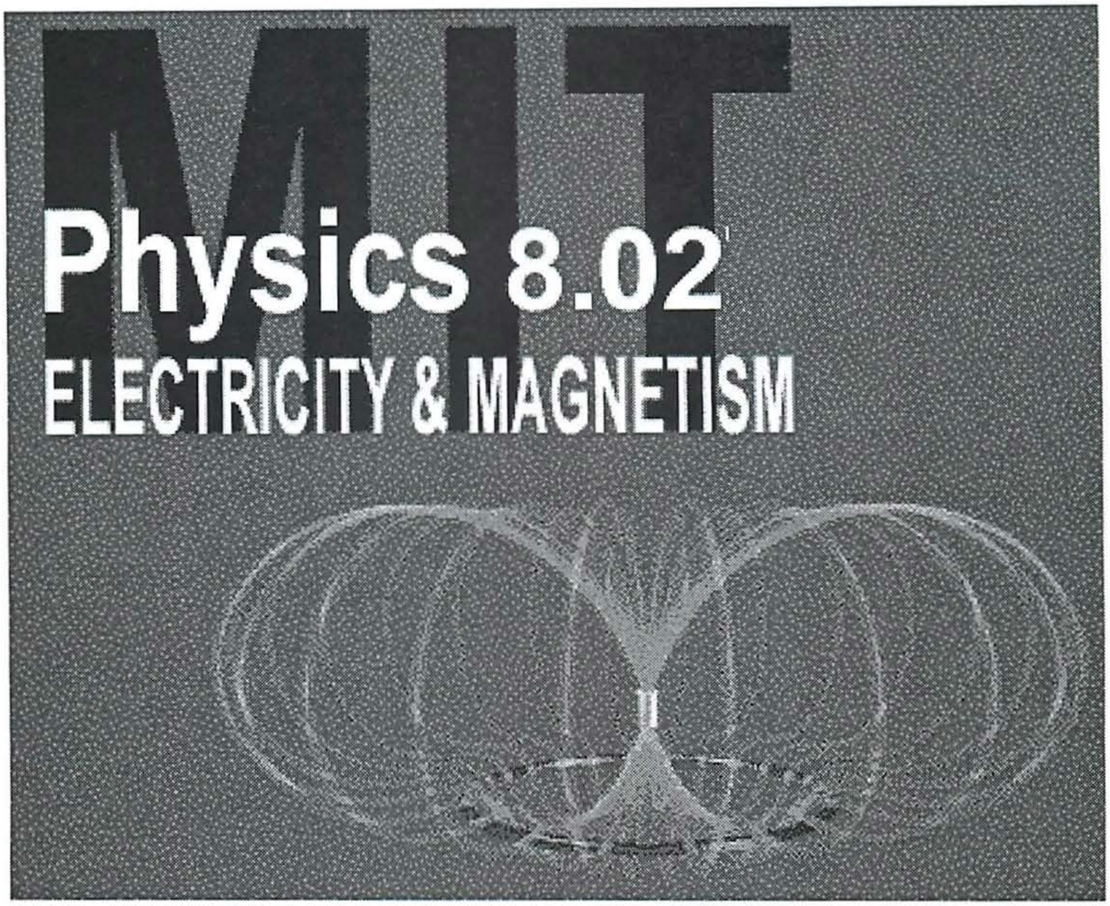


Spring 2010: Course Reader

Class Summaries
Presentation Notes
Problem Solving Sessions
Experiments

Website: http://web.mit.edu/8.02t/www/

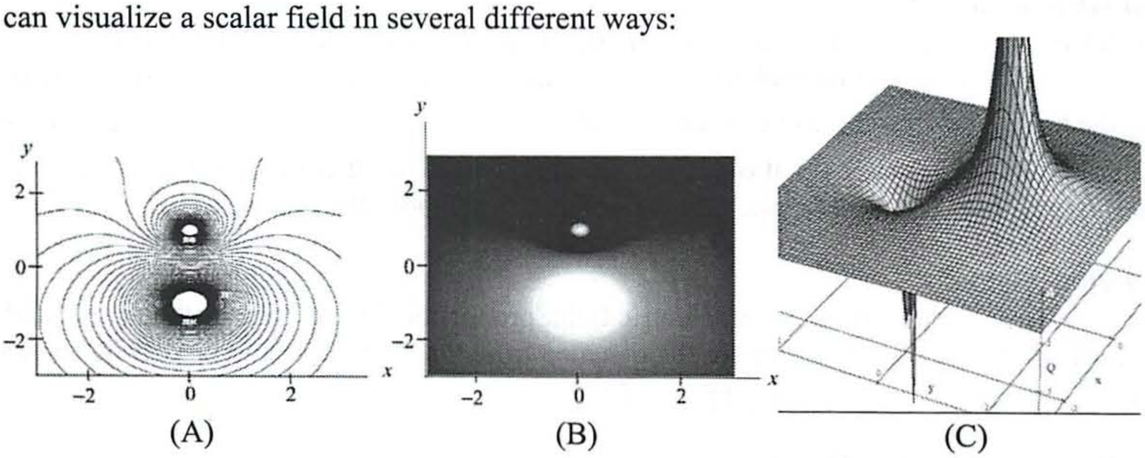
Topics: Introduction to TEAL; Fields; Review of Gravity; Electric Field
Related Reading:
Web Pages: Overview Section for test dates, cut lines, and grading guidelines
Course Notes: Sections 1.1 – 1.6; 1.8; Chapter 2

Topic Introduction

The focus of this course is the study of electricity and magnetism. Basically, this is the study of how charges interact with each other. We study these interactions using the concept of "fields" which are both *created* by and *felt* by charges. Today we introduce fields in general as mathematical objects, and consider gravity as our first "field." We then discuss how electric charges create electric fields and how those electric fields can in turn exert forces on other charges. The electric field is completely analogous to the gravitational field, where mass is replaced by electric charge, with the small exceptions that (1) charges can be either positive or negative while mass is always positive, and (2) while masses always attract, charges of the same sign repel (opposites attract).

Scalar Fields

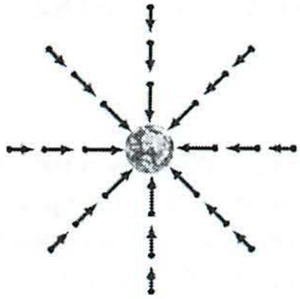
A *scalar field* is a function that gives us a single value of some variable for every point in space – for example, temperature as a function of position. We write a scalar field as a scalar function of position coordinates – e.g. $T(x, y, z)$, $T(r, \theta, \varphi)$, or, more generically, $T(\vec{\mathbf{r}})$. We can visualize a scalar field in several different ways:



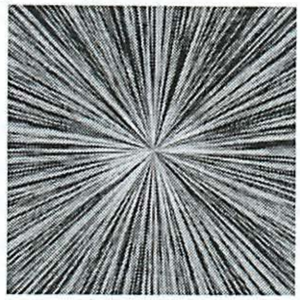
In these figures, the two dimensional function $\phi(x, y) = \dfrac{1}{\sqrt{x^2 + (y+d)^2}} - \dfrac{1/3}{\sqrt{x^2 + (y-d)^2}}$ has been represented in a (A) contour map (where each contour corresponds to locations yielding the same function value), a (B) color-coded map (where the function value is indicated by the color) and a (C) relief map (where the function value is represented by "height"). We will typically only attempt to represent functions of one or two spatial dimensions (these are 2D) – functions of three spatial dimensions are very difficult to represent.

Vector Fields

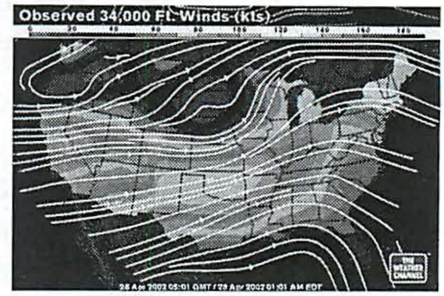
A vector is a quantity which has both a magnitude and a direction in space (such as velocity or force). A *vector field* is a function that assigns a vector value to every point in space – for example, wind speed as a function of position. We write a vector field as a vector function of position coordinates – e.g. $\vec{\mathbf{F}}(x, y, z)$ – and can also visualize it in several ways:



(A)



(B)



(C)

Here we show the force of gravity vector field in a 2D plane passing through the Earth, represented using a (A) vector diagram (where the field magnitude is indicated by the length of the vectors) and a (B) "grass seed" or "iron filing" texture. Although the texture representation does not indicate the absolute field direction (it could either be inward or outward) and doesn't show magnitude, it does an excellent job of showing directional details. We also will represent vector fields using (C) "field lines." A field line is a curve in space that is everywhere tangent to the vector field.

Gravitational Field

As a first example of a physical vector field, we recall the gravitational force between two masses. This force can be broken into two parts: the generation of a "gravitational field" **g** by the first mass, and the force that that field exerts on the second mass ($\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$). This way of thinking about forces – that objects create fields and that other objects then feel the effects of those fields – is a generic one that we will use throughout the course.

Electric Fields

Every charge creates around it an electric field, proportional to the size of the charge and decreasing as the inverse square of the distance from the charge. If another charge enters this electric field, it will feel a force $\left(\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}\right)$.

Important Equations

Force of gravitational attraction between two masses:	$\vec{\mathbf{F}}_g = -G\dfrac{Mm}{r^2}\hat{\mathbf{r}}$
Strength of gravitational field created by a mass *M*:	$\vec{\mathbf{g}} = \dfrac{\vec{\mathbf{F}}_g}{m} = -G\dfrac{M}{r^2}\hat{\mathbf{r}}$
Force on mass *m* sitting in gravitational field ***g***:	$\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$
Strength of electric field created by a charge *Q*:	$\vec{\mathbf{E}} = k_e\dfrac{Q}{r^2}\hat{\mathbf{r}}$
Force on charge *q* sitting in electric field ***E***:	$\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$

Welcome To

Physics 8.02T

http://web.mit.edu/8.02t/www

For now, please sit anywhere, 9 to a table

P01 - 1

Class 1: Outline

Hour 1:
- Why Physics?
- Course Overview
- Vector and Scalar Fields

Hour 2:
- Field Lines

P01 - 2

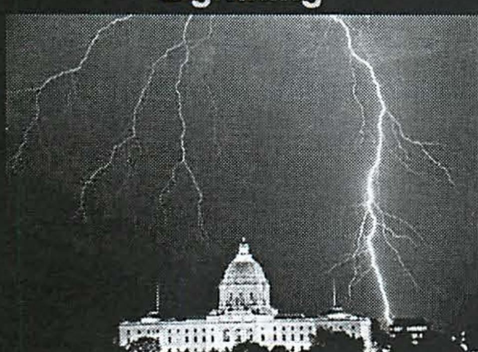
Why Physics?

P01 - 3

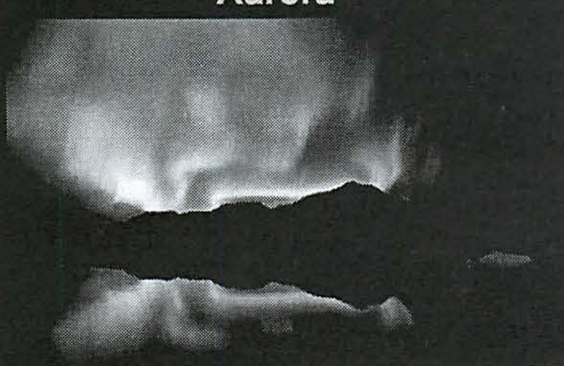
Why Study Physics?

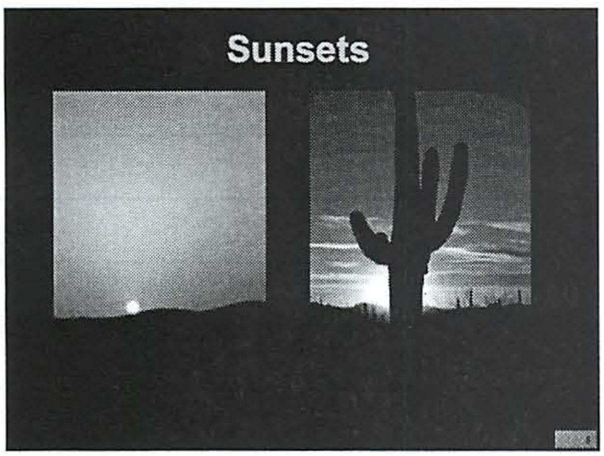
- Understand/appreciate nature

Lightning



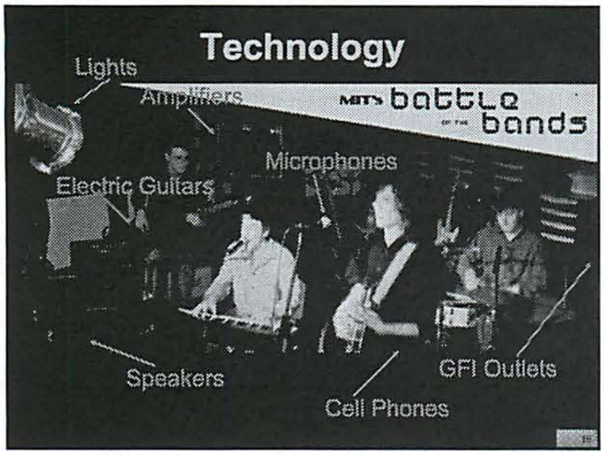
Aurora





Why Study Physics?

- Understand/Appreciate Nature
- Understand Technology



Why Study Physics?

- Understand/Appreciate Nature
- Understand Technology
- Learn to Solve Difficult Problems
- It's Required

Administrative Details
It's not that different from 8.01

all on web

go to web.mit.edu/8.02t/www/Student Info

Your Responsibilities

Before Class:
Read Course Notes and Summary (See current webpage for daily reading assignment)
In Class: (You must be present for credit)
Problem Solving, Desktop Experiments, Concept Tests
After Class:
Read Text, Review Visualizations
Mastering Physics (Due Sunday night at 10 pm
Homework (Due Tuesdays 9 pm),
Review Homework Solutions
Exams
3 Exams (45%) + Final (25%)

See "Overview/Grades" on http://web.mit.edu/8.02t

P01 - 13

Otherwise same

Psets this year
and Mastering Physics
and quizzes as well

Honesty Issues and Regrade Policy

Problem Sets:
The problems sets are to help you learn. You may work together BUT submit your own, uncopied work
In Class Assignments:
Must sign your own name to submitted work
Signing another's name is COD offense
Concept Questions:
Use only your own PRS device
Using another's PRS is COD offense
Regrade Policy:
You may submit any graded work for a regrade up to one week after the grades for that assignment have been posted

P01 - 14

Really wants people to
read pre-class

To Encourage Collaboration, Grades Are NOT Curved In 8.02:

	+		-
A	>=95	<95 & >=90	<90 & >=85
B	<85 & >=80	<80 & >=76	<76 & >=72
C	<72 & >=69	<69 & >=66	<66 & >=63
D		<63 & >=59	
F		<59	

See "Info: Grades" on http://web.mit.edu/8.02t

Grade Correction/Regrade Policy:
Grade changes **must** be requested within two weeks of the assignment due date (including 'no shows')

P01 - 15

don't wait till last min

his presentation does not align with this

Web Page
http://web.mit.edu/8.02t/

First Problem Set Due Tuesday February 10 9 PM in correct section box outside 32-082

Interactive On-Line Homework (Mastering Physics)

On-Line homework with hints and tutorials

Assignment due Sunday at 10 pm

Test review problems with hints

First Assignment due: Sun Feb 7 at 10 pm

Registering for Mastering Physics

- Go to http://www.masteringphysics.com
- Select MP for Young/Freedman if you already purchased that book.
- If you buy MP online, select MP stand alone
- Register with the access code.
- WRITE DOWN YOUR NAME AND PASSWORD
- Log on to Masteringphysics.com with your new name and password.
- The MIT zip code is 02139
- The class ID is MPMIT802SPRING2010

Course Reader

You MUST buy "8.02 Course Reader"

Copy Tech 11-004

And bring it with you to every class!

- Class Summaries
- Experiment Information

P01 -19

Textbook

Textbook:

"Introduction to E & M"

Liao, Dourmashkin, and Belcher

At the Coop and Online Version on website.

P01 -20

if never did E+M hard to read

-find another textbook

Common Questions & Answers

- Dysfunctional Group? • Tell Grad TA
- Must Miss Class? • Tell Grad TA
- Must Miss HW? • Tell Grad TA
- Must Miss Exam? • Tell admin. ASAP

Exam dates & times are online

Do NOT schedule early vacation departures, etc. without consulting these times!

Any Questions?

P01 -21

Physics is not Math…

P01 -22

…but we use concepts from 18.02

Gradients $\vec{\mathbf{E}} = -\nabla V$

Path Integrals $\Delta V \equiv -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$

Surface Integrals $\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$

Volume Integrals $Q = \iiint \rho \, dV$

P01 -23

PRS Questions:
Physics & Math Background

P01 -24

PRS: Math Background

Are you familiar with these concepts from vector calculus?

- 0% 1. I've never seen them before, and I am not so comfortable with math
- 0% 2. I've never seen them before, but I pick up new math concepts quickly
- 0% 3. I've seen them before, but definitely need some review
- 0% 4. I am comfortable with vector calculus

:20

PRS: Physics Experience

Have you taken a class in Electricity & Magnetism before?

- 0% 1. No, never
- 0% 2. Yes, here (8.02)
- 0% 3. Yes, here (8.02 TEAL)
- 0% 4. Yes, other college
- 0% 5. Yes, high school (regular)
- 0% 6. Yes, high school (AP)

:20

Don't Worry!

- For many this is new & I will introduce concepts before use (yell at me if not!)
- Concepts are VERY important –

Math introduction/review:

Th Feb ~~11~~ 4, 8:00-9:30 pm 32-082

Presentation slides are posted (top of Study Guide page)

P01-27

Same as 8.01

Integrals not hard
- just know the concepts

So what physics do we learn in 8.02 anyway????

What's the Physics?

8.01: Intro. to basic physics concepts: kinematics, force, momentum, energy, torque, angular momentum,...

How does matter interact?

Four Fundamental Forces:

Long range: Gravity (8.01 … Gen. Relativity)
Electromagnetic (8.02)

Both are inverse square forces. So all the results from gravitational forces can be easily adapted to electric forces

Short Range: Strong and Weak

Fields

8.02: Electricity and Magnetism

Also new way of thinking…

How do objects interact at a distance?

Fields We will learn about Electric & Magnetic Fields: how they are created & what they effect

Big Picture (Mathematical) Summary: Maxwell's Equations

$$\oint\!\!\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0} \qquad \oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\iint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

$$\oint\!\!\oint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0 \qquad \oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc} + \mu_0\varepsilon_0 \frac{d}{dt}\iint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

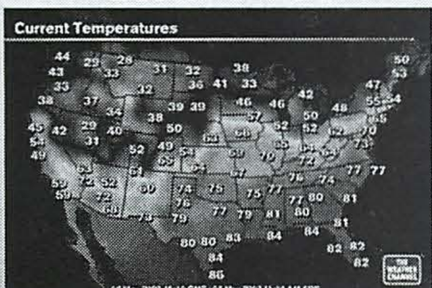
Lorentz Force: $\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$

All The class
the fundimental qu

Today: Fields
In General, then Gravitational & Electric

Review Vectors Analysis in Study Guide Online

Scalar Fields



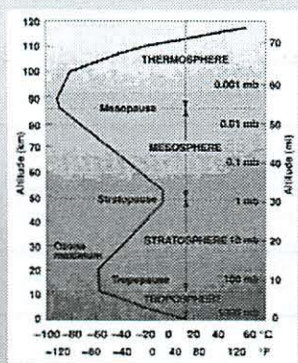
e.g. Temperature: Every location has associated value (number with units)

every point has a scalar (a #)

Scalar Fields - Contours

- Colors represent surface temperature
- Contour lines show constant temperatures

Fields are 3D



- T = T(x,y,z)
- Hard to visualize
→ Work in 2D

P01-34

Vector Fields

Vector (magnitude, direction) at every point in space

Example: Velocity vector field - jet stream

P01-35

Wind speed

lots of info to display

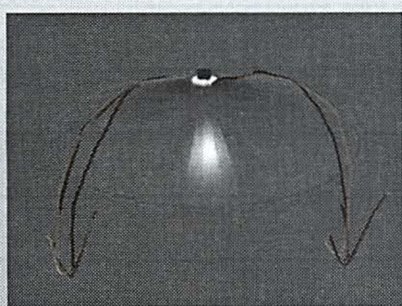
Vector Field Examples

Begin with Fluid Flow

P01-36

Vector Field Examples

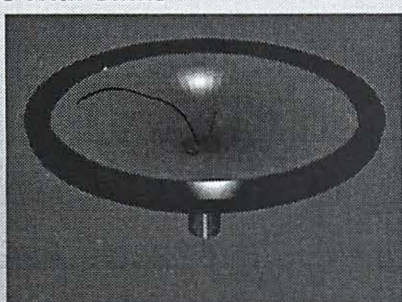
Flows With Sources



fluid flow

Vector Field Examples

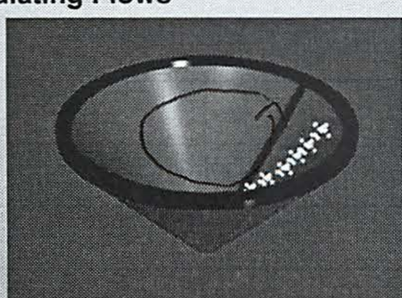
Flows With Sinks



what is source/sink
is big deal

Vector Field Examples

Circulating Flows



Visualizing Vector Fields: Three Methods

Vector Field Diagram

Arrows (different colors or length) in direction of field on uniform grid.

Field Lines

Lines tangent to field at every point along line

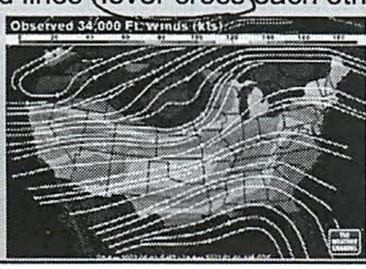
Grass Seeds

Textures with streaks parallel to field direction

All methods illustrated in Vector Field Diagram Java Applet

Vector Fields – Field Lines

- Direction of field line at any point is tangent to field at that point
- Field lines never cross each other



tangent to field

Or would be in 2 directions

can bump up againts each other

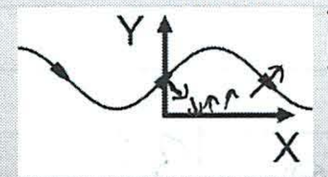
PRS Question: Vector Field

In General: Don't pick up unit until ready to answer

Then I'll know when class is ready

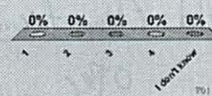
PRS: Vector Field

:20



The field line at left corresponds to the vector field:

1. $\vec{\mathbf{F}}(x,y)=\sin(x)\,\hat{\mathbf{i}}+\hat{\mathbf{j}}$
2. $\vec{\mathbf{F}}(x,y)=\hat{\mathbf{i}}+\sin(x)\,\hat{\mathbf{j}}$
3. $\vec{\mathbf{F}}(x,y)=\cos(x)\,\hat{\mathbf{i}}+\hat{\mathbf{j}}$
4. $\vec{\mathbf{F}}(x,y)=\hat{\mathbf{i}}+\cos(x)\,\hat{\mathbf{j}}$
5. I don't know

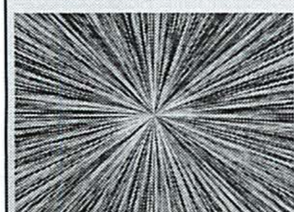


P01 -43

Y -alternating

not a plot
field line
tangent to field

Vector Fields – "Grass Seeds"



Source/Sink

Circulating

Although we don't know absolute direction, we can determine relative direction

P01 -44

which is it

direction missing

PRS Questions: "Grass Seed" Visualizations

P01 -45

20

PRS: Grass Seeds

The vector field at left is created by:

0% 1. Two sources (equal strength)
0% 2. Two sources (top stronger)
0% 3. Two sources (bottom stronger)
0% 4. Source & Sink (equal strength)
0% 5. Source & Sink (top stronger)
0% 6. Source & Sink (bottom stronger)
0% 7. I don't know

P01-46

could be 1 source + 1 sink

3

~~don't know direction~~ if it was

2 would flow from one to the

other

bottom stronger → fighting back

PRS: Grass Seeds

20

Here there is an initial downward flow.

told that

0% 1. The point is a source
0% 2. The point is a sink
0% 3. I don't know

P01-47

more flowing downward

top flows around it

because here changing direction

PRS: Circulation

20

These two circulations are in:

0% 1. The same direction (e.g. both clockwise)
0% 2. Opposite directions (e.g. one cw, one ccw)
0% 3. I don't know

P01-48

teacher:

both pushing to right

same dir

* remember tangent

PRS: Vector Field

20

The grass seeds field plot at left is a representation of the vector field:

- 0% 1. $\vec{\mathbf{F}}(x,y) = x^2\hat{\mathbf{i}} + y^2\hat{\mathbf{j}}$
- 0% 2. $\vec{\mathbf{F}}(x,y) = y^2\hat{\mathbf{i}} + x^2\hat{\mathbf{j}}$
- 0% 3. $\vec{\mathbf{F}}(x,y) = \sin(x)\,\hat{\mathbf{i}} + \cos(y)\,\hat{\mathbf{j}}$
- 0% 4. $\vec{\mathbf{F}}(x,y) = \cos(x)\,\hat{\mathbf{i}} + \sin(y)\,\hat{\mathbf{j}}$
- 0% 5. I don't know

P01-49

look at (+) x axis
vertical
so y-comp 0
x-comp not 0

?? is this right

Another Vector Field: Gravitational Field

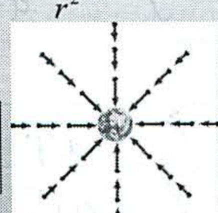
P01-50

Example Of Vector Field: Gravitation

Gravitational Force: $\vec{\mathbf{F}}_g = -G\dfrac{Mm}{r^2}\hat{\mathbf{r}}$

Gravitational Field:

$$\vec{\mathbf{g}} = \frac{\vec{\mathbf{F}}_g}{m} = -\frac{GMm/r^2}{m}\hat{\mathbf{r}} = -G\frac{M}{r^2}\hat{\mathbf{r}}$$



M: Mass of Earth

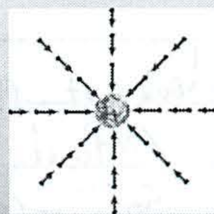
P01-51

Example Of Vector Field: Gravitation

Gravitational Field:

$\vec{\mathbf{g}} = -G\frac{M}{r^2}\hat{\mathbf{r}}$ | $\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$

Created by M | *Felt* by m



M: Mass of Earth

$\hat{\mathbf{r}}$: unit vector from M to m

$\vec{\mathbf{r}}$: vector from M to m

$\boxed{\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}}}{r}} \Rightarrow \boxed{\vec{\mathbf{g}} = -G\frac{M}{r^3}\vec{\mathbf{r}}}$ **USE THIS FORM!**

P01 -52

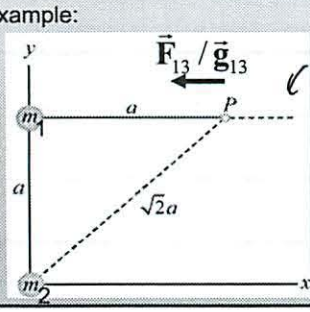
M - creating field
m - feels
↑ Simplistic

^ unit vector - most made mistake
so write like this

The Superposition Principle

Net force/field is vector sum of forces/fields

Example:



$\vec{\mathbf{F}}_3 = \vec{\mathbf{F}}_{13} + \vec{\mathbf{F}}_{23}$ ↖ on

In general:

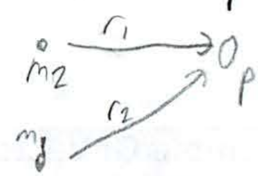
$$\boxed{\vec{\mathbf{F}}_j = \sum_{i=1}^{N}\vec{\mathbf{F}}_{ij}}$$

P01 -53

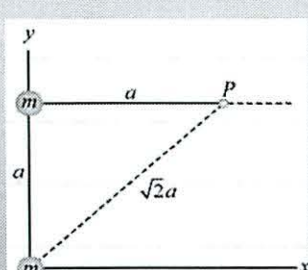
↑ n = items

$$F_j = \sum_{i=1}^{N}\vec{F}_{ij}$$

Steps
① Draw pic + label it



In Class Problem



Find the gravitational field $\vec{\mathbf{g}}$ at point P

Bonus: Where would you put another mass m to make the field $\vec{\mathbf{g}}$ become 0 at P?

Use $\vec{\mathbf{g}} = -G\frac{M}{r^3}\vec{\mathbf{r}}$

NOTE: Solutions will be posted within two days of class

P01 -54

leave units on

$$g = -\frac{Gm}{a^3}\vec{r}_1 - \frac{Gm}{(\sqrt{2}a)^3}\vec{r}_2$$

$$\vec{r}_1 = a\hat{i}$$
$$\vec{r}_2 = a(\hat{i} + \hat{j}) =$$

Don't simplify

$$g = -\frac{Gm}{a^3}a\hat{i} - \frac{Gm}{(\sqrt{2}a)^3}(\hat{i} + \hat{j})a$$

move a distance along $\hat{i}$ direction

From Gravitational to Electric Fields

Electric Charge (~Mass)

Two types of electric charge: positive and negative

Unit of charge is the ***coulomb*** [C]

Charge of electron (negative) or proton (positive) is

$$\pm e, \quad e = 1.602 \times 10^{-19} C$$

Charge is quantized

$$Q = \pm Ne$$

Charge is conserved

$$n \rightarrow p + e^- + \bar{\nu} \qquad e^+ + e^- \rightarrow \gamma + \gamma$$

r in fixed units of e

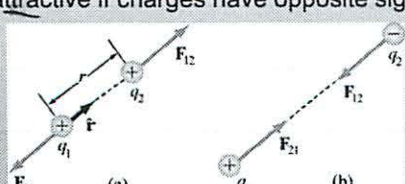
1 Coulomb is a lot of charge
ie 10 C is usually wrong

Electric Force (~Gravity)

The electric force between charges q_1 and q_2 is

(a) repulsive if charges have same signs

(b) attractive if charges have opposite signs



Like charges repel and opposites attract !!

has signs

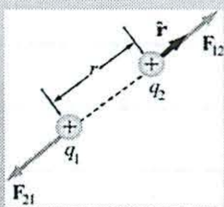
Yes - we know this

like gravitational law

Coulomb's Law

Coulomb's Law: Force on q_2 due to interaction between q_1 and q_2

$$\boxed{\vec{\mathbf{F}}_{12} = k_e \frac{q_1 q_2}{r^2}\hat{\mathbf{r}}}$$



$$k_e = \frac{1}{4\pi\varepsilon_0} = 8.9875\times10^9 \text{ N m}^2/\text{C}^2$$

$\hat{\mathbf{r}}$: unit vector from q_1 to q_2

$\vec{\mathbf{r}}$: vector from q_1 to q_2

$$\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}}}{r} \quad \Rightarrow \vec{\mathbf{F}}_{12} = k_e \frac{q_1 q_2}{r^3}\vec{\mathbf{r}}$$

P01-58

q instead of m

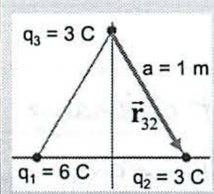
no minus sign ~~Repulsive~~

↳ the sign is in the q

Constant

Units are nasty in 8.02

Coulomb's Law: Example



$$\vec{\mathbf{F}}_{32} = ?$$

$$\vec{\mathbf{r}}_{32} = \left(\tfrac{1}{2}\hat{\mathbf{i}} - \tfrac{\sqrt{3}}{2}\hat{\mathbf{j}}\right) \text{ m}$$

$$r = 1\text{m}$$

$$\vec{\mathbf{F}}_{32} = k_e q_3 q_2 \frac{\vec{\mathbf{r}}}{r^3} = (9\times10^9 \text{N m}^2/\text{C}^2)(3\text{C})(3\text{C})\frac{\frac{1}{2}(\hat{\mathbf{i}} - \sqrt{3}\hat{\mathbf{j}})\text{m}}{(1\text{m})^3}$$

$$= \frac{81\times10^9}{2}(\hat{\mathbf{i}} - \sqrt{3}\hat{\mathbf{j}}) \text{ N}$$

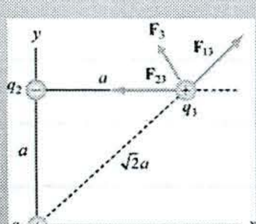
P01-59

Vector from generating it to what is feeling it

The Superposition Principle

Many Charges Present:
Net force on any charge is vector sum of forces from other individual charges

Example:



$$\vec{\mathbf{F}}_3 = \vec{\mathbf{F}}_{13} + \vec{\mathbf{F}}_{23}$$

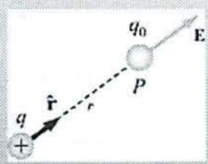
In general:

$$\boxed{\vec{\mathbf{F}}_j = \sum_{i=1}^{N} \vec{\mathbf{F}}_{ij}}$$

P01-60

Electric Field (~g)

The electric field at a point P due to a charge q is the force acting on a test charge q_0 at that point P, divided by the charge q_0 :



$$\vec{\mathbf{E}}_q(P) \equiv \frac{\vec{\mathbf{F}}_{qq_0}}{q_0}$$

For a point charge q: $\boxed{\vec{\mathbf{E}}_q(P) = k_e \frac{q}{r^2}\hat{\mathbf{r}}}$

Units: N/C, also Volts/meter

↑ main unit ↑ have not done Volts yet

q creates charge in every direction

other particle feels the charge

$\vec{F}_E = q\vec{E}$

Superposition Principle

The electric field due to a collection of N point charges is the vector sum of the individual electric fields due to each charge

$$\vec{\mathbf{E}}_{total} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \ldots\ldots = \sum_{i=1}^{N} \vec{\mathbf{E}}_i$$

Summary Thus Far

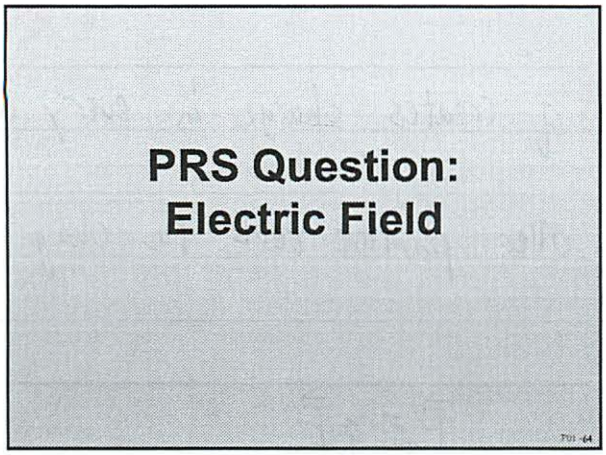
SOURCE:	Mass M_s	Charge q_s (±)
CREATE:	$\vec{\mathbf{g}} = -G\frac{M_s}{r^2}\hat{\mathbf{r}}$	$\vec{\mathbf{E}} = k_e\frac{q_s}{r^2}\hat{\mathbf{r}}$
FEEL:	$\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$	$\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$

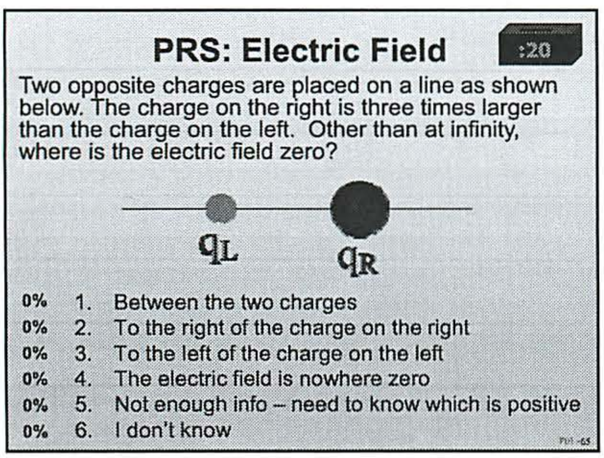
This is easiest way to picture field

think about the forces you would feel - as a person

* electrict field points in direction a ⊕ charge would want to move/acc *

- ask your self - what would ~~a~~ ⊕ do

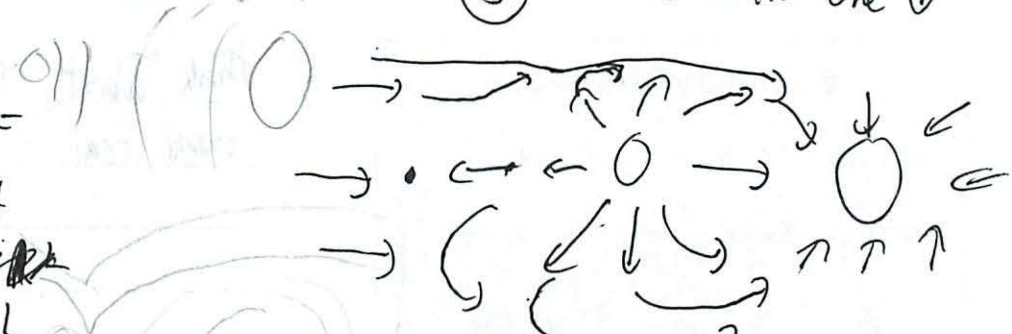




hard

assumed 1

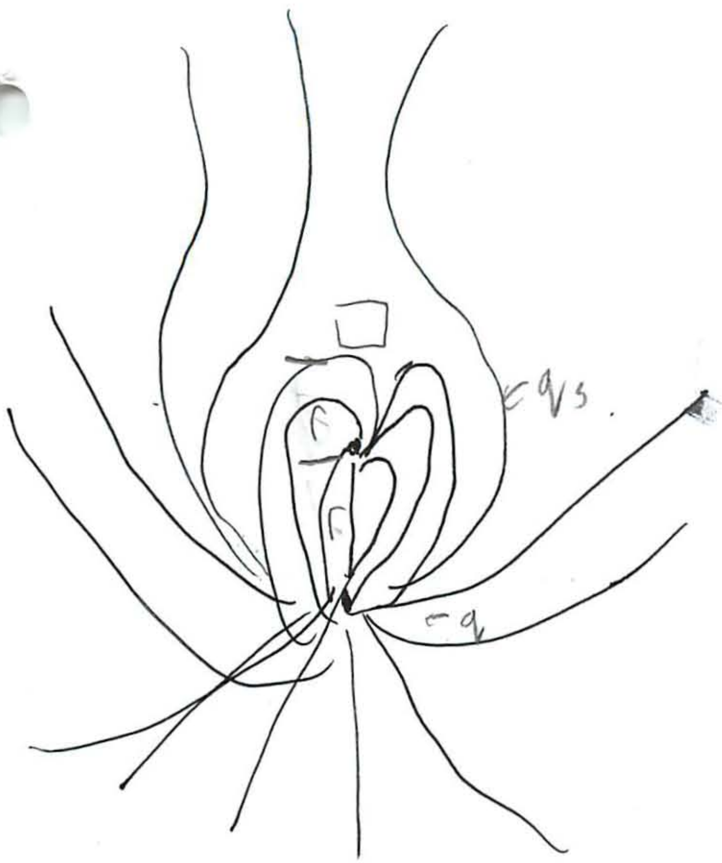
3 - just make one +



at the pt the fields from both are =
on the extreame left the field from ~~[illegible]~~ R > L so that dominates
(~~[illegible]~~)
- depends on size
- is there equation for field strength at a size
$F = k\frac{q_1 q_2}{r^3}\vec{r}$, could have put #s in

~~"Closeness of charge"~~

push left balanced by ballanced q_R's "pull" to right



bottom stronger

One source, one sink
(flows from one to oter)

know bottom is 4 x
as strong

$$F = \frac{k_E q_1 q_2}{r^2} \hat{r}$$

$$-k_E \frac{q_s}{8r^3}(r\hat{r}) + \frac{k_E q}{R^3} r\hat{r} = 0$$

$$\frac{2q_s}{8} = q$$

$$\boxed{q = \frac{q_s}{4}}$$

or

$$\frac{k_E q_1}{4r^2} = -\frac{k_E q_2}{r^2}\hat{r}$$

$$q_1 = 4q_2$$

know r diff is 2

$r^2 = 4$
$2^2 = 4$ ← 4 times stronger

Summary of Problem Solving Session 1 8.02 Class 2

Topics: Coordinate Systems; Gradients; Line and Surface Integrals ✓

Readings: **Course Notes: Chapter 2 Coulomb's Law Section 2.9-2.12**

Math Review: Spring 2006 Math Review Presentation
Hale Bradt's Spring 2001 8.02 Mathematics Supplement

There was reading for this and will be MP soon I think ↑Sun 9PM

Student info form

Start Qu

net charge in body

100 kg

2×10^{-27} kg/proton

2×10^{-19} C charge on proton -but have same # electrons so net charge $=0$

$\lambda = C/m$ unit

linear charge density

Topic Introduction

In this first problem solving session, you will learn how to solve for the electric field of a uniformly charged rod. This will involve setting up a vector integral. We will also introduce the concepts of understanding and calculating the electric field generated by a continuous distribution of charge.

We can find the electric field of a continuous distribution of charge using the superposition principle. Let's consider the system shown in Figure 1. Consider the infinitesimal element with charge Δq_i, contained in some small volume element ΔV_i.

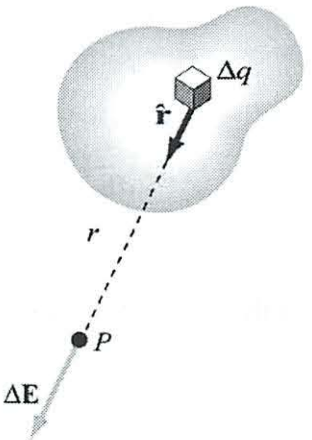


Figure 1 Electric field due to infinitesimal element with charge Δq_i

We shall assume the charge distribution is continuous. In the limit where ΔV_i shrinks to 0, the charge per unit volume, $\rho(\vec{\mathbf{r}}')$ (lowercase Greek letter *rho*) is called the volume charge density, and is defined as

$$\rho(\vec{\mathbf{r}}') = \lim_{\Delta V_i \to 0} \frac{\Delta q_i}{\Delta V_i} = \frac{dq}{dV} \qquad \text{(T0.1)}$$

The charge density may be uniform in space or may depend on the position $\vec{\mathbf{r}}'$ with respect to some choice of origin. The amount of charge, dq, in an infinitesimal volume element dV, located at the position $\vec{\mathbf{r}}'$, is

$$dq = \rho(\vec{\mathbf{r}}')dV \qquad \text{(T0.2)}$$

The electric field due to each infinitesimal charged element at a point P is given by Coulomb's Law:

$$d\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{\mathbf{r}} \tag{T0.3}$$

In this expression r is the distance from the infinitesimal charged element to the point P where we are determining the electric field. The unit vector $\hat{\mathbf{r}}$ points from the infinitesimal charged element to the point P (see Figure 2).

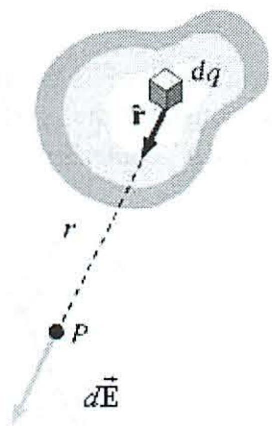


Figure 2 Electric field at the point P due to infinitesimal element of charge dq

The unit vector is given by

$$\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}'}{\left|\vec{\mathbf{r}} - \vec{\mathbf{r}}'\right|} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}'}{r} \tag{T0.4}$$

where $\vec{\mathbf{r}}$ is the position vector for the field point P with respect to the choice of origin, and $\vec{\mathbf{r}}'$ is the position vector for the infinitesimal element with charge dq, and $r = \left|\vec{\mathbf{r}} - \vec{\mathbf{r}}'\right|$ is the distance from the infinitesimal charged element to the point P.

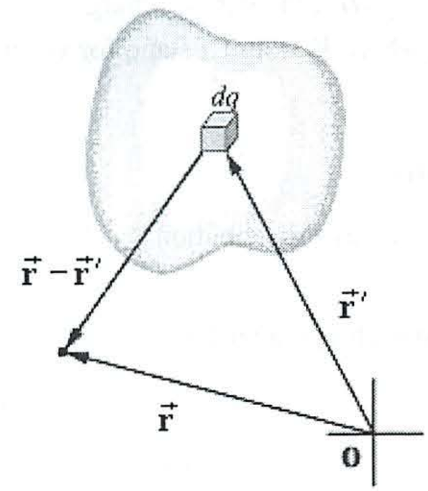


Figure 3 Vector geometry for the source and field point

We can use the superposition principle: the total electric field is the vector sum of all these infinitesimal contributions. This sum is just the integral

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0} \int_V \frac{dq}{r^2}\,\hat{\mathbf{r}} = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho(\vec{\mathbf{r}}')(\vec{\mathbf{r}} - \vec{\mathbf{r}}')dV}{\left|\vec{\mathbf{r}} - \vec{\mathbf{r}}'\right|^3} \tag{T0.5}$$

This integral is an example of a vector integral, which actually consists of three separate integrals, one for each direction in space that will give the component of the electric field in that direction. Each separate component integral is an integral over the volume where the charge is located.

Charge Density: We will regularly encounter in electrostatics three types of charge densities associated with 1-, 2-, or 3-dimensional charged objects that are defined as follows

$$\textit{volume charge density } \rho(\vec{\mathbf{r}}') = \frac{dq}{dV}$$

$$\textit{surface charge density } \sigma(\vec{\mathbf{r}}') = \frac{dq}{dA}$$

$$\textit{linear charge density } \lambda(\vec{\mathbf{r}}') = \frac{dq}{dL}$$

where dV, dA, dL are the infinitesimal volume, area, and line element respectively. These charge densities may be uniform or vary with position on the charged object.

Charge Density

When describing the amount of charge in a continuous charge distribution we often speak of the *charge density*. This function tells how much charge occupies a small region of space at any point in space. Depending on how the charge is distributed, we will either consider the volume charge density $\rho = dq/dV$, the surface charge density $\sigma = dq/dA$, or the linear charge density $\lambda = dq/d\ell$, where V, A and ℓ stand for volume, area and length respectively.

Important Equations

Electric field from a discrete charge distribution: $\vec{\mathbf{E}} = \dfrac{1}{4\pi\varepsilon_0}\sum_i \dfrac{q_i}{|r_i|^2}\hat{\mathbf{r}}_i = \dfrac{1}{4\pi\varepsilon_0}\sum_i \dfrac{q_i}{|r_i|^3}\vec{\mathbf{r}}_i$

Electric field from continuous charge distribution: $\vec{\mathbf{E}} = \dfrac{1}{4\pi\varepsilon_0}\int_V \dfrac{dq}{r^2}\hat{\mathbf{r}}$

Charge Densities: $dq = \begin{cases} \rho dV & \text{for a volume distribution} \\ \sigma dA & \text{for a surface (area) distribution} \\ \lambda d\ell & \text{for a linear distribution} \end{cases}$

Important Nomenclature:

A hat (e.g. $\hat{\mathbf{A}}$) over a vector means that that vector is a unit vector ($|\hat{\mathbf{A}}| = 1$)

The unit vector $\hat{\mathbf{r}}$ points ***from*** the charge creating ***to*** the observer measuring the field.

Contineous Charge

- you add multiple charges (super position)
- if you have a blob of charge
  - divide the blop into parts
  - each piece you know wat it is
  - add all the little pieces
  - make them (blobs) smaller + smaller until point
  - integrate

$$Q = \sum q_i \rightarrow \iiint dq$$

in 3D, triple integral
never really do one
multiply:
Charge density * lenght
* volume

$$\vec{E} = \iiint d\vec{E}$$

Today line of charge

$$dQ = \lambda dL$$

$$\lambda = \frac{Q}{L}$$

add/integrate to get total electric charge

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

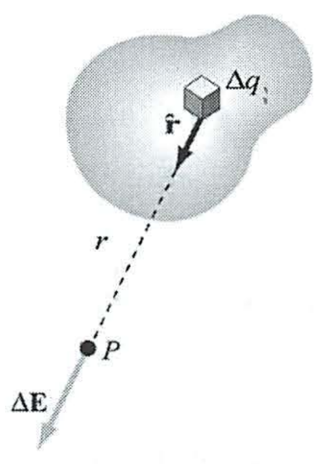
Problem Solving 1: Continuous Sources and Vector Calculus

Introduction: In this first problem solving session, you will learn how to solve for the electric field of a uniformly charged rod. This will involve setting up a vector integral.

Readings: Course Notes: Chapter 2 Coulomb's Law Section 2.9-2.12

When we charge up an object, through a physical transfer of charge or induction; we may typically place between a nano-coulomb and a micro-coulomb of charge, $10^{-9}C < Q < 10^{-6}C$, on the object. Since the charge of the electron is $e = 1.602 \times 10^{-19} C$, this means that we are placing between 10^{10} and 10^{13} electrons on the object. The electric field due to a small number of charged particles can readily be computed using the superposition principle. But what happens in our case when we have a very large charge distributed over some region in space? If we are trying to determine the electric field due to this charge distribution at a distance that is large compared to the distance between the charged objects for example electrons, then we can assume that the electrons form a continuous distribution of charge.

Let's consider the system shown in Figure 1. Consider the infinitesimal element with charge Δq_i, contained in some small volume element ΔV_i.



Changes

e) Only calculate $\vec{s} = \vec{r} - \vec{r}'$

f) Calculate $s = |\vec{s}| = |\vec{r} - \vec{r}'|$

Figure 1 Electric field due to infinitesimal element with charge Δq_i

We shall assume the charge distribution is continuous. In the limit where ΔV_i shrinks to 0, the charge per unit volume, $\rho(\vec{\mathbf{r}}')$ (lowercase Greek letter *rho*) is called the volume charge density, and is defined as

$$\rho(\vec{\mathbf{r}}') = \lim_{\Delta V_i \to 0} \frac{\Delta q_i}{\Delta V_i} = \frac{dq}{dV} \tag{T0.1}$$

The charge density may be uniform in space or may depend on the position $\vec{\mathbf{r}}'$ with respect to some choice of origin. The amount of charge, dq, in an infinitesimal volume element dV, located at the position $\vec{\mathbf{r}}'$, is

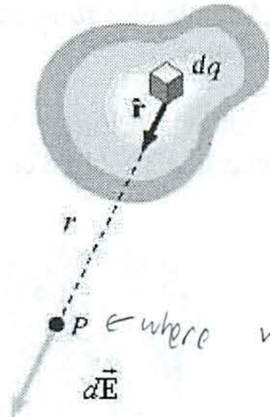
$$dq = \rho(\vec{\mathbf{r}}')dV \tag{T0.2}$$

The electric field due to each infinitesimal charged element at a point P is given by Coulomb's Law:

ke

$$d\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{\mathbf{r}} \tag{T0.3}$$

In this expression r is the distance from the infinitesimal charged element to the point P where we are determining the electric field. The unit vector $\hat{\mathbf{r}}$ points from the infinitesimal charged element to the point P (see Figure 2).



← where we are looking

Figure 2 Electric field at the point P due to infinitesimal element of charge dq

The unit vector is given by

$$\hat{\mathbf{r}} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}'}{|\vec{\mathbf{r}} - \vec{\mathbf{r}}'|} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}'}{r} \tag{T0.4}$$

where $\vec{\mathbf{r}}$ is the position vector for the field point P with respect to the choice of origin, and $\vec{\mathbf{r}}'$ is the position vector for the infinitesimal element with charge dq, and $r = |\vec{\mathbf{r}} - \vec{\mathbf{r}}'|$ is the distance from the infinitesimal charged element to the point P.

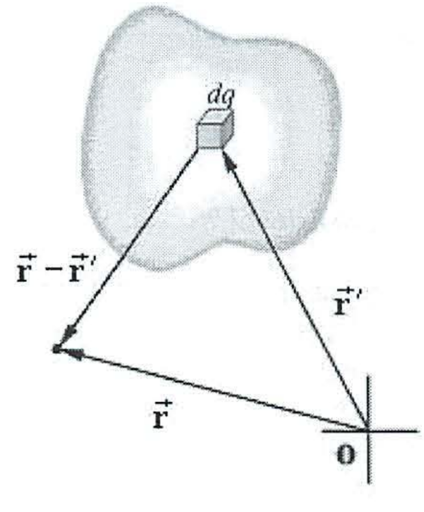


Figure 3 Vector geometry for the source and field point

We can use the superposition principle: the total electric field is the vector sum of all these infinitesimal contributions. This sum is just the integral /add "superposition"

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\int_V \frac{dq}{r^2}\,\hat{\mathbf{r}} = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\vec{\mathbf{r}}')(\vec{\mathbf{r}}-\vec{\mathbf{r}}')dV}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|^3} \qquad \text{(T0.5)}$$

This integral is an example of a vector integral, which actually consists of three separate integrals, one for each direction in space that will give the component of the electric field in that direction. Each separate component integral is an integral over the volume where the charge is located.

Charge Density: We will regularly encounter in electrostatics three types of charge densities associated with 1-, 2-, or 3-dimensional charged objects that are defined as follows

3D *volume charge density* $\rho(\vec{\mathbf{r}}') = \dfrac{dq}{dV}$

2D *surface charge density* $\sigma(\vec{\mathbf{r}}') = \dfrac{dq}{dA}$

1D *linear charge density* $\lambda(\vec{\mathbf{r}}') = \dfrac{dq}{dL}$

where dV, dA, dL are the infinitesimal volume, area, and line element respectively. These charge densities may be uniform or vary with position on the charged object.

PROBLEM 1: (*answer on the tear-sheet at the end*)

A hollow cylinder, of length L and radius a, is uniformly charged with total charge Q. There are no end caps on the cylinder.

(a) What is the *surface charge density* σ?

Find SA $= 2\pi a L$ $\sigma = \frac{Q}{2\pi a L}$ $\sigma = \frac{Q}{A}$

← says in notes can do

(b) What is the *linear charge density* λ?

$\lambda = \frac{Q}{L}$

(c) What is relationship between σ and λ?

λ rotated around in a circle

PROBLEM 2: (*answer on the tear-sheet at the end*)

A solid cylinder, of length L and radius a, is uniformly charged with total charge Q.

(a) What is the *volume charge density* ρ?

Volume cylindor $= \pi a^2 L$ $\sigma = \frac{Q}{\pi a^2 L}$

(b) What is the *linear charge density* λ?

? a slice

or still surface bar $\lambda = \frac{Q}{L}$

(c) What is relationship between ρ and λ?

Not rotated around circle

· Made up of linear bars

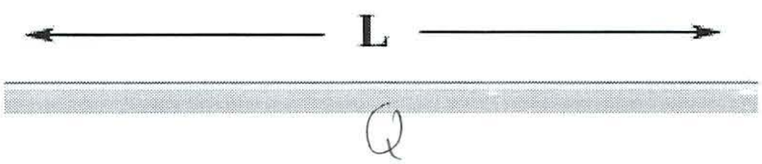
Like straws in a can

PROBLEM 3: Electric Field of Uniformly Charged Rod

In this problem you will learn how to set up an integral expression for the electric field

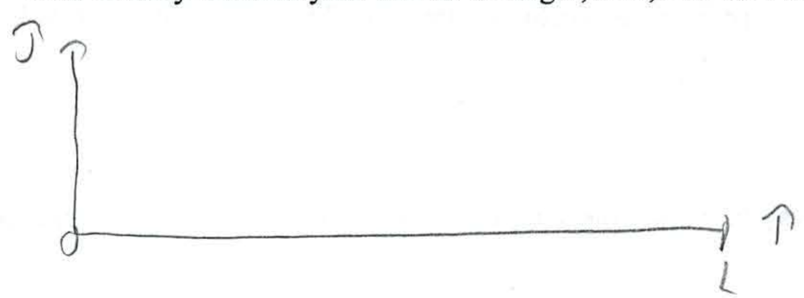
$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{dq}{r^2}\hat{\mathbf{r}}$$

of a rod of length L that has an amount of charge Q uniformly distributed. (You may assume the rod is a 1-dimensional object.)



Source Coordinates and Field Point Coordinates:

(a) Choose a 2-dimensional coordinate system and draw it in the space below for the wire. Clearly indicate your choice of origin, axis, and unit vectors.



(b) Choose an infinitesimal charge element dq. Clearly show where you located dq on the wire. Find an expression relating dq, Q, L and your choice of length for dq.

$\cancel{Q = \int_0^L dq}$

~~charge whole thing~~ ~~little pieces~~

know this

~~X = lenght dq~~

$dq \cancel{= \frac{Qx}{L}} = \lambda\, dl = \frac{Q}{L}\, dl$

$\hat{r}$ is where check field

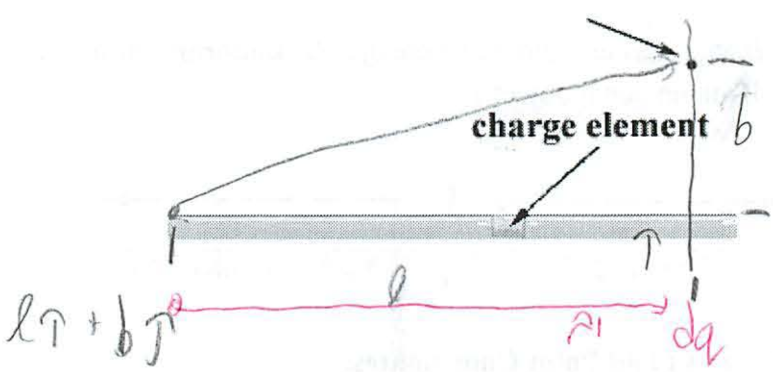
(c) Write down a vector expression for the source position vector $\vec{\mathbf{r}}'$ in terms of your source coordinates. These source coordinates will be your integration variables.

Distance along lenght

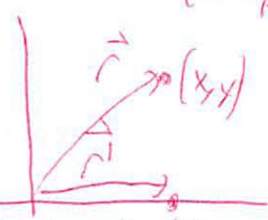
$\vec{r}\,' = dl\,\hat{i}$

dl is from start to point

(d) Consider a field point P that lies off both the axis of the wire and the perpendicular bisector of the wire. Using your same choice of origin, axis, coordinates, and unit vectors, write down an expression for the position vector $\vec{\mathbf{r}}(P)$ for the field point P.



life is easier if coord is at the center (always)



(e) Write down a vector expression for the vector from the source to the field point, $\vec{\mathbf{r}} - \vec{\mathbf{r}}'$. This is a problem in vector subtraction. Then find the magnitude of this vector, Only Calculate $\mathfrak{r} \equiv \vec{r} - \vec{r}'$

x' dq

$dq = dx'$

$$r = |\vec{\mathbf{r}} - \vec{\mathbf{r}}'|$$

- adding x' getting total distance moved

in the usual way by taking the square root of the square of the components of the vector $\vec{\mathbf{r}} - \vec{\mathbf{r}}'$.

arc of charge
- use θ'
- length (ds)

$ds = R\,d\theta'$

(f) Now find an expression for the unit vector, $\hat{\mathbf{r}}$, located at the field point that **points from the source to the field point,** in terms of both source and field point coordinates. The unit vector is given by

Only Calculate

$$\boxed{\hat{\mathbf{r}}} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}'}{\boxed{|\vec{\mathbf{r}} - \vec{\mathbf{r}}'|}} \qquad \text{(T0.6)}$$

prime = what is moving

(g) Using your results, find a vector expression for the infinitesimal electric field $d\vec{\mathbf{E}}$ (in terms of your unit vectors for the field point P) for the contribution of dq to the electric field using Coulomb's Law:

$$d\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{\mathbf{r}}$$

(h) Using your results from part (g), set up an expression for the vector integrals for the total electric field at P using

$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_0}\int_{wire}\frac{dq}{r^2}\hat{\mathbf{r}}.$$

Your expression should contain two separate integrals for the two directions that appear in the decomposition of $\hat{\mathbf{r}}$. You are integrating over the source dq, which means each separate integral is over the length of the rod. For each direction, set up an expression for the integral with the appropriate limits according to your choice of coordinates.

(i) If P lies on the perpendicular bisector of the wire, explain why any of you integrals should vanish. Can you show this explicitly by doing the integral?

(j) Integrate you're the integrals you found in part (i) to find an expression for the vector field $\vec{\mathbf{E}}$ as a function of your field point coordinates. (You may find this integral non-trivial in which case try to do it at home.)

Topics: Electric Charge; Electric Fields; Dipoles; Continuous Charge Distributions
Related Reading: Course Notes Section 1.6; Chapter 2

Topic Introduction

Today we review the concept of electric charge, and describe both how charges create electric fields and how those electric fields can in turn exert forces on other charges. Again, the electric field is completely analogous to the gravitational field, where mass is replaced by electric charge, with the small exceptions that (1) charges can be either positive or negative while mass is always positive, and (2) while masses always attract, charges of the same sign repel (opposites attract). We will also introduce the concepts of understanding and calculating the electric field generated by a continuous distribution of charge.

Electric Charge

All objects consist of negatively charged electrons and positively charged protons, and hence, depending on the balance of the two, can themselves be either positively or negatively charged. Although charge cannot be created or destroyed, it can be transferred between objects in contact, which is particularly apparent when friction is applied between certain objects (hence shocks when you shuffle across the carpet in winter and static cling in the dryer).

Electric Fields

Just as masses interact through a gravitational field, charges interact through an electric field. Every charge creates around it an electric field, proportional to the size of the charge and decreasing as the inverse square of the distance from the charge $\left(\vec{\mathbf{E}} = k_e \frac{Q}{r^2}\hat{\mathbf{r}}\right)$. If another charge enters this electric field, it will feel a force $\left(\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}\right)$. If the electric field becomes strong enough it can actually rip the electrons off of atoms in the air, allowing charge to flow through the air and making a spark, or, on a larger scale, lightening.

spelling?

Charge Distributions

Electric fields "superimpose," or add, just as gravitational fields do. Thus the field generated by a collection of charges is just the sum of the electric fields generated by each of the individual charges. If the charges are discrete, then the sum is just vector addition. If the charge distribution is continuous then the total electric field can be calculated by integrating the electric fields $d\vec{\mathbf{E}}$ generated by each small chunk of charge dq in the distribution.

Need to clarify - why vectors
and why $\frac{\vec{r}}{r^2}\hat{r}$ becomes r^3

Charge Density

When describing the amount of charge in a continuous charge distribution we often speak of the *charge density*. This function tells how much charge occupies a small region of space at any point in space. Depending on how the charge is distributed, we will either consider the volume charge density $\boxed{\rho = dq/dV}$, the surface charge density $\boxed{\sigma = dq/dA}$, or the linear charge density $\boxed{\lambda = dq/d\ell}$, where V, A and ℓ stand for volume, area and length respectively.

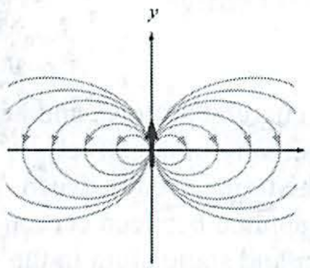
place where

Electric Dipoles

2 are seme d apart

The electric dipole is a very common charge distribution consisting of a positive and negative charge of equal magnitude q, placed some small distance d apart. We describe the dipole by its dipole moment $\mathbf{p}$, which has magnitude $p = qd$ and points ***from*** the negative ***to*** the positive charge. Like individual charges, dipoles both create electric fields and respond to them. The field created by a dipole is shown at left (its moment is shown as the purple vector). When placed in an external field, a dipole will attempt to rotate in order to align with the field, and, if the field is non-uniform in strength, will feel a force as well.

= magnitude
opposite sign



Important Equations

from ⊖ to ⊕

Electric force between two charges: $\left|\vec{\mathbf{F}}_E\right| = k_e \dfrac{qQ}{r^2}$,

Repulsive (attractive) if charges have the same (opposite) signs

Strength of electric field created by a charge Q: $\vec{\mathbf{E}} = k_e \dfrac{Q}{r^2}\hat{\mathbf{r}} = k_e \dfrac{Q}{r^3}\vec{\mathbf{r}}$,

$\hat{\mathbf{r}}$ points from charge to observer who is measuring the field

Force on charge q sitting in electric field $\boldsymbol{E}$: $\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$

Electric dipole moment: $\left|\vec{\mathbf{p}}\right| = qd$

Points from negative charge $-q$ to positive charge $+q$.

Torque on a dipole in an external field: $\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$

Dipole moment

Electric field from a discrete charge distribution: $\vec{\mathbf{E}} = \dfrac{1}{4\pi\varepsilon_0}\sum_i \dfrac{q_i}{\left|r_i\right|^2}\hat{\mathbf{r}}_i = \dfrac{1}{4\pi\varepsilon_0}\sum_i \dfrac{q_i}{\left|r_i\right|^3}\vec{\mathbf{r}}_i$

Electric field from continuous charge distribution: $\vec{\mathbf{E}} = \dfrac{1}{4\pi\varepsilon_0}\int_V \dfrac{dq}{r^2}\hat{\mathbf{r}}$

$\rho = \dfrac{dq}{dV}$ $\sigma = \dfrac{dq}{dA}$ $\lambda = \dfrac{dq}{d\ell}$

Charge Densities:
$$dq = \begin{cases} \rho dV & \text{for a volume distribution} \\ \sigma dA & \text{for a surface (area) distribution} \\ \lambda d\ell & \text{for a linear distribution} \end{cases}$$

Important Nomenclature:

$-d$ = displacement vector
⊖ → ⊕

A hat (e.g. $\hat{\mathbf{A}}$) over a vector means that that vector is a unit vector ($\left|\hat{\mathbf{A}}\right| = 1$)

The unit vector $\hat{\mathbf{r}}$ points ***from*** the charge creating ***to*** the observer measuring the field.

Class 3

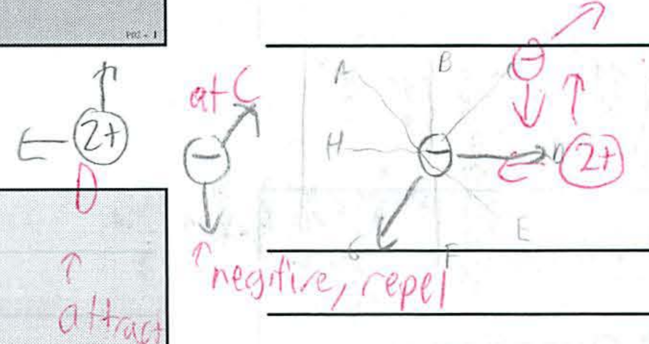
W1 D2

Class 02: Outline

Answer questions
Hour 1:
- Review: Electric Fields
- Charge
- Dipoles

Hour 2:
- Continuous Charge Distributions

Hardest Math is now
but Math is not the hardest



Last Time: Fields
Gravitational & Electric

Human body usually neutral
But if static charge then not equal

Gravitational & Electric Fields

SOURCE:	Mass M_s	Charge q_s (±)
CREATE:	$\vec{\mathbf{g}} = -G\frac{M_s}{r^2}\hat{\mathbf{r}}$	$\vec{\mathbf{E}} = k_e\frac{q_s}{r^2}\hat{\mathbf{r}}$
FEEL:	$\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$	$\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$

This is easiest way to picture field

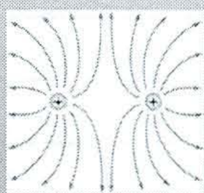
Make yourself a +, how would I move

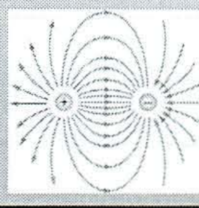
Depends on electronegativity of material - check a chart

When rod +, touch foil (foil attract) it becomes +, now foil to repel since both +

Electric Field Lines

1. Direction of field at any point is tangent to field line at that point
2. Field lines point away from positive charges and terminate on negative charges
3. Field lines never cross each other





P02 - 4

* tangent *

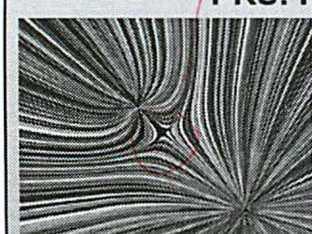
PRS Questions: Electric Field

P02 - 5

Diopole? No - charges are the same

PRS: Force

:15



The force between the two charges is:

0% 1. Attractive
0% 2. Repulsive
0% 3. Can't tell without more information
0% 4. I don't know

P02 - 6

PRS: Field Lines

15

Electric field lines show:

1. Directions of forces that exist in space at all times.
2. Directions in which charges on those lines will accelerate.
3. Paths that charges will follow.
4. More than one of the above.
5. I don't know.

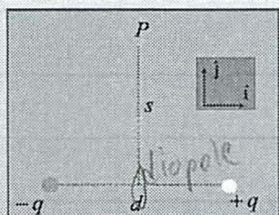
0% 0% 0% 0% 0%

Remember: Don't pick up until you are ready to answer

not an acc -
but the direction it would acc

*not flow lines objects follow
only dir it will accelerate

In-Class Problem

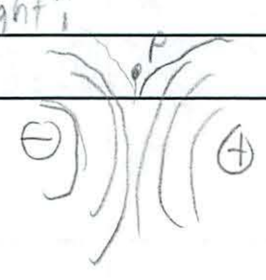


Consider two point charges of equal magnitude but opposite signs, separated by a distance d. Point P lies along the perpendicular bisector of the line joining the charges, a distance s above that line. What is the E field at P?

diopole

at pt d: diapole

But it asks what it is at point P? Not 0 right?



test charge always (+) so moves to left

$E = \frac{kq}{r^3}\vec{r}$

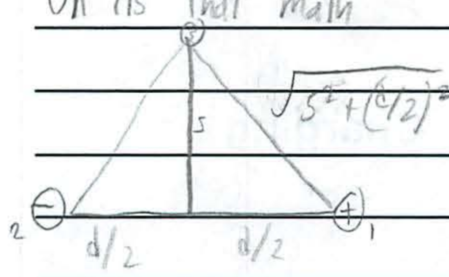
total distance is what matters

make sure have right lenght of r-vector

Two PRS Questions: E Field of Finite # of Point Charges

V: with limits $s \to 0$, $d \to 0$

Oh its that math



* denominator
* always the
* same
* put the distance
* in the numerator

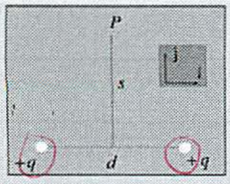
add for superposition

$\hat{j}$ cancle

watch signs

$$\vec{E} = \frac{-d\,kq}{\left(\sqrt{s^2+\left(\frac{d}{2}\right)^2}^{\,3}\right)}\hat{i} \quad \left(-\frac{kq_1q_3\,d/2}{\sqrt{(d/2)^2+s^2}}\hat{i} + -\frac{kq_2q_3\,d/2}{\sqrt{s^2+(d/2)^2}}\hat{i} + +\frac{kq_2q_3\,s}{\sqrt{s^2+d/2^2}}\hat{j} + -\frac{kq_2q_3\,s}{\sqrt{s^2+d/2^2}}\hat{j}\right)$$

PRS: Equal Charges



Electric field at P is:

1. $\vec{\mathbf{E}} = \dfrac{2k_e qs}{\left[s^2 + \dfrac{d^2}{4}\right]^{3/2}}\,\hat{\mathbf{j}}$
2. $\vec{\mathbf{E}} = -\dfrac{2k_e qd}{\left[s^2 + \dfrac{d^2}{4}\right]^{3/2}}\,\hat{\mathbf{i}}$
3. $\vec{\mathbf{E}} = \dfrac{2k_e qd}{\left[s^2 + \dfrac{d^2}{4}\right]^{3/2}}\,\hat{\mathbf{j}}$
4. $\vec{\mathbf{E}} = -\dfrac{2k_e qs}{\left[s^2 + \dfrac{d^2}{4}\right]^{3/2}}\,\hat{\mathbf{i}}$
5. I Don't Know

P02-10

When $d \to 0$ $= 2q$ charge

Candle in ↑ direction

PRS: 5 Equal Charges

15

Six equal positive charges q sit at the vertices of a regular hexagon with sides of length R. We remove the bottom charge. The electric field at the center of the hexagon (point P) is:

1. $\vec{\mathbf{E}} = \dfrac{2kq}{R^2}\,\hat{\mathbf{j}}$
2. $\vec{\mathbf{E}} = -\dfrac{2kq}{R^2}\,\hat{\mathbf{j}}$
3. $\vec{\mathbf{E}} = \dfrac{kq}{R^2}\,\hat{\mathbf{j}}$
4. $\vec{\mathbf{E}} = -\dfrac{kq}{R^2}\,\hat{\mathbf{j}}$
5. $\vec{\mathbf{E}} = 0$
6. I Don't Know

P02-11

Candles ↑ direction

Pretend to put ⊖ at bottom or which charge am I enabling

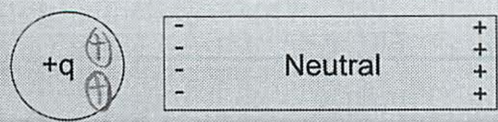
Field is ↓ so ⊖ distance R away

Charging

P02-12

How Do You Get Charged?

- Friction
- Transfer (touching)
- Induction



Charges split

Demonstrations: Instruments for Charging

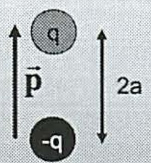
Electric Dipoles

A Special Charge Distribution

will see again + again

Electric Dipole

Two equal but opposite charges **+*q*** and **−*q***, separated by a distance **2a**



Dipole Moment

$\vec{\mathbf{p}} \equiv \text{charge} \times \text{displacement}$

$= q \times 2a\hat{\mathbf{j}} = 2qa\hat{\mathbf{j}}$

$\vec{\mathbf{p}}$ points from negative to positive charge

P02-16

opposite the field

moment = moment of inertia = mass

≈ charges

how hard to rotate
distribution of mass

* how things are distributed in space

Why Dipoles?

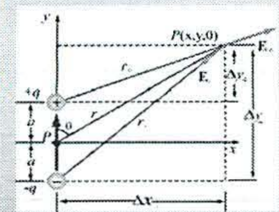
P02-17

want to link up in a chain
then still attract w/ less pull

Dipoles *make* Fields

P02-18

Electric Field Created by Dipole



Thou shalt use components!

$$\frac{\hat{\mathbf{r}}}{r^2} = \frac{\vec{\mathbf{r}}}{r^3} = \frac{\Delta x}{r^3}\hat{\mathbf{i}} + \frac{\Delta y}{r^3}\hat{\mathbf{j}}$$

$$E_x = k_e q\left(\frac{\Delta x}{r_+^3} - \frac{\Delta x}{r_-^3}\right) = k_e q\left(\frac{x}{\left[x^2+(y-a)^2\right]^{3/2}} - \frac{x}{\left[x^2+(y+a)^2\right]^{3/2}}\right)$$

$$E_y = k_e q\left(\frac{\Delta y_+}{r_+^3} - \frac{\Delta y_-}{r_-^3}\right) = k_e q\left(\frac{y-a}{\left[x^2+(y-a)^2\right]^{3/2}} - \frac{y+a}{\left[x^2+(y+a)^2\right]^{3/2}}\right)$$

P02 -19

never going to use this

but be able to calculate

PRS Question: Dipole Fall-Off

P02 -20

PRS: Dipole Field

As you move to large distances r away from a dipole, the electric field will fall-off as:

0% 1. $1/r^2$, just like a point charge

0% 2. More rapidly than $1/r^2$

0% 3. More slowly than $1/r^2$

0% 4. I Don't Know

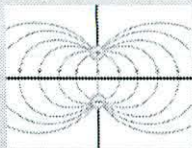
15

P02 -21

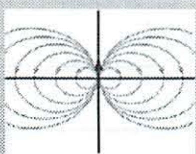
overrates it - but still some

~~? is this correct?~~ $\frac{1}{r^3}$

Point Dipole Approximation



Finite Dipole



Point Dipole

Take the limit $r >> a$

You can show…

$$E_x \rightarrow \frac{3p}{4\pi\varepsilon_0 r^3}\sin\theta\cos\theta$$

$$E_y \rightarrow \frac{p}{4\pi\varepsilon_0 r^3}\left(3\cos^2\theta - 1\right)$$

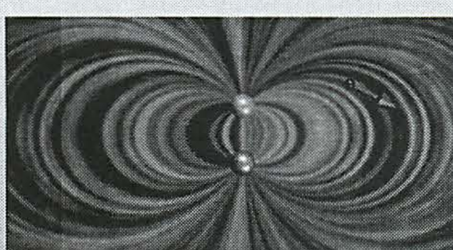
r gets really big

when look far away - look to cancle each other out

$\frac{1}{r^3}$

field looks to be weaker

Shockwave for Dipole



Dipole Visualization

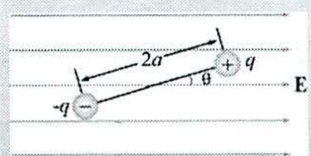
field a diapole makes

also

Dipoles *feel* Fields

Demonstration: Dipole in Field

Dipole in Uniform Field



$\vec{\mathbf{E}} = E\hat{\mathbf{i}}$

$\vec{\mathbf{p}} = 2qa(\cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}})$

↑ torques need angle

Total Net Force: $\vec{\mathbf{F}}_{net} = \vec{\mathbf{F}}_+ + \vec{\mathbf{F}}_- = q\vec{\mathbf{E}} + (-q)\vec{\mathbf{E}} = 0$

Torque on Dipole: $\boxed{\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}}$

$\tau = rF_+\sin(\theta) = (2a)(qE)\sin(\theta) = pE\sin(\theta)$

$\vec{\mathbf{p}}$ tends to align with the electric field

(+) → in direction of field

← (−)

Feels torque to rotate clockwise

total force = 0

want to rotate to align w/ external electric field

Torque on Dipole

Total Field (dipole + background) shows torque:

Animation

- Field lines transmit tension
- Connection between dipole field and constant field "pulls" dipole into alignment

field lines transmit tension

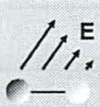
grab ends of field line + pull

dipole on a big vertical dipole

$\vec{p} \times \vec{E}$

PRS Question:
Dipole in Non-Uniform Field

PRS: Dipole in Non-Uniform Field



A dipole sits in a non-uniform electric field E

Due to the electric field this dipole will feel:

0% 1. force but no torque
0% 2. no force but a torque
0% 3. both a force and a torque
0% 4. neither a force nor a torque

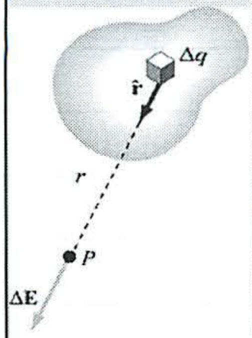
:15

force - does not cancle out

always a torque if not aligned with a field

Continuous Charge Distributions

Continuous Charge Distributions



Break distribution into parts:

$$Q = \sum_i \Delta q_i \rightarrow \iiint_V dq$$

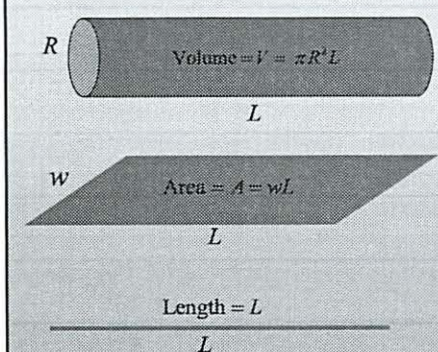
E field at P due to Δq

$$\Delta \vec{\mathbf{E}} = k_e \frac{\Delta q}{r^2} \hat{\mathbf{r}} \rightarrow d\vec{\mathbf{E}} = k_e \frac{dq}{r^2} \hat{\mathbf{r}}$$

Superposition:

$$\boxed{\vec{\mathbf{E}} = \sum \Delta \vec{\mathbf{E}} \rightarrow \int d\vec{\mathbf{E}}}$$

Continuous Sources: Charge Density

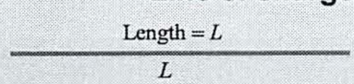


$$dQ = \rho\, dV \qquad \rho = \frac{Q}{V}$$

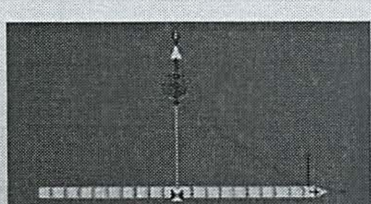
$$dQ = \sigma\, dA \qquad \sigma = \frac{Q}{A}$$

$$dQ = \lambda\, dL \qquad \lambda = \frac{Q}{L}$$

Examples of Continuous Sources: Line of charge



$$dQ = \lambda\, dL \qquad \lambda = \frac{Q}{L}$$



Link to applet

animation

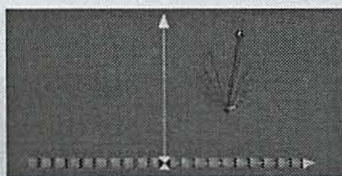
Examples of Continuous Sources: Line of charge

Length = L

L

$$dQ = \lambda\, dL$$

$$\lambda = \frac{Q}{L}$$



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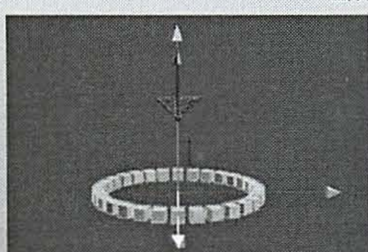
Examples of Continuous Sources: Ring of Charge

$$dQ = \lambda\, dL \qquad \lambda = \frac{Q}{2\pi R}$$



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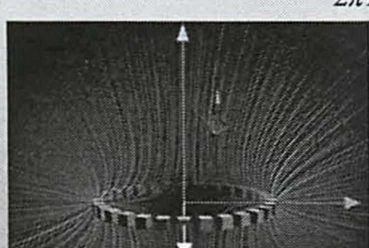
Examples of Continuous Sources: Ring of Charge

$$dQ = \lambda\, dL \qquad \lambda = \frac{Q}{2\pi R}$$



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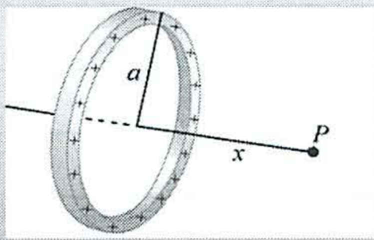
Example: Ring of Charge



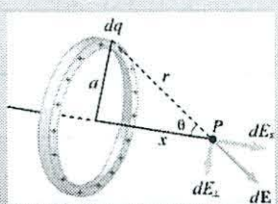
P on axis of ring of charge, *x* from center
Radius a, charge density λ.

Find **E** at P

Ring of Charge

1) Think about it
$E_\perp = 0$ Symmetry!

Mental Picture…



2) Define Variables

$$dq = \lambda\, dl = \lambda\left(a\, d\varphi\right)$$

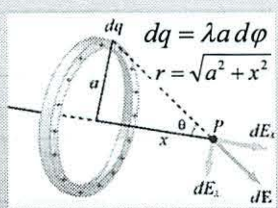
$$r = \sqrt{a^2 + x^2}$$

Ring of Charge

3) Write Equation

$$d\vec{\mathbf{E}} = k_e dq \frac{\hat{r}}{r^2} = k_e\, dq \frac{\vec{r}}{r^3}$$

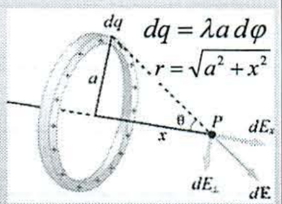
$$dE_x = k_e dq \frac{x}{r^3}$$



Ring of Charge

4) Integrate

$$E_x = \int dE_x = \int k_e\, dq \frac{x}{r^3} = k_e \frac{x}{r^3}\int dq$$



Very special case: everything except *dq* is constant

$$\int dq = \int_0^{2\pi} \lambda a\, d\varphi = \lambda a \int_0^{2\pi} d\varphi = \lambda \cdot a2\pi = Q$$

P02-40

$\lambda = \frac{dq}{dl}$

$dq = \lambda\, dl$

$dE = \frac{k\, dQ\, \vec{r}}{r^3}$

$\int$ take at constants

$E = \frac{kQ}{r^3}\vec{r}$

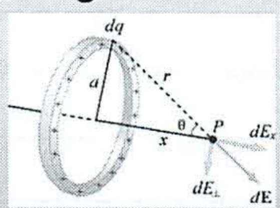
$E = \frac{kQ}{(a^2+x^2)^{3/2}}$

Ring of Charge

5) Clean Up

$$E_x = k_e Q \frac{x}{r^3}$$

$$E_x = k_e Q \frac{x}{\left(a^2+x^2\right)^{3/2}}$$



$$\boxed{\vec{\mathbf{E}} = k_e Q \frac{x}{\left(a^2+x^2\right)^{3/2}}\hat{\mathbf{i}}}$$

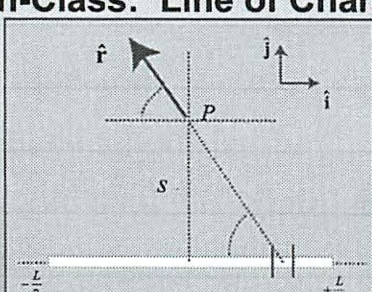
other parts of vector is 0

6) Check Limit $a \to 0$

$$E_x \to k_e Q \frac{x}{\left(x^2\right)^{3/2}} = \frac{k_e Q}{x^2}$$

P02-41

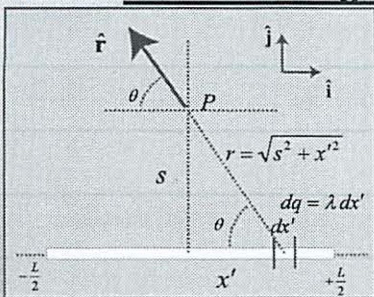
In-Class: Line of Charge



Point *P* lies on perpendicular bisector of uniformly charged line of length *L*, a distance *s* away. The charge on the line is *Q*. What is *E* at *P*?

P02-42

Hint: Line of Charge



Typically give the integration variable (x') a "primed" variable name. ALSO: Difficult integral (trig. sub.)

P02 -43

E Field from Line of Charge

$$\boxed{\vec{\mathbf{E}} = k_e \frac{Q}{s(s^2 + L^2/4)^{1/2}}\hat{\mathbf{j}}}$$

Limits: to test

$\lim_{s>>L} \vec{\mathbf{E}} \rightarrow k_e \frac{Q}{s^2}\hat{\mathbf{j}}$ Point charge ← did this

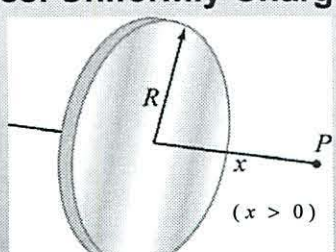
$\lim_{s<<L} \vec{\mathbf{E}} \rightarrow 2k_e \frac{Q}{Ls}\hat{\mathbf{j}} = 2k_e \frac{\lambda}{s}\hat{\mathbf{j}}$ Infinite charged line test if line is infinite

P02 -44

L very big $\frac{Q}{L} = \lambda$

Constant on a radius
distance is $\frac{1}{r}$
(diapole $\frac{1}{r^3}$) (pt charges $\frac{1}{r^2}$)

In-Class: Uniformly Charged Disk



P on axis of disk of charge, x from center
Radius R, charge density σ.

Find **E** at P

P02 -45

build a disk out of nested rings

Similar, but different

$\sigma = \frac{dq}{dA}$ $dq = \sigma\, dA$

$dE = \frac{k\sigma\, dA}{r^3}\vec{r}$

$E = \frac{k\sigma \iint dA}{(r^2+x^2)^{3/2}}\vec{r}$ $= E = \frac{k\sigma \pi r^2 \vec{r}}{(r^2+x^2)^{3/2}}$

$E = \frac{kQ}{(r^2+x^2)^{3/2}}\vec{r}$

Disk: Two Important Limits

messy $$\vec{\mathbf{E}}_{disk} = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{x}{\left(x^2+R^2\right)^{1/2}}\right]\hat{\mathbf{i}}$$

Limits:

$$\lim_{x>>R} \quad \vec{\mathbf{E}}_{disk} \xrightarrow{\cdots} \frac{1}{4\pi\varepsilon_o}\frac{Q}{x^2}\hat{\mathbf{i}}$$ Point charge

$$\lim_{x<<R} \quad \vec{\mathbf{E}}_{disk} \to \frac{\sigma}{2\varepsilon_o}\hat{\mathbf{i}}$$ Infinite charged plane is constant

P02-46

Scaling: E for Plane is Constant

general trend

1) Dipole: E falls off like $1/r^3$
2) Point charge: E falls off like $1/r^2$
3) Line of charge: E falls off like $1/r$
4) Plane of charge: E constant

P02-47

field is same -
does not change

white wall - how far are you from it
- don't know it's infinite

8.02 Notes
Columb's Law

2/7

$\rho = \frac{dq}{dV}$ Volume $Q = \int_V \rho(\vec{r})\,dV$

$\sigma = \frac{dq}{dA}$ surface $Q = \iint_S \sigma(\vec{r})\,dA$

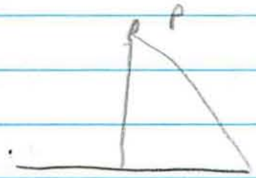
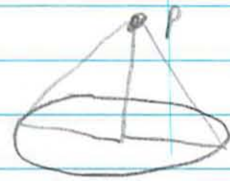
$\lambda = \frac{dq}{dl}$ line $Q = \int_{line} \lambda(\vec{r})\,dl$

$\vec{\tau} = \vec{p} \times \vec{E}$

diapole moment vector
- from $\ominus \to \oplus$
- $p = 2aq$

Coord systems
- cartesian $(x\, y\, z)$
- Cylindrical (ρ, ϕ, z)
- sphericcal (r, θ, ϕ)

nice summary notes 2-27 thats what I was looking for

	Line	Ring	Disk
			
Express dq terms charge density	$dq = \lambda\, dx'$	$dq = \lambda\, d\ell$	$dq = \sigma\, dA$
Write down dE	$dE = k_e \frac{\lambda\, dx'}{r'^2}$	$dE = k_e \frac{\lambda\, d\ell}{r^2}$	$dE = k_e \frac{\sigma\, dA}{r^2}$
Rewrite r and differentials w/ right coords	dx' $\cos\theta = \frac{y}{r'}$ $r' = \sqrt{x'^2+y^2}$	$d\ell = R\, d\phi'$ $\cos\theta = \frac{z}{r}$ $r = \sqrt{R^2+z^2}$	$dA = 2\pi r'\, dr'$ $\cos\theta = \frac{z}{r}$ $r = \sqrt{r'^2+z^2}$
Apply Symmetry to find non-vanishing dE	$dE_y = dE\cos\theta$ $= k_e \frac{\lambda y\, dx'}{(x'^2+y^2)^{3/2}}$	$dE_z = dE\cos\theta$ $= k_e \frac{\lambda R z\, d\phi'}{(R^2+z^2)^{3/2}}$	$dE_z = dE\cos\theta$ $= k_z \frac{2\pi\sigma z r'\, dr'}{(r'^2+z^2)^{3/2}}$
Integrate E	$E_y = k_e \lambda y \int_{-\ell/2}^{\ell/2} \frac{dx}{(x^2+y^2)^{3/2}}$ $= \frac{2k_e\lambda}{y} \frac{\ell/2}{\sqrt{(\ell/2)^2+y^2}}$	$E_z = k_e \frac{R\lambda z}{(R^2+z^2)^{3/2}} \int d\phi'$ $= k_e \frac{(2\pi R\lambda) z}{(R^2+z^2)^{3/2}}$ $= k_e \frac{Qz}{(R^2+z^2)^{3/2}}$	$E_z = 2\pi\sigma k_e z \int_0^R \frac{r'\, dr'}{(R'^2+z^2)^{3/2}}$ $= 2\pi\sigma k_e \left(\frac{z}{\lvert z\rvert} - \frac{z}{\sqrt{z^2+R^2}}\right)$

Summary of Class 04 **8.02**

Real, quality answers to paragraph (5 points)

Topics: Gauss's Law
Related Reading: Course Notes: Sections 4.1-4.2, 4.6

Topic Introduction

Did discrete + group fields

In this class we look at a new way of calculating electric fields – Gauss's law. Not only is Gauss's law (the first of four Maxwell's Equations) an exceptional tool for calculating the field from symmetric sources, it also gives insight into why E-fields have the r-dependence that they do.

Why interested - if put charge near it - will feel force $F = qE = ma$

The idea behind Gauss's law is that, pictorially, electric fields flow out of and into charges. If you surround some region of space with a closed surface (think bag), then observing how much field "flows" into or out of that surface tells you how much charge is enclosed by the bag. For example, if you surround a positive charge with a surface then you will see a net flow outwards, whereas if you surround a negative charge with a surface you will see a net flow inwards.

This is a easier way to calc E field -much more abstract -will teach surface integrals

Electric Flux

The picture of fields "flowing" from charges is formalized in the definition of the electric flux. For any flat surface of area A, the flux of an electric field $\vec{\mathbf{E}}$ through the surface is defined as $\Phi_E = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}}$, where the direction of $\vec{\mathbf{A}}$ is normal to the surface. This captures the idea that the "flow" we are interested in is *through* the surface – if $\vec{\mathbf{E}}$ is parallel to the surface then the flux $\Phi_E = 0$.

We can generalize this to non-flat surfaces by breaking up the surface into small patches which are flat and then integrating the flux over these patches. Thus, in general:

$$\Phi_E = \iint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

One of the most central laws

Gauss's Law

Gauss's law states that the electric flux through any closed surface is proportional to the total charge enclosed by the surface:

$$\Phi_E = \oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

A *closed* surface is a surface which completely encloses a volume, and the integral over a closed surface S is denoted by $\oiint_S$. < $\oiint$ square/rec does not matter

Symmetry and Gaussian Surfaces

Although Gauss's law is always true, as a tool for calculation of the electric field, it is only useful for highly symmetric systems. The reason that this is true is that in order to solve for the electric field $\vec{\mathbf{E}}$ we need to be able to "get it out of the integral." That is, we need to work with systems where the flux integral can be converted into a simple multiplication. Examples of systems that possess such symmetry and the corresponding closed *Gaussian surfaces* we will use to surround them are summarized below:

Symmetry	**System**	**Gaussian Surface**
Cylindrical	Infinite line	Coaxial Cylinder
Planar	Infinite plane	Gaussian "Pillbox"
Spherical	Sphere, Spherical shell	Concentric Sphere

Solving Problems using Gauss's law
Gauss's law provides a powerful tool for calculating the electric field of charge distributions that have one of the three symmetries listed above. The following steps are useful when applying Gauss's law:

(1) Identify the symmetry associated with the charge distribution, and the associated shape of "Gaussian surfaces" to be used.
(2) Divide space into different regions associated with the charge distribution, and determine the exact Gaussian surface to be used for each region. The electric field must be constant or known (i.e. zero) across the Gaussian surface.
(3) For each region, calculate q_{enc}, the charge enclosed by the Gaussian surface.
(4) For each region, calculate the electric flux Φ_E through the Gaussian surface.
(5) Equate Φ_E with $q_{\text{enc}} / \varepsilon_0$, and solve for the electric field in each region.

Important Equations

Electric flux through a surface *S*: $$\Phi_E = \iint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

Gauss's law: $$\Phi_E = \oint\!\!\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

Important Concepts

Gauss's Law applies to closed surfaces—that is, a surface that has an inside and an outside (e.g. a basketball). We can compute the electric flux through any surface, open or closed, but to apply Gauss's Law we must be using a closed surface, so that we can tell how much charge is inside the surface.

Gauss's Law is our first Maxwell's equations, and concerns closed surfaces. Another of Maxwell's equations, the magnetic Gauss's Law, $\Phi_B = \oint\!\!\oint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$, also applies to a closed surface. Our third and fourth Maxwell's equations will concern open surfaces, as we will see.

Class 04: Outline

Hours 1 & 2:

Working in Groups

Gauss' Law

P04 - 1

Groups

P04 - 2

Introduce Yourselves

Please discuss:

- What is your experience in E&M?
- What were the best group practices that you observed in 8.01?
- What do you expect/want from class?
- Did you have group issues in 8.01? If so, how to avoid them?

If you did not participate in TEAL style groups, please ask your group members to answer any questions you may have.

P04 - 3

Group Problem: Discovery Applet

Play with the applet and answer the worksheet questions

Electric field on surface d/t to both charge inside + outside
- always outward w/ ⊕ charge

but when outside total flux = 0
inside - based on sign so if have ⊕ ⊖ flux is 0
but E field still affected

Gauss's Law

The first Maxwell Equation!

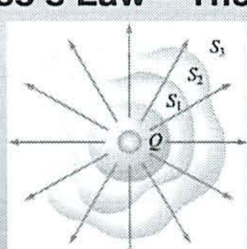
A very useful computational technique to find the electric field $\vec{E}$ when the source has 'enough symmetry'.

Only the total flux is affected by the inside only

the Gaussian surface is a tool to probe space

* Flux only due to charge inside
- everything outside goes in and out $\Sigma = 0$

Gauss's Law – The Idea



The total "flux" of field lines penetrating any of these closed surfaces is the same and depends only on the amount of charge inside

$\boxed{\text{Relationship b/w charges + field}}$

shape does not matter

≡ definition

Scalor equality
\# = #
dimensions =?

Gauss's Law – The Equation

flux →

$$\boxed{\Phi_E} \equiv \underset{\text{closed surface S}}{\oiint} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0}$$

Electric flux Φ_E (the surface integral of E over closed surface *S*) is proportional to charge inside the volume enclosed by *S*

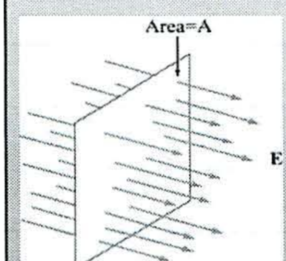
$\oint \vec{E} \cdot d\vec{a}$	$\frac{q_{in}}{\varepsilon_0}$
↑ will depend on surface	
=	charge inside surface you choose
can derive electric field	

Now the Details

open surface ~~charge~~

Electric Flux Φ_E

Case I: E is constant vector field perpendicular to planar surface S of area A



$$\Phi_E = \iint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

$$\Phi_E = +EA$$

Our Goal: Always reduce problem to this

constant - arrows same lenght
perpendicular to plane

how much charge perpendicular · area
↑ dot product
(how much of 1 vector in direction of other)

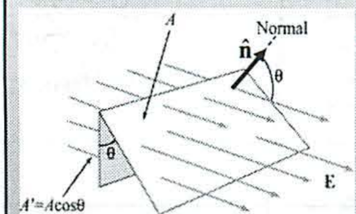
electric field = arrow in every pt in space

How to make A into a vector?

Choose a $\vec{N}$ perpendicular to area
Now an area is a vector
(helped in 18.02 today)

Electric Flux Φ_E

Case II: E is constant vector field directed at angle θ to planar surface S of area A



$$\Phi_E = \iint \vec{E} \cdot d\vec{A}$$

$$d\vec{A} = dA\,\hat{n}$$

$$\Phi_E = EA\cos\theta$$

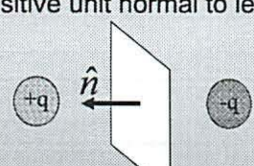
P04-10

E component that is perpendicular to surface (normal component)
• Area

PRS: Flux

15

The electric flux through the planar surface below (positive unit normal to left) is:



1. positive.
2. negative.
3. zero.
4. I don't know

P04-11

the arrow sets up what is (+) (−)

what is direction when have (+) (−)
- is a diapole
could set up field lines

* $\vec{E}$ goes from (+) → (−)
so dot product is (−)

No Guass Law since open

$\oiint$ = closed surface

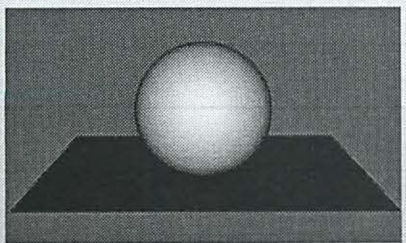
Gauss's Law

$$\Phi_E = \underset{\text{closed surface S}}{\oiint} \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

Note: Integral must be over closed surface

P04-12

Open and Closed Surfaces



A rectangle is an open surface — it does NOT contain a volume

A sphere is a closed surface — it DOES contain a volume

Convention: Normal vector always out

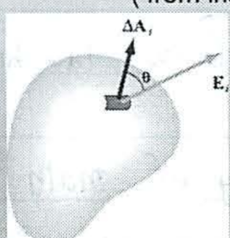
$$\oiint \vec{E} \cdot \hat{n}_{out}\, da$$

↑ if E points in (because ⊖ inside)
Flux is neg

$\boxed{⊕⊖}$ ← $\vec{E}$ Field $\neq 0$
ϕ Flux $= 0$

Area Element dA: Closed Surface

For closed surface, d**A** is normal to surface and points outward
(from inside to outside)



$\Phi_E > 0$ if E points out

$\Phi_E < 0$ if E points in

In this class $\oiint$ always 0 or EA
- never hard math

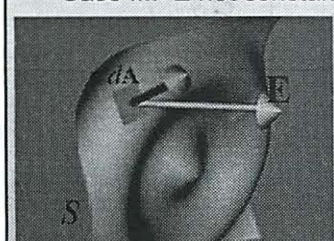
Abstract law
- sometimes right/left

$$\oiint E \cdot \hat{n}\, da = \frac{q_{in}}{\varepsilon_0}$$

$$E = \frac{q_{in}}{\varepsilon_0 A}$$

Electric Flux Φ_E

Case III: E not constant, surface curved



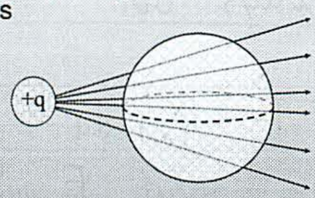
$$d\Phi_E = \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

$$\Phi_E = \iint d\Phi_E$$

PRS: Flux thru Sphere

15

The total flux through the below spherical surface is



0%	1. positive (net outward flux).
0%	2. negative (net inward flux).
0%	3. zero.
0%	4. I don't know

not inside

P04-16

Electric Flux: Sphere

Point charge Q at center of sphere, radius r

E field at surface:

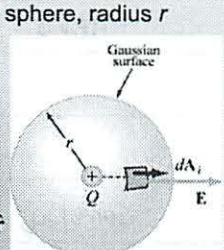
$$\vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_0 r^2}\,\hat{\mathbf{r}}$$

Electric flux through sphere:

$$\Phi_E = \oiint_S \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \oiint_S \frac{Q}{4\pi\varepsilon_0 r^2}\,\hat{\mathbf{r}}\cdot dA\,\hat{\mathbf{r}}$$

$$= \frac{Q}{4\pi\varepsilon_0 r^2}\oiint_S dA = \frac{Q}{4\pi\varepsilon_0 r^2}4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\boxed{d\vec{\mathbf{A}} = dA\,\hat{\mathbf{r}}}$$



P04-17

$4\pi r^2 = A$

field varries $\frac{1}{r^2}$ area has r^2

$\frac{r^2}{r^2}$ = constant

because of this - Gauss's law holds

$\frac{1}{r^2}$ is a cool thing of math

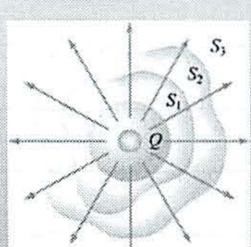
$\vec{E}$ outward, normal out) cos = 1 = 1

get integral of Area = Area

$$\frac{Q\,4\pi r^2}{4\pi\varepsilon_0 r^2} = \boxed{\frac{Q}{\varepsilon_0}}$$

sphere

Arbitrary Gaussian Surfaces



flux charge enclosed

$$\boxed{\Phi_E = \oiint_{\substack{\text{closed}\\ \text{surface S}}} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{Q}{\varepsilon_0}}$$

True for all surfaces such as S_1, S_2 or S_3

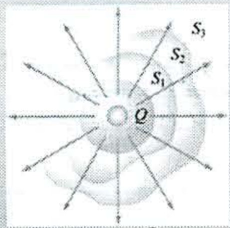
Why? As A gets bigger E gets smaller

P04-18

Why does it not depend on shape

~~th~~ - as area ↑ r^2 E field ↓ $\frac{1}{r^2}$

Choosing Gaussian Surface



$$\Phi_E = \underset{\substack{\text{closed} \\ \text{surface S}}}{\oiint} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q}{\varepsilon_0}$$

True for ALL surfaces

Useful (to calculate E) for SOME surfaces

✱ Desired **E**: Perpendicular to surface and constant on surface.
Flux is EA or -EA. ✱

Other **E**: Parallel to surface.
Flux is zero ← tangent

P04-19

Cylinder, sphere are examples

trying to find $\vec{E}$ field

-surface probes space

-want $\vec{E}$ perpendicular + constant

Symmetry & Gaussian Surfaces

Desired **E**: perpendicular to surface and constant on surface. So Gauss's Law useful to calculate E field from **highly symmetric sources**

Source Symmetry	Gaussian Surface
Spherical	Concentric Sphere
Cylindrical	Coaxial Cylinder
Planar	Gaussian "Pillbox"

P04-20

ρ, σ, λ

$Q = \rho \frac{4}{3}\pi R^3$

-spcically symetric source

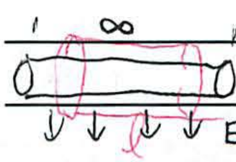
object charged - NOT surface

E is radially out/towards

If in the middle of sphere - good Gvassian

$\vec{E}$ perpendicular ✓
$\vec{E}$ constant ✓ so $\Phi = EA$

-answer is function of r/r variable



← endcap - ignore since ∞ long

↳ cylinder = guassian surface

Applying Gauss's Law

may need to calc multiple times →

1. Based on the source, identify regions in which to calculate E field.
2. Choose Gaussian surfaces S: Symmetry
3. Calculate $\Phi_E = \oiint \vec{E} \cdot d\vec{A}$
4. Calculate q_{in}, charge enclosed by surface S
5. Apply Gauss's Law to calculate E:

$$\Phi_E = \underset{\substack{\text{closed} \\ \text{surfaceS}}}{\oiint} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0}$$

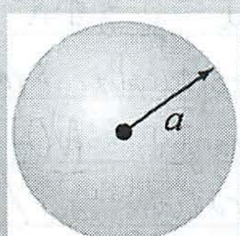
P04-21

Examples:
Spherical Symmetry
Cylindrical Symmetry
Planar Symmetry

PDI-22

Gauss: Spherical Symmetry

+Q uniformly distributed throughout non-conducting solid sphere of radius a. Find **E** everywhere



PDI-23

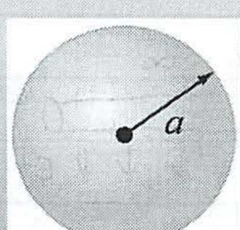
diff than 17 slide (- entire sphere
- charge distributed

Gauss: Spherical Symmetry

Symmetry is Spherical

$$\vec{\mathbf{E}} = E\,\hat{\mathbf{r}}$$

Use Gaussian Spheres



PDI-24

plane or slab

$+\sigma$ $\uparrow E$ $\downarrow E$

Only flux through end caps that matters

slab $\uparrow E$ $\downarrow E$ d A

Square pill box

Why not matter for plane - cylinder or pill box?

- ~~calcs proportional to area~~
~~and area~~ - area cancles

$E_{top} A + E_{bottom} A$

$2EA =$

- how much charge density enclosed?

$$2EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

$$\oint E\,dA = \frac{Q_{in}}{\epsilon_0} \qquad Q = \rho V \text{ - uniform}$$

don't need this one

$$E\,4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E = k\frac{Q}{r^2} \quad \rightarrow k = \frac{1}{4\pi\epsilon_0}$$

note

$$EA = \frac{Q}{\epsilon_0}$$

$$E\,4\pi a^2 = \frac{Q}{\epsilon_0}$$

$$E = k\frac{Q}{a^2}$$

answer slide 27

$\rho = br$ ← non uniform charge distribution

On p-set

$$E\,4\pi r^2 = \frac{\iiint \rho\, dV}{\varepsilon_0}$$

upper limit

$$\frac{\int_0 \rho\, 4\pi r^2\, dr}{\varepsilon_0}$$

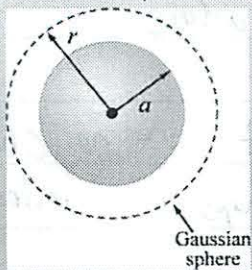
← must integrate 1 dimension

* special case

Gauss: Spherical Symmetry

Region 1: $r > a$

Draw Gaussian Sphere in Region 1 ($r > a$)

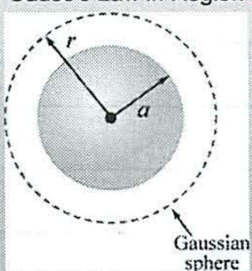


Note: r is arbitrary **but** is the radius for which you will calculate the E field!

Group Problem: Outside Sphere

Region 2: $r > a$

Use Gauss's Law in Region 2 ($r > a$)



Again: Remember that r is arbitrary **but** is the radius for which you will calculate the E field!

Gauss: Spherical Symmetry

Region 2: $r < a$

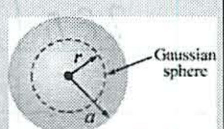
Total charge enclosed:

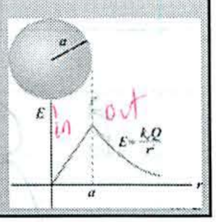
$$q_{in} = \left(\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi a^3}\right)Q = \left(\frac{r^3}{a^3}\right)Q \quad \text{OR} \quad q_{in} = \rho V$$

Gauss's law:

$$\Phi_E = E\left(4\pi r^2\right) = \frac{q_{in}}{\varepsilon_0} = \left(\frac{r^3}{a^3}\right)\frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0}\frac{r}{a^3} \quad \boxed{\Rightarrow \vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_0}\frac{r}{a^3}\hat{\mathbf{r}}}$$





people who get this right draw the gaussian surface

* ← graph * (note: $\vec{E}$ not flux)

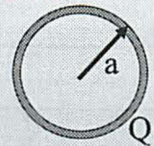
piecewise function

* r ↑ linearly

$-\frac{1}{r^2}$ ← when outside like a pt charge

PRS: Spherical Shell

We just saw that in a solid sphere of charge the electric field grows linearly with distance. Inside the charged spherical shell at right (r<a) what does the electric field do?



0% 1. Constant and Zero
0% 2. Constant but Non-Zero
0% 3. Still grows linearly
0% 4. Some other functional form (use Gauss' Law)
0% 5. Can't determine with Gauss Law

its like having a charge outside the surface

flux $= 0 = EA$ ← no charge inclosed → in and out

field $\neq 0$ perhaps

$\oint \vec{E} \cdot d\vec{a}$	$\frac{q_{in}}{\epsilon}$
0	0

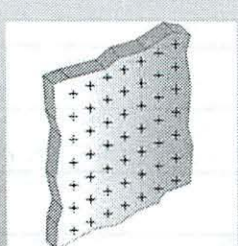
E field also Spherically symetric

Demonstration
Field Inside Spherical Shell (Grass Seeds):

Gauss: Planar Symmetry

Infinite slab with uniform charge density σ

Find **E** outside the plane



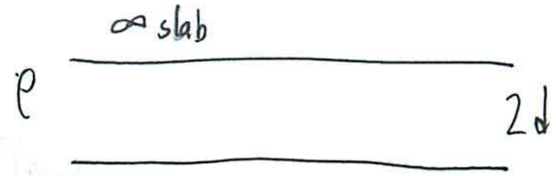
$+\sigma$ $x=0$

$2EA$	$\frac{\sigma A}{\epsilon_0}$

$$\vec{E} = \begin{cases} \frac{\sigma}{2\epsilon_0}\hat{i} & x>0 \\ -\frac{\sigma}{2\epsilon_0}\hat{i} & x<0 \end{cases}$$

↓ same problem

* you don't need a drawling in 3D look at cross section *



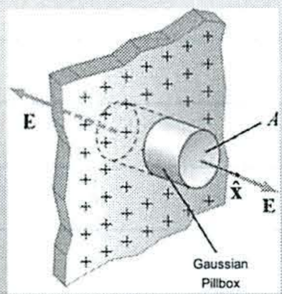
Gauss: Planar Symmetry

Symmetry is Planar

$$\vec{\mathbf{E}} = \pm E\,\hat{\mathbf{x}}$$

Use Gaussian Pillbox

Note: A is arbitrary (its size and shape) and should divide out



P04-31

↑ same problem

Gauss: Planar Symmetry

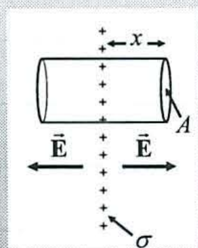
Total charge enclosed: $q_{in} = \sigma A$

NOTE: No flux through side of cylinder, only endcaps ← why?

$$\Phi_E = \oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E \oiint_S dA = EA_{Endcaps}$$

$$= E(2A) = \frac{q_{in}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0} \boxed{\Rightarrow \vec{\mathbf{E}} = \frac{\sigma}{2\varepsilon_0}\begin{Bmatrix} \hat{\mathbf{x}} & \text{to right} \\ -\hat{\mathbf{x}} & \text{to left} \end{Bmatrix}}$$



P04-32

↑ or up + down

E for Plane is Constant????

1) Dipole: E falls off like $1/r^3$
2) Point charge: E falls off like $1/r^2$
3) Line of charge: E falls off like $1/r$
4) Plane of charge: E constant

P04-33

E is a constant, infinite Electric field

but would have ∞ energy, so planes can not be ∞
will approx ∞ to make it eaiser to calculate

$\circ E$

$\circ E$ all constant, =

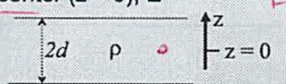
$\circ E$ q above

PRS: Slab of Charge

, not up/down/thickness

Consider positive, semi-infinite (in x & y) flat slab z-axis is perp. to the sheet, with center at z = 0. At the plane's center (z = 0), **E**

$E \uparrow$



$E \downarrow$

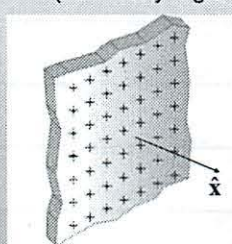
- 0% 1. points in the positive z-direction.
- 0% 2. points in the negative z-direction.
- 0% 3. points in some other (x,y) direction.
- 0% 4. is zero.
- 0% 5. I don't know

P04 -34

E 0 in center - reversing direction

Group Problem: Charge Slab

Infinite slab with uniform charge density ρ
Thickness is 2d (from x=-d to x=d).
Find **E** for x > 0 (how many regions is that?)



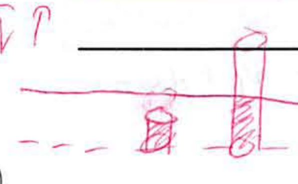
P04 -35

2 regions - inside + outside

$$Q_{in} = \rho V = \frac{\rho A x}{\varepsilon_0} = EA$$

$$\vec{E} = \frac{\rho x}{\varepsilon_0}\hat{x}$$

$\uparrow\uparrow$



Volume that encloses charge

$0 < x < d$	$x < d$
$EA = \frac{\rho x A}{\varepsilon_0}$	$EA = \frac{\rho d A}{\varepsilon_0}$
$\vec{E} = \frac{\rho x}{\varepsilon_0}\hat{\imath}$	$\vec{E} = \frac{\rho d}{\varepsilon_0}\hat{\imath}$

incs total charge

does not inc total charge

- trying to calc inside still don't really understand

Gauss: Cylindrical Symmetry

Infinitely long rod with uniform charge density λ
Find **E** outside the rod.

P04 -36

$$\vec{E} A = \frac{q_{in}}{\varepsilon_0} \qquad q_{in} = \lambda L$$

$$\vec{E} L(2\pi r) = \frac{\lambda L}{\varepsilon_0}$$

$$\vec{E} = \frac{\lambda}{2\pi r \varepsilon_0}\hat{r}$$

Guassian Surface should be Not -don't do it!

Know the areas of shapes
-sometimes stuff cancels

Gauss: Cylindrical Symmetry

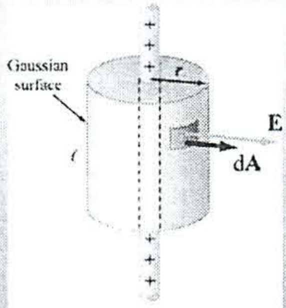
Symmetry is Cylindrical

$$\vec{\mathbf{E}} = E\,\hat{\mathbf{r}}$$

Use Gaussian Cylinder

Note: r is arbitrary **but** is the radius for which you will calculate the E field!

ℓ is arbitrary and should divide out



P04-37

Gauss: Cylindrical Symmetry

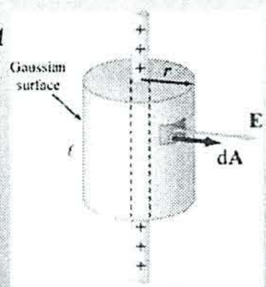
Total charge enclosed: $q_{in} = \lambda \ell$

$$\Phi_E = \oint\!\!\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E \oint\!\!\oint_S dA = EA$$

$$= E\left(2\pi r \ell\right) = \frac{q_{in}}{\varepsilon_0} = \frac{\lambda \ell}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \quad \boxed{\Rightarrow \vec{\mathbf{E}} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{\mathbf{r}}}$$

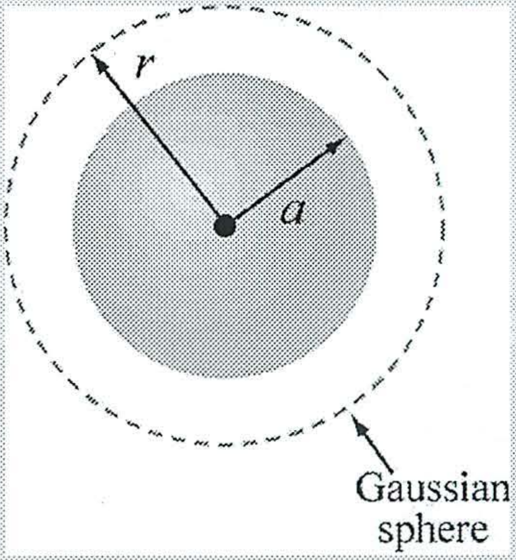
Side of guassian surface



P04-38

IC-Sol-W05D2-2

Gauss: Spherical Symmetry



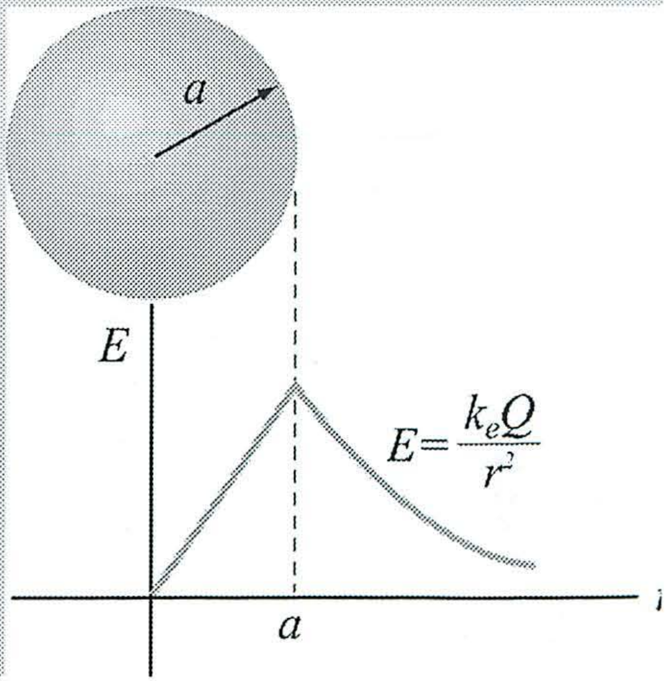
Region 2: $r > a$

Total charge enclosed $q_{in} = +Q$

$$\Phi_E = \oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E \oiint_S dA = EA$$
$$= E\left(4\pi r^2\right)$$

$$\Phi_E = 4\pi r^2 E = \frac{q_{in}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} \boxed{\Rightarrow \vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}}$$



MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics: 8.02

In Class W05D2_3 Solutions: Field from a Slab of Charge

Question:
A semi-infinite slab of charge with charge density ρ extends from $x = -d$ to $x = +d$. Find the electric field everywhere.

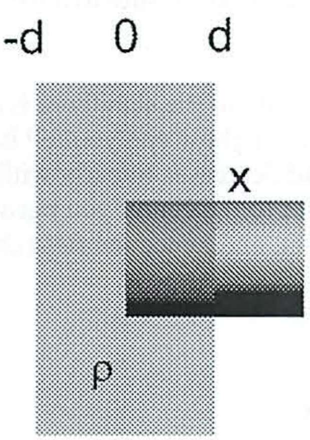
Solution:
1. Draw Picture
In the interest of saving space I only show the pictures with Gaussian surfaces drawn (see below)

2. Think
Considering symmetry, we note that the electric field at the center of the slab must be zero. To see this imagine putting a test charge right at the center of the slab. It will feel no net force (it would be pushed to the right by the charge to the left exactly as much as it would be pushed to the left by the charge to the right), so the electric field there must be zero.

The symmetry is planar so we will use Gaussian pillboxes (cylinders of cross-section A and height x) and will place one end of the pillbox at x=0 to take advantage of the fact that $\boldsymbol{E}$=0 there.

There are two distinct regions of space, inside and outside of the slab. By symmetry the magnitude of the field will be the same on the left as on the right of the slab, but will point in the opposite direction. We will only calculate explicitly for $x > 0$.

2. Calculate for Each Region
Region 1: Outside the slab ($x > d$)



The charge within this pillbox is $Q_{enc} = \rho V_{enc} = \rho A d$.
The flux (integral of the electric field over this pillbox) is zero on the sides (because **E** is perpendicular to the area normal there) and zero on the left end (because **E** is zero there). Thus:

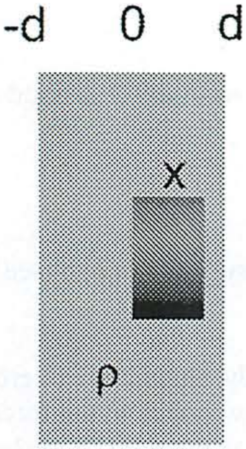
$$\oiint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \iint_{\text{sides}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \iint_{\text{left endcap}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \iint_{\text{right endcap}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 0 + 0 + EA$$

Applying Gauss's Law:

$$\oiint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EA = \frac{Q_{enc}}{\varepsilon_0} = \frac{\rho A d}{\varepsilon_0} \Rightarrow E = \frac{\rho d}{\varepsilon_0}$$

d is fixed

Region 2: Inside the slab ($x \le d$)



The charge within this pillbox is $Q_{enc} = \rho V_{enc} = \rho A x$.
As in region 1, the flux is given by:

$$\oiint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \iint_{\text{sides}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \iint_{\text{left endcap}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \iint_{\text{right endcap}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 0 + 0 + EA$$

Applying Gauss's Law:

$$\oiint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EA = \frac{Q_{enc}}{\varepsilon_0} = \frac{\rho A x}{\varepsilon_0} \Rightarrow E = \frac{\rho x}{\varepsilon_0}$$

X varies as far as you go in

Summarizing (and using symmetry to get **E** for $x < 0$):

$$\vec{\mathbf{E}} = \begin{cases} \dfrac{\rho d}{\varepsilon_o} \hat{\mathbf{i}} & \text{for } x \ge d \\ \dfrac{\rho x}{\varepsilon_o} \hat{\mathbf{i}} & \text{for } -d < x < d \\ -\dfrac{\rho d}{\varepsilon_o} \hat{\mathbf{i}} & \text{for } x \le d \end{cases}$$

Note that we explicitly insert the negative sign for x outside the slab on the left, but inside the slab on the left the negative sign of x itself takes care of the direction. Ignoring these signs is a common source of problems – always check a few concrete cases to make sure that the field as written points in the direction you think it should.

You should also check that the x-dependence makes sense. Outside of the slab there is no x-dependence. We have seen that this is the case for planes of charge (how can you tell how far away you are from a giant white wall?). Inside the slab the field decreases linearly with x as you approach the origin. This also makes sense – as you come closer to the center you become more and more balanced in the amount of charge on your left and right, and hence the field should decrease.

Michael Plasmeier

You don't have to staple your printed pset to your answers :)

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2010**

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Problem Set 1

Due: Tuesday, February 9 at 9 pm.

Hand in your problem set in your section slot in the boxes outside the door of 32-082. Make sure you clearly write your name and section on your problem set.

You might want to go to office hours to clear some concepts, Great job though! You'll get it.

Text: Liao, Dourmashkin, Belcher; Introduction to E & M MIT 8.02 Course Notes.

Buy 8.02 Course Reader at Copy Tech 11-004 and bring it with you to every class!

Reading Assignments:

Week One Introduction to Teal, Introduction Gravitational and Electric Fields

Class 1 TW Feb 2/3, Reading:	Introduction to Teal, Gravitational and Electric Fields Course Notes: Sections 1.1 – 1.6; 1.8; Chapter 2
Class 2 R/M Feb 4/8 Reading:	Electric Fields and Continuous Charge Distributions Course Notes Section 1.6; Chapter 2
Class 3 F Feb 5 Reading:	PS01: Math Review, Fields, Continuous Charge Distributions Course Notes: Chapter 2 Coulomb's Law Section 2.9-2.12

Optional Introduction/Review for Vector Calculus:
Spring 2006 Math Review Presentation,
Hale Bradt's Spring 2001 8.02 Mathematics Supplement

Week Two: Gauss's Law and Electric Potential

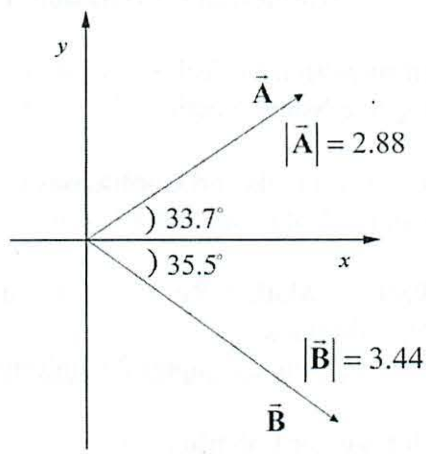
Class 4 T/W Feb 9/10 Reading:	Gauss' Law Course Notes: Sections 4.1-4.2, 4.6
Class 5 R/T Feb 11/16 Reading:	Electric Potential Course Notes: Sections 3.1-3.5
Class 6 F Feb 12 Reading:	PS02: Gauss's Law Course Notes: Sections 4.1-4.2, 4.7-4.8

Week Three: Electric Potential

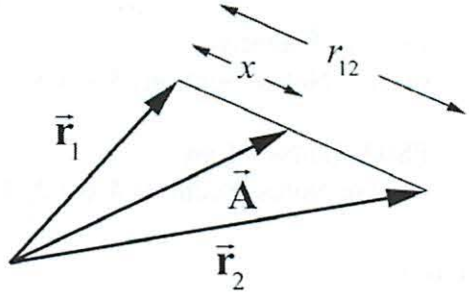
President's Day – M 2/15 / M Classes on T 2/16

Problem 1: Vectors (10 points) Consider the two vectors shown in the figure below. The magnitude of $|\vec{\mathbf{A}}| = 2.88$ and the vector $\vec{\mathbf{A}}$ makes an angle 33.7° with the positive x-axis. The magnitude of $|\vec{\mathbf{B}}| = 3.44$ and the vector $\vec{\mathbf{B}}$ makes an angle 35.5° with the positive x-axis pointing down to the right as shown in the figure below. Find the x and y components of

a) $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$;
b) $\vec{\mathbf{A}} + \vec{\mathbf{B}}$;
c) $\vec{\mathbf{A}} - \vec{\mathbf{B}}$;
d) a unit vector pointing in the direction of $\vec{\mathbf{A}}$;
e) a unit vector pointing in the direction of $\vec{\mathbf{B}}$.



Problem 2 Vectors (10 points) Consider two points located at $\vec{\mathbf{r}}_1$ and $\vec{\mathbf{r}}_2$, separated by distance $r_{12} = |\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2|$. Find a vector $\vec{\mathbf{A}}$ from the origin to the point on the line between $\vec{\mathbf{r}}_1$ and $\vec{\mathbf{r}}_2$ at a distance x from the point at $\vec{\mathbf{r}}_1$, where x is some number. Express your answer in terms of $\vec{\mathbf{r}}_1$, $\vec{\mathbf{r}}_2$, r_{12}, and x. Show your work.



Problem 3 Concept Questions (10 points)

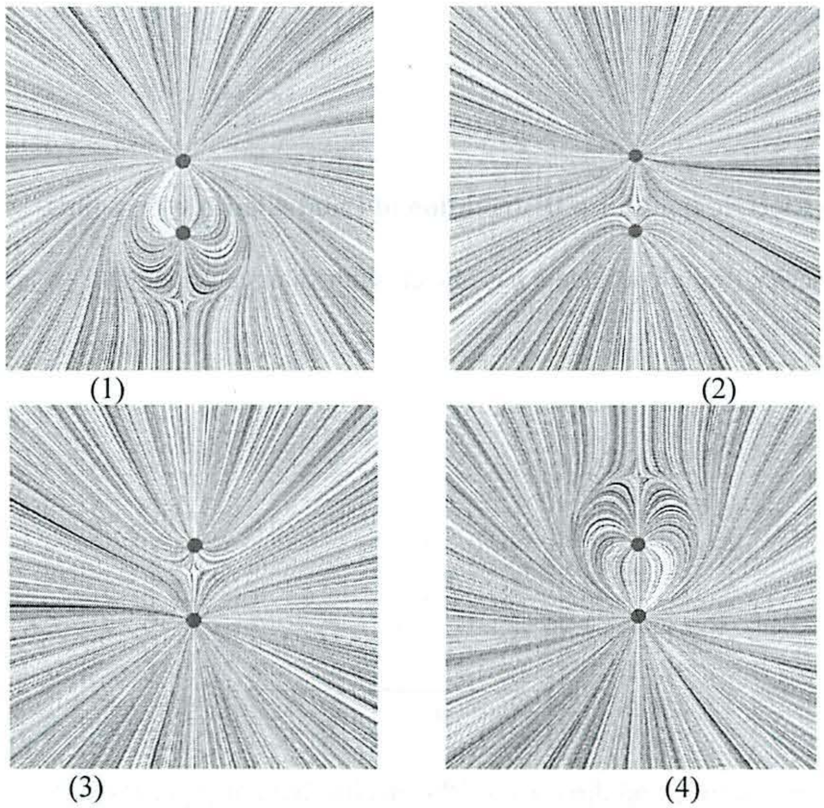
(a) (5 points) Two objects with charges $-q$ and $+3q$ are placed on a line as shown in the figure below.

Besides an infinite distance away from the charges, where else can the electric field possibly be zero?

1. Between the two charges.
2. To the right of the charge on the right.
3. To the left of the charge on the left.
4. The electric field is only zero an infinite distance away from the charges.

Explain your reasoning.

(b) (5 points). Two objects with charges $-4Q$ and $-Q$ lie on the y-axis. The object with the charge $-4Q$ is ***above*** the object with charge $-Q$. Below are four possible "grass seed" representations of the electric field of the two charges. Which of these representations is most nearly right for the two charges in this problem?



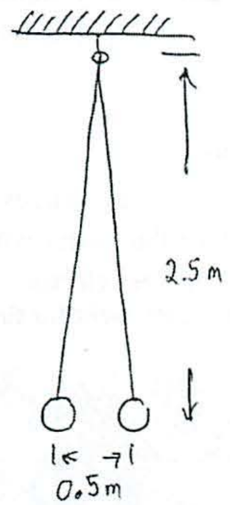
Explain your reasoning.

Problem 4: Ratio of Electric and Gravitational Forces (10 points)

What is the ratio of the magnitudes of the electric force and the gravitational force between two protons if the protons are separated by a distance r? In SI units the magnitude of the charge of the proton is $e = 1.6\times10^{-19}\ \text{C}$ and the mass of the proton is $m_p = 1.67\times10^{-27}\ \text{kg}$.

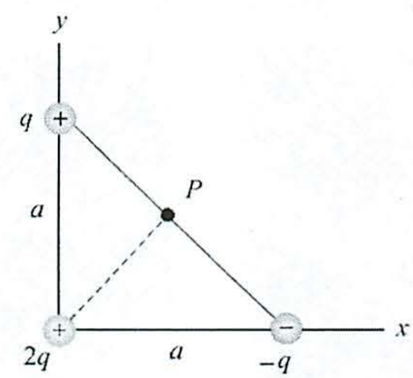
Problem 5: Coulomb's Law (10 points)

Two volley balls, each of mass $m = 0.2\ \text{kg}$, tethered by nylon strings and equally charged with an electrostatic generator, hang as shown in the figure such that the centers of the balls are a distance $r = 0.5\ \text{m}$ apart. The point equidistance between the two centers of the balls is a distance $d = 2.5\ \text{m}$ below the suspension point. What is the charge on each ball? Include your free-body force diagram in your solution.



Problem 6 Electric field for a Distribution of Point Charges (10 points)

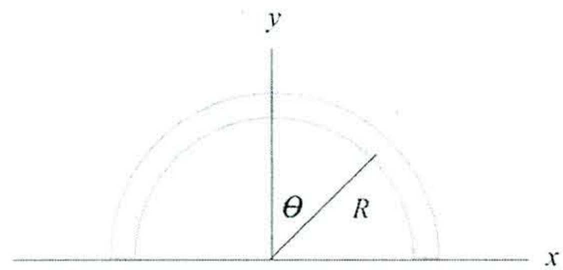
A right isosceles triangle of side a has objects with charges q, $+2q$ and $-q$ arranged on its vertices, as shown in the figure below.



What is the magnitude and direction of the electric field at point P due to the charges in the figure, midway between the line connecting the $+q$ and $-q$ charges?

Problem 7 Electric Field and Force (10 points)

A positively charged wire is bent into a semicircle of radius R, as shown in the figure below.



The total charge on the semicircle is Q. However, the charge per unit length along the semicircle is non-uniform and given by $\lambda = \lambda_0 \cos\theta$.

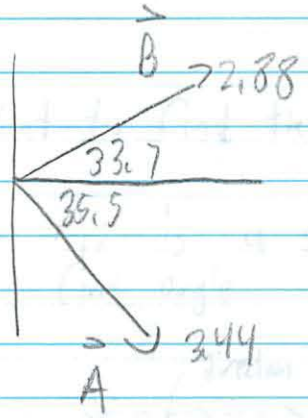
a) What is the relationship between λ_0, R and Q?

b) If a particle with a charge q is placed at the origin, what is the total force on the particle? Show all your work including setting up and integrating any necessary integrals.

8.02 P-Set 1

Michael Plaumeier 2/5

1a.



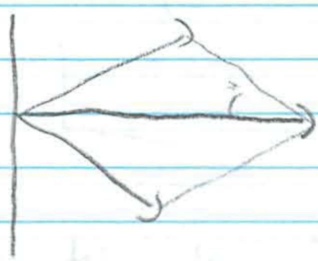
$\vec{A}$ in $x = 2.88 \sin 33.7 = 1.5979$

A in $y = 2.88 \cos 33.7 = 2.396$

$\vec{B}$ in $x = 3.44 \sin 35.5 = 1.9976$

$\vec{B}$ in $y = 3.44 \cos 35.5 = -2.800$

b) Parallelagram

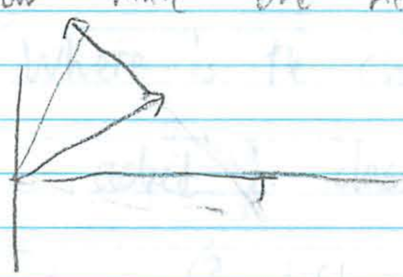


$\vec{C} = \langle a_1 + b_1, a_2 + b_2 \rangle$

$\langle 3.9939, -.8024 \rangle$

(direction angle can be found w/ tan-dub)

c) Now make one negitive, tail method



a - b - right

$\langle -.7981, 4.7976 \rangle$

d,e) unit vector is direction $\frac{\langle a_1, a_2 \rangle}{\sqrt{a_1^2 + a_2^2}}$

$\frac{\langle 1.5979, 2.396 \rangle}{\sqrt{1.59^2 + 2.39^2}}$ $\frac{\langle 1.9976, -2.800 \rangle}{\sqrt{1.9976^2 + (-2.8)^2}}$

2. Want to find the vector value

r_{12} is a scalar value
find angle of $r_1 - r_2 = \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1| - |\vec{r}_2|}$ = direction

$$\vec{r}_1 + \left(\underbrace{\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1| - |\vec{r}_2|}}_{\text{direction}} \quad \underbrace{x}_{\text{value}} \right) = \vec{A}$$

3a. Into this class

$-q$ ———— $3q$

Where is the charge 0?

- asked in class identically

It is 3 left of charge on left
Both fields are =

3q is stronger, but further away
q is closer

Only one point

b. $-4Q$ (#2) They are both $\ominus$, so should repel
$-Q$ The top should be stronger.

4. What is the ratio of magnitudes of electric force and gravitational force seperated by distance r

$e = 1.6 \times 10^{-19}\ C$
$m_p = 1.67 \cdot 10^{-27}\ kg$

Electric

Gravitation

Create

$k_e \frac{q_1}{r^3} \vec{r}$

$-G \frac{M}{r^3} \vec{r}$

$-\frac{9.8 \cdot 1.67 \cdot 10^{-27}}{r^3}$

k_e = Coulomb's Force Constant
$8.98 \cdot 10^{9}\ N \cdot m^2 \cdot C^{-2}$

$-\frac{8.98 \cdot 10^{9} \cdot 1.6 \cdot 10^{-19}}{r^3}$

$\cdot r^3 \cdot -1$

$1.43 \cdot 10^{-9}$

$\cdot r^3 \cdot -1$

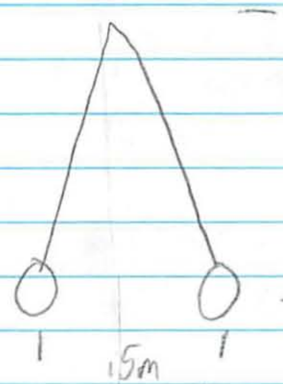
$1.63 \cdot 10^{-26}$

↑ much stronger

$\frac{1.43 \cdot 10^{-9}}{1.63 \cdot 10^{-26}} = 8.77 \cdot 10^{16}$ times larger
(Electric is)

$F = \frac{k q_1 q_2}{r^2}\hat{r} = \frac{k q_1 q_2}{r^3}\vec{r}$

5.



charged w/ electrostatic force

charge in each ball = q
but how know total F?
↑ the force of gravity on 1

$F = m \cdot 9.8$
$F = .2 \cdot 9.8$
$F = 1.96\ N$

$1.96 = \frac{k q_1 q_2}{.5^2}$

$.49 = k q_1 q_2$
$8.97 \cdot 10^9$

13

$q = \cancel{7.38 \cdot 10^{-6}\ C}$

seperate into a place where time does not flow

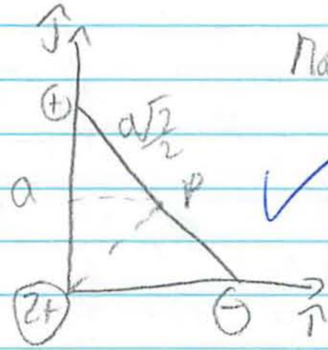
print 18.02 day-by-day

copy class summaries

I am going paper heavy this semester

Must make everyday count perfectly - be on top of game

6.



Magnitude + direction at point p

- its the superposition of all charges
- + test charge

* remember from class today: denom always the same - put distance in numerator

easy now 3 days later

$\frac{k \cdot 1 \cdot 1 \cdot \frac{a}{2}}{\left(\frac{a\sqrt{2}}{2}\right)^2}\hat{i} - \frac{k \cdot 1 \cdot 1 \cdot \frac{a}{2}}{\left(\frac{a\sqrt{2}}{2}\right)^2}\hat{j} + \frac{k \cdot 2 \cdot 1 \cdot \frac{a}{2}}{\left(\frac{a\sqrt{2}}{2}\right)^2}\hat{i} + \frac{k \cdot 2 \cdot 1 \cdot \frac{a}{2}}{\left(\frac{a\sqrt{2}}{2}\right)^2}\hat{j}$

$$-\frac{k\cdot 1\cdot -1\frac{a}{2}}{\left(\frac{a\sqrt{2}}{2}\right)^2}\hat{i} - \frac{k\cdot 1\cdot -1\frac{a}{2}}{\left(\frac{a\sqrt{2}}{2}\right)^2}\hat{j}$$

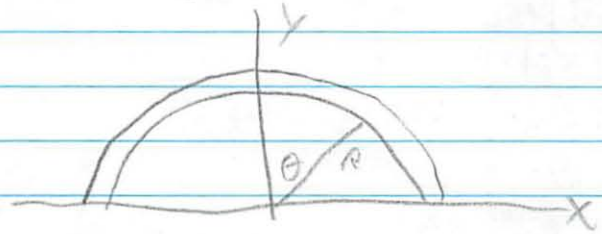
$$\frac{4k\frac{a}{2}a}{\left(\frac{a\sqrt{2}}{2}\right)^2}\hat{i} - \frac{2k\frac{a}{2}a}{\left(\frac{a\sqrt{2}}{2}\right)^2}\hat{j}$$

$$\vec{E} = \frac{4k\frac{a}{2}a}{\left(\frac{a\sqrt{2}}{2}\right)^3}\hat{i} - \frac{2k\frac{a}{2}a}{\left(\frac{a\sqrt{2}}{2}\right)^2}\hat{j}$$

? is that right
wish I had istant feedback

7. Posivitly charged wire in a semicircle of R

12



$\lambda = \lambda_0 \cos\theta$

$\lambda = \frac{dQ}{dL}$ $\quad dL = R\,d\theta$

$\lambda = R\frac{dQ}{d\theta} \rightarrow \frac{dQ}{R\,d\theta} = \lambda_0\cos\theta$

$dQ = \lambda_0 \cos\theta\, R\,d\theta$

total charge = Q
charge per unit lenght non uniform $\lambda = \lambda_0 \cos\theta$

$Q = 2\int_0^{\pi/2} \lambda_0 \cos\theta\, R\,d\theta$
$= 2\lambda_0 R$

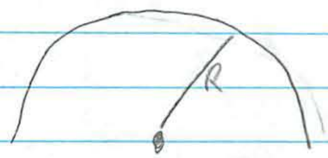
a) Relationship λ_0, R, Q ?

So λ is normally $\frac{Q}{L}$ but it is not even close,

lenght $= \frac{1}{2} 2\pi R = \pi R$
λ_0 = charge at $\cos\theta = 1$ or 0, 180°
Q = total charge $= \lambda \pi R$
λ = charge at a certain point

b. If a charge q is placed at the origin, what is the total force on the particle?

Wish more problems like this



The idea is to figure out $\frac{dQ}{dL} = \lambda_\theta$

$F =$

$$\lambda = \lambda_0 \cos\theta$$

$$F = \sum \frac{k q_r q}{r^2}$$

$$\frac{dQ}{dL} = 2\int_0^{\pi/2} \lambda_0 \cos\theta \, R \, d\theta$$

$$= 2\lambda_0 R$$

$$F = \frac{k}{r^2} \int dq \, q$$

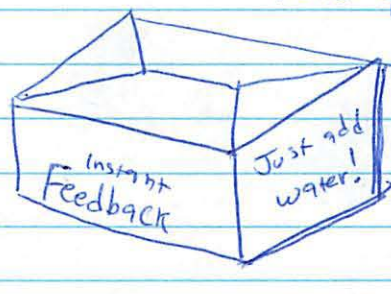
constants

$$F = \frac{kq}{R^2} \int \lambda_0 \cos\theta \, dL$$

wish I had instant feedback on this

here you go:

$$F = -\frac{kqQ}{R^2}$$



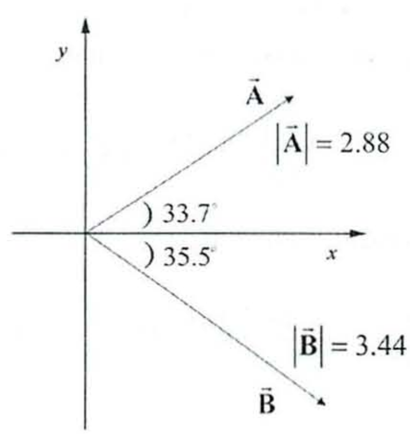
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2010**

Problem Set 1 Solutions

Problem 1: Vectors (10 points) Consider the two vectors shown in the figure below. The magnitude of $|\vec{\mathbf{A}}| = 2.88$ and the vector $\vec{\mathbf{A}}$ makes an angle 33.7° with the positive x-axis. The magnitude of $|\vec{\mathbf{B}}| = 3.44$ and the vector $\vec{\mathbf{B}}$ makes an angle 35.5° with the positive x-axis pointing down to the right as shown in the figure below. Find the x and y components of

a) $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$;
b) $\vec{\mathbf{A}} + \vec{\mathbf{B}}$;
c) $\vec{\mathbf{A}} - \vec{\mathbf{B}}$;
d) a unit vector pointing in the direction of $\vec{\mathbf{A}}$;
e) a unit vector pointing in the direction of $\vec{\mathbf{B}}$.



Solution: We need to use $\theta_A = 33.7^\circ$ in order to determine the x and y components of the vector $\vec{\mathbf{A}}$:

$$A_x = |\vec{\mathbf{A}}| \cos\theta_A = (2.88)(\cos(33.7^\circ) = 2.40,$$
$$A_y = |\vec{\mathbf{A}}| \sin\theta_A = (2.88)(\sin(33.7^\circ) = 1.60.$$

Thus

$$\vec{\mathbf{A}} = 2.40\,\hat{\mathbf{i}} + 1.60\,\hat{\mathbf{j}}\ .$$

We need to use $\theta_B = -35.5^\circ$ in order to determine the x and y components of the vector $\vec{\mathbf{B}}$:

$$B_x = \left|\vec{\mathbf{B}}\right| \cos\theta_B = (3.44)(\cos(-35.5^\circ) = 2.80\,,$$
$$B_y = \left|\vec{\mathbf{B}}\right| \cos\theta_B = (3.44)(\sin(-35.5^\circ) = -2.00\,.$$

Thus

$$\vec{\mathbf{B}} = 2.80\ \hat{\mathbf{i}} - 2.00\,\hat{\mathbf{j}}\,.$$

b) The vector sum is then

$$\begin{aligned}\vec{\mathbf{A}} + \vec{\mathbf{B}} &= (2.40\,\hat{\mathbf{i}} + 1.60\,\hat{\mathbf{j}}) + (2.80\ \hat{\mathbf{i}} - 2.00\,\hat{\mathbf{j}})\\ &= (5.20)\,\hat{\mathbf{i}} + (-.40)\,\hat{\mathbf{j}}\end{aligned}$$

c) The vector difference is

$$\begin{aligned}\vec{\mathbf{A}} - \vec{\mathbf{B}} &= (2.40\,\hat{\mathbf{i}} + 1.60\,\hat{\mathbf{j}}) - (2.80\ \hat{\mathbf{i}} - 2.00\,\hat{\mathbf{j}})\\ &= (-.40)\,\hat{\mathbf{i}} + (3.60)\,\hat{\mathbf{j}}\end{aligned}$$

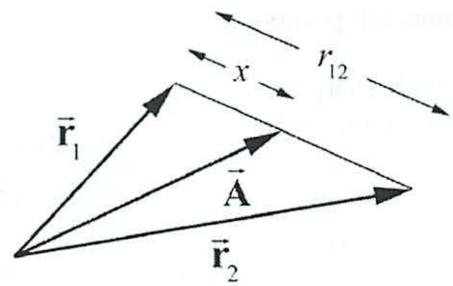
d) The unit vector pointing in the direction of $\vec{\mathbf{A}}$ is given by

$$\hat{\mathbf{A}} = \frac{\vec{\mathbf{A}}}{\left|\vec{\mathbf{A}}\right|} = \frac{\vec{\mathbf{A}} = 2.40\,\hat{\mathbf{i}} + 1.60\,\hat{\mathbf{j}}}{2.88} = 0.83\ \hat{\mathbf{i}} - 0.69\,\hat{\mathbf{j}}$$

e) The unit vector pointing in the direction of $\vec{\mathbf{B}}$ is given by

$$\hat{\mathbf{B}} = \frac{\vec{\mathbf{B}}}{\left|\vec{\mathbf{B}}\right|} = \frac{2.80\ \hat{\mathbf{i}} - 2.00\,\hat{\mathbf{j}}}{3.44} = 0.81\,\hat{\mathbf{i}} - 0.58\,\hat{\mathbf{j}}$$

Problem 2 Vectors (10 points) Consider two points located at $\vec{\mathbf{r}}_1$ and $\vec{\mathbf{r}}_2$, separated by distance $r_{12} = \left|\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2\right|$. Find a vector $\vec{\mathbf{A}}$ from the origin to the point on the line between $\vec{\mathbf{r}}_1$ and $\vec{\mathbf{r}}_2$ at a distance x from the point at $\vec{\mathbf{r}}_1$, where x is some number. Express your answer in terms of $\vec{\mathbf{r}}_1$, $\vec{\mathbf{r}}_2$, r_{12}, and x. Show your work.



Solution: Consider the unit vector pointing from $\vec{\mathbf{r}}_1$ and $\vec{\mathbf{r}}_2$ given by

$$\hat{\mathbf{r}}_{12} = \vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2 / \left|\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2\right| = \vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2 / r_{12}.$$

The vector $\vec{\alpha}$ in the figure connects $\vec{\mathbf{A}}$ to the point at $\vec{\mathbf{r}}_1$, therefore we can write

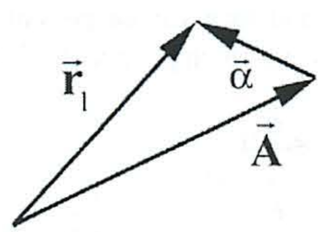
$$\vec{\alpha} = x\hat{\mathbf{r}}_{12} = \frac{x}{r_{12}}(\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2).$$

The vector

$$\vec{\mathbf{r}}_1 = \vec{\mathbf{A}} + \vec{\alpha}.$$

Therefore

$$\vec{\mathbf{A}} = \vec{\mathbf{r}}_1 - \vec{\alpha} = \vec{\mathbf{r}}_1 - \frac{x}{r_{12}}(\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2) = \vec{\mathbf{r}}_1\left(1 - \frac{x}{r_{12}}\right) + \frac{x}{r_{12}}\vec{\mathbf{r}}_2.$$



Problem 3 Concept Questions (10 points)

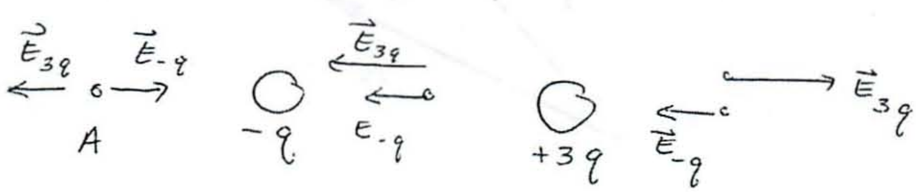
(a) (5 points) Two objects with charges $-q$ and $+3q$ are placed on a line as shown in the figure below.

Besides an infinite distance away from the charges, where else can the electric field possibly be zero?

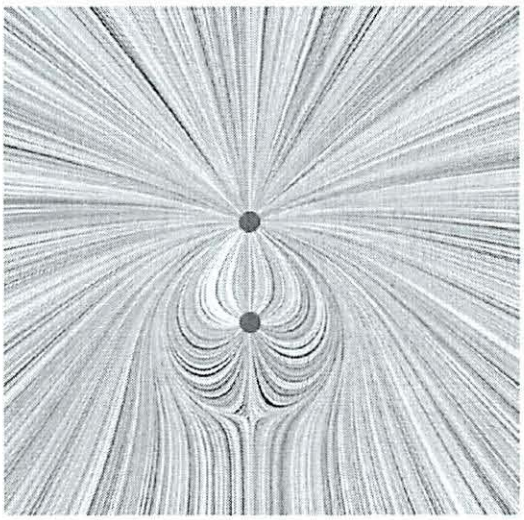
1. Between the two charges.
2. To the right of the charge on the right.
3. To the left of the charge on the left.
4. The electric field is only zero an infinite distance away from the charges.

Explain your reasoning.

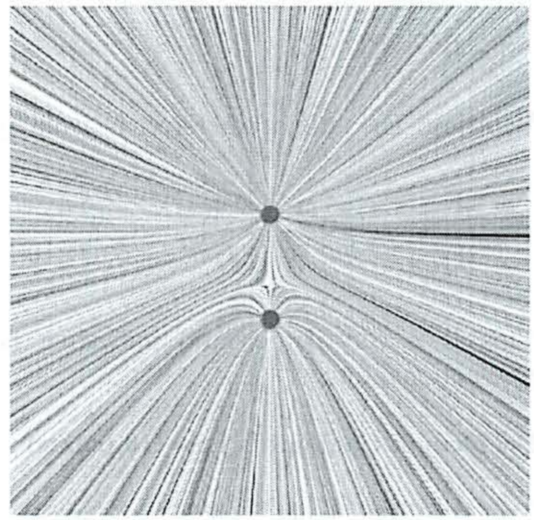
Answer 3. The electric field is the vector sum of the electric fields due to each charged object. There are two properties that determine the strength of the electric field, distance from the source (the strength of the field is proportional to $1/r^2$), and the magnitude of the charge (the strength of the field is proportional to q). In the figure below the electric fields of the two objects are shown at several points. At the point A to the left of the charged object on the left, the vectors point in opposite directions. Since the point A is closer to the object with charge $-q$ than the object with charge $+3q$, these two properties can balance and the vectors can add to zero. Whereas on the right, both properties contribute to making the field due to the object with charge $+3q$ larger than the field due to the object with charge $-q$, and then cannot possibly sum to zero. In the region between the objects the electric vectors both point to the left so they cannot sum to zero.



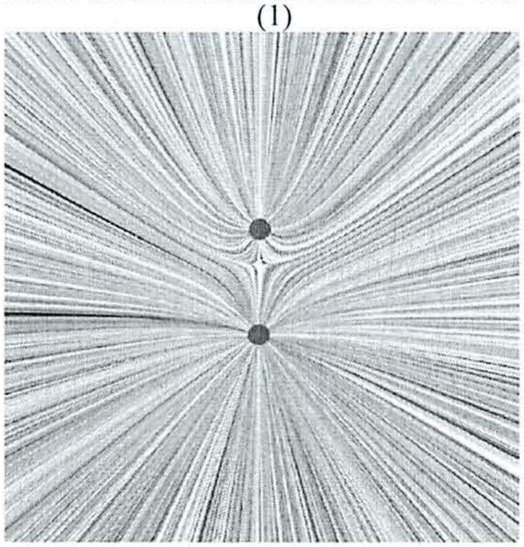
(b) (5 points). Two objects with charges $-4Q$ and $-Q$ lie on the y-axis. The object with the charge $-4Q$ is ***above*** the object with charge $-Q$. Below are four possible "grass seed" representations of the electric field of the two charges. Which of these representations is most nearly right for the two charges in this problem?



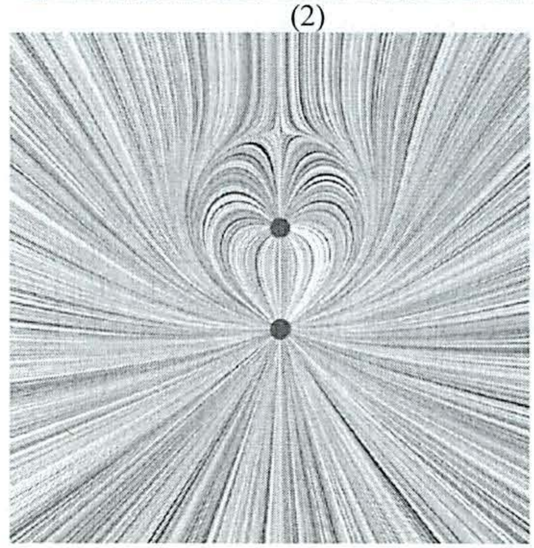
(1)



(2)



(3)



(4)

Explain your reasoning.

Answer (2) Both sources have negative charge so the field lines very near each source must point towards that source. Therefore there must be a point between the sources where the field is zero. (This eliminates figures (1) and (4).) The zero of the field must be closer to the weaker source in order to cancel the field from the stronger source that is further away. The weaker source is below the stronger source, so the figure (2) is the correct 'grass seed field' representation of the electric field of both sources.

Problem 4: Ratio of Electric and Gravitational Forces (10 points)

What is the ratio of the magnitudes of the electric force and the gravitational force between two protons if the protons are separated by a distance r? In SI units the magnitude of the charge of the proton is $e = 1.6 \times 10^{-19}\ \mathrm{C}$ and the mass of the proton is $m_p = 1.67 \times 10^{-27}\ \mathrm{kg}$.

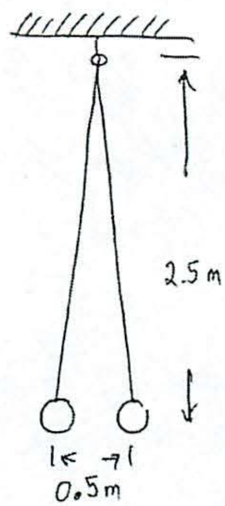
Solution: The ratio of the forces is given by

$$\frac{\left|\vec{\mathbf{F}}_{elec}\right|}{\left|\vec{\mathbf{F}}_{grav}\right|} = \frac{ke^2/r^2}{Gm_p^2/r^2} = \frac{ke^2}{Gm_p^2} = \frac{(9.0\times10^9\ \mathrm{N\cdot m^2\cdot C^{-2}})(1.6\times10^{-19}\ C)^2}{(6.67\times10^{-11}\ \mathrm{N\cdot m^2\cdot kg^{-2}})(1.67\times10^{-27}\ kg)^2} = 1.2\times10^{36}.$$

This is a very large ratio indicating how much stronger electric forces are than gravitational forces.

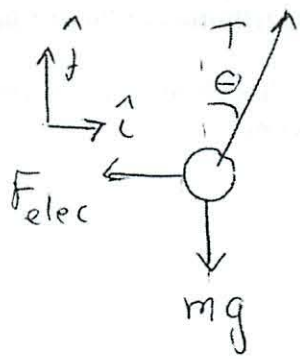
Problem 5: Coulomb's Law (10 points)

Two volley balls, each of mass $m = 0.2\ \mathrm{kg}$, tethered by nylon strings and equally charged with an electrostatic generator, hang as shown in the figure such that the centers of the balls are a distance $r = 0.5\ \mathrm{m}$ apart. The point equidistance between the two centers of the balls is a distance $d = 2.5\ \mathrm{m}$ below the suspension point. What is the charge on each ball? Include your free-body force diagram in your solution.



Solution:

Since the tetherballs are in static equilibrium, the sum of the forces must be zero. There are three forces acting on each ball, gravitation, tension from the rope, and the electric force that is proportional to q^2, where q is the charge on either tetherball.. We begin by drawing a free body diagram on one ball, then taking a vector decomposition of the forces on that ball, and setting each component equal to zero. Then we can solve for the charge on each tetherball.



The sum of the x-component of the forces is

$$F_x^T = \frac{kq^2}{r^2} - T\sin\theta = 0$$

where r is the distance between the centers of the tetherballs. The sum of the y-component of the forces is

$$F_y^T = T\cos\theta - mg = 0\,.$$

Solving for the tension we find that

$$T = \frac{mg}{\cos\theta}.$$

Substituting that back into the horizontal equation yields

$$\frac{kq^2}{r^2} - \frac{mg}{\cos\theta}\sin\theta = 0$$

which we can solve for the charge on the tetherball

$$q = \left(\sqrt{mg\tan\theta / k}\right) r\,.$$

Recall from the geometry of the set-up

$$\tan\theta = (0.25\text{ m}/2.5\text{ m}) = 0.1\,.$$

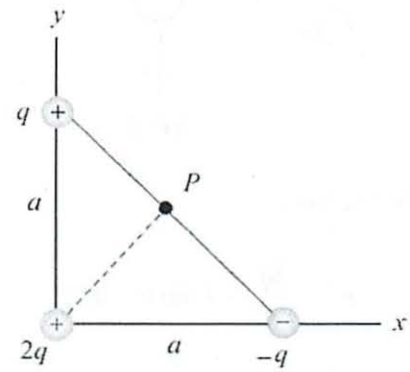
Thus the charge is

$$q = \left(\sqrt{mg\tan\theta / k}\right) r = \left(\sqrt{(0.2\text{ kg})(9.8\text{ m}\cdot\text{s}^{-2})(0.1)/(9.0\times10^{9}\text{ N}\cdot\text{m}^2\cdot\text{C}^{-2})}\right)(0.5\text{ m})$$

$$q = 2.3\times10^{-6}\text{ C}\,.$$

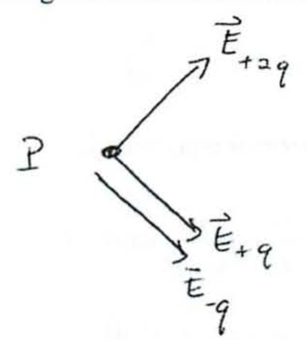
Problem 6 Electric field for a Distribution of Point *Charges* (10 points)

A right isosceles triangle of side a has objects with charges q, $+2q$ and $-q$ arranged on its vertices, as shown in the figure below.



What is the magnitude and direction of the electric field at point P due to the charges in the figure, midway between the line connecting the $+q$ and $-q$ charges?

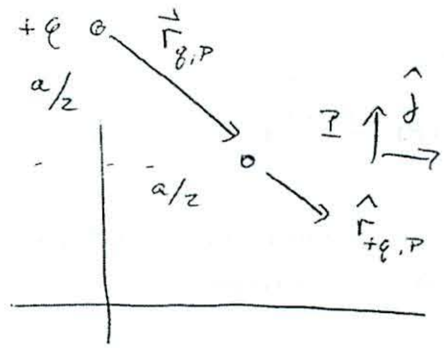
Solution: We can begin by drawing the three contributions to the electric field.



The total electric field is then

$$\vec{\mathbf{E}}(P) = \vec{\mathbf{E}}_{+q}(P) + \vec{\mathbf{E}}_{-q}(P) + \vec{\mathbf{E}}_{2q}(P).$$

We start with the field due to the charge $+q$:



The electric field is given by the expression

$$\vec{\mathbf{E}}_{+q}(P) = \frac{kq}{(r_{+q,P})^2}\hat{\mathbf{r}}_{+q,P} = \frac{kq}{(r_{+q,P})^3}\vec{\mathbf{r}}_{+q,P}.$$

Recall that the vector $\vec{\mathbf{r}}_{+q,P}$ is the vector that starts at the charge $+q$ and ends at the point P. From the figure above, we can write this vector as

$$\vec{\mathbf{r}}_{+q,P} = (a/2)\hat{\mathbf{i}} - (a/2)\hat{\mathbf{j}}.$$

The magnitude of this vector is

$$r_{+q,P} = \left|\vec{\mathbf{r}}_{+q,P}\right| = ((a/2)^2 + (a/2)^2)^{1/2} = a/\sqrt{2}.$$

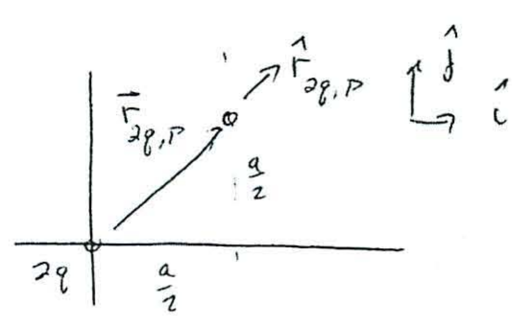
Thus

$$\vec{\mathbf{E}}_{+q}(P) = \frac{kq}{(r_{+q,P})^3}\vec{\mathbf{r}}_{+q,P} = \frac{kq((a/2)\hat{\mathbf{i}} - (a/2)\hat{\mathbf{j}})}{(a/\sqrt{2})^3} = \frac{\sqrt{2}kq(\hat{\mathbf{i}} - \hat{\mathbf{j}})}{a^2}.$$

Note that

$$\vec{\mathbf{E}}_{+q}(P) = \vec{\mathbf{E}}_{-q}(P).$$

The electric field due to the charge $2q$:



The electric field is given by

$$\vec{\mathbf{E}}_{2q}(P) = \frac{k(2q)}{(r_{2q,P})^2}\hat{\mathbf{r}}_{2q,P} = \frac{2kq}{(r_{2q,P})^3}\vec{\mathbf{r}}_{2q,P}.$$

Recall that the vector $\vec{\mathbf{r}}_{2q,P}$ is the vector that starts at the charge $2q$ and ends at the point P. From the figure above we can write this vector as

$$\vec{\mathbf{r}}_{2q,P} = (a/2)\hat{\mathbf{i}} + (a/2)\hat{\mathbf{j}}.$$

The magnitude of this vector is

$$r_{2q,P} = \left|\vec{\mathbf{r}}_{2q,P}\right| = ((a/2)^2 + (a/2)^2)^{1/2} = a/\sqrt{2}.$$

Thus

$$\vec{\mathbf{E}}_{2q}(P) = \frac{2kq}{(r_{2q,P})^3}\vec{\mathbf{r}}_{2q,P} = \frac{2kq((a/2)\hat{\mathbf{i}} + (a/2)\hat{\mathbf{j}})}{(a/\sqrt{2})^3} = \frac{2\sqrt{2}kq(\hat{\mathbf{i}} + \hat{\mathbf{j}})}{a^2}.$$

Thus the vector sum is

$$\vec{\mathbf{E}}(P) = \vec{\mathbf{E}}_{+q}(P) + \vec{\mathbf{E}}_{-q}(P) + \vec{\mathbf{E}}_{2q}(P) = 2\vec{\mathbf{E}}_{+q}(P) + \vec{\mathbf{E}}_{2q}(P).$$

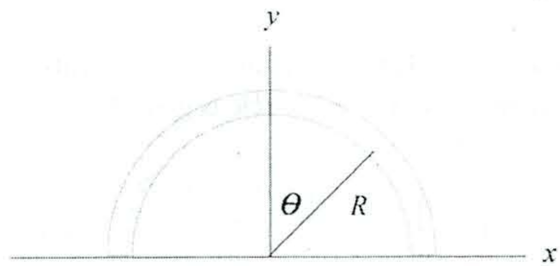
Adding together all three contributions, we get

$$\vec{\mathbf{E}}(P) = 2\frac{\sqrt{2}kq(\hat{\mathbf{i}} - \hat{\mathbf{j}})}{a^2} + \frac{2\sqrt{2}kq(\hat{\mathbf{i}} + \hat{\mathbf{j}})}{a^2} = \frac{4\sqrt{2}kq\hat{\mathbf{i}}}{a^2}$$

.

Problem 7 Electric Field and Force (10 points)

A positively charged wire is bent into a semicircle of radius R, as shown in the figure below.



The total charge on the semicircle is Q. However, the charge per unit length along the semicircle is non-uniform and given by $\lambda = \lambda_0 \cos\theta$.

a) What is the relationship between λ_0, R and Q?

b) If a particle with a charge q is placed at the origin, what is the total force on the particle? Show all your work including setting up and integrating any necessary integrals.

Solution:

(a) In order to find a relation between λ_0, R and Q it is necessary to integrate the charge density λ because the charge distribution is non-uniform

$$Q = \int_{wire} \lambda ds = \int_{\theta'=-\pi/2}^{\theta'=\pi/2} \lambda_0 \cos\theta' R d\theta' = R\lambda_0 \sin\theta' \Big|_{\theta'=-\pi/2}^{\theta'=\pi/2} = 2R\lambda_0 .$$

(b) The force on the charged particle at the center P of the semicircle is given by

$$\vec{\mathbf{F}}(P) = q\vec{\mathbf{E}}(P).$$

The electric field at the center P of the semicircle is given by

$$\vec{\mathbf{E}}(P) = \frac{1}{4\pi\varepsilon_0} \int_{wire} \frac{\lambda ds}{r^2} \hat{\mathbf{r}} .$$

The unit vector, $\hat{\mathbf{r}}$, located at the field point, is directed from the source to the field point and in Cartesian coordinates is given by

$$\hat{\mathbf{r}} = -\sin\theta'\,\hat{\mathbf{i}} - \cos\theta'\,\hat{\mathbf{j}}.$$

Therefore the electric field at the center P of the semicircle is given by

$$\vec{\mathbf{E}}(P) = \frac{1}{4\pi\varepsilon_0}\int_{wire}\frac{\lambda ds}{r^2}\hat{\mathbf{r}} = \frac{1}{4\pi\varepsilon_0}\int_{\theta'=-\pi/2}^{\theta'=\pi/2}\frac{\lambda_0\cos\theta' R d\theta'}{R^2}(-\sin\theta'\,\hat{\mathbf{i}} - \cos\theta'\,\hat{\mathbf{j}}).$$

There are two separate integrals for the x and y components. The x-component of the electric field at the center P of the semicircle is given by

$$E_x(P) = -\frac{1}{4\pi\varepsilon_0}\int_{\theta'=-\pi/2}^{\theta'=\pi/2}\frac{\lambda_0\cos\theta'\sin\theta'\,d\theta'}{R} = \left.\frac{\lambda_0\cos^2\theta'}{8\pi\varepsilon_0 R}\right|_{\theta'=-\pi/2}^{\theta'=\pi/2} = 0.$$

We expected this result by the symmetry of the charge distribution about the y-axis.

The y-component of the electric field at the center P of the semicircle is given by

$$\begin{aligned}
E_y(P) &= -\frac{1}{4\pi\varepsilon_0}\int_{\theta'=-\pi/2}^{\theta'=\pi/2}\frac{\lambda_0\cos^2\theta' d\theta'}{R} = -\frac{1}{4\pi\varepsilon_0}\int_{\theta'=-\pi/2}^{\theta'=\pi/2}\frac{\lambda_0(1+\cos 2\theta')d\theta'}{2R} \\
&= -\frac{\lambda_0}{8\pi\varepsilon_0 R}\theta'\Big|_{\theta'=-\pi/2}^{\theta'=\pi/2} - \frac{\lambda_0}{16\pi\varepsilon_0 R}\sin 2\theta\Big|_{\theta'=-\pi/2}^{\theta'=\pi/2} \\
&= -\frac{\lambda_0}{8\varepsilon_0 R}
\end{aligned}.$$

Therefore the force on the charged particle at the point P is given by

$$\vec{\mathbf{F}}(P) = q\vec{\mathbf{E}}(P) = -\frac{q\lambda_0}{8\varepsilon_0 R}\hat{\mathbf{j}}.$$

8.02 Math Review
Surface Integral

2/11

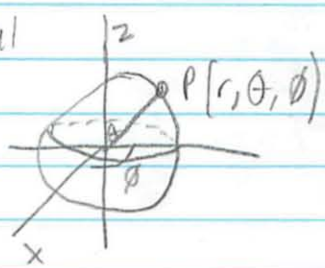
- physics POV
- how to use + what it means
- everything smooth + contineous in Physics

E+M is a 3D subject

Variables in 1, 2, 3 D

	Variable		Vol
1D	x		dx
2D			
	cartesean $(x\,y)$		$dx\,dy$
	polar (r, ϕ) ↑θ in math		$r\,dr\,d\theta$ ↑ won't say why this is here
3D	cartesean $(x\,y\,z)$		$dx\,dy\,dz$
	spherical (r, θ, ϕ)		$r^2 \sin\theta\, dr\, d\theta\, d\phi$
	cylindrical (r, ϕ, z)		$r\,dr\,d\phi\,dz$

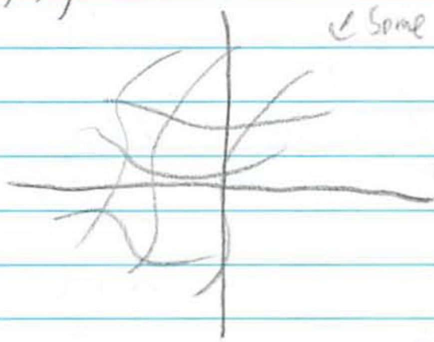
Spherical



$(\theta, \phi) \leftrightarrow (\phi, \theta)$

Math Physics

$f(x, y)$



Some surface in 3D
at each pt is a normal vector

(perpendicular to tangent of surface)

$\hat{n} = \hat{r}$

$$d\vec{A} = \hat{r} R^2 \underbrace{\sin\theta \, d\theta \, d\phi}_{d\Omega}$$

"Solid angle"
$\int d\Omega_2 = 4\pi$
patch drawn on ball

$x = r \cos\phi \sin\theta$

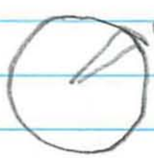
$y = r \sin\phi \sin\theta$

$z = r \cos\theta$

$\phi = \arctan\left(\frac{y}{x}\right)$

$\theta = \arctan\left(\frac{\sqrt{x^2+y^2}}{z}\right)$

$r = \sqrt{x^2+y^2+z^2}$



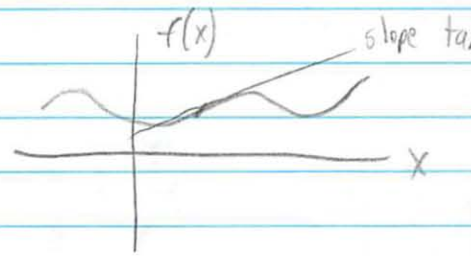
$$d\vec{A} = \hat{r} \cdot R \underbrace{d\phi}_{d\Omega_1 \text{ angle}}$$

$d\Omega_1 = d\phi$

$d\Omega_2 = \sin d\theta \, d\phi$] 8.02

$d\Omega_3 = \sin^2\psi \sin\theta \, d\psi \, d\theta \, d\phi$

$d\Omega_4 = \sin^3\eta \, \sin^2\psi \sin\theta \, d\eta \, d\psi \, d\theta \, d\phi$] Theoretical Physics

2D) 

3D) If changing 2 things at once want to find the gradiant

$$\vec{\nabla} f(x,y) = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$ cartisian

changing in x dir

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$\vec{\nabla} f(r,\theta,\phi) = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$ scalar

example

$$f(r,\theta,\phi) = \frac{B\cos\theta}{r^2}$$

$$\vec{\nabla} f = \frac{2B\cos\theta}{r^3}\hat{r} + \frac{B\sin\theta}{r^3} + 0\phi$$

diapole field spherical coords

Divergence

rate of change of a vector in the direction of a vector (getting bigger/smaller?)

$$\vec{\nabla}\cdot\vec{V}(x,y,z) = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$ how much stuff flowing around

$$\hookrightarrow \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\cdot\left(V_x, V_y, V_z\right)$$ "flux operator"

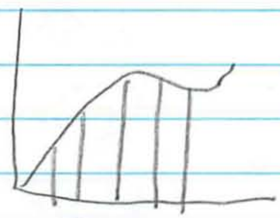
takes vector
Gradiant - scalar field
measures dir greastest change

know now

Divergence -
- gives a scalar

Contor maps = 2D Gradiants
Gradiant - how much changing in 3D

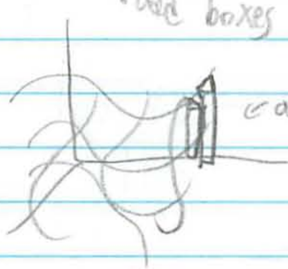
Integrals - add up stuff under it



$\sum f(x_i) \Delta x_i$ = area of rectangles

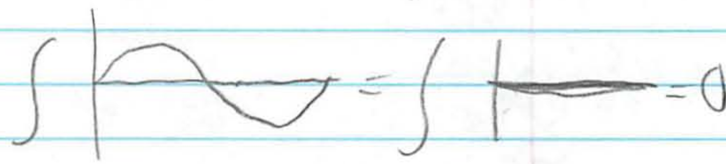
$\int f(x)\, dx$ = area under curve

add boxes



add all the towers

lose info when integrate



Can happen in Gauss Law
if charges =

$\int_{\text{limits}} [\text{stuff}]\, d\,\text{variables}$ = total stuff
contained in the limits
paramitized by the variables

Say it diff ways over & over So you get it

Gauss's Theorm (~~Not~~ his Law)

$$\int_{Vol} (\vec{\nabla}\cdot\vec{v})\, d^3x = \int_{Sur} \vec{v}\cdot\hat{n}\, d^2x$$

Short hands

$d^3x = dx\,dy\,dz = dV = d\tau$

$d^3r = r^2\,dr\,d\Omega$

$d^2x = dx\,dy = dA$

Left $\vec{\nabla}\cdot\vec{v}$ tells how much vector changing in that direction
add all up in some volume
= Total change of vector field $\vec{V}$

Right (Surface is boundry of volume)

Vol → Sur

d^2x is surface area element for sphere $R^2\,d\Omega$
flat $dx\,dy$
Cylinder $R\,d\phi\,dz$

$\hat{n}$ is the outward pointing normal
↳ sphere called $\hat{r}$
Flat plane $\hat{x}$
Cylinder $\hat{r}$ (polar)

$\vec{V}\cdot\hat{n}$ Magnitude of $\vec{V}$ in direction of $\hat{n}$
How much of $\vec{V}$ is getting in/out of surface

$\int \vec{V}\cdot\hat{n}$ is total amount of $\vec{V}$ flowing in/out of the surface

$$\boxed{\text{Gauss's Law: total charge of } \vec{V} \text{ inside} = \text{amt } \vec{V} \text{ flowing in/out}}$$

formally →

Gauss's Law

$$\vec{\nabla}\cdot\vec{E} = \frac{1}{\epsilon_0}\rho \quad \text{charge density}$$

$$\int_{Vol} \vec{\nabla}\cdot\vec{E}\, d^3x = \int \vec{E}\cdot d\vec{A}$$

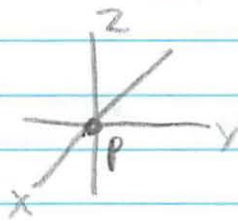
$$\parallel$$

$$\frac{1}{\epsilon_0}\int_{Vol} \rho\, d^3x = \frac{1}{\epsilon_0}\begin{bmatrix}\text{total } Q \\ \text{in vol}\end{bmatrix}$$

$$= \boxed{\int \vec{E}\cdot da = \frac{Q_{enc}}{\epsilon_0}}$$

Examples

point charge



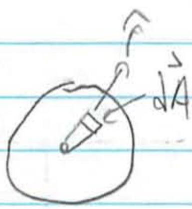
has spherical symmetry

RHS: $\frac{1}{\epsilon_0}\int \rho\, d^3x = \frac{Q_{enc}}{\epsilon_0} = +\frac{Q}{\epsilon_0}$

LHS: does not depend on angles (θ or ϕ)

$$\vec{E}(r) = \hat{r}\,|\vec{E}(r)|$$

$$\int \vec{E}\cdot d\vec{A} \longleftarrow \hat{r}\, r^2 \underbrace{\sin\theta\, d\theta\, d\phi}_{d\Omega}$$



$$\int (\hat{r}\,|\vec{E}(r)|)\cdot(\hat{r}\, r^2 d\Omega)$$

$$r^2\,|\vec{E}(r)| \int d\Omega$$

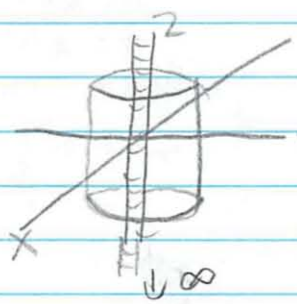
$$4\pi r^2\,|\vec{E}(r)|$$

$$4\pi r^2 |\vec{E}(r)| = \frac{Q}{\epsilon_0}$$

We found

$$\Rightarrow \boxed{\vec{E}(r) = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{\epsilon_0} \hat{r} \quad \text{Coulomb's Law!}}$$

Line of Charge



λ ← linear density

Cylindrical Symmetry

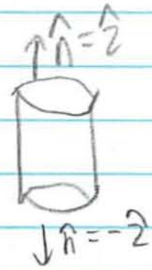
RHS: $\frac{1}{\epsilon_0}\int \rho \, d^3x$

$\frac{1}{\epsilon_0}\int \lambda \, dz = \frac{\lambda z}{\epsilon_0}$

LHS: can't depend on z
same up + down ← since
spinning ϕ → same

$$\vec{E}(r,\phi,z) = \hat{r}\,|\vec{E}(r)|$$

First do endcaps



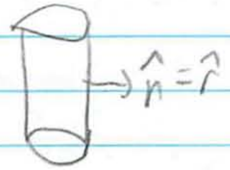
$\vec{E}\cdot\vec{n}\,\big|_{end} \propto \vec{r}\cdot\vec{z} = 0$

same up + down

can't have z dependence

now nothing to $\int$

Sides



$d\vec{A} = \hat{r} \cdot r \cdot d\phi \cdot dz$

$\int (\hat{r} |\vec{E}(r)|) \cdot (\hat{r}\, r\, d\phi dz)$

dot product should be 1 or 0

$r |\vec{E}(r)| \int d\phi\, dz$

$2\pi r z |\vec{E}(r)|$

put together

$$\frac{\lambda z}{\epsilon_0} = 2\pi r z |\vec{E}(r)|$$

$$\vec{E}(r) = \frac{\lambda}{2\pi \epsilon_0} \cdot \frac{1}{r}\, \hat{r}$$

don't forget that in $\hat{r}$ direction proportional to r for cylinder

Topics: Gauss's Law
Related Reading: Course Notes: Sections 4.1-4.2, 4.7-4.8

Topic Introduction

In this class we will practice calculating electric fields using Gauss's law by doing problem solving #3.2 Remember that the idea behind Gauss's law is that, pictorially, electric fields flow out of and into charges. If you surround some region of space with a closed surface (think bag), then observing how much field "flows" into or out of that surface tells you how much charge is enclosed by the bag. For example, if you surround a positive charge with a surface then you will see a net flow outwards, whereas if you surround a negative charge with a surface you will see a net flow inwards.

Gauss's Law

Gauss's law states that the electric flux through any closed surface is proportional to the total charge enclosed by the surface:

$$\Phi_E = \oint\!\!\!\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

A *closed* surface is a surface which completely encloses a volume, and the integral over a closed surface S is denoted by $\oint\!\!\!\oint$.

Symmetry and Gaussian Surfaces

Although Gauss's law is always true, as a tool for calculation of the electric field, it is only useful for highly symmetric systems. The reason for this is that in order to solve for the electric field $\vec{\mathbf{E}}$ we need to be able to "get it out of the integral." That is, we need to work with systems where the flux integral can be converted into a simple multiplication. Examples of systems that possess such symmetry and the corresponding closed *Gaussian surfaces* we will use to surround them are summarized below:

Symmetry	System	Gaussian Surface
Cylindrical	Infinite line	Coaxial Cylinder
Planar	Infinite plane	Gaussian "Pillbox"
Spherical	Sphere, Spherical shell	Concentric Sphere

Solving Problems using Gauss's law

Gauss's law provides a powerful tool for calculating the electric field of charge distributions that have one of the three symmetries listed above. The following steps are useful when applying Gauss's law:

(1) Identify the symmetry associated with the charge distribution, and the associated shape of "Gaussian surfaces" to be used.
(2) Divide space into different regions associated with the charge distribution, and determine the exact Gaussian surface to be used for each region. The electric field must be constant or known (i.e. zero) across the Gaussian surface.
(3) For each region, calculate q_{enc}, the charge enclosed by the Gaussian surface.
(4) For each region, calculate the electric flux Φ_E through the Gaussian surface.
(5) Equate Φ_E with $q_{\text{enc}} / \varepsilon_0$, and solve for the electric field in each region.

Important Equations

Electric flux through a surface S: $$\Phi_E = \iint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

Gauss's law: $$\Phi_E = \oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics
8.02

Problem Solving 2: Gauss's Law

REFERENCE: Section 4.2, 8.02 Course Notes.

Introduction: When approaching Gauss's Law problems, we described a problem solving strategy summarized below (see also, Section 4.7, 8.02 Course Notes):

$$\underset{\substack{\text{closed} \\ \text{surface S}}}{\oiint} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0}$$

Summary: Methodology for Applying Gauss's Law

Step 1: Identify the 'symmetry' properties of the charge distribution.

Step 2: Determine the direction of the electric field

Step 3: Decide how many different regions of space the charge distribution determines

For each region of space…

Step 4: Choose a Gaussian surface through each part of which the electric flux is either constant or zero

Step 5: Calculate the flux through the Gaussian surface (in terms of the unknown $\boldsymbol{E}$)

Step 6: Calculate the charge enclosed in the choice of the Gaussian surface

Step 7: Equate the two sides of Gauss's Law in order to find an expression for the magnitude of the electric field

Then…

Step 8: Graph the magnitude of the electric field as a function of the parameter specifying the Gaussian surface for all regions of space.

You should now apply this strategy to the following problem.

6. Supplemental)

end

$\vec{E} \cdot \hat{n}$ / end $\propto \hat{r} \cdot \hat{z} = 0$

~~so can't act~~

same up + down

can't have z dependence

nothing to $\int$

I don't really get

and it does not leak out there

sides

$d\vec{A} = \hat{r} \cdot r \cdot d\phi \cdot dz$

$\int (\hat{r} |\vec{E}(r)|) \cdot (\hat{r}\, r\, d\phi\, dz)$

r should be 1 or 0

$r |\vec{E}(r)| \int d\phi\, dz$

$2\pi r z |\vec{E}(r)|$

far simpler

$\oint \vec{E}\, dA = \vec{E} A_{sides}$

$\vec{E}\, 2\pi r L$

$\vec{E}\, 2\pi r z$

are =

7.) $\cancel{\frac{1}{\epsilon_0} \int \rho\, d^3x}$

$\cancel{\frac{\rho^2}{\epsilon_0}}$

far simpler

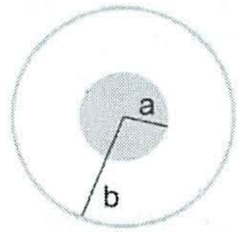
$Q_{enc} = \rho V$ know the shape volumes

$\rho (\pi r^2 4)$

$\rho (\pi r^2 z)$

↑ simpler easier to understand + get

Question: ***Concentric Cylinders***

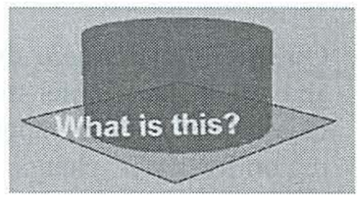


A long very thin non-conducting cylindrical shell of radius b and length L surrounds a long solid non-conducting cylinder of radius a and length L with $b > a$. The inner cylinder has a uniform charge $+Q$ distributed throughout its volume. On the outer cylinder we place an equal and opposite to charge, $-Q$. The region $a < r < b$ is empty.

↙ could not see

You can find a three dimensional visualization of this charge configuration and its fields at http://web.mit.edu/viz/EM/electrostatics/GaussLawProblems/filledCylinderShell/. Go to this *URL*, read the "Help" file, and try out the various Gaussian surfaces available in this applet. Then answer the following questions.

Question 1: (*Answer on the tear-sheet at the end!*) There is an icon in the applet as shown to the right. What does the height of the cylinder in this icon represent?



flux

Question 2 (this is Step 1 of your methodology above): (*Answer on the tear-sheet at the end!*) What is the 'symmetry' property of the charge distribution here (which of the three below)?

Spherical   Cylindrical   Planar

says in problem

Question 3 (Step 2 of your methodology): (*Answer on the tear-sheet at the end!*) What is the direction of the electric field (again, which of the three choices below)?

Radial (in/out)   Angular (CW/CCW)   Perpendicular to page

no endcaps

Question 4 (Step 3 of the methodology): (*Put your answer on the tear-sheet at the end!*) How many different regions of space does the charge distribution determine (in other words, how many different formulae for **E** are you going to have to calculate?)

~~1 if we assume no endcaps~~

not ∞ lenght

3- side and the 2 endcap

but does charge go out here

don't think so

Question 5 (Step 4 of your methodology): ***(Put your answer on the tear-sheet at the end!)*** For each region of space, describe your choice of a Gaussian surface. What variable did you choose to parameterize your Gaussian surface (for example, for a sphere you'd use the radius *r*)? What is the range of that variable?

(given)

r polar for cylinder $0 < r < b+)$

r, ϕ, z ← does not depend$_z$ same up + down

r does not depend on ϕ spinning

Question 6 (Step 5 of your methodology): ***(Put your answer on the tear-sheet at the end!)*** For the region for $r < a$, calculate the flux through your choice of the Gaussian surface (that is, just write down the left hand side of Gauss's Law). Your expression should include the unknown electric field for that region. $\vec{E}(r,\phi,z) = \hat{r}|\vec{E}(r)|$ $\vec{E} \cdot \hat{n}_{end} \perp \vec{r} \cdot \vec{z} = 0$

$\hat{n} = \hat{z}$

$\hat{n} = -\hat{z}$

→ see backwards back of 2-1

Question 7 (Step 6 of your methodology): ***(Put your answer on the tear-sheet at the end!)*** For the region for $r < a$, write the charge enclosed in your choice of Gaussian surface (this should be in terms of *Q*, *r* & *a*, NOT **E**).

→ other page

Question 8: ***(Put your answer on the tear-sheet at the end!)*** Go to the applet that you have used above. In that applet there is a measure of the charge enclosed inside the Gaussian surfaces. Qualitatively, in the applet, does the charge interior to the cylindrical Gaussian surface in the region for $r < a$ change with *r* in the way your formula given directly above indicates?

Yes - as you change r (when $r<a$) the flux changes

Question 9 (Step 7 of your methodology): ***(Put your answer on the tear-sheet at the end!)*** For the region for $r < a$, equate the two sides of Gauss's Law that you calculated in questions 6 and 7, and solve to find an expression for the magnitude of the electric field.

$$\vec{E}\, 2\pi r z = \frac{\rho \pi r^2 z}{\epsilon_0}$$

$$\vec{E} = \frac{\rho \pi r^2 z}{\epsilon_0 2\pi r z} = \boxed{E = \frac{\rho r}{2\epsilon_0}}$$ cylinder

Question 10 (Step 6 and 7 or your methodology): ***(Put your answer on the tear-sheet at the end!)*** Repeat the same procedure in order to calculate the electric field as a function of *r* for the regions $a < r < b$.

Same just that Q_{enc} is same for all r values $a < r < b$

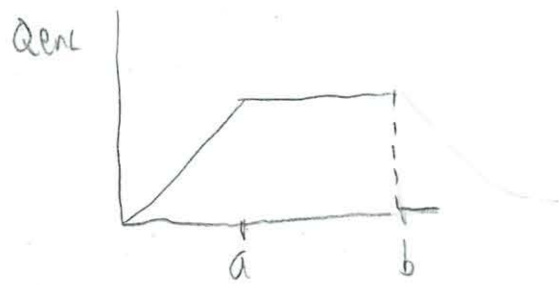
$r \to a$

$$E\, 2\pi a z = \frac{\rho \pi a^2 z}{\epsilon_0}$$

$$E = \frac{\rho a}{2\epsilon_0}$$

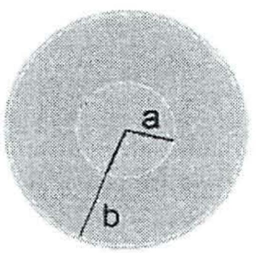
Question 11 (Step 8 of your methodology): ***(Put your answer on the tear-sheet at the end!)*** Make a graph of the magnitude of the electric field as a function of the parameter specifying the Gaussian surface for all regions of space.

Curved is as you go away when $\frac{1}{r}$



* memorize what it is for each shape

Sample Exam Questions (Try these yourself, closed notes. You'll need paper)



Problem 1: A very long non-conducting cylinder is constructed of two materials. The inner portion, radius a, has a non-uniform volume charge density given by:

$$\rho(r<a)=\frac{\sigma}{2\pi r} \quad \text{where } \sigma \text{ is a constant (what units?)}$$

The outer portion, with inner radius a and outer radius b has a uniform charge density.

(a) If the electric field outside the cylinder ($r > b$) is everywhere zero, what is the uniform charge density $\rho\,(a < r < b)$ of the outer portion of the cylinder?

(b) What is the electric field everywhere in space?

Problem 2:
Consider the following cylindrically symmetric electric field:

$$\vec{\mathbf{E}}(\vec{\mathbf{r}})=\hat{\mathbf{r}}\begin{cases} 0 & r<a \\ \dfrac{Q}{\varepsilon_0 a^2}\left(1-\dfrac{a}{r}\right) & a\le r\le 2a \\ \dfrac{Q}{\varepsilon_0 a r} & 2a<r \end{cases}$$

What is the charge distribution that creates this field? In other words, what is $\rho(r)$?

8.02 Week 1+2
Cheat Sheet

2/14

Fields - lies are tangent to Field

$g = k_e \frac{q_1 q_2}{r^3} \vec{r}$ $\qquad$ $F = qE$ $\qquad$ q = charge in Colombs

Create $\qquad$ Feel

superposition - add

1 charge = $1.602 \cdot 10^{-19}$

Charge Density

$\rho = \frac{dq}{dV}$ volume 3D

$\sigma = \frac{dq}{dA}$ surface 2D

$\lambda = \frac{dq}{dL}$ linear 1D

Colomb's Law

$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum \frac{dq}{r^2} \hat{r}$ discrete

$\vec{E} = \frac{1}{4\pi\epsilon_0} \int_V \frac{dq}{r^2} \hat{r}$ continuous

from the charge to observer points vector

Diapole moment - measure of seperation of ⊕⊖ charges

- measure of polarity
- d = displacement vector ⊖ → ⊕

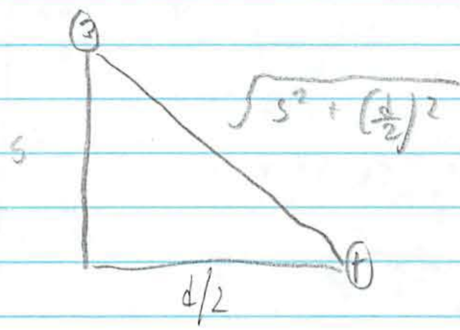
Diapole

← diapole moment $p = qd$

creates fields + responds to them
will rotate to align w/ field

$\vec{\tau} = \vec{p} \times \vec{E}$

When splitting x y denom is always the same
Put distance in numerator



confirm that is correct →

$$\frac{-k q_1 q_3 \frac{d}{2} \hat{\imath}}{\sqrt{(d/2)^2+s^2}} + - \frac{k q_1 q_3 s}{\sqrt{(d/2)^2+s^2}} \hat{\jmath}$$

⊖ due to direction

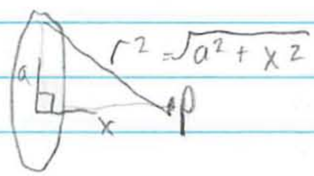
$$\vec{E} = \sum \Delta \vec{E} = \int d\vec{E}$$

Ring of charge

$$dQ = \lambda \, dL$$

$$\lambda = \frac{Q}{2\pi R} \leftarrow \text{know the circumference of a circle}$$

When at a point P



$E_{\hat{j}} = 0$ because of symatry

$$dq = \lambda \, dl = \lambda (a \, d\varphi)$$

$$d\vec{E} = \frac{k_e \, dq}{r^2} \hat{r} = \frac{k_e \, dq}{r^3} \vec{r}$$

$$dE = k_e \, dq \frac{x}{r^3}$$

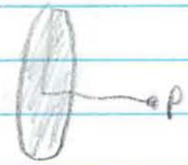
$$\int \qquad \int$$

$$E_x = k_e \frac{x}{r^3} \int dq$$

$$= k_e Q \frac{x}{r^3}$$

$$= \frac{k_e Q x}{\sqrt{a^2 + x^2}^3}$$

Disk



- uniformly charged

$dQ = \sigma\, dA$

$$dE = \frac{k\,\sigma\, dA}{r^3}\,\vec{r}$$

$$E = \frac{k\,\sigma\,\pi r^2}{\sqrt{r^2+x^2}^{\,3}}\,\vec{r}$$

← that was easy now
- just fill in

Falls Off

diapole	$\frac{1}{r^3}$
point	$\frac{1}{r^2}$
line	$\frac{1}{r}$
plane	1 / constant

Steps

1. Express dq in terms of charge density
2. Write dE
3. Write r w/ proper coords
4. Apply symmetry to find non vanishing E
5. Integrate

area circle $= \pi r^2$
circ circle $= 2\pi r$
Vol cyl $= \pi r^2 h$
area cyl sides $= 2\pi r h$
area cyl top $= \pi r^2$
Vol sphere $= \frac{4}{3}\pi r^3$
SA sphere $= 4\pi r^2$

Gauss' Law

$$\Phi_E = \underset{S}{\oiint} \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

↑closed surface

line → cylinder –
plane → pillbox
sphere → sphere

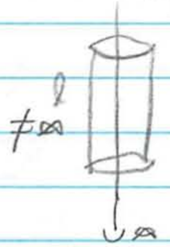
Line

$$EA = \frac{q_{in}}{\varepsilon}$$

$$\underbrace{E\,2\pi r l}_{} = \frac{\lambda l}{\varepsilon_0}$$

$q_{in} = \lambda l$

side area of cylinder



$$E = \frac{\lambda l}{\varepsilon_0 2\pi r l}$$

$$= \frac{\lambda}{\varepsilon_0 2\pi r}\hat{r}$$

$$= \frac{\lambda}{\varepsilon 2\pi r^2}\vec{r}$$

Seems far easier – understand better
– make sure to know the shape rules
+ peculiarities

Line → line is ∞ lenght
use a cylineder of <u>defined</u> lenght
don't care abound endcaps

<u>Plane</u> → $q = \rho V$
E constant any pt above plane
$\oplus$ / $\ominus$ ← 0 in the middle
divide into 2 regions inside + above
inside $\frac{\rho x}{\epsilon_0}$ outside $\frac{\rho d}{\epsilon_0}$
↑ includes total charge
↑??

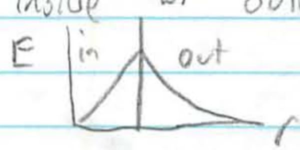
* Cylinder - endcaps only

$$E_{A\,endcap} = \frac{q_{inc}}{\epsilon_0}$$

$$E\,2A_{end} = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon}\begin{cases} \hat{x} & \text{up} \\ -\hat{x} & \text{down} \end{cases}$$

<u>Sphere</u> → inside or outside sphere



in $E = k\frac{Q}{a^2}$ out $E = \frac{Q}{4\pi\epsilon_0}\frac{r}{a^3}$

? where is 4π from? SA sphere & <u>know it</u>

$$q_{in} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi a^3}Q = \frac{r^3}{a^3}Q$$

Topics: Working in Groups, Electric Potential, E from V
Related Reading: Course Notes: Sections 3.1-3.5

Topic Introduction

We first discuss groups and what we expect from you in group work. We then turn to the concept of electric potential. Just as electric fields are analogous to gravitational fields, electric potential is analogous to gravitational potential. We introduce from the point of view of calculating the electric potential given the electric field. Next we consider the opposite process, that is, how to calculate the electric field if we are given the electric potential.

Potential Energy

Before defining potential, we first remind you of the more intuitive idea of potential energy. You are familiar with gravitational potential energy, U (= mgh in a uniform gravitational field g, such as is found near the surface of the Earth), which changes for a mass m only as that mass changes its position. To change the potential energy of an object by ΔU, one must do an equal amount of work W_{ext}, by pushing with a force $\boldsymbol{F}_{ext}$ large enough to move it:

$$\Delta U = U_B - U_A = \int_A^B \vec{\mathbf{F}}_{\text{ext}} \cdot d\vec{\mathbf{s}} = W_{ext}$$

How large a force must be applied? It must be equal and opposite to the force the object feels due to the field it is sitting in. For example, if a gravitational field g is pushing down on a mass m and you want to lift it, you must apply a force mg upwards, equal and opposite the gravitational force. Why equal? If you don't push enough then gravity will win and push it down and if you push too much then you will accelerate the object, giving it a velocity and hence kinetic energy, which we don't want to think about right now. to hold it steady - right

This discussion is generic, applying to both gravitational fields and potentials and to electric fields and potentials. In both cases we write:

$$\Delta U = U_B - U_A = -\int_A^B \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}}$$

where the force $\boldsymbol{F}$ is the force the field exerts on the object.

Finally, note that we have only defined *differences* in potential energy. This is because only differences are physically meaningful – what we choose, for example, to call "zero energy" is completely arbitrary.

Potential

Just as we define electric fields, which are created by charges, and which then exert forces on other charges, we can also break potential energy into two parts: (1) charges create an electric potential around them, (2) other charges that exist in this potential will have an associated potential energy. The creation of an electric potential is intimately related to the creation of an electric field: $\Delta V = V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$. As with potential energy, we only define a potential difference. We will occasionally ask you to calculate "the potential," but in these cases we must arbitrarily assign some point in space to have some fixed potential. A common assignment is to call the potential at infinity (far away from any charges) zero. In order to find the potential anywhere else you must integrate from this place where it is known (e.g. from $A=\infty$, $V_A=0$) to the place where you want to know it.

Once you know the potential, you can ask what happens to a charge q in that potential. It will have a potential energy $U = qV$. Furthermore, because objects like to move from high potential energy to low potential energy, as long as the potential is not constant, the object will feel a force, in a direction such that its potential energy is reduced. Mathematically that is the same as saying that $\vec{\mathbf{F}} = -\nabla U$ (where the gradient operator $\nabla \equiv \frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial y}\hat{\mathbf{j}} + \frac{\partial}{\partial z}\hat{\mathbf{k}}$) and hence, since $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$, $\vec{\mathbf{E}} = -\nabla V$. That is, if you think of the potential as a landscape of hills and valleys (where hills are created by positive charges and valleys by negative charges), the electric field will everywhere point the fastest way downhill.

field points to where charge is least

Configuration Energy

Since moving a charge through a potential difference takes energy (it changes the potential energy of the charge), we can also discuss the total amount of energy that it would take to assemble a collection of charges, assuming that they started a very far distance apart ("at infinity") and then were brought in to their final positions. A straight-forward way to think about, and calculate, this is to bring the charges in one at a time. The first one is "free" – it doesn't see a potential. The second charge is brought in through the potential created by the first. The third sees the potential from the first two, and so forth.

points to the ⊖ charge,

Important Equations

have to force to come?

Potential Energy (Joules) Difference: $\Delta U = U_B - U_A = -\int_A^B \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}}$

Electric Potential Difference (Joules/Coulomb = Volt): $\Delta V = V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$

Electric Potential (Joules/coulomb) created by point charge: $V_{\text{Point Charge}}(r) = \frac{kQ}{r}$

Potential energy U (Joules) of point charge q in electric potential V: $U = qV$

Configuration Energy: $U = \sum_{\text{all pairs}} \frac{q_i q_j}{4\pi\varepsilon_o \left|\vec{\mathbf{r}}_i - \vec{\mathbf{r}}_j\right|}$

2/16

only 1 D integrals in 8.02

Class 05: Outline

Hour 1:
Electric Potential
Hour 2:
Electric Potential

P05 - 1

Electric Potential Difference

$V(B) - V(A) = -\int_A^B \vec{E} \cdot d\vec{s}$

Potential Energy $= q \Delta V$

$V(x_B) - V(x_A) = -\int_{x_A}^{x_B} E_x \, dx$

then $E_x = \frac{-dV}{dx}$

if have potential - can get field
1 D gradiant

Summary: Gravitational & Electric Fields

↑ 8.01

P05 - 2

parrall ideas

Summary: Gravity - Electricity

SOURCE:	Mass M_s	Charge q_s (±)
	always attractive	both signs
CREATE:	$\vec{g} = -G\frac{M_s}{r^2}\hat{r}$	$\vec{E} = k_e\frac{q_s}{r^2}\hat{r}$
FEEL:	$\vec{F}_g = m\vec{g}$	$\vec{F}_E = q\vec{E}$

This is easiest way to picture field

P05 - 3

"create"
inverse square

field: an arrow at every point

Potential Energy and Potential

Start with Gravity

Gravity: Force and Work

Gravitational force on m due to M:

$$\vec{\mathbf{F}}_g = -G\frac{Mm}{r^2}\hat{\mathbf{r}}$$

Work done by gravity moving m from A to B:

$$\boxed{W_g = \int_A^B \vec{\mathbf{F}}_g \cdot d\vec{\mathbf{s}}}$$

PATH INTEGRAL

amt of Force in distance of motion
- add it up for ~~each~~ whole path

Work Done by Earth's Gravity

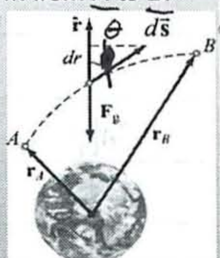
Work done by gravity moving m from A to B:

$$W_g = \int \vec{\mathbf{F}}_g \cdot d\vec{\mathbf{s}}$$

$$= \int_A^B \left(-\frac{GMm}{r^2}\hat{\mathbf{r}}\right)\cdot\left(dr\,\hat{\mathbf{r}} + \cancel{r\,d\theta\,\hat{\theta}}\right) \quad r\,d\phi\,\hat{\phi}$$

$$= \int_{r_A}^{r_B} -\frac{GMm}{r^2}\,dr = \left[\frac{GMm}{r}\right]_{r_A}^{r_B}$$

$$\boxed{= GMm\left(\frac{1}{r_B} - \frac{1}{r_A}\right)}$$

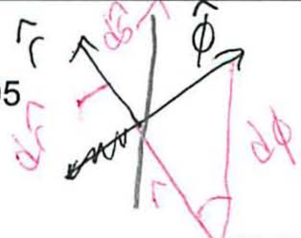


θ is ok

Think, try to say it in a sentance

Can move w/ or w/o external force
fuel in rocket

This problem: gravitational force



why ϕ

$\hat{r}\cdot\hat{r} = 1$

$\hat{r}\cdot\hat{\phi} = 0$

$d\vec{s} = dr\,\hat{r} + r\phi\,\hat{\phi}$

$F = -\frac{GMm}{r^2}\hat{r}$

$F\cdot d\vec{s} = -\frac{GmM}{r^2} + 0$

point: tengental part does not matter

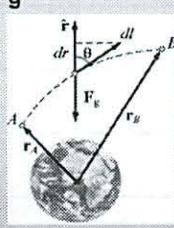
PRS Question:
Sign of W_g

All are conservitive forces
-how you get there
does not matter
-because radial force

PRS: Sign of W_g

Thinking about the sign and meaning of this…

$$W_g = GMm\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$



Moving from r_A to r_B:

0% 1. W_g is positive – we do work
0% 2. W_g is positive – gravity does work
0% 3. W_g is negative – we do work
0% 4. W_g is negative – gravity does work
0% 5. I don't know

:20

think conceptully

~~move~~ up gravity $-W_g$
always −

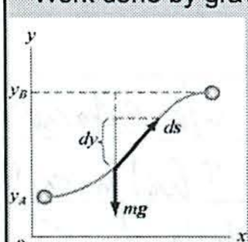
we ⊕ work

look at dot product
think of if $r_B < r_a$

Work Near Earth's Surface

G roughly constant: $\vec{g} \approx -\frac{GM}{r_E^2}\hat{y} = -g\hat{y}$

Work done by gravity moving m from A to B:



$$W_g = \int \vec{F}_g \cdot d\vec{s} = \int_A^B (-mg\hat{y}) \cdot d\vec{s}$$
$$= -\int_{y_A}^{y_B} mg\,dy = -mg(y_B - y_A)$$

W_g depends ***only*** on endpoints
– ***not*** on path taken –
Conservative Force

Only y direction after do dot product
I'm confused about

*always a difference

Potential Energy (Joules)

$$\boxed{\Delta U_g = U_B - U_A = -\int_A^B \vec{\mathbf{F}}_g \cdot d\vec{\mathbf{s}} = -W_g}$$

If lift thing up ↑U
have to apply work

(1) $\vec{\mathbf{F}}_g = -\frac{GMm}{r^2}\hat{\mathbf{r}} \rightarrow U_g = -\frac{GMm}{r} + U_0$ ref pt $y=0$

(2) $\vec{\mathbf{F}}_g = -mg\hat{\mathbf{y}} \rightarrow U_g = mgy + U_0$

- U_0: constant depending on reference point
- Only potential difference ΔU has physical significance

$U(y) = mgy + U(y=0)$
↑change in PE

$U(r) - U(\infty) = -\int_\infty^r -\frac{GmM}{r^2}\,dr$

$U(r) = -\frac{GmM}{r^2} + U(\infty)$
ref value at 0

*Difference in PE b/w r and ∞

Gravitational Potential (Joules/kilogram)

Define gravitational potential difference:

$$\boxed{\Delta V_g = \frac{\Delta U_g}{m} = -\int_A^B (\vec{\mathbf{F}}_g / m) \cdot d\vec{\mathbf{s}} = -\int_A^B \vec{\mathbf{g}} \cdot d\vec{\mathbf{s}}}$$

↑field

Just as $\underbrace{\vec{\mathbf{F}}_g}_{\text{Force}} \rightarrow \underbrace{\vec{\mathbf{g}}}_{\text{Field}}, \underbrace{\Delta U_g}_{\text{Energy}} \rightarrow \underbrace{\Delta V_g}_{\text{Potential}}$

That is, two particle interaction → single particle effect

$\frac{PE}{m}$ ← little mass

Only want M ← big mass/earth

*integral of field
large conceptual

PRS Question: Masses in Potentials

Something profound hidden here

PRS: Masses in Potentials

Consider 3 equal masses sitting in different gravitational potentials:

A) Constant, zero potential
B) Constant, non-zero potential
C) Linear potential ($V \propto x$) but sitting at $V = 0$

Which statement is true?

0%	1. None of the masses accelerate
0%	2. Only B accelerates
0%	3. Only C accelerates
0%	4. All masses accelerate, B has largest acceleration
0%	5. All masses accelerate, C has largest acceleration
0%	6. I don't know

:20

When you think about pontential think height

Constat potential = no acc

value of potential does NOT matter only slope matters

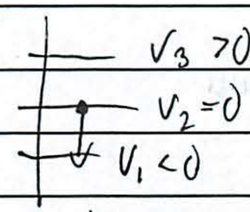
0 change in potential nothing would move

*electric potential - about location/field

*potential energy - about an object

Move to Electrostatics

P05 - 14



$\Delta V = V_1 - V_2$ ⊖

$\Delta U = m \Delta V$ ⊖

will acc downhill

* how potential changing in neighboorhood not just in 1 point

Gravity - Electrostatics

Mass M	Charge q (±)
$\vec{\mathbf{g}} = -G\dfrac{M}{r^2}\hat{\mathbf{r}}$	$\vec{\mathbf{E}} = k_e \dfrac{q}{r^2}\hat{\mathbf{r}}$
$\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$	$\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$

Both forces are conservative, so…

$\Delta V_g = -\int_A^B \vec{\mathbf{g}} \cdot d\vec{\mathbf{s}}$ $\quad$ $\Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$ — Potential Difference

$\Delta U_g = -\int_A^B \vec{\mathbf{F}}_g \cdot d\vec{\mathbf{s}}$ $\quad$ $\Delta U = -\int_A^B \vec{\mathbf{F}}_E \cdot d\vec{\mathbf{s}}$ — Potential Energy

P05 - 15

? q instead of m

$$\Delta V \overset{def}{=} \frac{\Delta U}{q_{test}} = -\int \frac{\vec{F}_E \cdot d\vec{s}}{q_t} = -\int \vec{E} \cdot d\vec{s}$$

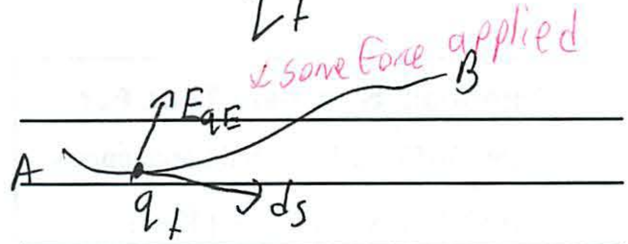
Potential & Potential Energy

$$\boxed{\Delta V \equiv -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}}$$

Units: Joules/Coulomb = Volts

Change in potential energy in moving the charged object (charge q) from A to B:

$$\Delta U = U_B - U_A = q\Delta V \quad \text{Joules}$$



If know field to get force multiply by q

Move charge from A → B $q \cdot \Delta V$ all the other charges

electric potential to U difference

Potential & External Work

Change in potential energy in moving the charged object (charge q) from A to B:

$$\Delta U = U_B - U_A = q\Delta V \quad \text{Joules}$$

The external work is

$$W_{ext} = \Delta K + \Delta U$$

If the kinetic energy of the charged object does not change, $\Delta K = 0$

then the external work equals the change in potential energy

$$W_{ext} = \Delta U = q\Delta V$$

he does not know why this is here

* Can describe changes in Potential Energy

Gradiant perpendicular to height

only if ends at same speed

Generally we will have no external forces $\Delta k + \Delta U = 0$

How Big is a Volt?

Know These!

- AA, C, D Batteries 1.5 V
- Car Battery 12 V
- US Outlet 120 V (AC)
- Residential Power Line
- Our Van de Graaf
- Big Tesla Coil $< 100{,}000$ V

Have to really know to estimate

Potential: Summary Thus Far

Charges *CREATE* Potential Landscapes

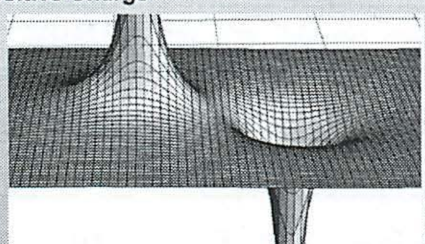
$$V(\vec{\mathbf{r}}) = V_0 + \Delta V \equiv V_{"0"} - \int_{"0"}^{\vec{\mathbf{r}}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

P05 - 19

charges - fields + potentials

Potential Landscape

Positive Charge



Negative Charge

P05 - 20

*difference in landscape

Potential: Summary Thus Far

Charges *CREATE* Potential Landscapes

$$V(\vec{\mathbf{r}}) = V_0 + \Delta V \equiv V_{"0"} - \int_{"0"}^{\vec{\mathbf{r}}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

Charges *FEEL* Potential Landscapes

$$U(\vec{\mathbf{r}}) = qV(\vec{\mathbf{r}})$$

We work with ΔU (ΔV) because ***only changes matter***

P05 - 21

2 PRS Questions: Potential & Potential Energy

PRS: Positive Charge

Place a positive charge in an electric field. It will accelerate from

0%	1. higher to lower *electric potential*; lower to higher *potential energy*
0%	2. higher to lower *electric potential*; higher to lower *potential energy* ✓
0%	3. lower to higher *electric potential*; lower to higher *potential energy*
0%	4. lower to higher *electric potential*; higher to lower *potential energy*

Objects move to reduce PE
(+) charges do this by going to lower potential

$\Delta U = q \Delta V$

$\ominus$ or $\ominus$ $\oplus$ $\oplus$ $\ominus$ $\ominus$

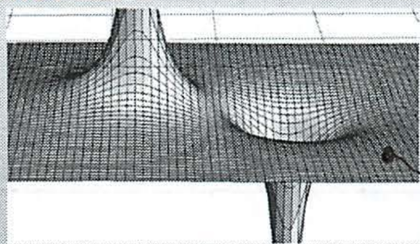
PRS: Negative Charge

Place a negative charge in an electric field. It will accelerate from

0%	1. higher to lower *electric potential*; lower to higher *potential energy*
0%	2. higher to lower *electric potential*; higher to lower *potential energy*
0%	3. lower to higher *electric potential*; lower to higher *potential energy*
0%	4. lower to higher *electric potential*; higher to lower *potential energy*

Potential Landscape

Positive Charge



Negative Charge

P05 - 25

If q (+), $\frac{kQ}{r}$ very (+) near (+) charge

(+) charge would go down the hole if on mountain

if (+) charge at flat area won't go anywhere

how batteries work – electrons (−) go from lower to higher (+) terminal

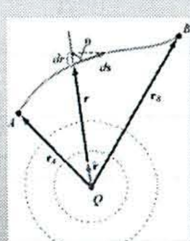
current is opposite ((+) charges)

Creating Potentials: Calculating from E, Two Examples

P05 - 26

* Very Important Calculation *

Gp Prob: Pt Charge Potential



Consider a SINGLE point charge Q.

What potential difference
$$\Delta V = V_B - V_A$$
does it create between point B and point A?

If $V_A \equiv 0$ for $r_A = \infty$, what is $V(r)$?

P05 - 27

Group Work

$V(\vec{r}) = V_0 + \Delta V$

$= V_0 - \int \vec{E} \cdot d\vec{s}$

$= -\int E\hat{r} \cdot d\vec{s}$

$= -\int_A^B E\hat{r} \cdot (dr\,\hat{r} + r\,d\phi\,\hat{\phi})$

$= -\int_A^B E\,dr$

$= -\int_A^B -k_e \frac{q}{r^2}\,dr$

$= \frac{k_e q}{r}\Big|_A^B$

$= k_e q\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$

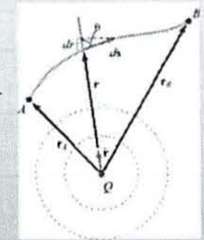
ans →

Potential Created by Pt Charge

$$\Delta V = V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$= -\int_A^B kQ\frac{\hat{\mathbf{r}}}{r^2} \cdot d\vec{\mathbf{s}} = -kQ\int_A^B \frac{dr}{r^2}$$

$$= kQ\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$



Take $V = 0$ at $r = \infty$:

$$\boxed{V_{\text{Point Charge}}(r) = \frac{kQ}{r}}$$

$$\vec{\mathbf{E}} = kQ\frac{\hat{\mathbf{r}}}{r^2}$$

$$d\vec{\mathbf{s}} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}}$$

P05 - 28

Started at ∞
now r from q

Meaning of dot product -
how much of 1 vector in direction of other - in this problem how much of it is in radial direction
- why the $\emptyset$ cancles in this problem

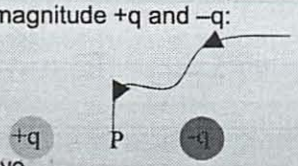
test charge at ∞
as move to distance r
calculate charge at origin
test charge

PRS Question: Point Charge Potential

P05 - 29

:20 PRS: Two Point Charges

The work done in moving a positive test charge from infinity to the point P midway between two charges of magnitude +q and –q:



0% 1. is positive.
0% 2. is negative.
0% 3. is zero.
0% 4. can not be determined – not enough info is given.
0% 5. I don't know

P05 - 30

* what matters is the charge
No work done
but particle would move if released

$W_{ext} = \Delta K + \Delta U$
$= 0$ start + end at rest $+\Delta U$
$= q\,\Delta V$

Superposition: add $\frac{kq}{d} - \frac{kq}{d} = 0$
difference b/w them $= 0$
so
$= 0$

force may vary in direction
sometimes with / against E
if came straight down, always 0

E field can only get so big or ionizes air

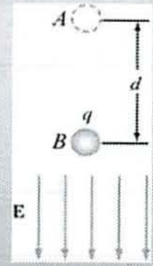
$E = \frac{kq}{r^2}$ for point charge/sphere

Potential in a Uniform Field

$$\Delta V = \underbrace{V_B}_{\text{to}} - \underbrace{V_A}_{\text{from}} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$= -\int_A^B -E\hat{\mathbf{j}} \cdot d\vec{\mathbf{s}} = E\int_A^B dy$$

$$= -Ed$$



Just like gravity, moving in field direction reduces potential

$\vec{\mathbf{E}} = -E\hat{\mathbf{j}}$

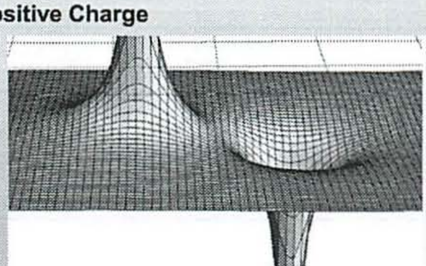
$d\vec{\mathbf{s}} = dy\hat{\mathbf{j}}$

P05 - 31

not Ed for point charge

Potential Landscape

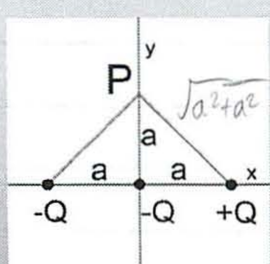
Positive Charge



Negative Charge

P05 - 32

Group Problem: Superposition



Consider the 3 point charges at left.

What total electric potential do they create at point P (assuming $V_\infty = 0$)

P05 - 33

Cancels out

$\cancel{\frac{kq}{\sqrt{a^2+a^2}}} - \cancel{\frac{kq}{\sqrt{a^2+a^2}}} \boxed{-\frac{kq}{a}}$

0 at ∞ - bring to P - find $\Delta PE^{\infty \to P}$ test charge

don't confuse w/ electric charge

Scalar quantity - not a vector

not $\frac{1}{r^2}$

$\frac{dr}{r^2} = \frac{1}{r}$

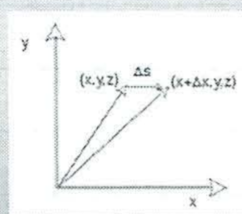
Not value of potential
But how it is changing

Deriving E from V

PHS - 34

Deriving E from V

$$\Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$



A = (x,y,z), B=(x+Δx,y,z)

$$\Delta \vec{\mathbf{s}} = \Delta x \hat{\mathbf{i}}$$

$$\Delta V = -\int_{(x,y,z)}^{(x+\Delta x,y,z)} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} \cong -\vec{\mathbf{E}} \cdot \Delta \vec{\mathbf{s}} = -\vec{\mathbf{E}} \cdot (\Delta x \hat{\mathbf{i}}) = -E_x \Delta x$$

$$\boxed{E_x \cong -\frac{\Delta V}{\Delta x} \rightarrow -\frac{\partial V}{\partial x}}$$

E_x = Rate of change in V with y and z held constant

PHS - 35

direction that it displaces (partial derivative)

⊖ points down potential landscape

Deriving E from V

If we do all coordinates: 3D

$$\vec{\mathbf{E}} = -\left(\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}} \right)$$

$$= -\underbrace{\left(\frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial y}\hat{\mathbf{j}} + \frac{\partial}{\partial z}\hat{\mathbf{k}} \right)}_{\text{Gradient (del) operator:}} V$$

Same for each

$$\boxed{\vec{\mathbf{E}} = -\nabla V}$$

Gradient (del) operator:

$$\nabla \equiv \frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial y}\hat{\mathbf{j}} + \frac{\partial}{\partial z}\hat{\mathbf{k}}$$

PHS - 36

Field perpendicular to equipotentials

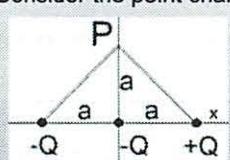
calc gradients for E Field
+ do Gauss Law

PRS Question: E from V

PRS: E from V

20

Consider the point charges you looked at earlier:



$$V(P) = -kQ/a$$

You calculated V(P). From that can you derive E(P)?

0% 1. Yes, its kQ/a^2 (up)
0% 2. Yes, its kQ/a^2 (down)
0% 3. Yes in theory, but I don't know how to take a gradient
0% 4. No, you can't get E(P) from V(P)
0% 5. I don't know

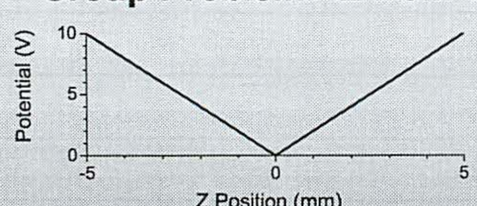
$E = -\nabla V$

- don't know how potential is changing - only know value at 1 point

need to - know how it is changing

like looking around w/ blindfolds - don't know what is uphill

Group Problem: E from V



A potential V(*x*,*y*,*z*) is plotted above. It does not depend on *x* or *y*.
What is the electric field everywhere?
Are there charges anywhere? What sign?

Demonstration:
Making & Measuring
Potential
(Lab Preview)

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2010**

Problem Set 2

Due: Tuesday, February 16 at 9 pm.

Hand in your problem set in your section slot in the boxes outside the door of 32-082. Make sure you clearly write your name and section on your problem set.

Text: Liao, Dourmashkin, Belcher; Introduction to E & M MIT 8.02 Course Notes.

Reading Assignments:

Week Two: Gauss's Law and Electric Potential

Class 4 T/W Feb 9/10	Gauss' Law
Reading:	Course Notes: Sections 4.1-4.2, 4.6
Class 5 R/T Feb 11/16	Electric Potential
Reading:	Course Notes: Sections 3.1-3.5, 3.7-3.8
Class 6 F Feb 12	PS02: Gauss's Law
Reading:	Course Notes: Sections 4.1-4.2, 4.7-4.8

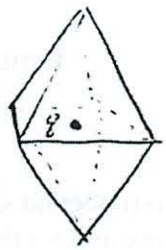
Week Three: Electric Potential

President's Day – M 2/15 / M Classes on T 2/16

Class 5 W03D1 T Feb 16	Electric Potential
Reading:	Course Notes Sections 3.1-3.5, 3.7-3.8
Class 7 W03D02 W/R Feb 17/18	Electric Potential; Equipotential Lines and Electric Fields Expt.1: Electric Potential; Configuration Energy;
Reading:	Course Notes: Sections 3.1-3.5
Experiment:	Expt. 1: Electric Potential
Class 8 W03D3 F Feb 19	PS03: Electric Potential
Reading:	Course Notes: Sections 3.1-3.5, 3.7-3.8

Problem 1 (10 points): Concept Questions. Explain your reasoning.

Concept Question 1: A pyramid has a square base of side a, and four faces which are equilateral triangles. A charge Q is placed on the center of the base of the pyramid. What is the net flux of electric field emerging from one of the triangular faces of the pyramid?



1. 0
2. $\dfrac{Q}{8\varepsilon_0}$
3. $\dfrac{Qa^2}{2\varepsilon_0}$
4. $\dfrac{Q}{2\varepsilon_0}$
5. Undetermined: we must know whether Q is infinitesimally above or below the plane?

Concept Question 2: A charge distribution generates a radial electric field

$$\vec{\mathbf{E}} = \frac{a}{r^2} e^{-r/b} \hat{\mathbf{r}}$$

where a and b are constants. The total charge giving rise to this electric field is

1. $4\pi\varepsilon_0 a$
2. 0
3. $4\pi\varepsilon_0 b$

Problem 2 (10 points): Non-uniformly charged sphere A sphere of radius R has a charge density $\rho = \rho_0 (r/R)$ where ρ_0 is a constant and r is the distance from the center of the sphere.

a) What is the total charge inside the sphere?

b) Find the electric field everywhere (both inside and outside the sphere).

Problem 3 (10 points): N-P Junction

When two slabs of N-type and P-type semiconductors are put in contact, the relative affinities of the materials cause electrons to migrate out of the N-type material across the junction to the P-type material. This leaves behind a volume in the N-type material that is positively charged and creates a negatively charged volume in the P-type material.

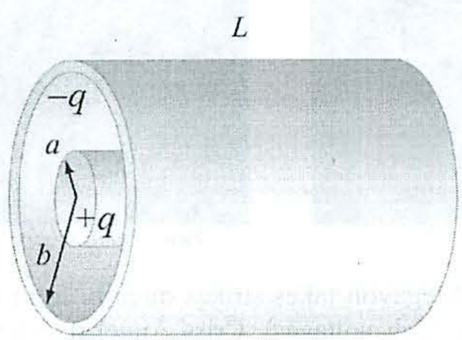
Let us model this as two infinite slabs of charge, both of thickness a with the junction lying on the plane $z = 0$. The N-type material lies in the range $0 < z < a$ and has uniform charge density $+\rho_0$. The adjacent P-type material lies in the range $-a < z < 0$ and has uniform charge density $-\rho_0$. Thus:

$$\rho(x,y,z) = \rho(z) = \begin{cases} +\rho_0 & 0 < z < a \\ -\rho_0 & -a < z < 0 \\ 0 & |z| > a \end{cases}$$

Find the electric field everywhere.

Problem 4 (10 points): Co-axial Cylinders

A very long conducting cylinder (length L and radius a) carrying a total charge $+q$ is surrounded by a thin conducting cylindrical shell (length L and radius b) with total charge $-q$, as shown in the figure.

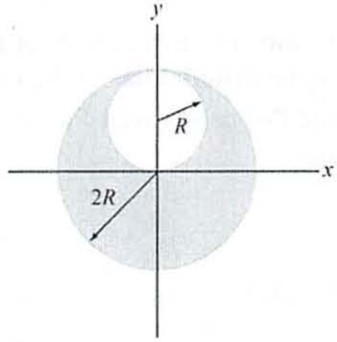


(a) Using Gauss's Law, find an expression for the direction and magnitude of the electric field $\vec{\mathbf{E}}$ for the region $r < a$.

(b) Similarly, find an expression for the direction and magnitude of the electric field $\vec{\mathbf{E}}$ for the region $a < r < b$.

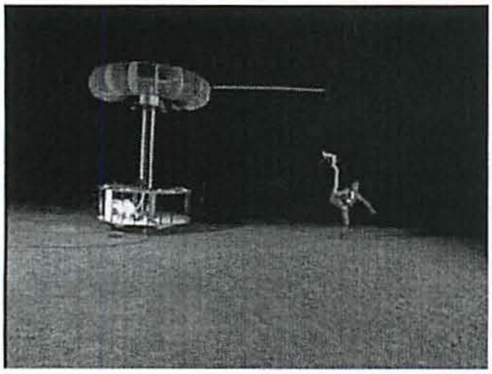
Problem 5 (10 points): Non-Conducting Solid Sphere with a Cavity

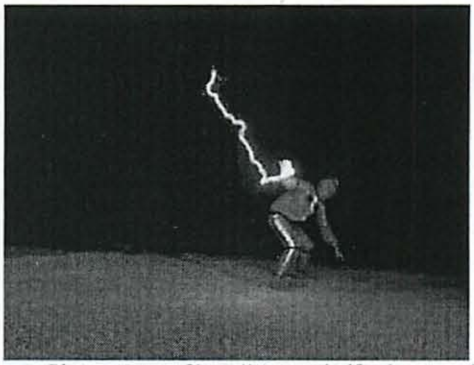
A sphere of radius $2R$ is made of a non-conducting material that has a uniform volume charge density ρ. (Assume that the material does not affect the electric field.) A spherical cavity of radius R is then carved out from the sphere, as shown in the figure below. Find the electric field within the cavity.



Problem 6 (10 points): Stupid Hobbies...

Some people like to do incredibly dangerous things. Like Austin Richards (also known as Dr. Megavolt or Criss Angel, who performed a similar stunt on the "Tesla Coil" episode of his show mindfreak. Here are some pictures.





Pictures care of http://www.mindfreakconnection.com/

You'll note that while Dr. Megavolt takes strikes directly from the Tesla Coil (a device capable of making insanely high voltages), Criss Angel decides to get shocked from a small ball attached to the coil instead – convenient for the purposes of answering this question. At about what voltage was the Tesla coil for the strikes pictured above and about how much excess charge was on his hand (in the right picture) the instant before the strike was initiated? (HINT: Dry air breaks down at an electric field strength of about 3×10^6 V/m)

Problem 7 (10 points): Expt. 1: Equipotential Lines and Electric Fields Pre-Lab Questions

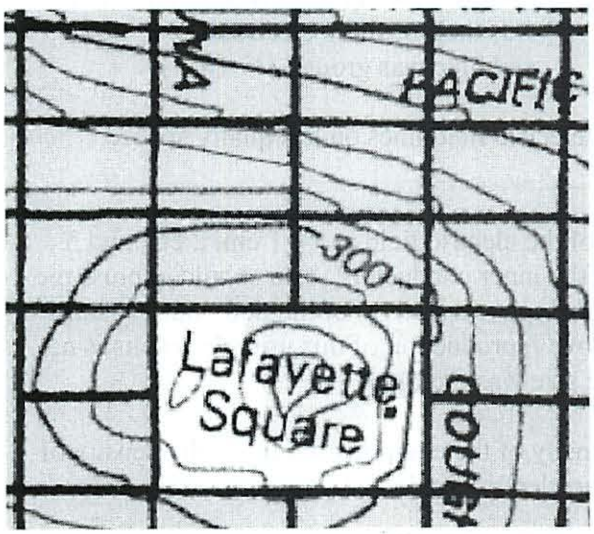
Read Experiment 1. The link is

> **http://web.mit.edu/8.02t/www/materials/Experiments/exp01.pdf.**

Then answer the following pre-lab questions.

1. Equipotentials Curves – Reading Topographic Maps

Below is a topographic map of a 0.4 mi square region of San Francisco. The contours shown are separated by heights of 25 feet (so from 375 feet to 175 feet above sea level for the region shown)



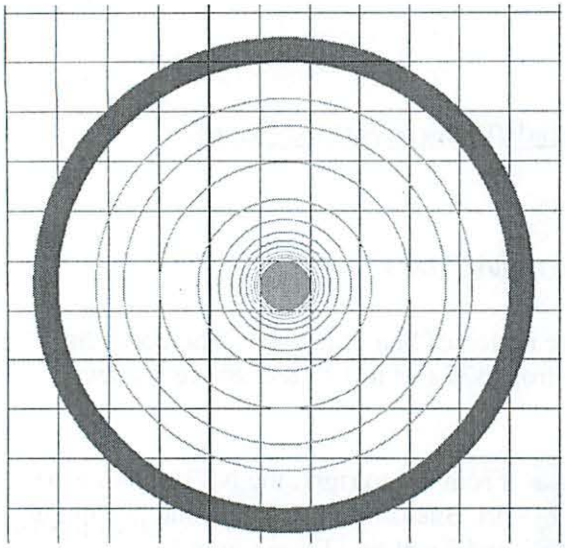
From left to right, the NS streets shown are Buchanan, Laguna, Octavia, Gough and Franklin. From top to bottom, the EW streets shown are Broadway, Pacific, Jackson, Washington, Clay (which stops on either side of the park) and Sacramento.

(a) In the part of town shown in the above map, which street(s) have the steepest runs? Which have the most level sections? How do you know?

(b) How steep is the steepest street at its steepest (what is its slope in ft/mi)?

(c) Which would take more work (in the physics sense): walking 3 blocks south from Laguna and Jackson or 1 block west from Clay and Franklin?

2. Equipotentials, Electric Fields and Charge



One group did this lab and measured the equipotentials for a slightly different potential landscape then the ones you have been given (although still on a 1 cm grid).

Note that they went a little overboard and marked equipotential curves (the magenta circles) at V = 0.25 V, 0.5V and then from V = 1 V to V = 10 V in 1 V increments.

They followed the convention that red was their positive electrode ($V = +10$ V) and blue was ground ($V = 0$ V).

(a) Copy the above figure and sketch eight electric field lines on it (equally spaced around the inner conductor).

(b) What, approximately, is the magnitude of the electric field at $r = 1$ cm, 2 cm, and 3 cm, where r is measured from the center of the inner conductor? You should express the field in V/cm. (HINT: The field is the local slope (derivative) of the potential. Also, if you choose to use a ruler realize that the above reproduction of this group's results is not the same size as the original, where the grid size was 1 cm).

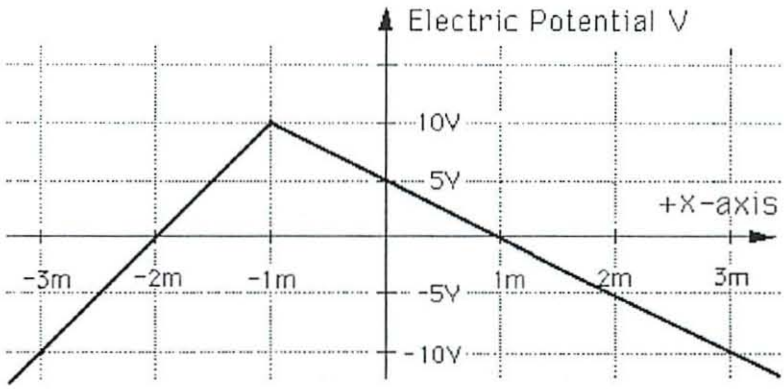
(c) What is the relationship between the density of the equipotential lines, the density of the electric field lines, and the strength of the electric field?

(d) Plot the field strength vs. $1/r^2$ for the three points from part (a). If the field were created by a single point charge what shape should this sketch be? Is it?

(e) Approximately how much charge was on the inner conductor when the group made their measurements?

3. Finding the Electric Field from the Electric Potential

The graph shows the variation of an electric potential V with distance x. The potential does not vary in the y or z directions. Be sure to include units as appropriate.



(a) What is E_x in the region $x > -1\text{ m}$? (Be careful to indicate the sign of E_x.)

(b) What is E_x in the region $x < -1\text{ m}$? (Be careful to indicate the sign of E_x.)

(c) A negatively charged dust particle with mass $m_q = 1\times10^{-13}\text{ kg}$ and charge $q = -1\times10^{-12}\text{ C}$ is released from rest at $x = +2\text{ m}$. Will it move to the left or to the right?

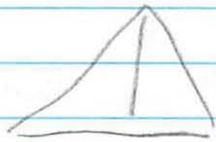
Michael Plasmeier 11C

8.02 Pset 2

72

You had a lot of good questions. I'm glad you're identifying what's confusing. be sure to ask a TA, go to office hours, or mail 8.02-help@mit.edu 2/14

1. Pyrmid w/ charge q. Net flux out of 1 side



$\phi = EA = \frac{q_{inc}}{\epsilon}$

Want $\frac{1}{4}$ of this since its 1/4 the area - right? - no square base

$r^2 + 4 \frac{1}{2} a^2$

round up to $\frac{1}{8}$

$\frac{2}{2}$

$\frac{1}{8} \frac{Q}{\epsilon_0} = EA$

#2

1b. Charge distributes radial electric field

$\vec{E} = \frac{a}{r^2} e^{-r/b} \hat{r}$ a/b = constants

Total charge? Q?

$\cancel{EA = \frac{q_{inc}}{\epsilon_0}}$ $\vec{E} = \frac{1}{4\pi\epsilon_0} \int_V \frac{dq}{r^2} \hat{r}$

if sub in for colombs law - does not make sense except for 0 →

$E = k_e \frac{Q}{r^3} \vec{r}$

$\frac{Er^3}{k_e} = Q$

$Q = \frac{a e^{-r/b} r^3}{r^2 k_e} = \frac{are^{-r/b}}{k_e} \to \cancel{0}$ $=0$ #2

$\boxed{k = \frac{1}{4\pi\epsilon_0}\ \text{Note}}$

$\lim_{r\to\infty}$ ← enclose whole world

Non uniformly charged sphere

2.  $\rho = \rho_0 \frac{r}{R}$ ← radius

↑ constant

a. total charge inside sphere

$$\rho = \int \frac{Q}{V}$$

$$Q = \int \rho V$$

$$Q = \int \rho_0 \frac{r}{R} \frac{4}{3}\pi r^3$$

don't know tripple integrals
- look at cons when comes out

$$Q = \int \frac{4\pi r^4}{3R}\rho_0 = \frac{4\pi r^3}{3}\rho_0 \int \frac{r}{R}$$

b. Electric field

$$EA = \frac{Q_{inc}}{\epsilon_0}$$

inside

$$E \cdot 4\pi r^2 = \int \frac{\frac{4}{3}\pi r^4}{R\epsilon_0}\rho_0$$

$$E = \frac{4\pi r^3 \rho_0}{3 \cdot 4\pi r^2 \epsilon_0}\int \frac{r}{R}$$

$$= \frac{\rho_0}{3\epsilon_0}\int \frac{r}{R}$$

does not reduce w/ distance - right?

not confused on concepts but may be missing subtilies

outside

$$E \cdot 4\pi R^2 = \frac{4\pi r^3}{3\epsilon_0}\rho_0$$

do we do some thing that $r = R$?

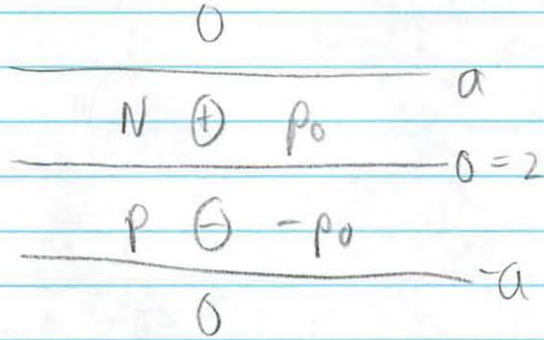
$$E = \frac{4\pi R^3 \rho_0}{3\epsilon_0 \cdot 4\pi R^2} = \frac{\rho_0 R}{3\epsilon_0}$$

or not?

3. N-P Junction

2 slabs N-type and p-type semiconductors in contact
electrons go from N to P
so N is $\oplus$ charged, P $\ominus$ charged

do as 2 slabs of infinite charge



$$\rho = \begin{cases} +\rho_0 & 0 < z < a \\ -\rho_0 & -a < z < 0 \\ 0 & |z| > a \end{cases}$$

E = constant

$$\Phi = E\,A_{endcaps} = \frac{q_{inc}}{\varepsilon_0}$$

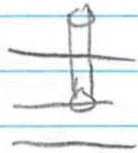
$$E\,2A = \frac{\rho_0 A}{\varepsilon_0}$$

$$E = \frac{\rho_0}{2\varepsilon_0}$$

Basic for plane - pillbox tops

I think I have to do the same for each region - 4 regions

$z > a$



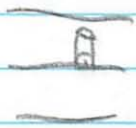
$\cancel{EA = \frac{\rho_0 z}{\epsilon_0}}$

$2EA = \frac{\rho_0 A}{\epsilon_0}$

$E = \frac{\rho_0}{2\epsilon_0}$

missing something to tie together

$0 < z < a$

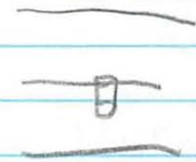


$EA = \frac{\rho_0 \pi z}{\epsilon_0}$

$E(2\pi r z) = \frac{\rho_0 z}{\epsilon_0}$

$E = \frac{\rho_0}{2\pi r \epsilon_0}$

$-a < z < 0$



$EA = -\frac{\rho_0 z}{\epsilon_0}$

$E = \frac{\rho_0}{-2\pi r \epsilon_0}$

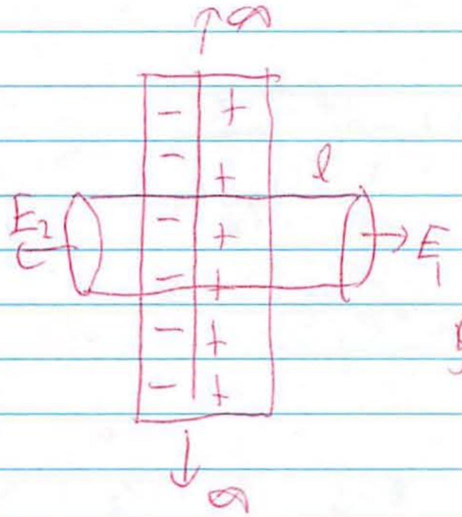
$-z > -a$

$\cancel{EA = -\frac{\rho_0 z}{\epsilon_0}}$

$E = -\frac{\rho_0}{2\epsilon_0}$

another try of this problem

missing some subtility



Pill box
ends only

$\oint \vec{E} \cdot d\vec{A} = E_1 A + E_2 A$

$E_1 = E_2$

so $2EA$

$2E\pi r^2 = \frac{\rho_0 \pi r^2 z}{\varepsilon_0}$

I know outside $= 0$ both ways

$E = \frac{\rho_0 z}{2\varepsilon_0}$ $\quad \vec{E}_2 = -\vec{E}_1$

$\uparrow z > 2a$

Inside

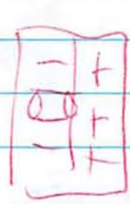
$\underbrace{EA}_{0?} + E_1 A = \frac{\rho A z}{\varepsilon_0}$

?! still don't really get

$\frac{\rho_0 z}{\varepsilon_0} - \frac{\rho_0 a}{\varepsilon_0} = \frac{\rho_0 (z-a)}{\varepsilon_0}$

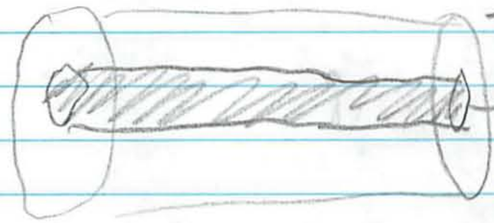
$E = -\frac{\rho_0}{\varepsilon_0}(a-z)\hat{z}$

and then



$\frac{\rho_0}{\varepsilon_0}(a-z) - \hat{z}$

4. Co-axial Cylinders



Same as problem solving except its non-conducting material so all charge is $r=a$ $r<a=0$

a) Find $\vec{E}$ for $r<a$

Hint: ↳ will find that is 0

$\phi = EA = \frac{q_{inc}}{\epsilon_0}$

surface area

1. Identify symetry
2. Determine direction
   - radially outward
3. 1 area of space
4. Cylinder
5. after math review - don't need

$d\vec{A} = \hat{r} \cdot r \cdot d\phi \cdot dz$

$\int (\vec{r} \cdot E)(\hat{r}\, r\, d\phi\, dz)$

Or simply EA_{sides}

$E\, 2\pi r z$

$E\, 2\pi r z = \rho V / \epsilon_0$

$\frac{\rho \pi r^2 z}{\epsilon_0}$

$E = \frac{\rho \cancel{\pi} r^{\cancel{2}} \cancel{z}}{\epsilon_0 2 \cancel{\pi} \cancel{r} \cancel{z}} = \frac{\rho r}{\epsilon_0 2} = 0$

is 0 since free conduction only on sides

non conducting material

b. This should be very similar - but does not depend on r

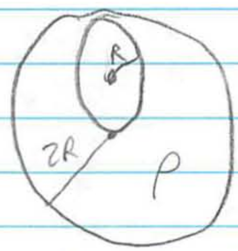
From problem solving: Q_{enc} is same for all r values
- so I guess rewrite $r \to a$

a is container size

$$E\, 2\pi a z = \frac{\rho_0 \pi a^2 z}{\varepsilon_0}$$

$$E = \frac{\rho_0 a}{2\varepsilon_0}$$

5. Non conducting Solid Sphere w/ a Cavity



Superposition Gauss' law

$$\rho = \frac{Q}{V}$$

General $EA = \frac{q_{inc}}{\varepsilon_0}$

$$E\, 4\pi r^2 = \frac{\rho \frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$E = \frac{\rho \cancel{4} \cancel{\pi} r^{\cancel{3}}}{3\varepsilon_0 \cancel{4}\cancel{\pi} \cancel{r^2}} = \frac{\rho r}{3\varepsilon_0}$$

Small $= \frac{\rho (R)}{3\varepsilon_0}$ ← sphere w/o cavity ✓

?, but that seems too simple

Small sphere has charge density $-\rho$

? $\ominus$ since opposite Charge

$$\vec{E}_2 = \frac{-\rho}{3\varepsilon}(\vec{r} - \vec{R}) \qquad r < 2R$$

Total $\vec{E}$ in cavity

$$E_1 + E_2 = \frac{\rho \vec{r}}{3\varepsilon_0} - \frac{\rho \vec{r}}{3\varepsilon_0} + \frac{\rho \vec{R}}{3\varepsilon_0} = \frac{\rho \vec{R}}{3\varepsilon_0}$$

Hint: answer should be remarkable
? don't get it

agreed

6. Stupid Hobbies of Criss Angel

What voltage + how much extra charge on hand?
Dry air breaks down at an Electric Field strenght of 3×10^6 V/m

after what we learned today

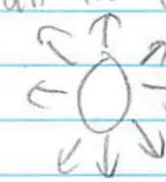
Hint: Height $\sim$ 2m (between 1-3m) assume charge on ballon of some radius (1-3cm)

So 2m = $V_\Delta = 6\cdot10^6$ V

$$V_\Delta = -\int_A^B E\cdot ds$$

$$6\cdot10^6 = -\int_0^6 \vec{E}\cdot d\vec{s} \quad \frac{dr}{r^2}$$

all radially outward of small sphere so all in direction



I don't really get it

$$V = \frac{kQ}{r} \rightarrow 6\cdot10^6 = \frac{kQ}{.02}$$

$$120{,}000 = kQ$$

Also asks about how much charge
- can find using flux

$$EA = \frac{\rho V}{\varepsilon_0}$$

$$E4\pi r^2 = \frac{\rho\frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$E = \frac{\rho 4\pi r^3}{3\varepsilon_0 4\pi r^2} = \frac{\rho r}{3\varepsilon_0}$$

Experiment 1 Pre-Lab

1. Topo Map of San Fransisco
   - lines seporated by 25 ft

   Steepest (lines closest together, perpendicular to lines
      Gough
      Washington
      Jackson

   Flattest (lines furtest apart, parrallel lines)
      Pacific
      Buchannan near to Park
      Broadway

b. How steep is steepest street
   Gough from Washington to Broadway
   ~ .27 miles
   125 ft

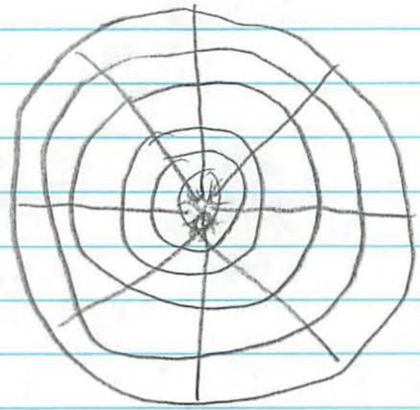
   $\frac{125}{.27} = 462.9$ ft/mile

c. Which greatest Physical work

   Laguna + Jackson 3 south
      would end you up at same elevation
      which physics calls no work

   Clay + Franklin west
      net elevation change, so done work

2.



Field lines go perpendicular to these equipotent lines

Go in the direction downhill to a lower potential

$\Delta V = V_B - V_A = \int_A^B E \, d\vec{s}$

b. Magnitude of field at r = 1, 2, 3 cm

What does this really mean? The # of lines in a cm cube?

1 5 V / 1 cm = 5 V/cm
2 8.5 V / 2cm = 4.25 V/cm case A
3 = 9 V / 3cm = 3 V/cm

Hint: The field is the local slope (derivative) of the potential.

case B

cm	# lines
1	6
2	2
3	1

c) Relationship
- density equipotent lines
- density field lines
- strenght field

Denser lines = Steeper potential change
Larger

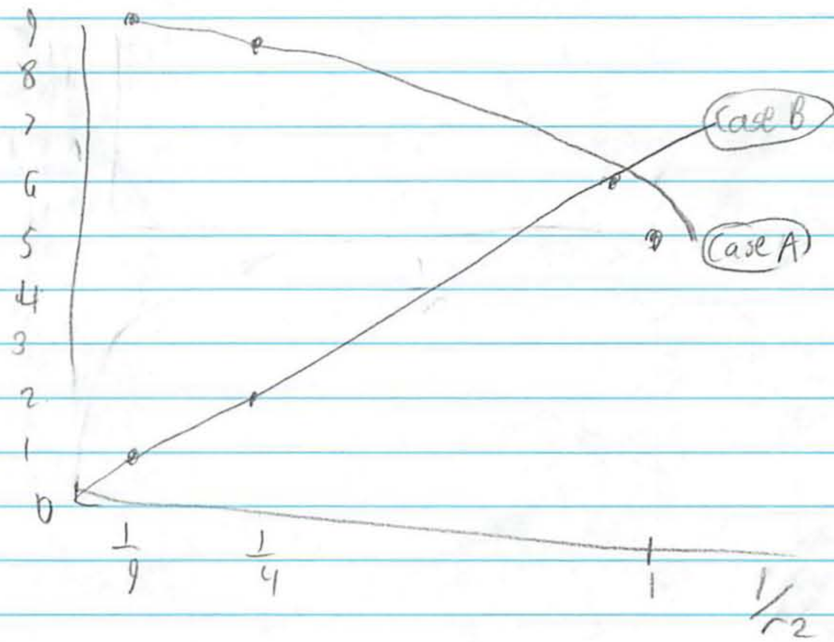
in which case d will be

Density electric field lines - ~~I don't thing mean anything~~ indicates strenght of field

See $\vec{d}$

d. Plot

field strength $\boxed{V/cm}$



What if it was a point charge?

- would be the same?

- ~~The curves would be closer~~

e) How much charge was on inner conductor?

Driviving E from V

$$V = -\int_A^B E \, ds$$

$$E = -\nabla V$$ ← but can't do gradient

or is it $Q = \rho V$

$$Q = \rho \pi r^2$$

or do Guass's Law and set =

$$EA = \frac{\rho V}{\varepsilon_0}$$

3. Finding the electric field from electric potential

Hint: Potential is linear, so E should be constant in each region

a. E_x when $x > -1m$

$$\Delta V = V_B - V_A = -\int_A^B E \cdot ds$$

$$-10 - 10 = -$$

$$\int E \cdot 3 - -1$$

? is this at all legal? confused on reverse of $\int$

$$E = -\nabla V$$

$$20 = \int_{10}^{-10} E \cdot 4$$

$E = -5$ ← or is it the slope which is same as what I calculated

b. E_x when $x < -1$ m

$$\frac{-10 - 10}{-3 - -1} = \frac{-20}{-2} = 10 \text{ units?}$$

C. A negitivly charged dust particle

$m_q = 1 \cdot 10^{-13}$ kg $\quad q = -1 \cdot 10^{-12}$ C

released from rest $x = 2m$

will it move left or right?

It will always (+ and − move to a lower potential energy. If it is −, it will move to a more + electric potential. In this case to left

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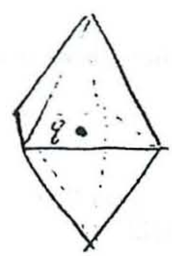
Problem Set 2 Solutions

Problem 1 (10 points): Concept Questions. Explain your reasoning.

Concept Question 1: A pyramid has a square base of side a, and four faces which are equilateral triangles. A charge Q is placed on the center of the base of the pyramid. What is the net flux of electric field emerging from one of the triangular faces of the pyramid?

1. 0
2. $\dfrac{Q}{8\varepsilon_0}$
3. $\dfrac{Qa^2}{2\varepsilon_0}$
4. $\dfrac{Q}{2\varepsilon_0}$
5. Undetermined: we must know whether Q is infinitesimally above or below the plane?

Answer 2: Explain your reasoning: Construct an eight faced closed surface consisting of two pyramids with the charge at the center. The total flux by Gauss's law is just Q/ε_0. Since each face is identical, the flux through each face is one eight the total flux or $Q/8\varepsilon_0$.



Concept Question 2: A charge distribution generates a radial electric field

$$\vec{\mathbf{E}} = \frac{a}{r^2} e^{-r/b}\hat{\mathbf{r}}$$

where a and b are constants. The total charge giving rise to this electric field is

1. $4\pi\varepsilon_0 a$
2. 0
3. $4\pi\varepsilon_0 b$

Answer 2: Explain your reasoning: In order to fine the total charge I choose a Gaussian surface that extends over all space. Because the electric field is radially symmetric, I choose my Gaussian surface to be a sphere of radius r and I will take the limit as $r \to \infty$. The flux is given by

$$\lim_{r\to\infty} \oiint_r \vec{\mathbf{E}}\cdot d\vec{\mathbf{a}} = \lim_{r\to\infty} \oiint_r \frac{a}{r^2} e^{-r/b}\hat{\mathbf{r}}\cdot da\hat{\mathbf{r}} = \lim_{r\to\infty} \oiint_r \frac{a}{r^2} e^{-r/b} da = \lim_{r\to\infty} \frac{a}{r^2} e^{-r/b} 4\pi r^2 = 4\pi a \lim_{r\to\infty} e^{-r/b} = 0$$

When I take the limit as $r \to \infty$, the exponential term goes to zero, and so the flux goes to zero. Therefore the charge enclosed is zero.

Problem 2 (10 points): Non-uniformly charged sphere A sphere of radius R has a charge density $\rho = \rho_0 (r/R)$ where ρ_0 is a constant and r is the distance from the center of the sphere.

a) What is the total charge inside the sphere?

Solution:

The total charge inside the sphere is the integral

$$Q = \int_{r'=0}^{r=R} \rho 4\pi r^2 dr = \int_{r'=0}^{r=R} \rho_0 (r/R) 4\pi r^2 dr = \frac{\rho_0 4\pi}{R} \int_{r=0}^{r=R} r^3 dr = \frac{\rho_0 4\pi}{R}\frac{R^4}{4} = \rho_0 \pi R^3$$

b) Find the electric field everywhere (both inside and outside the sphere).

Solution:

There are two regions of space: region I: $r < R$, and region II: $r > R$ so we apply Gauss' Law to each region to find the electric field.

For region I: $r < R$, we choose a sphere of radius r as our Gaussian surface. Then, the electric flux through this closed surface is

$$\oiint \vec{\mathbf{E}}_{\mathrm{I}} \cdot d\vec{\mathbf{A}} = E_I \cdot 4\pi r^2 .$$

Since the charge distribution is non-uniform, we will need to integrate the charge density to find the charge enclosed in our Gaussian surface. In the integral below we use the integration variable r' in order to distinguish it from the radius r of the Gaussian sphere.

$$\frac{Q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int_{r'=0}^{r'=r} \rho 4\pi r'^2 dr' = \frac{1}{\varepsilon_0}\int_{r'=0}^{r'=r} \rho_0 (r'/R) 4\pi r'^2 dr' = \frac{\rho_0 4\pi}{R\varepsilon_0}\int_{r'=0}^{r'=r} r'^3 dr' = \frac{\rho_0 4\pi r^4}{4R\varepsilon_0} = \frac{\rho_0 \pi r^4}{R\varepsilon_0}.$$

Notice that the integration is primed and the radius of the Gaussian sphere appears as a limit of the integral.

Recall that Gauss's Law equates electric flux to charge enclosed:

$$\oiint \vec{\mathbf{E}}_{\mathrm{I}} \cdot d\vec{\mathbf{A}} = \frac{Q_{enc}}{\varepsilon_0}.$$

So we substitute the two calculations above into Gauss's Law to arrive at:

$$E_I \cdot 4\pi r^2 = \frac{\rho_0 \pi r^4}{R\varepsilon_0}.$$

We can solve this equation for the electric field

$$\vec{\mathbf{E}}_{\mathrm{I}} = E_I \hat{\mathbf{r}} = \frac{\rho_0 r^2}{4R\varepsilon_0}\hat{\mathbf{r}}, \ 0 < r < R.$$

The electric field points radially outward and has magnitude $\left|\vec{\mathbf{E}}_{\mathrm{I}}\right| = \frac{\rho_0 r^2}{4\varepsilon_0}, \ 0 < r < R.$

For region II: $r > R$: we choose the same spherical Gaussian surface of radius $r > R$, and the electric flux has the same form

$$\oiint \vec{\mathbf{E}}_{\mathrm{II}} \cdot d\vec{\mathbf{A}} = E_{II} \cdot 4\pi r^2.$$

All the charge is now enclosed, $Q_{enc} = Q = \rho_0 \pi R^3$, so the right hand side of Gauss's Law becomes

$$\frac{Q_{enc}}{\varepsilon_0} = \frac{Q}{\varepsilon_0} = \frac{\rho_0 \pi R^3}{\varepsilon_0}.$$

Then Gauss's Law becomes

$$E_{II} \cdot 4\pi r^2 = \frac{\rho_0 \pi R^3}{\varepsilon_0}.$$

We can solve this equation for the electric field

$$\vec{\mathbf{E}}_{\mathbf{II}} = E_{II}\hat{\mathbf{r}} = \frac{\rho_0 R^3}{4\varepsilon_0 r^2}\hat{\mathbf{r}}, \; r > R.$$

In this region of space, the electric field points radially outward and has magnitude $\left|\vec{\mathbf{E}}_{\mathbf{II}}\right| = \frac{\rho_0 R^3}{4\varepsilon_0 r^2}$, $r > R$, so it falls off as $1/r^2$ as we expect since outside the charge distribution, the sphere acts as if it all the charge were concentrated at the origin.

Problem 3 (10 points): N-P Junction

When two slabs of N-type and P-type semiconductors are put in contact, the relative affinities of the materials cause electrons to migrate out of the N-type material across the junction to the P-type material. This leaves behind a volume in the N-type material that is positively charged and creates a negatively charged volume in the P-type material.

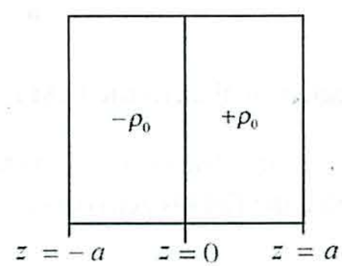
Let us model this as two infinite slabs of charge, both of thickness a with the junction lying on the plane $z = 0$. The N-type material lies in the range $0 < z < a$ and has uniform charge density $+\rho_0$. The adjacent P-type material lies in the range $-a < z < 0$ and has uniform charge density $-\rho_0$. Thus:

$$\rho(x, y, z) = \rho(z) = \begin{cases} +\rho_0 & 0 < z < a \\ -\rho_0 & -a < z < 0 \\ 0 & |z| > a \end{cases}$$

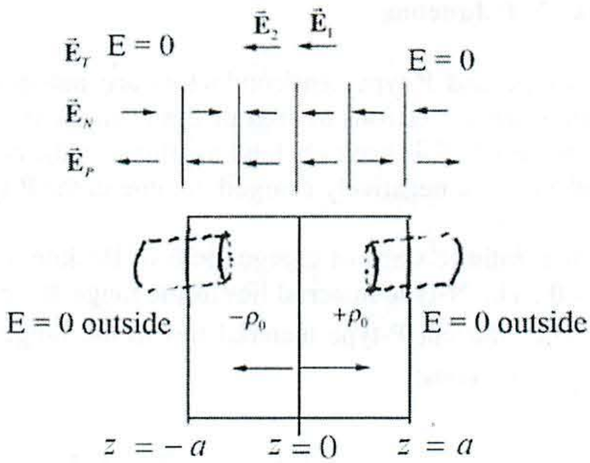
Find the electric field everywhere.

Solution:

In this problem, the electric field is a superposition of two slabs of opposite charge density.



Outside both slabs, the field of a positive slab $\vec{\mathbf{E}}_P$ (due to the P-type semi-conductor) is constant and points away and the field of a negative slab $\vec{\mathbf{E}}_N$ (due to the N-type semi-conductor)is also constant and points towards the slab, so when we add both contributions we find that the electric field is zero outside the slabs. The fields $\vec{\mathbf{E}}_P$ are shown on the figure below. The superposition of these fields $\vec{\mathbf{E}}_T$ is shown on the top line in the figure.



The electric field can be described by

$$\vec{\mathbf{E}}_T(z) = \begin{cases} \vec{\mathbf{0}} & z < -a \\ \vec{\mathbf{E}}_2 & -a < z < 0 \\ \vec{\mathbf{E}}_1 & 0 < z < a \\ \vec{\mathbf{0}} & |x| > d \end{cases}.$$

We shall now calculate the electric field in each region using Gauss's Law:

For region $-a < z < 0$: The Gaussian surface is shown on the left hand side of the figure below. Notice that the field is zero outside. Gauss's Law states that

$$\oiint_{\substack{closed\\surface}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{a}} = \frac{Q_{enclosed}}{\varepsilon_0}.$$

So for our choice of Gaussian surface, on the cap inside the slab the unit normal for the area vector points in the positive z-direction, thus $\hat{\mathbf{n}} = +\hat{\mathbf{k}}$. So the dot product becomes $\vec{\mathbf{E}}_2 \cdot \hat{\mathbf{n}} da = E_{2,z}\, \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} da = E_{2,z}\, da$. Therefore the flux is

$$\oiint_{\substack{closed\\surface}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{a}} = E_{2,z} A_{cap}$$

The charge enclosed is

$$\frac{Q_{enclosed}}{\varepsilon_0} = \frac{-\rho_0 A_{cap}(a+z)}{\varepsilon_0}$$

where the length of the Gaussian cylinder is $a+z$ since $z<0$.

Substituting these two results into Gauss's Law yields

$$E_{2,z} A_{cap} = \frac{-\rho_0 A_{cap}(a+z)}{\varepsilon_0}$$

Hence the electric field in the N-type is given by

$$E_{2,x} = \frac{-\rho_0 (a+z)}{\varepsilon_0}.$$

The negative sign means that the electric field point in the –z direction so the electric field vector is

$$\vec{\mathbf{E}}_2 = \frac{-\rho_0 (a+z)}{\varepsilon_0} \hat{\mathbf{k}}.$$

Note when $z=-a$ then $\vec{\mathbf{E}}_2 = \vec{\mathbf{0}}$ and when $z=0$, $\vec{\mathbf{E}}_2 = \frac{-\rho_0 a}{\varepsilon_0} \hat{\mathbf{k}}$.

We make a similar calculation for the electric field in the P-type noting that the charge density has changed sign and the expression for the length of the Gaussian cylinder is $a-z$ since $z>0$. Also the unit normal now points in the –z-direction. So the dot product becomes

$$\vec{\mathbf{E}}_1 \cdot \hat{\mathbf{n}} da = E_{1,z}(-\hat{\mathbf{k}}) \cdot \hat{\mathbf{k}} da = -E_{1,z}\, da$$

Thus Gauss's Law becomes

$$-E_{1,z} A_{cap} = \frac{+\rho_0 A_{cap}(a-z)}{\varepsilon_0}.$$

So the electric field is

$$E_{1,z} = -\frac{\rho_0 (a-z)}{\varepsilon_0}.$$

The vector description is then

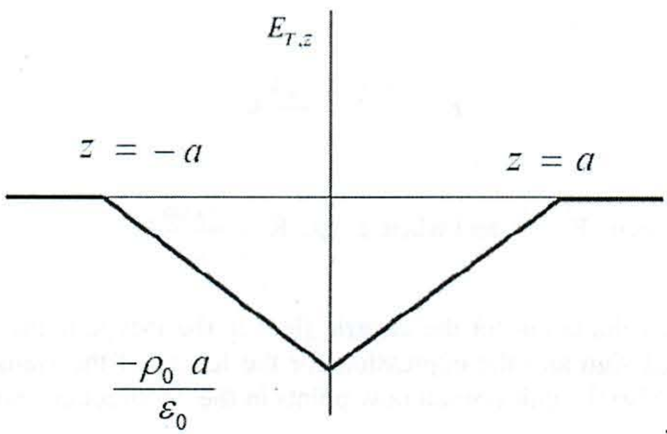
$$\vec{\mathbf{E}}_1 = \frac{-\rho_0 (a-a)}{\varepsilon_0} \hat{\mathbf{k}}$$

Note when $z = a$ then $\vec{\mathbf{E}}_1 = \vec{\mathbf{0}}$ and when $z = 0$, $\vec{\mathbf{E}}_1 = \dfrac{-\rho_0 a}{\varepsilon_0}\hat{\mathbf{k}}$.

So the resulting field is

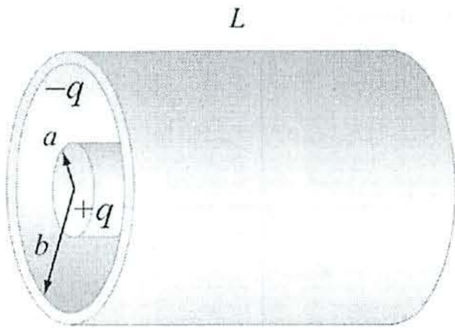
$$\vec{\mathbf{E}}_T(z) = \begin{cases} \vec{\mathbf{0}} & z < -a \\ \vec{\mathbf{E}}_2 = \dfrac{-\rho_0(a+z)}{\varepsilon_0}\hat{\mathbf{k}} & -a < z < 0 \\ \vec{\mathbf{E}}_1 = \dfrac{-\rho_0(a-z)}{\varepsilon_0}\hat{\mathbf{k}} & 0 < z < a \\ \vec{\mathbf{0}} & |z| > a \end{cases}.$$

The graph of the electric field is shown below



Problem 4 (10 points): Co-axial Cylinders

A very long conducting cylinder (length L and radius a) carrying a total charge $+q$ is surrounded by a thin conducting cylindrical shell (length L and radius b) with total charge $-q$, as shown in the figure.



(a) Using Gauss's Law, find an expression for the direction and magnitude of the electric field $\vec{\mathbf{E}}$ for the region $r < a$.

Solution: The electric field is zero inside the inner conducting cylinder.

(b) Similarly, find an expression for the direction and magnitude of the electric field $\vec{\mathbf{E}}$ for the region $a < r < b$.

Solution: We use a Gaussian cylinder of length l and radius $a < r < b$. Then, the flux is

$$\iint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E 2\pi r l .$$

The charge enclosed is given by

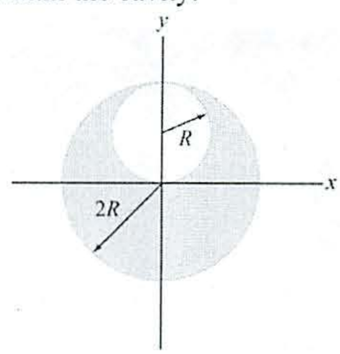
$$Q_{enc} = \lambda l = (q / L) l .$$

So Gauss' Law becomes

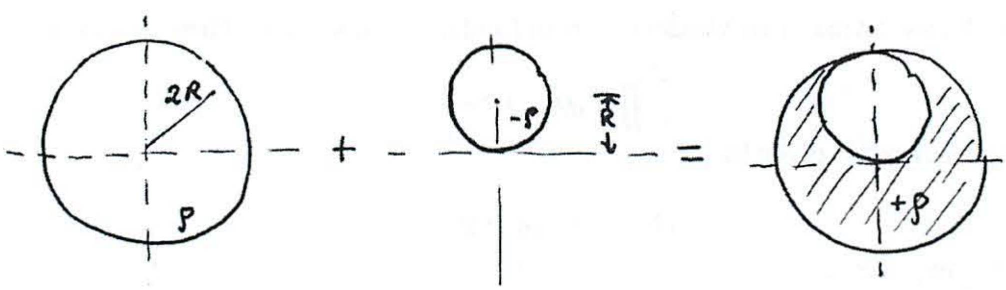
$$\oiint \vec{\mathbf{E}}_I \cdot d\vec{\mathbf{A}} = \frac{Q_{enc}}{\varepsilon_0} \Rightarrow E 2\pi r l = \frac{q l}{L \varepsilon_0} \Rightarrow \vec{\mathbf{E}} = \frac{q}{L 2\pi \varepsilon_0} \frac{1}{r} \hat{\mathbf{r}} \, ; a < r < b$$

Problem 5 (10 points): Solid Sphere with a Cavity

A sphere of radius $2R$ is made of a non-conducting material that has a uniform volume charge density ρ. (Assume that the material does not affect the electric field.) A spherical cavity of radius R is then carved out from the sphere, as shown in the figure below. Find the electric field within the cavity.

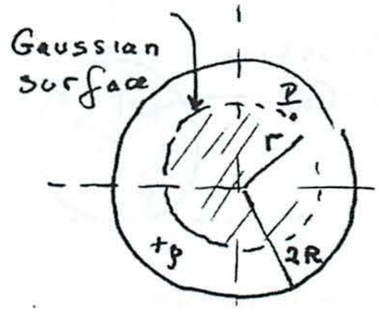


Solution: At first glance this charge distribution does not seem to have any of the symmetries that enable us to use Gauss's law. However we can consider this charge distribution as the sum of two uniform spherical distributions of charge. The first is a sphere of radius $2R$ centered at the origin with a uniform volume charge density ρ. The second is a sphere of radius R centered at the point along the y-axis a distance R from the origin (the center of the spherical cavity) with a uniform volume charge density $-\rho$.



When we add together these two distributions of charge we obtain the uniform charged sphere with a spherical cavity of radius R as described in the problem. We can then add together the electric fields from these two distributions at any point in the cavity to obtain the electric field of the original distribution at that point inside the cavity (superposition principle). Each of these two distributions are spherically symmetric and therefore we can use Gauss's Law to find the electric field associated with each of them.. We do need to be careful when we add together the electric fields. As you will see that process is somewhat subtle and a good vector diagram will help considerably.

So let's begin by choosing a point P inside the cavity. We will now apply Gauss's Law to our first distribution, the sphere of radius $2R$ centered at the origin with a uniform volume charge density ρ. The point P is a distance $r < 2R$ from the origin. We choose a sphere of radius r as our Gaussian surface with $r < 2R$.



Then, the electric flux through this closed surface is

$$\oiint \vec{\mathbf{E}}_\rho \cdot d\vec{\mathbf{A}} = E_\rho \cdot 4\pi r^2,$$

where E_ρ denotes the outward normal component of the electric field at the point P associated to the spherical distribution with uniform volume charge density ρ. Because the charge distribution is uniform, the charge enclosed in the Gaussisan surface is

$$\frac{Q_{enc}}{\varepsilon_0} = \frac{\rho(4\pi r^3/3)}{\varepsilon_0}.$$

Recall that Gauss' Law equates electric flux to charge enclosed:

$$\oiint \vec{\mathbf{E}}_\rho \cdot d\vec{\mathbf{A}} = \frac{Q_{enc}}{\varepsilon_0}.$$

So we substitute the two calculations above into Gauss' law to arrive at:

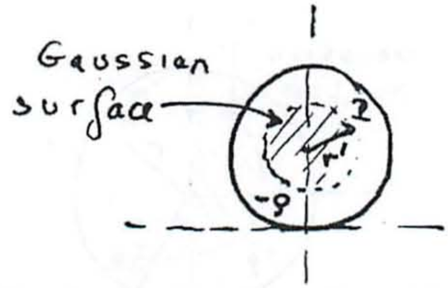
$$E_\rho \cdot 4\pi r^2 = \frac{\rho(4\pi r^3/3)}{\varepsilon_0}.$$

We can solve this equation for the electric field

$$\vec{\mathbf{E}}_\rho(P) = E_\rho \hat{\mathbf{r}} = \frac{\rho r}{3\varepsilon_0}\hat{\mathbf{r}}\ .$$

where $\hat{\mathbf{r}}$ is a unit vector at the point P pointing radially away from the origin.

We now apply Gauss's Law to our second distribution, a sphere of radius R centered at the point along the y-axis a distance R from the origin with a uniform volume charge density $-\rho$. The point P is a distance $r' < R$ from the center of the cavity.



We choose a sphere of radius r' as our Gaussian surface with $r' < R$. Then, the electric flux through this closed surface is

$$\oiint \vec{\mathbf{E}}_{-\rho} \cdot d\vec{\mathbf{A}} = E_{-\rho} \cdot 4\pi r'^2 ,$$

where $E_{-\rho}$ denotes the outward normal component of the electric field at the point P associated to the spherical distribution with uniform volume charge density $-\rho$. Because the charge distribution is uniform, the charge enclosed in the Gaussisan surface is

$$\frac{Q_{enc}}{\varepsilon_0} = -\frac{\rho(4\pi r'^3/3)}{\varepsilon_0} .$$

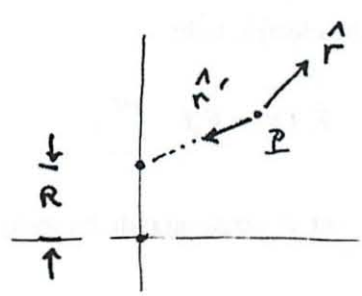
Therefore applying Gauss's Law yields

$$E_{-\rho} \cdot 4\pi r^2 = -\frac{\rho(4\pi r'^3/3)}{\varepsilon_0} .$$

We can solve this equation for the electric field

$$\vec{\mathbf{E}}_{-\rho}(P) = E_{-\rho}\hat{\mathbf{r}}' = -\frac{\rho r'}{3\varepsilon_0}\hat{\mathbf{r}}' .$$

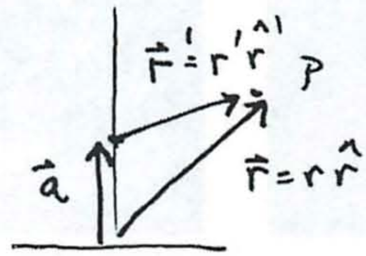
where $\hat{\mathbf{r}}'$ is a unit vector at the point P pointing radially away from the center of the cavity.



The electric field associated with our original distribution is then

$$\vec{\mathbf{E}}(P) = \vec{\mathbf{E}}_{\rho}(P) + \vec{\mathbf{E}}_{-\rho}(P) = E_{\rho}\hat{\mathbf{r}} + E_{-\rho}\hat{\mathbf{r}}' = \frac{\rho r}{3\varepsilon_0}\hat{\mathbf{r}} - \frac{\rho r'}{3\varepsilon_0}\hat{\mathbf{r}}' = \frac{\rho}{3\varepsilon_0}(r\hat{\mathbf{r}} - r'\hat{\mathbf{r}}') = \frac{\rho}{3\varepsilon_0}(\vec{\mathbf{r}} - \vec{\mathbf{r}}')$$

where $\vec{\mathbf{r}}$ is a vector from the origin to the point P and $\vec{\mathbf{r}}'$ is a vector from the center of the cavity to the point P. From our diagram we see that $\vec{\mathbf{a}} = \vec{\mathbf{r}} - \vec{\mathbf{r}}'$.



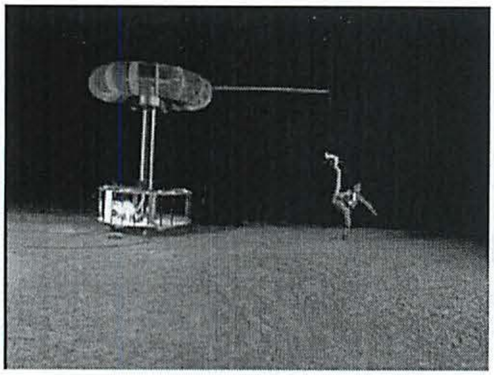
Therefore the electric field at the point P is given by

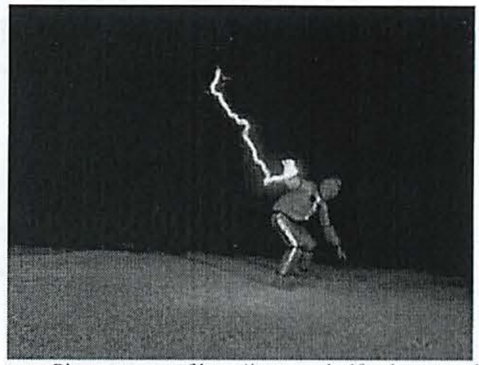
$$\vec{\mathbf{E}}(P) = \frac{\rho}{3\varepsilon_0}\vec{\mathbf{a}}.$$

This is a remarkable result. The electric field inside the cavity is uniform. The direction of the electric field points from the center of entire sphere to the center of the cavity. This direction is uniquely specified and is an example of 'broken symmetry'.

Problem 6 (10 points): Stupid Hobbies...

Some people like to do incredibly dangerous things. Like Austin Richards (also known as Dr. Megavolt or Criss Angel, who performed a similar stunt on the "Tesla Coil" episode of his show Mindfreak. Here are some pictures.





Pictures care of http://www.mindfreakconnection.com/

You'll note that while Dr. Megavolt takes strikes directly from the Tesla Coil (a device capable of making insanely high voltages), Criss Angel decides to get shocked from a small ball attached to the coil instead – convenient for the purposes of answering this question. At about what voltage was the Tesla coil for the strikes pictured above and about how much excess charge was on his hand (in the right picture) the instant before the strike was initiated? (HINT: Dry air breaks down at an electric field strength of about 3 x 10^6 V/m)

Solution:

Judging from the picture, Criss is about a meter away from the ball when it arcs. Could be two meters, but it is easier to work with one meter, so I'll use that. If we make a simple minded assumption that V = Ed then the potential difference is given by:

$$3 \text{ x } 10^6 \text{ V/m x 1 m} \qquad \boxed{3\times10^6 \text{ V}}$$

(hence Dr. Megavolt!). You may complain that clearly this is more like a ball of charge then a parallel plate capacitor so we should have used a point charge potential, kQ/r. But notice that even in this case V ~ Er, so the above is approximately correct. There is also a question of where the field equals the breakdown field. Fortunately, this is a back of the envelope question so the details don't matter so much.

We can determine a minimum charge by requiring the field to be at breakdown strength just outside his hand (or the ball). Let's make them spheres of radius 5 cm. Then:

$$E = kQ/r^2 \Rightarrow Q = r^2E/k \square (5\text{ cm})^2\, 3\times10^6 \text{ V m}^{-1}\left(9\times10^9 \text{ V m C}^{-1}\right)^{-1} \cong 8\times10^{-7} \text{ C} \cong \boxed{5\times10^{12} e}$$

I say that this is a minimum because the field is clearly breaking down a much further distance away (a meter) which would require a charge 400 ($=20^2$) times larger. The real charge has to be somewhere between these two extremes, so I'll estimate

$$\boxed{Q \cong 10^{-4}\ \mathrm{C} \cong 5\times 10^{14} e}$$

Problem 7 (10 points): Expt. 1: Equipotential Lines and Electric Fields Pre-Lab Questions

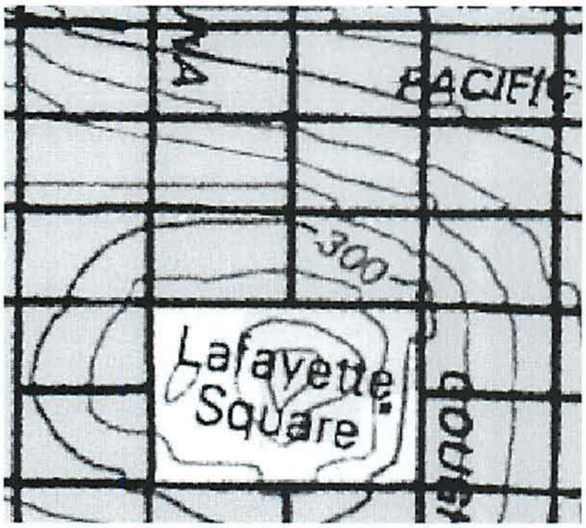
Read Experiment 1. The link is

> **http://web.mit.edu/8.02t/www/materials/Experiments/exp01.pdf.**

Then answer the following pre-lab questions.

1. Equipotentials Curves – Reading Topographic Maps

Below is a topographic map of a 0.4 mi square region of San Francisco. The contours shown are separated by heights of 25 feet (so from 375 feet to 175 feet above sea level for the region shown)



From left to right, the NS streets shown are Buchanan, Laguna, Octavia, Gough and Franklin. From top to bottom, the EW streets shown are Broadway, Pacific, Jackson, Washington, Clay (which stops on either side of the park) and Sacramento.

(a) In the part of town shown in the above map, which street(s) have the steepest runs? Which have the most level sections? How do you know?

Solution:
You can tell how steep something is by looking at how quickly it passes through constant height contours (~ equipotentials). The steepest section is along Octavia between Pacific and Washington. The most level street is Jackson between Buchanan and Octavia, which runs parallel to the 275 foot contour and hence is very flat.

(b) How steep is the steepest street at its steepest (what is its slope in ft/mi)?

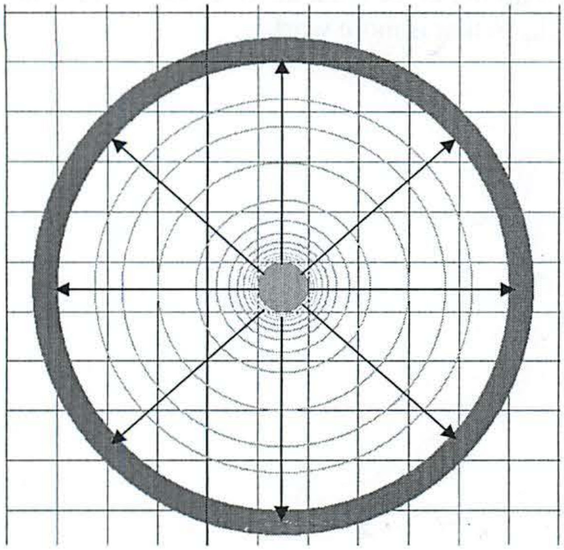
Solution:
Looking at Octavia, it passes through 5 contours (125 feet) in two blocks (about 0.12 miles) so it has a slope of ~1000 ft/mi.

(c) Which would take more work (in the physics sense): walking 3 blocks south from Laguna and Jackson or 1 block west from Clay and Franklin?

Solution:

Work is change in potential energy (and hence height). The change in height walking 3 blocks S on Laguna is almost nothing (you go up but come back down again). West on Clay from Franklin you rise 50 feet in the block, so that is more work.

2. Equipotentials, Electric Fields and Charge



One group did this lab and measured the equipotentials for a slightly different potential landscape then the ones you have been given (although still on a 1 cm grid).

Note that they went a little overboard and marked equipotential curves (the magenta circles) at V = 0.25 V, 0.5V and then from V = 1 V to V = 10 V in 1 V increments.

They followed the convention that red was their positive electrode ($V = +10$ V) and blue was ground ($V = 0$ V).

(a) Copy the above figure and sketch eight electric field lines on it (equally spaced around the inner conductor).

Solution: See black arrows

(b) What, approximately, is the magnitude of the electric field at $r = 1$ cm, 2 cm, and 3 cm, where r is measured from the center of the inner conductor? You should express the field in V/cm. (HINT: The field is the local slope (derivative) of the potential. Also, if you choose to use a ruler realize that the above reproduction of this group's results is not the same size as the original, where the grid size was 1 cm).

Solution:

At r = 1 cm, V ~ 4 V and we move 1 V in about 1/5 cm.	E ~ 5 V/cm
At r = 2 cm, V ~ 1.5 V and we move about 1/2 V in 1/2 cm.	E ~ 1 V/cm
At r = 3 cm, V ~ 0.7 V and we move about 0.2 V in 1/2 cm.	E ~ 0.4 V/cm

(c) What is the relationship between the density of the equipotential lines, the density of the electric field lines, and the strength of the electric field?

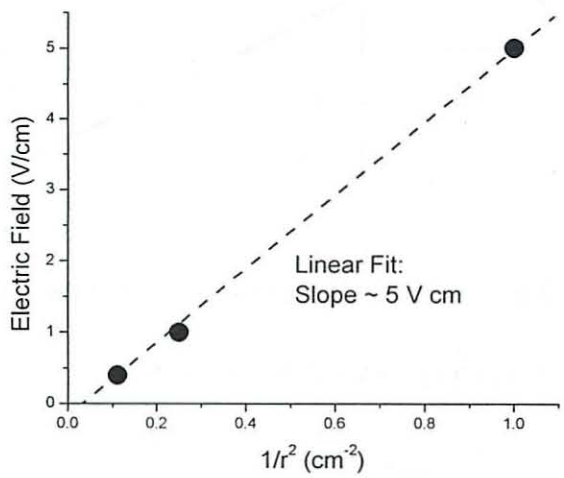
Solution:

The denser the equipotential lines and hence electric field lines, the stronger the field.

(d) Plot the field strength vs. $1/r^2$ for the three points from part (a). If the field were created by a single point charge what shape should this sketch be? Is it?

Solution:

It should be (and is!) a straight line



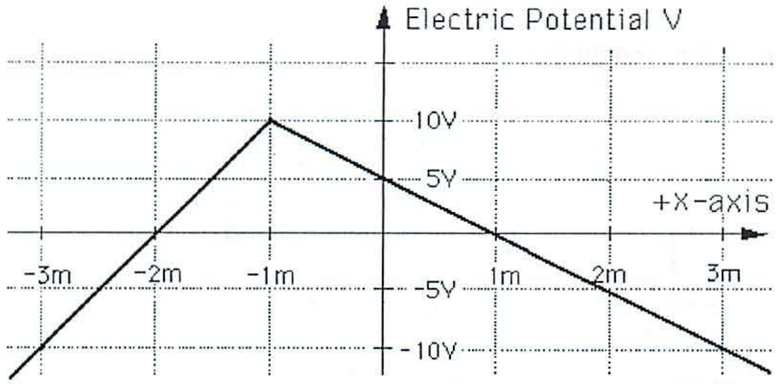
(e) Approximately how much charge was on the inner conductor when the group made their measurements?

Solution:

$E = k_e \frac{q}{r^2}$, so slope is $k_e q = 5$ V cm. $q \approx 5\times10^{-12}$ C

3. Finding the Electric Field from the Electric Potential

The graph shows the variation of an electric potential V with distance x. The potential does not vary in the y or z directions. Be sure to include units as appropriate.



(a) What is E_x in the region $x > -1\text{ m}$? (Be careful to indicate the sign of E_x.)

Solution: In the region $x > -1\text{ m}$, $V(x) = 5\text{ V} - (5\text{ V}\cdot\text{m}^{-1})\,x$. So

$$E_x = -\frac{d}{dx}V(x) = 5\text{ V}\cdot\text{m}^{-1}$$

(b) What is E_x in the region $x < -1\text{ m}$? (Be careful to indicate the sign of E_x.)

Solution: In the region $x < -1\text{ m}$, $V(x) = 20\text{ V}+(10\text{ V}\cdot\text{m}^{-1})\,x$. So

$$E_x = -\frac{d}{dx}V(x) = -10\text{ V}\cdot\text{m}^{-1}\ .$$

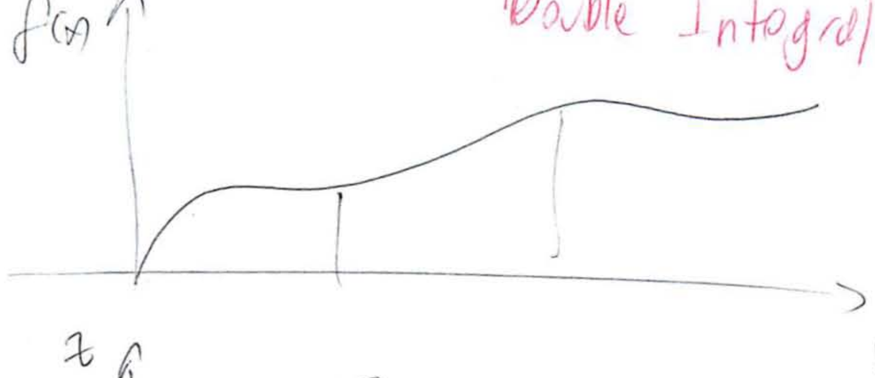
(c) A negatively charged dust particle with mass $m_q = 1\times10^{-13}\text{ kg}$ and charge $q = -1\times10^{-12}\text{ C}$ is released from rest at $x = +2\text{ m}$. Will it move to the left or to the right?

Solution: For $x > -1\text{ m}$, the electric field is pointing in the positive x-direction, so a negatively charged particle will experience a force pointing in the negative x-direction, hence it will move to the left.

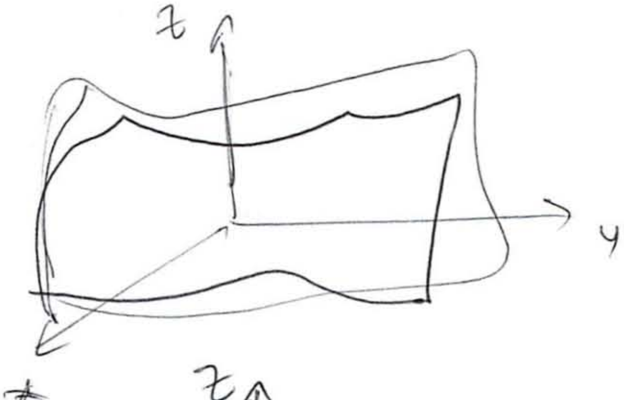
(5,0)

Double Integral Intro

2/15

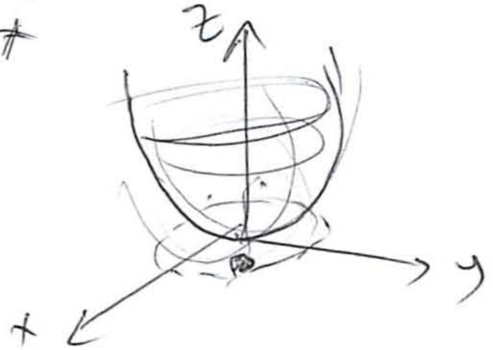


single integral



$$f(x,y) = x^2 + 2y - x + y^2$$

$$f(x,y) = x^2 + y^2$$



$$\iint_R f(x,y)\, \underbrace{dx\,dy}_{dA}$$

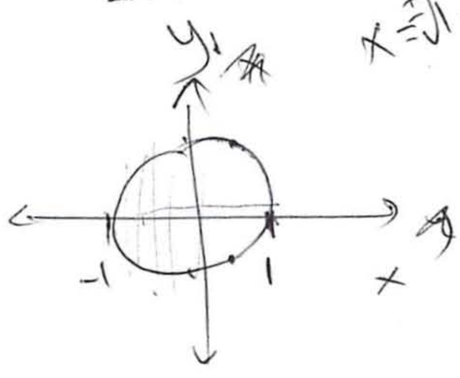
double integral

$$\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\,dx\,dy$$

$x^2+y^2=1$

$x = \pm\sqrt{1-y^2}$

do one at a time axis



← draw out projections

Topics: Electric Potential, Equipotentials
Related Reading: Course Notes: Sections 3.1-3.5
Experiments: (1) Equipotential Lines and Electric Fields

Topic Introduction

Today we continue our discussion of electric potentials and equipotentials, becoming more familiar with them and their relationship with charge and electric fields through our first experiment.

Equipotentials

Recall from our last class that when discussing potential and potential energy we only defined *differences*. This is because only differences are physically meaningful – what we choose, for example, to call "zero energy" is completely arbitrary. Today we will focus on the measurement of equipotential surfaces, that is, locations where the potential is the same, and will practice estimating electric field lines and charge distributions once those equipotential surfaces are known.

Experiment 1: Equipotential Lines and Electric Fields

Preparation: Read pre-lab and answer pre-lab questions
(Hand in pre-lab questions at the beginning of class)

Thus far in class we have talked about fields, both gravitational and electric, and how we can use them to understand how objects can interact at a distance. A charge, for example, creates an electric field around it, which can then exert a force on a second charge which enters that field. In this lab we will study another way of thinking about this interaction through electric potentials.

In particular, for several given charge configurations you will map out equipotential contours, that is, contours along which the potential is a constant. From these equipotentials you can determine both the direction and magnitude of the electric field.

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Experiment 1: Equipotential Lines and Electric Fields

OBJECTIVES

1. To develop an understanding of electric potential and electric fields
2. To better understand the relationship between equipotentials and electric fields
3. To become familiar with the effect of conductors on equipotentials and E fields

PRE-LAB READING

INTRODUCTION

Thus far in class we have talked about fields, both gravitational and electric, and how we can use them to understand how objects can interact at a distance. A charge, for example, creates an electric field around it, which can then exert a force on a second charge which enters that field. In this lab we will study another way of thinking about this interaction through electric potentials.

The Details: Electric Potential (Voltage)

Before discussing electric potential, it is useful to recall the more intuitive concept of potential energy, in particular *gravitational* potential energy. This energy is associated with a mass's position in a gravitational field (its height). The potential energy *difference* between being at two points is defined as the amount of work that must be done to move between them. This then sets the relationship between potential energy and force (and hence field):

$$\Delta U = U_B - U_A = -\int_A^B \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}} \quad \Rightarrow \quad (\text{in 1D}) \;\; F = -\frac{dU}{dz} \tag{1}$$

We earlier defined fields by breaking a two particle interaction, force, into two single particle interactions, the creation of a field and the "feeling" of that field. In the same way, we can define a potential which is created by a particle (gravitational potential is created by mass, electric potential by charge) and which then gives to other particles a potential energy. So, we define electric potential, V, and given the potential can calculate the field:

$$\Delta V = V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} \quad \Rightarrow \quad (\text{in 1D}) \;\; E = -\frac{dV}{dz}. \tag{2}$$

Noting the similarity between (1) and (2) and recalling that $\boldsymbol{F} = q\boldsymbol{E}$, the potential energy of a charge in this electric potential must be simply given by $U = qV$.

When thinking about potential it is convenient to think of it as "height" (for gravitational potential in a uniform field, this is nearly precise, since $U = mgh$ and thus the gravitational potential $V = gh$). Electric potential is measured in Volts, and the word "voltage" is often used interchangeably with "potential." You are probably familiar with this terminology from batteries, which maintain fixed potential differences between their two ends (e.g. 9 V in 9 volt batteries, 1.5 V in AAA-D batteries).

Equipotentials and Electric Fields

When trying to picture a potential landscape, a map of equipotential curves – curves along which the potential is equal – can be very helpful. For gravitational potentials these maps are called topographic maps. An example is shown in Fig. 1b.

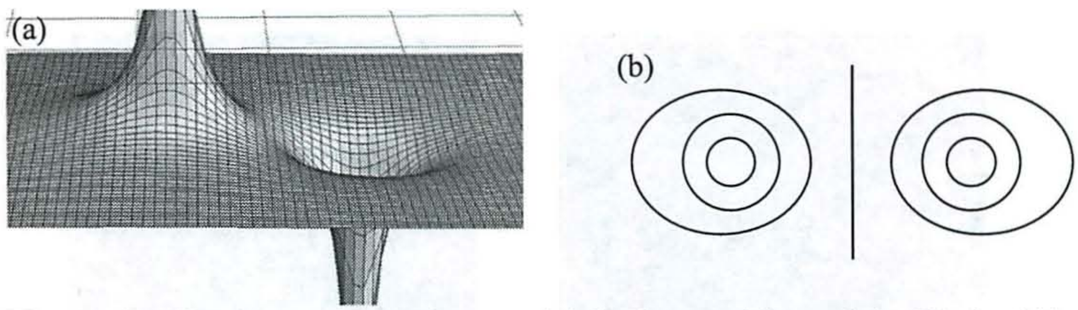


Figure 1: Equipotentials. A potential landscape (pictured in 3D in (a)) can be represented by a series of equipotential lines (b), creating a topographic map of the landscape. The potential ("height") is constant along each of the curves.

Now consider the relationship between equipotentials and fields. At any point in the potential landscape, the field points in the direction that a mass would feel a force if placed there (or that a positive charge would feel a force for electric potentials and fields). So, place a ball at the top of the hill (near the center of the left set of circles in the topographic map of Fig. 1b). Which way does it roll? Downhill! But what direction is that? Perpendicular to the equipotential lines. Why? Equipotential lines are lines of constant height, so moving along them at all does not achieve the objective of going downhill. So the force (and hence field) must point across them, pushing the object downhill. But why exactly perpendicular? Work done on an object changes its potential, so it can take no work to move along an equipotential line. Work is given by the dot product of force and displacement. For this to be zero, the force must be perpendicular to the displacement, that is, force (and hence fields) must be perpendicular to equipotentials.

Note: Potential vs. Potential Difference

Note that in equation (2) we only defined ΔV, the potential *difference* between two points, and not the potential V. This is because potential is like height – the location we choose to call "zero" is completely arbitrary. In this lab we will choose one location to call zero (the "ground"), and measure potentials relative to the potential at that location.

APPARATUS

1. Conducting Paper Landscapes

To get a better feeling for what equipotential curves look like and how they are related to electric field lines, we will measure sets of equipotential curves for several different potential landscapes. These landscapes are created on special paper (on which you can measure electric potentials) by fixing a potential difference between two conducting shapes on the paper. For reasons that we will discuss later, these conducting shapes are themselves equipotential surfaces, and their shape and relative position determines the electric field and potential everywhere in the landscape. One purpose of this lab is to develop an intuition for how this works. There are four landscapes to choose from (Fig. 2), and you will measure equipotentials on two of them (one from Fig. 1a, b and one from Fig. 1c, d).

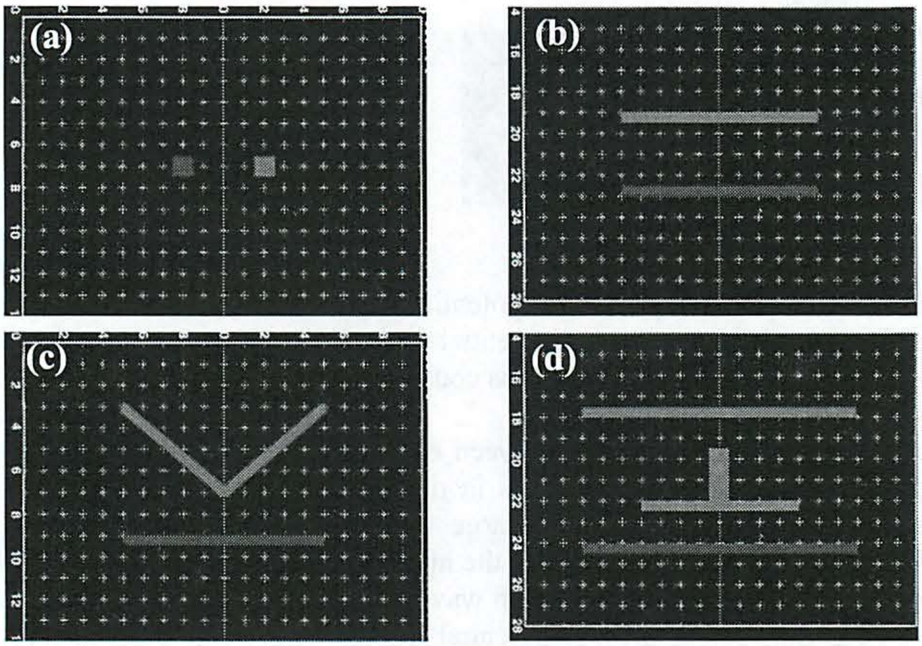


Figure 2 Conducting Paper Landscapes. Each of the four landscapes – the "standard" (a) dipole and (b) parallel plates, and the "non-standard" (c) bent plate and (d) filled plates – consists of two conductors which will be connected to the positive (red) and ground (blue) terminals of a battery. In (d) there is an additional conductor which is free to float to whatever potential is required. The pads are painted on conducting paper with a 1 cm grid.

Special paper

2. Science Workshop 750 Interface

In this lab we will again use the Science Workshop 750 interface both to create the potential landscapes (using the "OUPUT" connections that act like a battery which we will set to 5 V) and to measure the potential at various locations in that landscape using a voltage sensor.

3. Voltage Sensor

In order to measure the potential as a function of position we will once again use the voltage sensor, plugged into Channel A on the 750. When recording the "potential," you will really be measuring the potential difference between the two leads, (red minus black) and hence you should have the black lead connected to the output ground (what value of potential does this then assign to the output ground?)

GENERALIZED PROCEDURE

For each of the two landscapes that you choose, you will find at least four equipotential contours by searching for points in the landscape at the same potential using the voltage sensor. After recording these curves, you will draw several electric field lines, making use of the fact that they are everywhere perpendicular to equipotential contours.

END OF PRE-LAB READING

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

1. Download the LabView file from the web and save the file to your desktop (right click on the link and choose "Save Target As"). Start LabView by double clicking
2. Connect cables to the output of the 750 (red to the sin wave marked output, black to ground). One member of the group will hold these wires to the two conductors while another maps out the equipotentials.
3. Connect the Voltage Sensor to Analog Channel A on the 750 Interface
4. Connect the black lead of the voltage sensor to the black output (the ground). You will use the red lead to measure the potential around your landscapes.

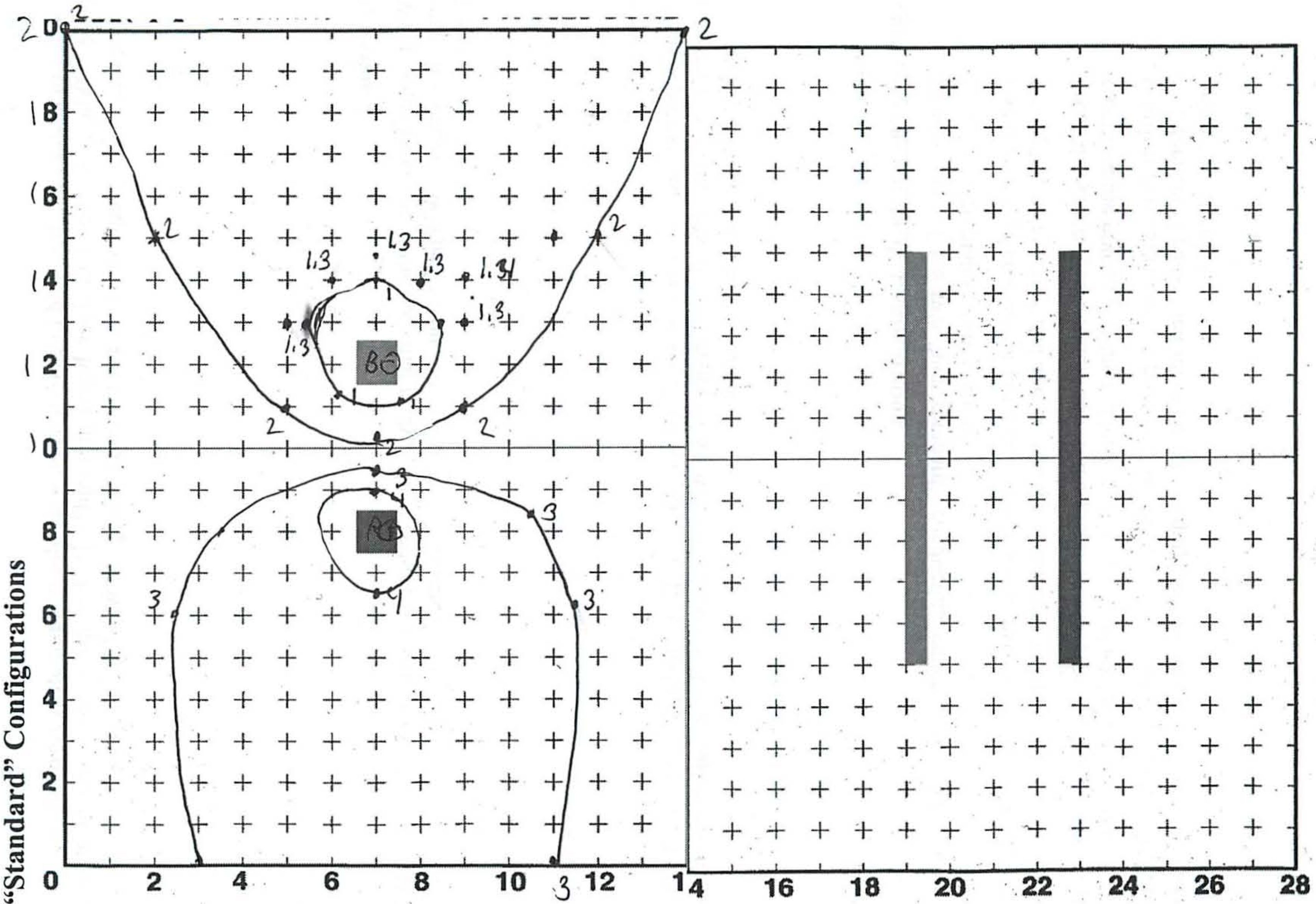
MEASUREMENTS

Part 1: "Standard" Configuration

1. Choose one of the two "standard" conducting paper landscapes (the dipole or parallel plate configuration)
2. Use the voltage connectors to make contacts to the two conducting pads
3. Press the green "Go" button above the graph to energize the battery and begin recording the potential of the red lead (relative to the black lead = ground).
4. Measure the potential of both conducting pads to confirm that they are properly connected (one should be at +5 V, the other at 0 V), and that they are indeed equipotential objects (we will explain why next week).
5. Now, try to find some location on the paper that is at about +1 V (don't worry about being too precise). Mark this point on the plot on the next page.

   Do NOT write on the conducting paper

6. Find another 1 V point, about 1 cm away. Continue until you have closed the curve or left the page. Sketch and label this equipotential curve.
7. Repeat this process to find equipotentials at 2 V, 3 V, and 4 V. Work pretty fast; it's more important to think about what these lines mean than it is to draw them perfectly. Think about what you are doing – are there symmetries that you can exploit to make this task easier?



"Standard" Configurations

field = $\frac{\text{Newtons}}{\text{Colomb}}$ $F = qE$ $\frac{kQQ}{R^2}$ Field $= -\frac{dV}{dx} = \frac{-5}{3} \frac{N}{C}$

Question 1:

Sketch in a set of electric field lines (~ ten) on your plot of equipotentials on the previous page. Where do the field lines begin and end? If they are equally spaced at their beginning, are they equally spaced at the end? Along the way? Why?

Question 2:

What, approximately, is the potential midway between the two conductors? REMINDER (just this once): Whenever you are asked for a numerical value DO NOT FORGET UNITS!

2.5 Volts

Question 3:

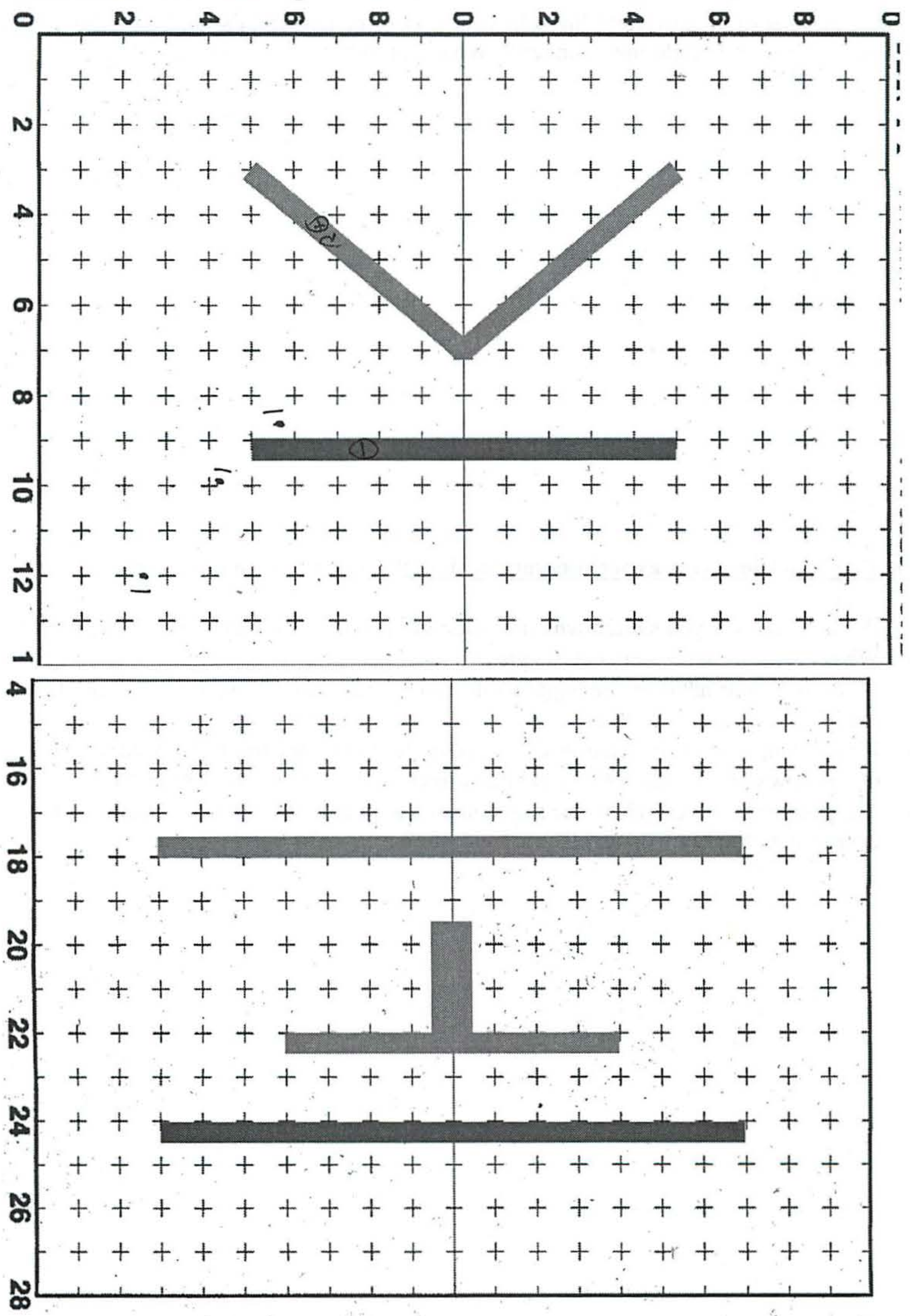
What, approximately, is the strength of the electric field midway between the two conductors? You may find it easier to answer this question if you just measure the potential at a few points near the center.

$$\text{field} = \frac{N}{C} = \frac{dV}{dt} = \frac{-5}{3}\frac{\text{Volts}}{\text{cm}}$$

__**Part 2: "Non-Standard" Configuration**__

1. Choose one of the two "non-standard" conducting paper landscapes (the bent plate or filled plates configuration)
2. Use the voltage connectors to make contacts to the two conducting pads (for the filled plates, the center pad does *not* have a connection to it)
3. Press the green "Go" button above the graph to energize the battery and begin recording the potential of the red lead (relative to the black lead = ground).
4. Confirm that everything is properly connected by measuring the potential on the two connected pads, then record a set of equipotential curves following the same procedure of part 1.

"Non-Standard" Configurations



Question 4:

Sketch in a set of electric field lines on your plot of equipotentials on the previous page. Where is the electric field the strongest? What, approximately, is its magnitude?

Field line stronger between the 2
Topo lines are closest
are $-\frac{5}{1\frac{3}{4}} = -3\frac{V}{cm}$

Further Questions (for experimentation, thought, future exam questions…)

- What changes if you switch which conducting pad is at +5 V and which is ground?
- What if you forget to connect the ground lead?
- If you rest your hand on the paper while making measurements, does it affect the readings? Why or why not?
- If you wanted to push a charge along one of the field lines from one conductor to the other, how does the choice of field line affect the amount of work required?
- The potential is everywhere the same on an equipotential line. Is the electric field everywhere the same on an electric field line?

Most uniform (can go w/ out charging)
- furthest away from both conductors
- low or no potential

Strongest - between the 2 conductors
- least uniform

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Experiment 1 *Solutions*: Equipotential Lines and Electric Fields

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

1. Download the LabView file from the web and save the file to your desktop (right click on the link and choose "Save Target As"). Start LabView by double clicking
2. Connect cables to the output of the 750 (red to the sin wave marked output, black to ground). One member of the group will hold these wires to the two conductors while another maps out the equipotentials.
3. Connect the Voltage Sensor to Analog Channel A on the 750 Interface
4. Connect the black lead of the voltage sensor to the black output (the ground). You will use the red lead to measure the potential around your landscapes.

MEASUREMENTS

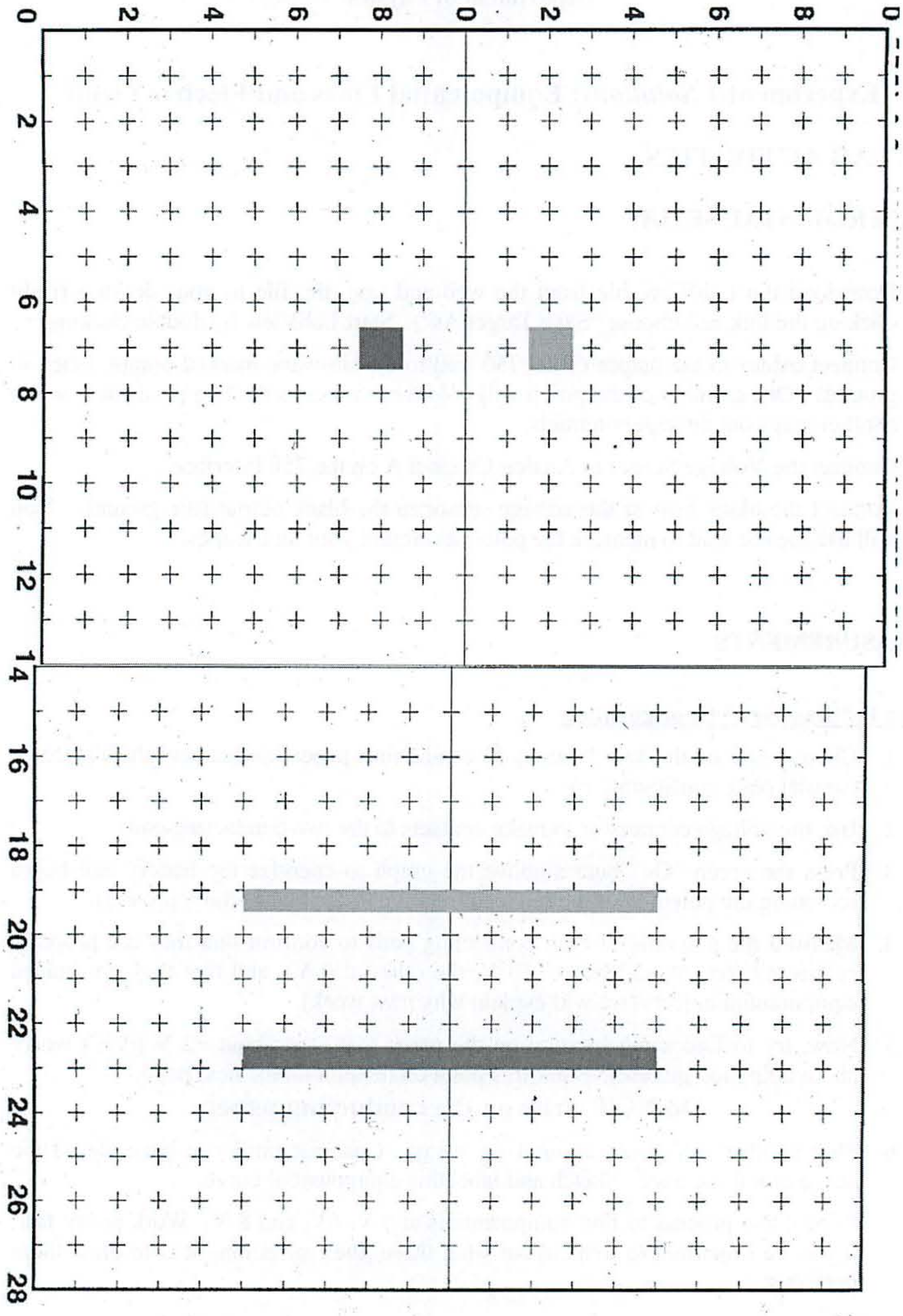
<u>Part 1: "Standard" Configuration</u>

1. Choose one of the two "standard" conducting paper landscapes (the dipole or parallel plate configuration)
2. Use the voltage connectors to make contacts to the two conducting pads
3. Press the green "Go" button above the graph to energize the battery and begin recording the potential of the red lead (relative to the black lead = ground).
4. Measure the potential of both conducting pads to confirm that they are properly connected (one should be at +10 V, the other at 0 V), and that they are indeed equipotential objects (we will explain why next week).
5. Now, try to find some location on the paper that is at about +2 V (don't worry about being too precise). Mark this point on the plot on the next page.

Do NOT write on the conducting paper

6. Find another 2 V point, about 1 cm away. Continue until you have closed the curve or left the page. Sketch and label this equipotential curve.
7. Repeat this process to find equipotentials at 4 V, 6V, and 8 V. Work pretty fast; it's more important to think about what these lines mean than it is to draw them perfectly.

“Standard” Configurations



Question 1:

Sketch in a set of electric field lines (~ ten) on your plot of equipotentials on the previous page. Where do the field lines begin and end? If they are equally spaced at their beginning, are they equally spaced at the end? Along the way? Why?

Yes, they are equally spaced at the end if they are at the beginning, by symmetry. The spacing changes along the way, spreading out significantly away from the sources.

Question 2:

What, approximately, is the potential midway between the two conductors?

By symmetry it must be half way between the two potentials, or 2.5 V

Question 3:

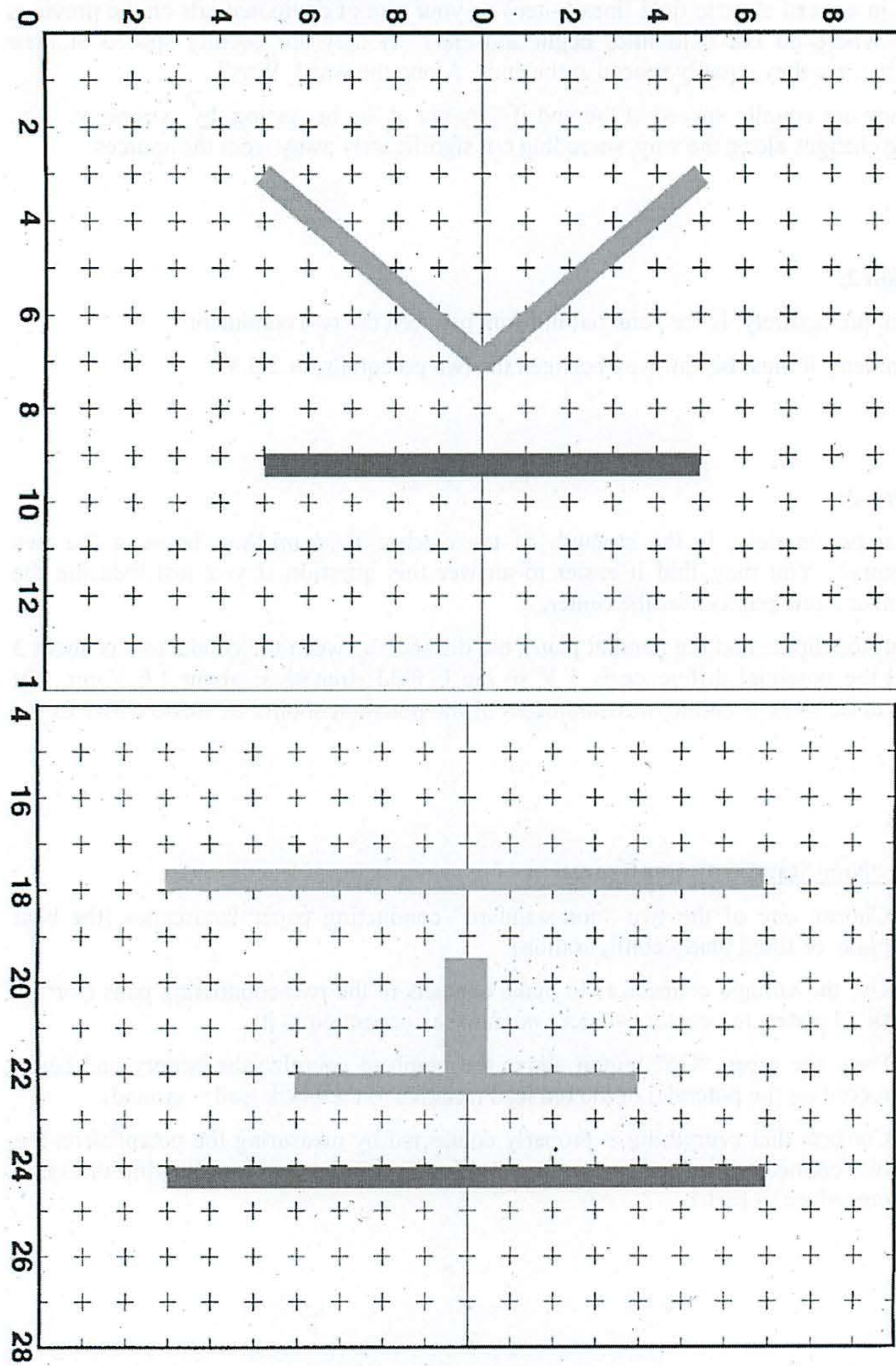
What, approximately, is the strength of the electric field midway between the two conductors? You may find it easier to answer this question if you just measure the potential at a few points near the center.

For both the dipole and the parallel plates the distance between the conductors is about 3 cm and the potential difference is 5 V so the E field strength is about 1.6 V/cm. Of course, to be more accurate, measurements of the potential should be made closer to the center.

Part 2: "Non-Standard" Configuration

1. Choose one of the two "non-standard" conducting paper landscapes (the bent plate or filled plates configuration)
2. Use the voltage connectors to make contacts to the two conducting pads (for the filled plates, the center pad does *not* have a connection to it)
3. Press the green "Go" button above the graph to energize the battery and begin recording the potential of the red lead (relative to the black lead = ground).
4. Confirm that everything is properly connected by measuring the potential on the two connected pads, the record a set of equipotential curves following the same procedure of part 1.

“Non-Standard” Configurations



Question 4:

Sketch in a set of electric field lines on your plot of equipotentials on the previous page. Where is the electric field the strongest? What, approximately, is its magnitude?

The electric field is the strongest near sharp points (where the conductors are the closest together).

Question 5:

Where is the electric field the most uniform? How can you tell?

The field is the most uniform outside of the plates, where the potential is nearly constant and the field is hence about zero.

Further Questions (for experimentation, thought, future exam questions…)

- What changes if you switch which conducting pad is at +10 V and which is ground?
- What if you forget to connect the ground lead?
- If you rest your hand on the paper while making measurements, does it affect the readings? Why or why not?
- If a charge were to move along one of your field lines from one conductor to the other, how does the choice of field line affect the amount of work required to move?
- The potential is everywhere the same on an equipotential line. Is the electric field everywhere the same on an electric field line?

2/17

Class 07: Outline

Hour 1:
Electric Potential

Hour 2:
Lab 1: Equipotentials

through electric potential diff

electron falls How much KE gains

$1.6 \cdot 10^{-10} \Delta V$ ~~Joules~~ Colombs

Colombs · Joules ↑ redundant
Colomb

ΔV has volts in it ΔV [Volts]

Last Time:
Potential and E Field

Exam 1 Thur 7:30-9:30 PM
- do a sample exam
- but no capacitence

Topics 1. Fields + Visualizations
2. Electric Field + Potential
- Discrete
- Contines
- Symmetric (Guass' Law)

1. Calc electric field, integrate to find electric potential
- sphere, Uniform + non Uniform
conducting + no conducting
↳ surface ↑ whole thing

mountain range

E Field and Potential: Creating

A point charge q creates a field and potential around it:

$\vec{\mathbf{E}} = k_e \frac{q}{r^2}\hat{\mathbf{r}}$; $V = k_e \frac{q}{r}$ Use superposition for systems of charges or integrate

They are related:

$\vec{\mathbf{E}} = -\nabla V$; $\Delta V \equiv V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$

gradiant ↑ arrow uphill
but (+) charges move ↓ so (−)

gradiant - fastest way ↑
field - fastest way ↓

E Field and Potential: Effects

If you put a charged particle, (charge *q*), in a field:

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

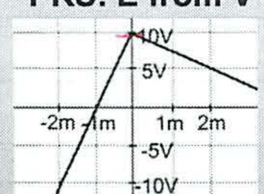
To move a charged particle, (charge *q*), in a field and the particle does not change its kinetic energy then:

$$W_{ext} = \Delta U = q\Delta V$$

Two PRS Questions: Potential & E Field

PRS: E from V

mountain



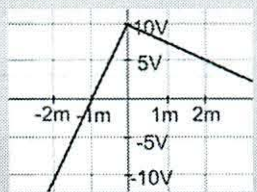
The graph above shows a potential V as a function of x. The *magnitude* of the electric field for x > 0 is

0% 1. larger than that for x < 0
0% 2. smaller than that for x < 0
0% 3. equal to that for x < 0
0% 4. I don't know

field = what force you feel when you are sitting there

left is steeper so magnitude is larger

20 PRS: E from V



The above shows potential V(x). Which is true?

0%	1. $E_{x>0}$ is > 0 and $E_{x<0}$ is > 0
0%	2. $E_{x>0}$ is > 0 and $E_{x<0}$ is < 0
0%	3. $E_{x>0}$ is < 0 and $E_{x<0}$ is < 0
0%	4. $E_{x>0}$ is < 0 and $E_{x<0}$ is > 0
0%	5. I don't know

P07- 7

or is it negative gradient
So it would be 2?

If put here move down
Field ⊖ ⊕ - way a ⊕ wants to move

$E = -\nabla V$

Potential from E

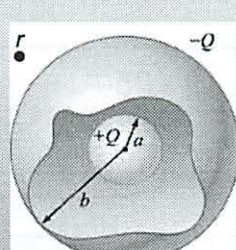
P07- 8

Potential for Nested Shells

From Gauss's Law

$$\vec{\mathbf{E}} = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}, & a<r<b \\ 0, & \text{elsewhere} \end{cases}$$

Use $V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$



Region 1: $r > b$

$$V(r) - \underbrace{V(\infty)}_{=0} = -\int_\infty^r 0\, dr = 0$$

No field →
No change in V!

P07- 9

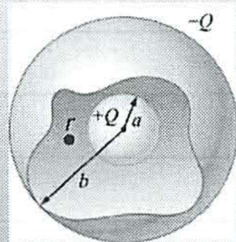
Potential for Nested Shells

Region 2: $a < r < b$

$$V(r) - \underbrace{V(r=b)}_{=0} = -\int_b^r dr \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$= \left. \frac{Q}{4\pi\varepsilon_0 r} \right|_{r=b}^{r}$$

$$= \frac{1}{4\pi\varepsilon_0} Q \left(\frac{1}{r} - \frac{1}{b} \right)$$



Electric field is just a point charge.
Electric potential is DIFFERENT – surroundings matter

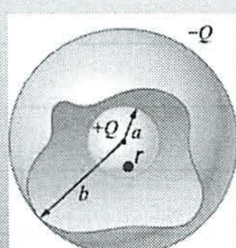
P07 - 10

Potential for Nested Shells

Region 3: $r < a$

$$V(r) - \underbrace{V(r=a)}_{=kQ\left(\frac{1}{a}-\frac{1}{b}\right)} = -\int_a^r dr\, 0 = 0$$

$$V(r) = V(a) = \frac{Q}{4\pi\varepsilon_0} \left(\frac{1}{a} - \frac{1}{b} \right)$$



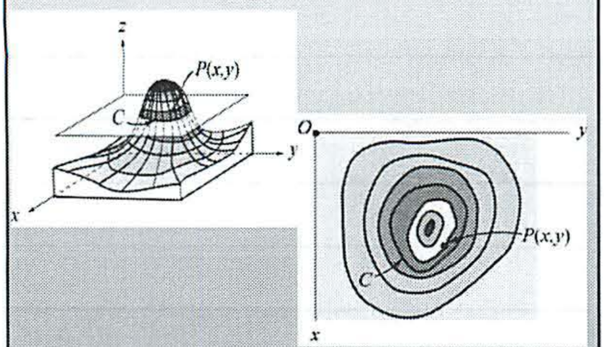
Again, potential is CONSTANT since E = 0.

P07 - 11

Equipotentials

P07 - 12

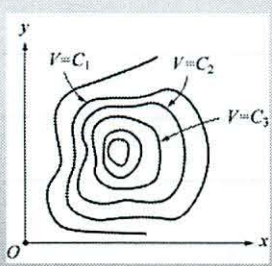
Topographic Maps



P07 - 13

every point on line has an equal potential

Equipotential Curves



All points on equipotential curve are at same potential.
Each curve represented by V(x,y) = constant

P07 - 14

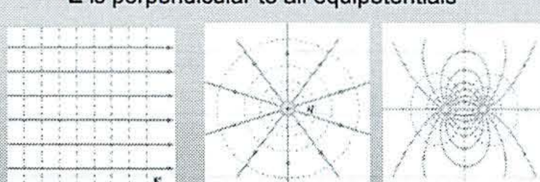
E field lines are perpendicular
↑ fastest way down mountain
for ⊕ charge

downhill = $-\nabla$

Law: measuring equal potential

Direction of Electric Field E

E is perpendicular to all equipotentials



Constant E field — Point Charge — Electric dipole

Ramp — Fancy shape

P07 - 15

field line: from high to low potential

Properties of Equipotentials

- E field lines point from high to low potential
- E field lines perpendicular to equipotentials
  - Have no component along equipotential
  - No work to move along equipotential

Demonstration: Kelvin Water Drop

20 PRS: Kelvin Water Dropper

A drop of water falls through the right can. If the can has positive charge on it, the separated water drop will have

0% 1. no net charge
0% 2. a positive charge
0% 3. a negative charge
0% 4. I don't know

Can

Water Drop

Equipotential Visualization

P07 - 19

Experiment 1: Equipotentials

Download LabView file (save to desktop) and run it

Log in to server and add each student to your group (enter your MIT ID)

Each group will do two of the four figures (your choice). We will break about half way through for some PRS

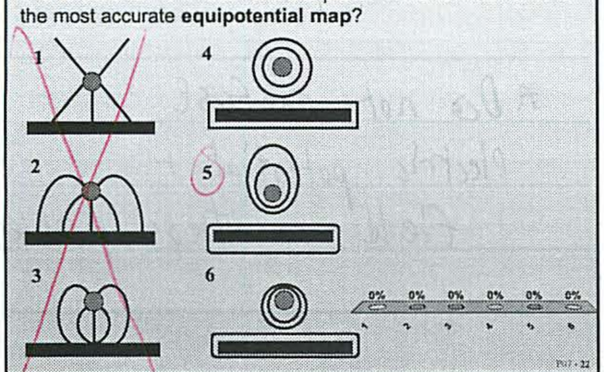
P07 - 20

PRS Questions: Midpoint Check

P07 - 21

PRS: Lab Midpoint: Equipotential

The circle is at +5 V relative to the plate. Which of the below is the most accurate **equipotential map**?



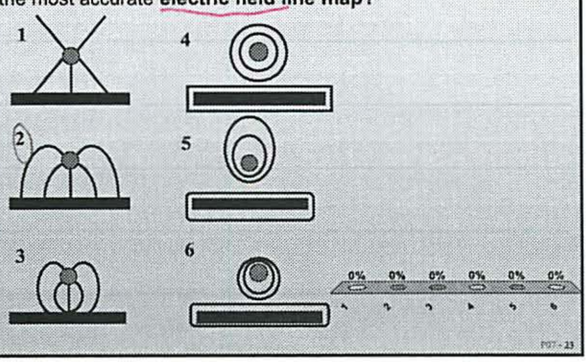
Can't be 1,2,3 - those are field lines - open

Closer = potential = steeper

PRS: Lab Midpoint: Field Lines

20

The circle is at +5 V relative to the plate. Which of the below is the most accurate **electric field line map?**



Experiment 1: Equipotentials

Continue with the experiment…

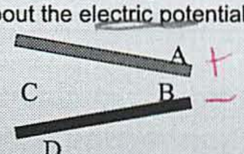
If you finish early make sure that you talk about the extra questions posed at the end of the lab. Labs will be asked about on the exams (see, for example, the final exam from Fall 2005)

PRS Questions: Lab Summary

* Do not confuse electric potential + field on these problems

PRS: Lab Summary: Potentials

Holding the red plate at +5 V relative to the ground of the blue plate, what is true about the electric potential at the following locations:



0%	1.	V(A) > V(B) > V(C) > V(D)
0%	2.	V(A) > V(B) ~ V(C) > V(D)
0%	3.	V(A) ~ V(B) > V(C) ~ V(D)
0%	4.	V(D) > V(C) ~ V(B) > V(A)
0%	5.	V(B) > V(C) > V(D) ~ V(A)
0%	6.	V(A) > V(D) ~ V(C) > V(B)

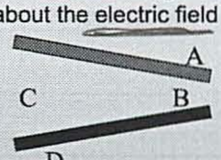
V(A) big - back of plate
not much change

Remember high near ⊕

B+C both at center line almost same

PRS: Lab Summary: E Field

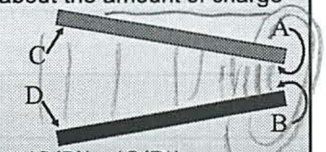
Holding the red plate at +5 V relative to the ground of the blue plate, what is true about the electric field at the following locations:



0%	1.	E(A) > E(B) > E(C) > E(D)
0%	2.	E(A) > E(B) ~ E(C) > E(D)
0%	3.	E(A) ~ E(B) > E(C) ~ E(D)
0%	4.	E(D) > E(C) ~ E(B) > E(A)
0%	5.	E(B) > E(C) > E(D) ~ E(A)
0%	6.	E(A) > E(D) ~ E(C) > E(B)

PRS: Lab Summary: Charge

Holding the red plate at +5 V relative to the ground of the blue plate, what is true about the amount of charge near the following points:



20

0%	1. $\lvert Q(A)\rvert \sim \lvert Q(C)\rvert > \lvert Q(B)\rvert \sim \lvert Q(D)\rvert$
0%	2. $\lvert Q(A)\rvert > \lvert Q(B)\rvert \sim \lvert Q(C)\rvert > \lvert Q(D)\rvert$
0%	3. $\lvert Q(A)\rvert \sim \lvert Q(B)\rvert > \lvert Q(C)\rvert \sim \lvert Q(D)\rvert$ ✓
0%	4. $\lvert Q(D)\rvert \sim \lvert Q(C)\rvert > \lvert Q(B)\rvert \sim \lvert Q(A)\rvert$
0%	5. $\lvert Q(B)\rvert \sim \lvert Q(D)\rvert > \lvert Q(A)\rvert \sim \lvert Q(C)\rvert$
0%	6. $\lvert Q(A)\rvert > \lvert Q(D)\rvert \sim \lvert Q(C)\rvert > \lvert Q(B)\rvert$

P07 - 28

absolute value

A+B strong electric field = large charges $E = \frac{kQ}{R}$

density of electric field = Stronger Field
denser field lines = stronger field = high amt of charge
A = −B   C = −D

Configuration Energy

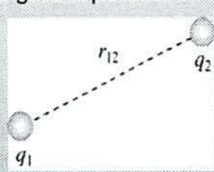
P07 - 29

Common topic on exam

Configuration Energy

How much energy to put two charges as pictured?

1) First charge is free
2) Second charge sees first:



$$U_{12} = W_2 = q_2 V_1 = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{12}}$$

P07 - 30

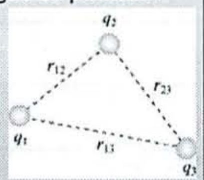
~~#2~~
From ∞
In the potential created by #1
↑ difference is the potential energy you need

Configuration Energy

How much energy to put three charges as pictured?

1) Know how to do first two
2) Bring in third:

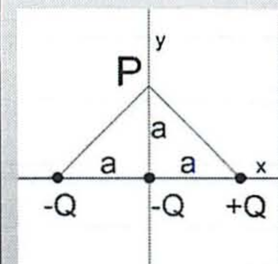
$$W_3 = q_3\left(V_1 + V_2\right) = \frac{q_3}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)$$



Total configuration energy:

$$U = W_2 + W_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right) = U_{12} + U_{13} + U_{23}$$

Group Problem: Build It



1) How much energy did it take to assemble the charges at left?

2) How much energy would it take to add a 4th charge +3Q at P?

solution posted

Topics: Electric Potential, E from V
Related Reading: Course Notes: Sections 3.1-3.5, 3.7-3.8

Topic Introduction

Today you will practice calculating potentials from charges and known field configurations in a problem solving. You will also play with the java applet "The Electric Potential Game" which should help solidify your understanding of the relationship between charge, field & potential.

Potential

Recall that the creation of an electric potential is intimately related to the creation of an electric field: $\Delta V = V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$. As with potential energy, we only define a potential difference. We will occasionally ask you to calculate "the potential," but in these cases we must arbitrarily assign some point in space to have some fixed potential. A common assignment is to call the potential at infinity (far away from any charges) zero. In order to find the potential anywhere else you must integrate from this place where it is known (e.g. from $A=\infty$, $V_A=0$) to the place where you want to know it.

Once you know the potential, you can ask what happens to a charge q in that potential. It will have a potential energy $U = qV$. Furthermore, because objects like to move from high potential energy to low potential energy, as long as the potential is not constant, the object will feel a force, in a direction such that its potential energy is reduced. Mathematically that is the same as saying that $\vec{\mathbf{F}} = -\nabla U$ (where the gradient operator $\nabla \equiv \frac{\partial}{\partial x}\hat{\mathbf{i}} + \frac{\partial}{\partial y}\hat{\mathbf{j}} + \frac{\partial}{\partial z}\hat{\mathbf{k}}$) and hence, since $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$, $\vec{\mathbf{E}} = -\nabla V$. That is, if you think of the potential as a landscape of hills and valleys (where hills are created by positive charges and valleys by negative charges), the electric field will everywhere point the fastest way downhill.

Important Equations

Potential Energy (Joules) Difference: $\Delta U = U_B - U_A = -\int_A^B \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}}$

Electric Potential Difference (Joules/Coulomb = Volt): $\Delta V = V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$

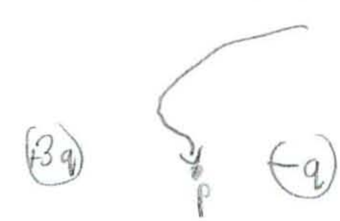
Electric Potential (Volts) created by point charge: $V_{\text{Point Charge}}(r) = \frac{kQ}{r}$

⊕ charge does not want to go to ⊕ so work done

Potential energy U (Joules) of point charge q in electric potential V: $U = qV$

Work done

- from ∞ away, looks like +2 is ⊕



$\frac{4kQ}{d}$

$\frac{k3q}{\frac{d}{2}}$ $\frac{kq}{\frac{d}{2}}$ ← add potentials

potential is $\frac{k(+3q-q)}{\frac{d}{2}}$ halfway b/w points

Work done is Q times that (since from ∞)

On final - tell about 4 Maxwell equations

Guass's Law

-counting field lines tells about enclosed charge

-charge makes diverging field

-fields start at ⊕ charges + go outwards

Mistakes

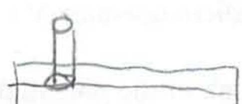
Uniform vs non uniform

↳ ρV ↳ must integrate (pratice)

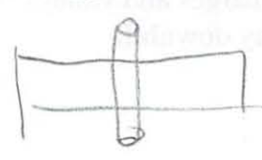
(fraction of sphere = fraction of charge)

E field must be constant or 0

only get relation of E_1 and E_2



$E_2 = 0$ so just E_1



E field 0 due to symmetry

surface
- must be at pt where want to calculate

- $\vec{E}$ is a vector (put vecter sign)

  - Guass Law gives E magnitude

  - write it as a vector (w/ direction)

- Don't forget units

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Problem Solving 3: Electric Potentials

REFERENCE: Chapter 3, 8.02 Course Notes.

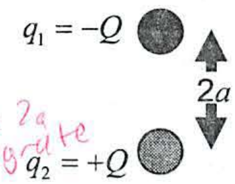
Consider two point-like charged objects with charges $q_1 = -Q$, and $q_2 = +Q$

Question 1: If the charges start out very far apart, how much energy it necessary to bring these charges together until they are a distance $2a$ apart? Give a physical reason for the sign of your answer. Does your answer depend on whether or not you choose infinity as a zero reference potential?

~~$U = qV$~~

~~$\int_0^{2a} F\,ds$~~) don't just write $2a$ need to integrate

~~$\int_0^{2a} -\frac{kqq}{r^2}\hat{r}$~~



$\boxed{U = -Q\frac{kq}{2a}}$

Easier superposition

0 work to move first particle in

2nd: $-\frac{kQ_2}{d}Q_1$ (potential difference) $\Delta V = V_B - V_A$.

ΔV charge since q

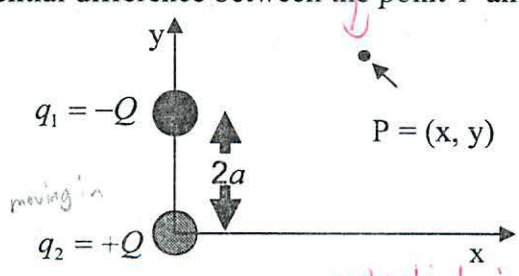
like work = distance · mass

if want 3rd do $-\frac{kq}{d}q$ for the 1 that is there + $-\frac{kq}{d}q$ for other

Choose a coordinate system such that the positively charged object is located at the origin and the negatively charged object is located a distance $2a$ along the positive y-axis (i.e. above it). Consider a point P that lies in the x-y plane with coordinates (x, y).

Question 2: What is the potential difference between the point P and infinity, $V(P) - V(\infty)$?

? does not mean anything just where 0



just potential diff

$-\frac{kq}{d} + -\frac{kq}{d}$ moving in

potential is not vectors

$\frac{-kq}{(x\hat{\imath}+y\hat{\jmath})} + -\frac{kq}{(x\hat{\imath}+(y-2a)\hat{\jmath})}$

$\frac{-kq}{\sqrt{x^2+y^2}} - \frac{kq}{\sqrt{x^2+(y-2a)^2}}$

partial deriv - hold everything else constant while doing that

Question 3: Use the fact that the electric field at the point P is given by

$$\vec{\mathbf{E}} = -\vec{\nabla} V = -\frac{\partial V}{\partial x}\hat{\mathbf{i}} - \frac{\partial V}{\partial y}\hat{\mathbf{j}}$$

↑ w/ respect to a certain variable

to find the x and y-components of the electric field at the point P from the potential you just calculated.

$$\vec{E} = -\nabla\left(\frac{-kq}{\sqrt{x^2+y^2}} - \frac{kq}{\sqrt{x^2+(y-2a)^2}}\right)$$

$$\frac{\partial V}{\partial x} = \left(-\frac{kq}{2}(x^2+y^2)^{-3/2}(2x) + \frac{kq}{2}\left(x^2+(y-2a)^2\right)^{3/2}(2x)\,\hat{i}\right)$$

$$= -kqx\left((x^2+y^2)^{-3/2} - \left(x^2+(y-2a)^2\right)^{-3/2}\hat{i}\right)$$

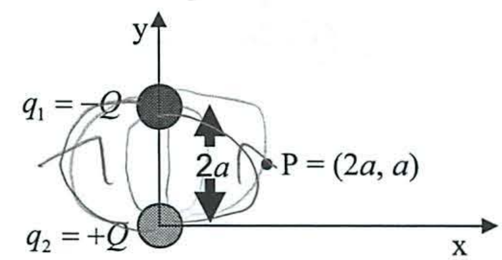
$$\frac{\partial V}{\partial y} = \frac{-kQ}{2}(x^2+y^2)^{-3/2}(2y) - \left(-\frac{kQ}{2}\left(x^2+(y-2a)^2\right)^{-3/2}2(y-2a)\right)$$

$$kQ\left(-(x^2+y^2)^{-3/2}y + \left(x^2+(y-2a)^2\right)^{-3/2}(y-2a)\,\hat{j}\right.$$

$$\vec{E} = -\frac{dV}{dx}\hat{i} - \frac{\partial V}{\partial y}\hat{j}$$

Question 4: Suppose the point P is located at $P = (2a, a)$. Using only symmetry considerations (i.e. without calculation), predict the direction of the electric field, and draw the direction on the sketch below.

"not including pt P"



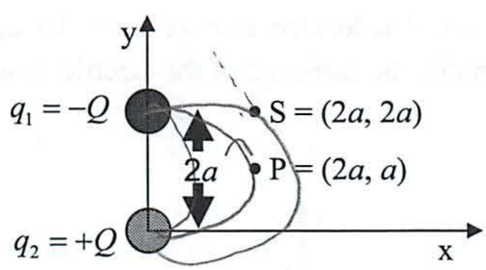
from ⊕ to ⊖

then field would just be "regular"

only up at that point
- due to vectors

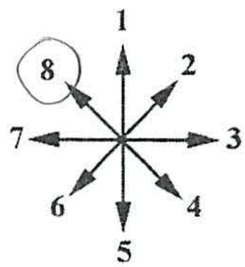
Question 5: Use the results of your calculations from part (b) and (c) to find an exact expression for the electric potential difference and the electric field at $P = (2a, a)$.

Question 6: Now move from $P = (2a, a)$ to $S = (2a, 2a)$. Without calculation answer the following: is the electric potential difference $V(S) - V(P)$ positive, zero, or negative? Why?



$\ominus$ moving in direction of the field

Question 7: Which arrow most closely represents the direction of the electric field at $S = (2a, 2a)$?



tangent to curve through line

Part Two: Electric potential game.

We next want you to look at an applet that shows you the electric potential due to two point charges, and how that is related to the electric field, using the examples from Part One above. We then want you to play a game where you explore bit by bit the electric potential due to two "invisible" point charges and guess the sign of the two invisible charges. You "win" the game by using the least number of moves to figure out what the signs of the charges are.

Question 8: Open up the landscape applet. When you open the application you will see the charge configuration you were given in Question 1 above. We also show the potentials due to these two charges. You can explore the electric field by moving your avatar around the xy plane in the scene using the keypad on the right. The vertical distance of the avatar above the xy plane is the electric potential at the avatar's location. We also show the electric field at the avatar's location below the avatar in the xy plane.

Using the application, confirm your answers to Questions (4), (6) and (7) above.

Question 9: Using the same application as above, create a potential landscape using two positively charged objects, using the controls on the right to change the sign of the charges. Find a point on the landscape where the electric field points away from both charged objects. Briefly describe your strategy.

Explore the region around your selected point and observe how the electric field changes direction. Move the charges around, and change their signs, to get an idea of what the potential landscape looks like for arbitrary placement of the charges and how the electric field varies as you move your avatar around the xy plane.

You will need the intuition developed here to do well in the game below!!

Question 10: Open up the electric potential game. You will have two charges which will be invisible, and located at random positions. You will only see that part of the electric potential that your avatar has explored. Move your avatar around the plane until you have enough information to guess the signs of the charges. Play the game and see which group at the table gets the lowest cumulative score for three tries.

Review

2/20

Electric Potential (Voltage)

$\vec{E} = -\nabla V$

derive each section individually

$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$

integrate

Point Charge

$\vec{E} = k_e \frac{q}{r^3}$

$EA = \frac{Q_{inc}}{\epsilon_0}$

$V = k_e \frac{q}{r}$

$F = qE$

$W = \Delta U = q \Delta V$

$U = qV$

Point charge potential

1st charge is 0
- just move it in

2nd charge - calculate
- if moving (+) charge into (+) field must do (+) work
- moving up hill (both move to lower U) $U = qV$

Can calculate via $\int$ or super position
- moving charge #2 in

$$\Delta V = V_B - V_A = -\int_{r_A}^{r_B} \vec{E}\, ds$$

$$-\int_{r_A}^{r_B} \frac{kQ}{r^3} \cdot ds$$

$$-kQ \int_{r_A}^{r_B} \frac{dr}{r^2}$$

$$kQ\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$ ← remember what it is to integrate!

$$\frac{kQ}{r}$$

Or can super position

$$\Delta V = V_B - \cancel{V_A} \nearrow 0 \text{ since } \infty$$

$$-\frac{kQ}{d}$$ point particle

$T \Delta V$

$$U = -\frac{kq}{d} \cdot Q$$

$\Delta V \cdot$ charge

like work = distance · mass

$$\boxed{\text{* Potential is Scalar *}}$$

Also

⊕ charge high → low potential
high → low U

$\Delta U = q\Delta V$

⊖ charge low → high potential
high → low U

Uniform
↳ ρV

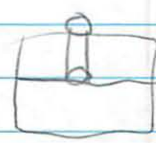
Non Uniform
↳ must integrate

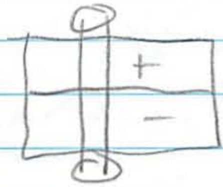
Conductive sphere
↳ all the charge lies on the surface
(Colomb's law - they try to go as far away as possible)
field inside must be 0, outside perpendicular

Non conductive sphere
↳ charge distributed throughout sphere

Read up more on what symmatries mean
- can only use Guass's law when symmatry

E field must constant or 0
- at one end where measuring
-





For example - here 0 due
to symmetry

Electric Potential
$V(0)$ $V(\infty)$

2/20

Potential energy
- changing position, configuration
- exerts a force on other charges
  - same repulsive
  - diff attractive

$$V = \frac{kQq}{r}$$

The 0 point is arbitrary, like origin coord system
- every point measured w/ respect to that 0

- ∞ best for single point / localized collection
- 0 best for ∞ line charge
  - otherwise local values would go to ∞
- ground best for real life circuits

~~∞~~

$$V = \frac{kQ}{r} = \frac{Q}{4\pi\epsilon_0 r}$$

electric field + force $\rightarrow 0$ at ∞