MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Tear off this page and turn it in at the end of class !!!!

Note:
Writing in the name of a student who is not present is a COD offense.

Problem Solving 5: RC Circuits

Group 11B (e.g. 6A Please Fill Out)

Names Jennifer Quintana
Michael Plasmeier

3 4

Question 1: What is the current I_C (through the capacitor) at $t=0^+$ (just after switch is closed)?

$I_C = \epsilon - R_1$ X $I_C = \frac{\epsilon}{R_1}$

$[A] \neq [V] - [\Omega]$

Question 2: What are the currents I_1 and I_2 (through R_1 and R_2 respectively) at $t=0^+$?

$I_1 = I_c = \epsilon - \frac{V}{R_1}$ X $I_2 = 0$ ✓

$\overline{[A]}\ \overline{[V]}$

look at your units, how can you get amps from voltage?

Question 3: What is the current I_C (through the capacitor) at $t=\infty$?

$I_C = 0$ ✓

Question 4: What are the currents I_1 and I_2 (through R_1 and R_2 respectively) at $t=\infty$? At $t=\infty$, assume battery $=0$

if $I_1 = 0$ then I everywhere is 0?

$I_1 = I_c = 0$ X $I_2\ \underbrace{\epsilon - \frac{V}{R_1}}_{c} - \frac{V}{R_2}$ X

Question 5: Using Kirchhoff's Loop Rules, obtain a differential equation for the charge q on the capacitor, assuming $R_1=R_2=R$ (in other words, the only current in the equation should be the current through the capacitor, which can be rewritten in terms of dq/dt). $I_c = I_1 - I_2$ ✓

how did you get this hard question + not get 1-4?

$\epsilon - I_1R + I_2R = \frac{2Q}{C}$

$\epsilon - R(I_1 + I_2) = \frac{2Q}{C}$

$\epsilon - \frac{dQ}{dt}R = \frac{2Q}{C}$

$\mathcal{E} - \frac{2Q}{C} = \frac{dQ}{dt}R$

$-\frac{dQ}{dt} = \frac{\mathcal{E}}{R} - \frac{2Q}{CR}$

$-\frac{2Q}{RC} + \frac{\epsilon}{R} = \frac{dQ}{dt}$

$\frac{dQ}{dt} = \frac{\mathcal{E}}{R} - \frac{2Q}{RC}$ $\frac{dQ}{dt} = \frac{1}{\tau}(Q - \overset{Q_{final}}{C\mathcal{E}})$ $Q(t) = \frac{C\mathcal{E}}{2}(1 - e^{-t/\tau})$ $\tau = RC/2$ $Q_{final} = \frac{\mathcal{E}C}{2}$

Question 6: What is the time constant for charging the capacitor?

$\tau = RC/2$ ✓

Question 7: Write an equation for the time dependence of the charge on the capacitor

$Q(t) = \frac{\mathcal{E}C}{2}\left(1 - e^{-t/RC/2}\right)$ ✓

Question 8: What is the current I_C (through the capacitor) at $t=T^+$ (just after switch is opened)? (BATTERY OFF)

$I_c = \frac{Q}{C} - R_2$? $\mathcal{E}$?

what is Q? it's not known

Question 9: What are the currents I_1 and I_2 (through R_1 and R_2 respectively) at $t=T^+$?

$I_1 = 0$ ✓ $I_2 = \cancel{-}I_c = \frac{Q}{C} - R_2$ ✗

Question 10: Using Kirchhoff's Loop Rules, obtain a differential equation for the charge q on the capacitor after the switch has been opened, assuming $R_1=R_2=R$ (in other words, the only current in the equation should be the current through the capacitor, which can be rewritten in terms of dq/dt).

$$\frac{Q}{C} - \frac{dQ}{dt}R = 0$$

$$\frac{Q}{CR} = \frac{dQ}{dt}$$ ✓

Question 11: What is the time constant for discharging the capacitor?

$\tau = RC$ ✓

Question 12: Write an equation for the time dependence of the charge on the capacitor after time T.

$Q(t) = \frac{\overset{Q_{final}}{VC}}{2}\left(1 - e^{-t/RC}\right)$

$\mathcal{E}$!

"t" $= t - T$

time switch opened

Topics: Magnetic Fields: Feeling Magnetic Fields – Charges and Dipoles
Related Reading: Course Notes: Chapter 8.1-8.3, 8.5-8.6, 8.8-8.9, 9.5
Experiments: (5) Magnetic Fields

Topic Introduction

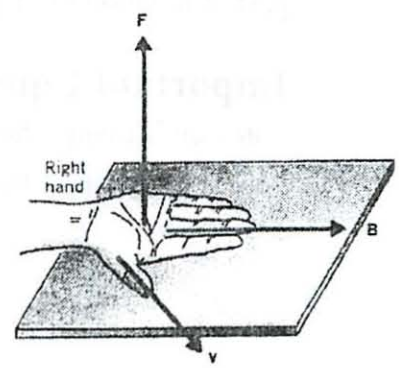
Here we go – another topic! that was fast (as always) – just getting started on other

Today we begin a major new topic in the course – magnetism. In some ways magnetic fields are very similar to electric fields: they are generated by and exert forces on electric charged particles. There are a number of differences though. First of all, magnetic fields only interact with (are created by and exert forces on) charged particles that are moving. Secondly, the simplest magnetic objects are not monopoles (like a point charged object) but are instead dipoles.

This week we begin by defining the magnetic field and studying the forces on moving charged objects in magnetic fields. In order to gain experience with magnetic fields we study the fields created by bar magnets and currents in a lab.

Lorenz Force

The magnetic field is defined by measuring the force exerted on a moving charged object. This force is called the Lorenz Force and is given by $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ (where q is the charge of the particle, **v** its velocity and **B** the magnetic field). We then study the motion of moving charged objects in a magnetic field. The fact that the force depends on a cross product of the particle velocity and the field can make forces from magnetic fields very non-intuitive.



The direction of the force $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ can be determined by the right hand rule pictured at right (thumb in direction of **v**, fingers in direction of **B**, palm shows direction of force). It is perpendicular to both the velocity of the charged particle and the magnetic field, and thus charged particles will follow curved trajectories while moving in a magnetic field, and can even move in circles (in a plane perpendicular to the magnetic field). The ability to make charged particles curve by applying a magnetic field is used in a wide variety of scientific instruments, from mass spectrometers to particle accelerators, and we will discuss some of these applications in class including studying the motion of the a moving charged particle in both an electric and magnetic field, $\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$.

Dipole Fields

We will note that the magnetic fields you are most familiar with, those generated by bar magnets and by the Earth, act like magnetic dipoles. Magnetic dipoles create magnetic fields identical in shape to the electric fields generated by electric dipoles. We even describe them in the same way, saying that they consist of a North pole (+) and a South pole (-) some distance apart, and that magnetic field lines flow from the North pole to the South pole. Despite these similarities, magnetic dipoles are different from electric dipoles, in that if you cut an electric dipole in half you will find a positive charge and a negative charge, while if you cut a magnetic dipole in half you will be left with two new magnetic dipoles. There is no such thing as an isolated "North magnetic charge" (a magnetic monopole).

Magnetic fields are also created by electric currents, i.e. by moving charged particles that form the current. We will study how moving charged particles create magnetic fields in detail in the next class. Today we gain some experience by measuring the fields created by bar magnets and current in Experiment 5.

Experiment 5: Magnetic Fields of a Bar Magnet and Current Loops

Preparation: Read pre-lab

In this lab you will measure the magnetic field generated by a bar magnet and two coils old currents, thus getting a feeling for magnetic field lines. Recall that as opposed to electric fields generated by charged particles, where the field lines begin and end at those charged particles, magnetic fields generated by dipoles have field lines that are closed loops (where part of the loop must pass through the dipole).

Important Equations

Force on Moving Charges in Magnetic Field: $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$

Force on Moving Charges in an Electric and Magnetic Field: $\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$

Class 17: Outline

Hour 1:
- Magnetic Field
- Magnetic Forces on Charges

Hour 2:
- Experiment 5: Magnetic Fields

P17- 1

q

$(F_q)_{elec} = q\vec{E}$ of moving electric

$q \rightarrow v_q$

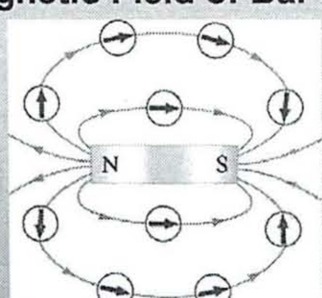
$(F_q)_{moving} = q\vec{v}_q \times \vec{B}$ magnetic

$\vec{F}_q = q\vec{F} + q\vec{v}_q \times \vec{B}$

Magnetic Fields

P17- 2

Magnetic Field of Bar Magnet



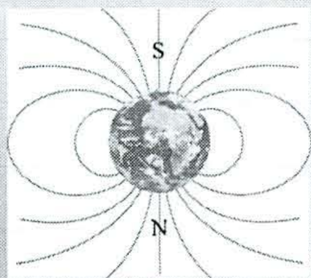
(1) A magnet has two poles, North (N) and South (S)
(2) Magnetic field lines leave from N, end at S

P17- 3

Compasses
Earth
Helm Holtz current

pharo magnetic = permanent
Special config of charges

Magnetic Field of the Earth



Also a magnetic dipole!

North magnetic pole located in southern hemisphere

P17- 4

moving charges → magnetic fields
- at macroscopic level

like current through a wire

Superimpose bar magnet over Earth's magnetic field

$\vec{A} \times \vec{B} = \vec{C}$

on plane

$\vec{C}$ perpendicular to plane

$|\vec{C}|$ = area parallelogram
$= |A||B|\sin\theta$
↑ angle b/w vectors

Recall: Cross Product

drives students crazy

P17- 5

$\begin{vmatrix} 1 & 0 & 4 \\ 1 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$ ~~to find volume 3D~~
in ~~this~~ class
somewhat
cross product

Notation Demonstration

OUT of page
"Arrow Head"

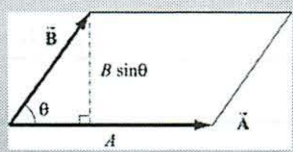
INTO page
"Arrow Tail"

P17- 6

Cross Product: Magnitude

Computing magnitude of cross product A x B:

$$\vec{C} = \vec{A} \mathrm{x} \vec{B} \qquad \left|\vec{C}\right| = \left|\vec{A}\right|\left|\vec{B}\right| \sin\theta$$



$|\vec{C}|$: area of parallelogram

P17- 7

Cross Product: Direction

Right Hand Rule #1:

$$\vec{C} = \vec{A} \mathrm{x} \vec{B}$$

For this method, keep your hand flat!

1) Put Thumb (of right hand) along **A**
2) Rotate hand so fingers point along **B**
3) Palm will point along **C**

P17- 8

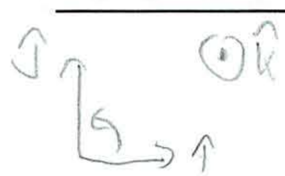
Cross Product: Signs

$$\hat{i} \times \hat{j} = \hat{k} \qquad \hat{j} \times \hat{i} = -\hat{k}$$
$$\hat{j} \times \hat{k} = \hat{i} \qquad \hat{k} \times \hat{j} = -\hat{i}$$
$$\hat{k} \times \hat{i} = \hat{j} \qquad \hat{i} \times \hat{k} = -\hat{j}$$

Cross Product is Cyclic (left column)
Reversing **A** & **B** changes sign (right column)

P17- 9

choose dirs w/ right hand rule



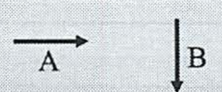
$\hat{i} \times \hat{j} = \hat{k}$

PRS Questions: Right Hand Rule

P17-10

PRS: Cross Product

What is the direction of A x B given the following two vectors?



- 0% 1. up
- 0% 2. down
- 0% 3. left
- 0% 4. right
- 0% 5. into page
- 0% 6. out of page
- 0% 7. Cross product is zero (so no direction)

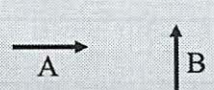
P17-11

95%

* always put tail to tail *

PRS: Cross Product

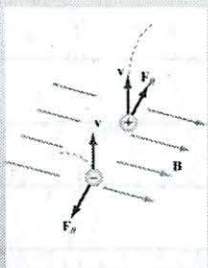
What is the direction of A x B given the following two vectors?

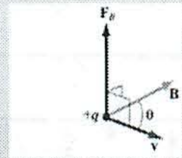


- 0% 1. up
- 0% 2. down
- 0% 3. left
- 0% 4. right
- 0% 5. into page
- 0% 6. out of page
- 0% 7. Cross product is zero (so no direction)

P17-12

Moving Charges Feel Magnetic Force





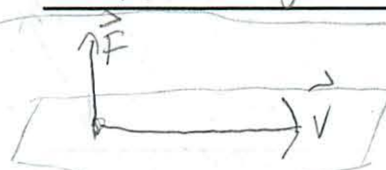
$$\vec{\mathbf{F}}_B = q\,\vec{\mathbf{v}}\times\vec{\mathbf{B}}$$

Magnetic force perpendicular both to:
Velocity **v** of charge and magnetic field **B**

like circ acc last year

Consistant force perpendicular to velocity *

only changes direction (velocity) → not magnitude (speed)



$\vec{v}$ on plane
$\vec{F}$ perpendicular

Reminder: B Field Units

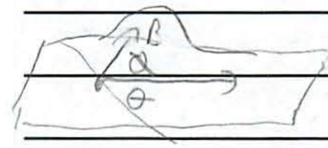
Since $\vec{\mathbf{F}}_B = q\,\vec{\mathbf{v}}\times\vec{\mathbf{B}}$

$$\text{B Units} = \frac{\text{newton}}{(\text{coulomb})(\text{meter/second})} = 1\frac{\text{N}}{\text{C}\cdot\text{m/s}} = 1\frac{\text{N}}{\text{A}\cdot\text{m}}$$

This is called 1 Tesla (T)

$$1\ \text{T} = 10^4\ \text{Gauss (G)}$$

$F = q\vec{v}\times\vec{B}$



$\vec{F}\perp\vec{v}$
$\vec{F}\perp\vec{B}$
~~$\vec{v}\perp\vec{B}$~~

Find ϕ

$|\vec{F}| = q|\vec{v}||\vec{B}|\sin\phi$

$B = \dfrac{|\vec{F}_q|}{q|\vec{v}_q|\sin\phi}$

How Big is a Tesla?

- Earth's Field — 5×10^{-5} T = 0.5 Gauss
- Brain (at scalp) — ~1 fT
- Refrigerator Magnet
- Inside MRI — 3 T
- Good NMR Magnet — 18 T
- Biggest in Lab — 150 T (pulsed)
- Biggest in Pulsars

$\vec{F}_q = q\vec{F} \rightarrow \vec{E} = \dfrac{\vec{F}_q}{q}$

$\phi = ?$

$\beta - \theta_1$ b/w vectors

may be P-set
or
2 experiments needed →

measure F_1 F_2 θ_1 θ_2
determine β

Gravitational – Electric Fields

	Mass m	Charge q (±)
Create:	$\vec{\mathbf{g}} = -G\dfrac{m}{r^2}\hat{\mathbf{r}}$	$\vec{\mathbf{E}} = k_e\dfrac{q}{r^2}\hat{\mathbf{r}}$
Feel:	$\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$	$\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$

Also saw… Dipole **p**

Creates:

Feels: $\vec{\boldsymbol{\tau}} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$

P17-15

neutral far away

dipole moment from ⊖ → ⊕

Magnetic field definition

More complicated than Electrical

$B = \dfrac{[\vec{F}]}{[q][\vec{v}]} = \dfrac{N}{C\,\frac{m}{s}} = \dfrac{\frac{kg\,m}{s^2}}{C\,\frac{m}{s}} = \dfrac{kg}{C\,s} = \text{Tesla}$

$\dfrac{N}{\frac{C}{s}\,m} = \dfrac{N}{A\,m} = \text{tesla}$

amp

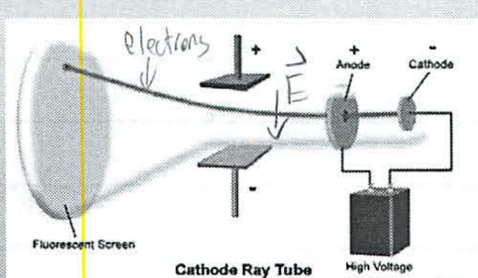
Demonstration: TV in Field

P17-17

feels torque in external field

will orient itself

How a CRT Works: It could…



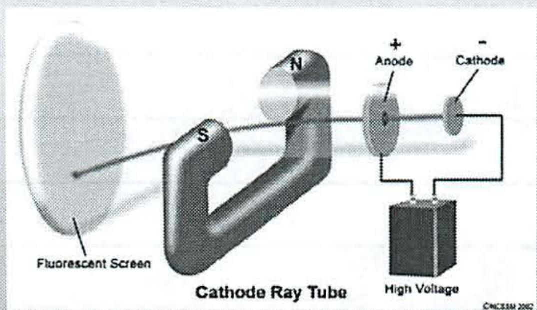
P17-18

electrons → higher place

deflected $q\vec{E}$

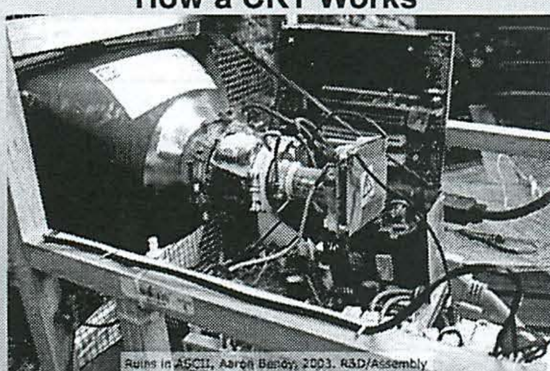
negative (opposite E)

How a CRT Works: More Typical



P17-19

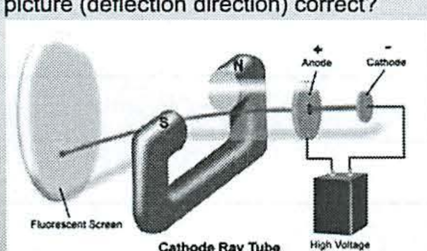
How a CRT Works



P17-20

PRS: Force Direction

Is this picture (deflection direction) correct?



20

0% 1. Yes
0% 2. No
0% 3. I don't know

P17-21

⊖ charge

magnetic S ⟵ W

fingers towards then thumb to ... thumb to V fingers to B

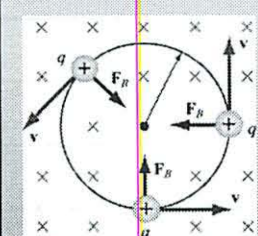
fingers towards V
curl to B
thumb pts to F

What Kind of Motion in Uniform B Field?

Exam April 1

simple magnetism on exam
how to compute analytical no
conceptual yes

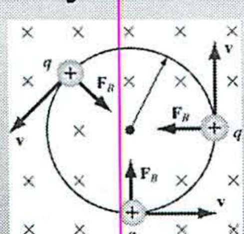
Group Problem: Cyclotron Motion



A charged particle with charge q is moving with speed v in a uniform magnetic field B pointing into the figure.

Find
(1) r : radius of the circle
(2) T : period of the motion
(3) ω : cyclotron frequency

Cyclotron Motion: Solution



(1) r : radius of the circle

$$qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$$

(2) T : period of the motion

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

(3) ω : cyclotron frequency $\boxed{\omega = 2\pi f = \frac{v}{r} = \frac{qB}{m}}$

does not depend on R

Putting it Together: Lorentz Force

Charges Feel...

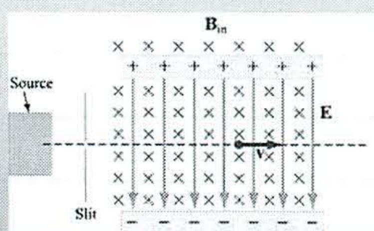
$\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$ Electric Fields

$\vec{\mathbf{F}}_B = q\,\vec{\mathbf{v}}\times\vec{\mathbf{B}}$ Magnetic Fields

$$\boxed{\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}}\times\vec{\mathbf{B}}\right)}$$

This is the final word on the force on a charge

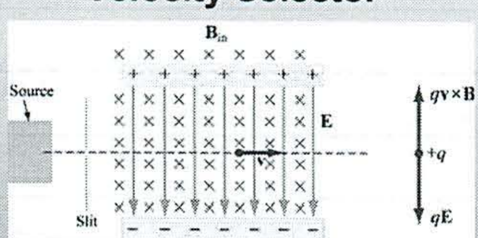
Application: Velocity Selector



What happens here?

Goes straight through if B and E cancle

Velocity Selector



Particle moves in a straight line when

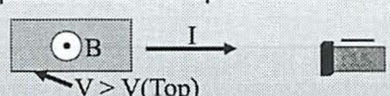
$$\vec{\mathbf{F}}_{net} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}}\times\vec{\mathbf{B}}) = 0 \Rightarrow \boxed{v = \frac{E}{B}}$$

only one where moves in straight through

PRS Question: Hall Effect

PRS: Hall Effect

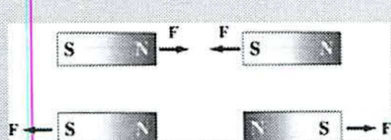
A conducting slab has current to the right. A B field is applied out of the page. Due to magnetic forces on the charge carriers, the bottom of the slab is at a higher electric potential than the top of the slab.



On the basis of **this** experiment, the sign of the charge carriers carrying the current in the slab is:

0%	1. Positive
0%	2. Negative
0%	3. Cannot be determined
0%	4. I don't know

Magnetism – Bar Magnet

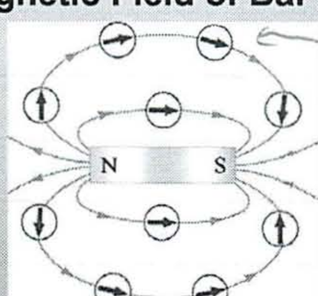


Like poles repel, opposite poles attract

Demonstration:
Magnetic Field Lines
from Bar Magnet

Demonstration:
Compass (bar magnet) in
Magnetic Field Lines
from Bar Magnet

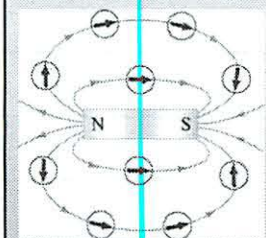
Magnetic Field of Bar Magnet



(1) A magnet has two poles, North (N) and South (S)
(2) Magnetic field lines leave from N, end at S

these are little compasses
show dir $\vec{B}$ field

Bar Magnets Are Dipoles!



- Create Dipole Field
- Rotate to orient with Field

Is there magnetic "mass" or magnetic "charge?"

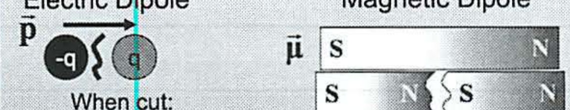
NO! Magnetic monopoles do not exist in isolation

P17-34

Magnetic Monopoles?

Electric Dipole

Magnetic Dipole



When cut: 2 monopoles (charges)

When cut: 2 dipoles

Magnetic monopoles do not exist in isolation
Another Maxwell's Equation! (2 of 4)

$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0}$$

Gauss's Law

$$\oiint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

Magnetic Gauss's Law

P17-35

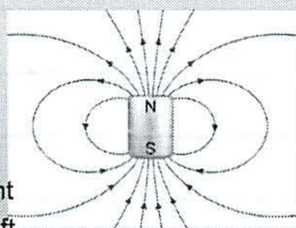
PRS:
B Field *inside* a Magnet

P17-36

PRS: Magnetic Field Lines

15

The picture shows the field lines outside a permanent magnet The field lines inside the magnet point:



0%	1. Up
0%	2. Down
0%	3. Left to right
0%	4. Right to left
0%	5. The field inside is zero
0%	6. I don't know

Fields: Grav., Electric, Magnetic

	Mass m	Charge q (±)	No Magnetic Monopoles!
Create:	$\vec{\mathbf{g}} = -G\dfrac{m}{r^2}\hat{\mathbf{r}}$	$\vec{\mathbf{E}} = k_e\dfrac{q}{r^2}\hat{\mathbf{r}}$	
Feel:	$\vec{\mathbf{F}}_g = m\vec{\mathbf{g}}$	$\vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$	
		Dipole **p**	Dipole μ
Create:		$\vec{\mathbf{E}} \rightarrow$	$\leftarrow \vec{\mathbf{B}}$
Feel:		$\vec{\boldsymbol{\tau}} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$	$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$

Experiment 5: Magnetic Fields: Bar Magnets & Wire Coils

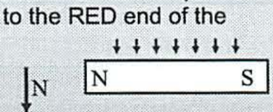
PRS Question:
Part I: B Field from Bar Magnet

P17-40

PRS: Bar Magnet B Field

15

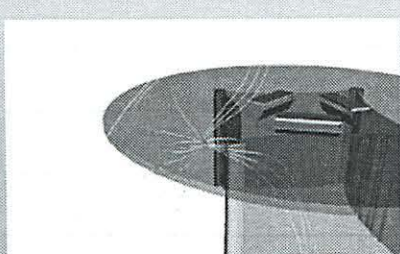
Imagine one of the small compasses sitting on a table with the Earth's geographic North as indicated. A bar magnet is slowly slid towards the compass as indicated. What happens to the RED end of the compass needle?



0% 1. Nothing
0% 2. Rotates from down CCW
0% 3. Rotates from down CW
0% 4. Rotates from up CCW
0% 5. Rotates from up CW
0% 6. I don't know

41

Visualization:
Bar Magnet &
Earth's Magnetic Field



P17-42

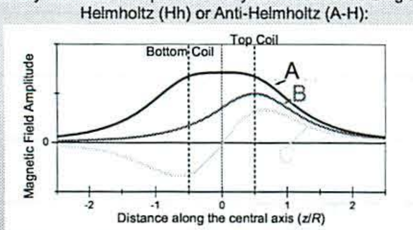
PRS Question:
Part 3: B Field from Helmholtz

P17-43

PRS: Helmholtz

Identify the three field profiles that you measured as Single (Sgl), Helmholtz (Hh) or Anti-Helmholtz (A-H):

15



The curves, A, B & C are respectively:

0%	1.	Sgl, Hh, A-H
0%	2.	Hh, A-H, Sgl
0%	3.	A-h, Sgl, Hh
0%	4.	Sgl, A-H, Hh
0%	5.	A-H, Hh, Sgl
0%	6.	Hh, Sgl, A-H

P17-44

Field Pressures and Tensions:
A Way To Understand the qVxB Magnetic Force

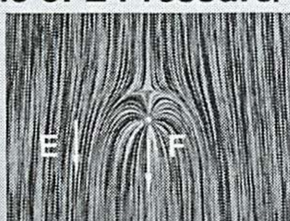
P17-45

Tension and Pressures Transmitted by E and B

E & B Fields:

- Transmit tension along field direction (Field lines want to pull straight)
- Exert pressure perpendicular to field (Field lines repel)

Example of E Pressure/Tension



(Animation)

Positive charge in uniform (downward) E field

Electric force on the charge is combination of

1. Pressure pushing down from top
2. Tension pulling down towards bottom

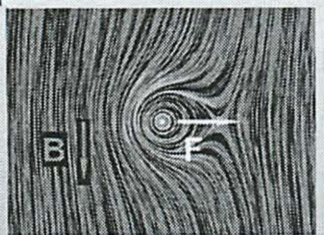
Example of B Pressure/Tension

(Animation)

Positive charge moving out of page in uniform (downwards) B field. Magnetic force combines:

1. Pressure pushing from left
2. Tension pulling to right

Example of B Pressure/Tension



(Animation)

Both cases: repelling "pressure" arises from HIGH field strength → HIGH energy density

P17-49

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics
8.02

Experiment 5: Magnetic Fields of a Bar Magnet and Helmholtz Coil

OBJECTIVES

1. To learn how to visualize magnetic field lines using compasses and a gauss meter
2. To examine the field lines from bar magnets and see how they add
3. To examine the field lines from a Helmholtz coil and understand the difference between using it in Helmholtz and anti-Helmholtz configurations.

PRE-LAB READING

INTRODUCTION

In this lab we will measure magnetic field lines using two methods. First, we will use small compasses that show the direction, but not magnitude, of the local magnetic field. Next we will use a gauss meter, which measures the magnitude of the magnetic field along a single, specific axis and thus does not allow as easy a visualization of the magnetic field direction. We will measure fields both from bar magnets and from a Helmholtz coil.

APPARATUS

1. Mini-Compass

You will receive a bag of mini-compasses (Fig. 1a) that indicate the magnetic field direction by aligning with it, with the painted end of the compass needle pointing away from magnetic north (i.e. pointing in the direction of the magnetic field). Conveniently, the magnetic south pole of the Earth is very close to its geographic north pole, so compasses tend to point North (Fig. 1b). Note that these compasses are cheap (though not necessarily inexpensive) and sometimes either point in the direction opposite the way they should, or get completely stuck. Check them out before using them.

(a)

(b)

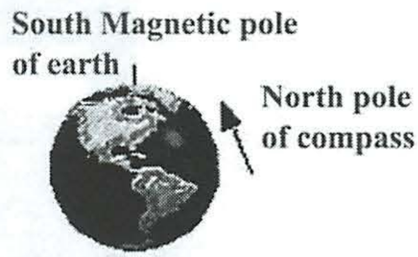


Figure 1 (a) A mini-compass like the ones we will be using in this lab. (b) The painted end of the compass points north because it points towards magnetic south.

2. Science Workshop 750 Interface

As always, we will use the Science Workshop 750 interface, this time for recording the magnetic field magnitude as measured by the magnetic field sensor (gauss meter).

3. Magnetic Field Sensor

The magnetic field sensor measures the strength of the magnetic field pointing into one of two white dots painted at its measurement end (far left in Fig. 2). Selecting "radial" mode records the strength of the field pointing into the dot on the side of the device, while "axial" records the strength of the field pointing into the dot on the end. There is also a tare button which sets the current field strength to zero (i.e. measures relative to it).

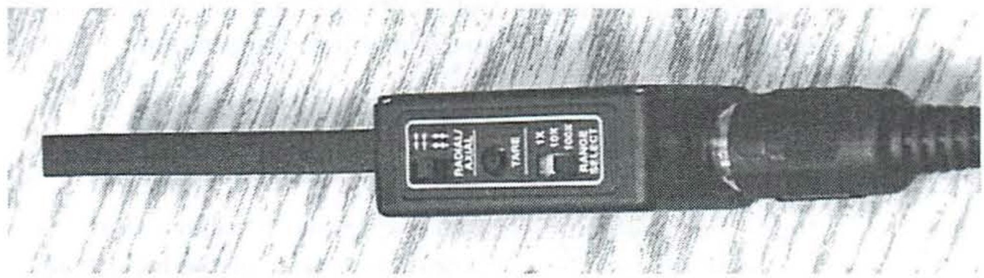


Figure 2 Magnetic field sensor, showing (from right to left) the range select switch, the tare button, and the radial/axial switch, which is set to radial.

4. Helmholtz Coil

Consider the Helmholtz Coil Apparatus shown in Fig. 3. It consists of two coaxial coils separated by a distance equal to their common radii. The coil can be operated in 3 modes. In the first, connections are made only to one set of banana plugs, pushing current through only one of the coils. In the second, a connection is made between the black plug from one coil to the red plug from the other. This sends current the same direction through both coils and is called "Helmholtz Mode." In the final configuration "Anti-Helmholtz Mode" a connection is made between the two black plugs, sending current in the opposite direction through the two coils.

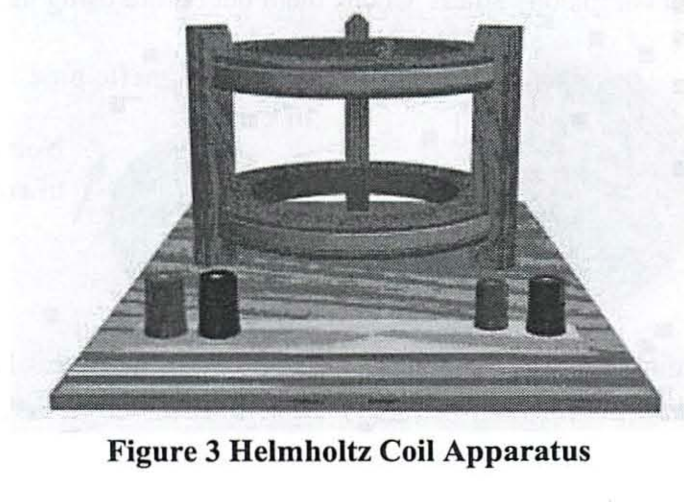


Figure 3 Helmholtz Coil Apparatus

4. Power Supply

Because the Helmholtz coils require a fairly large current in order to create a measurable field, we are unable to use the output of the 750 to drive them. For this reason, we will use an EZ dc power supply (Fig. 4). This supply limits both the voltage and the current, putting out the largest voltage possible consistent with both settings. That is, if the output is open (no leads connected, so no current) then the voltage output is completely determined by the voltage setting. On the other hand, if the output is shorted (a wire is placed between the two output plugs) then the voltage is completely determined by the current setting ($V = IR_{short}$).

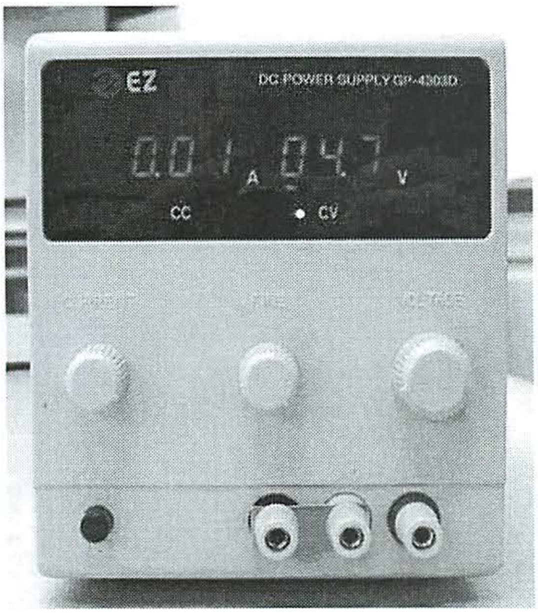


Figure 4 Power Supply for Helmholtz Coil The power supply allows independent control of current (left knob) and voltage (right knob) with whichever limits the output the most in control. The green light next to the "CV" in this picture means that we are in "constant voltage" mode – the voltage setting is limiting the output (which makes sense since the output at the bottom right is not hooked up so there is currently no current flow).

GENERALIZED PROCEDURE

This lab consists of three main parts. In each you will measure the magnetic field generated either by bar magnets or by current carrying coils.

Part 1: Mapping Magnetic Field Lines Using Mini-Compasses
Using a compass you will follow a series of field lines originating near the north pole of a bar magnet.

Part 2: Constructing a Magnetic Field Diagram
A pair of bar magnets are placed so that either their opposite poles or same poles are facing each other and you will map out the field lines from these configurations.

Part 3: Helmholtz Coil
In this part you will use the magnetic field sensor to measure the amplitude of the magnetic field generated from three different geometries of current carrying wire loops.

END OF PRE-LAB READING

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

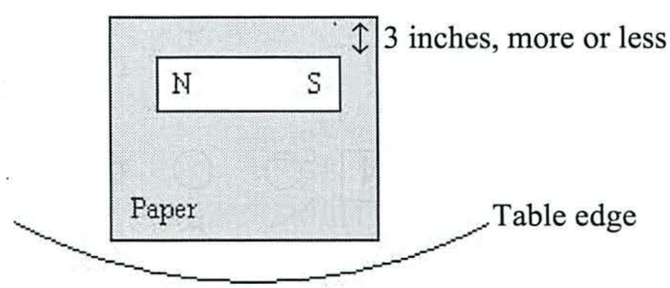
1. Download the LabView file and start up the program.
2. Connect the Magnetic Field Sensor to Analog Channel A on the 750 Interface
3. Without leads connected to the power supply, turn it on and set the voltage output to 2 V. Turn it off.

NOTE: When working with bar magnets, please do NOT force a north pole to touch a north pole (or force south poles to touch), as this will demagnetize the magnets.

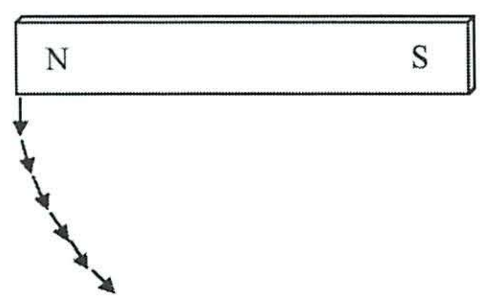
MEASUREMENTS

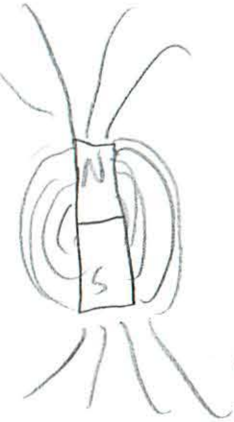
<u>Part 1: Mapping Magnetic Field Lines Using Mini-Compasses</u>

1. Tape a piece of brown paper (provided) onto your table.
2. Place a bar magnet about 3 inches from the far side of the paper, as shown below. Trace the outline of the magnet on the paper.



3. Place a compass near one end of the magnet. Make two dots on the paper, one at the end of the compass needle next to the magnet and the second at the other end of the compass needle. Now move the compass so that the end of the needle that was next to the magnet is directly over the second dot, and make a new dot at the other end of the needle. Continue this process until the compass comes back to the magnet or leaves the edge of the paper. Draw a line through the dots and indicate with an arrowhead the direction in which the North end of the needle pointed, as shown below





4. Repeat the process described above several more times (~4 field lines), starting at different locations on the magnet. Work fairly quickly – it is more important to get a feeling for the shape of the field lines than to map them precisely.

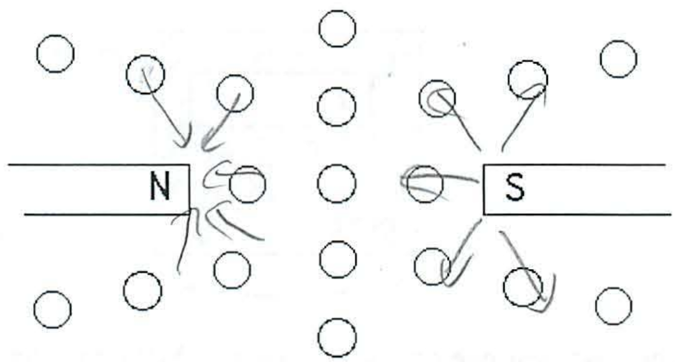
Question 1:

Mostly your field lines come back to the bar magnet, but some of them wander off and never come back to the bar magnet. Which part of your bar magnet do the ones that wander off never to return come from? Where are they going?

Part 2: Constructing a Magnetic Field Diagram

2A: Parallel Magnets

1. Arrange two bar magnets and a series of compasses as pictured here:



2. Sketch the compass needles' directions in the diagram. Based on these compass directions, sketch in some field lines.

Question 2:

Is there any place in this region where the magnetic field is zero? If so, where? How can you tell?

→ 0
goes
as go to ∞

but not inbetween magnets

; when ✓
same dir
as B

where 0 compass spin freely
(but then Earth's magnetic takes over)

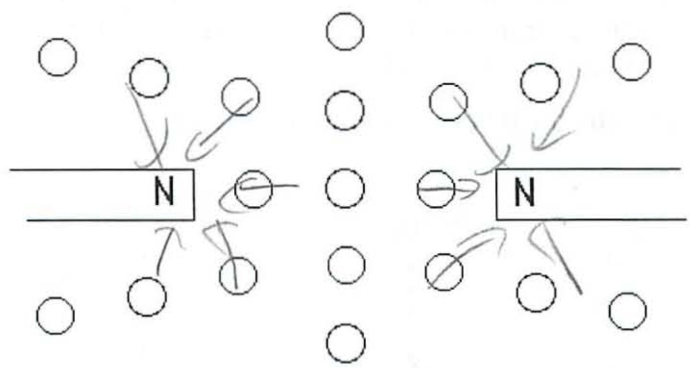
This answering on PC works well

Question 3:
Where is the magnetic field the strongest in this situation? How can you determine this from the field lines?

Between the 2 magnets - where they are the closest
Compasses vibrate somewhat

2B: Anti-Parallel Magnets

1. Arrange two bar magnets and a series of compasses as pictured here:



2. Sketch the compass needles' directions in the diagram. Based on these compass directions, sketch in some field lines.

Question 4:
Is there any place in this region where the magnetic field is zero? If so, where? How can you tell?

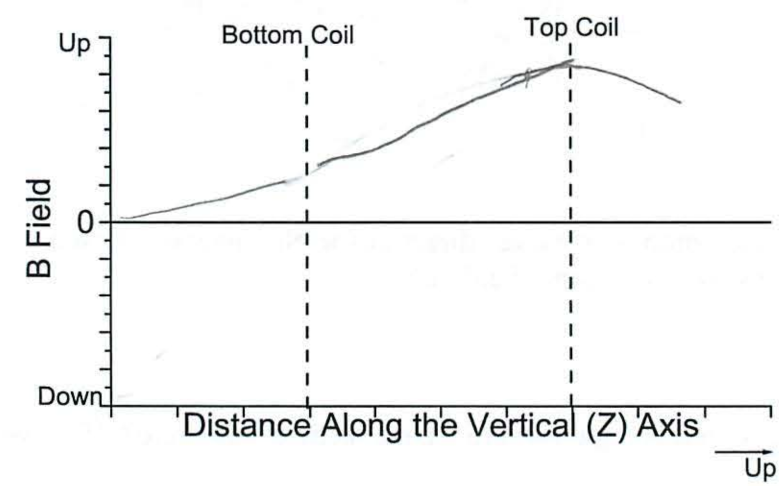
In the middle. The compass can change directions realitive quickly,

Part 3: Helmholtz Coil

In this part we are going to measure the z-component of the field along the z-axis (central axis of the coils)

3A: Using a Single Coil

1. With the power supply off, connect the red lead from the power supply to the red plug of the top coil, and the black lead to the black plug of the top coil. Turn the current knob fully counter-clockwise (i.e. turn off the current) then turn on the power supply and slowly turn the current up to ~ 0.6 A.
2. Put the magnetic field sensor in axial mode, set its gain to 10x and place it along the central axis of the Helmholtz coil, pushing into the indentation at the center of the holder. Tare it to set the reading to zero.
3. Start recording magnetic field (press Go) and raise the magnetic field sensor smoothly along the z-axis until you are above the top coil. Try raising at different rates to convince yourself that this only changes the time axis, and not the measured magnitude of the field.
4. Sketch the results for field strength vs. position



Question 5:

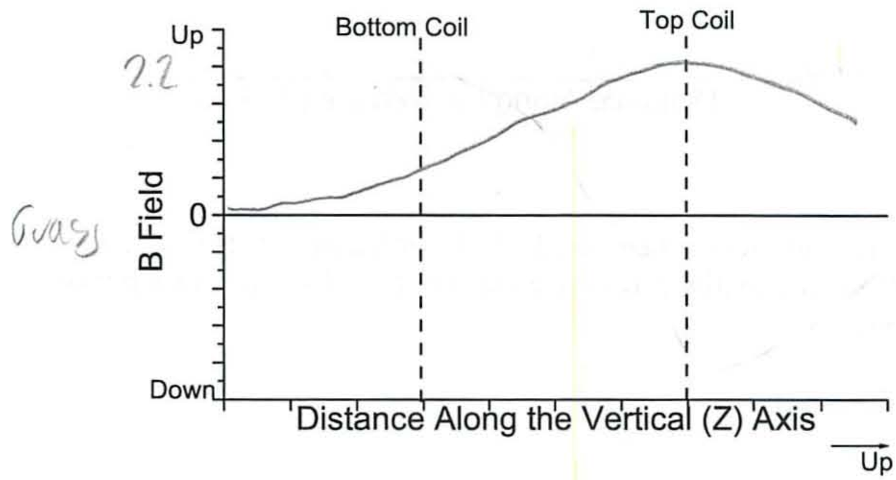
Where along the axis is the field from the single coil the strongest? What is its magnitude at this location? How does this compare to your pre-lab prediction?

Right at the top coil

5.5 Gauss

3B: Helmholtz Configuration

1. Move the black lead to the black terminal of the lower coil, and connect a lead from the black terminal of the upper to the red terminal of the lower, sending current in the same direction through both coils. Set the current to ~ 0.3 A
2. Follow the procedure in 3A to again measure field strength along the z-axis, plotting on the below figure.



Question 6:

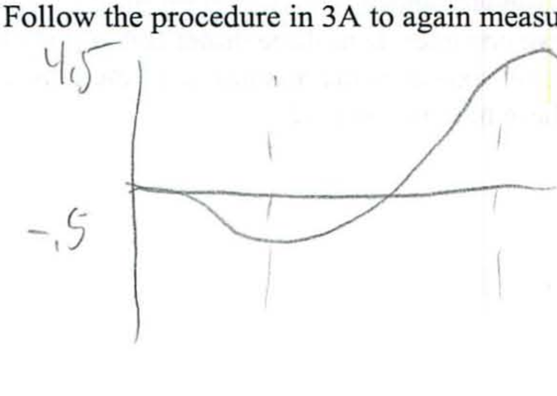
Where along the axis is the field from the strongest? What is its magnitude at this location? How does this compare to your pre-lab prediction. Aside from the location and strength of the maximum, is there a qualitative difference between the single coil and the Helmholtz coil field profile? If so, what is the difference?

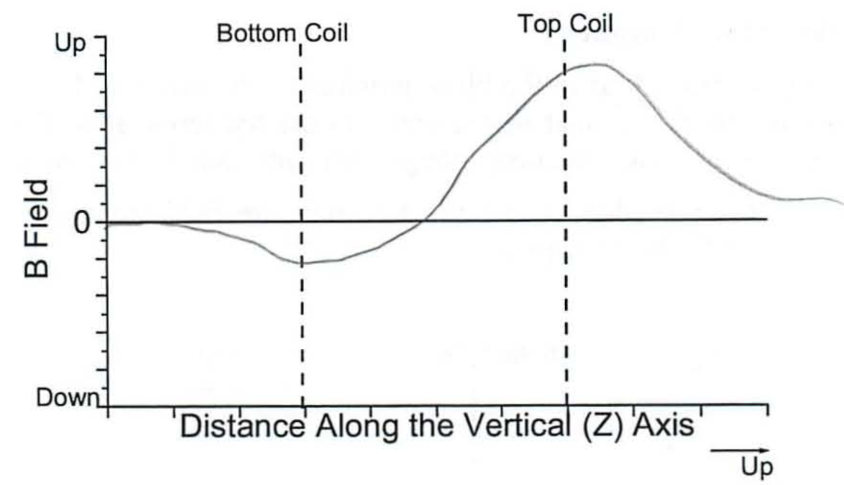
Same graph as last time

Magnitude is $\frac{1}{2}$

3C: Anti-Helmholtz Configuration

1. Swap the leads to the lower coil, keeping the current at ~ 0.3 A, although now running in opposite directions in the top and bottom coil.
2. Follow the procedure in 3A to again measure field strength along the z-axis.





Question 7:

What are two main differences between the field profile in Anti-Helmholtz configuration and in Helmholtz configuration? Does the maximum field strength match your prediction from the pre-lab?

Bottom negitive

Top more positive

Further Questions (for experiment, thought, future exam questions...)

- What does the field profile look like if we place two bar magnets next to each other rather than collinear with each other (either parallel or anti-parallel to each other).
- What does the radial field profile (e.g. the x component of the field) look like along the z-axis of the Helmholtz coil?
- What do the radial and axial field profiles look like moving across the top of the Helmholtz coil rather than down its central axis?
- It looks as though there is a local maximum of magnetic field strength at some point on the axis for both the single coil and Helmholtz coil configurations (at least looking at them along the z-axis only). If we consider them three dimensionally are they still local maxima? That is, if we move off axis does the magnitude of the field also decrease as we move away from these maxima points?

Topics: Magnetic Fields: Creating Magnetic Fields – Biot-Savart
Related Reading: Course Notes: Sections 8.3-8.4, 9.1-9.2, 9.10.1, 9.11.1-9.11.4
Experiments: (6) Torque on Magnetic Dipoless

Topic Introduction

Today we will focus on the *creation* of magnetic fields. The presentation is analogous to our discussion of charged particles creating electric fields. We first describe the magnetic field generated by a single charged particle and then proceed to collections of moving charges (currents), the fields from which we will calculate using superposition – just like for continuous charge distributions. We then calculate the forces that current carrying wires and loops feel in magnetic field. In particular we will perform a simple lab where we observe the motion of a dipole in the field created by two current carrying coils.

Field from a Single Moving Charge

Next we turn to the creation of magnetic fields. Just as a single electric charge creates an electric field which is proportional to charge q and falls off as r^{-2}, a single moving electric charge additionally creates a magnetic field given by

$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi} \frac{q \vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$$

Note the similarity to Coulomb's law for the electric field – the field is proportional to the charge q, obeys an inverse square law in r, and depends on a constant, the permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ T m/A. The difference is that the field no longer points along $\hat{\mathbf{r}}$ but is instead perpendicular to it (because of the cross product).

If you haven't worked with cross products in a while, you should read the vector analysis review module. Rapid calculation of at least the direction of cross-products will dominate the rest of the course you need to understand what they mean and how to compute them.

Field from a Current: Biot-Savart Law

We can immediately switch over from discrete charges to currents by replacing $q\vec{\mathbf{v}}$ with $Id\vec{\mathbf{s}}$:

$$d\vec{\mathbf{B}} = \frac{\mu_o}{4\pi} \frac{I\, d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

This is the Biot-Savart formula, and, like the differential form of Coulomb's Law, provides a generic method for calculating fields – here magnetic fields generated by currents. The *ds* in this formula is a small length of the wire carrying the current *I*, so that *I ds* plays the same role that *dq* did when we calculated electric fields from continuous charge distributions. To find the total magnetic field at some point in space you integrate over the current distribution (e.g. along the length of the wire), adding up the field generated by each little part of it *ds*.

Right Hand Rules

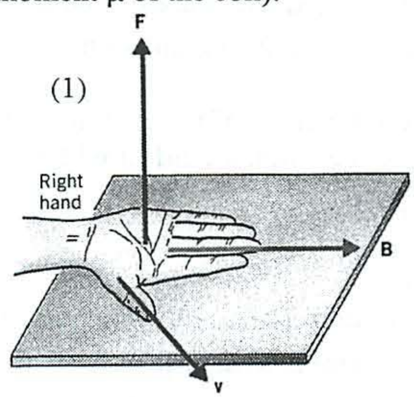
Because of the cross product in the Biot-Savart Law, the direction of the resulting magnetic field is not as simple as when we were working with electric fields. In order to quickly see what direction the field will be in, or what direction the force on a moving particle will be in, we can use a "Right Hand Rule." At times it seems that everyone has their own, unique, right hand rule. Certainly there are a number of them out there, and you should feel free to use whichever allow you to get the correct answer. Here I describe the three that I use (including one we won't come to until next week).

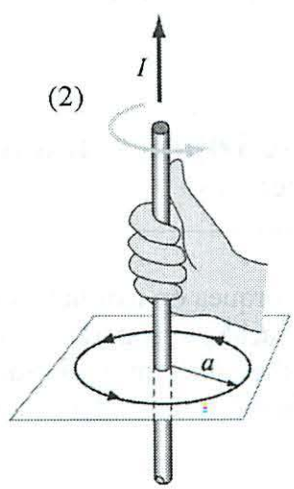
The important thing to remember is that cross-products yield a result which is perpendicular to both of the input vectors. The only open question is in which of the two perpendicular directions will the result point (e.g. if the vectors are in the floor does their cross product point up or down?). Using your RIGHT hand:

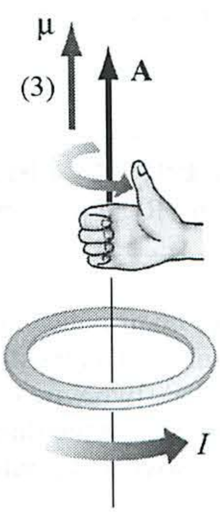
1) For a generic cross-product ($\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}}$): open your hand perfectly flat. Put your thumb along $\vec{\mathbf{A}}$ and your fingers along $\vec{\mathbf{B}}$. Your palm points along $\vec{\mathbf{C}}$.

2) For determining the direction of the magnetic field generated by a current: fields wrap around currents the same direction that your fingers wrap around your thumb. At any point the field points tangent to the circle your fingers will make as you twist your hand keeping your thumb along the current.

3) For determining the direction of the dipole moment of a coil of wire: wrap your fingers in the direction of current. Your thumb points in the direction of the North pole of the dipole (in the direction of the dipole moment μ of the coil).







Lorenz Force on Currents

Since a current is nothing more than moving charges, a current carrying wire will also feel a force when placed in a magnetic field: $\vec{\mathbf{F}} = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$ (where I is the current, and **L** is a vector pointing along the axis of the wire, with magnitude equal to the length of the wire).

Magnetic Dipole Moment
The rest of the class will be spent doing an experiment where you get to observe the motion of a magnetic dipole in an external field created by a Helmholtz coil (the same one you measured the field profile of last week). In thinking about the field profile of the Helmholtz coil, remember that coils are magnetic dipoles with the dipole moment $\vec{\boldsymbol{\mu}} = I\vec{\mathbf{A}}$, where I is the current in the loop and the direction of **A**, the area vector, is determined by a right hand rule:

Right Hand Rule for Direction of Dipole Moment
To determine the direction of the dipole moment of a coil of wire: wrap your fingers in the direction of current. Your thumb points in the direction of the North pole of the dipole (in the direction of the dipole moment μ of the coil).

Magnetic Dipole Moments in External Fields
You will observe the torque that a dipole can feel in an external field. Recall that the magnet will only feel a torque if not aligned with the external field – $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$ – and that the direction of the torque tends to align the dipole with the external field.

There will only be a force on the dipole if it is in a non-uniform field. If aligned with the external field, the dipole will seek higher field (it will climb the gradient) in order to minimize its energy $U = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}$.

Magnetic Dipole Moment
Next we turn our attention to loops of current, which act in the same way as magnetic dipoles, where the dipole moment is written $\vec{\boldsymbol{\mu}} = I\vec{\mathbf{A}}$, where I is the current in the loop and the direction of **A**, the area vector, is determined by a right hand rule:

Experiment 6: Torques on Magnetic Dipoles in a Magnetic Field
Preparation: Read pre-lab and answer pre-lab questions

This lab will be provide experience observing the torquea on magnetic dipoles in uniform magnetic fields. To investigate this we use the "TeachSpin apparatus," which consists of a Helmholtz coil (two wire coils that can produce either uniform or non-uniform magnetic fields depending on the direction of current flow in the coils) and a small magnet which is free both to move and rotate.

Important Equations

Biot-Savart – Field created by moving charge; current: $\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{q\,\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$; $d\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{I\,d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$

Force on Current-Carrying Wire of Length L: $\vec{\mathbf{F}} = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$

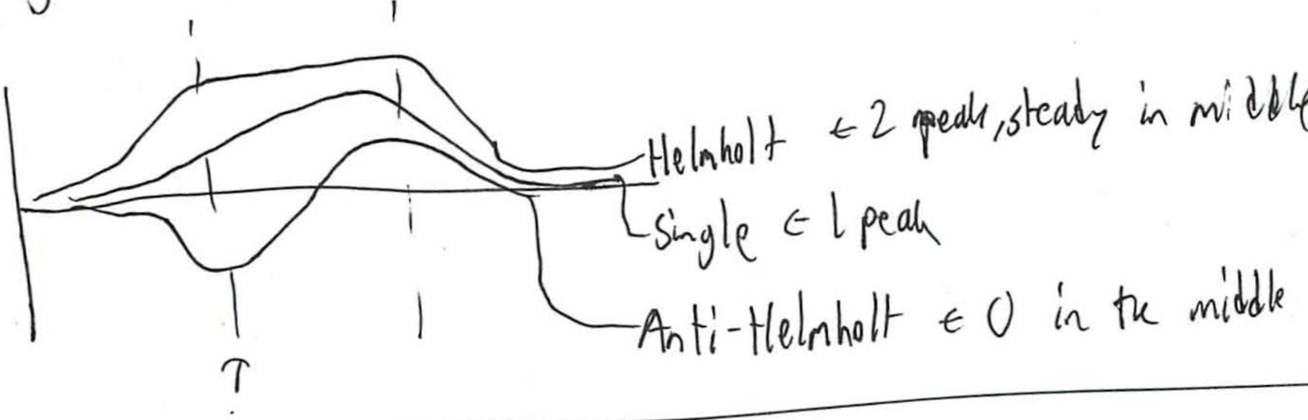
Magnetic Moment of Current Carrying Wire: $\vec{\boldsymbol{\mu}} = I\vec{\mathbf{A}}$ (direction from RHR above)

Torque on Magnetic Moment: $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$

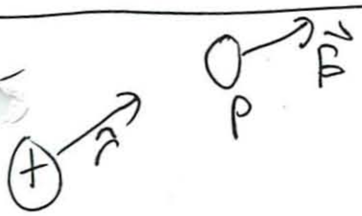
Energy of Moment in External Field: $U = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}$

Physics Class

Forgot book



What creates mag fields
- magnets, earth, moving charge



$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

$\mu_0 = 4\pi \times 10^{-7}\ T\cdot m/A$ permeability of free space

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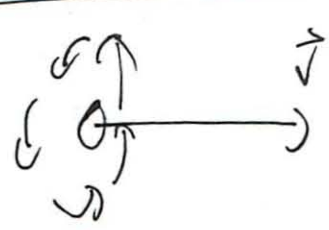
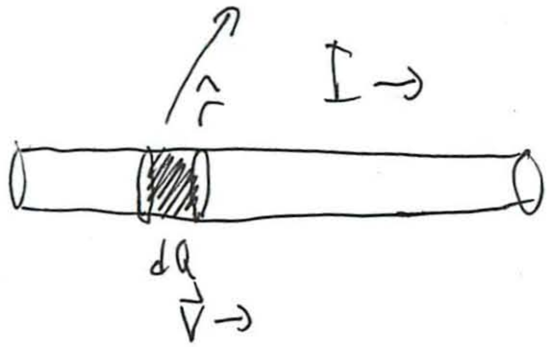


Thumb towards $\vec{v}$/current
fingers curl towards current
"2nd right hand rule"

- field point → $\hat{r}$ points to



add up all charges in wire (integral)

$$d\vec{B} = \int_{wire} \frac{\mu_0}{4\pi} dq \frac{\vec{v} \times \hat{r}}{r^2}$$

Say lenght of dq is ds

$$dq\,\vec{v} = dq \frac{d\vec{s}}{dt}$$

$$= \frac{dq}{dt} d\vec{s}$$

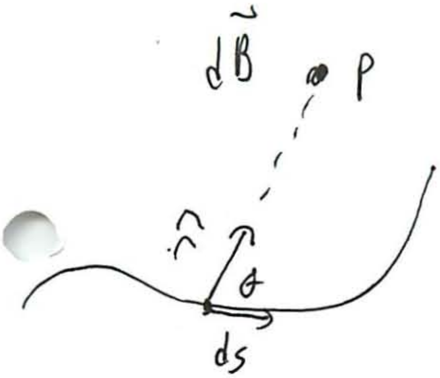
$$= I\, d\vec{s}$$

$$\boxed{dq\,\vec{v} = I\, ds}$$

$$\boxed{= \int \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2} \quad \text{Biot Savart Law}}$$

before brute force → Colomb's law
when have symmetry → Guass' Law] Electric field

brute force → Biot Savart Law
simple systems → Ampere's Law] Magnetics



$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{s} \times \hat{r}}{r^2}$$

will do conceptually and for ring

dir of B?

- thumb in dir of current
- curl towards ~~where I goes~~ from source to field point

⊙ into board

ignore this one - not perpendicular
competey 0

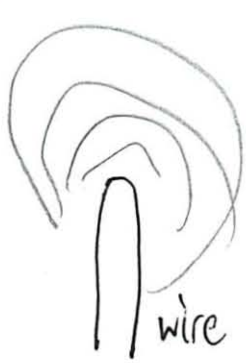
B at P?

Superposition principal

3rd right hand rule for loops
thumb in dir B field
curl fingers in dir of current
hand points to B field

into board

Semicircle + infinite wire (don't get this at all)



iron filing
make circle
around wire w/ current

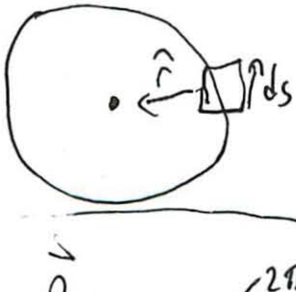
$\vec{B}(P) = ?$

— must make all of the choices

$$\vec{B}(P) = \frac{\mu_0}{4\pi}\int_{\text{wire}} \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

$\text{dir } d\vec{s} = \text{dir } I$

$\hat{r}$ points inward



$r = R$

$\text{dir}(d\vec{s} \times \vec{r}) = \odot$

$|d\vec{s}| = R\,d\theta$

$\hat{r}$ points from $d\vec{s}$ to P
source to
field point

thumb dir ds/v
fingers dir r
dir you curl in
is dir of cross
product!

$$\vec{B}_{(\text{center})} = \int_0^{2\pi} \frac{I\, r\, d\theta}{R^2}$$

$\frac{\mu_0}{4\pi}$

$$= \frac{2\pi \mu_0}{4\pi}\frac{I}{R} \odot = \frac{\mu_0 I}{2R} \odot$$

Helmholtz + this calculation

- say each ring gives contribution
- multiply ans we got by n

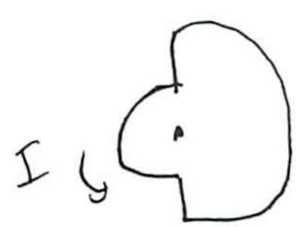
Example

1. Legs contribe

⋮ missed it

Grap problem

$B(P) = ?$



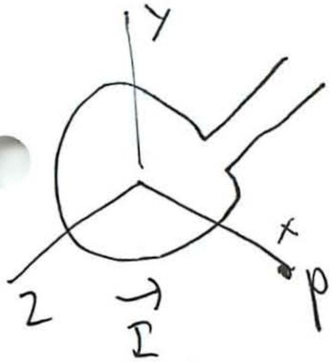
- just do both halves seportly

$$\frac{\mu_0}{4\pi}\int_0^{\pi} \frac{I\,R\,d\theta}{R^2} + I\,2R\,d\theta$$

$$\frac{I\pi}{R} + \frac{I\pi}{2R}$$

$$\frac{\mu_0}{4\pi} \cdot \frac{3I\pi}{2R}$$

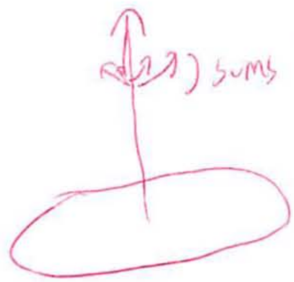
$$\frac{\mu_0 3I}{8R} \quad \odot$$



$B(P) = ?$

- much harder
- vector decomposition

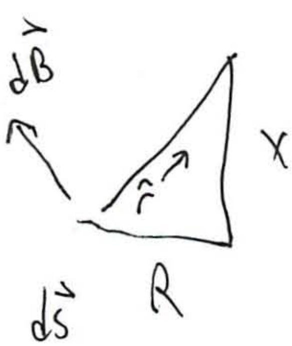
- from illustration



sums in a ring ↑ adds to be straight up

* * Label ds, r, etc *

is a really good diagram



all the horiz pieces cancle

all the vertical pieces add up

$r = \sqrt{R^2 + x^2}$

$d\vec{s} \times \hat{r}$

$ds = R\,d\theta$

$d\vec{s} \times \hat{r} = |d\vec{s}|$

$$\int \frac{\mu_0}{4\pi}\left(-(R\,d\theta\,\hat{k}) \times \frac{R(-\hat{i}) + x\hat{j}}{\sqrt{R^2 + x^2}}\right)$$

$\hat{k} \times \hat{i} = \hat{j}$

$-\hat{k} \times \hat{j} = +\hat{i}$

as you go around circle $\hat{j}$ term
cancles

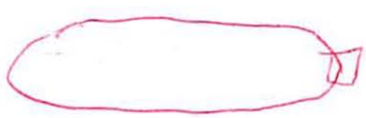
Only vertical piesse survives

Only difference r^2

Watch ring calculation simulation

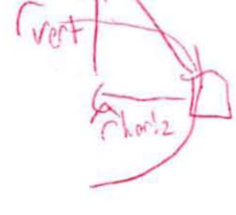
- all horiz cancle
- all vert add up

- so its exactly last calc w/ little mod



$\otimes\ d\vec{s}$

$\hookrightarrow d\vec{s} = R\,d\theta\,(-\hat{k})$



$$\frac{I R\,d\theta\,(-\hat{k}) \times (\hat{r}_{horiz} + \hat{r}_{vertical})}{R^2 + x^2}$$

$d\vec{s} = R\,d\theta\,(-\hat{k})$

$\hat{r}_{horiz} = \hat{r}\cos\phi$

$= \hat{r}\,\frac{R}{(R^2+x^2)^{1/2}}$

$* \quad d'r\,(-\hat{k} \times \hat{r}_{horiz}) = d'r\,\hat{j}$

$\uparrow \hat{r}_{vert}$

$\otimes d\vec{s}$ get cross product $\overrightarrow{\hat{r}_{vert} \times ds}$

↑ the vertical gets flipped to horiz
(this does not get cancled)

So left w/

$$\frac{I\,R\,d\theta\,(-\hat{k}) \times \hat{r}_{horiz}}{R^2+x^2}$$

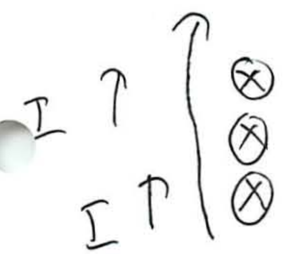
$$\frac{\mu_0}{4\pi}\,\frac{I R}{(R^2+x^2)}\,\frac{R}{(R^2+x^2)^{1/2}}\int_0^{2\pi} d\theta\ \hat{j}$$

$$= \frac{\mu_0 I R^2}{2\left(R^2 + x^2\right)^{3/2}} \hat{\jmath}$$

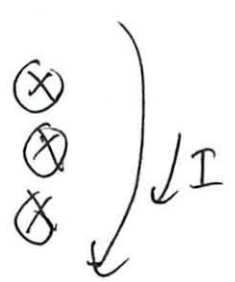
- like the helmholtz experiment

Torque

no current → no force



so pushes to left



reverse currenent

$d\vec{s}$ reversed

goes other way

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$= \text{charge}\ \frac{m}{s} \times \vec{B}$$

$$= \frac{\text{charge}}{\text{sec}}\ m \times \vec{B}$$

$$= I\left(\vec{L} \times \vec{B}\right)$$

see notes

note the $I\,d\vec{s} \times B$ quanity as go along curvey wire

$\int I\,d\vec{s} \times \vec{B}$

$(I\,d\vec{s}) \times \vec{B}$

↑ add the $d\vec{s}$ to get lenght

straight wire

$I = \int d\vec{s}$

So saying $I\,(\vec{L} \times \vec{B})$

means straight wire

Uniform $\vec{B}$ field

parallel wires attract

current in same directions

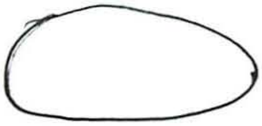
---

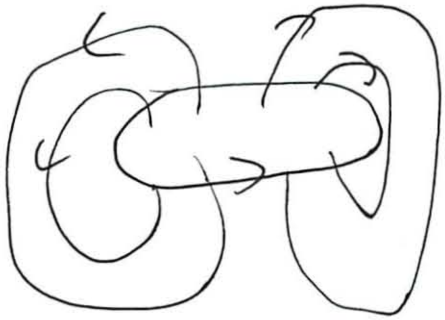
tension = pull in

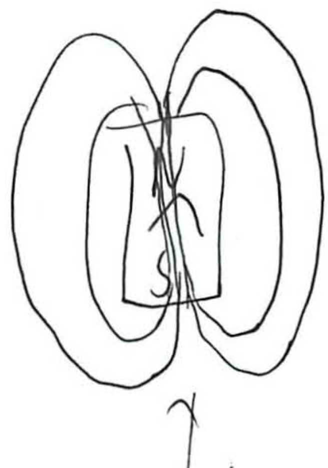
presure = push away

Magnetic Dipole Moment

$\vec{\mu} = IA\hat{n} = I\vec{A}$

 Ring





does not end inside

goes from south to north
inside magnet

$\oint\oint \vec{B} \cdot d\vec{a} = 0$ closed loop

called the magnetic moment

4th right hand rule

fingers dir I

$\vec{\mu}$ down the curl

$\vec{\mu} = (I)(Area)\hat{N}$ right hand rule 4

$= IA$

~~A~~ current loop is like N-S magnet

is like =

so attracts

like

repel

how compass needels torque to line up w/ B field

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

up = ~~tg~~ right straight

so will go right

Class 18: Outline

Hour 1 & 2:

Magnetic Fields, Creating Fields: Biot-Savart Law

P18- 1

Sources of Magnetic Fields

P18- 2

What creates fields?

Magnets – more about this later

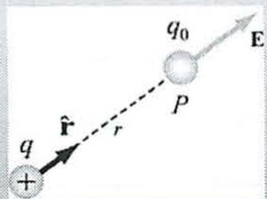
The Earth How's that work?

Moving charges!

P18- 3

Electric Field Of Point Charge

An electric charge produces an electric field:

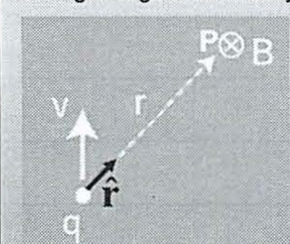


$$\vec{\mathbf{E}} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{\mathbf{r}}$$

$\hat{\mathbf{r}}$: unit vector directed from q to *P*

Magnetic Field Of Moving Charge

Moving charge with velocity v produces magnetic field:



$$\boxed{\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{q\,\vec{\mathbf{v}}\times\hat{\mathbf{r}}}{r^2}}$$

$\hat{\mathbf{r}}$: unit vector directed from q to P

$\mu_0 = 4\pi\times10^{-7}\ \mathrm{T\cdot m/A}$ permeability of free space

Animation: Field Generated by a Moving Charge

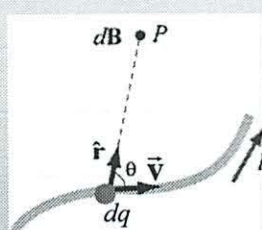
Continuous charge distributions: Currents & Biot-Savart

Moving

From Charges to Currents?

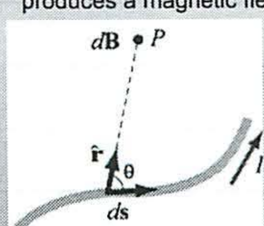
$$d\vec{\mathbf{B}} \propto dq\,\vec{\mathbf{v}} = (\text{charge})\frac{\text{m}}{\text{s}} = \frac{\text{charge}}{\text{s}}\text{m}$$

$$\boxed{d\vec{\mathbf{B}} \propto I d\vec{\mathbf{s}}}$$



The Biot-Savart Law

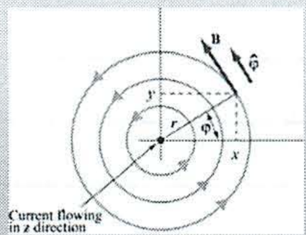
Current element of length ds carrying current I produces a magnetic field:



$$\boxed{d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}}$$

(Shockwave)

The Right-Hand Rule #2

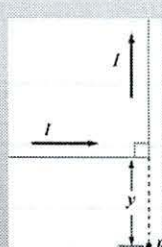


$$\hat{\mathbf{z}} \times \hat{\boldsymbol{\rho}} = \hat{\boldsymbol{\varphi}}$$

PRS Questions: B fields Generated by Currents

PRS: Biot-Savart

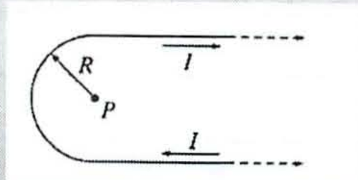
The magnetic field at P points towards the



0% 1. +x direction
0% 2. +y direction
0% 3. +z direction
0% 4. -x direction
0% 5. -y direction
0% 6. -z direction
0% 7. Field is zero (so no direction)

PRS: Bent Wire

The magnetic field at P is equal to the field of:

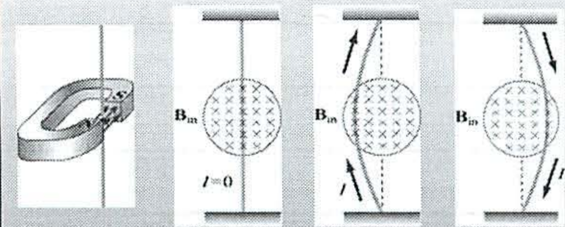


0%	1. a semicircle
0%	2. a semicircle plus the field of a long straight wire
0%	3. a semicircle minus the field of a long straight wire
0%	4. none of the above

Demonstration: Field Generated by Wire

Demonstration: Jumping Wire

Magnetic Force on Current-Carrying Wire



Current is moving charges, and we know that moving charges **feel** a force in a magnetic field

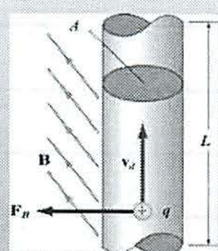
Magnetic Force on Current-Carrying Wire

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$$

$$= (\text{charge})\frac{\text{m}}{\text{s}}\times\vec{\mathbf{B}}$$

$$= \frac{\text{charge}}{\text{s}}\,\text{m}\times\vec{\mathbf{B}}$$

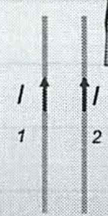
$$\boxed{\vec{\mathbf{F}}_B = I\left(\vec{\mathbf{L}}\times\vec{\mathbf{B}}\right)}$$



PRS Question: Parallel Current Carrying Wires

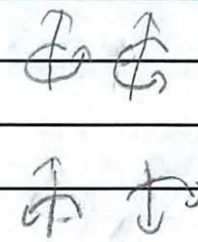
PRS: Parallel Wires

Consider two parallel current carrying wires. With the currents running in the same direction, the wires are



0% 1. attracted (likes attract?)
0% 2. repelled (likes repel?)
0% 3. pushed another direction
0% 4. not pushed – no net force
0% 5. I don't know

P18-19

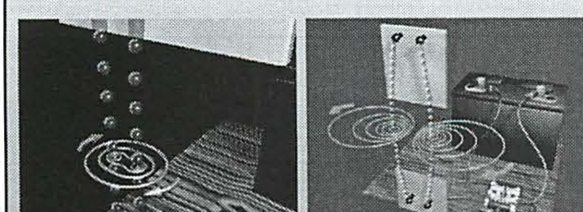


Demonstration: Parallel & Anti-Parallel Currents

P18-20

Can we understand why?

Whether they attract or repel can be seen in the shape of the created B field

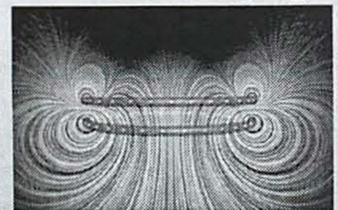


(Animation) (Animation)

P18-21

thumb towards Current
curl fingers in dir B field

PRS: Current Carrying Coils



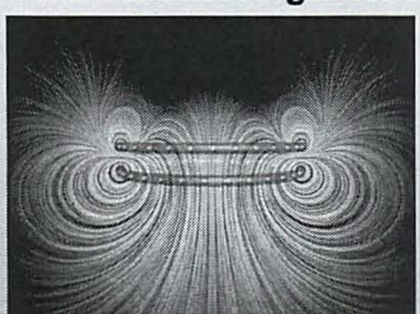
:10

The above coils have

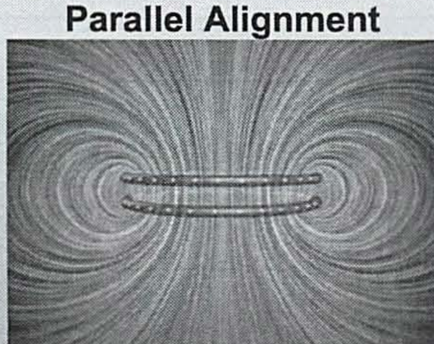
0%	1. parallel currents that attract
0%	2. parallel currents that repel
0%	3. opposite currents that attract
0%	4. opposite currents that repel

√

Force on Dipole from Dipole: Anti-Parallel Alignment

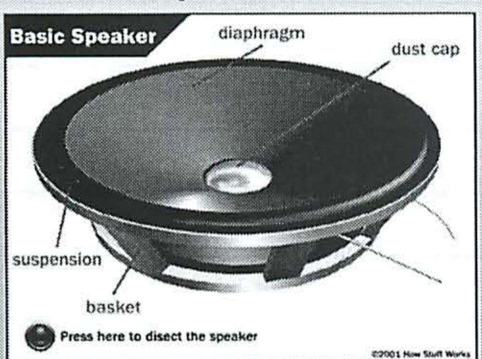


Force on Dipole from Dipole: Parallel Alignment

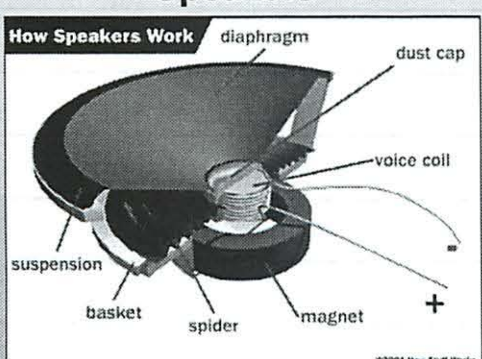


Applications

Speakers

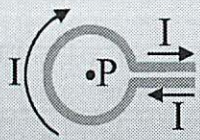


Speakers



Example : Coil of Radius *R*

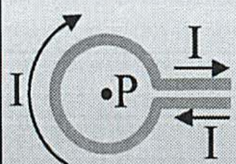
Consider a coil with radius *R* and current *I*



Find the magnetic field B at the center (P)

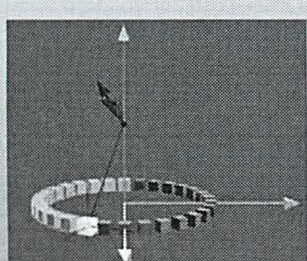
Example : Coil of Radius *R*

Consider a coil with radius *R* and current *I*



1) Think about it:
   - Legs contribute nothing
     I parallel to *r*
   - Ring makes field into page
2) Choose a ds
3) Pick your coordinates
4) Write Biot-Savart

Animation: Magnetic Field Generated by a Current Loop

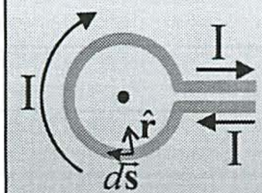


it all adds up

Example : Coil of Radius R

In the circular part of the coil…

$$d\vec{\mathbf{s}} \perp \hat{\mathbf{r}} \quad \rightarrow \quad |d\vec{\mathbf{s}} \times \hat{\mathbf{r}}| = ds$$

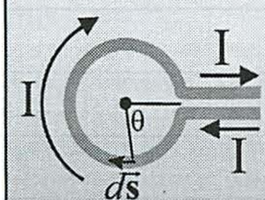


Biot-Savart:

$$dB = \frac{\mu_0 I}{4\pi}\frac{|d\vec{\mathbf{s}} \times \hat{\mathbf{r}}|}{r^2} = \frac{\mu_0 I}{4\pi}\frac{ds}{r^2} = \frac{\mu_0 I}{4\pi}\frac{R\,d\theta}{R^2} = \frac{\mu_0 I}{4\pi}\frac{d\theta}{R}$$

Example : Coil of Radius R

Consider a coil with radius R and current I



$$dB = \frac{\mu_0 I}{4\pi}\frac{d\theta}{R}$$

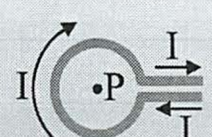
$$B = \int dB = \int_0^{2\pi} \frac{\mu_0 I}{4\pi}\frac{d\theta}{R} = \frac{\mu_0 I}{4\pi R}\int_0^{2\pi} d\theta = \frac{\mu_0 I}{4\pi R}(2\pi)$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2R} \text{ into page}$$

in current ($\vec{B}$ would be out of page

& don't forget to integrate!

Example : Coil of Radius R



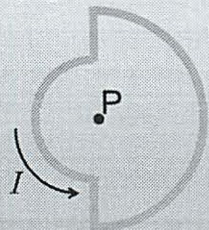
$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2R} \text{ into page}$$

Notes:
- This is an EASY Biot-Savart problem:
  - No vectors involved
- This is what I would expect on exam

Group Problem: B Field from Coil of Radius R

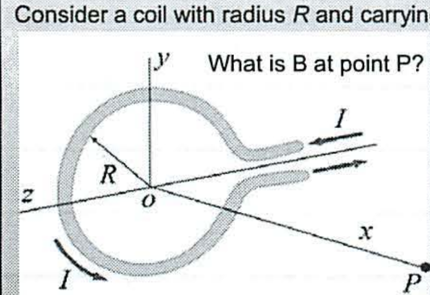
Consider a coil made of semi-circles of radii R and $2R$ and carrying a current I

What is **B** at point P?



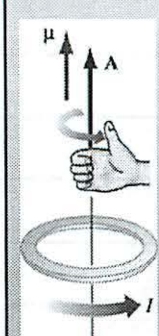
Group Problem: B Field from Coil of Radius R

Consider a coil with radius R and carrying a current I

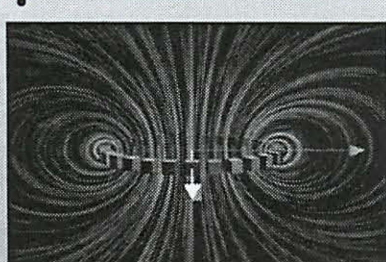


WARNING: This is much harder than the previous problem. Why??

Magnetic Dipole Moment



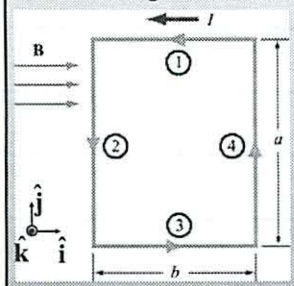
$$\vec{\mu} \equiv IA\hat{\mathbf{n}} \equiv I\vec{\mathbf{A}}$$



Torque on a Current Loop in a Uniform Magnetic Field

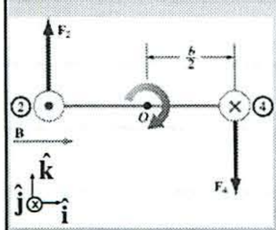
Group Problem: Current Loop

Place rectangular current loop in uniform B field



1) What is the net force on this loop?
2) What is the net torque on this loop?
3) Describe the motion the loop makes

Torque on Rectangular Loop



$$\vec{\boldsymbol{\tau}} = IAB\hat{\mathbf{j}}$$

$\vec{\mathbf{A}} = A\hat{\mathbf{n}} = ab\hat{\mathbf{n}}$: area vector

$\hat{\mathbf{n}} = +\hat{\mathbf{k}}, \quad \vec{\mathbf{B}} = B\hat{\mathbf{i}}$

$$\boxed{\vec{\boldsymbol{\tau}} = I\vec{\mathbf{A}} \times \vec{\mathbf{B}}}$$

Familiar? No net force but there is a torque

Magnetic Dipole Moment

Define Magnetic Dipole Moment:

$$\boxed{\vec{\mu} \equiv IA\hat{\mathbf{n}} \equiv I\vec{\mathbf{A}}}$$

Then:

$$\boxed{\vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}}}$$

Analogous to $\vec{\tau} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$

τ tends to align μ with B

Animation: Another Way To Look At Torque

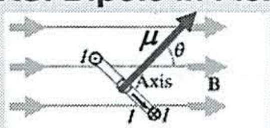
External field connects to field of magnet and "pulls" the dipole into alignment

Demonstration: Galvanometer

PRS Question: Force on Magnetic Dipole

PRS: Dipole in Field

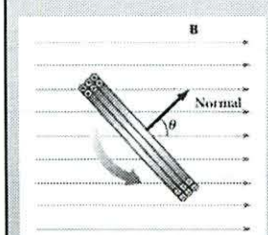
10



From rest, the coil above will:

0%	1.	rotate clockwise, not move
0%	2.	rotate counterclockwise, not move
0%	3.	move to the right, not rotate
0%	4.	move to the left, not rotate
0%	5.	move in another direction, without rotating
0%	6.	both move and rotate
0%	7.	neither rotate nor move
0%	8.	I don't know

DC Motor



Energy of Magnetic Dipole

$$U_{Dipole} = -\vec{\mu} \cdot \vec{B}$$

This equation gives you a general way to think about what dipoles will do in B fields

P18-46

Experiment 6: Magnetic Forces on Dipoles

This is a little tricky. We will lead you through with lots of PRS questions

P18-

First: Set up current supply

- Open circuit (disconnect a lead)
- Turn current knob full CCW (off)
- Increase voltage to ~12 V
  - This will act as a protection: V<12 V
- Reconnect leads in Helmholtz mode
- Increase current to ~1 A

P18-

Field Profiles: B vs. Height

VERY UNIFORM!

Single Coil

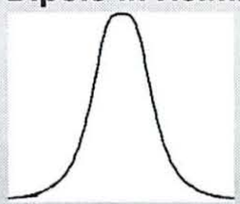
Helmholtz

Anti-Helmholtz

ZERO FIELD!

P18-

:10 PRS: Dipole in Helmholtz



A randomly aligned dipole at the center of a Helmholtz coil will feel:

0% 1. a force but not a torque
0% 2. a torque but not a force
0% 3. both a torque and a force
0% 4. neither force nor torque

P18-

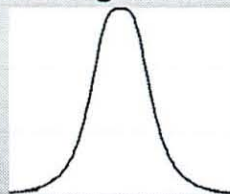
Next: Dipole in Helmholtz (Q1-2)

- Set in Helmholtz Mode (~1 A)
- Turn off current
- Put dipole in center (0 on scale)
- Randomly align using bar magnet
- Turn on current
- What happens?

P18-

PRS: Moving in Helmholtz

10



When moving through the above field profile, a dipole will:

0% 1. Never rotate

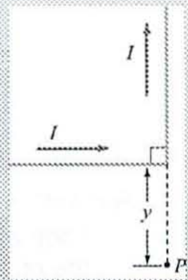
0% 2. Rotate once

0% 3. Rotate twice

Class 18

PRS: Biot-Savart

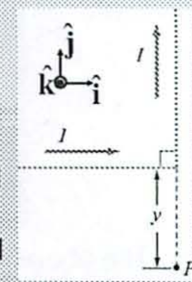
The magnetic field at P points towards the



0% 1. +x direction
0% 2. +y direction
0% 3. +z direction
0% 4. -x direction
0% 5. -y direction
0% 6. -z direction
0% 7. Field is zero (so no direction)

PRS Answer: Biot-Savart

Answer: 6. B(P) is in the -z direction (into page)

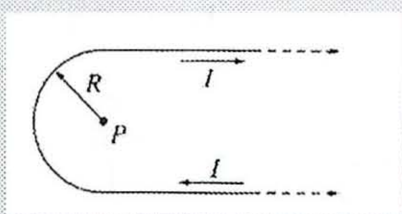


The vertical line segment contributes nothing to the field at P (it is parallel to the displacement). The horizontal segment makes a field into the page.

0

PRS: Bent Wire

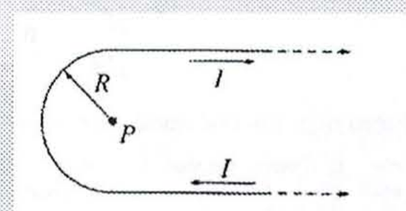
The magnetic field at P is equal to the field of:



0% 1. a semicircle
0% 2. a semicircle plus the field of a long straight wire
0% 3. a semicircle minus the field of a long straight wire
0% 4. none of the above

PRS Answer: Bent Wire

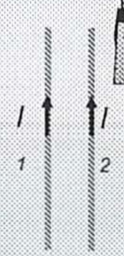
Answer: 2. Semicircle + infinite wire



All of the wire makes B into the page. The two straight parts, if put together, would make an infinite wire. The semicircle is added to this to get the complete field

PRS: Parallel Wires

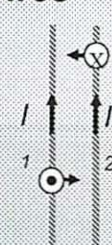
Consider two parallel current carrying wires. With the currents running in the same direction, the wires are



0% 1. attracted (likes attract?)
0% 2. repelled (likes repel?)
0% 3. pushed another direction
0% 4. not pushed – no net force
0% 5. I don't know

PRS Answer: Parallel Wires

Answer: 1. The wires are attracted

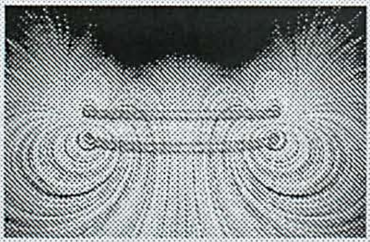


I_1 creates a field into the page at I_2. That makes a force on I_2 to the left.

I_2 creates a field out of the page at I_1. That makes a force on I_1 to the right.

PRS: Current Carrying Coils

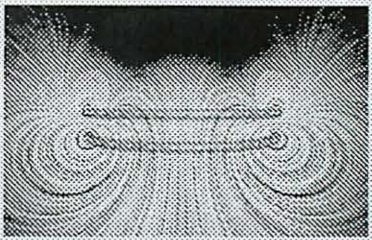
:00



The above coils have

0% 1. parallel currents that attract
0% 2. parallel currents that repel
0% 3. opposite currents that attract
0% 4. opposite currents that repel

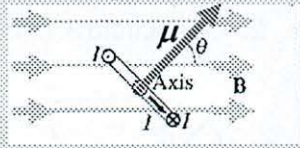
PRS Answer: I Carrying Coils



Answer: 4. Opposite currents that repel

Look at the field lines at the edge between the coils. They are jammed in, want to push out. Also, must be in opposite directions

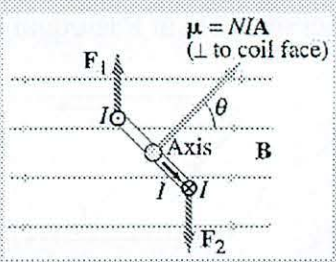
PRS: Dipole in Field



From rest, the coil above will:

0% 1. rotate clockwise, not move
0% 2. rotate counterclockwise, not move
0% 3. move to the right, not rotate
0% 4. move to the left, not rotate
0% 5. move in another direction, without rotating
0% 6. both move and rotate
0% 7. neither rotate nor move
0% 8. I don't know

PRS Answer: Dipole in Field

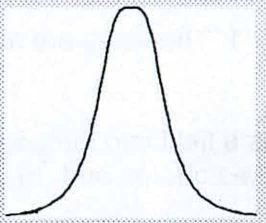


Answer: 1. Coil will rotate clockwise (not move)

No net force so no center of mass motion. BUT Magnetic dipoles rotate to align with external field (think compass)

PRS: Dipole in Helmholtz

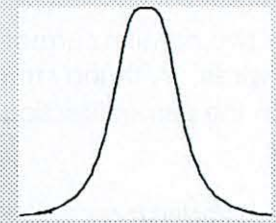
:10



A randomly aligned dipole at the center of a Helmholtz coil will feel:

0% 1. a force but not a torque
0% 2. a torque but not a force
0% 3. both a torque and a force
0% 4. neither force nor torque

PRS Answer: Dipole in Helmholtz

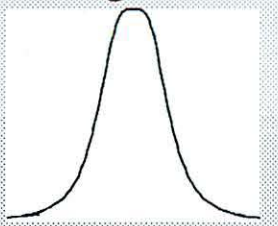


Answer: 2. a torque but not a force

- The Helmholtz coil makes a ***UNIFORM FIELD***
- Dipole feels only torque (need gradient for F)

PRS: Moving in Helmholtz

10



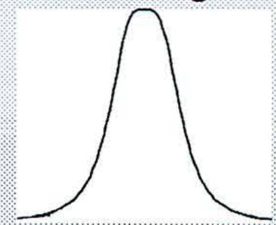
When moving through the above field profile, a dipole will:

0% 1. Never rotate

0% 2. Rotate once

0% 3. Rotate twice

PRS Answer: Moving in Helmholtz



Answer: 1. The dipole will never rotate

- The dipole is always aligned with the field so it will never rotate

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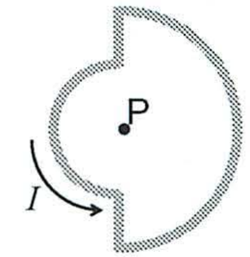
Solutions: Current Carrying Coil



Problem: Consider a coil consisting of two semi-circles of radii R and 2R carrying a current I. Calculate the magnetic field at the center, point P.

Remember it sums up (animation)

Solution:
This is very similar to the demonstration we did of a single circle, instead now we will integrate from 0 to π instead of 0 to 2π to get around each semi-circle. Each semi-circle creates a field in the same direction (out of the page) so we can just superimpose (add) them.

So, we will use the Biot-Savart formula, immediately getting rid of the vectors:

$$d\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\frac{d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2} \qquad \Rightarrow \qquad dB = \frac{\mu_0 I}{4\pi}\frac{ds}{r^2} = \frac{\mu_0 I}{4\pi}\frac{r d\theta}{r^2} = \frac{\mu_0 I}{4\pi r}d\theta$$

where r is either R or $2R$, depending on which side we are integrating and θ is the angular variable that let's us move around the circle.

The integral is easy since everything is a constant, and for each semi-circle we get:

$$B = \int_{\theta=0}^{\pi} \frac{\mu_0 I}{4\pi r}d\theta = \frac{\mu_0 I}{4\pi r}\pi = \frac{\mu_0 I}{4r}$$

Now we just add the fields together from the two semi-circles:

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4}\left(\frac{1}{R}+\frac{1}{2R}\right) = \boxed{\frac{3\mu_0 I}{8R} \text{ out of the page}}$$

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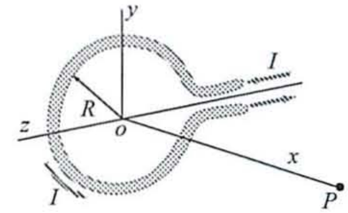
Solutions: Magnetic Field along Central Axis of Current Carrying Coil

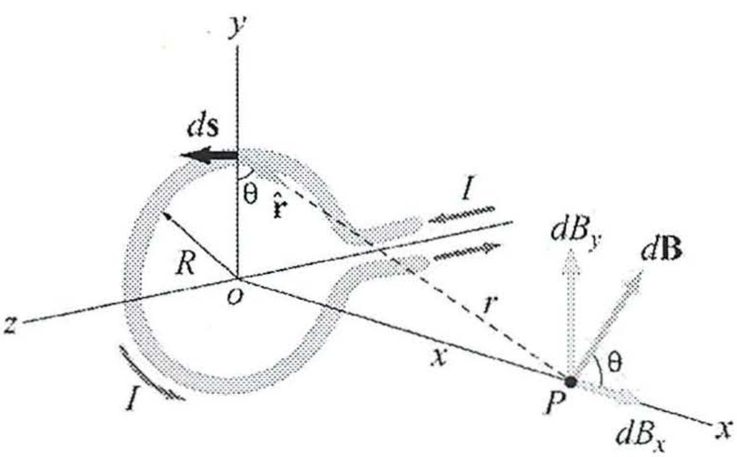


Problem: Calculate the magnetic field at a point P along the axis of a loop of current of radius R.

Solution:
As always, the first step is to think about the problem a little. Since we have a loop of current every point on that loop will contribute to the magnetic field at P. We can calculate the direction and how much using the Biot-Savart formula:

$$d\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\frac{d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$$

Now redraw the above picture, labeling everything in this formula – we will pick one $d\vec{\mathbf{s}}$ on the loop and see what $d\vec{\mathbf{B}}$ it makes:



Note that this is slightly more complicated than the case we looked at before – the field at the center of the loop. Here the $d\vec{\mathbf{B}}$ is at an angle to the axis (you can see this by using the right-hand rule). Also note that the $d\vec{\mathbf{s}}$ I chose is arbitrary. We have to do it for every $d\vec{\mathbf{s}}$ around the loop. If I had chosen the point at the bottom of the loop the $d\vec{\mathbf{B}}$ would be flipped across the x-axis (pointing in the negative y direction) but would be the same magnitude. If I had picked the point at the left or right of the loop then the field would be in the *xz* plane instead of the *xy* plane.

When we integrate around the loop we will see that all the non-x components will vanish (by symmetry) and that only the x-component survives. So we will only calculate the x-component:

$$dB_x = dB\cos\theta = \frac{\mu_0 I}{4\pi}\frac{|d\vec{\mathbf{s}}\times\hat{\mathbf{r}}|}{r^2}\cos\theta = \frac{\mu_0 IR}{4\pi}\frac{ds}{(x^2+R^2)^{3/2}} \quad \text{since } \cos\theta = \frac{R}{r} \text{ and } r = (x^2+R^2)^{1/2}$$

Note that there was no sin(θ) term from the cross product because $d\vec{\mathbf{s}}$ and $\hat{\mathbf{r}}$ are perpendicular. Finally, we just need to integrate around the loop:

$$B_x = \frac{\mu_0 IR}{4\pi(x^2+R^2)^{3/2}}\oint ds = \frac{\mu_0 IR^2}{2(x^2+R^2)^{3/2}}$$

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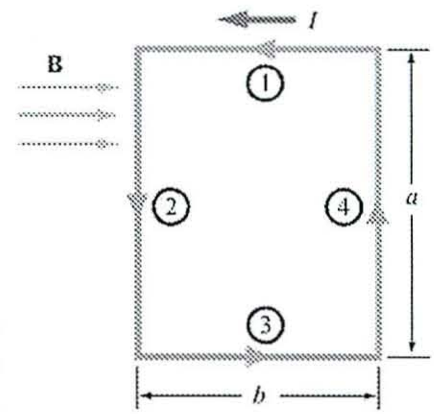
Solutions: Force and Torque on a Current Loop in a Uniform Magnetic Field

Problem: Consider the current carrying loop at right which sits in a uniform external magnetic field **B**. What is the force and torque on the loop? What motion does it make?

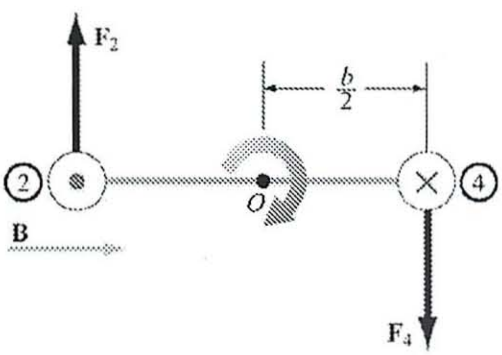


Solution:

1) What is the net force?

The forces on the top and bottom (legs 1 & 3) of the loop are both zero, as the current is parallel to the magnetic field. The forces on legs 2 & 4 are equal and opposite and hence cancel. So the net magnetic field is **zero**.

2) What is the net torque?



The force on leg 2 is out of the page and the force on leg 4 is into the page, so there will be a torque up (which means that the loop wants to rotate into the page on the right). To calculate its magnitude we need to arbitrarily choose an axis of rotation. I'll pick leg 2. Then the torque only comes from leg 4 and we have: $\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = b\hat{\mathbf{i}} \times IaB\left(-\hat{\mathbf{k}}\right) = IabB\hat{\mathbf{j}}$

So what happens to the loop due to this torque? It will rotate until it has rotated 90 degrees. At this point the forces on legs 2 and 4 will be "outward," making the loop want to expand rather than rotate. So at this point the loop won't want to rotate any more – it is an equilibrium position. Of course the loop will already have angular momentum from the motion it just underwent meaning that it will continue past this equilibrium point and then undergo harmonic motion about it.

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Experiment 6: Forces and Torques on Magnetic Dipoles

OBJECTIVES

1. To observe and measure the forces and torques acting on a magnetic dipole placed in an external magnetic field.

REMARK: We will only measure the torque on the dipole, (parts 1-3 below). If you have the time you may want to try and observe the forces on the dipole in parts 4-5 but it is not required.

PRE-LAB READING

INTRODUCTION

In this lab you will suspend a magnetic dipole (a small but strong bar magnet) in the field of a Helmholtz coil (the same apparatus you used in Expt. 5). You will observe the force and torque on the dipole as a function of position, and hence external field.

The Details: Magnetic Dipoles in External Fields

As we have discussed in class, magnetic dipoles are characterized by their dipole moment μ, a vector that points in the direction of the B field generated by the dipole (at the center of the dipole). When placed in an external magnetic field B, they have a potential energy

$$U_{Dipole} = -\vec{\mu}\cdot\vec{\mathbf{B}}$$

That is, they are at their lowest energy ("happiest") when aligned with a large external field

Torque

When in a non-zero external field the dipole will want to rotate to align with it. The magnitude of the torque which leads to this rotation is easily calculated:

$$\tau = \frac{dU}{d\theta} = -\frac{d}{d\theta}\mu B\cos(\theta) = \mu B\sin(\theta) = \left|\vec{\mu}\times\vec{\mathbf{B}}\right|$$

Again, the direction of the torque is such that the dipole moment rotates to align with the field (perpendicular to the plane in which μ and $\boldsymbol{B}$ lie, and obeying the right hand rule that if your thumb points in the direction of the torque, your fingers rotate from μ to $\boldsymbol{B}$.

Force

In order to feel a force, the potential energy of the dipole must change with a change in its position. If the magnetic field **B** is constant, then this will not happen, and hence *the dipole feels no force in a uniform field.* However, if the field is non-uniform, such as is created by another dipole, then there can be a force. In general, the force is quite complex, but for a couple of special cases it is simple:

1) If the dipole is aligned with the external field it seeks higher field
2) If the dipole is anti-aligned it seeks lower field

These rules can be easily remembered just by remembering that the dipole always wants to reduce its potential energy. They can also be remembered by thinking about the way that the poles of bar magnets interact – opposites attract while likes repel.

In one dimension, when the dipole is aligned with the field, a rather straight forward mathematical expression may also be derived:

$$F = -\frac{dU}{dz} = \frac{d}{dz}\mu B = \mu\frac{dB}{dz}$$

Here it is important to note that the magnitude of the force depends not on the field but on the derivative of the field. Aligned dipoles climb uphill. The steeper the hill, the more force they feel.

APPARATUS

1. Teach Spin Apparatus

(a) (b)

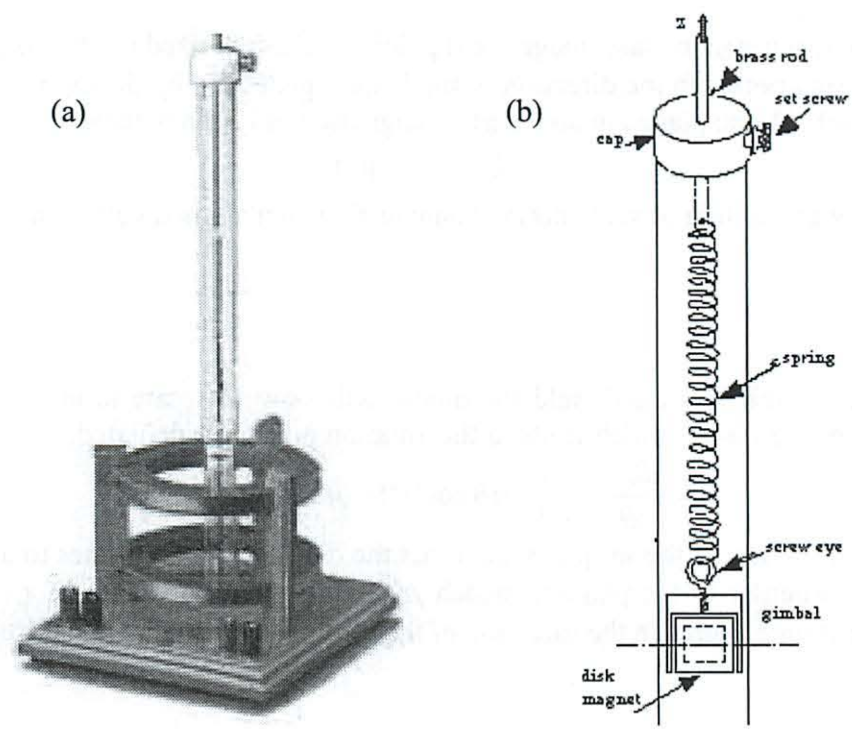


Figure 1 The Teach Spin Apparatus (a) The Helmholtz apparatus has a tower assembly (b) placed along its central axis. The tower contains a disk magnet which is free to rotate (on a gimbal) about an axis perpendicular to the tube and constrained to move vertically. The amount of motion can be converted into a force knowing the spring constant of the spring.

The central piece of equipment used in this lab is the Teach Spin apparatus (Fig. 1). It consists of the Helmholtz coil that you used in experiment 5, along with a Plexiglas tube containing a magnet on a spring. The magnet can both rotate and move vertically, allowing you to visualize both torques and the forces on dipoles.

It will be useful to recall some results from experiment 5 involving the Helmholtz coil. There are three different modes of operation – you can energize just a single coil, both coils in parallel (Helmholtz configuration) or both coils anti-parallel (anti-Helmholtz). The field profiles (as well as the derivatives of those profiles – necessary for thinking about force) look like the following:

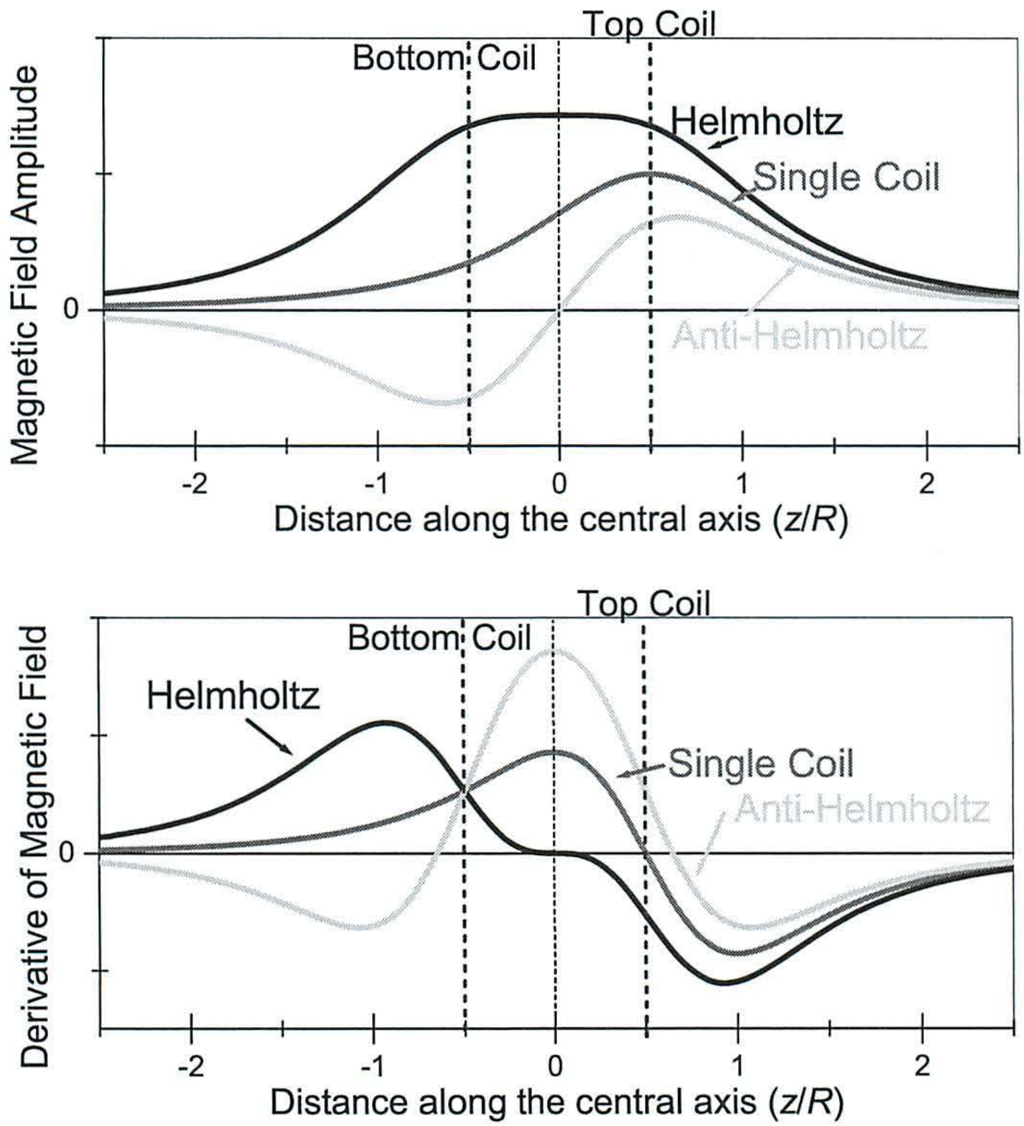


Figure 2: The z-component of the magnetic field and its derivative for the three modes of operation of the Helmholtz coil. See page the last page of this write-up for an "iron-filings" representation of these three field configurations.

2. Power Supply

We will also use the same power supply as in experiment 4 in order to create large enough fields in the Helmholtz apparatus to exert a measurable force on the magnet.

GENERALIZED PROCEDURE

This lab consists of five main parts. In each you will observe the effects (torque & force) of different magnetic field configurations on the disk magnet (a dipole).

Part 1: Dipole at center of Helmholtz Coil
You will move the disk magnet to the center of the Helmholtz apparatus and randomly align it and then see what happens when the coil is energized.

Part 2: Reversing the field
You will reverse the direction of the field and see what happens.

Part 3: Moving Through the Helmholtz Apparatus
Here you slowly pull the disk magnet up from the bottom of the Helmholtz apparatus (in Helmholtz mode) and out through the top, observing any torques or forces on the magnet.

Part 4: Dipole at center of Anti-Helmholtz Coil
Here you repeat part 1 in anti-Helmholtz configuration

Part 5: Moving Through the Anti-Helmholtz Apparatus
Here you slowly pull the disk magnet up from the bottom of the Helmholtz apparatus (in anti-Helmholtz mode) and out through the top, observing any torques or forces on the magnet.

END OF PRE-LAB READING

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

1. Download the LabView file and start up the program.
2. Without leads connected to the power supply, turn it on and set the voltage output to 12 V. Turn the current knob fully counter-clockwise (off).
3. Connect the leads to the Helmholtz apparatus, in Helmholtz mode.
4. Increase the current to approximately 1 A, then turn off the power supply (with the push button – do not change the voltage or current settings).

MEASUREMENTS

Part 1: Dipole in Helmholtz Mode

1. Slide the disk magnet to the center of the Helmholtz apparatus (0 on scale)
2. Randomly align the disk magnet using a bar magnet (try to make off axis)
3. Turn on the power supply, carefully watching the disk magnet

Question 1:

Did the disk magnet rotate? (Was there a torque on the magnet?)

Question 2:

Did the spring stretch or compress? (Was there a force on the magnet?)

Part 2: Reversing the Leads

1. Without touching the apparatus (or even bumping the table – be VERY careful) disconnect the leads from the power supply and insert them in the opposite direction (flip the current direction).
2. Carefully watch the dipole as you do this. Repeat the experiment several times.

Question 3:

What happened to the orientation of the disk magnet when you change the current direction in the coils in the Helmholtz configuration? Is this what you expect? Why?

Part 3: Moving a Dipole Along the Axis of the Helmholtz Apparatus

1. Now lower the disk magnet to bottom of the tube
2. Slowly pull the disk magnet up through the apparatus, until it is out the top. While pulling watch both the orientation of the magnet and the stretch or compression of the spring.

Question 4:

Starting from the bottom, describe the direction of the force (up or down) and the orientation of the disk magnet, paying careful attention to locations where they change.

Question 5:

Where does the force appear to be the largest? The smallest? How should you know this?

OPTIONAL

Part 4: Dipole in Anti-Helmholtz

1. Switch the apparatus to Anti-Helmholtz mode and increase the current to 2 A. Then turn off the power supply.
2. Move the disk magnet to the center (0 on scale) and randomly align it (off axis)
3. Turn on the power supply, carefully watching the disk magnet

Question 6:

Did the disk magnet rotate? (Was there a torque on the magnet?)

Question 7:

Did the spring stretch or compress? (Was there a force on the magnet?)

Part 5: Moving a Dipole Along the Axis of an Anti-Helmholtz Coil

1. Now lower the disk magnet to bottom of the tube
2. Slowly pull the disk magnet up through the apparatus, until it is out the top. While pulling watch both the orientation of the magnet and the stretch or compression of the spring.

Question 8:

Starting from the bottom, describe the direction of the force (up or down) and the orientation of the disk magnet, paying careful attention to locations where they change.

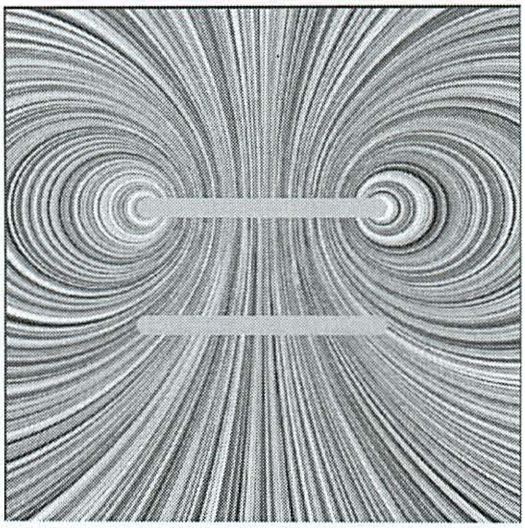
Question 9:

Where does the force appear to be the largest? The smallest? How should you know this?

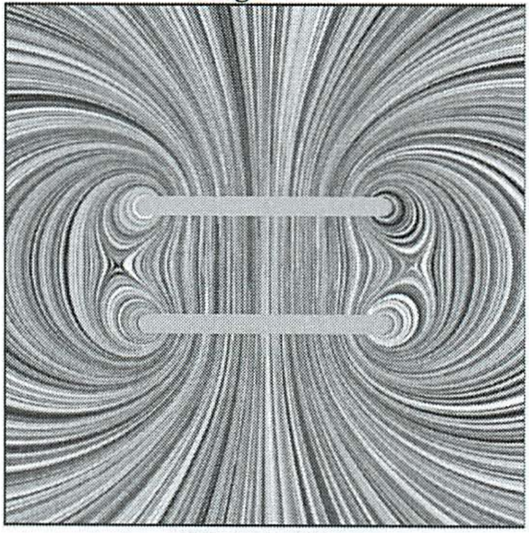
Further Questions (for experiment, thought, future exam questions…)

- What happens as we move through with just a single coil energized? Is it similar to the Helmholtz or anti-Helmholtz? How is it different?
- Are there places where we can put the disk magnet and then randomly orient it without either changing the force on it or having a torque rotate it back to alignment (in any of the 3 field configurations)?
- If you were to align the disk magnet with the x-axis (perpendicular the coil axis) and then center it in anti-Helmholtz mode, would there be a torque or force on it?

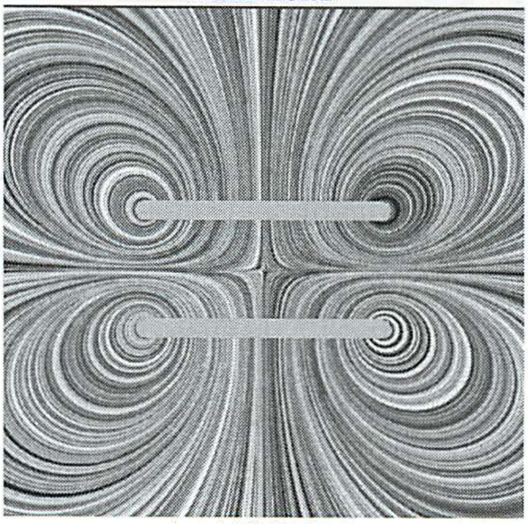
Iron Filings Patterns for Fields in the Helmholtz Apparatus



Single Coil



Helmholtz



Anti-Helmholtz

Week 4
Various Topics

Math Review for Physics

1. Cross Products
2. Line Integrals
3. Differential equations

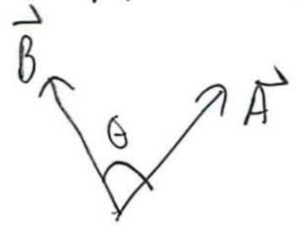
Cross Product

- scalars (#)
- vector (3 #)
- tensor (3^n #) $n \geq 1$ } not in this class

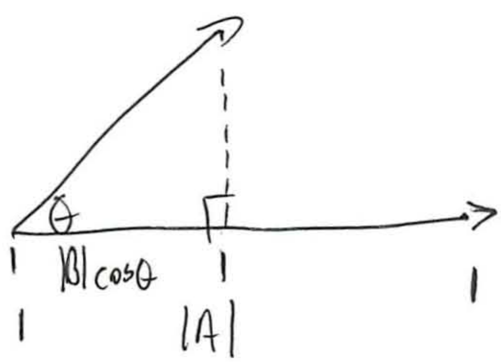
Scalar • ~~vector~~ sclar = scalar
" vector = vector } commute
Vector • scalar = same
Vector • vector = ~~a~~ scalar [dot product]
Vector x vector = ~~an~~ vector [cross product]

~~Scalar~~ Scalar product

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$



Measures how much the stuff lines up



$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$$

"measures projection of B onto A"

"how lined up they are"

$\vec{A}\cdot\vec{B} > 0$ more lined up

$= 0$ orthonal / perpendicular

any #d "perpendicular"

< 0 anti lined up

← same axis apart from each other
- will be θ

Vector / Cross product

$$\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{x} + (A_zB_x - A_xB_z)\hat{y} + (A_xB_y - A_yB_x)\hat{z}$$

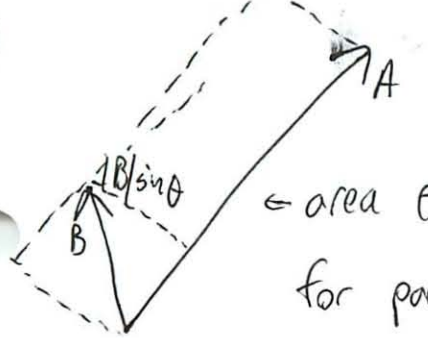
$$= -\vec{B}\times\vec{A}$$

does <u>not</u> commute (anticommuting)

$$= \det\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= |\vec{A}||\vec{B}|\sin\theta$$

(3)



← area enclosed by rectangle for part orthangonal to A of B

- gives you the line perpendicular to plane of A and B (except for sign)

$\vec{A} \cdot (\vec{A} \times \vec{B}) = 0$

$\vec{B} \cdot (\vec{A} \times \vec{B}) = 0$

measures how perpendicular they are

line up fingers w/ 1st vector
curl towards 2nd vector
thumb points to where

$\hat{x} \times \hat{y} = \hat{z}$ $\quad$ $\hat{r} \times \hat{\theta} = \hat{z}$

$\hat{y} \times \hat{z} = \hat{x}$ $\quad$ $\hat{\theta} \times \hat{z} = \hat{r}$

$\hat{z} \times \hat{x} = \hat{y}$ $\quad$ $\hat{z} \times \hat{r} = \hat{\theta}$

(4)

Line Integrals Biot-Savart Law

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \vec{r}}{|\vec{r}|^3}$$

- looks similar to Guass' Law $\frac{1}{4\pi\epsilon_0} \frac{dq\,\vec{r}}{|\vec{r}|^3}$

$\frac{\mu_0}{4\pi} \Leftrightarrow \frac{1}{4\pi\epsilon_0}$ constants to move units around

$\boxed{\text{not} =}$

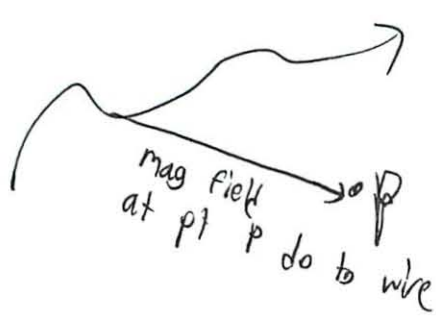
I some steady current

$(I_x)\, dl \Leftrightarrow dq$ stuff generating the field

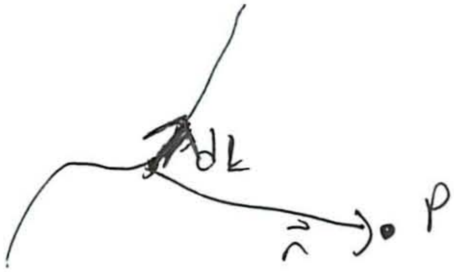
$E \sim Q,\ B \sim I \cdot L$

$\frac{\vec{r}}{|\vec{r}|^3} \Leftrightarrow \frac{\vec{r}}{|r|^3}$ → both go $\frac{1}{r^2}$ as you get away from it

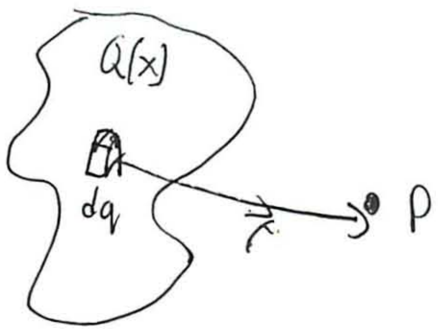
but magnetic is cross product
- direction matters



(5)



Magnetic



Electric

Must integrate along wire to find total magnetic field

the integral

$\cdot \leftarrow P$ want the B field there

along axis

circle in plane

- cylindrical symmetry

So use cylindrical coordinates

$$d\vec{B} = \underbrace{\frac{\mu_0 I}{4\pi}}_{\text{the constants}} \cdot \frac{d\vec{l} \times \vec{r}}{|\vec{r}|^3}$$

$\leftarrow$ I constant in most problems

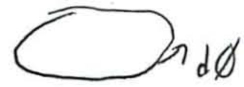
$d\vec{l} = R\, d\phi$

$\uparrow$ differential lenght arond circle

(6)

$d\vec{l} = \underbrace{R\, d\phi}_{\text{differential lenght along circle}} (+\hat{\phi})$

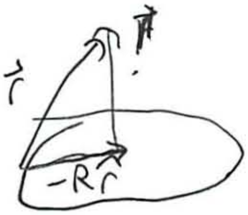
↑ direction of current is easiest to do



(fixed for a circle)

$\vec{r}$ = decompose into cylindrical vector components

$\vec{r}$ = diagonal



$\vec{r} = -R\hat{r}$ walk inside circle

$\vec{r} = -R\hat{r} + z\hat{z}$

$|\vec{r}|^3 = (R^2 + z^2)^{3/2}$

Now $d\vec{l} \times d\vec{r}$

$= R\, d\phi(\hat{\phi} \times -R\hat{r}) \xrightarrow{\hat{r} \times \hat{\phi} = \hat{z}} + R\, d\phi(\hat{\phi} \times z\hat{z})$

$= R\, d\theta\, (-R) - (\hat{z})$

Lenght of circle $= \int dl = \int R\, d\phi = 2\pi R$

(7)

$d\hat{l} \times d\hat{r} =$

$= (R\, d\phi\, \hat{\phi}) \times (-R\hat{r})$

$= (R\, d\phi)(-R)(\hat{\phi} \times \hat{r})$ the cross product section

$= -R^2\, d\phi\, (-\hat{z})$

$= R^2\, d\phi\, \hat{z} + Rz\, d\phi\, \hat{r}$

Now put it together

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{(R^2\, d\phi\, \hat{z} + Rz\, d\theta\, \hat{r})}{(R^2+z^2)^{3/2}}$$

by cylindrical symmatry

$\hat{r} = 0$

- that vid Dormashkin kept showing

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{R^2\, d\phi\, \hat{z} + Rz\, d\phi\, \hat{r}}{(R^2+z^2)^{3/2}}$$ ← that $\hat{r}$ depends on ϕ

Some where different w/ every ϕ

$\hat{r}$ term matters

$$\int_0^{2\pi} d\phi \cos\phi = \int_0^{2\pi} d\phi \sin\phi = 0$$

(8)

$$\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \frac{R z}{(R^2+z^2)^{3/2}} \cdot \overset{\text{doing integration}}{2\pi} \hat{z}$$

$$= \frac{\mu_0 I}{2} \cdot \frac{R^2 \hat{z}}{(R^2+z^2)^{3/2}}$$

$$= \underbrace{\frac{\mu_0}{4\pi}}_{\text{constant}} \underbrace{\frac{R}{(R^2+z^2)^{3/2}}}_{\text{Bio-Savart law}} \underbrace{\left(2\pi R I\right)}_{\text{total current}} \hat{z}$$

rewrite to collect terms

$$\vec{F} = \vec{I}\left(\vec{L} \times \vec{B}\right)$$ is like Q

~~mag~~ lorentz force on magnetic field

Is current a vector

- could write $\underset{\text{wire}}{L}(\underset{\text{dir current flowing in wire}}{\vec{I}} \times \vec{B})$

- or $(\vec{I \cdot L}) \times \vec{B}$

9 Exponential Differentials

$$\frac{dQ}{dt} = -\frac{1}{\tau}Q + \gamma$$

separable differential equation

$$\frac{dQ}{-\frac{1}{\tau}Q + \gamma} = dt$$

$$\downarrow \qquad \downarrow$$

$$\int \qquad \int$$

$$-\tau \int \frac{-\frac{1}{\tau}\,dQ}{-\frac{1}{\tau}Q + \gamma}$$

$$-\tau \ln\left(-\frac{1}{\tau}Q + \gamma\right) = t + C_0$$

$$-\frac{1}{\tau}Q + \gamma = \exp\left(-\frac{t}{\tau} - \frac{C_0}{\tau}\right) = e^{-t/\tau} \cdot e^{-C_0/\tau}$$

exponential

$$Q(t) = \underbrace{-\tau e^{-C_0/\tau}}_{Q_0}\, e^{-t/\tau} + \gamma\tau$$

$$\boxed{Q(t) = Q_0\, e^{-t/\tau} + \gamma\tau}$$

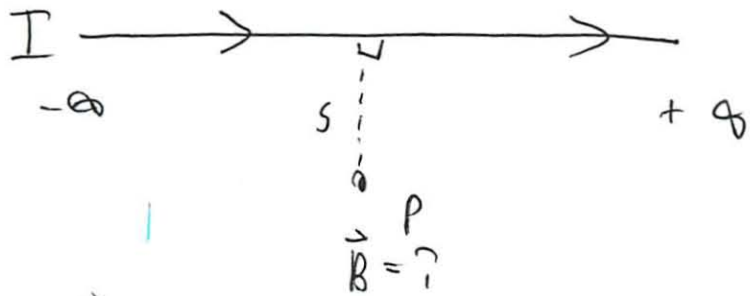
Solution to differential equation

"exponential differential equations"

- charge onto or off a capicator

(10)

One more line integral example

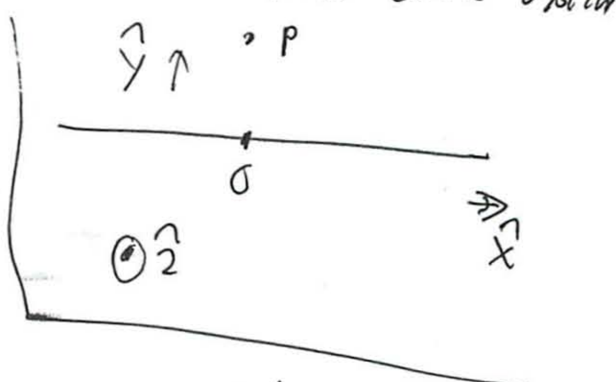


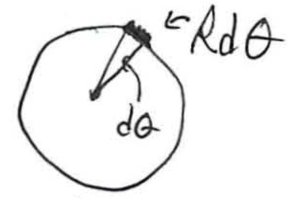
$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{r}}{|\vec{r}|^3}$$

$d\vec{l}$ = some little section of wire has some current

$= dx\,(+\hat{x})$

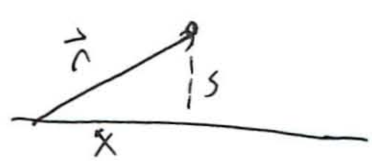
need to pick coord system





if more than curve add dr

$\vec{r}$ =



lenght $= \sqrt{x^2 + s^2} = |\vec{r}|$

$\vec{r} = -x'\hat{x} + s\,\hat{y}$

don't forget $\ominus$

(11)

$$|r|^3 = (x^2 + s^2)^{3/2}$$

$\otimes$ = cross product
since too many $\times$

$$d\vec{l} \otimes \vec{r} = \left(dx' \hat{x} \otimes -x'\hat{x} \right) + \left(dx' \hat{x} \otimes s\hat{y} \right)$$

$$= -x' dx' (\hat{x} \otimes \hat{x}) + s\, dx' (\hat{x} \otimes \hat{y})$$

$\hat{x} \otimes \hat{x} = 0$

remember its how perpendicular it is

$\hat{x} \otimes \hat{y} = \hat{z}$

$$= 0 + s\, dx' \hat{z}$$

(12)

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \frac{s\,dx'\,\hat{z}}{(s^2 + x'^2)^{3/2}}$$

now integrate

$$\vec{B} = \int_{-\infty}^{\infty}$$

whole wire

$$\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \int_{-\infty}^{\infty} \frac{s\,dx'}{(s^2 + x'^2)^{3/2}}\,\hat{z}$$

$$= \frac{\mu_0 I}{4\pi}\, s\,\hat{z} \int_{-\infty}^{\infty} \frac{dx'}{(s^2 + x'^2)^{3/2}}$$

take s^2 out b/c constant

$$\frac{1}{(s^2 + x'^2)^{3/2}} = \frac{1}{s^3} \cdot \frac{1}{\left(1 + \frac{x'^2}{s^2}\right)^{3/2}}$$

$$= \frac{\mu_0 I}{4\pi}\, \cancel{s}\,\hat{z}\, \frac{1}{s^{\cancel{2}}} \int_{-\infty}^{\infty} \frac{dx'/s}{\left(1 + \frac{x'^2}{s^2}\right)^{3/2}}$$

cancle

split to make eaiser

$$u = \frac{x'}{s} \quad du = \frac{dx'}{s}$$

$$= \frac{\mu_0 I}{4\pi}\, \frac{\hat{z}}{s} \underbrace{\int_{-\infty}^{\infty} \frac{du}{(1 + u^2)^{3/2}}}_{= \text{some \# that } = 2}$$

do trig sub where $u = \tan\theta$

$1 + u^2 = 1 + \tan^2\theta = \sec^2\theta$

(13)

$$du = \sec^2\theta \, d\theta \qquad \text{for} \quad \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sec^2\theta \, d\theta}{(\sec^2\theta)^{3/2}}$$

* limits of integration hath changed

$$\int_{-\frac{\pi}{2}}^{\pi/2} \frac{\sec^2\theta \, d\theta}{\sec^3\theta}$$

$$\int_{-\pi/2}^{\pi/2} \frac{1}{\sec\theta} \, d\theta$$

$$\int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta$$

$$\cos\left(\frac{\pi}{2}\right) - \cos\left(-\frac{\pi}{2}\right)$$

$$1 - -1$$

$$2$$

$$\vec{B} = \frac{\mu_0 I}{2\pi} \cdot \frac{1}{s} \hat{z}$$ straight line of charge

- ampere's law is short cut to find this

Topics: Magnetic Fields: Force and Torque on a Current Loop
Related Reading: Reading: Course Notes: Sections 8.8-8.9, 9.10.1, 9.11.1-9.11.4

Topic Introduction

In today's class we calculate the force on a charged particles moving in a magnetic field and the torque on a rectangular loop of wire.

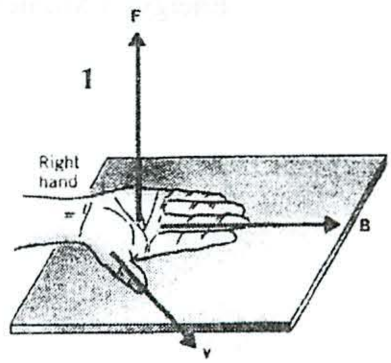
Lorenz Force on Moving Charges Currents

A charged particle moving in a magnetic field feels a force $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$. Similarly, a piece of current carrying wire placed in a magnetic field will feel a force: $d\vec{\mathbf{F}} = Id\vec{\mathbf{s}} \times \vec{\mathbf{B}}$ (where **ds** is a small segment of wire carrying a current I). We can integrate this force along the length of any wire to determine the total force on that wire.

Right Hand Rules

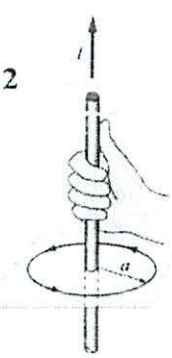
Because of the cross product in the Biot-Savart Law, the direction of the resulting magnetic field is not as simple as when we were working with electric fields. In order to quickly see what direction the field will be in, or what direction the force on a moving particle will be in, we can use a "Right Hand Rule." At times it seems that everyone has their own, unique, right hand rule. Certainly there are a number of them out there, and you should feel free to use whichever allow you to get the correct answer. Here I describe the four that I use.

1



The important thing to remember is that cross-products yield a result which is perpendicular to both of the input vectors. The only open question is in which of the two perpendicular directions will the result point (e.g. if the vectors are in the floor does their cross product point up or down?). Using your RIGHT hand:

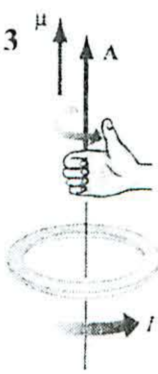
2



1) For determining the direction of the force of a field on a moving charge: open your hand perfectly flat. Put your thumb along v and your fingers along B. Your palm points along the direction of the force.

2) For determining the direction of the magnetic field generated by a current: fields wrap around currents the same direction that your fingers wrap around your thumb. At any point the field points tangent to the circle your fingers will make as you twist your hand keeping your thumb along the current.

3



3) For determining the direction of the dipole moment of a coil of wire: wrap your fingers in the direction of current. Your thumb points in the direction of the North pole of the dipole (in the direction of the dipole moment μ of the coil).

Torque Vector

I'll tack on one more right hand rule for those of you who don't remember what the direction of a torque τ means. If you put your thumb in the direction of the torque vector, the object being torque will want to rotate the direction your fingers wrap around your thumb (very similar to RHR #2 above).

Important Equations

Force on Moving Charges in Magnetic Field: $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$

Force on Current-Carrying Wire Segment: $d\vec{\mathbf{F}} = Id\vec{\mathbf{s}} \times \vec{\mathbf{B}}$

Magnetic Moment of Current Carrying Wire: $\vec{\boldsymbol{\mu}} = I\vec{\mathbf{A}}$ (direction for RHR #3 above)

Torque on Magnetic Moment: $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$

Force on Magnetic Moment: $\vec{\mathbf{F}}_{Dipole} = \left(\vec{\boldsymbol{\mu}} \cdot \vec{\nabla}\right)\vec{\mathbf{B}}$

Energy of Moment in External Field: $U = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}$

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Department of Physics

Problem Solving 6: Magnetic Force & Torque

OBJECTIVES

1. To look at the behavior of a charged particle in a uniform magnetic field by studying the operation of a mass spectrometer

2. To calculate the torque on a rectangular loop of current-carrying wire sitting in an external magnetic field.

3. To define the magnetic dipole moment of a loop of current-carrying wire and write the torque on the loop in terms of that vector and the external magnetic field.

REFERENCE: Sections 8.3 – 8.4, 8.02 Course Notes.

Mass Spectrometer

A mass spectrometer consists of an ionizer, which strips (ideally) a single electron from an atom whose mass you want to measure, an acceleration region, and a deflection region, as pictured in Fig. 1.

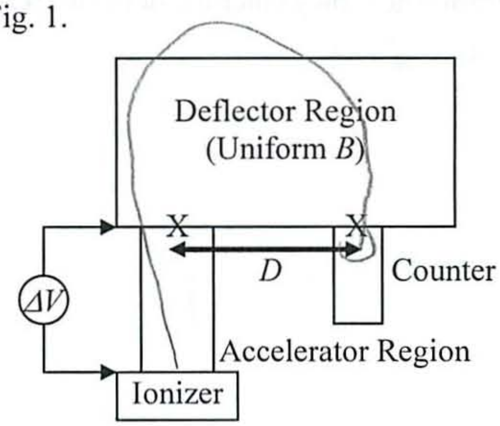


Figure 1: A mass spectrometer.

In Figure 1, the ions exit the ionizer at essentially zero velocity. They are accelerated by a potential difference through the accelerator region, where they enter the deflector region through a small orifice (X). The deflector region has a uniform B field which bends the ions around through another small orifice into the counter, where the ions are counted. By scanning the accelerating voltage ΔV a range of masses (or, more accurately, mass to charge ratios) can be measured, to determine the content of some unknown gas.

Question 1 (***Answer this and subsequent questions on the tear-off sheet at the end***): What should the polarity of the potential difference ΔV be (should potential be higher at the top near the deflector or at the bottom near the ionizer)?

+ ion (electrons stripped) so higher V at top of page

Question 2: In what direction should the B field point to guide the ions into the counter?

out of the page $\odot$

Question 3: Find an expression for the kinetic energy of the ions when they enter the deflector region (HINT: Why do they have kinetic energy? Where did it come from?)

$$KE = \frac{1}{2}mv^2 = \text{from the B field} \quad F = \frac{mv^2}{R} = I\, 2\pi R \times B$$

$$-2\pi I R^2 B\hat{r} + \frac{1}{2}I w^2 \quad \frac{mv^2}{R} = I\, 2\pi R \times B\hat{r}$$

$$= -e\Delta V \quad = \sqrt{\frac{2q\Delta V}{m}} \quad KE = \frac{1}{2}mv^2 = I 2\pi R^2 B\hat{r}$$

$$-2\pi I r^2 B\hat{r}$$

radial inward

Question 4: What path do the ions follow in the deflector region? Derive an expression for the magnetic field that is needed to make sure that ions of mass m end up in the counter (a distance D away from where they enter the deflector region).

a curved path

$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{s} \times \vec{r}}{r^2}$$

$$\vec{B} = \int_0^{D/2} \frac{\mu_0}{4\pi} \frac{I\, ds(-\hat{r})}{r^2} \rightarrow \vec{B} = \frac{-\mu_0 I D}{8\pi r^2}\hat{r}$$

Question 5: About what potential ΔV is needed to get singly ionized carbon-12 ions into the counter if $B = 1$ T and $D = 20$ cm? You can assume that protons and neutrons have about the same mass given by $mc^2 \sim 1$ GeV. (Do this as a back of the envelope calculation – NO CALCULATOR!)

Mass = 24 GeV

D = .2 m

B = 1 Telsa

$$F = \frac{mv^2}{R} = q\Delta V \times \vec{B}$$

$$\frac{mv^2}{R} = -qvB\hat{r}$$

$$v = \frac{-qRB\hat{r}}{m}$$

$$\rightarrow \frac{1}{2}mv^2 = -e\Delta V$$

$$\frac{\frac{1}{2}m\left(\frac{-qRB}{m}\right)^2}{-e} = \Delta V$$

$$=$$

Magnetic Dipole Moment

In class we determined that a planar loop of area A (with unit vector $\hat{\mathbf{n}}$ normal to the loop) and carrying current I, has a magnetic dipole moment $\vec{\boldsymbol{\mu}}$ given by:

$$\vec{\boldsymbol{\mu}} \equiv I\vec{\mathbf{A}} = IA\hat{\mathbf{n}}$$

The normal $\hat{\mathbf{n}}$ points in a direction defined by your thumb when you curl the fingers of your right hand in the direction of the current in the loop.

We calculated the torque on such a loop in a uniform magnetic field $\vec{\mathbf{B}}_{\text{ext}}$ to be

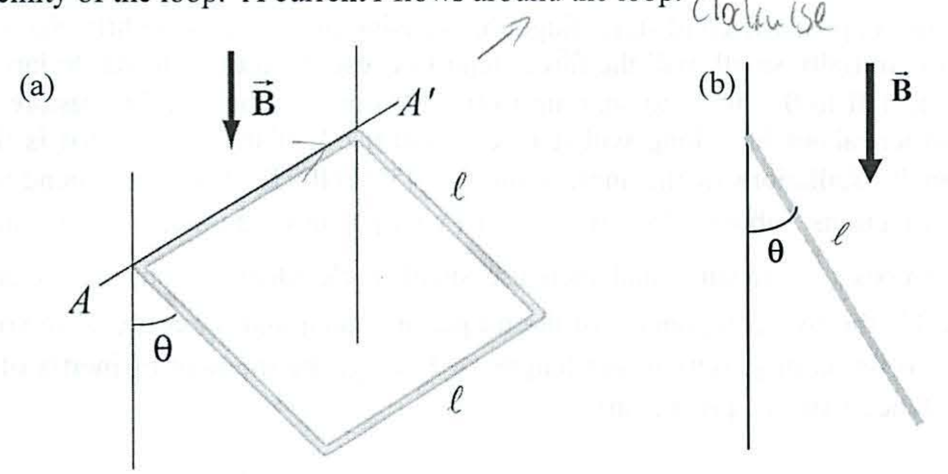
$$\vec{\boldsymbol{\tau}}_{\text{magnetic}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}_{\text{ext}}\,.$$

and the force on such a loop in a non-uniform magnetic field $\vec{\mathbf{B}}_{\text{ext}}$ to be:

$$\vec{\mathbf{F}}_{magnetic} = \left(\vec{\boldsymbol{\mu}} \cdot \vec{\nabla}\right)\vec{\mathbf{B}}_{\text{ext}}$$

(note that this evaluates to 0 if $\vec{\mathbf{B}}_{\text{ext}}$ is uniform – dipoles do not feel forces in uniform external fields).

Problem: A square loop of wire, of length ℓ on each side, pivots about an axis AA' that corresponds to a horizontal side of the square, as shown in Figure 4. A magnetic field of magnitude B is directed vertically downward, and uniformly fills the region in the vicinity of the loop. A current I flows around the loop. Clockwise



Question 6: Calculate the magnitude of the torque on this loop of wire in terms of the quantities given, using our expressions above.

$\tau = \mu \times B$ $\mu = I A \hat{n}$

$B = qvl\sin\theta$ ⊗

Question 7: In what direction does the current need to flow in order "levitate" the coil again the force of gravity (clockwise or counterclockwise viewed from above)?

want B field out of page ⊙

So counterclockwise

Question 8: Suppose that the loop (ℓ= 1 m) is essentially massless, but that a small child (m = 20 kg) wants to hang from the bottom rung of the loop and be supported at $\theta = 45°$. If we can push I = 100 A through the loop, about how large a B field will we need to support the child? child torquifying

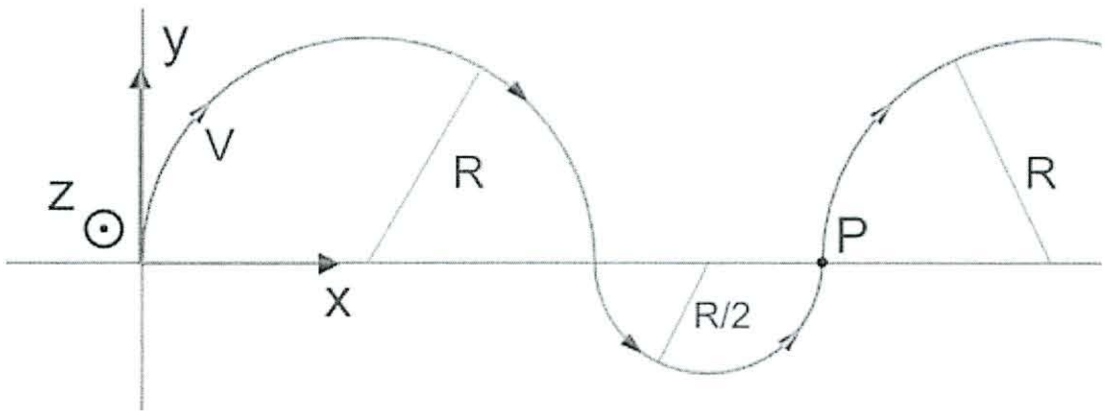
it will restore

Question 9: Now suppose the child starts fidgeting, causing the angle to slightly change. If the deviation is initially small, will the forces tend to cause the motion to run to larger excursions (i.e. to fall to $\theta = 0°$ or to snap up to $\theta = 90°$) or will they tend to restore to 45°? If the former, about how long will it take to fall/rise? If the latter, what is the frequency of small oscillations of the angle about $\theta = 45°$? HINT: You can expand the trigonometric functions about 45° using $\sin(x+y) = \sin x \cos y + \sin y \cos x$ and $\cos(x+y) = \cos x \cos y - \sin x \sin y$ and then use small angle identities sin(x) ~ x and cos(x) ~ 1. At 45° the two components of the torque are equal and opposite, so rewrite the total torque in terms of gravity g and length ℓ. Finally, the moment of inertia of a point mass a distance ℓ from a pivot is $m\ell^2$.

Question 10: If instead of balancing the child at 45° they wanted to ride at 60°, is it better to keep the field vertical or to switch to horizontal (i.e. which requires smaller B)?

Sample Exam Question (If time, try to do this by yourself, closed notes)

A charge of mass m and charge $q > 0$ is at the origin at $t = 0$ and moving upward with velocity $\vec{\mathbf{V}} = V\hat{\mathbf{j}}$. Its subsequent trajectory is shown in the sketch. The magnitude of the velocity $V = |\vec{\mathbf{V}}|$ is always the same, although the direction of $\vec{\mathbf{V}}$ changes in time.



(a) For $y > 0$, this positive charge is moving in a constant magnetic field which is either into the page or out of the page. Is that magnetic field for $y > 0$ into or out of the page?

(b) Derive an expression for the magnitude of the magnetic field for $y > 0$ in terms of the given quantities, that is in term of q, m, R, and V.

(c) For $y < 0$, the charge is moving in a *different* constant magnetic field. Is that field for $y < 0$ into or out of the page? What is the magnitude of that magnetic field in terms of in term of q, m, R, and V?

(d) How long does it take the charge to move from the origin to point P (see sketch) along the x-axis? Give your answer in terms of the given quantities.

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Tear off this page and turn it in at the end of class !!!!

Note: Writing in the name of a student who is not present is a COD offense.

Problem Solving 6: Magnetic Force & Torque

Group 11 | Everyone who botered to show up **(e.g. L02 6A Please Fill Out)**

Names Jennifer Quintana   Mary Zhuang

Michael Plasmeier

Melanie Alba

Kayla Meduna (A)

Mary Zhuang

3
5

Question 1: What should the polarity of the potential difference ΔV be?

Higher at the ionizer ✓

Question 2: In what direction should the magnetic field be pointed?

Out of the page ⊙ ✓

Question 3: Find the kinetic energy of the ions when they enter the deflector region.

$F = \frac{mv^2}{R} = I\,2\pi R \times B$

$KE = \frac{1}{2}mv^2 = -\pi I R^2 B\hat{r} = -e\Delta V$

not quite. These are ions, not electrons.

Question 4: What path do the ions follow in the deflector region? Derive an expression for the magnetic field that is needed to make sure that ions of mass m end up in the counter (a distance D away from where they enter the deflector region).

$\frac{mv^2}{R} = I\,2\pi R B\hat{r}$

$KE = \frac{1}{2}mv^2 = I\pi R^2 B\hat{r}$

$= 2\pi I r^2 B\hat{r}$

radially inward

A semicircular path

$$\vec{B} = \int_0^{D/2} \frac{\mu_0}{4\pi}\frac{I\,ds(-\hat{r})}{r^2} \Rightarrow \vec{B} = \frac{-\mu_0 I D}{8\pi r^2}\hat{r}$$

✗

Question 5: About what potential ΔV is needed to get singly ionized carbon-12 ions into the counter if $B = 1$ T and $D = 20$ cm? You can assume that protons and neutrons have about the same mass given by $mc^2 \sim 1$ GeV.

$\Delta V = 1.3 \times 10^{-27}$

$m = 24\ \text{GeV}$

$D = .2\text{m}$

$B = 1\text{T}$

$\frac{1}{2}mv^2 = -e\Delta V$

$$\frac{\frac{1}{2}m\left(\frac{-qRB}{m}\right)^2}{-q} = \Delta V$$

$\Delta V = \frac{1}{2}\frac{q}{m}R^2B^2$

✗

$F = qv \times B = \frac{mv^2}{R}$

$-qvB\hat{r} = \frac{mv^2}{R}$

$\frac{-qRB\hat{r}}{m} = v$

Question 6: Calculate the magnitude of the torque on this loop of wire in terms of the quantities given, using our expressions above.

$\mu = IA\hat{n}$ $\quad B = qvl\sin\theta \otimes$

$\cos\theta$

$\tau = \mu \times B_{ext}$

$\tau = IAqvl\sin\theta\,\hat{y}$

Question 7: In what direction does the current need to flow in order "levitate" the coil again the force of gravity (clockwise or counterclockwise viewed from above)?

Counterclockwise (opposites repel)

Question 8: Suppose that the loop ($\ell = 1$ m) is essentially massless, but that a small child ($m = 20$ kg) wants to hang from the bottom rung of the loop and be supported at $\theta = 45°$. If we can push $I = 100$ A through the loop, about how large a B field will we need to support the child?

Whose parents let their kids hang on wires anyway?

Good question.

Question 9: Now suppose the child starts fidgeting, causing the angle to slightly change. If the deviation is initially small, will the forces tend to cause the motion to run to larger excursions (i.e. to fall to $\theta = 0°$ or to snap up to $\theta = 90°$) or will they tend to restore to 45°? If the former, about how long will it take to fall/rise? If the latter, what is the frequency of small oscillations of the angle about $\theta = 45°$?

Restore

Question 10: If instead of balancing the child at 45° they wanted to ride at 60°, is it better to keep the field vertical or to switch it to horizontal (i.e. which requires smaller B)?

Switch to 60°

That wasn't an option.

8.02 Test 2

3/22/10

What concepts

Conductors + Capacitance 2/22

$$C = \frac{Q}{|\Delta V|} = \frac{\epsilon_0 A}{d}$$

Energy density

$$U_E = \frac{1}{2}\epsilon_0 E^2$$

for all shapes integrate E (gauss's law)

Work to charge capicator

$$W = \int dW = \frac{1}{C}\frac{Q^2}{2}$$

Should also review Test 1 material

Ice Pail + Sheilding + Conductors

- where the electrons go
- What $E = \Sigma_{sum}$ 0

$$E = \frac{kq_0 Q}{r^2} = \frac{F}{q} = \frac{kQ}{r^2}$$ superposition

(Think I should do pratice tests)

Dielectrics

- did not really do
- reduces potential difference in a capicator
- So $C \uparrow$

②

Circuit + Kirchoff's loop rules

- think I feel good about these
- Series + parallel
  - remember which is which!

Resistors series $R_{eq} = R_1 + R_2$ just add

parallel $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$ more doors in theater

When conductors are connected the potential =

Current Density

need to study this too!

$$J = \frac{I}{A} \hat{J}$$

$$I = \int_S \vec{J} \cdot \hat{n} \, dA$$

local direction charge is flowing

Ohm's Law

$E = \rho \vec{J}$ $\rho = \frac{1}{\sigma}$ no questions on this but important microscopic

Copper wire
- as more current flows through resistance drops

Current

$I = \frac{\Delta Q}{\Delta t}$ flow of charges

$$I = \frac{dQ}{dt}$$

(3)

Battery = skilift

Resistor = ski slope

~~Power = $I \Delta V$~~

Ohm's Law $V = IR = dQ\,R$

Work = U = Power = $Pt = I \Delta V = IVt = qV$

change in energy per unit time

$I = \frac{q}{t}$

Batt internal resistance

$\mathcal{E} - IR$

(yeah I think this is the stuff that was the hardest so I really learned it)

Capicators

series $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

parallel $C_{eq} = C_1 + C_2$ should make sense

Class 15

Circuits now time varrying

* know how to solve a differential equation
* know what everything means
  - still a bit foggy on

See esp 3/10 office hrs where it all became clear

(4)

Magnetic Field

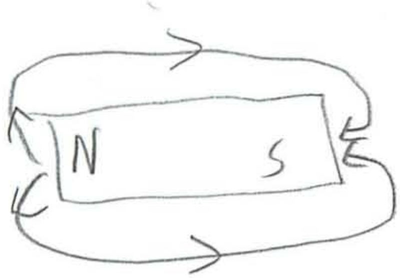
- did not really get

~~do P-set to again!~~

- well have not done P-Set for since so new
- hopefully doing That will help
- will make sheet on seperate sheet of paper

Magnetic Review

3/27



cross product $\vec{C} = \vec{A} \times \vec{B}$ $\quad \vec{C} = |\vec{A}||\vec{B}| \sin\theta$

$\vec{F}_B = q\vec{v} \times \vec{B}$

- tesla or Guass

like electrical fields

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{V} \times \vec{r}}{r^2}$$

Thumb towards v
fingers towards current
thumb points to B

- i don't really get - I have bad notes from this day

Re-read summaries

fields generated by + exert forces on electrically charged particles

Only for particles that are moving

and are diapoles

②

Magnetic Force = force exerted on moving charged object

Called Lorenz Force

$\vec{F} = q\vec{v} \times \vec{B}$

direction of force from right hand rule

perpendicular to vel and field

So particles will move in curves

$F = q(\vec{E} + \vec{v} \times \vec{B})$ if both types of fields

Diapoles always in pair N S

Fields created by current

Creation of Electric Fields

- from a single moving electron

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{V} \times \hat{r}}{r^2} \qquad \mu_0 = 4\pi \cdot 10^{-7} \frac{Tm}{A}$$

Perpendicular to $\hat{r}$

- from a current (Biot-Savart Law)
  - lots of charges

$$dB = \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

Ok lets get started on MP 7 - should help

- review ring example

3/29

Topics: Magnetic Fields: Creating Magnetic Fields – Ampere's Law
Related Reading: Course Notes: 9.3-9.4, 9.10.2, 9.11.5-9.11.8
Experiments: (8) Magnetic Fields

Colombs → B-S
Gauss → Ampère

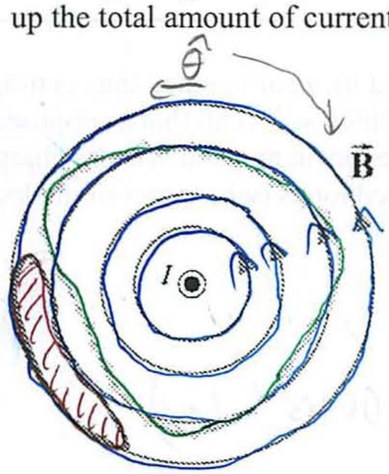
Topic Introduction

Today we cover two topics. At first, in experiment #8, we will measure the magnetic fields created by bar magnets. Then we will discuss Ampere's Law, the magnetic equivalent of Gauss's Law.

Ampere's Law

With electric fields we saw that rather than always using Coulomb's law, which gives a completely generic method of obtaining the electric field from charge distributions, when the distributions were highly symmetric it became more convenient to use Gauss's Law to calculate electric fields. The same is true of magnetic fields – Biot-Savart does not always provide the easiest method of calculating the field. In cases where the current source is very symmetric it turns out that Ampere's Law, another of Maxwell's four equations, can be used, greatly simplifying the task.

Ampere's law rests on the idea that if you have a curl in a magnetic field (that is, if it wraps around in a circle) the field must be generated by some current source inside that circle (at the center of the curl). So, if we walk around a loop and add up the magnetic field heading in our direction, then if, when we finish walking around, we have seen a net field wrapping in the direction we walked, there must be some current penetrating the loop we just walked around. Mathematically this idea is expressed as: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{\text{penetrate}}$, where on the left we are integrating the magnetic field as we walk around a closed loop, and on the right we add up the total amount of current penetrating the loop.



In the example pictured here, a single long wire carries current out of the page. As we discussed in class, this generates a magnetic field looping counter-clockwise around it (blue lines). On the figure we draw two "Amperian Loops." The first loop (~~yellow~~ green) has current I penetrating it. The second loop (red) has no current penetrating it. Note that as you walk around the yellow loop the magnetic field always points in roughly the same direction as the path: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} \neq 0$, whereas around the red loop sometimes the field points with you, sometimes against you: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = 0$.

so current there first out pg B Field

no current inclosd

then 2 test loops

We use Ampere's law in a very similar way to how we used Gauss's law. For highly symmetric current distributions, we know that the produced magnetic field is constant along certain paths. For example, in the picture above the magnetic field is constant around any blue circle. The integral then becomes simple multiplication along those paths $\left(\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \cdot \text{Path Length}\right)$, allowing you to solve for B.

Solving Problems using Ampere's law
Ampere's law provides a powerful tool for calculating the magnetic field of current distributions that have radial or rectangular symmetry. The following steps are useful when applying Ampere's law:

(1) Identify the symmetry associated with the current distribution, and the associated shape of "Amperian loops" to be used.
(2) Divide space into different regions associated with the current distribution, and determine the exact Amperian loop to be used for each region. The magnetic field must be constant, perpendicular to or known (e.g. zero) along each part of the loop.
(3) For each region, calculate $I_{\text{penetrate}}$, the current penetrating the Amperian loop.
(4) For each region, calculate the integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ around the Amperian loop.
(5) Equate $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ with $\mu_0 I_{\text{penetrate}}$, and solve for the magnetic field in each region.

Important Equations

Ampere's Law: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{\text{penetrate}}$

Experiment 8: Magnetic Fields of a Bar Magnet

Preparation: Read pre-lab

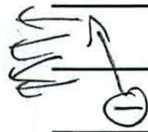
In this lab you will measure the magnetic field generated by a bar magnet, thus getting a feeling for magnetic field lines generated by magnetic dipoles. Recall that as opposed to electric fields generated by charges, where the field lines begin and end at those charges, fields generated by dipoles have field lines that are closed loops (where part of the loop must pass through the dipole).

Test 2 Grab bag (do exam 1, 2, and 3)
- capicators + conductors (inc Gauss' Law)
- DC/RL circuits
- feeling magnetic field
- Lorenz (moving charges)
- On diapoles (forces and Torque)

No B-S/Ampere

Class 20: Outline

Hour 1 & 2:
Ampere's Law



into pg
its ⊖

fingers to v
Curl towards B
thumb points

Last Time: Creating Magnetic Fields: Biot-Savart

rotate clockwise
not move

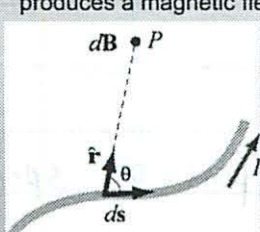
* think compass needle
N points right
Compass will rotate to point North

torque only

will feel force if B is
non uniform → diapoles want
to go to strong feel

The Biot-Savart Law

Current element of length *ds* carrying current *I* produces a magnetic field:



$$\vec{\mathbf{dB}} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

Moving charges are currents too…

$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{q\, \vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$$

like columb's law
except cross product

$U = -\vec{\mu} \cdot \vec{B}$ energy of diapole
wants to be reduced
- if aligned with B (lowest energy)
then go to highest field
you can find

Today:
3rd Maxwell Equation: Ampere's Law

Analog (in use) to Gauss's Law

P20- 4

Creating field

x

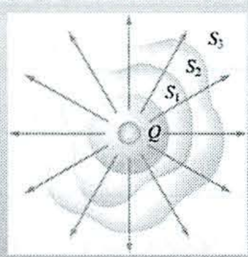
~~run current up~~

field ~~current~~ will point into pg

fingers up
curl towards X
thumb points into pg

or thumb towards current
fingers field

Gauss's Law – The Idea



The total "flux" of field lines penetrating any of these surfaces is the same and depends only on the amount of charge inside

P20- 5

electric field diverge from charges

Cycltron

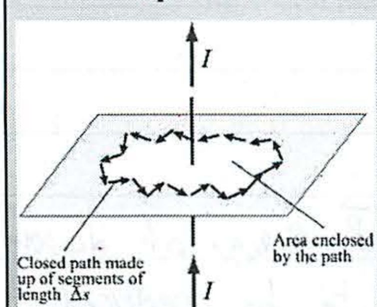
$qvB = \frac{mv^2}{r} \rightarrow \boxed{r = \frac{mv}{qB}}$

period

$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$

Does not depend on speed

Ampere's Law: The Idea



In order to have a B field around a loop, there must be current punching through the loop

P20- 6

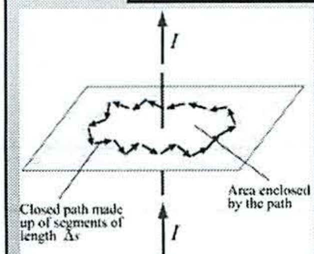
walk around a path
- add up magnetic field
- lets you know how much current you've walked around

*current makes magnetic fields that <u>curl</u>
how much follows you around as you go in a circle

Ampere's Law: The Equation

$$\boxed{\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}}$$

→ B times path lenght



The line integral is around any closed contour bounding an open surface S.

I_{enc} is current **through** S:

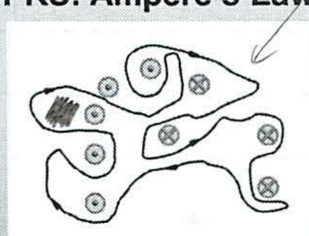
$$I_{enc} = \iint_S \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}$$

↑ sometimes integral → if current <u>not</u> uniform

PRS Questions: Ampere's Law

PRS: Ampere's Law

direction we are walking
how much charge are we enclosing?



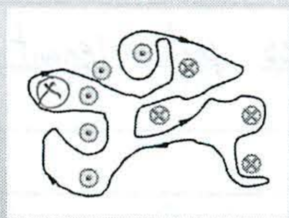
Integrating B around the loop shown gives us:

0% 1. a positive number
0% 2. a negative number
0% 3. zero

:15

More out of page
fingers up curl towards
where fingers go . points
out of page – we are
walking opposet so ⊖
(confused on this) *

PRS: Ampere's Law



Integrating B around the loop shown gives us:

0% 1. a positive number

0% 2. a negative number

0% 3. zero

15

balances at

Biot-Savart vs. Ampere

Biot-Savart Law	$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$	general current source ex: finite wire wire loop
Ampere's law	$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$	symmetric current source ex: infinite wire infinite current sheet

P20-11

Applying Ampere's Law

1. Identify regions in which to calculate B field
   Get B direction by right hand rule
2. Choose Amperian Loops S: Symmetry
   B is 0 or constant on the loop!
3. Calculate $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$
4. Calculate current enclosed by loop S
5. Apply Ampere's Law to solve for B

$$\boxed{\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}}$$

P20-12

Always True, Occasionally Useful

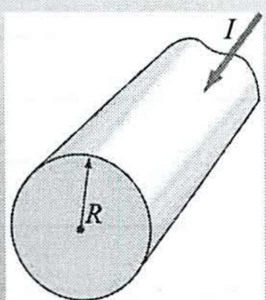
Like Gauss's Law,

Ampere's Law is **always** true

However, it is only useful for calculation in certain specific situations, involving highly symmetric currents.

Here are examples…

Example: Infinite Wire



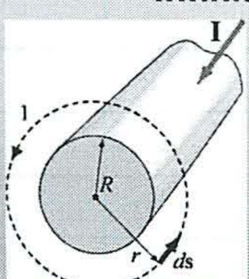
A cylindrical conductor has radius R and a uniform current density with total current I

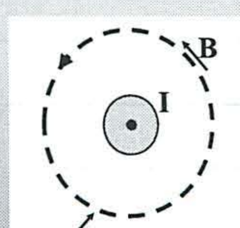
Find B everywhere

Two regions:
(1) outside wire ($r \geq R$)
(2) inside wire ($r < R$)

Symmetric
circle field lines
Field goes counter clockwise
↓
* thumb towards current
how do fingers curl
counter clockwisee

Ampere's Law Example: Infinite Wire



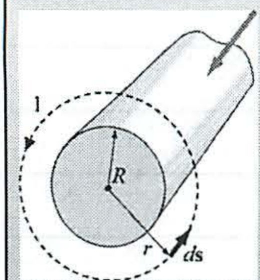


Amperian Loop:
B is Constant & Parallel
I Penetrates

Integrate $\oint B \cdot ds$
$= B\,2\pi r$

* Always redraw so current flows into or out of the page *
– so Amperian loop lies in plane of page

Example: Infinite Wire



Region 1: Outside wire (r ≥ R)

Cylindrical symmetry →

Amperian Circle

B-field counterclockwise

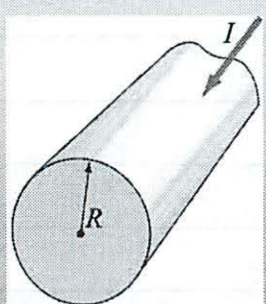
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \oint ds = B(2\pi r)$$
$$= \mu_0 I_{enc} = \mu_0 I$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r} \text{ counterclockwise}$$

P20-16

↑ since $\vec{B}$ is vector, needs direction

Group Problem: Inside

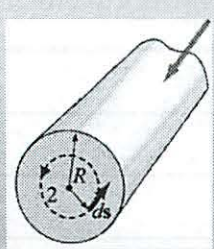


We just found B(r>R)

Now you find B(~~R~~ r<R)

P20-17

Example: Infinite Wire



Region 2: Inside wire (r < R)

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \oint ds = B(2\pi r)$$
$$= \mu_0 I_{enc} = \mu_0 I \boxed{\left(\frac{\pi r^2}{\pi R^2}\right)}$$

area is smaller that we have

$$\vec{\mathbf{B}} = \frac{\mu_0 I r}{2\pi R^2} \text{ counterclockwise}$$

Could also say: $\boxed{J = \frac{I}{A} = \frac{I}{\pi R^2};\ I_{enc} = J A_{enc} = \frac{I}{\pi R^2}(\pi r^2)}$

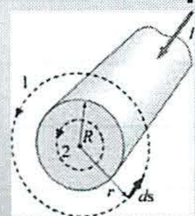
P20-18

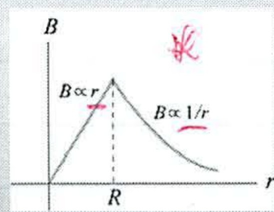
of the area that is enclosed

above it would be $\frac{\pi R^2}{\pi R^2}$

*The I that is given is for the whole wire need to figure from that *

Example: Infinite Wire



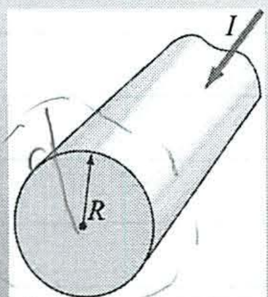


$$B_{in} = \frac{\mu_0 I r}{2\pi R^2} \qquad B_{out} = \frac{\mu_0 I}{2\pi r}$$

P20-19

as outside wire, B falls off $\frac{1}{r}$

Group Problem: Non-Uniform Cylindrical Wire



A cylindrical conductor has radius R and a non-uniform current density with total current:

$$\vec{\mathbf{J}} = J_0 \frac{R}{r}$$

Find B everywhere

P20-20

now need to integrate - right?

~~$B \cdot 2\pi r = \mu_0 I \; J A_{enc}$~~

~~$B \cdot 2\pi r = \mu_0 I \cdot J_0 \frac{R}{r} \cdot \pi R^2$~~

$r > R$ (O):

need a dA → build wire out of shapes where current density is constant

this problem: nested circles

$J = J_0 \frac{R}{r}$

$I = \int J \, dA$

$= \int J_0 \frac{R}{r} 2\pi r \, dr$ ← they did θ part of double integral + simplify

$= 2\pi J_0 R^2$

↓ plug it in

$B \, 2\pi r = \mu_0 (2\pi J_0 R^2)$

$B = \frac{\mu_0 J_0 R^2}{r}$ counter clockwise

Applying Ampere's Law

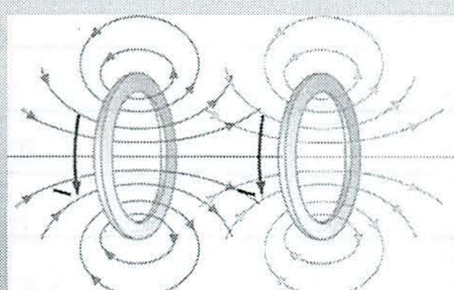
In Choosing Amperian Loop:

- Study & Follow Symmetry
- Determine Field Directions First
- Think About Where Field is Zero
- Loop Must
  - Be Parallel to (Constant) Desired Field
  - Be Perpendicular to Unknown Fields
  - Or Be Located in Zero Field

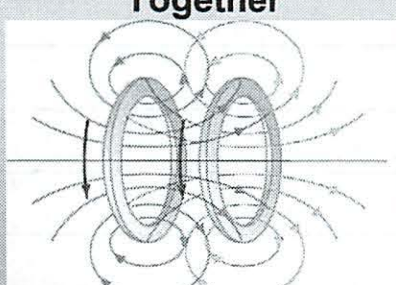
P20-21

Other Geometries

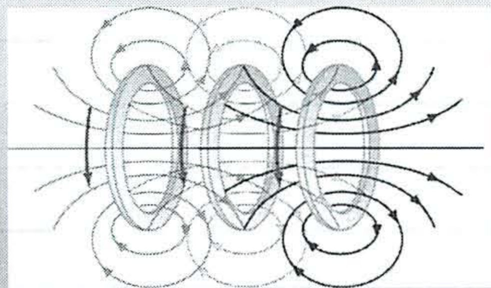
Two Loops



Two Loops Moved Closer Together

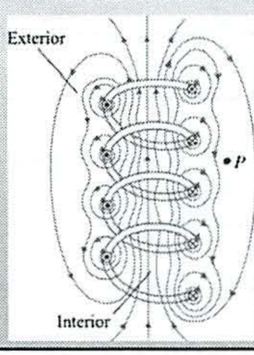


Multiple Wire Loops



P20-25

Multiple Wire Loops – Solenoid



P20-26

wires very close together
"slinky"
electromagnetic
very powerful magnetic field

B field ~~[illegible]~~ constant inside
"uniform field"
-reality: not quite true

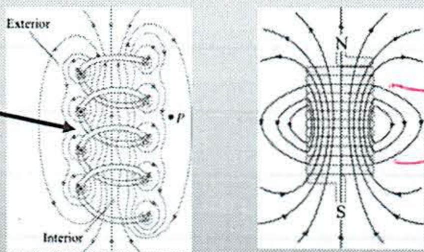
Demonstration: Long Solenoid

P20-27

not true at ends of solonoids

0 outside everywhere

Magnetic Field of Solenoid



Horiz. comp. cancel

loosely wound | tightly wound

For ideal solenoid, B is uniform inside & zero outside

P20-28

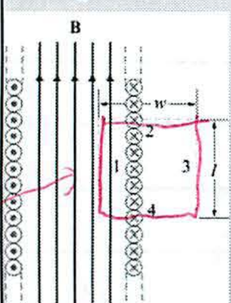
these

In on one out on oter

get longer + longer as it gets ∞ long – no longer matter

both true

Magnetic Field of Ideal Solenoid



amperian square

Using Ampere's law: Think!

$$\begin{cases} \vec{\mathbf{B}} \perp d\vec{\mathbf{s}} \text{ along sides 2 and 4} \\ \vec{\mathbf{B}} = 0 \text{ along side 3} \end{cases}$$ outside

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \int_1 \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} + \int_2 \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} + \int_3 \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} + \int_4 \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$$

$$= \boxed{Bl} + 0 + 0 + 0$$

only side 1 matters

$I_{enc} = nlI$ n: turn density

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = Bl = \mu_0 nlI$$

$n = N/L$: # turns/unit length

$$\boxed{B = \frac{\mu_0 nlI}{l} = \mu_0 nI}$$

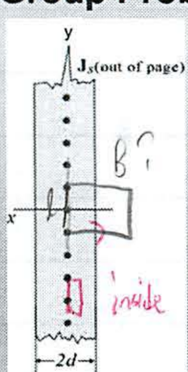
P20-29

square geometry

l = lenght of loop

lenght of solenoid so cancle
what matters is how tightly wound it is

Group Problem: Current Sheet



A sheet of current (infinite in the y & z directions, of thickness 2d in the x direction) carries a uniform current density:

$$\vec{\mathbf{J}}_s = J\hat{\mathbf{k}}$$

Find B for x > 0

P20-30

draw loop

$\oint B \cdot ds = \mu_0 I_{enc}$

$B \cdot l = \mu_0 \int J \hat{k} \, dA$

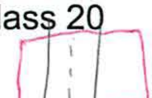
only 1 side

$\mu_0 J \hat{k} l d$

$B = \mu_0 J d \hat{J}$

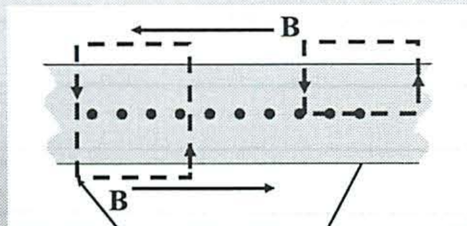
thickness

inside $B = \mu_0 J s \hat{J}$



Could also do

Ampere's Law: Infinite Current Sheet

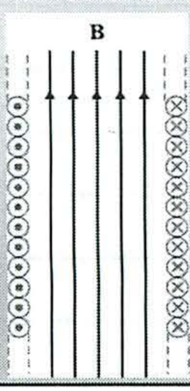


Amperian Loops:

B is Constant & Parallel OR Perpendicular OR Zero

I Penetrates

Solenoid is Two Current Sheets



Field outside current sheet should be half of solenoid, with the substitution:

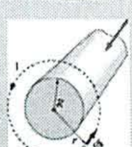
$$k = nI = 2dJ$$

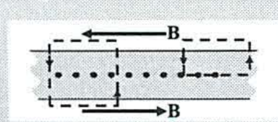
"linear current density"

This is current per unit length (equivalent of λ, but we don't have a symbol for it)

Ampere's Law: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$

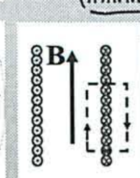
Long Circular Symmetry





(Infinite) Current Sheet

Solenoid = 2 Current Sheets



Torus

Solenoid put together used for inductors

Brief Review Thus Far...

P20-34

Maxwell's Equations (So Far)

Gauss's Law: $\oint\!\!\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$

Electric charges make diverging Electric Fields

Magnetic Gauss's Law: $\oint\!\!\oint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$

No Magnetic Monopoles! (No diverging B Fields) ← have not discovered any but for now no charge enclosed

Ampere's Law: $\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$

Currents make curling Magnetic Fields

P20-35

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8.02 **Spring 2010**

Problem Set 7

Due: Tuesday, March 30 at 9 pm.

Hand in your problem set in your section slot in the boxes outside the door of 32-082. Make sure you clearly write your name and section on your problem set.

Text: Liao, Dourmashkin, Belcher; Introduction to E & M MIT 8.02 Course Notes.

Week Seven Magnetic Fields

Class 17 W07D1 M/T Mar 15/16	Magnetic Fields; Magnetic Forces, Expt. 5: Bar Magnet
Reading:	Course Notes: Chapter 8.1-8.3, 8.5-8.6, 8.8-8.9, 9.5
Experiment:	Expt. 5: Bar Magnet
Class 18 W07D2 W/R Mar 17/18	Creating Fields: Biot-Savart Law, Currents & Dipoles; Expt. 6: Torque on Dipole
Reading:	Course Notes: Sections 8.3-8.4, 9.1-9.2, 9.10.1, 9.11.1-9.11.4
Experiment:	Expt. 6: Torque on Dipole
Class 19 W07D3 F Mar 19	PS06: Calculating Magnetic Fields and Magnetic Force
Reading:	Course Notes: Sections 8.8-8.9, 9.10.1, 9.11.1-9.11.4

Week Eight Spring Break

Week Nine Magnetic Fields; Exam 2

Class 20 W09D1 M/T Mar 29/30	Creating Fields: Ampere's Law
Reading:	Course Notes: 9.3-9.4, 9.10.2, 9.11.5-9.11.8
Class 21 W09D2 W/R Mar 31/Apr 1	PS07: Ampere's Law; Exam 2 Review
Reading:	Course Notes: 9.3-9.4, 9.10.2, 9.11.5-9.11.8
Exam 2 Thursday April 1	**7:30 pm –9:30 pm**
W09D3 F Apr 2	No class day after exam

Problem 1: Short Questions

(a) Can a constant magnetic field set into motion an electron which is initially at rest? Explain your answer.

(b) Is it possible for a constant magnetic field to alter the speed of a charged particle? What is the role of a magnetic field in a cyclotron?

(c) How can a current loop be used to determine the presence of a magnetic field in a given region of space?

(d) If a charged particle is moving in a straight line through some region of space, can you conclude that the magnetic field in that region is zero? Why or why not?

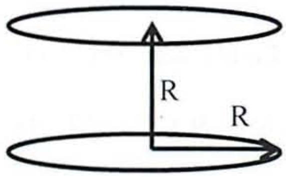
(e) List some similarities and differences between electric and magnetic forces.

Problem 2: Helmholtz Coil

The magnitude of the component of a magnetic field along the axis of a coil with N turns to be given by:

$$B_{axial} = \frac{N \mu_0 I R^2}{2} \frac{1}{(z^2 + R^2)^{3/2}}$$

where z is measured from the center of the coil.



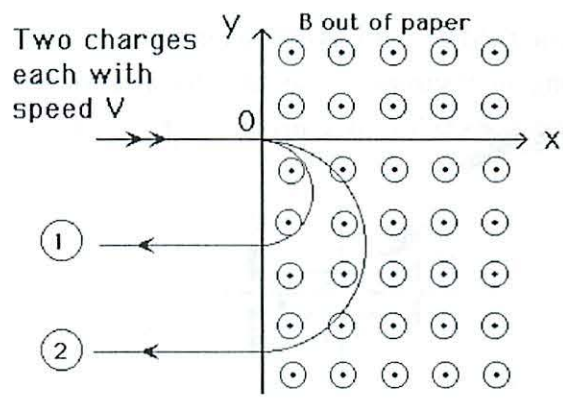
As pictured at left, a Helmholtz coil is created by placing two such coils (each of radius R and N turns) a distance R apart.

(a) If the current in the two coils is parallel (Helmholtz configuration), what is the magnitude of the magnetic field at the center of the apparatus (midway between the two coils)? How does this compare to the field strength at the center of the single coil configuration (e.g. what is the ratio)?

(b) In the anti-Helmholtz configuration the current in the two coils is anti-parallel. What is field strength at the center of the apparatus in this situation?

(c) Consider coils that have a radius R = 7 cm and N = 168 turns. Suppose I = 0.6 A runs in the single coil and 0.3 A runs in each in Helmholtz and anti-Helmholtz mode. What, approximately, are the largest on-axis fields we should expect in these three configurations? Where (approximately) are the fields the strongest?

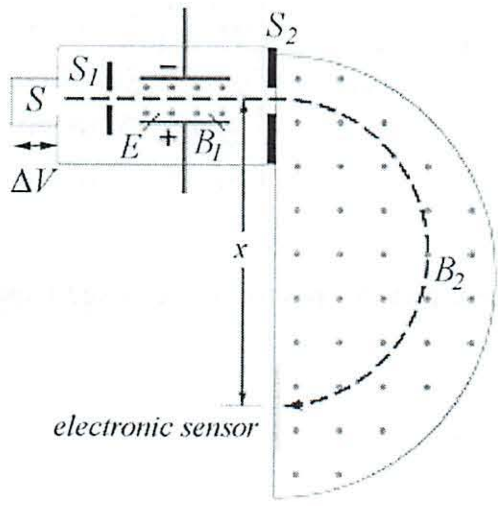
Problem 3: Particle Orbits in a Uniform Magnetic Field The entire x-y plane to the right of the origin O is filled with a uniform magnetic field of magnitude B pointing out of the page, as shown. Two charged particles travel along the negative x axis in the positive x direction, each with velocity $\vec{v}$, and enter the magnetic field at the origin O. The two particles have the same mass m, but have different charges, q_1 and q_2. When in the magnetic field, their trajectories both curve in the same direction (see sketch), but describe semi-circles with different radii. The radius of the semi-circle traced out by particle 2 is exactly twice as big as the radius of the semi-circle traced out by particle 1.



(a) Are the charges of these particles positive or negative? Explain your reasoning.

(b) What is the ratio q_2 / q_1?

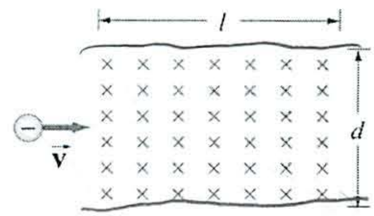
Problem 4 ***Mass Spectrometer*** Shown below are the essentials of a commercial mass spectrometer. This device is used to measure the composition of gas samples, by measuring the abundance of species of different masses. An ion of mass m and charge $q = +e$ is produced in source S, a chamber in which a gas discharge is taking place. The initially stationary ion leaves S, is accelerated by a potential difference $\Delta V > 0$, and then enters a selector chamber, S_1, in which there is an adjustable magnetic field $\vec{\mathbf{B}}_1$, pointing out of the page and a deflecting electric field $\vec{\mathbf{E}}$, pointing from positive to negative plate. Only particles of a uniform velocity $\vec{\mathbf{v}}$ leave the selector. The emerging particles at S_2, enter a second magnetic field $\vec{\mathbf{B}}_2$, also pointing out of the page. The particle then moves in a semicircle, striking an electronic sensor at a distance x from the entry slit. Express your answers to the questions below in terms of $E \equiv |\vec{\mathbf{E}}|$, e, x, m, $B_2 \equiv |\vec{\mathbf{B}}_2|$, and ΔV.



a) What magnetic field $\vec{\mathbf{B}}_1$ in the selector chamber is needed to insure that the particle travels straight through?

b) Find an expression for the mass of the particle after it has hit the electronic sensor at a distance x from the entry slit

Problem 5: Particle Trajectory A particle of charge $-e$ is moving with an initial velocity $\vec{\mathbf{v}}$ when it enters midway between two plates where there exists a uniform magnetic field pointing into the page, as shown in the figure below. You may ignore effects of the gravitational force.



(a) Is the trajectory of the particle deflected upward or downward?

(b) What is the magnitude of the velocity of the particle if it just strikes the end of the plate?

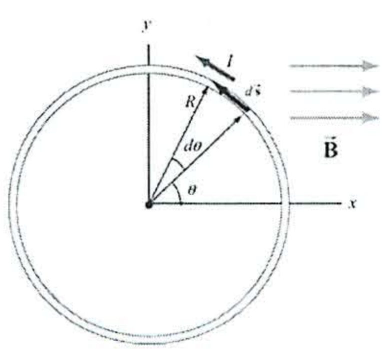
Problem 6: Levitating Wire A copper wire of diameter d carries a current density $\vec{\mathbf{J}}$ at the earth's equator where the earth's magnetic field is horizontal, points north, and has magnitude $\left|\vec{\mathbf{B}}_{earth}\right| = 0.5\times10^{-4}\ \mathrm{T}$. The wire lies in a plane that is parallel to the surface of the earth and is oriented in the east-west direction. The density of copper is $\rho_{Cu} = 8.9\times10^{3}\ \mathrm{kg\cdot m^{-3}}$. The resistivity of copper is $\rho_r = 1.7\times10^{-8}\,\Omega\cdot\mathrm{m}$.

a) How large must $\vec{\mathbf{J}}$ be, and which direction must it flow in order to levitate the wire? Use $g = 9.8\,\mathrm{m\cdot s^{-2}}$

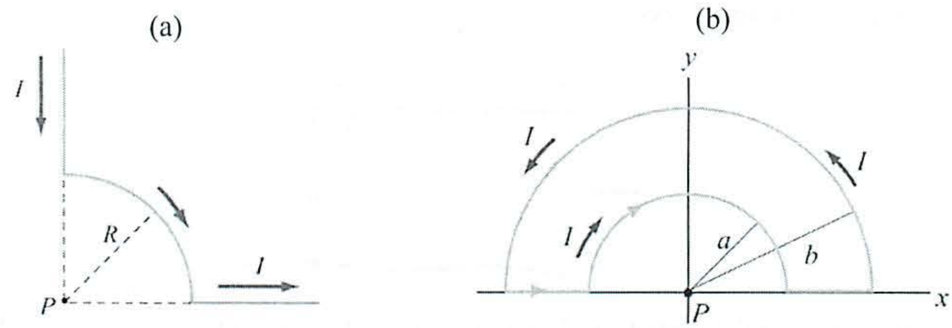
b) When the wire is floating how much power will be dissipated per cubic centimeter?

Problem 7: Torque on Circular Current Loop

A wire ring lying in the xy-plane with its center at the origin carries a counterclockwise current I. There is a uniform magnetic field $\vec{\mathbf{B}} = B\hat{\mathbf{i}}$ in the $+x$-direction. The magnetic moment vector $\vec{\boldsymbol{\mu}}$ is perpendicular to the plane of the loop and has magnitude $\mu = IA$ and the direction is given by right-hand-rule with respect to the direction of the current. What is the torque on the loop?



Problem 8: Magnetic Fields Find the magnetic field at point P due to the following current distributions:



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Department of Physics

8.02 Spring 2010

Problem Set 7 Solutions

Problem 1: Short Questions

(a) Can a constant magnetic field set into motion an electron which is initially at rest? Explain your answer.

Solution: No. Changing the velocity of a particle requires an accelerating force. The magnetic force is proportional to the speed of the particle. If the particle is not moving, there can be no magnetic force on it.

(b) Is it possible for a constant magnetic field to alter the speed of a charged particle? What is the role of a magnetic field in a cyclotron?

Solution: No, it is not possible. Because $\vec{\mathbf{F}}_B = q\left(\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right)$, the acceleration produced by a magnetic field on a particle of mass m is $\vec{\mathbf{a}}_B = \frac{q}{m}\left(\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right)$. For the acceleration to change the speed, a component of the acceleration must be in the direction of the velocity. The cross product tells us that the acceleration must be perpendicular to the velocity, and thus can only change the direction of the velocity.

The magnetic field in a cyclotron essentially keeps the charged particle in the electric field for a longer period of time, and thus experiencing a larger change in speed from the electric field, by forcing it in a spiral path. Without the magnetic field, the particle would have to move in a straight line through an electric field over a distance that is very large compared to the size of the cyclotron.

(c) How can a current loop be used to determine the presence of a magnetic field in a given region of space?

Solution: If the current loop feels a torque, it must be caused by a magnetic field. If the current loop feels no torque, try a different orientation—the torque is zero if the field is along the axis of the loop.

(d) If a charged particle is moving in a straight line through some region of space, can you conclude that the magnetic field in that region is zero? Why or why not?

Solution: Not necessarily. If the magnetic field is parallel or anti-parallel to the velocity of the charged particle, then the particle will experience no magnetic force. There may also be an electric force acting on the particle such that $\vec{\mathbf{F}}_q = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}}_q \times \vec{\mathbf{B}}\right) = 0$.

(e) List some similarities and differences between electric and magnetic forces.

Solution:

Similarities:

1. Both can accelerate a charged particle moving through the field.
2. Both exert forces directly proportional to the charge of the particle feeling the force.

Differences:

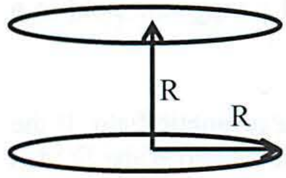
1. The direction of the electric force is parallel or anti-parallel to the direction of the electric field, but the direction of the magnetic force is perpendicular to the magnetic field and to the velocity of the charged particle.
2. Electric forces can accelerate a charged particle from rest or stop a moving particle, but magnetic forces cannot.

Problem 2: Helmholtz Coil

The magnitude of the component of a magnetic field along the axis of a coil with N turns to be given by:

$$B_{axial} = \frac{N \mu_0 I R^2}{2} \frac{1}{(z^2 + R^2)^{3/2}}$$

where z is measured from the center of the coil.



As pictured at left, a Helmholtz coil is created by placing two such coils (each of radius *R* and *N* turns) a distance *R* apart.

(a) If the current in the two coils is parallel (Helmholtz configuration), what is the magnitude of the magnetic field at the center of the apparatus (midway between the two coils)? How does this compare to the field strength at the center of the single coil configuration (e.g. what is the ratio)?

Solution: We use superposition principle to determine the magnetic field due to the two coils. We are the same distance $z = R/2$ from each coil and since the currents are parallel they both create a field in the same direction (for example, if both currents are counterclockwise they both create an upward magnetic field at the midpoint). The magnitude then is just twice that of a single coil:

$$B = 2\times\frac{N\,\mu_0\,I\,R^2}{2}\frac{1}{((R/2)^2+R^2)^{3/2}} = \frac{N\,\mu_0\,I}{R}\frac{1}{((1/2)^2+1)^{3/2}} = \frac{8N\,\mu_0\,I}{5^{3/2}R}$$

Comparing this to the field strength at the center of a single coil:

$$B_{\text{sgl coil}} = \frac{N\,\mu_0\,I}{2R}$$

We find that the field of a Helmholtz coil is slightly larger:

$$\frac{B_{\text{Helmholtz}}}{B_{\text{Sgl Coil}}} = \left(\frac{8N\,\mu_0\,I}{5^{3/2}R}\right)\Big/\left(\frac{N\,\mu_0\,I}{2R}\right) = \frac{16}{5^{3/2}} \approx 1.4$$

(b) In the anti-Helmholtz configuration the current in the two coils is anti-parallel. What is field strength at the center of the apparatus in this situation?

Solution: In this case the fields from the two coils are in opposite directions so they cancel each other out. That is, $B = 0$.

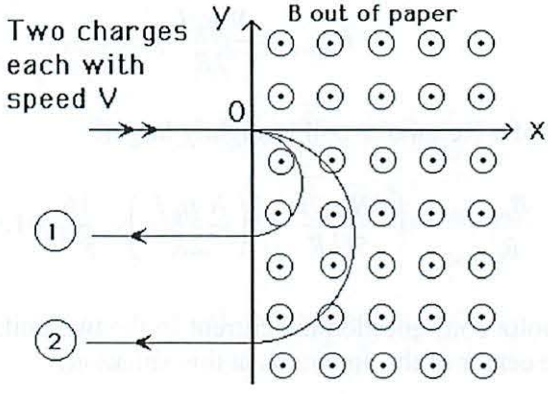
(c) Consider coils that have a radius $R = 7$ cm and $N = 168$ turns. Suppose $I = 0.6$ A runs in the single coil and 0.3 A runs in each in Helmholtz and anti-Helmholtz mode. What, approximately, are the largest on-axis fields we should expect in these three configurations? Where (approximately) are the fields the strongest?

Solution: For a single coil the maximum is at the center of the coil, for a Helmholtz at the center:

$$B^{\max}_{\text{sgl coil}} = \frac{(168)\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(0.6\ \text{A})}{2\left(7\times10^{-2}\ \text{m}\right)} = 9.0\times10^{-4}\ \text{T} = 9.0\ \text{Gauss;}$$

$$B^{\max}_{\text{Helmholtz}} = \frac{16}{5^{3/2}}\cdot 4.5\ \text{Gauss} = 6.5\ \text{Gauss}$$

Problem 3: Particle Orbits in a Uniform Magnetic Field The entire x-y plane to the right of the origin O is filled with a uniform magnetic field of magnitude B pointing out of the page, as shown. Two charged particles travel along the negative x axis in the positive x direction, each with velocity $\vec{v}$, and enter the magnetic field at the origin O. The two particles have the same mass m, but have different charges, q_1 and q_2. When in the magnetic field, their trajectories both curve in the same direction (see sketch), but describe semi-circles with different radii. The radius of the semi-circle traced out by particle 2 is exactly twice as big as the radius of the semi-circle traced out by particle 1.



(a) Are the charges of these particles positive or negative? Explain your reasoning.

Solution: Because $\vec{F}_B = q\vec{v} \times B$, the charges of these particles are POSITIVE.

(b) What is the ratio q_2 / q_1?

Solution: We first find an expression for the radius R of the semi-circle traced out by a particle with charge q in terms of q, $v \equiv |\vec{v}|$, B, and m. The magnitude of the force on the charged particle is qvB and the magnitude of the acceleration for the circular orbit is v^2 / R. Therefore applying Newton's Second Law yields

$$qvB = \frac{mv^2}{R}.$$

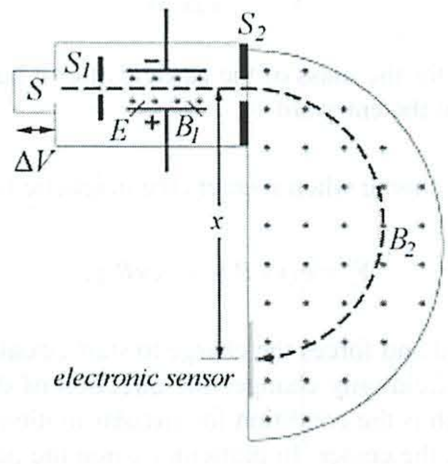
We can solve this for the radius of the circular orbit

$$R = \frac{mv}{qB}$$

Therefore the charged ratio

$$\frac{q_2}{q_1} = \left(\frac{mv}{R_2 B}\right) \Big/ \left(\frac{mv}{R_1 B}\right) = \frac{R_1}{R_2}.$$

Problem 4 ***Mass Spectrometer*** Shown below are the essentials of a commercial mass spectrometer. This device is used to measure the composition of gas samples, by measuring the abundance of species of different masses. An ion of mass m and charge $q = +e$ is produced in source S, a chamber in which a gas discharge is taking place. The initially stationary ion leaves S, is accelerated by a potential difference $\Delta V > 0$, and then enters a selector chamber, S_1, in which there is an adjustable magnetic field $\vec{\mathbf{B}}_1$, pointing out of the page and a deflecting electric field $\vec{\mathbf{E}}$, pointing from positive to negative plate. Only particles of a uniform velocity $\vec{\mathbf{v}}$ leave the selector. The emerging particles at S_2, enter a second magnetic field $\vec{\mathbf{B}}_2$, also pointing out of the page. The particle then moves in a semicircle, striking an electronic sensor at a distance x from the entry slit. Express your answers to the questions below in terms of $E \equiv |\vec{\mathbf{E}}|$, e, x, m, $B_2 \equiv |\vec{\mathbf{B}}_2|$, and ΔV.



a) What magnetic field $\vec{\mathbf{B}}_1$ in the selector chamber is needed to insure that the particle travels straight through?

Solution: We first find an expression for the speed of the particle after it is accelerated by the potential difference ΔV, in terms of m, e, and ΔV. The change in kinetic energy is $\Delta K = (1/2)mv^2$. The change in potential energy is $\Delta U = -e\Delta V$ From conservation of energy, $\Delta K = -\Delta U$, we have that

$$(1/2)mv^2 = e\Delta V .$$

So the speed is

$$v = \sqrt{\frac{2e\Delta V}{m}}$$

Inside the selector the force on the charge is given by

$$\vec{\mathbf{F}}_e = e(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}_1) .$$

If the particle travels straight through the selector then force on the charge is zero, therefore

$$\vec{\mathbf{E}} = -\vec{\mathbf{v}} \times \vec{\mathbf{B}}_1 .$$

Since the velocity is to the right in the figure above (define this as the $+\hat{\mathbf{i}}$ direction), the electric field points up (define this as the $+\hat{\mathbf{j}}$ direction) from the positive plate to the negative plate, and the magnetic field is pointing out of the page (define this as the $+\hat{\mathbf{k}}$ direction). Then

$$E\hat{\mathbf{j}} = -v\hat{\mathbf{i}} \times B_1\hat{\mathbf{k}} = vB_1\hat{\mathbf{j}} .$$

So

$$\vec{\mathbf{B}}_1 = \frac{E}{v}\hat{\mathbf{k}} = \sqrt{\frac{m}{2e\Delta V}}E\,\hat{\mathbf{k}}$$

b) Find an expression for the mass of the particle after it has hit the electronic sensor at a distance x from the entry slit

Solution: The force on the charge when it enters the magnetic field $\vec{\mathbf{B}}_2$ is given by

$$\vec{\mathbf{F}}_e = ev\hat{\mathbf{i}} \times B_2\hat{\mathbf{k}} = -evB_2\hat{\mathbf{j}} .$$

This force points downward and forces the charge to start circular motion. You can verify this because the magnetic field only changes the direction of the velocity of the particle and not its magnitude which is the condition for circular motion. When in circular motion the acceleration is towards the center. In particular when the particle just enters the field $\vec{\mathbf{B}}_2$, the acceleration is downward

$$\vec{\mathbf{a}} = -\frac{v^2}{x/2}\hat{\mathbf{j}} .$$

Newton's Second Law becomes

$$-evB_2\hat{\mathbf{j}} = -m\frac{v^2}{x/2}\hat{\mathbf{j}} .$$

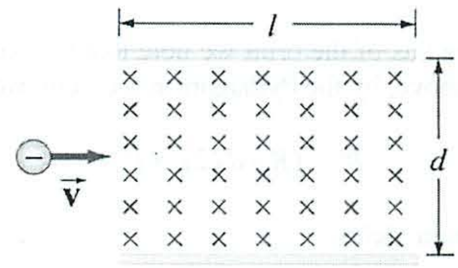
Thus the particle hits the electronic sensor at a distance

$$x = \frac{2mv}{eB_2} = \frac{2}{eB_2}\sqrt{2e\Delta Vm}$$

from the entry slit. The mass of the particle is then

$$m = \frac{eB_2{}^2x^2}{8\Delta V} .$$

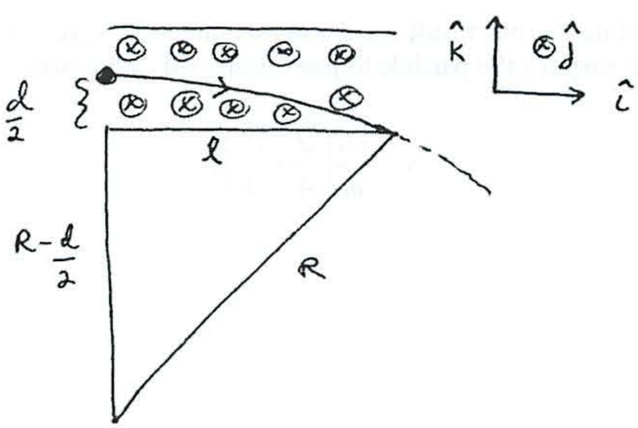
Problem 5: Particle Trajectory A particle of charge $-e$ is moving with an initial velocity $\vec{\mathbf{v}}$ when it enters midway between two plates where there exists a uniform magnetic field pointing into the page, as shown in the figure below. You may ignore effects of the gravitational force.



(a) Is the trajectory of the particle deflected upward or downward?

(b) What is the magnitude of the velocity of the particle if it just strikes the end of the plate?

Solution: Choose unit vectors as shown in the figure.



The force on the particle is given by

$$\vec{\mathbf{F}} = -e\left(v\,\hat{\mathbf{i}} \times B\hat{\mathbf{j}}\right) = -evB\hat{\mathbf{k}}\,.$$

so the direction of the force is downward. Remember that when a charged particle moves through a uniform magnetic field, the magnetic force on the charged particle only changes the direction of the velocity hence leaves the speed unchanged so the particle undergoes circular motion. Therefore we can use Newton's second law in the form

$$evB = m\frac{v^2}{R}.$$

The speed of the particle is then

$$v = \frac{eBR}{m}.$$

In order to determine the radius of the orbit we note that the particle just hits the end of the plate. From the figure above, by the Pythagorean theorem, we have that

$$R^2 = (R - d/2)^2 + l^2 .$$

Expanding the above equation yields

$$R^2 = R^2 - Rd + d^2/4 + l^2$$

which we can solve for the radius of the circular orbit:

$$R = \frac{d}{4} + \frac{l^2}{d}.$$

We can now substitute the our result for the radius into our expression for the speed and find the speed necessary for the particle to just hit the end of the plate:

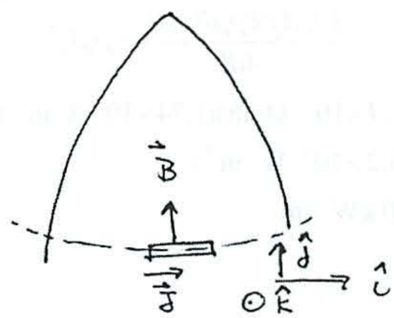
$$v = \frac{eB}{m}\left(\frac{d}{4} + \frac{l^2}{d}\right).$$

Problem 6: **Levitating Wire** A copper wire of diameter d carries a current density $\vec{\mathbf{J}}$ at the earth's equator where the earth's magnetic field is horizontal, points north, and has magnitude $\left|\vec{\mathbf{B}}_{earth}\right| = 0.5 \times 10^{-4}\ \mathrm{T}$. The wire lies in a plane that is parallel to the surface of the earth and is oriented in the east-west direction. The density of copper is $\rho_{Cu} = 8.9 \times 10^{3}\ \mathrm{kg \cdot m^{-3}}$. The resistivity of copper is $\rho_r = 1.7 \times 10^{-8}\ \Omega \cdot \mathrm{m}$.

a) How large must $\vec{\mathbf{J}}$ be, and which direction must it flow in order to levitate the wire? Use $g = 9.8\,\mathrm{m \cdot s^{-2}}$

b) When the wire is floating how much power will be dissipated per cubic centimeter?

Solution: At the equator, the magnetic field is pointing north. Choose unit vectors such that $\hat{\mathbf{i}}$ points east, $\hat{\mathbf{j}}$ points north, and $\hat{\mathbf{k}}$ points up. Let $\vec{\mathbf{J}} = J_x\,\hat{\mathbf{i}}$ (with the sign of J_x to be determined), $\vec{\mathbf{B}}_{earth} = B_{earth}\,\hat{\mathbf{j}}$. .



Then the magnetic force $d\vec{\mathbf{F}}_{mag}$ on the a small volume of wire dV_{vol} is

$$d\vec{\mathbf{F}}_{mag} = \vec{\mathbf{J}}dV_{vol} \times \vec{\mathbf{B}}_{earth} = J_x dV_{vol}\,\hat{\mathbf{i}} \times B_{earth}\hat{\mathbf{j}} = J_x B_{earth} dV_{vol}\,\hat{\mathbf{k}}.$$

In order to balance the gravitational force this must point upwards hence $J_x > 0$; the current flows from west to east in the wire. The total force on the small element of the wire is zero so

$$\vec{\mathbf{0}} = d\vec{\mathbf{F}}_{grav} + d\vec{\mathbf{F}}_{mag} = \rho_{Cu} dV_{vol} g(-\hat{\mathbf{k}}) + J_x B_{earth} dV_{vol}\,\hat{\mathbf{k}}.$$

We can solve the above equation for J_x:

$$J_x = \frac{\rho_{Cu} g}{B_{earth}}$$

$$J_x = \frac{(8.9\times10^3 \text{ kg}\cdot\text{m}^{-3})(9.8 \text{ m}\cdot\text{s}^{-2})}{(0.5\times10^{-4} \text{ T})} = 1.74\times10^9 \text{ A}\cdot\text{m}^{-2}$$

(b) Let $A = \pi(d/2)^2$ denote the cross-sectional area of the wire. The power dissipated per volume $dV_{vol} = Adl$ where dl is a unit length of wire is given by

$$\frac{P}{dV_{vol}} = \frac{I^2 R}{dV_{vol}}.$$

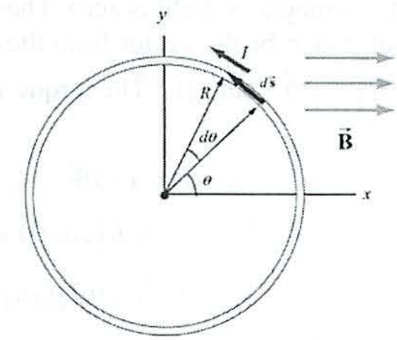
Let The current that flows in the wire is given by is given by $I = J_x A$. The resistance per unit length dl is given by $R = \rho_r dl / A$. So the above equation becomes

$$\begin{aligned}\frac{P}{dV_{vol}} &= \frac{(J_x A)^2(\rho_r dl / A)}{Adl} = \rho_r J_x{}^2 \\ &= (1.7\times10^{-8}\ \Omega\cdot\text{m})(1.74\times10^9 \text{ A}\cdot\text{m}^{-2})^2. \\ &= (5.2\times10^{10} \text{ W}\cdot\text{m}^{-3}) \\ &\cong 50 \text{ kW}\cdot\text{cm}^{-3}\end{aligned}$$

The wire will get very hot!

Problem 7: Torque on Circular Current Loop

A wire ring lying in the xy-plane with its center at the origin carries a counterclockwise current I. There is a uniform magnetic field $\vec{\mathbf{B}} = B\hat{\mathbf{i}}$ in the $+x$-direction. The magnetic moment vector $\vec{\boldsymbol{\mu}}$ is perpendicular to the plane of the loop and has magnitude $\mu = IA$ and the direction is given by right-hand-rule with respect to the direction of the current. What is the torque on the loop?



Solution: The torque on a current loop in a uniform field is given by

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}},$$

where $\mu = IA$ and the vector $\vec{\boldsymbol{\mu}}$ is perpendicular to the plane of the loop and right-handed with respect to the direction of current flow. The magnetic dipole moment is given by

$$\vec{\boldsymbol{\mu}} = I\vec{\mathbf{A}} = I(\pi R^2 \hat{\mathbf{k}}) = \pi I R^2 \hat{\mathbf{k}}.$$

Therefore,

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}} = \left(\pi I R^2 \hat{\mathbf{k}}\right) \times \left(B\hat{\mathbf{i}}\right) = \pi I R^2 B \hat{\mathbf{j}}.$$

Instead of using the above formula, we can calculate the torque directly as follows. Choose a small section of the loop of length $ds = R d\theta$. Then the vector describing the current-carrying element is given by

$$Id\vec{\mathbf{s}} = IRd\theta(-\sin\theta\,\hat{\mathbf{i}} + \cos\theta\,\hat{\mathbf{j}})$$

The force $d\vec{\mathbf{F}}$ that acts on this current element is

$$\begin{aligned} d\vec{\mathbf{F}} &= Id\vec{\mathbf{s}} \times \vec{\mathbf{B}} \\ &= I\,Rd\theta(-\sin\theta\,\hat{\mathbf{i}} + \cos\theta\,\hat{\mathbf{j}}) \times (B\hat{\mathbf{i}}) \\ &= -IRB\cos\theta\, d\theta\,\hat{\mathbf{k}} \end{aligned}$$

The force acting on the loop can be found by integrating the above expression.

$$\begin{aligned} \vec{\mathbf{F}} &= \oint d\vec{\mathbf{F}} = \int_0^{2\pi} (-IRB\cos\theta) d\theta\,\hat{\mathbf{k}} \\ &= -IRB\left[\sin\theta\right]_0^{2\pi} \hat{\mathbf{k}} = 0 \end{aligned}$$

We expect this because the magnetic field is uniform and the force ona current loop in a uniform magnetic field is zero. Therefore we can choose any point to calculate the torque about. Let $\vec{\mathbf{r}}$ be the vector from the center of the loop to the element $Id\vec{\mathbf{s}}$. That is, $\vec{\mathbf{r}} = R\left(\cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}}\right)$. The torque $d\vec{\tau} = \vec{\mathbf{r}} \times d\vec{\mathbf{F}}$ acting on the current element is then

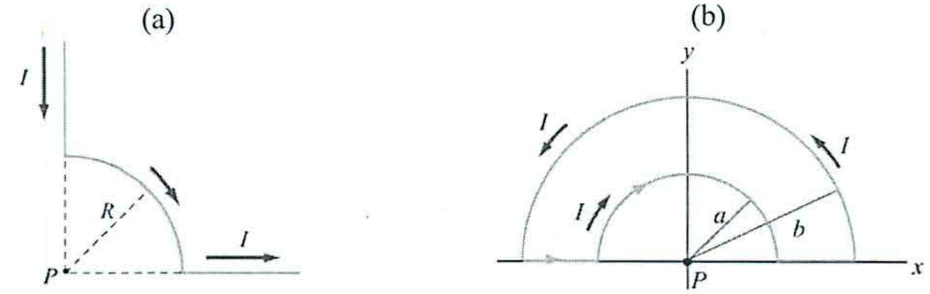
$$\begin{aligned}
d\vec{\tau} &= \vec{\mathbf{r}} \times d\vec{\mathbf{F}} \\
&= R\left(\cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}}\right) \times \left(-IRBd\theta\cos\theta\,\hat{\mathbf{k}}\right) \\
&= -IR^2Bd\theta\cos\theta\left(\sin\theta\,\hat{\mathbf{i}} - \cos\theta\,\hat{\mathbf{j}}\right)
\end{aligned}$$

Integrate $d\vec{\tau}$ over the loop to find the total torque $\vec{\tau}$.

$$\begin{aligned}
\vec{\tau} &= \oint d\vec{\tau} \\
&= \int_0^{2\pi} -IR^2Bd\theta\cos\theta\left(\sin\theta\,\hat{\mathbf{i}} - \cos\theta\,\hat{\mathbf{j}}\right) \\
&= -IR^2B\int_0^{2\pi}(\sin\theta\cos\theta\,\hat{\mathbf{i}} - \cos^2\theta\,\hat{\mathbf{j}})d\theta \\
&= \pi IR^2B\,\hat{\mathbf{j}}
\end{aligned}$$

This agrees with our result above.

Problem 8: Magnetic Fields Find the magnetic field at point P due to the following current distributions:



Solution:

(a) The fields due to the straight wire segments are zero at P because $d\vec{s}$ and $\hat{\mathbf{r}}$ are parallel or anti-parallel. For the field due to the arc segment, the magnitude of the magnetic field due to a differential current carrying element is given in this case by

$$d\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\frac{d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{R^2} = \frac{\mu_0}{4\pi}\frac{IRd\theta(\sin\theta\,\hat{\mathbf{i}} - \cos\theta\,\hat{\mathbf{j}})\times(-\cos\theta\,\hat{\mathbf{i}} - \sin\theta\,\hat{\mathbf{j}})}{R^2}$$
$$= -\frac{\mu_0}{4\pi}\frac{I(\sin^2\theta + \cos^2\theta)d\theta}{R}\hat{\mathbf{k}} = -\frac{\mu_0}{4\pi}\frac{Id\theta}{R}\hat{\mathbf{k}}$$

Therefore,

$$\vec{\mathbf{B}} = -\int_0^{\pi/2}\frac{\mu_0 I}{4\pi R}d\theta\,\hat{\mathbf{k}} = -\frac{\mu_0 I}{4\pi R}\left(\frac{\pi}{2}\right)\hat{\mathbf{k}} = -\left(\frac{\mu_0 I}{8R}\right)\hat{\mathbf{k}} \text{ (or, into the page).}$$

(b) There is no magnetic field due to the straight segments because point P is along the lines. Using the general expression for $d\vec{\mathbf{B}}$ obtained in (a), for the outer segment, we have

$$\vec{\mathbf{B}}_{\text{out}} = \int_0^{\pi}\frac{\mu_0}{4\pi}\frac{Id\theta}{b}\hat{\mathbf{k}} = \left(\frac{\mu_0 I}{4b}\right)\hat{\mathbf{k}}$$

Similarly, the contribution to the magnetic field from the inner segment is

$$\vec{\mathbf{B}}_{\text{in}} = \int_{\pi}^{0}\frac{\mu_0}{4\pi}\frac{Id\theta}{a}\hat{\mathbf{k}} = -\left(\frac{\mu_0 I}{4a}\right)\hat{\mathbf{k}}.$$

Therefore the net magnetic field at Point P is

$$\vec{\mathbf{B}}_{\text{net}} = \vec{\mathbf{B}}_{\text{out}} + \vec{\mathbf{B}}_{\text{in}} = -\frac{\mu_0 I}{4}\left(\frac{1}{a} - \frac{1}{b}\right)\hat{\mathbf{k}} \text{ (into the page since } a < b\text{).}$$

Review Dormaskin

9-10:30 PM

know what is on test

Properties of Conductors

Capicators + Capitance

- no dielectrics

Current, Resistance, Circuits

- no solving differential circuit :)
  - know graphically

Magnetism

- forces, torques current loops
- no Ampere's law
- B-S only conceptual

Old tests cover different things

Capitance

$Q = C\Delta V$

↑ charge appears when voltage diff applied

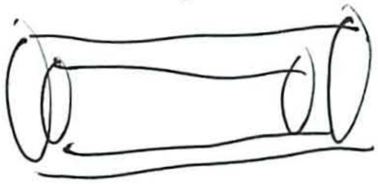
Calculate $C = \frac{Q}{|\Delta V|}$

1. How $|\Delta V|$? → $\int \vec{E} \cdot ds$ (gauss' law) to find E

(2)

3 types of capicators

- shells  (conducting)
- parallel plates  Area A
- concentric syllanders



→ infinite lenght
or neglect edge effects

lenght l

charges flow away from 1 plate to other
when voltage diff aplied

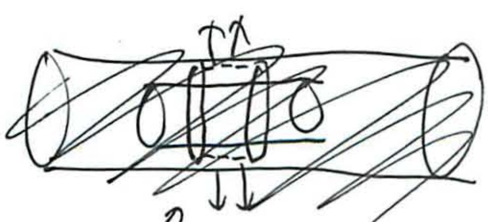
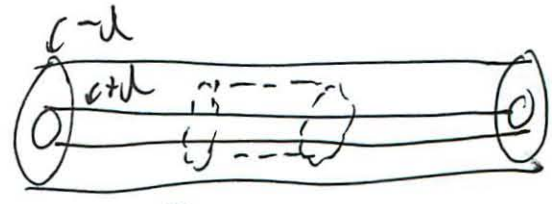
Q is where (+) charge is

$$C = \frac{Q}{|\Delta V|} = \frac{Q}{\left|\int \vec{E} \cdot d\vec{s}\right|}$$

---

ex coaxial conducting cylinders

need to assume they are charged up
- will cancle out later

lenght l radius r

(3)

$\oint\oint \vec{E} \cdot d\vec{a}$	$\frac{q_{enc}}{\epsilon_0}$
$E \cdot \text{area}$	$= \frac{\lambda l}{\epsilon_0}$
$E \cdot 2\pi r l$	$= \frac{\lambda}{\epsilon_0}$

$$\boxed{\vec{E} = \frac{\lambda}{2\pi\epsilon_0} \frac{1}{r} \hat{r} \quad a < r < b}$$

* Conductor so all charge on surface

← need to be able to do (from last test) → I need to look over + review

* Also ~~two~~ remember what pitfalls are

$$C = \frac{Q}{|\int \vec{E} \cdot d\vec{s}|}$$

$$\hookrightarrow -\int_{r=a}^{r=b} \vec{E} \cdot d\vec{s}$$

potential diff decreasing from inner to outer so − but abs value so does not matter

$$= -\int_a^b \frac{\lambda}{2\pi\epsilon_0} \frac{1}{r} \, dr$$

$$= -\frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{b}{a}\right)$$

← how the rule

$$C = \frac{Q}{\frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{b}{a}\right)}$$

but what is Q?

↳ should cancle

↳ $\lambda = \frac{Q}{L}$ ← known

$$C = \frac{\lambda L}{\frac{1}{2\pi\epsilon_0}\lambda \ln(b/a)} = \frac{L}{\frac{\ln(b/a)}{2\pi\epsilon_0}} = \boxed{\frac{2\pi\epsilon_0 L}{\ln(b/a)}}$$

Energy Stored in a Capicator

$$U_C = \frac{Q^2}{2C} \quad \leftarrow \text{memorize}$$

$$Q = C|\Delta V|$$

$$= \frac{1}{2}C|\Delta V|^2$$

$$= \frac{Q^2}{2\left(\frac{2\pi\epsilon_0 L}{\ln(b/a)}\right)} \quad \text{in this example}$$

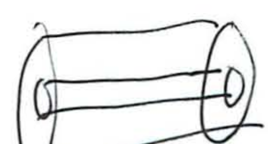
or if have voltage

$$= \frac{1}{2}\left(\frac{2\pi\epsilon_0 L}{\ln(b/a)}\right)\Delta V^2$$

also

$$= \frac{1}{2}\epsilon_0 \iiint E^2\, dV$$

$$= \frac{1}{2}\epsilon_0 \int_a^b \left(\frac{\lambda}{2\pi\epsilon_0}\frac{1}{r}\right)^2$$



$dV = L\,2\pi r\,dr$
cylindrical shell

will get same result

⑤

Try for all 3 cases (sphere, cylinder, plates)

know → if E non constant, must do <u>integral</u>

What are the electric fields

?must know figured out for last test

<u>Circuit Theory</u>

Current + Resistance

$$I = \iint \vec{J} \cdot d\vec{a}$$ how much J through an area

Ohm's Law

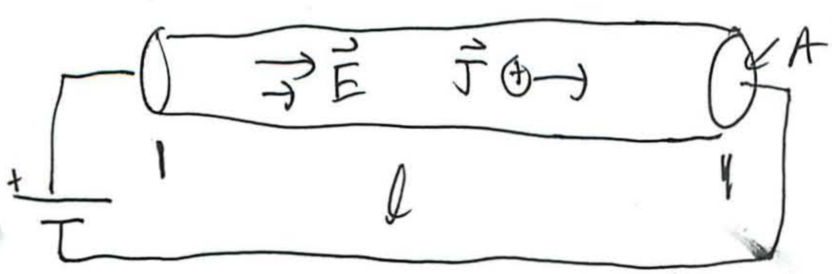
$$\Delta V_{element} = RI$$

$$R = \frac{|\Delta V_{element}|}{|I|} = \frac{|\int \vec{E} \cdot d\vec{s}|}{|\iint \vec{J} \cdot d\vec{a}|}$$

Resistivity (ρ_r)

$$\vec{E} = \rho_r \vec{J} \qquad \sigma_c = \frac{1}{\rho_r} = \text{conductivity}$$

$$\vec{J} = \sigma_c \vec{E} \qquad \text{(*) know}$$



a wire

$$R = \frac{|\Delta V|}{|I|} = \frac{|\int \vec{E} \cdot d\vec{s}|}{|\iint \vec{J} \cdot d\vec{a}|}$$

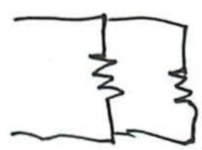
(6)

$$\boxed{E = \frac{\rho_c \cancel{A} J l}{J A} = \frac{\rho_c L}{A}}$$ ← do not know at all, study

Circuits

Resistors ~~Parallel~~ ~~Series~~

Series 
- Same current through each
- Voltage drops add
- replace w/ R_{eq}

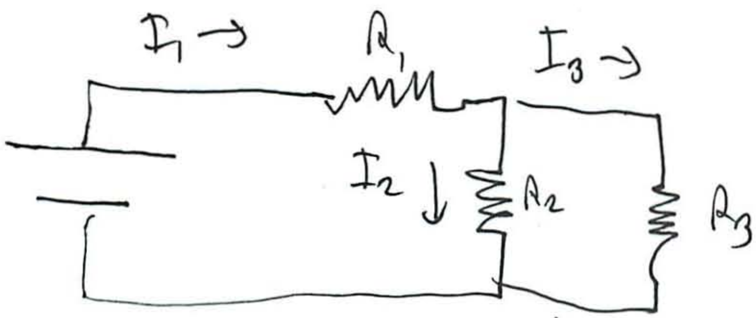
Parallel 
- Same voltage across them
- current added
- more doors to leave theater

$\boxed{\text{Can make things far simpler}}$

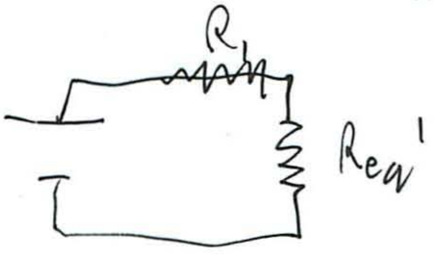
$\Delta V = I R_1$

$\Delta V = I R_2$

$$\frac{\Delta V}{R_{eq}} = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2}$$



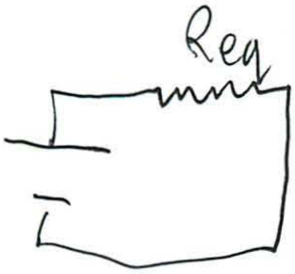
2 in parallel which together are in series is 1



$$\frac{1}{R_{eq}'} = \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_{eq} = \frac{R_2 R_3}{R_2 + R_3}$$ ← know

7



$$R_{eq} = R_1 + R_{eq}$$

$$I_1 = \frac{\Delta V}{R_{eq}}$$

to find I_2 or I_3

$$I_1 = I_2 + I_3$$

↑ just found

know ΔV 2 and 3 =

so $I_2 R_2 = I_3 R_3$

$$I_2 = \frac{I_3 R_3}{R_2}$$

$$I_1 = \frac{I_3 R_3}{R_2} + I_3 = \frac{\Delta V}{R_{eq}}$$

$$I_1 = I_3 \left(\frac{R_3}{R_2} + 1 \right)$$

$$I_2 = I_3 \frac{R_3}{R_2}$$

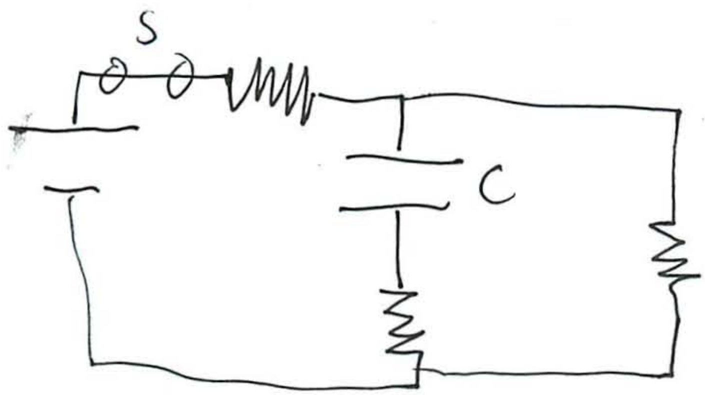
far better w/ algebra now thanks to Physics + problem solving

"solve a circuit" → find current in every branch

So what happens when you add a capicator

⟶

(8)



time dependent circuit
need loop laws + differential eqs to get full time dependence

but they just ask - switch just closed
- closed for really long time
- etc

$t=0$, close S → Capicator uncharged (like a wire)

$R_{eq}' = \frac{R}{2}$

$I = \frac{\Delta V}{\frac{3R}{2}}$

$R_{eq} = \frac{3R}{2}$

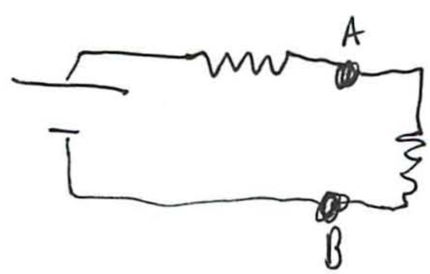
find I_2, I_3

$t=$ long time after s Closed

Capicator fully charged

$I_2 = $ zero

$V_f = ?$



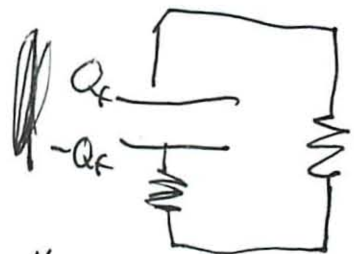
$= I_{1\,\text{final}} \cdot R$

$= \frac{\Delta V R}{2R} = \frac{\Delta V}{2}$ half the voltage of the battery

(4)

Now open switch ~~dig~~

-discharge capicator



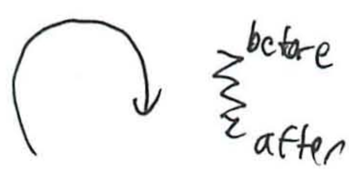
if solving differential equation

$$\Delta V = V_{after} - V_{Befor}$$

choice of current dir $\neq$ choice circulation dir

what after + before means

if going



So ~~the~~ Voltages are

+

−

$$\Delta V = IR$$

$$0 = -I_2 R - I_1 R + \frac{Q}{C}$$

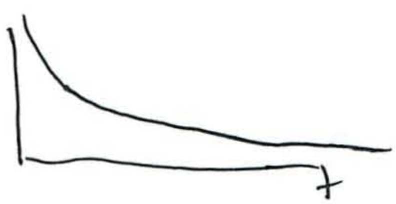
since going from ⊖ → ⊕ plate

$$\frac{Q}{C} = 2 I_2 R$$

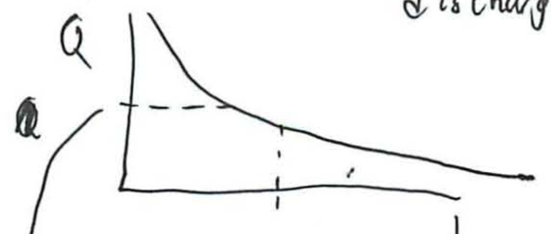
$$I_2 = -\frac{dQ}{dt} \quad \text{discharging}$$

$$\frac{Q}{C} = -2 \frac{dQ}{dt} R$$

$$\frac{Q}{-2RC} = \frac{dQ}{dt}$$



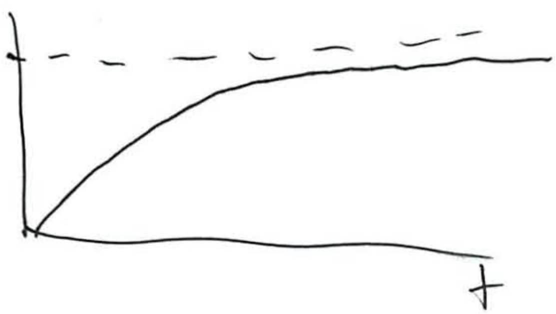
(10) $\frac{dQ}{Q} = \frac{dt}{2RC}$ discharging



$Q(t) = Q_0 e^{-1}$

charging

- bit more complicated



didn't do capicators in parallel

- do a bit more w/ graphs - like that lab

Magtism

- currents or moving charges produce magnetic fields

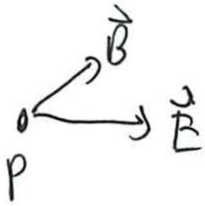
$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I\, d\vec{s} \times \hat{r}}{r^2}$ - come from sources

- 2 special cases
  - ring w/ field at center
  - ring w/ field some height z

- otherwise won't use B-S much

(1X) Given a $\vec{B}$ field + $\vec{E}$ field

$\vec{B}$ ← moving charges; $\vec{E}$ ← charges

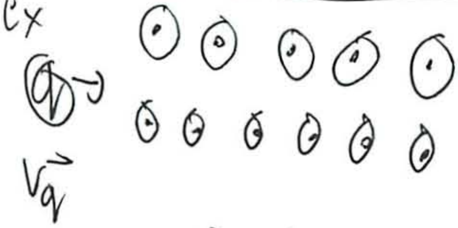


$$F = q(\vec{E}_{ext} + \vec{V}_q \times \vec{B}_{ext})$$

Place q at p

---

ex



$\vec{E} = 0$

$\vec{F}$	$m_q \vec{a}_q$
$q(\vec{V_q} \times \vec{B})$	

↑ cross product perpendicular to either vector
- velocity only change dir
"do no work"

If $\vec{B}$ is uniform
- uniform force
- will get circular motion
  - of radius r

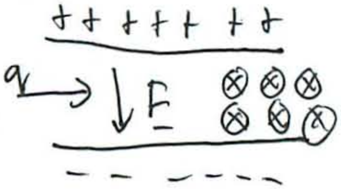
$$\boxed{VqB = \frac{mv^2}{R}}$$

← memorize
"cyclotron"

$$T = \frac{2\pi R}{V_q}$$

2)

q>0



$F = m\vec{a}$

$q(\vec{E} + \vec{v} \times \vec{B}) = m\vec{a}$

where it goes is up to the relative strenghts

straight → they cancle

if its <u>not straight</u> now force points a diff direction

So the problem has gotten a lot harder

Current loops in magnetic fields

$\vec{F}_{wire} = \int_{wire} I\, d\vec{s} \times \vec{B}$

$dF = dq\,\vec{v} \times B$

add that up over the wire → integrate

If $\vec{B}$ is uniform, then ~~the force on the wire~~ ↳ can pull out B

$\vec{F}_{wire} = (\int I \cdot ds) \times B$

If wire is straight just use lenght

$\vec{F}_{wire} = I(\int d\vec{s}) \times B = I\vec{L} \times B$

If wire is a loop (closed path) in <u>uniform $\vec{B}$ field</u>

$F_{wire} = 0 = (I \oint d\vec{s}) \times \vec{B}$

(13)

~~BR 201~~

Torque is easy to calculate

- Use concept of magnetic moment

$$\vec{\mu} = I A \hat{n}_{RHR}$$

$\odot$ $\hat{n}_{RHR}$ $\odot$ $\vec{\mu}$

↑ "screwdriver"

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Write some unit vectors

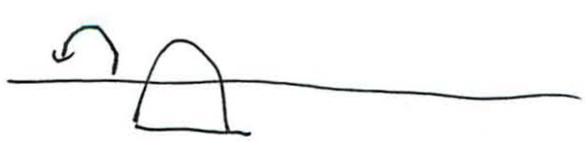
$\hat{j}$ ↑

$\odot\, \hat{k}$ → $\hat{i}$

$$\vec{\tau} = I A \hat{k} \times B \hat{j}$$

$$\vec{\tau} = I A B (-\hat{i})$$

- causes object to rotate
- points along ~~the~~ axis of rotation



at this instant
- magnetic moment changing di

(14)

Another problem "current in adj objects"

- 2 wires
  - parallel currents attract
  - antiparallel " repel
- current loops
  - same

Did not do much conductors

Think this test far better off

- then review session I where I learned a lot
- only real ~~new~~ thing I did not remember was Guass' Law
- and some formulas to memorize (but know how to use)

TEST TWO Thursday Evening April 1 7:30- 9:30 pm. The Friday class immediately following is canceled because of the evening exam.

What We Expect From You On The Exam

1. An understanding of capacitors, including the effects of dielectrics on them.

2. An understanding of current flow in a resistive material, e.g. how J is related to *I*, how E is related to J, how resistance is related to resistivity, and how to calculate it.

3. An understanding of simple circuits. For example, you should be able to set up the equations for multi-loop circuits, using Kirchhoff's Laws. You should be able to derive and guess the solution to differential equations for RC circuits, and should understand the meaning of time constants ($\tau = RC$).

4. An understanding of how to calculate the magnetic fields of moving charges or current elements using the Biot-Savart law, e.g.

5.

$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{q\,\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2} \qquad d\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{I\,d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

6. An understanding of how to calculate the force on a current element in an external magnetic field or on a charged particle moving in an external magnetic field or in both magnetic and electric fields, including the characteristics of cyclotron motion. That is, to understand and be able to apply the equations

$$q\vec{\mathbf{v}} \times \vec{\mathbf{B}} = m\vec{\mathbf{a}} \qquad q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}) = m\vec{\mathbf{a}} \qquad d\vec{\mathbf{F}} = I d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

7. An understanding of the magnetic moment vector of a current loop. A conceptual understanding of how the torque exerted on a magnetic dipole in an external field arises, and how to derive the expression for this torque ($\vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}}$). Also, a conceptual understanding of how the force exerted on a magnetic dipole in a non-uniform external magnetic field arises.

8. To be able to answer qualitative conceptual questions that require no calculation. These will be concept questions similar to those in lecture, where you will be asked to make a choice out of a multiple set of choices.

To study for this exam we suggest that you review your problem sets, in-class problems, Friday problem solving sessions, PRS in-class questions, and relevant parts of the study guide and class notes.

Class 21: Outline

Hour 1: Ampere's Law Problem Solving

Hour 2: Concept Review / Overview

PRS Questions – possible exam questions

Exam Thursday 7:30-9:30 pm

P21 - 1

Exam 2 Topics

Conductors

Capacitance

DC Circuits

Magnetic Fields...

Generating Magnetic Fields

~~Biot-Savart Law~~ no B-S

Feeling Magnetic Fields

Moving Charges (Lorentz Force)

On Dipoles (Force & Torque)

P21 - 2

General Exam Suggestions

- You should be able to complete every problem
  - If you are confused, ask
  - If it seems too hard, you aren't thinking enough
  - Look for hints in other problems
  - If you are doing math, you're doing too much
- Read directions completely (before & after)
- Write down what you know before starting
- Draw pictures, define (label) variables
  - Make sure that unknowns drop out of solution
- Don't forget units!

P21 - 3

No pages of math

Do everything it asks for

What You Should Study

- Review Friday Problem Solving (& Solutions)
- Review In Class Problems (& Solutions)
- Review PRS Questions (& Solutions)
- Review Problem Sets (& Solutions)
- Review PowerPoint Presentations
- Review Relevant Parts of Study Guide (& Included Examples)

- Do **Sample Exams** (online under Exam Prep)

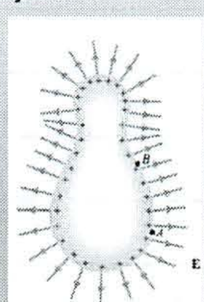
P21 - 4

Conductors in Equilibrium

Conductors are equipotential objects:
1) E = 0 inside
2) E perpendicular to surface
3) Net charge inside is 0
4) Excess charge on surface

$$E = \sigma / \varepsilon_0$$

5) Shielding – inside doesn't "talk" to outside



P21 - 5

net charge inside conductor is 0

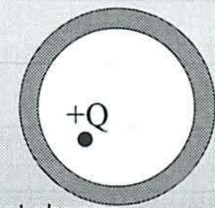
inside does not talk w/ outside

PRS Questions: Conductors

P21 - 6

PRS: Point Charge in Conductor

A point charge +Q is placed inside a neutral, hollow, spherical conductor. As the charge is moved around *inside*, the electric field ***outside***



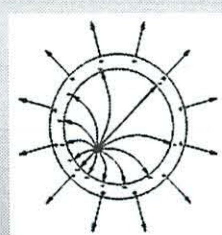
0% 1. is zero and does not change
0% 2. is non-zero but does not change
0% 3. is zero when centered but changes
0% 4. is non-zero and changes
0% 5. I don't know

:00

P21-7

PRS Answer: Q in Conductor

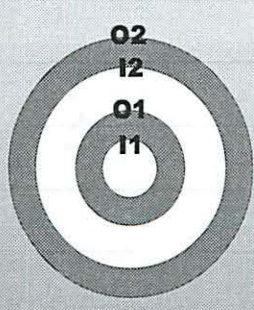
Answer: 2. is non-zero but does not change



E = 0 in conductor → -Q on inner surface
Charge conserved → +Q on outer surface
E = 0 in conductor → No "communication" between –Q & +Q → + Q uniformly distributed

P21-8

PRS Setup

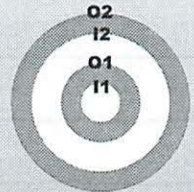


What happens if we put Q in the center of these nested (concentric) spherical conductors?

P21-9

PRS: Hollow Conductors

A point charge +Q is placed at the center of the conductors. The induced charges are:

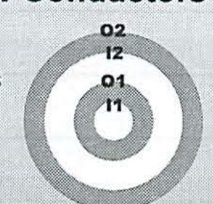


- 0% 1. $Q(I1) = Q(I2) = -Q$; $Q(O1) = Q(O2) = +Q$ ✓
- 0% 2. $Q(I1) = Q(I2) = +Q$; $Q(O1) = Q(O2) = -Q$
- 0% 3. $Q(I1) = -Q$; $Q(O1) = +Q$; $Q(I2) = Q(O2) = 0$
- 0% 4. $Q(I1) = -Q$; $Q(O2) = +Q$; $Q(O1) = Q(I2) = 0$

P21-10

PRS Answer: Hollow Conductors

Answer: 1. The inner faces are negative, the outer faces are positive.

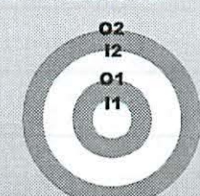


Looking in from each conductor, the total charge must be zero (this gives the inner surfaces as –Q). But the conductors must remain neutral (which makes the outer surfaces have induced charge +Q).

P21-11

PRS: Hollow Conductors

A point charge +Q is placed at the center of the conductors. The potential at O1 is:



- 0% 1. Higher than at I1
- 0% 2. Lower than at I1
- 0% 3. The same as at I1 ✓

P21-12

Need to look at all charges to see. No $\vec{E}$ field O1 → I1 - they are all tied up with $\vec{E}$ from (+) and I_2

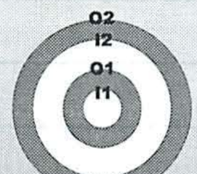
Same conductor = same potential

potential grows

does not change inside - no electric field

PRS Answer: Hollow Conductors

Answer: 3. O1 and I1 are at the same potential

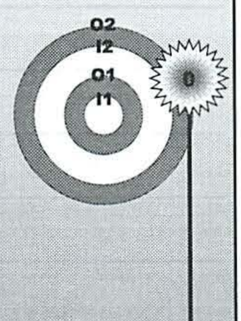


A conductor is an equipotential surface. O1 and I1 are on the same conductor, hence at the same potential

P21-43

PRS: Hollow Conductors

A point charge +Q is placed at the center of the conductors. The potential at O2 is:

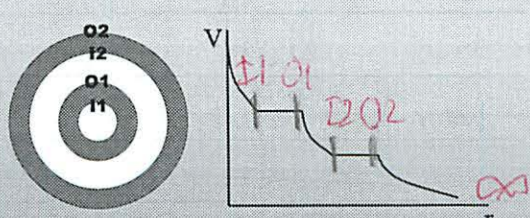


0% 1. Higher than at I1
0% 2. Lower than at I1
0% 3. The same as at I1

P21-44

PRS Answer: Hollow Conductors

Answer: 2. O2 is lower than I1

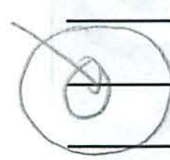


As you move away from the positive point charge at the center, the potential decreases.

P21-45

Well that is the group problem,

Calc V(b) from ∞



In the shell constant since they are = potentials

$V = \int E \cdot ds = \frac{kQ}{b}$

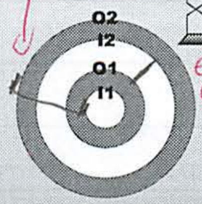
* Only from > O2
will need to $\int$

point charges + sphere of conductors

problem changed

PRS: Hollow Conductors

A point charge +Q is placed at the center of the conductors. If a wire is used to connect the two conductors, then current (positive charge) will flow



← original

0%	1.	from the inner to the outer conductor
0%	2.	from the outer to the inner conductor
0%	3.	not at all

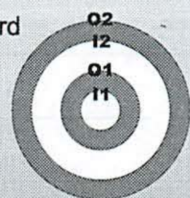
w/o charge

⊕ Would want to escape O1 to I2

↓⊖

PRS Answer: Hollow Conductors

Answer: 1. Current flows outward

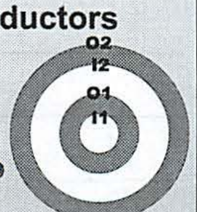


Positive charges always flow "downhill" – from high to low potential. Since the inner conductor is at a higher potential the charges will flow from the inner to the outer conductor.

Fields move to higher potential

PRS: Hollow Conductors

You connect the "charge sensor's" red lead to the inner conductor and black lead to the outer conductor. What does it actually measure?



0%	1.	Charge on I1
0%	2.	Charge on O1
0%	3.	Charge on I2
0%	4.	Charge on O2
0%	5.	Charge on O1 – Charge on I2
0%	6.	Average charge on inner – ave. on outer
0%	7.	Potential difference between outer & inner
0%	8.	I don't know

↑ $V(\infty)=0$ $V=\frac{kQ}{d}$

$r<a$ $E=\frac{kQ}{r^2}$

$a<r<b$ $E=0$

$b<r<c$ $E=\frac{kQ}{r^2}$ $\Delta V=-\int_a^b \frac{kQ}{r^2} = kQ\left(\frac{1}{b}-\frac{1}{c}\right)$

then if just want $V_B \to kQ\left(\frac{1}{b}-\frac{1}{c}\right)+\frac{kQ}{d}$

$\Delta V = V_B - V_C$

$\boxed{V_C = V_D = \frac{kQ}{d}}$

$c<r<d$ $E=0$ $V=\frac{kQ}{d}$

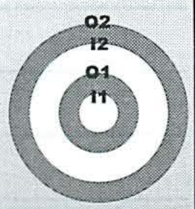
$r>d$ $E=\frac{kQ}{r^2}$ $V=-\int_\infty^d \frac{kQ}{r^2} = kQ\left(\frac{1}{d}\right)$

But with wire from I1 to O2

* When connected w/ wire → must be 0 equi potential - charges will still be able to move
* Think about potential
⊕ charge flows outwards
* Does not matter where attach wire - charges will find a way to move *

Don't remember

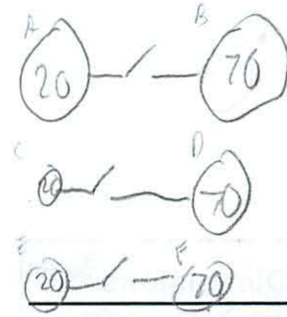
PRS Answer: Hollow Conductors



Answer: 7. "Charge Sensor" measures potential difference between outer & inner conductor

So what is the "charge axis?" From the capacitance and potential difference it can calculate Q = CΔV which is charge on O1 and negative charge on I2

P21-19



What is potential | switch closed

If attached by wire, potentials must =

$V_{sphere} = \frac{kQ}{r}$

$V_D > V_A$

Compare $\frac{kQ}{r}$ $\frac{kQ}{r}$

Uneven sizes = uneven charge distribution

45 A B 45

C $\frac{kQ_C}{r} = $ D $\frac{kQ_D}{r}$ k constant

find what r is

$Q_C + Q_D = 90$

conceptually D is larger

$V_E > V_C$

$V_C = V_D$

$V_D > V_A$

Capacitors

Capacitance

$$C = \frac{Q}{|\Delta V|}$$

To calculate:
1) Put on arbitrary ±Q
2) Calculate E
3) Calculate ΔV

In Series & Parallel

$$\frac{1}{C_{eq,\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} \qquad C_{eq,\text{parallel}} = C_1 + C_2$$

Energy

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q|\Delta V| = \frac{1}{2}C|\Delta V|^2 = \iiint u_E\, d^3r = \iiint \frac{\kappa\varepsilon_o E^2}{2} d^3r$$

P21-20

PRS Questions: Capacitors

P21-21

PRS: Changing Dimensions

A parallel-plate capacitor has plates with equal and opposite charges ±Q, separated by a distance d, and **is not** connected to a battery. The plates are pulled apart to a distance $D > d$. What happens?

0%	1.	V increases, Q increases
0%	2.	V decreases, Q increases
0%	3.	V is the same, Q increases
0%	4.	V increases,Q is the same
0%	5.	V decreases, Q is thesame
0%	6.	V is the same, Q is the same
0%	7.	V increases, Q decreases
0%	8.	V decreases, Q decreases
0%	9.	V is the same,Q decreases

P21 22

PRS Answer: Changing Dimensions

Answer: 4. V increases, Q is the same

With no battery connected to the plates the charge on them has no possibility of changing.

In this situation, the electric field doesn't change when you change the distance between the plates, so:

$$V = E\,d$$

As d increases, V increases.

P21 23

0 PRS: Changing Dimensions

A parallel-plate capacitor has plates with equal and opposite charges ±Q, separated by a distance d, and **is** connected to a battery. The plates are pulled apart to a distance $D > d$. What happens?

0%	1.	V increases, Q increases
0%	2.	V decreases, Q increases
0%	3.	V is the same, Q increases
0%	4.	V increases, Q is the same
0%	5.	V decreases, Q is the same
0%	6.	V is the same, Q is the same
0%	7.	V increases, Q decreases
0%	8.	V decreases, Q decreases
0%	9.	V is the same, Q decreases

P21 24

PRS Answer: Changing Dimensions

Answer: 9. V is the same, Q decreases

With a battery connected to the plates the potential V between them is held constant

In this situation, since

$$V = E\,d$$

As d increases, E must decrease.

Since the electric field is proportional to the charge on the plates, Q must decrease as well.

P21 25

PRS: Changing Dimensions

A parallel-plate capacitor, disconnected from a battery, has plates with equal and opposite charges, separated by a distance d.

Suppose the plates are pulled apart until separated by a distance $D > d$.

How does the final electrostatic energy stored in the capacitor compare to the initial energy?

0% 1. The final stored energy is smaller

0% 2. The final stored energy is larger

0% 3. Stored energy does not change.

P21 26

* Study pulling apart

← Energy came from you by pulling tem apart

PRS Answer: Changing Dimensions

Answer: 2. The stored energy increases

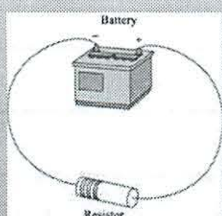
As you pull apart the capacitor plates you increase the amount of space in which the E field is non-zero and hence increase the stored energy. Where does the extra energy come from? From the work you do pulling the plates apart.

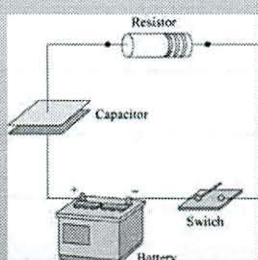
P21 27

DC Circuits

P21 - 28

Examples of Circuits





P21 - 29

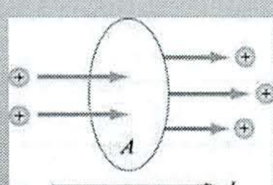
Current: Flow Of Charge

Average current I_{av}: Charge ΔQ flowing across area A in time Δt

$$I_{av} = \frac{\Delta Q}{\Delta t}$$

Instantaneous current: differential limit of I_{av}

$$I = \frac{dQ}{dt}$$



Units of Current: Coulomb/second = Ampere

P21 - 30

Add second light bulb in parallel - power (watt) output from batt is $\underline{2x}$ as much

$P = IV$

V: no change

I: doubles, eq resistance half

$= I^2 R$

Now add 2nd bulb in series

light from 1st bulb is

$P = IV$ * R has doubled

V: ~~same~~ half

I: ~~half~~ half

quarter = half · half

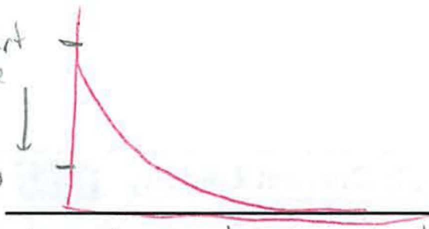
no light $\frac{1}{4}$

$P = I^2 R$ ← double

I decreases $\frac{1}{2}$ → $V = IR$

I: same; R: double; so half

* Did not copy RC graphs



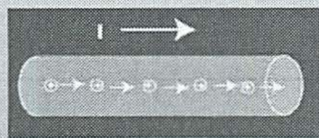
go down to ~ $\frac{1}{3}$ of original value (e)

now look where + is

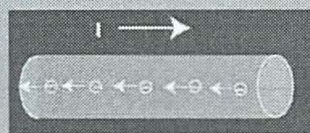
this is what J is

Direction of The Current

Direction of current is direction of flow of pos. charge



or, opposite direction of flow of negative charge



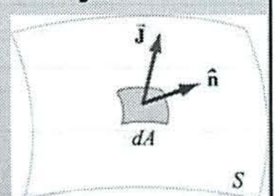
P21-31

Current Density J

J: current/unit area

$$\boxed{\vec{\mathbf{J}} \equiv \frac{I}{A}\hat{\mathbf{I}}}$$

$\hat{\mathbf{I}}$ points in direction of current



$$\boxed{I = \int_S \vec{\mathbf{J}} \cdot \hat{\mathbf{n}}\, dA = \int_S \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}}$$

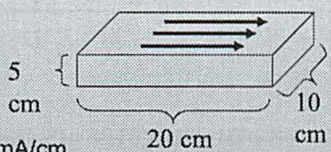
P21-32

PRS Question: Current Density

P21-33

PRS: Current Density

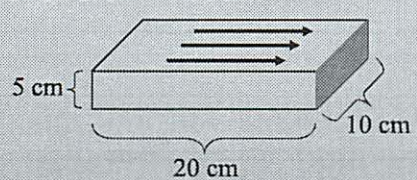
A current I = 200 mA flows in the above wire. What is the magnitude of the current density J?



0% 1. J = 40 mA/cm
0% 2. J = 20 mA/cm
0% 3. J = 10 mA/cm
0% 4. J = 1 mA/cm^2
0% 5. J = 2 mA/cm^2
0% 6. J = 4 mA/cm^2
0% 7. I don't know

PRS Answer: Current Density

Answer: 6. J = 4 mA/cm^2



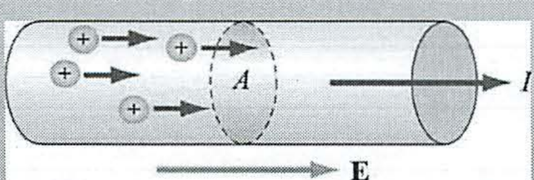
The area that matters is the cross-sectional area that the current is punching through – the 50 cm^2 area shaded grey.

So:

$J = I/A$ = 200 mA/50 cm^2 = 4 mA/cm^2

Why Does Current Flow?

If an electric field is set up in a conductor, charge will move (making a current in direction of E)

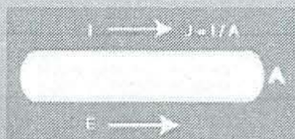


Note that when current is flowing, the conductor is not an equipotential surface (and $E_{inside} \neq 0$)!

Microscopic Ohm's Law

$$\boxed{\vec{\mathbf{E}} = \rho\vec{\mathbf{J}} \quad \text{or} \quad \vec{\mathbf{J}} = \sigma\vec{\mathbf{E}}}$$

$$\boxed{\rho \equiv \frac{1}{\sigma}}$$



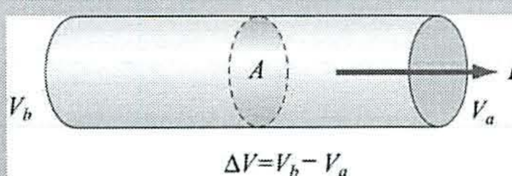
ρ and σ depend only on the microscopic properties of the material, not on its shape

σ: conductivity   ρ: resistivity

P21 - 37

Why Does Current Flow?

Instead of thinking of Electric Field, think of potential difference across the conductor

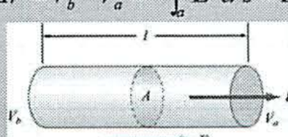


P21 - 38

Ohm's Law

What is relationship between ΔV and current?

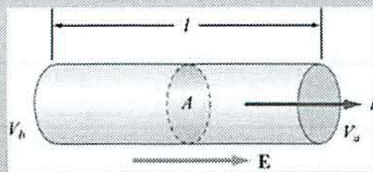
$$\Delta V = V_b - V_a = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E\ell$$



$$\left.\begin{array}{l} J = \dfrac{E}{\rho} = \dfrac{\Delta V / \ell}{\rho} \\ J = \dfrac{I}{A} \end{array}\right\} \Rightarrow \Delta V = I\left(\frac{\rho\ell}{A}\right) \equiv IR$$

P21 - 39

Ohm's Law



$$\boxed{\Delta V = IR}$$

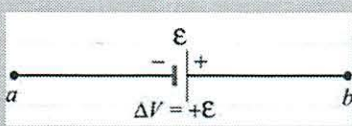
$$\boxed{R = \frac{\rho \ell}{A}}$$

R has units of Ohms (Ω) = Volts/Amp

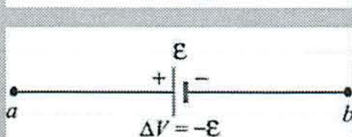
P21-40

Sign Conventions - Battery

Moving from the negative to positive terminal of a battery **increases** your potential



$$\boxed{\Delta V = V_b - V_a}$$

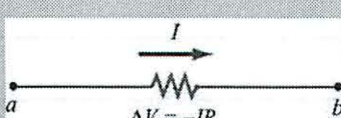


Think: Ski Lift

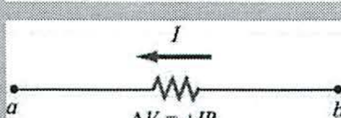
P21-41

Sign Conventions - Resistor

Moving across a resistor in the direction of current **decreases** your potential



$$\boxed{\Delta V = V_b - V_a}$$

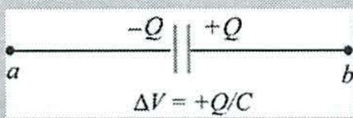


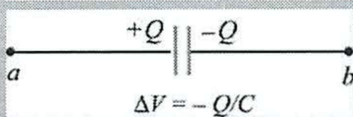
Think: Ski Slope

P21-42

Sign Conventions - Capacitor

Moving across a capacitor from the negatively to positively charged plate **increases** your potential



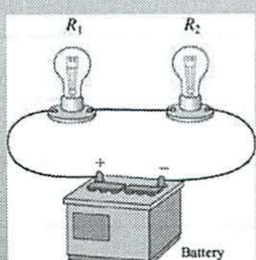


$$\boxed{\Delta V = V_b - V_a}$$

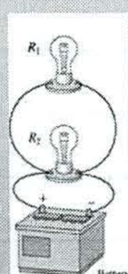
Think:
Ski Lodge

P21 - 43

Series vs. Parallel



Series

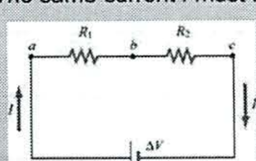


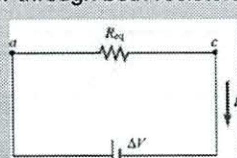
Parallel

P21 - 44

Resistors In Series

The same current *I* must flow through both resistors





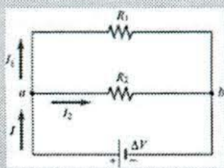
$$\Delta V = I\,R_1 + I\,R_2 = I(R_1 + R_2) = I\,R_{eq}$$

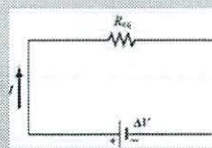
$$\boxed{R_{eq} = R_1 + R_2}$$

P21 - 45

Resistors In Parallel

Voltage drop across the resistors must be the same





$$\Delta V = \Delta V_1 = \Delta V_2 = I_1 R_1 = I_2 R_2 = I R_{eq}$$

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \frac{\Delta V}{R_{eq}} \qquad \boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

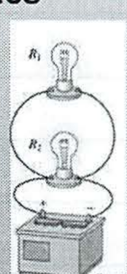
P21-46

PRS Questions: Two Light Bulbs

P21-47

0 PRS: Bulbs & Batteries

An ideal battery is hooked to a light bulb with wires. A second identical light bulb is connected in parallel to the first light bulb. After the second light bulb is connected, the current from the battery compared to when only one bulb was connected.

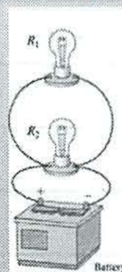


0% 1. Is Higher
0% 2. Is Lower
0% 3. Is The Same
0% 4. Don't know

P21-48

PRS Answer: Bulbs & Batteries

Answer: 1. More current flows from the battery



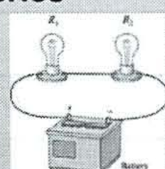
There are several ways to see this:

(A) The equivalent resistance of the two light bulbs in parallel is half that of one of the bulbs, and since the resistance is lower the current is higher, for a given voltage.

(B) The battery must keep two resistances at the same potential → I doubles.

:00 PRS: Bulbs & Batteries

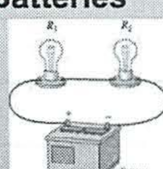
An ideal battery is hooked to a light bulb with wires. A second identical light bulb is connected in series with the first light bulb. After the second light bulb is connected, the current from the battery compared to when only one bulb was connected.



0% 1. Is Higher
0% 2. Is Lower
0% 3. Is The Same
0% 4. Don't know

PRS Answer: Bulbs & Batteries

Answer: 2. Less current flows from the battery



The equivalent resistance of the two light bulbs in series is twice that of one of the bulbs, and since the resistance is higher the current is lower, for the given voltage.

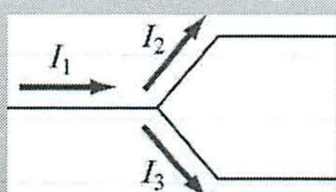
(Translation) The ski slope just got twice as hard so half as many skiers take it.

Kirchhoff's Loop Rules

P21 - 52

Kirchhoff's Rules

1. Sum of currents entering any junction in a circuit must equal sum of currents leaving that junction.



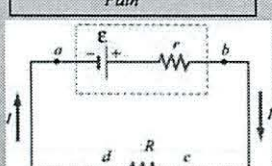
$$I_1 = I_2 + I_3$$

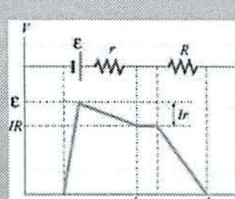
P21 - 53

Kirchhoff's Rules

2. Sum of potential differences across all elements around any closed circuit loop must be zero.

$$\boxed{\Delta V = -\oint_{\substack{\text{Closed} \\ \text{Path}}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0}$$

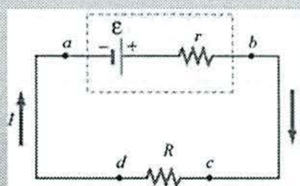




P21 - 54

Internal Resistance

Real batteries have an internal resistance, r, which is small but non-zero



Terminal voltage: $\Delta V = V_b - V_a = \mathcal{E} - I\,r$

(Even if you short the leads you don't get infinite current)

P21 -

Steps of Solving Circuit Problem

1. Straighten out circuit (make squares)
2. Simplify resistors in series/parallel
3. Assign current loops (arbitrary)
4. Write loop equations (1 per loop)
5. Solve

P21 - 36

Power

P21 - 37

Electrical Power

Power is change in energy per unit time

So power to move current through circuit elements:

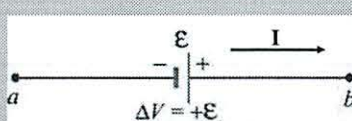
$$P = \frac{d}{dt}U = \frac{d}{dt}\left(q\Delta V\right) = \frac{dq}{dt}\Delta V$$

$$\boxed{P = I\,\Delta V}$$

P21-58

Power - Battery

Moving from the negative to positive terminal of a battery **increases** your potential. If current flows in that direction the battery **supplies** power

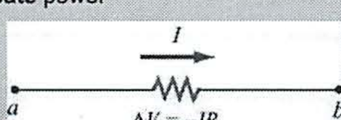


$$\boxed{P_{\text{supplied}} = I\,\Delta V = I\,\varepsilon}$$

P21-59

Power - Resistor

Moving across a resistor in the direction of current **decreases** your potential. Resistors **always dissipate** power



$$\boxed{P_{\text{dissipated}} = I\,\Delta V = I^2 R = \frac{\Delta V^2}{R}}$$

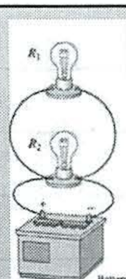
P21-60

PRS Questions: Two More Light Bulbs

P21-61

PRS: Power

An ideal battery is hooked to a light bulb with wires. A second identical light bulb is connected in parallel to the first light bulb. After the second light bulb is connected, the power output from the battery (compared to when only one bulb was connected)



0%	1.	Is four times higher
0%	2.	Is twice as high
0%	3.	Is the same
0%	4.	Is half as much
0%	5.	Is ¼ as much
0%	6.	Don't know

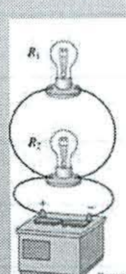
P21-62

PRS Answer: Power

Answer: 2. Is twice as high

The current from the battery must double (it must raise two light bulbs to the same voltage difference) and

$$P = I V$$

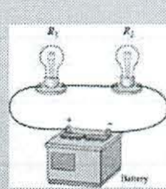


P21-63

PRS: Power

:00

An ideal battery is hooked to a light bulb with wires. A second identical light bulb is connected in series with the first light bulb. After the second light bulb is connected, the light (power) from the first bulb (compared to when only one bulb was connected)

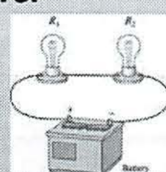


0% 1. Is four times higher
0% 2. Is twice as high
0% 3. Is the same
0% 4. Is half as much
0% 5. Is ¼ as much
0% 6. Don't know

P21 44

PRS Answer: Power

Answer: 5. Is 1/4 as bright



R doubles → current is cut in half. So power delivered by the battery is half what it was. But that power is further divided between two bulbs now.

Alternatively,

$$P = I^2R$$

P21 43

Right Hand Rules

1. Torque: Thumb = torque,
   Fingers show rotation
2. Feel: Thumb = I,
   Fingers = B,
   Palm = F
3. ~~3.~~ Create: Thumb = I
   Fingers (curl) = B
4. Moment: Fingers (curl) = I
   Thumb = Moment (=B inside loop)

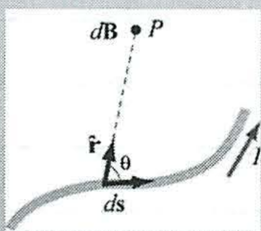
P21-56

May not use this exactly, but have one for these

Class 21

The Biot-Savart Law

Current element of length ds carrying current I (or equivalently charge q with velocity v) produces a magnetic field:

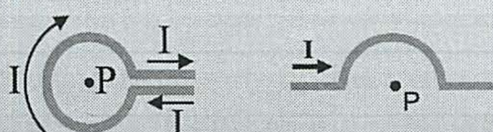


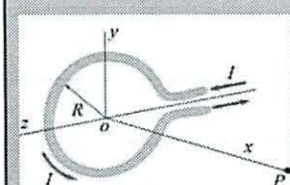
$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{q\,\vec{\mathbf{v}}\times\hat{\mathbf{r}}}{r^2}$$

$$\mathbf{d}\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$$

P21-67

Biot-Savart: 2 Problem Types





Notice that r is the same for every point on the loop. You don't really need to integrate (except to find path length)

P21-68

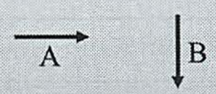
PRS Questions: Right Hand Rule

Class 15

P21-69

PRS: Cross Product

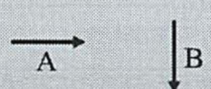
What is the direction of A x B given the following two vectors?



- 0% 1. up
- 0% 2. down
- 0% 3. left
- 0% 4. right
- 0% 5. into page
- 0% 6. out of page
- 0% 7. Cross product is zero (so no direction)

PRS Answer: Cross Product

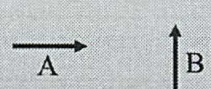
Answer: 5. A x B points into the page



Using your right hand, thumb along A, fingers along B, palm into page

PRS: Cross Product

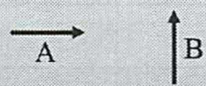
What is the direction of A x B given the following two vectors?



- 0% 1. up
- 0% 2. down
- 0% 3. left
- 0% 4. right
- 0% 5. into page
- 0% 6. out of page
- 0% 7. Cross product is zero (so no direction)

PRS Answer: Cross Product

Answer: 6. A x B points out of the page



Using your right hand, thumb along A, fingers along B, palm out of page

Also note from before, one vector flipped so result does too

P21-23

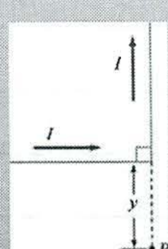
PRS Questions: Making B Fields

Classes 15-16

P21-24

PRS: Biot-Savart

The magnetic field at P points towards the

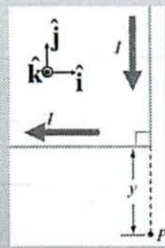


0% 1. +x direction
0% 2. +y direction
0% 3. +z direction
0% 4. -x direction
0% 5. -y direction
0% 6. -z direction
0% 7. Field is zero (so no direction)

P21-25

PRS Answer: Biot-Savart

Answer: 3. B(P) is in the +z direction (out of page)

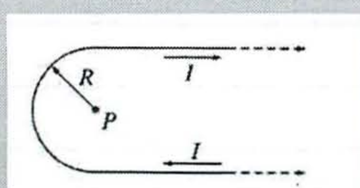


The vertical line segment contributes nothing to the field at P (it is parallel to the displacement). The horizontal segment makes a field out of the page.

P21-26

0 PRS: Bent Wire

The magnetic field at P is equal to the field of:

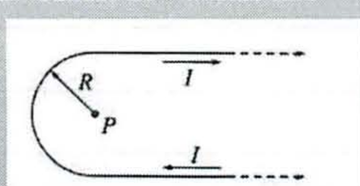


0%	1.	a semicircle
0%	2.	a semicircle plus the field of a long straight wire
0%	3.	a semicircle minus the field of a long straight wire
0%	4.	none of the above

P21-27

PRS Answer: Bent Wire

Answer: 2. Semicircle + infinite wire

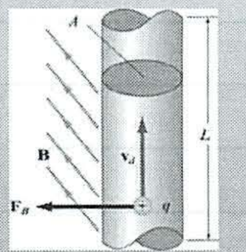


All of the wire makes B into the page. The two straight parts, if put together, would make an infinite wire. The semicircle is added to this to get the complete field

P21-28

Magnetic Force

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$



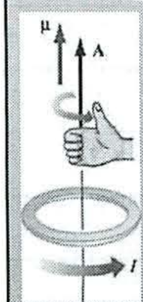
$$d\vec{\mathbf{F}}_B = Id\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

$$\vec{\mathbf{F}}_B = I\left(\vec{\mathbf{L}} \times \vec{\mathbf{B}}\right)$$

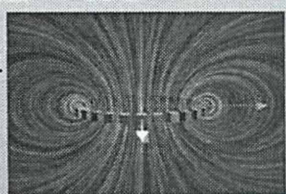
P21-79

Magnetic Dipole Moments

$$\vec{\mu} \equiv IA\hat{\mathbf{n}} \equiv I\vec{\mathbf{A}}$$



Generate:



Feel:

1) Torque to align with external field
2) Forces as for bar magnets $U = -\vec{\mu} \cdot \vec{\mathbf{B}}$

P21-80

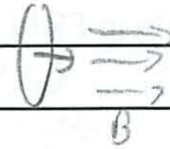
* No monopoles

* Diapoles are equivilant to bar magnet

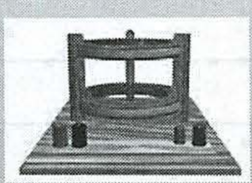
$\tau = \vec{\mu} \times \vec{B}$

$U_{Dipole} = -\vec{\mu} \cdot \vec{B}$

will want to reduce energy
- by alighing w/ $\vec{B}$



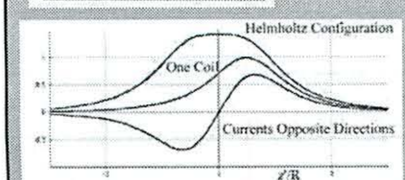
Helmholtz Coil



Common Concept Question

Parallel (Helmholtz) makes uniform field (torque, no force)

Anti-parallel makes zero, non-uniform field (force, no torque)



P21-81

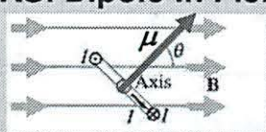
PRS Questions: Magnetic Dipole Moments

Class 19

P21 - 83

PRS: Dipole in Field

0



From rest, the coil above will:

0%	1.	rotate clockwise, not move
0%	2.	rotate counterclockwise, not move
0%	3.	move to the right, not rotate
0%	4.	move to the left, not rotate
0%	5.	move in another direction, without rotating
0%	6.	both move and rotate
0%	7.	neither rotate nor move
0%	8.	I don't know

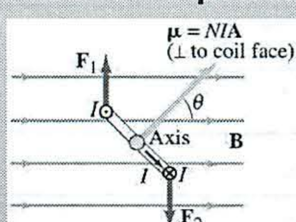
P21 43

~~torque to right~~
~~rotates clockwise~~

→ torqe into page ∈
rotates clockwise ✓

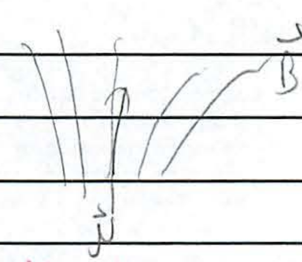
└ do right hand rule

PRS Answer: Dipole in Field



Answer: 1. Coil will rotate clockwise (not move)
No net force so no center of mass motion. BUT Magnetic dipoles rotate to align with external field (think compass)

P21 44



No torque
~~F = Il × B out~~

*B field stronger lower
dir of B field does not matter when it is aligned
force down

Class 21 more

$F_e = q\vec{E}$ $\qquad$ $F_q = q\vec{v} \times \vec{B}$

can do cross fields
- like the mass spectrometer

know cycltron

$$qvB = \frac{mv^2}{r}$$

Don't need to memorize equations
- helps if you do

8.02 Exam Two Spring 2010

P	L	A	S	M	E	I	E	R							

FAMILY (last) NAME

M	I	C	H	A	E	L									

GIVEN (first) NAME

9 2 1   6 4   5 2 6 1

Student ID Number

Your Section: X **L01 MW ~~F~~ 9 am** ____**L02 MW 11 am** ____**L03 MW 1 pm**
____**L04 MW 3 pm** ____**L05 TTh 9 am** ____**L06 TTh 11 am**
____**L07 TTh 1 pm** ____**L08 TTh 3 pm**

Your Group (e.g. 10A): 11 C

	Score	**Grader**
Problem 1 (25 points)	10	CM
Problem 2 (25 points)	17	+5 = 22
Problem 3 (25 points)	19	EJ
Problem 4 (25 points)	22	ADS
TOTAL	68	ASN

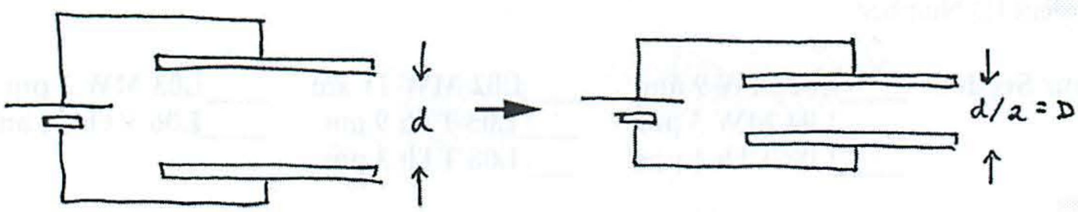
+5
73

Only studied 1 hr - very bad!

Problem 1: (25 points) Five Concept Questions. Please circle your answers.

Question 1: (5 points) Circle the correct answer.

Consider a simple parallel-plate capacitor whose plates are given equal and opposite charges and are separated by a distance d. The capacitor is connected to a battery. Suppose the plates are pushed together until they are separated by a distance D = d/2. How does the final electrostatic energy stored in the capacitor compare to the initial energy?



initial final

a) Final is half the initial.
b) Final is one fourth the initial.
c) Final is twice than initial.
d) Final is four times the initial.
e) They are the same.

Added energy pushed together

$$U = \frac{1}{2}\frac{Q^2}{C}$$

like problem on test

$$\boxed{\frac{1}{2}CV^2}$$

$$E \cdot 2\pi r^2 = \frac{Q}{\epsilon_0}$$

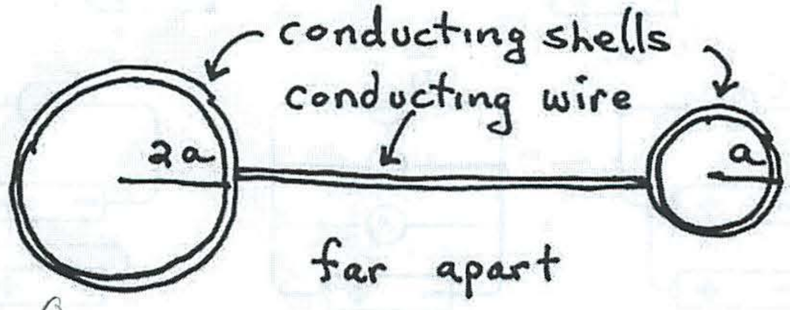
$$E = \frac{Q}{\epsilon_0 2\pi r^2} = \frac{\sigma\, 2\pi r^2}{\epsilon_0 2\pi r^2} = \frac{\sigma}{\epsilon_0}$$

$$V = -\int_0^d \frac{\sigma}{\epsilon_0}\, dr = \frac{\sigma}{\epsilon_0}\left(\frac{1}{d} - \frac{1}{0}\right) = \frac{\sigma}{\epsilon_0 d}$$

So should be twice

Question 2: (5 points) Circle the correct answer.

A conducting wire is attached to an initially charged spherical conducting shell of radius $2a$. The other end of the wire is attached to the outer surface of a neutral conducting spherical shell of radius a that is located a very large distance away (at infinity). When electrostatic equilibrium is reached, the charge on the shell of radius $2a$ is equal to



Q

a) one fourth the charge on the shell of radius a.

b) half the charge on the shell of radius a.

c) twice the charge on the shell of radius a.

d) four time the charge on the shell of radius a.

e) None of the above.

$$V = \frac{kQ_1}{2a} = \frac{kQ_2}{a}$$

$$10\ \frac{Q_1}{10} = 5\ \frac{Q_2}{5}$$

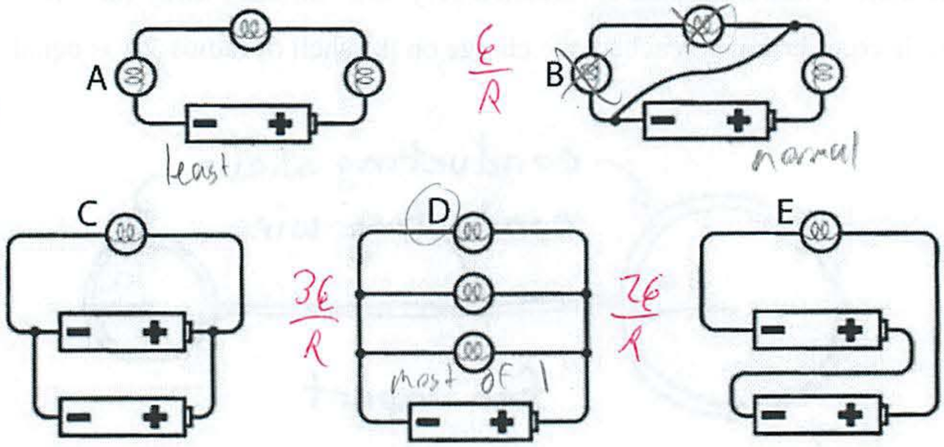
c

twice

Question 3: (5 points) Circle the correct answer.

What is the correct order for the *total power* dissipated in the following circuits, from least to greatest? Assume all bulbs and all batteries are identical. Ignore any internal resistance of the batteries.

$\frac{\mathcal{E}}{3R}$ $\frac{\mathcal{E}}{R}$



least normal

$\frac{\mathcal{E}}{R}$ $\frac{3\mathcal{E}}{R}$ $\frac{2\mathcal{E}}{R}$

most of I

2 batt in series

Lots of open paths

should never do

does B = C?

a) A < B < C < D < E

b) D < C < B < E < A

c) D < B < E < A < C

d) A = B < D < C < E

e) B < A < C = D < E

don't know that does not look like it

$P = IV$

where most current

$A \neq B$

Forgot batt in series parallel, guess adds

$\mathcal{E} + \mathcal{E} - IR$

oh want total power (not each light bulb)

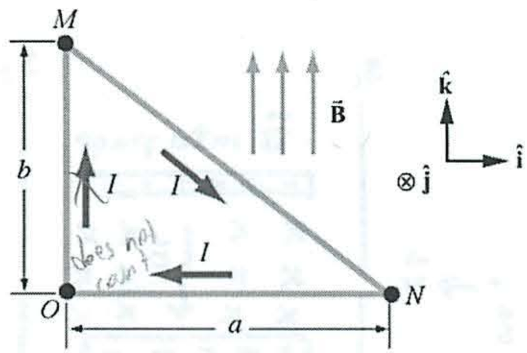
$A = B$

$A \to B \to D$

power not current

Question 4: (5 points) Circle the correct answer.

Consider a triangular loop of wire with sides a and b. The loop carries a current I in the direction shown, and is placed in a uniform magnetic field that has magnitude B and points in the same direction as the current in side OM of the loop.



At the moment shown in the figure the torque on the current loop

a) points in the $-\hat{\mathbf{i}}$-direction and has magnitude $IabB/2$.

b) points in the $+\hat{\mathbf{i}}$-direction and has magnitude $IabB/2$.

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

c) points in the $-\hat{\mathbf{j}}$-direction and has magnitude $IabB/2$.

← dipole moment

d) points in the $+\hat{\mathbf{j}}$-direction and has magnitude $IabB/2$.

all around loop

$$= I A \times \vec{B}$$

e) points in the $-\hat{\mathbf{i}}$-direction and has magnitude $IabB$.

f) points in the $+\hat{\mathbf{i}}$-direction and has magnitude $IabB$.

$$\vec{I} \cdot \frac{ab}{2} \times \vec{B}$$

g) points in the $-\hat{\mathbf{j}}$-direction and has magnitude $IabB$.

h) points in the $+\hat{\mathbf{j}}$-direction and has magnitude $IabB$.

I around closed loop

i) None of the above.

= 0 what?

wants to torque up

0 force?

k = moment $\qquad \vec{J} \times \vec{k} = \uparrow$

0 current?

0 field?

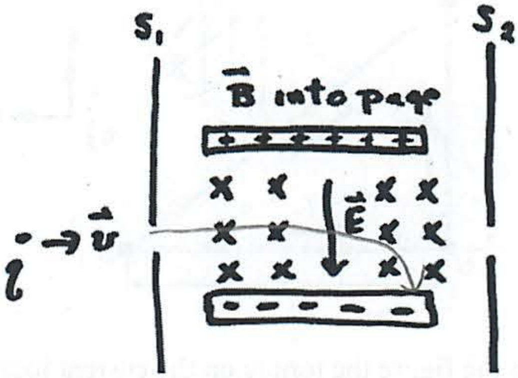
the one dipole qu

and of course I am unsure

Question 5: (5 points) Circle the correct answer.

A particle with charge q and velocity $\vec{v}$ enters through the hole in screen 1 and passes through a region with non-zero electric and magnetic fields (see sketch). If $q < 0$ and the magnitude of the electric field E is greater than the product of the magnitude of the initial velocity v and the magnitude of the magnetic field B, that is $E > vB$, then the force on the particle

A charge



$B \uparrow$

$E \downarrow$ ← bigger

a) is zero and the particle will move in a straight line and pass through the hole on screen 2.

b) is constant and the particle will follow a parabolic trajectory hitting the screen 2 above the hole.

c) is constant and the particle will follow a parabolic trajectory hitting screen 2 below the hole.

complex changing directory

d) is constant in magnitude but changes direction and the particle will follow a circular trajectory hitting the screen 2 above the hole.

e) is constant in magnitude but changes direction and the particle will follow a circular trajectory hitting the screen 2 below the hole.

f) changes magnitude and direction and the particle will follow a curved trajectory hitting the screen 2 above the hole. goes up → ⊖ particle E field bigger

g) changes magnitude and direction and the particle will follow a curved trajectory hitting the screen 2 below the hole.

E field so can change in magnitude

— but does it ??

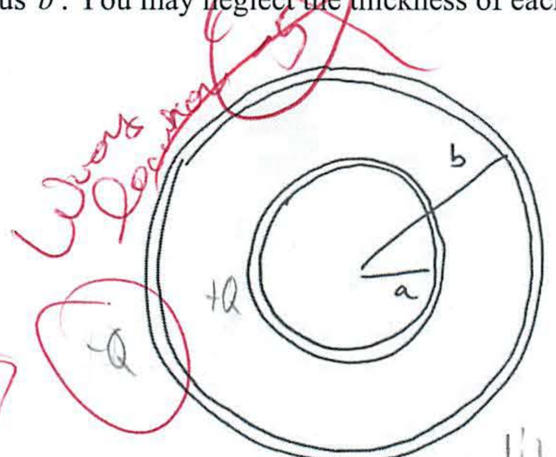
$\vec{B}$ changing as it curves complexly

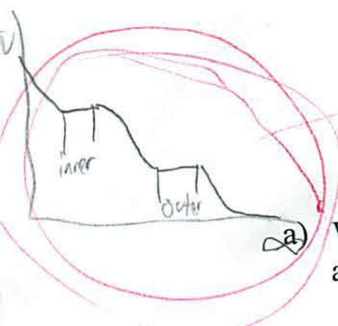
E also effects it as it curves → pulled down faster towards end ?

Problem 2 (25 points)

NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!) .

Consider a spherical vacuum capacitor consisting of inner and outer thin conducting spherical shells with charge $+Q$ on the inner shell of radius a and charge $-Q$ on the outer shell of radius b. You may neglect the thickness of each shell.





did in class
(reviewed somewhat today - remember

a) What are the magnitude and direction of the electric field everywhere in space as a function of r, the distance from the center of the spherical conductors?

$r < a$

$$\oint E \cdot dA = \frac{q_{inc}}{\epsilon_0}$$

$$E \cdot 4\pi r^2 = \cancel{\frac{\rho V}{\epsilon_0} = \rho \tfrac{4}{3}\pi r^3} \quad \frac{Q}{\epsilon_0}$$

$$E = \cancel{\frac{\rho 4\pi r^3}{\epsilon_0 4\pi r^3} = \frac{\rho r}{3\epsilon_0}} = \quad E = \frac{Q}{\epsilon_0 4\pi r^2} = \frac{kQ}{r^2} = 0$$

$$\boxed{k = \frac{1}{\epsilon_0 4\pi}}$$

nothing inclosed

$r = a$ $r = b$

$E = 0$ inside conductor

$a < r < b$

$$\oint E \cdot dA = \frac{q_{inc}}{\epsilon_0}$$

$$E = \frac{kQ}{r^2} \hat{r}$$

-2

$b > r$

$E = \frac{kQ}{r^2}$ looks like a point charge

falls off r^2

b) What is the capacitance of this capacitor?

$C = \frac{Q}{|V|}$

why so terrible (remembering formulas

$V = \left| -\int E \cdot ds \right| = \left| -\int_a^b \frac{kQ}{r^2}\, dr \right| = kQ\left(\frac{1}{a} - \frac{1}{b}\right)$

don't remember this

$C = \frac{Q}{kQ\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{1}{k\left(\frac{1}{a} - \frac{1}{b}\right)}$

c) Now consider the case that the dimension of the outer shell is doubled from b to $2b$. Assuming that the charge on the shells is not changed, how does the stored potential energy change? That is, find an expression for $\Delta U \equiv U_{after} - U_{before}$ in terms of a, b, and ε_0. and Q

$$U = \frac{1}{2}\frac{Q^2}{C}$$

$$U_{before} = \frac{1}{2}\frac{Q^2}{\left[\frac{1}{k\left(\frac{1}{a}-\frac{1}{b}\right)}\right]}$$

$$U_{before} = \frac{1}{2} Q^2 \cdot k\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$U_{after} = \frac{1}{2} Q^2 \cdot \left[k\left(\frac{1}{a}-\frac{1}{2b}\right) - k\left(\frac{1}{a}-\frac{1}{b}\right)\right]$$

Now if you really want in terms of ϵ_0

$$U_{before} = \frac{1}{2} \cdot Q^2 \cdot \frac{1}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$= \frac{Q^2}{8\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$U_{after} = \frac{Q^2}{8\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{2b}\right)$$

mcq -1

don't remember what test ans look like

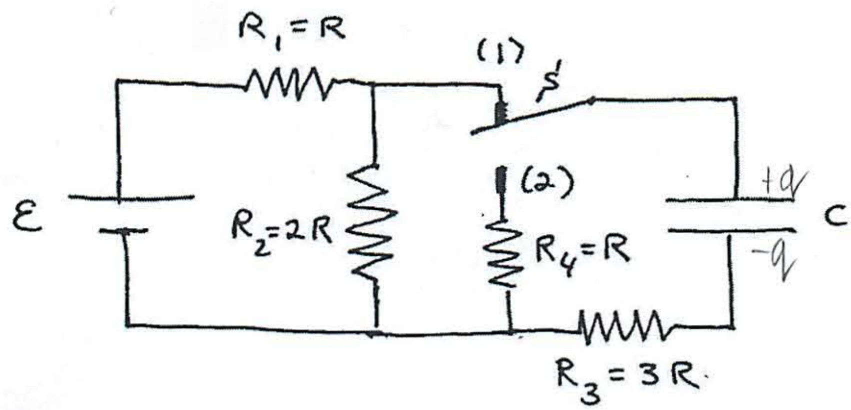
$$= \frac{kQ^2}{4b}$$

19

Problem 3 (25 points)

NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!) .

Consider the following circuit shown in the figure below. All questions can be answered without solving any differential equations.



a) Find the current through each of the four resistors, R_1, R_2, R_3, and R_4, a long time after the switch S has been in position (1).

Current through $R_4 = 0$ since not a complete branch

Current through resistor R_3 will be 0 because the capacitor has filled up after a long time and thus no current flows through this branch

Now we are left with

The 2 resistors could be combined to make 1 resistor

$$R_{eq} = R_1 + R_2$$
$$= 3R$$

$$\varepsilon - IR - I\,2R = 0$$
$$\varepsilon - I3R = 0$$
$$\varepsilon = I3R$$

$$\boxed{I = \frac{\varepsilon}{3R}}$$

b) Find the absolute value of the potential difference $|V_C|$ across the capacitor a long time after the switch S has been in position (1).

$Q = C|\Delta V|$

$\Delta V = \frac{Q}{C}$

When first connected

$$\begin{cases} \mathcal{E} - I3R = 0 \\ \mathcal{E} - \frac{Q}{C} - I4R = 0 \end{cases}$$

-3

~~$\mathcal{E} - I3R = \mathcal{E} - \frac{Q}{C} - I3R$~~

~~$-\mathcal{E} \quad +I3R \quad -\mathcal{E} \quad +I3R$~~

~~$0 = -Q/C$~~ ← initially - but after long time

easy

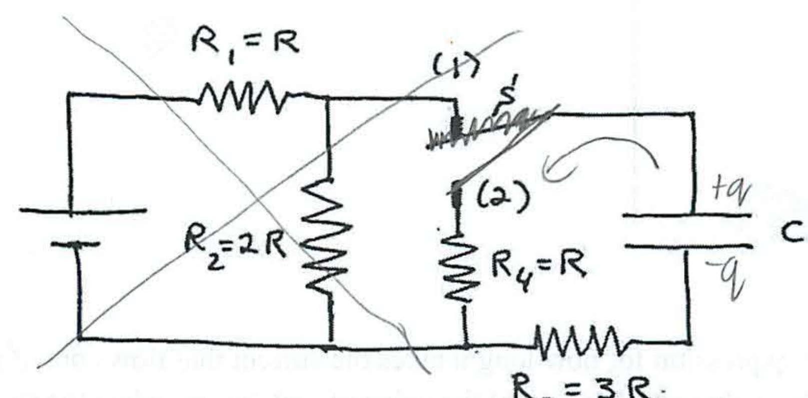
(see pg 21)

?

?, seems wrong - should have looked over

did not pay attention to +q -q

c) At $t = 0$ the switch is moved to position (2). What current will flow out of the capacitor at the instant the switch is moved to position (2)? Indicate whether the current will flow up or down in the branch of the circuit containing the capacitor.



Current will flow from +q plate on the capacitor around the loop

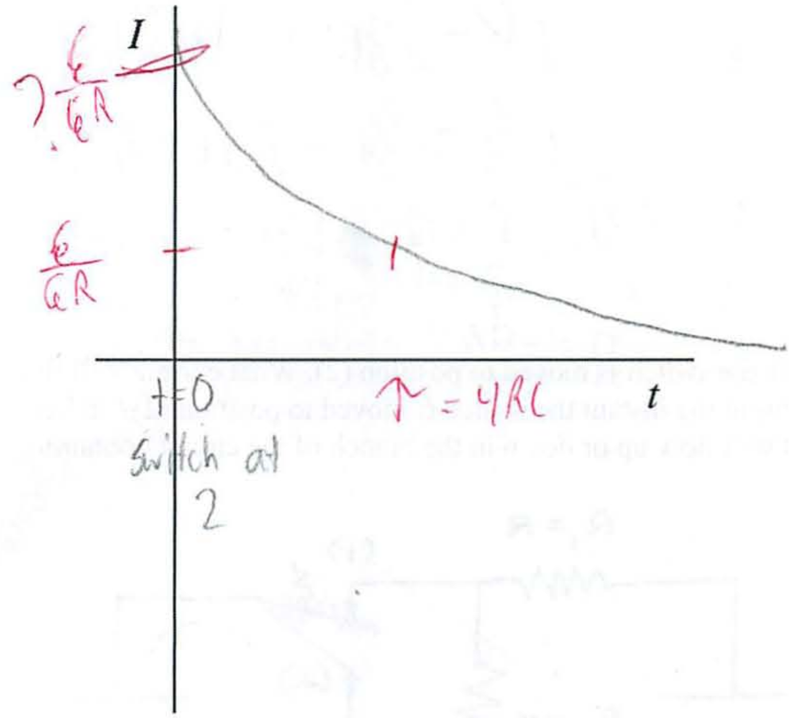
$\frac{Q}{C} - IR - I3R = 0$

$\frac{Q}{C} - 4IR = 0$

$\frac{Q}{C} = 4IR$ ✓

$I = \frac{Q}{4RC}$

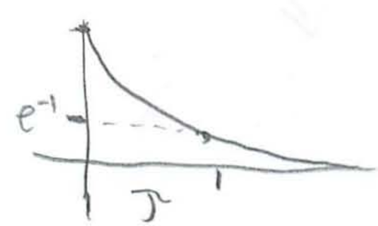
d) Make a graph of current vs. time for the current that flows out of the capacitor after the switch is moved to position (2) at $t = 0$. Indicate the value of the current at time $t = 0$ on your graph.



lable

e) Find an expression for how long it takes the current that flows out of the capacitor to reach a value equal to e^{-1} of the value of that current when the switch is moved to position (2) at $t = 0$. (You can answer this question without solving a differential equation.)

τ



e^{-1} will be the Δx

τ will be the Δy

Current will fall to be e^{-1} of current value after $\tau = 4RC$ seconds

f) After a long period in position (2), the switch is thrown to position (1) again. Immediately after the switch has been thrown to position (1), find the current through the battery.

This is the same as the inital problem part a except its now immeditly

~~$\mathcal{E} - I3R = 0$ $\boxed{?}$~~

~~$\mathcal{E} - IR - \frac{Q}{C} - I\,3R = 0$~~

~~$\mathcal{E} - I\,4R - \frac{Q}{C} = 0$~~

~~$\mathcal{E} - I_1 3R = \mathcal{E} - I_2 4R - \frac{Q}{C}$~~

Just did this in part b on pg 21

$\mathcal{E} - \frac{Q}{C} - \frac{11}{5} I R = 0$

like a wire

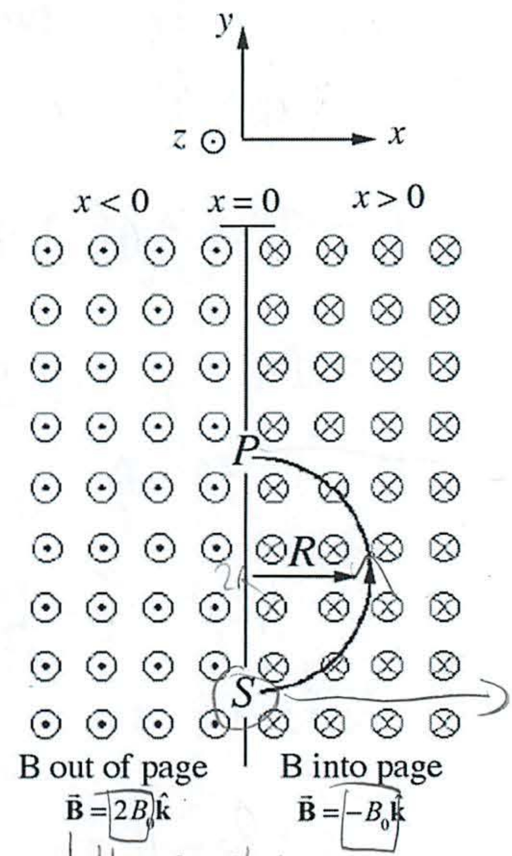
$\mathcal{E} = \frac{11}{5} I R$

$\frac{5\mathcal{E}}{11R} = I$

Problem 4 (25 points)

NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!) .

The $x-y$ plane for $x<0$ is filled with a uniform magnetic field pointing out of the page, $\vec{\mathbf{B}} = 2B_0\,\hat{\mathbf{k}}$ with $B_0 > 0$, as shown. The $x-y$ plane for $x>0$ is filled with a uniform magnetic field $\vec{\mathbf{B}} = -B_0\,\hat{\mathbf{k}}$, pointing into the page, as shown. A charged particle with mass m and charge q is initially at the point S at $x=0$, moving in the positive x-direction with speed v. It subsequently moves counterclockwise in a circle of radius R, returning to $x=0$ at point P, a distance $2R$ from its initial position, as shown in the sketch.



doubly as strong

a) Is the charge positive or negative? Briefly explain your reasoning.

So just deal w/ one field here

~~It is negative.~~ tricky - start at S not P

$\vec{V} \times \vec{B}$ would be up $(\hat{z})$ ~~but the particle~~ and the particle goes up so positive remember right? hopefully got right

b) Find an expression for the radius R of the trajectory shown, in terms of v, m, q, and B_0.

$$qvB = \frac{mv^2}{R}$$

$$RqvB = mv^2$$

$$R = \frac{mv^2}{qvB} = \frac{mv}{qB}$$

πr^2

$\frac{1}{3}\pi r$

c) How long does the particle take to return to the plane $x = 0$ at point P?

So it has gone $\frac{1}{2}$ of loop

traveled distance $\frac{d}{2} \cdot \frac{1}{3}\pi R$

$\frac{1}{6}\pi R$

$\cancel{v = \frac{\Delta x}{\Delta t}}$

$v = \frac{d}{t}$

$t = \frac{d}{v}$

$$\frac{RqvB}{v} = \frac{mv^2}{v}$$

$$RqB = mv$$

$$v = \frac{RqB}{m}$$

$$t = \frac{\frac{1}{6}\pi R}{\frac{RqB}{m}}$$

$\cancel{\frac{RqB}{m} = \frac{mv}{qB}\,/\,t}$

$$\frac{1}{6}\pi R \cdot \frac{m}{RqB}$$

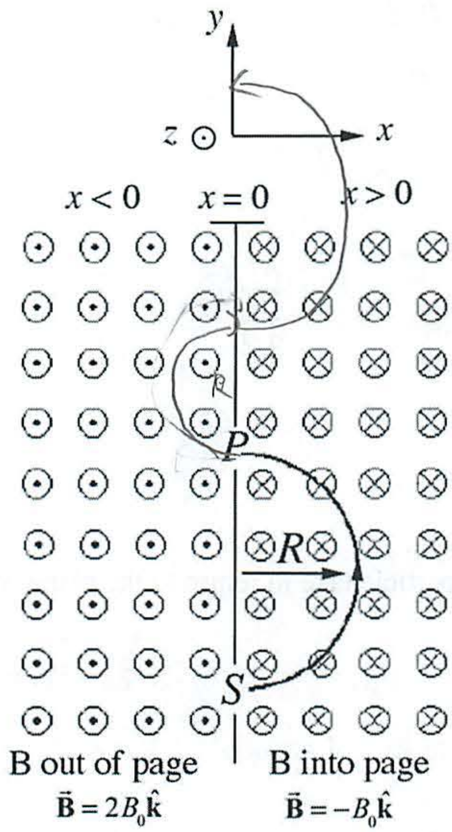
$$\boxed{t = \frac{\pi m}{6qB}}$$ -3

A r r

correct ???

did not remember circumference of circle

d) Describe and sketch the subsequent trajectory of the particle on the figure below after it passes point P. Be sure to define any relevant distances in terms of v, m, q, and B_0.



[assuming field continues

$r = \frac{mV}{qB}$ ← B is double, so radius will be ½ R

also B is different direction, so it will curve up

It will continue to make its way upward indefinitely

right

$R_R = \frac{mV}{q\,B_0}$

left

$R_L = \frac{mV}{q2B_0}$ which makes $R_L = \frac{1}{2} R_R$

3b after a long time

lets try R_{eq}

$$\frac{1}{R_{eq}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{2R} + \frac{1}{3R}$$

$$R_{eq} = \frac{R_2 R_3}{R_2 + R_3} = \frac{6R^2}{5R} = \frac{6R}{5} = \frac{6}{5}R$$

$$R_{eq}' = R_1 + R_{eq} = R + \frac{6}{5}R$$

$$= \frac{11}{5}R$$

$$\varepsilon - \frac{Q}{C} - \frac{11}{5}IR = 0$$

Solve for $\frac{Q}{C}$

$$\frac{Q}{C} = \varepsilon - \frac{11}{5}IR$$

↑ do we need to find I

this gets into differential,

should have reviewed

don't remember

what ans

looks like

w/ this type

$\boxed{\frac{26}{3}}$

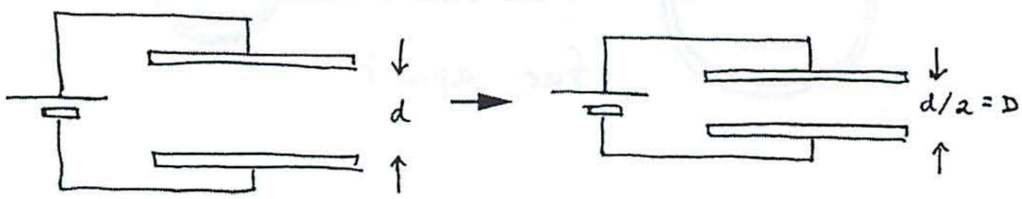
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 Spring 2010

8.02 Exam Two Spring 2010 Solutions

Question 1: (5 points) Circle the correct answer.

Consider a simple parallel-plate capacitor whose plates are given equal and opposite charges and are separated by a distance d. The capacitor is connected to a battery. Suppose the plates are pushed together until they are separated by a distance D = d/2. How does the final electrostatic energy stored in the capacitor compare to the initial energy?



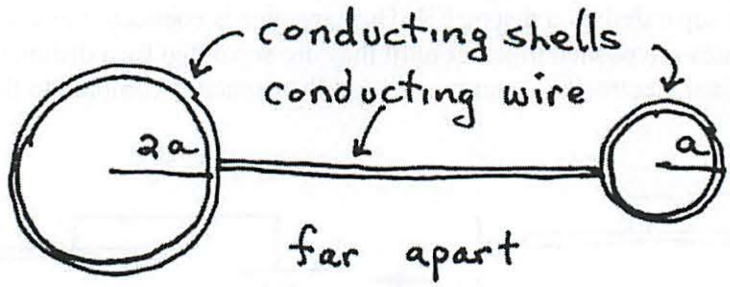
initial final

a) Final is half the initial.
b) Final is one fourth the initial.
c) Final is twice than initial.
d) Final is four times the initial.
e) They are the same.

Answer c. Because the capacitor is connected to the battery, the potential difference across the plates is constant. Therefore the ratio of the final stored energy to the initial stored energy is proportional ratio of the final capacitance to the initial capacitance, $U_f / U_i = \frac{1}{2} C_f \Delta V_C^2 / \frac{1}{2} C_i \Delta V_C^2 = C_f / C_i$. For a parallel plate capacitor, the capacitance is inversely proportional to the distance separating the plates, $C = \dfrac{Q}{\Delta V} = \dfrac{Q}{Ed} = \dfrac{\sigma A}{(\sigma / \varepsilon_0) d} = \dfrac{\varepsilon_0 A}{d}$. Therefore the ratio of the capacitance is $C_f / C_i = d_i / d_f = d / (d/2) = 2$. So $U_f / U_i = 2$.

Question 2: (5 points) Circle the correct answer.

A conducting wire is attached to an initially charged spherical conducting shell of radius $2a$. The other end of the wire is attached to the outer surface of a neutral conducting spherical shell of radius a that is located a very large distance away (at infinity). When electrostatic equilibrium is reached, the charge on the shell of radius $2a$ is equal to



a) one fourth the charge on the shell of radius a.

b) half the charge on the shell of radius a.

c) twice the charge on the shell of radius a.

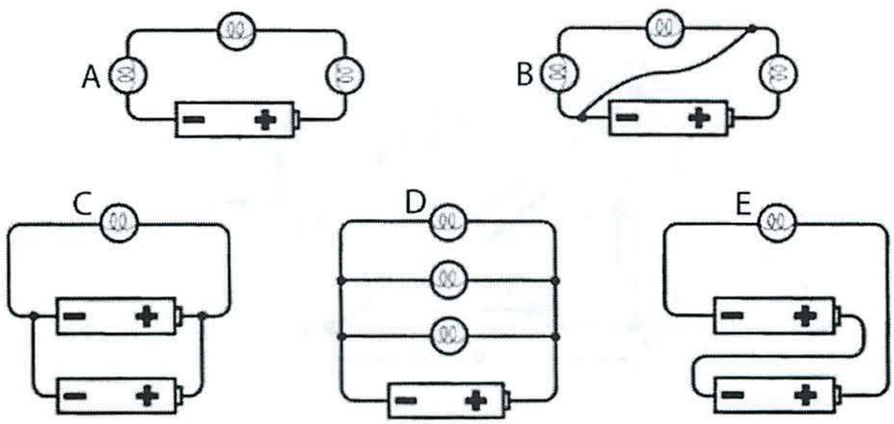
d) four time the charge on the shell of radius a.

e) None of the above.

Answer c. When electrostatic equilibrium is reached, the two shells form one conducting surface and hence the potential on that surface is constant. Because the two shells are very far apart, the potential of each shell with respect to infinity can be calculated separated. Because the electric field outside each charged shell is identical to the electric field of a point-like object with the same charge located at the center of the shell. The potential on each shell with respect to infinity is just $Q/4\pi\varepsilon_0 r$. The potential difference between the two shells is zero, or $Q_{2a}/4\pi\varepsilon_0 2a - Q_a/4\pi\varepsilon_0 a = 0$, or $Q_{2a} = 2Q_a$. Therefore the charge on the shell of radius $2a$ is equal to twice the charge on the shell of radius a.

Question 3: (5 points) Circle the correct answer.

What is the correct order for the *total power* dissipated in the following circuits, from least to greatest? Assume all bulbs and all batteries are identical. Ignore any internal resistance of the batteries.



a) $A < B = C < D < E$

b) $D < C < B = E < A$

c) $D < B < E < A < C$

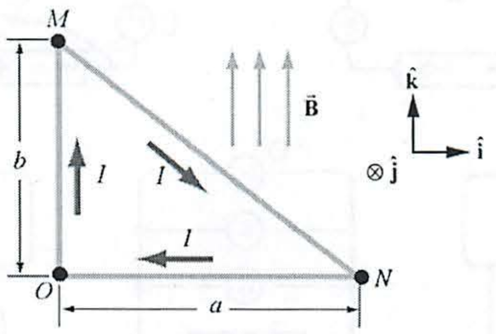
d) $A = B < D < C < E$

e) $B < A < C = D < E$

Answer a. The power dissipated in the circuits above is the equal to the power generated by the batteries. For a battery with a current I and an electromotive force V, the power generated by the battery is $P = IV$. The current from the battery in case A is $I_A = V/3R$, hence the power dissipated is $P_A = I_A V = V^2/3R$. The two resistors in series are shorted out in B, hence the current from the battery in case B is $I_B = V/R$, hence the power dissipated is $P_B = I_B V = V^2/R$. In case C, the current through the bulb is $I_C = V/R$. Because there is only one bulb, we can calculate the power dissipated across the bulb $P_C = I_C^2 R = V^2/R$. In case D the equivalent resistance is $R_D = R/3$. So the current from the battery in case D is $I_B = 3V/R$, hence the power dissipated is $P_D = I_D V = 3V^2/R$. Finally in case E, the electromotive force driving the current is $2V$, hence the current through the bulb is $I_E = 2V/R$. Because there is only one bulb, we can calculate the power dissipated across the bulb $P_E = I_E^2 R = 4V^2/R$. Therefore comparing our results we have that the correct order for the *total power* dissipated in the following circuits is $A < B = C < D < E$.

Question 4: (5 points) Circle the correct answer.

Consider a triangular loop of wire with sides a and b. The loop carries a current I in the direction shown, and is placed in a uniform magnetic field that has magnitude B and points in the same direction as the current in side OM of the loop.



At the moment shown in the figure the torque on the current loop

a) points in the $-\hat{\mathbf{i}}$-direction and has magnitude $IabB/2$.

b) points in the $+\hat{\mathbf{i}}$-direction and has magnitude $IabB/2$.

c) points in the $-\hat{\mathbf{j}}$-direction and has magnitude $IabB/2$.

d) points in the $+\hat{\mathbf{j}}$-direction and has magnitude $IabB/2$.

e) points in the $-\hat{\mathbf{i}}$-direction and has magnitude $IabB$.

f) points in the $+\hat{\mathbf{i}}$-direction and has magnitude $IabB$.

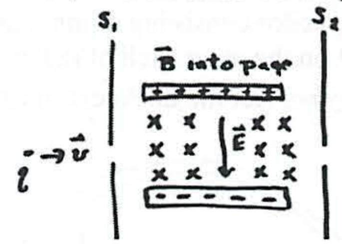
g) points in the $-\hat{\mathbf{j}}$-direction and has magnitude $IabB$.

h) points in the $+\hat{\mathbf{j}}$-direction and has magnitude $IabB$.

i) None of the above.

Answer b. The magnetic dipole moment vector is $\vec{\mu} = Iab/2\hat{j}$. The torque on the current loop is then $\vec{\tau} = \vec{\mu} \times \vec{B} = (Iab/2)\hat{j} \times B\hat{k} = (IabB/2)\hat{i}$.

Question 5: (5 points) Circle the correct answer. A particle with charge q and velocity $\vec{v}$ enters through the hole in screen 1 and passes through a region with non-zero electric and magnetic fields (see sketch). If $q < 0$ and the magnitude of the electric field E is greater than the product of the magnitude of the initial velocity v and the magnitude of the magnetic field B, that is $E > vB$, then the force on the particle



a) is zero and the particle will move in a straight line and pass through the hole on screen 2.

b) is constant and the particle will follow a parabolic trajectory hitting the screen 2 above the hole.

c) is constant and the particle will follow a parabolic trajectory hitting screen 2 below the hole.

d) is constant in magnitude but changes direction and the particle will follow a circular trajectory hitting the screen 2 above the hole.

e) is constant in magnitude but changes direction and the particle will follow a circular trajectory hitting the screen 2 below the hole.

f) changes magnitude and direction and the particle will follow a curved trajectory hitting the screen 2 above the hole.

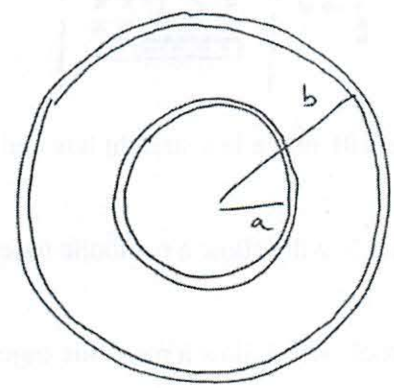
g) changes magnitude and direction and the particle will follow a curved trajectory hitting the screen 2 below the hole.

Answer f. When the particle enters the region where the fields are non-zero, the electric force is points upwards for a negatively charged particle and is greater in magnitude then the downward magnetic force. Both electric and magnetic forces are perpendicular to the particle's velocity and the particle starts to curve upwards. The electric force is always upwards but the magnetic force changes direction as the particle moves along a curved trajectory, so the direction of the force changes. It turns out that the magnitude of the force while the particle is between the plates is constant but does not point to a central point so the trajectory of the particle is not circular. Assuming that the time it takes the particle to cross the plates is smaller than $-\pi m / qB$, when the particle leaves the region between the plates the slope of the trajectory of the particle points upward, and so the particle will strike screen 2 above the hole. Because the fields in this region outside the plates are now zero, the force is zero so the magnitude of the force has changed.

Problem 2 (25 points)

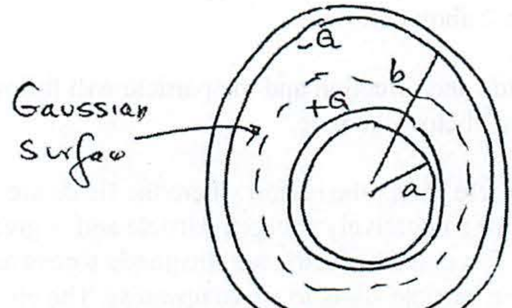
NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!) .

Consider a spherical vacuum capacitor consisting of inner and outer thin conducting spherical shells with charge $+Q$ on the inner shell of radius a and charge $-Q$ on the outer shell of radius b . You may neglect the thickness of each shell.



a) What are the magnitude and direction of the electric field everywhere in space as a function of r, the distance from the center of the spherical conductors?

Answer. There are three regions $r < a$, $a < r < b$, and $b < r$. The electric field is zero for $r < a$ and $b < r$ because the charge enclosed in a Gaussian sphere of radius r is zero for both of those regions.



For the region $a < r < b$, Gauss's Law implies that $E 4\pi r^2 = Q / \varepsilon_0$. Hence the magnitude of the electric field is $E = Q / 4\pi\varepsilon_0 r^2$ and the direction is radially outward.

$$\vec{E} = \begin{cases} \vec{0} & ; r < a \\ = \dfrac{Q}{4\pi\varepsilon_0 r^2}\hat{r} & ; a < r < b \\ \vec{0} & ; b < r \end{cases}$$

b) What is the capacitance of this capacitor?

Answer: The capacitance is given by

$$C = \frac{Q}{|\Delta V_C|} = \frac{Q}{-\int_{r=b}^{r=a} \vec{E} \cdot d\vec{s}} = \frac{Q}{-\int_{r=b}^{r=a} \frac{Q}{4\pi\varepsilon_0 r^2} dr} = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{4\pi\varepsilon_0 ab}{(b-a)}.$$

c) Now consider the case that the dimension of the outer shell is doubled from b to $2b$. Assuming that the charge on the shells is not changed, how does the stored potential energy change? That is, find an expression for $\Delta U \equiv U_{after} - U_{before}$ in terms of Q a, b, and ε_0 as needed.

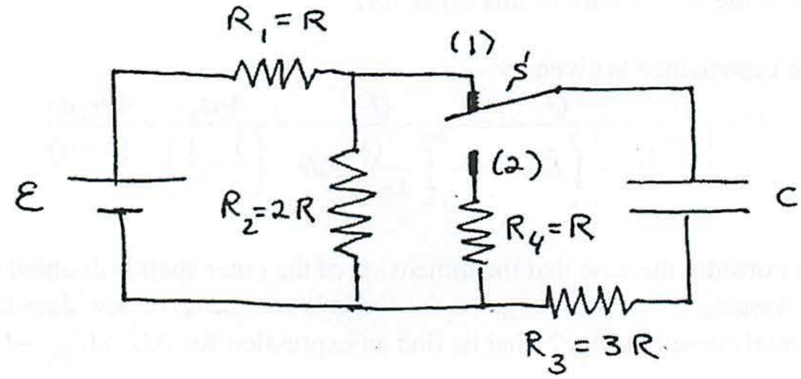
Answer: The energy stored in the capacitor is $U = Q^2 / 2C$. Therefore the change in stored energy is

$$\Delta U = \frac{Q^2}{2}\left(\frac{(2b-a)}{4\pi\varepsilon_0 a2b} - \frac{(b-a)}{4\pi\varepsilon_0 ab}\right) = \frac{Q^2}{2}\left(\frac{(2b-a) - 2(b-a)}{4\pi\varepsilon_0 a2b}\right) = \frac{Q^2}{16\pi\varepsilon_0 b}.$$

Problem 3 (25 points)

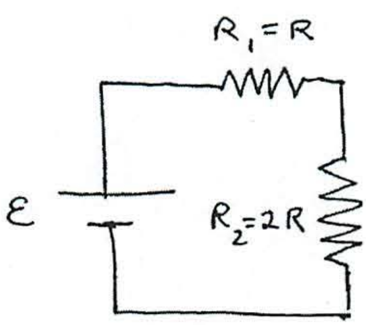
NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!) .

Consider the following circuit shown in the figure below. All questions can be answered without solving any differential equations.



a) Find the current through each of the four resistors, with resistances R_1, R_2, R_3, and R_4, a long time after the switch S has been in position (1).

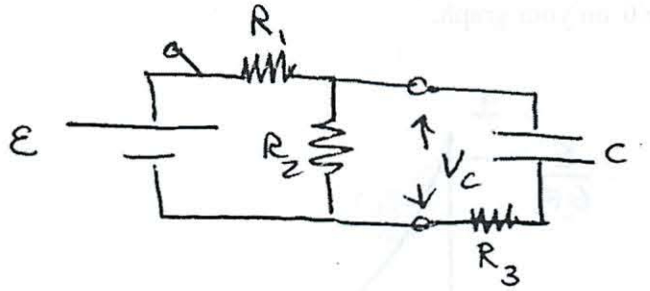
Answer: No current flows through resistor R_4. A long time after the switch as been closed no current flows through the branch of the circuit containing the capacitor and resistor R_3. So the circuit looks like the circuit diagram below.



In this single loop circuit with equivalent resistance $R_{eq} = R_1 + R_2 = 3R$, the current is the same through both resistors with resistances R_1, R_2, and is given by $I = \varepsilon/(3R)$.

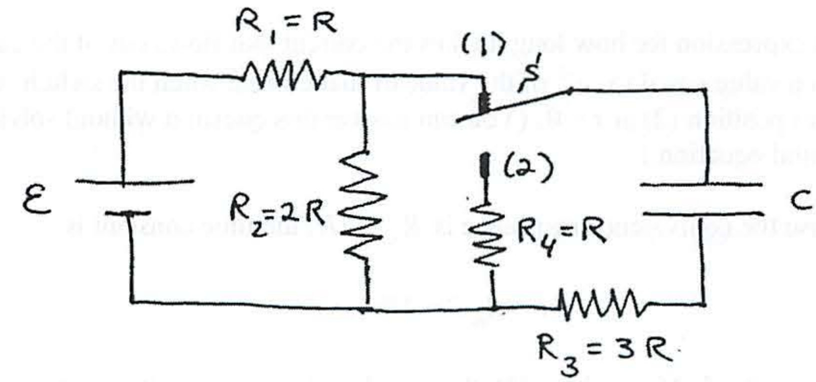
b) Find the absolute value of the potential difference $|V_C|$ across the capacitor a long time after the switch S has been in position (1).

Answer: The potential across the capacitor is the same as the potential across resistor 2, (see figure below)

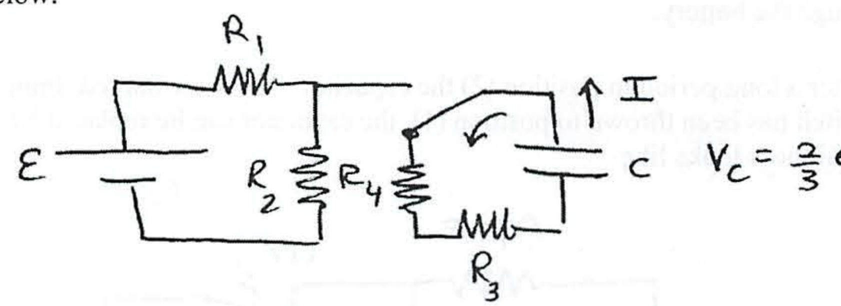


$$|V_C| = IR_2 = \frac{\mathcal{E}}{3R}2R = \frac{2\mathcal{E}}{3}.$$

c) At $t = 0$ the switch is moved to position (2). What current will flow out of the capacitor at the instant the switch is moved to position (2)? Indicate whether the current will flow up or down in the branch of the circuit containing the capacitor.



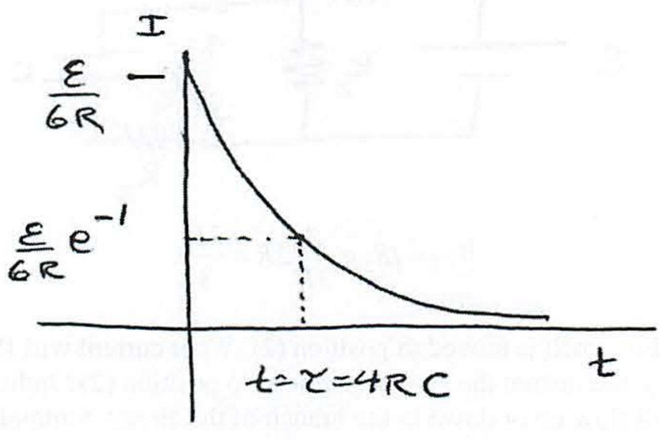
Answer: When the switch is moved to position 2 the circuit looks like the circuit diagram shown below.



The current flows counterclockwise (up from the capacitor). Because $|V_C| = (2/3)\mathcal{E}$ and the equivalent resistance is $R_{eq} = R_4 + R_3 = 4R$, the current is

$$I = \frac{|V_C|}{R_{eq}} = \frac{2\mathcal{E}}{3(4R)} = \frac{\mathcal{E}}{6R}.$$

d) Make a graph of current vs. time for the current that flows out of the capacitor after the switch is moved to position (2) at $t = 0$. Indicate the value of the current at time $t = 0$ on your graph.



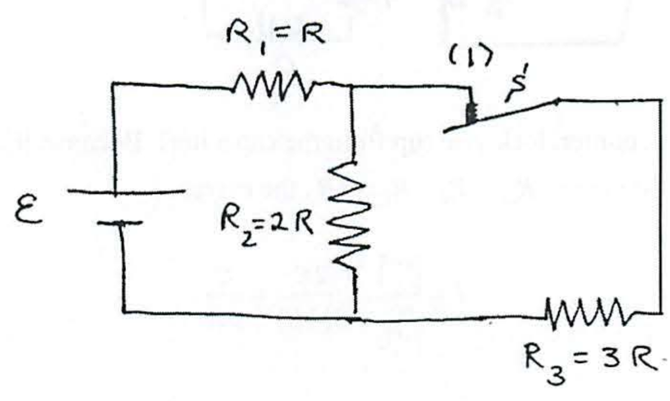
e) Find an expression for how long it takes the current that flows out of the capacitor to reach a value equal to e^{-1} of the value of that current when the switch is moved to position (2) at $t = 0$. (You can answer this question without solving a differential equation.)

Answer: Because the equivalence resistance is $R_{eq} = 4R$, the time constant is

$$\tau = R_{eq}C = 4RC.$$

f) After a long period in position (2), the switch is thrown to position (1) again. Immediately after the switch has been thrown to position (1), find the current through the battery.

Answer: After a long period in position (2) the capacitor is now uncharged. Immediately after the switch has been thrown to position (1), the capacitor can be replaced by a wire, and the circuit now looks like



Resistors 2 and 3 are now in parallel with equivalent resistance

$$(R_{eq})_{par} = \frac{R_2 R_3}{R_2 + R_3} = \frac{(2R)(3R)}{(2R+3R)} = \frac{6R}{5}\ .$$

Because resistor 1 is in series with the parallel pair of resistors 2 and 3, the equivalent resistance of the resistor network is

$$R_{eq} = R_1 + \frac{R_2 R_3}{R_2 + R_3} = R + \frac{6R}{5} = \frac{11R}{5}.$$

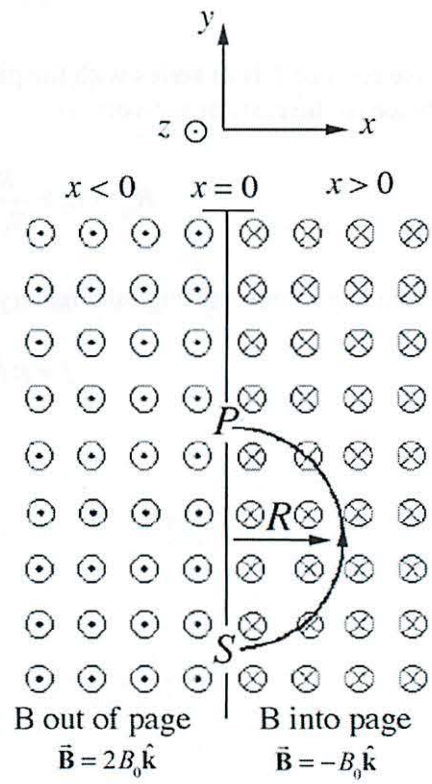
Therefore the current through the battery is

$$I = \varepsilon / R_{eq} = 5\varepsilon / 11R\ .$$

Problem 4 (25 points)

NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!) .

The $x-y$ plane for $x<0$ is filled with a uniform magnetic field pointing out of the page, $\vec{\mathbf{B}}=2B_0\,\hat{\mathbf{k}}$ with $B_0>0$, as shown. The $x-y$ plane for $x>0$ is filled with a uniform magnetic field $\vec{\mathbf{B}}=-B_0\,\hat{\mathbf{k}}$, pointing into the page, as shown. A charged particle with mass m and charge q is initially at the point S at $x=0$, moving in the positive x-direction with speed v. It subsequently moves counterclockwise in a circle of radius R, returning to $x=0$ at point P, a distance $2R$ from its initial position, as shown in the sketch.



a) Is the charge positive or negative? Briefly explain your reasoning.

Answer: Because the orbit is counterclockwise the force $\vec{F}=q\vec{v}\times\vec{B}$ must point up when the particle is at point S. The $\vec{v}\times\vec{B}=v\hat{i}\times(-B_0\,\hat{k})=vB_0\,\hat{j}$ points up therefore the charge of the particle must be positive in order for $\vec{F}=q\vec{v}\times\vec{B}$ also to point up.

b) Find an expression for the radius R of the trajectory shown, in terms of v, m, q, and B_0.

Answer: The orbit is circular, so Newton's second Law becomes $qvB_0=mv^2/R$. Thus the radius of the orbit is

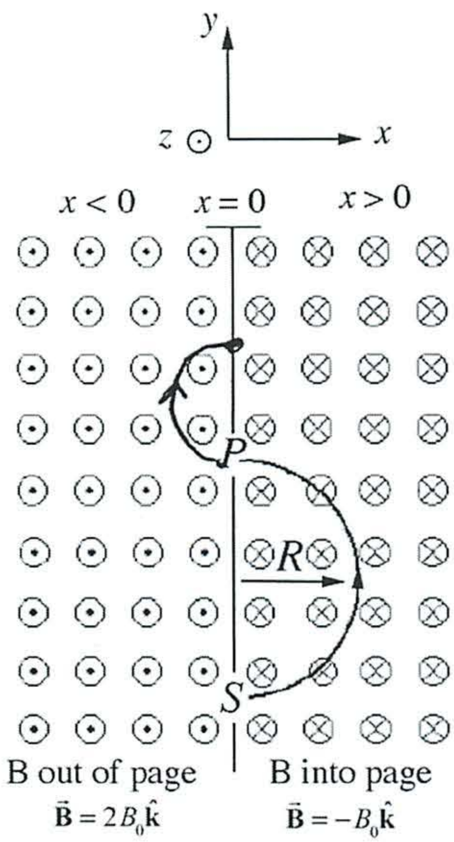
$$R=\frac{mv}{qB_0}.$$

c) How long does the particle take to return to the plane $x = 0$ at point P?

Answer: The time t_P it takes the particle to complete a semicircular path from S to P is

$$t_P = \frac{\pi R}{v} = \frac{\pi m}{qB_0}.$$

d) Describe and sketch the subsequent trajectory of the particle on the figure below after it passes point P. Be sure to define any relevant distances in terms of v, m, q, and B_0.



When the particle is at point P, the force is still up because both the velocity and the magnetic field now point in opposite directions. Hence

$$\vec{F} = q\vec{v} \times \vec{B} = v(-\hat{i}) \times (2B_0\,\hat{k}) = 2vB_0\,\hat{j}.$$

Newton's Second Law is now $2vB_0 = mv^2 / R_2$. Hence the radius is now

$$R_2 = mv / 2B_0 = R/2.$$