Summary of Class 22 **8.02**

Topics: Faraday's Law
Related Reading: Course Notes: Sections 10.1-10.3, 10.8-10.9
Experiments: (7) Faraday's Law of Induction

Topic Introduction

So far in this class magnetic fields and electric fields have been fairly well isolated. Electric fields are generated by static charges, magnetic fields by moving charges (currents). In each of these cases the fields have been static – we have had constant charges or currents making constant electric or magnetic fields. Today we make two major changes to what we have seen before: we consider the interaction of these two types of fields, and we consider what happens when they are not static. We will discuss the last of Maxwell's equations, Faraday's law, which explains that electric fields can be generated not only by charges but also by magnetic fields that vary in time and get a hands-on feeling for it in an expt.

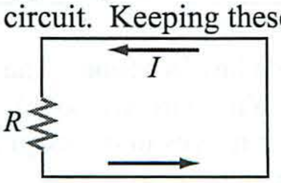
Faraday's Law
It is not entirely surprising that electricity and magnetism are connected. We have seen, after all, that if an electric field is used to accelerate charges (make a current) that a magnetic field can result. Faraday's law, however, is something completely new. We can now forget about charges completely. What Faraday discovered is that a changing magnetic flux generates an EMF (electromotive force). Mathematically:

no charge!

$$\mathcal{E} = -\frac{d\Phi_B}{dt}, \text{ where } \Phi_B = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} \text{ is the magnetic flux, and } \mathcal{E} = \oint \vec{\mathbf{E}}' \cdot d\vec{\mathbf{s}} \text{ is the EMF}$$

In the formula above, $\vec{\mathbf{E}}'$ is the electric field measured in the rest frame of the circuit, if the circuit is moving. The above formula is deceptively simple, so I will discuss several important points to consider when thinking about Faraday's law.

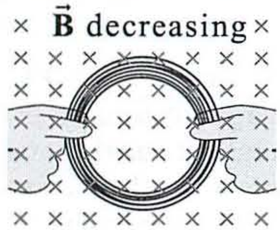
WARNING: First, a warning. Many students confuse Faraday's Law with Ampere's Law. Both involve integrating around a loop and comparing that to an integral across the area bounded by that loop. Aside from this mathematical similarity, however, the two laws are completely different. In Ampere's law the field that is "curling around the loop" is the magnetic field, created by a "current flux" $\left(I = \iint \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}} \right)$ that is penetrating the looping B field. In Faraday's law the electric field is curling, created by a *changing* magnetic flux. In fact, there need not be any currents at all in the problem, although as we will see below typically the EMF is measured by its ability to drive a current around a physical loop – a circuit. Keeping these differences in mind, let's continue to some details of Faraday's law. voltage

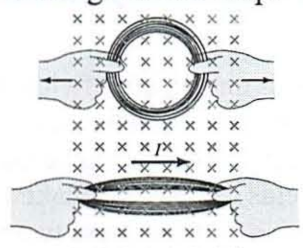


EMF: How does the EMF become apparent? Typically, when doing Faraday's law problems there will be a physical loop, a closed circuit, such as the one pictured at left. The EMF is then observed as an electromotive force that drives a current in the circuit: $\mathcal{E} = IR$. In this case, the path walked around in calculating the EMF is the circuit, and hence the associated area across which the magnetic flux is calculated is the rectangular area bordered by the circuit. Although this is the most typical initial use of Faraday's law, it is not the only one – we will see that it can be applied in "empty space" space as well, to determine the creation of electric fields.

like a battery

Changing Magnetic Flux: How do we get the magnetic flux Φ_B to change? Looking at the integral $\Phi_B = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = BA\cos(\theta)$, hints at three distinct methods: changing the strength of the field, the area of the loop, or the angle of the loop. These methods are shown below.





In each of the cases pictured above, the magnetic flux into the page is decreasing with time (because the (1) B field, (2) loop area or (3) projected area are decreasing with time). This decreasing flux creates an EMF. In which direction? We can use Lenz's Law to find out.

Lenz's Law
Lenz's Law is a non-mathematical statement of Faraday's Law. It says that systems will always act to *oppose* changes in magnetic flux. For example, in each of the above cases the flux into the page is decreasing with time. The loop doesn't want a decreased flux, so it will generate a clockwise EMF, which will drive a clockwise current, creating a B field into the page (inside the loop) to make up for the lost flux. This, by the way, is the meaning of the minus sign in Faraday's law. I recommend that you use Lenz's Law to determine the direction of the EMF and then use Faraday's Law to calculate the amplitude. By the way, just as with Faraday's Law, you don't need a physical circuit to use Lenz's Law. Just pretend that there is a wire in which current could flow and ask what direction it would need to flow in order to *oppose* the changing flux. In general, *opposing* a change in flux means *opposing* what is happening to change the flux (e.g. forces or torques *oppose* the change).

Applications
A number of technologies rely on induction to work – generators, microphones, metal detectors, and electric guitars to name a few.

Experiment 7: Faraday's Law of Induction
Preparation: Read pre-lab

In this lab you will have a chance to measure and even feel Faraday's law in action. The lab basically consists of moving a loop of wire over a magnetic dipole. You will (we hope) develop an intuition for how currents flow through the wire loop as it moves in the magnetic field of the dipole, and for the direction of the resultant force on the loop.

Important Equations

Faraday's Law: $\mathcal{E} = -\frac{d\Phi_B}{dt}$

Magnetic Flux: $\Phi_B = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$

EMF: $\mathcal{E} = \oint \vec{\mathbf{E}}' \cdot d\vec{\mathbf{s}}$ where $\vec{\mathbf{E}}'$ is the electric field measured in the rest frame of the circuit, if the circuit is moving.

all classes 68 78

Our class a lot better than the others. I did OK relative to everyone but pretty bad related to this class - what you get for studying 1 hr

4/5

Class 22: Outline

Hour 1:
Faraday's Law

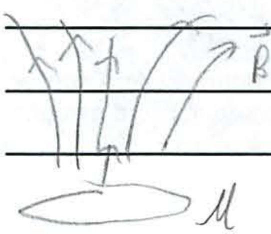
Hour 2:
Faraday's Law: Applications

While waiting today: Open applet

Group Problem Discovery: Faraday's Law Applets

bounds of flux don't depend on speed
- electric flux : guass' law

bigger current = faster



$\boxed{F = I(L \times B)}$

B field tilted + non uniform
Stronger field down below

Faraday's Law

Fourth (Final) Maxwell's Equation
Underpinning of Much Technology

x cool metals conduct more
- will jump higher

Demonstration: Falling Magnet

Magnet Falling Through a Ring

Falling magnet slows as it approaches a copper ring which has been immersed in liquid nitrogen.

← falls

← slows at ring,
then falls through

Demonstration: Jumping Rings

Jumping Ring

An aluminum ring jumps into the air when the solenoid beneath it is energized

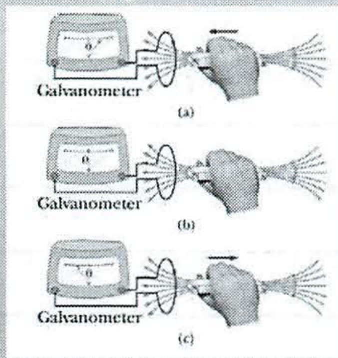
* cooler metals conduct more
- jumps higher

What is Going On?

It looks as though the conducting loops have current in them (they behave like magnetic dipoles) even though they aren't hooked up

Demonstration: Induction

Electromagnetic Induction



Faraday's Law of Induction

$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$$

A changing magnetic flux *induces* an EMF

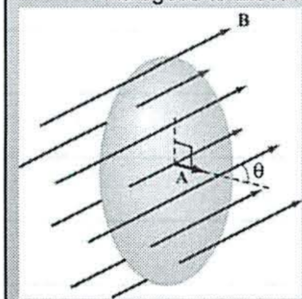
What is EMF?

$$\mathcal{E} = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

Looks like potential. It's a "driving force" for current

Magnetic Flux Thru Wire Loop

Analogous to Electric Flux (Gauss' Law)



(1) Uniform **B**

$$\Phi_B = B_\perp A = BA\cos\theta = \vec{\mathbf{B}}\cdot\vec{\mathbf{A}}$$

(2) Non-Uniform **B**

$$\boxed{\Phi_B = \iint_S \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}}}$$

Faraday's Law of Induction

$$\boxed{\mathcal{E} = \oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}}$$

A changing magnetic flux *induces* an EMF, a curling E field

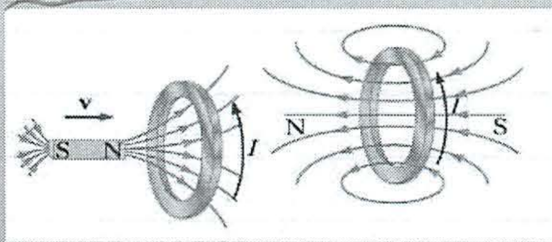
Faraday's Law of Induction

$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$$

A changing magnetic flux *induces* an EMF

Minus Sign? Lenz's Law

Induced EMF is in direction that ***opposes the change*** in flux that caused it

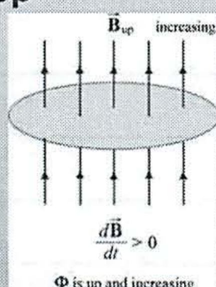


① magnetic flux → ② things hate change
wants flux to left
* so it drives current to ! right to oppose change

PRS: Loop

20

The magnetic field through a wire loop is pointed upwards and *increasing* with time. The induced current in the coil is



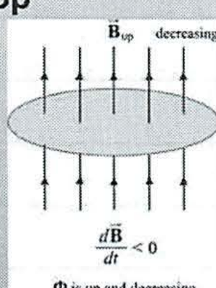
0% 1. Clockwise as seen from the top
0% 2. Counterclockwise

"screwdriver" rule

PRS: Loop

The magnetic field through a wire loop is pointed upwards and *decreasing* with time. The induced current in the coil is

20



0% 1. Clockwise as seen from the top
0% 2. Counterclockwise

The change is up field going away, want field pointing up so counter clockwise

* things fight change

Ways to Induce EMF

$$\boxed{\mathcal{E} = -\frac{d}{dt}\left(BA\cos\theta\right)}$$

if constant

Quantities which can vary with time:

- Magnitude of B
- Area A enclosed by the loop
- Angle θ between B and loop normal

P22-19

Ways to Induce EMF

$$\boxed{\mathcal{E} = -\frac{d}{dt}\left(BA\cos\theta\right)}$$

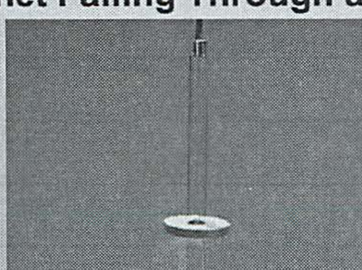
Quantities which can vary with time:

- **Magnitude of B**
- Area A enclosed by the loop
- Angle θ between B and loop normal

P22-20

3 things to change
- can do all 3

Group Discussion: Magnet Falling Through a Ring



Falling magnet slows as it approaches a copper ring which has been immersed in liquid nitrogen.

21

N

Define on I

Ring tries to repel it
wants to look like
a diapole opposite dir

N

needs to be

= clockwise current

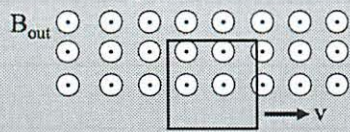
* Then it wants to flip current over to attract magnet after it falls through

Class 22

good demo

→ Where does KE go?
- current flowing - takes energy to oppose resistance
- goes to eddy current heating

PRS: Loop in Uniform Field



A rectangular wire loop is pulled thru a uniform B field penetrating its top half, as shown. The induced current and the force and torque on the loop are:

1. Current CW, Force Left, No Torque
2. Current CW, No Force, Torque Rotates CCW
3. Current CCW, Force Left, No Torque
4. Current CCW, No Force, Torque Rotates CCW
5. No current, force or torque

↳ why no force - no change (that's why I thought it looked weird)

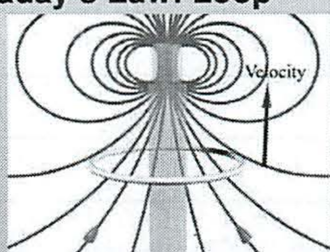
if super conductor - makes magnetic field
- actually both but relative values differ in each.

flux at middle is max
Force (derivative) = 0 = no current

So actually accelerates in middle of coil and then slows down again

PRS: Faraday's Law: Loop

A coil moves up from underneath a magnet with its north pole pointing upward. The current in the coil and the force on the coil:



0%	1.	Current clockwise; force up
0%	2.	Current counterclockwise; force up
0%	3.	Current clockwise; force down
0%	4.	Current counterclockwise; force down

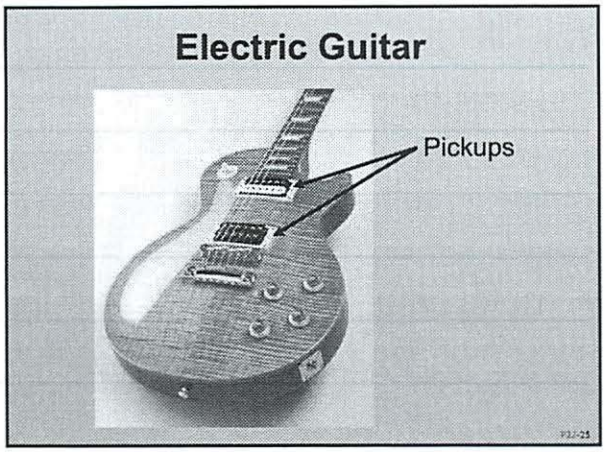
motion = change = force down

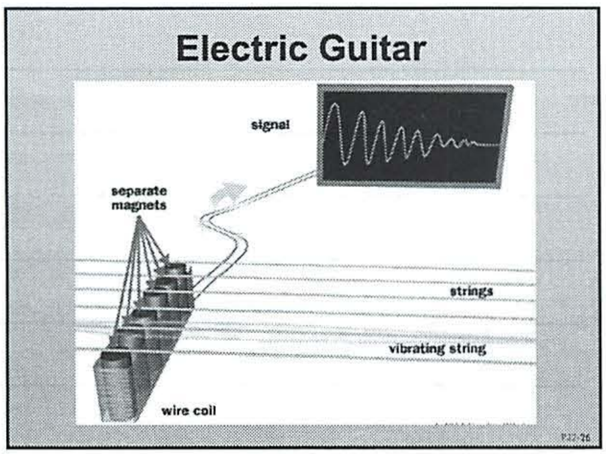
↳ flux is up, increasing
want flux down → clockwise current

Technology

Many Applications of Faraday's Law

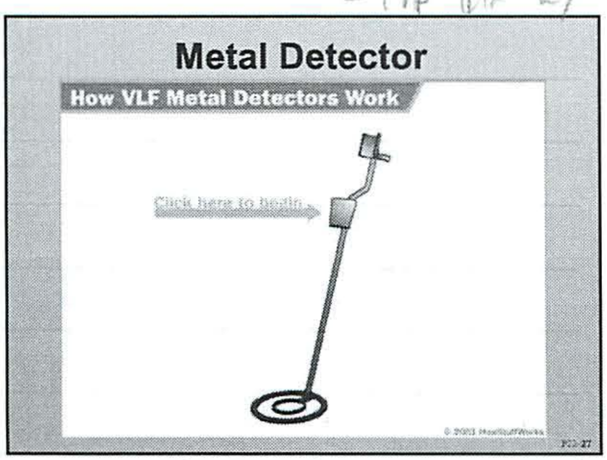
Speaker - drive current through solinoid - turns into a dipole - will make speaker move up + down
- is a perminant magnet solinoid is working with





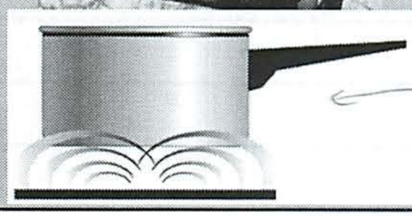
steel changes magnetic field
changes in magnet that measures
amplifyer amplifies it

DC motor - diapole moment
- wants to line up w/ field
- flip dir w/ current every half cycle



Motors + Generators almost all do this - turn coils of wire in magnetic field - Faraday's law makes power

Induction Stovetops



Not the ones that are standard but flat

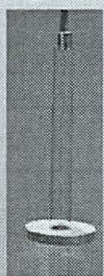
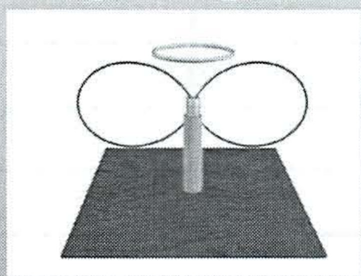
needs right cookware

GFI - current in wrong direction
current in must = current out
when net field it trips ring

Demonstration: Electric Guitar

Experiment 5: Faraday's Law of Induction

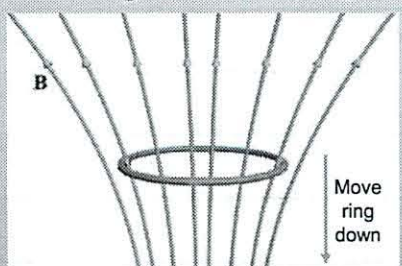
Example: Magnitude of B Magnet Falling Through a Ring

Falling magnet approaches a copper ring *or* Copper Ring approaches Magnet

P22-31

Moving Towards Dipole

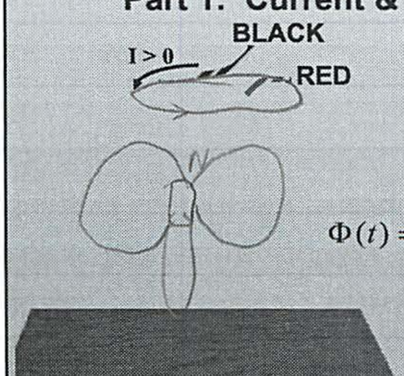


As ring approaches, what happens to flux?

Flux up increases

P22-32

Part 1: Current & Flux



Current?

Flux?

$$\Phi(t) = -R\int_0^t I(t')\,dt'$$

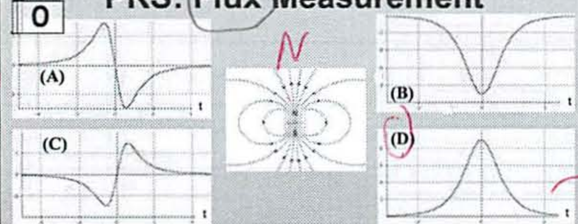
P22-33

PRS Predictions: Flux & Current

From Lab

PRS: Flux Measurement

0



Moving from above to below and back, you will measure a *flux* of:

1. A then A
2. C then C
3. A then C
4. C then A
5. B then B
6. D then D
7. B then D
8. D then B

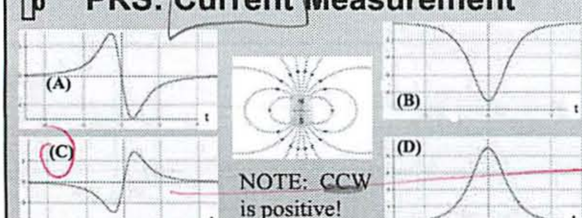
+ is up

field is up along axis

always +

low field → high → low

PRS: Current Measurement



Moving from above to below and back, you will measure a *current* of:

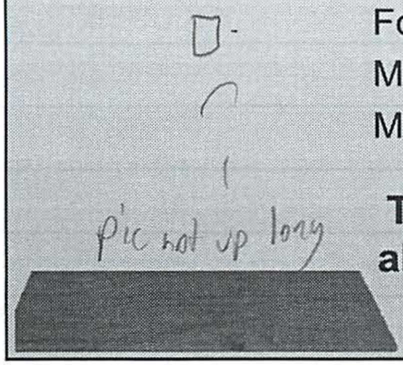
1. A then A
2. C then C
3. A then C
4. C then A
5. B then B
6. D then D
7. B then D
8. D then B

Clockwise → counter clockwise

Why does current have this shape?

current is deriv of flux

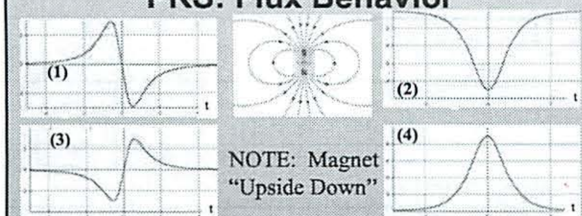
Part 2: Force Direction



Force when
Move Down?
Move Up?

Test with aluminum sleeve

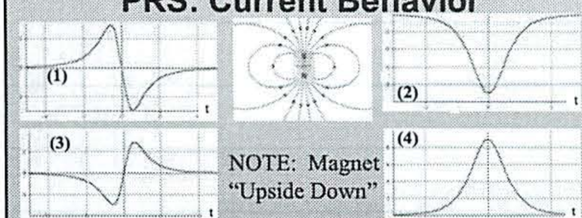
PRS: Flux Behavior



Moving from below to above, you would measure a *flux* best represented by which plot above (taking upward flux as positive)?

0% 0% 0% 0%

PRS: Current Behavior



Moving from *above* to *below*, you would measure a *current* best represented by which plot above (taking counterclockwise current as positive)?

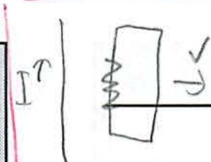
0% 0% 0% 0%

PRS Confirming Predictions?
Flux & Current

P22-40

PRS Question:
Wrap-Up
Faraday's Law

P22-41

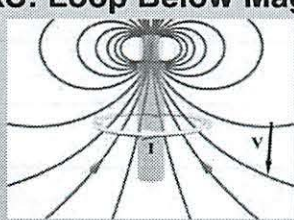


Field from wire = into pg
as v pulling away decreasing
Want field into pg
So drive current clockwise

Faster to think of force
- left leg most important
- want attractive
- So currents should be parallel

PRS: Loop Below Magnet

:20



A conducting loop is below a magnet and moving downwards. This induces a current as pictured. The *I ds x B* force on the coil is

0% 1. Up
0% 2. Down
0% 3. Zero

P22-42

Ways to Induce EMF

$$\boxed{\mathcal{E} = -\frac{d}{dt}\left(BA\cos\theta\right)}$$

Quantities which can vary with time:

- Magnitude of B
- Area A enclosed by the loop
- Angle θ between B and loop normal

The last of the Maxwell's Equations (Kind of)

Maxwell's Equations

Creating Electric Fields

$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$ (Gauss's Law)

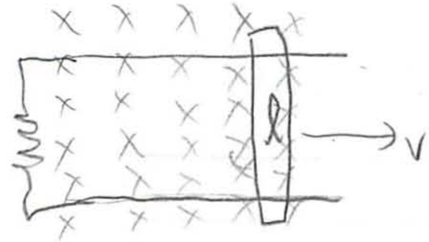
$\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$ (Faraday's Law)

Creating Magnetic Fields

$\oiint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$ (Magnetic Gauss's Law)

$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$ (Ampere's Law)

Group Problem



1. Dir induced current
2. Dir resultant force
3. Magnitude of emf
4. Magnitude of current
5. Power externally supplied to move at constant v?

$B \times \bar{F}$

1. up → oppose so down

2. want it to move not right but left

Counter clockwise current

3. $\text{emf} = -\frac{d}{dt}(BA\cos\theta) = B\frac{dA}{dt} = B\ell v$

4. $\Delta x = vt$

$B\ell vt$

$IR = \mathcal{E} = B\ell v$

$\mathcal{E} = IR = B\ell v$

$I = \frac{B\ell v}{R}$ ✓

5. Power

$\vec{F} \times v$ (velocity) $= I^2R$

$P_{\text{Resistor}} = IV = \frac{(B\ell v)^2}{R}$

$P_{\text{Mechanically}} = Fv = \underbrace{\left(\frac{B\ell v}{R}\right)}_{I} \cdot B\ell v = \frac{(B\ell v)^2}{R}$

* study this bigtime *

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics
8.02

Experiment 7: Faraday's Law

OBJECTIVES

1. To become familiar with the concepts of changing magnetic flux and induced current associated with Faraday's Law of Induction.

2. To see how and why the direction of the magnetic force on a conductor carrying an induced current is consistent with Lenz's Law. Lenz's Law says that the system always responds so as to try to keep things the same.

PRE-LAB READING

INTRODUCTION

In this lab you will develop an intuition for Faraday's and Lenz's Laws. By moving a coil of wire over a magnet you will change the magnetic flux through the coil, generating and EMF and hence current in the loop which you will measure using the 750.

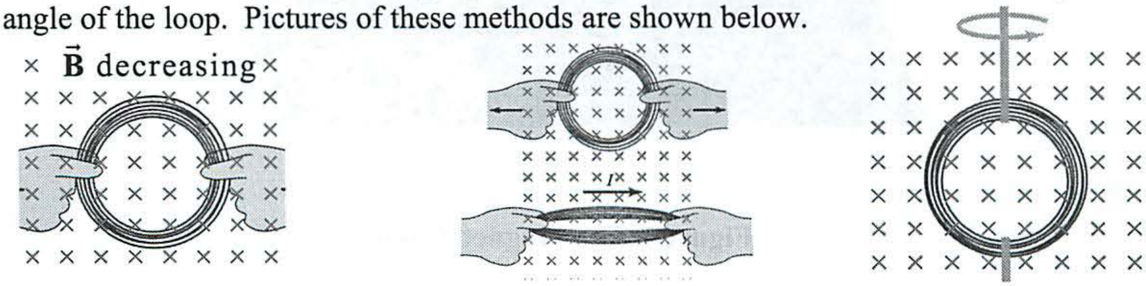
The Details: Faraday's Law

Faraday's Law states that a changing magnetic flux generates an EMF (electromotive force). Mathematically:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}, \text{ where } \Phi_B = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} \text{ is the magnetic flux, and } \mathcal{E} = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} \text{ is the EMF}$$

In the formula above, $\vec{\mathbf{E}}$ is the electric field measured in the rest frame of the circuit, if the circuit is moving.

Changing Magnetic Flux: How do we get the magnetic flux Φ_B to change? Looking at the integral in the case of a uniform magnetic field, $\Phi_B = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = BA\cos(\theta)$, hints at three distinct methods: by changing the strength of the field, the area of the loop, or the angle of the loop. Pictures of these methods are shown below.



In each of the cases pictured above, the magnetic flux into the page is decreasing with time (because the (1) B field, (2) loop area or (3) projected area are decreasing with time). This decreasing flux creates an EMF. In which direction? We can use Lenz's Law to find out.

Lenz's Law

Lenz's Law is a non-mathematical statement of Faraday's Law. It says that systems will always act to *oppose* changes in magnetic flux. For example, in each of the above cases the flux into the page is decreasing with time. The loop doesn't want a decreased flux, so it will generate a clockwise EMF, which will drive a clockwise current, creating a B field into the page (inside the loop) to make up for the lost flux. This, by the way, is the meaning of the minus sign in Faraday's law. I recommend that you use Lenz's Law to determine the direction of the EMF and then use Faraday's Law to calculate the amplitude. By the way, just as with Faraday's Law, you don't need a physical circuit to use Lenz's Law. Just pretend that there is a wire in which current could flow and ask in what direction it would need to flow to *oppose* the changing flux. In general, *opposing* a change in flux means *opposing* what is happening to change the flux (e.g. forces or torques *oppose* the change).

APPARATUS

1. Magnet Stand

The magnetic flux of Faraday's Law will be generated by a high field permanent magnet, sitting on a support beam so that you may move a coil from above to below and back.

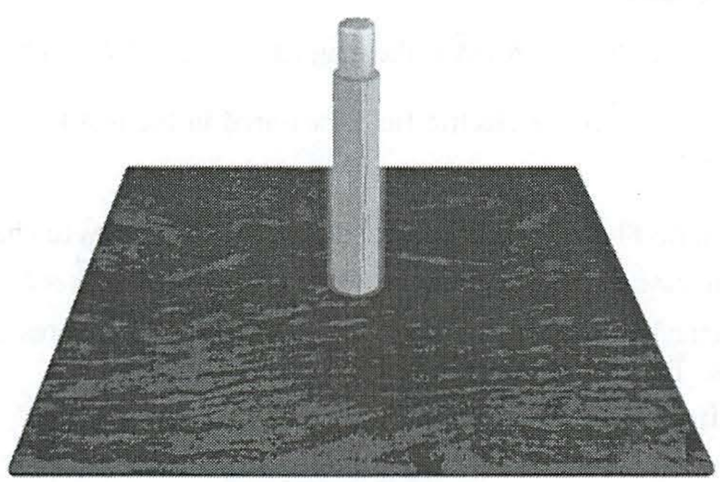


Figure 1 The Magnet Stand

2. Wire Loop, Current Sensor and Science Workshop 750 Interface

The magnetic field will penetrate a loop of wire, which you will plug into the current sensor, which is in turn plugged into channel A of the 750. In this lab we will use the convention that positive current flows counter-clockwise when observed from above. The current sensor records current that flows into its red terminal and out its negative terminal as positive, so make sure that you hook up the wire to the current sensor so that these two conventions are compatible with each other.

Figure 2 The Current Sensor

GENERALIZED PROCEDURE

This lab consists of two parts. In each you will observe the effects (current & force) of moving a loop around a dipole.

Part 1: Current and Flux through a Loop Moving Past a Dipole

You will move a wire loop from above to below a magnetic dipole, and observe plots of the current flowing through the loop (measured) and the flux through the loop (calculated).

Part 2: Feeling the Force

In this part you will repeat the motion, using a hollow aluminum cylinder instead of the wire loop. In doing so you will be able to feel the force on the cylinder due to Lenz's Law.

END OF PRE-LAB READING

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

1. Download the LabView file and start up the program.
2. Connect the current sensor to channel A of the 750.
3. Connect the wire loop to the current sensor so that, starting at the black terminal, the wire loops counterclockwise (when viewed from above) and then enters the red terminal of the current sensor

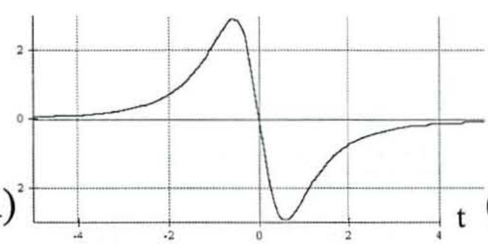
MEASUREMENTS

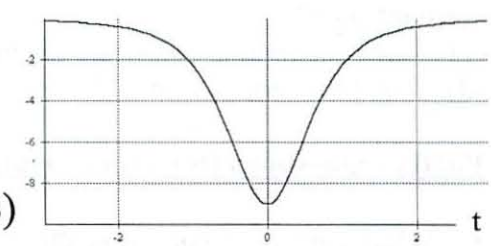
Part 1: Current and Flux through a Loop Moving Past a Dipole

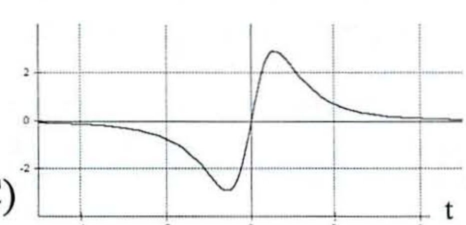
1. Press 'Go' to start recording current and flux
2. Move the wire loop from well above to well below the magnet and back again. Try to make the motion as smooth as possible and at a constant velocity.

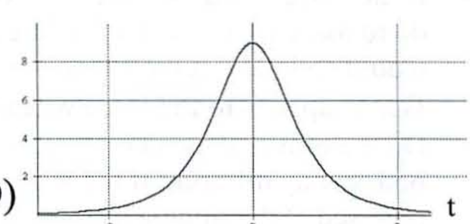
Question 1:

During the complete motion which of the following graphs (one for motion downwards, one for motion back upwards) most closely resembled the graph of:
(a) *magnetic flux through the loop* as a function of time?
(b) *current through the loop* as a function of time?

(A) 

(B) 

(C) 

(D) 

moving down up
flux D → D red
current C → C

Question 2:

Does the downward motion yield the same or different results from the upward motion? Why?

Same - has same orentation

Part 2: Feeling the Force

Although we could do this part of the lab with the same coil we just used, in order to better feel the force we will instead use an aluminum tube.

1. First hold the aluminum tube near the side of the magnet to convince yourself that Al is non-magnetic.
2. Place the tube over the Plexiglas and then push the tube downwards.
3. When you get to the bottom, pull the tube back up.

Question 3:

For each of the following four situations please indicate the direction of the magnetic force on the tube that you feel.

As you are moving the loop from well *above* the magnet to well *below* the magnet at a constant speed…
(a) … and the loop is *above* the magnet. upward
(b) … and the loop is *below* the magnet downward

As you are moving the loop from well *below* the magnet to well *above* the magnet at a constant speed…
(c) … and the loop is *below* the magnet. downward
(d) … and the loop is *above* the magnet upward

Further Questions (for experiment, thought, future exam questions…)

- What happens if you move the coil more quickly? Does the magnitude of the current change? Does the magnitude of the flux change? In part 2, does the force change?
- If the current, flux or force do not change in this situation, is there anything we could do to make them change? If they do change, what other changes could we make that would counter-act the change of moving more quickly?
- What happens to the force when the tube is exactly centered on the magnet? Why?
- Do the effects depend on history? In other words, is moving from the middle to the bottom any different if the motion started at the top than if it started at the bottom and reversed at the middle?
- What happens if we define the direction of positive current to be clockwise (in other words, if we flip the coil over)? Does this change have any affect on our definition of flux?

Spins from inconsisty in metals

Michael Plasmeier 11C

L01 87

4/3

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2010**

Problem Set 8

Due: Tuesday, April 6 at 9 pm.

Hand in your problem set in your section slot in the boxes outside the door of 32-082. Make sure you clearly write your name and section on your problem set.

Text: Liao, Dourmashkin, Belcher; Introduction to E & M MIT 8.02 Course Notes.

Week Ten Faraday's Law

Class 22 W10D1 M/T Apr 5/6	Faraday's Law; Expt.7: Faraday's Law
Reading:	Course Notes: Sections 10.1-10.3, 10.8-10.9
Experiment:	Expt.7: Faraday's Law
Class 23 W10D2 W/R Apr 7/8	Problem Solving Faraday's Law; Inductance & Magnetic Energy, RL Circuits
Reading:	Course Notes: 10.1-10.4,10.8-10.9, 11.1-11.4
Class 24 W10D3 F Apr 9	Special Lecture: Applications of Faraday's Law
Reading:	Course Notes: 10.1-10.4, 10.8-10.9, 11.1-11.4

Campus Preview Weekend

#1 on P-Set sheet →

rest on loose leaf

Problem 1: In this problem you will work through two examples from Problem Solving 7: Ampere's Law. Go it just Problem Solving 7; Ampere's Law

OBJECTIVES

1. To learn how to use Ampere's Law for calculating magnetic fields from symmetric current distributions
2. To find an expression for the magnetic field of a cylindrical current-carrying shell of inner radius a and outer radius b using Ampere's Law.
3. To find an expression for the magnetic field of a slab of current using Ampere's Law.

REFERENCE: Section 9-3, 8.02 Course Notes.

Summary: Strategy for Applying Ampere's Law (Section 9.10.2, 8.02 Course Notes)

Ampere's law states that the line integral of $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ around any closed loop is proportional to the total steady current passing through any surface that is bounded by the closed loop:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{\text{enc}}$$

To apply Ampere's law to calculate the magnetic field, we use the following procedure:

Step 1: Identify the 'symmetry' properties of the current distribution.

Step 2: Determine the direction of the magnetic field $\vec{B}$

Step 3: Decide how many different spatial regions the current distribution determines that you calc seperety

For each region of space...

Step 4: Choose an Amperian loop along each part of which the magnetic field is either constant or zero

Step 5: Calculate the current through the Amperian Loop

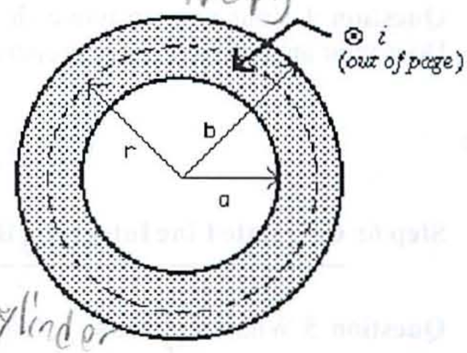
Step 6: Calculate the line integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ around the closed loop. isn't that always 0

Step 7: Equate $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ with $\mu_0 I_{\text{enc}}$ and solve for $\vec{\mathbf{B}}$.

* always redraw so current flows into or out of the pg

Example 1: Magnetic Field of a Cylindrical Shell

We now apply this strategy to the following problem. Consider the cylindrical conductor with a hollow center and copper walls of thickness $b - a$ as shown . The radii of the inner and outer walls are a and b respectively, and the current I is uniformly spread over the cross section of the copper (shaded region). We want to calculate the magnetic field in the region $a < r < b$.



in the cylinder

Question 1: Is the current density uniform or non uniform?

Problem Solving Strategy Step
Step 1: Identify Symmetry of Current Distribution
Either circular or rectangular

Step 2: Determine Direction of magnetic field
Clockwise or counterclockwise?

I out of page
"Screwdriver right hand rule"
Field is counterclockwise

Step 3: How many regions?
Three: r<a; a<r<b; r>b

don't we just want this?

Step 4: Draw Amperian Loop:
Here we take a loop that is a circle of radius r with $a < r < b$ (see figure).

Step 5: Current enclosed by Amperian Loop:

The next step is to calculate the current enclosed by this imaginary Amperian loop. There are typically two ways to do this. One way is to simply calculate it as a fraction of the total current. The second is to first calculate the current density J (current per unit area) and then multiply by the area enclosed. You should use both methods and compare.

Question 2 What is the magnitude of the current per unit area J in the region $a < r < b$? Remember we are assuming that the current I is uniformly spread over the area $a < r < b$, and also remember that current density J is defined as the current per unit area.

note uses big R for total area

$$J = \frac{I}{A} = \frac{I}{\pi R^2} \rightarrow I_{enc} = JA = \frac{I \cdot \pi r^2}{\pi R^2}$$

Question 3 What is the fraction of the total area that is enclosed by the Amperian Loop? What is the total current it encloses?

Well I looked off notes where infinite wire (solid)
Is it any different if the wire is hollow inside
Course notes has same problem

Do we use I in Conductor = 0 here???

Question 4 Your answer above should be zero when $r = a$ and I when $r = b$ (why?). Does your answer have these properties?

Yes, $\frac{\pi a^2}{\pi(b-a)^2}$ $\frac{\pi b^2}{\pi(b-a)^2}$

but why on this - is this right?

Step 6: Calculate Line Integral $\oint \vec{B} \cdot d\vec{s}$:

Question 5 What is $\oint \vec{B} \cdot d\vec{s}$? (That is, evaluate the integral, the left hand side of Ampere's law)

$$\oint B \cdot ds = B \cdot \oint ds = B(2\pi r) = \mu_0 I_{enc}$$

$$= \mu_0 \frac{I\pi r^2}{\pi(b-a)^2}$$

Step 7: Solve for $\vec{B}$:

Question 6 If you equate your answer to Question 5 to your answer to Question 3 times μ_o (i.e. use Ampere's Law), what do you get for the magnetic field in the region $a < r < b$?

Opps kind did that

$$B = \frac{\mu_0 I \pi r^2}{\pi(b-a)^2 \cdot 2\pi r} = \frac{\mu_0 I r}{2\pi(b-a)^2}$$ Counter clockwise

✓ matches in class
- This problem nicely broken down
- understand better

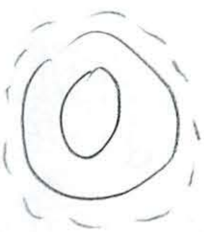
Question 7 Repeat the steps above to find the magnetic field in the region $r < a$.

~~$B(2\pi r) = \mu_0 \frac{I \pi r^2}{}$~~ $\qquad J = \frac{I}{A}$

there is no current ~~not~~ through wire

$= 0$!

Question 8 Repeat the steps above to find the magnetic field in the region $r > b$.



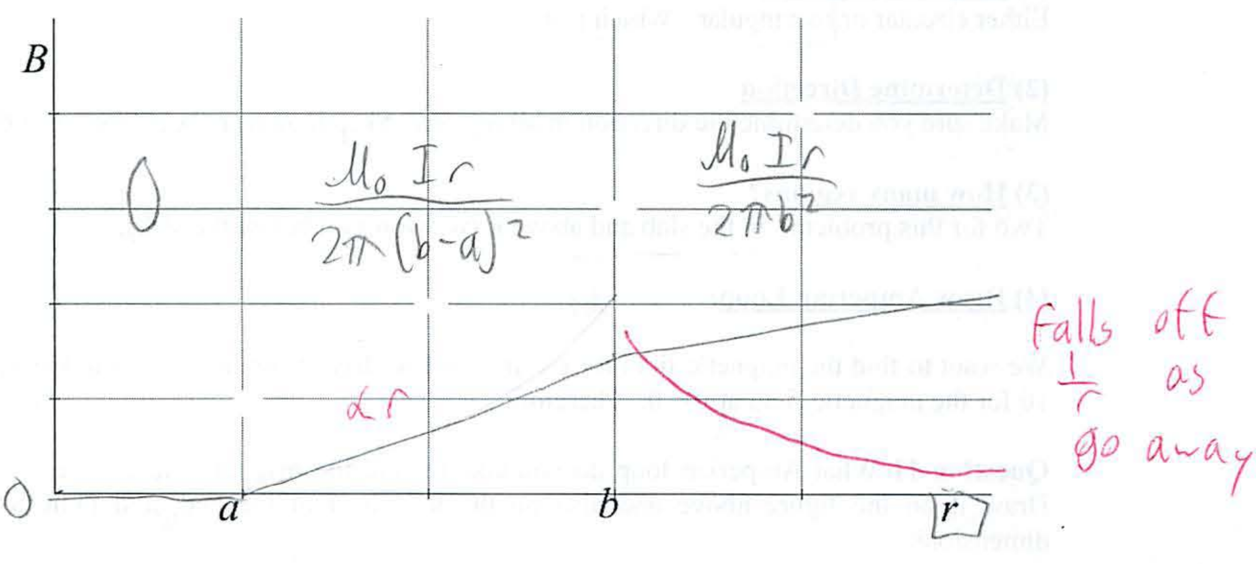
So outside it looks like normal wire

$$B(2\pi r) = \mu_0 \frac{I \cancel{\pi r^2}}{\pi b^2}$$

Course notes 2-18

$$B = \frac{\mu_0 I \cancel{\pi} r^2}{\pi b^2 \cdot 2\cancel{\pi} \cancel{r}} = \frac{\mu_0 I \cancel{r}}{2\pi \cancel{b^2}\, b}$$

Question 9 ***(put your answer on the tear-sheet at the end)*:** Plot B on the graph below.



I dk what it looks like

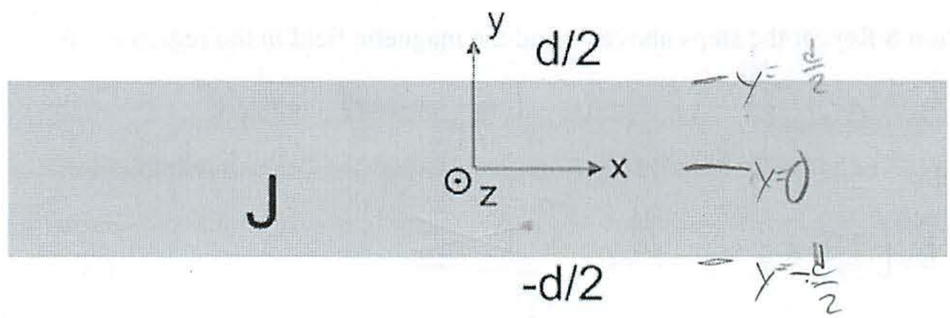
- tried graphing calc

$$\frac{r}{2\pi(2-1)^2}$$

graph from $0 \to 3$

Example 2: Magnetic Field of a Slab of Current

We want to find the magnetic field $\vec{\mathbf{B}}$ due to an infinite slab of current, using Ampere's Law. The figure shows a slab of current with current density $\vec{\mathbf{J}} = 2J_e|y|/d\,\hat{\mathbf{z}}$, where units of J_e are amps per square meter. The slab of current is infinite in the x and z directions, and has thickness d in the y-direction.



$J = \frac{2 J_e |y|}{d}$ $\frac{1}{2}$

So what is difference b/w d and y

- or d is fixed constant and y is actual position

Question 10 What is the magnetic field at y = 0, where y = 0 is the exact center of the slab?

$\oint B \cdot ds = \mu_0 I$

but $I = J \cdot A$

$J = \frac{2 J_e |0|}{d}$

$J = 0$

$I = 0$

$B = 0$

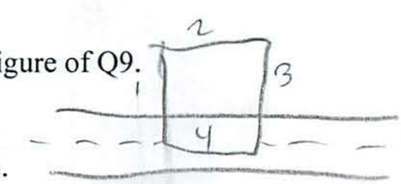
Problem Solving Strategy Step

(1) Identify Symmetry

Either circular or rectangular. Which is it?

(2) Determine Direction

Make sure you determine the direction in all regions. Sketch on tear sheet figure of Q9.



(3) How many regions?

Two for this problem: in the slab and above it (we won't do below the slab).

(4) Draw Amperian Loop: #1

1 and 3 cancel = 0

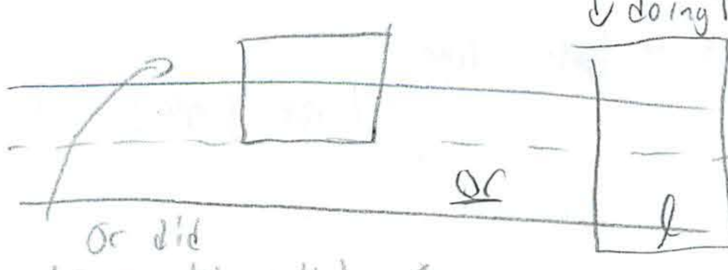
2 will be 0

4 will be with direction of I

We want to find the magnetic field for $y > d/2$, and we have from the answer to Question 10 for the magnetic field at y = 0. Therefore….

Question 11 What Amperian loop do you take to find the magnetic field for $y > d/2$? Draw it on the figure above and also on the tear-sheet at the end, and indicate its dimensions.

doing this



only want B field for

or did we say this would be 0

(5) **Current enclosed by Amperian Loop:**

The next step is to calculate the current enclosed by this imaginary Amperian loop. Hint: the current enclosed is the integral of the current density over the enclosed area.

$I_{enc} = JA$

Question 12 What is the total current enclosed by your Amperian loop from Question 11?

$$I_{enc} = JA = \iint \vec{J} \cdot d\vec{A}$$

$$= \cancel{\frac{2J_e|y|}{d}\hat{z}\cdot A} = \frac{2J_e|y|}{d}\cdot l$$

(6): Calculate Line Integral $\oint \vec{B}\cdot d\vec{s}$:

Question 13 What is $\oint \vec{B}\cdot d\vec{s}$?

$$\oint B\cdot ds = B(2l) = \mu_0 I$$

$$= \mu_0 \cdot \frac{2J_e|y|}{d} l$$

(7): Solve for B:

Question 14 If you equate your answers in Question 13 to your answer in Question 12 times μ_o using Ampere's Law, what do you get for the magnetic field in the region $y > d/2$?

$$B(2l) = \frac{\mu_0 \cdot 2J_e|y|l}{d}$$

$$B = \frac{\mu_0 2J_e|y|l}{2\cancel{l}d} = \frac{\mu_0 2J_e|y|}{2d}$$

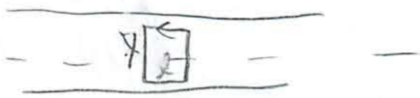
- constant, independent of distance from sheet (if J constant, right?)
- ; so not in this case?

We now want to find the magnetic field in the region $0 < y < d/2$.

(4) <u>Draw Amperian Loop</u>:

We want to find the magnetic field for $0 < y < d/2$, and we have from the answer to Question 10 for the magnetic field at $y = 0$. Therefore…

Question 15 What Amperian loop do you take to find the magnetic field for $0 < y < d/2$? Draw it on the figure above and on the tear-sheet at the end, and indicate its dimensions.



(5) <u>Current enclosed by Amperian Loop</u>:

The next step is to calculate the current enclosed by this imaginary Amperian loop.

Question 16 What is the total current enclosed by your Amperian loop from Question 15?

$$I_{enc} = \iint J \cdot dA = \frac{2J_e|y|}{d} \cdot 2|y|\ell$$

$$= \frac{4 J_e \ell |y^2|}{d}$$ ← constant

#17 $$\oint B \cdot ds = \mu_0 I_{enc}$$

$$B \cdot (2\ell) = \frac{\mu_0 4 J_e \ell y^2}{d}$$

$$B = \frac{\mu_0 \cancel{4}^{2} J_e \cancel{\ell} y^2}{d \cdot \cancel{2}\cancel{\ell}} = \frac{2\mu_0 J_e y^2}{d}$$

? Is that correct ?

(6) Calculate Line Integral $\oint \vec{B} \cdot d\vec{s}$:

Question 17 What is $\oint \vec{B} \cdot d\vec{s}$?

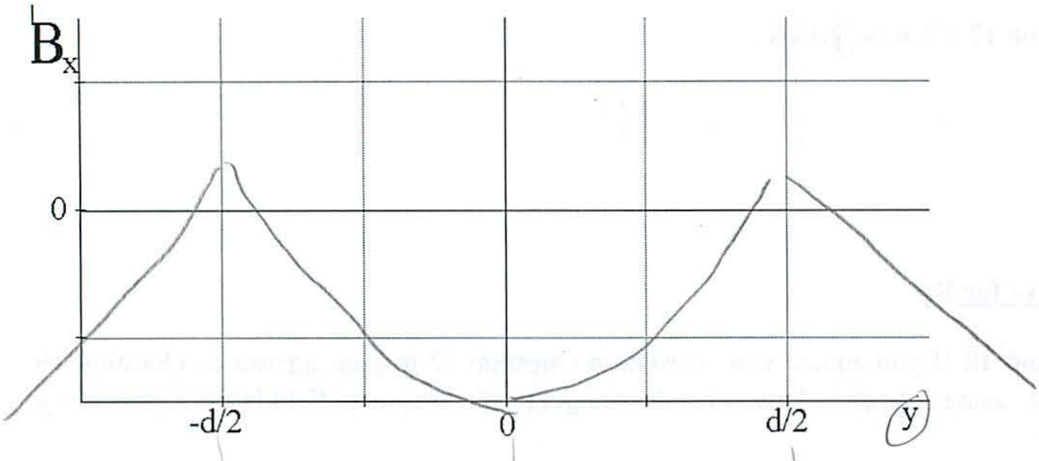
opps answered in 16

(7) **Solve for B:**

Question 18 If you equate you answers in Question 17 to your answer in Question 16 times μ_o using Ampere's Law, what do you get for the magnetic field in the region $0 < y < d/2$?

Opps answered in 16

Question 19 Plot B_x on the graph below. Use symmetry to determine B for y<0. Label the y-axis

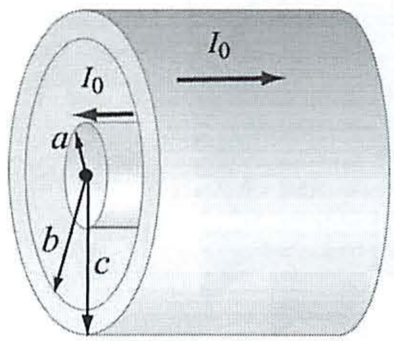


$\propto y$ | $\propto y^2$ | $\propto y^2$ | $\propto y$

Answering on paper now

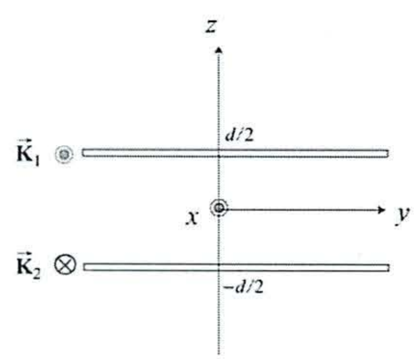
Problem 2 Co-axial Cable

A coaxial cable consists of a solid inner conductor of radius a, surrounded by a concentric cylindrical tube of inner radius b and outer radius c. The conductors carry equal and opposite currents I_0 *distributed uniformly across their cross-sections.* Determine the magnitude and direction of the magnetic field at a distance r from the axis. Make a graph of the magnitude of the magnetic field as a function of the distance r from the axis.



Problem 3: Two Current Sheets

Consider two infinitely large sheets lying in the xy-plane separated by a distance d carrying surface current densities $\vec{\mathbf{K}}_1 = K\hat{\mathbf{i}}$ and $\vec{\mathbf{K}}_2 = -K\hat{\mathbf{i}}$ in the opposite directions, as shown in the figure below (The extent of the sheets in the y direction is infinite.) Note that K is the current per unit width perpendicular to the flow.



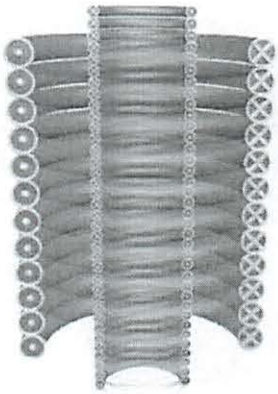
a) Find the magnetic field everywhere due to $\vec{\mathbf{K}}_1$.

b) Find the magnetic field everywhere due to $\vec{\mathbf{K}}_2$.

c) Applying superposition principle, find the magnetic field everywhere due to both current sheets.

d) How would your answer in (c) change if both currents were running in the same direction, with $\vec{\mathbf{K}}_1 = \vec{\mathbf{K}}_2 = K\hat{\mathbf{i}}$?

Problem 4 Nested Solenoids: Two long solenoids are nested on the same axis, as in the figure below. The inner solenoid has radius R_1 and n_1 turns per unit length. The outer solenoid has radius R_2 and n_2 turns per unit length. Each solenoid carries the same current I flowing in each turn, *but in opposite directions*, as indicated on the sketch.



Use Ampere's Law to find the direction and magnitude of the magnetic field in the following regions. Be sure to show your Amperian loops and all your calculations.

i) $0 < r < R_1$
ii) $R_1 < r < R_2$
iii) $R_2 < r$

Problem 5: Read Experiment 7 Faraday's Law.

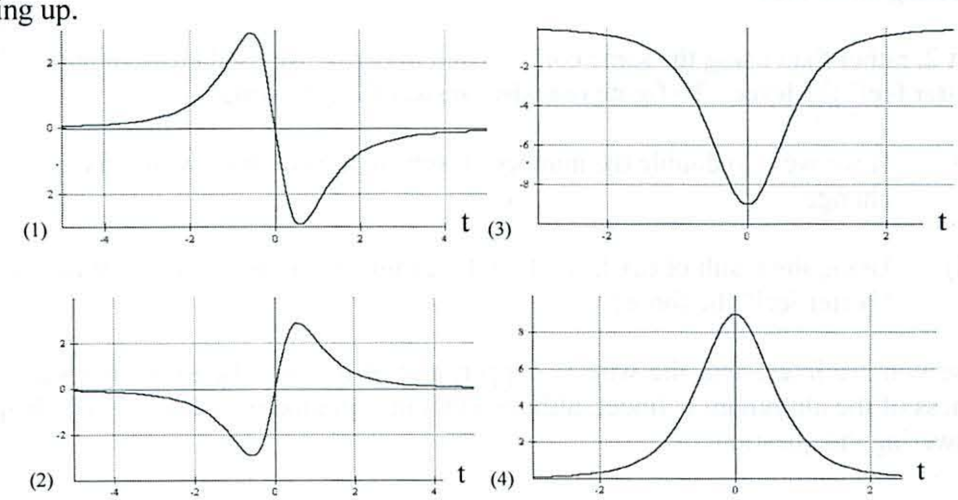
http://web.mit.edu/8.02t/www/materials/Experiments/exp05.pdf

(a) Calculating Flux from Current and Faraday's Law. In part 1 of the lab you moved a coil from well above to well below a strong permanent magnet. You measured the current in the loop during this motion using a current sensor. The program also displayed the flux "measured" through the loop, even though this value is never directly measured.

(i) Starting from Faraday's Law and Ohm's law, write an equation relating the current in the loop to the time derivative of the flux through the loop.

(ii) Now integrate that expression to get the time dependence of the flux through the loop $\Phi(t)$ as a function of current $I(t)$. What assumption must the software make before it can plot flux vs. time?

(b) Predictions: Coil Moving Past Magnetic Dipole
In moving the coil over the magnet, measurements of current and flux for each of several motions looked like one of the below plots. For current, counter-clockwise when viewed from above is positive. For flux, upwards is positive. The north pole of the magnet is pointing up.



Suppose you moved the loop from well *above* the magnet to well *below* the magnet at a constant speed. Which graph most closely resembles the graph of:

(i) *magnetic flux through the loop* as a function of time?
(ii) *current through the loop* as a function of time?

Suppose you moved the loop from well *below* the magnet to well *above* the magnet at a constant speed. Which graph most closely resembles the graph of:

(iii) *magnetic flux through the loop* as a function of time?

(iv) *current through the loop* as a function of time?

(c) Force on Coil Moving Past Magnetic Dipole

In part 2 of this lab you felt the force on a conducting loop as it moves past the magnet. For the following conditions, in what direction should the magnetic force point?

As you moved the loop from well *above* the magnet to well *below* the magnet at a constant speed…

(i) … and the loop is *above* the magnet.

(ii) … and the loop is *below* the magnet

As you moved the loop from well *below* the magnet to well *above* the magnet at a constant speed…

(iii) … and the loop is *below* the magnet.

(iv) … and the loop is *above* the magnet

(d) Feeling the Force

In part 2, rather than using the same coil we used in part 1, we used an aluminum cylinder to "better feel" the force. To figure out why, answer the following.

(i) If we were to double the number of turns in the coil how would the force change?

(ii) Using the result of (a), how should we think about the Al tube? Why do we "better feel" the force?

In case you are interested, the wire is copper, and of roughly the same diameter as the thickness of the aluminum cylinder, although this information won't necessarily help you in answering the question.

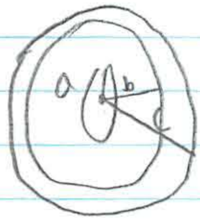
should do math two
lots of time wasted getting started
sleep well instead

8.02 P-Set 8

Michael Plasmeier 11C L01 4/3

#1. On P-Set sheet

#2 Co-axial cable



Inner $\leftarrow I_0$ $\odot$
Outer $\rightarrow I_0$ $\otimes$

twist so current into and out of the page

I_0 distributed evenly

$r < a$

Solid wire

$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$ ← around the circuit

$B \cdot 2\pi r = \mu_0 I_0$

$B = \frac{\mu_0 I_0}{2\pi r}$ counter

$a < r < b$

falls off in space

$I_{enc} = \left(\frac{\pi r^2}{\pi a^2}\right) I$ Counter

$$\oint B \cdot ds = \mu_0 I$$

$$B \cdot 2\pi r = \mu_0 I \left(\frac{\pi r^2}{\pi a^2}\right)$$

$$B = \frac{\mu_0 I \cancel{\pi} r^{\cancel{2}}}{\pi a^2 \cdot 2\cancel{\pi}\cancel{r}} = \frac{\mu_0 I r}{2\pi a^2}$$ counter

$\underline{b < r < c}$

Now inside hollow sphere

$$B(2\pi r) = \mu_0 I \left(\frac{\pi r^2}{\pi(c-b)^2}\right)$$ -4

$$B = \frac{\mu_0 I \cancel{\pi} r^{\cancel{2}}}{\pi(c-b)^2 \cdot 2\cancel{\pi}\cancel{r}} = \frac{\mu_0 I r}{2\pi(c-b)^2}$$ clockwise

$\underline{r > c}$

Now outside again

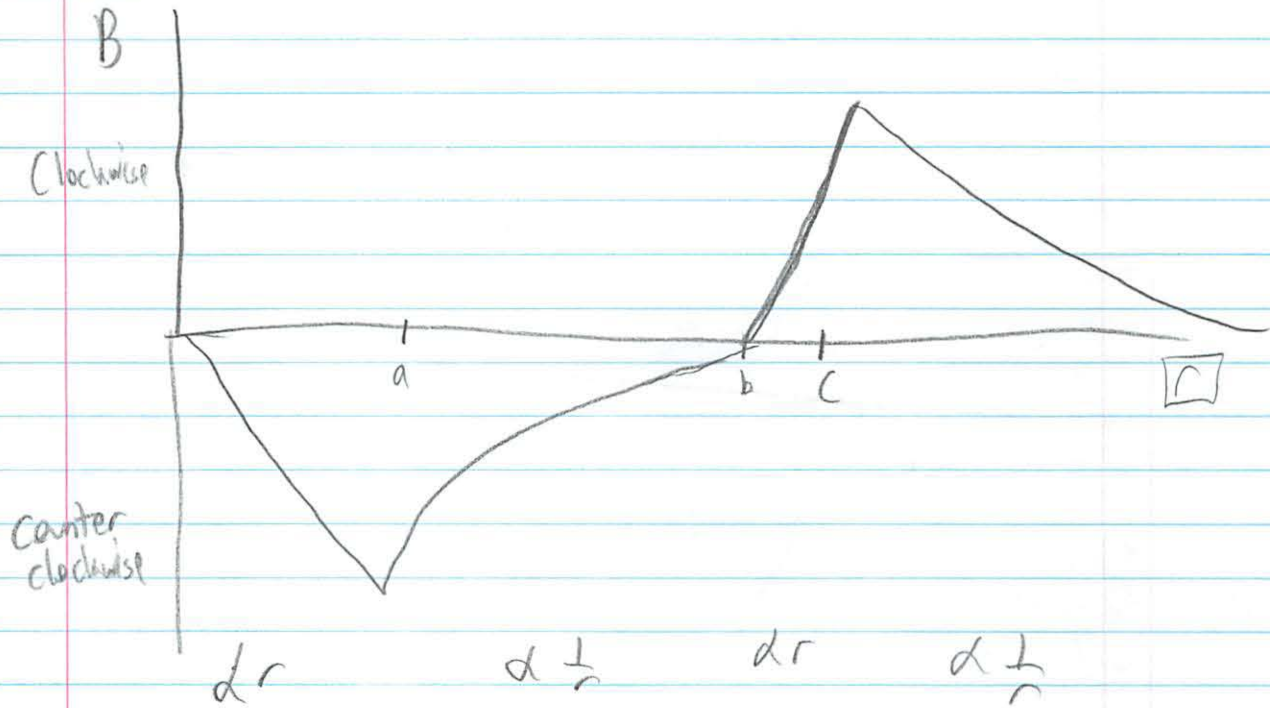
lots of practice!

$$B \cdot 2\pi r = \mu_0 I \left(\frac{\pi r^2}{\pi c^2}\right)$$

$$B = \frac{\mu_0 I \cancel{\pi} r^{\cancel{2}}}{\pi c^2 \cdot 2\cancel{\pi}\cancel{r}} = \frac{\mu_0 I r}{2\pi c^2}$$ clockwise

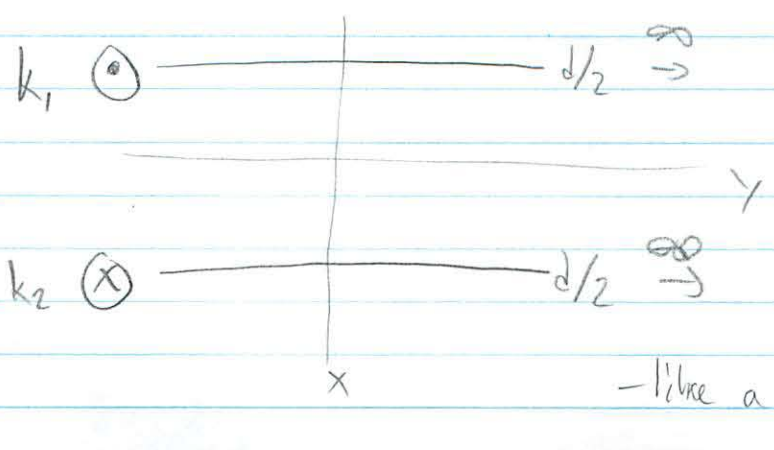
-4

when outside $I_{enc} = 0$, so $B = 0$



unless in region a-b we need effect from outer ring
-no I said that was 0 earlier

3. Two Current Sheets

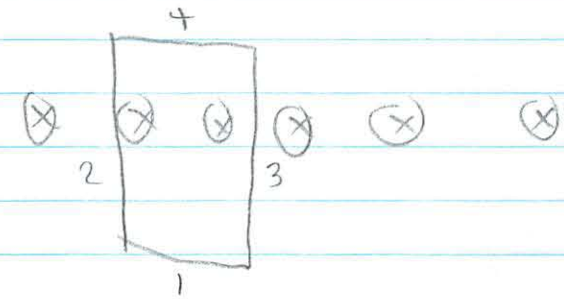


Why is it k and not J?
-current per unit width

-like a solenoid

b) Is the same except other way

Current still $\rightarrow$



$B\,l = \mu_0\, b\, l$

$B = \frac{\mu_0\, b\, l}{l} = \mu_0\, b$

Oh $n = \frac{N}{l}$ = # of turns per unit lenght

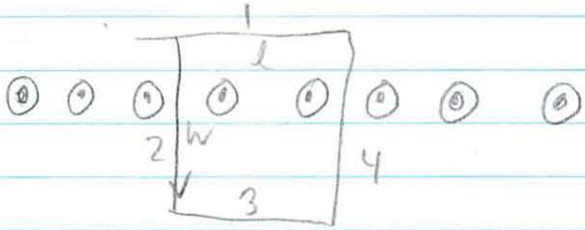
$k = n I$

So $B = \mu_0 k$

Is it − because it is down?

No current still to right

a. Find magnetic field everywhere due to $\vec{k}T$



$$\oint \vec{B}\cdot ds = \overset{1}{0} + \overset{2}{0} + \overset{3}{\int B \cdot d\vec{s}} + \overset{4}{0}$$

$$Bl$$

2 and 4 0 since perpendicular to current

Current is

$\longrightarrow$

Via screwdriver method

* 1 is 0 since $\vec{B}$ field is 0 outside solidnoid

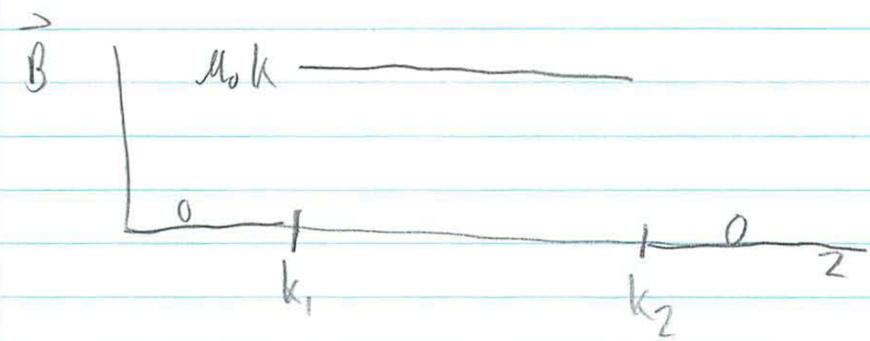
$$Bl = \mu_0 \underset{\uparrow \text{ \# of turns}}{N} l$$

$$= \mu_0 \underset{\uparrow \text{ for sheet}}{b} l$$

b = height
l = lenght

$$B = \frac{\mu_0 b l}{l} = \mu_0 b$$

c) Superposition ⇒ find magnetic field b/w both sheets

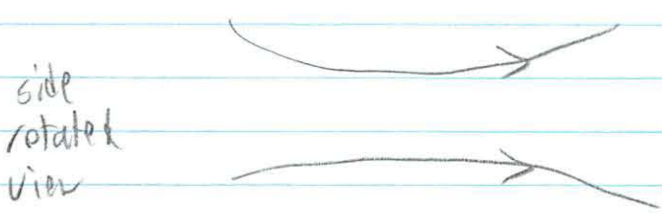


d) How would ans change if currents both in same direction?

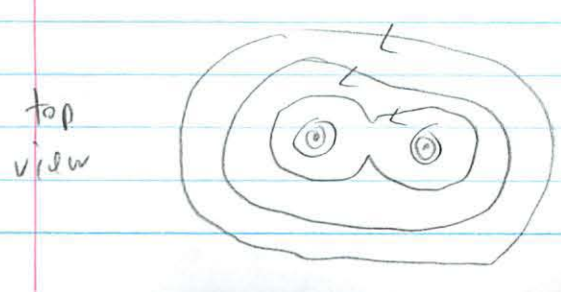
Redo bottom

Well bottom would be going ←

so B fields would pull togeter like we saw before



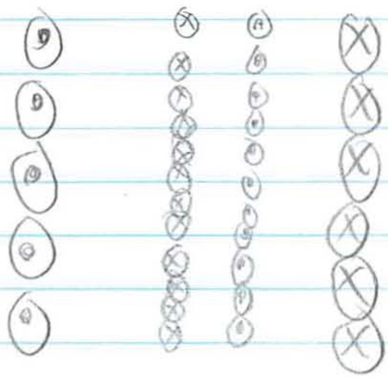
As for what would happen to B field?



Would merge into one big $\vec{B}$ field.

4. Nested Solidnoids

inner R_1 n_1
outer R_2 n_2



iii) $R_2 < r$ ← out of order

So going to do outside first since easiest

$\vec{B}$ field outside solonoid $= 0$

ii $R_1 < r < R_2$

So like inside of normal solidnoil

$$\oint B \cdot ds = \mu_0 I$$
$$B l = \mu_0 N_1 I$$
$$B = \mu_0 n_1 I$$

constant no matter position

Canter Clockwise B field up

i) $0 < r < R_1$

Is this the superposition of both

$$\oint B \cdot ds = \mu_0 I$$
$$B \, l = \mu_0 N I$$
$$B = \mu_0 n_2 I$$

clockwise so B field down

So if up is (+)

$$\vec{B}_{total} = \mu_0 n_1 I - \mu_0 n_2 I$$
$$\mu_0 I (n_1 - n_2)$$ simplify (doing easily now)

done -

- easy p-set to do
- got concepts
- worked well having course notes + working w/ them
- no stress
- took ~2 hrs
- no qu
- not going to OH

5. Calculating Flux from Current + Faraday's Law

So experiment measured current.
How was flux found?

$\Phi = B \cdot A$ $= \iint_S \vec{B} \cdot d\vec{A} =$
$= |B||A| \cos\theta$

~~[illegible]~~ o.K

$\mathcal{E} = -\frac{d\Phi}{dt} = \oint E \cdot ds = IR$ ~~[illegible]~~

↑ driving force for current

$I = \frac{\mathcal{E}}{R} = \frac{BA}{R}$ ← area

$I = \dfrac{-\frac{d\left(\iint B \cdot dA\right)}{dt}}{R} = \dfrac{-\frac{d\Phi}{dt}}{R}$

* current is the -derivitive of flux

ii Now integrate to find $\Phi(t)$ as a function of I

$IR = -\frac{d\Phi}{dt}$

$-\int IR = \int_S^E \frac{d\Phi}{dt} = \int_S^E \mathcal{E}$ R doesn't depend on time, it's constant.

??? think that's better - review!

$-\int IR = \boxed{\Phi} = \int \mathcal{E} = \boxed{-\int I(t)\, \cancel{R(t)}\, dt}$ -2

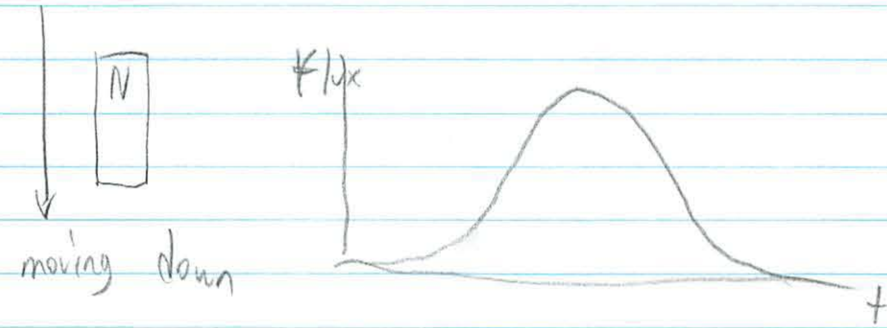
There is some resistence that needs to be there

Alex

b Predictions: Coil Moving Past Magnetic Diapole.

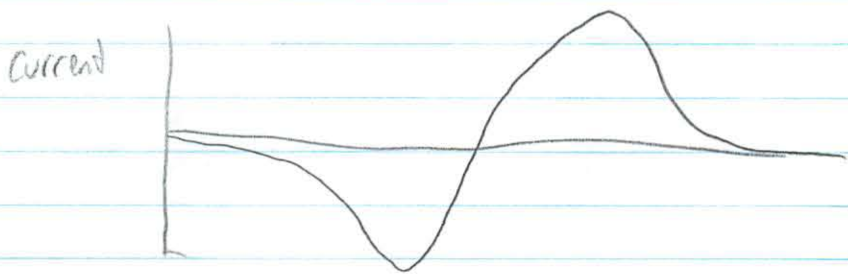
So I had already done the experiment at this point.

i) Magnetic flux through loop



field is always (+)

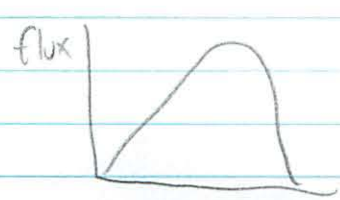
ii) current through the loop

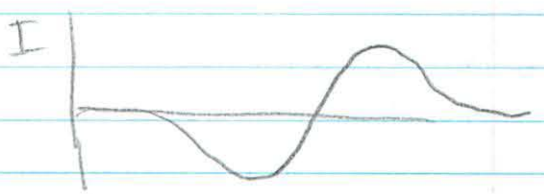


Current is – deriv of flux

c Now move from below up

Its the same





c) Force on coil moving past a dipole

What direction should magnetic force point?

a) Move loop from above → below and loop is above

(These were in the lab ...)

upward. Since it is trying to repel, push up the loop.

below magnet

downward - push magnet away

b) Below → above

loop below - downward - trying to push ring away

loop above → upward - trying to push ring away

d) Feeling the Force

Used an aluminum cylinder to feel the force in your hand

i) If we double the # of turns in the coil, the force changes?

→

well $\vec{B} = \mu_0 n I$

$\uparrow$ # of turns per unit area

$n = \frac{N}{\ell}$

$\vec{F} = I(\vec{L} \times \vec{B})$

$= q\vec{v} \times \vec{B}$

i) correct

not focusing much

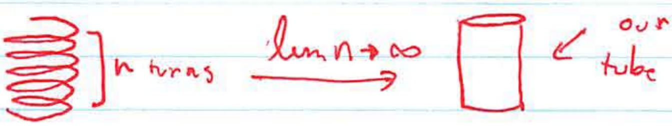
So twice as many coils would twice as much magnetic force ✓

ii) How should we think of the Al tube?

I don't know what this is asking. It just provides a large surface area which reacts to the magnetic field in a way that puts a force on your hand that you can feel
is affected by

−2

like a bunch of really tiny coils in a solenoid



MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 Spring 2010

Problem Set 8

Due: Tuesday, April 6 at 9 pm.

Hand in your problem set in your section slot in the boxes outside the door of 32-082. Make sure you clearly write your name and section on your problem set.

Text: Liao, Dourmashkin, Belcher; Introduction to E & M MIT 8.02 Course Notes.

Week Ten Faraday's Law

Class 22 W10D1 M/T Apr 5/6	Faraday's Law; Expt.7: Faraday's Law
Reading:	Course Notes: Sections 10.1-10.3, 10.8-10.9
Experiment:	Expt.7: Faraday's Law
Class 23 W10D2 W/R Apr 7/8	Problem Solving Faraday's Law; Inductance & Magnetic Energy, RL Circuits
Reading:	Course Notes: 10.1-10.4,10.8-10.9, 11.1-11.4
Class 24 W10D3 F Apr 9	Special Lecture: Applications of Faraday's Law
Reading:	Course Notes: 10.1-10.4, 10.8-10.9, 11.1-11.4

Campus Preview Weekend

Problem 1: In this problem you will work through two examples from Problem Solving 7: Ampere's Law.

OBJECTIVES

1. To learn how to use Ampere's Law for calculating magnetic fields from symmetric current distributions
2. To find an expression for the magnetic field of a cylindrical current-carrying shell of inner radius a and outer radius b using Ampere's Law.
3. To find an expression for the magnetic field of a slab of current using Ampere's Law.

REFERENCE: Section 9-3, 8.02 Course Notes.

Summary: Strategy for Applying Ampere's Law (Section 9.10.2, 8.02 Course Notes)

Ampere's law states that the line integral of $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ around any closed loop is proportional to the total steady current passing through any surface that is bounded by the closed loop:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{\text{enc}}$$

To apply Ampere's law to calculate the magnetic field, we use the following procedure:

Step 1: Identify the 'symmetry' properties of the current distribution.

Step 2: Determine the direction of the magnetic field

Step 3: Decide how many different spatial regions the current distribution determines

For each region of space...

Step 4: Choose an Amperian loop along each part of which the magnetic field is either constant or zero

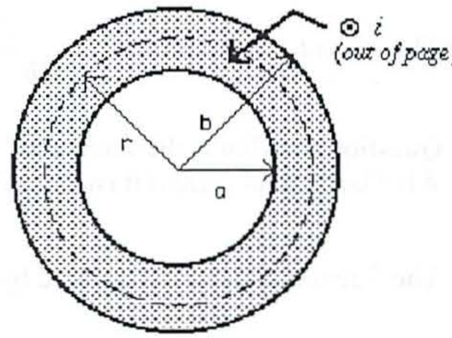
Step 5: Calculate the current through the Amperian Loop

Step 6: Calculate the line integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ around the closed loop.

Step 7: Equate $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ with $\mu_0 I_{\text{enc}}$ and solve for $\vec{\mathbf{B}}$.

Example 1: Magnetic Field of a Cylindrical Shell

We now apply this strategy to the following problem. Consider the cylindrical conductor with a hollow center and copper walls of thickness $b - a$ as shown . The radii of the inner and outer walls are a and b respectively, and the current I is uniformly spread over the cross section of the copper (shaded region). We want to calculate the magnetic field in the region $a < r < b$.



Question 1: Is the current density uniform or non uniform?

Answer: Uniform.

Problem Solving Strategy Step
Step 1: **<u>Identify Symmetry of Current Distribution</u>**
Either circular or rectangular

Step 2: **<u>Determine Direction of magnetic field</u>**
Clockwise or counterclockwise?

Step 3: **<u>How many regions?</u>**
Three: r<a; a<r<b; r>b

Step 4: **<u>Draw Amperian Loop</u>:**
Here we take a loop that is a circle of radius r with $a < r < b$ (see figure).

Step 5: **<u>Current enclosed by Amperian Loop</u>:**

The next step is to calculate the current enclosed by this imaginary Amperian loop. There are typically two ways to do this. One way is to simply calculate it as a fraction of the total current. The second is to first calculate the current density J (current per unit area) and then multiply by the area enclosed. You should use both methods and compare.

Question 2: What is the magnitude of the current per unit area J in the region $a < r < b$? Remember we are assuming that the current I is uniformly spread over the area $a < r < b$, and also remember that current density J is defined as the current per unit area.

The current density is $J = \dfrac{I}{A} = \dfrac{I}{\pi(b^2 - a^2)}$

Question 3: What is the fraction of the total area that is enclosed by the Amperian Loop? What is the total current it encloses?

The fraction of the area enclosed by the loop is $\left(\dfrac{r^2 - a^2}{b^2 - a^2}\right)$. The current enclosed is

$$I_{\text{enc}} = JA_{\text{enc}} = \frac{I}{\pi(b^2 - a^2)}\left(\pi r^2 - \pi a^2\right) = I\left(\frac{r^2 - a^2}{b^2 - a^2}\right)$$

Question 4: Your answer above should be zero when $r = a$ and I when $r = b$ (why?). Does your answer have these properties?

Yes. No current is enclosed when $r = a$. On the other hand, when $r = b$, the Amperian loop encloses all the current, so $I_{\text{enc}} = I$.

Step 6: Calculate Line Integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$:

Question 5: What is $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$? (That is, evaluate the integral, the left hand side of Ampere's law)

$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r)$.

Step 7: Solve for $\vec{\mathbf{B}}$:

Question 6: If you equate your answer to Question 5 to your answer to Question 3 times μ_o (i.e. use Ampere's Law), what do you get for the magnetic field in the region $a < r < b$?

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_0 I_{\text{enc}} = \mu_0 I\left(\frac{r^2 - a^2}{b^2 - a^2}\right) \Rightarrow \vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2 - a^2}{b^2 - a^2}\right) \text{ counter-clockwise}$$

Question 7: Repeat the steps above to find the magnetic field in the region $r < a$.

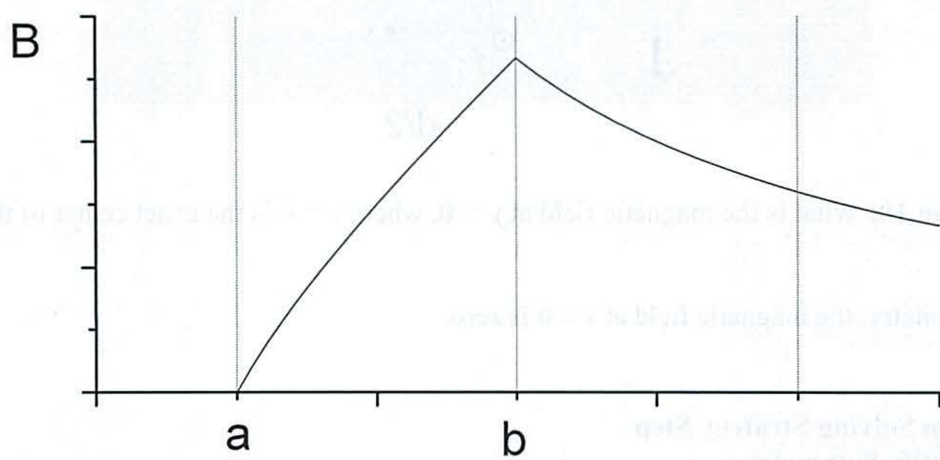
In the region $r < a$, $I_{\text{enc}} = 0$, and therefore $B = 0$.

Question 8: Repeat the steps above to find the magnetic field in the region $r > b$.

In the region $r > b$, $I_{enc} = I$. Therefore, we have

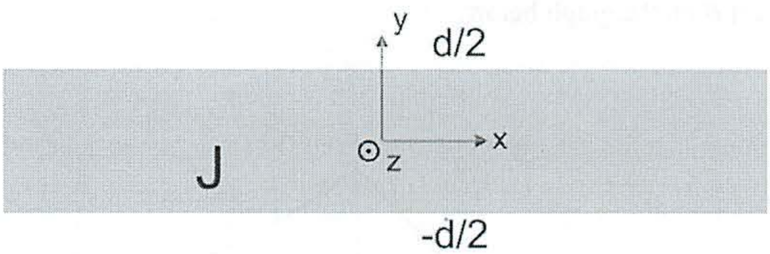
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_0 I_{enc} = \mu_0 I \Rightarrow \vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r} \text{ counter-clockwise.}$$

Question 9: Plot B on the graph below.



Example 2: Magnetic Field of a Slab of Current

We want to find the magnetic field $\vec{\mathbf{B}}$ due to an infinite slab of current, using Ampere's Law. The figure shows a slab of current with current density $\vec{\mathbf{J}} = 2J_e |y|/d\,\hat{\mathbf{z}}$, where units of J_e are amps per square meter. The slab of current is infinite in the x and z directions, and has thickness d in the y-direction.



Question 10: What is the magnetic field at $y = 0$, where $y = 0$ is the exact center of the slab?

By symmetry, the magnetic field at $y = 0$ is zero.

Problem Solving Strategy Step
(1) Identify Symmetry
Either circular or rectangular. Which is it?

(2) Determine Direction
Make sure you determine the direction in all regions. Sketch on tear sheet figure of Q9.

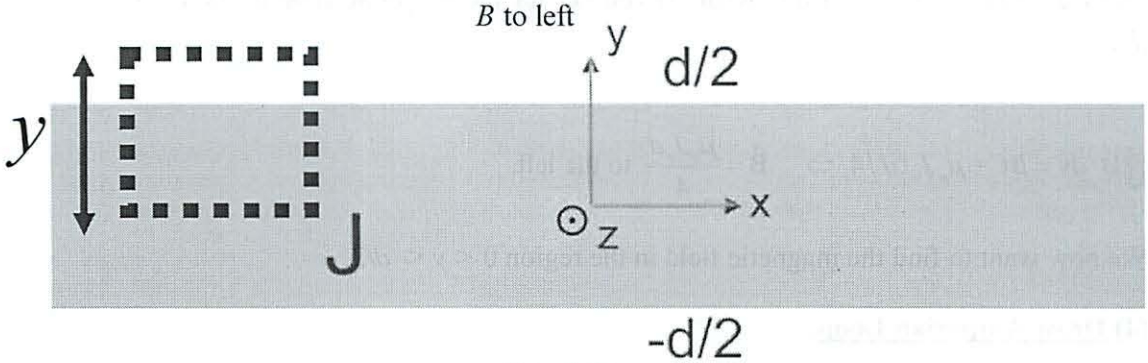
(3) How many regions?
Two for this problem: in the slab and above it (we won't do below the slab).

(4) Draw Amperian Loop:

We want to find the magnetic field for $y > d/2$, and we have from the answer to Question 10 for the magnetic field at $y = 0$. Therefore....

Question 11: What Amperian loop do you take to find the magnetic field for $y > d/2$? Draw it on the figure above and indicate its dimensions.

B to left



B to right

(5) __Current enclosed by Amperian Loop__:

The next step is to calculate the current enclosed by this imaginary Amperian loop. Hint: the current enclosed is the integral of the current density over the enclosed area.

Question 12: What is the total current enclosed by your Amperian loop from Question 11?

We take the above loop (in blue) in this case. We have to integrate the current density to get the enclosed current:

$$I_{\text{enc}} = \iint \frac{2J_e y}{d} dA = \frac{2J_e \ell}{d} \int_0^{d/2} y\, dy = \frac{2J_e \ell}{d} \frac{y^2}{2}\bigg|_0^{d/2} = \frac{J_e \ell d}{4}$$

(6): __Calculate Line Integral $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$:__

Question 13: What is $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$?

The loop has four segments. Along two of those (the sides) $\vec{\mathbf{B}}$ is perpendicular to $d\vec{\mathbf{s}}$ so $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = 0$. Along the center line $\vec{\mathbf{B}} = 0$. On the last side $\vec{\mathbf{B}}$ is parallel. Thus,

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\ell + 0 + 0 + 0 = B\ell$$

(7): <u>Solve for B:</u>

Question 14: If you equate your answers in Question 13 to your answer in Question 12 times μ_o using Ampere's Law, what do you get for the magnetic field in the region $y > d/2$?

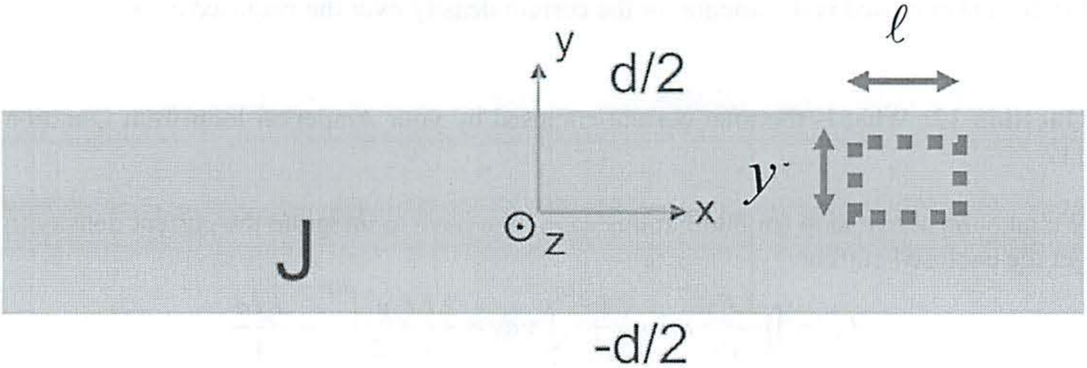
$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = B\ell = \mu_o J_e \ell d/4 \Rightarrow \quad \vec{\mathbf{B}} = \frac{\mu_o J_e d}{4} \text{ to the left}$$

We now want to find the magnetic field in the region $0 < y < d/2$.

(4) <u>Draw Amperian Loop</u>:

We want to find the magnetic field for $0 < y < d/2$, and we have from the answer to Question 10 for the magnetic field at $y = 0$. Therefore…

Question 15: What Amperian loop do you take to find the magnetic field for $0 < y < d/2$? Draw it on the figure above and on the tear-sheet at the end, and indicate its dimensions.



(5) <u>Current enclosed by Amperian Loop</u>:

The next step is to calculate the current enclosed by this imaginary Amperian loop.

Question 16: What is the total current enclosed by your Amperian loop from Question 15?

We take the above loop (in red) in this case. We have to integrate the current density to get the enclosed current:

$$I_{\text{enc}} = \iint \frac{2J_e y}{d} dA = \frac{2J_e \ell}{d} \int_0^y y\, dy = \frac{2J_e \ell}{d} \frac{y^2}{2}\bigg|_0^y = \frac{J_e \ell y^2}{d}$$

(6) Calculate Line Integral $\oint \vec{\mathbf{B}}\cdot d\vec{s}$:

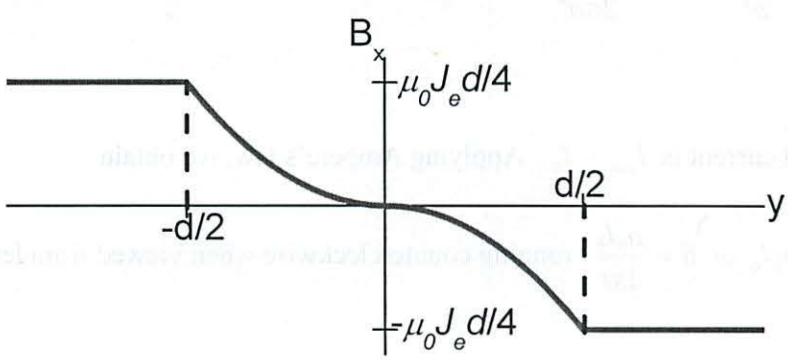
Question 17: What is $\oint \vec{\mathbf{B}}\cdot d\vec{s}$?
The loop has four segments. Along two of those (the sides) $\vec{\mathbf{B}}$ is perpendicular to ds so $\vec{\mathbf{B}}\cdot d\vec{s} = 0$. Along the centerline $\vec{\mathbf{B}} = 0$. Along the top side $\vec{\mathbf{B}}$ is parallel.
$\oint \vec{\mathbf{B}}\cdot d\vec{s} = B\ell + 0 + 0 + 0 = B\ell$.

(7) Solve for B:

Question 18: If you equate you answers in Question 17 to your answer in Question 16 times μ_o using Ampere's Law, what do you get for the magnetic field in the region $0 < y < d/2$?

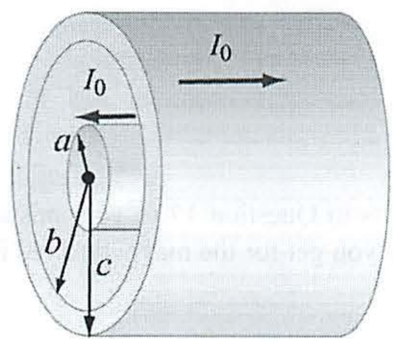
$$\oint \vec{\mathbf{B}}\cdot d\vec{s} = B\ell = \mu_0 J_e \ell y^2/d \Rightarrow B = \mu_0 J_e y^2/d$$

Question 19: Plot B_x on the graph below. Use symmetry to determine B for y<0. Label the y-axis



Problem 2 Co-axial Cable

A coaxial cable consists of a solid inner conductor of radius a, surrounded by a concentric cylindrical tube of inner radius b and outer radius c. The conductors carry equal and opposite currents I_0 *distributed uniformly across their cross-sections*. Determine the magnitude and direction of the magnetic field at a distance r from the axis. Make a graph of the magnitude of the magnetic field as a function of the distance r from the axis.



Solution:

(a) $r < a$;

The enclosed current is $I_{enc} = I_0\left(\frac{\pi r^2}{\pi a^2}\right) = \frac{I_0 r^2}{a^2}$. Applying Ampere's law, we have

$B(2\pi r) = \mu_0 \frac{I_0 r^2}{a^2}$ or $B = \frac{\mu_0 I_0}{2\pi a^2} r$, running counterclockwise when viewed from left

(b) $a < r < b$;

The enclosed current is $I_{enc} = I_0$. Applying Ampere's law, we obtain

$B(2\pi r) = \mu_0 I_0$ or $B = \frac{\mu_0 I_0}{2\pi r}$, running counterclockwise when viewed from left

(c) $b < r < c$;

$$I_{enc} = I_0 - I_0\left(\frac{\pi r^2 - \pi b^2}{\pi c^2 - \pi b^2}\right) = \frac{I_0 (c^2 - r^2)}{c^2 - b^2}$$

Applying Ampere's law,

$$B(2\pi r) = \mu_0 \frac{I_0(c^2 - r^2)}{c^2 - b^2}$$

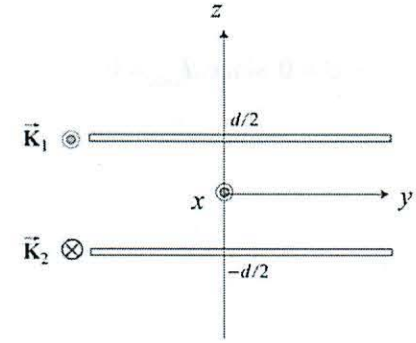
or $B = \dfrac{\mu_0 I_0(c^2 - r^2)}{2\pi(c^2 - b^2)r}$, running counterclockwise when viewed from left

(d) $r > c$.

$$B = 0 \text{ since } I_{enc} = 0$$

Problem 3: Two Current Sheets

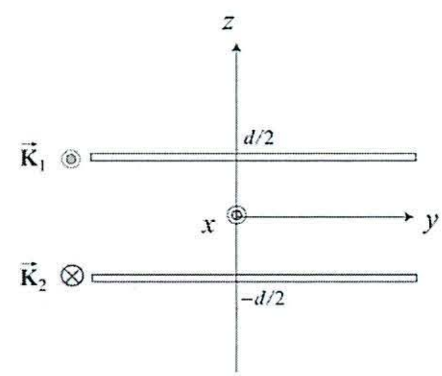
Consider two infinitely large sheets lying in the xy-plane separated by a distance d carrying surface current densities $\vec{\mathbf{K}}_1 = K\hat{\mathbf{i}}$ and $\vec{\mathbf{K}}_2 = -K\hat{\mathbf{i}}$ in the opposite directions, as shown in the figure below (The extent of the sheets in the y direction is infinite.) Note that K is the current per unit width perpendicular to the flow.



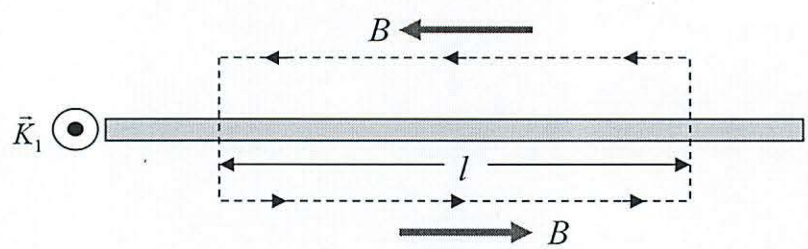
a) Find the magnetic field everywhere due to $\vec{\mathbf{K}}_1$.
b) Find the magnetic field everywhere due to $\vec{\mathbf{K}}_2$.
c) Applying superposition principle, find the magnetic field everywhere due to both current sheets.
d) How would your answer in (c) change if both currents were running in the same direction, with $\vec{\mathbf{K}}_1 = \vec{\mathbf{K}}_2 = K\hat{\mathbf{i}}$?

Solution:

Consider two infinitely large sheets lying in the xy-plane separated by a distance d carrying surface current densities $\vec{\mathbf{K}}_1 = K\hat{\mathbf{i}}$ and $\vec{\mathbf{K}}_2 = -K\hat{\mathbf{i}}$ in the opposite directions, as shown in the figure below (The extent of the sheets in the y direction is infinite.) Note that K is the current per unit width perpendicular to the flow.



(a) Find the magnetic field everywhere due to $\vec{\mathbf{K}}_1$.



Consider the Ampere's loop shown above. The enclosed current is given by

$$I_{\text{enc}} = \int \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}} = Kl$$

Applying Ampere's law, the magnetic field is given by

$$B(2l) = \mu_0 K l \ \text{ or } \ B = \frac{\mu_0 K}{2}$$

Therefore,

$$\vec{\mathbf{B}}_1 = \begin{cases} -\dfrac{\mu_0 K}{2}\hat{\mathbf{j}}, & z > \dfrac{d}{2} \\ \dfrac{\mu_0 K}{2}\hat{\mathbf{j}}, & z < \dfrac{d}{2} \end{cases}$$

(b) Find the magnetic field everywhere due to $\vec{\mathbf{K}}_2$.

The result is the same as part (a) except for the direction of the current:

$$\vec{\mathbf{B}}_2 = \begin{cases} \dfrac{\mu_0 K}{2}\hat{\mathbf{j}}, & z > -\dfrac{d}{2} \\ -\dfrac{\mu_0 K}{2}\hat{\mathbf{j}}, & z < -\dfrac{d}{2} \end{cases}$$

(c) Applying superposition principle, find the magnetic field everywhere due to both current sheets.

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = \begin{cases} \mu_0 K\,\hat{\mathbf{j}}, & -\dfrac{d}{2} < z < \dfrac{d}{2} \\ 0, & |z| > \dfrac{d}{2} \end{cases}$$

(d) How would your answer in (c) change if both currents were running in the same direction, with $\vec{\mathbf{K}}_1 = \vec{\mathbf{K}}_2 = K\,\hat{\mathbf{i}}$?

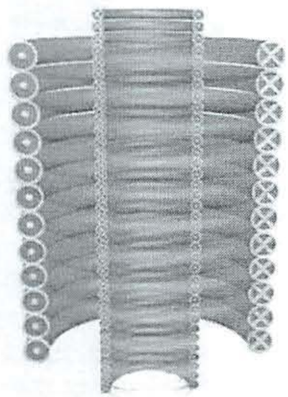
In this case, $\vec{\mathbf{B}}_1$ remains the same but

$$\vec{\mathbf{B}}_2 = \begin{cases} -\dfrac{\mu_0 K}{2}\,\hat{\mathbf{j}}, & z > -\dfrac{d}{2} \\ \dfrac{\mu_0 K}{2}\,\hat{\mathbf{j}}, & z < -\dfrac{d}{2} \end{cases}$$

Therefore,

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = \begin{cases} -\mu_0 K\,\hat{\mathbf{j}}, & z > \dfrac{d}{2} \\ 0, & -\dfrac{d}{2} < z < \dfrac{d}{2} \\ \mu_0 K\,\hat{\mathbf{j}}, & z < -\dfrac{d}{2} \end{cases}$$

Problem 4 Nested Solenoids: Two long solenoids are nested on the same axis, as in the figure below. The inner solenoid has radius R_1 and n_1 turns per unit length. The outer solenoid has radius R_2 and n_2 turns per unit length. Each solenoid carries the same current I flowing in each turn, *but in opposite directions*, as indicated on the sketch.



Use Ampere's Law to find the direction and magnitude of the magnetic field in the following regions. Be sure to show your Amperian loops and all your calculations.

i) $0 < r < R_1$

ii) $R_1 < r < R_2$

iii) $R_2 < r$

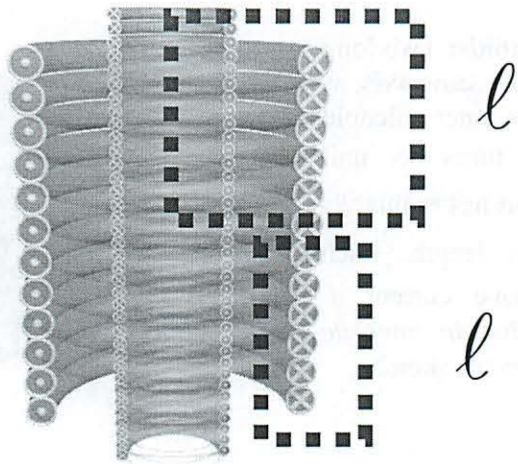
Solution: Nested Solenoids: Two long solenoids are nested on the same axis, as in the figure below. The inner solenoid has radius R_1 and n_1 turns per unit length. The outer solenoid has radius R_2 and n_2 turns per unit length. Each solenoid carries the same current I flowing in each turn, *but in opposite directions*, as indicated on the sketch.

Use Ampere's Law to find the direction and magnitude of the magnetic field in the following regions:

(a) $0 < r < R_1$;

To solve for the magnetic field in this case, we take the top rectangular loop shown in the figure. The current through the loop is

$$I_{\text{enc}} = -n_1 \ell I + n_2 \ell I = (-n_1 + n_2)\ell I$$



The loop has four segments. Along two of those (top and bottom, horizontal), $\vec{\mathbf{B}}$ is perpendicular to $d\vec{\mathbf{s}}$, and $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = 0$. On the other hand, along the outer vertical segment, $\vec{\mathbf{B}} = 0$. Thus, using Ampere's law $\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{\text{enc}}$, we have

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = B\ell + 0 + 0 + 0 = B\ell = \mu_0 \left(-n_1 \ell I + n_2 \ell I\right) \Rightarrow \vec{\mathbf{B}} = \mu_0 I \left(-n_1 + n_2\right) \hat{\mathbf{k}}$$

(b) $R_1 < r < R_2$

To solve for the magnetic field in this case, we take the bottom rectangular loop shown in the figure. The current through the loop is

$$I_{\text{enc}} = n_2 \ell I$$

The loop has four segments. Along two of those (top and bottom, horizontal), $\vec{\mathbf{B}}$ is perpendicular to $d\vec{\mathbf{s}}$, and $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = 0$. On the other hand, along the outer vertical segment, $\vec{\mathbf{B}} = 0$. Thus, using Ampere's law $\oint \vec{\mathbf{B}} \cdot d\vec{s} = \mu_0 I_{\text{enc}}$, we have

$$\oint \vec{\mathbf{B}} \cdot d\vec{s} = B\ell + 0 + 0 + 0 = B\ell = \mu_0 n_2 \ell I \Rightarrow \vec{\mathbf{B}} = \mu_0 n_2 I \hat{\mathbf{k}}$$

(c) $R_2 < r$

Since the net current enclosed by the Amperian loop is zero, the magnetic field is zero in this region.

Problem 5: Read Experiment 7 Faraday's Law.

http://web.mit.edu/8.02t/www/materials/Experiments/exp07.pdf

(a) Calculating Flux from Current and Faraday's Law. In part 1 of the lab you moved a coil from well above to well below a strong permanent magnet. You measured the current in the loop during this motion using a current sensor. The program also displayed the flux "measured" through the loop, even though this value is never directly measured.

(i) Starting from Faraday's Law and Ohm's law, write an equation relating the current in the loop to the time derivative of the flux through the loop.

$$\varepsilon = -\frac{d\Phi}{dt} = IR$$

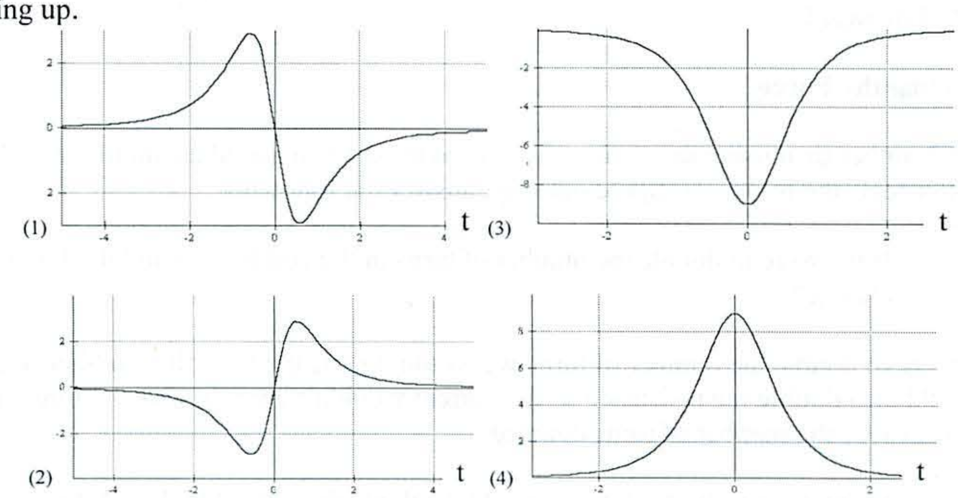
(ii) Now integrate that expression to get the time dependence of the flux through the loop $\Phi(t)$ as a function of current $I(t)$. What assumption must the software make before it can plot flux vs. time?

$$d\Phi = -IR\,dt \quad \Rightarrow \quad \Phi(t) = -R\int_{t=0}^{t} I(t')\,dt'$$

The software must assume (as I did above) that the flux at time t=0 is zero.

(b) Predictions: Coil Moving Past Magnetic Dipole

In moving the coil over the magnet, measurements of current and flux for each of several motions looked like one of the below plots. For current, counter-clockwise when viewed from above is positive. For flux, upwards is positive. The north pole of the magnet is pointing up.



Suppose you moved the loop from well *above* the magnet to well *below* the magnet at a constant speed. Which graph most closely resembles the graph of:

(i) *magnetic flux through the loop* as a function of time? 4

(ii) *current through the loop* as a function of time? 2

Suppose you moved the loop from well *below* the magnet to well *above* the magnet at a constant speed. Which graph most closely resembles the graph of:

(iii) *magnetic flux through the loop* as a function of time? 4

(iv) *current through the loop* as a function of time? 2

(c) Force on Coil Moving Past Magnetic Dipole

In part 2 of this lab you felt the force on a conducting loop as it moves past the magnet. For the following conditions, in what direction should the magnetic force point?

As you moved the loop from well *above* the magnet to well *below* the magnet at a constant speed…

(i) … and the loop is *above* the magnet.

(ii) … and the loop is *below* the magnet

As you moved the loop from well *below* the magnet to well *above* the magnet at a constant speed…

(iii) … and the loop is *below* the magnet.

(iv) … and the loop is *above* the magnet

In all of these cases the force opposes the motion. For (a) & (b) it points upwards, for (c) and (d) downwards.

(d) Feeling the Force

In part 2, rather than using the same coil we used in part 1, we used an aluminum cylinder to "better feel" the force. To figure out why, answer the following.

(i) If we were to double the number of turns in the coil how would the force change?

If we were to double the number of turns we would double the total flux and hence EMF, but would also double the resistance so the current wouldn't change. But the force would double because the number of turns doubled.

(ii) Using the result of (a), how should we think about the Al tube? Why do we "better feel" the force?

Going to the cylinder basically increases many times the number of coils (you can think about it as a bunch of thin wires stacked on top of each other). It also reduces the resistance and hence increases the current because the resistance is not through one very long wire but instead a bunch of short loops all in parallel with each other.

In case you are interested, the wire is copper, and of roughly the same diameter as the thickness of the aluminum cylinder, although this information won't necessarily help you in answering the question.

Topics: Faraday's Law
Related Reading: Course Notes: Sections 10.1-10.4, 10.8-10.9, 11.1-11.4
Experiments: (9) Faraday's Law of Induction

Topic Introduction

Today you will practice what you have learned about Faraday's Law and then we will study self-induction. in a problem solving session.

Faraday's Law & Lenz's Law

Recall: Faraday's Law says that a changing magnetic flux generates an EMF $\mathcal{E} = -d\Phi_B/dt$
Lenz's Law says that the direction of that EMF is so as to oppose the ***change*** in magnetic flux.

WARNING:
Because it bears repeating (especially with an upcoming exam on this material): many students confuse Faraday's Law with Ampere's Law. Both involve integrating around a loop and comparing that to an integral across the area bounded by that loop. Aside from this mathematical similarity, however, the two laws are completely different. In Ampere's law the field that is "curling around the loop" is the magnetic field, created by a "current flux" $\left(I = \iint \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}\right)$ that is penetrating the looping B field. In Faraday's law the electric field is curling, created by a *changing* magnetic flux. In fact, there need not be any currents at all in the problem, although as you will see in today's problem solving typically the EMF is measured by its ability to drive a current around a physical loop – a circuit.

Self Inductance

When a circuit has a current in it, it creates a magnetic field, and hence a flux, through itself. If that current changes, then the flux will change and hence an EMF will be induced in the circuit. The EMF obeys: $\mathcal{E} = -L\dfrac{dI}{dt}$, where L is a constant called the *self-inductance.* The action of that EMF will be to *oppose the change* in current (if the current is decreasing it will try to make it bigger, if increasing it will try to make it smaller). For this reason, we often refer to the induced EMF as the "back EMF." To calculate the self inductance (or inductance, for short) of an object, imagine that a current I flows through it, and determine how much magnetic field and hence flux Φ_B that makes through the object. The self inductance is then $L = \Phi_B/I$.

Inductors

When we worked with resistors in circuits, they 'resist' the flow of current. That is, you must supply a voltage drop across them to drive current through them.
Inductors (symbol L, measured in SI units of Henries), which we study today, instead *resist changes* in the current. That is, you must supply a potential drop across them if you want to change the current which is flowing through them. Another way to say this is that if you try to change the current the inductor will generate an EMF $\mathcal{E} = -L\frac{dI}{dt}$ to oppose the change.

So many things

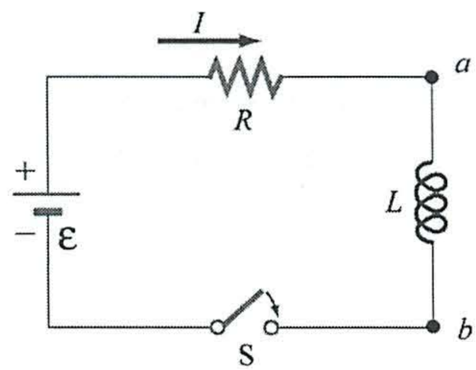
LR-circuit opposite

Energy in B Fields

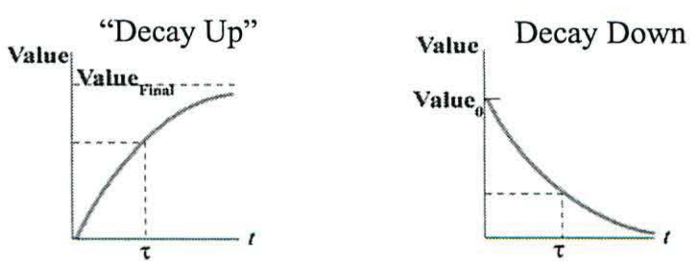
Remember that we defined the self inductance L by the amount of flux that an object generates through itself when a current I flows through it ($\Phi = LI$) and, from Faraday's Law, found that inductors will generate a back EMF: $\mathcal{E} = -L\,dI/dt$. They also store energy. In capacitors we found that energy was stored in the electric field between their plates. In inductors, energy is stored in the magnetic field. Just as with capacitors, where the electric field was created by a charge on the capacitor, we now have a magnetic field created when there is a current through the inductor. Thus, just as with the capacitor, we can discuss both the energy in the inductor, $U = \frac{1}{2}LI^2$, and the more generic energy density $u_B = \frac{B^2}{2\mu_0}$, stored in the magnetic field. Again, although we introduce the magnetic field energy density when talking about energy in inductors, it is a generic concept – whenever a magnetic field is created it takes energy to do so, and that energy is stored in the field itself.

RL Circuits

A simple RL circuit is shown below. When the switch is closed, if the inductor were not in the circuit, current would immediately flow in the circuit, with magnitude set by the resistance. The inductor, however, resists the change in current, letting it only gradually increase from $I = 0$.



We can quantify this behavior by writing down the differential equation for current flow using Kirchhoff's loop rules as well as $\mathcal{E} = -L\,dI/dt$ for an inductor. The solution to this differential equation shows that the current "decays upwards" towards a final value of the current in which the inductor is no longer doing anything. That is, at first, when the switch is closed and the current is trying to increase from 0, the inductor works hard to stop it. After a while the inductor stops fighting and no longer has an effect (when thinking about how much current is flowing in the circuit you can mentally remove it).



The rate at which this change happens is dictated by the "time constant" τ, which for this circuit is given by *L/R* (the bigger the inductance the slower that changes happen in the circuit, but the bigger the resistance, the smaller the current and hence changes in the current that the inductor will see).

We will speak about the solution to these types of differential equations in general, and you will see that all values either exponentially decay or "decay up," and hence that, at least at a conceptual level, you can usually determine what will happen to currents or voltages just by thinking about the behavior of the various circuit elements.

Important Equations

Faraday's Law: $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$

Magnetic Flux: $\Phi_B = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$

EMF: $\mathcal{E} = \oint \vec{\mathbf{E}}' \cdot d\vec{\mathbf{s}}$

Self Inductance, *L*: $L = \dfrac{\Phi_B}{I}$

Energy Stored in Inductor: $U = \dfrac{1}{2} L I^2$

EMF Induced by Inductor: $\mathcal{E} = -L \dfrac{dI}{dt}$

Exponential Decay: $Value = Value_{initial} e^{-t/\tau}$

Exponential "Decay Upwards": $Value = Value_{final} \left(1 - e^{-t/\tau}\right)$

Simple RL Time Constant: $\tau = L/R$

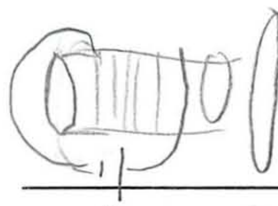
4/7

Class 23: Outline

Hour 1:
- Faraday's Law Problem Solving Session

Hour 2:
- Self Inductance
- Energy in Inductors
- Circuits with Inductors: RL Circuit

iron core - when put magnetic field on it it gets stronger



current induced immediatly

$\mathcal{E}$? what is current dir in ring

Faraday's Law of Induction

$$\boxed{\mathcal{E} = -\frac{d\Phi_B}{dt}}$$

Changing magnetic flux *induces* an EMF

Lenz: Induction ***opposes*** change

fighting charge

current in the core goes counterclockwise - flux left
in ring it wants to go the other way - clockwise - flux right
$\mathcal{E} = -\frac{d\Phi}{dt}$ wants to go opposite to stop change

Faraday's Law Problem Solving Session

Ways to Induce EMF

$$\mathcal{E} = -\frac{d}{dt}\left(BA\cos\theta\right)$$

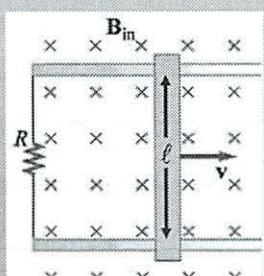
Quantities which can vary with time:

- Magnitude of B **e.g. Falling Magnet**
- **Area A enclosed by the loop**
- Angle θ between B and loop normal

P23- 4

Group Problem: Changing Area

Conducting rod pulled along two conducting rails in a uniform magnetic field B at constant velocity v



1. Direction of induced current?
2. Direction of resultant force?
3. Magnitude of EMF?
4. Magnitude of current?
5. Power externally supplied to move at constant v?

P23- 5

→ see solution

Ways to Induce EMF

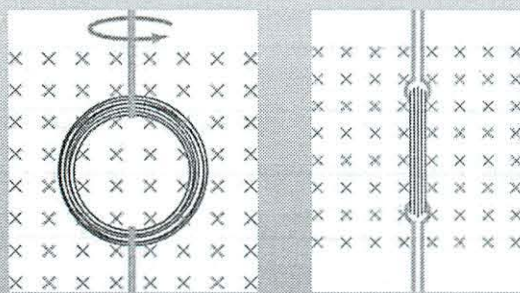
$$\mathcal{E} = -\frac{d}{dt}\left(BA\cos\theta\right)$$

Quantities which can vary with time:

- Magnitude of B **e.g. Moving Coil & Dipole**
- Area A enclosed **e.g. Sliding bar**
- **Angle θ between B and loop normal**

P23- 6

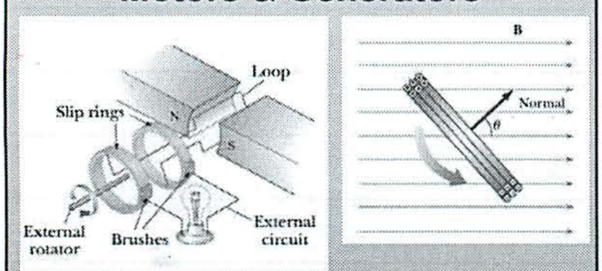
Changing Angle



$\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = BA$

$\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = 0$

Motors & Generators

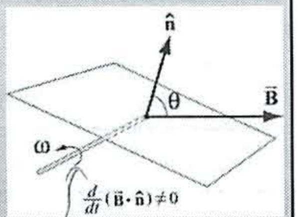


PRS Question: Generator

? all turned around

PRS: Generator

A square coil rotates in a magnetic field directed to the right. At the time shown, the current in the square, when looking down from the top of the square loop, will be



0%	1. Clockwise
0%	2. Counterclockwise
0%	3. Neither, the current is zero
0%	4. I don't know

:00

? wire

Point fingers towards B

fighting flux

$\vec{B}$ is through $\vec{A}$ → positive

as its rotating less flux going through

want to ↑ flux

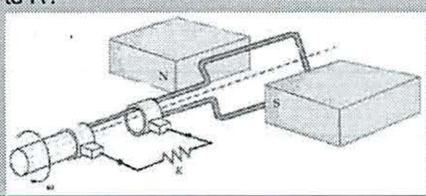
thumb to normal vector

fingers go counter clockwise

Group Problem: Generator

Square loop (side L) spins with angular frequency ω in a field of strength B. It is hooked to a load R.

1) Write an expression for current $I(t)$ assuming the loop is vertical at time $t = 0$.
2) How much work from generator per revolution?
3) To make it twice as hard to turn, what do you do to R?



P23-11

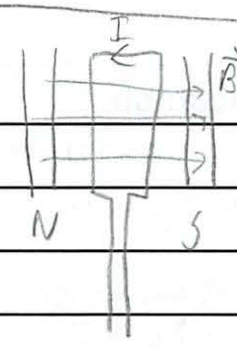
or is rotating, does not want

put diapole moment in dir of normal vector

same thumb to normal vector

fingers go counter-clockwise

current will flip direction every 180°



PRS Question: Wrap-Up Faraday's Law

P23-12

1) $\mathcal{E} = -\frac{d\Phi}{dt} = IR$ $\quad \Phi = BA\cos\omega t$

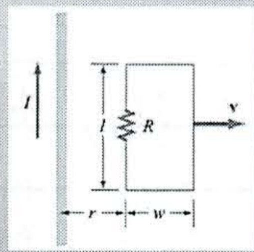
$I(t) = \frac{Bl^2\omega\sin\omega t}{R}$

→

PRS: Circuit

0

A circuit in the form of a rectangular piece of wire is pulled away from a long wire carrying current I in the direction shown in the sketch. The induced current in the rectangular circuit is



0%	1.	Clockwise
0%	2.	Counterclockwise
0%	3.	Neither, the current is zero

P23-11

Generator Problem

2. $P = IV$

$W = \int_0^{\frac{2\pi}{\omega}} P\,dt = \int_0^{\frac{2\pi}{\omega}} \frac{(BA\omega\sin\omega t)^2}{R}dt$

$= \left(\frac{BA}{R}\right)\int_0^{\frac{2\pi}{\omega}} \sin^2\omega t\,dt$

$= \frac{1}{2}\left(\frac{B^2A^2\omega^2}{R}\right)\left(\frac{2\pi}{\omega}\right) = \frac{B^2 l^4 \omega\pi}{R}$

Self Inductance

P23-14

3. decreasing resistance makes it harder to turn - increases load/current

Self Inductance

What if is the effect of putting current into coil 1?

There is "self flux": only 1 coil

$$\Phi \equiv LI$$

Faraday's Law → $\boxed{\mathcal{E} = -L\frac{dI}{dt}}$

P23-15

$\Phi = \Phi_{11} = M_{11} I_1 = LI$

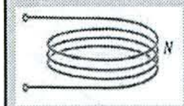
$\Phi_{12} = M_{12} I_2$

$\mathcal{E}_{12} = -M_{12}\frac{dI_2}{dt}$

don't need to know

coil translates music wirelessly
- current in 1 makes current in other
↳ magnetic field

Calculating Self Inductance



$$\boxed{L = \frac{\Phi_{\text{Total,self}}}{I}}$$

Unit: Henry $1\ \text{H} = 1\ \frac{\text{V} \cdot \text{s}}{\text{A}}$

1. Assume a current I is flowing in your device
2. Calculate the B field due to that I
3. Calculate the flux due to that B field
4. Calculate the self inductance (divide out I)

P23-16

↑ like capitance

amt of flux

amt of current

L = specific ring geometry

Group Problem: Solenoid

Calculate the self-inductance L of a solenoid (n turns per meter, length ℓ, radius R)

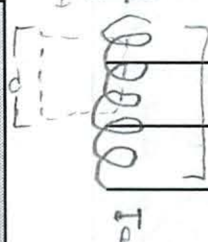
REMEMBER

1. Assume a current I is flowing in your device
2. Calculate the B field due to that I
3. Calculate the flux due to that B field
4. Calculate the self inductance (divide out I)

$$L = \Phi_{\text{Self, total}} / I$$

P23-17

↙ Amperian Loop



↙ dnI

$\oint \vec{B} \cdot d\vec{s} = I_{enc} \mu_0$

$Bd = \mu_0 d n I$

$\vec{B} = \mu_0 n I$

$\Phi = A|B|\cos\theta = \mu_0 n I (\pi r^2)$

↑ area of solinoid

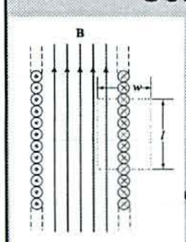
$\boxed{\cancel{L = \mu_0 n \pi r^2}}$

$L = \frac{N\Phi}{I} = N\mu_0 n \pi R^2$

$\boxed{= \mu_0 n^2 \pi R^2 \ell}$

↙ know this

Solenoid Inductance



$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\ell = \mu_0 I_{enc} = \mu_0 (n\ell) I$$

$$B = \mu_0 n I$$

$$\Phi_{B,sgl} = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = BA = \mu_0 n I \pi R^2$$

area of solenoid

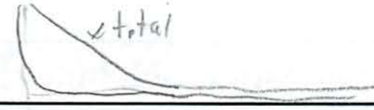
$$L = \frac{N\Phi_{B,sgl}}{I} = N\mu_0 n \pi R^2 = \mu_0 n^2 \pi R^2 l$$

P23-18

Mon preview

external flux

total flux = external + self induced



takes time for total flux to decay down

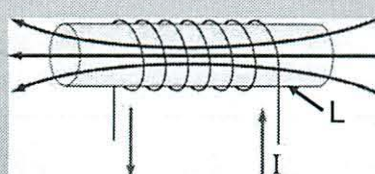
Energy in Inductors

when push current through have self flux

$\Phi = LI$

$\mathcal{E} = -L\frac{dI}{dt}$

Inductor Behavior



$$\mathcal{E} = -L\frac{dI}{dt}$$

Inductor with constant current does nothing

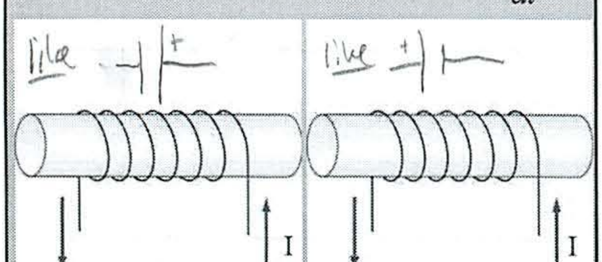
Steady state – current going through not changing – happy

if current is changing
- increasing

- unhappy w/ change
- will have $\mathcal{E}$ pushing back against current (like a battery)

* hate change – will make very big emfs – even could arc + make plasma balls

Back EMF $\varepsilon = -L\frac{dI}{dt}$



$\frac{dI}{dt} > 0,\ \varepsilon_L < 0$ | $\frac{dI}{dt} < 0,\ \varepsilon_L > 0$

Energy To "Charge" Inductor

1. Start with "uncharged" inductor
2. Gradually increase current. Must work:

$$dW = Pdt = \varepsilon I\, dt = L\frac{dI}{dt} I\, dt = LI\, dI$$

3. Integrate up to find total work done:

$$W = \int dW = \int_{I=0}^{I} LI\, dI = \tfrac{1}{2} L I^2$$

Energy Stored in Inductor

$$\boxed{U_L = \frac{1}{2} L I^2}$$

But where is energy stored?

Example: Solenoid

Ideal solenoid, length l, radius R, n turns/length, current I:

$$B = \mu_0 n I \qquad L = \mu_o n^2 \pi R^2 l$$

$$U_B = \tfrac{1}{2} L I^2 = \tfrac{1}{2}\left(\mu_o n^2 \pi R^2 l\right) I^2$$

$$U_B = \underbrace{\left(\frac{B^2}{2\mu_o}\right)}_{\text{Energy Density}} \underbrace{\pi R^2 l}_{\text{Volume}}$$

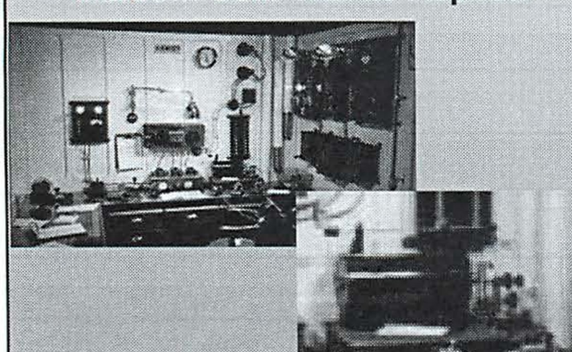
Demos: Breaking Circuits

Big Inductor
Marconi Coil

The Question:
What happens if big ΔI, small Δt

Marconi Coil: On the Titanic

Marconi Coil: Titanic Replica



make a spark
field can be read across Europe

Sollinoid in a circuit
makes very big emf
Sparks across 10 Cm
300,000 Volts

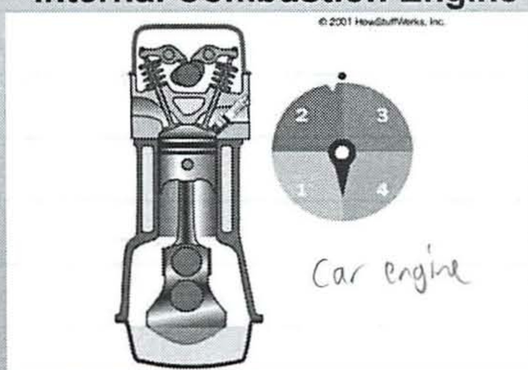
The Point: Big EMF

$$\boxed{\mathcal{E} = -L\frac{dI}{dt}}$$

Big L
Big dI $\longrightarrow$ Huge ε
Small dt

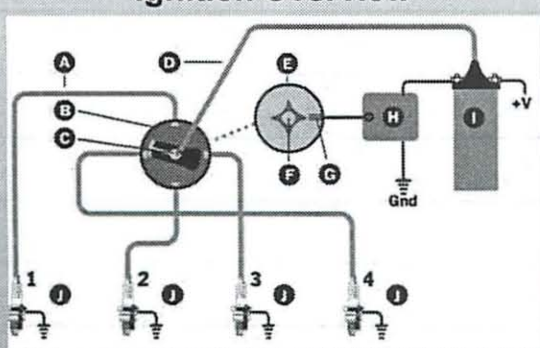
break circuit violently

Internal Combustion Engine

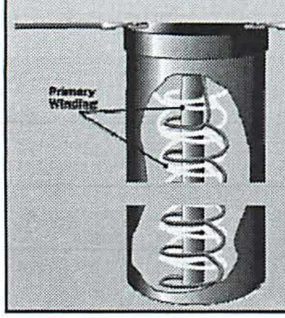


Car engine

Ignition Overview



The Workhorse: The Coil



Primary Coil:
~200 turns heavy Cu
DC (12 V) in to GND

Secondary Coil:
~20,000 turns fine Cu
Usually no voltage…
When primary breaks
up to ~45,000 V

P23-31

Energy Density

Energy is stored in the magnetic field!

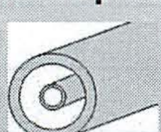
$$\boxed{u_B = \frac{B^2}{2\mu_o}}$$: Magnetic Energy Density

$$u_E = \frac{\varepsilon_o E^2}{2}$$: Electric Energy Density

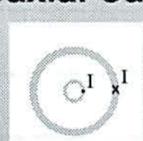
P23-32

$U_L = \frac{1}{2} L I^2$ – in the magnetic field

Group Problem: Coaxial Cable



Inner wire: $r = a$
Outer wire: $r = b$



1. How much energy is stored per unit length?
2. What is inductance per unit length?

HINTS: This does require an integral
The EASIEST way to do (2) is to use (1)

P23-33

does take time as circuits rises

Solenoid $B = \mu_0 n I$ $L = \mu_0 n^2 \pi R^2 l$

$U_B = \frac{1}{2} L I^2 = \frac{1}{2}(\mu_0 n^2 \pi R^2 l) I^2$

$U_B = \left(\frac{B^2}{2\mu_0}\right)(\pi R^2 l)$

energy density

volume

Think Harder about Faraday

P23-34

$$\mathcal{E} = \oint E \cdot ds = -\frac{d\Phi}{dt}$$

$$0 = -\oint E \cdot ds - \frac{d\Phi_B}{dt} \rightarrow 0 = 0$$

kirkoff's loop rule!

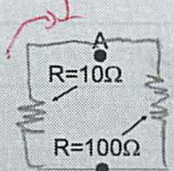
PRS Question: Faraday in Circuit

P23-35

PRS: Faraday Circuit

0

A magnetic field B penetrates this circuit outwards, and is increasing at a rate such that a current of 1 A is induced in the circuit (which direction?).



The potential difference VA-VB is: A B

0%	1.	+10 V
0%	2.	-10 V
0%	3.	+100 V
0%	4.	-100 V
0%	5.	+110 V
0%	6.	-110 V
0%	7.	+90 V
0%	8.	-90 V
0%	9.	None of the above

P23-36

$V = IR$

$V = 1 \cdot 110$

but what dir?

clockwise field out of page increasing

want it to go down so counter clockwise

$B \to A$ B higher by 10 V

$A \to B$ A higher by 100 V

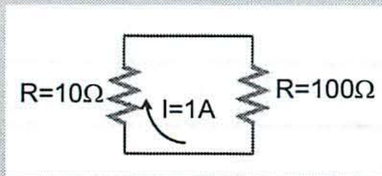
non conservative field

* None of above
- voltage no longer has meaning

$\boxed{V \neq \phi}$ $\sum \Delta V - \frac{d\Phi}{dt} = 0$

assume only place where changing magnetic field inside inductor

Non-Conservative Fields



$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

E is no longer a conservative field –
Potential now meaningless

$\Phi = LI \Rightarrow \sum V - I\frac{dL}{dt}$

ignoring magnetic flux is throughout entire circuit

Voltage no longer meaningful

Kirchhoff's Modified 2nd Rule

$$\sum_i \Delta V_i = -\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = +\frac{d\Phi_B}{dt}$$

$$\Rightarrow \sum_i \Delta V_i - \frac{d\Phi_B}{dt} = 0$$

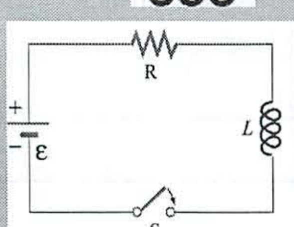
If all inductance is 'localized' in inductors then our problems go away – we just have:

$$\sum_i \Delta V_i - L\frac{dI}{dt} = 0$$

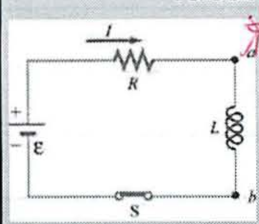
Inductors in Circuits

Inductor: Circuit element with self-inductance
Ideally it has zero resistance

Symbol: 



Ideal Inductor



*BUT, EMF generated *by* an inductor is ***not*** a voltage drop *across* the inductor!

$$\varepsilon = -L\frac{dI}{dt}$$

$$\Delta V_{\text{inductor}} \equiv -\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$$

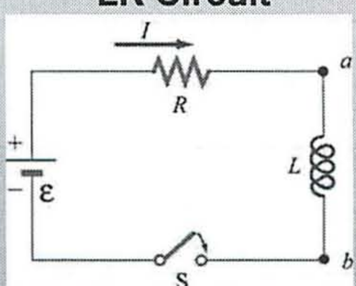
Because resistance is 0, E must be 0!

$V \neq \varepsilon$

Pretending ε only in an Inductor

Circuits:
Applying Modified Kirchhoff's
(Really Just Faraday's Law)

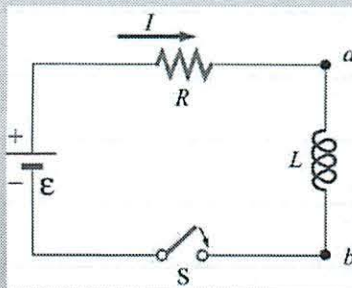
LR Circuit



$$\sum_i V_i = \varepsilon - IR - L\frac{dI}{dt} = 0$$

Time Dependent

LR Circuit



$$\varepsilon - IR - L\frac{dI}{dt} = 0 \;\Rightarrow\; \frac{dI}{dt} = -\frac{1}{\boxed{L/R}}\left(I - \frac{\varepsilon}{R}\right)$$

↳ τ

P23-43

Need Some Math: Exponential Decay

P23-44

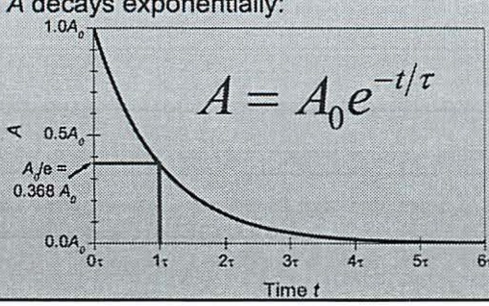
* general math forms

Important

Exponential Decay

Consider function A where: $\frac{dA}{dt} = -\frac{1}{\tau}A$

A decays exponentially:



$$A = A_0 e^{-t/\tau}$$

P23-45

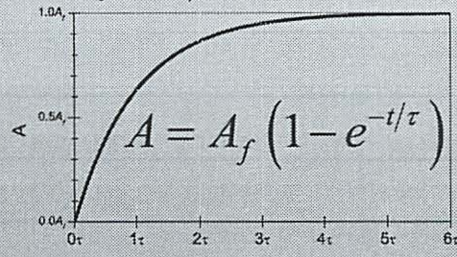
Important

* general math forms

Exponential Behavior

Slightly modify diff. eq.: $\dfrac{dA}{dt} = -\dfrac{1}{\tau}\left(A - A_f\right)$

A "decays" to A_f:



$$A = A_f\left(1 - e^{-t/\tau}\right)$$

P23-46

This is one of two differential equations we expect you to know how to solve (know the answer to).

The other is simple harmonic motion (more on that next week)

P23-47

Kirkoff Loop

solve for $\frac{dI}{dt}$

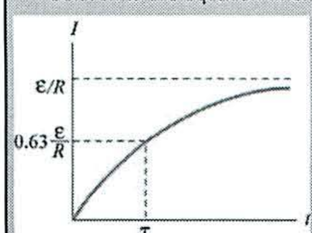
here

LR Circuit

$$\frac{dI}{dt} = -\frac{1}{L/R}\left(I - \frac{\varepsilon}{R}\right)$$

differential eq

Solution to this equation when switch is closed at t = 0:



$$\boxed{I(t) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)}$$

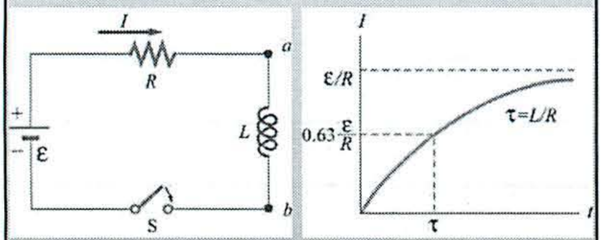
solution to diff. eq

$\tau = \dfrac{L}{R}$: time constant (units: seconds)

P23-48

Similar to RC - but different

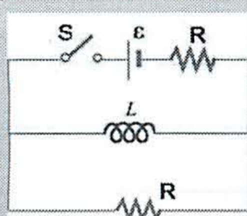
LR Circuit



$t=0^+$: Current is trying to change. Inductor works as hard as it needs to to stop it

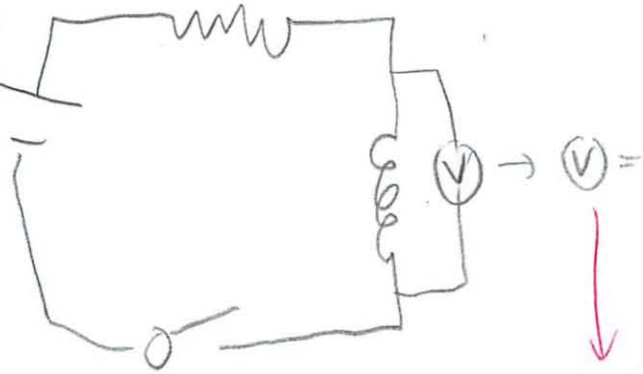
$t=\infty$: Current is steady. Inductor does nothing.

Group Problem: Circuits



For the above circuit sketch the currents through the two bottom branches as a function of time (switch closes at $t = 0$, opens at $t = T$). State values at $t = 0^+$, T^-, T^+

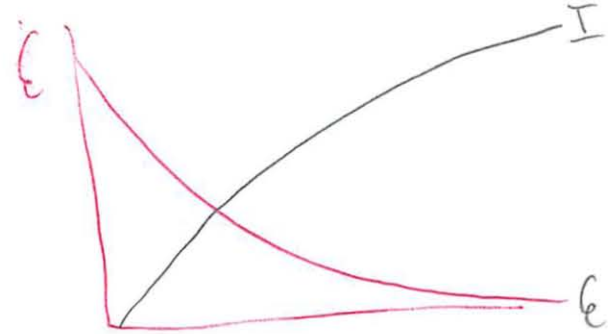
PRS

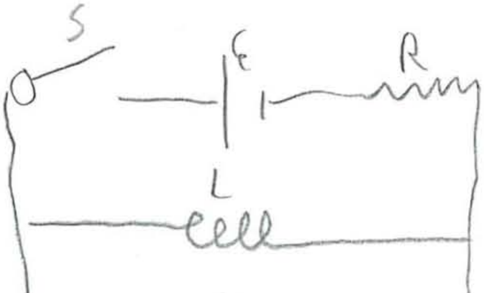


Will measure something
inductor working when there is change
biggest change is when just close
but inductor is conservative –
stops current from flowing

$$\boxed{V_L = \mathcal{E} e^{-t/\tau}}$$

after a long time
a lot of current flows
through





at $t=0$ switch closed

$I = \cancel{\frac{\mathcal{E}}{2R}}\ 0$

- Can not quickly change

* current takes time to change
at split second nothing changed

I Lower resistor right as switch closed

$I = \frac{\mathcal{E}}{2R}$ ✓

Now wait long time ∞

I through inductor $= \frac{6}{R}$ ✓

- like a wire

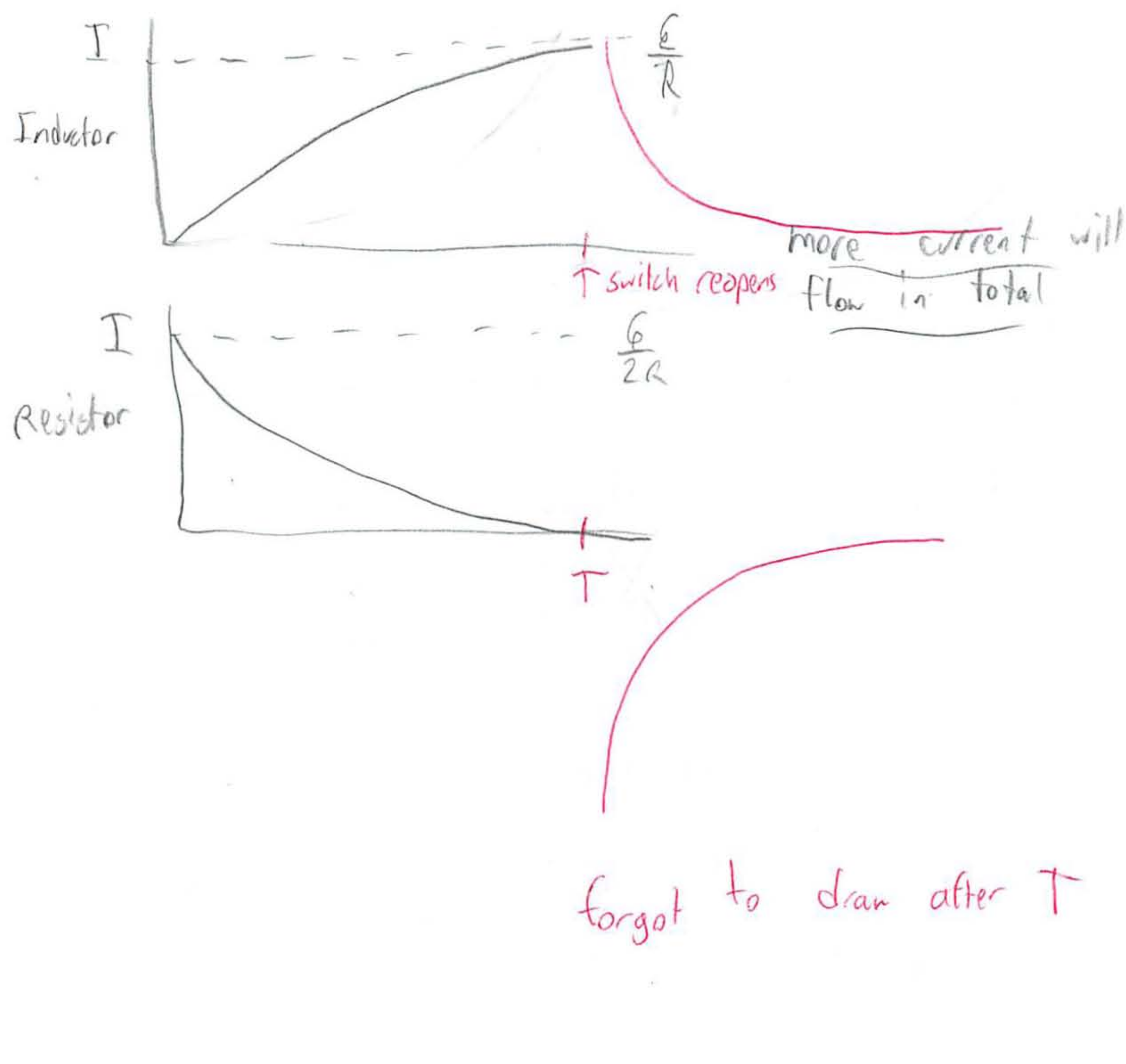
Now wait long time ∞

I through lower resistor

0 ← all goes through top wire

go slow + think about it

Sketch current as function of time



forgot to draw after T

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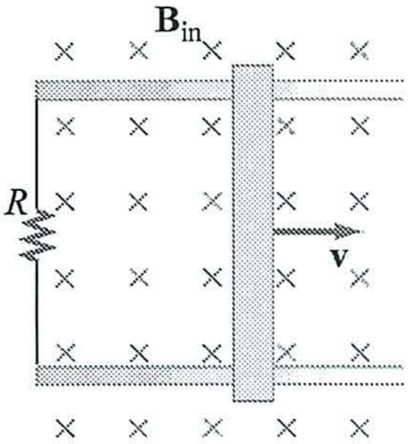
In Class W11D1-1 Solutions: Faraday's Law: Changing Area



Problem: A conducting rod is pulled along two conducting rails at a constant velocity v in a uniform magnetic field B. Find:

1. Direction of induced current
2. Direction of resultant force
3. Magnitude of EMF
4. Magnitude of current
5. Power externally supplied to move at constant v

Solution:
As always, the first step is to think about the problem a little. In Faraday's law problems, the thought should revolve along Lenz's law. But before we even get there, how do we recognize that this is a Faraday's law problem? There are several clues. We are asked about "induced current." Something is moving in a field that we are told about (rather than asked to calculate). And, as you will see, this is one of the few prototypical problems for this topic.

Back to the physics. Lenz tells us that the induced current will oppose the change. Since the area of the loop is increasing, the flux into the page is increasing, and the current will act to oppose it – it will flow **(1) counter-clockwise** to make a flux out of the page.

The resultant force can also be given by Lenz's law – it must oppose the change and hence **(2) be to the left**. Alternatively you could see this using the right hand rule on an upward current in a field into the page.

To find the magnitude we need to write down Faraday's law: $\mathcal{E} = -N\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA) = -B\frac{dA}{dt}$

We can jump to writing it like this because (1) there is only N=1 winding in the loop, (2) the field is perpendicular to the loop, and (3) the B field is uniform.
Now we just need an expression for A. If the distance between the rails is l and the distance from the resistor to the rod is x, then $A = lx;\ \frac{dA}{dt} = l\frac{dx}{dt} = lv$, so **(3)** $\mathcal{E} = Blv$ counter-clockwise.
Note that I have gotten rid of the minus sign since I tell what it means in words – much better!

The current is just determined by the EMF $\mathcal{E}$ and the resistance R: **(4)** $I = \frac{\mathcal{E}}{R} = \frac{Blv}{R}$

Finally, the power supplied by the force is all being dissipated in the resistor, so:

$$\textbf{(5)}\ P = I^2R = \left(\frac{Blv}{R}\right)^2 R = \frac{B^2l^2v^2}{R}$$

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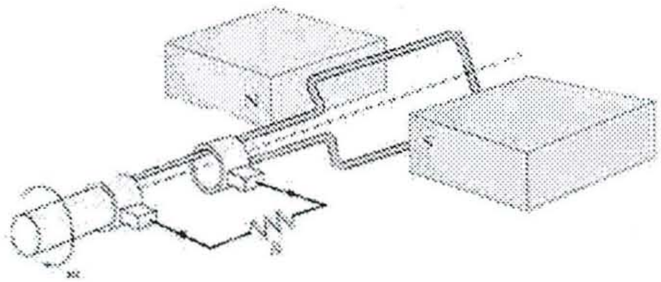
In Class W11D1-2 Solutions: Generator

Problem: Square loop (side L) spins with angular frequency w in field of strength B. It is hooked to a load R.



1) Write an expression for current I(t)
2) How much work from generator per revolution?
3) To make it twice as hard to turn, what do you do to R?

Solution:
This is a Faraday's Law problem. The flux is changing which generates and EMF which drives a current:

$$I(t) = \frac{\mathcal{E}(t)}{R} = \frac{1}{R}\frac{d\Phi_B}{dt} = \frac{1}{R}\frac{d(BA\cos\omega t)}{dt} = \boxed{\frac{BL^2}{R}\omega\sin(\omega t)}$$

I have dropped the sign because no direction was indicated. I also don't put in a phase, so the choice of sine instead of cosine is arbitrary.

The work that the generator done is the integral of the power:

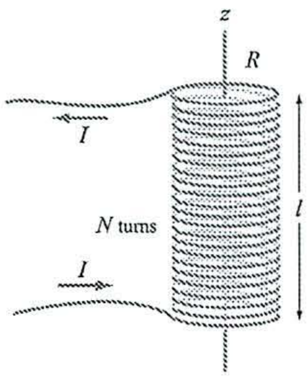
$$P = I^2R = \left(\frac{BL^2\omega}{R}\right)^2 R\sin^2(\omega t) \rightarrow W = \int_{t=0}^{2\pi/\omega} P(t)\,dt = \frac{B^2L^4\omega^2}{R}\int_{t=0}^{2\pi/\omega}\sin^2(\omega t)\,dt$$

Using the fact that the average value of $\sin^2(\omega t)$ is ½, (to see this, think $\sin^2(\omega t)+\cos^2(\omega t)=1$ and they both must have the same average value), we find:

$$W = \frac{B^2L^4\omega^2}{R}\left(\frac{1}{2}\cdot\frac{2\pi}{\omega}\right) = \boxed{\frac{\pi B^2L^4\omega}{R}}$$

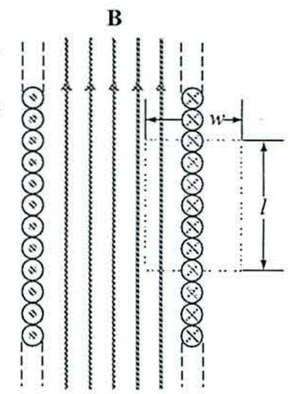
Finally, to make it twice as hard to turn that means twice as much work, which means that the resistance must be half as much. This is called "loading" the generator – where an increase in load is actually a *decrease* in the resistance.

In Class W11D2_1 Solutions: Inductance of Solenoid



Problem: Calculate the self-inductance of a solenoid of length ℓ, $n = N/\ell$ turns per meter and radius R

Solution:
To find the self inductance of an object, there are two typical methods. One is through the energy, which we will discuss later. The second method, shown here, is to push an arbitrary current I through the device and see what happens (what flux is created by that current).



To find the flux we first have to calculate the magnetic field. To do this for a solenoid it is easiest to use Ampere's Law. A solenoid is essentially two superimposed sheets of current, one going in to the page and the other coming out. By superposition we see that the field outside must be zero, and the field inside runs vertically. Hence we use the rectangular Amperian loop pictured and find:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\ell = \mu_0 I_{enc} = \mu_0 (n\ell) I$$

where $(n\ell)$ is the number of wires punching through our loop, each one carrying a current I. Solving we find $B = \mu_0 n I$ (up, as pictured).

Now we need to find the flux through any wire loop. Since the field is (approximately) uniform inside the solenoid, our flux integral becomes multiplication: $\Phi_{B,Sgl} = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = BA = \mu_0 n I \pi R^2$

Finally, we need to calculate the inductance, that is, how well the current produces a magnetic flux through the solenoid:

$$L = \frac{N\Phi_{B,Sgl}}{I} = N\mu_0 n \pi R^2 = \mu_0 n^2 \pi R^2 l$$

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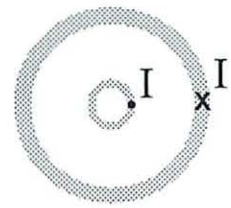
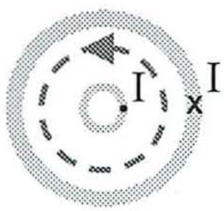
In Class W12D1_1 Solutions: Coaxial Cable



Problem: For the coaxial cable at left (inner radius a, outer radius b):
1) How much energy is stored per unit length?
2) What is inductance per unit length?

Solution:
There are several ways to find energy. One is to find the inductance and then use $U = \frac{1}{2}LI^2$. However, since they ask us to find the inductance after finding the energy, this is unlikely to be the way to approach this problem. Another way is to consider that the energy is stored in the magnetic field, and hence find the magnetic field then integrate the energy density to find the total energy. We take this approach.



To find the field use Ampere's law. Outside of b and inside of a the fields will be zero (because the contained current will be zero). Using the Amperian loop pictured (radius r), we find that in between the two current shells: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B2\pi r = \mu_0 I_{enc} = \mu_0 I \rightarrow B = \frac{\mu_0 I}{2\pi r}$ (CCW, as pictured)

The energy density is then given by: $u_B = \frac{B^2}{2\mu_o} = \frac{1}{2\mu_o}\left(\frac{\mu_0 I}{2\pi r}\right)^2 = \frac{\mu_0 I^2}{8\pi^2 r^2}$

Now we just need to integrate this energy density over the volume of space where we found there to be a magnetic field – in between the two shells. This is a volume integral (since u_B is an energy per unit volume), which we will do by integrating over cylindrical shells of radius r and length l. We can do this because the field and hence the energy density will be constant on these shells. Also, the length is arbitrary, because we are asked to find the energy per unit length. So:

$$U_B = \iiint u_B\left(d\text{Volume}\right) = \int_a^b \frac{\mu_0 I^2}{8\pi^2 r^2} \cdot 2\pi r l dr = \frac{\mu_0 I^2 l}{4\pi} \int_a^b \frac{1}{r} dr = \frac{\mu_0 I^2 l}{4\pi} \ln\left(\frac{b}{a}\right)$$

This gives us energy per unit length of: **(1)** $U_{B,\text{ per length}} = \frac{U_B}{l} = \frac{\mu_0 I^2}{4\pi} \ln\left(\frac{b}{a}\right)$

To find the inductance (per unit length) we simply use the equation that relates energy and inductance: $U = \frac{1}{2}LI^2$, except that in this case it is actually energy per unit length on the left and inductance per unit length on the right. So

$$U_B = \frac{1}{2}LI^2 \rightarrow L_{\text{per length}} = \frac{2U_{B,\text{ per length}}}{I^2} = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right)$$

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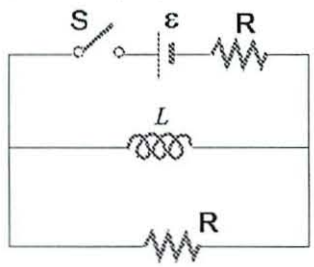
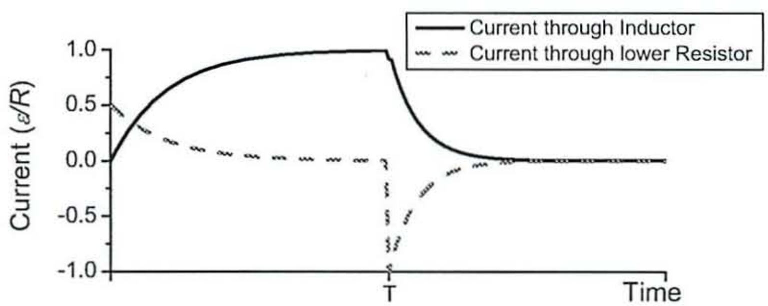
Department of Physics: 8.02

In Class W02D2_1 Solutions: LR Circuit

Problem: For the below circuit sketch the current through the two bottom branches as a function of time (if the switch closes at t = 0 and reopens at t = T, where T is a very long time). State the values of the currents at times $t = 0^+, T^-, T^+$



Solution:
The inductor fights change. So it will act as an open circuit (no current) initially when the switch closes and then after a long time, when the current has reached steady state, it will look like a short (zero resistance). Thus all the current will go through it, and none through the bottom resistor.



$T=0^+$: $I_L = 0$; $I_R = \varepsilon/2R$

$T=T^-$: $I_L = \varepsilon/R$; $I_R = 0$

$T=T^+$: $I_L = \varepsilon/R$; $I_R = -\varepsilon/R$

Note that the time constant is longer in the "charging" phase than in the "discharging" phase by a factor of two (from 2L/R to L/R), because in the charging phase the two resistors are essentially in parallel, cutting the effective resistance in half, but while discharging only the bottom resistor does anything.

Topics: Mutual Inductance & Transformers; Inductors
Related Reading: Course Notes: Sections 10.1-10.4, 10.8-10.9, 11.1-11.4

Topic Introduction

Today we have a special lecture in honor of Campus Preview Weekend.

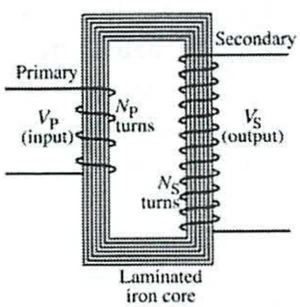
Faraday's Law & Lenz's Law

Recall: Faraday's Law says that a changing magnetic flux generates an EMF $\mathcal{E} = -d\Phi_B/dt$
Lenz's Law says that the direction of that EMF is so as to oppose the change in magnetic flux

Mutual Inductance

Since magnetic fields are typically generated by currents, Faraday's law implies that changing currents also generate EMFs. This is the idea of mutual inductance: given any two circuits, a changing current in one will induce an EMF in the other, or, mathematically, $\mathcal{E}_2 = -M\, dI_1/dt$, where M is the mutual inductance of the two circuits. How does this work? The current in loop 1 produces a magnetic field (and hence flux) through loop 2. If that current changes in time, the flux through 2 changes in time, creating an EMF in loop 2. The mutual inductance, *M*, depends on geometry, both on how well the current in the first loop can create a magnetic field and on how much magnetic flux through the second loop that magnetic field will create.

Transformers



A major application of mutual inductance is the transformer, which allows the easy modification of the voltage of AC (alternating current) signals. At left is the schematic of a step up transformer. An input voltage V_P on the primary coil creates an oscillating magnetic field, which is "steered" through the iron core (recall that ferromagnets like iron act like wires for magnetic fields) and through the secondary coils, which induces an EMF in them. In the ideal case, the amount of flux generated and received is proportional to the number of turns in each coil. Hence the ratio of the output to input voltage is the same as the ratio of the number of turns in the secondary to the number of turns in the primary. As pictured we have more turns in the secondary, hence this is a "step up transformer," with a larger output voltage than input.

The ease of creating transformers is a strong argument for using AC rather than DC power. Why? Before sending power across transmission lines, voltage is stepped way up (to 240,000 V), leading to smaller currents and losses in the lines. The voltage is then stepped down to 240 V before going into your home.

Self Inductance

Recall that we defined self inductance L by the amount of flux that an object generates through itself when a current I flows through it ($\Phi = LI$) and, from Faradays Law, found that inductors will generate a back EMF: $\mathcal{E} = -L\,dI/dt$. Self inductance is very similar to mutual inductance, obeying a similar equation: $\mathcal{E} = -L\,dI/dt$, and the same concept: when a circuit has a current in it, it creates a magnetic field, and hence a flux, through itself. If that current changes, then the flux will change and hence an EMF will be induced in the circuit. The action of that EMF will be to *oppose the change* in current (if the current is decreasing it will try to make it bigger, if increasing it will try to make it smaller). For this reason, we often refer to the induced EMF as the "back EMF."

To calculate the self inductance (or inductance, for short) of an object, imagine that a current I flows through it, and determine how much magnetic field and hence flux Φ_B that makes through the object. The self inductance is then $L = \Phi_B / I$.

An inductor is a circuit element whose main characteristic is its inductance, L. It is drawn as a coil in circuit diagrams. The strong resemblance to a solenoid is intentional – solenoids make very good inductors both because of their ability to make a strong field inside themselves, and also because the field they produce is fairly well contained, and hence doesn't produce much flux (and induce EMFs) in other, nearby circuits.

The role of an inductor is to oppose changing currents. At steady state, in a DC circuit, an inductor is off – it induces no EMF as long as the current through it is constant. As soon as you try to change the current through an inductor though, it will fight back. In this sense an inductor is the opposite of a capacitor. If a capacitor is placed in a steady state current it will eventually fill up and "open" the circuit, whereas an inductor looks like a short in this case. On the other hand, when starting from its uncharged state, a capacitor looks like a short when you first try to move current through it, while an inductor looks like an open circuit, as it prevents the change (from no current to some current).

Applications

A number of technologies rely on induction to work – generators, microphones, metal detectors, and electric guitars to name a few. Another common application is eddy current braking. A magnetic field penetrating a metal spinning disk (like a wheel) will induce eddy currents in the disk, currents which circle inside the disk and exert a torque on the disk, trying to stop it from rotating. This kind of braking system is commonly used in trains. Its major benefit (aside from eliminating costly service to maintain brake pads) is that the braking torque is proportional to angular velocity of the wheel, meaning that the ride smoothly comes to a halt.

Important Equations

Faraday's Law: $$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Magnetic Flux: $$\Phi_B = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

EMF: $\mathcal{E} = \oint \vec{\mathbf{E}}' \cdot d\vec{\mathbf{s}}$

Mutual Inductance: $\mathcal{E}_2 = -M \dfrac{dI_1}{dt}$

Self Inductance, L: $L = \dfrac{\Phi_B}{I}$

EMF Induced by Inductor: $\mathcal{E} = -L \dfrac{dI}{dt}$

CPW Fri 4/9

- I did have this PPT - must have skipped it somehow

See added pages after

Class 24: Outline

Hour 1:

Applications of Faraday's Law

Faraday's Law of Induction

$$\boxed{\varepsilon = -\frac{d\Phi_B}{dt}}$$

Changing magnetic flux *induces* an EMF

Lenz: Induction ***opposes*** change

Technology

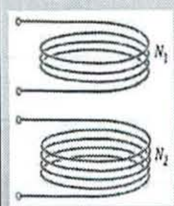
Many Applications of Faraday's Law

Today:
Using Inductance

First:
Mutual Inductance

Demonstration:
Remote Speaker

Mutual Inductance



Current I_2 in coil 2, induces magnetic flux Φ_{12} in coil 1. "Mutual inductance" M_{12}:

$$\Phi_{12} \equiv M_{12} I_2$$

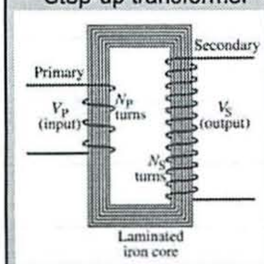
$M_{12} = M_{21} = M$

Change current in coil 2?
Induce EMF in coil 1:

$$\boxed{\varepsilon_{12} \equiv -M_{12} \frac{dI_2}{dt}}$$

Transformer

Step-up transformer



Flux Φ through each turn same:

$$\varepsilon_p = N_p \frac{d\Phi}{dt}; \quad \varepsilon_s = N_s \frac{d\Phi}{dt}$$

$$\boxed{\frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p}}$$

$N_s > N_p$: step-up transformer
$N_s < N_p$: step-down transformer

See added pages after

Demonstrations:

One Turn Secondary: Nail

Many Turn Secondary: Jacob's Ladder

PRS: Residential Transformer

If the transformer in the can looks like the picture, how is it connected?

20

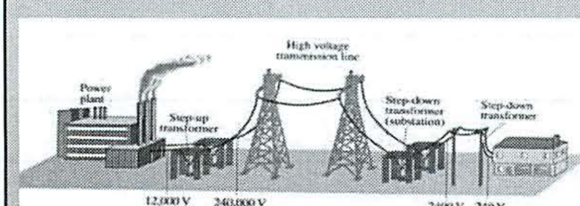
- 0% 1. House=Left, Line=Right
- 0% 2. Line=Left, House=Right
- 0% 3. I don't know

Answer: Residential Transformer

Answer: 1. House on left, line on right

The house needs a lower voltage, so we step down to the house (fewer turns on house side)

Transmission of Electric Power



Power loss can be greatly reduced if transmitted at high voltage

Example: Transmission lines

An average of 120 kW of electric power is sent from a power plant. The transmission lines have a total resistance of 0.40 Ω. Calculate the power loss if the power is sent at (a) 240 V, and (b) 24,000 V.

(a) $I = \frac{P}{V} = \frac{1.2\times10^5 W}{2.4\times10^2 V} = 500A$

$P_L = I^2R = (500A)^2(0.40\Omega) = 100kW$

83% loss!!

(b) $I = \frac{P}{V} = \frac{1.2\times10^5 W}{2.4\times10^4 V} = 5.0A$

$P_L = I^2R = (5.0A)^2(0.40\Omega) = 10W$

0.0083% loss

Group Discussion: Transmission lines

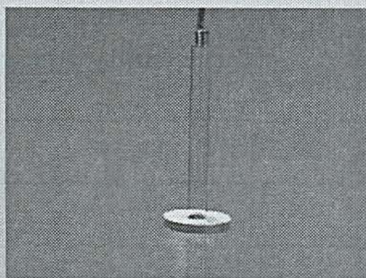
We just calculated that I^2R is smaller for bigger voltages.

What about V^2/R? Isn't that bigger?

Why doesn't that matter?

Brakes

Magnet Falling Through a Ring



What happened to kinetic energy of magnet?

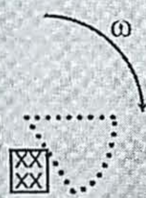
Demonstration: Eddy Current Braking

Eddy Current Braking

What happened to kinetic energy of disk?

Eddy Current Braking

The magnet induces currents in the metal that dissipate the energy through Joule heating:

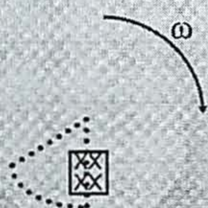


1. Current is induced counter-clockwise (out from center)
2. Force is opposing motion (creates slowing torque)

See added pages afterwards

Eddy Current Braking

The magnet induces currents in the metal that dissipate the energy through Joule heating:



1. Current is induced clockwise (out from center)
2. Force is opposing motion (creates slowing torque)
3. EMF proportional to ω
4. $F \propto \dfrac{\mathcal{E}^2}{R}$

Demonstration: Levitating Magnet

$\mathcal{E} = -\frac{d\Phi}{dt}$

rotation → Faraday's Law would torque opposit
always slows down

Mutual Induction

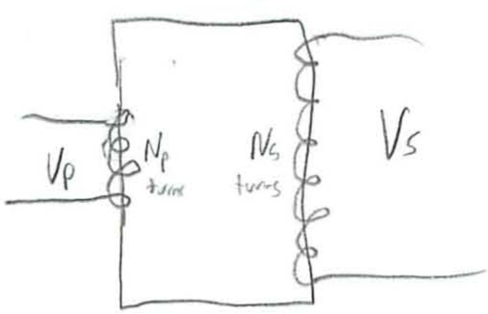
$\Phi_{12} \equiv M_{12} I_2$

the speakers example

$\mathcal{E}_{12} = -M_{12} \frac{dI_2}{dt}$

Transformer

AC circuit



idealized no magnetic losses
in real life loses energy

$\mathcal{E}_p = N_p \frac{d\Phi}{dt}$

$\mathcal{E}_s = N_s \frac{d\Phi}{dt}$

Φ through each turn the same

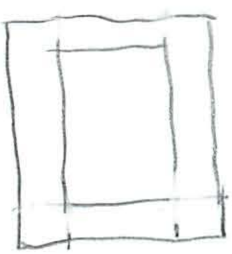
(2)

$$L = \frac{\Phi_{\text{total solenoid}}}{I}$$

$$= \frac{N\Phi_{\text{loop}}}{I}$$

$$= \frac{NBA}{I}$$ (B does depend on N)

$$B = \frac{\mu_0 N I}{l}$$



$$\Phi_{P_1 \text{ loop}} \overset{\text{ideal}}{=} \Phi_{P_2 \text{ loop}}$$

$$\mathcal{E}_p = N_p \frac{d\Phi_{P_1 \text{ loop}}}{dt}$$

$$\mathcal{E}_s = N_s \frac{d\Phi_{P_2 \text{ loop}}}{dt}$$

ideal

$$\frac{\mathcal{E}_p}{N_p} = \frac{\mathcal{E}_s}{N_s}$$

Voltage over secondary proportional to # of turns

$$\mathcal{E}_s = \frac{N_s}{N_p} \mathcal{E}_p$$

$N_p < N_s$ Step up (Voltage) transformer

$N_p > N_s$ Step down

③

Power

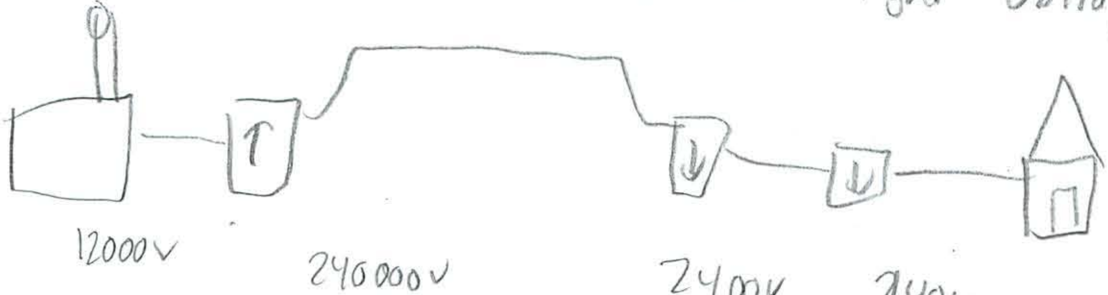
ideal $\quad i_s \mathcal{E}_s = i_p \mathcal{E}_p$

$P_s = P_p \qquad$ no power loss

$$i_s = i_p \frac{\mathcal{E}_p}{\mathcal{E}_s} = i_p \left(\frac{N_p}{N_s}\right)$$

Step down transformer ↑ current to keep same power

Power loss via wires reduced at higher voltages



Power Loss

$$P = I^2 R$$

240V

$$I = \frac{P}{V} = \frac{1.2 \cdot 10^5\ W}{2.4 \cdot 10^2\ V} = 500\ A$$

$$P_L = I^2 R = (500\ A)^2 \cdot .4\ \Omega = 100\ kW \leftarrow \text{lose } 80\%$$

?0,000V
↑
? or 24,000

$$I = \frac{P}{V} = \frac{1.2 \cdot 10 W}{2.4 \cdot 10^5} = 5A$$

$$P_L = I^2 R = (5A)^2 \cdot .4\Omega = 10W \leftarrow \text{much less}$$

(4)

What about $V^2 R$? Isn't it higher?

$$\Delta V = 24000V$$

$$\Delta V_{wire} = iR = 500A \cdot .4\Omega$$
$$= 200W$$

$$\frac{\Delta V_{wire}^2}{R} = i^2 R$$

$$\Delta V_{wire} = 5A \cdot .4\Omega = 2V$$

$$\frac{\Delta V_{wire}^2}{R} = \frac{(2V)^2}{.4\Omega} = 10W$$

Eddy Braking is similar

wants to oppose movement - so slows what you have

as section crosses magnet flux into board

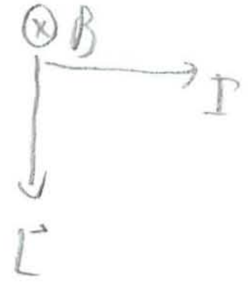
- increasing

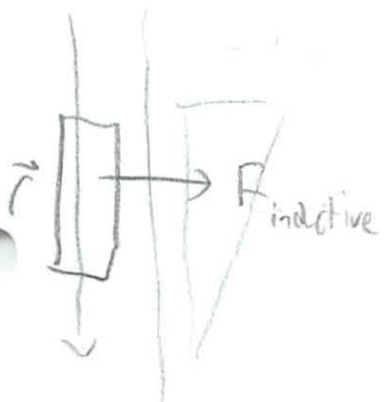
current will flow out of board (thumb)

fingers curl counter clockwise

(5)

$I\vec{L} \times \vec{B}$

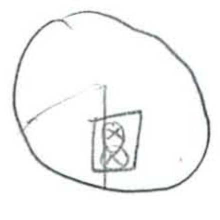




τ_{ind} = ⊙ slows it down

inductive counterforce

What happens on other side



current has to flow down from center
this is clockwise
flux ↓ decreasing
still counter torque

Review

4/11

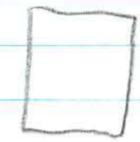
What is inductance
- like palm pre
- moving magnet induces electric current
- which moves to oppose motion

Mutual inductance
- the speakers wireless demo

Really liked How stuff works water wheel analogy
- always opposes current
- starting or stopping

How motor works

Measure with a loop you choose



Time dependent like resistor
with those charts

Transformer given
- increases current/voltage w/ # of loops

New stuff now →

★ 2 ways to think about inductors
- magnetically how it works - last week ←
- as circuit element - this week →

4/12

Topic: RL Circuits and undriven RLC Circuits
Related Reading: Course Notes: Sections 11.5-11.11
Experiments: (8) RL Circuits and Undriven RLC Circuits

Topic Introduction

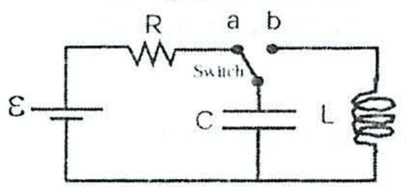
Today we will investigate the behavior of circuits containing resistors and capacitors and inductors (RL & RLC circuits). We have previously discussed RL (last week) and RC behavior in the class We now put them together in an undriven RLC circuit and observe that the current in these circuits oscillates, in a fashion completely analogous to the oscillation of a mass on a spring. In experiment 8, you will have a chance to measure their behavior yourself. Cool

? undriven

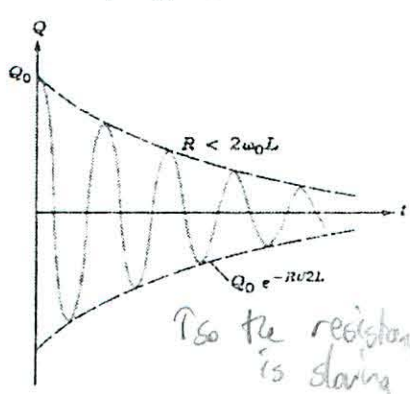
Mass on a Spring: Simple Harmonic Motion
In a simple system consisting of a mass hanging on a spring, when the mass is pulled down and released it oscillates up and down. We think about this in a couple of ways. One way is to look at the forces on the mass and write a differential equation for its motion, $F = m\ddot{x} = -kx$, where $\ddot{x}$ means two time derivatives of the displacement (acceleration). The solution to this is simple harmonic motion: $x = x_0 \cos(\omega t)$ where $\omega = \sqrt{k/m}$.

We can also think about the energy in the system. As the mass moves, energy oscillates between kinetic energy of the mass and potential energy stored in the spring. If there is no damping (friction) in the system to dissipate energy, the oscillation will continue forever.

Undriven L(R)C Circuits



Consider the LC circuit at left, where the switch is at "a" until the capacitor is fully charged and then thrown to "b." This is analogous to pulling down a mass and releasing it. Here the capacitor will want to discharge and will drive a current through the inductor. Eventually all the charges will run off of the capacitor (spring), so it won't "push" anymore, but now the inductor will want to keep the current flowing through it that it already has (inductors, like masses, have inertia). It will keep the current flowing, but that will eventually fill up the capacitor which will stop the current and send it back the other direction. Our differential equation is thus analogous, $V = -L\ddot{q} = q/C$, and has the same solution: $q = q_0 \cos(\omega t)$ where $\omega = \sqrt{1/LC}$.



? So the resistance is slowing it down

We can also think about energy here, where it oscillates between being stored in the electric field in the capacitor and the magnetic field in the inductor. As long as there is no dissipation (resistance) is the circuit the oscillations will continue forever.

If we add a resistor in series with the capacitor and inductor we provide a method of energy loss, through joule heating in the resistor as current flows. The oscillations will thus damp out to zero. The exact path the charge will take as it oscillates to zero depends on the relative sizes of L, R and C, but will typically look something like the curve above, where the oscillations are bounded by an "envelope" which is exponentially decaying to zero as a function of time.

Important Equations

Self Inductance, L:	$L = \dfrac{\Phi_B}{I}$
EMF Induced by Inductor:	$\mathcal{E} = -L\dfrac{dI}{dt}$
Exponential Decay:	$Value = Value_{initial} e^{-t/\tau}$
Exponential "Decay" Upwards:	$Value = Value_{final}\left(1 - e^{-t/\tau}\right)$
Simple RC/RL Time Constant:	$\tau = L/R$
Natural Frequency of LC Circuit:	$\omega_0 = \dfrac{1}{\sqrt{LC}}$

Experiment 8: RL and Undriven LRC Circuit

Preparation: Read pre-lab and answer pre-lab questions.

This lab has two parts. In the first part you will observe the exponential behavior of RL circuits as they are "charged" and "discharged" using a battery which periodically turns on and off. You will measure the time constant of several circuits and investigate how it changes as resistance and inductance are modified.

In the second part you will study an undriven LRC circuit and determine its natural frequency.

Its cool how this all comes together
But looks like a lot of math

4/12

Class 25: Outline

Hour 1:

Expt. 8: Part 1: LR Circuits

Hour 2:

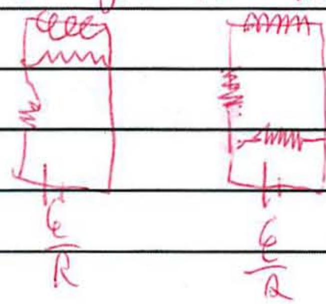
Expt. 8: Part 2: Undriven RLC Circuits

Download LabView file

PRS Quiz like first MP quiz

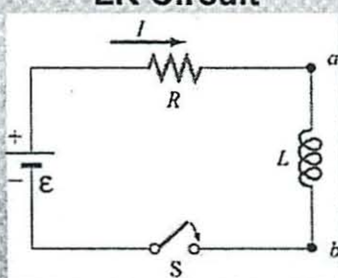
$\frac{\mathcal{E}}{2R}$ $\frac{\mathcal{E}}{R}$ ∞

Wait got it wrong



what L sees not what battery sees

LR Circuit



$$\sum_i V_i = \varepsilon - IR - L\frac{dI}{dt} = 0$$

$\mathcal{E} = -L \frac{dI}{dt}$

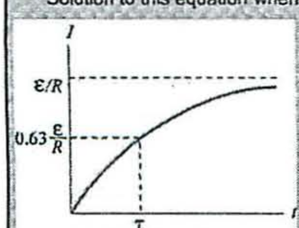
fights changes in current w/ back emf

$U = \frac{1}{2} L I^2$

LR Circuit

$$\frac{dI}{dt} = -\frac{1}{L/R}\left(I - \frac{\varepsilon}{R}\right)$$

Solution to this equation when switch is closed at t = 0:



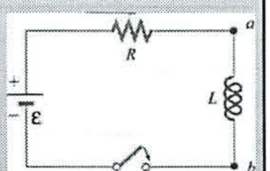
$$\boxed{I(t) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)}$$

$\tau = \frac{L}{R}$: time constant (units: seconds)

PRS Question: Voltage Across Inductor

PRS: Voltage Across Inductor

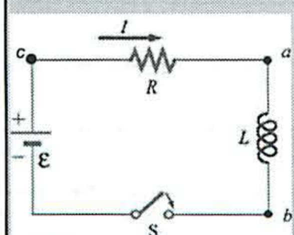
In the circuit at right the switch is closed at $t = 0$. A voltmeter hooked across the inductor will read:

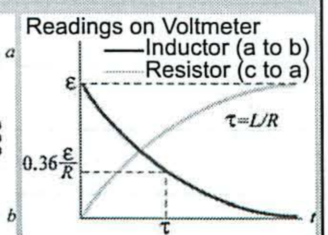


0%	1. $V_L = \varepsilon e^{-t/\tau}$
0%	2. $V_L = \varepsilon(1-e^{-t/\tau})$
0%	3. $V_L = 0$
0%	4. I don't know

0

LR Circuit





$t=0^+$: Current is trying to change. Inductor works as hard as it needs to to stop it

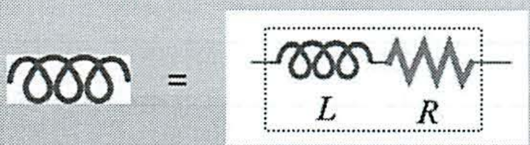
$t=\infty$: Current is steady. Inductor does nothing.

inductor is fighting change

lab today is like RC lab

Non-Ideal Inductors

Non-Ideal (Real) Inductor: Not only L but also some R



In direction of current: $\varepsilon = -L\dfrac{dI}{dt} - IR$

Experiment 8: Part 1
Inductance & LR Circuits

PRS Questions:
LR Circuits

PRS: Inserting a Core

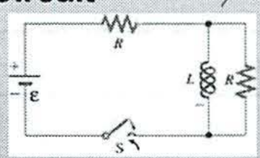
When you insert the iron core what happens?

0%	1. B Increases so L does too
0%	2. B Decreases so L does too
0%	3. B Increases so L Decreases
0%	4. B Decreases so L Increases
0%	5. I don't know
0%	

:20

PRS: RL Circuit

In the circuit at right the switch S has been closed a very long time. At t = 0, the switch is opened. Taking downward current as positive, immediately after the switch is opened the current in the inductor is equal to



0%	1. ε / R
0%	2. $\varepsilon / 2R$
0%	3. $-\varepsilon / R$
0%	4. $-\varepsilon / 2R$
0%	5. Zero
0%	6. I don't know

20

just before switch opened

$V = 0$

takes time for current to slow

what about closed for long time then opened

~~Z still Z I = 0~~

$V = \frac{\varepsilon}{2}$ ~~at long time after~~

misread question

Circuits that Oscillate (LRC)

P25-12

Mass on a Spring: Simple Harmonic Motion (Demonstration)

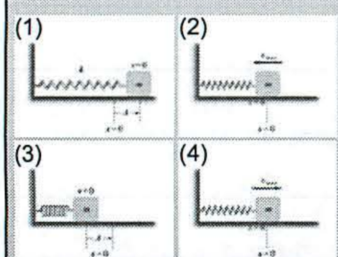
P25-11

"Burn the picture into your mind"
2nd order differential equation
know the solution for

Spring

Mass on a Spring

(1) (2) (3) (4)



What is Motion?

$$F = -kx = ma = m\frac{d^2x}{dt^2}$$

$$m\frac{d^2x}{dt^2} + kx = 0$$

Simple Harmonic Motion

$$x(t) = x_0 \cos(\omega_0 t + \phi)$$

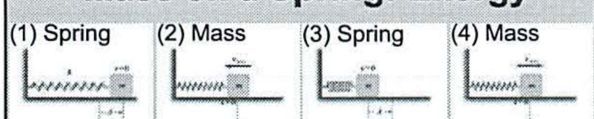
x_0: Amplitude of Motion
ϕ: Phase (time offset)

$\omega_0 = \sqrt{\dfrac{k}{m}}$ = Angular frequency

P25-14

Mass on a Spring: Energy

(1) Spring (2) Mass (3) Spring (4) Mass



$x(t) = x_0 \cos(\omega_0 t + \phi)$ $\quad x'(t) = -\omega_0 x_0 \sin(\omega_0 t + \phi)$

Energy has 2 parts: (Mass) Kinetic and (Spring) Potential

$$K = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 = \frac{1}{2}kx_0^2 \sin^2(\omega_0 t + \phi)$$

$$U_s = \frac{1}{2}kx^2 = \frac{1}{2}kx_0^2 \cos^2(\omega_0 t + \phi)$$

Energy sloshes back and forth

P25-15

opposite

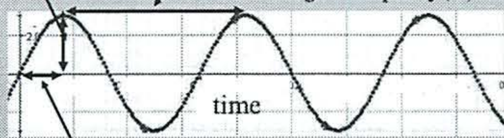
$K + U_s = 1$

out of phase with each other

Simple Harmonic Motion

$$\text{Period } (T) = \frac{1}{\text{frequency } (f)} = \frac{2\pi}{\text{angular frequency } (\omega)}$$

Amplitude (x_0)



$$x(t) = x_0 \cos(\omega_0 t + \phi)$$

Phase Shift $(\varphi) = \frac{-\pi}{2}$

P25-16

Electronic Analog: LC Circuits

P25-17

Analog: LC Circuit

Mass doesn't like to accelerate

Kinetic energy associated with motion

$$F = ma = m\frac{dv}{dt} = m\frac{d^2x}{dt^2}; \quad E = \frac{1}{2}mv^2$$

inertia

Inductor doesn't like to have current change

Energy associated with current

$$\varepsilon = -L\frac{dI}{dt} = -L\frac{d^2q}{dt^2}; \quad E = \frac{1}{2}LI^2$$

P25-18

$F \to \varepsilon$ $\quad m \to L$

$x \to q$ $\quad k \to C^{-1}$

$v \to I$

Analog: LC Circuit

Spring doesn't like to be compressed/extended

Potential energy associated with compression

$$F = -kx; \quad E = \frac{1}{2}kx^2$$

Capacitor doesn't like to be charged (+ or -)

Energy associated with stored charge

$$\varepsilon = \frac{1}{C}q; \quad E = \frac{1}{2}\frac{1}{C}q^2$$

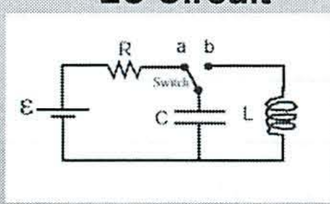
$$\boxed{F \to \varepsilon; \; x \to q; \; v \to I; \; m \to L; \; k \to C^{-1}}$$

$F \to \varepsilon$ $m \to L$

$x \to q$ $k \to C^{-1}$ ← defined backwards

$v \to I$ bigger capacitor is eaiser to put charge on

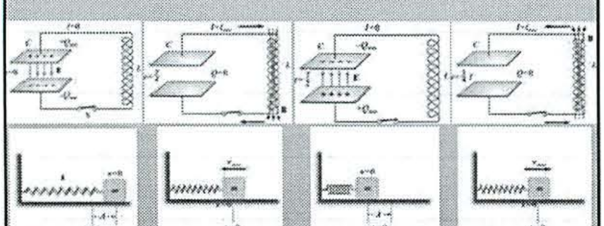
LC Circuit



1. Set up the circuit above with capacitor, inductor, resistor, and battery.
2. Let the capacitor become fully charged.
3. Throw the switch from a to b
4. What happens?

LC Circuit

It undergoes simple harmonic motion, just like a mass on a spring, with trade-off between charge on capacitor (Spring) and current in inductor (Mass)



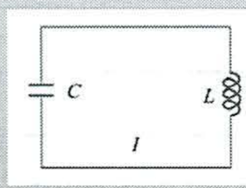
Whoever has the energy makes more continue

Current fills inductor   flows capacitor   back

PRS Questions: LC Circuit

PRS: LC Circuit

Consider the LC circuit at right. At the time shown the current has its maximum value. At this time



0%	1. The charge on the capacitor has its maximum value
0%	2. The magnetic field is zero /inductor
0%	3. The electric field has its maximum value
0%	4. The charge on the capacitor is zero
0%	5. Don't have a clue

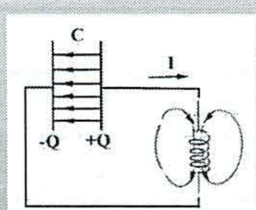
:20

all of energy in inductor

PRS: LC Circuit

20

In the LC circuit at right the current is in the direction shown and the charges on the capacitor have the signs shown. At this time,



0%	1. I is increasing and Q is increasing
0%	2. I is increasing and Q is decreasing
0%	3. I is decreasing and Q is increasing
0%	4. I is decreasing and Q is decreasing
0%	5. Don't have a clue

Q decreasing - leaving capicator

I still increasing

running off capicator

- off the + plate

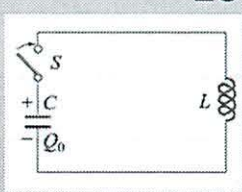
these I don't really get how to do

1+4 always wrong

as one ↑ the other ↓

energy is conserved

LC Circuit



$$\frac{Q}{C} - L\frac{dI}{dt} = 0 \ ; \ I = -\frac{dQ}{dt}$$

$$\frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$$

Simple Harmonic Motion

$$Q(t) = Q_0\cos(\omega_0 t + \phi) \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

Q_0: Amplitude of Charge Oscillation

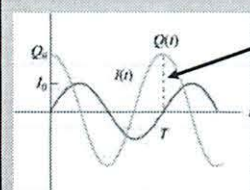
ϕ: Phase (time offset)

P25-25

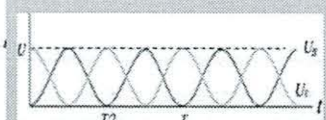
*

memorize

LC Oscillations: Energy



Notice relative phases



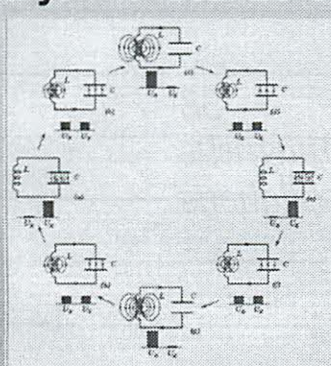
$$U_E = \frac{Q^2}{2C} = \left(\frac{Q_0^2}{2C}\right)\cos^2\omega_0 t \qquad U_B = \frac{1}{2}LI^2 = \frac{1}{2}LI_0^2\sin^2\omega_0 t = \left(\frac{Q_0^2}{2C}\right)\sin^2\omega_0 t$$

$$U = U_E + U_B = \frac{Q^2}{2C} + \frac{1}{2}LI^2 = \frac{Q_0^2}{2C}$$

Total energy is conserved !!

P25-26

Summary: The Ideal LC Circuit



P25-

Adding Damping: RLC Circuits

add a resistor

P15-28

The Real RLC Circuit: Energy Considerations

Include finite resistance: $\dfrac{Q}{C} + IR + L\dfrac{dI}{dt} = 0$

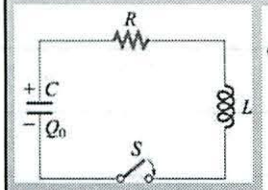
Multiply by I and after a little work:

$$\frac{d}{dt}\left[\frac{Q^2}{2C} + \frac{1}{2}LI^2\right] = -I^2R$$

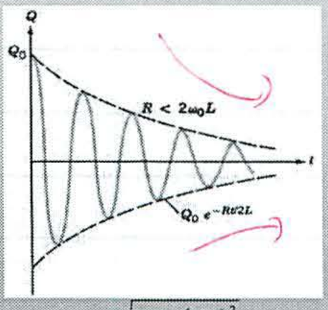
$$\frac{d}{dt}[\text{Total Energy}] = -I^2R$$

P15-

Damped LC Oscillations



Resistor dissipates energy and system rings down over time



Also, frequency decreases: $\omega' = \sqrt{\omega_0^2 - \left(\dfrac{R}{2L}\right)^2}$

P15-30

← don't need equations

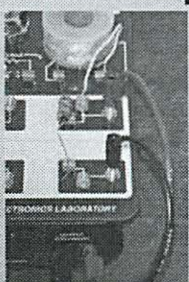
--- dashed line = envelope

Experiment 8:
Part 2 Undriven RLC Circuits

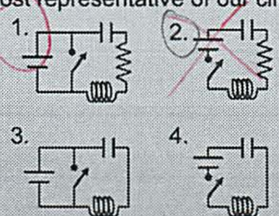
P25-31

PRS: Expt. 8

:20



In today's lab the battery turns on and off. Which circuit diagram is most representative of our circuit?



0% 0% 0% 0%

Load lab while waiting…

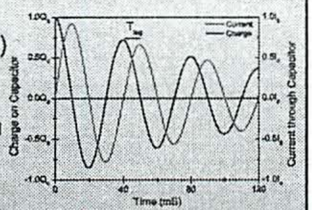
P25-

PRS Questions:
Undriven Circuits

P25-33

PRS: LC Circuit

The plot shows the charge on a capacitor (black curve) and the current through it (red curve) after you turn off the power supply. If you put a core into the inductor what will happen to the time T_{lag}? period



0% 1. It will increase ✓
0% 2. It will decrease
0% 3. It will stay the same
0% 4. I don't know

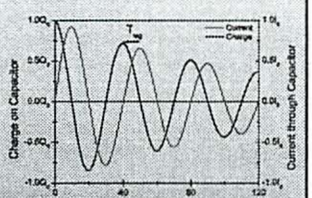
20

Period

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad T = 2\pi\sqrt{LC}$$

PRS: LC Circuit

If you increase the resistance in the circuit what will happen to rate of decay of the pictured amplitudes?



0% 1. It will increase (decay more rapidly)
0% 2. It will decrease (decay less rapidly)
0% 3. It will stay the same
0% 4. I don't know

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics
8.02

Experiment 8: RL Circuits and Undriven RLC Circuits

OBJECTIVES

1. To explore the time dependent behavior of RC and RL Circuits
2. To understand how to measure the time constant of such circuits
3. To explore the time dependent behavior of Undriven RLC Circuits

PRE-LAB READING

INTRODUCTION

In the first two parts of this lab we will continue our investigation of DC circuits, now including, along with our "battery" and resistors, inductors (RL circuits). We will measure the very different relationship between current and voltage in an inductor, and study the time dependent behavior of RL circuits.

In the second two parts of the lab we will study a circuit that includes a "battery", resistor, capacitor and inductor (undriven RLC circuits).

As most children know, if you get a push on a swing and just sit still on it, you will go back and forth, gradually slowing down to a stop. If, on the other hand, you move your body back and forth you can drive the swing, making it swing higher and higher. This only works if you move at the correct rate though – too fast or too slow and the swing will do nothing. ? what is this called

This is an example of resonance in a mechanical system. In the second two parts of this lab we will explore its electrical analog – the RLC (resistor, inductor, capacitor) circuit – and better understand what happens when it is undriven. In the next lab we will consider what happens when it is driven above, below and at the resonant frequency.

The Details: Inductors

ok so this lab is just sitting still on swing no resonance

Inductors store energy in the form of an internal magnetic field, and find their behavior dominated by Faraday's Law. In any circuit in which they are placed they create an EMF ε proportional to the time rate of change of current I through them: $\varepsilon = L\, dI/dt$. The constant of proportionality L is the inductance (measured in Henries = Ohm s), and determines how strongly the inductor reacts to current changes (and how large a self energy it contains for a given current). Typical circuit inductors range from nanohenries to hundreds of millihenries. The direction of the induced EMF can be determined by Lenz's Law: it will always oppose the change (inductors try to keep the current constant)

I really like water wheel example

RL Circuits

Consider the circuit shown in figure 1. The inductor is connected to a voltage source of constant emf $\mathcal{E}$. At $t = 0$, the switch S is closed.

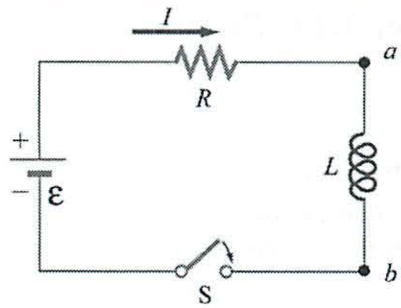


Figure 1 RL circuit. For t<0 the switch S is open and no current flows in the circuit. At t=0 the switch is closed and current *I* can begin to flow, as indicated by the arrow.

As we saw in class, before the switch is closed there is no current in the circuit. When the switch is closed the inductor wants to keep the same current as an instant ago – none. Thus it will set up an EMF that opposes the current flow. At first the EMF is identical to that of the battery (but in the opposite direction) and no current will flow. Then, as time passes, the inductor will gradually relent and current will begin to flow. After a long time a constant current ($I = V/R$) will flow through the inductor, and it will be content (no changing current means no changing B field means no changing magnetic flux means no EMF). The resulting EMF and current are pictured in Fig. 2.

much more behind scenes of waterfall analogy

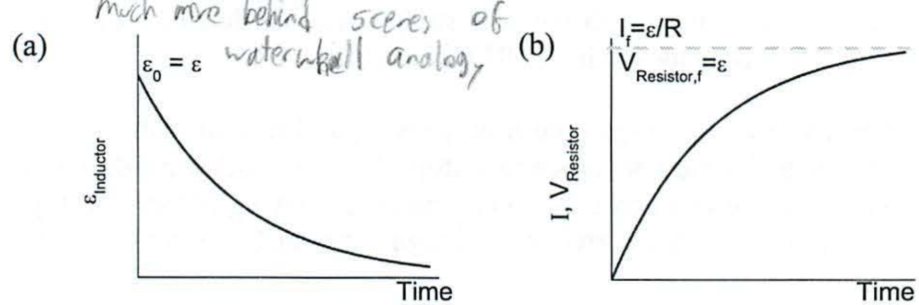


Figure 2 (a) "EMF generated by the inductor" decreases with time (this is what a voltmeter hooked in parallel with the inductor would show) (b) the current and hence the voltage across the resistor increase with time, as the inductor 'relaxes.'

After the inductor is "fully charged," with the current essentially constant, we can shut off the battery (replace it with a wire). Without an inductor in the circuit the current would instantly drop to zero, but the inductor does not want this rapid change, and hence generates an EMF that will, for a moment, keep the current exactly the same as it was before the battery was shut off. In this case, the EMF generated by the inductor and voltage across the resistor are equal, and hence EMF, voltage and current all do the same thing, decreasing exponentially with time as pictured in fig. 3.

Lot of pre-lab reading

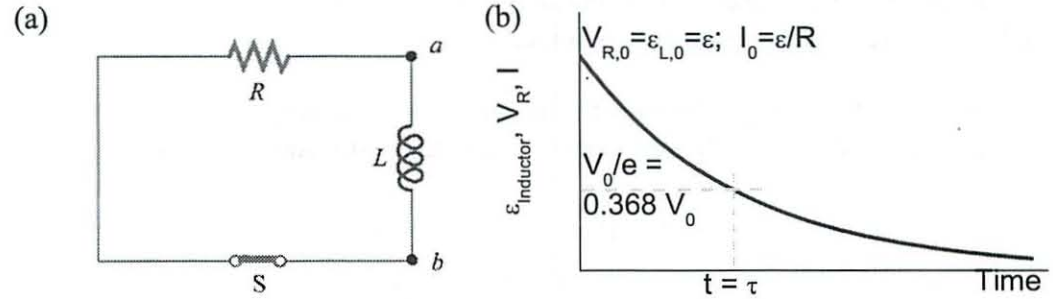


Figure 3 Once (a) the battery is turned off, the EMF induced by the inductor and hence the voltage across the resistor and current in the circuit all (b) decay exponentially.

The Details: Non-Ideal Inductors

So far we have always assumed that circuit elements are ideal, for example, that inductors only have inductance and not capacitance or resistance. This is generally a decent assumption, but in reality no circuit element is truly ideal, and today we will need to consider this. In particular, today's "inductor" has both inductance and resistance (real inductor = ideal inductor in series with resistor). Although there is no way to physically separate the inductor from the resistor in this circuit element, with a little thought you will be able to measure both the resistance and inductance.

The Details: Measuring the Time Constant τ

In this lab you will be faced with an exponentially decaying current $I = I_0 \exp(-t/\tau)$ from which you will want to extract the time constant τ. We will do this in two different ways, using the "two-point method" or the "logarithmic method," depicted in Fig. 4.

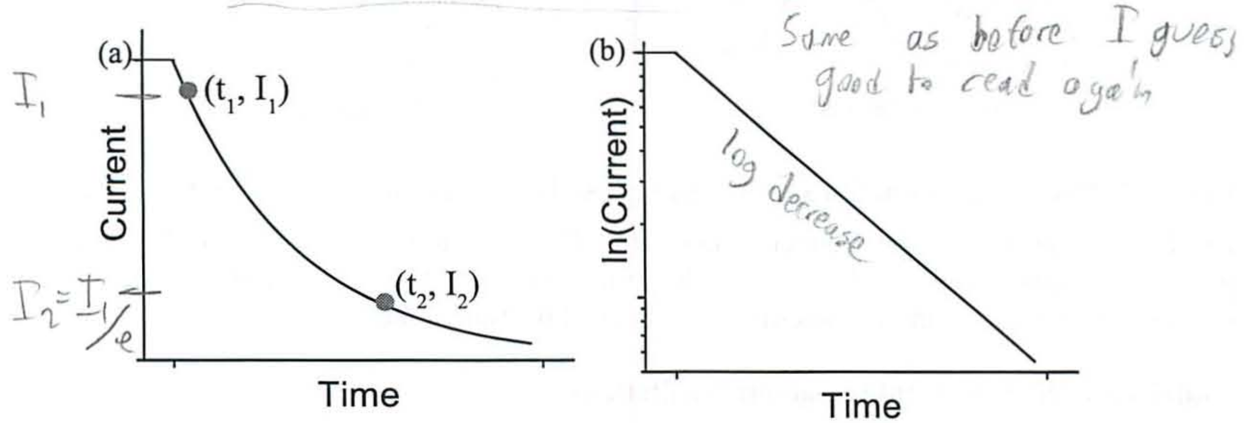


Figure 4 The (a) two-point and (b) logarithmic methods for measuring time constants

In the two-point method (Fig. 4a) we choose two points on the curve (t_1,I_1) and (t_2, I_2). Because the current obeys an exponential decay, $I = I_0 \exp(-t/\tau)$, we can extract the time constant τ most easily by picking I_2 such that $I_2 = I_1/e$. We should, in theory, be able to find this for any t_1, as long as we don't switch the battery off (or on) before enough time has passed. In practice the current will eventually get low enough that we won't be able to accurately measure it. Having made this selection, $\tau = t_2 - t_1$.

In the logarithmic method (Fig. 4b) we fit a line to the natural log of the current plotted vs time and obtain the slope m, which will give us the time constant as follows:

$$m = \frac{\text{rise}}{\text{run}} = \frac{\ln\left(I(t_2)\right) - \ln\left(I(t_1)\right)}{t_2 - t_1} = \frac{1}{t_2 - t_1}\ln\left(\frac{I(t_2)}{I(t_1)}\right)$$
$$= \frac{1}{t_2 - t_1}\ln\left(\frac{I_0 e^{-t_2/\tau}}{I_0 e^{-t_1/\tau}}\right) = \frac{1}{t_2 - t_1}\ln\left(e^{-(t_2 - t_1)/\tau}\right) = \frac{1}{t_2 - t_1}\left(\frac{-(t_2 - t_1)}{\tau}\right) = \boxed{-\frac{1}{\tau}}$$

That is, from the slope (which the software can calculate for you) you can obtain the time constant: $\boxed{\tau = -1/m.}$ m = slope

In using both of these methods you must take care to use points well into the decay (i.e. not on the flat part before the decay begins) and try to avoid times where the current has fallen close to zero, which are typically dominated by noise.

The Details: Oscillations hence oscillascope ?

In this lab you will be investigating current and voltages (EMFs) in RLC circuits. These oscillate as a function of time, either continuously (Fig. 5a) or in a decaying fashion (Fig. 5b).

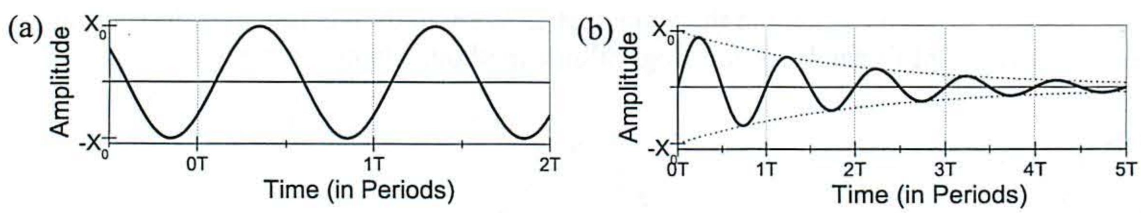


Figure 5 Oscillating Functions. (a) A purely oscillating function $x = x_0 \sin(\omega t + \varphi)$ has fixed amplitude x_0, angular frequency ω (period $T = 2\pi/\omega$ and frequency $f = \omega/2\pi$), and phase ϕ (in this case $\phi = -0.2\pi$). (b) The amplitude of a damped oscillating function decays exponentially (amplitude *envelope* indicated by dotted lines)
b/c resistance

Undriven Circuits: Thinking about Oscillations

Consider the RLC circuit of Fig. 6 below. The capacitor has an initial charge Q_0 (it was charged by a battery no longer in the circuit), but it can't go anywhere because the switch is open. When the switch is closed, the positive charge will flow off the top plate of the capacitor, through the resistor and inductor, and on to the bottom plate of the capacitor. This is the same behavior that we saw in RC circuits. In those circuits, however, the current flow stops as soon as all the positive charge has flowed to the negatively charged plate, leaving both plates with zero charge. The addition of an inductor, however, introduces inertia into the circuit, keeping the current flowing even when the capacitor is completely discharged, and forcing it to charge in the opposite polarity (Fig 6b).

↑ current flows other way

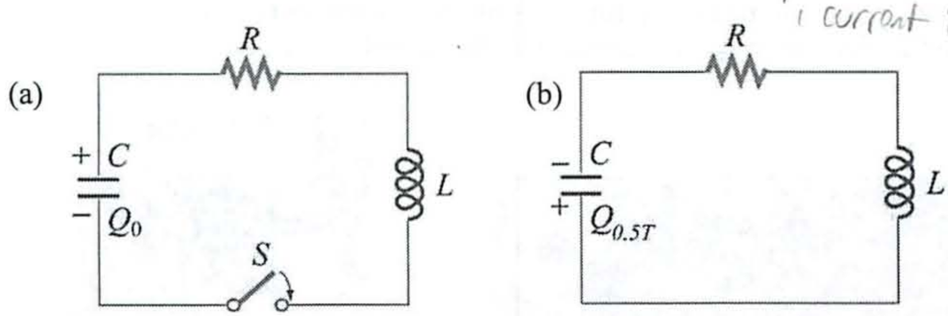


Figure 6 Undriven RLC circuit. (a) For t<0 the switch S is open and although the capacitor is charged ($Q = Q_0$) no current flows in the circuit. (b) A half period after closing the switch the capacitor again comes to a maximum charge, this time with the positive charge on the lower plate.

This oscillation of positive charge from the upper to lower plate of the capacitor is only one of the oscillations occurring in the circuit. For the two times pictured above (t=0 and t=0.5 T) the charge on the capacitor is a maximum and no current flows in the circuit. At intermediate times current is flowing, and, for example, at t = 0.25 T the current is a maximum and the charge on the capacitor is zero. Thus another oscillation in the circuit is between charge on the capacitor and current in the circuit. This corresponds to yet another oscillation in the circuit, that of energy between the capacitor and the inductor. When the capacitor is fully charged and the current is zero, the capacitor stores energy but the inductor doesn't ($U_C = Q^2/2C$; $U_L = \frac{1}{2}LI^2 = 0$). A quarter period later the current I is a maximum, charge Q = 0, and all the energy is in the inductor: $U_C = Q^2/2C = 0$; $U_L = \frac{1}{2}LI^2$. If there were no resistance in the circuit this swapping of energy between the capacitor and inductor would be perfect and the current (and voltage across the capacitor and EMF induced by the inductor) would oscillate as in Fig. 5a. A resistor, however, damps the circuit, removing energy by dissipating power through Joule heating ($P=I^2R$), and eventually ringing the current down to zero, as in Fig. 5b. Note that only the resistor dissipates power. The capacitor and inductor both store energy during half the cycle and then completely release it during the other half.

peak

peak or $y=0$

animation would be nice

APPARATUS

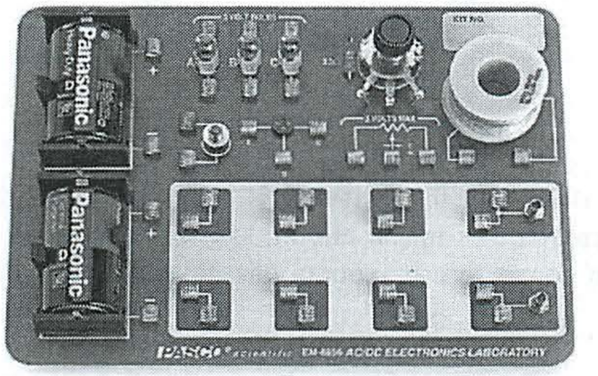
1. Science Workshop 750 Interface

In this lab we will again use the 750 interface to create a "variable battery" which we can turn on and off, whose voltage we can change and whose current we can measure. In the first two parts of this lab we will again use the Science Workshop 750 interface as an AC function generator, whose voltage we can set and current we can measure. We will also use it to measure the voltage across the capacitor using a voltage probe.

AC is kinda like this right?
but think it is completly different

2. AC/DC Electronics Lab Circuit Board

We will also again use the circuit board of Fig. 7a. This time we will use the inductor (E) as well as the connector pads (F) for resistors and capacitors, and the banana plug receptacles in the right-most pads to connect to the output of the 750.


(a)

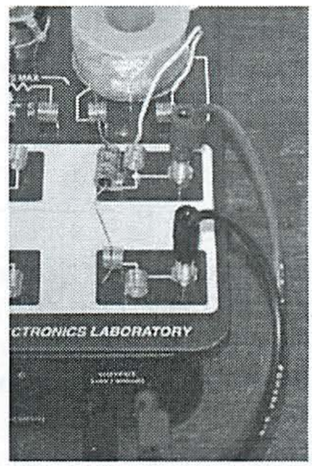

(b)

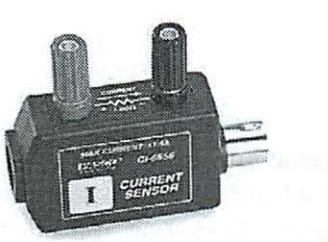
Figure 7 The AC/DC Electronics Lab Circuit Board (a) with (A) Battery holders, (B) light bulbs, (C) push button switch, (D) potentiometer, (E) inductor and (F) connector pads (b) Setup of the AC/DC Electronics Lab Circuit Board. In addition, in parallel with the capacitor you will connect a voltage probe (not pictured).

In the second two parts of this lab we will set up the circuit board with a 100 μF capacitor in series with the coil (which serves both as the resistor and inductor in the circuit), as pictured in Figure 7b .

3. Current & Voltage Sensors

Recall that both current and voltage sensors follow the convention that red is "positive" and black "negative." That is, the current sensor (Figure 8a) records currents flowing in the red lead and out the black as positive. The voltage sensor (Figure 8b) measures the potential at the red lead minus that at the black lead.

(a)



(b)

Figure 8 (a) Current and (b) Voltage Sensors

4. Capacitors

because not ideal

We will work with capacitors (and a coil which acts as both an inductor and a resistor.) Capacitors (Fig. 9) are typically stamped with a numerical value.

Figure 9 Example of a capacitor. Capacitors on the other hand come in a wide variety of packages and are typically stamped both with their capacitance and with a maximum working voltage.

GENERALIZED PROCEDURE

This lab consists of four main parts. In each you will set up a circuit and measure voltage and current while the battery periodically turns on and off. In the first two parts you are encouraged to develop your own methodology for measuring the resistance and inductance of the coil on the AC/DC Electronics Lab Circuit Board both with and without a core inserted. The core is a metal cylinder which is designed to slide into the coil and affect its properties in some way that you will measure.

Part 1: Measure Resistance and Inductance Without a Core
The battery will alternately turn on and turn off. You will need to hook up this source to the coil and, by measuring the voltage supplied by and current through the battery, determine the resistance and inductance of the coil.

Part 2: Measure Resistance and Inductance With a Core
In this section you will insert a core into the coil and repeat your measurements from part 3 (or choose a different way to make the measurements).

In the second two parts you will measure the behavior of an undriven series RLC circuit.

Part 3: Free Oscillations in an Undriven RLC Circuit
The capacitor is charged with a DC battery which is then turned off, allowing the circuit to ring down.

Part 4: Energy Ringdown in an Undriven RLC Circuit
Part 1 is repeated, except that the energy is reported instead of current and voltage.

END OF PRE-LAB READING

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP Parts One and Two

1. Download the LabView file from the web and save the file to your desktop (right click on the link and choose "Save Target As" to the desktop. Overwrite any file by this name that is already there). Start LabView by double clicking on this file.
2. Connect the Voltage Sensor to Analog Channel A on the 750 Interface. We will obtain the current directly from the "battery" reading.
3. Connect cables from the output of the 750 to the banana plug receptacles on the lower right side of the circuit board (red to the sin wave marked output, black to ground).

MEASUREMENTS

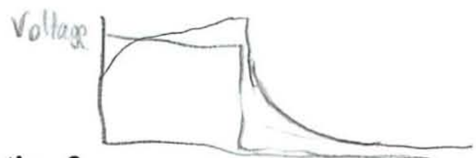
Part 1: Measure Resistance and Inductance Without a Core

1. Connect cables from the output of the 750 to either side of the coil (using the clip attachments over the usual banana plug connectors)
2. Make sure that the core is removed from the coil
3. Record the current through and voltage across the battery for a fraction of a second. (Press the green "Go" button above the graph).

Question 1:

What is the maximum current during the cycle? What is the EMF generated by the inductor at the time this current is reached?

I = yellow line = 175mA $\mathcal{E}$ = voltage = 1 V



Question 2:

What is the time constant τ of the circuit?

Read from the different points

$\frac{70.8}{e} = 26.0$

26.1   27.6 $= \boxed{1.5 ms}$

Question 3:

What are the resistance *r* and inductance *L* of the coil?

$\mathcal{E} = -L \frac{dI}{dt} - IR$

5 | 5 - but not supposed to solve but copy from notes

$I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$

$.175 = \frac{1}{R}\left(1 - e^{.024/1.5}\right)$

Solve for R

$5.71\ \Omega$

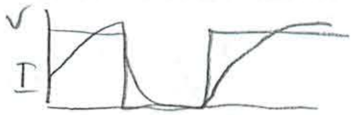
Now solve for L $\tau = L/R$

8.57 mH

* fix milli-amp + seconds

Part 2: Measure Resistance and Inductance With a Core

1. Insert the core into the center of the coil
2. Record the current through and voltage across the battery for a fraction of a second. (Press the green "Go" button above the graph).



Question 4:

Does the maximum current in the circuit change due to the introduction of the core? If it does, try to explain as clearly as possible why this happens (including why the change to bigger or smaller makes sense).

No, it does not change

Question 5:

Does the time constant τ of the circuit change due to the introduction of the core? If it does, try to explain as clearly as possible why this happens (including why the change to longer or shorter makes sense).

~~No, it does not change~~
~~It is a constant~~
~~(we didn't calculate though)~~

$\frac{121\text{ mA}}{e} = 44.7$

$57.4 - 51 = \boxed{6.4\text{ ms}}$

It makes sense that it is longer because ....

like a tranformer concentrates a $\vec{B}$ field kinda like a dielectric

Question 6

What are the new resistance *r* and inductance *L* of the coil?

Calculate like before

Has same R, longer τ, so bigger L

$R = 5.71\ \Omega$
$L = 36.544\text{ mH}$

complicated physics - will go over

field bigger → flux bigger →

$\boxed{\text{Could increase resistance - dissipating energy - AC circuit}}$

EXPERIMENTAL SETUP Parts 3 and 4

1. Set up the circuit pictured in Fig. 7b of the pre-lab reading (no core in the inductor!)
2. Connect a voltage probe to channel A of the 750 and connect it across the capacitor.

MEASUREMENTS

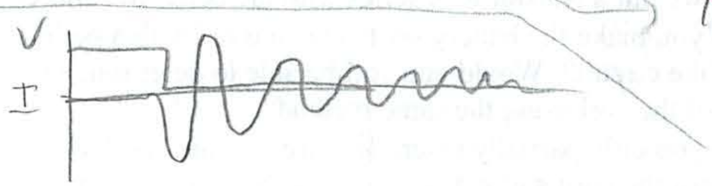
Part 3: Free Oscillations in an Undriven RLC Circuit

In this part we turn on a battery long enough to charge the capacitor and then turn it off and watch the current oscillate and decay away.

1. Press the green "Go" button above the graph to perform this process.

Before you begin, for the circuit as given (with a 10 μF capacitor and a coil with resistance ~ 5 Ω and inductance ~ 8.5 mH as measured in parts 1 and 2), what is the frequency at which the circuit should ring down?

~~10 846 oscillations /sec~~ Hz



$\sqrt{\frac{1}{LC}} = \omega \quad f = \frac{\omega}{2\pi}$

mili $= 10^{-3}$

micro $= 10^{-6} = \mu$

Question 7:

What is the period of the oscillations (measure the time between distant zeroes of the current and divide by the number of periods between those zeroes)? What is the frequency?

1.8 ms period

freq = 555.5 Hz

$\frac{\sqrt{8.57\cdot 10^{-3} \cdot 10 \cdot 10^{-6}}}{2\pi}$

now 545 Hz

Question 8:

Is this experimentally measured frequency the same as, larger than or smaller than what you calculated it should be? If it is not the same, why not?

The experimentally frequency was basically the same - slightly larger - since this is real life and not ideal

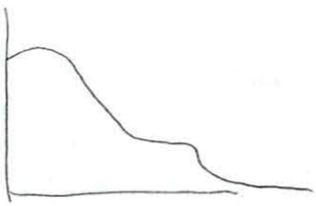
Part 4: Energy Ringdown in an Undriven RLC Circuit

1. Insert the core into the inductor for this part.
2. Repeat the process of part 3, this time recording the energy stored in the capacitor $\left(U_C = \frac{1}{2}CV^2\right)$ and inductor $\left(U_L = \frac{1}{2}LI^2\right)$, and the sum of the two.

Question 9:

more spaced out

The circuit is losing energy most rapidly at times when the slope of total energy is steepest. Is the electric (capacitor) or magnetic (inductor) energy a local maximum at those times? Briefly explain why.



others fluxuate
– very cool graph

Slope steepest when current is going into the inductor, Because the charge is leaving the capicator, adding to the current. The change in current is 0 at the maximum, so the inductor is happy, and current is flowing through it,

Further Questions (for experiment, thought, future exam questions…)

- What happens if we put a resistor R in series with the coil? In parallel with the coil?
- What happens if you make the battery switch on and off with a period shorter than the time constant of the circuit? Would you still be able to determine the inductance L and resistance r of the coil using the same method?
- What happens if you only partially insert the core into the coil? Can you continuously adjust the core's effects or there an abrupt jump from one behavior to another? Would another core (like your finger) have the same effects?
- If the coil were made of some superconducting material, what would its resistance be? Would the EMF you measure be any different? Would the potential difference from one side of the inductor to the other $\left(\Delta V = -\int_a^b \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}\right)$ be any different?
- What happened when you inserted the core into the coil? Why did we ask you to do that in part 4?
- What happens to the resonant frequency of the circuit if a resistor is placed in series with the capacitor and coil? In parallel? NOTE: You can use the variable resistor, called a potentiometer or "pot" (just to the left of the coil, connect to the center and right most contacts, allowing you to adjust the extra resistance from $0\,\Omega$ to $3.3\,\Omega$ by simply turning the knob).

Topics: LC, and Undriven LRC Circuits
Related Reading: Course Notes: Sections 12.1-12.7

Topic Introduction

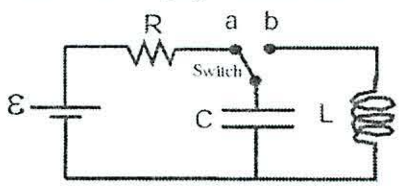
Today we investigate LRC circuits. We will see that the current in these circuits oscillates, in a fashion completely analogous to the oscillation of a mass on a spring

Mass on a Spring: Simple Harmonic Motion

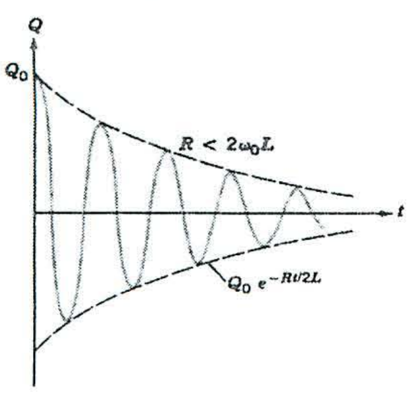
In a simple system consisting of a mass hanging on a spring, when the mass is pulled down and released it oscillates up and down. We think about this in a couple of ways. One way is to look at the forces on the mass and write a differential equation for its motion, $F = m\ddot{x} = -kx$, where $\ddot{x}$ means two time derivatives of the displacement (acceleration). The solution to this is simple harmonic motion: $x = x_0 \cos(\omega t)$ where $\omega = \sqrt{k/m}$.

We can also think about the energy in the system. As the mass moves, energy oscillates between kinetic energy of the mass and potential energy stored in the spring. If there is no damping (friction) in the system to dissipate energy, the oscillation will continue forever.

Undriven L(R)C Circuits



Consider the LC circuit at left, where the switch is at "a" until the capacitor is fully charged and then thrown to "b." This is analogous to pulling down a mass and releasing it. Here the capacitor will want to discharge and will drive a current through the inductor. Eventually all the charges will run off of the capacitor (spring), so it won't "push" anymore, but now the inductor will want to keep the current flowing through it that it already has (inductors, like masses, have inertia). It will keep the current flowing, but that will eventually fill up the capacitor which will stop the current and send it back the other direction. Our differential equation is thus analogous, $V = -L\ddot{q} = q/C$, and has the same solution: $q = q_0 \cos(\omega t)$ where $\omega = \sqrt{1/LC}$.



We can also think about energy here, where it oscillates between being stored in the electric field in the capacitor and the magnetic field in the inductor. As long as there is no dissipation (resistance) is the circuit the oscillations will continue forever.

If we add a resistor in series with the capacitor and inductor we provide a method of energy loss, through joule heating in the resistor as current flows. The oscillations will thus damp out to zero. The exact path the charge will take as it oscillates to zero depends on the relative sizes of L, R and C, but will typically look something like the curve above, where the oscillations are bounded by an "envelope" which is exponentially decaying to zero as a function of time.

P-set Review

w/ Sahar

4/12

Loop rule w/ induced emf

- non conservative
- no potential diff
- no kirkoff
  - so say $L \frac{dI}{dt}$

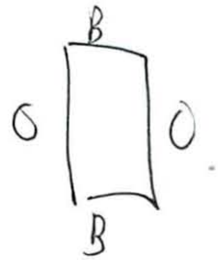
Current

~~2nd law~~ force $\vec{F} = I(\ell \times B)$ ← totally forgot about in this p-set

dir current

need to fix like everything

- depends on which side



direction up
- opposes gravity

when $F_g = F_c$ → terminal velocity

① Moving charge → Magnetic field

② External magnetic field → moving charge

✓ simplification
- goes around in a circle
  but gets too small

Went back + fixed the 2 questions

4/13

Redwine
Office Hrs

Current through B or add 0 L

external

3 e does not ~~not~~ induced so $F \leftarrow I(\ell \times B)$

- does not ask - but it is here there

Need to use words precisly

- changing flux → ℰ around loop
- Field → 0 everything stops

Was going too fast in OH

Step back + look

What is happening

- go slower
- read over my ans
- do this when I am only one
- ~~me~~

Terminal Velocity = no net force

I like don't have many qu

- Saler helped
- and I understand now

Differential eqs

- actually here big, big qu that good it got answered

He wants me to speak far more precisly

- heed to
- slower, but that is Ok
- a general problem I have
  - more recently than before

Being fully preped for each class

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2010**

Problem Set 9

Due: Tuesday, April 13 at 9 pm.

Hand in your problem set in your section slot in the boxes outside the door of 32-082. Make sure you clearly write your name and section on your problem set.

Text: Liao, Dourmashkin, Belcher; Introduction to E & M MIT 8.02 Course Notes.

Week Ten Faraday's Law

Class 22 W10D1 M/T Apr 5/6	Faraday's Law; Expt.7: Faraday's Law
Reading:	Course Notes: 10.1-10.3, 10.8-10.9
Experiment:	Expt.7: Faraday's Law
Class 23 W10D2 W/R Apr 7/8	Problem Solving Faraday's Law; Inductance & Magnetic Energy, RL Circuits
Reading:	Course Notes: 10.1-10.4, 10.8-10.9, 11.1-11.4
Class 24 W10D3 F Apr 9	Special Lecture: Applications of Faraday's Law
Reading:	Course Notes: 10.1-10.4, 10.8-10.9, 11.1-11.4

Campus Preview Weekend

Week Eleven AC Circuits

Class 25 W11D1 M/T Apr 12/13	Undriven RLC Circuits; Expt. 8: RL Circuits and Undriven RLC Circuits
Reading:	Course Notes: 11.5-11.11
Experiment:	Expt. 8: RL Circuits and Undriven RLC Circuits
Class 26 W11D2 W/R Apr 14/15	Driven RLC Circuits
Reading:	Course Notes: 12.1-12.7
Class 27 W11D3 F Apr 16	PS08: RLC Circuits
Reading:	Course Notes: 12.8-12.9

Problem 1: Short Questions

(a) When a small magnet is moved toward a solenoid, an emf is induced in the coil. However, if the magnet is moved around inside a toroid, no measurable emf is induced. Explain.

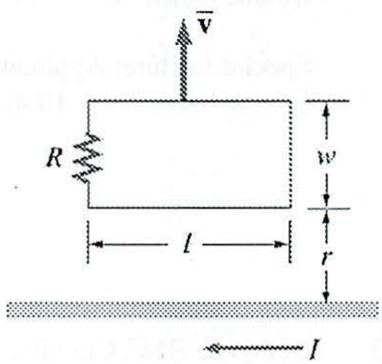
(b) A piece of aluminum is dropped vertically downward between the poles of an electromagnet. Does the magnetic field affect the velocity of the aluminum? Explain.

(c) What happens to the generated current when the rotational speed of a generator coil is increased?

(d) If you pull a loop through a non-uniform magnetic field that is perpendicular to the plane of the loop which way does the induced force on the loop act?

Problem 2: Moving Loop

A rectangular loop of dimensions l and w moves with a constant velocity $\vec{\mathbf{v}}$ away from an infinitely long straight wire carrying a current I in the plane of the loop, as shown in the figure. The total resistance of the loop is R.



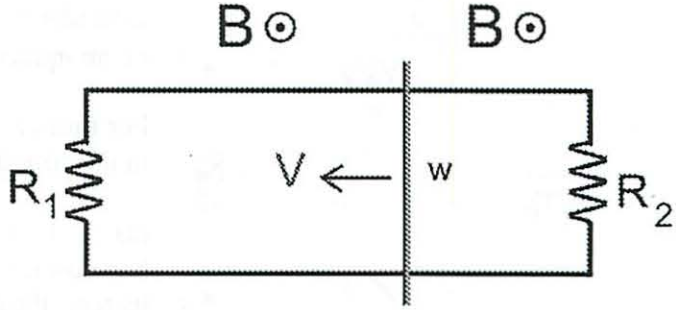
(a) Using Ampere's law, find the magnetic field at a distance s away from the straight current-carrying wire.

(b) What is the magnetic flux through the rectangular loop at the instant when the lower side with length l is at a distance r away from the straight current-carrying wire, as shown in the figure?

(c) At the instant the lower side is a distance r from the wire, find the induced emf and the corresponding induced current in the rectangular loop. Which direction does the induced current flow?

Problem 3: Faraday's Law

A conducting rod with zero resistance and length w slides without friction on two parallel perfectly conducting wires. Resistors R_1 and R_2 are connected across the ends of the wires to form a circuit, as shown. A constant magnetic field **B** is directed out of the page. In computing magnetic flux through any surface, take the surface normal to be out of the page, parallel to **B**.



(a) The magnetic flux in the right loop of the circuit shown is (circle one)
   1) decreasing
   2) increasing

What is the magnitude of the rate of change of the magnetic flux through the right loop?

(b) What is the current flowing through the resistor R_2 in the right hand loop of the circuit shown? Gives its magnitude and indicate its direction on the figure.

(c) The magnetic flux in the left loop of the circuit shown is (circle one)
   1) decreasing
   2) increasing

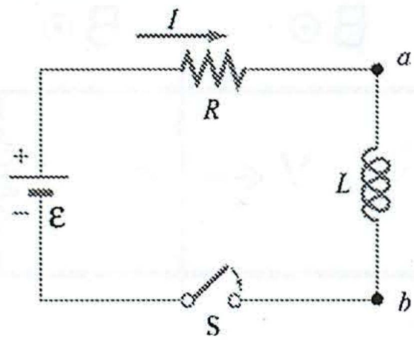
What is the magnitude of the rate of change of the magnetic flux through the left loop?

(d) What is the current flowing through the resistor R_1 in the left hand loop of the circuit shown? Gives its magnitude and indicate its direction on the figure.

(e) What is the magnitude and direction of the magnetic force exerted on this rod?

Problem 4: Read Experiment 8: Inductance and RL Circuits Pre-Lab Questions

1. RL Circuits



Consider the circuit at left, consisting of a battery (emf ε), an inductor L, resistor R and switch S.

For times $t<0$ the switch is open and there is no current in the circuit. At $t=0$ the switch is closed.

(a) Using Kirchhoff's loop rules (really Faraday's law now), write an equation relating the emf on the battery, the current in the circuit and the time derivative of the current in the circuit.

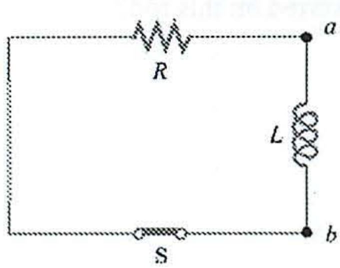
We know from thinking about it above that the results should look very similar to RC circuits. In other words:

$$I = A(X - \exp(-t/\tau))$$

(b) Plug this expression into the differential equation you obtained in (a) in order to confirm that it indeed is a solution and to determine what the time constant τ and the constants A and X are. What would be a better label for A? (HINT: You will also need to use the initial condition for current. What is $I(t=0)$?)

(c) Now that you know the time dependence for the current I in the circuit you can also determine the voltage drop V_R across resistor and the EMF generated by the inductor. Do so, and confirm that your expressions match the plots in Fig. 2a or 2b.

2. 'Discharging' an Inductor



After a long time T the current will reach an equilibrium value and inductor will be "fully charged." At this point we turn off the battery ($\varepsilon=0$), allowing the inductor to 'discharge,' as pictured at left. Repeat each of the steps a-c in problem 1, noting that instead of $\exp(-t/\tau)$, our expression for current will now contain $\exp(-(t-T)/\tau)$.

(a) Faraday's law:

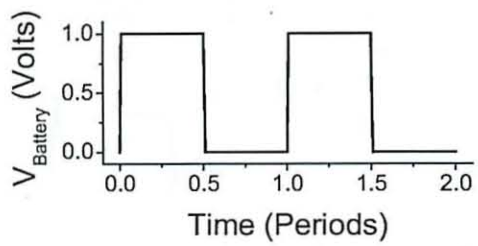
(b) Confirm solution:

(c) Determine V_R across resistor and the EMF generated by the inductor.

3. A Real Inductor

As mentioned above, in this lab you will work with a coil that does not behave as an ideal inductor, but rather as an ideal inductor in series with a resistor. For this reason you have no way to independently measure the voltage drop across the resistor or the EMF induced by the inductor, but instead must measure them together. None-the-less, you want to get information about both. In this problem you will figure out how.

(a) In the lab you will hook up the circuit of problem 1 (with the ideal inductor L of that problem now replaced by a coil that is a non-ideal inductor – an inductor L and resistor r in series). The battery will periodically turn on and off, displaying a voltage as shown here:



Sketch the current through the battery as well as what a voltmeter hooked across the coil would show versus time for the two periods shown above. Assume that the period of the battery turning off and on is comparable to but longer than several time constants of the circuit.

(b) How can you tell from your plot of the voltmeter across the coil that the coil is not an ideal inductor? Indicate the relevant feature clearly on the plot. Can you determine the resistance of the coil, r, from this feature?

(c) In the lab you will find it easier to make measurements if you do NOT use an additional resistor R, but instead simply hook the battery directly to the coil. (Why? Because the time constant is difficult to measure with extra resistance in the circuit). Plot the current through the battery and the reading on a voltmeter across the coil for this case. We will only bother to measure the current. Why?

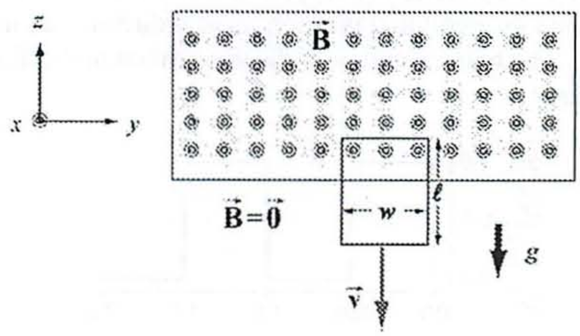
(d) For this case (only a battery & coil) how will you determine the resistance of the coil, r? How will you determine its inductance L?

4. The Coil

The coil you will be measuring has is made of thin copper wire (**radius** ~ 0.25 mm) and has about 600 turns of average diameter 25 mm over a length of 25 mm. What approximately should the resistance and inductance of the coil be? The resistivity of copper at room temperature is around 20 nΩ-m. Note that your calculations can only be approximate because this is not at all an ideal solenoid (where length >> diameter).

Problem 5 Falling Loop

A rectangular loop of wire with mass m, width w, vertical length l, and resistance R falls out of a magnetic field under the influence of gravity, as shown in the figure below. The magnetic field is uniform and out of the paper ($\vec{\mathbf{B}} = B\hat{\mathbf{i}}$) within the area shown and zero outside of that area. At the time shown in the sketch, the loop is exiting the magnetic field at speed $\vec{\mathbf{v}} = -v\hat{\mathbf{k}}$.



(a) What is the direction of the current flowing in the circuit at the time shown, clockwise or counterclockwise? Why did you pick this direction?

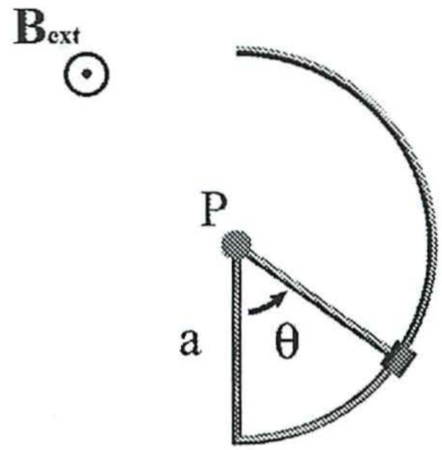
(b) Using Faraday's law, find an expression for the magnitude of the emf in this circuit in terms of the quantities given. What is the magnitude of the current flowing in the circuit at the time shown?

(c) Besides gravity, what other force acts on the loop in the $\pm\hat{\mathbf{k}}$ direction? Give its magnitude and direction in terms of the quantities given.

(d) Assume that the loop has reached a "terminal velocity" and is no longer accelerating. What is the magnitude of that terminal velocity in terms of given quantities?

(e) Show that at terminal velocity, the rate at which gravity is doing work on the loop is equal to the rate at which energy is being dissipated in the loop through Joule heating.

Problem 6: Generator



A "pie-shaped" circuit is made from a straight vertical conducting rod of length a welded to a conducting rod bent into the shape of a semi-circle with radius a (see sketch). The circuit is completed by a conducting rod of length a pivoted at the center of the semi-circle, *Point P*, and free to rotate about that point. This moving rod makes electrical contact with the vertical rod at one end and the semi-circular rod at the other end. The angle θ is the angle between the vertical rod and the moving rod, as shown. The circuit sits in a constant magnetic field $\mathbf{B}_{ext}$ pointing out of the page.

(a) If the angle θ is increasing with time, what is the direction of the resultant current flow around the "pie-shaped" circuit? What is the direction of the current flow at the instant shown on the above diagram? To get credit for the right answer, you must justify your answer.

For the next two parts, assume that the angle θ is increasing at a constant rate, $d\theta(t)/dt = \omega$.

(b) What is the magnitude of the rate of change of the magnetic flux through the "pie-shaped" circuit due to $\mathbf{B}_{ext}$ only (do **not** include the magnetic field associated with any induced current in the circuit)?

(c) If the "pie-shaped" circuit has a constant resistance R, what is the magnitude and direction of the magnetic force due to the external field on the moving rod in terms of the quantities given. What is the direction of the force at the instant shown on the above diagram?

P-Set 9

Michael Plasmeier L01 11C

P3 23
P5 20
Others 50

93

4/11

6a Short questions; When a small magnet is moved toward a solenoid, an emf is induced in a coil. However, if the magnet is moved in a toroid, no emf is induced.

There is a $\vec{B}$ field inside of a solenoid. In a toroid all of the B field is confined to the core - making it largely self sheilding. The flux is parallel to core of the toroid. Magnetic field is only on the toroid - so it does not oppose the movement

wp

other

$\frac{r}{2}$

is that all

b A piece of Al is dropped between an electromagnetic Is it affected

The electomagnet will induce a current in the Al making it slow down - just like one of those labs we did with the sleve - it will always oppose motion.

The flux will always oppose the motion - correct?

peer

c What happens to generated current when speed of generator is increased?

It would increase I am guessing. We did this in class 29 W11 D1

~~[illegible]~~
mag field changing more quickly

$$I(t) = \frac{\mathcal{E}(t)}{R} = \frac{1}{R}\frac{d\Phi_B}{dt} = \frac{1}{R}\frac{d(BA\cos\omega t)}{dt}$$

$$= \frac{BL^2}{R}\omega\sin(\omega t)$$

The magnetic field changes more quickly when you spin it faster. This means the flux is changing faster, producing more current.

You can see this in the ω angular velocity If it is higher, more current is produced.

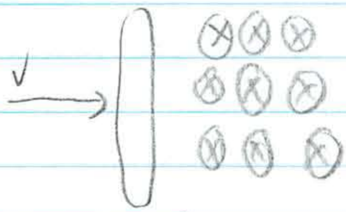
You can remember this from HS where when you spin the crank more, the bulb gets brighter.

But what is it that the resistance decreases? Something that more of the work goes to the load, not internal resistance

↳ well does not matter here

↓ If you pull a loop through a <u>non uniform</u> $\vec{B}$
? important, otherwise 0
that is $\perp$ to the plane of the loop which way
does the induced force on the loop act,

Ok direction question.



$\mathcal{E} = \int \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$

The current in the loop is opposet the way
the motion

So $v \times B$ = clockwise

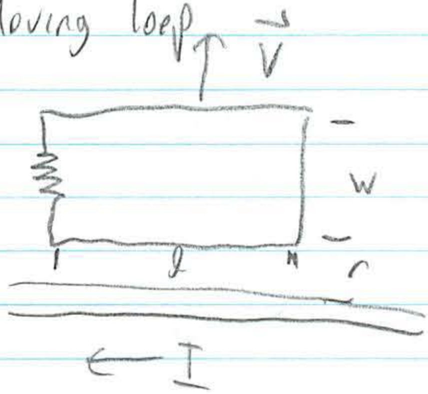
So current will flow in loop counterclockwise

peers

* will always oppose change in flux

Can't give certain direction since qu is vague

2. Moving loop



R = resistance of wire
moving away from wire

a) Using Ampere's law find the magnetic field at distance s away from the straight current carrying wire.

Pset 8
ans

~~- So rectangular~~
~~- only want region outside of wire~~
~~- but nothing is inside wire~~
~~- but that is 0~~
~~- So~~

~~$2+3=0$~~

~~1 and 4 = Bl~~

~~$2Bl = \frac{\mu_0 J e l d}{4}$~~

Peer

Do it circular as outside of wire
Not as slab

P-set 8
ans

$B(2\pi r) = \mu_0 I_{enc}$

$\vec{B} = \frac{\mu_0 I}{2\pi r}$

Now need direction

I should go always in or out of page
lets say ⊙ so CCW

So now we have the $\vec{B}$ field there
So now what?
Well that is answer to a

$$\vec{B} = \frac{\mu_0 I}{2\pi s}$$

b) What is the magnetic flux through the rectangular loop at the instant when the lower side is r distance away.

peer

$\int B$ field over region
- guass' law!

Own

Φ = well look this up
$\iint \vec{B} \cdot d\vec{A}$ or $BA\cos\theta$ if constant $\vec{B} \cdot \vec{A}$

~~$\Phi = \frac{\mu_0 I}{2\pi s} (B l)$~~ not constant

~~Oh Guass's law - From the very begining~~
~~but what it is~~
~~- ∞ long rod~~
~~- plane of charge~~

peers

No not Guassian's surface, and $\vec{B}$ field not constant, Area is the area enclosed by the loop

$$\Phi = l \cdot \int_r^{w+r} \vec{B}$$

own

$$\Phi = l \cdot \int \frac{\mu_0 I}{2\pi s} ds$$

$$\Phi = \frac{\mu_0 I l}{2\pi} \int_r^{w+r} \frac{1}{s} ds$$

remember what $\int \frac{1}{s}$

$$\Phi = \frac{\mu_0 I l}{2\pi} \ln\left(\frac{w+r}{r}\right)$$

So why did I not get that
- need to know rules + pratice more

c At the instant it is r, what is induced emf + current and dir?

Here is Fariday's law

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

So derivitive of Φ
- s is a function of time

peers

$* \; s = v \cdot t = r$

own

so $t = \frac{s}{v}$ or $s = tv = r$

$$\frac{d\left(\frac{\mu_0 I l}{2\pi} \ln\left(\frac{w+r}{r}\right)\right)}{dt}$$

$$\frac{\mu_0 I l}{2\pi} \cdot \frac{d \ln\left(\frac{w+vt}{vt}\right)}{dt}$$

$$\frac{\mu_0 I l}{2\pi} \cdot 0$$

$$0$$

think I did something wrong

peers

$$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln b$$

own

will try that

$$\frac{d \ln(w + vt)}{dt} - \frac{d \ln(vt)}{dt}$$

$$\cancel{\frac{1}{w+t} - \frac{1}{t}}$$

think calc not differentiating well - need to be able to do this easily

* know rules $\ln(ab) = \ln a + \ln b$
$\ln\left(\frac{a}{b}\right) = \ln a - \ln(b)$
$\ln(a^m) = m \ln a$
$\ln(e^x) = x$

$(\ln uv)' = (\ln u + \ln v)' = (\ln u)' + (\ln v)'$

$d(\ln x) = \frac{1}{x}$ $\quad d(\ln(x+5)) = \frac{1}{x+5}$

$d(\ln 5x) = \frac{1}{x}$ $\quad d(\ln(x+y) = \frac{1}{x+y}$

$d\left(\ln \frac{1}{x}\right) = -\frac{1}{x}$

So try again

$d \ln(w + vt) = \frac{1}{w+t}$ $\quad v = \text{constant}$

$d \ln(vt) = \frac{1}{x}$

$\frac{1}{w+t} + \frac{1}{t}$

kinda what I got before
need to turn back $s = tv$ $\quad t = \frac{s}{v}$

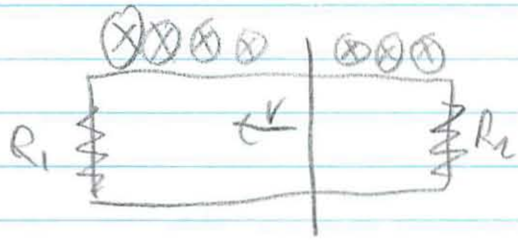
$\frac{1}{w+s} \cdot v - \frac{v}{s}$

which is the same as peer got $\frac{v}{r+w} - \frac{v}{r}$

must be able to do all of this - pratice this problem

That was horrible - math was ok except for d(ln) but physics always need point in right d'r - good working w/ others for once

3. A conducting rod w/ 0 resistant slides on wire w/o Friction



Normal vector = out of page

- so this is like an inclass problem day 29 #1

a) Magnetic flux in right loop is

Class 29 #1 in class

- area of right loop is increasing
- So flux into page is increasing
- current wants to oppose so it will flow counter CW so flux is out of page

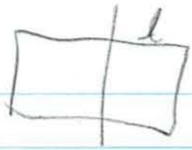
so Why? $\Phi = \vec{B} \cdot \vec{A}$

not changing ↑ ↑ increasing

how is area a vector?
- well dA a small piece
- but is the normal vector

fingers toward V
curl towards field
flux is into page ⊗

flux increasing ✓



own a2) What is rate of change of flux?

$$\frac{d\phi}{dt} = \frac{d(B \cdot A)}{dt}$$

not enough variables to describe

$$\frac{d(\vec{B} \cdot w \cdot \ell)}{dt}$$

$$B \cdot w \frac{d\ell}{dt} \qquad \boxed{* \frac{d\ell}{dt} = v} \quad \text{so } \underline{Bwv}$$

✓

b) What is current flowing through R_2

helpful p-set

not too easy not impossible

direction first → from a → CCW

amount $\quad \mathcal{E} = -\frac{d\phi}{dt} - RI$

$$\frac{d\phi}{dt} = RI \qquad * \boxed{I = \frac{\mathcal{E}}{R}}$$

$$\frac{d\phi}{dt\,R} = I$$

~~What did in class problem do?~~

$\boxed{\text{well solve it}}$ $\quad I = -\frac{Bwv}{R_2}$

✓

⊖ CCW is same as ⊕ CW :)

be careful to specify its R_2.

∅ agrees w/ inclass problem

c) Magnetic flux is

- decreasing because other increasing ✓
- rate is the opposet

Well is $\phi_R + \phi_L = \text{constant}$???

??? is it

So if do it fully

$$\phi = \frac{d(B \cdot A)}{dt}$$

$$\phi = B \frac{dA}{dt}$$

$$B w \frac{d(x-\ell)}{dt}$$

$$B w -v$$

yeah opposet

So $\frac{d\phi}{dt} = -Bwv$ ✓

d) Current through resistor 1

$$\frac{\mathcal{E}}{R} = I_R$$

$I_R = \frac{Bwv}{R_1}$ and this one ~~clockwise~~ CCW, right

need flux out of page to make up for loss of flux.

e) What is the magnitude + direction of the magnetic force on the rod?

So what is $\vec{B}$

Is this not given that it is out of page?
Its not induction - does that make a new magnetic field? - yeah that is the point
or is it?

↓

experiment & we felt a force opposite to motion
can feel the magnetic force which is due to the B field from perm magnet - moving over it - not $\vec{B}$ created from wire moving
Plus here magnetic field is constant

↓

so pretty sure constant $\vec{B}$ field from given only

But what is B?
- B-S
- Ampiere

But can't measure B field since its a given

↓

or I could work backwards if measured I

$$I = \frac{Bwv}{R}$$

$$\vec{B} = \frac{IR}{wv} \otimes$$

really confused

see

~~Well course notes 10-10~~

$$\boxed{\oint E_{nc} \cdot d\vec{s} = -\frac{d\Phi}{dt}}$$

~~induced electric Field~~
~~– not magnetic~~

~~that is what was confusing me~~

Sahar

$$F = I(\ell \times \vec{B})$$

majorly forgot
– why no example problems?

$$= \left(\frac{Bwv}{R_1} + \frac{Bwv}{R_2}\right) wB \uparrow$$

$$= Bwv\left(\frac{1}{R_1} + \frac{1}{R_2}\right) wB \uparrow$$

$$= B^2 w^2 \cancel{v^2} \left(\frac{1}{R_1} + \frac{1}{R_2}\right) \uparrow$$

not squared

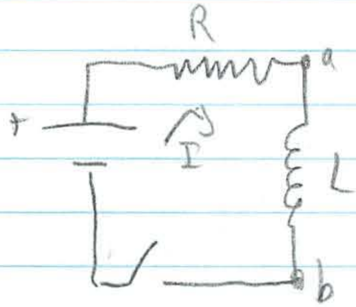
v. close

23/25

Doing after lab

4. Pre-lab questions

- just like pre lab and/or mastering physics



t = 0 switch closed

a) Write a kirchhoff loop

$$\mathcal{E} - IR - \frac{d\Phi}{dt} = 0$$

$$\mathcal{E} - IR - \frac{d(B \cdot A)}{dt} = 0$$

↑ what is charging - only current
- so how to represent that?

$$\boxed{* \ \mathcal{E} = -L\frac{dI}{dt}}$$

$$\boxed{L = \frac{\Phi}{I}} \quad \boxed{X = \omega L}$$

$$\mathcal{E} - IR - L\frac{dI}{dt} = 0$$

↑ reduce: direction correct - but missed why/how

Differential eq I don't know how to solve
learned it once - forgot
don't need to learn, but should

$$I = A\left(X - e^{-t/\tau}\right)$$

↑

$X = \omega L$

b) Plug differential eq in A to confirm it is solution and find τ, A, x. Need inital condition

$I(t=0) = 0$

Ok it is this math that really confuses me
Go back to differenital eq review

Ok saw some of it
A is $I(t=0)$
- or is it, then it would always be 0

course notes 12-6

First step sole for dI

$$\mathcal{E} - IR = L \frac{dI}{dt}$$

still DC source

$$\rightarrow \frac{\mathcal{E}}{L} - \frac{IR}{L} = \frac{dI}{dt}$$

→ skip a few pgs

~~now~~ $\cancel{\int \quad \int}$

L depends on t - but how

$$\cancel{I = \int \frac{\mathcal{E}}{L}\, dt - \int \frac{IR}{L}\, dt}$$

constant

$$\cancel{\frac{\mathcal{E}}{L} t - \ldots}$$

? how do you get this
on 12-10 course notes they just assume an answer via magic differential eq handwaving

Well it is the solution they give

(2012 er - its a lot of guessing, need to know tricks)

$$I(t) = I_0 (\cos L - e^{t/\gamma})$$

Although in course notes they find a which
is one step below

But again I_0 is ?

Course notes $I_0 = -Q_0 \omega$

$$I_0 = -\frac{V_0}{\sqrt{R^2 + X_L^2}}$$

?, I still confused

This is described in the prelab - but only
graphically - what are the ??

Seem to be only one confused

Redwine OH
- can't solve directly
- if can guess - only non trivial solution
- say try this form
  - what are constants
- depends on initial conditions

(i) won't instantaneously change
graphs in prelab
eq in PPTs

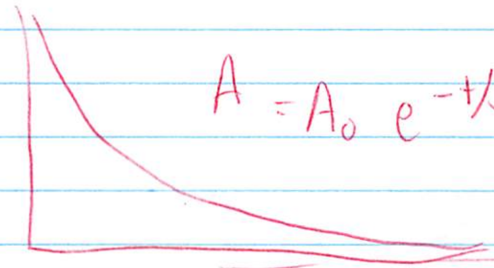
Redwire OH

Ok so Redwire points out class 23
p 15 + 16 slides

$$\frac{dA}{dt} = -\frac{1}{\tau} A$$ consider function ←

↑ where constant times A

the exponential decay eqs



$$\frac{dA}{dt} = -\frac{1}{\tau}(A - A_f)$$ bit harder

↑ constant

$$A = A_f\left(1 - e^{-t/\tau}\right)$$

own at oh

So basically know these general solutions and be able to plug in

So for LR circuit

Kirkoff loop rule
- they solved for $\frac{dI}{dt}$

$$\frac{dI}{dt} = -\frac{1}{L/R}\left(I - \frac{\mathcal{E}}{R}\right)$$ is the differential ← eq

Then they recognized this fit the pattern

$$\frac{dA}{dt} = -\frac{1}{\tau}(A - A_f)$$

So they knew generic solution to diff eq was

$$A = A_f\left(1 - e^{-t/\tau}\right)$$

So now plug stuff in

$$I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$$

---

own in
off

Oh so back to qu

$$\mathcal{E} - IR = L\frac{dI}{dt}$$

$$\frac{dI}{dt} = \frac{\mathcal{E}}{L} - \frac{IR}{L}$$

Redwte
help w/
getting in
right form

now for some reason they want us to factor out

$$-\frac{1}{L/R} \rightarrow \text{note is} \rightarrow -\frac{R}{L}$$

So want $-\frac{R}{L} \cdot ? = \frac{\mathcal{E}}{L} \rightarrow ? = -\frac{\mathcal{E}}{R}$

$-\frac{R}{L} \cdot ? = -\frac{IR}{L} \rightarrow ? = I$

Ok so that is where they got that

$$\frac{dI}{dt} = -\frac{1}{L/R}\left(I - \frac{6}{R}\right)$$

- yeah what they had
  - you have to know they want it in that weird form

Now we notice it fits our differential pattern
- same as example, so

$$I(t) = \frac{6}{R}\left(1 - e^{-t/\gamma}\right)$$

Now lets compare with what they gave us.

$$I = A\left(x - e^{-t/T}\right)$$

so $A = \frac{6}{R}$

* note this is the final current
depends on situation
reduce - stay away for generalizations
either initial or final

$x = 1 = \omega L$

Think I get this much better now

c) Now that you know the time dependence for current I can also find V_R and $\mathcal{E}$ generated by inductor

Yeah had a lot of trouble on MP
need to review more

So in course notes 12-11 they give instantaneous voltages for each element from a phasor diagram

$$V_R(t) = I_0 R = V_R$$
$$V_L(t) = I_0 X_L \sin\left(\omega t + \frac{\pi}{2}\right) = V_L \cos \omega t$$

Or $V_{L0} = I_0 X_L$

$$X_L = \omega L$$

but this is further confused because we had DC, not AC same

?: still dont get

And $\mathcal{E}$ of batt $= V_R(t) + V_L(t))$

Redwire OH

Ok so fixed B
$$I = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$$

For inductor
$$\mathcal{E} = L \frac{dI}{dt}$$

Take expression for current, differentiate multiply by L

$\underline{\text{Resistor}} = IR$

own
w/ help
oh

- plug in $\longrightarrow$
- see
- should all add back to 0

Inductor

$$I = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{tR}{L}}\right)$$

$$\cancel{\frac{dI}{dt} = \frac{R\mathcal{E}}{LI} \cdot t \, e^{\left(-\frac{tR}{L}\right)}}$$

duh had it before
- thought it sounded familiar

$$\cancel{\frac{dI}{dt} = -\frac{R}{L}\left(I - \frac{\mathcal{E}}{R}\right)}$$

$$\cancel{\text{So } V_{\text{Inductor}} = L \cdot \frac{R}{L}\left(I - \frac{\mathcal{E}}{R}\right) = IR - \mathcal{E}}$$

circular

; isn't that a circular argument

Do have to actually differentiate

- don't be afraid its easy

$$\frac{d}{dx}\left(e^{Ax}\right) = Ae^{Ax}$$

$$\frac{dI}{dt} = \frac{\mathcal{E}}{R} \cdot \frac{R}{L} e^{\left(-\frac{tR}{L}\right)}$$

cnahcry

$$= \frac{\mathcal{E}}{L} e^{-\frac{R}{L}t}$$

$$V_{\text{inductor}} = L \cdot \frac{\mathcal{E}}{L} e^{-\frac{R}{L}t} = \mathcal{E} e^{-\frac{Rt}{L}}$$

own

And for a resistor voltage is IR

$$V_R = \frac{\mathcal{E}}{\cancel{R}}\left(1 - e^{-\frac{tR}{L}}\right)\cdot\cancel{R}$$

$$V_R = \mathcal{E}\left(1 - e^{-\frac{tR}{L}}\right)$$

$$\mathcal{E} - \mathcal{E}e^{-\frac{tR}{L}}$$

And can check it adds to $\mathcal{E}$

$$\mathcal{E} - \mathcal{E}e^{-\frac{tR}{L}} + \mathcal{E}e^{-\frac{tR}{L}}$$

$$\mathcal{E}$$

Prelab 2. Discharging an inductor

After a long time T at equallibrium
$\mathcal{E}$ turned off
will discharge

$$I = A\left(x - e^{-\frac{t-T}{\tau}}\right)$$

a) Fariady's Law

Well kirkoff still same

$$\cancel{\mathcal{E} - IR - L\frac{dI}{dt} = 0}$$

$$\cancel{I = I_0\left(\ln L - e^{\frac{-t-T}{\tau}}\right)}$$

Now what?

After
Redwte OH
on own
↑
OH really
helped!

Ok - so same as before -

$$\cancel{\frac{dI}{dt} = -\frac{1}{L/R}\left(I - \frac{\mathcal{E}}{R}\right)}$$

No can't be same - inductor is pushing current other way - and there is no $\mathcal{E}$

$$L\frac{dI}{dt} - IR = 0$$

$$\frac{dI}{dt} = \frac{IR}{L}$$

Now does that fit some sort of pattern

$$\frac{dA}{dt} = -\frac{1}{\tau} A \rightarrow A = A_0 e^{-t/\tau}$$

$$\frac{dI}{dt} = -\frac{1}{L/R} I \rightarrow I = A_0 e^{-t/\frac{L}{R}}$$

$$I = I_0 e^{-\frac{tR}{L}}$$

? so what is I_0

so we know from last time $\frac{\varepsilon}{R}$

$$I = \frac{\varepsilon}{R} e^{-\frac{tR}{L}}$$

Not so hard now!

b) Confirm solution
- yeah same thing
- but what was I supposed to do

after redwire OH

So how do you confirm?
What do you confirm?

that it is $e^{-\frac{(t-T)}{a}}$

well that is just special time expression

Well I developed the equation with the kirkoff - so how do I confirm

$$e^{-\frac{(t-T)R}{L}}$$

(?
still confused if I have everything

c) Determine V_R and $\mathcal{E}$ by inductor

$$V_R = I_0 R$$

$$V_L = I_0 \omega L$$

$$\mathcal{E} = V_R + V_L$$

Or do they just want us to describe it

As soon as battery removed, inductor will put out same current as before for imediate time The current is dissipated by the resistor

$$I_0 \omega L = I_0 R$$
$$\omega L = R$$

Well they want time dependence eq which is a differential

Just don't get these qu

---

after Redwle OH

Resistor $V = IR$

$$V_R = \frac{\mathcal{E}}{R} e^{-\frac{tR}{L}} \cdot R$$

$$= \mathcal{E} e^{-\frac{tR}{4}}$$

Inductor

$\frac{dI}{dt}$ and actually differentiate

$$I = \frac{\mathcal{E}}{R} e^{-\frac{tR}{L}}$$

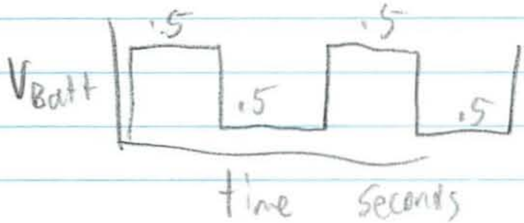
$$\frac{dI}{dt} = \frac{\mathcal{E}}{\cancel{R}} \cdot \frac{\cancel{R}}{L} e^{-\frac{tR}{L}}$$

$$V_L = \cancel{L} \cdot \frac{\mathcal{E}}{\cancel{L}} e^{-\frac{tR}{L}}$$
$$= \mathcal{E} e^{-\frac{tR}{L}}$$

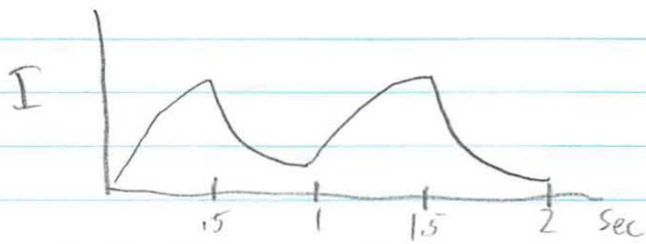
What is this about V_L being $\mathcal{E}$

Prelab 3. A real inductor
- is in series w/ an resistor

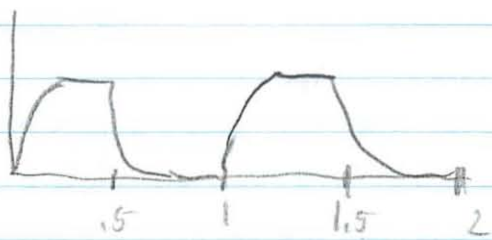
a) Will hook up circuit from Prelab 1 and battery will turn off and on



Sketch current through battery



? so is this a "driven" circuit ?



or if faster

b) How you can tell that it is not an indeal inductor

- Some current will always be lost
- with more I lost when more I is flowing
- So what does mean for graph
  - that its not a perfect log graph
- won't be as tall as otherwise

Also $\mathcal{E} - IR - L\frac{dI}{dt} = 0$

could solve for $I(t)$
like in past problem

and find R
-which is a constant

Does not say we need to actually do

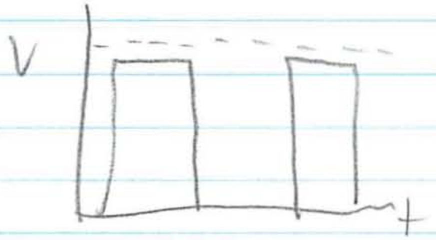
c) Do not use an extra resistor - why?
Well because it adds even more resistance
futer disturbing the graph.

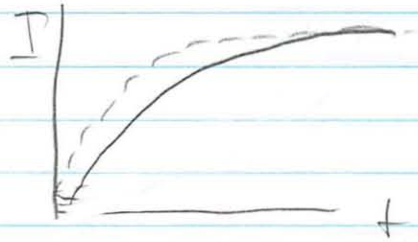
~~So Volt meter would not change~~
Voltmeter across current would not go as high

$$\mathcal{E} = \underbrace{L\frac{dI}{dt}}_{\text{here}} + \underbrace{IR}_{\text{would not include}}$$

but difference would depend on I

Current through battery is same as
through whole circuit, and would be less





d) For this case how would you determine resistance of coil itself only?

You could measure voltage drop over capicator and then remove it from equation

$$\mathcal{E} - L\frac{dI}{dt} - IR + IR = 0$$

$$\mathcal{E} - L\frac{dI}{dt} = 0$$

$$\mathcal{E} = L\frac{dI}{dt}$$

here from measuring voltage across battery you could solve a differential equation for L

Pre Lab 4 The coil has 600 turns of wire radius .25 mm
diameter is 25m
lenght is 25m (height?)

What is wire resistance of coil?

So what are the pertient measurements

Circumference $= 2\pi r$

$2\pi \cdot 12.5 \cdot 600$ = lenght

~~cross sectional area~~ volume $= \cancel{2\pi \cdot 12.5 \cdot 600 \cdot \pi .25^2}$ mm

$= \cancel{9243\ mm^3}$

$\cancel{9.2\ cm^3}$ don't figure

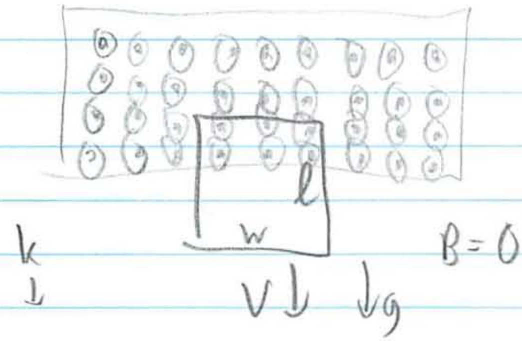
$$R = \frac{\rho L}{A}$$

$$\frac{20 \cdot 10^{-6} \cdot 2\pi \cdot 12.5 \cdot 600}{\pi .25^2} = \frac{.942}{.19625} = 4.8\Omega$$

Is that reasonable?
May be math error - used cell calculator

5. Falling Loop

Back to the real problems



So it is moving out of the B field means there is a flux

Is this like #1? Remember doing something like this in class one day

Ampere's law - amt in loop is changing at a constant acceleration = g

Flux also changing as area decreasing and amt area is ↓ is ↑ (acc)

So what is it asking?

a) Which dir is current flowing?
So first $v \times B$
will be CCW ✓
So current will flow clockwise
explain more
(Lenz Law) ^

b) Using Faraday's Law find an expression for emf

$$\mathcal{E} = -\frac{d\phi}{dt}$$

$$-\frac{d(B \cdot A)}{dt}$$

$$-B\frac{d\,lw}{dt}$$

$$-Bw\frac{dl}{dt}$$

$-Bwv$ but v is the $\int acc$

$$-Bw\int g\,dt$$

$$I = \frac{Bwv}{R}$$

c) Besides gravity what other forces act in the $\pm k$ direction

Well does induced current?
And does the current have an effect?
Did we go over this before

Yeah like in the expeiment it will slow it leaving the B field

But is there any sample qu w/ that

$$\mathcal{E} = \oint E \cdot ds$$

$$\boxed{\oint E \cdot ds = -\frac{d\phi}{dt}}$$

But does this act in k direction?
- just produces current

Sahar

$\boxed{\text{* Major forget}}$

$$F = I(\ell \times \vec{B})$$

It ossilates back and forth but it gets less + less
but don't need to worry about

$$F_B = \frac{Bwv}{R}\left(w \rightarrow \times \vec{B} \uparrow\right)$$

$$F = \frac{B^2 w^2 v}{R} \hat{k}$$

- remember seeing that - but I thought that was power
- don't forget directions
- write it out w/ vector signs

Againts the motion aka ↑

d) Assume loop has reached terminal velocity
- what is that

Well what is terminal velocity to start with?
Or do we just want $v = \text{constant}$
$a = 0$
Or need to find terminal velocity
- but that is air/fluid dynamics which we have not done

?

Unless I got an above problem wrong →

e) The next question gives a hint: rate at which gravity does work = rate energy dissipated in loop through Joule heating.

So is this energy which I don't think of

$U = mgh$ potential energy
$U_k = \frac{1}{2}mv^2$

$mgh + \frac{1}{2}mv^2 = \text{constant}$
So where does heat come from →
- well slowing

$mgh + \cancel{\frac{1}{2}mv^2} = J_{\text{heating}}$

- ~~not moving, so no change in~~ PE
- is moving - not accelerating
- so KE constant
- PE changing
$J = mgh$ ← h is dependent on time

Saher after c fix d) $F_B = F_g$ When force of electric ↑ = force gravity ↓

$$mg = \frac{B^2 w^2 v}{R}$$

$$v = \frac{Rmg}{(Bw)^2}$$ ✓ terminal velocity

own e) Now show this is given off by Joule heating

? How do this ?
Find energy dissipated by resistor ?

Would it not just be v ?

why does velocity = $P = I^2R$?

wp $$v = \frac{Rmg}{(Bw)^2} = I^2 R$$ ↑ resistance

$v mg = I^2 R$

−3

own $$v = \frac{Rmg}{(Bw)^2} = \frac{B^2 w^2 v^2}{R^2} R^2$$

$$\frac{Rmg}{B^2 w^2} = B^2 w^2 v^2$$

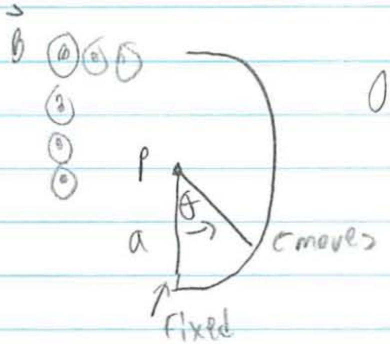
$$v = Rmg = v^2$$

terminal $v = \frac{Rmg}{(Bw)^2}$

So what does that mean

20/25

6. Generator



Oh this is a riostat or something like that - variable resistor

a) If θ ↑ with time, what is direction of current flow around pie shaped current?

So this is what dir is flux

$B \times V = \nearrow \times \odot$ = clockwise

So induced current is CCW

b) Assume θ increasing at constant rate $\frac{d\theta(t)}{dt} = w$

What is magnitude of rate of changing of magnetic flux due to Bext only

$\phi = BA$ ← what is area? what is

$\frac{d\phi}{dt} = \frac{d(B \cdot A)}{dt}$

$A(\theta = 0) = 0$

$A(2\pi) = \pi r^2$

$A(\pi) = \frac{\pi r^2}{2}$

$A(\pi/2) = \frac{\pi r^2}{4}$

$A(\theta) = \frac{\pi r^2}{\left(\frac{2\pi}{\theta}\right)} = \frac{\pi r^2 \theta}{2\pi} = \frac{\theta r^2}{2}$

$\theta = B \cdot \frac{\theta r^2}{2}$

$dQ = B \cdot \frac{d\left(\frac{\theta r^2}{2}\right)}{dt}$

θ depends on t

$d\theta = B w \frac{d}{dt}\left(\frac{r^2}{2}\right)$ $\quad \frac{d\theta}{dt} = w$

$B wr$

c) If pie shaped circuit has constant resistance R what is the magnitude + direction of magnetic force due to external field

- So R does not changed where it is?
  I thought that was the whole point?
- an how would that work if it was all metal
- But that is not point of qu

?, reduce; they could have to make problem less complex

$$\cancel{\mathcal{E}} - L\frac{dI}{dt} - IR = 0$$

$$-L\frac{dI}{dt} = IR$$

$$I = \frac{-L\frac{dI}{dt}}{R}$$

? do I need to $\int$

Or is it ampere's law

$$\oint B \cdot ds = \mu_0 I_{enc}$$

$$B \cdot (2a + \mu) = \mu_0 I_{enc}$$

what is this

section circumference $(\theta = 0) = 0$

$\theta = 2\pi = 2\pi r$

$(\pi) = \pi r$

$\left(\frac{\pi}{2}\right) = \frac{\pi}{2} r$

$(\theta) = \theta r$

$= w$

$\mathcal{E} = V = -B\mu r$

$\mathcal{E} = IR$

$-B\mu r = IR$

$\cancel{R = \frac{-B\mu r}{I_{enc}}}$ R is given

$I = \frac{-B\mu r}{R}$

GP

$B \cdot (2a + \mu) = \mu_0 \frac{-B\mu r}{R}$

$B = \frac{\mu_0 - B\mu r}{R(2a + \mu)}$

- Why a B in I
  - makes sense since it is $\mathcal{E}$
  - in these type of problems

Was an awful P-set
- very confusing
- reasonable
- 4-5 hrs all today

Need to go to OH!

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2010**

Problem Set 9 Solutions

Problem 1: Short Questions

(a) When a small magnet is moved toward a solenoid, an emf is induced in the coil. However, if the magnet is moved around inside a toroid, no measurable emf is induced. Explain.

Moving a magnet inside the hole of the doughnut-shaped toroid will not change the magnetic flux through any turn of wire in the toroid, and thus not induce any current.

(b) A piece of aluminum is dropped vertically downward between the poles of an electromagnet. Does the magnetic field affect the velocity of the aluminum? Explain.

Yes. The induced eddy currents on the surface of the aluminum will slow the descent of the aluminum. It may fall very slowly.

(c) What happens to the generated current when the rotational speed of a generator coil is increased?

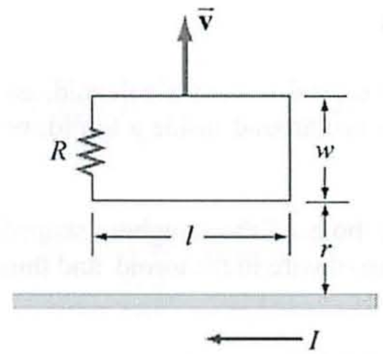
The maximum induced emf will increase, increasing the terminal voltage of the generator resulting in a larger amplitude for the current.

(d) If you pull a loop through a non-uniform magnetic field that is perpendicular to the plane of the loop which way does the induced force on the loop act?

The direction of the induced force is opposite the direction of the pulling force.

Problem 2: Moving Loop

A rectangular loop of dimensions l and w moves with a constant velocity $\vec{\mathbf{v}}$ away from an infinitely long straight wire carrying a current I in the plane of the loop, as shown in the figure. The total resistance of the loop is R.



(a) Using Ampere's law, find the magnetic field at a distance s away from the straight current-carrying wire.

Consider a circle of radius s centered on the current-carrying wire. Then around this Amperian loop, $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi s) = \mu_0 I$

which gives

$$B = \frac{\mu_0 I}{2\pi s} \text{ (into the page)}$$

(b) What is the magnetic flux through the rectangular loop at the instant when the lower side with length l is at a distance r away from the straight current-carrying wire, as shown in the figure?

$$\Phi_B = \iint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \int_r^{r+w} \left(\frac{\mu_0 I}{2\pi s} \right) l\, ds = \frac{\mu_0 I l}{2\pi} \ln\left(\frac{r+w}{r} \right) \text{ (into the page)}$$

(c) At the instant the lower side is a distance r from the wire, find the induced emf and the corresponding induced current in the rectangular loop. Which direction does the induced current flow?

The induce emf is

$$\varepsilon = -\frac{d}{dt}\Phi_B = -\frac{\mu_0 I l}{2\pi} \frac{r}{(r+w)} \left(\frac{-w}{r^2} \right) \frac{dr}{dt} = \frac{\mu_0 I l}{2\pi} \frac{vw}{r(r+w)}$$

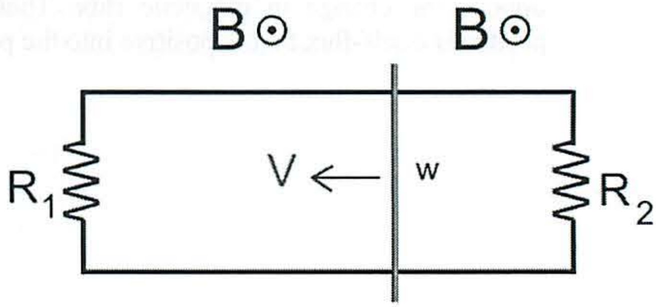
The induced current is

$$I = \frac{|\varepsilon|}{R} = \frac{\mu_0 I l}{2\pi R} \frac{vw}{r(r+w)}$$

The flux into the page is decreasing as the loop moves away because the field is growing weaker. By Lenz's law, the induced current produces magnetic fields which tend to oppose the change in magnetic flux. Therefore, the current flows clockwise, which produces a self-flux that is positive into the page.

Problem 3: Faraday's Law

A conducting rod with zero resistance and length w slides without friction on two parallel perfectly conducting wires. Resistors R_1 and R_2 are connected across the ends of the wires to form a circuit, as shown. A constant magnetic field **B** is directed out of the page. In computing magnetic flux through any surface, take the surface normal to be out of the page, parallel to **B**.



(a) The magnetic flux in the right loop of the circuit shown is (circle one)
   1) decreasing
   2) increasing.

What is the magnitude of the rate of change of the magnetic flux through the right loop?

$$\frac{d\Phi(t)}{dt} = \frac{d}{dt}BA = B\frac{d}{dt}A = BwV$$

(b) What is the current flowing through the resistor R_2 in the right hand loop of the circuit shown? Gives its magnitude and indicate its direction on the figure.

The flux out of the page is increasing so the current is clockwise to make a flux into the page. The magnitude we can get from Faraday:

$$I = \frac{|\varepsilon|}{R_2} = \frac{1}{R_2}\frac{d\Phi(t)}{dt} = \frac{BwV}{R_2}$$

(c) The magnetic flux in the left loop of the circuit shown is (circle one)
   1) decreasing
   2) increasing.

What is the magnitude of the rate of change of the magnetic flux through the right loop?

$$\frac{d\Phi(t)}{dt} = \frac{d}{dt}BA = B\frac{d}{dt}A = -BwV$$

"Magnitude" is ambiguous – either a positive or negative number will do here. I use the negative sign to indicate that the flux is decreasing.

(d) What is the current flowing through the resistor R_1 in the left hand loop of the circuit shown? Gives its magnitude and indicate its direction on the figure.

The flux out of the page is decreasing so the current is counterclockwise to make a flux out of the page to make up for the loss. The magnitude we can get from Faraday:

$$I = \frac{|\varepsilon|}{R_1} = \frac{1}{R_1}\frac{d\Phi(t)}{dt} = \frac{BwV}{R_1}$$

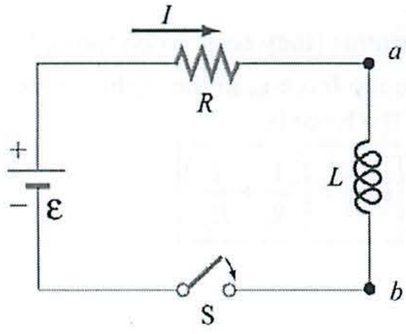
(e) What is the magnitude **and direction** of the magnetic force exerted on this rod?

The total current through the rod is the sum of the two currents (they both go up through the rod). Using the right hand rule on $\vec{\mathbf{F}} = I\vec{\mathbf{L}} \times \vec{\mathbf{B}}$ we see the force is to the **right**. You could also get this directly from Lenz. The magnitude of the force is:

$$F = \left|I\vec{\mathbf{L}} \times \vec{\mathbf{B}}\right| = ILB = \left(BwV\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right)wB = \boxed{B^2w^2V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

Problem 4: Read Experiment 8: Inductance and RL Circuits Pre-Lab Questions

1. RL Circuits



Consider the circuit at left, consisting of a battery (emf ε), an inductor L, resistor R and switch S.

For times $t<0$ the switch is open and there is no current in the circuit. At $t=0$ the switch is closed.

(a) Using Kirchhoff's loop rules (really Faraday's law now), write an equation relating the emf on the battery, the current in the circuit and the time derivative of the current in the circuit.

Walking in the direction of current, starting at the switch

$$\varepsilon - IR - L\frac{dI}{dt} = 0$$

We know from thinking about it above that the results should look very similar to RC circuits. In other words:

$$I = A(X - \exp(-t/\tau))$$

(b) Plug this expression into the differential equation you obtained in (a) in order to confirm that it indeed is a solution and to determine what the time constant τ and the constants A and X are. What would be a better label for A? (HINT: You will also need to use the initial condition for current. What is $I(t=0)$?)

$$0 = \varepsilon - A\left(X - e^{-t/\tau}\right)R - L\frac{Ae^{-t/\tau}}{\tau} = \left(\varepsilon - ARX\right) + \left(AR - L\frac{A}{\tau}\right)e^{-t/\tau}$$

Both the constant and time dependent part must equal zero, giving us two equations. The third (because there are three unknowns) we can get from initial conditions:

$$I(t=0) = A(X-1) = 0 \qquad \Rightarrow X = 1$$

$$\varepsilon - ARX = 0 \qquad \Rightarrow A = \frac{\varepsilon}{RX} = \frac{\varepsilon}{R}$$

$$\left(AR - L\frac{A}{\tau}\right)e^{-t/\tau} = 0 \qquad \Rightarrow \tau = \frac{L}{R}$$

A better label for A would be I_f, the final current.

(c) Now that you know the time dependence for the current I in the circuit you can also determine the voltage drop V_R across resistor and the EMF generated by the inductor. Do so, and confirm that your expressions match the plots in Fig. 2a or 2b.

We find:

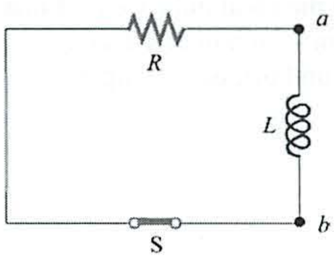
$$I(t)=A\left(X-e^{-t/\tau}\right)=\frac{\varepsilon}{R}\left(1-e^{-t/\tau}\right) \qquad \text{(Fig. 2a)}$$

$$V_R(t)=IR=\varepsilon\left(1-e^{-t/\tau}\right) \qquad \text{(Fig. 2a)}$$

$$\varepsilon_L(t)=-L\frac{dI}{dt}=-L\frac{\varepsilon}{R\tau}e^{-t/\tau}=-\varepsilon e^{-t/\tau} \qquad \text{(Fig. 2b)}$$

Looking at the EMF from the inductor you see that it starts the same as the battery (but in the opposite direction) which explains why no current initially flows. Then as time goes on it relaxes.

2. 'Discharging' an Inductor



After a long time T the current will reach an equilibrium value and inductor will be "fully charged." At this point we turn off the battery (ε=0), allowing the inductor to 'discharge,' as pictured at left. Repeat each of the steps a-c in problem 1, noting that instead of exp($-t/\tau$), our expression for current will now contain exp($-(t-T)/\tau$).

(a) Faraday's law:
Walking in the direction of current, starting at the switch

$$-IR-L\frac{dI}{dt}=0$$

(b) Confirm solution:

$$0=-A\left(X-e^{-(t-T)/\tau}\right)R-L\frac{Ae^{-(t-T)/\tau}}{\tau}=\left(-ARX\right)+\left(AR-L\frac{A}{\tau}\right)e^{-(t-T)/\tau}$$

Both the constant and time dependent part must equal zero, giving us two equations. The third (because there are three unknowns) we can get from initial conditions:

$$-ARX=0 \qquad \Rightarrow X=0$$

$$\left(AR-L\frac{A}{\tau}\right)e^{-t/\tau}=0 \qquad \Rightarrow \tau=\frac{L}{R}$$

$$I(t=T)=A(X-1)=\frac{\varepsilon}{R} \qquad \Rightarrow A=-\frac{\varepsilon}{R}$$

A better label for A would be I_0, the initial current.

(c) Determine V_R across resistor and the EMF generated by the inductor.

Everything is exponentially decaying with time:

$$I(t) = A\left(X - e^{-t/\tau}\right) = \frac{\varepsilon}{R}e^{-t/\tau} \qquad \text{(Fig. 2b)}$$

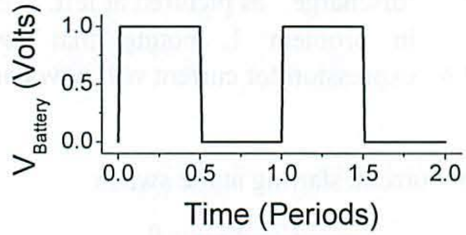
$$V_R(t) = IR = \varepsilon e^{-t/\tau} \qquad \text{(Fig. 2b)}$$

$$\varepsilon_L(t) = -L\frac{dI}{dt} = L\frac{\varepsilon}{R\tau}e^{-t/\tau} = \varepsilon e^{-t/\tau} \qquad \text{(Fig. 2b)}$$

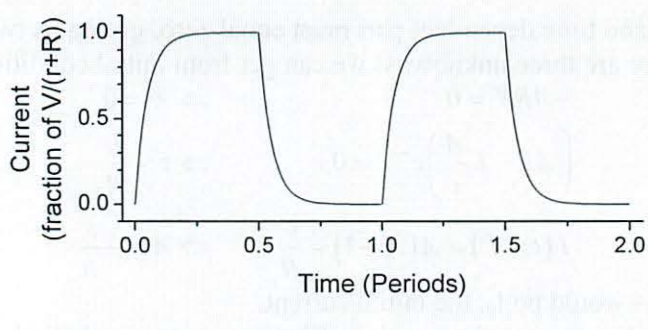
3. A Real Inductor

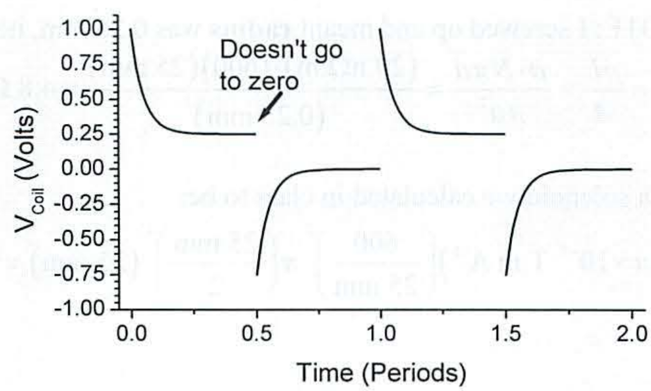
As mentioned above, in this lab you will work with a coil that does not behave as an ideal inductor, but rather as an ideal inductor in series with a resistor. For this reason you have no way to independently measure the voltage drop across the resistor or the EMF induced by the inductor, but instead must measure them together. None-the-less, you want to get information about both. In this problem you will figure out how.

(a) In the lab you will hook up the circuit of problem 1 (with the ideal inductor L of that problem now replaced by a coil that is a non-ideal inductor – an inductor L and resistor r in series). The battery will periodically turn on and off, displaying a voltage as shown here:



Sketch the current through the battery as well as what a voltmeter hooked across the coil would show versus time for the two periods shown above. Assume that the period of the battery turning off and on is comparable to but longer than several time constants of the circuit.





(b) How can you tell from your plot of the voltmeter across the coil that the coil is not an ideal inductor? Indicate the relevant feature clearly on the plot. Can you determine the resistance of the coil, r, from this feature?

The voltage measured across the coil doesn't go to zero because even when the inductor is "off" the coil resistance still has a voltage drop across it. You can determine r from this voltage – $r = V/I$ (in this case I made r ¼ of the total resistance, that is, 1/3 of R).

(c) In the lab you will find it easier to make measurements if you do NOT use an additional resistor R, but instead simply hook the battery directly to the coil. (Why? Because the time constant is difficult to measure with extra resistance in the circuit). Plot the current through the battery and the reading on a voltmeter across the coil for this case. We will only bother to measure the current. Why?

The current is the same as the current above (although the time constant will be longer because of the lower resistance). The voltage measured across the coil will be the same as the voltage measured across the battery because they are the only two things in the circuit, so there is no need to measure it.

(d) For this case (only a battery & coil) how will you determine the resistance of the coil, r? How will you determine its inductance L?

In this case we can determine the resistance from the final current ($r = V/I$) and the inductance from the time constant.

4. The Coil

The coil you will be measuring has is made of thin copper wire (**radius** ~ 0.25 mm) and has about 600 turns of average diameter 25 mm over a length of 25 mm. What approximately should the resistance and inductance of the coil be? The resistivity of copper at room temperature is around 20 nΩ-m. Note that your calculations can only be approximate because this is not at all an ideal solenoid (where length >> diameter).

The resistance (NOTE: I screwed up and meant **radius** was 0.25 mm, not diameter)

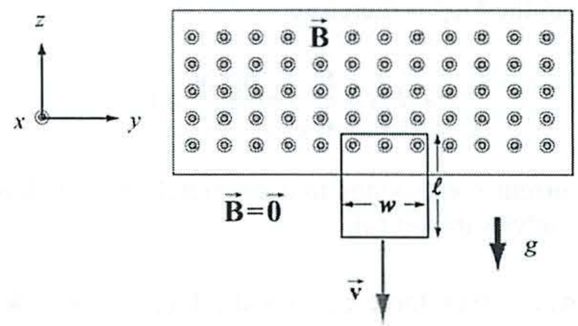
$$R = \frac{\rho L}{A} = \frac{\rho \cdot N\pi d}{\pi a^2} \approx \frac{(20\ \text{n}\Omega\ \text{m})\cdot(600)(25\ \text{mm})}{(0.25\ \text{mm})^2} \approx 4.8\ \Omega$$

The inductance of a solenoid we calculated in class to be:

$$L = \mu_0 n^2 \pi R^2 l \approx \left(4\pi \times 10^{-7}\ \text{T m A}^{-1}\right)\left(\frac{600}{25\ \text{mm}}\right)^2 \pi \left(\frac{25\ \text{mm}}{2}\right)^2 (25\ \text{mm}) \approx 9\ \text{mH}$$

Problem 5 Falling Loop

A rectangular loop of wire with mass m, width w, vertical length l, and resistance R falls out of a magnetic field under the influence of gravity, as shown in the figure below. The magnetic field is uniform and out of the paper ($\vec{\mathbf{B}} = B\hat{\mathbf{i}}$) within the area shown and zero outside of that area. At the time shown in the sketch, the loop is exiting the magnetic field at speed $\vec{\mathbf{v}} = -v\hat{\mathbf{k}}$.



(a) What is the direction of the current flowing in the circuit at the time shown, clockwise or counterclockwise? Why did you pick this direction?

Solution: As the loop falls down, the magnetic flux is pointing out of the page and decreasing. Therefore an induced current flows in the counterclockwise direction. The effect of this induced current is to produce magnetic flux out of page through the surface enclosed by the loop, and thus opposing the change of the external magnetic flux.

(b) Using Faraday's law, find an expression for the magnitude of the emf in this circuit in terms of the quantities given. What is the magnitude of the current flowing in the circuit at the time shown?

Solution: For the loop, we choose out of the page ($+\hat{\mathbf{i}}$ -direction) as the positive direction for the unit normal to the area of the loop. This means that a current flowing in the counterclockwise direction (looking at the page) has positive sign.

Choose the plane $z = 0$ at the bottom of the area where the magnetic field is non-zero. Then at time t, the top of the loop is located at $z(t)$. The area of the loop at time t is then

$$A(t) = z(t)w .$$

where w is the width of the loop. The magnetic flux through the loop is then given by

$$\Phi_{magnetic} = \iint \vec{\mathbf{B}} \cdot \hat{\mathbf{n}}\, da = \iint B_x\, \hat{\mathbf{i}} \cdot \hat{\mathbf{i}}\, da = \iint B_x\, da = B_x A(t) = B_x z(t) w .$$