$ reactance

So voltage shrinks

-when did we learn that? -is it on the formula sheet

(d) I said above that you would want to assume that the load had a very large resistance R. Typically when we say that something is very large, what we mean is that it is much larger than something else. At non-zero frequencies, what should R be much larger than (give an equation here)? Why?

~~larger than inductance or capicatence~~

larger than the resistence of the wire

if want to ignore resistence when thinking about current want ~~it~~ to

$$R >> \frac{1}{\omega C}$$

$\uparrow$ reactance of capicator

Yeah let me check on that 5/16

Course notes 12

$Q = Q_0 \cos(\omega t - \phi)$

$I = \frac{dQ}{dt} = I_0 \sin(\omega t - \phi)$

impedance = effective resistance

$P = I^2 R = IV$

$\langle \quad \rangle$ time average

Power max when current is max

Play w/ mathlet

as freq ↑

V amplitude same

but period gets smaller

(how fast it spins)

Still don't see it

5/16

Power

$$P = I\Delta V = I^2 R = \frac{\Delta V^2}{R} = \vec{F} \cdot \vec{v}$$

= Ohmic heating = $I|\mathcal{E}|$

can equal other power calculations (what?)

$$= \iint S \cdot dA = SA = \frac{\mathcal{E}^2}{R}$$

$$U_E = u_E \cdot V$$

$dU_E = S \cdot A$ = power consumed by capacitor

Wikipedia

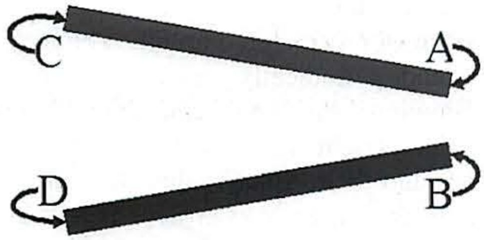
$$P = \frac{\Delta W}{\Delta t}$$ amt of work performed in a certain time

$$W = Pt$$

measured in watts = $\frac{\text{joules}}{\text{sec}}$ = energy/current

Problem 1: Ten Short Questions. Circle your choice for the correct answer
Each problem is worth 4 points for the correct answer or 1 point for admitting that you don't know how to do it. Unanswered or incorrectly answered problems will earn a 0.

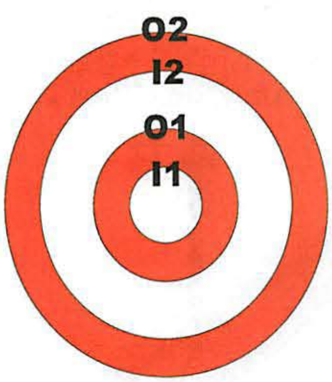
Question A (4 points out of 40 points):



In lab 1 you fixed the potential difference between two plates and measured equipotential lines from which you determined electric field lines and approximate charge distributions. You are given the two conducting plates at left, with the top plate held at +5 V relative to the bottom plate. What can you say about the relative magnitude of the charge densities near the four locations indicated?

1. $|Q(A)| \sim |Q(C)| > |Q(B)| \sim |Q(D)|$
2. $|Q(A)| > |Q(B)| \sim |Q(C)| > |Q(D)|$
3. $|Q(A)| \sim |Q(B)| > |Q(C)| \sim |Q(D)|$
4. $|Q(D)| \sim |Q(C)| > |Q(B)| \sim |Q(A)|$
5. $|Q(B)| \sim |Q(D)| > |Q(A)| \sim |Q(C)|$
6. $|Q(A)| > |Q(D)| \sim |Q(C)| > |Q(B)|$
7. $|Q(C)| > |Q(A)| \sim |Q(B)| > |Q(D)|$
8. I don't know (this answer is worth 1 point)

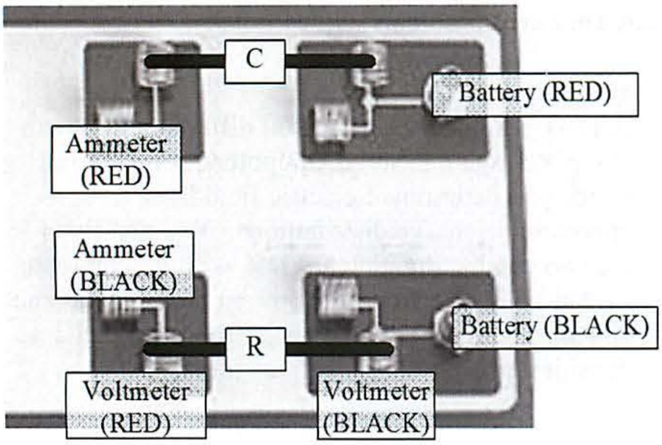
Question B (4 points out of 40 points):



In the second lab you worked with a Faraday pail, two nested conducting cylinders as pictured at left. You held the outer cylinder at ground (i.e. at the same potential as infinity) and measured the potential of the inner cylinder relative to the outer cylinder. For one of the measurements you started from a condition where both cylinders were uncharged, introduced a positive charge producer into the central region, briefly connected the inner at outer cylinders with a conductor (your finger) and, after removing the connection, removed the positive charge producer. The positive charge producer never touched either of the cylinders during this measurement. Which of the following statements about the surface charges at the end of this measurement is true?

1. $Q(I1) = 0$; $Q(O1) = 0$; $Q(I2) = 0$; $Q(O2) = 0$
2. $Q(I1) = 0$; $Q(O1) < 0$; $Q(I2) > 0$; $Q(O2) = 0$
3. $Q(I1) = 0$; $Q(O1) < 0$; $Q(I2) > 0$; $Q(O2) < 0$
4. $Q(I1) = 0$; $Q(O1) > 0$; $Q(I2) < 0$; $Q(O2) = 0$
5. $Q(I1) = 0$; $Q(O1) > 0$; $Q(I2) < 0$; $Q(O2) > 0$
6. $Q(I1) < 0$; $Q(O1) > 0$; $Q(I2) < 0$; $Q(O2) > 0$
7. I don't know (this answer is worth 1 point)

Question C (4 points out of 40 points):

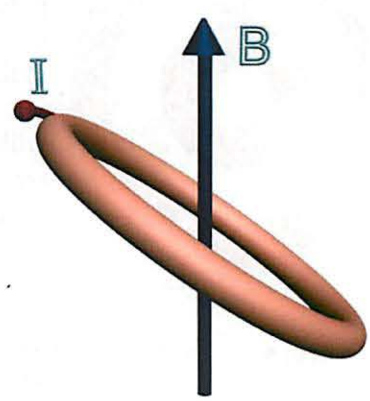


In the third lab you constructed the circuit at left in order to study the effects of capacitors in circuits. You measured the current through and voltage across the resistor R using the ammeter and voltmeter as pictured at left. The battery would periodically switch on and off, allowing you to measure their "initial" values (right after the battery switched on) and their "final" values (a long time after the battery was switched on). After measuring the behavior in this circuit you had the opportunity to add a second resistor **in parallel** with the capacitor C. After adding the second resistor which of the following statements is true?

1. Neither the initial current nor the final voltage changed
2. The initial current was smaller but the final voltage was the same
3. The initial current was larger but the final voltage was the same
4. The initial current was the same but the final voltage was smaller
5. The initial current was the same but the final voltage was larger
6. I don't know (this answer is worth 1 point)

Question D (4 points out of 40 points):

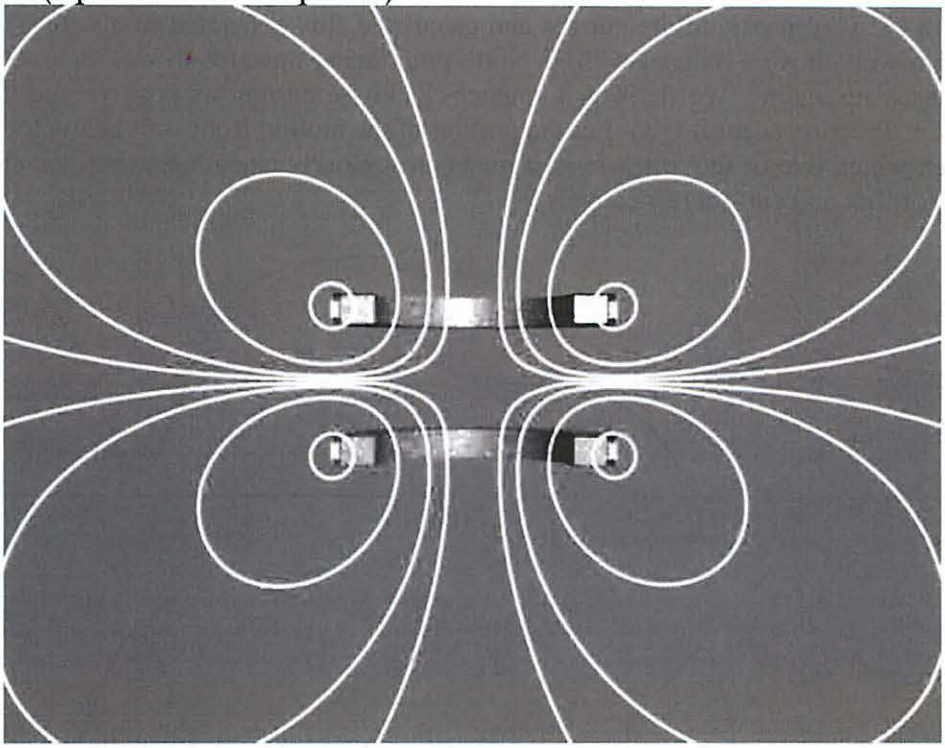
In the fourth lab you studied the effects of magnetic fields. A current-carrying coil is placed in a uniform magnetic field pointing upward. The current flows as shown, out of the page in the upper left and in on the lower right.



What are the force and torque on the coil?

1. No force or torque
2. No force, torque to rotate clockwise
3. No force, torque to rotate counterclockwise
4. Force up, no torque
5. Force up, torque to rotate clockwise
6. Force up, torque to rotate counterclockwise
7. Force down, no torque
8. Force down, torque to rotate clockwise
9. Force down, torque to rotate counterclockwise
10. I don't know (this answer is worth 1 point)

Question E (4 points out of 40 points):

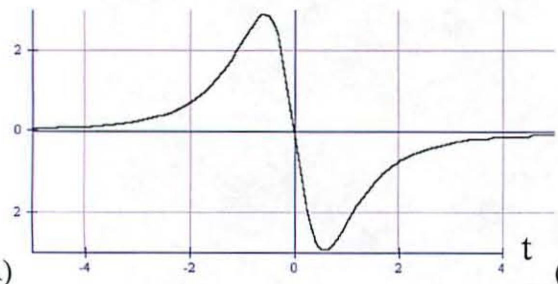


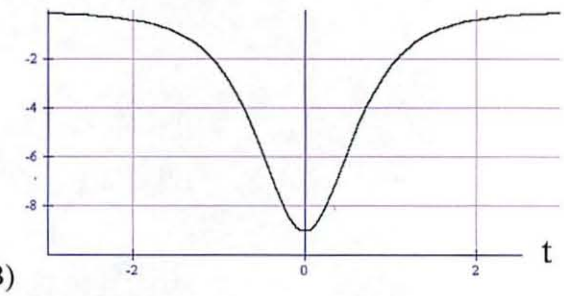
In the fifth lab you measured the force and torque on a magnetic dipole in the field of a Helmholtz coil (which you could energize in either Helmholtz or Anti-Helmholtz mode). The picture above shows the field configuration of the coils after you have energized them in one of these two ways.

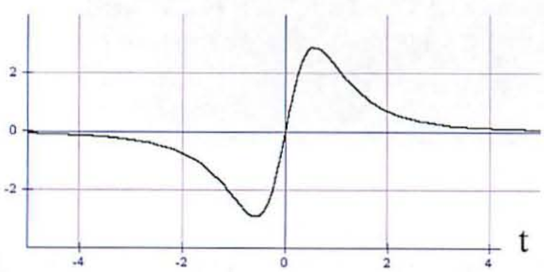
If, before the above field is turned on, you place a dipole so that it is very slightly above center and points very slightly away from alignment with the eventual field, what force and torque will it feel when the coils are energized?

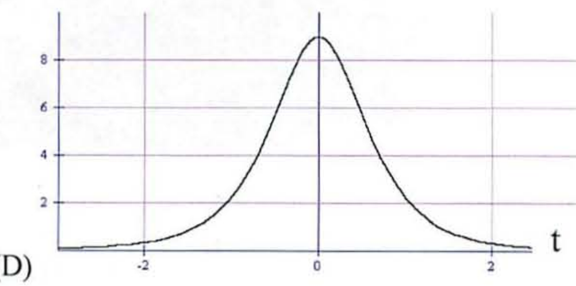
1. It will feel no force or torque
2. It will feel a force down (towards the center) but no torque
3. It will feel a force up (away from the center) but no torque
4. It will feel no force but a torque to align with the field
5. It will feel a force down (towards the center) and a torque to align with the field
6. It will feel a force up (away from the center) and a torque to align with the field
7. It will feel no force but a torque to anti-align with the field
8. It will feel a force down (towards the center) and a torque to anti-align with the field
9. It will feel a force up (away from the center) and a torque to anti-align with the field
10. I don't know (this answer is worth 1 point)

Question F (4 points out of 40 points):
In the sixth lab you measured the current and calculated flux generated in a wire coil that was moved from well above a magnet with its North pole facing upwards to well below the magnet and then back up again. We defined a counter-clockwise current as positive and defined the positive flux direction accordingly. For the portion of the motion from well below to well above the magnet, which two of the following diagrams most closely resembles what you should have measured for flux and current respectively?

(A)


(B)


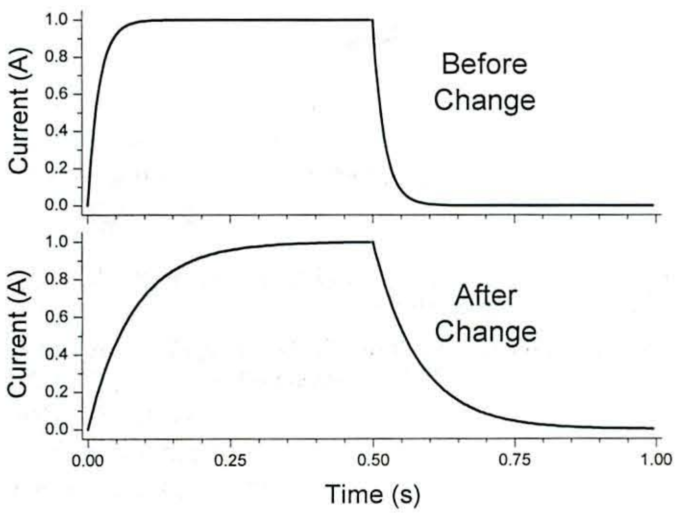
(C)


(D)


1. A (flux) & B (current)
2. A & D
3. C & B
4. C & D
5. B & A
6. B & C
7. D & A
8. D & C
9. I don't know (this answer is worth 1 point)

Question G (4 points out of 40 points):

In experiment seven you set up a simple series LR circuit which consisted of the 750 function generator and the coil (which as you may recall has both a resistance and an inductance). The 750 power supply was used as a "variable battery" which would periodically turn on and off, and the current through the battery was plotted vs. time. In this experiment you had the opportunity to measure the effect of inserting and removing an iron core from the coil as well as the effect of adding an additional resistor either in series or in parallel with the coil. In moving between the two plots below, which of those four things was done?



1) Core was added
2) Core was removed
3) Resistor was added in parallel
4) Resistor was added in series
5) I don't know (1 point)

Question H (4 points out of 40 points):

In experiment eight you studied an undriven series LRC circuit and made a plot of energy stored in the capacitor and in the inductor vs. time, which, in addition to oscillating with time, also decayed with time. The total energy (the sum of these two) also decayed in time, but not always at the same rate. When did the total energy in the system decrease most rapidly?

1. When the voltage across the capacitor was a maximum
2. When the voltage across the resistor was a maximum
3. When the voltage across the inductor was a maximum
4. More than one of the above
5. All of the above
6. None of the above
7. I don't know (1 point)

Question I (4 points out of 40 points):

In experiment nine you measured the angular dependence of the radiation from a spark gap antenna by moving your receiver either horizontally or vertically around the transmitter.

Angular dependence - Horizontal

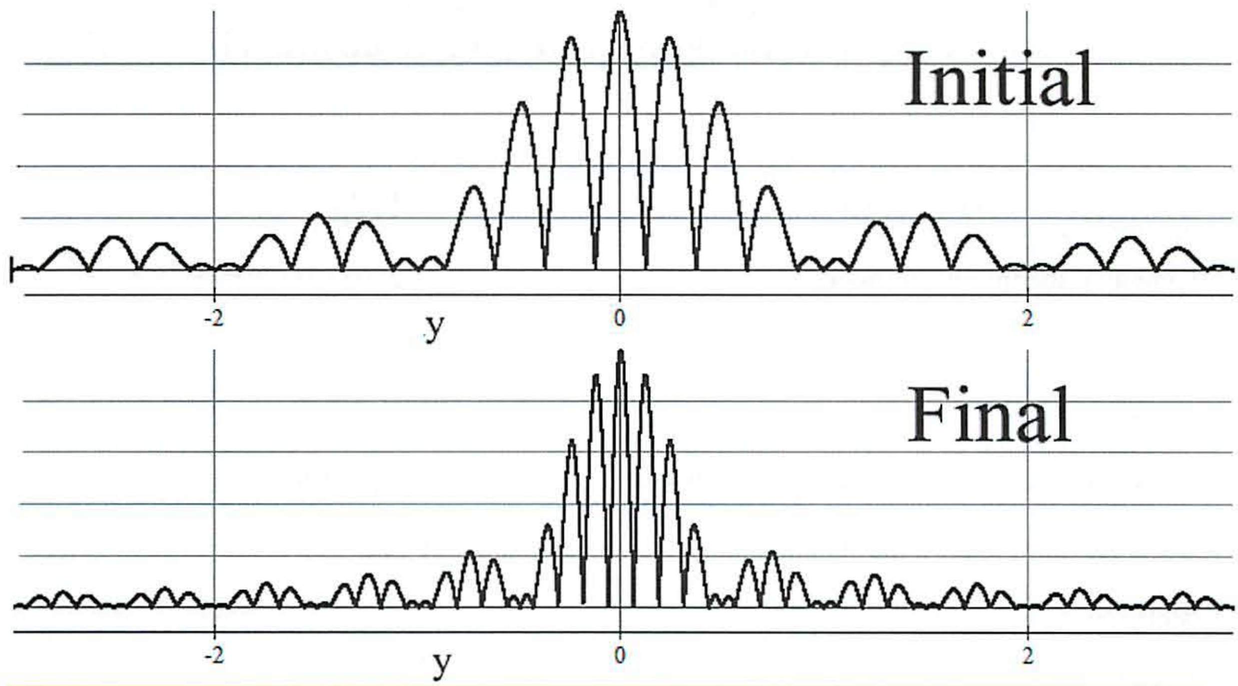
Angular dependence - Vertical

Which kind of motion, horizontal or vertical, shows a larger change in radiation intensity over the range of motion?

1. Horizontal
2. Vertical
3. Both show same range of change
4. I don't know (1 point)

Question J (4 points out of 40 points):

In experiment ten you observed an "Initial" intensity pattern for light coming from two slits and hitting a screen. If you had used a green laser rather than a red one, would you have seen a pattern similar to "Final" below?



1. Yes
2. No, the distance d between the slits must have changed in going from Initial to Final
3. No, the width a of the slits must have changed in going from Initial to Final
4. No, the change depicted results from a change in wavelength the other direction (as if we had started with green light in Initial and moved to red light in Final)
5. I don't know (this answer is worth 1 point)

Problem 2: Back of the Envelope Calculation – Numerical Estimation (15 Pts)
I hope that the back of the envelope calculations this semester have given you the confidence to make numerical estimates about things that you aren't exactly sure about (in addition to improving your Google skills). To see if this is true, please estimate the following values. NOTE: I know that you don't necessarily know the answers to these (for example, the radius of the Earth might not be stored in your brain). That is the point. You should be able to make good estimates based on what you do know. **For credit, give all answers in SI units.**

Length

Thickness of notebook paper .... 100 μm

Length of the infinite corridor: .... 250 m

Radius of the Earth: .... 6×10^6 m

Time

Time for Earth to rotate once about its own axis:................ 1 day = 86,400 s

Time for Earth to orbit the sun:... 1 year = 32×10^6 s

Velocity

Speed of a commercial airplane:... 200 m/s

Speed of sound in air at atmospheric pressure:.................... 340 m/s

Acceleration

Peak acceleration of a good car: .... 5 m/s^2

0 to 60 mph in 5 sec. Note that this 0 to 60 mph is not really a good determination of peak acceleration because the gear change from 1st to 2nd slows things a little. Actually, many cars intentionally put the 2nd to 3rd gear shift just above 63 mph (100 km/hr) in order to make this reported "acceleration time" shorter, even though it compromises the overall performance of the car.

Energy

Energy stored in a cell phone battery:.................................. 3×10^4 J

Lots of different kinds, but typical is 4 V for 2 A-Hr

Power

Power consumed by a light bulb.. 100 W

Power generated by a power plant:...................................... 1 GW

Electric Field

Electric field at your face from the lamp on your desk.......... 80 V/m

$$I = \frac{1}{2\mu_0} E_0 B_0 = \frac{E_0^2}{2c\mu_0} \approx \frac{100\text{ W}}{4\pi(1\text{ m})^2} \Rightarrow E_0 = \sqrt{2c\frac{\mu_0}{4\pi}\frac{100\text{ W}}{(1\text{ m})^2}} \approx \sqrt{2\left(3\times10^8\text{m/s}\right)\left(10^{-7}\frac{\text{N}}{\text{A}^2}\right)\frac{100\text{ W}}{(1\text{ m})^2}}$$

Voltage

Voltage between finger and door when getting a shock .... 10^4 V

Magnetic Field

Magnetic field generated by an MRI magnet .... 3 T

And finally…

Number of times (it might be less that one) you'd need to run from the bottom to the top of the Green building to burn the calories in a candy bar

.... 10 times

1 candy bar ≈ 200 Calories $\approx 8\times10^5$ J

Green Building = 21 stories ≈ 80 m

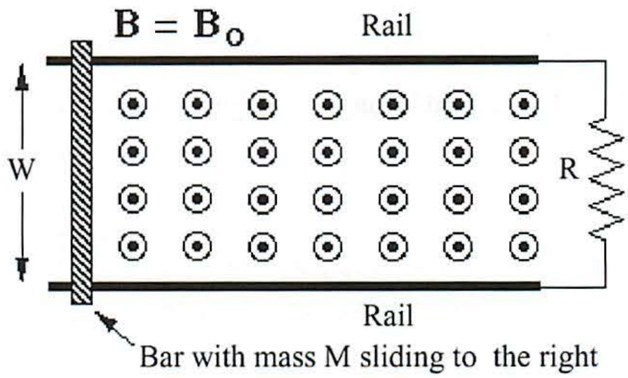
$$\text{Energy to climb } \approx \text{mgh}\cdot\text{N} \Rightarrow \text{N} \approx \frac{8\times10^5\text{ J}}{(100\text{ kg})\left(10\ {}^{\text{m}}\!/_{\text{s}^2}\right)(80\text{ m})} = 10\text{ times}$$

Of course, you are doing more work than just mgh, you probably don't really weigh 100 kg and the body only converts about half the food energy into useful energy, so this is all approximate, but that's the point after all.

Problem 3 Sliding Bar (25 points): A conducting bar has mass M. It slides to the right along two frictionless horizontal rails separated by a distance W, as shown in the sketch. The rails are connected on the far right by a resistor of resistance R. The bar itself and the rails have zero resistance.

At time t = 0 the bar has slid (under its own inertia – no one is pushing it anymore) to the point pictured below, where everywhere to its right there is a constant magnetic field $\mathbf{B}_0$ directed out of the page. This is the ***only*** B field that you are to think about it this problem. At this time it has a velocity $\vec{\mathbf{v}}(t=0)=V_0\hat{\mathbf{i}}$,

where $\hat{\mathbf{i}}$ points to the right in the below picture.



(a) As time goes on what will happen to the velocity of the conducting bar? (Circle ans)

1. Increases without limit
2. Increases to limiting value
3. Remains constant
4. Decreases to zero
5. Decreases to zero, then reverses directions

(b) Briefly explain why this happens (use words, ***not*** equations). If you will need to use a Maxwell equation to determine subsequent motion of the bar then explicitly state which equation (by name), write the equation, and briefly explain what it means.

As the bar slides in the magnetic field the loop that it forms the right hand leg of shrinks, so the magnetic flux decreases. By Lenz's law, nature doesn't want this to happen so it exerts a magnetic force on the bar, slowing and eventually stopping it.

We will use Faraday's law, $\varepsilon=-\frac{d\Phi_B}{dt}$, to calculate the EMF and hence current around the loop.

Faraday's Law says that changing magnetic fields are accompanied by ("generate") electric fields.

(c) Assume that at some later time t the speed of the bar is $v(t)$. What is the current, if any, in the circuit?

$$I=\frac{\varepsilon}{R}=\frac{1}{R}\frac{d\Phi_B}{dt}=\frac{1}{R}\frac{d}{dt}\left(B_0 wx\right)=\frac{B_0\,w\,v(t)}{R}\quad\text{counter-clockwise}$$

Problem 3: Sliding Bar ***continued***

(d) What, if any, is the total magnetic force $\vec{\mathbf{F}}_B$ on the moving bar at this time?

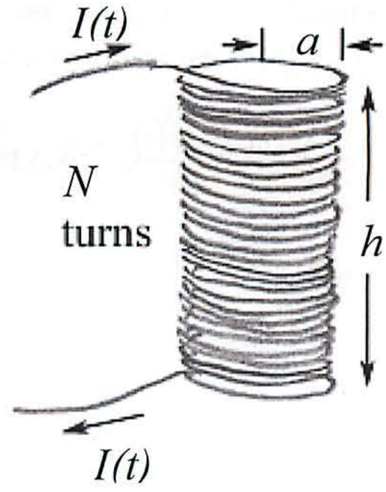
$\vec{\mathbf{F}} = I\vec{\mathbf{w}} \times \vec{\mathbf{B}} = \dfrac{B_0^2\, w^2\, v(t)}{R}$ to the left

(e) Is the kinetic energy of the bar changing? If so, where is that energy going to or coming from? Do a calculation to demonstrate that your answer to this question is true. If not, simply write down an expression for the kinetic energy of the bar.

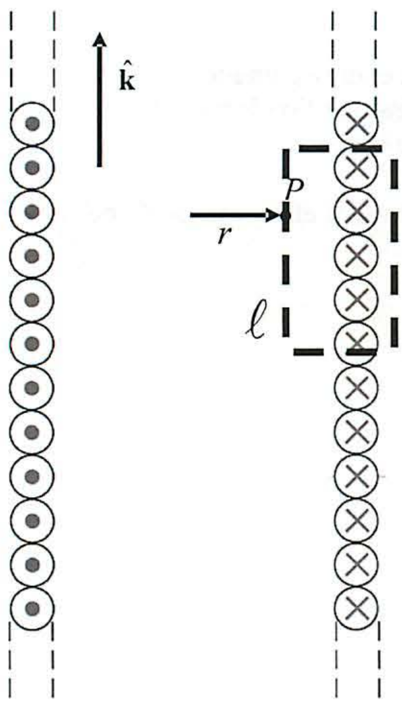
Yes, there is a force on the bar causing it to slow, and hence the kinetic energy is changing (decreasing). The energy is being dissipated by the resistor:

$$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}(t) = \frac{B_0^2\, w^2\, v^2(t)}{R} = I^2 R$$

Problem 4: Solenoid (45 pts)
Consider a very long solenoid of N turns, radius a, and length h ($h >> a$) (as pictured at right). It is coaxial with the z-axis.



(a) By looking at the solenoid in cross-section (as I do below) you can calculate the magnetic field at an arbitrary point P, at a radius $r < a$. If the current through the solenoid is $I(t)$, explicitly calculate the magnetic field $\vec{\mathbf{B}}(t)$ at point P. Make sure that you state and briefly explain which Maxwell's equation you are using, and that you draw and label anything needed to do the calculation on the below image. Be completely clear about every step you take (for example, if anything in the calculation is zero, explain why).



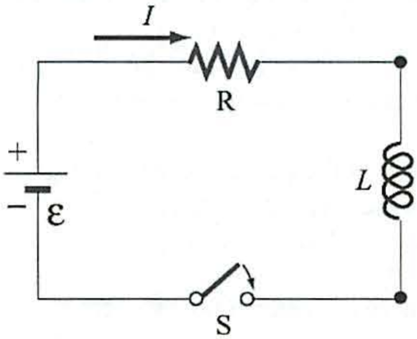
We calculate the magnetic field using Ampere's law: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_o I_{\text{enc}}$ which means that currents create magnetic fields that circulate around them.

We need to draw an Amperian loop, as pictured at left. When we do the integral, the leg outside gives zero contribution because the field outside the solenoid is zero (the field from the right half cancels the field from the left half). The top and bottom legs also give zero contribution because there B is perpendicular to ds (B is "up" while ds is horizontal). So:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\ell = \mu_o I_{\text{enc}} = \mu_o N \frac{\ell}{h} I \Rightarrow \boxed{\vec{\mathbf{B}} = \frac{\mu_o N I}{h} \text{ up}}$$

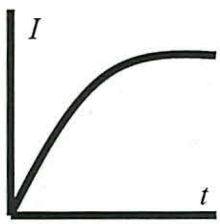
Problem 4: Solenoid *continued*

This solenoid is an inductor of inductance L (which I'm sure you can calculate so I won't ask you to). We put it in a series LR circuit (pictured below) consisting of a battery with EMF ε, a resistor of resistance R and a switch S. At time $t = 0$ we close the switch in the circuit.



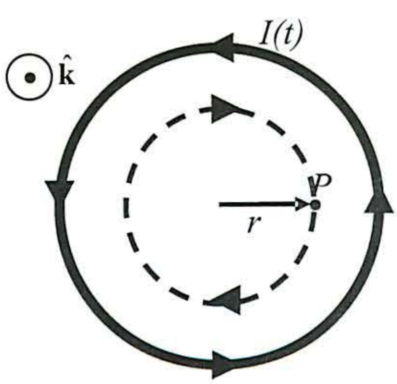
(b) Sketch the time dependence of the current in the circuit, and write an equation for $I(t)$. Clearly identify the initial (t=0) and final (t=∞) values of the current. Briefly explain why the current behaves the way it does.

The current increases to a final value of ε/R after starting at 0. It behaves like that because the inductor initially fights the change of current then eventually gives up.



$$I(t) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right) \text{ where } \tau = \frac{L}{R}$$

(c) From part (a) you know the magnetic field $\vec{\mathbf{B}}(t)$ as a function of the current $I(t)$, which you have just calculated in part (b). Now let's look at the electric field that is induced inside the solenoid. It is easiest to do that in a top view of the solenoid, which is provided below. Calculate the induced electric field $\vec{\mathbf{E}}(t)$ at point P. If you need to use one of Maxwell's equations then name and briefly explain it before using it. Be very explicit about how you do the calculation, drawing and labeling anything that you need on the figure below. Feel free to leave your answer in terms of $\vec{\mathbf{B}}(t)$ or $I(t)$ as you find convenient – there is no need to substitute your results from part (a) or (b).



From Faraday's law a changing magnetic flux will induce an electric field that "loops around" the flux:

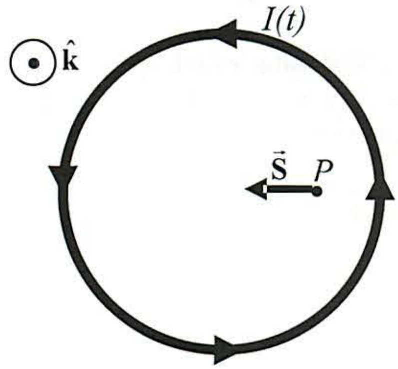
$$\oint \vec{\mathbf{E}} \cdot \mathbf{d}\vec{\mathbf{s}} = E \cdot 2\pi r = -\frac{d\Phi_B}{dt} = -\frac{d\left(\vec{\mathbf{B}} \cdot \vec{\mathbf{A}}\right)}{dt} = \pi r^2 \frac{d}{dt} B(t)$$

$$\boxed{\vec{\mathbf{E}} = \frac{r}{2}\frac{d}{dt} B(t) \text{ down}}$$

The loop is clockwise (down at P) by Lenz's law: the E field is created so that if it were to drive a current and hence create a B field it would oppose the changing B field. Here the B field is increasing out of the page so out E field needs to be clockwise to try to make a B field in.

Problem 4: Solenoid *continued*

(d) You now have an electric field $\vec{\mathbf{E}}(t)$ and a magnetic field $\vec{\mathbf{B}}(t)$ at point *P*, meaning that there is a Poynting vector there. Briefly explain the meaning of the Poynting vector (for example, what units does it have?) and then calculate its value at point *P*. Feel free to leave the answer in terms of the field magnitudes *E(t)* and *B(t)* (you do *not* need to plug in your answers from previous parts) but **do explicitly state its direction**. To be clear please indicate the direction of the Poynting vector at point *P* on the diagram below. What does the direction indicate about this system?



The Poynting vector tells you the power flow per unit area (W/m^2) and point in the direction that power is flowing. At P E is down and B is out of the page so the cross product (the Poynting vector) is radially inward (to the left), meaning that energy is entering the system.

$$S = \frac{1}{\mu_o}\vec{\mathbf{E}}\times\vec{\mathbf{B}}$$

(e) To demonstrate the meaning of the Poynting vector we typically have you integrate it over some area and show that it is equal to something else. In the case of this solenoid, over what area should we integrate? Be very clear here – probably the easiest thing to do is to state an equation for the area. What should that integral be equal to (state this both in words and as an equation)? There is no need to do the actual integral or to plug in values you calculated above to demonstrate that this is indeed the case.

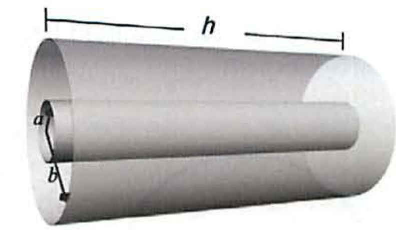
You would integrate it over the cylindrical surface of the solenoid ($A = 2\pi ah$). That integral tells you the rate at which energy is entering the system, so it is the time rate of change of the energy stored in the solenoid, that is:

$$\iint \vec{\mathbf{S}}\cdot d\vec{\mathbf{A}} = \frac{d}{dt}\left(\frac{1}{2}LI^2\right)$$

Problem 5: Transmission Line (75 pts)

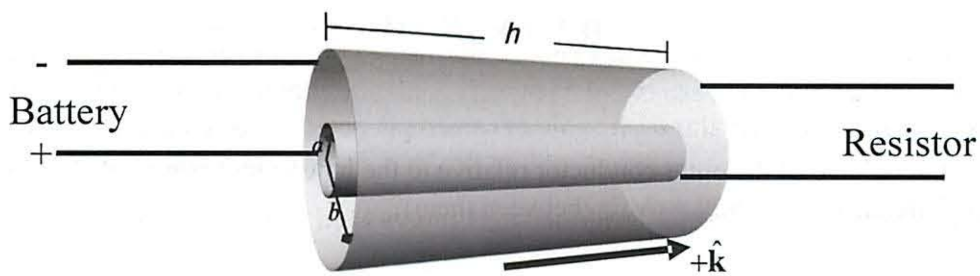
The rest of this exam is an extended question dealing with transmission lines. There are a variety of transmission lines used in the world. A simple example is two wires running next to each other with current flowing one direction in one and the opposite in the other.

In this problem you will calculate the properties of a coaxial cable. It consists of a solid core or radius a and a thin outer "shield" conductor of radius b, both of length h. They are typically held apart by a dielectric, but to make your life simple let's just pretend there is vacuum between the conductors. It is shown in perspective at right.



The dimensions are such that **you should assume that any fields created by the transmission line are confined to the region between to two conductors.**

We use transmission lines to carry power from batteries or power supplies to loads (typically modeled as resistors):



In this problem you will calculate the capacitance per unit length and inductance per unit length of the microstrip transmission line and then study energy flow at DC. Finally, you will describe its behavior when driven by an AC function generator.

NOTE: PLEASE READ THIS CAREFULLY

In several parts of this problem you will be asked to calculate something that will require the use of one of Maxwell's equations. Make sure that you state the name of the equation and the write it in the form that you plan to use it before you do that part. You do not need to describe the equation as you were asked to do in earlier parts of this exam, but you do need to be explicit in the calculations and draw and label anything that you need to use to do the calculation. I will not provide any further drawings. Please duplicate drawings from this page (simplified to remove the perspective of course) when you think they will be useful.

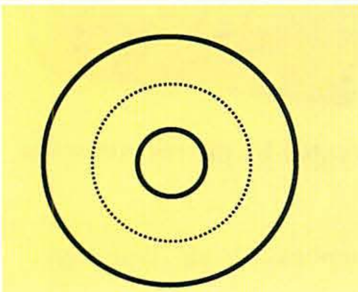
Do not forget to give both magnitude and direction of vector quantities.

Feel free to tear out this page so that you do not have to continually turn back to it.

Problem 5A: Capacitance of the Coaxial Cable

In this part we will consider the transmission line in isolation (no battery or load resistor). Assume that the inner conductor has a charge $+Q$ and the outer conductor has a charge $-Q$.

(a) What is the electric field between the conductors?



We have a charge +Q on the inner conductor so an electric field will be created pointing outwards. We will Gauss's Law to calculate the electric field between the plates: $\oiint_{\text{Pillbox}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{enc}}}{\varepsilon_o}$

We use a Gaussian cylinder with radius r and length L ($L < h$). The only surface of the cylinder we care about is the rounded surface (nothing penetrates the endcaps).

$$\oiint_{\text{Pillbox}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E2\pi rL = \frac{Q_{\text{enc}}}{\varepsilon_o} = \frac{Q}{\varepsilon_o}\frac{L}{h} \Rightarrow \boxed{\vec{\mathbf{E}} = \frac{Q}{2\pi\varepsilon_o rh}\hat{\mathbf{r}}}$$

(b) What is the voltage of the outer conductor relative to the inner conductor (that is, what is the voltage difference $\Delta V = V_{\text{outer}} - V_{\text{inner}}$ between them)?

$$\Delta V = V_{\text{outer}} - V_{\text{inner}} = -\int_{\text{inner}}^{\text{outer}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\int_{r=a}^{b} \frac{Q}{2\pi\varepsilon_o rh} dr = -\frac{Q}{2\pi\varepsilon_o h}\ln(r)\Big|_a^b = -\frac{Q}{2\pi\varepsilon_o h}\ln\left(\frac{b}{a}\right)$$

Note the sign! It is negative because the outer conductor is at a lower potential.

(c) What is the capacitance of the transmission line?

$$C = \frac{Q}{\Delta V} = \frac{2\pi\varepsilon_o h}{\ln(b/a)}$$

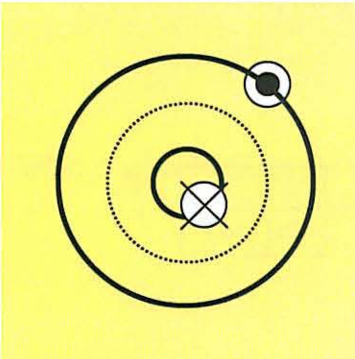
(d) What is the capacitance per unit length of the transmission line? Note that h should not appear in this answer – what I mean by "per unit length" is that you need to multiply by the length h to get the total capacitance.

$$c = \frac{2\pi\varepsilon_o}{\ln(b/a)}$$

Problem 5B: Inductance of the Coaxial Cable

In this part we will assume that the transmission line has a constant current I traveling down the inner conductor (in the $+\hat{\mathbf{k}}$ direction, to the right) and back along the outer conductor (in the $-\hat{\mathbf{k}}$ direction).

(a) What is the magnetic field between the conductors?



We have a current I flowing down the center and back on the outer conductor. The field is zero outside by cancellation. We use Ampere's Law with the Amperian loop pictured at left:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \cdot 2\pi r = \mu_o I_{\text{enc}} = \mu_o I \Rightarrow \boxed{\vec{\mathbf{B}} = -\frac{\mu_o I}{2\pi r}\hat{\varphi} \text{ (clockwise)}}$$

(b) What is the inductance of the transmission line? NOTE: There are two ways to do this. If you don't recall either of them then I suggest that you at least calculate the magnetic energy between the conductors.

We will use energy to calculate the inductance:

$$U_B = \iiint u_B dV = \iiint \frac{B^2}{2\mu_o} dV = \int_{y=0}^{h} \int_{\varphi=0}^{2\pi} \int_{r=a}^{b} \frac{1}{2\mu_o}\left(\frac{\mu_o I}{2\pi r}\right)^2 dr\,(r d\varphi)\,dy = \frac{2\pi h}{2\mu_o}\left(\frac{\mu_o I}{2\pi}\right)^2 \int_{r=a}^{b} \frac{dr}{r}$$

$$= \frac{h\mu_o I^2}{4\pi}\ln\left(\frac{b}{a}\right) = \frac{1}{2}LI^2 \Rightarrow \boxed{L = \frac{h\mu_o}{2\pi}\ln\left(\frac{b}{a}\right)}$$

(c) What is the inductance per unit length of the transmission line?

$$\frac{L}{\text{length}} = \frac{\mu_o}{2\pi}\ln\left(\frac{b}{a}\right)$$

Problem 5C: DC Power Transmission with the Coaxial Cable

We now connect the transmission line to a battery (EMF ε) on the left and a resistor (resistance R) on the right, as pictured at the beginning of this problem. We are interested in what happens a long time after this connection has been made (after any transient behavior has passed). In answering the below questions feel free to use the results from previous sections of this problem (you do not need to derive them again) but express your answers only in terms of variables given here and at the beginning of the problem (NOT in terms of Q or I from parts 5A and 5B).

(a) What is the electric field between the conductors?

From before,

$$\vec{\mathbf{E}} = \frac{Q}{2\pi\varepsilon_o rh}\hat{\mathbf{r}} \text{ and } \Delta V = \varepsilon = \frac{Q}{2\pi\varepsilon_o h}\ln\left(\frac{b}{a}\right) \Rightarrow \vec{\mathbf{E}} = \frac{2\pi\varepsilon_o h\Delta V}{2\pi\varepsilon_o rh\ln(b/a)}\hat{\mathbf{r}} = \boxed{\frac{\varepsilon}{r\ln(b/a)}\hat{\mathbf{r}}}$$

(b) What is the magnetic field between the plates?

$$\vec{\mathbf{B}} = -\frac{\mu_o}{2\pi r}I\hat{\varphi} = -\frac{\mu_o}{2\pi r}\frac{\varepsilon}{R}\hat{\varphi}$$

(c) What is the Poynting vector between the plates?

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}} = \frac{1}{\mu_0}\left(\frac{\varepsilon}{r\ln(b/a)}\hat{\mathbf{r}}\right)\times\left(-\frac{\mu_o}{2\pi r}\frac{\varepsilon}{R}\hat{\varphi}\right) = \frac{\varepsilon^2}{2\pi r^2 R\ln(b/a)}\hat{\mathbf{k}}$$

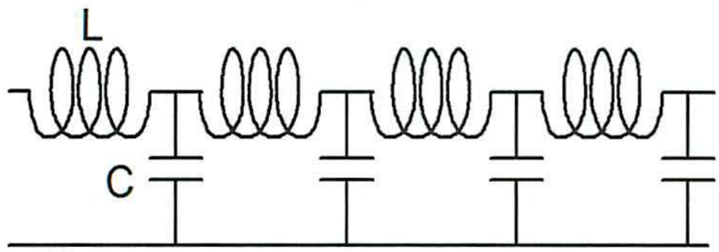
(d) Integrate the Poynting vector over a relevant area and show that the result simplifies to what you would expect given the meaning of the Poynting vector.

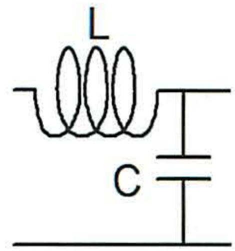
The relevant area is the cross-sectional area of the transmission line. There is no angular dependence so we just integrate in rings:

$$\iint \vec{\mathbf{S}}\cdot d\vec{\mathbf{A}} = \int_{r=a}^{b}\frac{\varepsilon^2}{2\pi r^2 R\ln(b/a)}\cdot 2\pi r dr = \frac{\varepsilon^2}{R}\int_{r=a}^{b}\frac{dr}{r\ln(b/a)} = \frac{\varepsilon^2}{R} = \text{Power dissipated by the resistor}$$

Problem 5D: Transients and AC Transmission in the Coaxial Cable

You calculated that the transmission line has an inductance per unit length and a capacitance per unit length. A typical way to model the behavior of the transmission line is as a collection of inductors and capacitors, as pictured below left, or even more simply as just a single inductor and capacitor, as pictured below right.





These are "lossless" models – we are ignoring the resistance of the transmission line itself.

(a) Let's first think about the transient behavior of this circuit. The instant after you attach a battery (ε) on the left and a resistive load (R) on the right, what is the current through the load? Why? Describe what the inductive and capacitive parts of the transmission are behaving like at this instant.

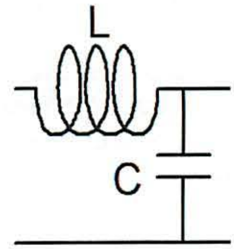
The instant the switch is closed no current will flow because the inductor acts like an open circuit preventing the flow of current (and the capacitor looks like a short circuit since it is uncharged).

(b) A long time after the battery ε and load resistor R have been connected what is the current through R? Why? Describe what the inductive and capacitive parts of the transmission are behaving like at this instant.

A long time after the switch is closed the current will be ε/R because the inductor acts like a short circuit (constant current so the inductor does nothing) and the capacitor looks like an open circuit (because it is "fully charged").

Problem 5D: Transients and AC Transmission *continued*

Now instead of attaching a battery on the left, let's attach a function generator, driving a voltage $V = V_0 \sin \omega t$. On the right we still have a resistive load, but to make life simpler let's assume that it is a very large resistor R.



(c) You have already discussed the very low frequency (DC) behavior of the transmission line. As we turn up the frequency of the power supply, **QUALITATIVELY** describe (no equations) what happens to the voltage that the load sees. Why?

As we turn up the frequency the inductor progressively get a larger reactance and the capacitor gets a smaller reactance, meaning that the voltage across the load will shrink.

(d) I said above that you would want to assume that the load had a very large resistance R. Typically when we say that something is very large, what we mean is that it is much larger than something else. At non-zero frequencies, what should R be much larger than (give an equation here)? Why?

If you want to be able to ignore its resistance when thinking about current in the circuit then you want it to have a large resistance compared to the reactance of the capacitor that it is in parallel with:

$$R \gg \frac{1}{\omega C}$$

Sources of Magnetic Fields

9.1 Biot-Savart Law

Currents which arise due to the motion of charges are the source of magnetic fields. When charges move in a conducting wire and produce a current I, the magnetic field at any point P due to the current can be calculated by adding up the magnetic field contributions, $d\vec{\mathbf{B}}$, from small segments of the wire $d\vec{\mathbf{s}}$, (Figure 9.1.1).

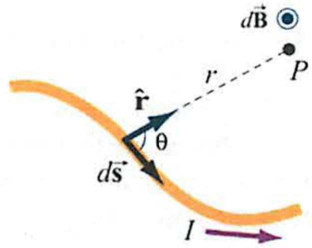


Figure 9.1.1 Magnetic field $d\vec{\mathbf{B}}$ at point P due to a current-carrying element $I\,d\vec{\mathbf{s}}$.

These segments can be thought of as a vector quantity having a magnitude of the length of the segment and pointing in the direction of the current flow. The infinitesimal current source can then be written as $I\,d\vec{\mathbf{s}}$.

Let r denote as the distance form the current source to the field point P, and $\hat{\mathbf{r}}$ the corresponding unit vector. The Biot-Savart law gives an expression for the magnetic field contribution, $d\vec{\mathbf{B}}$, from the current source, $Id\vec{\mathbf{s}}$,

$$\boxed{d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}} \tag{9.1.1}$$

where μ_0 is a constant called the *permeability of free space*:

$$\mu_0 = 4\pi\times10^{-7}\,\mathrm{T\cdot m/A} \tag{9.1.2}$$

Notice that the expression is remarkably similar to the Coulomb's law for the electric field due to a charge element dq:

$$d\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{\mathbf{r}} \tag{9.1.3}$$

$q_1 q_2$ columb $k\,\frac{q_1 q_2}{r^2}$

Adding up these contributions to find the magnetic field at the point P requires integrating over the current source,

$$\vec{\mathbf{B}} = \int_{\text{wire}} d\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi} \int_{\text{wire}} \frac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2} \tag{9.1.4}$$

The integral is a vector integral, which means that the expression for $\vec{\mathbf{B}}$ is really three integrals, one for each component of $\vec{\mathbf{B}}$. The vector nature of this integral appears in the cross product $I\,d\vec{\mathbf{s}} \times \hat{\mathbf{r}}$. Understanding how to evaluate this cross product and then perform the integral will be the key to learning how to use the Biot-Savart law.

Interactive Simulation 9.1: Magnetic Field of a Current Element

Figure 9.1.2 is an interactive ShockWave display that shows the magnetic field of a current element from Eq. (9.1.1). This interactive display allows you to move the position of the observer about the source current element to see how moving that position changes the value of the magnetic field at the position of the observer.

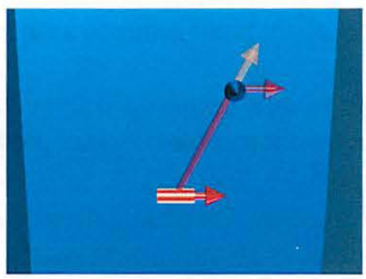
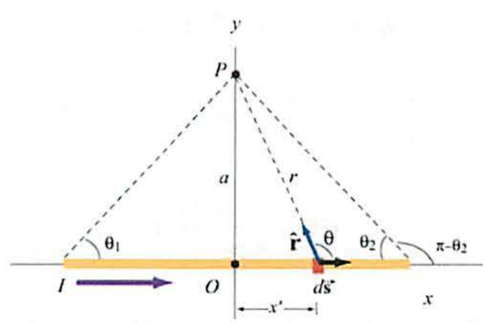


Figure 9.1.2 Magnetic field of a current element.

Example 9.1: Magnetic Field due to a Finite Straight Wire

A thin, straight wire carrying a current I is placed along the x-axis, as shown in Figure 9.1.3. Evaluate the magnetic field at point P. Note that we have assumed that the leads to the ends of the wire make canceling contributions to the net magnetic field at the point P.



add up all the points

Figure 9.1.3 A thin straight wire carrying a current I.

Solution:

This is a typical example involving the use of the Biot-Savart law. We solve the problem using the methodology summarized in Section 9.10.

(1) Source point (coordinates denoted with a prime)

Consider a differential element $d\vec{\mathbf{s}} = +dx'\hat{\mathbf{i}}$ carrying current I in the x-direction. The location of this source is represented by $\vec{\mathbf{r}}' = x'\hat{\mathbf{i}}$.

(2) Field point (coordinates denoted with a subscript "P")

Since the field point P is located at $(x, y) = (0, a)$, the position vector describing P is $\vec{\mathbf{r}}_P = a\hat{\mathbf{j}}$.

(3) Relative position vector

The vector $\vec{\mathbf{r}} = \vec{\mathbf{r}}_P - \vec{\mathbf{r}}'$ is a "relative" position vector which points from the source point to the field point. In this case, $\vec{\mathbf{r}} = a\hat{\mathbf{j}} - x'\hat{\mathbf{i}}$, and the magnitude $r = |\vec{\mathbf{r}}| = \sqrt{a^2 + x'^2}$ is the distance from between the source and P. The corresponding unit vector is given by

$$\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}}}{r} = \frac{a\hat{\mathbf{j}} - x'\hat{\mathbf{i}}}{\sqrt{a^2 + x'^2}} = \sin\theta\,\hat{\mathbf{j}} - \cos\theta\,\hat{\mathbf{i}}$$

Vector point of unit lenght

(4) The cross product $\boxed{d\vec{\mathbf{s}}} \times \hat{\mathbf{r}}$

differential current element

The cross product is given by

$$d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = (+dx'\hat{\mathbf{i}}) \times (-\cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}}) = (dx'\sin\theta)\,\hat{\mathbf{k}}$$

(5) Write down the contribution to the magnetic field due to $Id\vec{\mathbf{s}}$

The expression is

$$d\vec{\mathbf{B}} = \boxed{\frac{\mu_0 I}{4\pi}\frac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}} = \frac{\mu_0 I}{4\pi}\frac{dx'\,\sin\theta}{r^2}\,\hat{\mathbf{k}}$$

which shows that the magnetic field at P will point in the $+\hat{\mathbf{k}}$ direction, or out of the page.

(6) Simplify and carry out the integration

SS

The variables θ, x' and r are not independent of each other. In order to complete the integration, let us rewrite the variables x' and r in terms of θ. From Figure 9.1.3, we have

$$\begin{cases} r = a/\sin(\pi-\theta) = a\csc\theta \\ x' = a\cot(\pi-\theta) = -a\cot\theta \quad\Rightarrow\quad dx' = a\csc^2\theta\, d\theta \end{cases}$$

Upon substituting the above expressions, the differential contribution to the magnetic field is obtained as

$$d\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\frac{(a\csc^2\theta\, d\theta)\sin\theta}{(a\csc\theta)^2}\hat{\mathbf{k}} = \frac{\mu_0 I}{4\pi a}\sin\theta\, d\theta\,\hat{\mathbf{k}}$$

Integrating over all angles subtended from θ_1 to $\pi-\theta_2$ (note our definition of θ_2), we obtain

$$\boxed{\begin{aligned}\vec{\mathbf{B}} &= \frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\pi-\theta_2}\sin\theta\, d\theta\,\hat{\mathbf{k}} = -\frac{\mu_0 I}{4\pi a}\left[\cos(\pi-\theta_2)-\cos\theta_1\right]\hat{\mathbf{k}} \\ &= \frac{\mu_0 I}{4\pi a}(\cos\theta_2+\cos\theta_1)\,\hat{\mathbf{k}}\end{aligned}} \tag{9.1.5}$$

The first term involving θ_2 accounts for the contribution from the portion along the $+x$ axis, while the second term involving θ_1 contains the contribution from the portion along the $-x$ axis. The two terms add!

Let's examine the following cases:

(i) In the symmetric case where $\theta_2 = \theta_1$, the field point P is located along the perpendicular bisector. If the length of the rod is $2L$, then $\cos\theta_1 = L/\sqrt{L^2+a^2}$ and the magnetic field is

$$B = \frac{\mu_0 I}{2\pi a}\cos\theta_1 = \frac{\mu_0 I}{2\pi a}\frac{L}{\sqrt{L^2+a^2}} \tag{9.1.6}$$

(ii) The infinite length limit $L\to\infty$

This limit is obtained by choosing $(\theta_1,\theta_2)=(0,0)$. The magnetic field at a distance a away becomes

$$\boxed{B = \frac{\mu_0 I}{2\pi a}} \tag{9.1.7}$$

Note that in this limit, the system possesses cylindrical symmetry, and the magnetic field lines are circular, as shown in Figure 9.1.4.

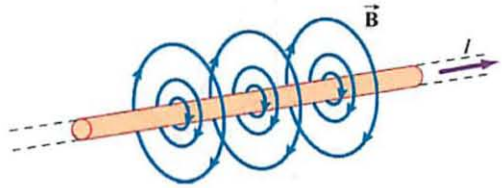


Figure 9.1.4 Magnetic field lines due to an infinite wire carrying current I.

In fact, the direction of the magnetic field due to a long straight wire can be determined by the right-hand rule (Figure 9.1.5).

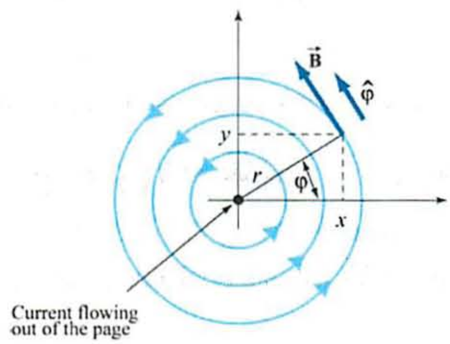


Figure 9.1.5 Direction of the magnetic field due to an infinite straight wire

If you direct your right thumb along the direction of the current in the wire, then the fingers of your right hand curl in the direction of the magnetic field. In cylindrical coordinates (r, φ, z) where the unit vectors are related by $\hat{\mathbf{r}} \times \hat{\boldsymbol{\varphi}} = \hat{\mathbf{z}}$, if the current flows in the $+z$-direction, then, using the Biot-Savart law, the magnetic field must point in the φ-direction.

Example 9.2: Magnetic Field due to a Circular Current Loop

A circular loop of radius R in the xy plane carries a steady current I, as shown in Figure 9.1.6.

(a) What is the magnetic field at a point P on the axis of the loop, at a distance z from the center?

(b) If we place a magnetic dipole $\vec{\mu} = \mu_z \hat{\mathbf{k}}$ at P, find the magnetic force experienced by the dipole. Is the force attractive or repulsive? What happens if the direction of the dipole is reversed, i.e., $\vec{\mu} = -\mu_z \hat{\mathbf{k}}$

part of a soloid

left + right adds to 0 so only up

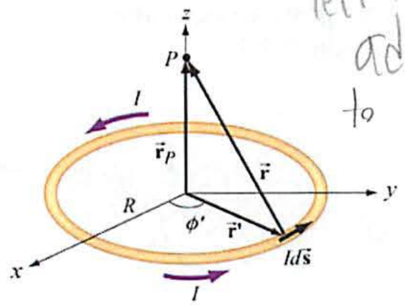


Figure 9.1.6 Magnetic field due to a circular loop carrying a steady current.

Solution:

(a) This is another example that involves the application of the Biot-Savart law. Again let's find the magnetic field by applying the same methodology used in Example 9.1.

(1) Source point

In Cartesian coordinates, the differential current element located at $\vec{\mathbf{r}}\,'=R(\cos\phi'\,\hat{\mathbf{i}}+\sin\phi'\,\hat{\mathbf{j}})$ can be written as $Id\vec{\mathbf{s}}=I(d\vec{\mathbf{r}}\,'/d\phi')d\phi'=IRd\phi'(-\sin\phi'\,\hat{\mathbf{i}}+\cos\phi'\,\hat{\mathbf{j}})$.

(2) Field point

Since the field point P is on the axis of the loop at a distance z from the center, its position vector is given by $\vec{\mathbf{r}}_P = z\hat{\mathbf{k}}$.

(3) Relative position vector $\vec{\mathbf{r}}=\vec{\mathbf{r}}_P-\vec{\mathbf{r}}\,'$

The relative position vector is given by

$$\vec{\mathbf{r}}=\vec{\mathbf{r}}_P-\vec{\mathbf{r}}\,'=-R\cos\phi'\,\hat{\mathbf{i}}-R\sin\phi'\,\hat{\mathbf{j}}+z\,\hat{\mathbf{k}} \tag{9.1.8}$$

and its magnitude

$$r=|\vec{\mathbf{r}}|=\sqrt{(-R\cos\phi')^2+(-R\sin\phi')^2+z^2}=\sqrt{R^2+z^2} \tag{9.1.9}$$

is the distance between the differential current element and P. Thus, the corresponding unit vector from $Id\vec{\mathbf{s}}$ to P can be written as

$$\hat{\mathbf{r}}=\frac{\vec{\mathbf{r}}}{r}=\frac{\vec{\mathbf{r}}_P-\vec{\mathbf{r}}\,'}{|\vec{\mathbf{r}}_P-\vec{\mathbf{r}}\,'|}$$

(4) Simplifying the cross product

The cross product $d\vec{\mathbf{s}}\times(\vec{\mathbf{r}}_P-\vec{\mathbf{r}}\,')$ can be simplified as

$$\begin{aligned} d\vec{\mathbf{s}}\times(\vec{\mathbf{r}}_P-\vec{\mathbf{r}}\,') &= R\,d\phi'\left(-\sin\phi'\,\hat{\mathbf{i}}+\cos\phi'\,\hat{\mathbf{j}}\right)\times[-R\cos\phi'\,\hat{\mathbf{i}}-R\sin\phi'\,\hat{\mathbf{j}}+z\,\hat{\mathbf{k}}] \\ &= R\,d\phi'[z\cos\phi'\,\hat{\mathbf{i}}+z\sin\phi'\,\hat{\mathbf{j}}+R\,\hat{\mathbf{k}}] \end{aligned} \tag{9.1.10}$$

(5) Writing down $d\vec{\mathbf{B}}$

Using the Biot-Savart law, the contribution of the current element to the magnetic field at P is

$$\begin{aligned} d\vec{\mathbf{B}} &= \frac{\mu_0 I}{4\pi}\frac{d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2} = \frac{\mu_0 I}{4\pi}\frac{d\vec{\mathbf{s}}\times\vec{\mathbf{r}}}{r^3} = \frac{\mu_0 I}{4\pi}\frac{d\vec{\mathbf{s}}\times(\vec{\mathbf{r}}_P-\vec{\mathbf{r}}\,')}{|\vec{\mathbf{r}}_P-\vec{\mathbf{r}}\,'|^3} \\ &= \frac{\mu_0 IR}{4\pi}\frac{z\cos\phi'\,\hat{\mathbf{i}}+z\sin\phi'\,\hat{\mathbf{j}}+R\,\hat{\mathbf{k}}}{(R^2+z^2)^{3/2}}\,d\phi' \end{aligned} \tag{9.1.11}$$

(6) Carrying out the integration

Using the result obtained above, the magnetic field at P is

$$\vec{\mathbf{B}} = \frac{\mu_0 IR}{4\pi}\int_0^{2\pi}\frac{z\cos\phi'\,\hat{\mathbf{i}}+z\sin\phi'\,\hat{\mathbf{j}}+R\,\hat{\mathbf{k}}}{(R^2+z^2)^{3/2}}\,d\phi' \tag{9.1.12}$$

The x and the y components of $\vec{\mathbf{B}}$ can be readily shown to be zero:

$$B_x = \frac{\mu_0 IRz}{4\pi(R^2+z^2)^{3/2}}\int_0^{2\pi}\cos\phi'\,d\phi' = \frac{\mu_0 IRz}{4\pi(R^2+z^2)^{3/2}}\sin\phi'\Big|_0^{2\pi} = 0 \tag{9.1.13}$$

$$B_y = \frac{\mu_0 IRz}{4\pi(R^2+z^2)^{3/2}}\int_0^{2\pi}\sin\phi'\,d\phi' = -\frac{\mu_0 IRz}{4\pi(R^2+z^2)^{3/2}}\cos\phi'\Big|_0^{2\pi} = 0 \tag{9.1.14}$$

On the other hand, the z component is

$$B_z = \frac{\mu_0}{4\pi}\frac{IR^2}{(R^2+z^2)^{3/2}}\int_0^{2\pi}d\phi' = \frac{\mu_0}{4\pi}\frac{2\pi IR^2}{(R^2+z^2)^{3/2}} = \frac{\mu_0 IR^2}{2(R^2+z^2)^{3/2}} \tag{9.1.15}$$

Thus, we see that along the symmetric axis, B_z is the only non-vanishing component of the magnetic field. The conclusion can also be reached by using the symmetry arguments.

I doubt this will be on test

The behavior of B_z / B_0 where $B_0 = \mu_0 I / 2R$ is the magnetic field strength at $z = 0$, as a function of z / R is shown in Figure 9.1.7:

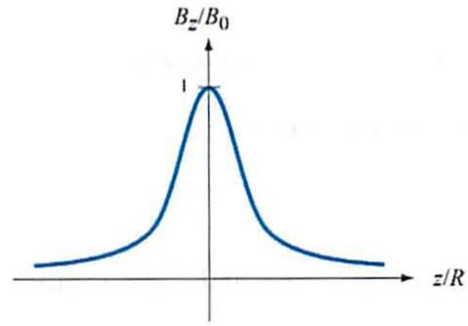


Figure 9.1.7 The ratio of the magnetic field, B_z / B_0, as a function of z / R

(b) If we place a magnetic dipole $\vec{\boldsymbol{\mu}} = \mu_z \hat{\mathbf{k}}$ at the point P, as discussed in Chapter 8, due to the non-uniformity of the magnetic field, the dipole will experience a force given by

$$\vec{\mathbf{F}}_B = \nabla(\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}) = \nabla(\mu_z B_z) = \mu_z \left(\frac{dB_z}{dz} \right) \hat{\mathbf{k}} \qquad (9.1.16)$$

like up w/

- forget specifics go back + study!

Upon differentiating Eq. (9.1.15) and substituting into Eq. (9.1.16), we obtain

$$\vec{\mathbf{F}}_B = -\frac{3\mu_z \mu_0 I R^2 z}{2(R^2 + z^2)^{5/2}} \hat{\mathbf{k}} \qquad (9.1.17)$$

Thus, the dipole is attracted toward the current-carrying ring. On the other hand, if the direction of the dipole is reversed, $\vec{\boldsymbol{\mu}} = -\mu_z \hat{\mathbf{k}}$, the resulting force will be repulsive.

9.1.1 Magnetic Field of a Moving Point Charge

Suppose we have an infinitesimal current element in the form of a cylinder of cross-sectional area A and length ds consisting of n charge carriers per unit volume, all moving at a common velocity $\vec{\mathbf{v}}$ along the axis of the cylinder. Let I be the current in the element, which we define as the amount of charge passing through any cross-section of the cylinder per unit time. From Chapter 6, we see that the current I can be written as

$$n A q |\vec{\mathbf{v}}| = I \qquad (9.1.18)$$

The total number of charge carriers in the current element is simply $dN = n A\, ds$, so that using Eq. (9.1.1), the magnetic field $d\vec{\mathbf{B}}$ due to the dN charge carriers is given by

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{(nAq\,|\,\vec{\mathbf{v}}\,|)\,d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2} = \frac{\mu_0}{4\pi}\frac{(n\,A\,ds)q\,\vec{\mathbf{v}}\times\hat{\mathbf{r}}}{r^2} = \frac{\mu_0}{4\pi}\frac{(dN)q\,\vec{\mathbf{v}}\times\hat{\mathbf{r}}}{r^2} \tag{9.1.19}$$

where r is the distance between the charge and the field point P at which the field is being measured, the unit vector $\hat{\mathbf{r}} = \vec{\mathbf{r}}/r$ points *from* the source of the field (the charge) *to* P. The differential length vector $d\vec{\mathbf{s}}$ is defined to be parallel to $\vec{\mathbf{v}}$. In case of a single charge, $dN = 1$, the above equation becomes

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{q\,\vec{\mathbf{v}}\times\hat{\mathbf{r}}}{r^2} \tag{9.1.20}$$

Note, however, that since a point charge does not constitute a steady current, the above equation strictly speaking only holds in the non-relativistic limit where $v \ll c$, the speed of light, so that the effect of "retardation" can be ignored.

The result may be readily extended to a collection of N point charges, each moving with a different velocity. Let the ith charge q_i be located at (x_i, y_i, z_i) and moving with velocity $\vec{\mathbf{v}}_i$. Using the superposition principle, the magnetic field at P can be obtained as:

$$\vec{\mathbf{B}} = \sum_{i=1}^{N}\frac{\mu_0}{4\pi}q_i\vec{\mathbf{v}}_i \times \left[\frac{(x-x_i)\hat{\mathbf{i}} + (y-y_i)\hat{\mathbf{j}} + (z-z_i)\hat{\mathbf{k}}}{\left[(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2\right]^{3/2}}\right] \tag{9.1.21}$$

Animation 9.1: Magnetic Field of a Moving Charge

Figure 9.1.8 shows one frame of the animations of the magnetic field of a moving positive and negative point charge, assuming the speed of the charge is small compared to the speed of light.

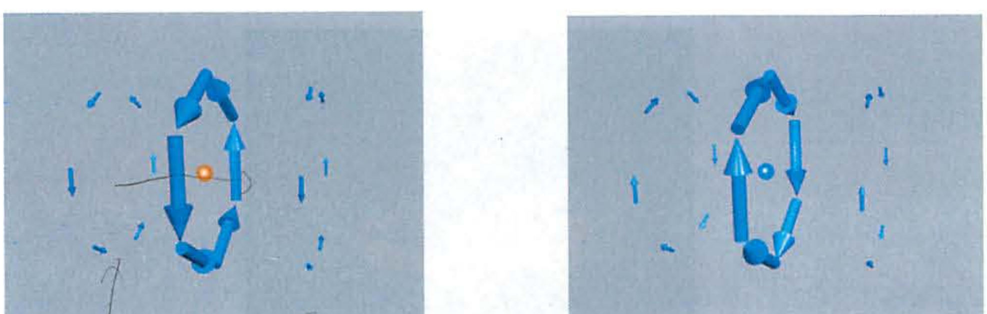


Figure 9.1.8 The magnetic field of (a) a moving positive charge, and (b) a moving negative charge, when the speed of the charge is small compared to the speed of light.

getting thinner/smaller

getting thicker/bigger

Animation 9.2: Magnetic Field of Several Charges Moving in a Circle

Suppose we want to calculate the magnetic fields of a number of charges moving on the circumference of a circle with equal spacing between the charges. To calculate this field we have to add up vectorially the magnetic fields of each of charges using Eq. (9.1.19).

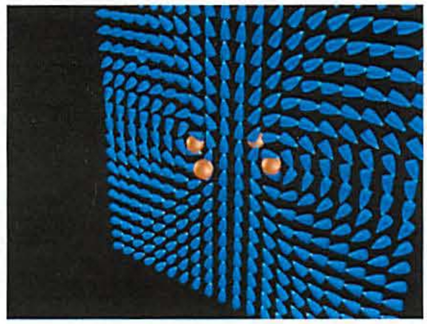
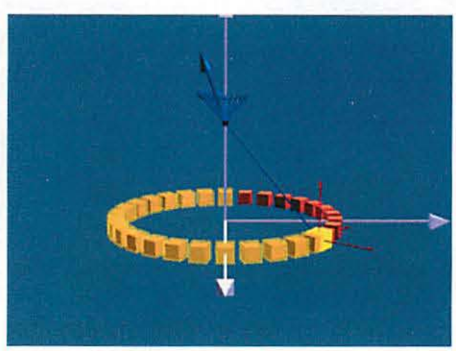


Figure 9.1.9 The magnetic field of four charges moving in a circle. We show the magnetic field vector directions in only one plane. The bullet-like icons indicate the direction of the magnetic field at that point in the array spanning the plane.

Figure 9.1.9 shows one frame of the animation when the number of moving charges is four. Other animations show the same situation for $N=1$, 2, and 8. When we get to eight charges, a characteristic pattern emerges--the magnetic dipole pattern. Far from the ring, the shape of the field lines is the same as the shape of the field lines for an electric dipole.

Interactive Simulation 9.2: Magnetic Field of a Ring of Moving Charges

Figure 9.1.10 shows a ShockWave display of the vectoral addition process for the case where we have 30 charges moving on a circle. The display in Figure 9.1.10 shows an observation point fixed on the axis of the ring. As the addition proceeds, we also show the resultant up to that point (large arrow in the display).



↑ adds up

Figure 9.1.10 A ShockWave simulation of the use of the principle of superposition to find the magnetic field due to 30 moving charges moving in a circle at an observation point on the axis of the circle.

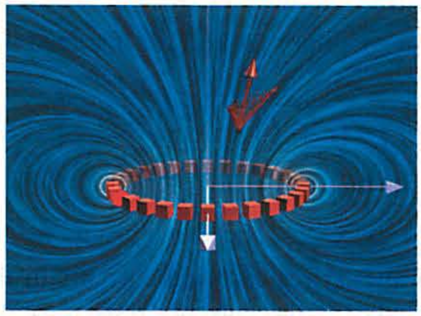


Figure 9.1.11 The magnetic field due to 30 charges moving in a circle at a given observation point. The position of the observation point can be varied to see how the magnetic field of the individual charges adds up to give the total field.

In Figure 9.1.11, we show an interactive ShockWave display that is similar to that in Figure 9.1.10, but now we can interact with the display to move the position of the observer about in space. To get a feel for the total magnetic field, we also show a "iron filings" representation of the magnetic field due to these charges. We can move the observation point about in space to see how the total field at various points arises from the individual contributions of the magnetic field of to each moving charge.

9.2 Force Between Two Parallel Wires

We have already seen that a current-carrying wire produces a magnetic field. In addition, when placed in a magnetic field, a wire carrying a current will experience a net force. Thus, we expect two current-carrying wires to exert force on each other.

Consider two parallel wires separated by a distance a and carrying currents I_1 and I_2 in the $+x$-direction, as shown in Figure 9.2.1.

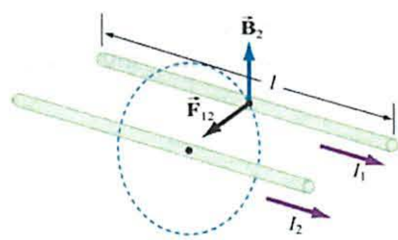


Figure 9.2.1 Force between two parallel wires

The magnetic force, $\vec{\mathbf{F}}_{12}$, exerted on wire 1 by wire 2 may be computed as follows: Using the result from the previous example, the magnetic field lines due to I_2 going in the $+x$-direction are circles concentric with wire 2, with the field $\vec{\mathbf{B}}_2$ pointing in the tangential direction. Thus, at an arbitrary point P on wire 1, we have $\vec{\mathbf{B}}_2 = -(\mu_0 I_2 / 2\pi a)\hat{\mathbf{j}}$, which points in the direction perpendicular to wire 1, as depicted in Figure 9.2.1. Therefore,

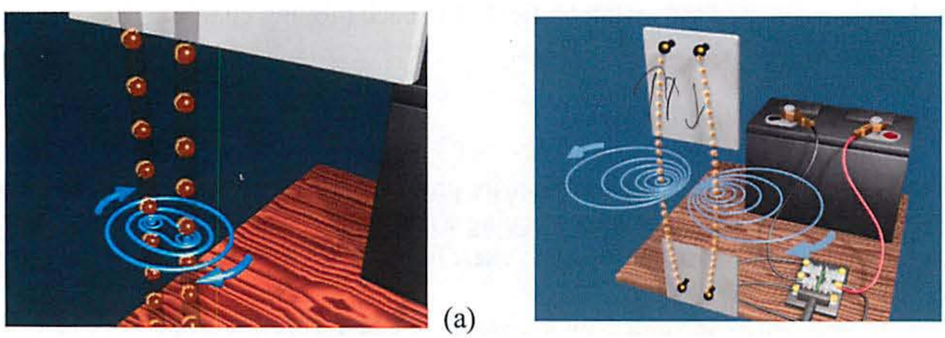
$$\vec{\mathbf{F}}_{12} = I_1 \vec{l} \times \vec{\mathbf{B}}_2 = I_1 \left(l\hat{\mathbf{i}} \right) \times \left(-\frac{\mu_0 I_2}{2\pi a}\hat{\mathbf{j}} \right) = -\frac{\mu_0 I_1 I_2 l}{2\pi a}\hat{\mathbf{k}} \tag{9.2.1}$$

Clearly $\vec{\mathbf{F}}_{12}$ points toward wire 2. The conclusion we can draw from this simple calculation is that two parallel wires carrying currents in the same direction will attract each other. On the other hand, if the currents flow in opposite directions, the resultant force will be repulsive.

Animation 9.3: Forces Between Current-Carrying Parallel Wires

Figures 9.2.2 shows parallel wires carrying current in the same and in opposite directions. In the first case, the magnetic field configuration is such as to produce an attraction between the wires. In the second case the magnetic field configuration is such as to produce a repulsion between the wires.

↑ ↑ → ← attract



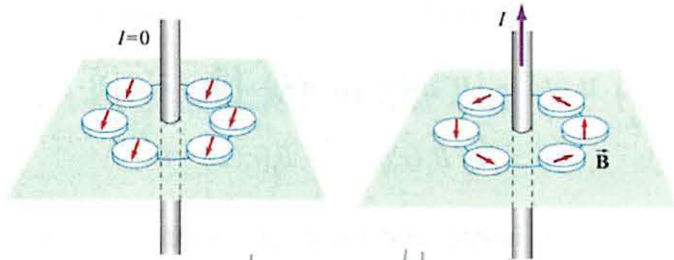
(a) (b)

↑ ↓ ← → repel

Figure 9.2.2 (a) The attraction between two wires carrying current in the same direction. The direction of current flow is represented by the motion of the orange spheres in the visualization. (b) The repulsion of two wires carrying current in opposite directions.

9.3 Ampere's Law

We have seen that moving charges or currents are the source of magnetism. This can be readily demonstrated by placing compass needles near a wire. As shown in Figure 9.3.1a, all compass needles point in the same direction in the absence of current. However, when $I \neq 0$, the needles will be deflected along the tangential direction of the circular path (Figure 9.3.1b).



Only matters if current flowing

Figure 9.3.1 Deflection of compass needles near a current-carrying wire

Let us now divide a circular path of radius r into a large number of small length vectors $\Delta\vec{\mathbf{s}} = \Delta s\hat{\boldsymbol{\varphi}}$, that point along the tangential direction with magnitude Δs (Figure 9.3.2).

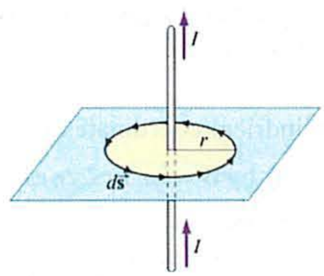
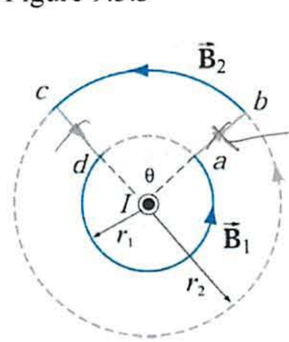


Figure 9.3.2 Amperian loop

In the limit $\Delta\vec{\mathbf{s}} \to \vec{0}$, we obtain

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\oint ds = \left(\frac{\mu_0 I}{2\pi r}\right)(2\pi r) = \mu_0 I \qquad (9.3.1)$$

The result above is obtained by choosing a closed path, or an "Amperian loop" that follows one particular magnetic field line. Let's consider a slightly more complicated Amperian loop, as that shown in Figure 9.3.3



Does not count

Figure 9.3.3 An Amperian loop involving two field lines

The line integral of the magnetic field around the contour *abcda* is

$$\oint_{abcda} \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \int_{ab} \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} + \int_{bc} \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} + \int_{cd} \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} + \int_{cd} \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}}$$
$$= 0 + B_2(r_2\theta) + 0 + B_1[r_1(2\pi - \theta)] \tag{9.3.2}$$

where the length of arc bc is $r_2\theta$, and $r_1(2\pi-\theta)$ for arc da. The first and the third integrals vanish since the magnetic field is perpendicular to the paths of integration. With $B_1 = \mu_0 I / 2\pi r_1$ and $B_2 = \mu_0 I / 2\pi r_2$, the above expression becomes

$$\oint_{abcda} \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \frac{\mu_0 I}{2\pi r_2}(r_2\theta) + \frac{\mu_0 I}{2\pi r_1}[r_1(2\pi-\theta)] = \frac{\mu_0 I}{2\pi}\theta + \frac{\mu_0 I}{2\pi}(2\pi-\theta) = \mu_0 I \tag{9.3.3}$$

We see that the same result is obtained whether the closed path involves one or two magnetic field lines.

As shown in Example 9.1, in cylindrical coordinates (r,φ,z) with current flowing in the $+z$-axis, the magnetic field is given by $\vec{\mathbf{B}} = (\mu_0 I / 2\pi r)\hat{\boldsymbol{\varphi}}$. An arbitrary length element in the cylindrical coordinates can be written as

$$d\vec{\mathbf{s}} = dr\,\hat{\mathbf{r}} + r\,d\varphi\,\hat{\boldsymbol{\varphi}} + dz\,\hat{\mathbf{z}} \tag{9.3.4}$$

which implies

$$\oint_{\text{closed path}} \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \oint_{\text{closed path}} \left(\frac{\mu_0 I}{2\pi r}\right) r\,d\varphi = \frac{\mu_0 I}{2\pi} \oint_{\text{closed path}} d\varphi = \frac{\mu_0 I}{2\pi}(2\pi) = \mu_0 I \tag{9.3.5}$$

In other words, the line integral of $\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}}$ around any closed Amperian loop is proportional to I_{enc}, the current encircled by the loop.

but circle must be neat enough to =0

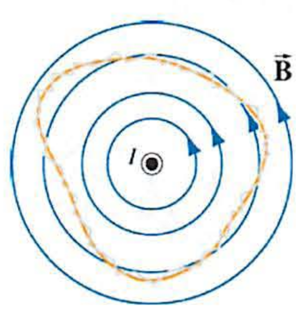


Figure 9.3.4 An Amperian loop of arbitrary shape.

The generalization to any closed loop of arbitrary shape (see for example, Figure 9.3.4) that involves many magnetic field lines is known as Ampere's law:

$$\boxed{\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{\text{enc}}} \tag{9.3.6}$$

Ampere's law in magnetism is analogous to Gauss's law in electrostatics. In order to apply them, the system must possess certain symmetry. In the case of an infinite wire, the system possesses cylindrical symmetry and Ampere's law can be readily applied. However, when the length of the wire is finite, Biot-Savart law must be used instead.

Biot-Savart Law	$\vec{\mathbf{B}} = \dfrac{\mu_0 I}{4\pi} \displaystyle\int \dfrac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$	general current source ex: finite wire
Ampere's law	$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{\text{enc}}$	current source has certain symmetry ex: infinite wire (cylindrical)

Ampere's law is applicable to the following current configurations:

1. Infinitely long straight wires carrying a steady current I (Example 9.3)

2. Infinitely large sheet of thickness b with a current density J (Example 9.4).

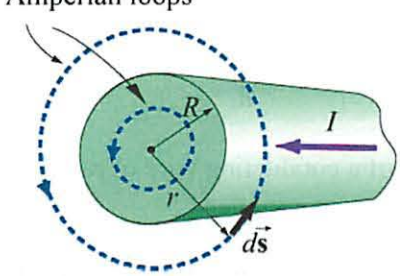
3. Infinite solenoid (Section 9.4).

4. Toroid (Example 9.5).

We shall examine all four configurations in detail.

Example 9.3: Field Inside and Outside a Current-Carrying Wire

Consider a long straight wire of radius R carrying a current I of uniform current density, as shown in Figure 9.3.5. Find the magnetic field everywhere.



must do both

Figure 9.3.5 Amperian loops for calculating the $\vec{\mathbf{B}}$ field of a conducting wire of radius R.

Solution:

(i) Outside the wire where $r \geq R$, the Amperian loop (circle 1) completely encircles the current, i.e., $I_{\text{enc}} = I$. Applying Ampere's law yields

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\oint ds = B(2\pi r) = \mu_0 I$$

which implies

$$B = \frac{\mu_0 I}{2\pi r}$$

(ii) Inside the wire where $r < R$, the amount of current encircled by the Amperian loop (circle 2) is proportional to the area enclosed, i.e.,

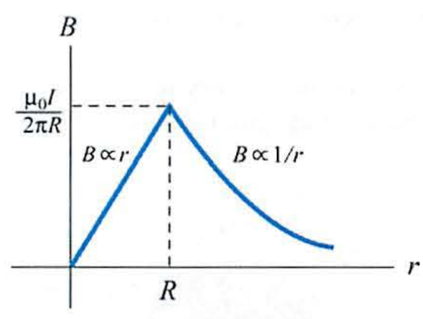
$$I_{\text{enc}} = \left(\frac{\pi r^2}{\pi R^2}\right) I$$

? fraction of wire

Thus, we have

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_0 I\left(\frac{\pi r^2}{\pi R^2}\right) \quad \Rightarrow \quad \boxed{B = \frac{\mu_0 I r}{2\pi R^2}}$$

We see that the magnetic field is zero at the center of the wire and increases linearly with r until r=R. Outside the wire, the field falls off as $1/r$. The qualitative behavior of the field is depicted in Figure 9.3.6 below:



k memorize these charts

Figure 9.3.6 Magnetic field of a conducting wire of radius R carrying a steady current I.

Example 9.4: Magnetic Field Due to an Infinite Current Sheet

Consider an infinitely large sheet of thickness b lying in the xy plane with a uniform current density $\vec{\mathbf{J}} = J_0\hat{\mathbf{i}}$. Find the magnetic field everywhere.

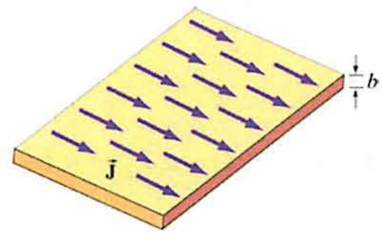


Figure 9.3.7 An infinite sheet with current density $\vec{\mathbf{J}} = J_0\hat{\mathbf{i}}$.

Solution:

We may think of the current sheet as a set of parallel wires carrying currents in the $+x$-direction. From Figure 9.3.8, we see that magnetic field at a point P above the plane points in the $-y$-direction. The z-component vanishes after adding up the contributions from all wires. Similarly, we may show that the magnetic field at a point below the plane points in the $+y$-direction.

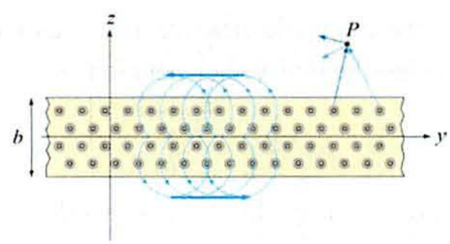


Figure 9.3.8 Magnetic field of a current sheet

We may now apply Ampere's law to find the magnetic field due to the current sheet. The Amperian loops are shown in Figure 9.3.9.

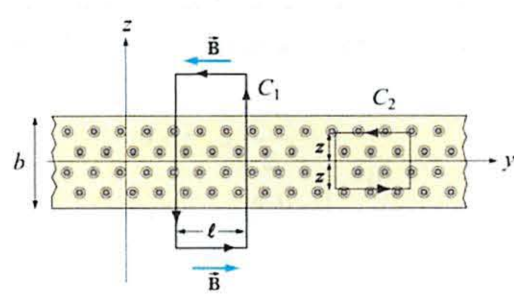


Figure 9.3.9 Amperian loops for the current sheets

For the field outside, we integrate along path C_1. The amount of current enclosed by C_1 is

$$I_{\text{enc}} = \iint \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}} = J_0(b\ell) \tag{9.3.7}$$

Applying Ampere's law leads to

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\ell) = \mu_0 I_{\text{enc}} = \mu_0(J_0 b\ell) \tag{9.3.8}$$

or $B = \mu_0 J_0 b/2$. Note that the magnetic field outside the sheet is constant, independent of the distance from the sheet. Next we find the magnetic field inside the sheet. The amount of current enclosed by path C_2 is

$$I_{\text{enc}} = \iint \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}} = J_0(2|z|\ell) \tag{9.3.9}$$

Applying Ampere's law, we obtain

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\ell) = \mu_0 I_{\text{enc}} = \mu_0 J_0(2|z|\ell) \tag{9.3.10}$$

or $B = \mu_0 J_0 |z|$. At $z = 0$, the magnetic field vanishes, as required by symmetry. The results can be summarized using the unit-vector notation as

$$\vec{\mathbf{B}} = \begin{cases} -\dfrac{\mu_0 J_0 b}{2}\hat{\mathbf{j}}, & z > b/2 \\ -\mu_0 J_0 z\,\hat{\mathbf{j}}, & -b/2 < z < b/2 \\ \dfrac{\mu_0 J_0 b}{2}\hat{\mathbf{j}}, & z < -b/2 \end{cases} \tag{9.3.11}$$

Let's now consider the limit where the sheet is infinitesimally thin, with $b \to 0$. In this case, instead of current density $\vec{\mathbf{J}} = J_0\hat{\mathbf{i}}$, we have surface current $\vec{\mathbf{K}} = K\hat{\mathbf{i}}$, where $K = J_0 b$. Note that the dimension of K is current/length. In this limit, the magnetic field becomes

$$\vec{\mathbf{B}} = \begin{cases} -\dfrac{\mu_0 K}{2}\hat{\mathbf{j}}, & z > 0 \\ \dfrac{\mu_0 K}{2}\hat{\mathbf{j}}, & z < 0 \end{cases} \tag{9.3.12}$$

9.4 Solenoid

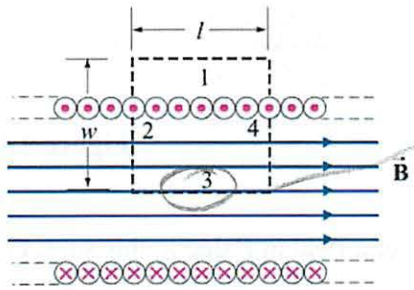
A solenoid is a long coil of wire tightly wound in the helical form. Figure 9.4.1 shows the magnetic field lines of a solenoid carrying a steady current I. We see that if the turns are closely spaced, the resulting magnetic field inside the solenoid becomes fairly uniform, provided that the length of the solenoid is much greater than its diameter. For an "ideal" solenoid, which is infinitely long with turns tightly packed, the magnetic field inside the solenoid is uniform and parallel to the axis, and vanishes outside the solenoid.

ignore edge effects

Figure 9.4.1 Magnetic field lines of a solenoid

We can use Ampere's law to calculate the magnetic field strength inside an ideal solenoid. The cross-sectional view of an ideal solenoid is shown in Figure 9.4.2. To compute $\vec{\mathbf{B}}$, we consider a rectangular path of length l and width w and traverse the path in a counterclockwise manner. The line integral of $\vec{\mathbf{B}}$ along this loop is

$$\begin{aligned}\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} &= \int_1 \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} + \int_2 \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} + \int_3 \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} + \int_4 \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} \\ &= \quad 0 \quad + \quad 0 \quad + \quad Bl \quad + \quad 0\end{aligned} \tag{9.4.1}$$



Only that matters, constant

Figure 9.4.2 Amperian loop for calculating the magnetic field of an ideal solenoid.

In the above, the contributions along sides 2 and 4 are zero because $\vec{\mathbf{B}}$ is perpendicular to $d\vec{\mathbf{s}}$. In addition, $\vec{\mathbf{B}} = \vec{\mathbf{0}}$ along side 1 because the magnetic field is non-zero only inside the solenoid. On the other hand, the total current enclosed by the Amperian loop is $I_{\text{enc}} = NI$, where N is the total number of turns. Applying Ampere's law yields

$$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = Bl = \mu_0 NI \tag{9.4.2}$$

one side

or

N= # of turns

n = # turns
per unit lenght

$$B = \frac{\mu_0 NI}{l} = \mu_0 nI \tag{9.4.3}$$

where $\boxed{n = N/l}$ represents the number of turns per unit length., In terms of the surface current, or current per unit length $K = nI$, the magnetic field can also be written as,

$$B = \mu_0 K \tag{9.4.4}$$

What happens if the length of the solenoid is finite? To find the magnetic field due to a finite solenoid, we shall approximate the solenoid as consisting of a large number of circular loops stacking together. Using the result obtained in Example 9.2, the magnetic field at a point P on the z axis may be calculated as follows: Take a cross section of tightly packed loops located at z' with a thickness dz', as shown in Figure 9.4.3

The amount of current flowing through is proportional to the thickness of the cross section and is given by $dI = I(n\,dz') = I(N/l)dz'$, where $n = N/l$ is the number of turns per unit length.

edge effets
don't need
to know

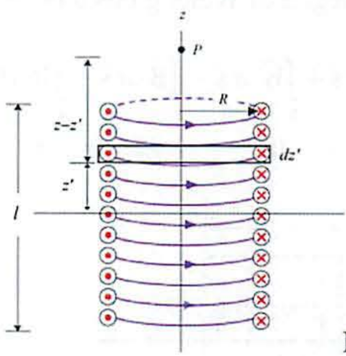


Figure 9.4.3 Finite Solenoid

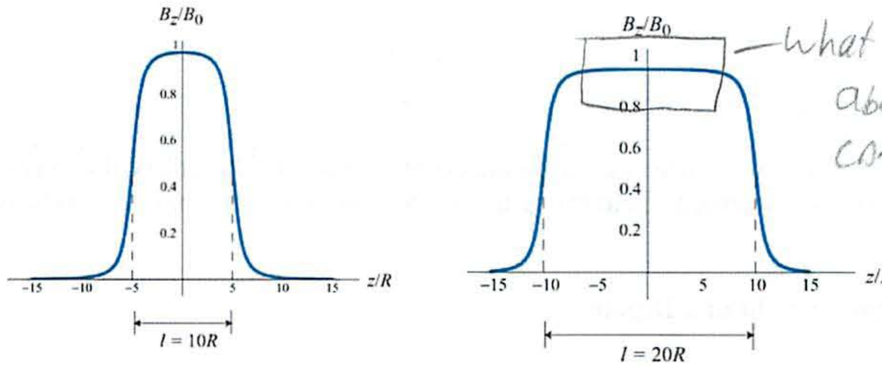
The contribution to the magnetic field at P due to this subset of loops is

$$dB_z = \frac{\mu_0 R^2}{2[(z-z')^2 + R^2]^{3/2}} dI = \frac{\mu_0 R^2}{2[(z-z')^2 + R^2]^{3/2}} (nI\,dz') \tag{9.4.5}$$

Integrating over the entire length of the solenoid, we obtain

$$\begin{aligned} B_z &= \frac{\mu_0 nIR^2}{2} \int_{-l/2}^{l/2} \frac{dz'}{[(z-z')^2 + R^2]^{3/2}} = \frac{\mu_0 nIR^2}{2} \frac{z'-z}{R^2\sqrt{(z-z')^2 + R^2}} \Bigg|_{-l/2}^{l/2} \\ &= \frac{\mu_0 nI}{2}\left[\frac{(l/2)-z}{\sqrt{(z-l/2)^2+R^2}} + \frac{(l/2)+z}{\sqrt{(z+l/2)^2+R^2}}\right] \end{aligned} \tag{9.4.6}$$

A plot of B_z / B_0, where $B_0 = \mu_0 nI$ is the magnetic field of an infinite solenoid, as a function of z / R is shown in Figure 9.4.4 for $l = 10R$ and $l = 20R$.



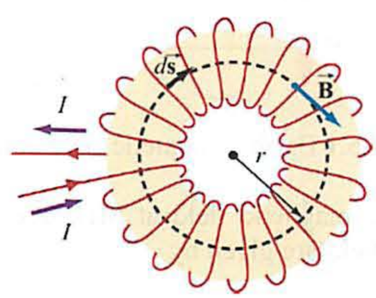
—what we care about constant

Figure 9.4.4 Magnetic field of a finite solenoid for (a) $l = 10R$, and (b) $l = 20R$.

Notice that the value of the magnetic field in the region $|z| < l/2$ is nearly uniform and approximately equal to B_0.

Examaple 9.5: Toroid

Consider a toroid which consists of N turns, as shown in Figure 9.4.5. Find the magnetic field everywhere.



Solidnoid wrapped around

Figure 9.4.5 A toroid with N turns

Solutions:

One can think of a toroid as a solenoid wrapped around with its ends connected. Thus, the magnetic field is completely confined inside the toroid and the field points in the azimuthal direction (clockwise due to the way the current flows, as shown in Figure 9.4.5.)

Applying Ampere's law, we obtain

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \oint B\,ds = B\oint ds = B(2\pi r) = \mu_0 NI \tag{9.4.7}$$

or

$$B = \frac{\mu_0 NI}{2\pi r} \tag{9.4.8}$$

where r is the distance measured from the center of the toroid.. Unlike the magnetic field of a solenoid, the magnetic field inside the toroid is non-uniform and decreases as $1/r$.

9.5 Magnetic Field of a Dipole

Let a magnetic dipole moment vector $\vec{\boldsymbol{\mu}} = -\mu\hat{\mathbf{k}}$ be placed at the origin (*e.g.*, center of the Earth) in the yz plane. What is the magnetic field at a point (*e.g.*, MIT) a distance r away from the origin?

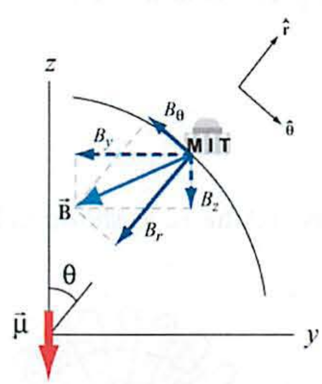


Figure 9.5.1 Earth's magnetic field components

In Figure 9.5.1 we show the magnetic field at MIT due to the dipole. The y- and z-components of the magnetic field are given by

$$B_y = -\frac{\mu_0}{4\pi}\frac{3\mu}{r^3}\sin\theta\cos\theta, \qquad B_z = -\frac{\mu_0}{4\pi}\frac{\mu}{r^3}(3\cos^2\theta - 1) \tag{9.5.1}$$

Readers are referred to Section 9.8 for the detail of the derivation.

In spherical coordinates (r, θ, ϕ), the radial and the polar components of the magnetic field can be written as

$$B_r = B_y \sin\theta + B_z\cos\theta = -\frac{\mu_0}{4\pi}\frac{2\mu}{r^3}\cos\theta \tag{9.5.2}$$

and

$$B_\theta = B_y \cos\theta - B_z \sin\theta = -\frac{\mu_0}{4\pi}\frac{\mu}{r^3}\sin\theta \tag{9.5.3}$$

respectively. Thus, the magnetic field at MIT due to the dipole becomes

$$\vec{\mathbf{B}} = B_\theta\,\hat{\boldsymbol{\theta}} + B_r\,\hat{\mathbf{r}} = -\frac{\mu_0}{4\pi}\frac{\mu}{r^3}(\sin\theta\,\hat{\boldsymbol{\theta}} + 2\cos\theta\,\hat{\mathbf{r}}) \tag{9.5.4}$$

Notice the similarity between the above expression and the electric field due to an electric dipole $\vec{\mathbf{p}}$ (see Solved Problem 2.13.6):

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}(\sin\theta\,\hat{\boldsymbol{\theta}} + 2\cos\theta\,\hat{\mathbf{r}})$$

The negative sign in Eq. (9.5.4) is due to the fact that the magnetic dipole points in the $-z$-direction. In general, the magnetic field due to a dipole moment $\vec{\boldsymbol{\mu}}$ can be written as

$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{3(\vec{\boldsymbol{\mu}}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \vec{\boldsymbol{\mu}}}{r^3} \tag{9.5.5}$$

The ratio of the radial and the polar components is given by

$$\frac{B_r}{B_\theta} = \frac{-\dfrac{\mu_0}{4\pi}\dfrac{2\mu}{r^3}\cos\theta}{-\dfrac{\mu_0}{4\pi}\dfrac{\mu}{r^3}\sin\theta} = 2\cot\theta \tag{9.5.6}$$

9.5.1 Earth's Magnetic Field at MIT

The Earth's field behaves as if there were a bar magnet in it. In Figure 9.5.2 an imaginary magnet is drawn inside the Earth oriented to produce a magnetic field like that of the Earth's magnetic field. Note the South pole of such a magnet in the northern hemisphere in order to attract the North pole of a compass.

It is most natural to represent the location of a point P on the surface of the Earth using the spherical coordinates (r,θ,ϕ), where r is the distance from the center of the Earth, θ is the polar angle from the z-axis, with $0 \le \theta \le \pi$, and ϕ is the azimuthal angle in the xy plane, measured from the x-axis, with $0 \le \phi \le 2\pi$ (See Figure 9.5.3.) With the distance fixed at $r = r_E$, the radius of the Earth, the point P is parameterized by the two angles θ and ϕ.

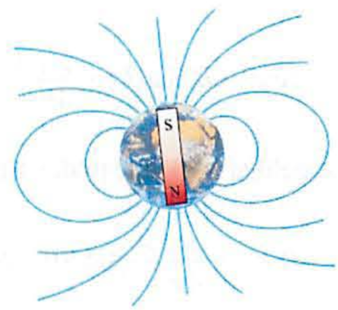


Figure 9.5.2 Magnetic field of the Earth

In practice, a location on Earth is described by two numbers – latitude and longitude. How are they related to θ and ϕ? The latitude of a point, denoted as δ, is a measure of the elevation from the plane of the equator. Thus, it is related to θ (commonly referred to as the colatitude) by $\delta = 90° - \theta$. Using this definition, the equator has latitude $0°$, and the north and the south poles have latitude $\pm 90°$, respectively.

The longitude of a location is simply represented by the azimuthal angle ϕ in the spherical coordinates. Lines of constant longitude are generally referred to as *meridians*. The value of longitude depends on where the counting begins. For historical reasons, the meridian passing through the Royal Astronomical Observatory in Greenwich, UK, is chosen as the "prime meridian" with zero longitude.

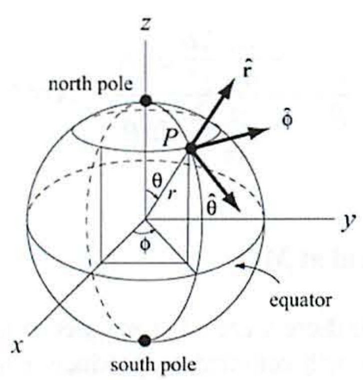
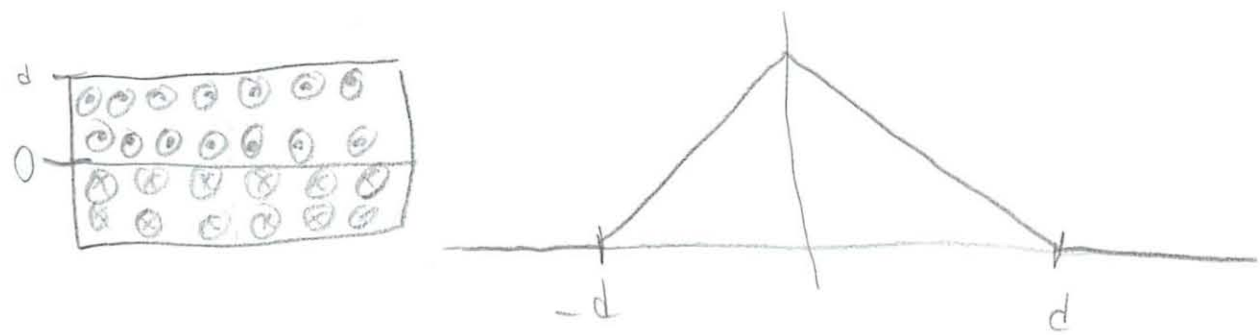


Figure 9.5.3 Locating a point P on the surface of the Earth using spherical coordinates.

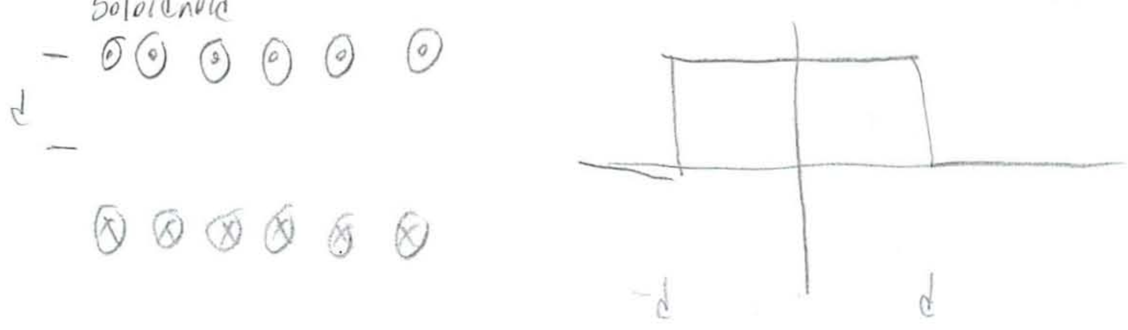
Let the z-axis be the Earth's rotation axis, and the x-axis passes through the prime meridian. The corresponding magnetic dipole moment of the Earth can be written as

$$\begin{aligned} \vec{\boldsymbol{\mu}}_E &= \mu_E (\sin\theta_0 \cos\phi_0 \,\hat{\mathbf{i}} + \sin\theta_0 \sin\phi_0 \,\hat{\mathbf{j}} + \cos\theta_0 \,\hat{\mathbf{k}}) \\ &= \mu_E (-0.062\,\hat{\mathbf{i}} + 0.18\,\hat{\mathbf{j}} - 0.98\,\hat{\mathbf{k}}) \end{aligned} \tag{9.5.7}$$

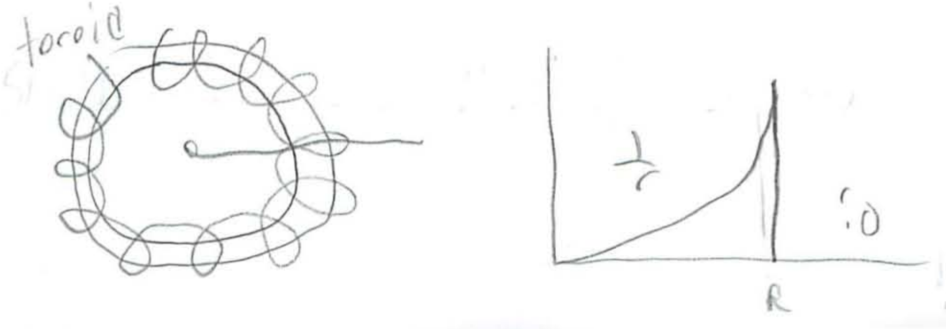
$\vec{B}$ vs Distance



Soloidnoid



toroid



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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

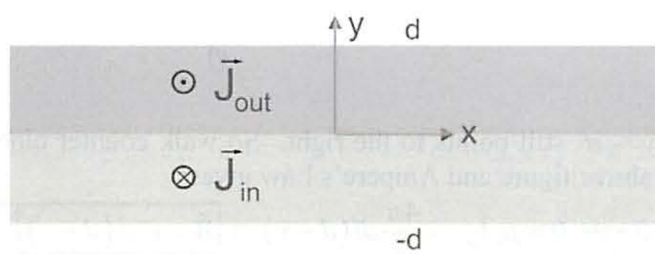
8.02 **Spring 2010**

Problem Set 11 Solutions

Problem 1: Current Slabs

The figure below shows two slabs of current. Both slabs of current are infinite in the x and z directions, and have thickness d in the y-direction. The top slab of current is located in the region $0 < y < d$ and has a constant current density $\vec{\mathbf{J}}_{out} = J\,\hat{\mathbf{z}}$ out of the page. The bottom slab of current is located in the region $-d < y < 0$ and has a constant current density $\vec{\mathbf{J}}_{in} = -J\,\hat{\mathbf{z}}$ into the page.

One thing I was never good at was where to start the loop

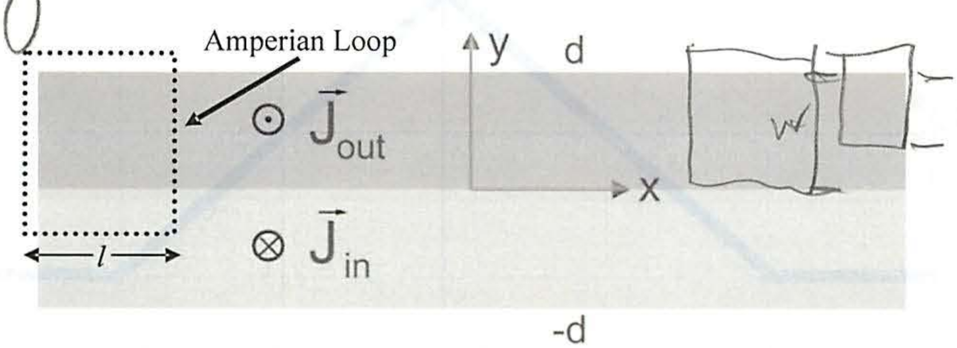


(a) What is the magnetic field for $|y| > d$? Justify your answer.

Zero. The two parts of the slab create equal and opposite fields for $|y| > d$.

b) Use Ampere's Law to find the magnetic field at $y = 0$. Show the Amperian Loop that you use and give the magnitude and direction of the magnetic field.

always have one side $= 0$ (and of course 2 sides $= 0$)



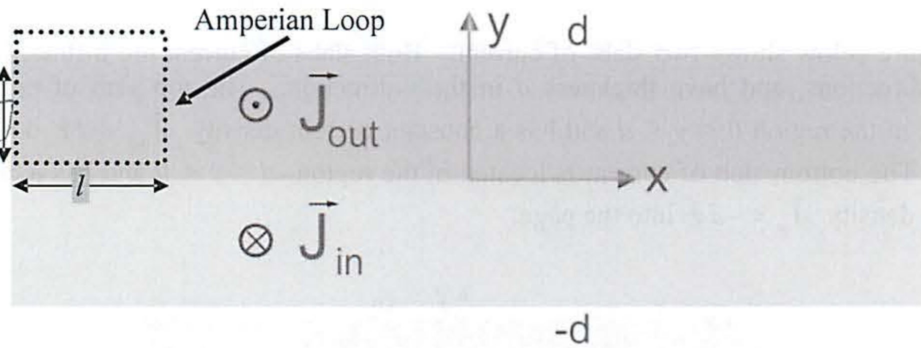
h how far you want to measure in

The field at $y = 0$ points to the right (both slabs make it point that way). So walk counter clockwise around the loop shown in the above figure and Ampere's Law gives:

$$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = Bl + 0 + 0 + 0 = \frac{4\pi}{c} I_{enc} = \mu_0 \left(Jld \right) \Rightarrow \boxed{\vec{\mathbf{B}} = \mu_0 Jd\,\hat{\mathbf{i}}\ \text{(to the right)}}$$

c) Use Ampere's Law to find the magnetic field for $0 < y < d$. Show the Amperian Loop that you use and give the magnitude and direction of the magnetic field.

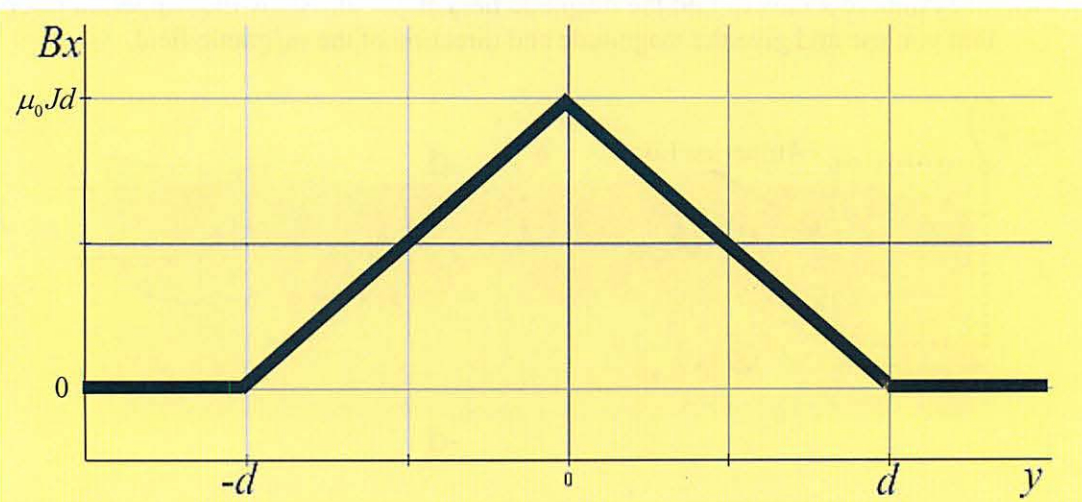
pick sensical variables



The field for $0 < y < d$ still points to the right. So walk counter clockwise around the loop shown in the above figure and Ampere's Law gives:

$$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = Bl + 0 + 0 + 0 = \mu_o I_{enc} = \frac{4\pi}{c} Jl\left(d-y\right) \Rightarrow \boxed{\vec{\mathbf{B}} = \mu_0 J\left(d-y\right)\hat{\mathbf{i}}\ \text{(to the right)}}$$

(d) Plot the x-component of the magnetic field as a function of the distance y on the graph below. Label your vertical axis.

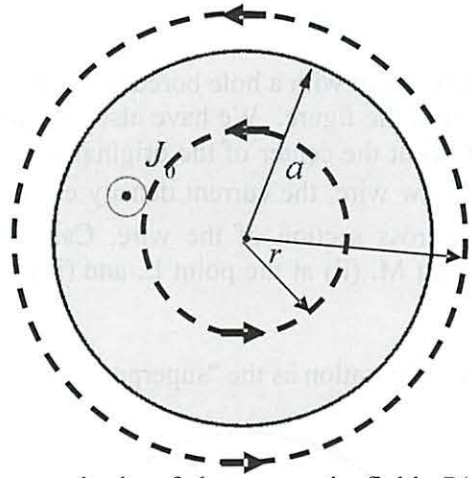


remember that chart I drew

Single

Problem 2:

An infinitely long wire of radius a carries a current density J_0 which is uniform and constant. The current points "out of" the page, as shown in the figure.



a) Calculate the magnitude of the magnetic field $B(r)$ for (i) $r < a$ and (ii) $r > a$. For both cases show your Amperian loop and indicate (with arrows) the direction of the magnetic field.

well r changes w/ something

The dashed lines above are the Amperian loops I will use for (i) and (ii). They both have a radius of r, and in both cases the paths are counterclockwise, as is the B field, due to a current out of the page (right hand rule).

(i) $r < a$.

From Ampere's Law:

only portion that does

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = 2\pi r B = \mu_0 I_{penetrate} = \mu_0 J_0 \pi r^2 \Rightarrow \boxed{B = \frac{\mu_0 J_0 \pi r^2}{2\pi r} = \frac{\mu_0 J_0 r}{2} \text{ counterclockwise}}$$

(ii) $r > a$.

Now we just contain all of the current:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = 2\pi r B = \mu_0 I_{penetrate} = \mu_0 J_0 \pi a^2 \Rightarrow \boxed{B = \frac{\mu_0 J_0 \pi a^2}{2\pi r} = \frac{\mu_0 J_0 a^2}{2r} \text{ counterclockwise}}$$

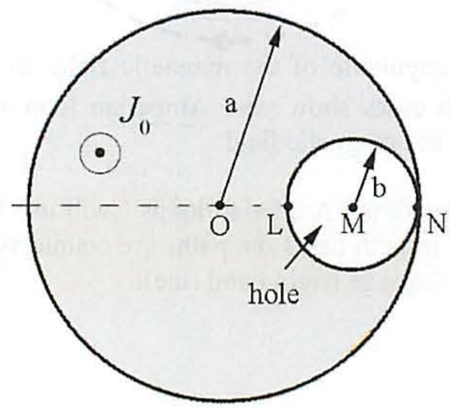
this is too hard to be on test
– think Hudson said that or other prof

(b) What happens to the answers above if the direction of the current is reversed so that it flows "into" the page ?

If the direction of current flips then so does the direction of the magnetic field, so it is clockwise rather than counterclockwise. The magnitude of the field remains the same.

c) Consider now the same wire but with a hole bored throughout. The hole has radius b (with $2b < a$) and is shown in the figure. We have also indicated four special points: O, L, M, and N. The point O is at the center of the original wire and the point M is at the center of the hole. In this new wire, the current density exists and remains equal to J_0 over the remainder of the cross section of the wire. Calculate the magnitude of the magnetic field at (i) the point M, (ii) at the point L, and (iii) at the point N. Show your work.

Hint: Try to represent the configuration as the "superposition" of two types of wires.



The point here is that we have two wires superimposed on top of each other. The large (radius a) wire carries current out of the page while the smaller (radius b) wire carries current into the page (with the same current density). At all point L, M and N we are inside the large wire and on the right, so the counterclockwise B field is pointing up the page. What is happening from the small wire changes from place to place

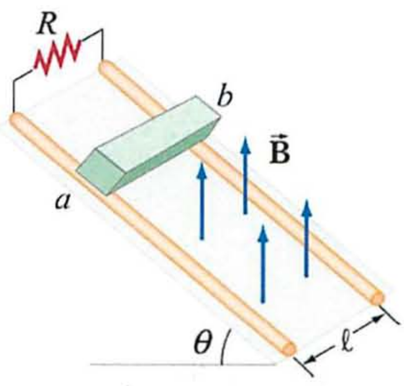
(i) the point M:

Here we are at the center of the small wire, so it contributes nothing. We are at a radius r = a - b inside the big wire, so from part (a.i) of this problem we have:

$$\boxed{B = \frac{\mu_0 J_0 (a-b)}{2} \text{ up}}$$

Problem 3: Sliding Bar on Wedges

A conducting bar of mass m slides down two frictionless conducting rails which make an angle θ with the horizontal, separated by a distance ℓ and connected at the top by a resistor R, as shown in the figure. In addition, a uniform magnetic field $\vec{\mathbf{B}}$ is applied vertically upward. The bar is released from rest and slides down. At time t the bar is moving along the rails at speed $v(t)$.



(a) Find the induced current in the bar at time t. Which way does the current flow, from a to b or b to a?

The flux between the resistor and bar is given by

$$\Phi_B = B\,\ell\,x(t)\cos\theta$$

where $x(t)$ is the distance of the bar from the top of the rails.

Then,

$$\varepsilon = -\frac{d}{dt}\Phi_B = -\frac{d}{dt}B\,\ell\,x(t)\cos\theta = -B\,\ell\,v(t)\cos\theta$$

Because the resistance of the circuit is R, the magnitude of the induced current is

$$I = \frac{|\varepsilon|}{R} = \frac{B\,\ell\,v(t)\cos\theta}{R}$$

By Lenz's law, the induced current produces magnetic fields which tend to oppose the change in magnetic flux. Therefore, the current flows clockwise, from b to a across the bar.

(b) Find the terminal speed v_T of the bar.

At terminal velocity, the net force along the rail is zero, that is gravity is balanced by the magnetic force:

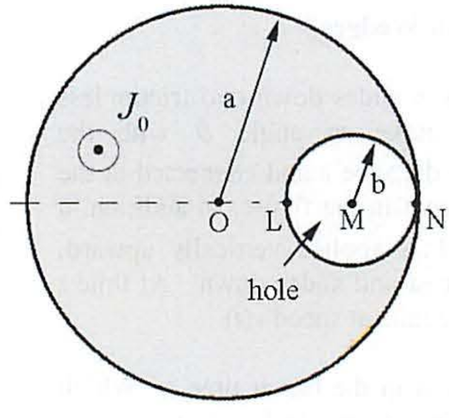
$$mg\sin\theta = IB\ell\cos\theta = \left(\frac{B\,\ell\,v_t(t)\cos\theta}{R}\right)B\ell\cos\theta$$

or

$$v_t(t) = \frac{Rmg\sin\theta}{(B\ell\cos\theta)^2}$$

After the terminal speed has been reached,

(c) What is the induced current in the bar?



(ii) at the point L:

Here we are to the left of the small wire (at a radius r = b), so the clockwise field (as we said in part b) is pointing up, just like the CCW field from the big wire We are at a radius r = a - 2b inside the big wire, so:

$$\boxed{B = \frac{\mu_0 J_0 (a-2b)}{2} + \frac{\mu_0 J_0 b}{2}\text{ up} = \frac{\mu_0 J_0 (a-b)}{2}\text{ up}}$$

(iii) at the point N:

Here we are to the right of the small wire (at a radius r = b), so the clockwise field is pointing down, opposite the CCW field from the big wire so they subtract rather than add We are at a radius r = a inside the big wire, so:

$$\boxed{B = \frac{\mu_0 J_0 a}{2} - \frac{\mu_0 J_0 b}{2}\text{ up} = \frac{\mu_0 J_0 (a-b)}{2}\text{ up}}$$

Of Course $\vec{B}$ field Same everywh

A comment about people's work on this problem: I was stunned at how many people tried to do Ampere's law on the wire with a hole in it. Since the hole breaks the cylindrical symmetry of the problem you just can't do this. That is, since B is no longer constant around an Amperian centered on O, $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} \neq 2\pi r B$. B isn't constant, so you can't just pull it out!

$$I = \frac{B\ell v_t(t)\cos\theta}{R} = \frac{B\ell\cos\theta}{R}\left(\frac{Rmg\sin\theta}{(B\ell\cos\theta)^2}\right) = \frac{mg\sin\theta}{B\ell\cos\theta} = \frac{mg}{B\ell}\tan\theta$$

(d) What is the rate at which electrical energy is being dissipated through the resistor?

The power dissipated in the resistor is

$$P = I^2R = \left(\frac{mg}{B\ell}\tan\theta\right)^2 R$$

never forget

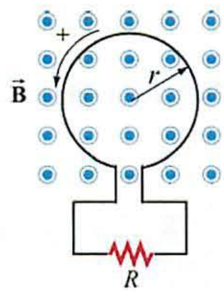
(e) What is the rate of work done by gravity on the bar? The rate at which work is done is $\vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$. How does this compare to your answer in (d)? Why?

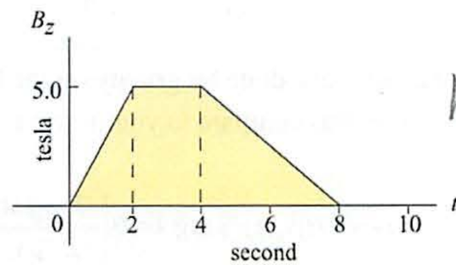
$$\vec{F} \cdot \vec{V} = (mg\sin\theta)v_t(t) = mg\sin\theta\left(\frac{Rmg\sin\theta}{(B\ell\cos\theta)^2}\right) = \left(\frac{mg}{B\ell}\tan\theta\right)^2 R = P$$

That is, they are equal. All of the work done by gravity is dissipated in the resistor, which is why the rod isn't accelerating past its terminal velocity.

Problem 4 EMF Due to a Time-Varying Magnetic Field

A uniform magnetic field $\vec{\mathbf{B}}$ is perpendicular to a one-turn circular loop of wire of negligible resistance, as shown in the figure below. The field changes with time as shown (the z direction is out of the page). The loop is of radius $r = 50\,\text{cm}$ and is connected in series with a resistor of resistance $R = 20\,\Omega$. The "+" direction around the circuit is indicated in the figure. **In order to obtain credit you must show your work; partial answers without work will not be accepted.**





here it is $\vec{B}$ that changes

(a) What is the expression for EMF in this circuit in terms of $B_z(t)$ for this arrangement?

Solution: When we choose a "+" direction around the circuit shown in the figure above, then we are also specifying that magnetic flux out of the page is positive. (The unit vector $\hat{\mathbf{n}} = +\hat{\mathbf{k}}$ points out of the page). Thus the dot product becomes

$$\vec{\mathbf{B}} \cdot \hat{\mathbf{n}} = \vec{\mathbf{B}} \cdot \hat{\mathbf{k}} = B_z . \qquad (0.1)$$

$d\Phi = A \frac{dB}{dt}$

From the graph, the z-component of the magnetic field B_z is given by

$$B_z = \begin{cases} (2.5\,\text{T}\cdot\text{s}^{-1})t\,;\, 0 < t < 2\,\text{s} \\ 5.0\,\text{T}\,;\, 2\,\text{s} < t < 4\,\text{s} \\ 10\,\text{T} - (1.25\,\text{T}\cdot\text{s}^{-1})t\,;\, 4\,\text{s} < t < 8\,\text{s} \\ 0\,;\, t > 8\,\text{s} \end{cases} . \qquad (0.2)$$

do sepertly at each time

The derivative of the magnetic field is then

$$\frac{dB_z}{dt} = \begin{cases} 2.5\,\text{T}\cdot\text{s}^{-1}\,;\, 0 < t < 2\,\text{s} \\ 0\,;\, 2\,\text{s} < t < 4\,\text{s} \\ -1.25\,\text{T}\cdot\text{s}^{-1}\,;\, 4\,\text{s} < t < 8\,\text{s} \\ 0\,;\, t > 8\,\text{s} \end{cases} . \qquad (0.3)$$

The magnetic flux is therefore

$$\Phi_{magnetic} = \iint \vec{\mathbf{B}} \cdot \hat{\mathbf{n}}\, d\vec{\mathbf{A}} = \iint B_z dA = B_z \pi r^2 .(0.4)$$

The electromotive force is

$$\mathcal{E} = -\frac{d}{dt}\Phi_{magnetic} = -\frac{dB_z}{dt}\pi r^2 . \qquad (0.5)$$

So we calculate the electromotive force by substituting Eq. (0.3) into Eq. (0.5) yielding

$$\mathcal{E} = \begin{cases} -(2.5\,\mathrm{T\cdot s^{-1}})\pi r^2 \,;\, 0 < t < 2\,\mathrm{s} \\ 0 \,;\, 2\,\mathrm{s} < t < 4\,\mathrm{s} \\ (1.25\,\mathrm{T\cdot s^{-1}})\pi r^2 \,;\, 4\,\mathrm{s} < t < 8\,\mathrm{s} \\ 0 \,;\, t > 8\,\mathrm{s} \end{cases} . \qquad (0.6)$$

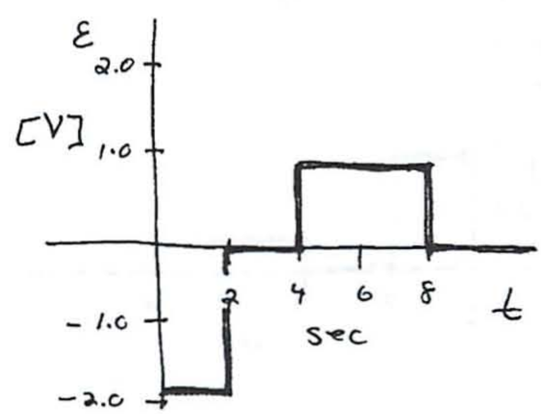
Using $r = 0.5\ \mathrm{m}$, the electromotive force is then

$$\mathcal{E} = \begin{cases} -1.96\,\mathrm{V} \,;\, 0 < t < 2\,\mathrm{s} \\ 0 \,;\, 2\,\mathrm{s} < t < 4\,\mathrm{s} \\ 0.98\,\mathrm{V} \,;\, 4\,\mathrm{s} < t < 8\,\mathrm{s} \\ 0 \,;\, t > 8\,\mathrm{s} \end{cases} \qquad (0.7)$$

Solution:

(b) Plot the EMF in the circuit as a function of time. Label the axes quantitatively (numbers and units). Watch the signs. Note that we have labeled the positive direction of the emf in the left sketch consistent with the assumption that positive $\vec{\mathbf{B}}$ is out of the paper.

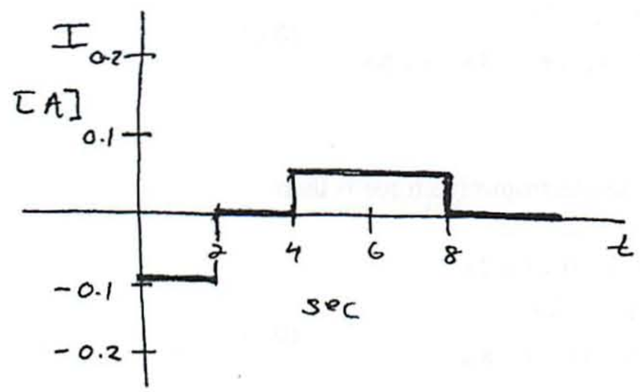
Solution:



(c) Plot the current I through the resistor R. Label the axes quantitatively (numbers and units). Indicate with arrows on the sketch the *direction* of the current through R during each time interval.

Solution: The current through the resistor ($R = 20\,\Omega$) is given by

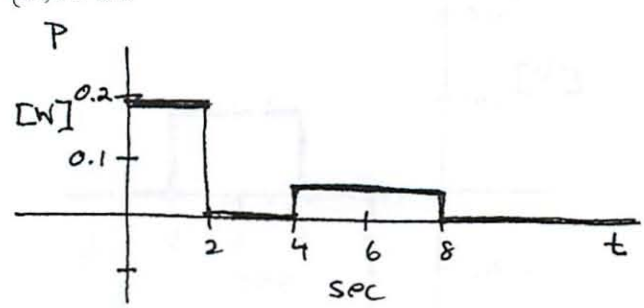
$$I = \frac{\mathcal{E}}{R} = \begin{cases} -9.8\times10^{-2}\ \mathrm{A}\,;0<t<2\ \mathrm{s} \\ 0\,;2\ \mathrm{s}<t<4\ \mathrm{s} \\ 4.9\times10^{-2}\ \mathrm{A}\,;4\ \mathrm{s}<t<8\ \mathrm{s} \\ 0\,;t>8\ \mathrm{s} \end{cases}. \tag{0.8}$$



(d) Plot the power dissipated in the resistor as a function of time.

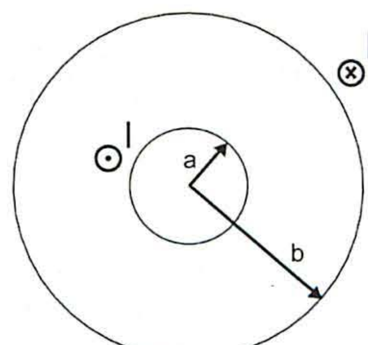
Solution: The power dissipated in the resistor is given by

$$P = I^2R = \begin{cases} 1.9\times10^{-1}\ \mathrm{W}\,;0<t<2\ \mathrm{s} \\ 0\,;2\ \mathrm{s}<t<4\ \mathrm{s} \\ 4.8\times10^{-2}\ \mathrm{W}\,;4\ \mathrm{s}<t<8\ \mathrm{s} \\ 0\,;t>8\ \mathrm{s} \end{cases}. \tag{0.9}$$



Problem 5: Inductor

An inductor consists of two very thin conducting cylindrical shells, one of radius a and one of radius b, both of length h. Assume that the inner shell carries current I out of the page, and that the outer shell carries current I into the page, distributed uniformly around the circumference in both cases. The z-axis is out of the page along the common axis of the cylinders and the r-axis is the radial cylindrical axis perpendicular to the z-axis.



a) Use Ampere's Law to find the magnetic field between the cylindrical shells. Indicate the direction of the magnetic field on the sketch. What is the magnetic energy density as a function of r for $a < r < b$?

like that pratic test power line

The enclosed current I_{enc} in the Ampere's loop with radius r is given by

$$I_{enc} = \begin{cases} 0, & r < a \\ I, & a < r < b \\ 0, & r > b \end{cases}$$

Applying Ampere's law, $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_0 I_{enc}$, we obtain

$$\vec{\mathbf{B}} = \begin{cases} 0, & r < a \\ \dfrac{\mu_0 I}{2\pi r}\hat{\boldsymbol{\varphi}}, & a < r < b \text{ (counterclockwise in the figure)} \\ 0, & r > b \end{cases}$$

The magnetic energy density for $a < r < b$ is

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r}\right)^2 = \boxed{\frac{\mu_0 I^2}{8\pi^2 r^2}}$$

It is zero elsewhere.

b). Calculate the inductance of this long inductor recalling that $U_B = \frac{1}{2}LI^2$ and using your results for the magnetic energy density in (a).

The volume element in this case is $2\pi r h dr$. The magnetic energy is :

$\int r \quad L = \frac{N\Phi}{I}$

$$U_B = \int_V u_B d\text{Vol} = \int_a^b \left(\frac{\mu_0 I^2}{8\pi^2 r^2}\right) 2\pi h r dr = \frac{\mu_0 I^2 h}{4\pi} \ln\left(\frac{b}{a}\right)$$

must integrate over r

Since $U_B = \dfrac{\mu_0 I^2 l}{4\pi} \ln\left(\dfrac{b}{a}\right) = \dfrac{1}{2} L I^2$, the inductance is

$$\boxed{L = \frac{\mu_0 h}{2\pi} \ln\left(\frac{b}{a}\right)}$$

$\frac{r}{r^2} = r$

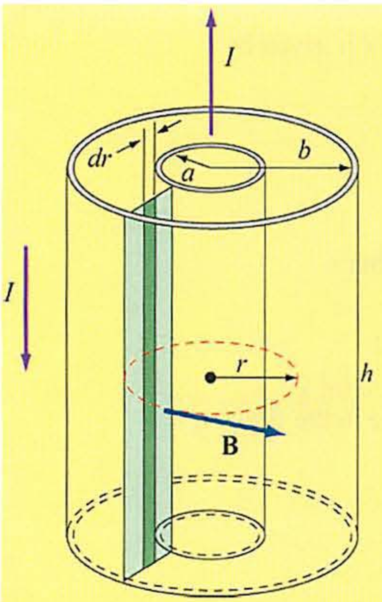
$\int_a^b r = \ln\left(\frac{b}{a}\right)$

do not forget!

here are the 2 ways to calculate

c) Calculate the inductance of this long inductor by using the formula $\Phi = LI = \int_{open\ surface} \vec{\mathbf{B}} \cdot \mathbf{d}\vec{\mathbf{A}}$ and your results for the magnetic field in (a). To do this you must choose an appropriate open surface over which to evaluate the magnetic flux. Does your result calculated in this way agree with your result in (b)?

yeah nope



The magnetic field is perpendicular to a rectangular surface shown in the figure. The magnetic flux through a thin strip of area $dA = ldr$ is

$$d\Phi_B = BdA = \left(\frac{\mu_0 I}{2\pi r}\right)(h\,dr) = \frac{\mu_0 I h}{2\pi r} dr$$

Thus, the total magnetic flux is

$$\Phi_B = \boxed{\int d\Phi_B} = \int_a^b \frac{\mu_0 I h}{2\pi r} dr = \frac{\mu_0 I h}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 I h}{2\pi} \ln\left(\frac{b}{a}\right)$$

again must integrate

Thus, the inductance is

$$L = \frac{\Phi_B}{I} = \boxed{\frac{\mu_0 h}{2\pi} \ln\left(\frac{b}{a}\right)}$$

which agrees with that obtained in (b).

yeah same

How does an inductor actually work again
Builds $\vec{B}$ field

next pg

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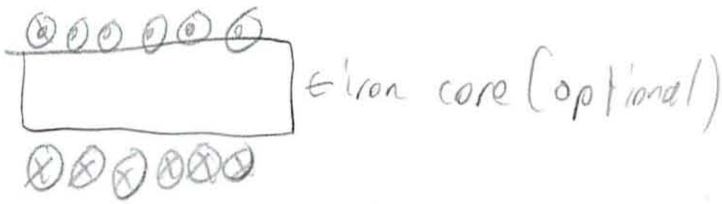
Inductor

Coil of wire



builds up $\vec{B}$ field

when field building coil inhibits current flowing

Switch closed

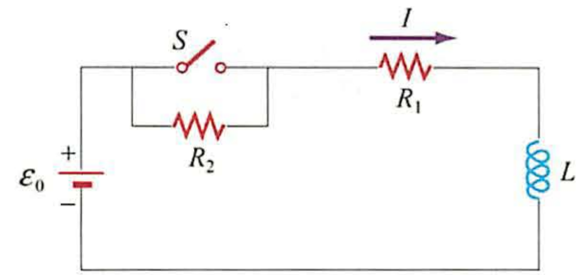
$\vec{B}$ field gets current to flow

base in this is Faraday's Law

Stores energy

Problem 6: Trying to open the switch on an *RL* Circuit

The *LR* circuit shown in the figure contains a resistor R_1 and an inductance L in series with a battery of emf ε_0. The switch S is initially closed. At t = 0, the switch S is opened, so that an additional very large resistance R_2 (with $R_2 \gg R_1$) is now in series with the other elements.



(a) If the switch has been *closed* for a long time before t = 0, what is the steady current I_0 in the circuit?

There is no induced emf before t = 0. Also, no current is flowing on R_2. Therefore,

$$I_0 = \frac{\varepsilon_0}{R_1}$$

(b) While this current I_0 is flowing, at time t = 0, the switch S is opened. Write the differential equation for $I(t)$ that describes the behavior of the circuit at times $t \geq 0$. Solve this equation (by integration) for $I(t)$ under the approximation that $\varepsilon_0 = 0$. (Assume that the battery emf is negligible compared to the total emf around the circuit for times just after the switch is opened.) Express your answer in terms of the initial current I_0, and R_1, R_2, and L.

The differential equation is

$$\varepsilon_0 - I(t)(R_1 + R_2) = L\frac{dI(t)}{dt}$$

Under the approximation that $\varepsilon_0 = 0$, the equation is

$$-I(t)(R_1 + R_2) = L\frac{dI(t)}{dt}$$

The solution with the initial condition $I(0) = I_0$ is given by

$$I(t) = I_0 \exp(-\frac{(R_1 + R_2)}{L} t)$$

(c) Using your results from (b), find the value of the total emf around the circuit (which from Faraday's law is $-LdI/dt$) just after the switch is opened. Is your assumption in (b) that ε_0 could be ignored for times just after the switch is opened OK?

$$\varepsilon = -L \left.\frac{dI(t)}{dt}\right|_{t=0} = I_0 (R_1 + R_2)$$

Since $I_0 = \dfrac{\varepsilon_0}{R_1}$,

$$\varepsilon = \frac{\varepsilon_0}{R_1}(R_1 + R_2) = \left(1 + \frac{R_2}{R_1}\right)\varepsilon_0 >> \varepsilon_0 \quad (\because R_2 >> R_1)$$

Thus, the assumption that ε_0 could be ignored for times just after the switch is open is OK.

(d) What is the magnitude of the potential drop across the resistor R_2 at times $t > 0$, just after the switch is opened? Express your answers in terms of ε_0, R_1, and R_2. How does the potential drop across R_2 just after $t = 0$ compare to the battery emf ε_0, if $R_2 = 100R_1$?

The potential drop across R_2 is given by

$$\Delta V_2 = \frac{R_2}{R_1 + R_2}\varepsilon = \left(\frac{R_2}{R_1 + R_2}\right)\left(1 + \frac{R_2}{R_1}\right)\varepsilon_0 = \frac{R_2}{R_1}\varepsilon_0$$

If $R_2 = 100R_1$,

$$\Delta V_2 = 100\,\varepsilon_0$$

This is why you have to open a switch in a circuit with a lot of energy stored in the magnetic field very carefully, or you end up very dead!!

Problem 7: *LC* Circuit

An inductor having inductance L and a capacitor having capacitance C are connected in series. The current in the circuit increase linearly in time as described by $I = Kt$. The capacitor initially has no charge. Determine

(a) the voltage across the inductor as a function of time,

The voltage across the inductor is

$$\varepsilon_L = -L\frac{dI}{dt} = -L\frac{d}{dt}(Kt) = -LK$$

(b) the voltage across the capacitor as a function of time, and

Using $I = \dfrac{dQ}{dt}$, the charge on the capacitor as a function of time may be obtained as

$$Q(t) = \int_0^t I dt' = \int_0^t Kt' dt' = \frac{1}{2}Kt^2$$

Thus, the voltage drop across the capacitor as a function of time is

$$\Delta V_C = -\frac{Q}{C} = -\frac{Kt^2}{2C}$$

(c) the time when the energy stored in the capacitor first exceeds that in the inductor.

The energies stored in the capacitor and the inductor are

$$U_C = \frac{1}{2}C(\Delta V_C)^2 = \frac{1}{2}C\left(-\frac{Kt^2}{2C}\right)^2 = \frac{K^2t^4}{8C}$$

$$U_L = \frac{1}{2}LI^2 = \frac{1}{2}L(Kt)^2 = \frac{1}{2}LK^2t^2$$

The two energies are equal when

$$\frac{K^2t'^4}{8C} = \frac{1}{2}LK^2t'^2 \quad \Rightarrow \quad t' = 2\sqrt{LC}$$

Therefore, $U_C > U_L$ when $t > t'$.

Problem 8: *LC* Circuit

(a) Initially, the capacitor in a series *LC* circuit is charged. A switch is closed, allowing the capacitor to discharge, and after time T the energy stored in the capacitor is one-fourth its initial value. Determine L if C and T are known.

The energy stored in the capacitor is given by

$$U_C(t) = \frac{Q(t)^2}{2C} = \frac{(Q_0 \cos \omega_0 t)^2}{2C} = \frac{Q_0^2}{2C} \cos^2 \omega_0 t$$

Thus,

$$\frac{U_C(T)}{U_C(0)} = \frac{\cos^2 \omega_0 T}{\cos^2 (0)} = \frac{\cos^2 \omega_0 T}{1} = \frac{1}{4} \quad \Rightarrow \cos \omega_0 T = \frac{1}{2}$$

which implies that $\omega_0 T = \frac{\pi}{3} \text{rad} = 60°$. Therefore, with $\omega_0 = \frac{1}{\sqrt{LC}}$, we obtain

$$T = \frac{\pi}{3\omega_0} = \frac{\pi}{3}\sqrt{LC} \quad \Rightarrow \quad L = \frac{1}{C}\left(\frac{3T}{\pi}\right)^2$$

(b) A capacitor in a series *LC* circuit has an initial charge Q_0 and is being discharged. The inductor is a solenoid with N turns. Find, in terms of L and C, the flux through each of the N turns in the coil at time t, when the charge on the capacitor is $Q(t)$.

We can do this two ways, either is acceptable. First,we can make the explicit assumption that $Q(t) = Q_0 \cos \omega_0 t$ and the total flux through the inductor is $LI = L\frac{dQ}{dt} = -L\omega_0 Q_0 \sin \omega_0 t$

Therefore the flux through one turn of the inductor at time t is $\Phi_{\text{one turn}} = -\frac{L\omega_0 Q_0}{N} \sin \omega_0 t$

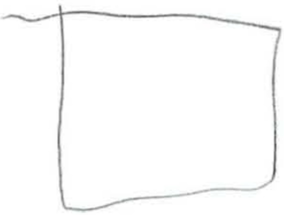
or in terms of L and C, $\Phi_{\text{one turn}} = -\sqrt{\frac{L}{C}}\frac{Q_0}{N} \sin \omega_0 t$. Or second, we can simply leave $Q(t)$ as an unspecified function of time and write (using the same arguments as above) that $\Phi_{\text{one turn}} = \frac{L}{N}\frac{dQ}{dt}$.

(c) An *LC* circuit consists of a 20.0-mH inductor and a 0.500-μF capacitor. If the maximum instantaneous current is 0.100 A, what is the greatest potential difference across the capacitor?

The greatest potential difference across the capacitor when $U_{C\max} = U_{L\max}$, or

$$\frac{1}{2}CV_{C\max}^2 = \frac{1}{2}LI_{\max}^2 \quad \Rightarrow \quad V_{C\max} = \sqrt{\frac{L}{C}}I_{\max} = \sqrt{\frac{(20.0\,\text{mH})}{(0.500\,\mu\text{F})}}\left(0.100\text{A}\right) = 20\text{ V}$$

Conductor (class 9)   5/16



charges are free to move

Conduct current

metals

$\vec{E}$ Field inside $= 0$

always no net field after infatesemal time

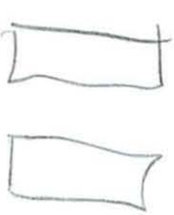
(p Capicatiance

- E field b/w

$C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}$

Energy

shells -remember inside + outside of shell is different!



$EA = \frac{q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon}$ $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$ ← makes more sense to me now

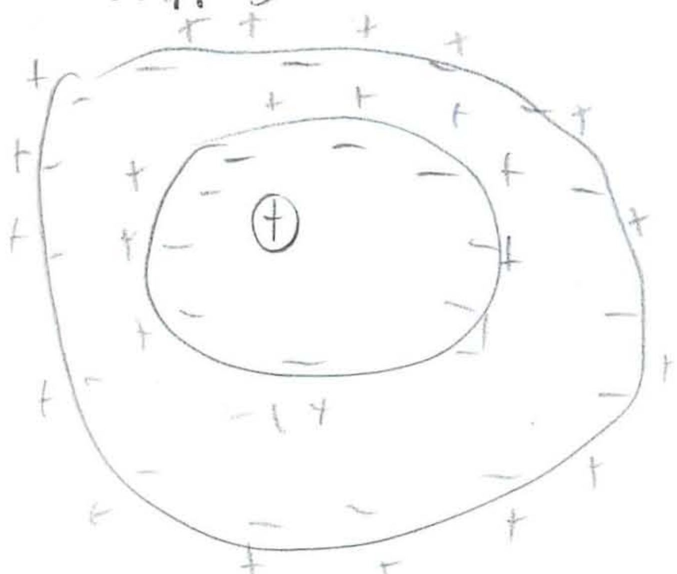
Spherical capicators

$$\Delta = -\int_{in}^{out} E \cdot ds$$

$$= -\int_a^b \frac{Q}{4\pi\epsilon_0 r^2} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$$

- could I do this on my ~~art~~ own?

Skipping now to 3/10 OH

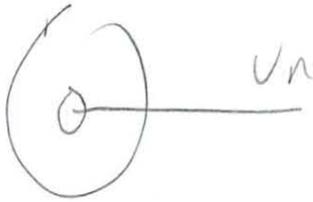


$\vec{E}$ field 0 since conducctor

V ~ here is what was confused on

$V = \int E \cdot ds$

- if can't calc E → then can't calc V

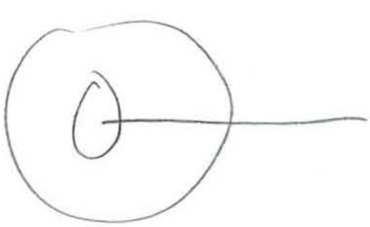


un calcuable

$\vec{E}\ \frac{dV}{ds}$

$V = I R$

so back to that proplem



define at ∞ $V=0$

So $\vec{E}$ fixed E same inside everywhere

$E \circ \hat{r}$

? distance

$\int$ integrate

then $b \to a$

$$\int_a^b \frac{1}{r^2} = \left(\frac{1}{b} - \frac{1}{a}\right)$$

? this makes so much more

sense now

learn the field lines thing

field

equipotential

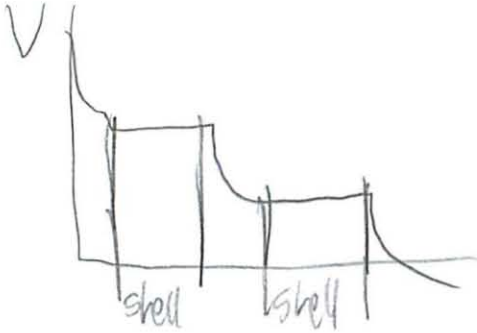
I'm getting confused on same stuff over + over again

potential same inside conductor

key question of experiment

what is meaning of potential difference?

$V(B) - V(A) = -\int E \cdot ds$



not to scale whatsoever

I thought I understood Faraday Ice pail well - but got that qu wrong

$I = I_0 \; e$

$I = I_0 e^{-t/\tau}$ - $\tau = RC$ τ right

$\boxed{\tau = RC}$

will never get this

-well ~~…~~ if take 10,03

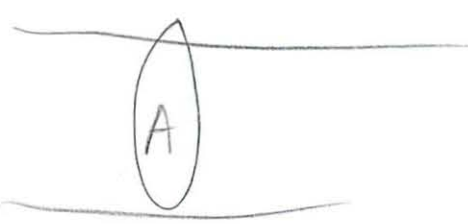
$I_0 (1 - e^{t/\tau})$ for declining

* When conductors are connected - potential equalizes $\frac{kqq}{r^2}$

Current density

$J = \frac{200}{10 \cdot 5}$

current going through area of $5 \cdot 10$
what sahar helped me w/



$J = \frac{I}{A}$

$I = \int J\,dA$

$W = \Delta PE$

$= q\Delta V$

I need q $W = It\Delta V$

q $I = \frac{q}{t}$

I def

Formulas not on formulas Sheet

~~Phal~~

$\vec{F} = \frac{kqQ}{r^2}$ electric attraction

$\vec{F} = I(\vec{w} \times \vec{B})$

- sliding block

$\Phi_E = \int E\,dA$

$\Phi_B = \int B\,dA = BA\cos\theta$

$\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \vec{r}}{r^2}$

$\tau = \mu_0 \times B$

$\quad n\hat{n}$

$\tau = IA\hat{n} \times B$

going through 1st pratice test again
realize what I did wrong on them :)

Redo Pratice Tests

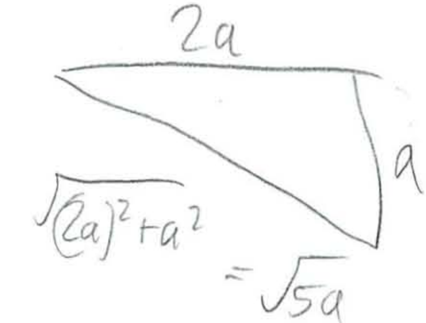
5/16

(2Q) (-Q)

$\downarrow p$

(+3Q) (-2Q)

$$k\frac{qq}{r^2}\hat{r}$$



↓ forgot to write

$$\frac{-kq}{(a)^2}\hat{\imath} + \frac{-2kq}{(a)^2}-\hat{\imath} + \frac{k2q}{\sqrt{5}a}\,a\hat{\imath}-2a\hat{\jmath} + \frac{k3q}{\sqrt{5}a}-a\hat{\imath}-2a\hat{\jmath}$$

↑ minus for ⊖ ←↓ dir's

~~cancle~~ diff magnitudes

try defined ←↓ as ⊕ and multiplied magnitude to direction

~~cancle~~ diff magnitudes don't cancle

$$\frac{k\cancel{5}q}{\cancel{5}a}-4\hat{\jmath} = \frac{-4kq}{a}\hat{\jmath}$$

---

try again

$$\frac{kq}{}\left(-\frac{1}{a^2}\hat{\imath}+\frac{2}{a2}\hat{\imath}+\frac{2}{\sqrt{5}}\hat{\imath}-\frac{4}{\sqrt{5}}\hat{\jmath}-\frac{3}{\sqrt{5}}\hat{\imath}-\frac{6}{\sqrt{5}}\hat{\jmath}\right)$$

~~Ra~~ ~~wh~~ what to do w/ a^2

they factor a^2 out

$\cancel{\frac{a}{a^2}}$

oh I wrote wrong too - mixed methods

So I see why they did a^3

$\boxed{\text{either}}$ $\frac{1}{a^3}a$ ↑ or $\frac{1}{a^2}$ ↑

I did the simple ones ~~w~~

and the hard ones $\frac{1}{a^2}a$

not a^3

~~Anan~~ And it was

$\sqrt{(2a)^2+a^2} = \sqrt{4a^2+a^2} = \sqrt{5a^2} = \sqrt{5}\,a$

was sloppy when writing

$\cancel{\frac{kq}{a^2}}$

$-\frac{kq}{a^2}\hat{i} + \frac{-2kq}{a^2}-\hat{i} + \frac{k2q}{(\sqrt{5}a)^3}\,a\hat{i} - \cancel{2}a\hat{j}$

also $2\hat{i}$ is in $\hat{i}$ not $\hat{j}$

$+\frac{k3q}{(\sqrt{5}a)^3} - a\hat{i} - 2a\hat{j}$

$\frac{kq}{a^2}\left(-\hat{i} + 2\hat{i} +\right.$

how to modify

$\frac{kq}{a^2}\left[\frac{4}{5\sqrt{5}}\hat{i} - \frac{2}{5\sqrt{5}}\hat{j}\right]$

$\frac{kq}{a^2}\left[\frac{6}{5\sqrt{5}}\hat{i} + \frac{3}{5\sqrt{5}}\hat{j}\right]$

So what did they do

$\frac{2}{(\sqrt{5}a)^3}\cdot a = \sqrt{5}^3 = 5^{3/2}\,\frac{\cancel{a}}{a^{\cancel{3}}} = \frac{2}{5^{3/2}} = \frac{2}{5\sqrt{5}}$

know $5^{3/2} = 5\sqrt{5}$

guess can't leave $5\sqrt{5}$

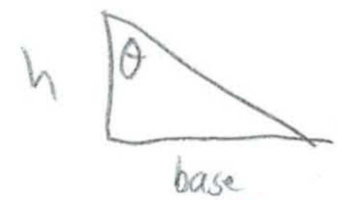
then I think I solved it
but this was supposed to be do whole
problem
- but that is to focus

Resistive Bar Pivot

A) $\Phi_B = BA$

↑ leave

$\theta = \omega t$



$A = \frac{1}{2} h \cdot \text{base}$

$\cancel{\sin}\ \theta = \frac{\text{base}}{h}$

tan

$h \cancel{\sin}(\omega t)$ tan

$\Phi = B \frac{dA}{dt}$

$\frac{1}{2} B h^2 \cancel{\sin}(\omega t)$

tan

duh wrong trig identity (stupid)

- remembered otherwise

b) Resistance

r ^ surface lenght

- frame has 0 resistance

$\cos \omega t = \frac{h}{r}$

$r \cos \omega t = h$

$r = \frac{h}{\cos \omega t}$

$\frac{r h}{\cos \omega t}$

✓ Bingo

Faradays

↓ carry over mistake

$$- \frac{d\Phi}{dt} = \frac{-d\left(B\frac{1}{2}h^2\, d\tan(\omega t)\right)}{dt}$$

$$= -B\frac{1}{2}h^2 \frac{d\tan(\omega t)}{dt}$$

use given rule

$$= -B\frac{1}{2}h\, \cancel{\omega}$$

And that was not right either

$$\mathcal{E} = \frac{-\frac{1}{2}Bh}{\cos^2(\omega t)} \rightarrow \text{simplify once more} = \frac{-Bh}{2\cos^2(\omega t)}$$

Also need to find current (read whole qu!)

$$I = \frac{\mathcal{E}}{R} = \frac{-Bh}{2\cos^2(\omega t)} \cdot \frac{\cancel{\cos(\omega t)}}{r\cancel{h}}$$

$$I = \frac{+Bh}{2\cos(\omega t)\, r}$$

✓ except sign

well think about direction seperatly + write

I will oppose changing flux
B getting bigger now
will want to get smaller
) clockwise ✓

d) Bar's resistive force

$$F = I(\vec{v} \times \vec{B})$$

dir of I

$\uparrow \times \odot = \rightarrow$

well

$\searrow \times \odot = \swarrow$ so <u>harder</u> to move ✓

opposite dir of μ

Magnitude

at right angle so multiply across

$I \cdot L \cdot B$

previous

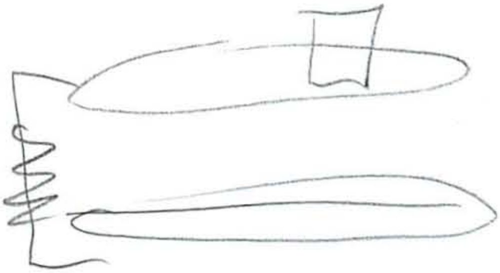
leave as variable

lenght of bar they say $\frac{h}{\cos(\omega t)}$

(same thing I found for l before – just bar)

4. Discharging capicator

1. Guass



$$\oint \underline{E} \cdot d\underline{a} = \frac{Q}{\epsilon_0}$$

$$EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{\epsilon_0}$$ ~~#~~ direction up obviously

b) Here is that strange inside surface always think it is for induction current

What is it actually asking for

Condiction current

- means normal (not induction) current?

$$J = \frac{I}{A} = \frac{-\frac{dQ}{dt}}{\pi a^2}$$

$$\text{current} = J \pi r^2 = -\frac{dQ}{dt}\frac{r^2}{a^2}$$

right only a part of the current (fraction) goes through

their explination of ~~potial~~ current density is weird

extra things not really needed

c) time rate of change of Φ_E

$$\Phi = E A$$

$$\Phi = \frac{1}{\epsilon_0} \frac{dQ}{dt} \cdot \pi r^2 = \frac{\pi r^2}{\pi \epsilon_0 a^2}\frac{dQ}{dt}$$

why did area not cancle out of E in their #a answer

Oh density = $\frac{Q}{\pi a^2}$ good job reading UV!

d) What is the $\int$ of magnetic field

$$\int B \cdot 2\pi r = \mu_0 \frac{\pi r^2}{\pi a^2} I$$

$$B = \frac{\mu_0 r^2 I}{2\pi \cancel{r} a^2} = \frac{\mu_0 r I}{2\pi a^2}$$

Oh opps no current inclosed

and I did not find displacement current

e) Yes goes to resistor heating

3. RC circuit

Better be much better now that remember what a capicator does

capicator like a short

So $\frac{1}{2} I$

$\mathcal{E} = I R_{eq}$

$I = \frac{\mathcal{E}}{R_{eq}}$

Find Req

$R + \frac{1}{R} + \frac{1}{R}$

$\frac{1}{R_{eq}} = \frac{R}{R} = \frac{R}{2}$

$R + \frac{R}{2} = \frac{3}{2} R$ ✓ actualy find on test

Put it together

$\frac{1}{2} \frac{\mathcal{E}}{\frac{3}{2} R} = \frac{1}{\cancel{2}} \frac{\cancel{2}}{3} \frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{3R}$ ✓ bingo

~~Now open S_1~~
~~diff eq~~

b) What is full charge on C

$V = \frac{Q}{C}$ $\quad CV = Q$

? voltage is ? w/h $\frac{\mathcal{E}}{2}$

? did we find C.
? leave in terms of

$V_C = V_R = RI_{tot} = \frac{\mathcal{E}}{2R} R$

? current down to
$I_{tot} = \frac{\mathcal{E}}{2R}$ (one branch)

think I am confused ~~as~~ as to what they want the answer in terms of

c) Now open S_1 diff eq

$\frac{Q}{C} - I2R = 0$

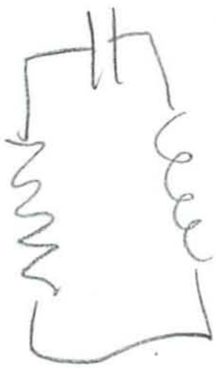
$\frac{Q}{C} = I2R \quad I = \frac{Q}{C2R} = \frac{Q}{2RC}$ ← τ time constant
remember

d) Now going pure inductor

Close S_1

No current → inductor resists

e) Now RLC damped



$$\frac{Q}{C} - L\frac{dI}{dt} - IR = 0$$

what do we write

$I = I_0 \sin(\omega t + \phi)$

$$I = \frac{Q}{RC} - \frac{L\,dI}{dt}$$

$$\frac{Q}{RC} - L\frac{d^2Q}{dt}$$

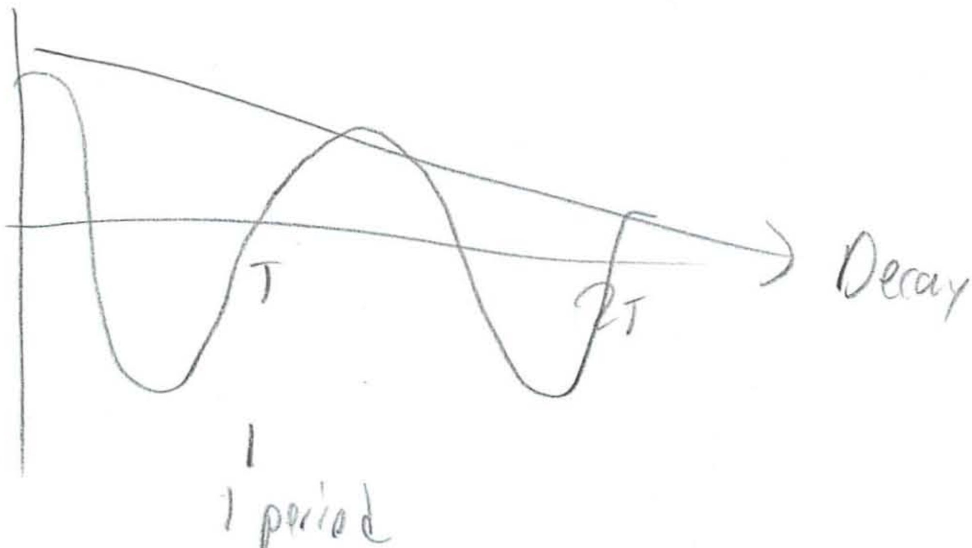
and what is freq of occilation

read whole qu

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

- always right?

f) Draw



4. Black box

m different so both

- explain more on test

b) what is current

$I = \frac{\cancel{R}\mathcal{E}}{R}$ ← driven, ← impedance $= \frac{1V}{R\sqrt{\left(1 + \left(\frac{X_L - X_C}{R}\right)^2\right.}}$ ← given

ϕ = what again?

$\tan\phi = \frac{X_L - X_C}{R}$ $= \frac{1V}{2\Omega\sqrt{1 + \tan^2\phi}}$

c) Find C, L

$\cancel{C = \frac{Q}{V}}$ Q

$V = \frac{Q}{C}$

$CV = Q$

$C = \frac{Q}{V}$ was right

$L = \frac{N\Phi_B}{I}$

but I think they found it a different way

V given

but Q how are you

I found

but Φ how find

key eq $\tan \phi R = X_L - X_C$

$\omega L - \frac{1}{\omega C}$

$\tan\phi = \sqrt{3}$

$-\sqrt{3}$

Then plug + chug

I would have never remembered that relation!

Now find angular freq current at max

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

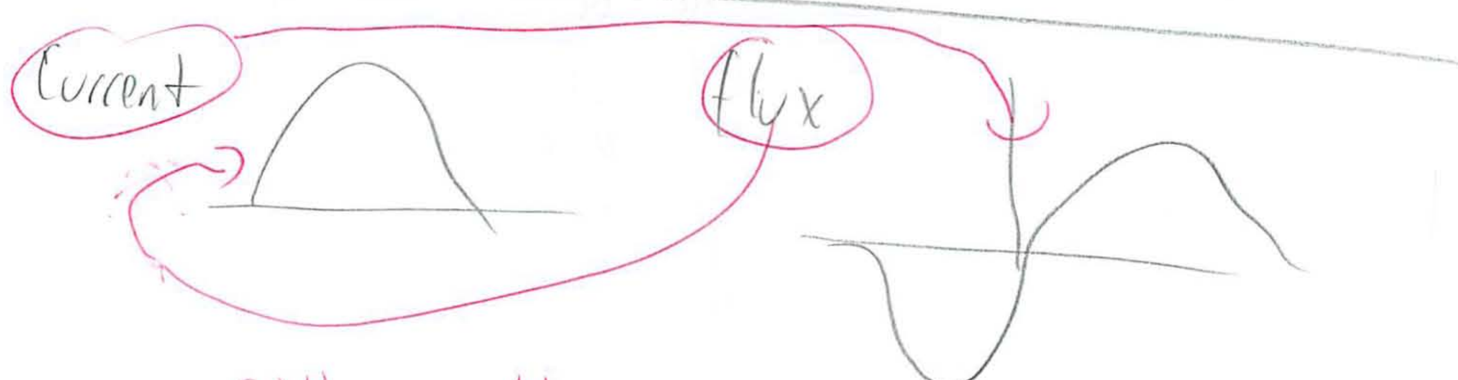
plug in

Remember ~~[illegible]~~ voltage drop = voltage $= V_L$

$= V_C$

$$V_L = L\frac{dI}{dt}$$

$V_C = \frac{Q}{C}$ now that have those can plug in



Still can't get this

Remember current alternates and I wonder how pre deals w/

there this one is colombs more closly

~~other~~ and evens out more

$$\frac{kqq}{r^2}\hat{r} + \frac{kqq}{r^2}-\hat{r}$$

now I get it

Generator

am I better at it now?

I max

$$I = \frac{\varepsilon}{R} = \frac{\frac{-d(BA)}{dt}}{R} = \frac{-B\frac{dA}{dt}}{R}$$

rotates around

max change in area

? idk where in turn

at 45°??

$$A = nS\sin(2\pi f t)$$

area

$$max = \underbrace{nS}$$

wait that is rotor

-guess just property of generator

Now anotor Circuit problem

in parallel don't forget

(this is kinda biased since I remember exect problems

$\omega = 0 \rightarrow \quad I_C = 0$ nothing goes through conductor

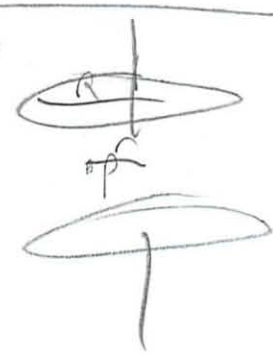
$I_R = I_L = \frac{V_0}{R}$

$\omega = \infty \quad I_L = 0$ nothing goes through inductor

$I_R = I_C = \frac{V_0}{R}$

---

6. Capicator



can I finally get this right?

$U = \iint u_E \, dV = u_E \cdot Vol$

on Formula sheet

$= \frac{1}{2} \epsilon_0 E^2 \pi R^2 d$ ✓

b) Now suppose E field $\uparrow$ w/ time

$\frac{dE}{dt}$

p is b/w

Find B

is $\frac{dE}{dt} = I$?

no $\frac{dQ}{dt}$

how are

$\oint B \cdot ds = \mu_0 \, I_{\cancel{enc}\,disp}$

$B \cdot 2\pi r = \mu_0 \frac{\pi r^2}{\pi R^2} I$

don't forget displacement current

do anyway

$$\boxed{I_{disp} = \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \frac{d(E\pi r^2)}{dt} = \epsilon_0 \pi r^2 \frac{dE}{dt}}$$

$B = \frac{1}{2} \mu_0 \epsilon_0 r \frac{dE}{dt}$

Poynting vector easy

magnitude + direction!

Energy $\iiint$ pointing S $\cdot\, dV$

should be same

---

7. Transmission lines

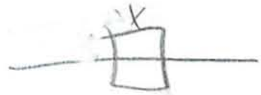
What I forgot on this was depth

$$J = \frac{I}{A_{surface}} \qquad \frac{x}{w} \leftarrow \text{width (3D)}$$

~~than~~ $C = \frac{Q}{V}$ ← q is just there
← and no v drop ??? → wrong

$\boxed{V = Ed}$

↑ find



$$\oint E \cdot ds = \frac{Q}{\epsilon_0} = E \cdot x = \frac{\cancel{x}/w}{\epsilon_0}$$

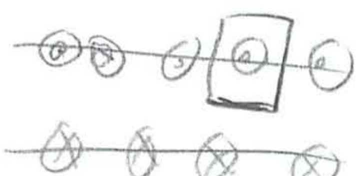
this is not that problem

$$= E = \frac{1}{w\,\epsilon_0}$$

$EA = \frac{Q_{en}}{\epsilon_0} = \frac{Q}{xy\,\epsilon_0}$ ← why did they leave it at Q – was my trick only for I?

Q cancles out later

b) $L = \frac{N\Phi}{I}$



$\Phi = BA$

remember to use core where drawing loop

$\oint B\cdot ds = \mu_0 I_{enc}$

here

$B\cdot x = \mu_0 \frac{x}{w} I$

$B = \frac{\mu_0 I}{w}$ ✓

$\Phi = \frac{\mu_0 I}{w}\cdot xy$

$L = \frac{\mu_0 \cancel{I} xy N}{w \cancel{I}} = \frac{\mu_0 xy \cancel{N}}{w}$ forget this otherwise didn't work out

c) Now Connect what is $\vec{E}$ field

$\cancel{\#}\ V = Ed$

$E = \cancel{\#} = \cancel{\#}$ division error

$\frac{V}{d} = \frac{6}{d}$

d) Mag field

alreay found

Redid 2 ~~a~~ pratice tests - should I try a third - or would fustrate me more?

-Read over quickly

5/16

Physics 8.02 **Final Exam** **Spring 2005**

Some (possibly useful) Relations:

$$\oint\!\!\oint_{\substack{closed\\surface}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{Q_{enclosed}}{\varepsilon_o}$$

$$\oint_{\substack{closed\\path}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\iint_{\substack{open\\surface}} \vec{\mathbf{B}}_{total}\cdot d\vec{\mathbf{A}}$$

$$= -\frac{d}{dt}\iint_{\substack{open\\surface}} \left[\vec{\mathbf{B}}_{external} + \vec{\mathbf{B}}_{self}\right]\cdot d\vec{\mathbf{A}}$$

$$\oint\!\!\oint_{\substack{closed\\surface}} \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}} = 0$$

$$\oint_{\substack{closed\\path}} \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \mu_0 I_{thru} + \mu_0\varepsilon_o \frac{d}{dt}\Phi_E$$

$$where \quad \Phi_E = \iint_{\substack{open\\surface}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}}$$

$$u_{elec} = \frac{1}{2}\varepsilon_0 E^2$$

$$u_{mag} = \frac{1}{2\mu_0}B^2$$

$$\vec{\mathbf{B}} = \int_{source} d\vec{\mathbf{B}} = \int_{source} \frac{\mu_0}{4\pi}\frac{Id\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$$

$$\vec{\mathbf{F}}_q = q\vec{\mathbf{E}} + q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$$

$$d\vec{\mathbf{F}} = I\, d\vec{\mathbf{s}}\times\vec{\mathbf{B}}$$

$$\mathbf{d}\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{\mathbf{r}}$$

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\int_V \frac{dq}{r^2}\hat{\mathbf{r}}$$

The electric potential at point P_2 minus that at point P_1 is given by

$$V(P_2) - V(P_1) = -\int_{P_1}^{P_2} \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}$$

$$\Delta V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{i=N}\frac{q_i}{r_{i,P}}$$

$$\Delta V = IR$$

$$P_{Joule\ Heating} = I^2 R = \Delta V^2 / R$$

$$|Q| = C|\Delta V|$$

$$U_{\text{capacitor}} = \frac{1}{2}C|\Delta V|^2 = \frac{Q^2}{2C}$$

$$N \iint_{one\ turn} \vec{\mathbf{B}}_{self\ field}\cdot d\vec{\mathbf{A}} = L_{self} I$$

$$U_{\text{inductor}} = \frac{1}{2}LI^2$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$c = \sqrt{\frac{1}{\mu_0\varepsilon_0}}$$

$$f = 1/T$$

$$\omega = 2\pi f = 2\pi/T$$

$$k = 2\pi/\lambda$$

$$c = \lambda/T = \lambda f = \omega/k$$

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}}$$

DOUBLE SLIT:

constructive:

$d\sin\theta = m\lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots$

destructive:

$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, ..$

SINGLE SLIT:

destructive:

$a\sin\theta = n\lambda, \quad n = \pm 1, \pm 2, \pm 3, \ldots$

Areas, Volumes, etc.:

1) The area of a circle of radius r is πr^2 Its circumference is $2\pi r$
2) The surface area of a sphere of radius r is $4\pi r^2$. Its volume is $\frac{4}{3}\pi r^3$
3) The area of the sides of a cylinder of radius r and height h is $2\pi r h$. Its volume is $\pi r^2 h$

USEFUL INTEGRALS:

$$\int_c^d dx = d - c$$

$$\int_c^d r\, dr = \frac{1}{2}\left(d^2 - c^2\right)$$

$$\int_c^d \left[\frac{1}{r}\right] dr = \ln\frac{d}{c}$$

$$\int_c^d \left[\frac{1}{r^2}\right] dr = \left(\frac{1}{c} - \frac{1}{d}\right)$$

8.02 Final Exam Spring 2005

FAMILY (last) NAME

GIVEN (first) NAME

Student ID Number

Your Section (check one): ___**MW 10 am** ____**MW 12 noon** ____**MW 2 pm**
___**TTh 10 am** ____**TTh 12 noon** ____**TTh 2 pm**

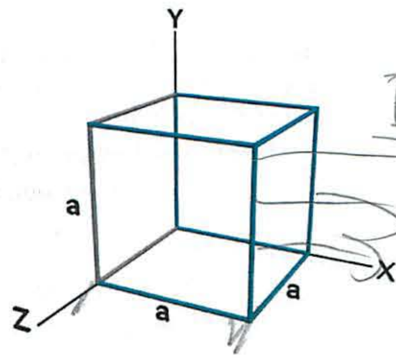
Your Group (e.g. 10A): _________

Problem	**Score**	**Grader**
1 (40 points)		
2 (40 points)		
3 (40 points)		
4 (40 points)		
5 (40 points)		
6 (40 points)		
TOTAL		

Problem 1: Eight Conceptual Questions (40 points out of 240 total). Circle your choice for the correct answer to the question.

Question A (5 points):

Consider a cubic volume with sides of length a oriented as shown in the figure. An electric field fills the space both inside and outside the cube and this electric field everywhere points in the +x direction. The magnitude of the field is independent of y and z but varies with x. The value of the electric field at $x = 0$ is $\mathbf{E}\big|_{x=0} = \hat{\mathbf{i}}\dfrac{3}{\varepsilon_o a^2}$ *Volts / meter* and the value of the electric field at $x = a$ is $\mathbf{E}\big|_{x=0} = \hat{\mathbf{i}}\dfrac{7}{\varepsilon_o a^2}$ *Volts / meter*. The total charge contained inside the cube is



$\vec{E}$

7-4 = 3 E=∫ Q

(a) -10 Coulombs
(b) -4 Coulombs
(c) 0 Coulombs
(d) +4 Coulombs
(e) +7 Coulombs
(f) Not enough information given to answer

? did math wrong

7-3=4 duh opps

$\int E \cdot dA = \dfrac{Q}{\varepsilon_0}$

$E = \dfrac{Q}{\varepsilon_0 A}$

$Q_0 = 3$

$Q_a = 7$

Question B (5 points):

Consider three equal charges, A, B, and C. Each one sits at the origin ($x = 0$) but in a different electric potential, as follows:

Charge A sits at $x = 0$ in a potential which is constant and identically zero
Charge B sits at $x = 0$ in a potential which is constant and non zero
Charge C sits at $x = 0$ in a linear potential (V proportional to x).

Which statement is true:

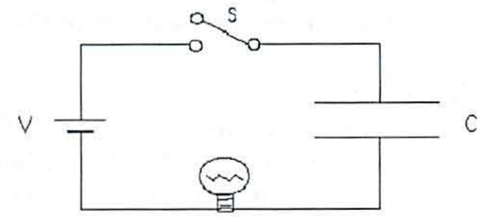
(a) None of the charges will accelerate
(b) Only B will accelerate
(c) Only C will accelerate
(d) All charges will accelerate, but B will have the largest acceleration
(e) All charges will accelerate, but C will have the largest acceleration

Potential does not matter?

only changing V causes it to move

fully read qu

Question C (5 points):
The circuit contains a battery, a capacitor, a bulb and a switch. The switch is initially open as shown in the diagram, and the capacitor is uncharged.

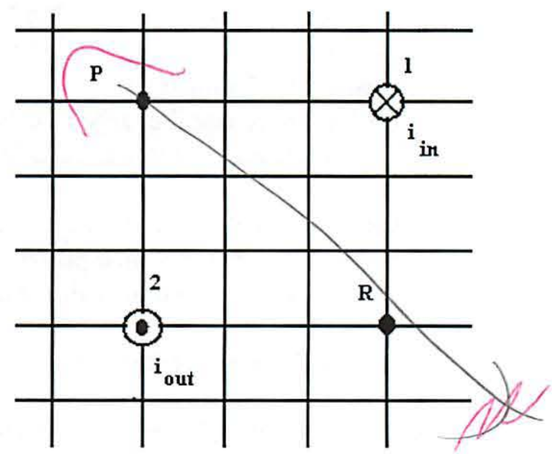


Which correctly describes what happens to the bulb when the switch is closed?

(a) The bulb is dim and remains dim.
(b) At first the bulb is dim and it gets brighter and brighter until its brightness levels off.
(c) The bulb is bright and remains bright.
(d) At first the bulb is bright and it gets dimmer and dimmer until it goes off.
(e) None of these is correct.

✓

Question D (5 points):
Wire 1 carries a current I flowing into the page, as shown in the diagram. Wire 2 has the same current I, but flowing out of the page. Which direction are the magnetic fields at positions P and R?



	P	R
(a)	↖	↖
(b)	↖	↘
(c)	↘	↖
(d)	↘	↘
(e)	None of these	

why

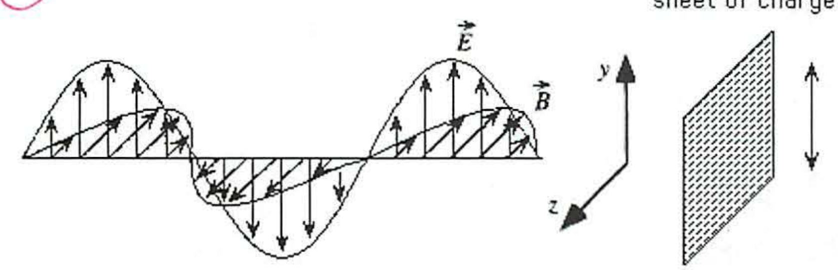
points

messed up symbols! gm

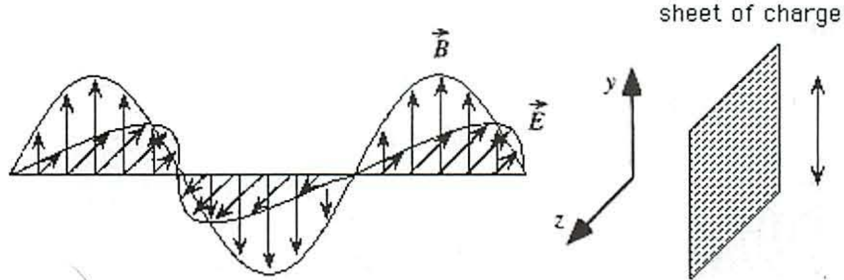
Question E (5 points):
An infinite sheet of positive charge in the *yz* plane (see figure below) is shaken up and down in the *y* direction. This shaking of the sheet generates a plane wave to the left of the sheet. Which of the following is the correct representation of the electromagnetic wave generated to the left of the sheet of charge as a result of this shaking.

(a)

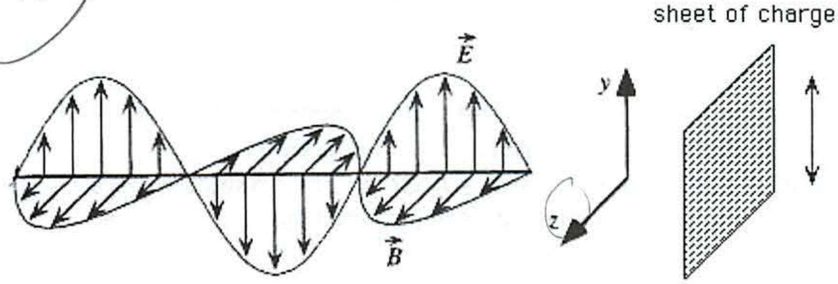
how in all world do know again?



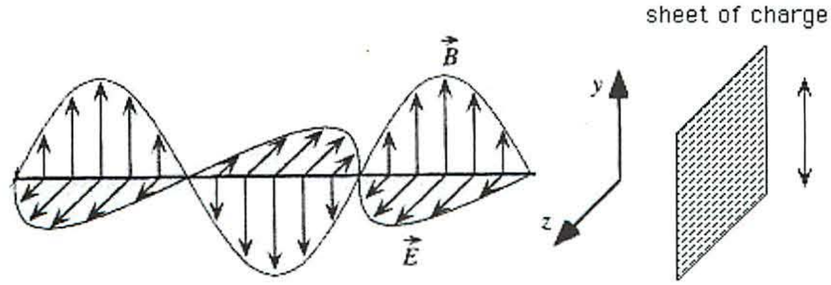
(b)



(c)



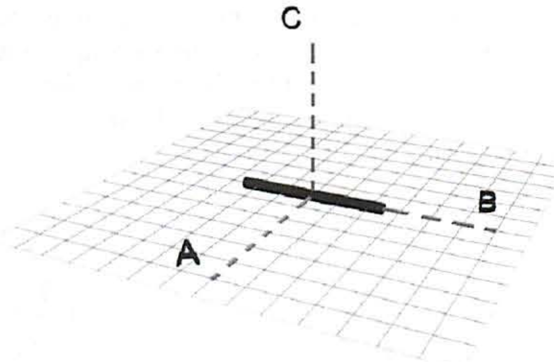
(e)



(e) None of these is correct.

Question F (5 points):

An antenna is oriented as shown and is emitting electric dipole radiation. The observations points A, B, and C are all located at the same distance from the center of the antenna, and are very far away. Let E_A be the positive amplitude of the radiation electric field at observation point A, and so on.

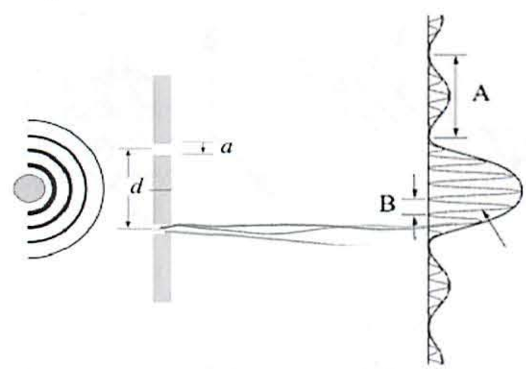


Which of the following is true?

(a) $E_A = E_C = E_B$
(b) $E_A > E_C = E_B$
(c) $E_A = E_C > E_B$
(d) $E_A < E_C = E_B$
(e) $E_A = E_C < E_B$

A = C > B

Question G (5 points):



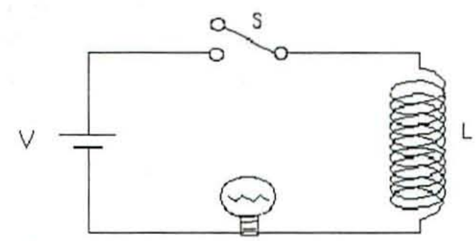
what was it again

Monochromatic light waves impinge on two long narrow apertures (slits) that are separated by a distance d. Each aperture has width a, with $a << d$. The resulting pattern on a screen far away is shown above. The distance A between the zeroes of the envelope (see figure) is determined by

(a) d
(b) d^2 / a
(c) a
(d) a^2 / d
(e) $\sqrt{ad}$

Check on this

Question H (5 points):
The circuit contains a battery, an inductor, a bulb and a switch. The switch is initially open as shown in the diagram.



Which correctly describes what happens to the bulb when the switch is closed?

(1) The bulb is dim and remains dim.
(2) At first the bulb is dim and it gets brighter and brighter until its brightness levels off at a constant level. ✓
(3) The bulb is bright and remains bright.
(4) At first the bulb is bright and it gets dimmer and dimmer until it goes off.
(5) None of these is correct.

Problem 2 (40 points out of 240 total):

An electromagnetic wave has a wave length λ of 3 meters and a frequency f of 10^8 Hertz (Hertz = cycle per second). The time averaged value of the Poynting flux vector is given by

$$\left\langle \vec{\mathbf{S}} \right\rangle_{\text{time averaged}} = -\hat{\mathbf{k}}\,\frac{cB_o^2}{2\mu_o}.$$

(a) In which direction does this wave propagate? Be sure to indicate the direction of propagation with a unit vector ($\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, or $\hat{\mathbf{k}}$) and an appropriate sign (+ or –). ***Briefly explain why you choose this direction***

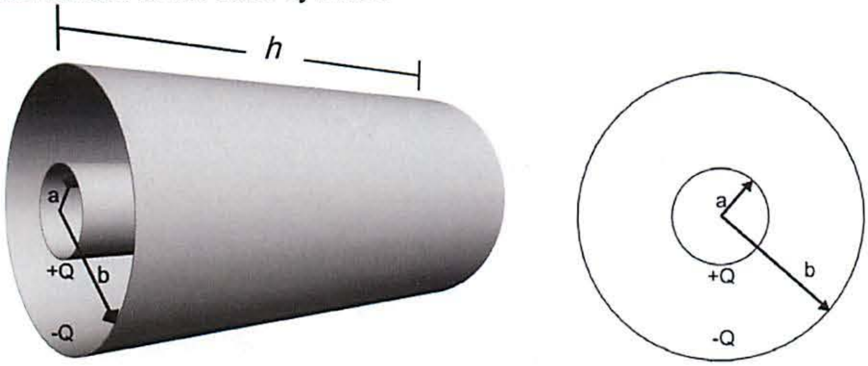
(b) The electric field of the wave is along the x-axis. Write a vector equation for the electric field of this wave, in terms of π, x, y, z, t, c, B_o, μ_o, and/or ε_0 (your equation should involve ***only*** these quantities, and numbers, but you do not have to use all of these in your answer). Is the equation you write for the electric field determined uniquely by the information you have been given? Explain.

(c) What is the corresponding magnetic field vector equation in terms of $\pi, x, y, z, t, c, B_o, \mu_o$, and/or ε_0 (your equation should involve ***only*** these quantities, and numbers, but you do not have to use all of these in your answer)?

(d) Suppose this wave is perfectly reflected by a conducting plane at $z = 0$. Write a vector equation for the electric field of the reflected wave, given the expression for the electric field you gave in (b).

(e) Write a vector equation for the magnetic field of this reflected wave, given the expression for the magnetic field you gave in (c).

Problem 3: (40 points out of 240 total)
Consider two long concentric hollow conducting cylinders. The inner cylinder has radius a, and the outer cylinder has radius b, and the length of both is h, with $h >> b$, as shown in the figures. The *inner* conducting cylinder carries a total charge $+Q$ spread uniformly on the outer part of the inner surface (giving an effective change per unit length of $\lambda = Q/h$), and the *outer* conducting cylinder carries a charge $-Q$ spread uniformly on the inner surface of the outer cylinder.

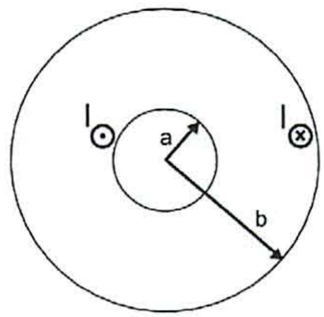


(a) Find the direction and magnitude of the electric field $\vec{\mathbf{E}}$ in the region $a < r < b$, ignoring end effects, in terms of $r, Q, \varepsilon_o, \pi, h, a,$ and b (your expression should involve ***only*** these quantities, but you do not have to use all of these in your answer). State which Maxwell equation you use (write the equation, do not just give the name) and show explicitly how you find this electric field.

(b) What is the potential difference $\Delta V = V_a - V_b$ between the inner and outer cylinder. Write your answer in terms of $Q, \varepsilon_o, \pi, h, a,$ and b (your expression should involve ***only*** these quantities, but you do not have to use all of these in your answer)

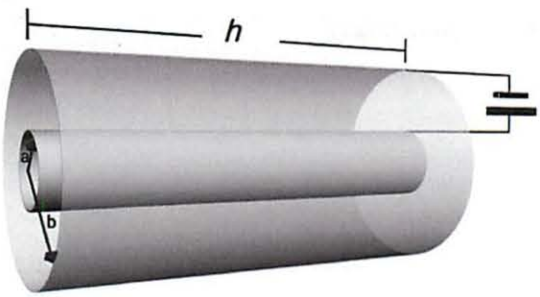
(c) Using your result (b), rewrite your expression for the electric field obtained in (a) in terms of r, ΔV, a, and b (your expression should involve ***all of these quantities and only these quantities***)

Now consider the same two long concentric hollow conducting cylinders. Suppose a current I is uniformly distributed over the surface of the inner conductor and flows *out of the page* on the inner conductor. The same current flows *into the page* on the outer conductor, and is also distributed uniformly over its surface.



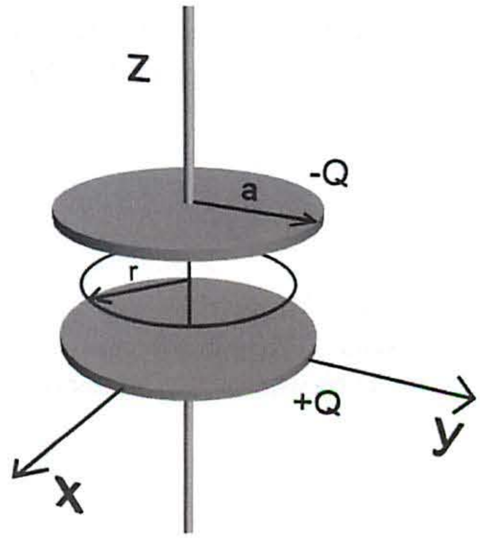
(d) Find the direction and magnitude of the magnetic field $\vec{\mathbf{B}}$ in the region $a < r < b$, ignoring end effects, in terms of $r, I, \mu_o, h, a,$ and b (your expression should involve *only* these quantities, but you do not have to use all of these in your answer). State which Maxwell equation you use (write the equation, do not just give the name) and show explicitly how you find this magnetic field.

(e) Now we combine the two cases above. That is, our conducting cylinders carry both current and charge exactly as described above. We do this by hooking up a battery with voltage ΔV as shown in the figure. What is the Poynting flux vector $\vec{\mathbf{S}}$ for $a < r < b$ in terms of $r, \Delta V, I, \pi, a,$ and b (your expression should involve ***all of these quantities and only these quantities***)?. Calculate the magnitude and clearly indicate the direction $\vec{\mathbf{S}}$ on the figure.



(f) By integrating $\vec{\mathbf{S}}$ over the appropriate surface, find the rate at which energy flows in the region $a < r < b$. What should you expect this value to be in terms of ΔV and the other quantities given above? You will get significant credit if you answer this last question correctly, even if you do not have the right expression for $\vec{\mathbf{S}}$ and/or do not do the integral correctly.

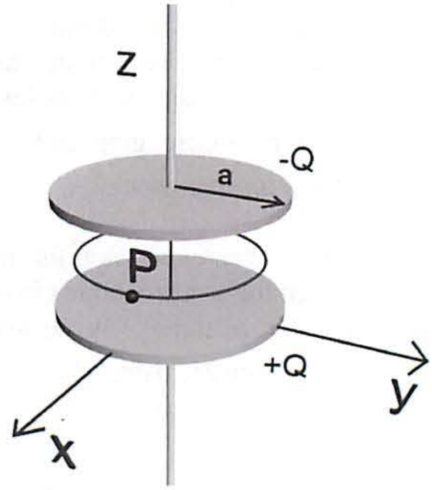
Problem 4 (40 points out of 240 total)
Consider a discharging capacitor made out of two identical circular conducting plates of radius a. One plate is placed on the xy plane centered at the origin, and the second is at a distance d up the z-axis at $z = d$ (see Figure). The bottom plate carries charge $+Q(t)$ and the top plate carries a charge $-Q(t)$. The capacitor is discharging, and $Q(t) = Q_o e^{-t/\tau}$, where τ is the time constant.



(a) Derive an expression for the electric field between the plates at time t in terms of $t, Q_o, a, \tau, \pi,$ and ε_o. Write down the Maxwell's equation you are using and show your steps in obtaining this expression.

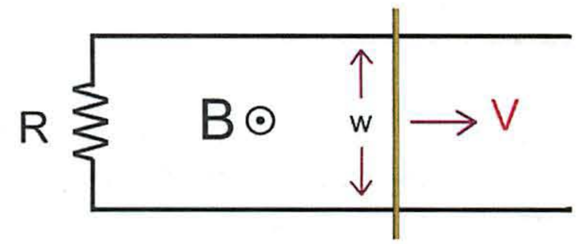
(b) Calculate the (time-dependent) displacement current $I_d(r,t)$ through a loop of radius $r < a$ as shown in the figure (the normal to this loop is in the positive z-direction). Give your answer in terms of $r, t, Q_o, a, \tau, \pi,$ and ε_o. Is this displacement current upward or downward?

(c) Use Ampere-Maxwell's law to calculate the induced magnetic field $\vec{\mathbf{B}}(r,t)$ inside the capacitor for $r < a$, in terms of r, t, Q_o, μ_o, a, τ, and π. Draw the direction of the magnetic field at point P on the figure below.



(d) In what direction is the Poynting flux at $r = a$? State in words the relationship between the surface integral of the Poynting flux over the sides of the capacitor and the electrostatic energy stored inside the capacitor.

Problem 5 (40 points out of 240 total): A conducting rod with zero resistance slides without friction on two parallel perfectly conducting rails. The distance between the rails is w. An external agent forces the rod to move at constant speed V to the right. Resistor R is connected across the ends of the rails to form a circuit, as shown. A constant magnetic field **B** is directed out of the page.



(a) What is the rate of change of the magnetic flux through the loop formed by the bar, the resistor, and the rails? In calculating this flux, ignore any self-magnetic field due to the induced current in the loop.

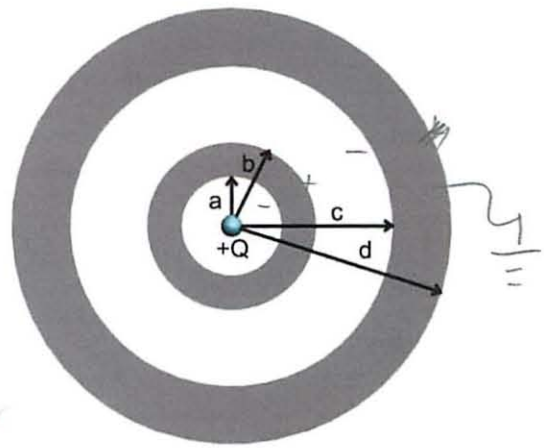
(b) Starting from a Maxwell equation (indicate which one), what is the current flowing through the resistor R? Gives its magnitude and indicate its direction on the figure. Why did you choose this direction for the current?

(c) What is the magnitude and direction of the magnetic force that is exerted on the sliding rod?

(d) If the external agent moving the rod only has to provide enough force to counterbalance the magnetic force, show that the rate at which the external agent does work ($\vec{\mathbf{F}}_{agent} \cdot \vec{\mathbf{V}}$) is equal to the rate at which energy is being dissipated in the resistor.

Problem 6 (40 points out of 240 total):
Consider two nested, spherical, conducing shells. The first conducting shell has inner radius a and outer radius b. The second conducting shell has inner radius c and outer radius d. The outer surface of the outer conductor is tied to ground, which means that it can bring in as much positive or negative charge as needed in order to make the potential of the outer surface of the outer conductor zero, the same as the potential at $r =$ *infinity*.

I. The inner conductor is initially uncharged. A charge $+Q$ is then fixed at the origin (see figure).



(a) What is the net charge in each of the following regions?

Inner surface of inner conductor: $-$ ✓

Interior of the inner conductor: 0 ✓

Outer surface of the inner conductor: + ✓

Inner surface of the outer conductor: $-$ ✓

Interior of outer conductor: 0 ✓

Outer surface of the outer conductor: 0 ✓

(b) What is the electric field in the following regions. Write your answers in terms of $r, Q, \varepsilon_o, \pi, a, b, c$ and d (your expression should involve ***only*** these quantities, but you do not have to use all of these in your answer)

$r < a$:

Guass cylinders

$\int E \cdot ds = \frac{Q}{\mu_0}$ Gadd q

$a < r < b$:

$b < r < c$:

(c) What is the potential V_c at the inner surface of the outer conductor?

travel distance

Zero

S

at ground

Space to write small so guess ()

(d) What is the potential V_b at the outer surface of the inner conductor?

clarify at ()

$$V_b - V_c = -\int_B^C E \cdot ds$$

$$\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{b} - \frac{1}{c}\right]$$

study more

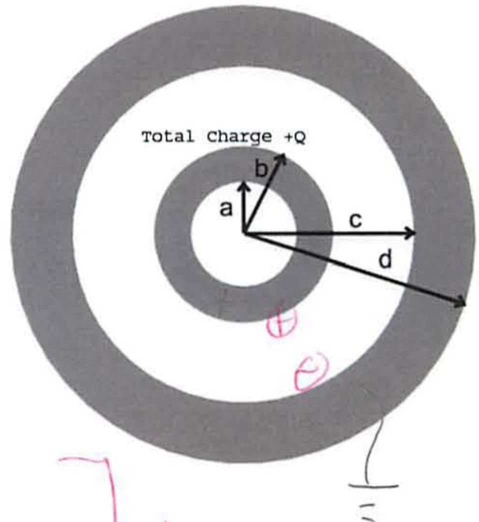
(d) What is the potential V_a at the inner surface of the inner conductor (you may use V_b in your answer)?

S1

same since conductor

Same as above

II. Now the charge $+Q$ is moved from the origin and placed on the inner conductor, and allowed to redistribute itself on that conductor. The outside of the outer conductor is still maintained at zero potential.



(e) What now is the net charge in each of the following regions?

Inner surface of inner conductor: 0

Interior of the inner conductor: 0

Outer surface of the inner conductor:

0 +

Inner surface of the outer conductor: 0 −

two unsure about inkl'g

? that is still there

So this is here

Interior of outer conductor: 0

Outer surface of the outer conductor: 0

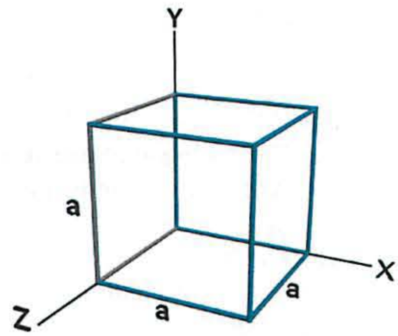
Solutions

Problem 1: Eight Conceptual Questions (40 points out of 240 total). Circle your choice for the correct answer to the question.

Question A (5 points):

Consider a cubic volume with sides of length a oriented as shown in the figure. An electric field fills the space both inside and outside the cube and this electric field everywhere points in the +x direction. The magnitude of the field is independent of y and z but varies with x. The value of the electric field at $x = 0$ is

$\mathbf{E}\big|_{x=0} = \hat{\mathbf{i}}\dfrac{3}{\varepsilon_o a^2}$ *Volts / meter* and the value of the electric field at $x = a$ is $\mathbf{E}\big|_{x=0} = \hat{\mathbf{i}}\dfrac{7}{\varepsilon_o a^2}$ *Volts / meter*. The total charge contained inside the cube is



(a) -10 Coulombs
(b) -4 Coulombs
(c) 0 Coulombs
(d) +4 Coulombs **CORRECT**
(e) +7 Coulombs
(f) Not enough information given to answer

Question B (5 points):

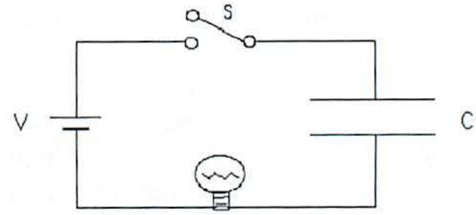
Consider three equal charges, A, B, and C. Each one sits at the origin ($x = 0$) but in a different electric potential, as follows:

Charge A sits at $x = 0$ in a potential which is constant and identically zero
Charge B sits at $x = 0$ in a potential which is constant and non zero
Charge C sits at $x = 0$ in a linear potential (V proportional to x).

Which statement is true:

(a) None of the charges will accelerate
(b) Only B will accelerate
(c) Only C will accelerate **CORRECT**
(d) All charges will accelerate, but B will have the largest acceleration
(e) All charges will accelerate, but C will have the largest acceleration

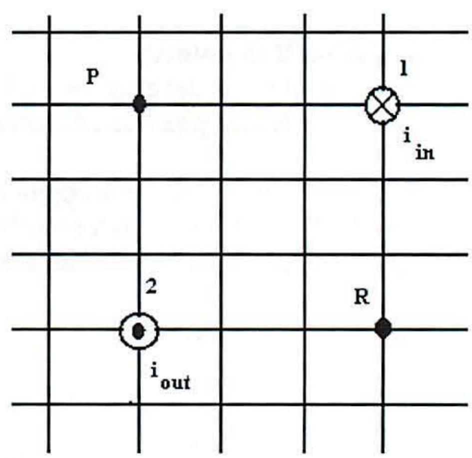
Question C (5 points):
The circuit contains a battery, a capacitor, a bulb and a switch. The switch is initially open as shown in the diagram, and the capacitor is uncharged.



Which correctly describes what happens to the bulb when the switch is closed?

(a) The bulb is dim and remains dim.
(b) At first the bulb is dim and it gets brighter and brighter until its brightness levels off.
(c) The bulb is bright and remains bright.
(d) At first the bulb is bright and it gets dimmer and dimmer until it goes off. **CORRECT**
(e) None of these is correct.

Question D (5 points):
Wire 1 carries a current *I* flowing into the page, as shown in the diagram. Wire 2 has the same current *I*, but flowing out of the page. Which direction are the magnetic fields at positions P and R?



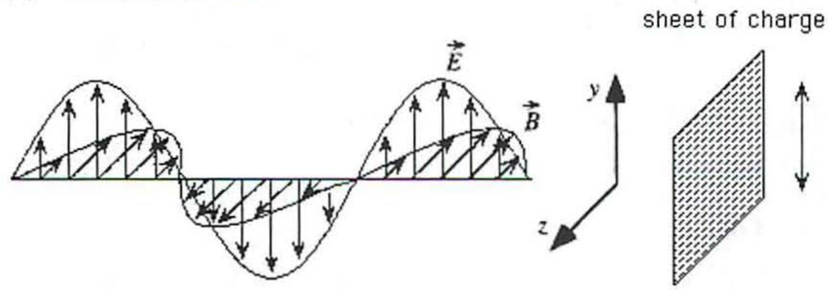
	P	R
(a)	↖	↖
(b)	↖	↘
(c)	↘	↖
(d)	↘	↘
(e)	None of these	

(a) CORRECT

Question E (5 points):

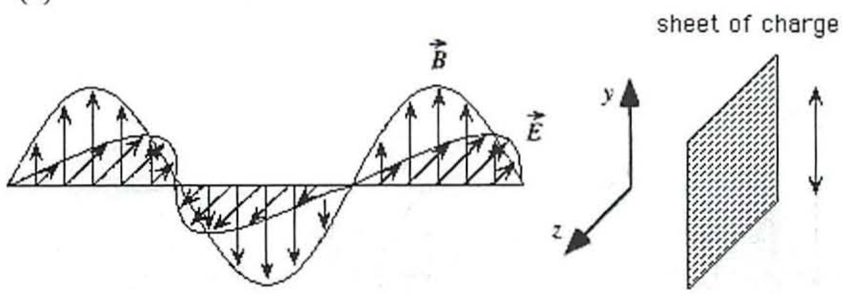
An infinite sheet of positive charge in the yz plane (see figure below) is shaken up and down in the y direction. This shaking of the sheet generates a plane wave to the left of the sheet. Which of the following is the correct representation of the electromagnetic wave generated to the left of the sheet of charge as a result of this shaking.

(a) **CORRECT**



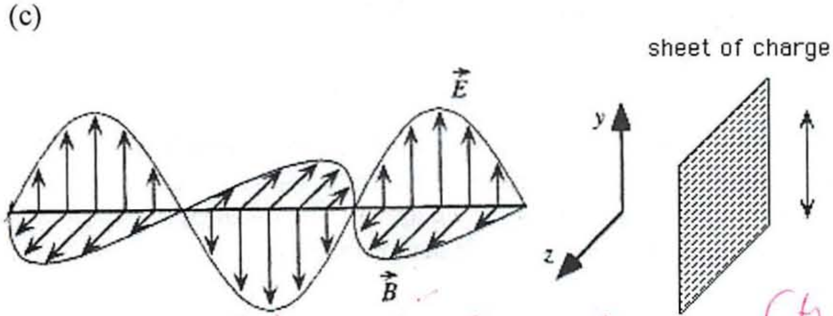
propagating ←

(b)



E×B false

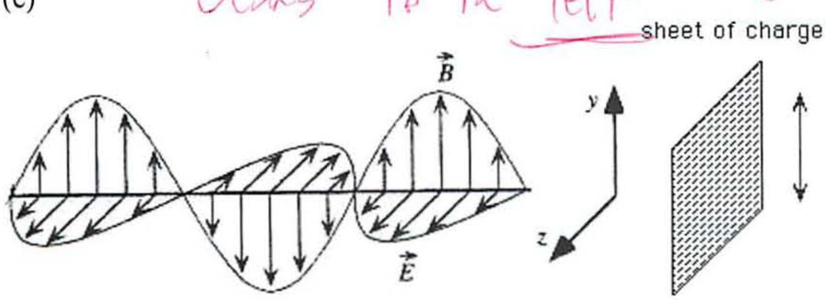
(c)



propagating →

(this is to right of sheet)

(e) asks to the left

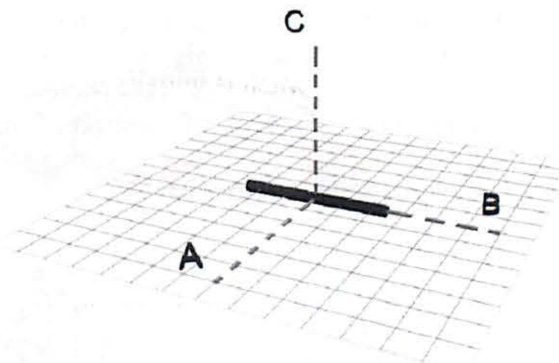


B not up

(e) None of these is correct.

Question F (5 points):

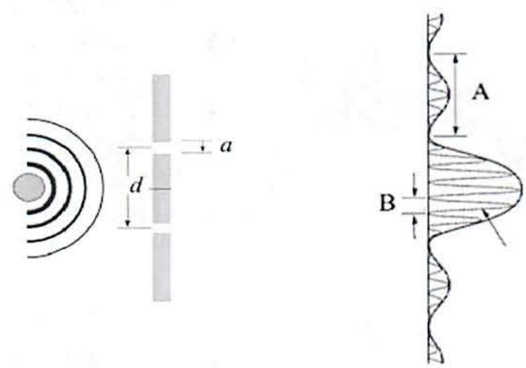
An antenna is oriented as shown and is emitting electric dipole radiation. The observations points A, B, and C are all located at the same distance from the center of the antenna, and are very far away. Let E_A be the positive amplitude of the radiation electric field at observation point A, and so on.



Which of the following is true?

(a) $E_A = E_C = E_B$
(b) $E_A > E_C = E_B$
(c) $E_A = E_C > E_B$ **CORRECT**
(d) $E_A < E_C = E_B$
(e) $E_A = E_C < E_B$

Question G (5 points):

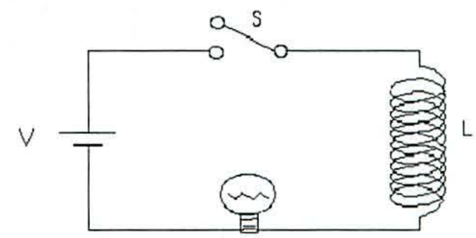


Monochromatic light waves impinge on two long narrow apertures (slits) that are separated by a distance d. Each aperture has width a, with $a << d$. The resulting pattern on a screen far away is shown above. The distance A between the zeroes of the envelope (see figure) is determined by

(a) d
(b) d^2 / a
(c) a **CORRECT**
(d) a^2 / d
(e) $\sqrt{a d}$

Question H (5 points):

The circuit contains a battery, an inductor, a bulb and a switch. The switch is initially open as shown in the diagram.



Which correctly describes what happens to the bulb when the switch is closed?

(1) The bulb is dim and remains dim.
(2) At first the bulb is dim and it gets brighter and brighter until its brightness levels off at a constant level. **CORRECT**
(3) The bulb is bright and remains bright.
(4) At first the bulb is bright and it gets dimmer and dimmer until it goes off.
(5) None of these is correct. **CORRECT AS WELL**

Problem 2 (40 points out of 240 total):

An electromagnetic wave has a wave length λ of 3 meters and a frequency f of 10^8 Hertz (Hertz = cycle per second). The time averaged value of the Poynting flux vector is given by

$$\left\langle \vec{\mathbf{S}} \right\rangle_{\text{time averaged}} = -\hat{\mathbf{k}}\,\frac{cB_o^2}{2\mu_o}.$$

(a) In which direction does this wave propagate? Be sure to indicate the direction of propagation with a unit vector ($\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, or $\hat{\mathbf{k}}$) and an appropriate sign (+ or –). ***Briefly explain why you choose this direction***

In the $-\hat{\mathbf{k}}$ direction. Since the energy flow is in this direction, the wave must propagate in this direction.

(b) The electric field of the wave is along the x-axis. Write a vector equation for the electric field of this wave, in terms of π, x, y, z, t, c, B_o, μ_o, and/or ε_0 (your equation should involve ***only*** these quantities, and numbers, but you do not have to use all of these in your answer). Is the equation you write for the electric field determined uniquely by the information you have been given? Explain.

$\vec{\mathbf{E}}(z,t) = \hat{\mathbf{i}}\, cB_o \cos(\frac{2\pi}{3}z + 2\pi \times 10^8 t)$ Only determined to a phase, i.e. we can add any phase angle to this expression.

(c) What is the corresponding magnetic field vector equation in terms of $\pi, x, y, z, t, c, B_o, \mu_o$, and/or ε_0 (your equation should involve ***only*** these quantities, and numbers, but you do not have to use all of these in your answer)?

$$\vec{\mathbf{B}}(z,t) = -\hat{\mathbf{j}}\, B_o \cos(\frac{2\pi}{3} z + 2\pi \times 10^8 t)$$

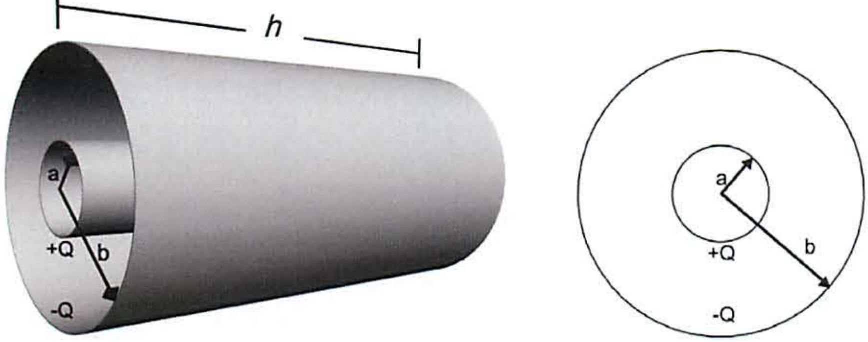
(d) Suppose this wave is perfectly reflected by a conducting plane at $z = 0$. Write a vector equation for the electric field of the reflected wave, given the expression for the electric field you gave in (b).

$$\vec{\mathbf{E}}(z,t) = -\hat{\mathbf{i}}\, cB_o \cos(\frac{2\pi}{3} z - 2\pi \times 10^8 t)$$

(e) Write a vector equation for the magnetic field of this reflected wave, given the expression for the magnetic field you gave in (c).

$$\vec{\mathbf{B}}(z,t) = -\hat{\mathbf{j}}\, B_o \cos(\frac{2\pi}{3} z - 2\pi \times 10^8 t)$$

Problem 3: (40 points out of 240 total)
Consider two long concentric hollow conducting cylinders. The inner cylinder has radius a, and the outer cylinder has radius b, and the length of both is h, with $h >> b$, as shown in the figures. The *inner* conducting cylinder carries a total charge $+Q$ spread uniformly on the outer part of the inner surface (giving an effective change per unit length of $\lambda = Q/h$), and the *outer* conducting cylinder carries a charge $-Q$ spread uniformly on the inner surface of the outer cylinder.



(a) Find the direction and magnitude of the electric field $\vec{\mathbf{E}}$ in the region $a < r < b$, ignoring end effects, in terms of $r, Q, \varepsilon_o, \pi, h, a,$ and b (your expression should involve ***only*** these quantities, but you do not have to use all of these in your answer). State which Maxwell equation you use (write the equation, do not just give the name) and show explicitly how you find this electric field.

$2\pi rhE = Q/\varepsilon_o \qquad \vec{\mathbf{E}} = \dfrac{Q}{2\pi rh\varepsilon_o}\hat{\mathbf{r}}$

cylinder

(b) What is the potential difference $\Delta V = V_a - V_b$ between the inner and outer cylinder. Write your answer in terms of $Q, \varepsilon_o, \pi, h, a,$ and b (your expression should involve ***only*** these quantities, but you do not have to use all of these in your answer)

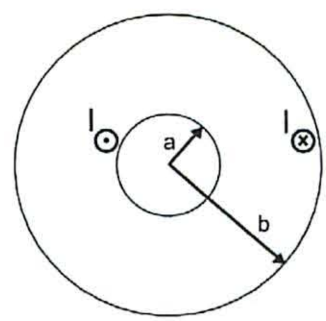
$$\Delta V = V_a - V_b = -\int_b^a \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\int_b^a \frac{Q}{2\pi rh\varepsilon_o} dr = \frac{Q}{2\pi h\varepsilon_o}\ln\left(\frac{b}{a}\right)$$

✓ remember

(c) Using your result (b), rewrite your expression for the electric field obtained in (a) in terms of r, ΔV, a, and b (your expression should involve ***all of these quantities and only these quantities***)

$$\vec{\mathbf{E}} = \hat{\mathbf{r}} \frac{\Delta V}{\ln\left(\dfrac{b}{a}\right)} \frac{1}{r}$$

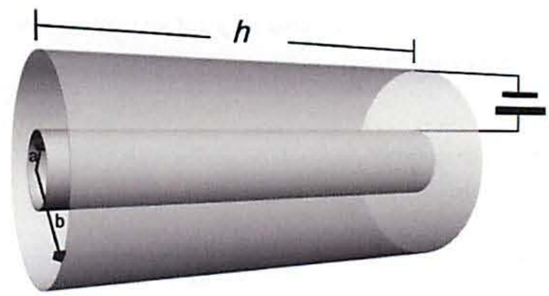
Now consider the same two long concentric hollow conducting cylinders. Suppose a current I is uniformly distributed over the surface of the inner conductor and flows *out of the page* on the inner conductor. The same current flows *into the page* on the outer conductor, and is also distributed uniformly over its surface.



(d) Find the direction and magnitude of the magnetic field $\vec{\mathbf{B}}$ in the region $a < r < b$, ignoring end effects, in terms of r, I, μ_o, h, a, and b (your expression should involve ***only*** these quantities, but you do not have to use all of these in your answer). State which Maxwell equation you use (write the equation, do not just give the name) and show explicitly how you find this magnetic field.

$$\oint_{\substack{closed \\ path}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_o I_{thru} \qquad 2\pi r B = \mu_o I \qquad B = \frac{\mu_o I}{2\pi r}$$ counterclockwise

(e) Now we combine the two cases above. That is, our conducting cylinders carry both current and charge exactly as described above. We do this by hooking up a battery with voltage ΔV as shown in the figure. What is the Poynting flux vector $\vec{\mathbf{S}}$ for $a < r < b$ in terms of $r, \Delta V, I, \pi, a,$ and b (your expression should involve ***all of these quantities and only these quantities***)?. Calculate the magnitude and clearly indicate the direction $\vec{\mathbf{S}}$ on the figure.

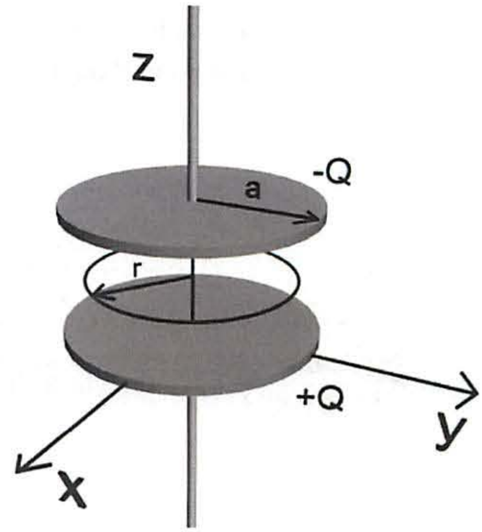


$$\vec{\mathbf{S}} = \frac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_o} \qquad \left|\vec{\mathbf{S}}\right| = \frac{1}{\mu_o} \frac{\Delta V}{\ln\left(\frac{b}{a}\right)} \frac{1}{r}\left[\frac{\mu_o I}{2\pi r}\right] = \frac{\Delta V\, I}{2\pi\, r^2 \ln\left(\frac{b}{a}\right)}$$ to the left, away from the battery

(f) By integrating $\vec{\mathbf{S}}$ over the appropriate surface, find the rate at which energy flows in the region $a < r < b$. What should you expect this value to be in terms of ΔV and the other quantities given above? You will get significant credit if you answer this last question correctly, even if you do not have the right expression for $\vec{\mathbf{S}}$ and/or do not do the integral correctly.

$$\int \vec{\mathbf{S}} \cdot d\vec{\mathbf{A}} = \int_a^b S\, 2\pi r\, dr = \int_a^b \frac{\Delta V\, I}{2\pi\, r^2 \ln\left(\frac{b}{a}\right)} 2\pi r\, dr = \frac{\Delta V\, I}{\ln\left(\frac{b}{a}\right)} \int_a^b \frac{dr}{r} = \Delta V\, I$$

Problem 4 (40 points out of 240 total)
Consider a discharging capacitor made out of two identical circular conducting plates of radius a. One plate is placed on the xy plane centered at the origin, and the second is at a distance d up the z-axis at $z = d$ (see Figure). The bottom plate carries charge $+Q(t)$ and the top plate carries a charge $-Q(t)$. The capacitor is discharging, and $Q(t) = Q_o e^{-t/\tau}$, where τ is the time constant.



(a) Derive an expression for the electric field between the plates at time t in terms of $t, Q_o, a, \tau, \pi,$ and ε_o. Write down the Maxwell's equation you are using and show your steps in obtaining this expression.

$$\vec{\mathbf{E}} = \hat{\mathbf{z}}\frac{Q_o}{\varepsilon_o \pi a^2}e^{-t/\tau}$$

(b) Calculate the (time-dependent) displacement current $I_d(r,t)$ through a loop of radius $r < a$ as shown in the figure (the normal to this loop is in the positive z-direction). Give your answer in terms of $r, t, Q_o, a, \tau, \pi,$ and ε_o. Is this displacement current upward or downward?

√ diff eq - are they going an
extra
step
you really
don't need

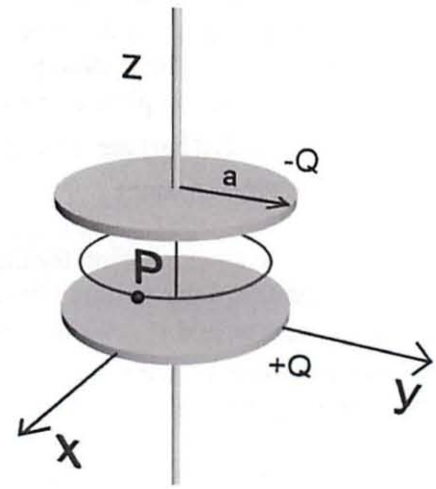
$I_d(r,t) = \pi r^2 \varepsilon_o \dfrac{dE}{dt} = -\dfrac{r^2}{a^2}\dfrac{Q_o}{\tau}e^{-t/\tau}$ where the minus sign indicates the current is downward

this problem again

(c) Use Ampere-Maxwell's law to calculate the induced magnetic field $\vec{\mathbf{B}}(r,t)$ inside the capacitor for $r < a$, in terms of r, t, Q_o, μ_o, a, τ, and π. Draw the direction of the magnetic field at point P on the figure below.

$$2\pi r B = \mu_o I_d(r,t) = -\mu_0 \frac{r^2}{a^2}\frac{Q_o}{\tau}e^{-t/\tau}$$

$$B = -\frac{\mu_o r}{2\pi a^2}\frac{Q_o}{\tau}e^{-t/\tau}$$ direction clockwise as viewed from above

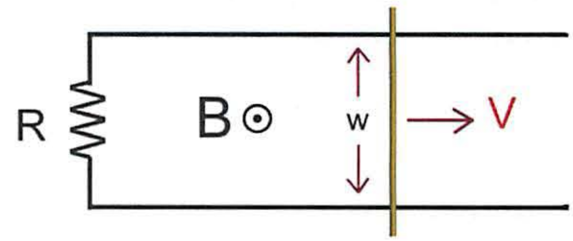


(d) In what direction is the Poynting flux at $r = a$? State in words the relationship between the surface integral of the Poynting flux over the sides of the capacitor and the electrostatic energy stored inside the capacitor.

The Poynting flux is outward. Its surface integral is equal to the time rate of change of the energy stored inside the capacitor.

Problem 5 (40 points out of 240 total)**:** A conducting rod with zero resistance slides without friction on two parallel perfectly conducting rails. The distance between the rails is w. An external agent forces the rod to move at constant speed V to the right. Resistor R is connected across the ends of the rails to form a circuit, as shown. A constant magnetic field **B** is directed out of the page.

Here is basic rod problem



(a) What is the rate of change of the magnetic flux through the loop formed by the bar, the resistor, and the rails? In calculating this flux, ignore any self-magnetic field due to the induced current in the loop.

$$\frac{d}{dt}\Phi_B = VwB$$

(b) Starting from a Maxwell equation (indicate which one), what is the current flowing through the resistor R? Gives its magnitude and indicate its direction on the figure. Why did you choose this direction for the current?

$IR = emf = -\frac{d}{dt}\Phi_B \quad I = -\frac{VwB}{R}$ direction is clockwise, so that the self flux tries as to counter the increasing flux as the loop expands

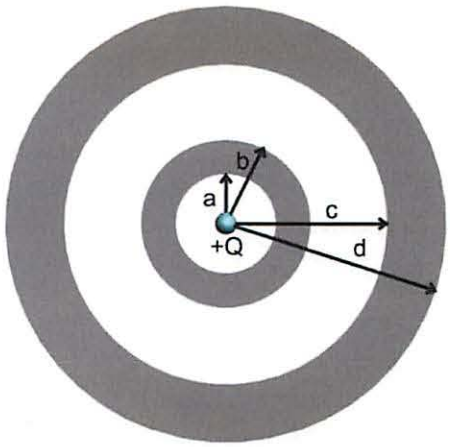
(c) What is the magnitude and direction of the magnetic force that is exerted on the sliding rod?

Direction to the left, $F = IwB = \frac{Vw^2B^2}{R}$

(d) If the external agent moving the rod only has to provide enough force to counterbalance the magnetic force, show that the rate at which the external agent does work ($\vec{\mathbf{F}}_{agent} \cdot \vec{\mathbf{V}}$) is equal to the rate at which energy is being dissipated in the resistor.

$$FV = \frac{V^2 w^2 B^2}{R} = I^2 R$$

Problem 6 (40 points out of 240 total):
Consider two nested, spherical, conducing shells. The first conducting shell has inner radius a and outer radius b. The second conducting shell has inner radius c and outer radius d. The outer surface of the outer conductor is tied to ground, which means that it can bring in as much positive or negative charge as needed in order to make the potential of the outer surface of the outer conductor zero, the same as the potential at $r =$ *infinity*.



I. The inner conductor is initially uncharged. A charge $+Q$ is then fixed at the origin (see figure).

(a) What is the net charge in each of the following regions?

Inner surface of inner conductor: ____-Q________

Interior of the inner conductor: _______0________

Outer surface of the inner conductor: _____+Q_________

Inner surface of the outer conductor: ________-Q____________

Interior of outer conductor: ________0__________

Outer surface of the outer conductor: _______0__________

(b) What is the electric field in the following regions. Write your answers in terms of $r, Q, \varepsilon_o, \pi, a, b, c$ and d (your expression should involve ***only*** these quantities, but you do not have to use all of these in your answer)

$r < a$:

$$\vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_o r^2}\hat{\mathbf{r}}$$

$a < r < b$:

$$\vec{\mathbf{E}} = 0\hat{\mathbf{r}}$$

$b < r < c$:

$$\vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_o r^2}\hat{\mathbf{r}}$$

(c) What is the potential V_c at the inner surface of the outer conductor?

zero

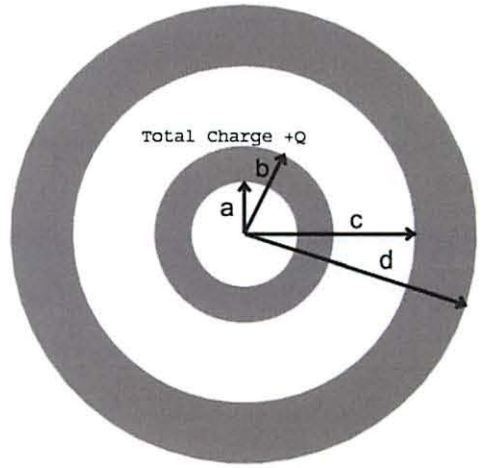
(d) What is the potential V_b at the outer surface of the inner conductor?

$$V_b - V_c = V_b = -\int_c^b \vec{\mathbf{E}}\cdot d\vec{\mathbf{r}} = -\int_c^b \frac{Qdr}{4\pi\varepsilon_o r^2} = \left.\frac{Q}{4\pi\varepsilon_o r}\right|_c^b = \frac{Q}{4\pi\varepsilon_o}\left[\frac{1}{b} - \frac{1}{c}\right]$$

(d) What is the potential V_a at the inner surface of the inner conductor (you may use V_b in your answer)?

The same as above, $\frac{Q}{4\pi\varepsilon_o}\left[\frac{1}{b} - \frac{1}{c}\right]$

II. Now the charge $+Q$ is moved from the origin and placed on the inner conductor, and allowed to redistribute itself on that conductor. The outside of the outer conductor is still maintained at zero potential.



(e) What now is the net charge in each of the following regions?

Inner surface of inner conductor: ______0_______

Interior of the inner conductor: ________0_______

Outer surface of the inner conductor:

____+Q__________

Inner surface of the outer conductor: _____-Q_______________

Interior of outer conductor: _______0__________

Outer surface of the outer conductor: _______0_________

So paging through ans seems easy
— all concepts did before

Just hoping remember calculations

Do this exam's PRS + go to bed

Did not do super awesome

— review ones missed