2/22

$Q = C\Delta V$

Reordered to fit topics

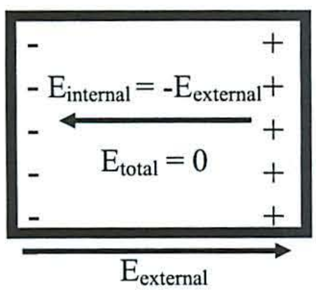
Topics: Conductors & Capacitors
Related Reading: Course Notes: Sections 4.3-4.4; 5.1-5.4, 5.9

double ΔV, Q will double

Topic Introduction

Today we introduce the concept of conductors and put the idea of capacitance, which you have already played with in circuits, on firm ground. Conductors are materials in which charge is free to move. That is, they can *conduct* electrical current (the flow of charge). Metals are conductors. For many materials, such as glass, paper and most plastics this is not the case. These materials are called insulators. For the rest of the class we will try to understand what happens when conductors are put in different configurations, when potentials are applied across them, and so forth.

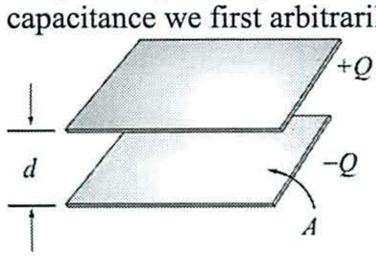
Conductors



Since charges are free to move in a conductor, the electric field inside of an isolated conductor must be zero. Why? Assume that the field were not zero. The field would apply forces to the charges in the conductor, which would then move. As they move, they begin to set up a field in the opposite direction. An easy way to picture this is to think of a bar of metal in a uniform external electric field (from left to right in the picture below). A net positive charge will then appear on the right of the bar, a net negative charge on the left. This sets up a field opposing the original. As long as a net field exists, the charges will continue to flow until they set up an equal and opposite field, leaving a net zero field inside the conductor.

Capacitance

You already know much about capacitors, for example, that they store electric charge and that they are characterized by the amount of charge they can store for a given potential difference ($C \equiv Q/|\Delta V|$), that is, that a large capacitance capacitor can store a lot of charge with little "effort" – little potential difference between the two plates.
Today we begin taking a second look at capacitors, namely learning how to calculate the capacitance of various configurations of conductors. A simple example is pictured at left – the parallel plate capacitor, consisting of two plates of area A, a distance d apart. To find its capacitance we first arbitrarily place charges $\pm Q$ on the plates. We calculate the electric field between the plates (using Gauss's Law) and integrate to obtain the potential difference between them. Finally we calculate the capacitance: $C = Q/|\Delta V| = \varepsilon_0 A/d$. Note that the capacitance depends only on geometrical factors, not on the amount of charge stored (which is why we were justified in starting with an arbitrary amount of charge).



Energy

As you already know, in the process of storing charge, a capacitor also stores electric energy. Today we derive the formula you have been using by considering how you "charge" a capacitor. Imagine that you start with an uncharged capacitor. Carry a small amount of positive charge from one plate to the other (leaving a net negative charge on the first plate). Now a potential difference exists between the two plates, and it will take work to move over subsequent charges. Reversing the process, we can release energy by giving the charges a method of flowing back where they came from. So, in charging a capacitor we put energy into the system, which can later be retrieved. Where is the energy stored? In the process of charging the capacitor, we also create an electric field, and it is in this electric field that the energy is stored. We assign to the electric field a "volume energy density" u_E, which, when integrated over the volume of space where the electric field exists, tells us exactly how much energy is stored.

Important Equations

Capacitance: $C \equiv Q/|\Delta V|$ $= \frac{\epsilon_0 A}{d}$

Energy Stored in a Capacitor: $U = Q^2/2C = \frac{1}{2}Q|\Delta V| = \frac{1}{2}C|\Delta V|^2$

Energy Density in Electric Field: $u_E = \frac{1}{2}\varepsilon_o E^2$

General formula

So is a battery a type of capicitor?

Volume energy density (U_E) = electric field in capicitor where energy is stored

2/22

Class 09: Outline

Hour 1

Conductors and Insulators;

Hour 2

Capacitance and Capacitors

* $Q = C\Delta V$

the greater the potential difference, the greater the capacity to store charge

Conductors

Conductors and Insulators

Conductor: Charges are free to move

Electrons weakly bound to atoms

Example: metals

Insulator: Charges are NOT free to move

Electrons strongly bound to atoms

Examples: plastic, paper, wood

can move from their lattice site

-weakly bound

semiconductors in middle

Charges try to get far away from each other

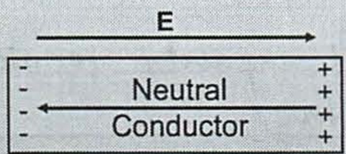
-migrate to surface in conducting sphere

Conductors

Conductors have free charges

→ E must be zero inside the conductor

→ Conductors are equipotential objects



charges move b/c force is on them

Until E field inside cancled
- happens very quickly
- hard to calc

if not 0 charges would move more

↑ external field - induced charge) produces own field to left cancles external field

$E \rightarrow \oplus$

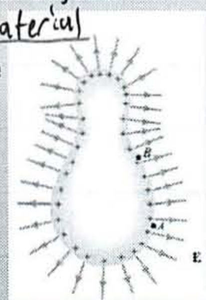
Some external charge density

Conductors in Equilibrium

Conductors are equipotential objects:

1) E = 0 inside conducting material
2) E perpendicular to surface
3) Net charge inside is 0
4) Excess charge on surface

if tangent → would still move



⊕ charges down
⊖ charges up) $-\int \vec{E} \cdot d\vec{s}$

← diff b/w 2 pts on surface = 0
↑ same potential

Inside

non 0 field inside (Faraday ice cage)

but flux of whole thing = 0

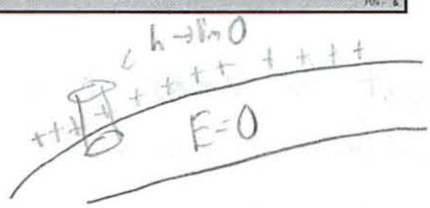
Conductors in Equilibrium: Free Charges Move To Surface

Put net charge inside conductor

It moves to get away from other charges

Java applet link

← Electrons trying to get as far away as possible



no flux on bottom

flux on top σ surface charge density

E ⊥ surface $= \frac{\sigma}{\epsilon_0}$

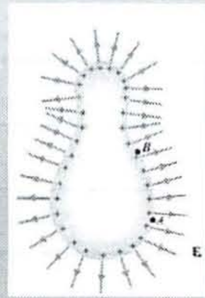
magnitude of E on surface would vary

Conductors in Equilibrium

Conductors are equipotential objects:

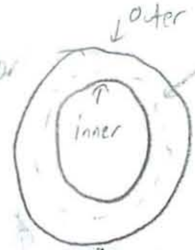
1) E = 0 inside
2) E perpendicular to surface
3) Net charge inside is 0
4) Excess charge on surface

$$E = \sigma / \varepsilon_0$$



P09 - 7

+Q on conductor

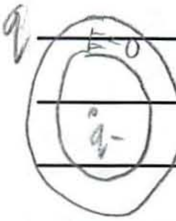


$\oiint E \cdot da$

E = 0 on conductor – so no flux

Q distributes itself on the outer surface

– 0 charge on inner surface



$\oiint E \cdot d\vec{A}$ = still 0

– field on surface

– no E field

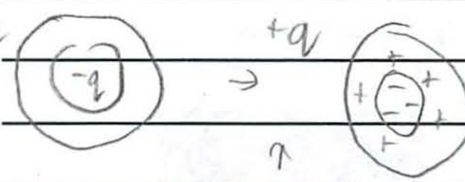
migrates

Capacitors and Capacitance

Our first of 3 standard electronics devices
(Capacitors, Resistors & Inductors)

P09 - 8

└ circuit elements

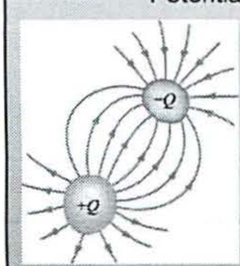


has no idea anything inside

will distribute itself as normal

Capacitors: Store Electric Charge

Capacitor: Two isolated conductors
Equal and opposite charges ±Q
Potential difference ΔV between them.



$$C = \frac{Q}{|\Delta V|}$$

Units: Coulombs/Volt or Farads

C is Always Positive

P09 - 9

* Charges move everywhere until $\vec{E}$ field = 0 *

could bring charge to 1

– now potential difference b/w them

– one has (+) other (–)

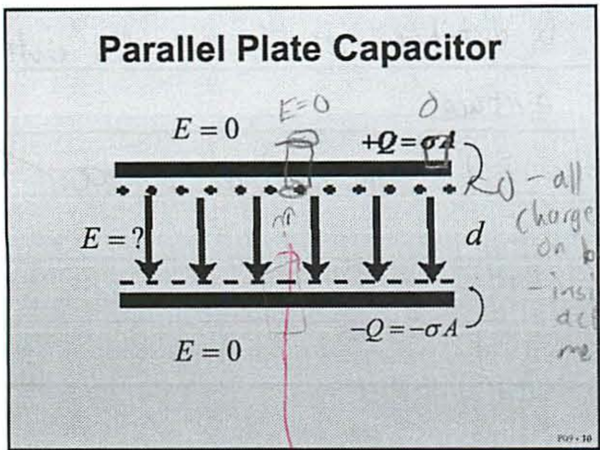
humans can be capacitors

C is (+) constant

so Q (+) as well

(+) at higher potential

have some thickness



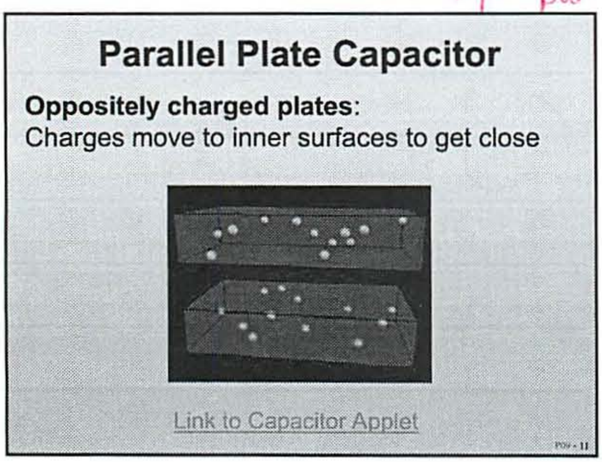
Calculate E field b/w plates

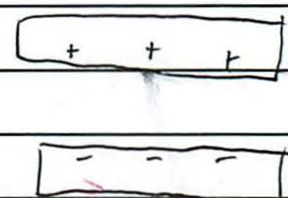
$EA = \frac{\sigma A}{\varepsilon_0}$

* if no bottom bar
charges go to top + bottom

(on slide: E=0; σ; all charges on bottom; inside actual metal = 0)

E only on bottom of top bar → $E = \frac{\sigma}{\varepsilon_0}$ due to bottom bar

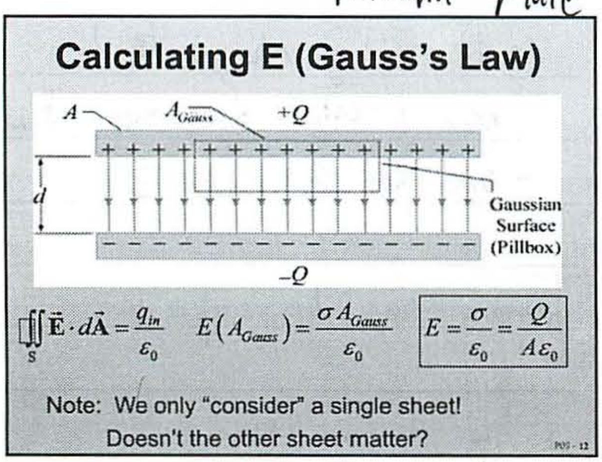




$\vec{E} = \vec{E}_+ + \vec{E}_-$

If outside guassian surface
– does not contribute to flux

Parrallel Plate

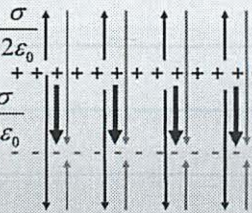


* know

← only for parrel plate

Super position argument

Alternate Calculation Method



Top Sheet: $E = -\frac{\sigma}{2\varepsilon_0}$ (above), $E = \frac{\sigma}{2\varepsilon_0}$ (below)

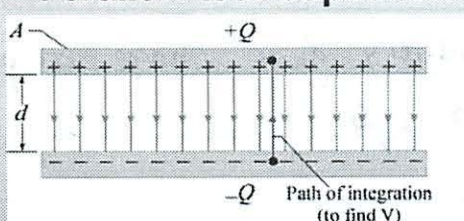
Bottom Sheet: $E = \frac{\sigma}{2\varepsilon_0}$ (above), $E = -\frac{\sigma}{2\varepsilon_0}$ (below)

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

P09 - 13

Each sheet seperatly
Add together

Parallel Plate Capacitor



$$\Delta V = -\int_{bottom}^{top} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = Ed = \frac{Q}{A\varepsilon_0} d \qquad C = \frac{Q}{|\Delta V|} = \frac{\varepsilon_0 A}{d}$$

C depends only on geometric factors *A* and *d*

P09 - 14

? easy integral here
not a point charge

but E constant

(+) at higher potential
− bottom → top = + top → bottom

So $\frac{Q}{\left(\frac{Qd}{\varepsilon_0 A}\right)} = \frac{\varepsilon_0 A}{d}$ Know the math + order opps that is going on here

PRS Questions: Changing C Dimensions

P09 - 15

* Capatiance depends on ~~capa~~ area of capacitors

3 main problems - parralel plates
- cylindrical shells
- shells

- Since can use Guass' Law
other shapes still capacitors, can't measure

* know what changes + what does not
know the formulas

PRS: Changing Dimensions

:20

A parallel-plate capacitor has plates with equal and opposite charges ±Q, separated by a distance *d*, and **is not** connected to a battery. The plates are pulled apart to a distance *D* > *d*. What happens?

0%	1.	V increases, Q increases
0%	2.	V decreases, Q increases
0%	3.	V is the same, Q increases
0%	4.	V increases,Q is the same
0%	5.	V decreases, Q is thesame
0%	6.	V is the same, Q is the same
0%	7.	V increases, Q decreases
0%	8.	V decreases, Q decreases
0%	9.	V is the same,Q decreases

P09 - 16

$C = \frac{Q}{V} = \frac{\epsilon_0 A}{d}$ ← won't know from formula

↑2x

Work done by moving the plates across

Q nowhere to go

but V increases as work added

If Q same, E same

$V = \frac{E}{D}$ ← same; E increases; ↑ increases

20 PRS: Changing Dimensions

A parallel-plate capacitor has plates with equal and opposite charges ±Q, separated by a distance *d*, and **is** connected to a battery. The plates are pulled apart to a distance *D* > *d*. What happens?

1. V increases, Q increases
2. V decreases, Q increases
3. V is the same, Q increases
4. V increases, Q is the same
5. V decreases, Q is the same
6. V is the same, Q is the same
7. V increases, Q decreases
8. V decreases, Q decreases
9. V is the same, Q decreases

P09 - 17

* battery gives charge a place to go

battery – keeps it to same potential (the potential of battery)

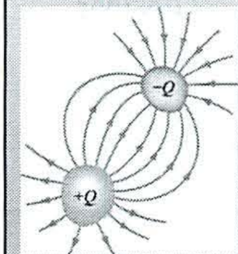
$V = \frac{E}{D}$ ← smaller → q gets smaller; D ← bigger; ↑ same

Demonstration: Changing C Dimensions

P09 - 18

Capacitors: Review

Capacitor: Two isolated conductors
Equal and opposite charges ±Q
Potential difference ΔV between them.



$$C = \frac{Q}{|\Delta V|}$$

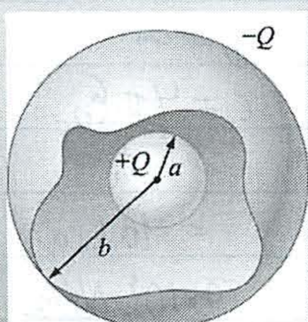
Units: Coulombs/Volt or Farads

C is Always Positive

P09 - 19

* for simple symmetry

Group Problem: Spherical Shells



These two spherical shells have equal but opposite charge.

Find E everywhere

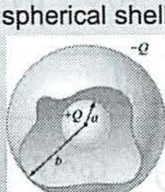
Find V everywhere (assume V(∞) = 0)

P09 - 20

3. Step E → V → C

Spherical Capacitor

Two concentric spherical shells of radii *a* and *b*



What is E?

Gauss's Law → $E \neq 0$ only for $a < r < b$, where it looks like a point charge:

$$\vec{\mathbf{E}} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}$$

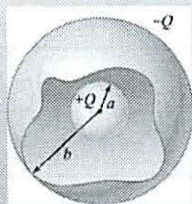
P09 - 21

⊖ is lower
make sure know which is which

Spherical Capacitor

$$\Delta V = -\int_{inside}^{outside} \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = -\int_a^b \frac{Q\hat{\mathbf{r}}}{4\pi\varepsilon_0 r^2} \cdot dr\,\hat{\mathbf{r}} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$$

Is this positive or negative? Why?



$$C = \frac{Q}{|\Delta V|} = \frac{4\pi\varepsilon_0}{(a^{-1} - b^{-1})}$$

For an isolated spherical conductor of radius *a*:

$$C = 4\pi\varepsilon_0 a$$

P09 - 22

Outside = E constant = V_0

Inside = same everywhere inside

↑ derivative of constant = 0

$E \neq 0$ b/w spheres

Capacitance of Earth

For an isolated spherical conductor of radius *a*:

$$C = 4\pi\varepsilon_0 a$$

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{F/m} \qquad a = 6.4 \times 10^6\ \text{m}$$

$$C = 7 \times 10^{-4}\ \text{F} = 0.7\text{mF}$$

A Farad is REALLY BIG! We usually use pF (10^{-12}) or nF (10^{-9})

P09 - 23

Human capacitor
- you ⊕ sphere
- ∞ ⊖ sphere

$\frac{1}{\infty}$ $b \to \infty$

$C = \frac{4\pi\varepsilon_0}{\frac{1}{a}}$

$= 4\pi\varepsilon_0 a$

$\frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^9 \frac{N/m}{C}$

$a = 1m$

picofaron $= 10^{-12}$ F

$C = \frac{1}{10^{10}}$ N·m^2

C = 100 pF

so ground yourself before touching computer

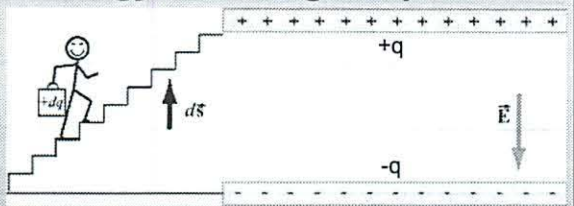
Energy Stored in Capacitor

Start charging capacitor

- Something must charge them

P09 - 24

Energy To Charge Capacitor



1. Capacitor starts uncharged.
2. Carry $+dq$ from bottom to top.
   Now top has charge $q = +dq$, bottom $-dq$
3. Repeat
4. Finish when top has charge $q = +Q$, bottom $-Q$

P09 - 25

person carrying charge

← intermediate integration
- going up potential hill
- adding energy

$Q = C\Delta V$

increasing as going up

∫ that

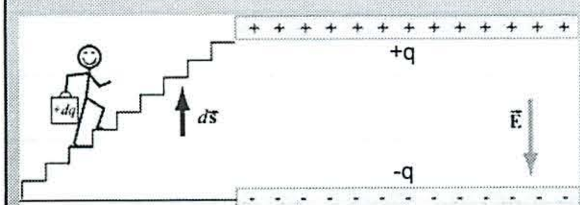
work being done

Work Done Charging Capacitor

At some point top plate has $+q$, bottom has $-q$

Potential difference is $\Delta V = q / C$

Work done lifting another dq is $dW = dq\,\Delta V$



P09 - 26

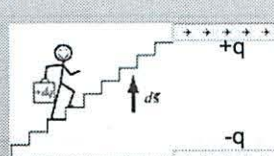
Work Done Charging Capacitor

So work done to move dq is:

$$dW = dq\,\Delta V = dq\frac{q}{C} = \frac{1}{C}q\,dq$$

Total energy to charge to $q = Q$:

$$W = \int dW = \frac{1}{C}\int_0^Q q\,dq = \boxed{\frac{1}{C}\frac{Q^2}{2}}$$



P09 - 27

↑ total

(That P-set qu everyone has trouble with

Energy Stored in Capacitor

Since $C = \frac{Q}{|\Delta V|}$

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q|\Delta V| = \frac{1}{2}C|\Delta V|^2$$

Where is the energy stored???

its how much changes that matters

→ stored in Electric field

P09 - 28

Energy Stored in Capacitor

Energy stored in the E field!

Parallel-plate capacitor: $C = \frac{\varepsilon_o A}{d}$ and $V = Ed$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\varepsilon_o A}{d}(Ed)^2 = \frac{\varepsilon_o E^2}{2} \times (Ad) = u_E \times (volume)$$

$$u_E = E \text{ field energy density} = \frac{\varepsilon_o E^2}{2}$$

P09 - 29

energy stored in electric field

PRS Question:
Changing C Dimensions
Energy Stored

P09 - 30

PRS: Changing Dimensions

A parallel-plate capacitor, disconnected from a battery, has plates with equal and opposite charges, separated by a distance *d*.

Suppose the plates are pulled apart until separated by a distance *D* > *d*.

How does the final electrostatic energy stored in the capacitor compare to the initial energy?

0% 1. The final stored energy is smaller
0% 2. The final stored energy is larger
0% 3. Stored energy does not change.

P09 - 31

Q constant
E same
Volume increased
did work to ↑ d
~~Q is constant~~ thats stored in E field
- remember seperating plates

Demonstration: Big Capacitor

P09 - 37

Dumashkin

Office Hrs

$\frac{1}{2}\epsilon_0 E^2$ = energy density for all shapes

- to get energy = -Ed

have to integrate for other shapes

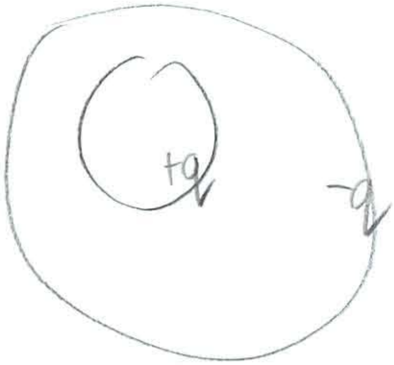
$\int dr'$

? capitance

$\int e$ $\frac{1}{2}C(\Delta V)^2$ -

See Mon slides
what is E density
when E field

Where is $\frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}C\Delta V^2$
- all space, density



Calc energy density

$\int \frac{1}{2}\epsilon_0 E^2 \cdot 2\pi R L \, dr'$

Guass Law
$\left(\frac{1}{r}\right)^2$
$\frac{1}{r^2}$

R

end w/ ln

What is energy stored?
$\frac{1}{2}\epsilon_0 E^2$ - same for resistance

← | | →

move apart

large volume

E field same

had to have more energy

- have to add some E to move them apart

---

know how to integrate over each type of area

Capitance - learn for non uniform E density

- sphere + cylinder

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2010**

Problem Set 4

Due: Tuesday, March 2 at 9 pm.

Hand in your problem set in your section slot in the boxes outside the door of 32-082. Make sure you clearly write your name and section on your problem set.

Text: Liao, Dourmashkin, Belcher; Introduction to E & M MIT 8.02 Course Notes.

Reading Assignments:

Week Five Conductors as Shields; Current and Ohm's Law

Class 11 W05D1 M/T Mar 1/2	Conductors as Shields; Expt. 2: Faraday Ice Pail; Capacitors and Dielectrics
Reading:	Course Notes: Sections 4.3-4.4; 5.5, 5.9, 5.10.2
Experiment:	Experiment 2: Faraday Ice Pail

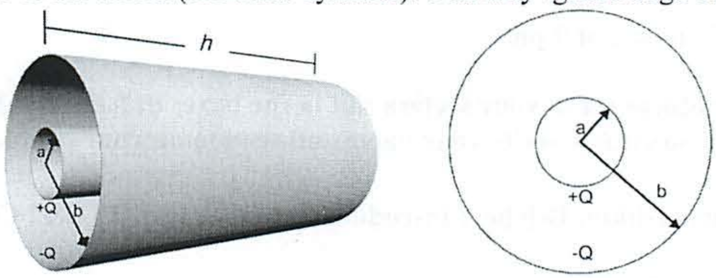
http://web.mit.edu/8.02t/www/materials/Experiments/exp02.pdf

Class 12 W05D2 W/R Mar 3/4	Current, Current Density, and Resistance and Ohm's Law; DC Circuits
Reading:	Course Notes: Sections 6.1-6.5; 7.1-7.4
Class 13 W05D3 F Mar 5:	PS04: PHET: Building a Simple DC Circuit
Reading:	Course Notes: Sections 6.1-6.5; 7.1-7.4

Add Date Mar 5

Problem 1: Read Experiment 2: Faraday Ice Pail
http://web.mit.edu/8.02t/www/materials/Experiments/exp02.pdf

Consider two nested cylindrical conductors of height h and radii a & b respectively. A charge $+Q$ is evenly distributed on the outer surface of the pail (the inner cylinder), $-Q$ on the inner surface of the shield (the outer cylinder). You may ignore edge effects.



a) Calculate the electric field between the two cylinders ($a < r < b$).

b) Calculate the potential difference between the two cylinders:

c) Calculate the capacitance of this system, $C = Q/\Delta V$

d) Numerically evaluate the capacitance for your experimental setup, given: $h \cong 15$ cm, $a \cong 4.75$ cm and $b \cong 7.25$ cm.

e) Find the electric field energy density at any point between the conducting cylinders. How much energy resides in a cylindrical shell between the conductors of radius r (with $a < r < b$), height h, thickness dr, and volume $2\pi rh\,dr$? Integrate your expression to find the total energy stored in the capacitor and compare your result with that obtained using $U_E = (1/2)C(\Delta V)^2$.

Problem 2: Experiment 2 Faraday Ice Pail Predictions

A. Prediction: Charging by Contact Sketch your prediction for the graph of potential difference vs. time for part 2 of this experiment. Indicate the following events on the time axis:

(a) Insert positive charge producer into pail
(b) Rub charge producer against inner surface of pail
(c) Remove charge producer

B. Prediction: Charging by Induction Sketch your prediction for the graph of potential difference vs. time for part 3 of this experiment. Indicate the following events on the time axis:

(a) Insert positive charge producer into pail
(b) Ground pail to shield
(c) Remove ground contact between pail and shield
(d) Remove charge producer

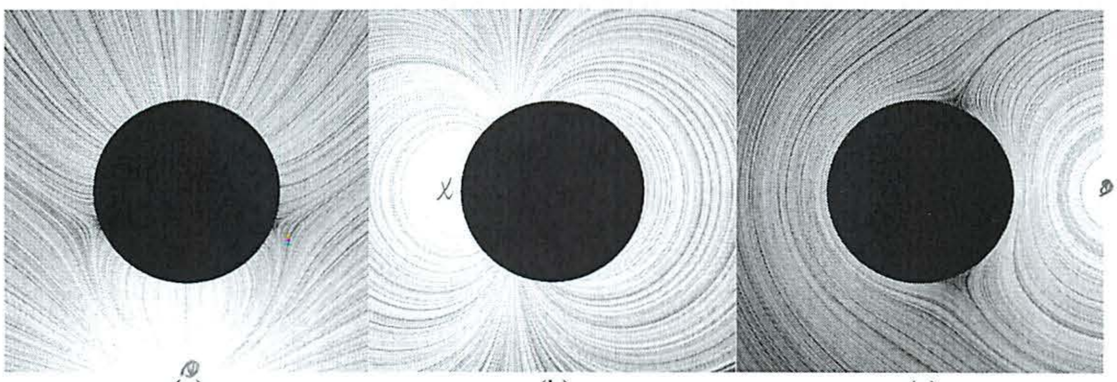
Problem 3: Electrostatic Shielding

Part of the lab this week involves shielding. We have a visualization to help you better understand this. Open it up:

http://web.mit.edu/viz/EM/visualizations/electrostatics/ChargingByInduction/shielding/shielding.htm

and play with it for a while. You can move the charge around the outside of the shield (or even inside) using the parameters "radius pc" and "angle pc." You can change which field you are looking at – the total field, just the field of the external charge ("Free charge") or just the field of the induced charge (on the shield). You can visualize it with grass seeds or display equipotential streaks by clicking "Electric Potential."

Below are three captured images. I've blanked out the center so that you can't see what is going on inside the conductor. For each describe where the charge is (ROUGH angle and distance), tell whether I am looking at field lines (grass seeds) or equipotential streaks ("Electric Potential") and indicate whether I am doing so for the total field, or just the external or induced field. Also briefly explain HOW you know this (not just "I looked around until I was able to repeat the pattern").



(a) (b) (c)

Problem 4: Parallel Plate Capacitor

A potential difference V_0 is applied across the plates of a parallel-plate capacitor resulting in charges $+Q_0$ and $-Q_0$ on the plates. The source of the potential difference is then disconnected from the plates. You then halve the distance between the plates. What happens to

a) the charge on the plates?

b) the electric field?

c) the energy stored in the electric field?

d) the potential?

e) How much work did you do in halving the distance between the plates?

Problem 5: Human Capacitor

What, approximately, is the capacitance of a typical MIT student? Check out the exhibit in Strobe Alley (4^{th} floor of building 4) for a hint or just to check your answer.

8.02 PSet 4

100 − 15 − 12 = 73

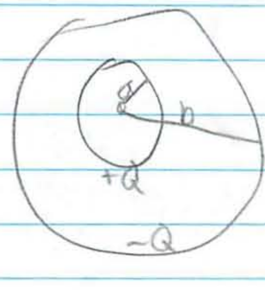
Michael Plasmeier L01 11C

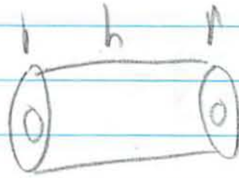
2/28

1. 2 nested cylinders

ignore edge effects

So I started on my own and then noticed an example in the book





a. Calculate the electric field b/w the cyliner

≠0 because in the middle off
So calculate the inside + outside seperatly?

$$EA = \frac{Q_{inc}}{\epsilon_0}$$

- endcaps only ?
- or sides only
  - but this is not infinite
  - but ignore edge effects
  - yeah intersted in r

$$E\,2A_{end} = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0} \begin{cases} \hat{x} & \text{up} \\ -\hat{x} & \text{down} \end{cases}$$

like problem solving 2

$$EA = \frac{Q_{inc}}{\epsilon_0}$$

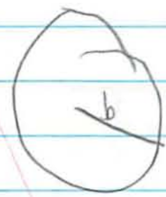
$$E(2\pi r h) = \frac{+Q - \lambda h}{\epsilon_0}$$

$$E = \frac{Q\ \lambda}{\epsilon_0 2\pi r h}$$ ← dependent on $\frac{1}{r}$

So actally I did get it

E from other one

- inside



$$EA = \frac{Q_{inc}}{\epsilon_0}$$

$$E(2\pi r h) = \frac{-Q}{\epsilon_0}$$

$$E = \frac{-Q}{\epsilon_0 2\pi r h}$$

I had gotten it on first page

Only care from $a < r < b$

No need to add anything - its all there

(why?)

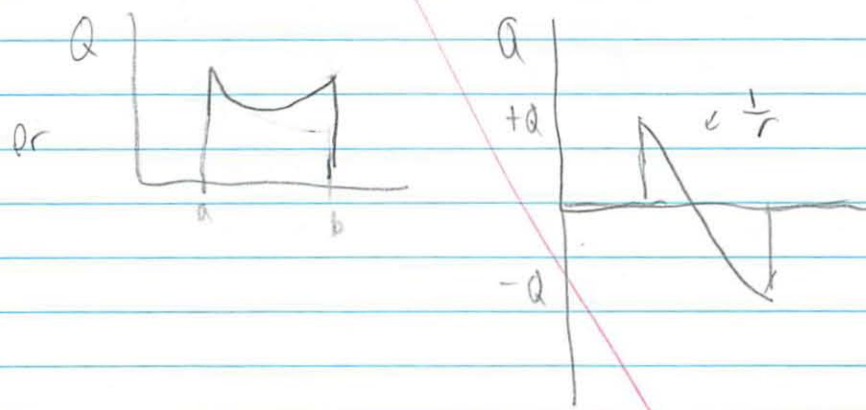
?

$$E = \frac{Q}{\epsilon_0 2\pi r h} - \frac{Q}{\epsilon 2\pi r h} \qquad a < r < b$$

r r

is this the same r

or is it distance from something



$$E = \frac{Q}{\epsilon_0 2\pi (r - r_a) h} - \frac{Q}{\epsilon 2\pi (r_b - r) h}$$

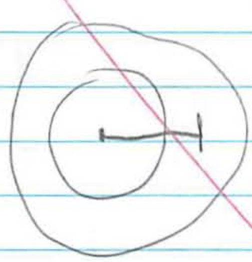
? I guess

I'm glad to see you workin so hard and writing questions and stuff, thats great. But could you put a big box around your final attempt? It's kind of hard to find with so much work but I don't want to discourage your involvement with learning. Keep up the good work!

b. Calculate Potential Difference

$V = V(P) - V(Q)$
$V(P) - 0$

$V \quad a < r < b$



Have to sum the inside
What is the actual shell E
- just ignore?

$V = -\int E \cdot ds$

since I did a) too complex

$-\int \frac{Q}{\epsilon_0 2\pi(r - r_a)h} - \frac{Q}{\epsilon_0 2\pi(r_b - r)h} \, dr$

see redo

---

actually course notes has example just like this one

$L > b - a$ so edge effects neglicted

a(redo) Guassian surface $l < L \quad a < r < b$

$EA = \frac{Q_{inc}}{\epsilon_0}$

$E(2\pi r l) = \frac{\lambda l}{\epsilon_0}$

$E = \frac{\lambda}{2\pi \epsilon_0 r}$

~~not quite.~~ O.K. see solns. You called "h" "L". given as h in the problem.

$\lambda = \frac{Q}{L}$ charge per unit lenght

- So I was kinda right - its just in the middle - simplest ans bredo

$$\Delta V = V_b - V_a = -\int_a^b E_r \, dr$$

$$= -\frac{\lambda}{2\pi\epsilon_0} \int_a^b \frac{dr}{r}$$

which one is your answer? I'll give partial credit. I can't tell.

simple to do when simple e

−3

$$= -\frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{b}{a}\right) \quad \text{← what is } b-a \quad -\frac{\lambda}{2\pi\epsilon_0} \ln(r)$$

remember $\int_a^b \frac{1}{r} = \ln\left(\frac{b}{a}\right)$

- conductor w/ ⊖ charge has lower potential

c. Capatiance $C = \frac{Q}{|\Delta V|} = \frac{\lambda L}{\lambda \ln(r) / 2\pi\epsilon_0} = \frac{2\pi\epsilon_0 L}{\ln(\text{x})}$

$\ln\left(\frac{b}{a}\right)$

- Capatiance depends only on L, a, b

the voltage difference ΔV between the two plates is not a function of the distance; are fixed at a and b. −3

d. Now with numbers

h = 15cm

a = 4.75 cm

b = 7.25cm

$$C = \frac{2\pi\epsilon_0(.15)}{\ln(7.25/4.75)}$$

$$\epsilon_0 = 8.8 \cdot 10^{-12} \; m^{-3} kg^{-1} s^4 A^2$$

$$C = 1.96 \cdot 10^{-11}$$ ✓

e. Find the electric field energy density

How much Energy with $a < r < b$

h

dr

$2\pi r h\, dr$

Integrate to find the total energy stored in capacitor. Compare w/

$U_E = \frac{1}{2} C (\Delta V)^2$

So we want $\lambda = \frac{Q}{L}$

$U = \frac{1}{2} C \Delta V^2$ have total E

know know have volume

integrate find energy / volume

$\frac{1}{2} \left(\frac{2\pi \epsilon_0 L}{\ln(b/a)} \right) \left(\frac{-\lambda}{2\pi \epsilon_0} \ln(b/a) \right)^2$

E in each shell

$\int \frac{1}{2} \epsilon_0 E^2$ ←?

- integral of cylindrical shells $2\pi r h\, dr$

Jay's not-helpful help

↑ if that's it, then I agree :)

Ok - after Dumaskin's Office Hrs

from class notes day 9 slide 28

⟶

$$W = \frac{1}{C}\int_0^Q q\,dq = \boxed{\frac{1}{C}\frac{Q^2}{2}}$$

day 9 slide 22 for parrall plate

Since $C = \frac{Q}{|\Delta V|}$ $V = Ed$ (parrallel plate capicators)

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q|\Delta V| = \frac{1}{2}C|\Delta V|^2$$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\epsilon_0 A}{d}(Ed)^2 = \frac{\epsilon_0 E^2}{2}\cdot(Ad)$$

$= U_E \cdot$ volume

U_E = E field density $= \frac{1}{2}\epsilon_0 E^2$

Darmastein OH

Where does $V\frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}C\Delta V^2$
-all space all density

$$U_E = \int \overbrace{\frac{1}{2}\epsilon_0 E^2}^{U_E} \cdot \overbrace{2\pi r l\,dr}^{\text{around cylinder}}$$

guass law

$\left(\frac{1}{r}\right)^2 \cdot r = \frac{1}{r}$ so will have ln

$$\frac{1}{2}\epsilon_0 \frac{\lambda^2}{2^2\pi^2\epsilon_0^2}\cdot 2\pi l\int_0 \frac{1}{r^2}\,r\,dr$$

$$\boxed{U_E = \frac{\lambda^2 l}{4\pi\epsilon_0}\ln(r)}$$

not a function of r. ΔV between plates is fixed, because their distance is fixed!

$a < r < b$ ✗

~~Careful! You've been using $\frac{Q}{l}$ as λ. That means you're numerator reduces to~~

O.K. You're right. I didn't see the exponent. ✗✗ ◡

-3

Now compare w/ $\frac{1}{2} C (\Delta V)^2$

$$\frac{1}{2} \cdot \frac{2\pi\epsilon_0 L}{\ln(1/r)} \cdot \left(\frac{-\lambda \ln(r)}{2\pi\epsilon_0}\right)^2$$

$$\frac{1}{8} \cdot \frac{8\pi\epsilon_0 L}{\ln(r)} \cdot \frac{-\lambda^2 \ln^2(r)}{4\pi^2\epsilon_0^2}$$ -3

$$\boxed{U_E = \frac{L \cdot \lambda^2 \ln(r)}{4\pi\epsilon_0}}$$

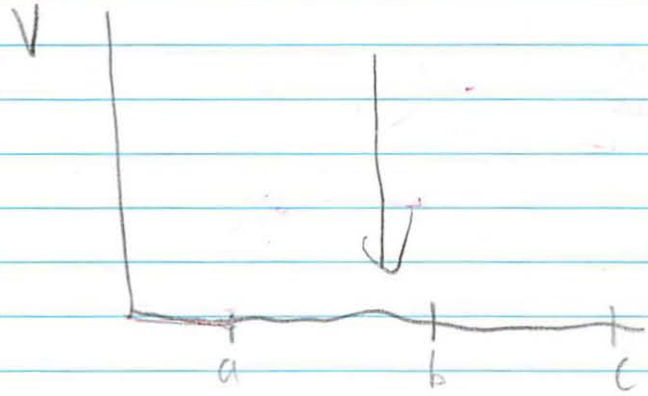
again, not a function of r, but I think this is just error propagation.

works!

-thanks to Domaskin's OH ✓

Good to see you're getting help when you're confused. You can also email 8.02.help@gmail.com any time you have a question.

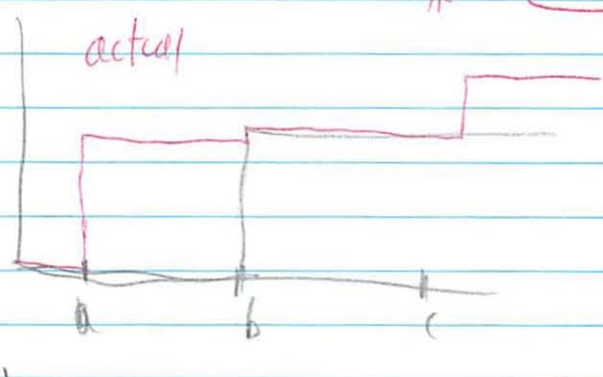
2A Prediction: potential diff vs time



(+) Charge creates high potential

a = insert (+) charge
b = rub charge producer on pail
c = remove charge potential

a, c = no change of the entire pail if not connected to ground
* ^ it was connected to ground *

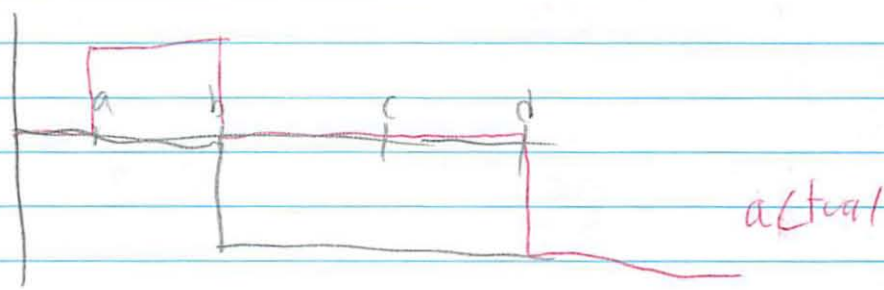


b = electrons flow from pail → ball trying to even out
pail left w/ (+) charge
thus higher pontential

2B

a = insert ⊕ ball
b = ground
c = remove ground
d = remove ⊕ ball

a = no effect on whole pail
b = ⊕ charges escape to ground, leaving ⊖ potential
c = no change on whole pail
d = the ⊖ charge rearrange, no charge on whole pail



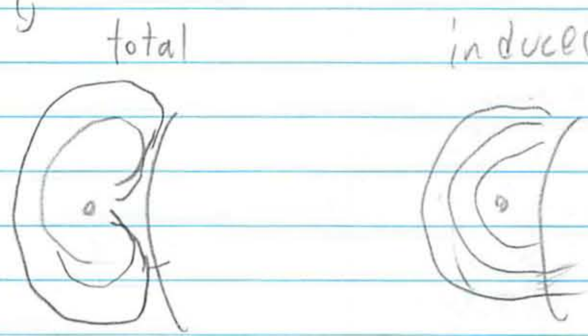
* outer pail connected to ground always *

3. Electric sheilding

a) know angle is 270° since charge is at botton
distance is bottom of pic (10)
know it is field lines not circle equipotential lines
know it is showing both fields because of te
conflicts at edges
T: better word

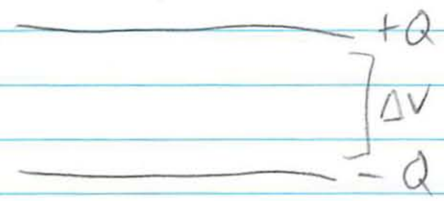
✓

b) know this is 180° and charge close to outer edge (6)
know it is equipotential since lines connect all of
te orange lines on te far side
it is showing only te induced field because
there are no lines going to te charge (pic is better)



c) 0° and ~10 to be at te edge
equipotential for same reason as b
total charge since charge is surranding charge

4. Parallel Plate Capacitor Moving



Source disconnected
half distance

- like PRS Qu from day 9 ~~or 10~~

V ↑ Q stays the same

$C = \frac{Q}{|\Delta V|} = \frac{\epsilon_0 A}{d}$ ⊕ at higher potential

$V = \frac{E}{D}$ ← same b/c Q same, ← $\frac{1}{2}$
↑ increases

Q is same b/c charge has no where to go
if connected to batt Q has somewhere to go
so Voltage stays same

a) ✓ Q same because charge has no where to go

b) Electric field is same b/c Q is the same ✓

memorize
c) Energy stored in E field = capitance ? ✗ halves −5
$C = \frac{\epsilon_0 A}{d}$ ← same, ← $\frac{1}{2}$ $C = \frac{1}{(\frac{1}{2})} = 2$ doubles

d) Potential $V = \frac{E}{d}$ ← same, ← $\frac{1}{2}$ $V = \frac{1}{\frac{1}{2}} = 2$ ✗ doubles halves −5

e) How much work did you do?

$$dW = dq\,\Delta V = dq = \frac{q}{C} = \frac{1}{C} q\, dq$$

$$W = \int dW = \frac{1}{C}\int_0^Q q\, dQ$$

-5

$$= \frac{1}{C}\frac{Q^2}{2}$$

I'm a little confused by what you did...

try $U = \frac{1}{2} C \Delta V^2 = \frac{1}{2} QV$.

then $W = \Delta U$. If the energy halves $\rightarrow W = -\frac{1}{2} U = -\frac{1}{4} Q \cdot V$. J

So $\frac{1}{2} \cdot \frac{1^2}{2} = \frac{1}{4}$ units? → Joules

$\frac{1}{4}$ what? $\frac{1}{4}$ Joules? that's not right. it depends on Q and V.

5. Human Capacitor
- grr estimate qv
- Hudson did in class

you are the ⊕ sphere
∞ is the ⊖ sphere

✓

need more practice w/ in upcoming days

$\frac{1}{\infty}$

$b \to \infty$

$$C = \frac{4\pi\epsilon_0}{\frac{1}{a}} = 4\pi\epsilon_0 a$$

$$\frac{1}{4\pi\epsilon_0} = 9 \cdot 10^9 \frac{Nm}{C}$$

$a = 1m$

pico faron $= 10^{-12} F$

$$C = \frac{1}{10^{10}\, N \cdot m^2} = 100\ pF$$

- ground yourself before touching PC

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

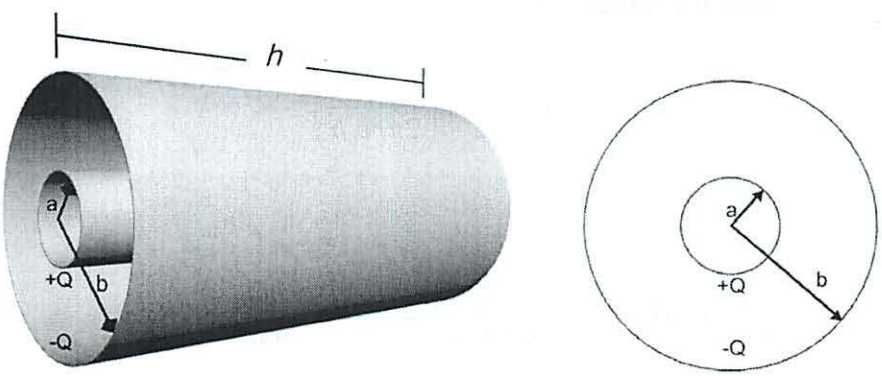
8.02 Spring 2010

Problem Set 4 Solution

Problem 1: Experiment: **Expt. 2: Faraday Ice Pail**

Capacitance of our Experimental Set-Up

Part 1 Consider two nested cylindrical conductors of height h and radii a & b respectively. A charge $+Q$ is evenly distributed on the outer surface of the pail (the inner cylinder), $-Q$ on the inner surface of the shield (the outer cylinder).



(a) Calculate the electric field between the two cylinders ($a < r < b$).

For this we use Gauss's Law, with a Gaussian cylinder of radius r, height l

$$\oiint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 2\pi r l E = \frac{Q_{inside}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\frac{Q}{h} l \;\Rightarrow\; E(r)_{a<r<b} = \frac{Q}{2\pi r \varepsilon_0 h}$$

(b) Calculate the potential difference between the two cylinders:

The potential difference between the outer shell and the inner cylinder is

$$\Delta V = V(a) - V(b) = -\int_b^a \frac{Q}{2\pi r' \varepsilon_0 h} dr' = -\frac{Q}{2\pi \varepsilon_0 h} \ln r' \Big|_b^a = \frac{Q}{2\pi \varepsilon_0 h} \ln\left(\frac{b}{a}\right)$$

(c) Calculate the capacitance of this system, $C = Q/\Delta V$

$$C = \frac{|Q|}{|\Delta V|} = \frac{|Q|}{\dfrac{|Q|}{2\pi\varepsilon_0 h}\ln\left(\dfrac{b}{a}\right)} = \frac{2\pi\varepsilon_0 h}{\ln\left(\dfrac{b}{a}\right)}$$

(d) Numerically evaluate the capacitance for your experimental setup, given:

$h \cong 15$ cm, $a \cong 4.75$ cm and $b \cong 7.25$ cm

$$C = \frac{2\pi\varepsilon_o h}{\ln\left(\frac{b}{a}\right)} = \frac{1}{2\cdot 9\times 10^9 \text{ m F}^{-1}} \frac{15 \text{ cm}}{\ln\left(\frac{7.25 \text{ cm}}{4.75 \text{ cm}}\right)} \cong 20 \text{ pF}$$

e) Find the electric field energy density at any point between the conducting cylinders. How much energy resides in a cylindrical shell between the conductors of radius r (with $a < r < b$), height h, thickness dr, and volume $2\pi rh\,dr$? Integrate your expression to find the total energy stored in the capacitor and compare your result with that obtained using $U_E = (1/2)C(\Delta V)^2$.

The total energy stored in the capacitor is

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{Q}{2\pi r\varepsilon_0 h}\right)^2$$

Then

$$dU = u_E dV = \frac{1}{2}\varepsilon_0\left(\frac{Q}{2\pi r\varepsilon_0 h}\right)^2 2\pi rh\,dr = \frac{hQ^2}{4\pi\varepsilon_0}\frac{dr}{r}$$

Integrating we find that

$$U = \int_a^b dU = \int_a^b \frac{hQ^2}{4\pi\varepsilon_0}\frac{dr}{r} = \frac{hQ^2}{4\pi\varepsilon_0}\ln(b/a).$$

From part d) $C = 2\pi\varepsilon_o h/\ln(b/a)$, therefore

$$U = \int_a^b dU = \int_a^b \frac{hQ^2}{4\pi\varepsilon_0}\frac{dr}{r} = \frac{hQ^2}{4\pi\varepsilon_0}\ln(b/a) = \frac{Q^2}{2C} = \frac{1}{2}C\Delta V^2$$

which agrees with that obtained above.

Part 2 Experimental Predictions

A. Prediction: Charging by Contact

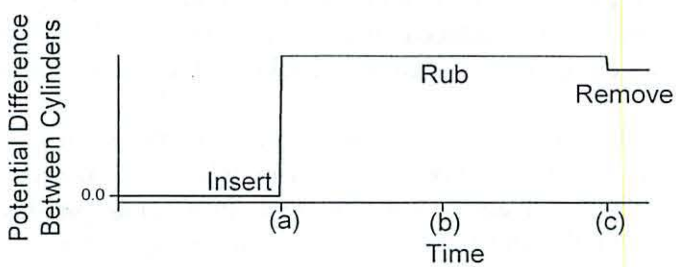
Sketch your prediction for the graph of potential difference vs. time for part 2 of this experiment. Indicate the following events on the time axis:

(a) Insert positive charge producer into pail

(b) Rub charge producer against inner surface of pail

(c) Remove charge producer

Solution:



I picture the potential dropping a little as you remove the charge producer because it is likely that you still have some charge on the producer when you remove it (the transfer wasn't perfect).

B. Prediction: Charging by Induction

Sketch your prediction for the graph of potential difference vs. time for part 3 of this experiment. Indicate the following events on the time axis:

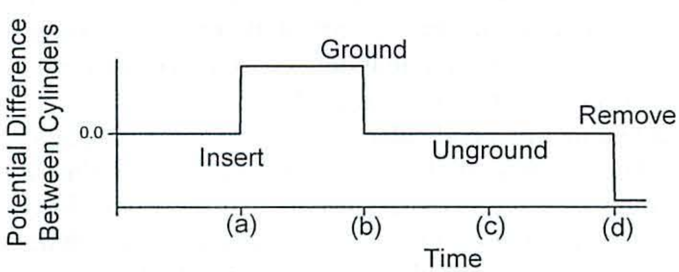
(a) Insert positive charge producer into pail

(b) Ground pail to shield

(c) Remove ground contact between pail and shield

(d) Remove charge producer

Solution:



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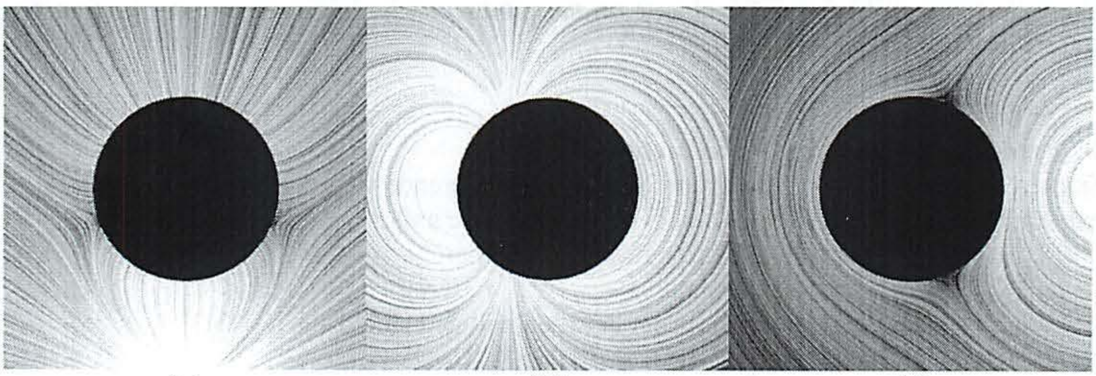
Part of the lab this week involves shielding. We have a visualization to help you better understand this. Open it up:

http://web.mit.edu/viz/EM/visualizations/electrostatics/ChargingByInduction/shielding/shielding.htm

and play with it for a while. You can move the charge around the outside of the shield (or even inside) using the parameters "radius pc" and "angle pc." You can change which field you are looking at – the total field, just the field of the external charge ("Free charge") or just the field of the induced charge (on the shield). You can visualize it with grass seeds or display equipotential streaks by clicking "Electric Potential."

Below are three captured images. I've blanked out the center so that you can't see what is going on inside the conductor. For each describe where the charge is (ROUGH angle and distance), tell whether I am looking at field lines (grass seeds) or equipotential streaks ("Electric Potential") and indicate whether I am doing so for the total field, or just the external or induced field. Also briefly explain HOW you know this (not just "I looked around until I was able to repeat the pattern").



(a) (b) (c)

(a) These are electric fields lines (grass seeds) of the entire field. We can tell because they come in perpendicular to the equipotential surface of the conductor, which is only true for the total field (not the individual parts). The charge is clearly below the conductor ($\theta = 270°$) and just off the screen ($R = 11.5$).

(b) Here the lines are neither perpendicular nor parallel to the conductor, so it can't be for the entire field. They loop around, looking like a dipole, so they are associated with the induced charges, not the external charge. Are they field lines or equipotentials though? Without seeing the center this is non-trivial. If the charge were below, the field lines would look very much like this. But since the left and right "lobes" are not symmetric, it must be equipotentials created by a charge on the left ($R = 6$, $\theta = 180°$).

(c) This one is easier. The lines wrap around the conductor, so they are clearly equipotential lines associated with the entire field. The charge is on the right ($R=11$, $\theta=0°$)

Problem 4: Parallel Plate Capacitor

A parallel-plate capacitor is charged to a potential V_0, charge Q_0 and then disconnected from the battery. The separation of the plates is then halved. What happens to

(a) the charge on the plates?
No Change. We aren't attached to a battery, so the charge is fixed.

(b) the electric field?
No Change. The charge is constant so, in the planar geometry, so is the field.

(c) the energy stored in the electric field?
Halves. The volume in which we have field halves, so the energy does too.

(d) the potential?
Halves. $V = E\,d$, so if d halves, so does V

(e) How much work did you do in halving the distance between the plates?
The work done is the change in energy. Energy, given the charge and potential, is:

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV$$

The energy halves, so the change is half the initial energy: $\boxed{W = \Delta U = -\tfrac{1}{4}Q_0V_0}$

Notice the sign – you did negative work bringing the plates together because that is the way they naturally want to move; the field did positive work.

Problem 5: Human Capacitor

What, approximately, is the capacitance of a typical MIT student? Check out the exhibit in Strobe Alley (4^{th} floor of building 4) for a hint or just to check your answer.
There are lots of ways to do this. The note in strobe alley tells us to use a cylinder of dimensions such that when filled with water it would be your mass. Personally I feel more like a sphere, of which we have already calculated the capacitance in class. All I need to know is my radius, a. As a first approximation, probably it's a meter (I'm certainly less than 10 m and more than 10 cm). So my capacitance should be about:

$$C \approx 4\pi\varepsilon_0 a \approx 1\,\text{m}/9\times10^9\ \text{F}^{-1}\ \text{m} \approx 100\ \text{pF}$$

Not a bad approximation – according to the measurement I'm really ~170 pF.
Note that for simplicity I used the value for k_E rather than ε_0. Always look for ways to recombine constants into things that you know.

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Department of Physics
8.02

Experiment 2 *Solutions*: Faraday Ice Pail

EXPERIMENTAL SETUP

1. Download the LabView file from the web and save the file to your desktop (right click on the link and choose "Save Target As" to the desktop. Overwrite any file by this name that is already there). Start LabView by double clicking on this file.
2. Using the multi-pin cable, connect the Charge Sensor to Analog Channel A on the 750 Interface. The cable runs from the left end of the sensor (in Fig. 5) to Channel A.
3. Connect the lead assembly to the BNC port on the Charge Sensor (right end of the sensor in Fig. 5). Line up the connector on the end of the cable with the pin on the BNC port. Push the connector onto the port and twist it clockwise about one-quarter turn until it clicks into place. Set the Charge Sensor gain to 1x.
4. Connect the charge sensor input lead (red alligator clip) to the pail (the inner wire mesh cylinder), and the ground lead (black alligator clip) to the shield (the outer wire mesh cylinder).

MEASUREMENTS

Important Notes:

The charge producers are delicate. When rubbing them together do so briskly but gently.

Each experiment should begin with completely discharged cylinders. **To discharge them, ground the pail by touching both it and the shield at the same time with a conductor (e.g. the finger of one hand). You also will always want to zero the charge sensor before starting by pressing the "Zero" button.**

Finally, note that the amount of charge measured is small and hence there will be fluctuations in the signal as well as small features due to the person holding the charge producers. In answering questions focus on the BIG features (sign of potential, ...) not the noise.

Part 1: Polarity of the Charge Producers

1. Ground the pail and zero the charge sensor
2. Start recording data. (Press the green "Go" button above the graph).
3. Rub the blue and white surfaces of the charge producers together several times.
4. Without touching the pail, lower the white charge producer into the pail.
5. Remove the white charge producer and then lower in the blue charge producer

Question 1 (Don't forget to submit answers in the software!):

What are the polarities of the white and the blue charge producers?

The white producer is positive, the blue producer negative

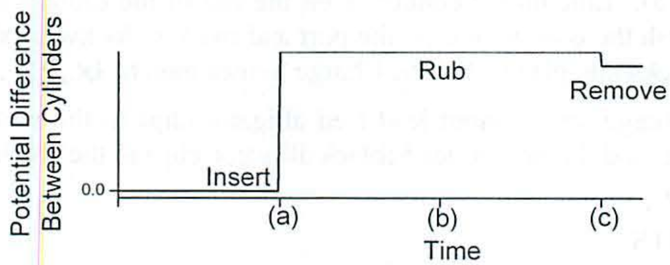
Part 2: Charging By Contact

Part 2A: Using the White Charge Producer

1. Ground & zero; Start recording; Rub the producers
2. Lower the *white* charge producer into the pail
3. Rub the charge producer against the inner surface of the pail
4. Remove the charge producer

Question 2: Sketch the plot of voltage vs. time and indicate the charge on the inner and outer surfaces of the pail after each of the above steps

Answer:



Charge on inner & outer surfaces of the inner cylinder (indicate sign, and use a variable like q for non-zero magnitudes)

After Step 1:	$Q_{inner} = 0$	$Q_{outer} = 0$
After Step 2:	$Q_{inner} = -q$	$Q_{outer} = q$
After Step 3:	$Q_{inner} = -0.1\ q$	$Q_{outer} = q$
After Step 4:	$Q_{inner} = 0$	$Q_{outer} = 0.9\ q$

Part 2B: Using the Blue Charge Producer

1. Ground & zero; Start recording; Rub the producers
2. Lower the *blue* charge producer into the inner cylinder
3. Rub the charge producer against the inner surface of the inner cylinder
4. Remove the charge producer

Question 3:

What happens to the charge on the pail when you rub it with the blue charge producer?

You transfer negative charge to the pail, which neutralizes some of the positive charge that had been attracted there by the negative charge.

Part 3: Charging By Induction

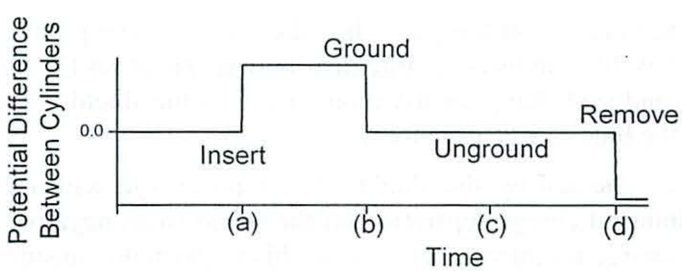
Part 3A: Using the White Charge Producer

1. Ground & zero; Start recording; Rub the producers
2. Lower the *white* charge producer into the pail, without touching it
3. Ground the pail by connecting it to the shield with your finger
4. Remove the ground connection (your finger)
5. Remove the charge producer

Question 4:

Sketch the plot of voltage vs. time and indicate the charge on the inner and outer surfaces of the pail after each of the above steps

Answer:



Charge on inner & outer surfaces of the inner cylinder (indicate sign, and use a variable like q for non-zero magnitudes)

After Step 1:	$Q_{inner} = 0$	$Q_{outer} = 0$
After Step 2:	$Q_{inner} = -q$	$Q_{outer} = q$
After Step 3:	$Q_{inner} = -q$	$Q_{outer} = 0$
After Step 4:	$Q_{inner} = -q$	$Q_{outer} = 0$
After Step 5:	$Q_{inner} = 0$	$Q_{outer} = -q$

3B: Using the Blue Charge Producer

1. Ground & zero; Start recording; Rub the producers
2. Lower the *white* charge producer into the pail, without touching it
3. Ground the pail by connecting it to the shield with your finger
4. Remove the ground connection (your finger)
5. Remove the charge producer

Question 5:

What happens to the charge on the pail when you do the above steps?

You end up inducing a positive charge on the inner pail (it is pulled over through your finger from the shield when the negative blue producer is in the pail).

Part 4: Testing the shield

1. Ground & zero; Start recording; Rub the producers
2. Bring the *white* charge producer to just outside the shield (the outer cylinder) ***Do Not Touch it!***
3. Repeat, bringing the *blue* charge producer just outside the shield.

Question 6:

What happens to the charge on the pail when the white charge producer is placed just outside the shield? Will an induced charge distribution appear on the pail? Explain your reasoning. Will an induced charge distribution appear on the shield? Are we sensitive to this? What about the blue charge producer?

Because the pail is shielded by the shield, almost no charge will appear on the pail. There will be an induced charge separation on the shield (with negative charges running towards the white charge producer), but because this is all on the outside of the shield we are not at all sensitive to it in our measurement. The same is true of the blue charge producer.

Further Questions (for experiment, thought, future exam questions...)

- What happens if we repeat the above measurements with the ground (black clip) attached to the pail and the red clip attached to the shield? Does anything change aside from the sign of the voltage difference?
- What happens if in part 2 we touch the charge producer to the outside of the pail rather than the inside?
- What happens if we place the charge producer between the pail & shield rather than inside the pail?
- What happens if we put both the white & blue charge producers inside the pail together (not touching, just both inside). Is the cancellation exact? Should it be?
- What if in part 2 we touch the white producer and then the blue producer to the pail? What if we touch the white producer, then recharge it and touch again? Doing this repeatedly, is there a difference between touching the inside of the pail and the outside of the pail?

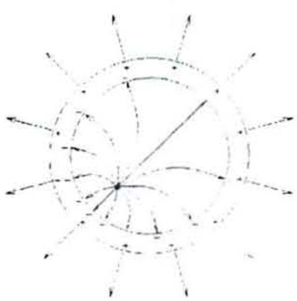
3/1

Topics: Electrostatic Shielding
Related Reading: Course Notes: Sections 4.3-4.4; 5.5, 5.9, 5.10.2
Experiments: (2) Faraday Ice Pail

Topic Introduction

Today we return to our discussion of conductors & capacitors, now focusing on the idea of electrostatic shielding by conductors. This is also the focus of our next lab, the Faraday Ice Pail experiment.

Conductors & Shielding



Last class we noted that conductors were equipotential surfaces, and that all charge moves to the surface of a conductor so that the electric field remains zero inside. Because of this, a hollow conductor very effectively separates its inside from its outside. For example, when charge is placed inside of a hollow conductor an equal and opposite charge moves to the inside of the conductor to shield it. This leaves an equal amount of charge on the outer surface of the conductor (in order to maintain neutrality). How does it arrange itself? As shown in the picture at left, the charges on the outside don't know anything about what is going on inside the conductor. The fact that the electric field is zero in the conductor cuts off communication between these two regions. The same would happen if you placed a charge outside of a conductive shield – the region inside the shield wouldn't know about it. Such a conducting enclosure is called a Faraday Cage, and is commonly used in science and industry in order to eliminate the electromagnetic noise ever-present in the environment (outside the cage) in order to make sensitive measurements inside the cage.

oh cool – charges auto redistribute themselves

Experiment 2: Faraday Ice Pail

Preparation: Read pre-lab reading

In this lab we will study electrostatic shielding, learning how charges move on conductors when other charges are brought near them. The idea of the experiment is quite simple. We will have two concentric cylindrical cages, and can measure the potential difference between them. We can bring charges (positive or negative) into any of the three regions created by these two cylindrical cages. And finally, we can connect either cage to "ground" (e.g. the Earth), meaning that it can pull on as much charge as it wants to respond to your moving around charges. The point of the lab is to get a good understanding of what the responses are to you moving around charges, and how the potential difference changes due to these responses.

Dielectrics

A dielectric is a piece of material that, when inserted into an electric field, has a reduced electric field in its interior. Thus, if a dielectric is placed into a capacitor, the electric field in that capacitor is reduced, as is hence the potential difference between the plates, thus increasing the capacitor's capacitance (remember, $C \equiv Q/|\Delta V|$). The effectiveness of a dielectric is summarized in its "dielectric constant" κ. The larger the dielectric constant, the more the field is reduced (paper has κ=3.7, Pyrex κ=5.6). Why do we use dielectrics? Dielectrics increase capacitance, which is something we frequently want to do, and can also prevent breakdown inside a capacitor, allowing more charge to be pushed onto the plates before the capacitor "breaks down" (before charge jumps from one plate to the other).

Dielectrics ↓ E $\quad Q = CV$

↑ Capacitance

~~Voltage~~

↑ capacitance same V ↑ charge capacity/charge

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Department of Physics
8.02

Experiment 2: Faraday Ice Pail

OBJECTIVES

1. To explore the charging of objects by friction and by contact.
2. To explore the charging of objects by electrostatic induction.
3. To explore the concept of electrostatic shielding.

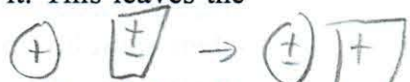
PRE-LAB READING

INTRODUCTION

When a charged object is placed near a conductor, electric fields exert forces on the free charge carriers in the conductor which cause them to move. This process occurs rapidly, and ends when there is no longer an electric field inside the conductor ($\mathbf{E}_{\text{inside conductor}}=0$). The surface of the conductor ends up with regions where there is an excess of one type of charge over the other. For example, if a positive charge is placed near a metal, electrons will move to the surface nearest the charge, leaving a net positive charge on the opposite surface[1]. This charge distribution is called an *induced charge distribution.* The process of separating positive from negative charges on a conductor by the presence of a charged object is called *electrostatic induction.* but still total = 0

Michael Faraday used a metal ice pail as a conducting object to study how charges distributed themselves when a charged object was brought inside the pail. Suppose we lower a positively charged metal ball into the pail ***without touching it to the pail***. When we do this, positive charges move as far away from the ball as possible – to the outer surface of the pail – leaving a net negative charge on the inner surface. If at this point we provide some way for the positive charges to flow away from the pail, for example by touching our hand to it, they will run off through our hand. If we then remove our hand from the pail and then remove the positively charged metal ball from inside the pail, the pail will be left with a net negative charge. This is called *charging by induction.*

In contrast, if we touch the positively charged ball to the uncharged pail, electrons flow from the pail into the ball, trying to neutralize the positive charge on it. This leaves the pail with a net positive charge. This is called *charging by contact.*



Finally, when a positively charged ball approaches the ice pail from outside of the pail, charges will redistribute themselves on the outside surface of the pail and will exactly cancel the electric field inside the pail. This is called *electrostatic shielding.*

still somewhat (+) - shortage of (-) ... sxs of both

??? read more about

[1] We will typically say that "positive charge flows outward" even though in metals it's really electrons moving inward. This is a completely equivalent way of thinking about it for our purposes.

its the electrons that move

* keep clear what moves where

You will investigate all three of these phenomena—charging by induction, charging by contact, and electrostatic shielding—in this experiment.

The Details: Gauss's Law

In the above situations, the excess charge on the conductor resides entirely on the surface, a fact that may be explained by Gauss's Law. Gauss's Law[2] states that the electric flux through any closed surface is proportional to the charge enclosed inside that surface,

$$\oiint_{\substack{\text{closed}\\ \text{surface}}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{enc}}}{\varepsilon_0}. \tag{2.1}$$

Consider a mathematical, closed Gaussian surface that is *inside* the ice pail:

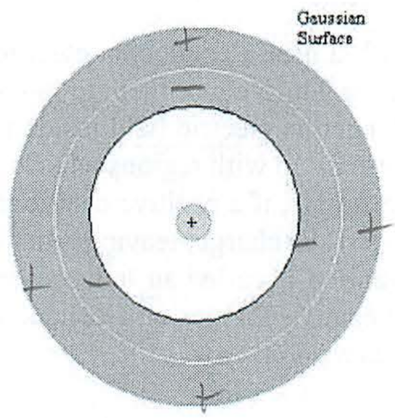


Figure 1 Top View of Gaussian surface for the Faraday Ice Pail (a thick walled cylinder)

Once static equilibrium has been reached, the electric field inside the conducting metal walls of the ice pail is zero. Since the Gaussian surface is in a conducting region where there is zero electric field, the electric flux through the Gaussian surface is zero. Therefore, by Gauss's Law, the net charge inside the Gaussian surface must be zero. For the Faraday ice pail, the positively charged ball is inside the Gaussian surface. Therefore, there must be an additional induced negative charge on the inner surface of the ice pail that exactly cancels the positive charge on the ball. It must reside on the surface because we could make the same argument with any Gaussian surface, including one which is just barely outside the inner surface. Since the pail is uncharged, by charge conservation there must be a positive induced charge on the pail which has the same magnitude as the negative induced charge. This positive charge must reside outside the Gaussian surface, hence on the outer surface of the ice pail.

Note that the electric field in the hollow region inside the ice pail is not zero due to the presence of the charged ball, and that the electric field outside the pail is also not zero, due to the positive charge on its outer surface.

[2] For more details on Gauss's Law, see Chapter 4 of the *Course Notes*, Section 4.3 for info on conductors.

Now suppose the ice pail is connected to a large conducting object ("ground"):

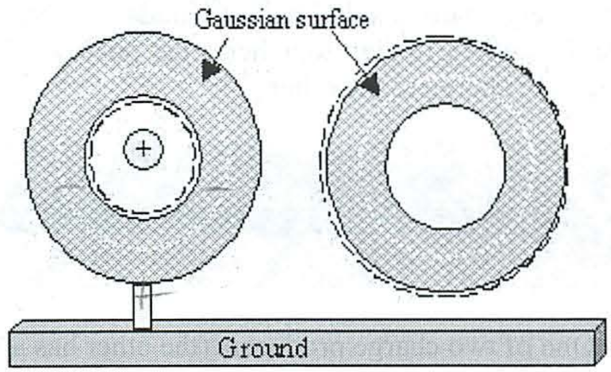


Figure 2 Grounding the ice pail (left) and after removing the ground & ball (right)

Now the positive charges that had moved to the surface of the ice pail can get even further away from each other by flowing into the ground. Now that there are no charges on the outer surface of the pail, the electric field outside the pail is zero and the pail is at the same "zero" potential as the ground (and infinity). If the wire to ground is then disconnected, the pail will be left with an overall negative charge. Once the positively charged ball is removed, this negative charge will redistribute itself over the outer surface of the pail.

Finally, when a charged ball approaches the ice pail from outside of the pail, charges will redistribute themselves *on the outside surface* of the pail while the electric field inside the pail will remain zero, cut-off from any knowledge of what is going on outside by the enforced zero electric field inside the conductor. This effect is called shielding or "screening" and explains popular science demonstrations in which a person sits safely inside a cage while an enormous voltage is applied to the cage. This same effect explains why metal boxes are used to screen out undesirable electric fields from sensitive equipment.

APPARATUS

1. Ice Pail

Our primary apparatus consists of two concentric wire-mesh cylinders. The inner cylinder (the "pail") is electrically isolated by three insulating rods. The outer cylinder (the "shield") will be attached to ground – charge can flow to or from it as necessary. This cylinder will act both as a screen to eliminate the effect of any external charges and other external fields and as a "zero potential" point, relative to which you will measure the potential of the pail.

Figure 3 The Ice Pail

2. Charge Producers

To replace the positively charged metal ball of Faraday's experiment, you will use charge producers (Figure 4). When rubbed together a net positive charge will move to one of them and a net negative charge to the other.

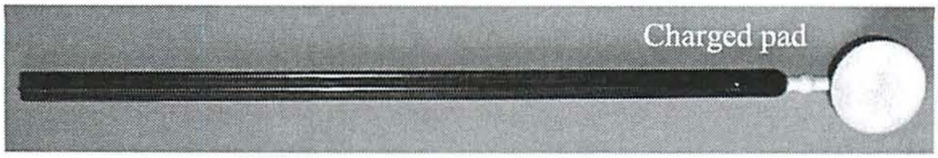


Figure 4 One of two charge producers (the other has a blue charged pad)

3. Charge Sensor

The Charge Sensor does not directly measure charge, but instead measures the voltage difference between its positive (red) and negative (black) leads. Furthermore, it connects the black lead to ground, meaning that as much charge can flow into or out of that lead as is necessary to keep it at "zero potential" (ideally the same voltage as at infinity).

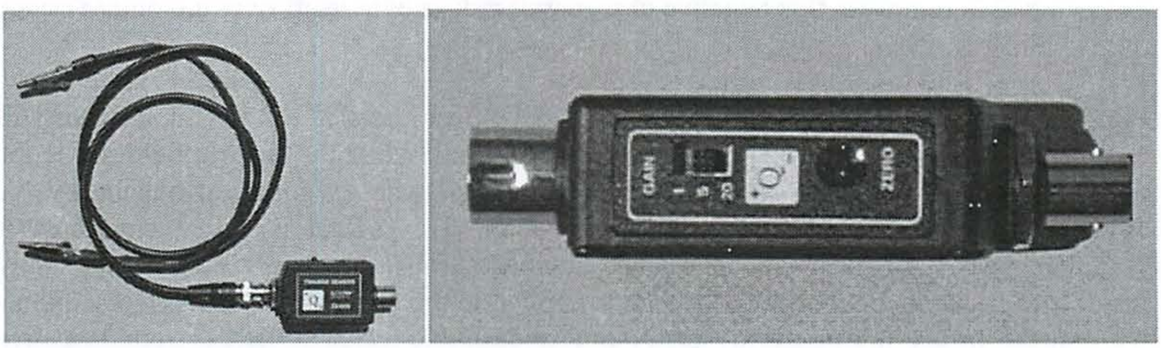


Figure 5 Charge Sensor – measures voltage difference between its red and black leads. Left: Shown attached to the lead assembly. Right: The gain switch (used to amplify small signals) should be set at 1. The zero button sets the output signal to zero.

The red lead is free to be at any potential, although by pushing the "zero" button on the sensor (Fig. 5, right), it too can be attached to ground (the potential difference between the red and black leads is set to zero).

Even though this is really a potential difference sensor, we none-the-less call it a "Charge Sensor" because the voltages measured arise from the presence of charges on the ice pail.

GENERALIZED PROCEDURE

This lab consists of four main parts. In each you will measure the voltage between the inner and outer cylinder to determine what is happening on the inner cylinder.

Part 1: Determine Polarity of (Sign of Charge on) Charge Producers
Here you will lower the charge producers into the center of the pail (the inner cylinder) and determine which producer is positively charged and which is negatively charged

Part 2: Charging by Contact
You will now rub the charge producer against the inner surface of the pail and see if the charge is transferred to it.

Part 3: Charging by Induction
In this part you will not let the charge producer touch the pail, but will instead briefly ground the pail by connecting it to the shield (the outer cylinder) while the charge producer is inside. Then you will remove the charge producer and observe the induced charge on the pail.

Part 4: Electrostatic Shielding
In this part you will measure the effects of placing a charge producer outside of the grounded shield.

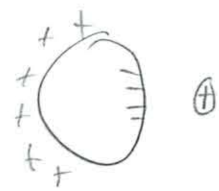
END OF PRE-LAB READING

$E = \frac{F}{q}$ $\vec{E}$ = same dir $\vec{F}$ if q ⊕ at that pos

Behind shellding - the sheild has ⊕ ⊖ needs to be taken into account

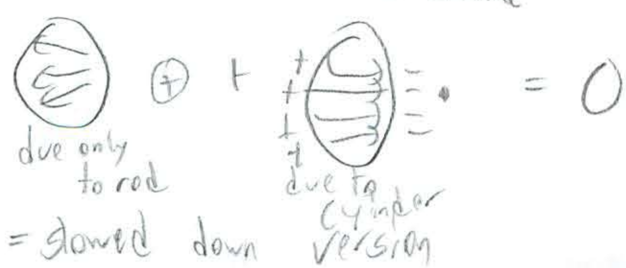
$F = \frac{kqQ}{r^2}$

$E = \frac{F}{q} = \frac{kQ}{r^2}$ - superposition



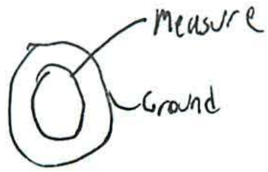
So inside (op) cal the field sums to 0

negitive of rod alone



Sometimes reading about experiment better - more relaxing focuses on the why

paper = slowed down version



Measuring potential surface between the 2 cylinders

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

1. Download the LabView file from the web and save the file to your desktop (right click on the link and choose "Save Target As" to the desktop. Overwrite any file by this name that is already there). Start LabView by double clicking on this file.
2. Using the multi-pin cable, connect the Charge Sensor to Analog Channel A on the 750 Interface. The cable runs from the left end of the sensor (in Fig. 5) to Channel A.
3. Connect the lead assembly to the BNC port on the Charge Sensor (right end of the sensor in Fig. 5). Line up the connector on the end of the cable with the pin on the BNC port. Push the connector onto the port and twist it clockwise about one-quarter turn until it clicks into place. Set the Charge Sensor gain to 1x.
4. Connect the charge sensor input lead (red alligator clip) to the pail (the inner wire mesh cylinder), and the ground lead (black alligator clip) to the shield (the outer wire mesh cylinder).

MEASUREMENTS

Important Notes:

The charge producers are delicate. When rubbing them together do so briskly but gently.

Each experiment should begin with completely discharged cylinders. **To discharge them, ground the pail by touching both it and the shield at the same time with a conductor (e.g. the finger of one hand). You also will always want to zero the charge sensor before starting by pressing the "Zero" button.**

Finally, note that the amount of charge measured is small and hence there will be fluctuations in the signal as well as small features due to the person holding the charge producers. In answering questions focus on the BIG features (sign of potential, …) not the noise.

Part 1: Polarity of the Charge Producers

1. Ground the pail and zero the charge sensor
2. Start recording data. (Press the green "Go" button above the graph).
3. Rub the blue and white surfaces of the charge producers together several times.
4. Without touching the pail, lower the white charge producer into the pail.
5. Remove the white charge producer and then lower in the blue charge producer

Question 1 (Don't forget to submit answers in the software!):

What are the polarities of the white and the blue charge producers?

Note: There may be some variations in this from group to group.

white fuzzy = (+)
blue smooth = (−)

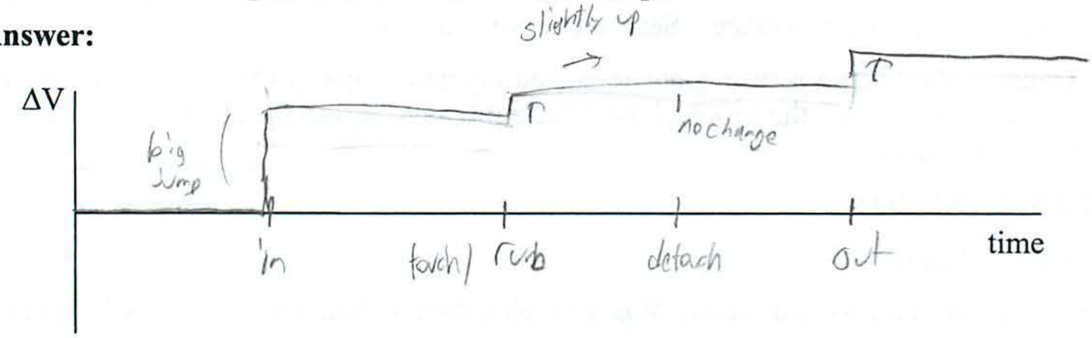
Part 2: Charging By Contact

Part 2A: Using the White Charge Producer

1. Ground & zero; Start recording; Rub the producers
2. Lower the *white* charge producer into the pail
3. Rub the charge producer against the inner surface of the pail
4. Remove the charge producer

Question 2: Sketch the plot of voltage vs. time and indicate the charge on the inner and outer surfaces of the pail after each of the above steps

Answer:



Charge on inner & outer surfaces of the inner cylinder (**indicate sign, and use a variable like q for non-zero magnitudes – do NOT simply record numerical values)**

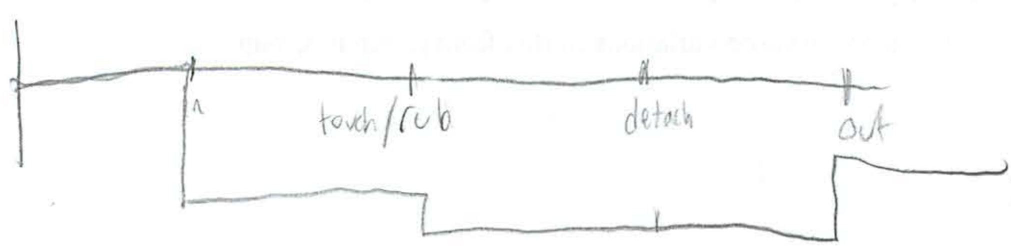
After Step 1:	Q_{inner} = 0	Q_{outer} = 0
After Step 2:	Q_{inner} = −q	Q_{outer} = +q
After Step 3:	Q_{inner} = −q	Q_{outer} = +q
After Step 4:	Q_{inner} = −q	Q_{outer} = +q

Part 2B: Using the Blue Charge Producer

1. Ground & zero; Start recording; Rub the producers
2. Lower the *blue* charge producer into the inner cylinder
3. Rub the charge producer against the inner surface of the inner cylinder
4. Remove the charge producer

Question 3:

What happens to the charge on the pail when you rub it with the blue charge producer?



always did labs in school haphazardly

Part 3: Charging By Induction

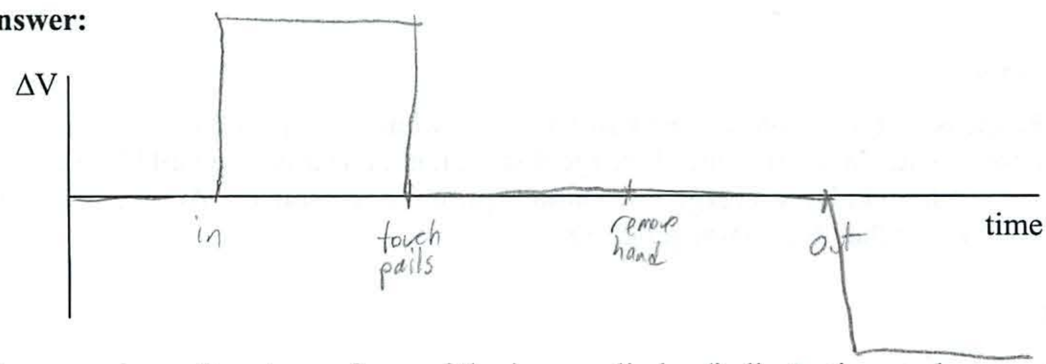
Part 3A: Using the White Charge Producer

1. Ground & zero; Start recording; Rub the producers
2. Lower the *white* charge producer into the pail, without touching it
3. Ground the pail by connecting it to the shield with your finger
4. Remove the ground connection (your finger)
5. Remove the charge producer

Question 4:

Sketch the plot of voltage vs. time and indicate the charge on the inner and outer surfaces of the pail after each of the above steps

Answer:



Charge on inner & outer surfaces of the inner cylinder (indicate sign, and use a variable like q for non-zero magnitudes – do NOT simply record numerical values)

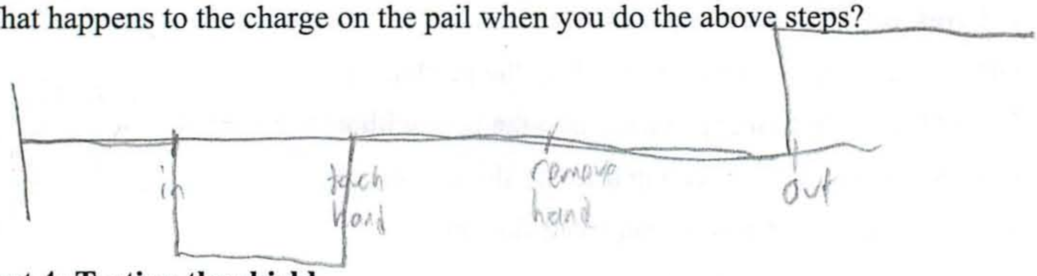
After Step 1:	Q_{inner} = 0	Q_{outer} = 0
After Step 2:	Q_{inner} = −q	Q_{outer} = +q
After Step 3:	Q_{inner} = −q	Q_{outer} = 0) grounded
After Step 4:	Q_{inner} = −q	Q_{outer} = 0
After Step 5:	Q_{inner} = +q	Q_{outer} = −q

3B: Using the Blue Charge Producer

1. Ground & zero; Start recording; Rub the producers
2. Lower the *blue* charge producer into the pail, without touching it
3. Ground the pail by connecting it to the shield with your finger
4. Remove the ground connection (your finger)
5. Remove the charge producer

Question 5:

What happens to the charge on the pail when you do the above steps?



Part 4: Testing the shield

1. Ground & zero; Start recording; Rub the producers
2. Bring the *white* charge producer to just outside the shield (the outer cylinder) ***Do Not Touch it!***
3. Repeat, bringing the *blue* charge producer just outside the shield.

Question 6:

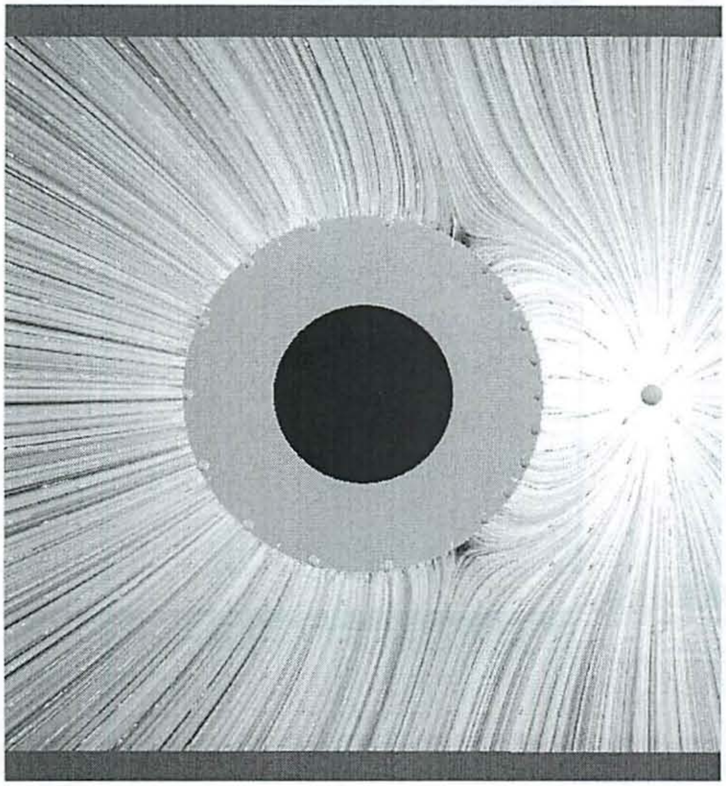
What happens to the charge on the pail when the white charge producer is placed just outside the shield? Will an induced charge distribution appear on the pail? Explain your reasoning. Will an induced charge distribution appear on the shield? Are we sensitive to this? What about the blue charge producer?

No negligible change - no induced charge distribution,
The sheild makes every thing inside look like 0
No induced charge distribution on sheild since it is grounded
Same for blue

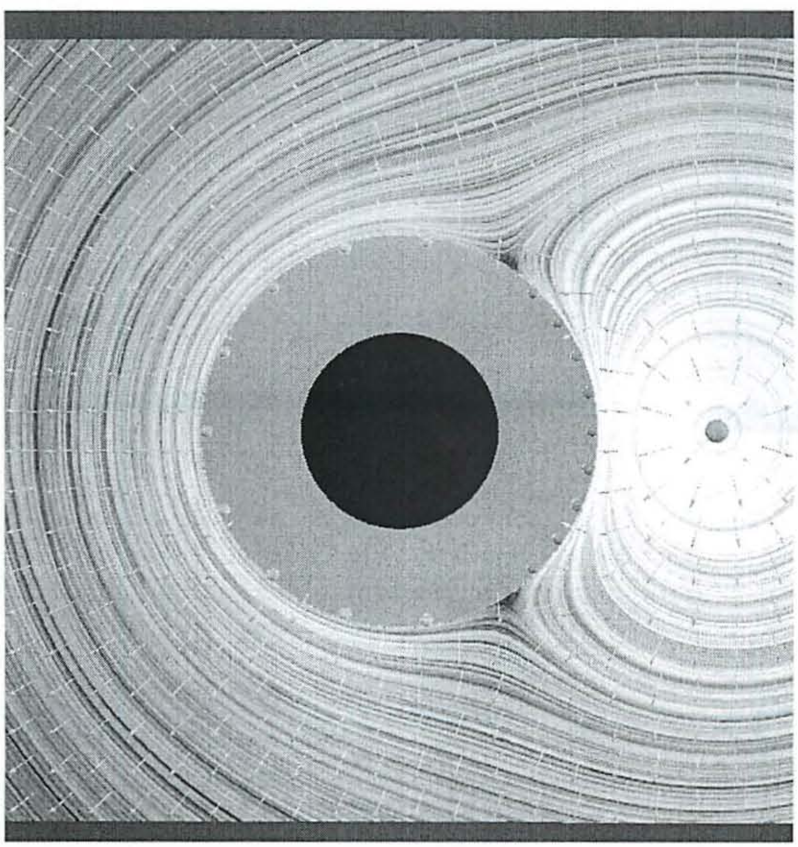
Further Questions (for experiment, thought, future exam questions...)

- What happens if we repeat the above measurements with the ground (black clip) attached to the pail and the red clip attached to the shield? Does anything change aside from the sign of the voltage difference?
- What happens if in part 2 we touch the charge producer to the outside of the pail rather than the inside?
- What happens if we place the charge producer between the pail & shield rather than inside the pail?
- What happens if we put both the white & blue charge producers inside the pail together (not touching, just both inside). Is the cancellation exact? Should it be?
- What if in part 2 we touch the white producer and then the blue producer to the pail? What if we touch the white producer, then recharge it and touch again? Doing this repeatedly, is there a difference between touching the inside of the pail and the outside of the pail?

all lines 2/28



field lines



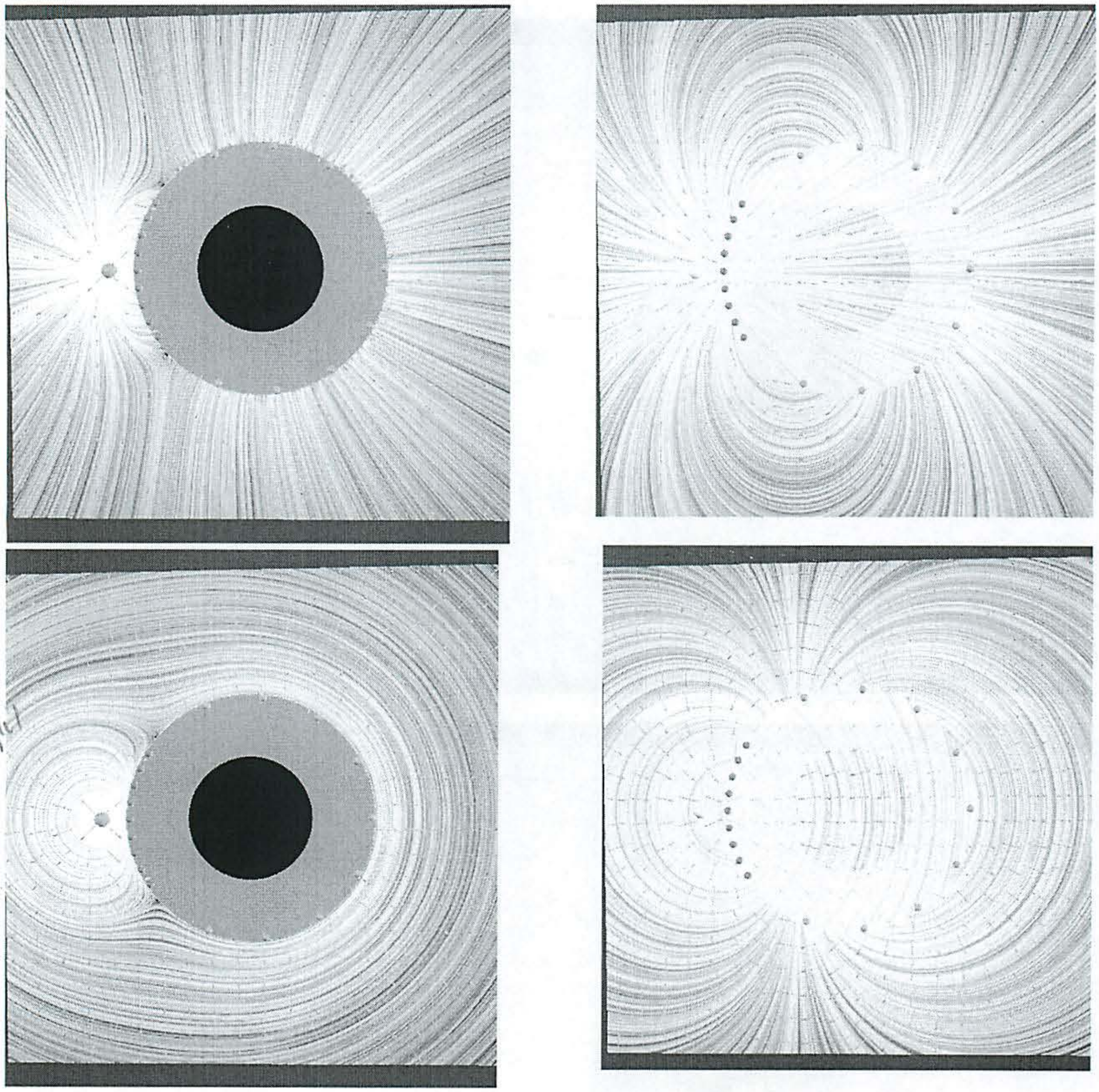
equipotential

all

induced

field lines

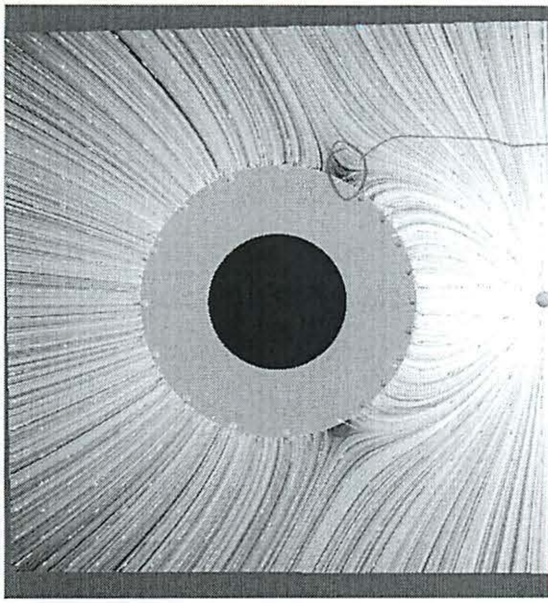
equipotential



all

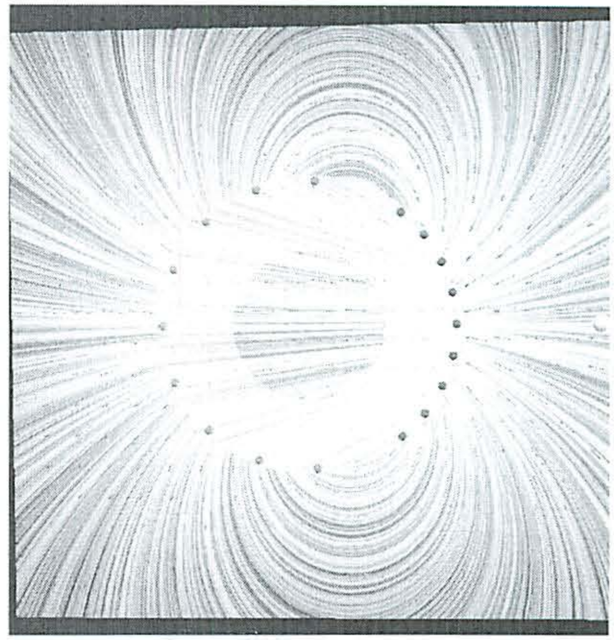
induced

Field

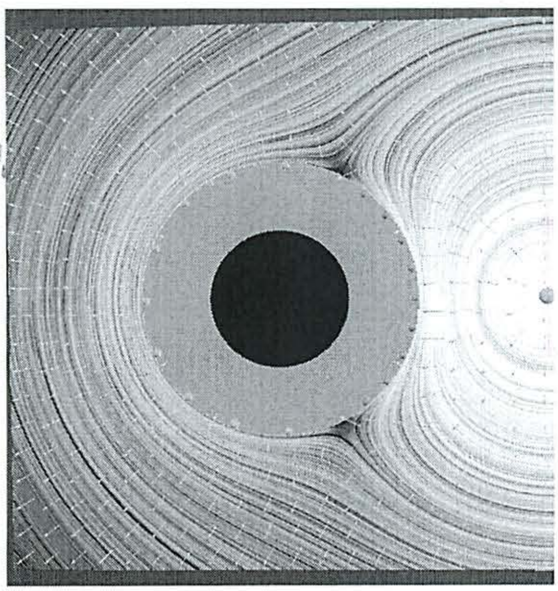


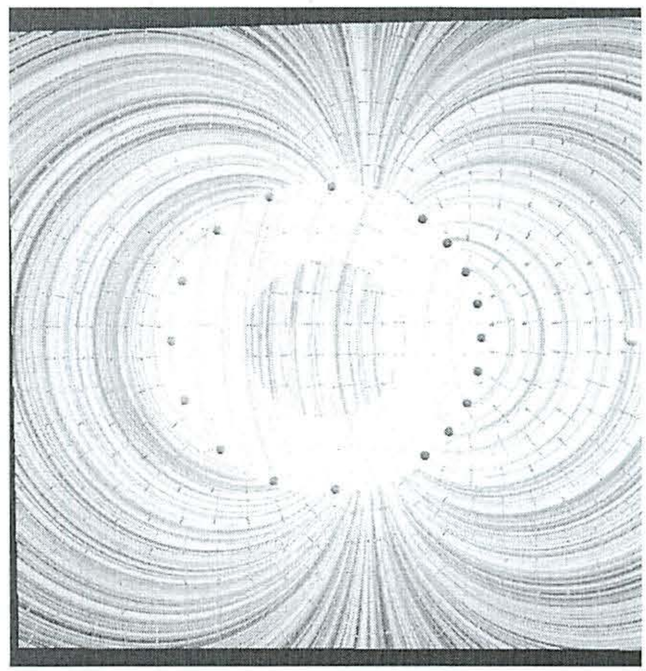
conflict

e lines
Charge
to near



equipotential





T ignores charges

Class 11: Outline

Hour 1:
- Last Time: Conductors
- Conductors as Shields
- Expt. 2: Faraday Ice Pail

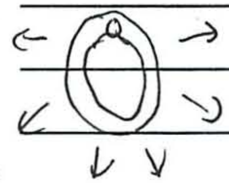
Hour 2:
- Capacitors & Dielectrics

Conductor - electrons free to move

PRS

neutral conductor & Completely screens inside
Outside non-0 + does not change
E field
- conductor's charge = +Q

$E_{outer} = \frac{\sigma_0}{\varepsilon_0} = \frac{Q}{Area\,\varepsilon_0}$



$E = \frac{\sigma}{\varepsilon_0}$ ⊥ to surface

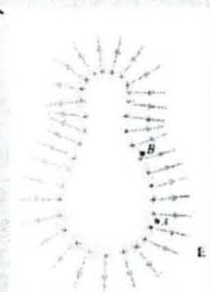
σ inner not uniform
- auto adjusts

Last Time: Conductors

Conductors in Equilibrium

Conductors are equipotential objects:
1) E = 0 inside (Does V=0?)
2) E perpendicular to surface
3) Net charge inside is 0
4) Excess charge on surface

$E = \sigma / \varepsilon_0$



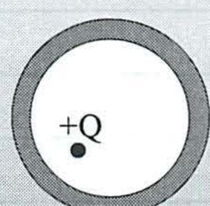
whole cage at 5 Volts
charges move around as long as there is a voltage
will even out quickly

Conductors as Shields

PRS Question: Point Charge Inside Conductor

PRS: Point Charge in Conductor

A point charge +Q is placed inside a neutral, hollow, spherical conductor. As the charge is moved around *inside*, the electric field ***outside***

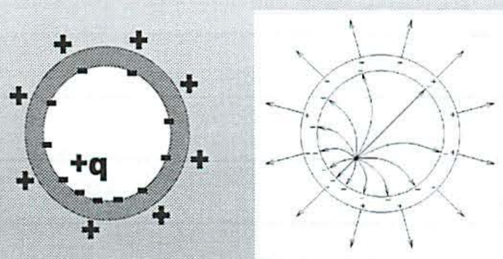


0%	1. is zero and does not change
0%	2. is non-zero but does not change
0%	3. is zero when centered but changes
0%	4. is non-zero and changes
0%	5. I don't know

:00

Hollow Conductors

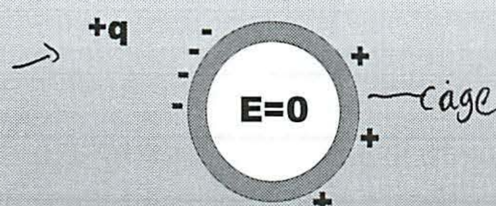
Charge placed INSIDE induces balancing charge ON INSIDE



Hollow Conductors

Charge placed OUTSIDE induces charge separation ON OUTSIDE

Van der Graff

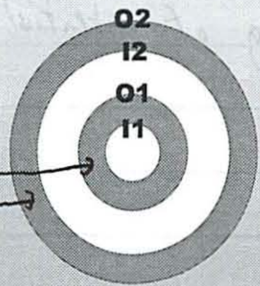


The Ben Franklin experiment

computers work b/c of screening - why computers are in metal

PRS Questions: Point Charge Inside Conductor

PRS Setup



What happens if we put Q in the center of these nested (concentric) spherical conductors?

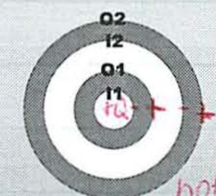
P11-40

We are assuming no edge effects

PRS: Hollow Conductors

0

A point charge +Q is placed at the center of the conductors. The induced charges are:



0% 1. $Q(I1) = Q(I2) = -Q$; $Q(O1) = Q(O2) = +Q$

0% 2. $Q(I1) = Q(I2) = +Q$; $Q(O1) = Q(O2) = -Q$

0% 3. $Q(I1) = -Q$; $Q(O1) = +Q$; $Q(I2) = Q(O2) = 0$

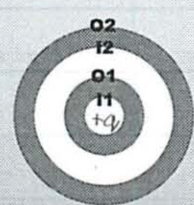
0% 4. $Q(I1) = -Q$; $Q(O2) = +Q$; $Q(O1) = Q(I2) = 0$

P11-41

PRS: Hollow Conductors

A point charge +Q is placed at the center of the conductors. The potential at O1 is:

w/ respect to ∞



0% 1. Higher than at I1

0% 2. Lower than at I1

0% 3. The same as at I1 74% ✓

P11-42

Is a conductor at equilibrium

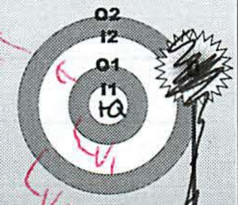
potential same inside conductor

{ electrons move around till equilibrium

* Can have +q −q at same potential

PRS: Hollow Conductors

A point charge +Q is placed at the center of the conductors. The potential at O2 is:



0% 1. Higher than at I1
0% (2.) Lower than at I1
0% 3. The same as at I1

* key Qs of Experiment *

what is meaning of potential diff

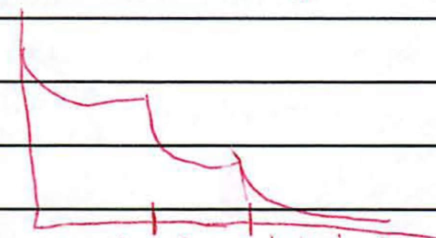
$V(B) - V(A) = -\int E \cdot ds$

← field points out

→ following path from ∞ inward

E field points to lower potential so V_1 at higher potential

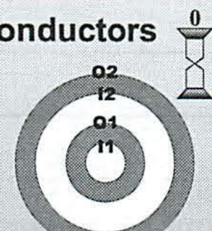
if ⊖ charge that V_1 lower potential



graph from lab today

PRS: Hollow Conductors

A point charge +Q is placed at the center of the conductors. If a wire is used to connect the two conductors, then current (positive charge) will flow



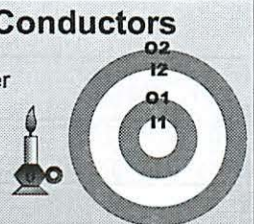
0% 1. from the inner to the outer conductor
0% (2.) from the outer to the inner conductor
0% 3. not at all

Will ground / neutralize

think of 2nd ring as ground

PRS: Hollow Conductors

You connect the "charge sensor's" red lead to the inner conductor and black lead to the outer conductor. What does it actually measure?



0% 1. Charge on I1
0% 2. Charge on O1
0% 3. Charge on I2
0% 4. Charge on O2
0% 5. Charge on O1 – Charge on I2
0% 6. Average charge on inner – ave. on outer
0% (7.) Potential difference between outer & inner
0% 8. I don't know

95%

← this experiment

↳ like last lab where measuring V between the 2 points

"Charge sensor"

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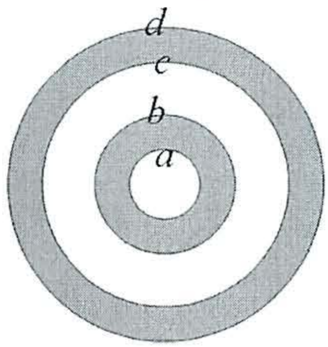
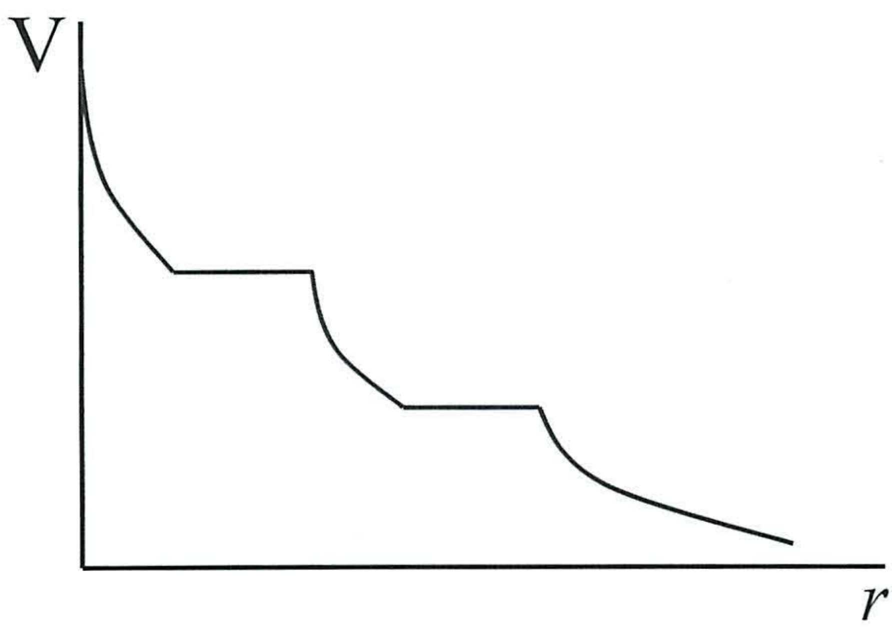
Department of Physics: 8.02

In Class W07D2-1 Solutions: Potential from Concentric Spheres



Question:
Two concentric hollow spherical conductors have inner and outer radii as pictured. A positive charge +Q (not pictured) is placed at the center of the setup. Sketch the electric potential everywhere.

Solution:
We know that the conductors act as equipotential surfaces. In order for that to be the case, negative charges must be induced on the inner surfaces of both conductors ($r = a$ and $r = c$) and by charge conservation positive charges must be induced at their outer surfaces ($r = b$ and $r = d$). Everywhere else the electric field will be as from a point charge ($1/r^2$) and hence the potential will decay as $1/r$. So, since all we need to do is sketch (rather than give exact equations, which you would need to calculate by integrating from a known potential – at $r = \infty$), we have:



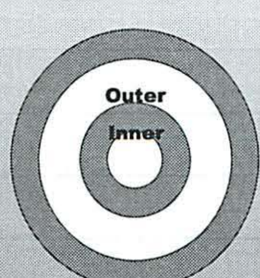
where the 'terraces' (the flat regions) are the equipotential surfaces of the two conductors, and everything else is changing as $1/r$.

Demonstration: Conductive Shielding

Visualization and Lab Prep: Inductive Charging

Palm Pre Touch stone

Experiment 2: Faraday Ice Pail



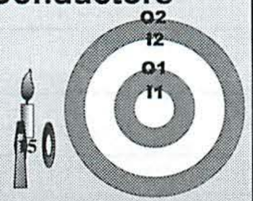
find C - capitance
then find Q from that

if put hand on top - now a
3 conductor system

our model ideal - real
life more complex

PRS: Hollow Conductors

You connected the "charge sensor's" red lead to the inner conductor and black lead to the outer conductor. What does it actually measure?



0%	1. Charge on I1
0%	2. Charge on O1
0%	3. Charge on I2
0%	4. Charge on O2
0%	5. Charge on O1 – Charge on I2
0%	6. Average charge on inner – ave. on outer
0%	7. Potential difference between inner & outer
0%	8. I don't know

Appendix: Dielectrics

Demonstration: Dissectible Capacitor

Dielectrics

A dielectric is a non-conductor or insulator
Examples: rubber, glass, waxed paper

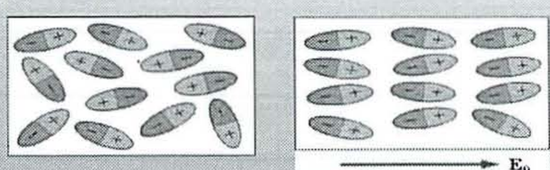
When placed in a charged capacitor, the dielectric reduces the potential difference between the two plates

HOW???

Molecular View of Dielectrics

Polar Dielectrics :

Dielectrics with permanent electric dipole moments
Example: Water

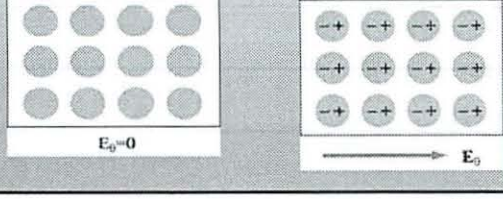


Only line up when charge right?

Molecular View of Dielectrics

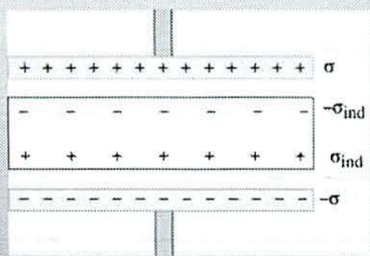
Non-Polar Dielectrics

Dielectrics with induced electric dipole moments
Example: CH_4



No charge untill induced

Dielectric in Capacitor



Potential difference decreases because dielectric polarization decreases Electric Field!

P11-25

Dielectric Constant κ

Dielectric *weakens* original field by a factor κ

$$\varepsilon = \kappa\varepsilon_0 \longrightarrow E = \frac{E_0}{\kappa}$$

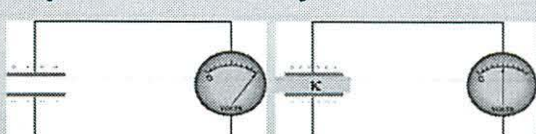
Dielectric Constant

Dielectric constants	
Vacuum	**1.0**
Paper	**3.7**
Pyrex Glass	**5.6**
Water	**80**

P11-26

Dielectric in a Capacitor

Q_0= constant after battery is disconnected



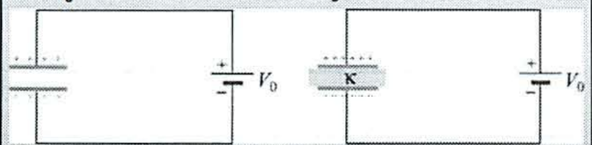
Upon inserting a dielectric: $V = \frac{V_0}{\kappa}$

$$C = \frac{Q}{V} = \frac{Q_0}{V_0/\kappa} = \kappa\frac{Q_0}{V_0} = \kappa C_0$$

P11-27

Dielectric in a Capacitor

V_0 = constant when battery remains connected



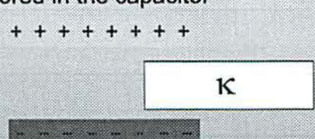
$$Q = CV = \kappa C_0 V_0$$

Upon inserting a dielectric: $Q = \kappa Q_0$

PRS Questions: Dielectric in a Capacitor

PRS: Dielectric

A parallel plate capacitor is charged to a total charge Q and the battery removed. A slab of material with dielectric constant κ in inserted between the plates. The **charge** stored in the capacitor

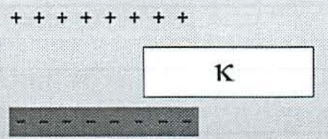


0 Seconds Remaining

0% 1. Increases
0% 2. Decreases
0% 3. Stays the Same

PRS Answer: Dielectric

Answer: 3. Charge stays the same



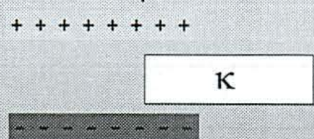
Since the capacitor is disconnected from a battery there is no way for the amount of charge on it to change.

P11B1

PRS: Dielectric

:00

A parallel plate capacitor is charged to a total charge Q and the battery removed. A slab of material with dielectric constant κ in inserted between the plates. The **energy** stored in the capacitor



0% 1. Increases

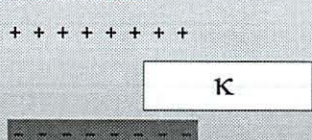
0% 2. Decreases

0% 3. Stays the Same

P11B2

PRS: Dielectric

A parallel plate capacitor is charged to a total charge Q and the battery removed. A slab of material with dielectric constant κ in inserted between the plates. The **force on the dielectric**



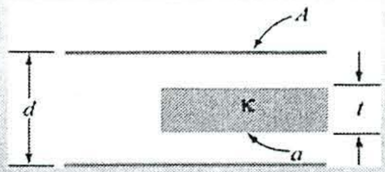
0% 1. pulls in the dielectric

0% 2. pushes out the dielectric

0% 3. is zero

P11B3

Group: Partially Filled Capacitor



What is the capacitance of this capacitor?

Gauss's Law with Dielectrics

$$\oint\!\!\oint_S \kappa \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{free,in}}}{\varepsilon_0}$$

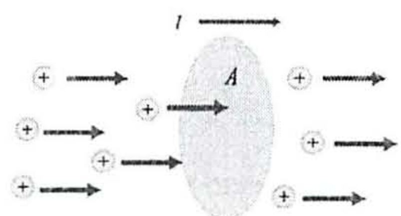
Topics: Current and Simple DC Circuits
Related Reading: Course Notes: Sections 6.1-6.5; 7.1-7.4

Topic Introduction

In today's class we will review *current*, *current density*, and *resistance* and discuss how to analyze simple DC (constant current) circuits using Kirchhoff's Circuit Rules.

Current and Current Density

Electric currents are flows of electric charge. Suppose a collection of charges is moving perpendicular to a surface of area *A*, as shown in the figure



The electric current I is defined to be the rate at which charges flow across the area A. If an amount of charge ΔQ passes through a surface in a time interval Δt, then the current I is given by $I = \frac{\Delta Q}{\Delta t}$ (coulombs per second, or amps). The current density $\vec{\mathbf{J}}$ (amps per square meter) is a concept closely related to current. The magnitude of the current density $\vec{\mathbf{J}}$ at any point in space is the amount of charge per unit time ***per unit area*** flowing pass that point. That is, $\left|\vec{\mathbf{J}}\right| = \frac{\Delta Q}{\Delta t\, \Delta A}$. The current I is a scalar, but $\vec{\mathbf{J}}$ is a vector, the direction of which is the direction of the current flow.

Microscopic Picture of Current Density

If charge carriers in a conductor have number density n, charge q, and a drift velocity $\vec{\mathbf{v}}_d$, then the current density $\vec{\mathbf{J}}$ is the product of n, q, and $\vec{\mathbf{v}}_d$. In *Ohmic* conductors, the drift velocity $\vec{\mathbf{v}}_d$ of the charge carriers is proportional to the electric field $\vec{\mathbf{E}}$ in the conductor. This proportionality arises from a balance between the acceleration due the electric field and the deceleration due to collisions between the charge carriers and the "lattice." In steady state these two terms balance each other, leading to a steady drift velocity (a "terminal" velocity) proportional to $\vec{\mathbf{E}}$. This proportionality leads directly to the "microscopic" Ohm's Law, which states that the current density $\vec{\mathbf{J}}$ is equal to the electric field $\vec{\mathbf{E}}$ times the conductivity σ. The conductivity σ of a material is equal to the inverse of its resistivity ρ.

Current and Voltage

Electric currents (symbol I) are flows of electric charge (symbol Q, typically electrons, but because of sign conventions we will almost always consider positive charges). You can think

2nd law - sum of voltages must = 0

- travels in circle = 0

charge review = conserved property of particles

- determines electrostatic interaction
- influences + produces field
- source of electromagnetic force

of charges moving as balls rolling on a mountain side. The height of this 'electronic mountain' is the voltage (symbol V), so positive charges move to get down the mountain, from high to low potential. We will define these terms more accurately (and more mathematically) later in the course, but for the next several weeks you should try to gain a good conceptual feeling for how voltage and current is related and how circuit elements (resistors, capacitors and inductors) effect this relationship.

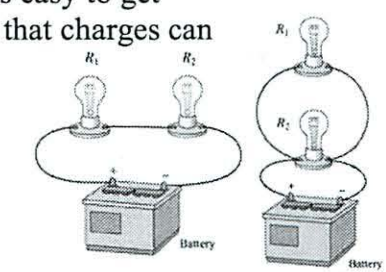
Electromotive Force

A source of electric energy is referred to as an electromotive force, or emf (symbol ε). Batteries are an example of an emf source. They can be thought of as a "charge pump" that moves charges from lower potential to the higher one, opposite the direction they would normally flow. In doing this, the emf creates electric energy (typically from chemical energy), which then flows to other parts of the circuit. The emf ε is defined as the work done to move a unit charge in the direction of higher potential. The SI unit for ε is the volt (V), i.e. Joules/coulomb. * batteries = charge pump ⊖ → ⊕

Resistance & Ohm's Law

but does not convert excess to heat

The first circuit elements we will work are the battery and resistor (symbol R). If the battery is thought of as a "charge pump" we can continue the water analogy and think of the resistor as a pipe, through which the charge is flowing. A "high resistance" is a small pipe (one it is difficult to get through). A "low resistance" is a large pipe that is easy to get through. We will pretend that wires have zero resistance, that is, that charges can freely move through them. Just like pressure drops in a pipe, voltage drops in a resistor, as given by Ohm's law: $\Delta V = IR$. Another way to think of this is that if you want current to flow through a resistor you need to push on it (supply a potential difference across the resistor). *



Series Parallel

Series vs. Parallel

Now that we have batteries and resistors we can consider hooking them together to make circuits. When we do that we have two choices for hooking two elements together – they can either be hooked in series (with the 'end' of one hooked to the 'beginning' of the next) or in parallel (with the 'beginning' and 'end' of each element tied together). An example of light bulbs in series and parallel is show at right. For elements in series, any charges (current) that flow through one element must also flow through the second. In parallel the voltage drop across two elements must be the same (they are 'at the same height' at both their 'beginning' and 'end' and hence the drop across both must be the same). Using these ideas we will derive relationships for resistors in parallel and in series. memorize

Kirchhoff's Circuit Rules

In analyzing circuits, there are two fundamental (Kirchhoff's) rules: (1) The junction rule states that at any point where there is a junction between various current carrying branches, the sum of the currents into the node must equal the sum of the currents out of the node

In = Out at branches

(otherwise charge would build up at the junction); (2) The loop rule states that the sum of the voltage drops ΔV across all circuit elements that form a closed loop is zero (this is the same as saying the electrostatic field is conservative).

If you travel through a battery from the negative to the positive terminal, the voltage drop ΔV is $+\varepsilon$, because you are moving against the internal electric field of the battery; otherwise ΔV is $-\varepsilon$. If you travel through a resistor in the direction of the assumed flow of current, the voltage drop is $-IR$, because you are moving parallel to the electric field in the resistor; otherwise ΔV is $+IR$.

Important Equations

Relation between $\vec{\mathbf{J}}$ and I: $I = \iint \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}$

Microscopic Ohm's Law: $\vec{\mathbf{J}} = \sigma \vec{\mathbf{E}} = \vec{\mathbf{E}} / \rho$

Macroscopic Ohm's Law: $V = IR$

Resistance of a conductor with resistivity ρ, cross-sectional area A, and length l: $R = \rho\, l \,/\, A$

Resistors in series: $R_{eq} = R_1 + R_2$

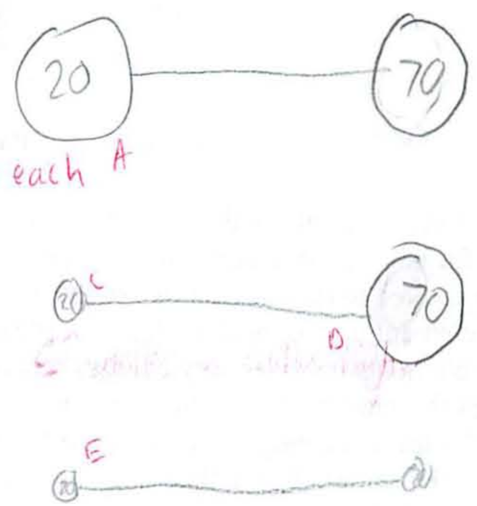
Resistors in parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$

Power: $P = \Delta V\, I$

PRS

Connected conductors

charge = $\frac{90}{2} = 45$ on each A



Charge total same

When conductors are connected - same potential ← that's what equalizes!

→ $V = \frac{kq}{r}$

↑ can see how voltage changes

charge is proportional to radius

$\frac{k q_c}{r_c} = \frac{k q_d}{r_d}$

$q_c \neq q_d$

$V_A < V_C = V_D < V_E$

Class 12: Outline

Hour 1:

Current, Current Density, and Ohm's Law

Hour 2:

DC Circuits and Kirchhoff's Loop Rules

P12-1

Flow of Charge

New Topics: Current, Current Density, Resistance, Ohm's Law

P12-2

Current: Flow Of Charge

Average current I_{av}: Charge ΔQ flowing across area A in time Δt

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

Instantaneous current: differential limit of I_{av}

$$I = \frac{dQ}{dt}$$

Units of Current: Coulomb/second = Ampere

P12-3

Watch how much current goes thragh your gate at a time

be cause 2 in 3 out
A is getting a charge

How Big is an Ampere?

- Household Electronics ~1 A
- Battery Powered ~100 mA (1-10 A-Hr)
- Household Service 100 A
- Lightning Bolt 10 to 100 kA
- To hurt you 40 (5) mA DC(AC)
- To throw you 60 (15) mA DC(AC)
- To kill you 0.5 (0.1) A DC(AC)
- Fuse/Circuit Breaker 15-30 A

P12-4

how power is sold

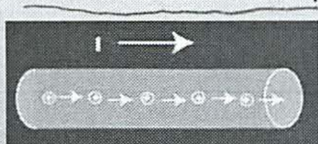
amp = charge / time

amp hrs = charge / time · time = charge

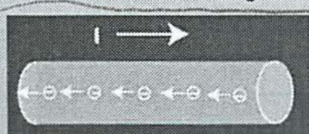
eaiser for wall power to hurt you
depends on freq of current - US 60 Hz
which happens to be most dangerous

Direction of The Current

Direction of current is direction of flow of pos. charge



or, opposite direction of flow of negative charge



P12-5

← what actually flows in most materials

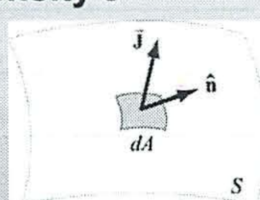
* current dir ⊕ travels

Current Density J

J: current/unit area

$$\vec{\mathbf{J}} \equiv \frac{I}{A}\hat{\mathbf{I}}$$

$\hat{\mathbf{I}}$ points in direction of current



$$I = \int_S \vec{\mathbf{J}} \cdot \hat{\mathbf{n}}\, dA = \int_S \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}$$

P12-6

like charge density

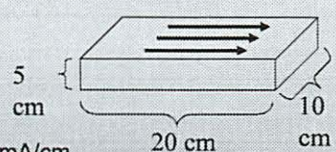
has local direction charge is flowing

PRS Question: Current Density

PRS: Current Density

:00

A current I = 200 mA flows in the above wire. What is the magnitude of the current density J?



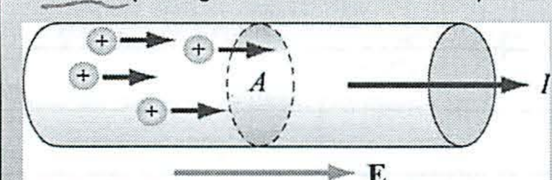
0% 1. J = 40 mA/cm
0% 2. J = 20 mA/cm
0% 3. J = 10 mA/cm
0% 4. J = 1 mA/cm²
0% 5. J = 2 mA/cm²
0% 6. J = 4 mA/cm²
0% 7. I don't know

area = 5 · 10 cm²

$J = \frac{200 mA}{50 cm^2} = 4 mA/cm^2$

Why Does Current Flow?

If an electric field is set up in a conductor, charge will move (making a current in direction of E)



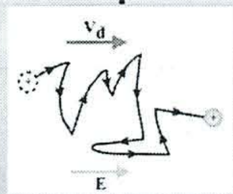
Note that when current is flowing, the conductor is not an equipotential surface (and $E_{inside} \neq 0$)!

but conductors should have no E field inside

* but there is a voltage difference *

- charges try to eliminate electric field, but field continues to be applied

Microscopic Picture



Drift speed is velocity forced by applied electric field in the presence of collisions.

It is typically 4×10^{-5} m/sec, or 0.04 mm/second!

To go one meter at this speed takes about 10 hours!

How Can This Be?

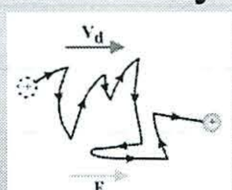
P12-40

bounce off impurities, lattice vibrations

like desk toys - first one pushes last one off

but E field is pushing them

Conductivity and Resistivity



Ability of current to flow depends on density of charges & rate of scattering

Two quantities summarize this:

σ: conductivity

ρ: resistivity

P12-41

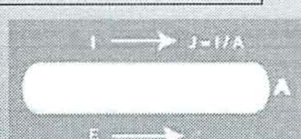
depends on the material

inverses of each other

Microscopic Ohm's Law

$$\vec{\mathbf{E}} = \rho\vec{\mathbf{J}} \quad \text{or} \quad \vec{\mathbf{J}} = \sigma\vec{\mathbf{E}}$$

$$\rho \equiv \frac{1}{\sigma}$$



ρ and σ depend only on the microscopic properties of the material, not on its shape

P12-42

velocity should be ↑ and current as well
- but its not

* Field determines velocity not acceleration *

- no qu about this
- but conceptually importance

Demonstrations:
Water
Temperature Effects on ρ

copper wire

as gets colder + colder
it gets brighter
- more current flowing through
- resistance goes down

Or over insulator glass
no current at room temp
heat it up + becomes conductor and glowing

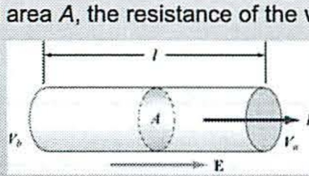
PRS Question:
Resistance?

lattice "balls" vibrate
- cool → slows down → now a path opens

- insulator all electrons stuck but when heat given they break free of bonds and get current flow

PRS: Resistance

When a current flows in a wire of length L and cross sectional area A, the resistance of the wire is



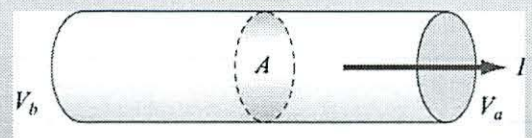
1. Proportional to A; inversely proportional to L.
2. Proportional to both A and L.
3. Proportional to L; inversely proportional to A.
4. Inversely proportional to both L and A
5. Do Not Know

pure water is not a conductor
- but salt dissolves in it it ionizes and can carry current
- resistance drops as add salt

important to get intuitive feel
- pipe analogy: make it shorter and fatter
thicker for stuff to get through

Why Does Current Flow?

Instead of thinking of Electric Field, think of potential difference across the conductor



$\Delta V = V_b - V_a$

Apply a potential difference

macroscopic: voltage drives current

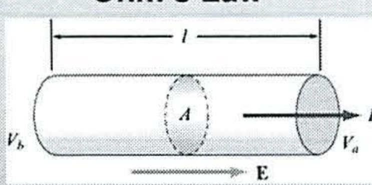
Ohm's Law

What is relationship between ΔV and current?

$$\Delta V = V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E\ell$$

$$\left.\begin{aligned} J &= \frac{E}{\rho} = \frac{\Delta V / \ell}{\rho} \\ J &= \frac{I}{A} \end{aligned}\right\} \Rightarrow \Delta V = I\left(\frac{\rho \ell}{A}\right) \equiv IR$$

Ohm's Law



$$\Delta V = IR$$

$$R = \frac{\rho \ell}{A}$$

R has units of Ohms (Ω) = Volts/Amp

→ Remember Ohm's law

R = resistance
V = voltage
I = current

How Big is an Ohm?

- Short Copper Wire — milliohms (mΩ)
- Notebook paper (thru) — ~1 GΩ
- Typical resistors — Ω to 100 MΩ

- You (when dry) — 100 kΩ
- You (when wet) — 1 kΩ
- Internally (hand to foot) — 500 Ω

Stick your wet fingers in an electrical socket:

$I = V / R \approx 120\,\text{V} / 1\text{k}\Omega \approx 0.1\,\text{A}$ You're dead!

P12-19

very different in materials

higher Ω = less current

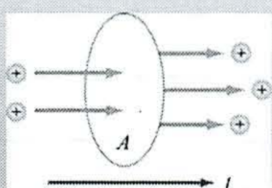
Current: Flow Of Charge

Average current I_{av}: Charge ΔQ flowing across area A in time Δt

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

Instantaneous current: differential limit of I_{av}

$$I = \frac{dQ}{dt}$$



Units of Current: Coulomb/second = Ampere

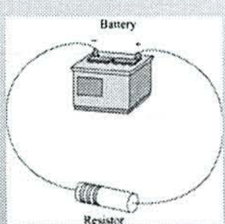
P12-20

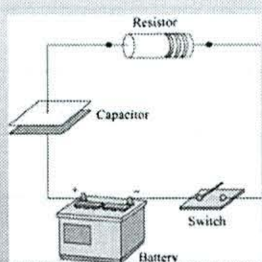
Batteries & Elementary Circuits

P12-21

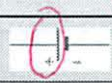
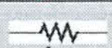
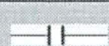
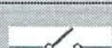
DC Circuits

Examples of Circuits



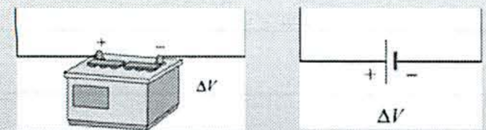


Symbols for Circuit Elements

Battery	
Resistor	
Capacitor	
Switch	

— (+) is at higher voltage so line is longer

Ideal Battery

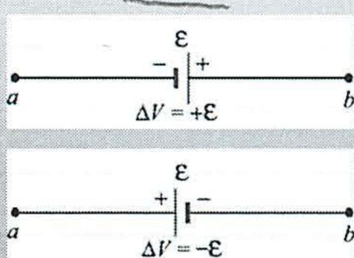


Fixes potential difference between its terminals
Sources as much charge as necessary to do so

Think: Makes a mountain

Sign Conventions - Battery

Moving from the negative to positive terminal of a battery **increases** your potential



$$\Delta V = V_b - V_a$$

Think: Ski Lift

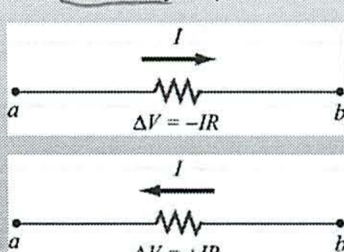
$\varepsilon \approx V$ (at least for this class)

battery like a pump

Sign Conventions - Resistor

Moving across a resistor in the direction of current → direction of flow of ⊕ charge

decreases your potential



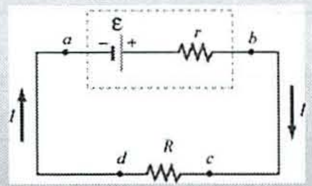
$$\Delta V = V_b - V_a$$

Think: Ski Slope

Capacitor
Ski Lodge - people flow in
every time someone flows in, someone flows out

Internal Resistance

Real batteries have an internal resistance, r, which is small but non-zero



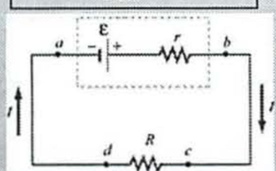
Terminal voltage: $\Delta V = V_b - V_a = \mathcal{E} - I r$

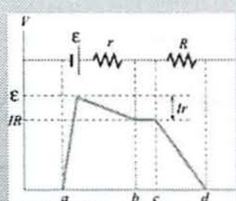
(Even if you short the leads you don't get infinite current)

Potential Difference Around a Closed Path

Sum of potential differences across all elements around any closed circuit loop must be zero.

$$\Delta V = -\oint_{\text{Closed Path}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$$

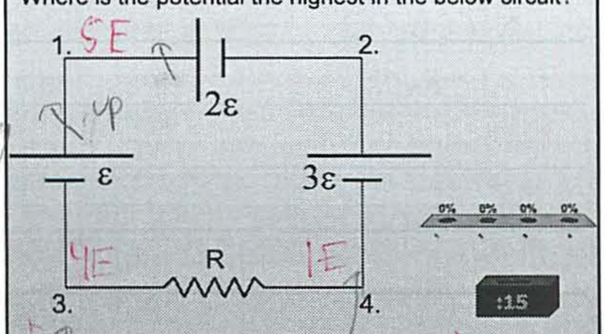




that law

PRS: Potential in Circuits

Where is the potential the highest in the below circuit?



higher than everything

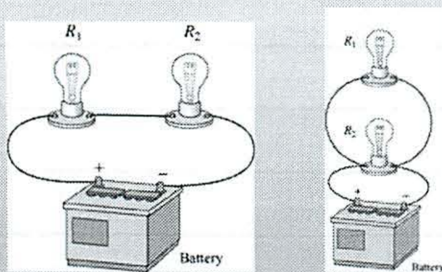
higher potential than 4 so know right IR 4E

(+) higher potential

(+) charge leaves from (+) terminal

for our purposes current all the way around

Series vs. Parallel



Series — current is same throughout

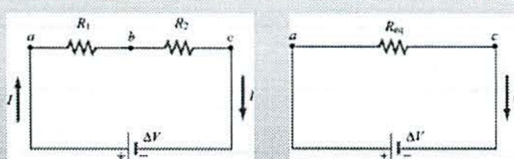
Parallel — can go through one or the other

Multiple paths for charges to flow through

* Voltage must be same on both

Resistors In Series

The same current I must flow through both resistors



$$\Delta V = I R_1 + I R_2 = I(R_1 + R_2) = I R_{eq}$$

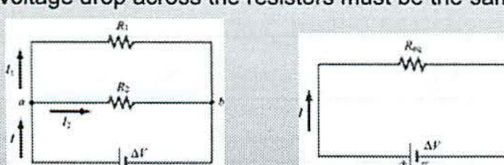
$$R_{eq} = R_1 + R_2$$

(+) top of mountain
(−) bottom "
drop must be the same.

current same through them (ie series)

Resistors In Parallel

Voltage drop across the resistors must be the same



$$\Delta V = \Delta V_1 = \Delta V_2 = I_1 R_1 = I_2 R_2 = I R_{eq}$$

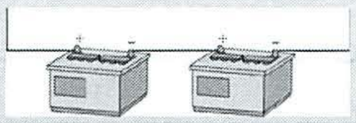
$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \frac{\Delta V}{R_{eq}} \qquad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

add inversely

* reduces resistance
  - another exit door from theater

$I_1 + I_2 = I$

Batteries in Series



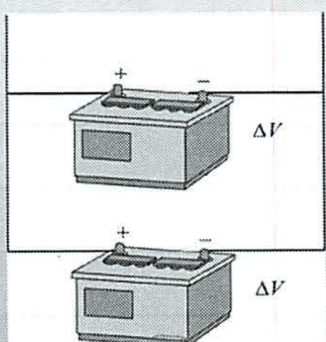
Net voltage change is $\Delta V = \Delta V_1 + \Delta V_2$

Think: Two Mountains Stacked

P12-34

just add

Batteries in Parallel



Net voltage still ΔV

P12-35

Bad idea unless charging

-each battery wants to define potential, if they are not the same - depending on battery chemistry

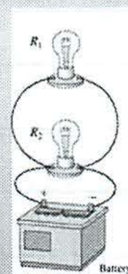
usually a wall source for charging

PRS Questions: Two Light Bulbs

P12-36

PRS: Bulbs & Batteries

An ideal battery is hooked to a light bulb with wires. A second identical light bulb is connected in parallel to the first light bulb. After the second light bulb is connected, the current from the battery compared to when only one bulb was connected.



0%	1. Is Higher
0%	2. Is Lower
0%	3. Is The Same
0%	4. Don't know

P18-37

$V = IR$

↑ ↑ decreases in parallel

must increase

$I = I_1 + I_2$ = more current

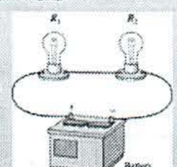
you just add another bulb

twice as much current

PRS: Bulbs & Batteries

:00

An ideal battery is hooked to a light bulb with wires. A second identical light bulb is connected in series with the first light bulb. After the second light bulb is connected, the current from the battery compared to when only one bulb was connected.



0%	1. Is Higher
0%	2. Is Lower
0%	3. Is The Same
0%	4. Don't know

P18-38

$V = IR$

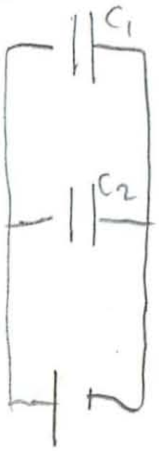
$V = I(R_1 + R_2)$

↑ doubling

must decrease

because voltage stays same

Capacitators in Parallel



$$Q = Q_1 + Q_2 = C_1 \Delta V + C_2 \Delta V$$
$$= (C_1 + C_2)\Delta V$$

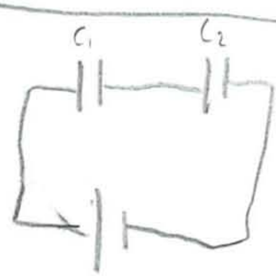
$$C_{eq} = \frac{Q}{\Delta V} = C_1 + C_2$$

* basically just pushing togetor to get more surface area *

Capicator

- to store charge
- penalty: must supply potential to store charge
- a good capicator stores a lot of charge w/o requiring a lot of voltage

Capacitators in Series

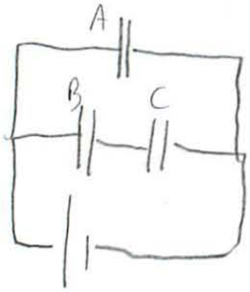


Charge on capicators same
Potential can be diff b/c capicatance different

$$\Delta V = \Delta V_1 + \Delta V_2$$
$$= \frac{Q}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} \quad \boxed{Q = Q_2}$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

- ability to store charge decreases (have to pay "potential" price)



Before + after battery removed

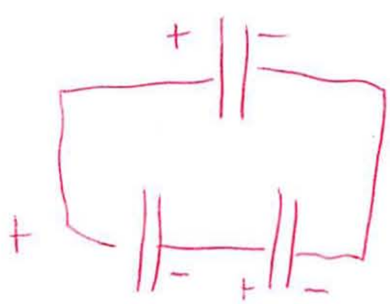
Initial $Q_A > \underbrace{Q_B = Q_C}$

are not as good, don't store enough charge

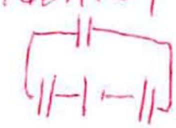
After ~ is charge there, so still potential difference
- nothing gobbles up charge, so no change

OR take batt out now 3 capacitors in series

no nothing changes
- the 3 capicators are not identical



set up so not identical
not like



potential drop across A and B+C same
so no reason for charge to move

Is the battery still doing anything?
- no, capicators loaded, no current flowing
- not doing anything, removing it does nothing

Power $= \frac{dU}{dt} = \frac{d(q\Delta V)}{dt} = \frac{dq}{dt}(\Delta V)$

$= I\Delta V$ for circuit devices

battery: energy being supplied

$P = I\Delta V$

? missed notes in class 14 slides

resistor - dissipate power

$P = I\Delta V = \frac{I^2}{R}$

? missed notes

capacitor - asorb energy

$P = I\Delta V = \frac{dQ}{dt}\frac{Q}{C} = \frac{d}{dt}\frac{Q^2}{2C} = \frac{dU}{dt}$

Topics: PHET Simulation: Building Simple DC Circuits
Related Reading: Course Notes: Sections 6.1-6.5; 7.1-7.4

Topic Introduction

In today's class we will use a PHET simulation to build simple DC circuits.

Current and Voltage

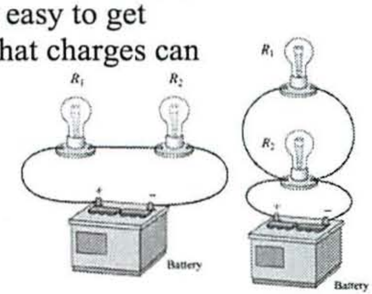
Electric currents (symbol *I*) are flows of electric charge (symbol *Q*, typically electrons, but because of sign conventions we will almost always consider positive charges). You can think of charges moving as balls rolling on a mountain side. The height of this 'electronic mountain' is the voltage (symbol *V*), so positive charges move to get down the mountain, from high to low potential. We will define these terms more accurately (and more mathematically) later in the course, but for the next several weeks you should try to gain a good conceptual feeling for how voltage and current is related and how circuit elements (resistors, capacitors and inductors) effect this relationship.

Electromotive Force

A source of electric energy is referred to as an electromotive force, or emf (symbol ε). Batteries are an example of an emf source. They can be thought of as a "charge pump" that moves charges from lower potential to the higher one, opposite the direction they would normally flow. In doing this, the emf creates electric energy (typically from chemical energy), which then flows to other parts of the circuit. The emf ε is defined as the work done to move a unit charge in the direction of higher potential. The SI unit for ε is the volt (V), i.e. Joules/coulomb.

Resistance & Ohm's Law

The first circuit elements we will work are the battery and resistor (symbol *R*). If the battery is thought of as a "charge pump" we can continue the water analogy and think of the resistor as a pipe, through which the charge is flowing. A "high resistance" is a small pipe (one it is difficult to get through). A "low resistance" is a large pipe that is easy to get through. We will pretend that wires have zero resistance, that is, that charges can freely move through them. Just like pressure drops in a pipe, voltage drops in a resistor, as given by Ohm's law: $\Delta V = IR$. Another way to think of this is that if you want current to flow through a resistor you need to push on it (supply a potential difference across the resistor).



Series Parallel

Series vs. Parallel

Now that we have batteries and resistors we can consider hooking them together to make circuits. When we do that we have two choices for hooking two elements together – they can either be hooked in series (with the 'end' of one hooked to the 'beginning' of the next) or in parallel (with the 'beginning' and 'end' of each element tied together). An example of light bulbs in series and parallel is show at right. For elements in series, any charges (current) that flow through one element must also flow through the second. In parallel the voltage drop across two elements must be the same (they are 'at the same height' at both their 'beginning' and 'end' and hence the drop across both must be the same). Using these ideas we will derive relationships for resistors in parallel and in series.

Kirchhoff's Circuit Rules

In analyzing circuits, there are two fundamental (Kirchhoff's) rules: (1) The junction rule states that at any point where there is a junction between various current carrying branches, the sum of the currents into the node must equal the sum of the currents out of the node (otherwise charge would build up at the junction); (2) The loop rule states that the sum of the voltage drops ΔV across all circuit elements that form a closed loop is zero (this is the same as saying the electrostatic field is conservative).

If you travel through a battery from the negative to the positive terminal, the voltage drop ΔV is $+\varepsilon$, because you are moving against the internal electric field of the battery; otherwise ΔV is $-\varepsilon$. If you travel through a resistor in the direction of the assumed flow of current, the voltage drop is $-IR$, because you are moving parallel to the electric field in the resistor; otherwise ΔV is $+IR$.

Important Equations

Macroscopic Ohm's Law:	$V = IR$
Resistance of a conductor with resistivity ρ, cross-sectional area A, and length l:	$R = \rho\, l\, /\, A$
Resistors in series:	$R_{eq} = R_1 + R_2$
Resistors in parallel:	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$
Power:	$P = \Delta V\, I$

Resistors = voltage across termals proportional to current
$V = IR$

Current through them same
Voltage difference
(voltage = pressure)

Capicator = 2 conductors seperated by dielectric
when potential diff / voltage → E field presents
field stores energy

$C = \frac{Q}{V} = \frac{\varepsilon A}{d}$ parallel

measure how good it is ?

Parallel $C_1 + C_2$

Series $\frac{1}{C_1} + \frac{1}{C_2}$

Simulation

3/4

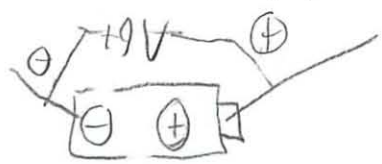
The simulation I wanted!

~~Current same over entire circuit~~

Volt meter = voltage drop across 2 pt 

red ⊕ - touch to ⊕ of DC circuit
black ⊖
↑ the ⊕ end of battery

then battery is +9V



if flip ⊖ ⊕ leades then # -

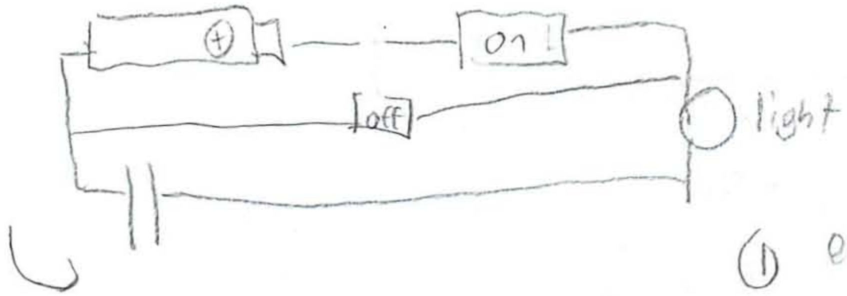
Remember electrons move from ⊖ to ⊕
but E field from ⊕ to ⊖

A light bulb filament is just a resistor, right?
- slows charge
- like a smaller r and longer pipe
  - lowers pressure (voltage)
  - makes it drop

If just had large tank 
small pipe
water comes out at lower pressure

But if trying to <u>force</u> more water into pipe it will speed up, right?

Found one w/ capicator

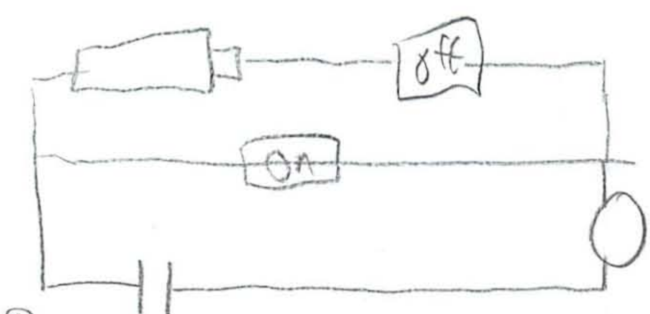


elections move

<u>charging</u>

1. electrons flow, light lights 0V diff
2. charge builds on capicator electons slow, light dims Volt diff grows
3. Then electrons stop flowing, light is off, capicator fully charged & Volt diff on capicator

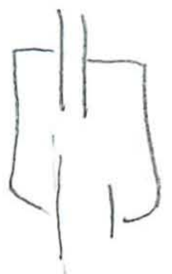
<u>discharging</u>



electrons move

1. light on bright 9V diff
2. Then light dims, electrons slow down to a stop, V diff falls
3. lights off, 0 V diff

Speed of electrons = current (amps)

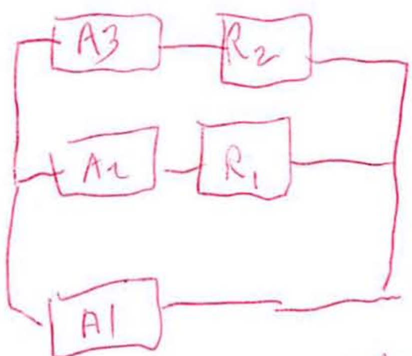


quickly charges (fire?)

quickly decharges

Class 13 Quiz

$R_2 > R_1$



$A_1 > A_2 > A3$

when circuit in parrallel
then ampage not everywhere same

Michael Plasmeier

From: Eric Hudson [8.02.help@gmail.com]
Sent: Saturday, March 06, 2010 10:28 PM
To: Michael Plasmeier
Subject: RE: MP Question M

Hi Michael,

Work is change in potential energy, which as you'll recall is q*delta V (for shortness I'll write qV). You know V. You need to know q. You are told 1 minute, so that must be important. To get charge from a time, you'll also need to know a current. Because I=q/t to q=It.

Hope that helps.

From: Michael Plasmeier [mailto:plaz@theplaz.com]
Sent: Saturday, March 06, 2010 7:01 PM
To: 8.02.help@gmail.com
Subject: MP Question M

Hi,

Can someone please help me understand how you arrive at Part M of An Introduction to EMF and Circuits. I looked around the web and only got more confused. Thanks -Michael

How much work W does the battery connected to the 21.0-ohm resistor perform in one minute?
Express your answer in joules. Use three significant figures.
W =360 J

Work = U = Power = Pt = IVt = qV

Topics: Simple DC Circuits
Related Reading: Course Notes: Sections 7.1-7.5, 7.8-7.9
Experiments: (3) Building Simple Circuits with Resistors

Topic Introduction

In today's class we will study multi-loop circuits, power and energy, measuring devices, capacitors in circuits, review *current*, and build simple circuits in a lab.

Kirchhoff's Circuit Rules

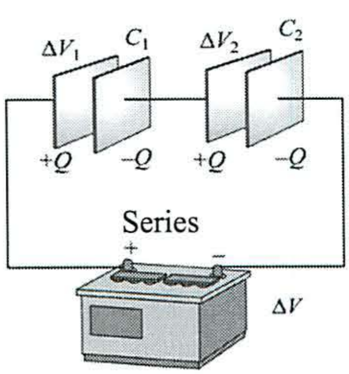
In analyzing circuits, there are two fundamental (Kirchhoff's) rules: (1) The junction rule states that at any point where there is a junction between various current carrying branches, the sum of the currents into the node must equal the sum of the currents out of the node (otherwise charge would build up at the junction); (2) The loop rule states that the sum of the voltage drops ΔV across all circuit elements that form a closed loop is zero (this is the same as saying the electrostatic field is conservative).

If you travel through a battery from the negative to the positive terminal, the voltage drop ΔV is $+\varepsilon$, because you are moving against the internal electric field of the battery; otherwise ΔV is $-\varepsilon$. If you travel through a resistor in the direction of the assumed flow of current, the voltage drop is $-IR$, because you are moving parallel to the electric field in the resistor; otherwise ΔV is $+IR$.

Steps for Solving Multi-loop DC Circuits

1) Draw a circuit diagram, and label all the quantities;
2) Assign a direction to the current in each branch of the circuit--if the actual direction is opposite to what you have assumed, your result at the end will be a negative number;
3) Apply the junction rule to the junctions;
4) Apply the loop rule to the loops until the number of independent equations obtained is the same as the number of unknowns.

Capacitance



Next we will discuss what happens when multiple capacitors are put together. There are two distinct ways of putting circuit elements (such as capacitors) together: in *series* and in *parallel.* Elements in series (such as the capacitors and battery at left) are connected one after another. As shown, the charge on each capacitor must be the same, as long as everything is initially uncharged when the capacitors are connected (which is always the case unless otherwise stated). In parallel, the capacitors have the same potential drop across them (their bottoms and tops are at the same potential). From these setups we will calculate the equivalent capacitance of the system – what one capacitor could

Parallel

replace the two capacitors and store the same amount of charge when hooked to the same battery. It turns out that in parallel capacitors add ($C_{eq} \equiv C_1 + C_2$) while in series they add inversely ($C_{eq}^{-1} \equiv C_1^{-1} + C_2^{-1}$).

Experiment 3: Resistors and Simple Circuits

Preparation: Read pre-lab

In this lab you learn how to build simple circuits with a battery and resistors, and how to make and measure current through the circuit. This is an introduction to the experimental materials you will use for the next several weeks, but also a chance to understand Ohm's law and to see how resistors add in series and in parallel.

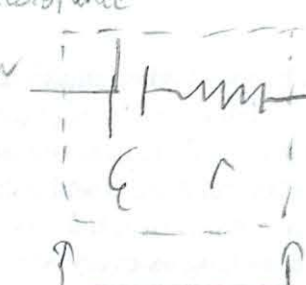
$P = IV = I^2 R$

Important Equations

Macroscopic Ohm's Law:	$V = IR$
Resistors in series:	$R_{eq} = R_1 + R_2$
Resistors in parallel:	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$
Power:	$P = \Delta V\, I$
Capacitors in Series:	$\frac{1}{C_{eq}} \equiv \frac{1}{C_1} + \frac{1}{C_2}$
Capacitors in Parallel:	$C_{eq} \equiv C_1 + C_2$

Internal Resistance

Draw



terminal voltage

As it starts sourcing current internal resistance droping voltage

$\mathcal{E} - Ir$

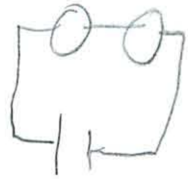
"open loop" - no load on circuit / no internal resistance

Power = IV - Disipated / supplying / store = watt = $\frac{dU}{dt}$ ← Energy

Energy over lifetime $U = Pt = IVt = qV$

↑ can find batt lenght

3/8



Class 14: Outline

Hour 1:

DC Circuits and Kirchhoff's Loop Rules

Hour 2: Experiment 3 Building a Circuit with Resistors

P14- 1

Brightness based on power = IV

of lightbulb

P = IV across the lightbulb not the battery = I^2R

Battery has fixed voltage across it
Current changes (in ideal battery) linearly?

Kirchhoff's Loop Rules

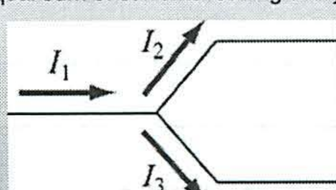
P14- 2

Lots of voltage - say which voltage
- batt voltage does not change
- but voltage across 1 bulb is $\frac{1}{2}$ when anoter added

Complex circuit

Kirchhoff's Rules

1. Sum of currents entering any junction in a circuit must equal sum of currents leaving that junction.



$$I_1 = I_2 + I_3$$

P14- 3

Conservation of current

Ammeter = in series
Voltmeter = parrallel

* Series = current same *
* Parallel = voltage drop same * across all elements

Formulas

$C = \frac{Q}{V}$ ← price to pay

$V = IR$

$X = X_1 + X_2$ resistors in series - have to go through all

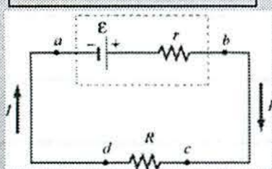
$\frac{1}{X} = \frac{1}{X_1} + \frac{1}{X_2}$ - resistors in parallel = more ways to go

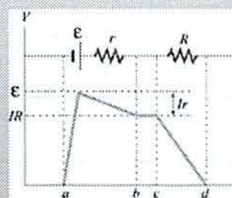
$P = IV$

Kirchhoff's Rules

2. Sum of potential differences across all elements around any closed circuit loop must be zero.

$$\Delta V = -\oint_{\text{Closed Path}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$$





P14-4

× Make the table

Capitance in parallel adds - get more capitance

" in series adds inversly

Sum of voltage = 0

battery lifts you up mountain

resistor drops you back down

now you are back where you started!

Steps of Solving Circuit Problem

1. Straighten out circuit (make squares) — eaiser to read, wires are free
2. Simplify resistors in series/parallel
3. Assign current loops (arbitrary)
4. Write loop equations (1 per loop)
5. Solve

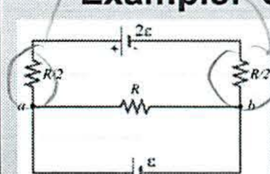
n currents solving for

n equations

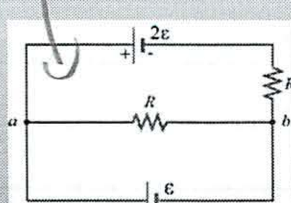
P14-5

In series (same current flows through both) - no branches

Example: Simple Circuit



You can simplify resistors in series (but don't need to)

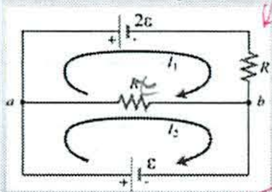


What is current through the bottom battery?

P14-6

Pick random loops ; dir of walking

Example: Simple Circuit



Start at a in both loops

Walk in direction of current

$-2\varepsilon - I_1 R - (I_1 - I_2)R = 0$

down down what dir

$-(I_2 - I_1)R + \varepsilon = 0$

Add these: $-2\varepsilon - I_1 R + \varepsilon = 0 \rightarrow I_1 = \frac{-\varepsilon}{R}$

We wanted I_2: $(I_2 - I_1)R = \varepsilon \rightarrow I_2 = \frac{\varepsilon}{R} + I_1$

$\boxed{I_2 = 0}$

← having 2 loops = no node rule

resistors always down in current's dir

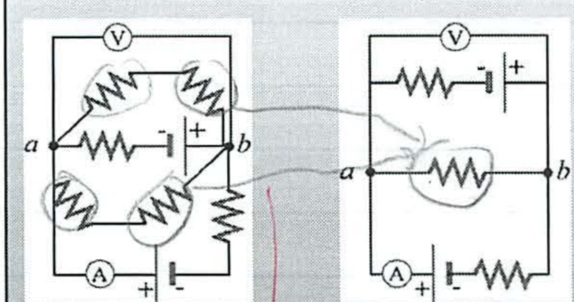
bottom branch

Way current going is same as dir of voltage drop

One time left/other right/ so sign

Group Problem: Circuit

Find meters' values. All resistors are R, batteries are ε



- same thing could do in parallel when +IR ← uphill opposit dir current but walk in dir of current (easier)

← Voltmeter $V = \varepsilon - Ir$

ampmeter $(V = IR)$ $I = \frac{V}{R} = \frac{\varepsilon - Ir}{R}$

$I = \frac{\varepsilon}{(r+R)}$

Series and parrallel

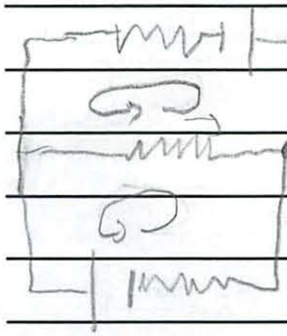
$\frac{1}{2R} + \frac{1}{2R} = \frac{2}{2R}$ ← take reciprocal

Current through Vmeter = 0

Power

$\frac{1}{\text{eq resistance}} = \sum \frac{1}{R}$

eq resistance = R



$-\varepsilon - I_1 R - (I_1 + I_2)R = \varepsilon$

↑ opposit dir = (+)

$-(I_1 - I_2)R - \varepsilon - I_2 R = 0$

$I_1 = I_2 = \frac{\varepsilon}{R}$ so current cancles

current between A + B is 0.

Electrical Power

Power is change in energy per unit time

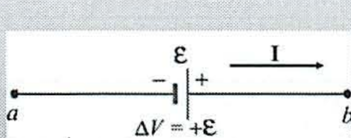
So power to move current through circuit elements:

$$P = \frac{d}{dt}U = \frac{d}{dt}(q\Delta V) = \frac{dq}{dt}\Delta V$$

$$\boxed{P = I\,\Delta V}$$

Power - Battery

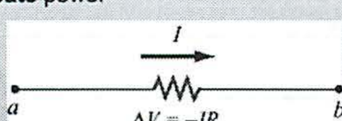
Moving from the negative to positive terminal of a battery **increases** your potential. If current flows in that direction the battery **supplies** power



$$\boxed{P_{\text{supplied}} = I\,\Delta V = I\,\varepsilon}$$

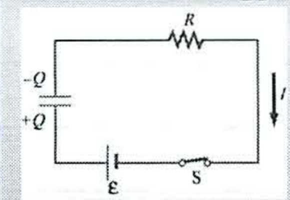
Power - Resistor

Moving across a resistor in the direction of current **decreases** your potential. Resistors **always dissipate** power



$$\boxed{P_{\text{dissipated}} = I\,\Delta V = I^2 R = \frac{\Delta V^2}{R}}$$

Energy Balance



$$\mathcal{E} - \frac{Q}{C} - IR = 0$$

Multiplying by *I*:

$$\mathcal{E}I = I^2R + \frac{Q}{C}\frac{dQ}{dt} = I^2R + \frac{d}{dt}\left(\frac{1}{2}\frac{Q^2}{C}\right)$$

(power delivered by battery) = (power dissipated through resistor) + (power absorbed by the capacitor)

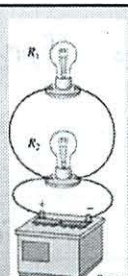
P14-13

PRS Questions: Two More Light Bulbs

P14-14

PRS: Power

An ideal battery is hooked to a light bulb with wires. A second identical light bulb is connected in parallel to the first light bulb. After the second light bulb is connected, the power output from the battery (compared to when only one bulb was connected)



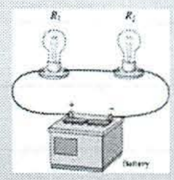
0%	1.	Is four times higher
0%	2.	Is twice as high
0%	3.	Is the same
0%	4.	Is half as much
0%	5.	Is ¼ as much
0%	6.	Don't know

P14-15

:20

PRS: Power

An ideal battery is hooked to a light bulb with wires. A second identical light bulb is connected in series with the first light bulb. After the second light bulb is connected, the light (power) from the first bulb (compared to when only one bulb was connected)



0%	1.	Is four times higher
0%	2.	Is twice as high
0%	3.	Is the same
0%	4.	Is half as much
0%	5.	Is ¼ as much
0%	6.	Don't know

P13-16

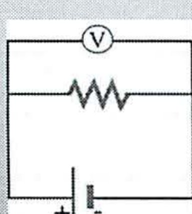
Measuring Voltage & Current

P13-17

Be careful in language!

Measuring Potential Difference

A voltmeter must be hooked in *parallel* across the element you want to measure the potential difference across



$$\frac{1}{R_{effective}} = \frac{1}{R} + \frac{1}{R_{voltmeter}}$$

0

Voltmeters have a very large resistance, so that they don't affect the circuit too much

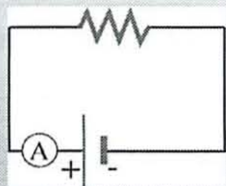
P13-18

Across something

- So it has a large internal resistance

Measuring Current

An ammeter must be hooked in *series* with the element you want to measure the current through



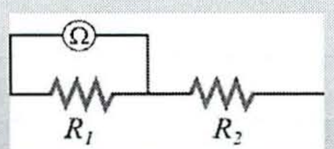
$R_{effective} = R + R_{ammeter}$ (arrow to $R_{ammeter}$: 0)

Ammeters have a very low resistance, so that they don't affect the circuit too much

goes through something

Measuring Resistance

An ohmmeter must be hooked in *parallel* across the element you want to measure the resistance of



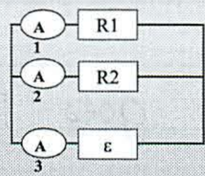
Here we are measuring R_1

Ohmmeters apply a voltage and measure the current that flows. They typically won't work if the resistor is powered (connected to a battery)

PRS Question: Ammeters and Resistors

20 PRS: Measuring Current

If R1 > R2, compare the currents measured by the three ammeters:

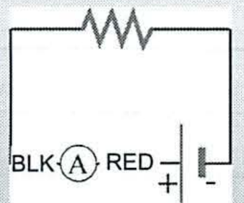


0% 1. A1 > A2 > A3
0% 2. A2 > A1 > A3
0% 3. A3 > A1 > A2
0% 4. A3 > A2 > A1
0% 5. A3 > A1 = A2
0% 6. None of the above
0% 7. I don't know

Experiment 3: Building a Circuit with Resistors

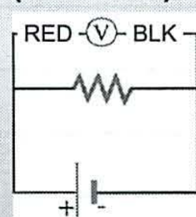
From Diagrams to Reality: Measuring I & V

Measuring Current (THRU)



1. Hook in SERIES: current must go thru to measure
2. "Positive" if runs from Red to Black
3. Note: Not ideal – 1 Ω resistance. Does it matter?

Measuring Voltage (ACROSS)

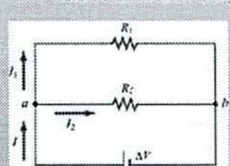


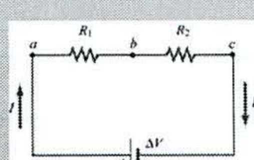
1. Hook in PARALLEL: reads $V_{Red} - V_{Black}$
2. Note: Not ideal – 1 MΩ resistance. Does it matter?

Red ~~[illegible]~~ – black

E3: Two Resistors

1. Set up resistors in (2) parallel and (3) series
2. Compare voltage and current from battery to voltage across and current through ONE resistor





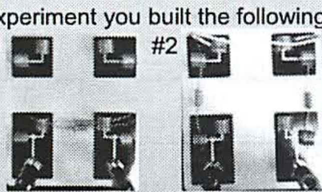
Experiment result

2 resistors series ↓ 2 parallel ↓

PRS: Expt. 1

In the experiment you built the following circuits:

#1 #2



How much current flowed in circuit 1 relative to circuit 2?

0%	1. Four times as much
0%	2. Twice as much
0%	3. The same
0%	4. Half as much
0%	5. One quarter as much

P14-28

1 $R_1 + R_2 = 2R$

2 $\frac{1}{R_1} + \frac{1}{R_2} = \frac{2}{R}$ ~~= 2R~~ $\frac{R}{2}$

$\frac{2}{R}$ inverse, looking for total

~~not important~~

$2R$ vs $\frac{R}{2}$ $= 4$

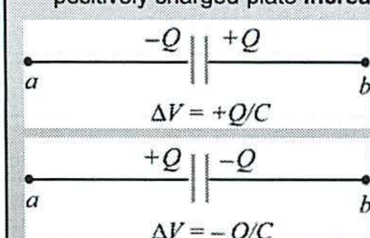
DC Circuits with Capacitors

P14-29

Did this class 12 end

Sign Conventions - Capacitor

Moving across a capacitor from the negatively to positively charged plate **increases** your potential

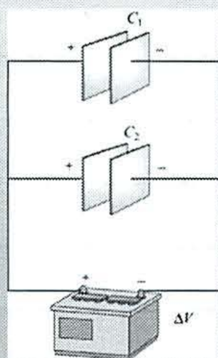


$\Delta V = V_b - V_a$

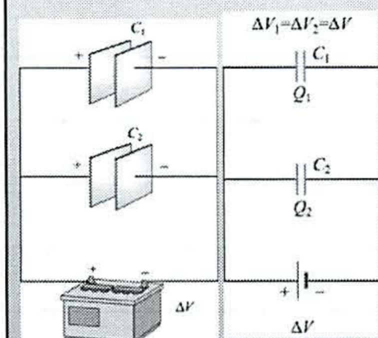
Think: Ski Lodge

P14-30

Capacitors in Parallel



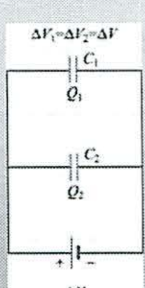
Capacitors in Parallel



Same potential!

$$C_1 = \frac{Q_1}{\Delta V},\ C_2 = \frac{Q_2}{\Delta V}$$

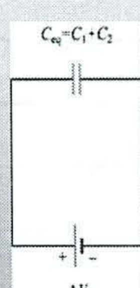
Equivalent Capacitance



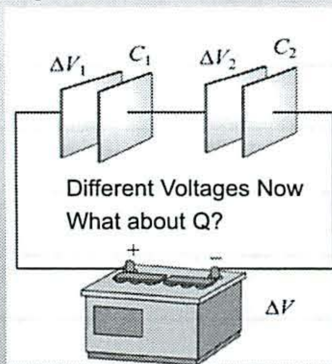
?

$$Q = Q_1 + Q_2 = C_1 \Delta V + C_2 \Delta V$$
$$= (C_1 + C_2)\Delta V$$

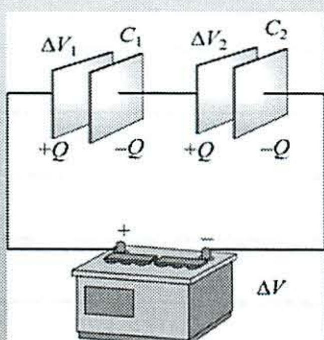
$$C_{eq} = \frac{Q}{\Delta V} = C_1 + C_2$$



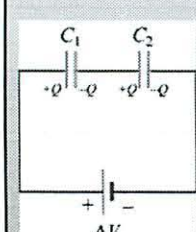
Capacitors in Series



Capacitors in Series



Equivalent Capacitance



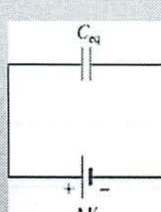
$$\Delta V_1 = \frac{Q}{C_1},\ \Delta V_2 = \frac{Q}{C_2}$$

$$\Delta V = \Delta V_1 + \Delta V_2$$

(voltage adds in series)

$$\Delta V = \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

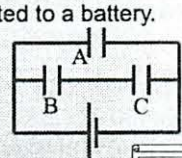
$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$



PRS Question: Capacitors in Series and Parallel

PRS: Capacitors

Three identical capacitors are connected to a battery. The battery is then disconnected. How do the charge on A, B & C compare before and after the battery is removed?

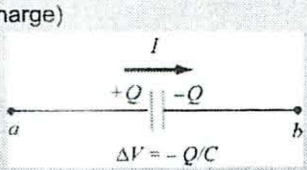


20

		BEFORE:	AFTER
0%	1.	$Q_A = Q_B = Q_C$;	No Change
0%	2.	$Q_A = Q_B = Q_C$;	$Q_A > Q_B = Q_C$
0%	3.	$Q_A = Q_B = Q_C$;	$Q_A < Q_B = Q_C$
0%	4.	$Q_A > Q_B = Q_C$;	No Change
0%	5.	$Q_A > Q_B = Q_C$;	$Q_A = Q_B = Q_C$
0%	6.	$Q_A < Q_B = Q_C$;	No Change
0%	7.	$Q_A < Q_B = Q_C$;	$Q_A = Q_B = Q_C$

Power - Capacitor

Moving across a capacitor from the positive to negative plate **decreases** your potential. If current flows in that direction the capacitor **absorbs** power (stores charge)



$$P_{\text{absorbed}} = I\,\Delta V = \frac{dQ}{dt}\frac{Q}{C} = \frac{d}{dt}\frac{Q^2}{2C} = \frac{dU}{dt}$$

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Experiment 3: Ohm's Law & DC Circuits

OBJECTIVES

1. To explore the measurement of voltage & current in circuits
2. To see Ohm's law in action for resistors
3. To learn how to translate circuit diagrams to physical circuits on a board

PRE-LAB READING

INTRODUCTION

When a battery is connected to a circuit consisting of wires and other circuit elements like resistors and capacitors, voltages can develop across those elements and currents can flow through them. In this lab we will investigate simple circuits with only resistors in them. We will confirm that there is a linear relationship between current through and potential difference across resistors (Ohm's law: $V = IR$).

The Details: Measuring Voltage and Current

Imagine you wish to measure the voltage drop across and current through a resistor in a circuit. To do so, you would use a voltmeter and an ammeter – similar devices that measure the amount of current flowing in one lead, through the device, and out the other lead. But they have an important difference. An ammeter has a very low resistance, so when placed in series with the resistor, the current measured is not significantly affected (Fig. 1a). A voltmeter, on the other hand, has a very high resistance, so when placed in parallel with the resistor (thus seeing the same voltage drop) it will draw only a very small amount of current (which it can convert to voltage using Ohm's Law $V_R = V_{meter} = I_{meter}R_{\text{meter}}$), and again will not appreciably change the circuit (Fig. 1b).

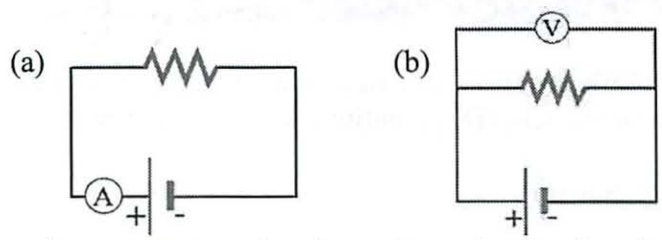


Figure 1: Measuring current and voltage in a simple circuit. To measure current *through* the resistor (a) the ammeter is placed in series with it. To measure the voltage drop *across* the resistor (b) the voltmeter is placed in parallel with it.

APPARATUS

1. Science Workshop 750 Interface

In this lab we will again use the Science Workshop 750 interface to create a "variable battery" which we can turn on and off, whose voltage we can change and whose current we can measure.

2. AC/DC Electronics Lab Circuit Board

We will also use, for the first of several times, the circuit board pictured in Fig. 2. This is a general purpose board, with (A) battery holders, (B) light bulbs, (C) a push button switch, (D) a variable resistor called a potentiometer, and (E) an inductor. It also has (F) a set of 8 isolated pads with spring connectors that circuit components like resistors can easily be pushed into. Each pad has two spring connectors connected by a wire (as indicated by the white lines). The right-most pads also have banana plug receptacles, which we will use to connect to the output of the 750.

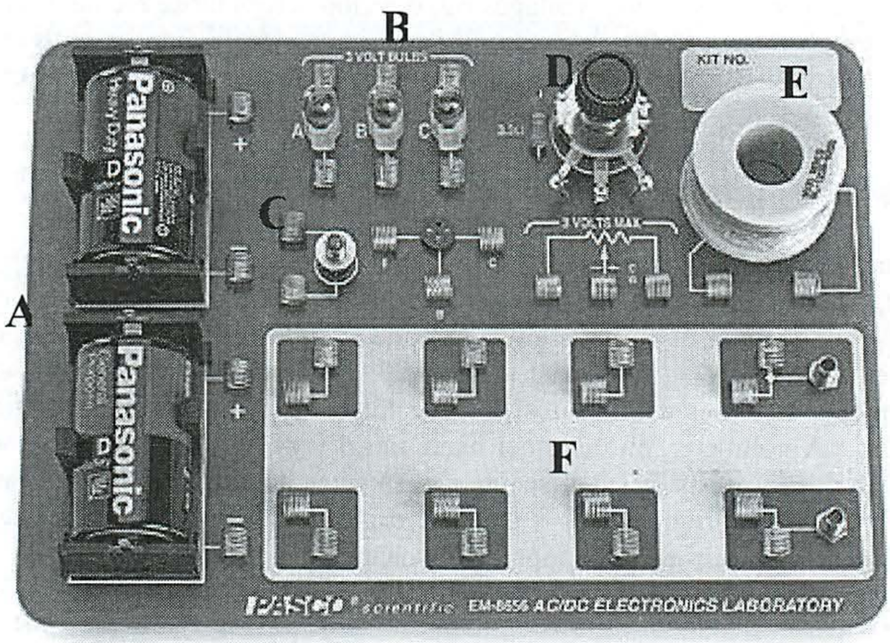


Figure 2 The AC/DC Electronics Lab Circuit Board, with (A) Battery holders, (B) light bulbs, (C) push button switch, (D) potentiometer, (E) inductor and (F) connector pads

3. Current & Voltage Sensors

Recall that both current and voltage sensors follow the convention that red is "positive" and black "negative." That is, the current sensor records currents flowing in the red lead and out the black as positive. The voltage sensor measures the potential at the red lead minus that at the black lead.

red (+)
black (−)

(a) 

(b) 

Figure 3 (a) Current and (b) Voltage Sensors

4. Resistors

Resistors (Fig. 4) have color bands that indicate their value. In this lab we ask you to ignore the bands – even if you know how to read them please do not do so.

Figure 4 Example of a resistor. Aside from their size, most resistors look the same, with 4 or 5 colored bands indicating the resistance.

GENERALIZED PROCEDURE

This lab consists of two main parts. In each you will set up a circuit and measure voltage and current.

Part 1: Measure Voltage Across & Current Through a Resistor
Here you will measure the voltage drop across and current through a single resistor attached to the output of the 750.

Part 2: Resistors in Parallel
Now attach a second resistor in parallel to the first and see what happens to the voltage drop across and current through the first.

END OF PRE-LAB READING

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

1. Download the LabView file from the web and save the file to your desktop (right click on the link and choose "Save Target As" to the desktop. Overwrite any file by this name that is already there). Start LabView by double clicking on this file.
2. Connect the Voltage Sensor to Analog Channel A on the 750 Interface and the Current Sensor to Analog Channel B.
3. Connect cables from the output of the 750 to the banana plug receptacles on the lower right side of the circuit board (red to the sin wave marked output, black to ground).

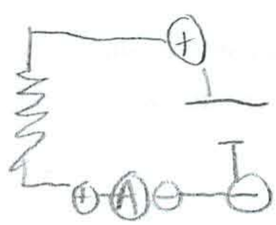
MEASUREMENTS

Part 1: Measuring the Resistance of a Single Resistor

1. Hook up a circuit to measure the voltage across and current through a single resistor driven by the "battery."
2. Record *V* and *I* for 1 second. (Press the green "Go" button above the graph). During this time the battery will switch between putting out 1 Volt and 0 Volts.

Question 1:

When the battery is "on" what is the voltage drop across the resistor and what is the current through it? What is the resistance of the resistor (calculate it from what you just measured, do NOT figure it out from the color code, which can be inaccurate).



$I = 10.059\, mA$

$V = .984\, V$

$R = \frac{V}{I} = \frac{.984}{10 \cdot 10^{-3}} = 97.8$ ohms

Part 2: Testing Ohm's Law

1. Use the same circuit from part 1
2. Choose signal generation parameters (waveform, frequency and amplitude) that you think will help you test Ohm's law $V = IR$
3. Record *V* and *I* for 1 second. (Press the green "Go" button above the graph). During this time the battery will output the waveform that you have selected.

– will change voltage w/ time

∴ So resistance does not change – so change in current

– Voltage + current in direct proportion → Slope is resistance

resistance is slope - does not change

Question 2:

Given the possibilities you are presented with, what do you think is the best way to test Ohm's law? What waveform, frequency, amplitude and plot do you use? Is Ohm's law valid for your resistor? How do you extract the resistance of the resistor using your method? What is it?

V vs I linear relationship
R is the slope (does not change)
- first part: current at one voltage
- this part: changing V - ohm's law still true

Part 3: Resistors in Parallel

1. Hook up a circuit to measure the voltage across and current through the first resistor connected in parallel to a second resistor
2. Record V and I for 1 second. (Press the green "Go" button above the graph). During this time the battery will switch between putting out 1 Volt and 0 Volts.

Only V of 1 resistor

Question 3:

When the battery is "on" what is the voltage drop across the first resistor and what is the current through it? Did these values change from Part 1? Why or why not?

V = .493 V (half)
I = 5.8 mA (half as well) ← why?

Question 4:

If it did change: is there something you could measure that wouldn't change?

If it didn't change: is there something you could measure that would change?

Voltage drop across both resistors
~~Current across both should be same~~
↳ V = I R (same, ½, ×2) Everywhere current will be ½ (moves slower)

Further Questions (for experiment, thought, future exam questions...)

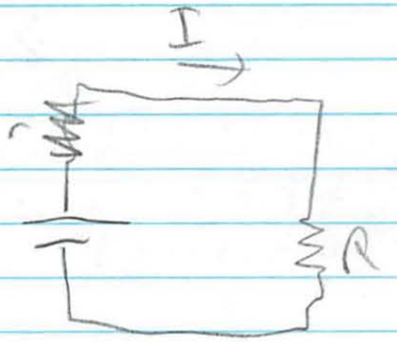
- The ammeter is marked as having a 1 ohm resistance, small, but not tiny. Can you see the effects of the ammeter resistance in the circuits of part 1 and 2? Can you measure the voltage drop across the ammeter? Does this make the measurement of the current through the resistor inaccurate?
- What happens if we instead put the second resistor in series with the first?

Office Hrs
on P-set 5

3/8

eeeda';

this is lab



$\mathcal{E} - Ir - IR = 0$

5b on pset half started

$I = \frac{\mathcal{E}}{r+R}$

↑ of course this should be 0 for max

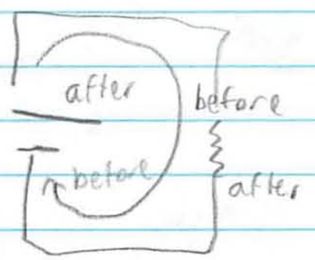
durmation office hrs

conventions: pick circulation ↻ dir

$\Sigma V = 0$

$\Delta V = V_{after} - V_{before}$

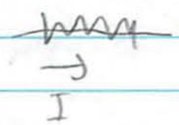
determined by circ direction



direction shows which is before + after

2. Choose a ⊕ direction for current in each branch
- if get a − sign that just means go other way

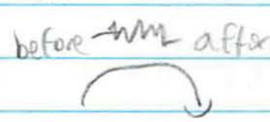
3. Resistors

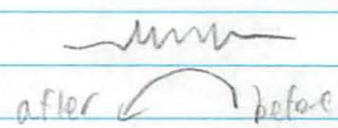


so ⊕ ⊖

Voltage = V(after) − V(before)
↑ ✗ have not chosen a circulation direction

So





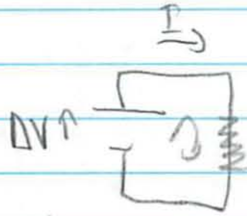
I →

$= -IR$

I →

$= +IR$

If go in same dir current voltage ↓
opposit " " " ↑



← ⊖ since crossing in <u>same</u> dir of current

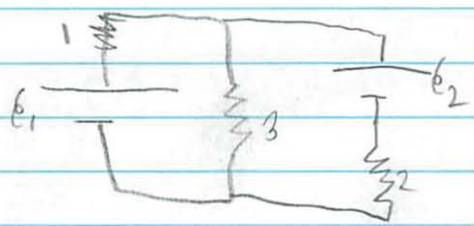
does not depend on current

$$\mathcal{E} - I r_i - I R_L = 0$$

$$I = \frac{\mathcal{E}}{r_i + R_L}$$

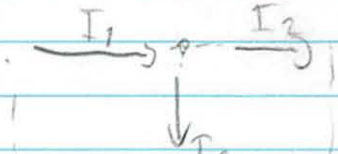
For multiloop circuits

If go around closed path - field is path independent - no work done - back to 0
Kirchif's law



1. Choose circ dir

3 currents flow in this loop



Whatever comes into a junction pt
every thing must add to 0

$$I_1 = I_3 + I_2$$

$$\varepsilon_1 - I_1 R_1 - I_3 R_3 = 0$$

$$-\varepsilon_2 - I_2 - R_2 + I_3 R_3 = 0$$

3 eq
3 unknown

going from (+) to (−)

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8.02 **Spring 2010**

Problem Set 5

Due: Tuesday, March 9 at 9 pm.

Hand in your problem set in your section slot in the boxes outside the door of 32-082. Make sure you clearly write your name and section on your problem set.

Text: Liao, Dourmashkin, Belcher; Introduction to E & M MIT 8.02 Course Notes.

Week Five Conductors as Shields; Current and Ohm's Law

Class 11 W05D1 M/T Mar 1/2	Conductors as Shields; Expt. 2: Faraday Ice Pail; Capacitors and Dielectrics
Reading:	Course Notes: Sections 4.3-4.4; 5.5, 5.9, 5.10.2
Experiment:	Expt. 2: Faraday Ice Pail
Class 12 W05D2 W/R Mar 3/4	Current, Current Density, and Resistance and Ohm's Law; DC Circuits
Reading:	Course Notes: Sections 6.1-6.5; 7.1-7.4
Class 13 W05D3 F Mar 5:	PS04: PHET: Building a Simple DC Circuit
Reading:	Course Notes: Sections 6.1-6.5; 7.1-7.4

Add Date Mar 5

Week Six DC Circuits

Class 14 W06D1 M/T Mar 8/9	Expt. 3 Building a Circuit with Resistors, DC Circuits & Kirchhoff's Loop Rules;
Reading:	Course Notes: Sections 7.1-7.5, 7.8-7.9
Experiment:	Expt. 3 Building a Circuit with Resistors
Class 15 W06D2 W/R Mar 10/11	RC Circuits; Expt. 4: RC Circuits
Reading:	Course Notes: Sections 7.5 – 7.6
Experiment:	Expt. 4: RC Circuits
Class 16 W06D3 F Mar 12	PS05: RC Circuits
Reading:	Course Notes: Sections 7.1 – 7.6, 7.8-7.9

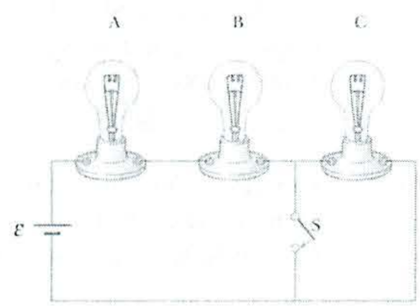
Problem 1: Short Questions

a) Why is it possible for a bird to stand on a high-voltage wire without getting electrocuted?

b) If your car's headlights are on when you start the ignition, why do they dim while the car is starting?

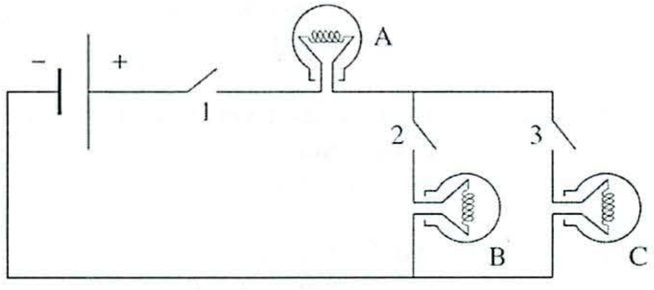
c) Suppose a person falling from a building on the way down grabs a high-voltage wire. If the wire supports him as he hangs from it, will he be electrocuted? If the wire then breaks, should he continue to hold onto the end of the wire as he falls?

d) A series circuit consists of three identical lamps connected to a battery as shown in the figure below. When the switch S is closed, what happens to the brightness of the light bulbs? Explain your answer.



Problem 2: Circuit

The circuit below consists of a battery (with negligible internal resistance), three incandescent light bulbs (A, B & C) each with exactly the same resistance, and three switches (1, 2, & 3). In what follows, you may assume that, regardless of how much current flows through a given light bulb, its resistance remains unchanged. Assume that when current flows through a light bulb that it glows. The higher the current, the brighter the light will be.



In each situation (a, b, c) as described below, we want to know which light bulbs are glowing (and which are not) and how bright they are (relative to each other). *Always briefly discuss your reasoning.*

a) Switch #1 is closed; the others are open.

b) Switches #1 & #2 are closed; #3 is open.

c) All three switches are closed.

d) Now compare situations a, b & c. Which bulb is brightest of all, and which is faintest of all (bulbs which are off don't count).

Now replace bulb A by a wire of negligible resistance. We still have three switches and now two light bulbs (B & C).

e) Answer the questions b) through d) again for this situation.

Problem 3: Ohm's Law

A straight cylindrical wire lying along the *x*-axis has a length L and a diameter d. It is made of a material described by Ohm's law with a resistivity ρ. Assume that a potential V is maintained at $x = 0$, and that $V = 0$ at $x = L$. In terms of L, d, V, ρ, and physical constants, determine expressions for

a) the electric field in the wire.

b) the resistance of the wire.

c) the electric current in the wire.

d) the current density in the wire. Express vectors in vector notation.

e) Show that $\vec{\mathbf{E}} = \rho\vec{\mathbf{J}}$.

Problem 4: Resistance of Conductor in Telegraph Cable

.The first telegraphic messages crossed the Atlantic Ocean in 1858, by a cable 3000 km long laid between Newfoundland and Ireland. The conductor in this cable consisted of seven copper wires, each of diameter 0.73 mm, bundled together and surrounded by an insulating sheath. Calculate the resistance of the conductor. Use $3\times10^{-8}\ \Omega\cdot\text{m}$ for the resistivity of copper, which was of somewhat dubious purity.

Problem 5: Current, Energy and Power A battery of emf $\mathcal{E}$ has internal resistance R_i, and let us suppose that it can provide the emf to a total charge Q before it expires. Suppose that it is connected by wires with negligible resistance to an external (load) with resistance R_L.

a) What is the current in the circuit?

b) What value of R_L maximizes the current extracted from the battery, and how much chemical energy is generated in the battery before it expires?

c) What value of R_L maximizes the total power delivered to the load, and how much energy is delivered to the load before it expires? How does this compare to the energy generated in the battery before it expires?

d) What value for the resistance in the load R_L would you need if you want to deliver 90% of the chemical energy generated in the battery to the load? What current should flow? How does the power delivered to the load now compare to the maximum power output you found in part c)?

Problem 6: Battery Life AAA, AA, …, D batteries have an open circuit voltage (emf) of 1.5 V. The difference between different sizes is in their lifetime (total energy storage). A AAA battery has a life of about 0.5 A-hr while a D battery has a life of about 10 A-hr. Of course these numbers depend on how quickly you discharge them and on the manufacturer, but these numbers are roughly correct. One important difference between batteries is their internal resistance – alkaline (now the standard) D cells are about 0.1Ω. Suppose that you have a multi-speed winch that is 50% efficient (50% of energy used does useful work) run off a D cell, and that you are trying to lift a mass of 60 kg (hmmm, I wonder what mass that would be). The winch acts as load with a variable resistance R_L that is speed dependent.

a) Suppose the winch is set to super-slow speed. Then the load (winch motor) resistance is much greater than the battery's internal resistance and you can assume that there is no loss of energy to internal resistance. How high can the winch lift the mass before discharging the battery?

b) To what resistance R_L should the winch be set in order to have the battery lift the mass at the fastest rate? What is this fastest rate (m/sec)? HINT: You want to maximize the power delivery to the winch (power dissipated by R_L).

found V and Power

c) At this fastest lift rate how high can the winch lift the mass before discharging the battery?

have energy power, find time · V = distance

d) Compare the cost of powering a desk light with D cells as opposed to plugging it into the wall. Does it make sense to use rechargeable batteries? Residential electricity costs about $0.1/kwh.

Problem 7: Faraday Cage

Consider two nested, spherical conducting shells. The first has inner radius a and outer radius b. The second has inner radius c and outer radius d.

In the following four situations, determine the total charge on each of the faces of the conducting spheres (inner and outer for each), as well as the electric field and potential everywhere in space (as a function of distance r from the center of the spherical shells). In all cases the shells begin uncharged, and a charge is then instantly introduced somewhere.

a) Both shells are floating – that is, their net charge will remain fixed. A positive charge +Q is introduced into the center of the inner spherical shell. Take the zero of potential to be at infinity.

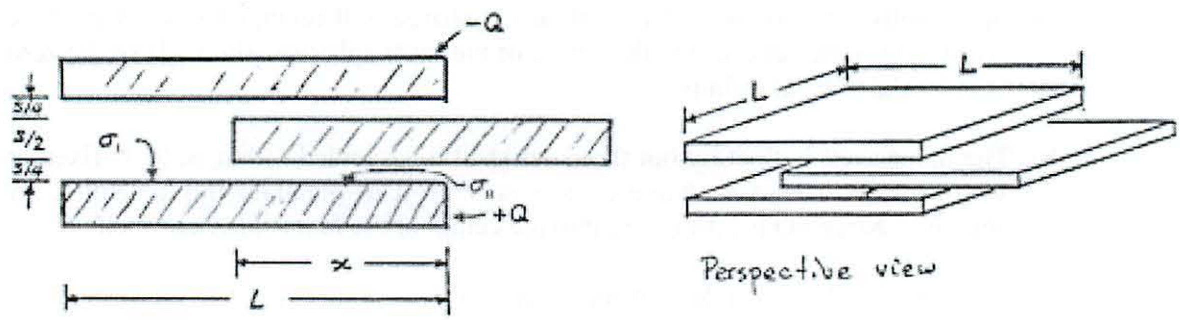
b) The inner shell is floating but the outer shell is grounded – that is, it is fixed at V=0 and has whatever charge is necessary on it to maintain this potential. A negative charge –Q is introduced into the center of the inner spherical shell.

c) The inner shell is grounded but the outer shell is floating. A positive charge +Q is introduced into the center of the inner spherical shell.

d) Finally, the outer shell is grounded and the inner shell is floating. This time the positive charge +Q is introduced into the region in between the two shells. In this case the questions "What is **E**(r)/V(r)?" are not well defined in some regions of space. In the regions where these questions can be answered, answer them. In the regions where they can't be answered, explain why, and give as much information about the potential as possible (is it positive or negative, for example).

Problem 8: Capacitance, Work and Energy

Two flat, square metal plates have sides of length L, and thickness $s/2$, are arranged parallel to each other with a separation of s, where $s \ll L$ so you may ignore fringing fields. A charge Q is moved from the upper plate to the lower plate. Now a force is applied to a third uncharged conducting plate of the same thickness $s/2$ so that it lies between the other two plates to a depth x, maintaining the same spacing $s/4$ between its surface and the surfaces of the other two. You may neglect edge effects.



a) Using the fact that the metals are equipotential surfaces, what are the surface charge densities σ_L on the lower plate adjacent to the wide gap and σ_R on the lower plate adjacent to the narrow gap?

b) What is the electric field in the wide and narrow gaps? Express your answer in terms of L, x, and s.

c) What is the potential difference between the lower plate and the upper plate?

d) What is the capacitance of this system?

e) How much energy is stored in the electric field?

1 8.02 Pset 5 Hint

Hi L08 problem-solvers,
========
Hints for Pset 5
========

prob 1-a) small distance thus small voltage difference.
comparing the resistance.

prob 1-b)
starter motor needs energy

prob 1-c)
i) comparing resistance, ii) check whether there is a close circuit for current. iii) grounded makes current flow to the ground.

prob 1-d)
resistance is the same, the power($P = I^2R$)proportional to the resistance

prob 2) again, compare the current

prob 3) Here we have two exact physical formulas:
(i) $E_x = -dV/dx$, (ii) $I = JA$,
two empirical formulas:
(iii)$R = V/I$ (i.e. the definition of resistance, macroscopic view of resistance),
(iv)$\rho = E/J$ (i.e. definition of resistivity, microscopic view of resistivity).
From (iii) and (iv) together give us a relation between R and ρ.

The above 4 eqs constraint 4 degrees of freedom, thus 4 unknowns E, R, I, J can be written as the remained parameters L, d, V, ρ.

prob 4) i) resistors in parallel, ii)resistance proportional to cross section area

prob 7) if there is no net charge initially, floating shell remains zero net charge, grounded metal may bot be neutral.

a) no E field inside the conductor interior
b), c) grounded implies V=0
d) Think about the potential landscape. Think inner and outer shells separately, once you understand each case, then combine two cases together. Find: Potential V=0 for r¿c. potential is highest at the source charge Q. potential V=positive constant for r¡b.

prob 8) The below i) ii) iii) give you sets of equation, you can then solve charge density distribution, thus solve a) b) c) d) e) in order.

i) By equipotential of the conductor, so that potential difference for LHS of two plates is the same as the potential difference for RHS of three plates. Note: E=0 inside the conductor.

ii) By symmetry of (-Q on the top and +Q on the bottom). You know the charge distribution on top plate is the same as charge distribution on bottom plate, up to a minus sign.

iii) Sum over charge density equals to total charge, given as -Q and +Q for top and bottom plates.

e) two methods:
i) charging up process, $U_E = \int dqV$, integration
ii) E^2 volume integration, ie. $U_E = \int \frac{1}{2}\epsilon_0 E^2 d(Volume)$

If you have free time, challenge yourself with the following.

[Hard] prob 7-d)
It will be a challenging problem to find out the analytic form of potential and electric field between r=b and r=c for prob 7d. the exact potential(and thus its negative gradient, the E field), can be obtained from "Method of Image" + "Superposition principle". There will be a series of image charge. since we have two mirrors here(inner shell and outer shell), there are many images of image($\hat{n}$) charge. One can expect certain geometric series sum of potential can lead to the exact analytic full potential. Normally we will start from assuming inner shell and outer shell are grounded for simplicity, but here specially need to be aware that the inner shell is not grounded, so the inner shell must be neutral, need to artificially provide a the same negative amount of surface charge well-distributed on the surface to cancel the amount of total charge on the outer surface of inner shell(which charge sum is equal to the sum of image charges inside the inner shell).

Good Ref: Chap 3-2, Method of Images Griffiths, Introduction to Electrodynamics.
(Indeed to sovel prob 8 analytically is a bit above Griffiths level.)

prob 8-e) You find out the minimum stored energy is at x=L, then you know it is stable for inserting the 3rd plate entirely into the middle of two plates. you also know perturbing around a stable equilibrium point would experience a restoring force. You can ask what's the motion and the periodicity T_{period} for this motion.

You can find out: For small Δx perturbation($|\Delta x| \ll L$), $U = \frac{Q^2 s}{4\epsilon_0 L^2(1-\frac{|\Delta x|}{2L})} \simeq \frac{Q^2 s}{4\epsilon_0 L^2}(1+\frac{|\Delta x|}{2L})$;
thus $F_x = -dU/dx = -\frac{Q^2 s}{8\epsilon_0 L^3}$, thus $a_x = F_x/m = -\frac{Q^2 s}{8m\epsilon_0 L^3}$.

We find it is indeed a constant acceleration! (surprisingly, not Simple Harmonic Motion). Where, $\Delta x = 1/2at^2$, so $t = \sqrt{\frac{2\Delta x}{a}}$
$T_{period} = 4t = 4\sqrt{\frac{2\Delta x}{a}} = 4\sqrt{\frac{16m\epsilon_0 L^3}{Q^2 s}\Delta x}$. (You can check by plugging in a by yourself.)

PSet 5 100-3 = (97)

Michael Plasmeier 11C L01 3/6/10

1. Short Question

a) Why can a bird stand on high voltage wire without dying?

✓

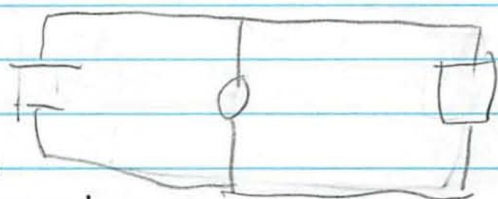
Because they are not touching the ground.

The current takes the path of least resistance which is the wire, not the bird
↑ right

b) Why do your car headlights dim when you start the car?

The car starting draws new power

$$\text{Power} = \frac{\Delta W}{\Delta t} = IV$$



Voltage drop across same for both

Like another exit from theater

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

each item different resistance

current adds

It's great that you work so hard and try so many different things, but could you somehow show your final answer and work? box it, hilight it, — whatever. it's hard to keep track of what's going on. Sometimes I think you did something wrong only to see you figured it out 3 pages later. It would help both of us.
— Chris

So voltage is same

$P = I V$

↑ ↑ ↑same

increases | must theirfore also increase

= current · voltage

Light from bulb ~~proportional to voltage~~

Some new current brought to new branch

(margin: ?! asked same one from last semester / looked online)

alternators take > 100 amps (high current)
lots of energy needed to pass this voltage through battery
potential diff drops + lights dim

due to batt has low internal resistance

(margin, in red: exactly the key →) if (high internal resistance) this is more noticible at lower current drops

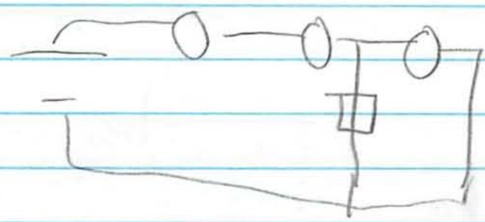
★ So it looks like what I was missing was batt internal resistance ★

$V_T = \mathcal{E} - Ir$

c) A person falls and grabs high voltage wire. If it supports him as he hangs, does he die? If it breaks and he lands, does he die?

No in both cases. He does not die if the wire is connected and he does not touch the ground. Also once the wire breaks he is no longer in danger. The field no longer pushes the charge, so he is safe.

b) A series circuit 3 lamps



Bulbs 1 and 2 increase in brightness
3 goes off

The current now finds a path of lower resistance by going through the switch instead through bulb 3

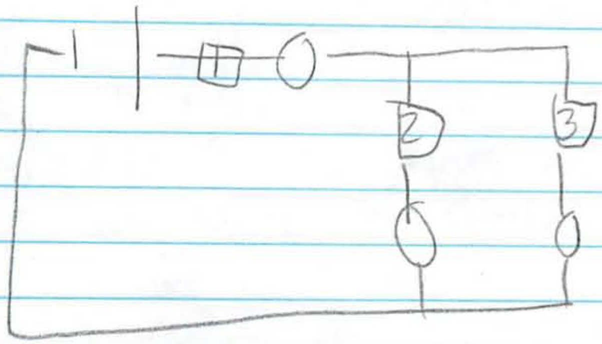
These lights are in series

$$V = I(R_1 + R_2)$$

↑ same ↑ increases, thus lights brighter; ↖ lowers when R_3 goes to 0

* Brightness = current · voltage = watt *
= power

2.



Resistance in all 3 bulbs = and unchanging

a) Switch 1 closed otters open

No closed circuit, no flow, no light

b) Switch 1 + 2 closed 3 open

We have 2 bulbs in series

$V = I(R_1 + R_2)$

1 and 2 glowing at same rate

c) All switches

All 3 bulbs are on
Bulb 1 is twice the brightness (current) of
2 and 3 which are =

d Which was the brightest across situations?
↳ power = voltage · current

(Built test curicit)

3A = .6 amp ← greatest
3BC = .3 Amp ← least

# = problem
letter = bulb

2A = .45 amp
2B = .45 amp

e) Now replace A w/ wire

a) Still no complete circut

b) only bulb B is on at .9 amps

* height diff = voltage drop *

c) Bulb B and C both on at = brightness of .9 amp
(batt can give as much current as need - it ideal)

d) They are all same brightness (parallel circuit)
"more exits to a theater"

3. Ohm's Law: A straight wire has lenght l & ρ

but lenght of wire matters

Voltage at x=0 and 0 at x=L

d) Express field on wire L, d, V, ρ

(review this

$$\vec{J} = \sigma \vec{E}$$

↑ external electric field

$$\rho = \frac{1}{\sigma}$$

that should be $\vec{\text{end}}$)

charge density → $\vec{J} = \frac{E}{\rho}$ → $\vec{E} = \vec{J}\rho$

b) Resistance of wire

$$\sigma = \frac{n e^2 \tau}{m_e} \qquad \rho = \frac{m_e}{n e^2 \tau}$$

↑ from finding drift velocity

$$I = \iint \vec{J} \cdot d\vec{A}$$

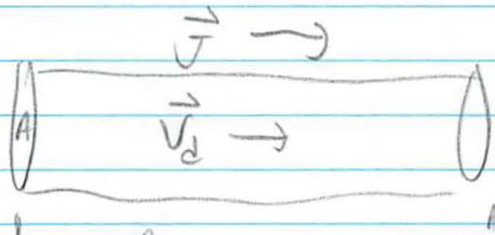
↑ current density A/m^2

q = charge of carriers

n = # of carriers moving at V_d

Charge $\Delta Q = q(nA\Delta x)$

↑ $V_d \Delta t$



$\Delta x = V_d t$

$$I_{avg} = \frac{\Delta Q}{\Delta t} = n q V_d A$$

but does not travel in a straight line

$$\vec{J} = n q \vec{v_d}$$

Well electrons feel force $\vec{a} = \frac{\vec{F_e}}{m_e} = \frac{-e\vec{E}}{m_e}$

velocity before next collision

$$\vec{V_f} = \vec{V_i} + \vec{a}t = V_i - \frac{e\vec{E}}{m_e}t$$

So overall average v_f

$$\langle v_f \rangle = \langle v_i \rangle - \frac{eE}{m_e} \langle t \rangle = v_d$$

When no field $\langle v_i \rangle = 0$

When $\tau = \langle t \rangle$ = mean time before collisions

$$v_d = \langle v_f \rangle = -\frac{eE}{m_e} \tau$$

So currend density

$$\vec{J} = -ne\vec{v_d} = -ne\left(\frac{e\vec{E}}{m_e} \tau\right) = \frac{ne^2\tau}{m_e} \vec{E}$$

c Current in wire

$$I_{avg} = n q v_d A \qquad I = \frac{dQ}{dt}$$

d Current density in wire

$$\vec{J} = n q \vec{v_d} = \sigma \vec{E}$$

e Show $\vec{E} = \rho \vec{J}$

$$\vec{J} = \frac{ne^2\tau}{m_e} \vec{E} \qquad \vec{E} = \rho J$$

$$\rho = \frac{m_e}{ne^2\tau} \qquad \left(\frac{m_e}{ne^2\tau}\right)\left(\frac{ne^2\tau}{m_e}\right)\vec{E}$$

$$\vec{E} = \vec{E}$$

Hints for 3

Two Formulas

$$E_x = -\frac{dV}{dx} \qquad I = JA$$

$$R = \frac{V}{I} \text{ (resistance macroscopic)} \qquad \rho = \frac{E}{J} \text{ (resistivity microscopic)}$$

I don't really get this one

$E = -\nabla V$ ← Says we should start with

(+) downhill

qu hour

(redo)

potential linear

but that goes w/ $I = \iint \vec{J} \cdot d\vec{A}$ right?

? what do we start from?

3. $V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{s}$

retry

$V_{x=L} - V_{x=0} = \int_0^L \vec{E} \cdot ds = -EL$

$V_{x=L} = 0$

missed that step in book

$-V_{x=0} = -V = -EL$

$$|E| = \frac{V}{L} \hat{x}$$

✓

b $R = \frac{\rho L}{A} = \frac{\rho L}{\pi (\frac{d}{2})^2} = \frac{\rho L}{\frac{\pi d^2}{4}} = \frac{4 \rho L}{\pi d^2}$

duh - obvious

c $I = \frac{V}{R} = \frac{V \pi d^2}{4 \rho L}$

d. $|\vec{J}| = \frac{I}{A} = \frac{\frac{V \pi d^2}{4 \rho L}}{\left(\frac{\pi d^2}{4}\right)} = \frac{V \cancel{\pi d^2}}{\cancel{4} \rho L} \cdot \frac{\cancel{4}}{\cancel{\pi d^2}} = \frac{V}{\rho L} \hat{x}$

e $\frac{\vec{E}}{\vec{J}} = \frac{\frac{V}{L}}{\frac{V}{\rho L}} = \frac{\cancel{V}}{\cancel{L}} \cdot \frac{\rho \cancel{L}}{\cancel{V}} = \rho$

much better!

$E = \rho \vec{J}$

4. Resistance of Conductor in Telegraph

7 wires diameter .77 mm
Find resistance $3 \cdot 10^{-8}\ \Omega$
3000 km

$$R = \frac{\rho L}{A}$$

$$\frac{3 \cdot 10^{-8} \cdot 3000\,000}{\pi \cdot \frac{.00773^2}{4}}$$

2150 each wire

$$\frac{1}{2150} + \frac{1}{2150} + \ldots$$

$$2150^{-1} \cdot 7$$

.003255

Course Notes
Book $A = N\pi r^2 = N\frac{\pi d^2}{4}$

so directly 7 times area here

$$R = \frac{\rho l}{A} = \frac{3 \cdot 10^{-8}\ \Omega \cdot m \cdot 3000\,000\ m}{7 \cdot \pi \frac{(.00073^2)}{4}}$$ ← unit error

so basically only mistake is where the 7
area 7 times

$= 30719\ \Omega$

5 - general
6 - application

internal

5. Battery $\mathcal{E}$ r R_L ← load

a) Current in circuit
at terminals

① draw
② Pick a circ dir

$V_t = \mathcal{E} - Ir$

$IR = \mathcal{E} - Ir$

reveiw

$IR + Ir = \mathcal{E}$

$I = \frac{\mathcal{E}}{R+r}$

$I = \frac{\mathcal{E}}{R+r}$ ✓

b) What value of R_L maximises current from battery and how much energy before it expires.

$I = \frac{\mathcal{E}}{R+r}$ see solns. -3

Work = $\Delta U = U_E$
$= q\Delta V$

$I = \frac{q}{t}$

the smaller the value, the greater the current, (max current when no internal resistance)

Energy $U_E = \Delta W = q\Delta V = IVt = Pt$

~~$\mathcal{E}q = \int I$~~

~~$U_E = \Delta V \int I \cdot dt$~~

well Q given

~~$U_E = \int \frac{\mathcal{E}}{2R+2r} dt \int E\, ds$~~ or is it

no.

(you need more quantities to solve)

- well property of battery

$U_E = \frac{\mathcal{E}}{(R+r)} \cdot V \cdot t = QV$ $Q = \frac{\mathcal{E}}{(R+r)} t$

c) What value of R_L maximizes total power to load before it expires?

power = $\frac{\Delta W}{\Delta t}$ =

$\Delta V I$
= rate at which energy is disabled

$U = \Delta W = q\Delta V = IVt = Pt$

$P_{light bulb} = I V_{across bulb} = I^2 R$ of load

↑ maximize ↑ of bulb

$P = I^2 R$

↑ dependent on R! (internal resistance)

like problem 6B - take deriv to find maximum or find critical pt / max on calc

$$\frac{dP}{dR} = \left(\frac{\varepsilon}{(R+r)}\right)^2 R$$

$$= R\varepsilon^2 (r+R)^{-2}$$

$$P = \varepsilon^2 \cdot 1 (r+R)^{-2} + -2(r+R)^{-3} \cdot 1 \cdot R\varepsilon^2$$

$$P' = \frac{\varepsilon^2}{(r+R)^2} - \frac{2R\varepsilon^2}{(r+R)^2}$$

↑ set = to 0 to find critical pts

$$0 = \frac{\varepsilon^2}{(r+R)^2} - \frac{2R\varepsilon^2}{(r+R)^2}$$

calc solve

$$0 = \varepsilon^2 - 2R\varepsilon^2$$

$$\frac{\varepsilon^2}{\varepsilon^2} = \frac{2R\varepsilon^2}{\varepsilon^2}$$

$$1 = 2R$$

$$R = \frac{1}{2}$$ r?

Are we using a D battery?

$$\Delta h = .5 \frac{I^2 R_L}{mg}$$

10 ~~amp hours~~ = Q

10 amp = $\frac{1 \text{ colomb}}{\text{seconds}} \cdot \frac{60 \cdot 60 \text{ seconds}}{\text{hour}} = \frac{36000 \text{ colombs}}{\text{~~hour~~}}$

amp-hour unit of electric charge Q
1 amp hr = 3600 colombs
charge transfered by steady current of 1amp for 1hr

$$\Delta h = .5 \frac{\left(\frac{Q}{t}\right)^2 R t}{mg}$$

$$\Delta h = .5 \frac{\left(\frac{10 \text{Amp hrs}}{t}\right)^2 R}{mg}$$

? are h and t not both related
- need to calc when batt runs out of charge

$$Q = \int_0^{\text{inf}} I \, dt$$

ATA

or howmuch energy in batt

$E = \frac{1}{2} QV$
$P = I \varepsilon$

Work $= q \int_i^f E \, dr$
$= q \Delta V$

q ↑ 36000 colombs; ΔV ↑ 1.5V

How much energy is this?

$$U = \frac{\mathcal{E}}{R+r} Vt = Q \cdot V$$

$\uparrow t_{1/2}$; so could find time

↑ ↑ given from battery

Also it asks energy of battery vs energy of load.

? ask about 5 and 6

- voltage is the same
- but is more energy used up in battery (due to internal resistance) than powers the load
- but how to represent that in a formula?

Dormashkin

Conservation of energy

- taking charges ⊕ moving inside batt raising PE $\frac{dq}{dt} \cdot \mathcal{E} = q\mathcal{E}$
- gets disapated over circuit

$$P = \frac{dU}{dt} = \frac{dq}{dt}\mathcal{E} = I\mathcal{E}$$

↑ can figure out how long it will run

$$U = Pt = I\mathcal{E}t$$

- some E → load
  ↳ internal resistance

~~~~
~~~~

5Bb redo

dormaskin OH

$$I = \frac{\mathcal{E}}{r_i + R_L}$$

↑ largest when no resistance ✓ got this

Power in battery

$$\frac{du}{dt} = \frac{dq}{dt}\mathcal{E} = \text{Power}_{\text{battery}} = I\mathcal{E}$$

Power dissipated by load
- can lose to thermal
- or motor can lift weight

) efficency

$$\frac{du}{dt} = \left|\frac{dq}{dt}\Delta V_{load}\right| = I|V_{load}| = I^2R$$

↑ how much PE going to something else

$$U_{batt} = \text{Power}_{\text{battery}} \cdot \text{time}$$

$$U_{gen} = (I\mathcal{E})t = Q\mathcal{E}$$

~~↑ not interested in voltage~~

Dormaskin's writing

$$* \ \mathcal{E} = V_{batt} \ *$$

5Bc redo

$$P_L = I^2R_L = \frac{\mathcal{E}^2}{(r_i + R_L)^2}R_L$$

↑ maximize

take deriv R_L
Product rule

$$r_i = R_L$$

↑ maximizing power to load "interesting"
half and half

$$\text{Power}_{load} = \frac{\mathcal{E}^2}{4R_L}$$

Energy
- batt generated
- some dissipated internal resistances
- rest dissipated to load

since $r_i = R_L$ half and half
Energy disipated

2

so E to load can find

$$I^2 r = I^2 R$$

- when not same next qu 2

d Condition

90% load $P = I^2 R$

10% internal resistance $P = I^2 r$

↑ same I both

$R_L = 9 r_i$

If resistances $= \frac{\mathcal{E}}{2} = I_{max}$

If resistances not =

$$I = \frac{\mathcal{E}}{r_i + R_L} = \frac{\mathcal{E}}{10 R_i}$$

only getting 20% current before

review energy argument
- moving charges across potential
- lots of Bio is this

Power to load $I^2 R_L = \frac{\mathcal{E}^2}{(r_i + 9 r_i)} 9 r_i$

fast = lots of energy wasted internally
slow = less energy wasted internally

day after review
So confused
Sun confused one more
I don't want think about this
-will see when results posted

d

redo

$$\Delta V_{load} = Q\mathcal{E}\left(\frac{R_L}{R_L+R_i}\right) = V_c = \left(\frac{R_L}{R_L+R_i}\right) = 90\% V_c$$

$$\frac{9}{10} = \frac{R_L}{R_L+R_i}$$

$$9R_i + 9R_L = 10R_L$$

$$R_L = 9R_i$$

$$I = \frac{\mathcal{E}}{10R_i} \qquad P = I^2R = \frac{\mathcal{E}^2}{100R_i^2}(9R_i)$$

Max power in c

$$\frac{\mathcal{E}^2}{(R_i+R_L)^2}R_L \quad \text{when } R_L = R_i$$

$$= \frac{\mathcal{E}^2 R_i}{4R_i^2} = \frac{\mathcal{E}^2}{4R_i}$$

$$\frac{P_{now}}{P_{max}} = \frac{\frac{.09\mathcal{E}^2}{R_i}}{\frac{\mathcal{E}^2}{4R_i}} = \frac{.09\mathcal{E}^2}{R_i} \cdot \frac{4R_i}{\mathcal{E}^2} = .36$$

study!

36% of P_{max} ✓

6. AAA, AA, D have $\mathcal{E}$ 1.5 V
difference is lifetime → energy storage
AAA .5 Amp hour
D 10 Amp hours

D internal resistance .1 Ω

Have 50% efficent winch
Trying to lift 60 kg
Winch is R_L (speed dependent)

a) Suppose winch is super slow speed
Winch motor R_L > r internal so no
loss of energy to internal resistance.
How high can it lift?

$$\Delta V = \mathcal{E} - Ir$$

$$\mathcal{E} - Ir - IR = 0 \quad \text{complete loop}$$

$$I = \frac{\mathcal{E}}{R+r}$$

$$\text{Power} = I\mathcal{E} = I(IR + Ir) = I^2R + I^2r$$

no internal resistance so Power $= I^2R$

$$= \frac{\Delta W}{\Delta t} = \Delta mgh$$

~~$I^2 R \Delta t = mg\Delta h$~~

~~$\Delta h = \frac{I^2 R \Delta t}{mg}$~~

Amp · hour
→ charge (colomb)

$$1\,\text{Amp} = 1\,\frac{\text{Col}}{\text{sec}}$$

$$Q = It$$

$$mg\Delta h = \Delta V = \Delta E = \text{work}$$

$$36000\ C \cdot 1.5V = mg\Delta h$$

$$\Delta h = \frac{36000\ C \cdot 1.5V}{mg}$$

* so basically know the quantites + how they relate

I'm just pushing to finish - not learning and not really getting it - takes more time

b) To what resistance R_L should set to get batt lift at fastest rate (maximize power)

$$\text{Power} = I\mathcal{E} = I(IR + Ir) = I^2R + I^2 r$$

to find max take 1st deriv and find critical pt

Only want power in load

$$P = I^2 R$$

↑ I depends on R

$$0 = \mathcal{E} - Ir - IR$$

$$I = \frac{\mathcal{E}}{r+R}$$

Only interested in power disipated by load

$$\frac{dP}{dR} = \left(\frac{\mathcal{E}}{r+R}\right)^2 R$$

* must chain rule it

$$\frac{dP}{dR} = R\mathcal{E}^2 (r+R)^{-2}$$

$$\frac{d}{dx}\left(\frac{x}{(a+x)^2}\right) =$$

$$P' = (\mathcal{E}^2 \cdot 1)(r+R)^{-2} + -2(r+R)^{-3} \cdot 1 \cdot R\mathcal{E}^2$$

$$\frac{\mathcal{E}^2}{(r+R)^2} - \frac{2R\mathcal{E}^2}{(r+R)^2}$$

$$\frac{\mathcal{E}^2 R}{(r_i + R)^2}$$

$$\text{Rate} = \frac{\Delta h}{\Delta t} \qquad \text{Power} = \frac{\Delta W}{\Delta t} \qquad \Delta W = mgh$$

↑ deriv of work

find v $mg\frac{d}{dt}$ h = set = to

Pratice

$\frac{d}{dx} \frac{x}{(a+x)^2}$ $x(a+x)^{-2}$

$1 \cdot (a+x)^{-2} + -2(a+x)^{-3} \cdot 1 \cdot x$

$\frac{1}{(a+x)^2} + \frac{-2x}{(a+x)^3}$ ✓

$\frac{a+x-2x}{(a+x)^3} =$

$\frac{a-x}{(a+x)^3}$

$$\text{Power} = \frac{\Delta W}{\Delta t} \qquad \Delta W = mgh$$

$$P = mg\frac{dh}{dt} = mgV = \frac{\mathcal{E}^2}{(R+r)^2} - \frac{2R\mathcal{E}^2}{(R+r)^2}$$

$$V = \frac{\frac{\mathcal{E}^2}{(R+r)^2} - \frac{2R\mathcal{E}^2}{(R+r)^2}}{mg}$$

c) At this fastest lift rate - how long before discharging

$$36000\,C \cdot 1.5V = mg\Delta h$$

$$\Delta h = \frac{36000\,C \cdot 1.5V}{mg}$$

doing this before we learn about power! - am completely lost the first time this semester

~~$v = \frac{dh}{dt} \quad V = \int_0^{\frac{36000\,C \cdot 1.5V}{mg}} \frac{\frac{\mathcal{E}^2}{(R+r)^2} - \frac{2R\mathcal{E}^2}{(R+r)^2}}{mg}\, dt$~~

in B found V and Power
have energy and power, find time
· velocity = distance

$$W = qV = Ed$$

$$\text{Power} = \frac{qV}{t} = \frac{Ed}{t}$$

$$mgV = \frac{36000 \cdot 1.5}{t}$$

$$\frac{G^2}{(R+r)^2} - \frac{2RG^2}{(R+r)^2} = \frac{36000 \cdot 1.5}{t}$$

$$t = \frac{36000 \cdot 1.5}{\frac{G^2}{(R+r)^2} - \frac{2RG^2}{(R+r)^2}}$$

can · velocity to find distance

$$d = vt$$

$$d = \frac{36000 \cdot 1.5}{\frac{G^2}{(R+r)^2} - \frac{2RG^2}{(R+r)^2}} \cdot \frac{\frac{G^2}{(R+r)^2} - \frac{2RG^2}{(R+r)^2}}{mg}$$

or using a

cushion in w/o reading course notes

$$d = \frac{36000 \cdot 1.5}{a} \cdot \frac{a}{mg}$$

$$d = \frac{36000 \cdot 1.5}{mg}$$

d. Compare the cost of powering desk lights with D batteries instead of .1 kwh

So ~~power of a battery~~

~~$P = \frac{qV}{t} = \frac{36000\,C \cdot 1.5\,V}{1\,hr} =$~~

kwh is energy

1 kwh = 3.6 mega joules

Power = watts

Amp hours · Volts = watt hrs

watt hr = Power · t = $\frac{qV}{\Delta t}$ · t = qV = Work = Energy

$P = \frac{qV}{\Delta t}$

So 10 Amp hours · 1.5 V = 10.5 Watt hours ← that is total Energy it can deliver

$\frac{10.5}{1000}$ = .0105 kWh for $3

vs 1 kWh for .1

$\frac{.0105 \text{ kwh}}{\$3}$ = .0035 kwh for $1

.00035 kwh for $.1

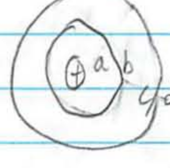
vs 1

$\frac{1}{.00035}$ = 2857 times more expensive

and that is w/ perfect efficency

7. Faraday Cage

Two nested spherical shells



should be easier

a) Both shells are floating - so net charge fixed (ie not grounded?)
+q in middle

just here today 3 hrs but feel really bad - not too much of a waste of time

- so at a all (−)
b all (+)
c all (−)
d all (+)

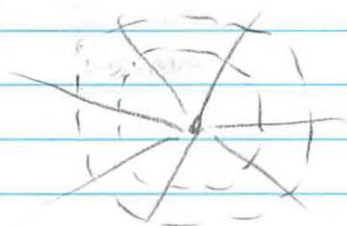
Fields are like that visualization

but the E field circles inside
no lines right - just empty space
simulation: if q directly in middle no lines
- q directly in middle no induced field

b) Inner shell is floating - outer shell grounded
⊖Q added

a all ⊕
b all ⊖
c all ⊕
d nothing all of the ⊖ charge disappears

E field will be same



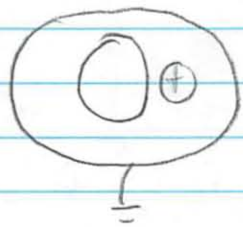
c) Inner grounded outer Floating ⊕ in middle

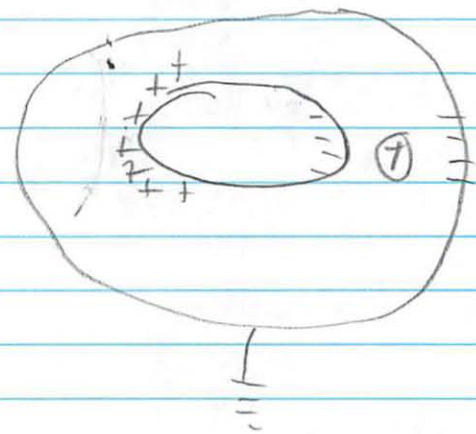
a all ⊖
b nothing - all of the ⊕ disappear
c all ⊖
d all ⊕ (assuming grounded since last problem

same/similar E Field

d. Outer shell grounded inner shell floating

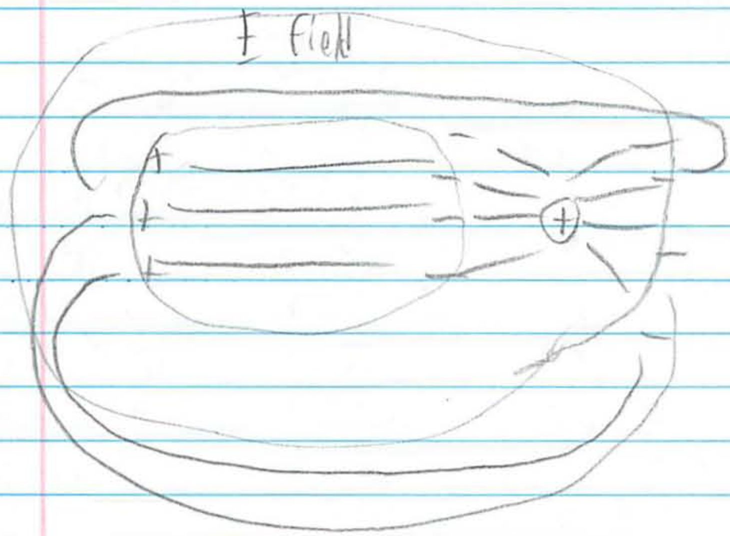
⊕ Q added between cage



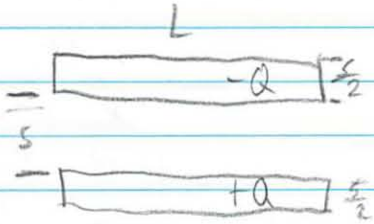


$E = \frac{\sigma}{\epsilon_0}$ ⊥ to surface

What is $\frac{E(r)}{V(r)}$? ← what is this asking



8. Capacitance, Work, Energy

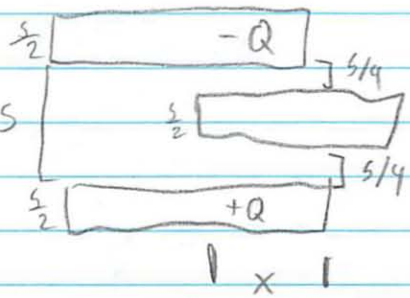


$s \ll L$ so no edge effects aka frindging field

+Q charge moved from top plate to lower plate

what exactly does that mean?

Now Force is applied to ? ?!



a) Use the fact that metals are equipotential what is σ_L and σ_R

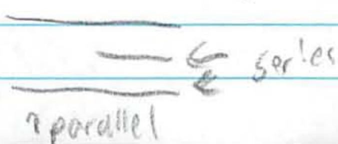
$$EA = \frac{\sigma A}{\epsilon}$$

parallel $E = \frac{\sigma}{\epsilon_0}$ so $\sigma = E\epsilon_0$

? now what ?

? So what is going on here?

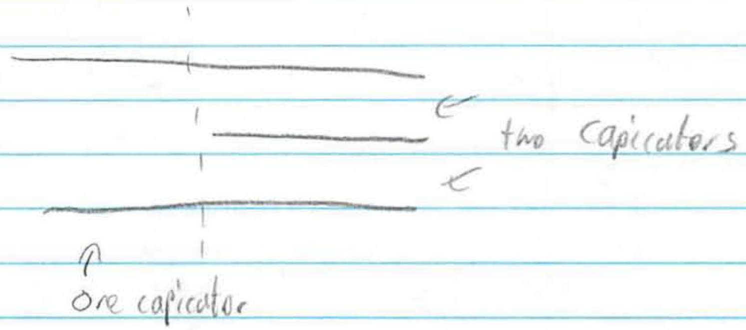
I wish I knew...



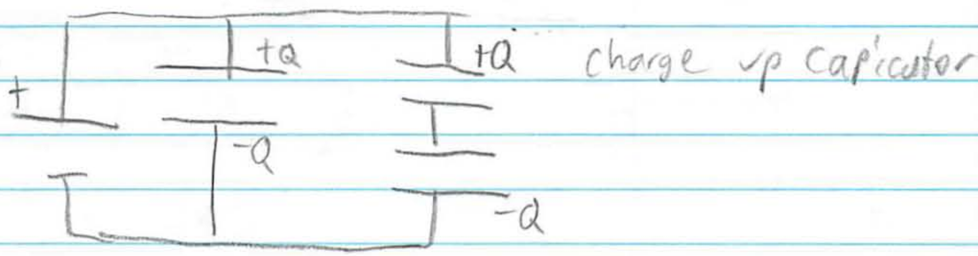
8

dormaskin
on

Ideas



potential = (



potential =

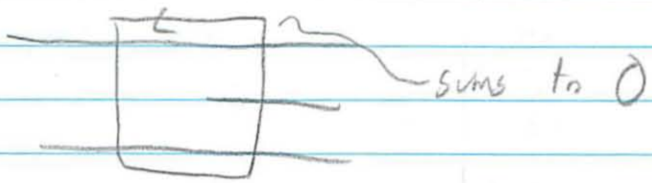
(ours flipped
+ -)

same potential difference, except 2 capicators on right

but does not want us to do this way

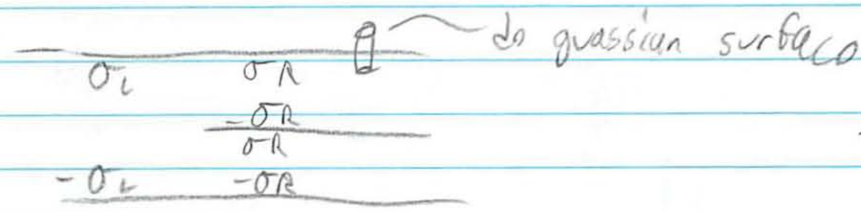
Whats happening?

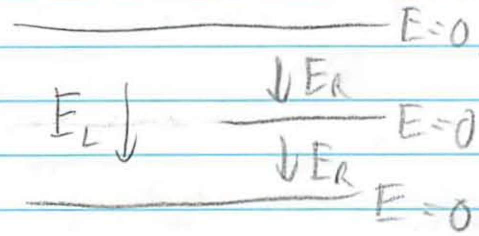
$C = \frac{Q}{\Delta V}$ ← both sides $= \frac{Q}{|\int E \cdot ds|}$ ← ds does not matter



Potential diff where non-0 E field

Suppose σ_L and σ_R





$$Q = \sigma_L A_L + \sigma_R A_R$$

Relationship E_L and E_R
And σ w/ guass's law
potentials = (path independent)

Can you follow ideas through in symbols

Solve for σ_L and σ_R
E_L and E_R
and V
and C

Can check; $\frac{1}{2} \epsilon_0 E_L^2$
solve for E_R
should get $\frac{Q^2}{2C}$

Could do normal capitance parallel + in series

8 on own

Yeah do normal Guassian Surface

$$EA = \frac{\sigma A}{\epsilon_0}$$

$$EA\epsilon_0 = \sigma A$$

$$E\epsilon_0 = \sigma \qquad \sigma = \frac{Q}{A} \begin{matrix} \leftarrow \text{charge} \\ \leftarrow \text{area} \end{matrix}$$

$$E_L = \frac{\sigma_L}{\epsilon_0}$$

$$C = \frac{Q}{\Delta V} \leftarrow \text{same potential drop} \quad = \frac{Q}{|\int E \cdot ds|}$$

$$E_L = 2E_R$$

$$Q = \sigma_L A_L + \sigma A_R$$

$$\frac{\sigma_L}{\epsilon_0} = 2\frac{\sigma_R}{\epsilon_0} \qquad \Delta V_L = \Delta V_R$$

$$C = \frac{Q}{V}$$

or can do parral and series

Series $\Delta V = \Delta V_1 + \Delta V_2$

$V_L = 2V_R$ (each section)

$$= \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$E_L 8 = 2 E_R \frac{8}{4}$$

$$E_L = \frac{1}{2} E_R$$

$$\sigma_L = \frac{\sigma_R}{2}$$

have to define an area + solve for !

$$Q = \sigma_L (L-x)L + 2\sigma_L xL$$
$$Q = \sigma_L L^2 - \sigma_L Lx + 2\sigma_L xL$$
$$= \sigma_L L + \sigma_L L^2$$

✓

$$\sigma_L = \frac{Q}{L^2+xL} \quad \checkmark \quad \sigma_R = \frac{2Q}{L^2+xL}$$

b $E_L = \frac{\sigma_L}{\epsilon_0}\hat{j}$ $\quad E_R = \frac{2\sigma_L}{\epsilon_0}\hat{j}$

$= \frac{Q}{\epsilon_0(L^2+xL)}\hat{j}$ $\quad = \frac{2Q}{\epsilon_0(L^2+xL)}\hat{j}$ ✓

c $\Delta V = \text{same} = E_L s = \frac{Qs}{\epsilon_0(L^2+xL)}$ ✓

d $C = \frac{Q}{\Delta V} = \frac{Q\epsilon_0(L^2+xL)}{Qs} = \frac{\epsilon_0(L^2+xL)}{s}$ ✓

e energy $= \frac{1}{2}QV = \frac{1}{2}\frac{Q^2 s}{\epsilon_0(L^2+xL)}$ ✓

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2010**

Problem Set 5 Solutions

Problem 1: Short Questions

(a) Why is it possible for a bird to stand on a high-voltage wire without getting electrocuted?

The reason is because the potential on the entire wire is nearly uniform, and the potential difference between the bird's feet is approximately zero. Thus, the amount of current flowing through the bird is negligible, since the resistance through the bird's body between its feet is much greater than the resistance through the wire between the same two points.

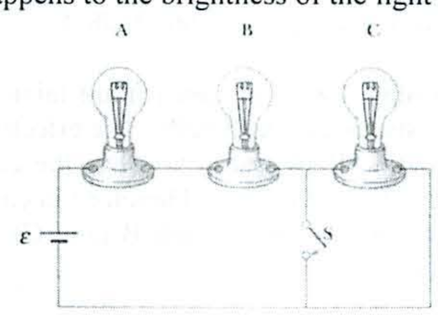
(b) If your car's headlights are on when you start the ignition, why do they dim while the car is starting?

The starter motor draws a significant amount of current from the battery while it is starting the car. This, coupled with the internal resistance of the battery, decreases the output voltage of the battery below its the nominal 12 V. This decrease in voltage decreases the current through (and brightness of) the headlights.

(c) Suppose a person falling from a building on the way down grabs a high-voltage wire. If the wire supports him as he hangs from it, will he be electrocuted? If the wire then breaks, should he continue to hold onto the end of the wire as he falls?

As long as he only grabs one wire and does not touch anything that is grounded, he will be safe. If the wire breaks, *let go!* If he continues to hold on to the wire, there will be a large—and rather lethal—potential difference between the wire and his feet when he hits the ground.

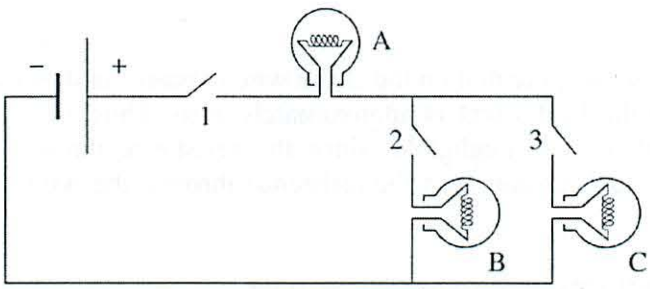
(d) A series circuit consists of three identical lamps connected to a battery as shown in the figure below. When the switch S is closed, what happens to the brightness of the light bulbs? Explain your answer.



Closing the switch makes the switch and the wires connected to it a zero-resistance branch. All of the current through A and B will go through the switch and lamp C goes out, with zero voltage across it. With less total resistance, the current in the battery becomes larger than before and lamps A and B get brighter.

Problem 2: Circuit

The circuit below consists of a battery (with negligible internal resistance), three incandescent light bulbs (A, B & C) each with exactly the same resistance, and three switches (1, 2, & 3). In what follows, you may assume that, regardless of how much current flows through a given light bulb, its resistance remains unchanged. Assume that when current flows through a light bulb that it glows. The higher the current, the brighter the light will be.



In each situation (a, b, c) as described below, we want to know which light bulbs are glowing (and which are not) and how bright they are (relative to each other). *Always briefly discuss your reasoning.*

a. Switch #1 is closed; the others are open.

No bulbs glowing; no closed circuit anywhere and hence no current anywhere

b. Switches #1 & #2 are closed; #3 is open

A & B glow with equal brightness as they are connected in series to the battery and thus the same current passes through each. C is still off.

c. All three switches are closed

A, B & C all glow. A is brightest, for all current flows through it. B & C glow with equal but lesser brightness, as the current through A is split equally between B & C.

d. Now compare situations a, b & c. Which bulb is brightest of all, and which is faintest of all (bulbs which are off don't count).

Bulb A in case (c) is brightest of all; effective resistance of the bulb combination is decreased from that of part (b) by the addition of light bulb C in parallel with bulb B. By Ohm's law, more current is then drawn from the battery in case (c) as compared to case (b) leading to a brighter bulb A.

Bulbs B & C in case (c) are faintest of all. Let V be the battery voltage and R be the resistance of each bulb. The effective resistance of the circuit as a whole is 2R in case (b) and 1.5R in case (c). Thus the current through A is V/2R in case (b) and V/1..5R = 2V/3R in case (c). Therefore in case (b) the current through B is also V/2R, but in case (c) the current through B (and C) is half of 2V/3R or V/3R. This latter current is the smallest.

Now replace bulb A by a wire of negligible resistance. We still have three switches and now two light bulbs (B & C).

e. Answer the questions b through d again for this situation.

(e-b) B glowing, C off
(e-c) B & C glowing with equal brightness
(e-d) All on-bulb brightnesses are equal, for all bulbs have the full battery voltage across themselves, and therefore the same current goes through each.

Problem 3: Ohm's Law

A straight cylindrical wire lying along the x-axis has a length L and a diameter d. It is made of a material described by Ohm's law with a resistivity ρ. Assume that a potential V is maintained at $x = 0$, and that $V = 0$ at $x = L$. In terms of L, d, V, ρ, and physical constants, determine expressions for

(a) the electric field in the wire.

This problem is simply the review of the Chapter 6 of the *Course Notes*. You should read it if you have anything unfamiliar with.

$$\vec{\mathbf{E}} = \frac{V}{L}\hat{\mathbf{x}}$$

(b) the resistance of the wire.

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi (d/2)^2} = \frac{4\rho L}{\pi d^2}$$

(c) the electric current in the wire.

$$\vec{I} = \frac{V}{R}\hat{x} = V \Big/ \left(\frac{4\rho L}{\pi d^2}\right)\hat{x} = \frac{\pi d^2 V}{4\rho L}\hat{x}$$

(d) the current density in the wire. Express vectors in vector notation.

$$\vec{\mathbf{J}} = \frac{\vec{\mathbf{I}}}{A} = \left(\frac{\pi d^2 V}{4\rho L}\hat{\mathbf{x}}\right) \Big/ \pi (d/2)^2 = \frac{V}{\rho L}\hat{\mathbf{x}}$$

(e) Show that $\vec{\mathbf{E}} = \rho\vec{\mathbf{J}}$.

$$\rho\vec{\mathbf{J}} = \rho\left(\frac{V}{\rho L}\hat{\mathbf{x}}\right) = \frac{V}{L}\hat{\mathbf{x}} = \vec{\mathbf{E}}$$

Problem 4: Resistance of Conductor in Telegraph Cable

The first telegraphic messages crossed the Atlantic Ocean in 1858, by a cable 3000 km long laid between Newfoundland and Ireland. The conductor in this cable consisted of seven copper wires, each of diameter 0.73 mm, bundled together and surrounded by an insulating sheath. Calculate the resistance of the conductor. Use $3\times10^{-8}\ \Omega\cdot\text{m}$ for the resistivity of copper, which was of somewhat dubious purity.

Solution: When current flows in the cable, the ends of each of the seven copper wires are held at the same voltage difference, so the wires are in parallel. Recall that when resistors are in parallel, the equivalent resistance adds inversely:

$$\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}+\cdots.$$

Since resistance is inversely proportional to area, we have that

$$\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}+\cdots=\frac{A_1}{\rho L_1}+\frac{A_2}{\rho L_2}+\cdots.$$

The wires are all the same length and area so for seven wires

$$\frac{1}{R_{eq}}=\frac{7A}{\rho L}.$$

Thus the equivalent resistance is

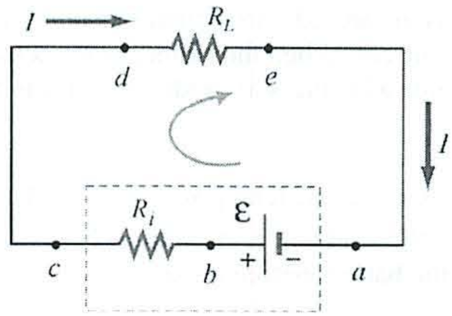
$$R_{eq}=\frac{\rho L}{7A}=\frac{(3\times10^{-8}\ \Omega\cdot\text{m})(3\times10^{6}\ \text{m})}{(7)(\pi)(7.3\ \times10^{-4}\text{m}/2)^2}=3.0\times10^{4}\ \Omega.$$

Check: Since resistance is inversely proportional to area, the effective area is seven times the area of one wire.

Problem 5: Current, Energy and Power A battery of emf ε has internal resistance R_i, and let us suppose that it can provide the emf to a total charge Q before it expires. Suppose that it is connected by wires with negligible resistance to an external (load) with resistance R_L.

a) What is the current in the circuit?

Solution:



The Kirchoff loop law (the sum of the voltage differences across each element around a closed loop is zero) yields

$$\varepsilon - I R_i - I R_L = 0 .$$

Solving for the current we find that

$$I = \frac{\varepsilon}{R_i + R_L} .$$

b) What value of R_L maximizes the current extracted from the battery, and how much chemical energy is generated in the battery before it expires?

Solution: The current is maximized when $R_L = 0$.

The chemical energy generated in the battery is given by

$$U_{emf} = \int_0^{\Delta t} \varepsilon I dt = \varepsilon I \Delta t$$

During this time interval, the battery delivers a charge

$$Q = \int_0^{\Delta t} I dt = I \Delta t .$$

Therefore the chemical energy generated is

$$U_{emf} = \varepsilon I \Delta t = \varepsilon I \frac{Q}{I} = \varepsilon Q$$

This result is independent of the current and only depends on the charge Q that is transferred across the EMF. So for all the following parts, this quantity is the same.

All of this chemical energy is dissipated into thermal energy due to the internal resistance of the battery to the flow of current. When the battery stops delivering current, the battery will reach thermal equilibrium with the surroundings and this thermal energy will flow into the surroundings.

c) What value of R_L maximizes the total power delivered to the load, and how much energy is delivered to the load before it expires? How does this compare to the energy generated in the battery before it expires?

Solution: The power delivered to the load is

$$P_L = I^2 R_L = \left(\frac{\varepsilon}{R_i + R_L} \right)^2 R_L .$$

We can maximize this by considered the derivative with respect to R_L:

$$\frac{dP_L}{dR_L} = \varepsilon^2 \left(\left(\frac{1}{R_i + R_L} \right)^2 - 2R_L \left(\frac{1}{R_i + R_L} \right)^3 \right) = 0 .$$

Solve this equation for R_L:

$$\left(\frac{1}{R_i + R_L} \right)^2 = 2R_L \left(\frac{1}{R_i + R_L} \right)^3 ,$$

$$R_i + R_L = 2R_L ,$$

$$R_L = R_i .$$

The current is then

$$I = \frac{\varepsilon}{R_i + R_L} = \frac{\varepsilon}{2R_i} .$$

The power delivered to the load is

$$P_{L,\max} = I^2 R_L = \left(\frac{\varepsilon}{2R_i} \right)^2 R_i = \frac{1}{4} \frac{\varepsilon^2}{R_i}$$

The energy delivered to the load is then

$$U_L = I\,R_L\,Q = \frac{\varepsilon}{2R_i} R_i\,Q = \frac{\varepsilon Q}{2} = \frac{1}{2} U_{chem}\,.$$

So exactly half the chemical energy is delivered to the load.

d) What value for the resistance in the load R_L would you need if you want to deliver 90% of the chemical energy generated in the battery to the load? What current should flow? How does the power delivered to the load now compare to the maximum power output you found in part c)?

Solution: Even though we maximized the power delivered to the load in part cc), we are wasting one half the chemical energy. Suppose you want to waste only 10% of the chemical energy. What current should flow?

$$U_L = 0.9\,U_{chem} = 0.9\,\varepsilon Q = I'\,R_L\,Q\,.$$

This implies that

$$I'\,R_L = \frac{\varepsilon}{R_i + R_L} R_L = 0.9\varepsilon\,.$$

This is satisfied when

$$R_L = 9R_i\,.$$

So the current is

$$I' = \frac{\varepsilon}{10R_i}\,.$$

The power output is then

$$P_L = I'^2 R_L = \left(\frac{\varepsilon}{10R_i}\right)^2 9R_i = \frac{9}{25}\left(\frac{1}{4}\frac{\varepsilon^2}{R_i}\right) = \frac{9}{25} P_{L,\max}\,.$$

So we waste 10% of the energy and still maintain 36% of the maximum power output.

Problem 6: Battery Life

AAA, AA, … D batteries have an open circuit voltage (EMF) of 1.5 V. The difference between different sizes is in their lifetime (total energy storage). A AAA battery has a life of about 0.5 A-hr while a D battery has a life of about 10 A-hr. Of course these numbers depend on how quickly you discharge them and on the manufacturer, but these numbers are roughly correct. One important difference between batteries is their internal resistance – alkaline (now the standard) D cells are about 0.1Ω.

Suppose that you have a multi-speed winch that is 50% efficient (50% of energy used does useful work) run off a D cell, and that you are trying to lift a mass of 60 kg (hmmm, I wonder what mass that would be). The winch acts as load with a variable resistance R_L that is speed dependent.

a) Suppose the winch is set to super-slow speed. Then the load (winch motor) resistance is much greater than the battery's internal resistance and you can assume that there is no loss of energy to internal resistance. How high can the winch lift the mass before discharging the battery?

This is just a question of energy. The battery has an energy storage of (1.5 V)(10 A-hr) = 15 W-hr or 54 kJ. So it can lift the mass:

$$U = mgh \Rightarrow \quad h = \frac{U}{mg} = \frac{54\,\text{kJ} \cdot \frac{1}{2}}{(60\text{ kg})(9.8\text{ m/s})} = \boxed{46\text{ m}}$$

The factor of a half is there because the winch is only 50% efficient.

b) To what resistance R_L should the winch be set in order to have the battery lift the mass at the fastest rate? What is this fastest rate (m/sec)? HINT: You want to maximize the power delivery to the winch (power dissipated by R_L).

First we need to determine how to maximize power delivery. If a battery V is connected to two resistances, r_i (the internal resistance) and R, the load resistance, the power dissipated in the load is:

$$P = I^2 R = \left(\frac{V_0}{R + r_i}\right)^2 R = V_0^2 \frac{R}{(R + r_i)^2}$$

We want to maximize this by varying R:

$$\frac{dP}{dR} = \frac{d}{dR}\left(V_0^2 R (R + r_i)^{-2}\right) = V_0^2\left[(R + r_i)^{-2} - 2R(R + r_i)^{-3}\right] = 0$$

Multiply both sides by $V_0^{-2}(R + r_i)^3 : \left[(R + r_i) - 2R\right] = r_i - R = 0 \quad \Rightarrow \quad \boxed{R = r_i}$

So, to get the fastest rate of lift (most power dissipation in the winch) we need the winch resistance to equal the battery internal resistance, $R_L = r_i = 0.1\ \Omega$.

Using this we can get the lift rate from the power:

$$P = I^2 R_L = \left(\frac{V_0}{R_L + r_i}\right)^2 R_L = \frac{V_0^2}{4r_i} \overset{50\%\,eff}{=} 2\,\frac{d}{dt}(mgh) \Rightarrow v = \frac{dh}{dt} = \frac{V_0^2}{8r_i mg}$$

Thus we find a list rate of $\boxed{v = 4.8\text{ mm/s}}$

c) At this fastest lift rate how high can the winch lift the mass before discharging the battery?

This is just part a over again, except now we waste half the energy in the internal resistor, so the winch will only rise half as high, to **23 m**

d) Compare the cost of powering a desk light with D cells as opposed to plugging it into the wall. Does it make sense to use rechargeable batteries? Residential electricity costs about $0.1/kwh.

A D cell has a battery life of 10 A-hr, meaning a total energy storage of (1.5 V)(10 A-hr) = 15 Watt-hrs. We could convert that to about 50 kJ but Watt-hours are a useful unit to use because electricity is typically charged by the kW-hour so this will make comparison easier. A D battery costs about $1 (you can pay more, but why?) So D batteries cost about $1/0.015 kwh or $70/kwh.
Residential electricity costs about $0.1/kwh. So the battery is nearly three orders of magnitude more expensive. It definitely makes sense to use rechargeable batteries – even though the upfront cost is slightly more expensive you will get it back in a couple recharges. As for your desk light, or anything that can run on batteries or wall power, plug it in. If it is 60 Watts, for every hour you pay only 0.6¢ with wall power but run through $4 in D batteries.

Problem 7: Faraday Cage

Consider two nested, spherical conducting shells. The first has inner radius a and outer radius b. The second has inner radius c and outer radius d.

In the following four situations, determine the total charge on each of the faces of the conducting spheres (inner and outer for each), as well as the electric field and potential everywhere in space (as a function of distance r from the center of the spherical shells). In all cases the shells begin uncharged, and a charge is then instantly introduced somewhere.

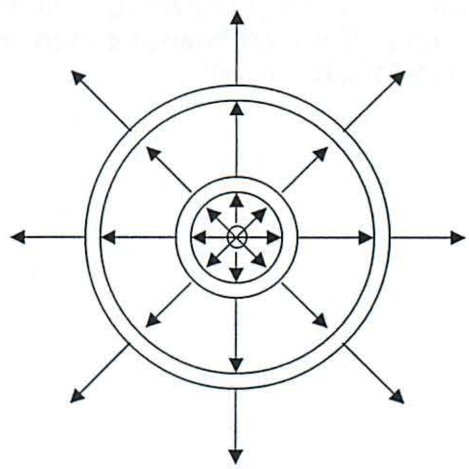
(a) Both shells are floating – that is, their net charge will remain fixed. A positive charge +Q is introduced into the center of the inner spherical shell. Take the zero of potential to be at infinity.

There is no electric field inside a conductor. Also, the net charge on an isolated conductor is zero (i.e. $Q_a + Q_b = Q_c + Q_d = 0$).

$Q_a = -Q$, $Q_b = -Q_a = Q$, $Q_c = -Q$, $Q_d = -Q_c = Q$

Using the Gauss's law,

$$\vec{E}(r) = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}, r > d \\ 0, c < r < d \\ \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}, b < r < c \\ 0, a < r < b \\ \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}, r < a \end{cases}$$



Since $V(r) = -\int_{\infty}^{r} E(r)dr$,

$$V(r)=\begin{cases}\dfrac{Q}{4\pi\varepsilon_0 r}, r>d\\ \dfrac{Q}{4\pi\varepsilon_0 d}, c<r<d\\ \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{r}-\dfrac{1}{c}+\dfrac{1}{d}\right), b<r<c\\ \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{b}-\dfrac{1}{c}+\dfrac{1}{d}\right), a<r<b\\ \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{r}-\dfrac{1}{a}+\dfrac{1}{b}-\dfrac{1}{c}+\dfrac{1}{d}\right), r<a\end{cases}$$

(b) The inner shell is floating but the outer shell is grounded – that is, it is fixed at V=0 and has whatever charge is necessary on it to maintain this potential. A negative charge –Q is introduced into the center of the inner spherical shell.

Since the outer shell is now grounded, $Q_d = 0$ to maintain $\vec{E}(r) = 0$ outside the outer shell. We have.

$Q_a = Q$, $Q_b = -Q_a = -Q$, $Q_c = Q$, $Q_d = 0$

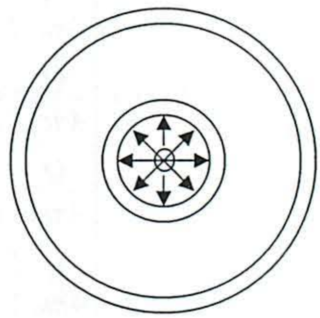
$$\vec{E}(r)=\begin{cases}0, r>c\\ -\dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}, b<r<c\\ 0, a<r<b\\ -\dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}, r<a\end{cases}$$

$$V(r)=\begin{cases}0, r>c\\ -\dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{r}-\dfrac{1}{c}\right), b<r<c\\ -\dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{b}-\dfrac{1}{c}\right), a<r<b\\ -\dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{r}-\dfrac{1}{a}+\dfrac{1}{b}-\dfrac{1}{c}\right), r<a\end{cases}$$

(c) The inner shell is grounded but the outer shell is floating. A positive charge +Q is introduced into the center of the inner spherical shell.

Since the inner shell is grounded and $Q_b = 0$ to maintain $\vec{E}(r) = 0$ outside the inner shell. Since there is no electric field on the outer shell, $Q_c = Q_d = 0$.

$Q_a = -Q$, $Q_b = Q_c = Q_d = 0$



$$\vec{E}(r) = \begin{cases} 0, r > a \\ \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r}, r < a \end{cases}$$

$$V(r) = \begin{cases} 0, r > a \\ \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{r} - \dfrac{1}{a}\right), r < a \end{cases}$$

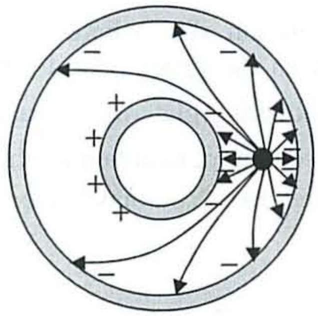
(d) Finally, the outer shell is grounded and the inner shell is floating. This time the positive charge +Q is introduced into the region in between the two shells. In this case the questions "What is **E**(r)/V(r)?" are not well defined in some regions of space. In the regions where these questions can be answered, answer them. In the regions where they can't be answered, explain why, and give as much information about the potential as possible (is it positive or negative, for example).

The electric field within the cavity is zero. If there is any field line that began and ended on the inner wall, the integral $\oint \mathbf{E} \cdot d\vec{\mathbf{s}}$ over the closed loop that includes the field line would not be zero. This is impossible since the electrostatic field is conservative, and therefore the electric field must be zero inside the cavity. The charge Q between the two conductors pulls minus charges to the near side on the inner conducting shell and repels plus charges to the far side of that shell. However, the net charge on the outer surface of the inner shell (Q_b) must be zero since it was initially uncharged (floating). Since the outer shell is grounded, $Q_d = 0$ to maintain $\vec{E}(r) = 0$ outside the outer shell. Thus,

$$Q_a = Q_b = Q_d = 0,\ Q_c = -Q \text{ and } \vec{E}(r) = 0,\ r < b \text{ or } r > c$$

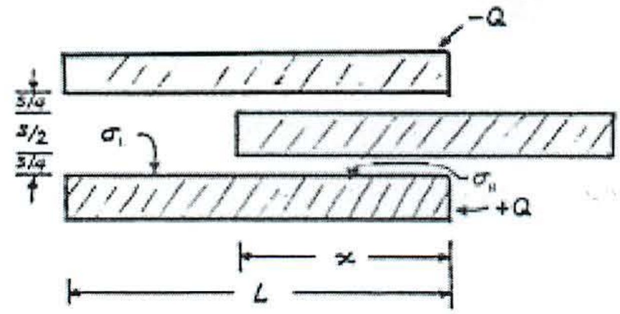
For $b < r < c$, $\vec{E}(r)$ is in fact well defined but it is very complicated. The filed lines are shown in the figure below.

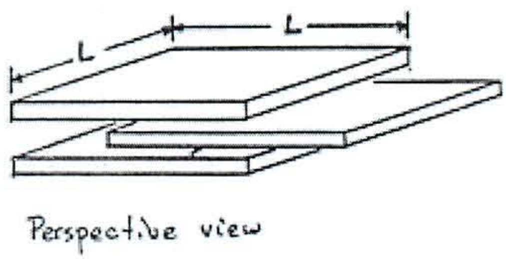
What can we say about the electric potential? $V(r) = 0$ **for** $r > c$, **and** $V(r) = \text{constant}$ **for** $r < a$ **but the potential is very complicated defined between the two shells.**



Problem 8: Capacitance, Work and Energy

Two flat, square metal plates have sides of length L, and thickness $s/2$, are arranged parallel to each other with a separation of s, where $s << L$ so you may ignore fringing fields. A charge Q is moved from the upper plate to the lower plate. Now a force is applied to a third uncharged conducting plate of the same thickness $s/2$ so that it lies between the other two plates to a depth x, maintaining the same spacing $s/4$ between its surface and the surfaces of the other two. You may neglect edge effects.





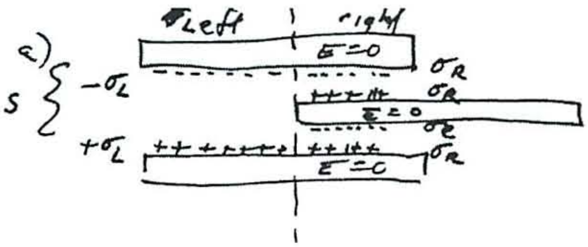
a) Using the fact that the metals are equipotential surfaces, what are the surface charge densities σ_L on the lower plate adjacent to the wide gap and σ_R on the lower plate adjacent to the narrow gap?

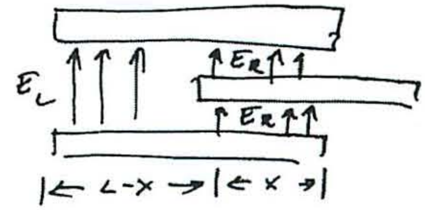
b) What is the electric field in the wide and narrow gaps? Express your answer in terms of L, x, and s.

c) What is the potential difference between the lower plate and the upper plate?

d) What is the capacitance of this system?

e) How much energy is stored in the electric field?





a) $\Delta V_L = \Delta V_R$ since upper and lower plates are held at same potential difference

$$\Delta V_L = E_L s = \frac{\sigma_L}{\varepsilon_0} s$$

$$\Delta V_R = E_R \frac{s}{4} + E_R \frac{s}{4} = E_R \frac{s}{2} = \frac{\sigma_R}{\varepsilon_0} \frac{s}{2}$$

$$\Delta V_L = \Delta V_R \Rightarrow \frac{\sigma_L s}{\varepsilon_0} = \frac{\sigma_R}{2} \frac{s}{\varepsilon_0} \Rightarrow \sigma_L = \frac{\sigma_R}{2}$$

$$Q^T = \sigma_L (L-x) L + \sigma_R x L = \frac{\sigma_R}{2} (L-x) L + \sigma_R x L$$

$$Q^T = \sigma_R \frac{L^2}{2} + \sigma_R \frac{xL}{2} = \sigma_R \frac{L}{2} (L+x)$$

$$\sigma_R = \frac{Q^T}{\frac{L}{2}(L+x)} \Rightarrow \sigma_L = \frac{\sigma_R}{2} = \frac{Q^T}{L(L+x)}$$

b) $$E_L = \frac{\sigma_L}{\varepsilon_0} = \frac{Q^T}{\varepsilon_0 L (L+x)} \qquad E_R = \frac{2 Q^T}{(L)(L+x)\varepsilon_0}$$

c) $$\Delta V = E_L s = \frac{Q^T}{\varepsilon_0 L (L+x)} s$$

d) = $$C = \frac{Q^T}{\Delta V} = \frac{Q^T}{\frac{Q^T s}{\varepsilon_0 (L(L+x))}} = \frac{\varepsilon_0 (L)(L+x)}{s}$$

e) $$U = \frac{(Q^T)^2}{2C} = \frac{1}{2} \frac{(Q^T)^2 s}{\varepsilon_0 L (L+x)}$$

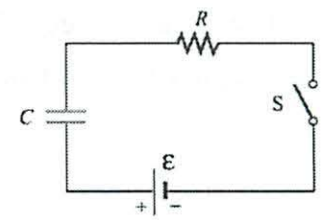
Topic: RC Circuits
Related Reading: Course Notes: Sections 7.5 – 7.6
Experiments: (4) RC Circuits

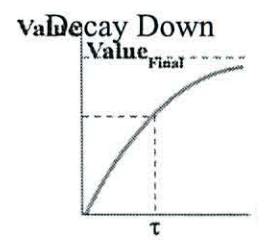
Topic Introduction

Today we will investigate the behavior of DC circuits containing resistors and capacitors (RC circuits). We will then measure voltage, current and across various RC circuit elements and the time constant for an RC circuit in experiment 4.

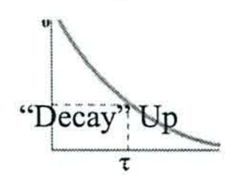
RC Circuits

A simple RC circuit is shown at right. When the switch is closed, current will flow in the circuit, but as time goes on this current will decrease. We can write down the differential equation for current flow by writing down Kirchhoff's loop rules, recalling that $|\Delta V| = Q/C$ for a capacitor and that the charge Q on the capacitor is related to current flowing in the circuit by $I = \pm dQ/dt$, where the sign depends on whether the current is flowing into the positively charged plate (+) or the negatively charged plate (-). The solution to this differential equation shows that the current decreases exponentially from its initial value while the potential on the capacitor grows exponentially to its final value. The rate at which this change happens is dictated by the "time constant" τ, which for this circuit is given by $\tau = RC$.





Interestingly, in RC circuits any value that you could ask about (current, potential drop across the resistor, across the capacitor, …) "decays" exponentially (either down or up). You should be able to determine which of the two plots at right will follow just by thinking about it.



Measuring Voltage and Current Circuits
In the first experiment you relied on the battery voltage and an internal current sensor to tell you the voltage and current in the circuit. In this lab we will want to record the voltage not only across the battery but also, separately, across the capacitor. We also will have some parallel branches which we want to measure current through. In order to make these measurements you will need to use a voltmeter and ammeter. Details of the use may be found in the experimental write-up, but more generally, when thinking about current and voltage there is an important difference you should keep in mind. Current is a value associated with the flow of charges THRU some surface (some point in the wire). Voltage measurements, on the other hand, are only meaningful as differences, and hence are measured ACROSS a circuit element or BETWEEN two points in a circuit.

Experiment 4: RC Circuits

Preparation: Read pre-lab and answer pre-lab questions

This extended lab will introduce you to the techniques of measuring current and voltage in a circuit and then allow you to observe the exponential behavior of RC circuits as they are "charged" and "discharged" using a battery which periodically turns on and off. You will measure the time constant of several circuits and investigate how it changes as resistance, or capacitance are modified.

Important Equations

Exponential Decay: $Value = Value_{initial} e^{-t/\tau}$

Exponential "Decay" Upwards: $Value = Value_{final}\left(1 - e^{-t/\tau}\right)$

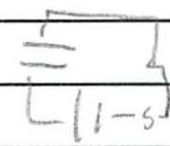
Simple RC Time Constant: $\tau = RC$;

Class 15: Outline

Hour 1:
RC Circuits

Hour 2:
Expt 4: RC Circuits

After switch is closed, current

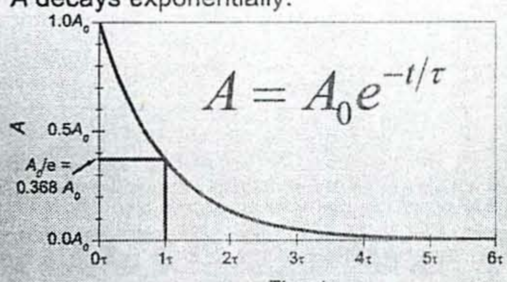


Nerely zero

Exponential Decay

Consider function A where: $\frac{dA}{dt} = -\frac{1}{\tau}A$

A decays exponentially:



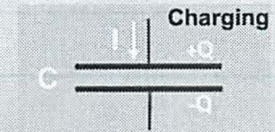
$$A = A_0 e^{-t/\tau}$$

RC Circuits

(Dis)Charging a Capacitor

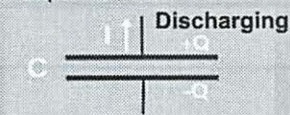
1. When the direction of current flow is toward the positive plate of a capacitor, then

$$I = +\frac{dQ}{dt}$$



2. When the direction of current flow is away from the positive plate of a capacitor, then

$$I = -\frac{dQ}{dt}$$

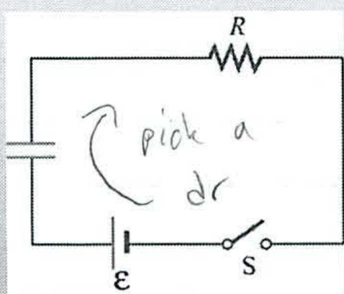


P15- 4

current torads (+)

current torads (−)
away from (+)

Charging A Capacitor



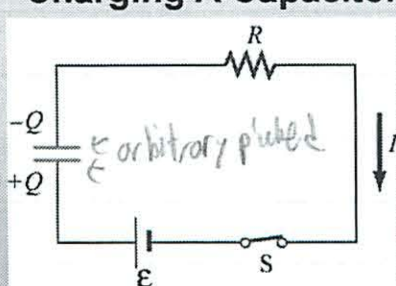
PRS

What happens when we close switch S?

P15- 5

Arbitrary assign dir
- if choose wrong q will be (−)
- but relationship is fixed

Charging A Capacitor



$$\sum_i \Delta V_i = \varepsilon - \frac{Q}{C} - \frac{dQ}{dt}R = 0$$

batt capacitor Resistor

P15- 6

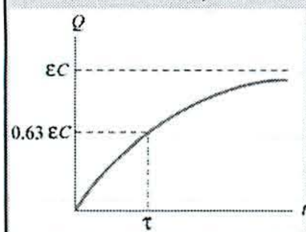
kirkoff's loop rule

differential eq
- has derivitive

RC Circuit

$$\frac{dQ}{dt} = -\frac{1}{RC}\left(Q - C\varepsilon\right)$$

Solution to this equation when switch is closed at t = 0:



$$Q(t) = C\mathcal{E}\left(1 - e^{-t/\tau}\right)$$

$\tau = RC$: time constant

(units: seconds)

P15- 7

need exponential decay

function $\frac{dA}{dt} = -\frac{1}{\tau}A$ = So don't do math

$A = A_0 e^{-t/\tau}$

$A = A_f\left(1 - e^{-t/\tau}\right)$

$\frac{dA}{dt} = \frac{-1}{\tau}\left(A - A_f\right)$

Solve Diferential Equation for Charging RC Circuits

P15- 8

$\frac{dQ}{dt} = \frac{1}{RC}\left(Q - C\varepsilon\right)$

RC = τ = time constant

$Q(t) = C\varepsilon\left(1 - e^{-t/\tau}\right)$

τ = RC time constant

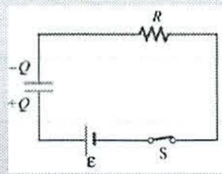
PRS Question: Current in RC Circuit

P15- 9

PRS: RC Circuit

20

An uncharged capacitor is connected to a battery, resistor and switch. The switch is initially open but at t = 0 it is closed. A very long time after the switch is closed, the current in the circuit is

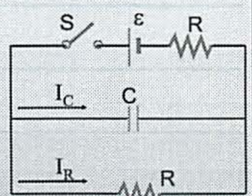


0%	1. Nearly zero
0%	2. At a maximum and decreasing
0%	3. Nearly constant but non-zero
0%	4. I don't know

P15-10

PRS: RC Circuit

Consider the circuit at right, with an initially uncharged capacitor and two identical resistors. At the instant the switch is closed:



0%	1. $I_R = I_C = 0$
0%	2. $I_R = \varepsilon/2R$; $I_C = 0$
0%	3. $I_R = 0$; $I_C = \varepsilon/R$
0%	4. $I_R = \varepsilon/2R$; $I_C = \varepsilon/R$
0%	5. I don't know

20

Can not solve seprtly like that

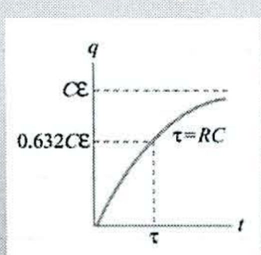
at instant switch closes C is like a wire $I_R = 0$

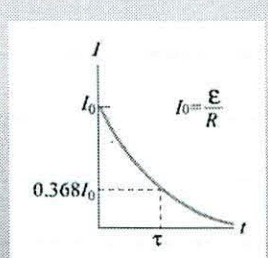
I_C ← still have resistor on top

$\varepsilon = IR$

$\frac{\varepsilon}{R} = \frac{IR}{R} = \frac{\varepsilon}{R} = I$

Charging A Capacitor





$$Q = C\mathcal{E}\left(1 - e^{-t/RC}\right) \qquad I = \frac{dQ}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC}$$

P15-12

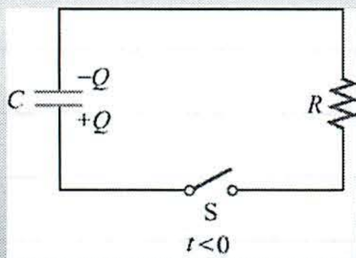
What is current doing?

decreasing exponentially

Uncharged → 0V drop (like a wire) bilds up a charge, so blocks current which falls to 0 - voltage drop at

Short it out = lower resistive path

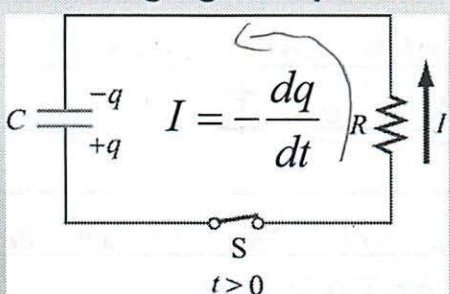
Discharging A Capacitor



What happens when we close switch S?

take out battery

Discharging A Capacitor



$$I = -\frac{dq}{dt}$$

$$\sum_i \Delta V_i = \frac{q}{C} - IR = 0 \qquad \sum_i \Delta V_i = \frac{q}{C} + \frac{dq}{dt}R = 0$$

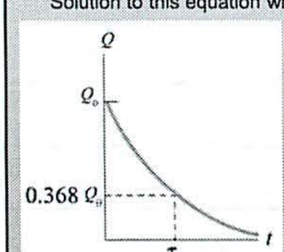
↑I can't represent in terms of t

must do this to solve for

RC Circuit: Discharging

$$\frac{dQ}{dt} = -\frac{1}{RC}Q$$

Solution to this equation when switch is closed at t = 0:

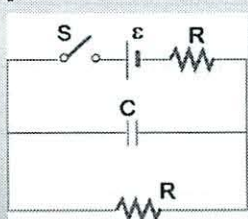


$$Q(t) = Q_o e^{-t/\tau}$$

$\tau = RC$: time constant

Demonstrations:
RC Time Constants

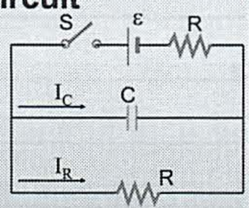
Group Problem: Circuits



For the above circuit sketch the currents through the two bottom branches as a function of time (switch closes at $t = 0$, opens at $t = T$). State values at $t = 0^+$, T^-, T^+

PRS Questions:
RC Circuit

PRS: RC Circuit

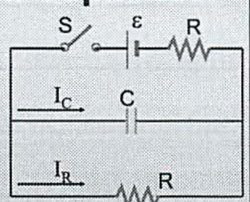


Now, after the switch has been closed for a very long time, it is opened. What happens to the current through the lower resistor?

0% 1. It stays the same
0% 2. Same magnitude, flips direction
0% 3. It is cut in half, same direction
0% 4. It is cut in half, flips direction
0% 5. It doubles, same direction
0% 6. It doubles, flips direction
0% 7. None of the above

20

PRS: Current Thru Capacitor



In the circuit at right the switch is closed at $t = 0$. At $t = \infty$ (long after) the *current through the capacitor* will be:

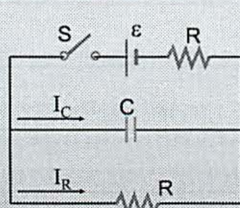
1. $I_C = 0$
2. $I_C = \varepsilon/R$
3. $I_C = \varepsilon/2R$
4. I don't know

P15-20

0 since blocks it out ✓

like wire clipped

PRS: Current Thru Resistor



In the circuit at right the switch is closed at $t = 0$. At $t = \infty$ (long after) the *current through the lower resistor* will be:

1. $I_R = 0$
2. $I_R = \varepsilon/R$
3. $I_R = \varepsilon/2R$
4. I don't know

P15-21

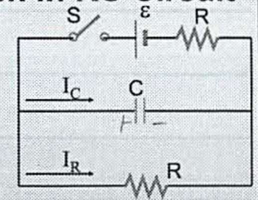
ignore capicator - like its not there

2 resistors

$\frac{\varepsilon}{R} = \frac{IR}{R}$ $\quad \frac{\varepsilon}{R} = I$

PRS: Opening Switch in RC Circuit

Now, after the switch has been closed for a very long time, it is opened. What happens to the current through the lower resistor?



1. It stays the same
2. Same magnitude, flips direction
3. It is cut in half, same direction
4. It is cut in half, flips direction
5. It doubles, same direction
6. It doubles, flips direction
7. None of the above.

P15-22

I Before $= \frac{\varepsilon}{2R}$

* current flows out of the capacitor the opposite dir it flew in *

top resistor snipped away
* assuming capacitor has voltage of ε
but always matching V across Resistor

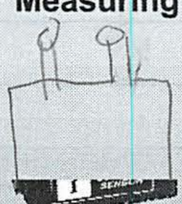
* Voltage of capicator always match capicator & resistor

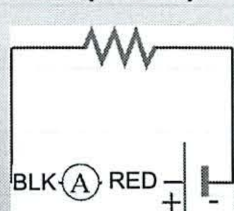
Not the voltage of the battery nessarily

Experiment 4: RC Circuits

P15-23

Measuring Current (THRU)

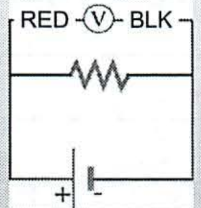




1. Hook in SERIES: current must go thru to measure
2. "Positive" if runs from Red to Black
3. Note: Not ideal – 1 Ω resistance. Does it matter?

P15-24

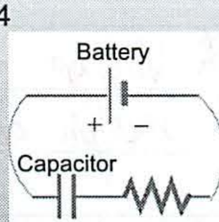
Measuring Voltage (ACROSS)



1. Hook in PARALLEL: reads $V_{Red} - V_{Black}$
2. Note: Not ideal – 1 MΩ resistance. Does it matter?

Expt. 4, Part I: RC Circuits

- Download and run Lab 4
- Build an RC circuit:
- Measure **current thru** and **voltage across** capacitor
- As battery 'turns on and off,' what happens to the capacitor? WHY?



PRS: Voltage/Current in RC

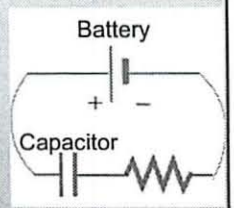
Starting from a point in time where the voltage across the battery (V_B) & across the capacitor (V_C) as well as the current (I) are all zero, what happens when the battery is 'turned on'?

1. I jumps up then decays as V_C rises (circled)
2. V_C jumps up then decays as I rises
3. I & V_C both jump up then decay
4. I & V_C both gradually rise
5. I don't know

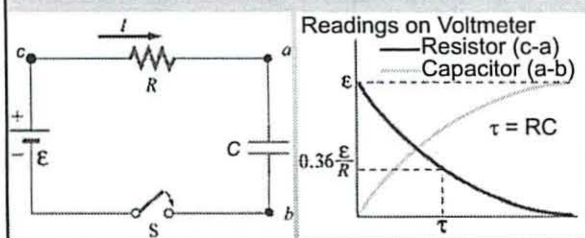
Current can jump
Voltage on capacitor changes smoothly

Expt. 4, part II: RC Circuits

- Same RC circuit
- Determine the resistance
- Measure the time constant to determine the capacitance
- You have a 2nd identical resistor. Where do you put it to make the TC as SHORT as possible?



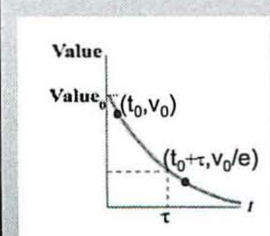
RC Circuit



$t=0^+$: Capacitor is uncharged so resistor sees full battery potential and current is largest

$t=\infty$: Capacitor is "full." No current flows

Measuring Time Constant



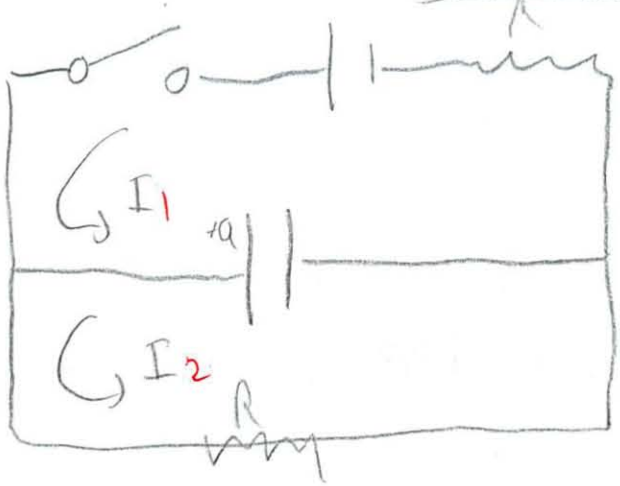
$$\text{Value}(t) = \text{Value}_0 e^{-t/\tau}$$

How do you measure τ?

1) a) Pick a point
   b) Find point with "value" down by e
   c) Time difference is τ
2) Plot semi-log and fit curve (make sure you exclude data at both ends)

Read instructions about cursors. Right click to fit

In Class Problem



$\sum V = 0$

Not correct

top I = $\mathcal{E} - \frac{Q}{C} - \frac{dQ}{dt} R = 0$

← depend on each other

bottom I = $+\frac{Q}{C} - \frac{dQ}{dt} R = 0$

add these

should not cancel

$\mathcal{E} - \frac{Q}{C} - \frac{dQ}{dt} R + \frac{Q}{C} - \frac{dQ}{dt} R = 0$

$\mathcal{E} - 2\frac{dQ}{dt} R = 0$

$\frac{-\mathcal{E}}{-2R} = \frac{-2\frac{dQ}{dt} R}{-2R}$

$\frac{dQ}{dt} = \frac{\mathcal{E}}{2R}$ ← is what I found for capicator bottom after long time

↑ should be a Q and C

- did not think of time dependence

pratice in OH some time

Sketch

- see website - missed

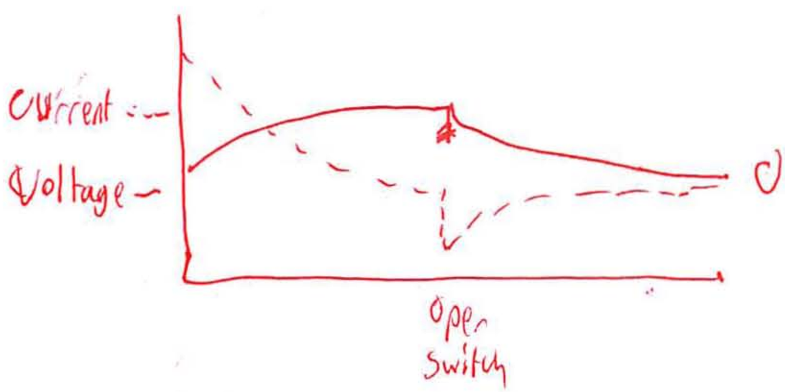
capicator ~~full~~ fills up → current drops

Why current through resisistor
- b/c can't go through capicator

Voltage across resistor
- Capicator making that

Current flowing

Current = time deriv of Charge



current can be discontineous
⊖ = opposite dir

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics
8.02

Experiment 4: RC Circuits

OBJECTIVES

1. To explore the time dependent behavior of RC Circuits
2. To understand how to measure the time constant of such circuits

PRE-LAB READING

INTRODUCTION

In this lab we will continue our investigation of DC circuits, now including, along with our "battery" and resistors, capacitors (RC circuits). We will measure the relationship between current and voltage in a capacitor, and study the time dependent behavior of RC circuits.

The Details: Capacitors

Capacitors store charge, and develop a voltage drop V across them proportional to the amount of charge Q that they have stored: $V = Q/C$. The constant of proportionality C is the capacitance (in Farads = Coulombs/Volt), and determines how easily the capacitor can store charge. Typical circuit capacitors range from picofarads (1 pF = 10^{-12} F) to millifarads (1 mF = 10^{-3} F). In this lab we will use microfarad capacitors (1 μF = 10^{-6} F).

RC Circuits

Consider the circuit shown in Figure 1. The capacitor (initially uncharged) is connected to a voltage source of constant emf $\mathcal{E}$. At $t = 0$, the switch S is closed.

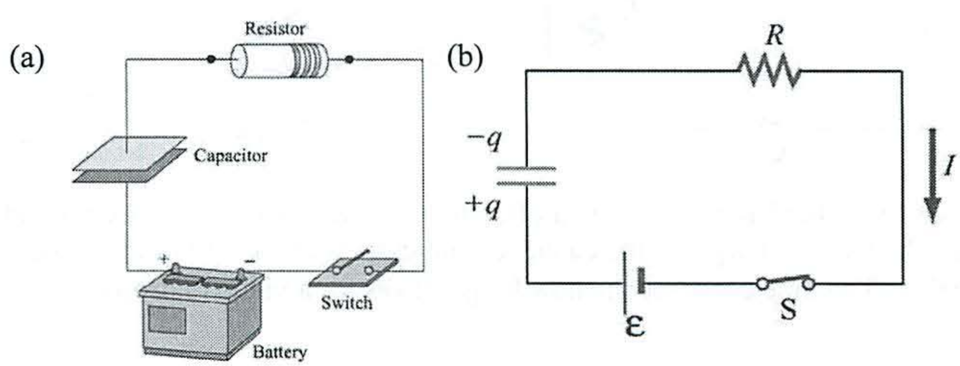


Figure 1 (a) *RC* circuit (b) Circuit diagram for $t > 0$

In class we derived expressions for the time-dependent charge on, voltage across, and current through the capacitor, but even without solving differential equations a little thought should allow us to get a good idea of what happens. Initially the capacitor is uncharged and hence has no voltage drop across it (it acts like a wire or "short circuit"). This means that the full voltage rise of the battery is dropped across the resistor, and hence current must be flowing in the circuit ($V_R = IR$). As time goes on, this current will "charge up" the capacitor – the charge on it and the voltage drop across it will increase, and hence the voltage drop across the resistor and the current in the circuit will decrease. This idea is captured in the graphs of Fig. 2.

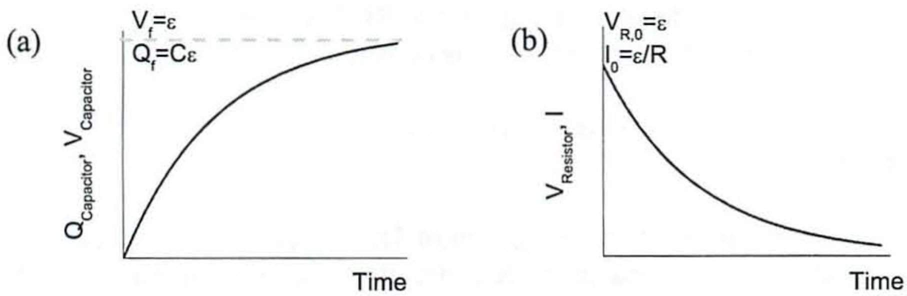


Figure 2 (a) Voltage across and charge on the capacitor increase as a function of time while (b) the voltage across the resistor and hence current in the circuit decrease.

After the capacitor is "fully charged," with its voltage essentially equal to the voltage of the battery, the capacitor acts like a break in the wire or "open circuit," and the current is essentially zero. Now we "shut off" the battery (replace it with a wire). The capacitor will then release its charge, driving current through the circuit. In this case, the voltage across the capacitor and across the resistor are equal, and hence charge, voltage and current all do the same thing, decreasing with time. As you saw in class, this decay is exponential, characterized by a time constant t, as pictured in fig. 3.

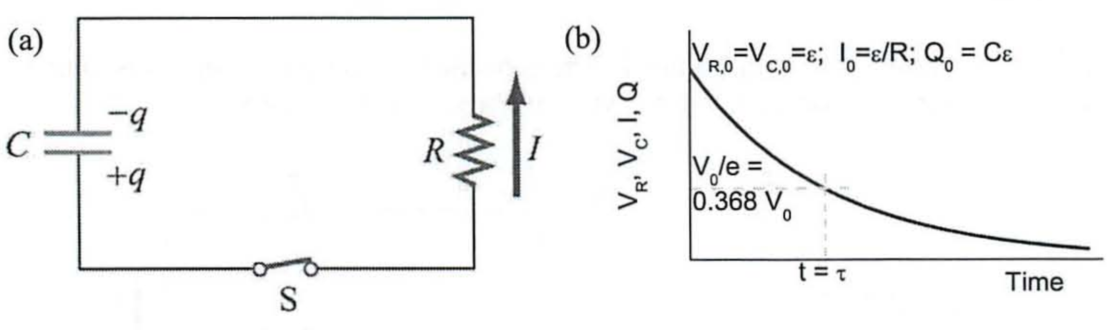


Figure 3 Once (a) the battery is "turned off," the voltages across the capacitor and resistor, and hence the charge on the capacitor and current in the circuit all (b) decay exponentially. The time constant τ is how long it takes for a value to drop by e (~2.7).

The Details: Measuring the Time Constant τ

In this lab you will be faced with an exponentially decaying current $I = I_0 \exp(-t/\tau)$ from which you will want to extract the time constant τ. We will do this in two different ways, using the "two-point method" or the "logarithmic method," depicted in Fig. 7.

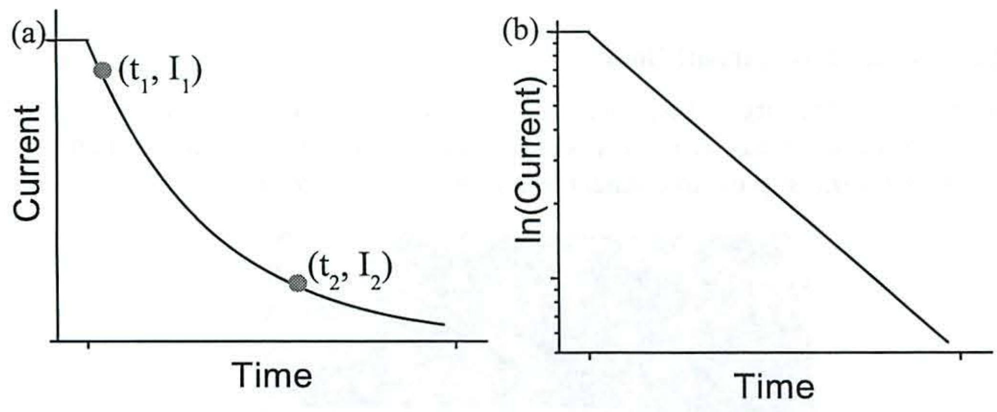


Figure 7 The (a) two-point and (b) logarithmic methods for measuring time constants

In the two-point method (Fig. 7a) we choose two points on the curve (t_1,I_1) and (t_2, I_2). Because the current obeys an exponential decay, $I = I_0 \exp(-t/\tau)$, we can extract the time constant τ most easily by picking I_2 such that $I_2 = I_1/e$. We should, in theory, be able to find this for any t_1, as long as we don't switch the battery off (or on) before enough time has passed. In practice the current will eventually get low enough that we won't be able to accurately measure it. Having made this selection, $\tau = t_2 - t_1$.

In the logarithmic method (Fig. 7b) we fit a line to the natural log of the current plotted vs time and obtain the slope *m*, which will give us the time constant as follows:

$$m = \frac{\text{rise}}{\text{run}} = \frac{\ln\left(I(t_2)\right) - \ln\left(I(t_1)\right)}{t_2 - t_1} = \frac{1}{t_2 - t_1}\ln\left(\frac{I(t_2)}{I(t_1)}\right)$$

$$= \frac{1}{t_2 - t_1}\ln\left(\frac{I_0 e^{-t_2/\tau}}{I_0 e^{-t_1/\tau}}\right) = \frac{1}{t_2 - t_1}\ln\left(e^{-(t_2-t_1)/\tau}\right) = \frac{1}{t_2 - t_1}\left(\frac{-(t_2 - t_1)}{\tau}\right) = -\frac{1}{\tau}$$

That is, from the slope (which the software can calculate for you) you can obtain the time constant: $\tau = -1/m$.

In using both of these methods you must take care to use points well into the decay (i.e. not on the flat part before the decay begins) and try to avoid times where the current has fallen close to zero, which are typically dominated by noise.

APPARATUS

1. Science Workshop 750 Interface

In this lab we will again use the 750 interface to create a "variable battery" which we can turn on and off, whose voltage we can change and whose current we can measure.

2. AC/DC Electronics Lab Circuit Board

We will also again use the circuit board of Fig. 8. This time we will use the inductor (E) as well as the connector pads (F) for resistors and capacitors, and the banana plug receptacles in the right-most pads to connect to the output of the 750.

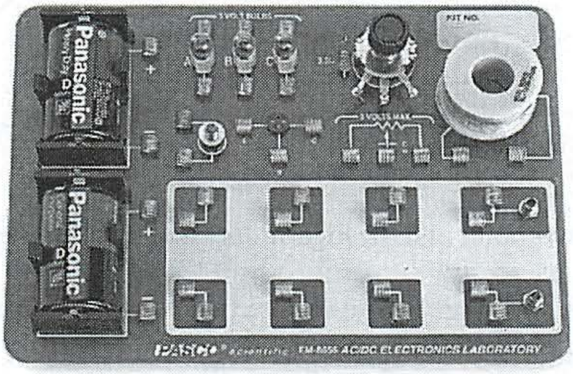


Figure 8 The AC/DC Electronics Lab Circuit Board, with (A) Battery holders, (B) light bulbs, (C) push button switch, (D) potentiometer, (E) inductor and (F) connector pads

3. Current & Voltage Sensors

Recall that both current and voltage sensors follow the convention that red is "positive" and black "negative." That is, the current sensor records currents flowing in the red lead and out the black as positive. The voltage sensor measures the potential at the red lead minus that at the black lead.

(a) **(b)**

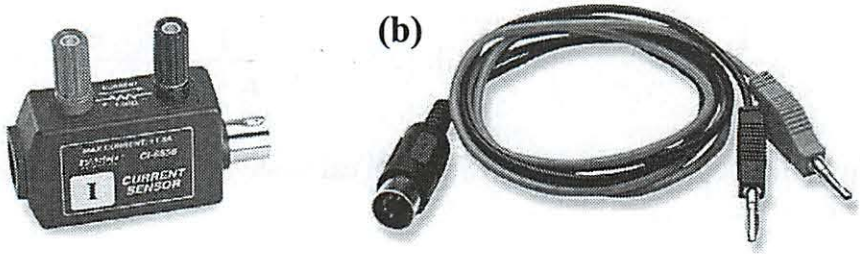
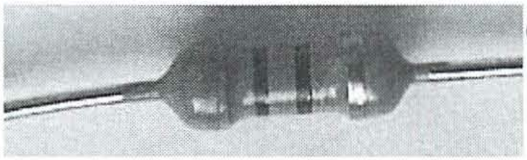


Figure 9 (a) Current and (b) Voltage Sensors

4. Resistors & Capacitors

We will work with resistors and capacitors in this lab. While resistors (Fig. 10a) have color bands that indicate their value, capacitors (Fig. 10b) are typically stamped with a numerical value.

(a)



(b)

Figure 10 Examples of a (a) resistor and (b) capacitor. Aside from their size, most resistors look the same, with 4 or 5 colored bands indicating the resistance. Capacitors on the other hand come in a wide variety of packages and are typically stamped both with their capacitance and with a maximum working voltage.

GENERALIZED PROCEDURE

This lab consists of two main parts. In each you will set up a circuit and measure voltage and current while the battery periodically turns on and off.

Part 1: Measuring Voltage and Current in an *RC* Circuit
In this part you will create a series RC (resistor/capacitor) circuit with the battery turning on and off so that the capacitor charges then discharges. You will measure the time constant using both methods described above and use this measurement to determine the capacitance of the capacitor.

Part 2: Measuring Voltage and Current in an *RC* Circuit
In this part you will add a second resistor in parallel with the capacitor to confirm your understanding of the in class problem worked before this part of the lab.

END OF PRE-LAB READING

2 pt method
1. Be in linear mode
2. Be in curve
3. Green I / e = place red where I = that
4. Subtract 2 times = τ

(t₀, V₀) → (t₀+τ, V₀/e)

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

1. Download the LabView file from the web and save the file to your desktop (right click on the link and choose "Save Target As" to the desktop. Overwrite any file by this name that is already there). Start LabView by double clicking on this file.
2. Connect the Voltage Sensor to Analog Channel A on the 750 Interface. We will obtain the current directly from the "battery" reading.
3. Connect cables from the output of the 750 to the banana plug receptacles on the lower right side of the circuit board (red to the sin wave marked output, black to ground).

MEASUREMENTS

Part 1: Measuring Voltage and Current in an *RC* Circuit

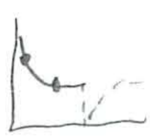
1. Quickly measure the resistance of the resistors (how can you do that?)
2. Create a circuit with the first resistor and the capacitor in series with the battery
3. Connect the voltage sensor (channel A) across the capacitor
4. Record the voltage across the capacitor V and the current sourced by the battery I (Press the green "Go" button above the graph). During this time the battery will switch between putting out 1 Volt and 0 Volts.

Question 1:

What is the resistance of the resistor? Using the two-point method, what is the time constant of this circuit? Using this time constant and the typical expression for an RC time constant, what is the capacitance of the capacitor?

I is it time varying

$V = IR \quad R = \frac{V}{I} \quad \frac{.97V}{198mA} \quad \frac{.97}{.198} = 4.89\ \Omega$ ohms



$I = I_0 \exp(-t/\tau)$
$I_2 = I_1/e$
$\tau = t_2 - t_1$

$\tau = 11\,ms$

$C = \frac{Q}{V}$

$I = \frac{dQ}{dt} = \frac{1}{\tau}(Q - C\mathcal{E})$

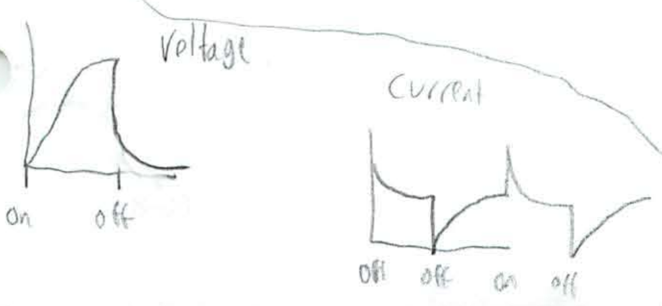
Question 2:

Using the logarithmic method, what is the time constant of this circuit? Using this time constant, what is the capacitance of the capacitor?

$Q(t) = C\mathcal{E}(1 - e^{-t/\tau})$

$C = \frac{\mathcal{E}(1 - e^{-t/\tau})}{Q(t)}$

linear



$\tau = RC$

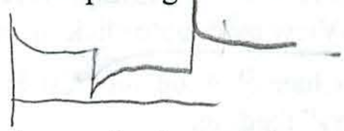
$C = \frac{\tau}{R} = \frac{11}{4.89} = .0022$ farhads

& Can use either voltage or current to find the constant

Part 2: Measuring Voltage and Current in a parallel *RC* Circuit

1. Add the second resistor in parallel with the capacitor
2. Record the voltage across the capacitor *V* and the current sourced by the battery *I* (Press the green "Go" button above the graph). During this time the battery will switch between putting out 1 Volt and 0 Volts.

current



batt Voltage changes slightly

Question 3:

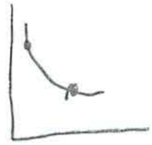
Using one of the two methods used above, what is the time constant of this new circuit? Is there any difference between this circuit (where the battery "turns off") and the one you solved analytically in class (where a switch opens next to the battery)? If so, what? If not, why not?

18.6 mA

$\gamma = 5.6$ ms

No, there is no difference because opening a switch is like turning off a battery

current



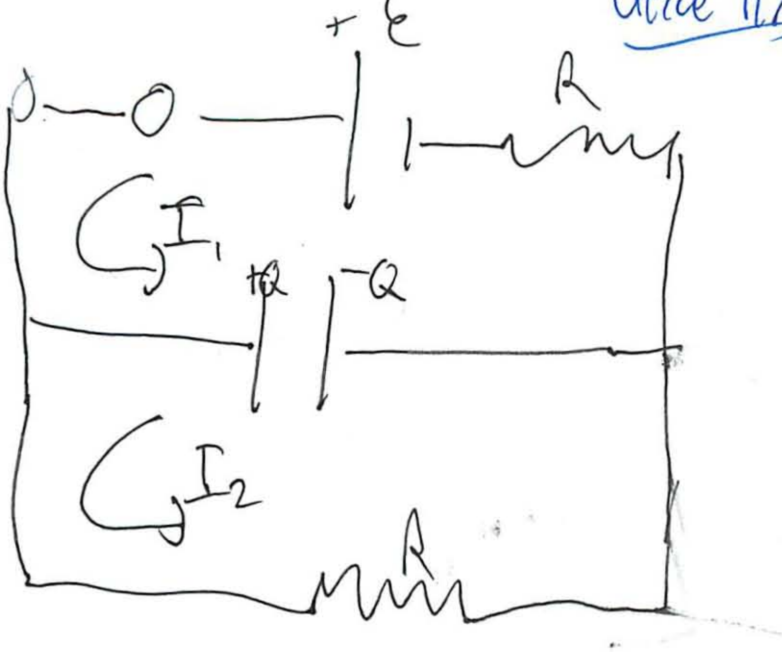
Further Questions (for experiment, thought, future exam questions…)

- What happens if we instead put the second resistor in series with the capacitor?
- What if we change the order of the elements in the circuit (e.g. put the capacitor between the two resistors, or switch the leads from the battery)?

- w/ capicator time is involved
  - simple is RC
  - complex need differential capicator
- takes time for charge to build up
- current can jump (discontinous)
- Voltage constant

Office Hrs alone (very helpful)

3/20



could do either

$$0 = \mathcal{E} - \frac{Q}{C} - I_1 R$$

$$0 = \frac{Q}{C} - I_2 R$$

← not yet!

$I = \frac{dQ}{dt}$

I_1, I_2, Q unknowns
need 3rd eq

$$\frac{dQ}{dt} = I_1 - I_2$$

$$\mathcal{E} - \frac{Q}{C} - I_1 R = \frac{Q}{C} - I_2 R$$

$$+\frac{Q}{C} \quad +I_2 R \qquad +\frac{Q}{C} \quad +I_2 R$$

$$\mathcal{E} - I_1 R + I_2 R = 2\frac{Q}{C}$$

$$\mathcal{E} - R(I_1 - I_2) = 2\frac{Q}{C}$$

$$\mathcal{E} - R\frac{dQ}{dt} = 2\frac{Q}{C}$$

Solve $\frac{dQ}{dt}$

$$\frac{2\frac{Q}{C} - \mathcal{E}}{-R} = \frac{dQ}{dt}$$

$$\frac{-2\frac{Q}{C} + \mathcal{E}}{R} = \frac{-2\frac{Q}{C}}{R} + \frac{\mathcal{E}}{R} = \frac{dQ}{dt}$$

$$-2\frac{Q}{C} \cdot \frac{1}{R} + \frac{\mathcal{E}}{R}$$

$$\boxed{-\frac{2Q}{RC} + \frac{\mathcal{E}}{R} = \frac{dQ}{dt}}$$ is differential eq

match

$\frac{RC}{2} = \tau = \frac{1}{\text{coefficent of } Q}$ ← only thing that changes

How does Q change w/ time

Solution
- exponential decay
- look up

$$\frac{dQ}{dt} = \frac{1}{\tau}(Q - \boxed{C\mathcal{E}})$$ generic

$C\mathcal{E}$: Q_{final}

$Q - Q_{final}$

Looking Q function of time $= Q(t)$

$$Q(t) = Q_{final}\left(1 - e^{-t/\tau}\right)$$

$\tau \to \frac{RC}{2}$

t: variable

$Q_{final} = $ ↓

$$\frac{-2}{RC}\left(Q + \frac{\mathcal{E}C}{-2}\right)$$

↑ $\frac{\mathcal{E}}{R}$ ← constant

$$\frac{\mathcal{E}}{R} = \frac{-2}{RC} \cdot \frac{\mathcal{E}C}{-2}$$

↑ find

$$Q_{final} = [illegible] - \frac{\mathcal{E}C}{-2} = \frac{\mathcal{E}C}{2}$$

$$Q(t) = \frac{\mathcal{E}C}{2}\left(1 - e^{-t/\frac{RC}{2}}\right)$$

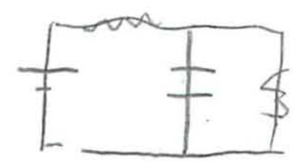
- more complex than exam
- how it works

$\tau \neq$ always RC
- something like

Q_{final} = max charge on capacitor when it was charging
VC =

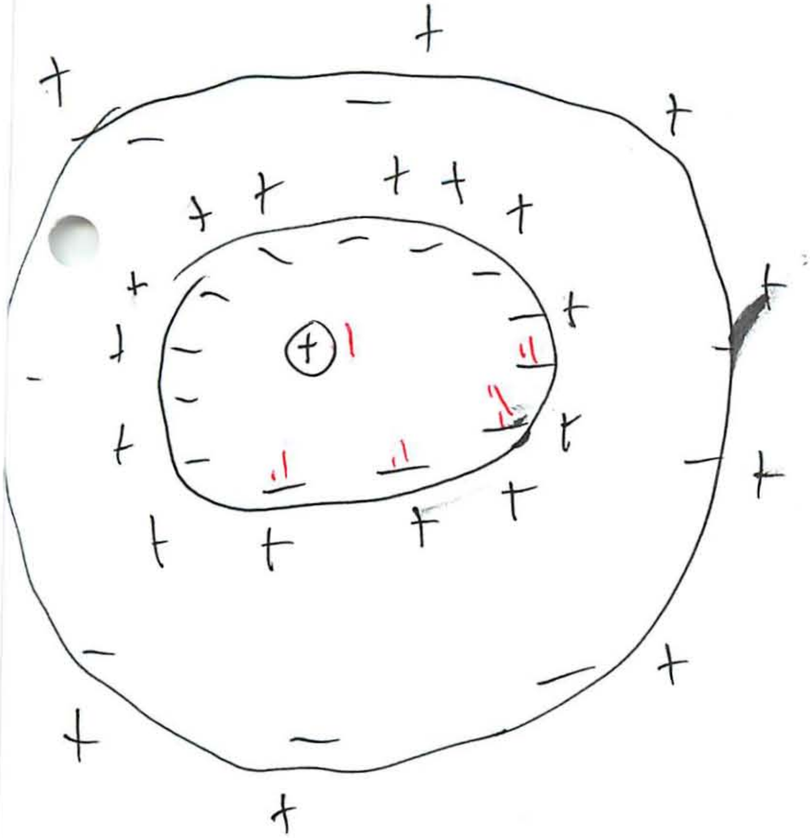
- would be $Q_{initial}$ if discharging
$V = \mathcal{E}$

So



↑ when it was charging 2 resistors so

$Q_{final} = \frac{VC}{2} = \frac{\mathcal{E}C}{2}$ still



E field

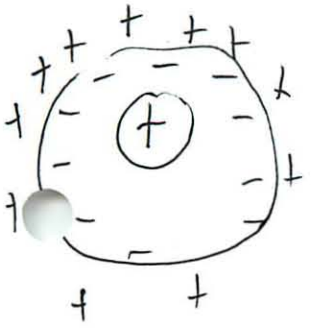
- guassian surface - inside line

$$EA = \frac{q_{total}}{\epsilon_0}$$

- E field inside conductor 0
  - q total = 0
  - have − charges on rim ← far smaller sum to 1q
  - but also ⊕ in middle

take away
⊕ ⊖ hook up
q total still 0

don't think discretly
Jam- smoothly coat inside
- disappears



(+) on outside is $+q$

$+q_{outside} - q_{inside} = $ still 0 charge conservation

Guass' law + charge conservation

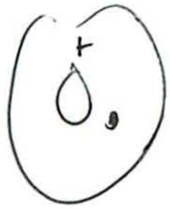
7d

$E(r)$ and $V(r)$ not divided!

$E(r)$

 $\vec{E} = 0$ outside grounded shell

 $\vec{E} = 0$ conductor



- no symmatry no Gauss law

E field a mess, non uniform, can't calc simply

Conductor $E = 0$

Sheilding $E = 0$

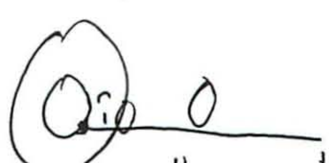
$V(r)$

- last test

~~V(0) = ∞~~ $V(\infty) = 0$

all 0 - no E field = no ΔV

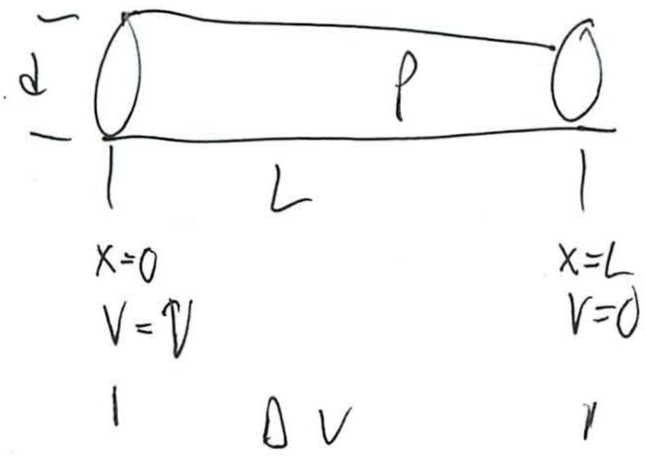
if we can't calc E field
can't calc ΔV



can't walk to in a way we know = un calcable

un calcable

Ohm's Law Problem 3



a) $\vec{E} = ?$ have potential

$$V = -\int E \cdot ds$$

$$-\frac{dV}{ds} = E \quad \text{(1 dimension)}$$

$$E = -\vec{\nabla} V \quad \text{(multi)}$$

$$\frac{\Delta V}{\Delta s} = \frac{V}{L} = \vec{E}$$

b) resistance = R = geometry and material

$\left(\frac{L}{A}\right) \rightarrow A = \pi\left(\frac{d}{2}\right)^2$ ρ (resistivity)

$$R = \frac{L \rho}{\pi\left(\frac{d}{2}\right)^2}$$

c) current $= I = \frac{V}{R}$

current $= \frac{dQ}{dt} = I$

$P = IV$

$V = IR$ ← this problem

$$\frac{V}{\frac{L\rho}{\pi\left(\frac{d}{2}\right)^2}}$$

$$\frac{V}{1} \cdot \frac{\pi\left(\frac{d}{2}\right)^2}{L\rho} = \boxed{\frac{V\pi\left(\frac{d}{2}\right)^2}{L\rho}} = I$$

d) current density $= \vec{J} = \frac{I}{A}$

current density $= \frac{\text{current}}{\text{unit area}} = \frac{I}{A}$

$$\vec{J} = \frac{\frac{V\pi\left(\frac{d}{2}\right)^2}{L\rho}}{\pi\left(\frac{d}{2}\right)^2} = \frac{V\pi\left(\frac{d}{2}\right)^2}{L\rho} \cdot \frac{1}{\pi\left(\frac{d}{2}\right)^2} = \frac{V}{L\rho}$$

$$\boxed{\vec{J} = \frac{V}{L\rho} \uparrow}$$ ← don't forget dir

e) Show $\vec{E} = \rho \vec{J}$

$$\rho \frac{V}{L\rho}\hat{\imath} = \frac{V}{L}\hat{\imath} = \frac{V}{L}\hat{\imath}$$

✓

Math Review
Differentials

3/11

Why - Dynamics

what → equations that involve derivitives

example → time varing circuits, capicator

in 8.01 → $\sum F = ma$

$$a = \frac{dv}{dt} = \frac{\partial^2 x}{dt^2}$$

$F(x, v, t, \text{constants})$

Solve for $x(t)$ or $v(t)$

8.02

$\sum V = 0$

kirkoff's 2nd law

$V(t), I(t), Q(t)$

Makeup

- ordinary = only involves 1 derivitive
- partial = many kinds of derivitives

$m\frac{dv}{dt} = g - bv$ ordinary $\frac{dv}{dt}$

$\frac{\partial^2 y}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial x^2}$ partial $\frac{\partial}{\partial t}$ $\frac{\partial}{\partial x}$ ← not in 8.02

②

<u>Linear</u> - each term has the object of interest to the 0th or 1st power

<u>Non-linear</u> - everything else

$m\frac{dv}{dt} = -bv$ linear power of v ← 8.02

$m\frac{dv}{dt} = -cv^2$ non linear

<u>Homogenious</u> - each term contains exactly one power of the object (1st power only)
no constants

<u>Non homogenious</u> - everything else

$m\frac{dv}{dt} = -bv$ homogenous

$m\frac{dv}{dt} = g - bv$ non homogenious

<u>Order</u> - highest power derivitive in the equation

$m\frac{dv}{dt} = -bv$ 1st order

$m\frac{d^2x}{dt^2} = -bv$ 2nd order

(3)

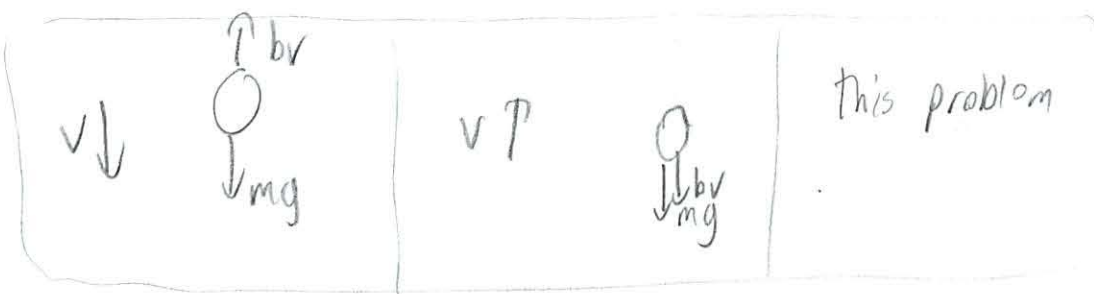
$m\frac{dv}{dt} = -mg - bv$ velocity of an object falling w/ drag

$m\frac{dv}{dt}$: amt accelerating | $-mg$: constant gravity ↓ | $-bv$: drag

Sign? Does it drag the change up or down?

Magnitude? Does it get bigger or smaller?

This is not obvious at first glance



$\frac{dv}{dt} = -g - \frac{b}{m}v$

Verify that is an answer

$v(t) = \frac{mg}{b}\left(e^{-\frac{b}{m}t} - 1\right)$

$\frac{mg}{b}\left(-\frac{b}{m}e^{-\frac{b}{m}t}\right) = -g - \frac{b}{m}\cdot\frac{mg}{b}\left(e^{-\frac{b}{m}t} - 1\right)$

$-ge^{-b/m\,t} = -g - g\left(e^{-\frac{b}{m}t} - 1\right)$

$-ge^{-b/m\,t} = -ge^{-b/m\,t}$

✓

④ $\frac{dQ}{dt} = \frac{Q}{RC} - \frac{\mathcal{E}}{R}$ Sample RC circuit

---

Grandma gives you \$500
5% interest compounded daily
deposit \$ 5/day
In 30 years, how much do you have

$\frac{d\$}{dt} = r\$ + d$ deposit $=$ change in \$ per day
↑ interest rate

$\$(t) = \$_0 + \frac{d}{r}\left(e^{rt} - 1\right)$
↑ so how do you find this?

These are seperable solutions
- can seperate the derivitive

$\frac{dx}{dt} = \beta x + \gamma$

$\frac{dx}{dt} = f(x,t) \rightarrow g(x)dx = h(t)\,dt$

↑ wanted to seperate
x on one side
t on other
) seperable solution

$\frac{dx}{\beta x + \gamma} = dt$

how $\int$ integrate

$\int \frac{dx}{\beta x + \gamma} = \int dt$

$\frac{1}{\beta} \int \frac{\beta\, dx}{\beta x + \gamma}$

$u = \beta x + \gamma$) u substitution to make it eaiser
$du = \beta dx$

$\frac{1}{\beta}\int \frac{du}{u} = \int dt$

$\frac{1}{\beta} \ln u + C_1 = t + C_2$

$\frac{1}{\beta} \ln u = t + C_2 - C_1$

$\frac{1}{\beta} \ln u = t + C$

$\ln(\beta x + \gamma) = \beta t + \beta C$ $\qquad e^{A+B} = e^A e^B$

$\beta x + \gamma = e^{\beta t} \cdot e^{\beta C}$ $\qquad$ set $A_0 = e^{\beta C}$

$x(t) = \frac{1}{\beta}(e^{\beta C} \cdot e^{\beta t} - \gamma)$

$$\boxed{x(t) = \frac{A_0}{\beta} e^{\beta t} - \frac{\gamma}{\beta}}$$ general solution

this all moves very fast
- like switches

this was the seperability solutions
each one has its own trick to solve one

We did not know the inital position in problems
besides the A (interst rate)
this is the inital value problem

$x(0) = X_0 = \frac{A_0}{\beta} e^{\beta t} - \frac{\gamma}{\beta}$

$\beta X_0 = A_0 - \gamma$

$A_0 = \beta X_0 + \gamma$

(6) $$\boxed{x(t) = \frac{\beta x_0 + \gamma}{\beta} e^{\beta t} - \frac{\gamma}{\beta}}$$

exact answer

β, γ given in equation
x_0 = initial condition given

$x(t) \rightarrow Q(t)$
$\beta \rightarrow -\frac{1}{RC} = \tau$ (time constant)
$\gamma \rightarrow -\frac{\mathcal{E}}{R}$
$x_0 \rightarrow 0$

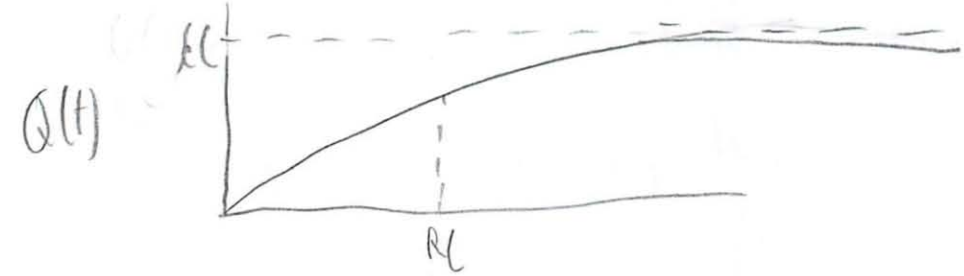
$Q(0) = 0$

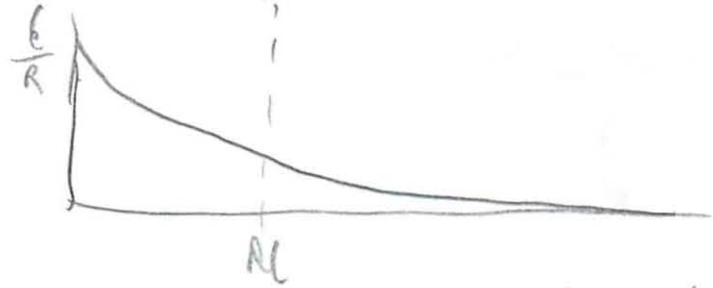
$$Q(t) = \frac{\left(-\frac{\mathcal{E}}{R}\right)}{\left(-\frac{1}{RC}\right)} e^{-t/RC} - \frac{\left(-\frac{\mathcal{E}}{R}\right)}{\left(-\frac{1}{RC}\right)}$$

$$Q(t) = \mathcal{E} C \left(e^{-t/RC} - 1\right)$$

$$I(t) = \frac{dQ}{dt} = -\frac{\mathcal{E}}{R} e^{-t/RC}$$

$\mathcal{E}$ = Script E = emf
ϵ_0 - epsilon = dielectric constant

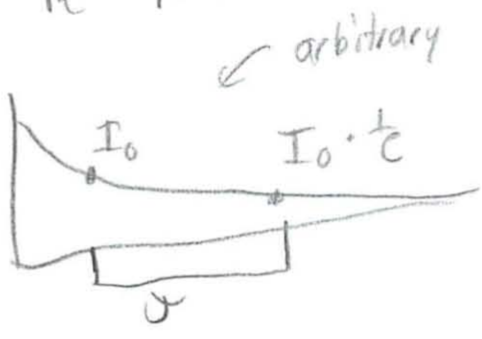




↑ when $t = RC$ its e^{-1} this is where you look } what they have defined as ↑ time constant

Fully charged is like $10 \cdot \tau$

You find 1 pt and then pt $\cdot e^{-1}$ → The time in between is the time constant



$\frac{1}{e} = e^{-1} \approx \frac{1}{3}$

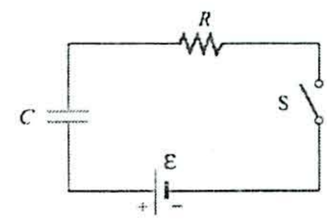
Topic: RC Circuits
Related Reading: Course Notes: Sections 7.1 – 7.6, 7.8-7.9
Experiments: (4) RC Circuits

Topic Introduction

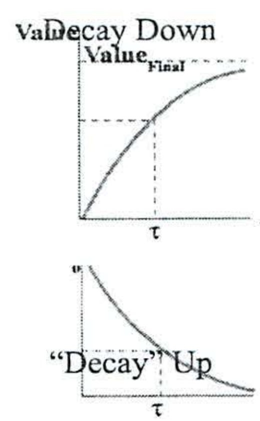
In the last couple of classes you had the chance to hear about and then investigate the behavior of RC circuits. In today's problem solving session you will practice solving analytic and answering short conceptual questions about these circuits.

RC Circuits

A simple RC circuit is shown at right. When the switch is closed, current will flow in the circuit, but as time goes on this current will decrease. We can write down the differential equation for current flow by writing down Kirchhoff's loop rules, recalling that $|\Delta V| = Q/C$ for a capacitor and that the charge Q on the capacitor is related to current flowing in the circuit by $I = \pm dQ/dt$, where the sign depends on whether the current is flowing into the positively charged plate (+) or the negatively charged plate (-). The solution to this differential equation shows that the current decreases exponentially from its initial value while the potential on the capacitor grows exponentially to its final value. The rate at which this change happens is dictated by the "time constant" τ, which for this circuit is given by $\tau = RC$.



Interestingly, in RC circuits any value that you could ask about (current, potential drop across the resistor, across the capacitor, …) "decays" exponentially (either down or up). You should be able to determine which of the two plots at right will follow just by thinking about it.



Important Equations

Exponential Decay: $Value = Value_{initial} e^{-t/\tau}$

Exponential "Decay" Upwards: $Value = Value_{final}\left(1 - e^{-t/\tau}\right)$

) derived formula

Simple RC Time Constant: $\tau = RC$;

Test - week after break

Conductors

Capacitors (Guass' Law)

Circuits

Magnetic Force (in class next week)

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Problem Solving 5: RC Circuits

OBJECTIVES

1. To gain intuition for the behavior of DC circuits with both resistors and capacitors or inductors. In this particular problem solving you will be working with an RC circuit. You should carefully consider what would change if the capacitor were replaced with an inductor.

2. To calculate the time dependent currents in such circuits

what is RC?
- resistance + capacitor

REFERENCE: Chapter 7, 8.02 Course Notes.

An RC circuit consists of both resistors and capacitors, and typically a battery to get the current flowing. Capacitors, when uncharged, act like pieces of wire ("shorts") as they have no voltage drop across them. However, once charge has flowed on to them for a while, they "charge up," eventually reaching a potential equal and opposite that trying to charge them and effectively preventing the further flow of current.

This problem solving consists of two parts. In the first you will answer a series of short questions developing your intuition for the behavior of these circuits on short and long time scales. In the second part you will work through a quantitative problem.

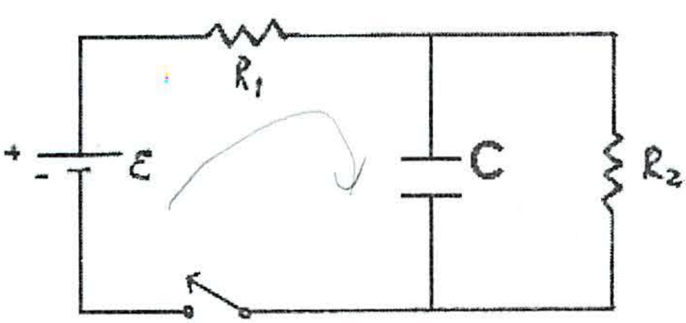


Figure 1: RC Circuit
An RC circuit consists of two resistors, R_1 and R_2, a capacitor C, a battery ε, and a switch. The switch has been open for a very long time before it is closed at time t=0.

Write your answer to this and all following questions on the tear-sheet at the end!
What is/are...
Question 1: the current I_C (through the capacitor) at $t=0^+$ (just after switch is closed)?

$I_C = +\varepsilon - R_1 - C V = 0$

$I_C = \varepsilon - \frac{V}{R_1}$

Question 2: the currents I_1 and I_2 (through R_1 and R_2 respectively) at $t=0^+$?

$I_1 = I_c = \mathcal{E} - \frac{V}{R_1}$

~~$I_2 = \mathcal{E} - R_1$~~

$I_2 = 0$ } all goes through capacitor

Question 3: the current I_C (through the capacitor) at $t=\infty$?

$\lim_{t \to \infty} I_c = 0$

Question 4: the currents I_1 and I_2 (through R_1 and R_2 respectively) at $t=\infty$?

$I_1 = I_c = 0$ $\quad I_2 = \mathcal{E} - \frac{V}{R_1} - \frac{V}{R_2}$ battery alive

batt does not die

~~$I_2 = (V - \frac{V}{R_2} =$~~ batt dead

~~$I_2 = 0$~~ batt / capacitor dead

At intermediate time *t* assume there is a charge *q* on the capacitor.

Question 5: Using Kirchhoff's Loop Rules, obtain a differential equation for the charge *q* on the capacitor, assuming $R_1=R_2=R$ (in other words, the only current in the equation should be the current through the capacitor, which can be rewritten in terms of d*q*/dt).

$\mathcal{E} - I_1 R - \frac{Q}{C} = \frac{Q}{C} - I_2 R$ $\qquad I_c = I_1 - I_2 = \frac{dQ}{dt}$

solve for $\frac{dQ}{dt}$

$\mathcal{E} - I_1 R + I_2 R = \frac{2Q}{C}$

$\mathcal{E} - R(I_1 + I_2) = \frac{2Q}{C}$

$\mathcal{E} - \frac{dQ}{dt} R = \frac{2Q}{C}$

solve for $\frac{dQ}{dt}$

$\frac{\frac{2Q}{C} - \mathcal{E}}{-R} = \frac{dQ}{dt}$

$-\frac{2Q}{RC} + \frac{\mathcal{E}}{R} = \frac{dQ}{dt}$

Question 6: What is the time constant for charging the capacitor?

$\tau = \frac{1}{\text{coefficent of } Q} = \frac{RC}{2}$

Question 7: Write an equation for the time dependence of the charge on the capacitor

$\frac{dQ}{dt} = \frac{1}{\tau}(Q - Q_{final})$

$Q(t) = Q_{final}\left(1 - e^{-t/\tau}\right)$ $\to \frac{RC}{2}$

$Q_{final} = \frac{\mathcal{E}C}{2}$?

$V=IR$

$R=\frac{I}{V}$

$I=\frac{dV}{dt}\cdot C$

After a long time *T* the switch is opened.

What is/are…

Question 8: the current I_C (through the capacitor) at $t=T^+$ (just after switch is opened)?

$I_C = VC - \frac{V}{R_2}$

Question 9: the currents I_1 and I_2 (through R_1 and R_2 respectively) at $t=T^+$?

$I_1 = 0$ $I_2 = I_C = \frac{Q}{C} - R_2$

Question 10: Using Kirchhoff's Loop Rules, obtain a differential equation for the charge *q* on the capacitor after the switch has been opened, assuming $R_1=R_2=R$ (in other words, the only current in the equation should be the current through the capacitor, which can be rewritten in terms of d*q*/dt).

$VC = Q$

$I = \frac{dQ}{dt}$

$VC - R_2 = 0$

$\frac{Q}{C} - \frac{dQ}{dt} R = 0$

$\frac{Q}{C} = \frac{dQ}{dt} R$

$\frac{dQ}{dt} = \frac{Q}{RC}$

Question 11: What is the time constant for discharging the capacitor?

$\tau = \frac{1}{\text{coefficient of } Q} = RC$

Question 12: Write an equation for the time dependence of the charge on the capacitor after time T.

$Q(t) = Q_{final}\left(1 - e^{-t/RC}\right)$

? solve for final charge question 6

VC = max charge

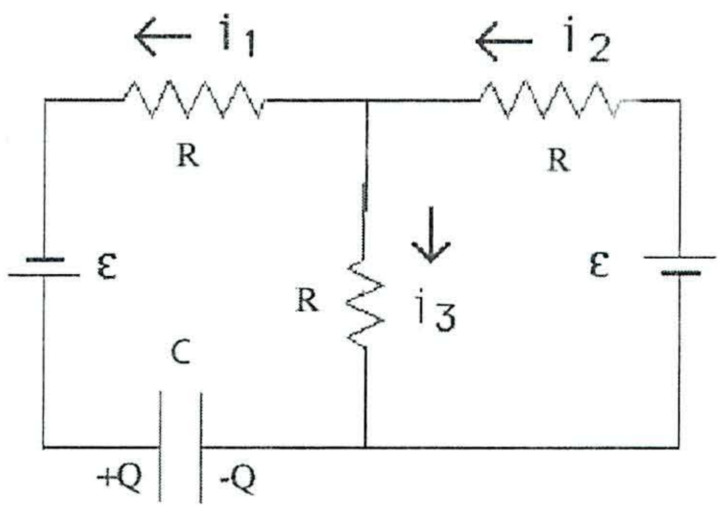
$Q(t) = \frac{VC}{2}\left(1 - e^{-t/RC}\right)$

? ↑ 2 resistors when it was charging

$V ? \mathcal{E}$

Q initial if discharging

Sample Exam Question (If time, try to do it by yourself, closed notes)



(a) From Kirchoff's first rule, what is the relation between i_1, i_2, and i_3?

(b) What does the loop theorem (Kirchhoff's second rule) yield if we traverse the left loop of the above circuit *moving counterclockwise*, in terms of the quantities shown, with the directions of the currents as shown?

(c) What does the loop theorem (Kirchhoff's second rule) yield if we traverse the right loop of the above circuit *moving counterclockwise*, in terms of the quantities shown, with the directions of the currents as shown?

(d) After a very long time, $t >> RC$, what is the current i_1?

(e) After a very long time, $t >> RC$, what are the currents i_2 and i_3?

(f) After a very long time, $t >> RC$, what is the voltage across the capacitor in terms of the quantities given? (Hint: use your results from part (b)-(e)).

pset 6 after

Durmashkin OH

3/15

Ideas → methodology

- keep a list of concepts
- conventions

Multiloop

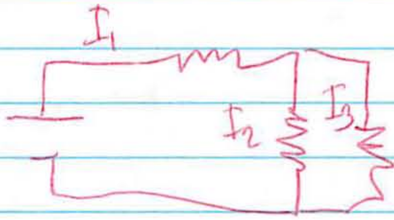
$\sum V = 0$

at any node I goes in and out

$P = IV = I^2 R$

How does Voltage change over time between points

branches, nodes + loops



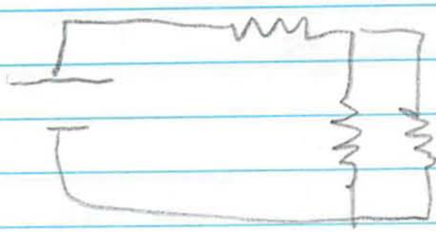
current same everywhere in branch

3 branches

1. Choose a current + dir in each branch

2. Node = junction
   - Current conserved at nodes

$I_1 = I_2 + I_3$



Loops
2 independent ← always n − 1
3 total

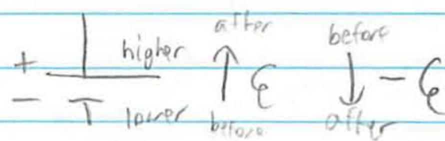
He uses interior ones

For each loop $\sum V_i = 0$

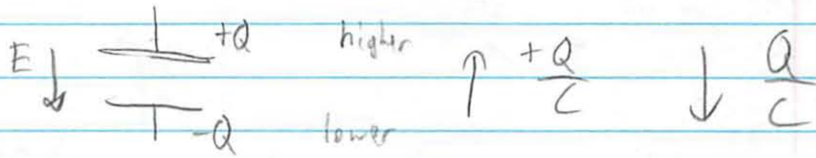
Choose a travel direction
- not nesserally the current
- draw an arrow

- multiple currents in loop
- tells us ΔV_i → after − before

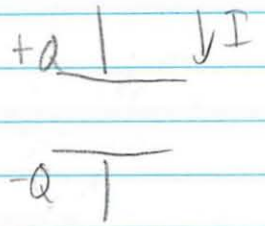
Convention for each element

Battery 

Capicator 
- choose a +q, −q



relation I Q



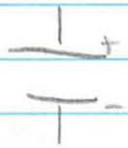
I → ⊕ charges flowing

I = charge per time

$$I = \frac{dQ}{dt}$$

? ⊕ ? so ⊕

If I↑

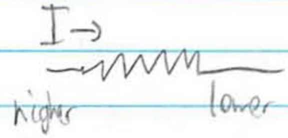


Then discharging

$$\frac{dQ}{dt} = \ominus$$

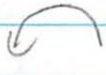
If want I ⊕ → $I = -\frac{dQ}{dt}$

<u>Resistor</u>



resisitor always ↓ V

$V = -IR$



opposit current $V = IR$

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8.02 **Spring 2010**

Problem Set 6

Due: Tuesday, March 16 at 9 pm.

Hand in your problem set in your section slot in the boxes outside the door of 32-082. Make sure you clearly write your name and section on your problem set.

Text: Liao, Dourmashkin, Belcher; Introduction to E & M MIT 8.02 Course Notes.

Week Six DC Circuits

Class 14 W06D1 M/T Mar 8/9	Expt. 3 Building a Circuit with Resistors, DC Circuits & Kirchhoff's Loop Rules;
Reading:	Course Notes: Sections 7.1-7.5, 7.8-7.9
Experiment:	Expt. 3 Building a Circuit with Resistors
Class 15 W06D2 W/R Mar 10/11	RC Circuits; Expt. 4: RC Circuits
Reading:	Course Notes: Sections 7.5 – 7.6
Experiment:	Expt. 4: RC Circuits
Class 16 W06D3 F Mar 12	PS05: RC Circuits
Reading:	Course Notes: Sections 7.1 – 7.6, 7.8-7.9

Week Seven Magnetic Fields

Class 17 W07D1 M/T Mar 15/16	Magnetic Fields; Magnetic Forces, Expt. 5: Bar Magnet
Reading:	Course Notes: Chapter 8.1-8.3, 8.5-8.6, 8.8-8.9, 9.5
Experiment:	Expt. 5: Bar Magnet
Class 18 W07D2 W/R Mar 17/18	Creating Fields: Biot-Savart Law, Currents & Dipoles; Expt. 6: Torque on Dipole
Reading:	Course Notes: Sections 8.3-8.4, 9.1-9.2, 9.10.1, 9.11.1-9.11.4
Experiment:	Expt. 6: Torque on Dipole
Class 19 W07D3 F Mar 19	PS06: Calculating Magnetic Fields and Magnetic Force
Reading:	Course Notes: Sections 8.8-8.9, 9.10.1, 9.11.1-9.11.4

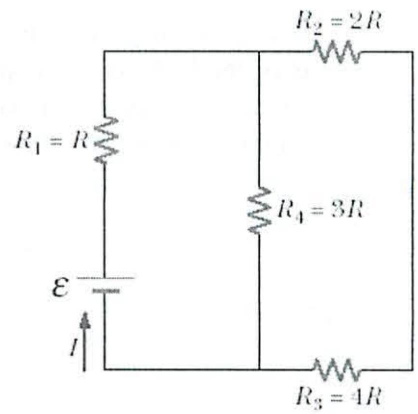
Week Eight Spring Break

Week Nine Magnetic Fields; Exam 2

Class 20 W09D1 M/T Mar 29/30	Creating Fields: Ampere's Law
Reading:	Course Notes: 9.3-9.4, 9.10.2, 9.11.5-9.11.8
Class 21 W09D2 W/R Mar 31/Apr 1	PS07: Ampere's Law; Exam 2 Review
Reading:	Course Notes: 9.3-9.4, 9.10.2, 9.11.5-9.11.8
Exam 2 Thursday April 1	**7:30 pm –9:30 pm**
W09D3 F Apr 2	No class day after exam

Problem 1: Four Resistors

Four resistors are connected to a battery as shown in the figure. The current in the battery is I, the battery emf is $\mathcal{E}$, and the resistor values are $R_1 = R$, $R_2 = 2R$, $R_3 = 4R$, $R_4 = 3R$.



(a) Rank the resistors according to the potential difference across them, from largest to smallest. Note any cases of equal potential differences.

(b) Determine the potential difference across each resistor in terms of $\mathcal{E}$.

(c) Rank the resistors according to the current in them, from largest to smallest. Note any cases of equal currents.

(d) Determine the current in each resistor in terms of I.

(e) If R_3 is increased, what happens to the current in each of the resistors?

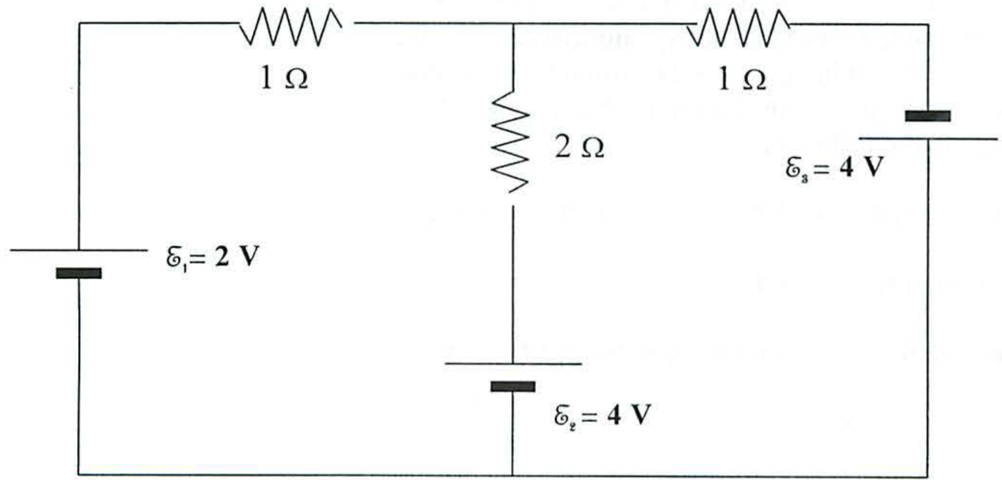
(f) In the limit that $R_3 \to \infty$, what are the new values of the current in each resistor in terms of I, the original current in the battery?

Problem 2 Multi-loop Circuit

In the circuit below, you can neglect the internal resistance of all batteries.

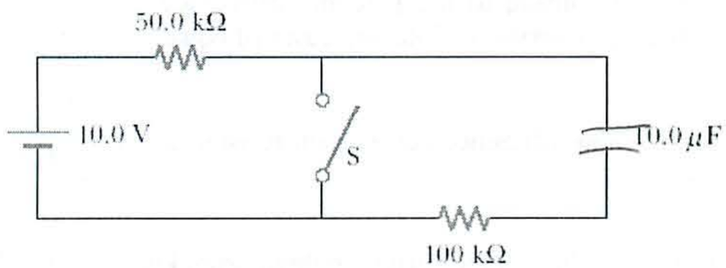
(a) Calculate the current through each battery

(b) Calculate the power delivered or used (specify which case) by each battery



Problem 3: RC Circuit

In the circuit shown, the switch S has been closed for a long time. At time t=0 the switch is opened. It remains open for "a long time" T, at which point it is closed again. Write an equation for (a) the voltage drop across the 100 kΩ resistor and (b) the charge stored on the capacitor as a function of time.

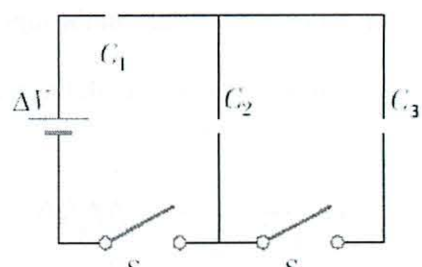


Problem 4: Energy stored in a capacitor

You know that the power supplied by a battery is given by P = VI (the battery voltage times the current it is supplying). You also know that a resistor dissipates power (turns it into heat) at a rate given by $P = I^2R$.

Consider a simple RC circuit (battery, resistor R, capacitor C). Determine an expression for the energy stored in the capacitor by integrating the difference between the power supplied by the battery and that consumed by the resistor. Should the energy be related to the current through the capacitor or the potential across it?

Problem 5: Capacitors



In the circuit shown at right $C_1 = 2.0\ \mu F$, $C_2 = 6.0\ \mu F$, $C_3 = 3.0\ \mu F$ and $\Delta V = 10.0\,\mathrm{V}$. Initially all capacitors are uncharged and the switches are open. At time $t = 0$ switch S_2 is closed. At time $t = T$ switch S_2 is then opened, proceeded nearly immediately by the closing of S_1. Finally at $t = 2T$ switch S_1 is opened, proceeded nearly immediately by the closing of S_2. Calculate the following:

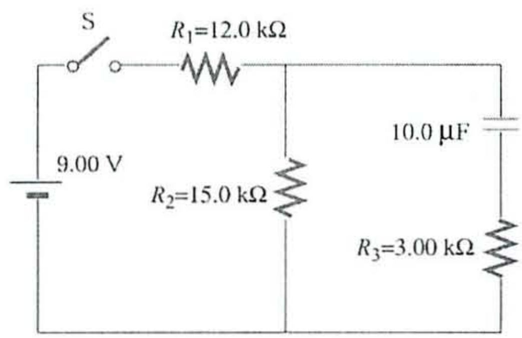
(a) the charge on C_2 for $0 < t < T$ (after S_2 is closed)

(b) the charge on C_1 for $T < t < 2T$

(c) the final charge on each capacitor (for $t > 2T$)

Problem 6: RC Circuit

Consider the *RC* circuit shown in the figure. Suppose that the switch has been closed for a length of time sufficiently long enough for the capacitor to be fully charged.



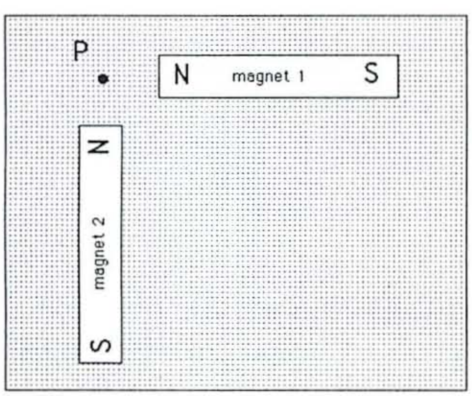
(a) Find the steady-state current in each resistor.

(b) The switch is opened at $t = 0$. Write an equation for the current I_2 in R_2 as a function of time.

(c) Find the time that it takes for the charge on the capacitor to fall to 1/e of its initial value.

Problem 7: Experiment 5: Magnetic Fields of a Bar Magnet and Helmholtz Coil Pre-Lab Questions

Read Experiment 5 before answering these questions



Consider two bar magnets placed at right angles to each other, as pictured at left.

(a) If a small compass is placed at point P, what direction does the painted end of the compass needle point?

(b) If the compass needle instead pointed 15 degrees clockwise of where you predicted in (a), what could you *qualitatively* conclude about the relative strengths of the two magnets?

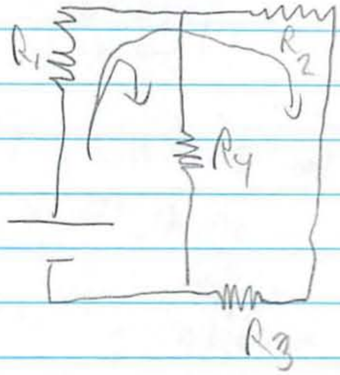
P-Set 6

Michael Plasmeier L01 11C — 3/14

100-8-8= (84)

1. Four Resistors

Fairly short all RC circuits - which I want to practice



$R_1 = R$
$R_2 = 2R$] $= 6R$ ⇒ just add
$R_3 = 4R$
$R_4 = 3R$

$\mathcal{E} - I_1 R_1 - I_4 R_4 = 0$
$\mathcal{E} - I_1 R - I_4 4R = 0$

✓

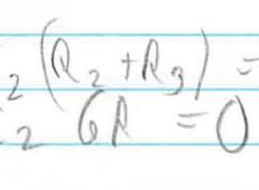
$\mathcal{E} - I_1 R_1 - I_2 (R_2 + R_3) = 0$
$\mathcal{E} - I_1 R_1 - I_2 6R = 0$

$\mathcal{E} = I_1 R + I_4 4R = I_1 R + I_2 6R$

? so the question is where from here

unknowns I_1, I_2, I_4,
need a 3rd equation relating 2 of I_1, I_2, I_4

also draw



$$\frac{1}{R_{total}} = \frac{1}{R_2} + \frac{1}{R_4}$$

$R_{total} = R_2 + R_4$

anyway that is not the question!

a) Rank V across resistors in terms of $\mathcal{E}$

$V = \frac{I}{R}$ $\quad V_1 = \frac{I_1}{R}$ $\quad V_3 = \frac{I_2}{4R}$

$V_2 = \frac{I_2}{2R}$ $\quad V_4 = \frac{I_3}{3R}$

think back to that MP problem
think of adding

$$V = IR + \left(\frac{1}{I\,6R} + \frac{1}{I\,3R}\right)$$

$\frac{1}{2}\mathcal{E}$ $\quad \frac{1}{2}\frac{6}{2}$ $\quad \frac{1}{2}\frac{6}{2}$ if = Resistance

Voltage drop $\quad \frac{1}{2}\mathcal{E}$ $\quad \frac{1}{2}\cdot\frac{2}{3}\mathcal{E}$ $\quad \frac{1}{2}\cdot\frac{1}{3}\mathcal{E}$ in this case?

so R_1 is largest drop $\frac{1}{2}\mathcal{E}$
R_2 is $\frac{1}{2}\cdot\frac{2}{9} = \frac{1}{9}$ 4th
R_3 is $\frac{1}{2}\cdot\frac{4}{9} = \frac{2}{9}$ 3rd largest
R_4 is $\frac{1}{2}\cdot\frac{3}{9} = \frac{1}{6}$ 2nd

b) +C) Current $V = IR$

R_1) $\frac{1}{2}\mathcal{E} = I\cdot 1$ so $\frac{1}{2} = I$ $\quad I = \frac{1}{2}$ what's relative unit

R_2) $= \frac{1}{9} = I\cdot 2$ $\quad I = \frac{1}{18}$

R_3) $\frac{2}{9} = I\cdot 4$ $\quad I = \frac{1}{18}$
R_4) $\frac{1}{6} = I\cdot 3$ $\quad I = \frac{1}{18}$

? So does this make sense

- same current through R_2 and R_3 does make sence

but <u>does</u> not have to be same current through 4

Let's run simulation to ✓

but less current should go through R_2 + R_3 since its so much less

So Amps

2/3 through R_4 .2A	1/3 though $R_2 + R_3$ .1A	.3A total

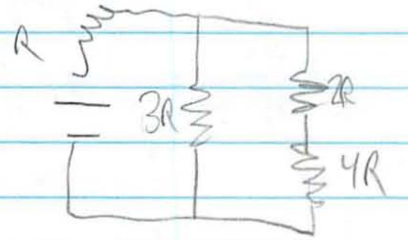
Volts

3V drop R_1
6V drop R_4
2V drop R_2
4V drop R_3
) batt = 9V

So my V calc was wrong

$R_1 = \frac{1}{3}\varepsilon$ $R_2 = \frac{2}{3}\varepsilon$ $R_3 = \frac{2}{9}\varepsilon$ $R_4 = \frac{4}{9}\varepsilon$

Try again - no loop rule trying



Find R_{eq}

$$R_{eq} = R + \frac{1}{\frac{1}{3R} + \frac{1}{(2+4)R}}$$

~~$R + \frac{1}{3R + 6R}$~~

~~$R_{eq} = R + \frac{1}{9R}$~~

~~$R_{total} = \frac{10}{9} R$~~ (like the first MP problem)

? so then what find?

* So this is like a battery w/ 1 resistor of value $\frac{10}{9} R$ where R is a constant

$V = IR$
$6 = I \cdot \frac{10}{9} R$
$I = \frac{6}{\frac{10}{9} R}$

Now lets try ~~it~~ to match sim $\frac{9V}{\frac{10}{9} \cdot 10\,\Omega} = .81$

which is not the .3 I was looking for

No did addition wrong above ⟶

$$R + \frac{1}{\frac{1}{3R} + \frac{1}{6R}}$$

need to get common denom

$$\frac{2}{6R} + \frac{1}{6R} = \frac{3}{6R} = \frac{1}{2R}$$

$$R + \frac{1}{\frac{1}{2}R}$$

$$R + 2R$$

$$R_{eq} = 3R$$

$$V = I \cdot 3R$$

$$I = \frac{6}{3R}$$

try again w/ #

$$\frac{9V}{3 \cdot 10\Omega} = .3 \text{ amps}$$ ✓ matches simulation

So where do I stand on the question

Now just the R_4 branch

~~$V = 6 - IR - I3R$~~

~~$6 = I4R$~~

~~$I = \frac{6}{4R} \approx \frac{9V}{4 \cdot 10\Omega} = .225 \text{ amps}$~~

missing a branch

$\mathcal{E} = I_1 R + I_2 6R$

↑ .3 amps in our example

$9V = .3 \cdot 10\Omega + I_2 \cdot 6 \cdot 10\Omega$

$9 = 3 + 60 I_2$

$-3 \quad -3$

$6 = +60 I_2$

$I_2 = .1$ amps or $\frac{1}{3} I$

↳ matches simulation ⓪ or $\frac{1}{3} I$

$\mathcal{E} = IR + I_4 3R$

$9 = .3 \cdot 10 + I_4 \cdot 3 \cdot 10$

$6 = 30 I_4$

$I_4 = .2$ amps or $\frac{2}{3} I$

↳ matches simulation ⓪ or

e) If R_3 was ↑

current through it would furter ↓

current through R_4 ~~would not change~~ } see next problem

- instead less current drawn from batt

as this equation shows

f) If $R_3 \to \infty$ what is new current everywhere

so $R \to \infty$ is like break in circuit

$I_2 = 0$

I_4 would be one circuit

$\mathcal{E} - IR - I3R = 0$

$\mathcal{E} = I4R$

with simulation #

$9V = I \cdot 4 \cdot 10\,\Omega$

$I = .225$ amps

or 75% I

? but I though changing R_3 would not effect on other one

- but doesn't it → its like closing door to teater

Yes → it decreases a very small amt

But why??

I think I need to go to Office Hrs

- really confused

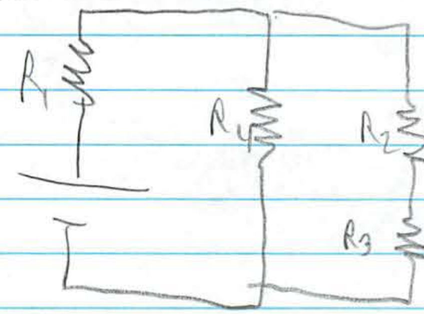
I is the same ($I_1 = I_4$) in 2nd problem so did not have to figure seperatly

this was the change from $R_3 \to \infty$ vs before

- mirrored simulation
- but kinda weird

Method 1 Series + Parrall

#1 redo Dormation OH



Could do series parallel to find I_1

} take each seperatly

find $R_{eq\,2+3}$

then $R_{eq\,4+(2+3)}$

then $R_{eq\,1+(4+(2+3))}$ → $R_{eq\,final}$

$$I_1 = \frac{\mathcal{E}}{R_{eq\,final}}$$

$$I_1 = I_2 + I_3 \leftarrow I_3 R_4 = I_2(R_2 + R_3)$$

$$I_3 = \frac{I_2(R_2 + R_3)}{R_4}$$

complex + long but doable

Method 2

But could also do w/o reducing kirkoff rules

$$I_1 = I_2 + I_3$$

$$\mathcal{E} - I_1 R_1 - I_3 R_4 = 0$$

$$-I_2 R_2 - I_2 R_3 + I_3 R_4 = 0$$

3 eq w/ 3 unknowns (I_1, I_2, I_3)
Put #s in to make it easier + solve

* Trying to find current in each branch *

- resistors + batts known

If ↑ R_3

Ratio b/w 2 branches changes
^of resistance

$$\frac{I_3}{I_2} = \frac{R_2 + R_3}{R_4}$$

More current through path of less resistance

R_4 ∞ → R_2 goes to 0

Then do the 1 loop problem

Current does not automatically change

$$\frac{1}{R_{eq}} = \frac{1}{R_4} + \frac{1}{(R_2 + R_3)}$$

$$R_{eq} = \frac{(R_2 + R_3) R_4}{R_2 + R_3 + R_4}$$ ← combine denoms + flip

like $\frac{1}{10} + \frac{1}{100} \approx \frac{11}{100} \rightarrow R_{eq} = \frac{100}{11} = \frac{200}{22}$

increase R_3 $\frac{1}{10} + \frac{1}{200}$ $\frac{21}{200} \rightarrow R_{eq}$ $\frac{200}{21}$

bigger by a very small amt

Resistance of everything ↑
Current ↓

Don't know ratios
- they may change

Need to do equivilancy calculation

Complex (exactly what I found)

---

as $R_3 \to \infty$

Ratio changes
I_1 ↓
solve for I_2

$$I_1 = I_2\left(1 + \frac{R_2 + R_3}{R_4}\right)$$

$$I_2 = \frac{I_1}{1 + \frac{R_2 + R_3}{R_4}}$$ solve for I_1

not obvious if it ↑ or ↓

I_1 must ↓

$$\frac{1}{\infty} = 0$$

$R_{eq} = R_4$ when $R_2 + R_3 = \infty$

$R_{eq} \leq R_4$ always

have to do more algebra
- solve

1, self relook at after OH and more pratice

So I had it fairly right

but confused b/w the 2 methods
& id have to use a combo of both inorder to solve

* Also I screwed up adding $\frac{1}{\text{stuff}}$

So $\frac{1}{eq} = \frac{1}{R_4} + \frac{1}{(R_2+R_3)}$

need to get common denom

$$\frac{1}{eq} = \frac{R_2+R_3}{R_4(R_2+R_3)} + \frac{R_4}{(R_2+R_3)R_4}$$

$$\frac{1}{eq} = \frac{R_2+R_3+R_4}{R_4(R_2+R_3)}$$

$$eq = \frac{R_4(R_2+R_3)}{R_2+R_3+R_4}$$ ✓checks out

Now pratice solving system

$$I_1 = I_2 + I_3$$
$$6 - I_1 R_1 - I_3 R_4 = 0$$
$$-I_2 R_2 - I_2 R_3 + I_3 R_4 = 0$$

Plug # in

$$I_1 = I_2 + I_3$$
$$6 - I_1 R - I_3 3R = 0$$
$$-I_2 2R - I_2 4R + I_3 3R = 0$$

So last time I got that far and gave up
and switched to eq resistors the wrong way

Don't nessarily set them = to each other
Could solve w/ matrixes like in math - but
that gets long + complex
Try ~~...~~ solving + replacing
└ to reduce # of variables

First group terms + get in terms of $I_1 + I_2$

$$6 - I_1 R - (I_1 - I_2) 3R = 0$$
$$6 - R(I_1 + 3(I_1 - I_2)) = 0$$

$$-I_2 6R + (I_1 - I_2) 3R = 0$$
$$R(3(I_1 - I_2) - 6 I_2) = 0$$

$$6 - R(I_1 + 3(I_1 - I_2)) = R(3(I_1 - I_2) - 6 I_2)$$
$$+R \qquad \qquad + R$$
$$6 = R(3(I_1 - I_2) - 6 I_2 + I_1 + 3(I_1 - I_2))$$
$$6 = R(I_1 - 6 I_2 + 6 I_3)$$
now can solve for each
$$6 = I_1 R - 6 I_2 R + 6 I_3 R$$

$$I_1 = \frac{6 + 6 I_2 R + 6 I_3 R}{R}$$

$$I_2 = -\frac{6 - I_1 R - 6 I_3 R}{6R}$$

$$I_3 = \frac{6 - I_1 R - 6 I_2 R}{6R}$$

But the current is still in terms of something else - is there a way to avoid that?

? Write I_2 in for I_1 ?? - well there are no #s

? Some sort of differential eq

Or should I go w/ plain equivilant resistance

$$R_{eq_{2+3}} = R_2 + R_3$$

$$R_{eq_{4+(2+3)}} = \text{did before} = \frac{R_4(R_2+R_3)}{R_2+R_3+R_4}$$

$$R_{eq_{1+[4+(2+3)]}} = R_1 + \frac{R_4(R_2+R_3)}{R_2+R_3+R_4}$$

$$I_{total} = \frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{R_1 + \frac{R_4(R_2+R_3)}{R_2+R_3+R_4}}$$

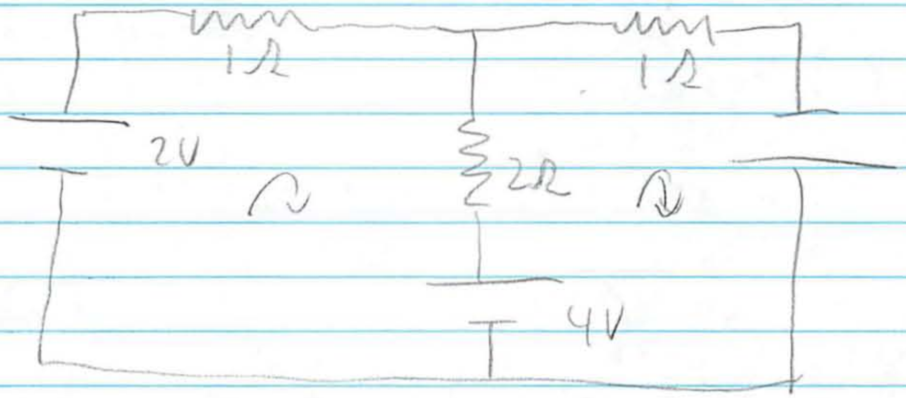
that is really complex as well but does not depend on other Is

So I guess I could have subbed in Is and solved simultaneously

2. Multi-Loop Circuit

No internal resistance

a) Calc current through battery



numbers! makes it eaiser, when the numbers are right :)

$2 - I_1 \cdot 1 - 2I_1 - 4 = 0 = 4 - 2I_2 - I_2 + 4$

$-3I_1 = 2$ $\qquad -3I_2 = 8$

-8

$I_1 = -\frac{2}{3}$ amps $\qquad I_2 = \frac{8}{-3}$ amps

? batt 1 $\qquad$? batt 3

batt 2 $= -\frac{8}{3} - -\frac{2}{3} = -\frac{6}{3} = -2$ amps ← This could be right or wrong depending on how you drew it.

b) Calc power delivered by

$P = IV$

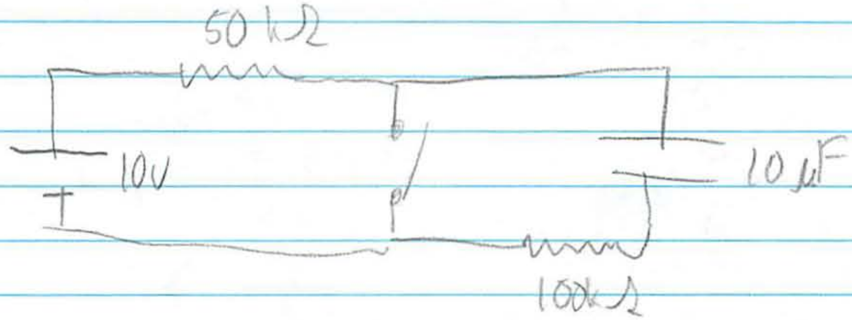
P = 1V ✓

$P_1 = \frac{2}{3} \cdot 2 = \frac{4}{3}$ watts

$P_2 = 2 \cdot 4 = 8$ watts

$P_3 = \frac{8}{3} \cdot 2 = \frac{16}{3} = 5.333$ watts

3. RC circuit



Switch closed long time
$t = 0$ switch opened
Remains open for long time
Closed

a) Voltage drop acrosss 100 kΩ resistor

Instant closed - bypasses batt
Starts de-charging

$$\frac{Q}{C} - I \cdot 100\,k\Omega = 0$$

I need to find
Q from when it was open

$$10 - I \cdot 50 - I \cdot 100 - \frac{Q}{C} = 0$$

$$10 = 150 \frac{dQ}{dt} + \frac{Q}{C}$$

$$\frac{dQ}{dt} = \frac{10 - \frac{Q}{C}}{150}$$

$$\frac{dQ}{dt} = \frac{1}{15} - \frac{Q}{150C}$$ ← well that leads to b

Just the voltage drop

$\Delta V = \frac{Q}{C} = 100 I$

↑ time varring

so $\Delta V = 100 \left(\frac{1}{15} - \frac{Q}{100C} \right)$

at t switch closed it is fully
need to fine the current
but to find $\frac{Q}{C}$ need to find original

$\frac{Q}{C}$ of device ~~where did I hear that~~

can't find it again.

$Q(t) = Q_{final} \left(1 - e^{-t/R_{eq}C} \right)$

↑ max charge on capicator when it was charging

$V = \mathcal{E}$

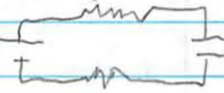
$Q_{final} = VC$ $\mathcal{E}C$

↙ write R_{eq} since $R_1 + R_2$ not $\frac{R}{2}$ ← depends how problem defined

$Q(t) = \mathcal{E}C \left(1 - e^{-t/R_{eq}C} \right)$

↑ so is that right – kinda?

So write



$\mathcal{E} - I(R_1 + R_2) - \frac{Q}{C} = 0$

↑ goes to 0

$\mathcal{E} - \frac{Q}{C} = 0$

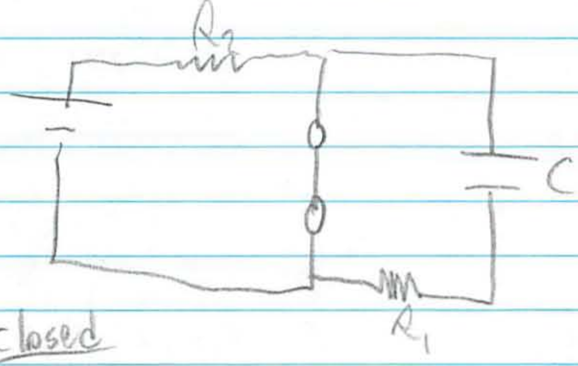
$Q = \mathcal{E}C$

↑ how to find Q final
pay close attention

3 reds

Dormashkin OH

When S is closed for a long time



closed

Short all current goes through [Ω | 0]

opened

t=0 current flows

V across resistor

$\Delta V_R = -IR_1$

$\Delta V_C = -\frac{Q}{C}$

Choose ± Q placement, direction

$\mathcal{E} - IR_2 - \frac{Q}{C} - IR_1 = 0$

$\mathcal{E} - I(R_1 + R_2) - \frac{Q}{C} = 0$

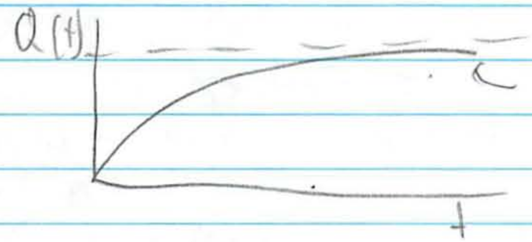
R_{eq}

$\frac{Q}{C}$ charging

$I = +\frac{dQ}{dt}$

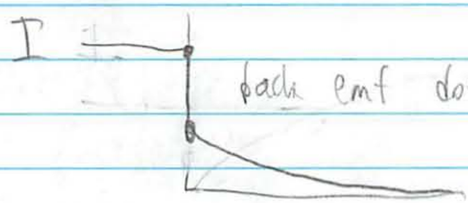
$\mathcal{E} - \frac{dQ}{dt}(R_{eq}) - \frac{Q}{C} = 0$

$\frac{\mathcal{E}}{R_{eq}} - \frac{Q}{R_{eq}C} = \frac{dQ}{dt}$



$Q(t) = C\mathcal{E}(1 - e^{-t/\tau})$

$\Delta V = -\frac{Q}{C}$



will be instantanous drop in current

when not charged → just circuit 2 resistors

→ $C\mathcal{E}$ how to find

$\mathcal{E} - I(R_1 + R_2) - \frac{Q}{C} = 0$

current goes to 0 when fully charged

$\mathcal{E} - \frac{Q}{C} = 0$

$\mathcal{E} = \frac{Q}{C}$

$Q = \mathcal{E}C$

4. Energy stored in a capacitor

$P = IV = I^2 R$

Simple RC circuit

Get expression for energy stored in capicator

$I = \frac{V}{R}$

$P = \left(\frac{V}{R}\right)^2 R$ ✓

$P = \frac{V^2}{R^2} \cdot R = \frac{V^2}{R}$

But they said by $\int$ diff of power supplied and power consumed. Is energy related to current through capicator or potential accross

$P_{batt} = \mathcal{E} I$

$P_{Resisitor} = R I^2$ ← current through it

$= \frac{V^2}{R}$ ← voltage accross it

But doesn't $P_{batt} = P_{Resistor}$ → if no internal resistance

? So what should I $\int$
well find energy

Remember last P-set

Energy → Joules → killowatt hours

So $E = \int_0^t P$

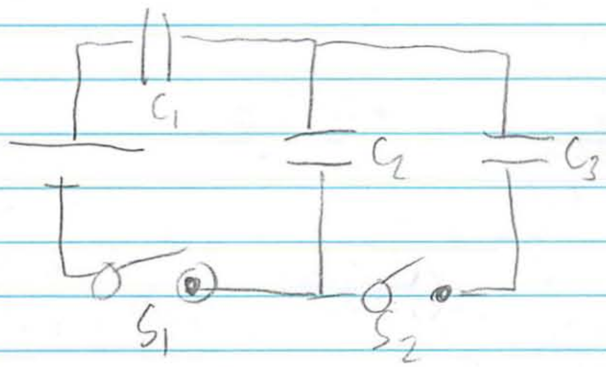
$Pt - P \cdot 0$

$E = Pt$

$E = \mathcal{E} I t = R I^2 t = \frac{V^2 t}{R}$

But what is difference b/w supplied by batt + consumed by resistor
- batt internal resistance?
- wire resistance?

5. Capicators



$C_1 = 2\ \mu F$
$C_2 = 6\ \mu F$
$C_3 = 3\ \mu F$
$V = 10V = \mathcal{E}$

Start all open + uncharged

$t = 0 \rightarrow S_2$ closed
$t = T \rightarrow S_2$ opened S_1 closed
$t = 2T \rightarrow S_1$ opened S_2 closed

a) Charge on C_2 for $0 < t < T$

∴ not a complete circuit ✓

$0 = Q_2$

b) Charge on C_1 for $T < t < 2T$

We now have 2 capicatores in series

$$\Delta V = V_1 + V_2$$
$$= \frac{Q}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$10V = \frac{Q_1}{2} + \frac{Q_2}{6}$$

In order to add need to get same denom

$$10\,V = \frac{3Q_1}{6} + \frac{Q_2}{6}$$

$$10V = \frac{3Q_1 + Q_2}{6\mu F}$$

$$.0000006F \cdot 10\,V = 3Q_1 + Q_2$$

? but how to attribute to 1, not the other

Q_1 ? Q_2

not = right

* no they are *

* Capicators in series have same charge *

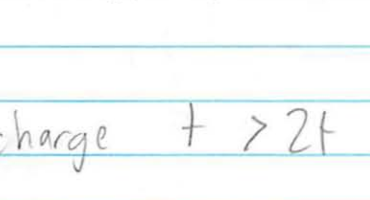
$$Q = 1.5 \cdot 10^{-6}\ C$$ ✓

please box your answers, there's so much work it's hard for me to find.

c) Final charge $t > 2t$

So C_1, C_2 charged
Switch 2 closed

what about C_1?. −5



? so C_2 discharges half way

? do we need to find Q(t)
- need more pratice w/ this no.
don't think I can do

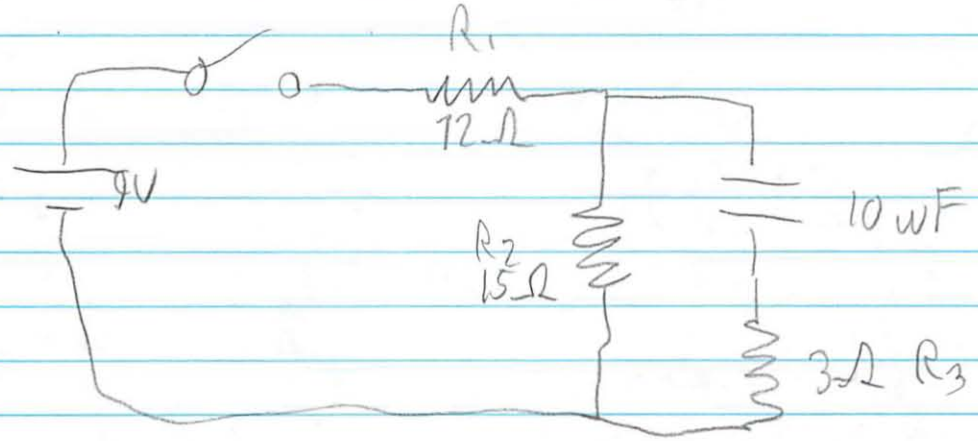
see sdns

$Q_2 \neq Q_3$

but won't each go to $\frac{1}{2}$ Q
or $7.5 \cdot 10^{-7}$ C

−3

6.



Swich closed long time

Find current through each resisistor

$R_3 = 0$ amp since capacitor fully charged

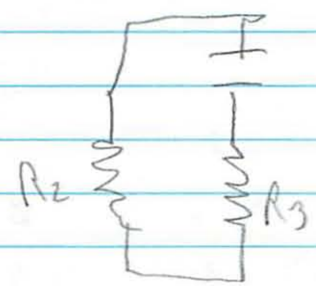
R_1 and R_2 are in series so same current

$$R_{eq} = R_1 + R_2 = 12 + 15 = 27\ \Omega$$

$$V = IR \qquad I = \frac{9V}{27\ \Omega} = \frac{1}{3} \text{ amps}$$

b Switch opened at $t = 0$

Find $I_2(R)$



$$\frac{Q}{C} - I_2 R_2 - I_2 R_3 = 0$$

? but what is this initally?

$$\frac{Q}{C} = I_2 (R_2 + R_3)$$

$$\frac{dQ}{dt} = \frac{Q}{C(R_2 + R_3)}$$

$$\frac{dQ}{dt} = \frac{1}{\tau}\left(Q - Q_{final}\right)$$

↳ $C(R_2 + R_3)$

Q_{final} = max charge on capicator when charging

$= VC = \cancel{6C}$

↳ but what about the other resistors they do something right?

So what voltage was it getting

resistors in parallel = same V drop b/w

" " series $IR_1 + IR_2$

↑ want this current same

$= I(R_1 + R_2)\,C$

$= \frac{1}{3}\text{amp}\,(15\,\Omega)\,C$

$$I = \frac{dQ}{dt} = \frac{1}{C(R_2 + R_3)}\left(Q - I(R_1 + R_2)C\right)$$

$$= \frac{1}{17C}\left(Q - 5C\right)$$

$C = .000\,001$

$$= \frac{1}{.000017}\left(Q - .000005\right)$$

$$= 58823\left(Q - .000005\right)$$ does not seme right

c Find the time to fall to $\frac{1}{e}$ of initial value

This is like experiment and differential review

$$Q(t) = Q_{final}\left(1 - e^{-t/\tau}\right)$$

$$Q(t) = 5C\left(1 - e^{-t/17C}\right)$$

Q (1) = lets say .29

↓ so 1/e value

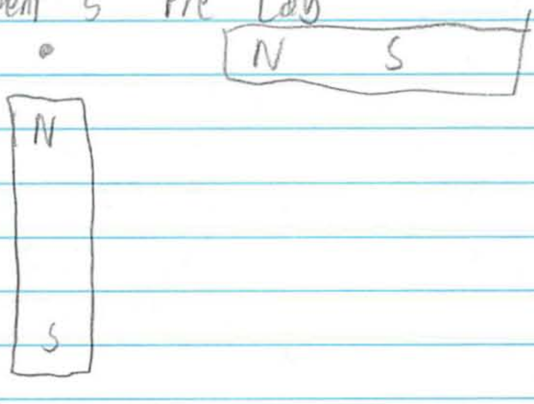
when → what time does q = that

$t = -.3665$

↑ does not make sense

- can not exist

7. Experiment 5 Pre Lab



a) If compass placed at P
What dir does it point?

well pulled on by equal strenght magnites
wants to point to magnetic south

so ↑ + ← = ↖ 135°

b. If needle at 120° instead

↑ that means bottom (#2) magnet stronger

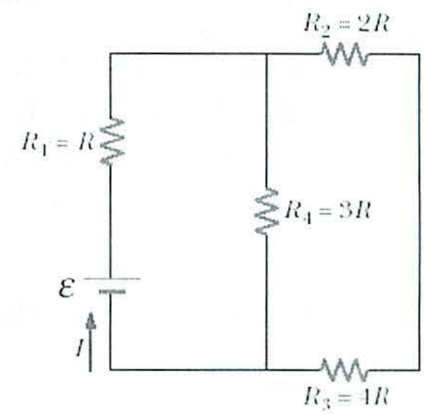
that I got but the other stuff need to see ans/ office hrs to sort out

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 Spring 2010

Problem Set 6 Solutions

Problem 1: Four Resistors



Four resistors are connected to a battery as shown in the figure. The current in the battery is I, the battery emf is $\mathcal{E}$, and the resistor values are $R_1 = R$, $R_2 = 2R$, $R_3 = 4R$, $R_4 = 3R$.

(a) Rank the resistors according to the potential difference across them, from largest to smallest. Note any cases of equal potential differences.

Resistors 2 and 3 can be combined (in series) to give $R_{23} = R_2 + R_3 = 2R + 4R = 6R$. R_{23} is in parallel with R_4 and the equivalent resistance R_{234} is

$$R_{234} = \frac{R_{23}R_4}{R_{23}+R_4} = \frac{(6R)(3R)}{6R+3R} = 2R$$

Since R_{234} is in series with R_1, the equivalent resistance of the whole circuit is $R_{1234} = R_1 + R_{234} = R + 2R = 3R$. In series, potential difference is shared in proportion to the resistance, so R_1 gets 1/3 of the battery voltage ($\Delta V_1 = \varepsilon/3$) and R_{234} gets 2/3 of the battery voltage ($\Delta V_{234} = 2\varepsilon/3$). This is the potential difference across R_4 ($\Delta V_4 = 2\varepsilon/3$), but R_2 and R_3 must share this voltage: 1/3 goes to R_2 ($\Delta V_2 = (1/3)(2\varepsilon/3) = 2\varepsilon/9$) and 2/3 to R_3 ($\Delta V_3 = (2/3)(2\varepsilon/3) = 4\varepsilon/9$). The ranking by potential difference is $\Delta V_4 > \Delta V_3 > \Delta V_1 > \Delta V_2$.

(b) Determine the potential difference across each resistor in terms of $\mathcal{E}$.

As shown from the reasoning above, the potential differences are

$$\Delta V_1 = \frac{\varepsilon}{3}, \quad \Delta V_2 = \frac{2\varepsilon}{9}, \quad \Delta V_3 = \frac{4\varepsilon}{9}, \quad \Delta V_4 = \frac{2\varepsilon}{3}$$

(c) Rank the resistors according to the current in them, from largest to smallest. Note any cases of equal currents.

All the current goes through R_1, so it gets the most ($I_1 = I$). The current then splits at the parallel combination. R_4 gets more than half, because the resistance in that branch is less than in the other branch. R_2 and R_3 have equal currents because they are in series. The ranking by current is $I_1 > I_4 > I_2 = I_3$.

(d) Determine the current in each resistor in terms of *I*.

R_1 has a current of *I*. Because the resistance of R_2 and R_3 in series ($R_{23} = R_2 + R_3 = 2R + 4R = 6R$) is twice that of $R_4 = 3R$, twice as much current goes through R_4 as through R_2 and R_3. The current through the resistors are

$$I_1 = I,\quad I_2 = I_3 = \frac{I}{3},\quad I_4 = \frac{2I}{3}$$

(e) If R_3 is increased, what happens to the current in each of the resistors?

Since

$$R_{1234} = R_1 + R_{234} = R_1 + \frac{R_{23}R_4}{R_{23} + R_4} = R_1 + \frac{(R_2 + R_3)R_4}{R_2 + R_3 + R_4}$$

increasing R_3 increases the equivalent resistance of the entire circuit. The current in the circuit, which is the current through R_1, decreases. This decreases the potential difference across R_1, increasing the potential difference across the parallel combination. With a larger potential difference the current through R_4 is increased. With more current going through R_4, and less in the circuit to start with, the current through R_2 and R_3 must decrease. Thus,

$$I_4 \text{ increases and } I_1,\ I_2, \text{ and } I_3 \text{ decrease}$$

(f) In the limit that $R_3 \to \infty$, what are the new values of the current in each resistor in terms of *I*, the original current in the battery?

If R_3 has an infinite resistance, it blocks any current from passing through that branch and the circuit effectively is just R_1 and R_4 in series with the battery. The circuit now has an equivalent resistance of $R_{14} = R_1 + R_4 = R + 3R = 4R$. The current in the circuit drops to 3/4 of the original current because the resistance has increased by 4/3. All this current passes through R_1 and R_4, and none passes through R_2 and R_3. Therefore,

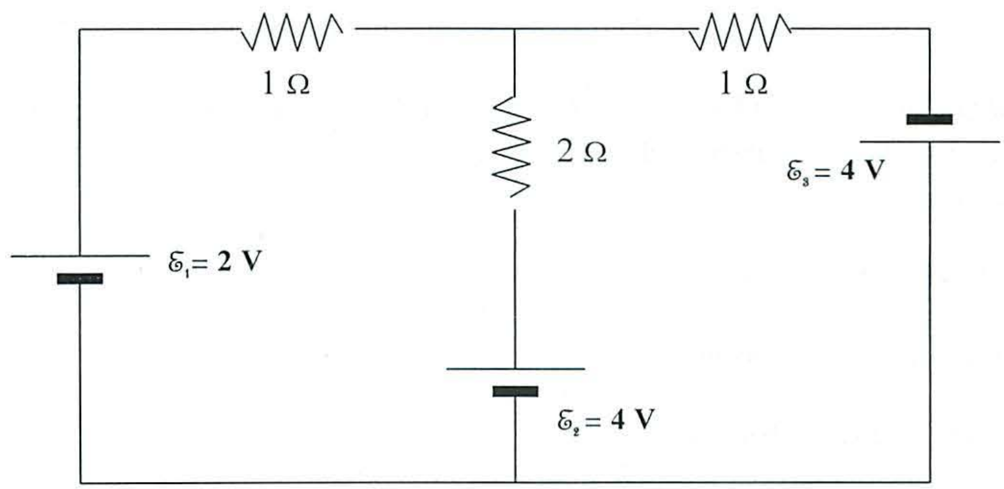
$$I_1 = \frac{3I}{4},\ I_2 = I_3 = 0,\ I_4 = \frac{3I}{4}$$

Problem 2 Multiloop Circuit

In the circuit below, you can neglect the internal resistance of all batteries.

(a) Calculate the current through each battery

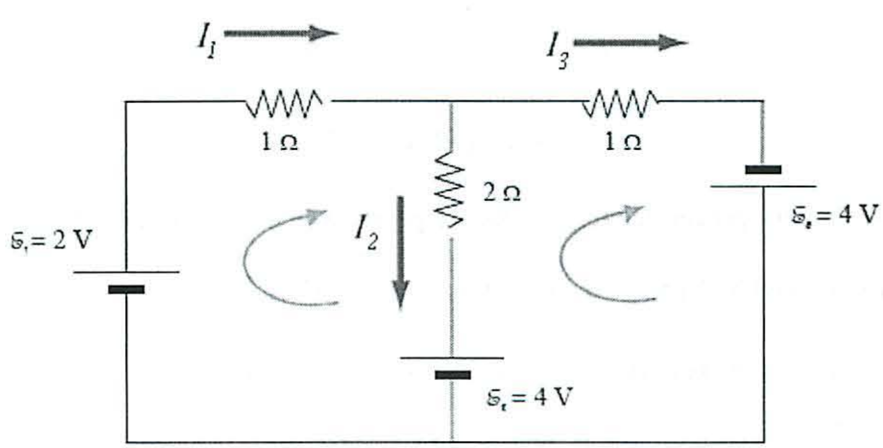
(b) Calculate the power delivered or used (specify which case) by each battery



Solution:

(a) Calculate the current through each battery.

We begin by choosing currents in every branch and travel directions in the two loops as shown below.



Current conservation is given by the condition that the current into a junction of branches is equal to the current that leaves that junction:

$$I_1 = I_2 + I_3 .$$

The two loop laws for the voltage differences are:

$$2 \text{ V} - (I_1)(1\ \Omega) - (I_2)(2\ \Omega) - 4 \text{ V} = 0 .$$

$$-(I_3)(1\ \Omega) + 4 \text{ V} + 4 \text{ V} + (I_2)(2\ \Omega) = 0 .$$

Strategy: Solve the first loop law for I_1 in terms of I_2. Solve the second loop law for I_3 in terms of I_2. Then substitute these results into the current conservation and solve for I_2. Then determine I_1 and I_3.

The first loop law becomes

$$I_1 = -2 \text{ A} - 2I_2 .$$

The second loop law becomes

$$I_3 = 8 \text{ A} + 2I_2 .$$

Current conservation becomes

$$-2 \text{ A} - 2I_2 = I_2 + 8 \text{ A} + 2I_2 .$$

Solve for I_2:

$$I_2 = -2 \text{ A} .$$

Note that the negative sign means the I_2 is flowing in a direction opposite the direction indicated by the arrow. This means that battery 2 is supplying current.

Solve for I_1:

$$I_1 = -2 \text{ A} - 2(-2 \text{ A}) = 2 \text{ A}$$

Solve for I_3:

$$I_3 = 8 \text{ A} + 2(-2 \text{ A}) = 4 \text{ A} .$$

(b) Calculate the power delivered or used (specify which case) by each battery.

The power delivered by battery 1 is $P_1 = (\varepsilon_1)(I_1) = (2 \text{ V})(2 \text{ A}) = 4 \text{ W}$.

The power delivered by battery 2 is $P_2 = (\varepsilon_2)(I_2) = (4 \text{ V})(2 \text{ A}) = 8 \text{ W}$.

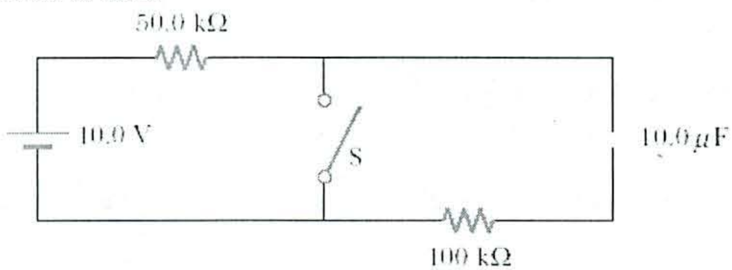
The power delivered by battery 3 is $P_3 = (\varepsilon_3)(I_3) = (4 \text{ V})(4 \text{ A}) = 16 \text{ W}$.

The total power delivered by the batteries is 28 W

Check: The power delivered to the resistors:
The power delivered to resistor 1 (in left branch) $P_1 = (I_1^2)(R_1) = (2\text{ A})^2(1\ \Omega) = 4\text{ W}$.
The power delivered to resistor 2 (in center branch) $P_2 = (I_2^2)(R_2) = (2\text{ A})^2(2\ \Omega) = 8\text{ W}$.
The power delivered to resistor 3 (in right branch) $P_3 = (I_3^2)(R_3) = (4\text{ A})^2(1\ \Omega) = 16\text{ W}$.
The total power delivered to the resistors is also 28 W.

Problem 3: RC Circuit

In the circuit shown, the switch S has been closed for a long time. At time t=0 the switch is opened. It remains open for "a long time" T, at which point it is closed again. Write an equation for (a) the charge stored on the capacitor and (b) the current through the switch as a function of time.



(a) The capacitor begins uncharged. When the switch is opened at t=0 we have an RC circuit with R = 150 kΩ and C = 10.0 μF, so τ = RC = 1.50 s. The final voltage (after an infinite time) on the capacitor will be the battery voltage (10.0V) so we can write the equation for voltage on the capacitor during charging as:

$$V_C = V_F\left(1 - e^{-t/\tau}\right) = 10.0\text{ V}\left(1 - e^{-t/1.50\text{ s}}\right) \quad [\text{for } t < T]$$

During discharge the capacitor starts at its value at $t = T$ (which we can get with the equation above) and then drives through the 100 kΩ resistor and the switch. The time constant is thus now only 1.00 s. So the voltage goes like:

$$V_C = V_0 e^{-(t-T)/\tau} = 10.0\text{ V}\left(1 - e^{-T/1.50\text{ s}}\right)e^{-(t-T)/1.00\text{ s}} \quad [\text{for } t \geq T]$$

Of course, we were asked for charge, not voltage, for which we use $Q = CV$.

(b) When the switch is open (between t = 0 and T) there is no current through it. When it is closed, however, current flows both from the battery AND from the capacitor, both in the same direction (from top to bottom). So they add. The battery just drives a current by ohm's law through the 50.0 kΩ resistor. The capacitor current we can get from the above voltage and the 100 kΩ resistor. So add them and we have:

$$I = \frac{10.0\text{ V}}{100\text{ k}\Omega}\left(1 - e^{-T/1.50\text{ s}}\right)e^{-(t-T)/1.00\text{ s}} + \frac{10.0\text{ V}}{50\text{ k}\Omega} \quad [\text{for } t \geq T]$$

Note that we could replace V/kΩ with mA, but there is no particular need to do so.

Problem 4: Energy stored in a capacitor
You know that the power supplied by a battery is given by P = VI (the battery voltage times the current it is supplying). You also know (from the Friday problem solving) that a resistor dissipates power (turns it into heat) at a rate given by $P = I^2R$.

Consider a simple RC circuit (battery, resistor R, capacitor C). Determine an expression for the energy stored in the capacitor by integrating the difference between the power supplied by the battery and that consumed by the resistor. Should the energy be related to the current through the capacitor or the potential across it?

We know that the current that flows in the circuit decays exponentially:

$$I = I_0 e^{-t/\tau} = \frac{\varepsilon}{R} e^{-t/RC}.$$

We can integrate the power supplied by the battery minus the power consumed by the resistor then to get:

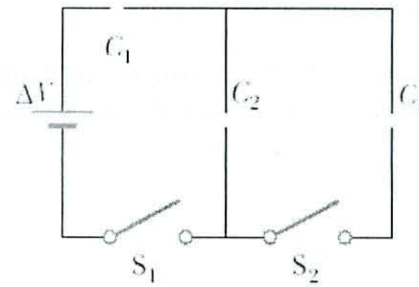
$$U_C = \int_{t'=0}^{t} P_B(t') - P_R(t')\,dt' = \int_{t'=0}^{t} \frac{\varepsilon}{R} e^{-t'/RC} \cdot \varepsilon - \left(\frac{\varepsilon}{R} e^{-t'/RC}\right)^2 R\,dt'$$

$$= \frac{\varepsilon^2}{R} \int_{t'=0}^{t} e^{-t'/\tau} - e^{-2t'/\tau}\,dt' = \frac{\varepsilon^2}{R}\left[-\tau e^{-t'/\tau} + \frac{\tau}{2} e^{-2t'/\tau}\right]_0^t = \frac{\varepsilon^2}{R}\frac{\tau}{2}\left[e^{-2t'/\tau} - 2e^{-t'/\tau}\right]_0^t.$$

$$= \frac{1}{2} C\varepsilon^2 \cdot \left[e^{-2t/\tau} - 2e^{-t/\tau} + 1\right] = \frac{1}{2} C\left[\varepsilon\left(1 - e^{-t/\tau}\right)\right]^2 = \boxed{\frac{1}{2} C V_C^2}$$

That is, the energy stored in the capacitor depends on the voltage across the capacitor (which makes sense, as that is a feature of the capacitor, while the current through it depends more on what resistor it happens to be hooked to).

Problem 5: Capacitors



In the circuit shown at right $C_1 = 2.0\ \mu$F, $C_2 = 6.0\ \mu$F, $C_3 = 3.0\ \mu$F and $\Delta V = 10.0\ \text{V}$. Initially all capacitors are uncharged and the switches are open. At time $t = 0$ switch S_2 is closed. At time $t = T$ switch S_2 is then opened, proceeded nearly immediately by the closing of S_1. Finally at $t = 2T$ switch S_1 is opened, proceeded nearly immediately by the closing of S_2. Calculate the following:

(a) the charge on C_2 for $0 < t < T$ (after S_2 is closed)

As long as S1 is open the battery is out of the circuit and hence none of the capacitors will have any charge on them.

(b) the charge on C_1 for $T < t < 2T$
When S_1 is closed, the battery is in series with C_1 and C_2. The charge on them will thus be equal, and equal to the charge that an equivalent capacitor would have.

$$C_{eqiv} = \left(C_1^{-1} + C_2^{-1}\right)^{-1} = \left(\frac{1}{2.0\,\mu\text{F}} + \frac{1}{6.0\,\mu\text{F}}\right)^{-1} = 1.5\,\mu\text{F}$$

$$Q_2\left(T < t < 2T\right) = Q_{equiv} = C_{equiv}\Delta V_{equiv} = (1.5\,\mu\text{F})(10.0\text{ V}) = \boxed{15\ \mu\text{C}}$$

(c) the final charge on each capacitor (for $t > 2T$)
When S_1 is opened, the battery and C_1 are removed from the circuit. This means that the charge on C1 is fixed at the value it was at, $\boxed{Q_1 = 15\ \mu\text{C}}$.

The charge on C_2 will be shared with C_3, so that their potentials will be the same (since they are now in parallel). So:

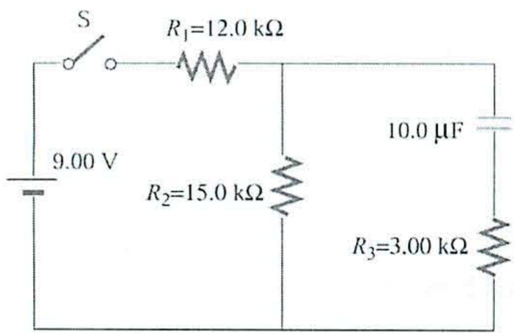
$$V_2 = Q_2/C_2 = V_3 = Q_3/C_3; \qquad Q_2 + Q_3 = Q_2\left(t = 2T^-\right)$$

$$\frac{Q_2}{C_2} = \frac{Q_3}{C_3} = \frac{Q_2\left(t = 2T^-\right) - Q_2}{C_3} \Rightarrow Q_2 C_3 = C_2\left(Q_2\left(t = 2T^-\right) - Q_2\right) \Rightarrow$$

$$Q_2 = \frac{C_2 Q_2\left(t = 2T^-\right)}{C_2 + C_3} = \frac{6.0\ \mu\text{F}\cdot 15\ \mu\text{C}}{6.0\ \mu\text{F}+3.0\ \mu\text{F}} = \boxed{10\ \mu\text{C} = Q_2} \Rightarrow \boxed{Q_3 = 5\ \mu\text{C}}$$

Problem 6: *RC* Circuit

Consider the *RC* circuit shown in the figure. Suppose that the switch has been closed for a length of time sufficiently long enough for the capacitor to be fully charged.



(a) Find the steady-state current in each resistor.

Since the capacitor represents an open circuit, there is no current through R_3. Therefore, all the charges flowing through R_1 goes through R_2: hence $I_1 = I_2$ and $I_3 = 0$. Now, all you need to do is to find a current flowing through the two resistors in series.

$$I_1 = I_2 = \frac{\varepsilon}{R_{12}} = \frac{\varepsilon}{R_1 + R_2} = \frac{9.00V}{12.0k\Omega + 15.0k\Omega} = 0.333\text{mA} = 3.33\times 10^{-4}\text{ A}$$

(b) Find the charge Q on the capacitor.

At equilibrium, the capacitor is fully charged and ΔV_{cap} is equal to the voltage drop across R_2 since there is no current through R_3 (and therefore the voltage drop across it is zero).

$$\Delta V_{\text{cap}} = I_2 R_2 = \frac{R_2}{R_1 + R_2}\varepsilon = \frac{15.0k\Omega}{12.0k\Omega + 15.0k\Omega}(9.00\text{V}) = 5.00\text{V}$$

Thus, the charge on the capacitor is given by

$$Q = C\Delta V_{\text{cap}} = C\varepsilon = (10.0\mu\text{F})(5.00\text{V}) = 50.0\mu\text{C} = 5.00\times 10^{-5}\text{ C}$$

(c) The switch is opened at $t = 0$. Write an equation for the current I_2 in R_2 as a function of time.

With the switch opened, the capacitor discharges through the resistors, R_2 and R_3. There is no emf in the circuit. You also need to notice R_1 is no longer a part of the closed circuit and there is no current through it. Now, you should follow the discussion in Section 7.6.2 of the *Course Notes* with $R = R_{23} = R_2 + R_3$ and $I = I_2 = -I_3$. You'll then obtain

$$q(t) = Qe^{-t/R_{23}C}$$

and

$$I_2(t) = -\frac{dq_2}{dt} = \left(\frac{Q}{R_{23}C}\right)e^{-t/R_{23}C} = \left(\frac{C\Delta V_{\text{cap}}}{(R_2 + R_3)C}\right)e^{-t/(R_2+R_3)C} = 0.278e^{-t/180\text{ms}}\text{ milliamps}$$

(d) Find the time that it takes for the charge on the capacitor to fall to 1/e of its initial value.

$$\frac{I_2(t)}{I_2(0)} = \frac{0.278e^{-t/180\text{ms}}}{0.278e^{-0/180\text{ms}}} = e^{-t/180\text{ms}} = e^{-1}$$

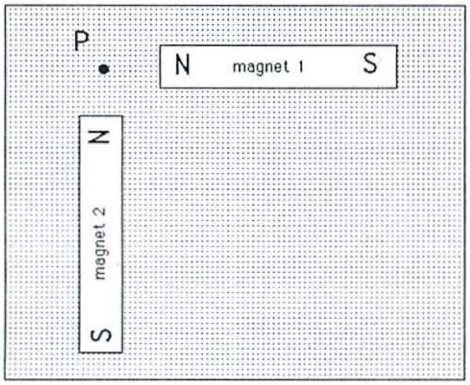
Thus,

$$\frac{-t}{180\,\text{ms}} = -1 \text{ or } t = 180 \text{ ms}$$

which is called "time constant (τ).

Problem 7: Experiment 4: Magnetic Fields of a Bar Magnet and Helmholtz Coil
Pre-Lab Questions

Read Experiment 5 before answering these questions

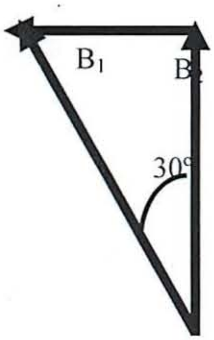


Consider two bar magnets placed at right angles to each other, as pictured at left.

(a) If a small compass is placed at point P, what direction does the painted end of the compass needle point?

It points away from each magnetic North, which means toward the upper left corner (45 degrees if they are the same magnitude).

(b) If the compass needle instead pointed 15 degrees clockwise of where you predicted in (a), what could you qualitatively conclude about the relative strengths of the two magnets?



In order for it to point 15 degrees clockwise the second magnet must be stronger than the first. Since the total field is just a vector sum of the two we can see how much stronger.

$$\tan 30^\circ = \frac{B_1}{B_2} = \frac{1}{\sqrt{3}} \Rightarrow \boxed{B_2 = \sqrt{3}B_1}$$