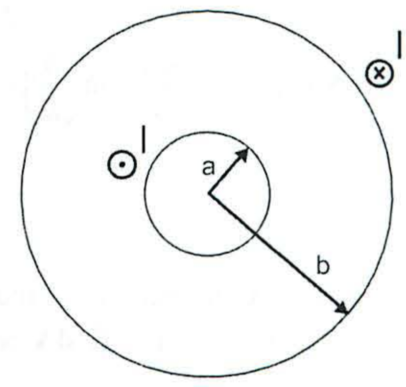
Problem 5: Inductor

An inductor consists of two very thin conducting cylindrical shells, one of radius *a* and one of radius *b*, both of length *h*. Assume that the inner shell carries current *I* out of the page, and that the outer shell carries current *I* into the page, distributed uniformly around the circumference in both cases. The *z*-axis is out of the page along the common axis of the cylinders and the *r*-axis is the radial cylindrical axis perpendicular to the *z*-axis.



a) Use Ampere's Law to find the magnetic field between the cylindrical shells. Indicate the direction of the magnetic field on the sketch. What is the magnetic energy density as a function of *r* for $a < r < b$?

The enclosed current I_{enc} in the Ampere's loop with radius *r* is given by

$$I_{\text{enc}} = \begin{cases} 0, & r < a \\ I, & a < r < b \\ 0, & r > b \end{cases}$$

Applying Ampere's law, $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_0 I_{\text{enc}}$, we obtain

$$\vec{\mathbf{B}} = \begin{cases} 0, & r < a \\ \dfrac{\mu_0 I}{2\pi r}\hat{\boldsymbol{\varphi}}, & a < r < b \text{ (counterclockwise in the figure)} \\ 0, & r > b \end{cases}$$

The magnetic energy density for $a < r < b$ is

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r}\right)^2 = \boxed{\frac{\mu_0 I^2}{8\pi^2 r^2}}$$

It is zero elsewhere.

b). Calculate the inductance of this long inductor recalling that $U_B = \frac{1}{2}LI^2$ and using your results for the magnetic energy density in (a).

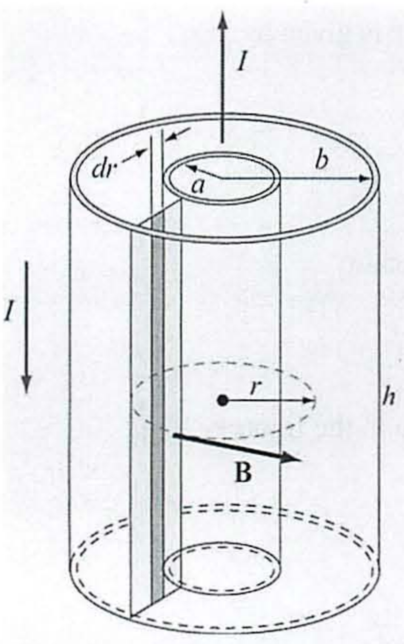
The volume element in this case is $2\pi r h dr$. The magnetic energy is :

$$U_B = \int_V u_B d\text{Vol} = \int_a^b \left(\frac{\mu_0 I^2}{8\pi^2 r^2}\right) 2\pi h\, r dr = \frac{\mu_0 I^2 h}{4\pi} \ln\left(\frac{b}{a}\right)$$

Since $U_B = \dfrac{\mu_0 I^2 l}{4\pi} \ln\left(\dfrac{b}{a}\right) = \dfrac{1}{2} L I^2$, the inductance is

$$\boxed{L = \frac{\mu_0 h}{2\pi} \ln\left(\frac{b}{a}\right)}$$

c) Calculate the inductance of this long inductor by using the formula $\Phi = LI = \int_{open\ surface} \vec{\mathbf{B}} \cdot \mathbf{d}\vec{\mathbf{A}}$ and your results for the magnetic field in (a). To do this you must choose an appropriate open surface over which to evaluate the magnetic flux. Does your result calculated in this way agree with your result in (b)?



The magnetic field is perpendicular to a rectangular surface shown in the figure. The magnetic flux through a thin strip of area $dA = l dr$ is

$$d\Phi_B = B dA = \left(\frac{\mu_0 I}{2\pi r}\right)(h\, dr) = \frac{\mu_0 I h}{2\pi r} dr$$

Thus, the total magnetic flux is

$$\Phi_B = \int d\Phi_B = \int_a^b \frac{\mu_0 I h}{2\pi r} dr = \frac{\mu_0 I h}{2\pi} \int_a^b \frac{dr}{r} = \frac{\mu_0 I h}{2\pi} \ln\left(\frac{b}{a}\right)$$

Thus, the inductance is

$$L = \frac{\Phi_B}{I} = \boxed{\frac{\mu_0 h}{2\pi} \ln\left(\frac{b}{a}\right)}$$

which agrees with that obtained in (b).

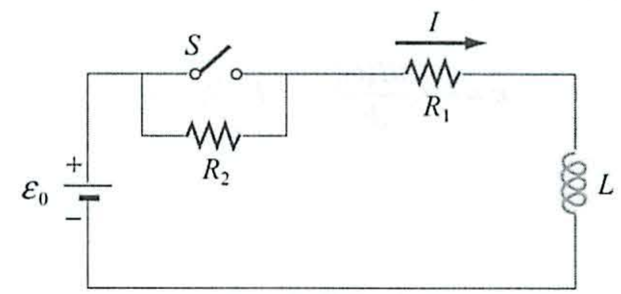
don't think I got that

Review notes

- go to review session
- do math hw real fast
→ - do pratice problems & correct instantly
- review PRS for today
- dir of Lenz's law

Problem 6: Trying to open the switch on an *RL* Circuit

The *LR* circuit shown in the figure contains a resistor R_1 and an inductance L in series with a battery of emf ε_0. The switch *S* is initially closed. At $t = 0$, the switch *S* is opened, so that an additional very large resistance R_2 (with $R_2 \gg R_1$) is now in series with the other elements.



(a) If the switch has been *closed* for a long time before $t = 0$, what is the steady current I_0 in the circuit?

There is no induced emf before $t = 0$. Also, no current is flowing on R_2. Therefore,

$$I_0 = \frac{\varepsilon_0}{R_1}$$

(b) While this current I_0 is flowing, at time $t = 0$, the switch *S* is opened. Write the differential equation for $I(t)$ that describes the behavior of the circuit at times $t \geq 0$. Solve this equation (by integration) for $I(t)$ under the approximation that $\varepsilon_0 = 0$. (Assume that the battery emf is negligible compared to the total emf around the circuit for times just after the switch is opened.) Express your answer in terms of the initial current I_0, and R_1, R_2, and L.

The differential equation is

$$\varepsilon_0 - I(t)(R_1 + R_2) = L\frac{dI(t)}{dt}$$

Under the approximation that $\varepsilon_0 = 0$, the equation is

$$-I(t)(R_1 + R_2) = L\frac{dI(t)}{dt}$$

The solution with the initial condition $I(0) = I_0$ is given by

$$I(t) = I_0 \exp(-\frac{(R_1 + R_2)}{L} t)$$

(c) Using your results from (b), find the value of the total emf around the circuit (which from Faraday's law is $-LdI/dt$) just after the switch is opened. Is your assumption in (b) that ε_0 could be ignored for times just after the switch is opened OK?

$$\varepsilon = -L \left. \frac{dI(t)}{dt} \right|_{t=0} = I_0 (R_1 + R_2)$$

Since $I_0 = \frac{\varepsilon_0}{R_1}$,

$$\varepsilon = \frac{\varepsilon_0}{R_1}(R_1 + R_2) = \left(1 + \frac{R_2}{R_1}\right)\varepsilon_0 >> \varepsilon_0 \quad (\because R_2 >> R_1)$$

Thus, the assumption that ε_0 could be ignored for times just after the switch is open is OK.

(d) What is the magnitude of the potential drop across the resistor R_2 at times $t > 0$, just after the switch is opened? Express your answers in terms of ε_0, R_1, and R_2. How does the potential drop across R_2 just after $t = 0$ compare to the battery emf ε_0, if $R_2 = 100R_1$?

The potential drop across R_2 is given by

$$\Delta V_2 = \frac{R_2}{R_1 + R_2}\varepsilon = \left(\frac{R_2}{R_1 + R_2}\right)\left(1 + \frac{R_2}{R_1}\right)\varepsilon_0 = \frac{R_2}{R_1}\varepsilon_0$$

If $R_2 = 100R_1$,

$$\Delta V_2 = 100\,\varepsilon_0$$

This is why you have to open a switch in a circuit with a lot of energy stored in the magnetic field very carefully, or you end up very dead!!

Problem 7: *LC* Circuit

An inductor having inductance L and a capacitor having capacitance C are connected in series. The current in the circuit increase linearly in time as described by $I = Kt$. The capacitor initially has no charge. Determine

(a) the voltage across the inductor as a function of time,

The voltage across the inductor is

$$\varepsilon_L = -L\frac{dI}{dt} = -L\frac{d}{dt}(Kt) = -LK$$

(b) the voltage across the capacitor as a function of time, and

Using $I = \dfrac{dQ}{dt}$, the charge on the capacitor as a function of time may be obtained as

$$Q(t) = \int_0^t I dt' = \int_0^t Kt' dt' = \frac{1}{2}Kt^2$$

Thus, the voltage drop across the capacitor as a function of time is

$$\Delta V_C = -\frac{Q}{C} = -\frac{Kt^2}{2C}$$

(c) the time when the energy stored in the capacitor first exceeds that in the inductor.

The energies stored in the capacitor and the inductor are

$$U_C = \frac{1}{2}C(\Delta V_C)^2 = \frac{1}{2}C\left(-\frac{Kt^2}{2C}\right)^2 = \frac{K^2t^4}{8C}$$

$$U_L = \frac{1}{2}LI^2 = \frac{1}{2}L(Kt)^2 = \frac{1}{2}LK^2t^2$$

The two energies are equal when

$$\frac{K^2t'^4}{8C} = \frac{1}{2}LK^2t'^2 \quad \Rightarrow t' = 2\sqrt{LC}$$

Therefore, $U_C > U_L$ when $t > t'$.

Problem 8: *LC* Circuit

(a) Initially, the capacitor in a series *LC* circuit is charged. A switch is closed, allowing the capacitor to discharge, and after time *T* the energy stored in the capacitor is one-fourth its initial value. Determine *L* if *C* and *T* are known.

The energy stored in the capacitor is given by

$$U_C(t) = \frac{Q(t)^2}{2C} = \frac{(Q_0 \cos\omega_0 t)^2}{2C} = \frac{Q_0^2}{2C}\cos^2\omega_0 t$$

Thus,

$$\frac{U_C(T)}{U_C(0)} = \frac{\cos^2\omega_0 T}{\cos^2(0)} = \frac{\cos^2\omega_0 T}{1} = \frac{1}{4} \quad \Rightarrow \cos\omega_0 T = \frac{1}{2}$$

which implies that $\omega_0 T = \frac{\pi}{3}\text{rad} = 60°$. Therefore, with $\omega_0 = \frac{1}{\sqrt{LC}}$, we obtain

$$T = \frac{\pi}{3\omega_0} = \frac{\pi}{3}\sqrt{LC} \quad \Rightarrow L = \frac{1}{C}\left(\frac{3T}{\pi}\right)^2$$

(b) A capacitor in a series *LC* circuit has an initial charge Q_0 and is being discharged. The inductor is a solenoid with N turns. Find, in terms of *L* and *C*, the flux through each of the *N* turns in the coil at time *t*, when the charge on the capacitor is *Q*(*t*).

We can do this two ways, either is acceptable. First,we can make the explicit assumption that $Q(t) = Q_0 \cos\omega_0 t$ and the total flux through the inductor is $LI = L\frac{dQ}{dt} = -L\omega_0 Q_0 \sin\omega_0 t$

Therefore the flux through one turn of the inductor at time *t* is $\Phi_{\text{one turn}} = -\frac{L\omega_0 Q_0}{N}\sin\omega_0 t$

or in terms of *L* and *C*, $\Phi_{\text{one turn}} = -\sqrt{\frac{L}{C}}\frac{Q_0}{N}\sin\omega_0 t$. Or second, we can simply leave $Q(t)$ as an unspecified function of time and write (using the same arguments as above) that $\Phi_{\text{one turn}} = \frac{L}{N}\frac{dQ}{dt}$.

(c) An *LC* circuit consists of a 20.0-mH inductor and a 0.500-μF capacitor. If the maximum instantaneous current is 0.100 A, what is the greatest potential difference across the capacitor?

The greatest potential difference across the capacitor when $U_{C\max} = U_{L\max}$, or

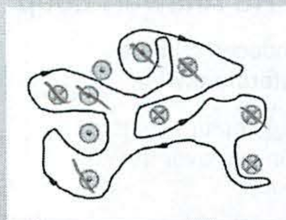
$$\frac{1}{2}CV_{C\max}^2 = \frac{1}{2}LI_{\max}^2 \quad \Rightarrow \quad V_{C\max} = \sqrt{\frac{L}{C}}I_{\max} = \sqrt{\frac{(20.0\,\text{mH})}{(0.500\,\mu\text{F})}}\,(0.100\text{A}) = 20\text{ V}$$

PRS Review

4/27 class

Redo for practice 4/28

PRS: Ampere's Law



nets out

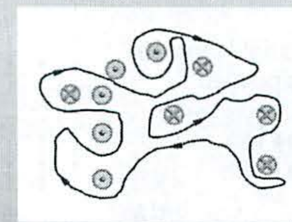
Integrating B around the loop shown gives us:

- 0% 1. a positive number
- 0% 2. a negative number
- 0% (3.) zero

:15

Plain screwdriver method

PRS Answer: Ampere's Law

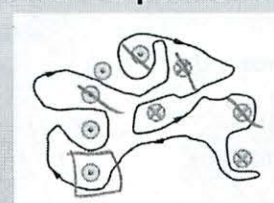


Answer: 3. Total penetrating current is zero, so

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc} = 0$$

✓

PRS: Ampere's Law



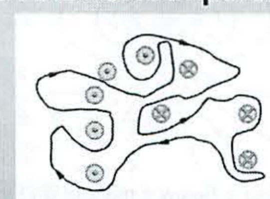
Integrating B around the loop shown gives us:

- 0% 1. a positive number
- 0% (2.) a negative number
- 0% 3. zero

15

thumb up ↪ but arrow other way

PRS Answer: Ampere's Law



Answer: 2. $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} < 0$

Net penetrating current is out of the page, so field is counter-clockwise (opposite path)

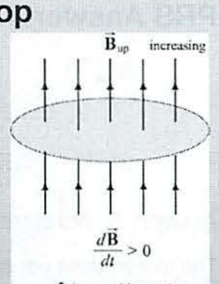
✓

much better since review session

PRS: Loop

0

The magnetic field through a wire loop is pointed upwards and *increasing* with time. The induced current in the coil is

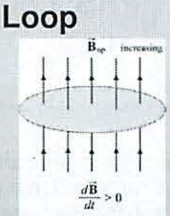


- 0% (1) Clockwise as seen from the top
- 0% 2. Counterclockwise

matters what dir it is going

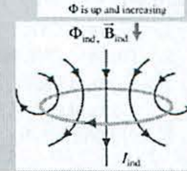
PRS Answer: Loop

Answer: 1. Induced current is **clockwise**



This produces an "induced" B field pointing down over the area of the loop.

The "induced" B field opposes the increasing flux through the loop – Lenz's Law



✓

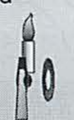
↑ don't get this pic

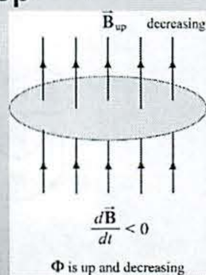
So B↑ will cause ↻

So flux will be ↺

PRS: Loop

The magnetic field through a wire loop is pointed upwards and *decreasing* with time. The induced current in the coil is





0% 1. Clockwise as seen from the top

0% 2. Counterclockwise

P31- 7

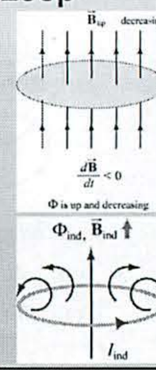
decreasing is like ↓ ↻
want other way ↺

PRS Answer: Loop

Answer: 2. Induced current is **counterclockwise**

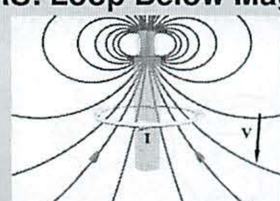
This produces an "induced" B field pointing up over the area of the loop.

The "induced" B field opposes the decreasing flux through the loop – making up for the loss – Lenz's Law



P31- 8

PRS: Loop Below Magnet



A conducting loop is below a magnet and moving downwards. This induces a current as pictured. The *I ds x B* force on the coil is

0% 1. Up

0% 2. Down

0% 3. Zero

I(L×B)

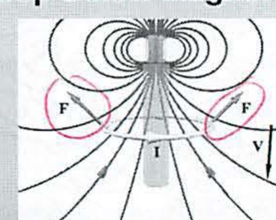
P31- 9

~~B field ↑~~ ~~but~~ decreasing
will always move up

PRS Answer: Loop Below Magnet

Answer: 1. Force is Up

Lenz' Law:
Must oppose motion – force is up

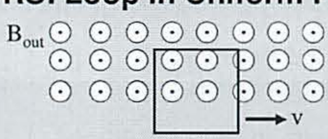


More detail:
Induced current is counter-clockwise to oppose drop in upward flux.
This looks like a dipole facing upward, so it is attracted to the other dipole

P31-10

I don't know all the explanation
just that falling experiment

PRS: Loop in Uniform Field



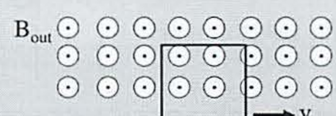
A rectangular wire loop is pulled thru a uniform B field penetrating its top half, as shown. The induced current and the force and torque on the loop are:

% 1. Current CW, Force Left, No Torque
% 2. Current CW, No Force, Torque Rotates CCW
% 3. Current CCW, Force Left, No Torque
% 4. Current CCW, No Force, Torque Rotates CCW
% 5. No current, force or torque

P31-11

even, no change

PRS Answer: Loop in Uniform Field



Answer: 5. No current, force or torque

The motion does not change the magnetic flux, so Faraday's Law says there is no induced EMF, or current, or force, or torque.

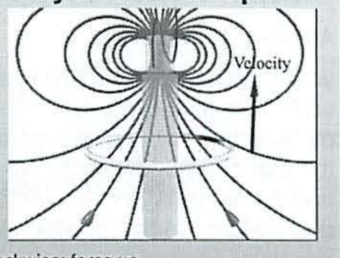
Of course, if we were pulling at all up or down there would be a force to oppose that motion.

P31-12

PRS: Faraday's Law: Loop

100

A coil moves up from underneath a magnet with its north pole pointing upward. The current in the coil and the force on the coil:



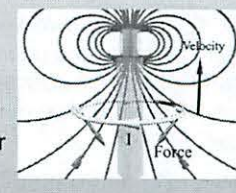
0% 1. Current clockwise; force up
0% 2. Current counterclockwise; force up
0% 3. Current clockwise; force down
0% 4. Current counterclockwise; force down

P31-13

PRS Answer: Faraday's Law: Loop

Answer: 3. Current is clockwise; force is down

The clockwise current creates a self-field downward, trying to offset the increase of magnetic flux through the coil as it moves upward into stronger fields (Lenz's Law).



The I dl x B force on the coil is a force which is trying to keep the flux through the coil from increasing by slowing it down (Lenz's Law again).

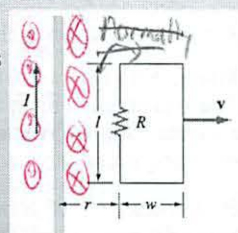
P31-14

B field ↑ increasing ↺
opposet ↻ force opposes ↓

PRS: Circuit

0

A circuit in the form of a rectangular piece of wire is pulled away from a long wire carrying current I in the direction shown in the sketch. The induced current in the rectangular circuit is



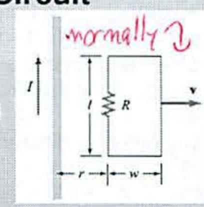
0% 1. Clockwise
0% 2. Counterclockwise
0% 3. Neither, the current is zero

P31-15

PRS Answer: Circuit

Answer: 1. Induced current is **clockwise**

B due to I is into page; the flux through the circuit due to that field decreases as the circuit moves away. So the induced current is clockwise (to make a B into the page) obvious



Note: I_{ind} dl x B force is left on the left segment and right on the right, but the force on the left is bigger. So the net force on the rectangular circuit is to the left, again trying to keep the flux from decreasing by slowing the circuit's motion

P31-16

B-S law ✓ ✗

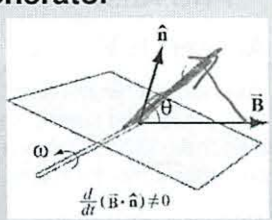
yes is normally current through

if not moving 0 current
Well what is B from wire? ⊗ Screwdriver method - duh
~~⊥ to current~~ ~~guessing CCW~~ like 1/r away from wire

$F = I(l \times B)$ - decreasing - so still ↻

PRS: Generator

A square coil rotates in a magnetic field directed to the right. At the time shown, the current in the square, when looking down from the top of the square loop, will be



0% 1. Clockwise
0% 2. Counterclockwise
0% 3. Neither, the current is zero
0% 4. I don't know

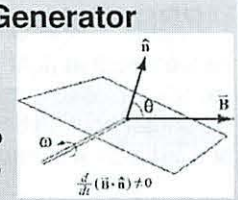
100

P31-17

PRS Answer: Generator

Answer: 1. Induced current is **counterclockwise**

Flux through loop *decreases* as normal rotates away from B. To try to keep flux from decreasing, induced current will be CCW, trying to keep the magnetic flux from decreasing (Lenz's Law)



Note: I_{ind} dl x B force on the sides of the square loop will be such as to produce a torque that tries to stop it from rotating (Lenz's Law).

P31-18

✗

Normally ↺ from B does not ask induced
will be induced current
But is it ↑ or ↓ [must always ask]

PRS: Stopping a Motor

Consider a motor (a loop of wire rotating in a B field) which is driven at a constant rate by a battery through a resistor.
Now grab the motor and prevent it from rotating. What happens to the current in the circuit?

0%	1. Increases
0%	2. Decreases
0%	3. Remains the Same
0%	4. I don't know

Burns out

PRS Answer: Stopping a Motor

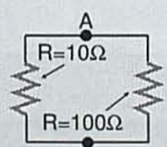
Answer: 1. **Increases**

When the motor is rotating in a magnetic field an EMF is generated which opposes the motion, that is, it reduces the current. When the motor is stopped that back EMF disappears and the full voltage of the battery is now dropped across the resistor – the current increases. For some motors this increase is very significant, and a stalled motor can lead to huge currents that burn out the windings (e.g. your blender).

✓

PRS: Faraday Circuit

A magnetic field B penetrates this circuit outwards, and is increasing at a rate such that a current of 1 A is induced in the circuit (which direction?).
The potential difference VA-VB is:



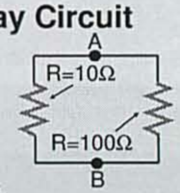
0%	1. +10 V
0%	2. -10 V
0%	3. +100 V
0%	4. -100 V
0%	5. +110 V
0%	6. -110 V
0%	7. +90 V
0%	8. -90 V
0%	9. None of the above

Remembering a lot from class

Obi hated this qu skip

PRS Answer: Faraday Circuit

Answer: 9. None of the above



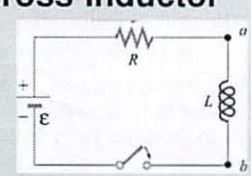
The question is meaningless. There is no such thing as potential difference when a changing magnetic flux is present.

By Faraday's law, a non-conservative E is induced (that is, its integral around a closed loop is non-zero). Non-conservative fields can't have potentials associated with them.

O Omit

PRS: Voltage Across Inductor

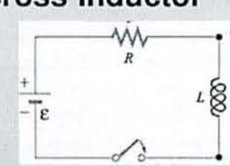
In the circuit at right the switch is closed at $t = 0$. A voltmeter hooked across the inductor will read:



0%	1. $V_L = \varepsilon e^{-t/\tau}$ increasing
0%	2. $V_L = \varepsilon(1-e^{-t/\tau})$ decreasing
0%	3. $V_L = 0$
0%	4. I don't know

PRS Answer: V Across Inductor

Answer: 1. $V_L = \varepsilon e^{-t/\tau}$



The inductor "works hard" at first, preventing current flow, then "relaxes" as the current becomes constant in time.

Although "voltage differences" between two points isn't completely meaningful now, we certainly can hook a voltmeter across an inductor and measure the EMF it generates.

X

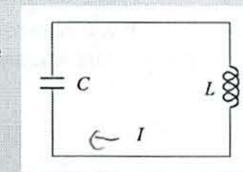
Wrong direction

$\mathcal{E} - IR - L\frac{dI}{dt} = 0$ current will slowly ↑

$L\frac{dI}{dt} = V_L = \mathcal{E} - IR$

PRS: LC Circuit

Consider the LC circuit at right. At the time shown the current has its maximum value. At this time

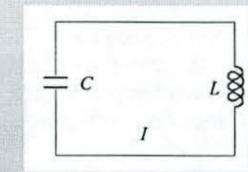


0% 1. The charge on the capacitor has its maximum value *min value*
0% 2. The magnetic field is zero
0% 3. The electric field has its maximum value
0% 4. The charge on the capacitor is zero ✓
0% 5. Don't have a clue

:00

PRS Answer: LC Circuit

Answer: 4. The current is maximum when the charge on the capacitor is zero



Current and charge are exactly 90 degrees out of phase in an ideal LC circuit (no resistance), so when the current is maximum the charge must be identically zero.

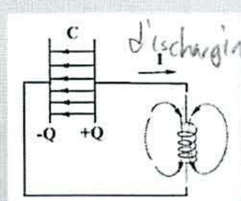
P31-26

✓

kinda remember
3 remember is if capacitor is full — max mag field max value

PRS: LC Circuit

In the LC circuit at right the current is in the direction shown and the charges on the capacitor have the signs shown. At this time,



discharging

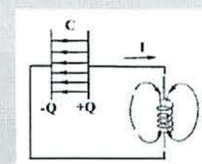
0% 1. I is increasing and Q is increasing
0% 2. I is increasing and Q is decreasing
0% 3. I is decreasing and Q is increasing
0% 4. I is decreasing and Q is decreasing
0% 5. Don't have a clue

P31-27

PRS Answer: LC Circuit

Answer: 2. I is increasing; Q is decreasing

With current in the direction shown, the capacitor is discharging (Q is decreasing).



But since Q on the right plate is positive, I must be increasing. The positive charge *wants* to flow, and the current will increase until the charge on the capacitor changes sign. That is, we are in the first quarter period of the discharge of the capacitor, when Q is decreasing and positive and I is increasing and positive.

P31-28

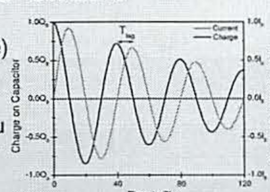
✓

I ↑ Q ↓

They tell you anyway →

PRS: LC Circuit

The plot shows the charge on a capacitor (black curve) and the current through it (red curve) after you turn off the power supply. If you put a core into the inductor what will happen to the time T_{Lag}?

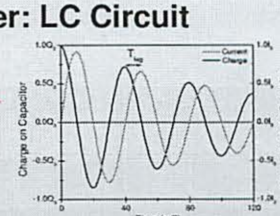


0% 1. It will increase
0% 2. It will decrease
0% 3. It will stay the same
0% 4. I don't know

PRS Answer: LC Circuit

Answer:

1. T_{Lag} will increase



Putting in a core increases the inductor's inductance and hence decreases the natural frequency of the circuit. Lower frequency means longer period. The phase will remain at 90º (a quarter period) so T_{Lag} will increase.

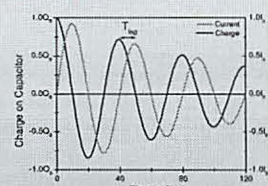
P31-30

✗

current lagging charge
Q ← dominates → I, L ↓ ← 0
core ↑ L so what happens to I
well T_{Lag} → stays same
↑L ↓ω
$\omega = \frac{1}{\sqrt{LC}}$
so ↓ω = ↑T — remember this

PRS: LC Circuit

If you increase the resistance in the circuit what will happen to rate of decay of the pictured amplitudes?



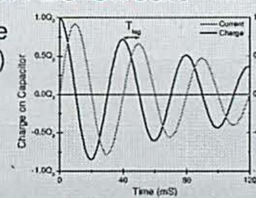
0%	1. It will increase (decay more rapidly)
0%	2. It will decrease (decay less rapidly)
0%	3. It will stay the same
0%	4. I don't know

:00

Pratice test now

PRS Answer: LC Circuit

Answer: 1. It will increase (decay more rapidly)



Resistance is what dissipates power in the circuit and causes the amplitude of oscillations to decrease. Increasing the resistance makes the energy (and hence amplitude) decay more rapidly.

TEST THREE Thursday Evening April 29 from 7:30-9:30 pm.

The Friday class immediately following is canceled because of the evening exam. Please see announcements for room assignments for Exam 3.

What We Expect From You On The Exam

1. An understanding of how to calculate magnetic fields in highly symmetric situations using Ampere's Law, e.g. as in the Ampere's Law Problem Solving Session.

2. An understanding of how to use Faraday's Law in problems involving the generation of induced EMF's. You should be able to formulate quantitative answers to questions about energy considerations in Faraday's Law problems, e.g. the power going into ohmic dissipation comes from the decreasing kinetic energy of a rolling rod, etc.

3. The ability to calculate the inductance of specific circuit elements, for example that of a long solenoid with N turns, radius a, and length L.

4. An understanding of simple circuits. For example, you should be able to set up the equations for multi-loop circuits, using Kirchhoff's Laws that include inductors. You should be able to understand and graph the solution to the differential equations for a circuit involving a battery, resistor, and inductor, and a circuit just involving a resistor and inductor. You should be able to compare and contrast RL and RC circuits, and should understand the meaning of time constants ($\tau = L/R$, $\tau = RC$)s

5. An understanding of the concept of energies stored in magnetic fields, that is $U = \frac{1}{2}LI^2$ for the total magnetic energy stored in an inductor, and $u_B = \frac{1}{2\mu_0}B^2$ for the energy density in magnetic fields. You also should review the concept of energies stored in electric fields, that is $U = \frac{1}{2}CV^2 = \frac{1}{2C}Q^2$ for the total electric energy stored in a capacitor, and $u_E = \frac{1}{2}\varepsilon_0 E^2$ for the energy density in electric fields.

6. An understanding of the nature of the *free* oscillations of an LC circuit.

To study for this exam we suggest that you review your problem sets, in-class problems, Friday problem solving sessions, PRS in-class questions, and relevant parts of the study guide and class notes.

Note: This exam will not include questions regarding undriven and driven RLC circuits but will include questions about free oscillations of LC circuits.

4/28

Class 31: Outline

Hour 1:
- Concept Review / Overview
- PRS Questions – possible exam questions

Hour 2:
- Sample Exam

Yell if you have any questions

P31- 1

Exam 3 Topics

- Ampere's Law
- Faraday's Law of Induction
- Self Inductance
  - Energy Stored in Inductor/Magnetic Field
- Circuits
  - LR & RC Circuits
  - Undriven (R)LC Circuits
  - Driven RLC Circuits
- Energy Flow and Poynting Vector: Resistors, Inductors, Capacitors

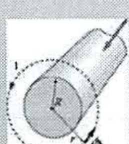
NO: Transformers, Mutual Inductance, EM Waves

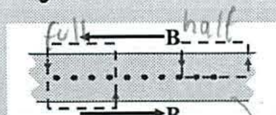
P31- 2

Driven RLC
→ Displacement current

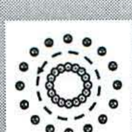
Ampere's Law: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$

Long Circular Symmetry



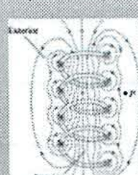


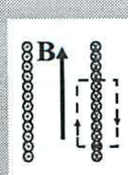
(Infinite) Current Sheet



Torus/Coax

Solenoid = 2 Current Sheets





P31- 3

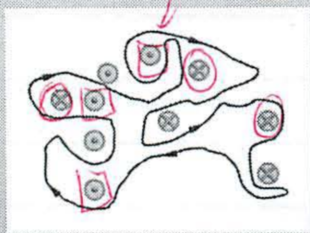
make a current loop
field along loop = current inclosed

if 2 currents opposite dir then 0 use half

~~if don't know outside~~ → use full eq
could ~~you~~ use either one in any case

did not see

PRS: Ampere's Law



Integrating B around the loop shown gives us:

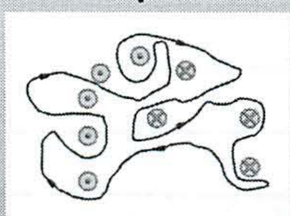
0% 1. a positive number
0% 2. a negative number
0% 3. zero

:15

It does balance out

if it was not here would be ⊕
right

PRS: Ampere's Law



Integrating B around the loop shown gives us:

0% 1. a positive number
0% 2. a negative number
0% 3. zero

15

B·dr says ↺ ⊗
but more out of pg so ⊙

or if current other dir ⊕

Faraday's Law of Induction

$$\mathcal{E} = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

Moving bar, entering field

$$= -\frac{d}{dt}\left(BA\cos\theta\right)$$

Ramp B   Rotate area in field

Lenz's Law:

Induced EMF is in direction that ***opposes*** the change in flux that caused it

P31- 46

The minus sign
Opposes change in flux

PRS Questions: Faraday's & Lenz's Law

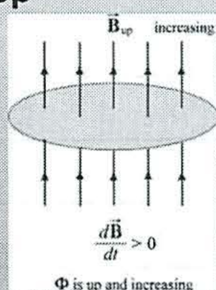
Classes 21 & 23

Can change size, angle, position

0

PRS: Loop

The magnetic field through a wire loop is pointed upwards and *increasing* with time. The induced current in the coil is



0% 1. Clockwise as seen from the top

0% 2. Counterclockwise

$-\frac{d}{dt}(BA\cos\theta)$

$-$ $+$

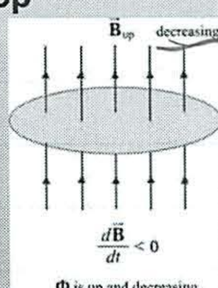
⊖ CCW

use right hand rule
current would go ↺
but current opposes
wants to go other
way ↻

forgot that last part

PRS: Loop

The magnetic field through a wire loop is pointed upwards and *decreasing* with time. The induced current in the coil is



0% 1. Clockwise as seen from the top

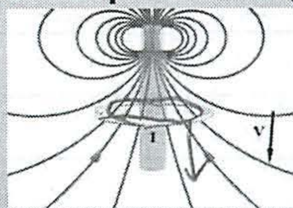
0% 2. Counterclockwise

So other way
current would go ↻
but wants opposite way ↺

But from start

current wants to increase, point up
hand goes CCW
(This is simpler 2nd explanation)

PRS: Loop Below Magnet



A conducting loop is below a magnet and moving downwards. This induces a current as pictured. The $I\,ds \times B$ force on the coil is

0% 1. Up
0% 2. Down
0% 3. Zero

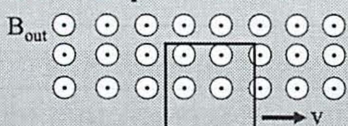
current CCW

* loop falling, not rod

it is moving down
wants to slow it
current moves up
$I\,ds \times B$ ← force
∴ still up
–makes it resist falling
B field not ⊥ so components ↑ can do cross product
Lenz's law

PRS: Loop in Uniform Field

0



A rectangular wire loop is pulled thru a uniform B field penetrating its top half, as shown. The induced current and the force and torque on the loop are:

1. Current CW, Force Left, No Torque
2. Current CW, No Force, Torque Rotates CCW
3. Current CCW, Force Left, No Torque
4. Current CCW, No Force, Torque Rotates CCW
5. No current, force or torque

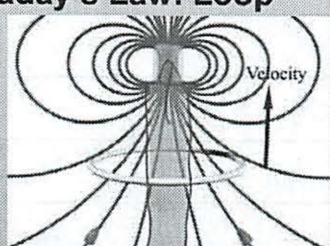
don't know these as well as I should

~~current will want ↺ but will go opposite ↻ $I(L \times B)$ ← no torque~~

no change in flux
no current, no force
uniform field

PRS: Faraday's Law: Loop

A coil moves up from underneath a magnet with its north pole pointing upward. The current in the coil and the force on the coil:



0% 1. Current clockwise; force up
0% 2. Current counterclockwise; force up
0% 3. Current clockwise; force down
0% 4. Current counterclockwise; force down

will want to oppose

Maxwell's Equations

$$\oint\!\!\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$$ (Gauss's Law)

$$\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$ (Faraday's Law)

$$\oint\!\!\oint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$ (Magnetic Gauss's Law)

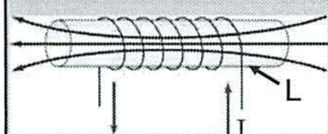
$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$ (Ampere-Maxwell Law)

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$$ (Lorentz force Law)

P31-13

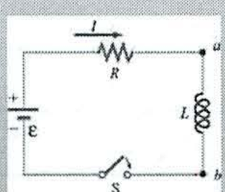
All E+M from these problems

Self Inductance & Inductors



$$L = \frac{\Phi_{Self}}{I} = \frac{N\Phi_{Sgl}}{I}$$

(handwritten: "total # of loops" pointing to N; "each loop" under $\frac{\Phi_{Self}}{I}$)



When traveling in direction of current:

$$\varepsilon = -L\frac{dI}{dt}$$

Notice: This is called "Back EMF"
It is just Faraday's Law!

P31-14

Energy Stored in Inductor

$$U_L = \frac{1}{2} L I^2$$

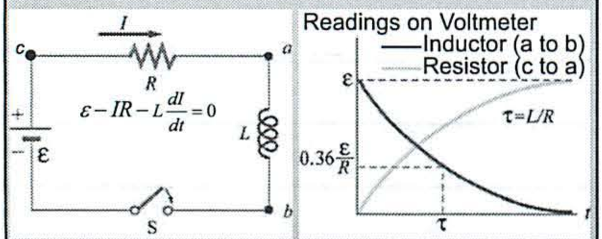
Energy is stored in the magnetic field:

$$u_B = \frac{B^2}{2\mu_o}$$: Magnetic Energy Density

P31-15

Memorize these energy eq

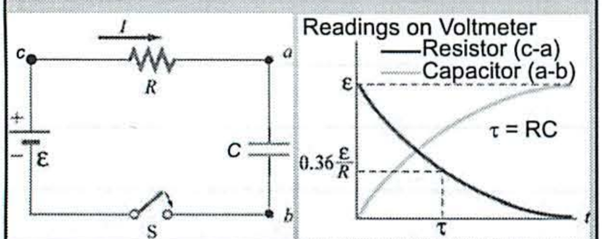
LR Circuit



t=0⁺: Current is trying to change. Inductor works as hard as it needs to in order to stop it

t=∞: Current is steady. Inductor does nothing.

RC Circuit



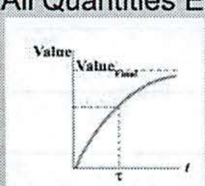
t=0⁺: Capacitor is uncharged so resistor sees full battery potential and current is largest

t=∞: Capacitor is "full." No current flows

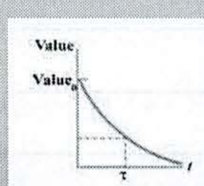
capicators don't allow instantenous voltage changes

General Comment: LR/RC

All Quantities Either:



$\text{Value}(t) = \text{Value}_{\text{Final}}\left(1 - e^{-t/\tau}\right)$



$\text{Value}(t) = \text{Value}_0 e^{-t/\tau}$

τ can be obtained from differential equation (prefactor on d/dt) e.g. τ = L/R or τ = RC

PRS Questions: Inductors & Circuits

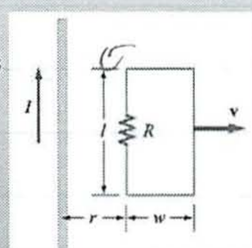
Classes 22, 23 & 25

P31-19

PRS: Circuit

0

A circuit in the form of a rectangular piece of wire is pulled away from a long wire carrying current *I* in the direction shown in the sketch. The induced current in the rectangular circuit is



0%	1. Clockwise
0%	2. Counterclockwise
0%	3. Neither, the current is zero

P31-20

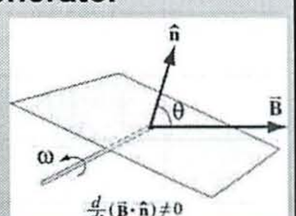
wants to counter move away ↺

Wants an attractive force
flux decreasing - want current to maintain same flux

Why am I getting these wrong!
need major review
Only 15% getting it wrong

PRS: Generator

A square coil rotates in a magnetic field directed to the right. At the time shown, the current in the square, when looking down from the top of the square loop, will be



0%	1. Clockwise
0%	2. Counterclockwise
0%	3. Neither, the current is zero
0%	4. I don't know

21

wants to ↑ B
by making ↺ current

PRS: Stopping a Motor

Consider a motor (a loop of wire rotating in a B field) which is driven at a constant rate by a battery through a resistor.
Now grab the motor and prevent it from rotating. What happens to the current in the circuit?

0%	1. Increases
0%	2. Decreases
0%	3. Remains the Same
0%	4. I don't know

✓

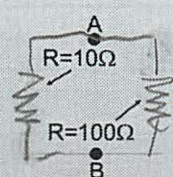
wants to keep moving
"burn out"

Rotating before caused Δ in B field - no more changing flux
battery just run current

PRS: Faraday Circuit

0

A magnetic field B penetrates this circuit outwards, and is increasing at a rate such that a current of 1 A is induced in the circuit (which direction?).
The potential difference VA-VB is:



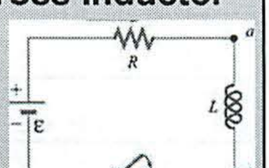
0%	1. +10 V
0%	2. -10 V
0%	3. +100 V
0%	4. -100 V
0%	5. +110 V
0%	6. -110 V
0%	7. +90 V
0%	8. -90 V
0%	9. None of the above

very split answers

Very bad qv
have E, curl is non 0
trick qv
no such thing as magnetic flux when no potential difference

PRS: Voltage Across Inductor

In the circuit at right the switch is closed at $t = 0$. A voltmeter hooked across the inductor will read:



0%	1. $V_L = \varepsilon e^{-t/\tau}$
0%	2. $V_L = \varepsilon(1-e^{-t/\tau})$
0%	3. $V_L = 0$
0%	4. I don't know

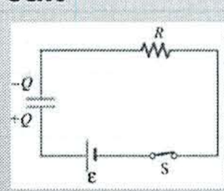
$V = L\frac{dI}{dt}$

when first turn on inductor does not let any current through
0 current
So potential of resistor is 0

inductors can change initial value instantaneously

PRS: RC Circuit

An uncharged capacitor is connected to a battery, resistor and switch. The switch is initially open but at t = 0 it is closed. A very long time after the switch is closed, the current in the circuit is

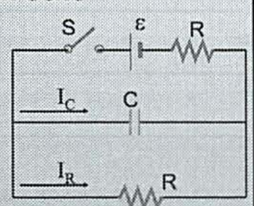


- 0% 1. Nearly zero
- 0% 2. At a maximum and decreasing
- 0% 3. Nearly constant but non-zero
- 0% 4. I don't know

Like a mass on a spring

PRS: RC Circuit

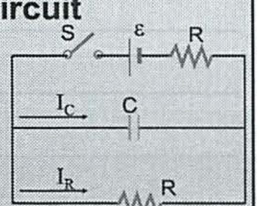
Consider the circuit at right, with an initially uncharged capacitor and two identical resistors. At the instant the switch is closed:



- 0% 1. $I_R = I_C = 0$
- 0% 2. $I_R = \varepsilon/2R$; $I_C = 0$
- 0% 3. $I_R = 0$; $I_C = \varepsilon/R$
- 0% 4. $I_R = \varepsilon/2R$; $I_C = \varepsilon/R$
- 0% 5. I don't know

PRS: RC Circuit

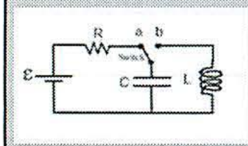
Now, after the switch has been closed for a very long time, it is opened. What happens to the current through the lower resistor?



- 0% 1. It stays the same
- 0% 2. Same magnitude, flips direction
- 0% 3. It is cut in half, same direction
- 0% 4. It is cut in half, flips direction
- 0% 5. It doubles, same direction
- 0% 6. It doubles, flips direction
- 0% 7. None of the above

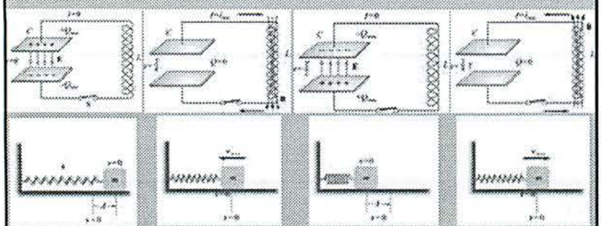
my main thing is work on direction

Undriven LC Circuit



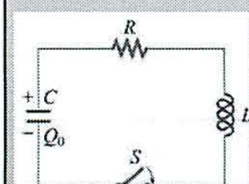
Oscillations: From charge on capacitor (Spring) to current in inductor (Mass)

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

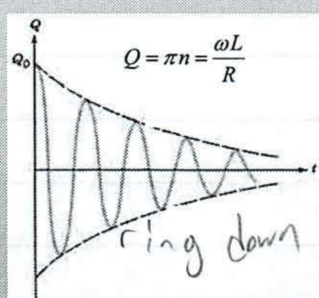


different time

Damped LC Oscillations



Resistor dissipates energy and system rings down over time

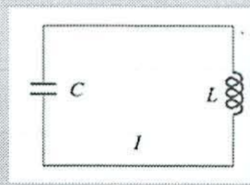


PRS Questions: Undriven RLC Circuits

Classes 27 & 28

PRS: LC Circuit

Consider the LC circuit at right. At the time shown the current has its maximum value. At this time



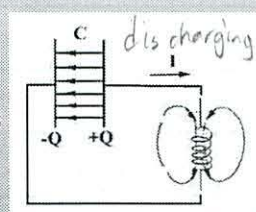
- 0% 1. The charge on the capacitor has its maximum value X
- 0% 2. The magnetic field is zero X
- 0% 3. The electric field has its maximum value ← in capacitor
- 0% 4. The charge on the capacitor is zero √ better ans
- 0% 5. Don't have a clue

:00

PRS: LC Circuit

0

In the LC circuit at right the current is in the direction shown and the charges on the capacitor have the signs shown. At this time,

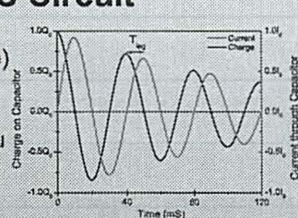


I ↑
Q ↓

- 0% 1. I is increasing and Q is increasing
- 0% 2. I is increasing and Q is decreasing √
- 0% 3. I is decreasing and Q is increasing
- 0% 4. I is decreasing and Q is decreasing
- 0% 5. Don't have a clue

PRS: LC Circuit

The plot shows the charge on a capacitor (black curve) and the current through it (red curve) after you turn off the power supply. If you put a core into the inductor what will happen to the time T_{Lag}?



- 0% 1. It will increase
- 0% 2. It will decrease
- 0% 3. It will stay the same
- 0% 4. I don't know

0

Whats lagging / leading ?

know the rules

current L ↑ C ↓ lagging

Core ↑ L √

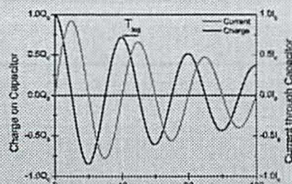
~~So T lag same~~

↑L means ω ↓

time b/w pts ↑ (inverse!)

PRS: LC Circuit

If you increase the resistance in the circuit what will happen to rate of decay of the pictured amplitudes?



0% 1. It will increase (decay more rapidly) ✓ 94%

0% 2. It will decrease (decay less rapidly)

0% 3. It will stay the same

0% 4. I don't know

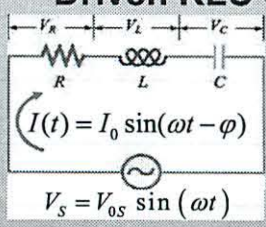
:00

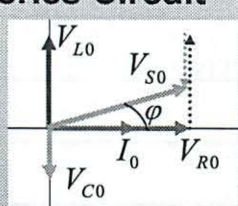
AC Circuits: Summary

Element	V vs I_0	Current vs. Voltage	Resistance-Reactance (Impedance)
Resistor	$V_{0R} = I_0 R$	In Phase	$R = R$
Capacitor	$V_{0C} = \frac{I_0}{\omega C}$	Leads (90°)	$X_C = \frac{1}{\omega C}$
Inductor	$V_{0L} = I_0 \omega L$	Lags (90°)	$X_L = \omega L$

L32 - 35

Driven RLC Series Circuit





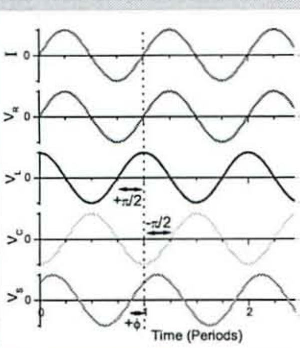
$$V_{S0} = \sqrt{V_{R0}^2 + (V_{L0} - V_{C0})^2} = I_0 \sqrt{R^2 + (X_L - X_C)^2} \equiv I_0 Z$$

$$I_0 = \frac{V_{S0}}{Z} \qquad Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Impedance

P33-36

Plot I, V's vs. Time



$$I(t) = I_0 \sin(\omega t)$$

$$V_R(t) = I_0 R \sin(\omega t)$$

$$V_L(t) = I_0 X_L \sin\left(\omega t + \tfrac{\pi}{2}\right)$$

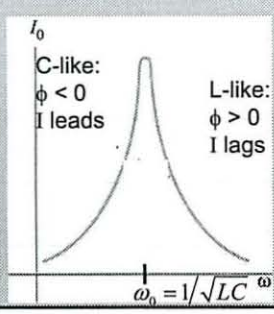
$$V_C(t) = I_0 X_C \sin\left(\omega t - \tfrac{\pi}{2}\right)$$

$$V_S(t) = V_{S0} \sin(\omega t + \varphi)$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Resonance

$$I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + (X_L - X_C)^2}}; \quad X_L = \omega L, \quad X_C = \frac{1}{\omega C}$$



On resonance:
I_0 is max; X_L=X_C; Z=R;
φ=0; Power to R is max

Average Power: Resistor

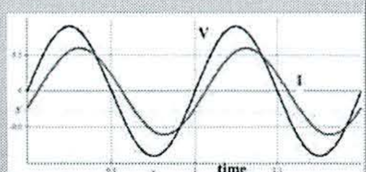
$$\begin{aligned} \langle P \rangle &= \langle I^2(t) R \rangle \\ &= \langle I_0^2 \sin^2(\omega t - \varphi) R \rangle \\ &= I_0^2 R \langle \sin^2(\omega t - \varphi) \rangle \\ &= I_0^{\,2} R\left(\tfrac{1}{2}\right) \end{aligned}$$

PRS Questions: Driven RLC Circuits

Classes 27 & 28

PRS: Leading or Lagging?

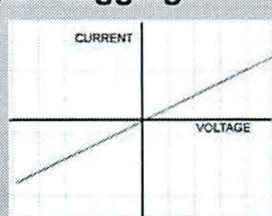
The plot shows the driving voltage V (black curve) and the current I (red curve) in a driven RLC circuit. In this circuit,



- 0% 1. The current leads the voltage
- 0% 2. The current lags the voltage
- 0% 3. Don't have a clue

PRS: Leading or Lagging?

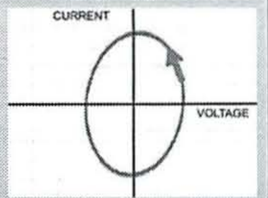
The graph shows current versus voltage in a driven RLC circuit at a given driving frequency. In this plot



- 0% 1. The current leads the voltage by about 45°
- 0% 2. The current lags the voltage by about 45°
- 0% 3. The current and the voltage are in phase
- 0% 4. Don't have a clue.

PRS: Leading or Lagging?

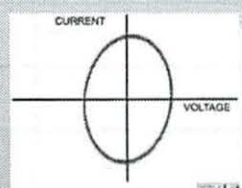
The graph shows current versus voltage in a driven RLC circuit at a given driving frequency. In this plot



- 0% 1. Current lags voltage by ~90°
- 0% 2. Current leads voltage by ~90°
- 0% 3. Current and voltage are almost in phase
- 0% 4. Not enough info (but they aren't in phase!)
- 0% 5. I don't know

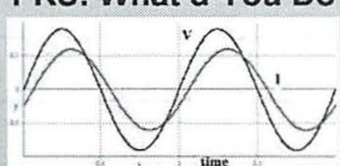
PRS: Leading or Lagging

The graph shows the current versus the voltage in a driven RLC circuit at a given driving frequency. In this plot



- 0% 1. Current lags voltage by ~90°
- 0% 2. Current leads voltage by ~90°
- 0% 3. Current and voltage are almost in phase
- 0% 4. We don't have enough information (but they aren't in phase!)
- 0% 5. I don't know

PRS: What'd You Do?



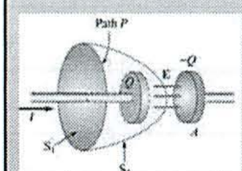
The graph shows current & voltage vs. time in a driven RLC circuit. We had been in resonance a second ago but then either put in or took out the core from the inductor. Which was it?

- 0% 1. Put in the core
- 0% 2. Took out the core
- 0% 3. I don't know

Displacement Current

P31-46

Displacement Current



$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

Direction is same as E field (opposite if negative)

So we have to modify Ampere's Law:

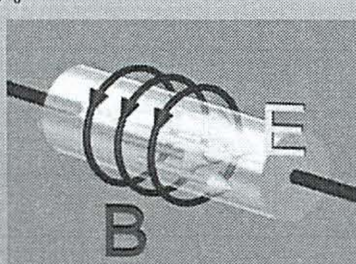
$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 (I_{encl} + I_d)$$

P31-47

Also in Circuit Elements…

$$\vec{\mathbf{S}} = \frac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0}$$

On surface of resistor is INWARD

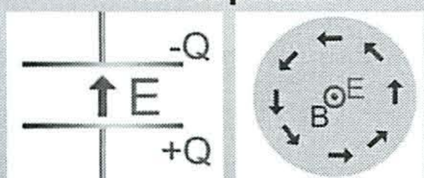


P31-48

PRS Questions: Poynting Vector

Class 33

PRS: Capacitor

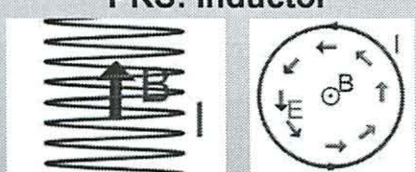


The figures above show a side and top view of a capacitor with charge Q and electric and magnetic fields E and B at time *t*. At this time the charge *Q* is:

0% 1. Increasing in time
0% 2. Constant in time.
0% 3. Decreasing in time.
0% 4. I don't know

0

PRS: Inductor



The figures above show a side and top view of a solenoid carrying current I with electric and magnetic fields E and B at time t. In the solenoid, the current I is:

0% 1. Increasing in time
0% 2. Constant in time.
0% 3. Decreasing in time.
0% 4. I don't know

:00

SAMPLE EXAM:

Review Session Test 3

Dormashkin

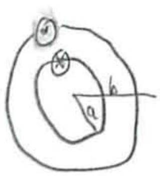
Read equation sheet - putting up differential eq

Straight foward problems - need to know which are which

~ combining ideas
- stratigies

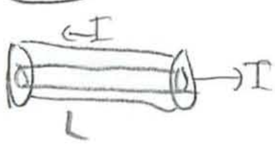
Coverage on past exam different than before

Long coaxial cylinder
- very thin



always have current into + out of board

2D art better than 3D



Find L

self

$$L = \frac{N\phi}{I}$$

$$\phi = BA = B \cdot \pi b^2$$ (which - a loop)

$$\int B\,ds = \mu_0 I_{enc}$$

for each size shape

2 ways to get L

(1.) $L = \frac{\phi_{\text{obbed}}}{I}$ ← surface integral $= \frac{\iint B \cdot da}{I}$ harder

(2) $\frac{1}{2} L I^2 = U = \iiint u_B \, dV$

(2)

$$= \iiint \frac{B^2}{2\mu_0} \, dV$$

$$L = \frac{2U}{I}$$

need B for both methods → Ampere's Law

$\oint B \cdot ds$	$= \mu_0 I_{enc}$
line integral	surface integral
	$\mu_0 \iint J \cdot dA$
B · lenght of loop	harder

know this fairly well - its dir and when is what

(1) Cyclinders
(2) solineids
(3) Torods
(4) Plances

different regions

(1) $r < a$ ← will be 0
(2) $a < r < b$
(3) $r > b$

(know this better P-sett # 1 I got wrong)

$\vec{B}$ fields tangental

Slabs → parallel to surfaces ← did not know

(3)

Make a good drawling

$r < a$

How much current goes through <u>this</u> loop

$\oint \vec{B} \cdot d\vec{s}$	$\mu_0 I$
part that is $\perp$	
$B\,2\pi r$	$= 0$ nothing inside

$\vec{B} = 0$

$a < r < b$

$\int B \cdot ds$	$\mu_0 I$
$B\,2\pi r$	$\mu_0 I$

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\,\hat{\theta} \quad \curvearrowright$$

direction

$I \otimes$

$B \curvearrowright$ "screwdriver method"

right hand rule

$r > b$

$\int B \cdot ds$	$\mu_0 I$
$B\,2\pi r$	$\mu_0\, I - I$

$\vec{B} = 0$

Make sure to work it through
knew it all
- but must be able to do

How harder?
- toroid (review)
- non constant $\vec{B}$ field
- solid
- B field falls off away from it

Thinking about solid vs hollow
- never thought of this
- but big diff
- these are the types of things I would confuse

Non solid $\vec{B}$ field

$J = cr\hat{k}$ $r < a$

Pick an amperian loop inside

$\oint B \cdot ds$	$\mu_0 \iint J \cdot da$
	must $\iint J$ pick da' which we integrate

r' - a ring w/ small thickness

(5)

Add the small rings

$$\mu_0 \int_0^r c r' \underbrace{2\pi r' \, dr'}_{da'}$$

$$\mu_0 \, c \, 2\pi \int_0^r r'^2 \, dr'$$

$$\mu_0 \, c \, \frac{2\pi r^3}{3}$$

Be prepared when it is $\int_a^b = \ln\left(\frac{b}{a}\right)$

$B \cdot 2\pi r$	$\mu_0 \, c \, \frac{2\pi r^3}{3}$

$$B = \frac{\mu_0 c r^2}{3} \theta \quad r < a$$

I is ⊗ so ↻ $\vec{B}$ (Clockwise)

---

(2) Energy method

$$L = \frac{2U}{I^2} \qquad u_B = \frac{B^2}{2\mu_0} \qquad a < r < b$$

$$= \left(\frac{\mu_0 I}{2\pi r}\right)^2 \frac{1}{2\mu_0} = \frac{\mu_0 I^2}{8\pi^2 r^2}$$

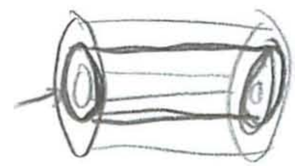
energy density
need to $\int$

6

$$L = \frac{2U}{I^2}$$

$$= \frac{2}{I^2} \int \frac{\mu I^2}{8\pi^2 r^2} \, dV$$

volume integral
complex to $\iiint$
pick a volume element
how much energy in there?



$$dV = 2\pi r \, dr \, l$$

$$= \frac{2}{I^2} \int_a^b \frac{\mu_0 I^2}{8\pi^2 r^2} 2\pi r \, dr \, l$$

$$= \frac{\mu_0 l}{2\pi} \int_a^b \frac{dr}{r}$$

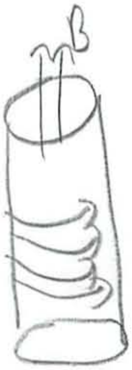
$$= \frac{\mu_0}{2\pi} l \ln\left(\frac{b}{a}\right)$$

---

(1) Normal method
- far harder

$$L = \frac{\Phi_{object}}{I} = \frac{\iint \vec{B} \cdot d\vec{a}}{I}$$

②



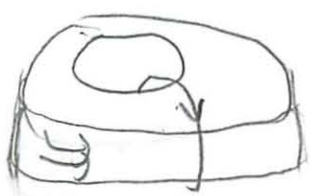
$$\Phi_{\substack{total \\ solidnoid}} = \underset{\substack{\#of \\ turns}}{N} \; \Phi_{loop}$$

$$= N \iint_{loop} \vec{B} \cdot d\vec{a}$$

$\vec{B}$ field through loops

toroid

(review)



bunch of wrappings

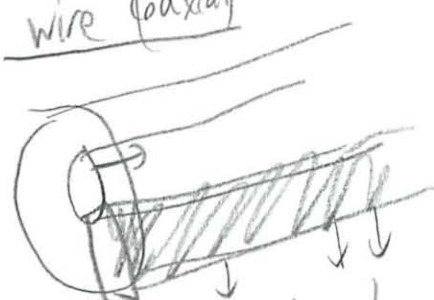
$$\Phi_{total} = N \, \Phi_{loop}$$

$$= N \iint \vec{B} \cdot d\vec{a}$$

but non uniform

must choose an area element

wire (coaxial)



$\vec{B}$ field is tangental

coming down

since perpendicular

Side View

$\vec{B}$ field non uniform

must $\int$ it

$$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\theta}$$

$da = l \, dr$

(8)

$$\frac{\Phi_{coaxial}}{I} = \frac{\int_a^b \frac{\mu_0 I}{2\pi r} l \, dr}{I}$$

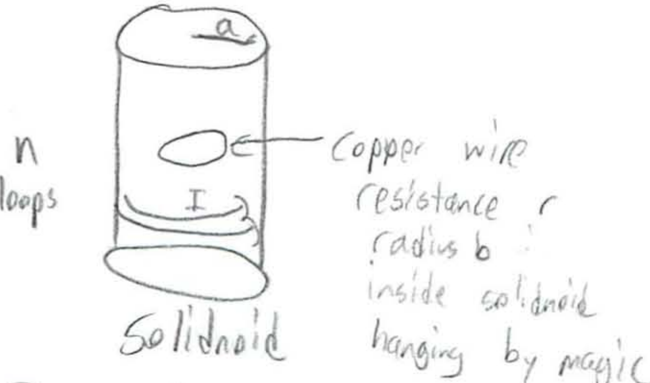
not hard integral

$$L = \frac{\mu_0 \, l \, \ln\left(\frac{b}{a}\right)}{2\pi}$$ ← same answer

Challenge your self to do $\vec{B}$ field of a toroid

2 diff approches to self inductance
- 4 cases
- energy density key concept
  - scalar
  - $\int$ over space

#2 Fariday's Law + Induced current



$I(t) = ct^2$ ccw
↑ time dependent

find $I_{induced}$ (+ dir)
- combo of ideas

[will not consider induced current's affect on $\vec{B}$ field - total current
we are making approx]

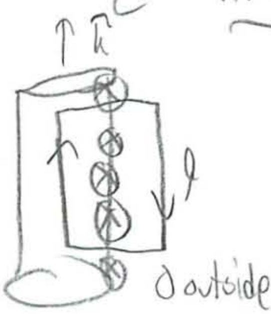
Fariday's Law

$$\mathcal{E} = \frac{-d\Phi}{dt}$$

$$I_{end} R = -\frac{d}{dt}\iint_{\text{copper loop}} \vec{B} \cdot d\vec{a}$$

for solidnoid have n
but we are looking at flux
through ring

need B field of solidnoid



Ampere's Law (assuming solidnoid ∞ long)

$$\oint B \cdot ds = \mu_0 I_{enc}$$

$$Bl = \mu_0 \frac{N}{d} l I$$

only the inside one

↑ count wires

$n = \frac{N}{d}$ = # wires / unit length

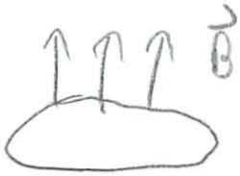
$$\vec{B} = \mu_0 \frac{N}{d} I \hat{k} \text{ up}$$

which way does induced current flow (really study this)
well how is it moving?
will oppose motion

Lenz' Law

(10)

2 step argument



B field is ↑

Φ is ↑ (same dir as B field)

flux not always ↓

must look at increasing/decreasing

(knew this from today)

(getting bored - sometimes memorable)

Φ is increasing

$\Phi_{induced}$ ↓ to oppose change

So current goes ↻ cw

screwdriver rule

(got this now!)

Now for magnitude

- don't worry about ⊖ sign
- we delt w/ it

...

$\vec{B}$ field is uniform inside solidnoid

But I changing so $\vec{B}$ not constant

$$|\mathcal{E}_{ind} R| = \left(\frac{dB}{dt}\right) \pi b^2$$

(11)

$$I_{ind} = \frac{d}{dt}\left(\mu_0 \frac{N}{d} \frac{ct^2}{R} \right) \pi b^2$$

$$= \frac{2\mu_0 NC}{dR} \pi b^2 \; t$$

differenitate

Faraday's Law → changing flux

Use Ampere's Law to find $\vec{B}$ field

- is it uniform and/or constant

take deriv of $\vec{B}$

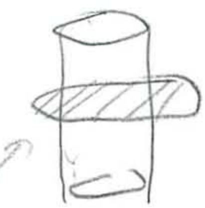
Lenz's Law direction

<u>To make harder</u>

if loop was outside

~~- no induced current~~

~~- no B field~~



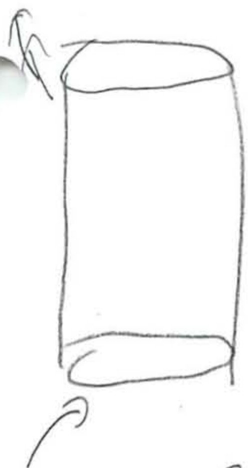
Surface area is area

- <u>is</u> $\vec{B}$ field inside solidnoid
- <u>will</u> be $\vec{B}$ field

Voltage produces $\vec{E}$ field

- drive charge in wire

(12)



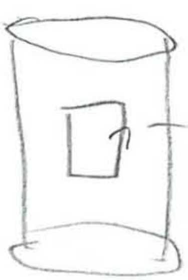
$I(t) = ct^2$

find $\vec{E}$ everywhere

$E \leftrightarrow \frac{\partial B}{\partial t}$

still fariday's law, $\frac{d\phi}{dt}$

$\mathcal{E}$	$= -\frac{d\phi}{dt}$ mag
$\oint \vec{E} \cdot d\vec{s}$	$= -\frac{d}{dt} \iint_{loop} \vec{B} \cdot d\vec{a}$
fariday loop not ampere's loop	
line S	surface S

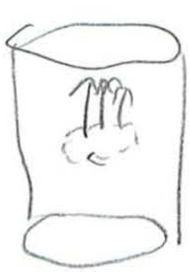


no I

but still $\vec{E}$ field

E field targental

- chose circle
- just like ampere's law



→ is magnetic flux

$E\, 2\pi r = \left(-\frac{d}{dt} B\right) \pi r^2$

$r < a$
inside solidnoid

(13) Is E field outside?

Yes

$r > a$

$$E\, 2\pi r = -\frac{dB}{dt} \pi a^2$$

↑ could solve

Even w/o copper wire, E field still there

- due to Δ B field
- keep current steady and no induced current

$E \propto \frac{dB}{dt}$
proportional

- cal calc w/ line integral

Line integral

- field $\perp$ path

Could also do loop changing area

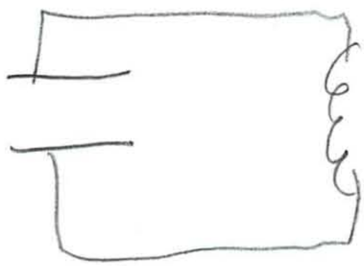
$-B \frac{dA}{dt}$ $\quad a(t) = a_0 t \rightarrow r = \pi a^2 t^2$

Or changing angle

$-BA \frac{d \cos\theta}{dt}$

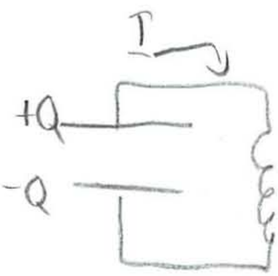
(14) LC circuits

- also do LR
- open + close switches
(ok w/ this, but not differential eq)



- 2 ways to think about
    - energy (easiest)
        - stored in capacitor or inductor
          (E field) (B field)
(need to get better at energy approaches)
(I've never been good at energy)

dis. Charging



$I = \frac{-dQ}{dt}$

charging

$I = \frac{dQ}{dt}$

) I (+) both pictures
- always is

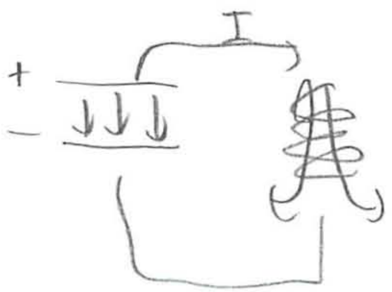
If there is a current, there is a B field

(15)

Go through a full cycle

Some energy (inital) must be added some how

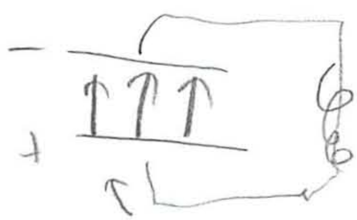
Start w/ all Energy in capicator



Current max when no charge on capicator

starts charging other plane

Capicator reverses sign



Now half way through cycle

Now current goes other way

Back to where started

$U =$ Constant

$\frac{dU}{dt} = 0$

$Q(t)$

$I(t)$

$$\frac{dU}{dt} = \frac{2Q}{2C}\frac{dQ}{dt} + \frac{2}{2} L I \frac{dI}{dt}$$

(... on board - very good - not TEAL)

$I = \frac{dQ}{dt}$

(know all of the cases
- lead / lag
- be able to do)

$= -\frac{Q}{C} I + L I \, dI \, dt = 0$

I's cancle

$\frac{Q}{C} - L \frac{dI}{dt} = 0$

↑ same as if kirkoff loop rule

Now how to get differential eq?

$\frac{dI}{dt} = \frac{-d^2Q}{dt}$ (discharging)

$\frac{Q}{C} - L \frac{d^2Q}{dt} = 0$

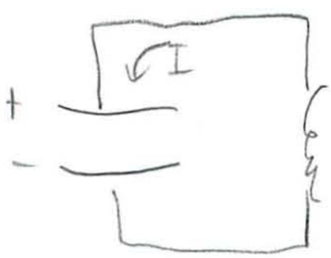
↑ SHM ossilator eq

$$\boxed{\frac{d^2Q}{dt^2} + \frac{1}{LC} Q = 0}$$

→ like Spring $\frac{d^2x}{dt^2} + \frac{k}{m} x = 0$

↑ $\omega = \frac{1}{\sqrt{LC}}$

know differential eq



$I = \frac{dQ}{dt}$

$\frac{du}{dt} = \frac{Q}{C} \frac{dQ}{dt} + L I \frac{dI}{dt} = 0$ ← know energy eq

(17)

$$-\frac{Q}{C} I + L I \frac{dI}{dt} = 0$$

$$\frac{Q}{C} + L \frac{dI}{dt} = 0$$

$$-\frac{Q}{C} - L \frac{dI}{dt} = 0$$

Same eq

$$\frac{Q}{C} + L \frac{d^2Q}{dt} = 0$$

Same eq

- does not matter which way you do it

Be careful to lable stuff

Make sure

1. Understand energy
2. Can go through a whole cycle
3. Talk way through cycle

After Review
W/ Dermashkin

~~ΔL~~

$-L \frac{dI}{dt}$ in dir of current

Loop generator $= 2 \frac{BA}{\text{sor}}$

$BA - -BA$

and Δt is constant

So when S

its $\frac{\Delta t}{\Delta t}$ Δt does not matter

$\frac{1}{2} L I^2$

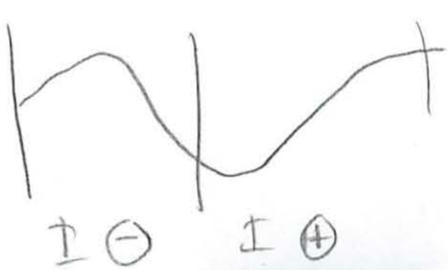
↑ derivitive

$Q(t) = Q_0 \cos(\omega t + \phi)$

$I = \frac{dQ}{dt}$ → deriv of this function

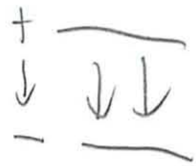
$= \omega_0 Q_0 \sin(\omega t + \phi)$ ← know

just know the directions



depends on part of cycle

Choose correct convention

$\vec{E}$ 

~~$\vec{E}$ field from ⊕ plate~~

~~I current~~

		Fake thing
Charging	I → (towards) ⊕ plate	away ⊖ plate
discharging	I ← (away) ⊕ plate	toward ⊕ plate

$\mathcal{E} = L \frac{dI}{dt}$

~~big~~ biggest when no current

inductor gives

inductor like inertia

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2010**

8.02 Exam Three Spring 2010

P	L	A	S	M	E	I	E	R							

FAMILY (last) NAME

M	I	C	H	A	E	L									

GIVEN (first) NAME

9	2	1		6	4		5	2	6	1

Student ID Number

Your Section: X **L01 MW 9 am** ____**L02 MW 11 am** ____**L03 MW 1 pm**
____**L04 MW 3 pm** ____**L05 TTh 9 am** ____**L06 TTh 11 am**
____**L07 TTh 1 pm** ____**L08 TTh 3 pm**

Your Group (e.g. 10A): 11C

	Score	**Grader**
Problem 1 (25 points)	15	[illegible]
Problem 2 (25 points)	22	JDL
Problem 3 (25 points)	25	ADR
Problem 4 (25 points)	14	EF
TOTAL	76	

Section mean 82
Dormaskin mean 86

deserving of a B

Problem 1: (25 points) Five Concept Questions. Please circle your answers.

Question 1 (5 points):

A very long solenoid consisting of N turns has radius R and length d $(d >> R)$. Suppose the number of turns is halved keeping all the other parameters fixed. The self inductance

total

a) remains the same.

b) doubles.

c) is halved.

$L = \frac{N^2 \Phi}{I}$

d) is four times as large.

e) is four times as small.

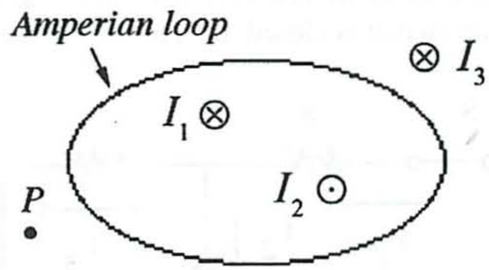
f) None of the above.

total self inductance

5

Question 2 (5 points):

The sketch below shows three wires carrying currents I_1, I_2 and I_3, with an Ampèrian loop drawn around I_1 and I_2. The wires are all perpendicular to the plane of the paper.



Which currents produce the magnetic field at the point P shown in the sketch (circle one)?

a) I_3 only.

b) I_1 and I_2.

c) I_1, I_2 and I_3.

d) None of them.

e) It depends on the size and shape of the Amperian Loop.

loop does what ???
- very confused

-5

→ loop given

poorly worded

$B \cdot ds = \mu_0 I_{enc}$

no enc

and that is not p

- weird q a

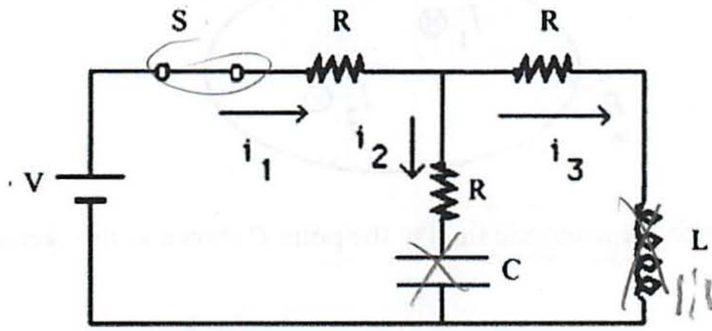
Yeah loop worthless
all matter

tricked

All of them affect P
- but why the loop
- must be in loop as well

Question 3 (5 points):

A circuit consists of a battery with emf V, an inductor with inductance L, a capacitor with capacitance C, and three resistors, each with resistance R, as shown in the sketch. The capacitor is initially uncharged and there is no current flowing anywhere in the circuit. The switch S has been open for a long time, and is then closed, as shown in the diagram. If we wait a long time after the switch is closed, the currents in the circuit are given by:



like a wire

fully charge

$I_2 = 0$

a) $i_1 = \frac{2V}{3R}$ $i_2 = \frac{V}{3R}$ $i_3 = \frac{V}{3R}$.

b) $i_1 = \frac{V}{2R}$ $i_2 = 0$ $i_3 = \frac{V}{2R}$.

$I_1 = I_3$

c) $i_1 = \frac{V}{3R}$ $i_2 = 0$ $i_3 = \frac{V}{3R}$.

$6 - I\,2R = 0$

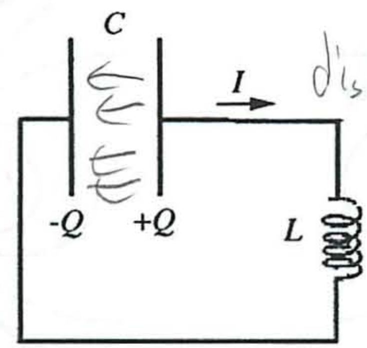
d) $i_1 = \frac{V}{2R}$ $i_2 = \frac{V}{2R}$ $i_3 = 0$.

$I = \frac{6}{2R}$

e) None of the above.

Question 4 (5 points):

At the moment depicted in the LC circuit the current is non-zero and the capacitor plates are charged (as shown in the figure below). The energy in the circuit is stored



a) only in the electric field and is decreasing.

b) only in the electric field and is constant.

c) only in the magnetic field and is decreasing.

d) only in the magnetic field and is constant.

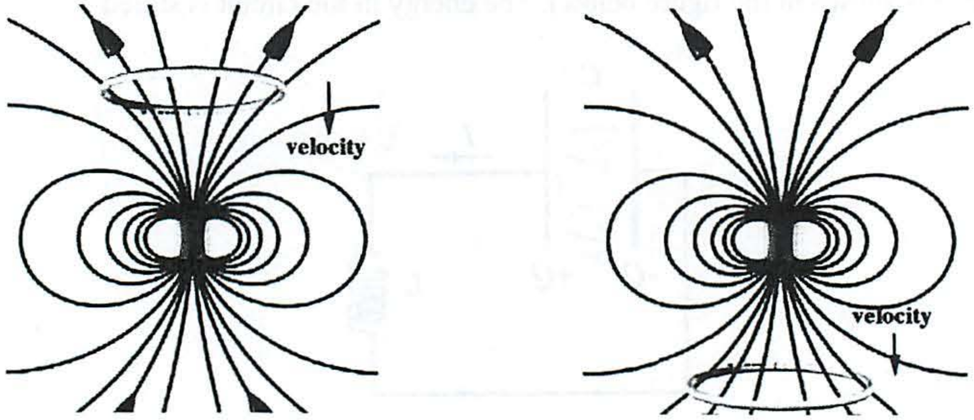
e) in both the electric and magnetic field and is constant.

f) in both the electric and magnetic field and is decreasing.

Question 5 (5 points): CCW (+)

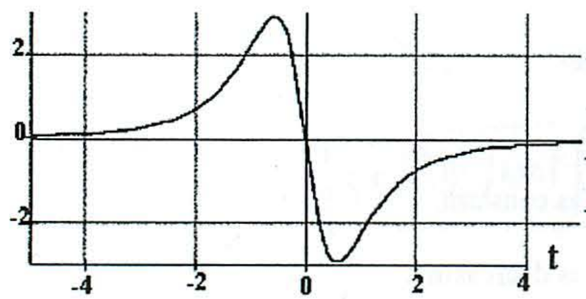
A coil of wire is above a magnet whose north pole is pointing up. For current, counter-clockwise when viewed from above is positive. For flux, upwards is positive.

B↑ dB↑ will go ↻ want ↺ ACW (−)

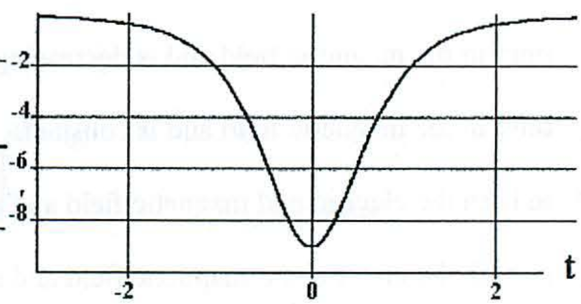


B↑ dB↓ will go ↻ want ↺ CCW (+)

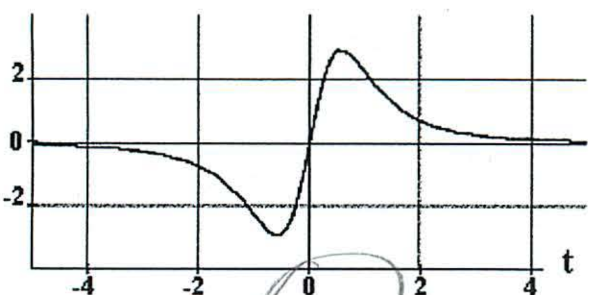
Suppose you moved the loop from well ***above*** the magnet to well ***below*** the magnet at a constant speed. Which graph most closely resembles the graph of ***current through the loop*** as a function of time?



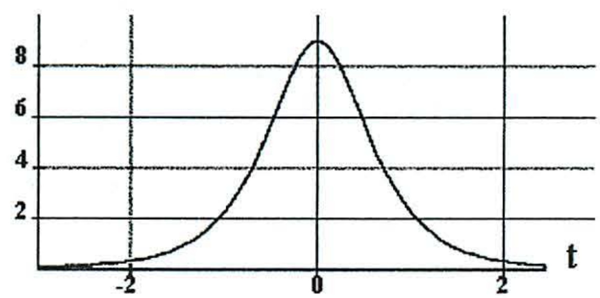
(a)



(b)



(c)



(d)

(e) None of the above.

doing ok I think — relaxed studying more helpful — less frustration

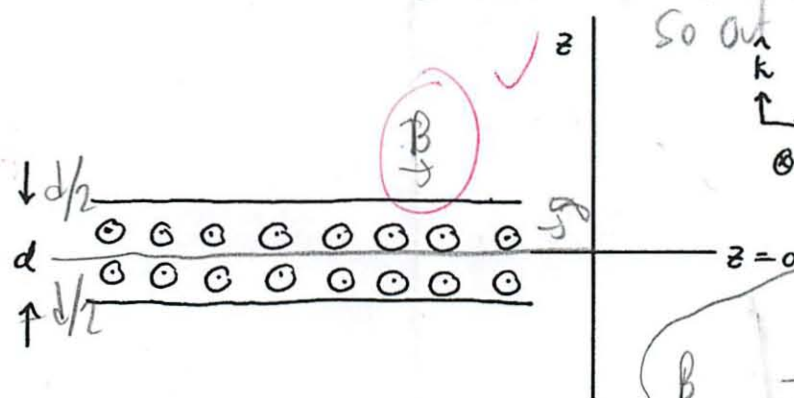
Problem 2 (25 points)

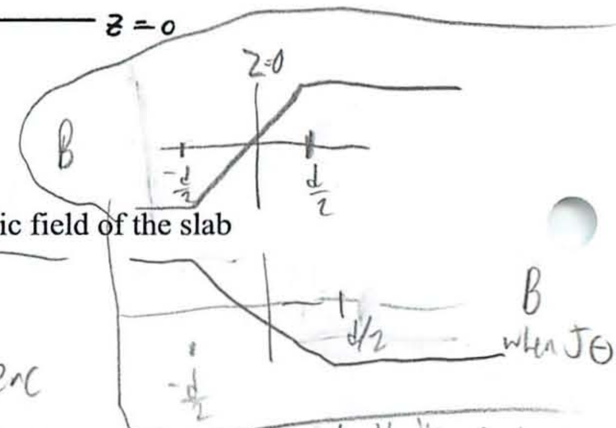
NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!) **Clearly show all Ampèrian loops that you use.**

slab

An infinitely large (in the $x-$ and $y-$directions) conducting slab of thickness d is centered at $z = 0$. The current density $\vec{\mathbf{J}} = -J_0\,\hat{\mathbf{j}}$ in the slab is uniform and points out of the page in the diagram below.

Constant - and ⊖ So out of page



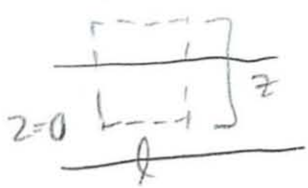


$I_{enc} = JA$

$J = \frac{I}{A}$

a) Calculate the direction and magnitude of the magnetic field of the slab

i) above the slab, $z > d/2$.



$\oint B \cdot ds = \mu_0 I_{enc}$

$B \cdot l = \mu_0 J l \frac{d}{2}$ ← used since that is limit of charge

$B = \frac{\mu_0(-J_0) l \frac{d}{2}}{l} = \mu_0(-J_0)\frac{d}{2}$

left ? ↑ direction? drew it but did not write

3/4

ii) below the slab, $z < -d/2$.

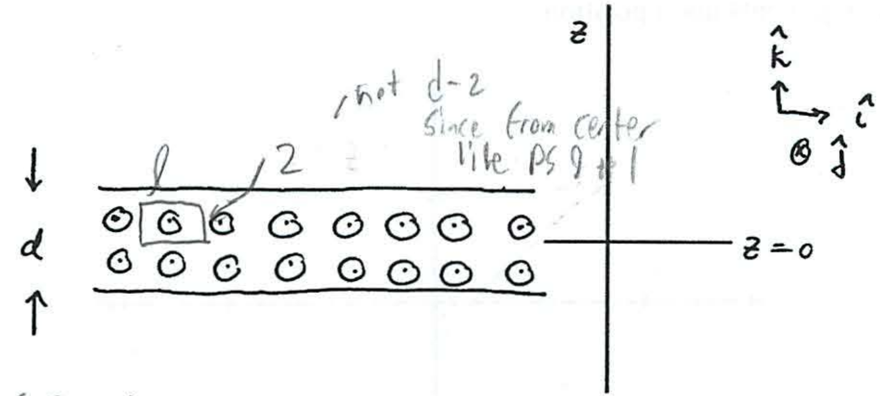
$B = \mu_0 - J_0 - \frac{d}{2}$

$B = J_0 \mu_0 \left|\frac{d}{2}\right|$

direction right ↑

3/4

iii) inside the slab, $-d/2 < z < d/2$.



$\oint B \cdot ds = \mu_0 I_{enc}$

$B l = \mu_0 (-J_0) l z$

directions?

$B = \mu_0 - J_0 z$ ↑ $\quad 0 < z < d/2$

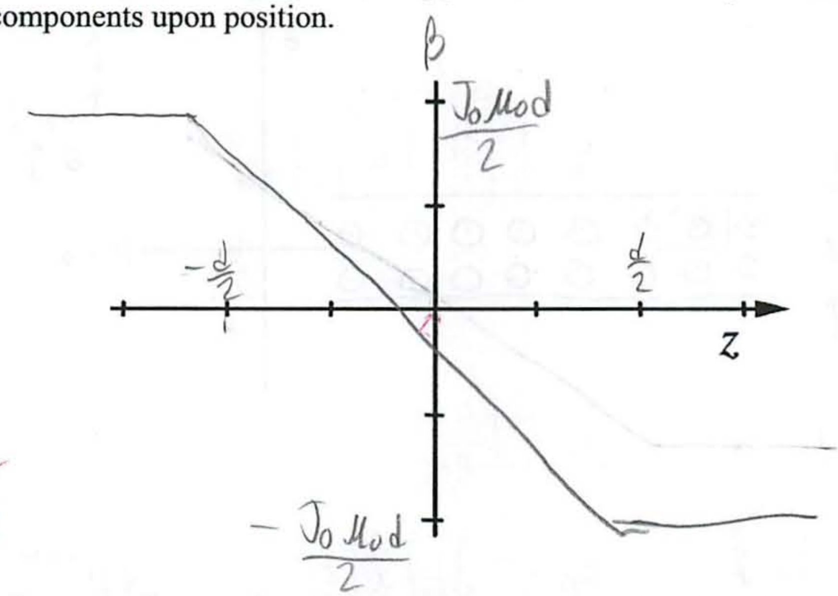
$B = \mu_0 - J_0 |z|$

$= \mu_0 J_0 z$ ↑ $\quad \frac{d}{2} < z < 0$

#6/7

b) Make a carefully labeled graph showing your results for the dependence of the field components upon position.

No part C

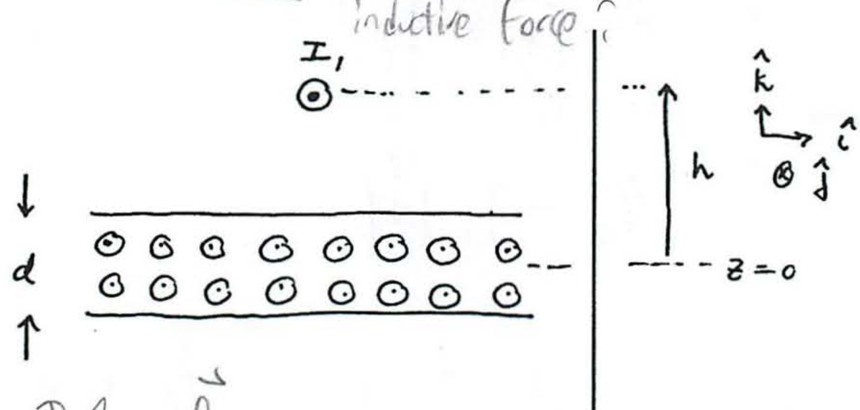


thank you dormaskin - exactly the problem here

5/5

d) A very long wire is now placed at a height $z = h$ above the slab. The wire carries a current I_1 pointing out of the page in the diagram below. What is the direction and magnitude of the force per unit length on the wire?

inductive force ?



What dir B? What a'i I so forgot! - never good at these! l?

$F = I\vec{l} \times \vec{B}$

I_1 [l?] $-\frac{J_0 \mu_0 d}{2}$

dir $\odot \times \rightarrow$ is down $-\hat{k}$

did dir here

$I\,l\,B$

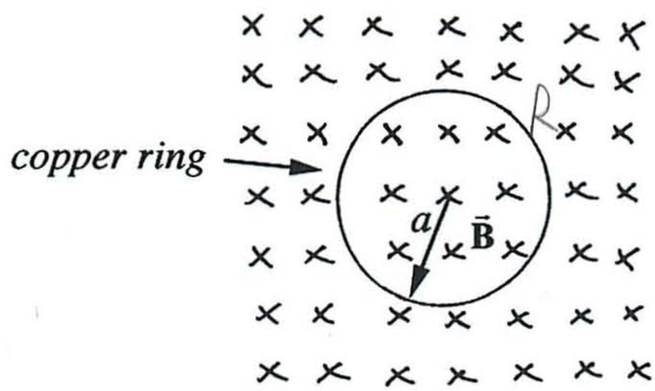
5/5

$$\vec{F} = -\frac{I_1\, l\, J_0 \mu_0 d}{2}\,\hat{k}$$

Problem 3 (25 points)

NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!) .

Consider a copper ring of radius a and resistance R. The loop is in a constant magnetic field $\vec{\mathbf{B}}$ of magnitude B_0 perpendicular to the plane of the ring (pointing into the page, as shown in the diagram).



(a) What is the magnetic flux Φ through the ring? Express your answer in terms of B_0, a, R, and μ_0 as needed.

$\Phi = BA$

~~$\oint B \cdot ds = \mu_0 I_{enc}$~~

B given

$\Phi = B_0 \cdot \pi a^2$

✓

r, seems too simple

Now, the magnitude of the magnetic field is decreased during a time interval from $t = 0$ to $t = T$ according to

$$B(t) = B_0\left(1 - \frac{t}{T}\right), \text{ for } 0 < t \le T$$

✓ Changing magnetic field

(b) What are the magnitude and direction (draw the direction on the figure above) of the current I in the ring? Express your answer in terms of B_0, T, a, R, t, and μ_0 as needed.

$$\frac{d\Phi}{dt} = \pi r^2 \cdot \frac{dB}{dt}$$

$$B(t) = B_0\left(1 - \frac{t}{T}\right) = B_0 - \frac{B_0 t}{T}$$

$$\frac{dB}{dt} = 0 - \frac{B_0}{T}$$

← seems wrong

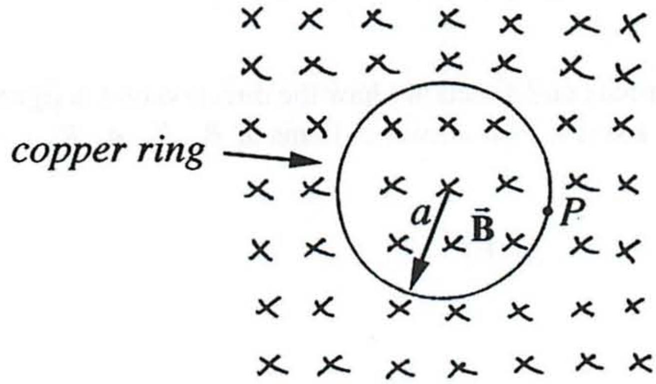
$$\mathcal{E} = -\frac{d\Phi}{dt} = -\pi r^2 \cdot -\frac{B_0}{T}$$

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{\frac{\pi r^2 B_0}{T}}{R_{ring}}$$

2

$$= \frac{\pi r^2 B_0}{T R_{ring}}$$

(c) What is the total charge Q that has moved past a fixed point P in the ring during the time interval that the magnetic field is changing? Express your answer in terms of B_0, T, a, R, t, and μ_0 as needed.



$I = \frac{dQ}{dt}$ $Q = \int I\,dt$

$\frac{dI}{dt} = \frac{d^2Q}{dt^2}$

$Q = \int_0^T \frac{\pi r^2 B_0}{TR} dt$

$\frac{\pi r^2 B_0 t}{TR} \Big|_0^T$

b

$= \frac{\pi r^2 B_0}{R_{ring}}$

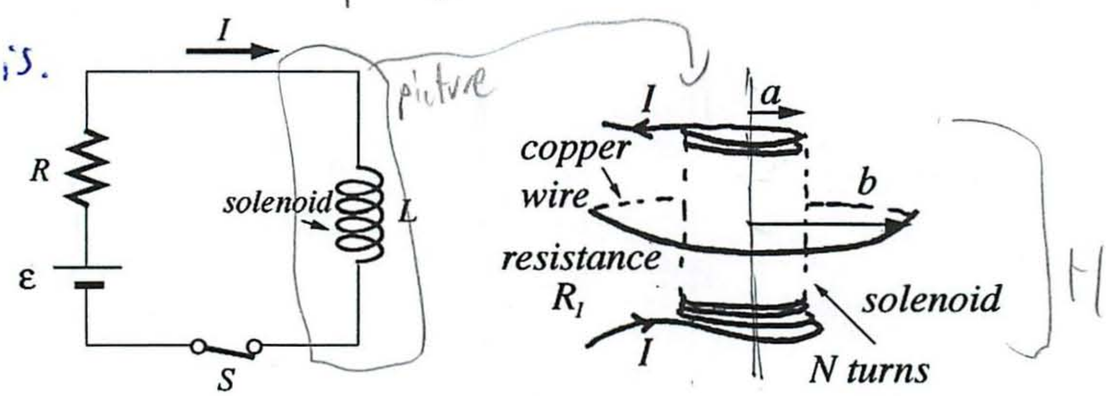
Problem 4 (25 points)

NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!) .

Consider the circuit shown in the figure, consisting of a battery (emf $\mathcal{E}$), a resistor with resistance R, a long solenoid of radius a, height H that has N turns and a switch S. Coaxial with the solenoid at the center of the solenoid is a circular copper ring of wire of radius b with $b > a$ and resistance R_1. At $t = 0$ the switch S is closed.

not English
yes it is.

part c



(a) What is the rate that the current is changing the instant the switch is closed at $t = 0$? Express your answer in terms of R, $\mathcal{E}$, and L, the self-inductance of the solenoid, as needed.

You want a rate at an instant?

$\mathcal{E} - IR - L\frac{dI}{dt} = 0$ ✓ well $\frac{dI}{dt}$

$\frac{dI}{dt} = \frac{\mathcal{E} - IR}{L}$ ← ans has I — how do you get rid of — never figured this out — diff eq

diff eq

$I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{tR}{L}}\right) = \frac{\mathcal{E}}{R} - \frac{\mathcal{E}}{R}e^{-\frac{tR}{L}}$

$I(t)$ | $I_f\left(1 - e^{-t/\tau}\right)$

$I_f = \frac{\mathcal{E}}{R}$

$\tau = \frac{L}{R}$

forget differentiating this

Prob wrong

$\frac{dI}{dt} = 0 - \frac{\mathcal{E}}{R}\frac{tR}{L}e^{-\frac{tR}{L}}$ (−1) ?

$= -\frac{\mathcal{E}}{L}e^{-\frac{tR}{L}}$ @ t=0 ? (−1)

e^x ~~xe^x~~ $d(e^x) = e^x$

can't differentiate

$\frac{di}{dt} = \frac{\mathcal{E}}{L}$

Feels like going on forever

(b) What is the self-inductance $\underline{L}$ of the solenoid? You may assume that the solenoid is very long and so can ignore edge effects. Express your answer in terms of μ_0, a, b, H, N, R_1, R, and ε as needed. **Answers without any work shown will receive no credit.**

can use $L = \frac{N\phi}{I}$ $U_L = \frac{1}{2} L I^2$

But have to find B either way

$\oint B \cdot ds = \mu_0 I_{enc}$

- 2 regions
- only $r < a$ matters

why?

$B \cdot 2\pi r = \mu_0 I_0 \pi r^2$

← more?

(-3)

$\vec{B} = \frac{\mu_0 I_0 \pi r^2}{2\pi r} = \frac{\mu_0 I_0 r}{2}$

$B = \mu_0 \frac{N}{H} I$

$\phi = B \cdot A$

$= \frac{\mu_0 I_0 r}{2} \cdot \pi a^2 = \frac{\mu_0 I_0 \pi r^3}{2}$

r = a, right? yes...

Same B field anywhere inside

(-3)

$L = \frac{N^2 \frac{\mu_0 I \pi r^3}{2}}{I} = \frac{N^2 \mu_0 \pi r^3}{2} = \frac{N^2 \mu_0 \pi a^3}{2}$

?

screwed up in first qu as well

$$\mathcal{E} = \frac{-dQ}{dt} \qquad I = \frac{\mathcal{E}}{R}$$

I knew it would come here

(c) What is the induced current in the copper ring at the instant the switch is closed at $t = 0$? Express your answer in terms of μ_0, a, b, H, N, R_1, R, and $\mathcal{E}$ as needed.

Should have reviewed closer how inductors work

$$\mathcal{E}_{ind} = -\frac{d\Phi}{dt}$$

$$\Phi = \frac{\mu_0 I_{supp} \pi a^3}{2}$$ wrong area (-2)

What is changing?
Current as switch closed

dr

$$\mathcal{E}_{ind} = \frac{\mu_0 \pi a^3}{2} \frac{d I_{supp}}{dt}$$

$$\mathcal{E}_{ind} = \frac{\mu_0 \pi a^3}{2} \left(-\frac{\mathcal{E} t}{L} e^{-tR/L} \right)$$

$$I_{ind} = \frac{\frac{\mu_0 \pi a^2}{2} \left(-\frac{\mathcal{E} t}{L} e^{\frac{-tR}{L}} \right)}{R_1}$$

@ $t=0$ (-1)

Got through fairly fast - good ability

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2010**

8.02 Exam Three Spring 2010 Solutions

Problem 1: (25 points) Five Concept Questions. Please circle your answers.

Question 1 (5 points):

A very long solenoid consisting of N turns has radius R and length d $(d >> R)$. Suppose the number of turns is halved keeping all the other parameters fixed. The self inductance

a) remains the same.

b) doubles.

c) is halved.

d) is four times as large.

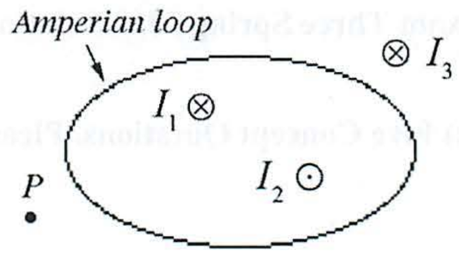
e) is four times as small.

f) None of the above.

Solution e. The self-induction of the solenoid is equal to the total flux through the object which is the product of the number of turns time the flux through each turn. The flux through each turn is proportional to the magnitude of magnetic field. By Ampere's Law the magnitude of the magnetic field is proportional to the number of turns per unit length or hence proportional to the number of turns. Hence the self-induction of the solenoid is proportional to the square of the number of turns. If the number of turns is halved keeping all the other parameters fixed then he self inductance is four times as small.

Question 2 (5 points):

The sketch below shows three wires carrying currents I_1, I_2 and I_3, with an Ampèrian loop drawn around I_1 and I_2. The wires are all perpendicular to the plane of the paper.



Which currents produce the magnetic field at the point P shown in the sketch (circle one)?

a) I_3 only.

b) I_1 and I_2.

c) I_1, I_2 and I_3.

d) None of them.

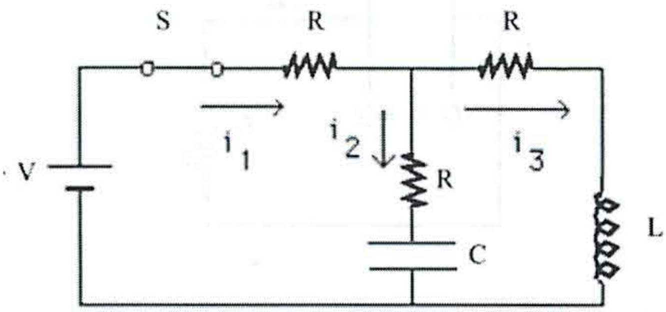
e) It depends on the size and shape of the Amperian Loop.

Solution c. All there currents I_1, I_2 and I_3 contribute to the magnetic field at the

at the point P.

Question 3 (5 points):

A circuit consists of a battery with emf V, an inductor with inductance L, a capacitor with capacitance C, and three resistors, each with resistance R, as shown in the sketch. The capacitor is initially uncharged and there is no current flowing anywhere in the circuit. The switch S has been open for a long time, and is then closed, as shown in the diagram. If we wait a long time after the switch is closed, the currents in the circuit are given by:



a) $i_1 = \dfrac{2V}{3R} \quad i_2 = \dfrac{V}{3R} \quad i_3 = \dfrac{V}{3R}$.

b) $i_1 = \dfrac{V}{2R} \quad i_2 = 0 \quad i_3 = \dfrac{V}{2R}$.

c) $i_1 = \dfrac{V}{3R} \quad i_2 = 0 \quad i_3 = \dfrac{V}{3R}$.

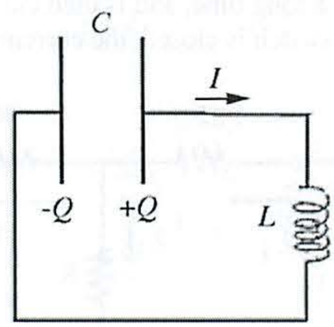
d) $i_1 = \dfrac{V}{2R} \quad i_2 = \dfrac{V}{2R} \quad i_3 = 0$.

e) None of the above.

Solution b.: If we wait a long time after the switch is closed, the capacitor is completely charged and no current flows in that branch, $i_2 = 0$. Also the current has reached steady state and is not changing in time so there is no effect from the self-inductance. Hence the inductor acts like a resistance-less wire. (Note that real inductors do have finite resistance as you saw in your lab.) Therefore the same current flow through resistors 1 and 3 and is given by $i_1 = i_3 = V/2R$.

Question 4 (5 points):

At the moment depicted in the LC circuit the current is non-zero and the capacitor plates are charged (as shown in the figure below). The energy in the circuit is stored



a) only in the electric field and is decreasing.

b) only in the electric field and is constant.

c) only in the magnetic field and is decreasing.

d) only in the magnetic field and is constant.

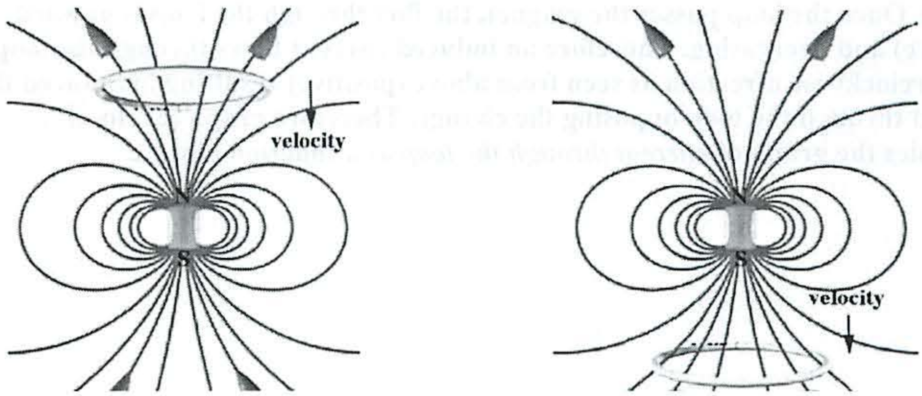
e) in both the electric and magnetic field and is constant.

f) in both the electric and magnetic field and is decreasing.

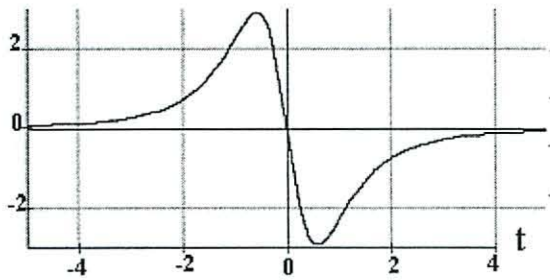
Solution e. Since there is no resistance there is no dissipation of energy so energy is constant in time. At the moment depicted in the figure, the capacitor is charged so there is a non-zero electric field associated with the capacitor. There is a non-zero current in the circuit and so there is a non-zero magnetic field. Therefore the energy in the circuit is stored in both the electric and magnetic field and is constant.

Question 5 (5 points):

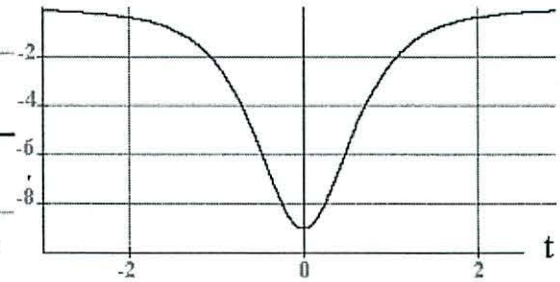
A coil of wire is above a magnet whose north pole is pointing up. For current, counter-clockwise when viewed from above is positive. For flux, upwards is positive.



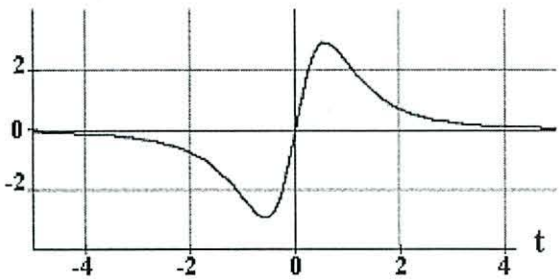
Suppose you moved the loop from well ***above*** the magnet to well ***below*** the magnet at a constant speed. Which graph most closely resembles the graph of ***current through the loop*** as a function of time?



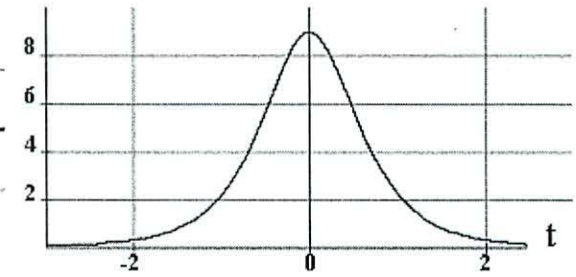
(a)



(b)



(c)



(d)

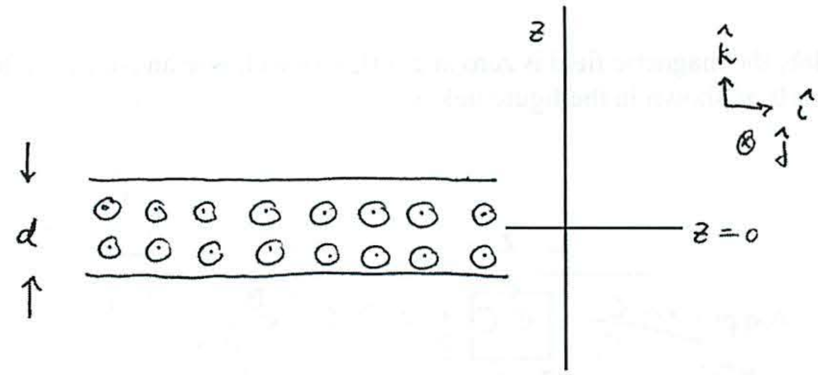
(e) None of the above.

Solution c. If you moved the loop from well *above* the magnet to well *below* the magnet at a constant speed, then as the loops approaches the magnet from below the flux through the loop is upward (positive) and increasing. Therefore an induced current flows through the loop in a clockwise direction as seen from above (negative) resulting in induced flux downward through the loop opposing the change. Once the loop passes the magnet, the flux through the loop is upward (positive) and decreasing. Therefore an induced current flows through the loop in a counterclockwise direction as seen from above (positive) resulting in induced flux upward through the loop opposing the change. Therefore graph (c) closely resembles the graph of *current through the loop* as a function of time.

Problem 2 (25 points)

NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!) **Clearly show all Ampèrian loops that you use.**

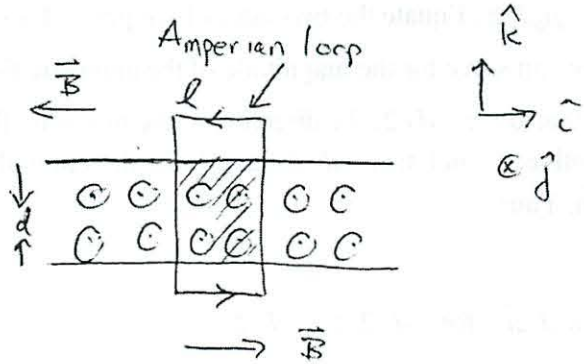
An infinitely large (in the $x-$ and $y-$directions) conducting slab of thickness d is centered at $z=0$. The current density $\vec{\mathbf{J}} = -J_0\,\hat{\mathbf{j}}$ in the slab is uniform and points out of the page in the diagram below.



a) Calculate the direction and magnitude of the magnetic field of the slab

   i) above the slab, $z > d/2$.

Solution: I choose an Amperian loop circulating counterclockwise as shown in the figure above.

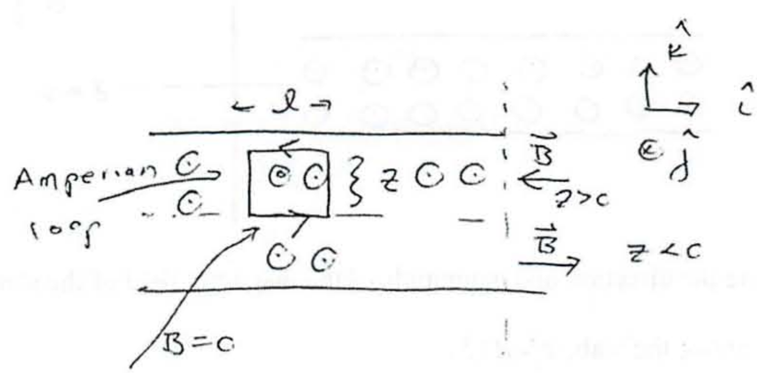


By symmetry, the magnitude of the magnetic field is the same on the upper and lower legs of the loop. Therefore with our choice of circulation direction the left-hand-side of Ampere's Law $\oint \vec{B}\cdot d\vec{s} = \mu_0 \iint \vec{J}\cdot d\vec{a}$ becomes $\oint \vec{B}\cdot d\vec{s} = 2Bl$. The current density is uniform and with the unit normal pointing out of the page ($-\hat{\mathbf{j}}$-direction) consistent with the choice of counterclockwise circulation direction, the right-hand side of Ampere's Law becomes $\mu_0 \iint \vec{J} \cdot d\vec{a} = \mu_0 J_0 l d$. Equate the two sides of Ampere's Law, we have that $2Bl = \mu_0 J_0 l d$ which we can solve for the magnitude of the magnetic field $B = \mu_0 J_0 d / 2$. The direction of the magnetic field is the same as the circulation direction on the upper and lower legs. Thus

i) $\vec{B} = -\mu_0 J_0 d / 2\hat{i}$ above the slab, $z > d/2$.

ii) $\vec{B} = \mu_0 J_0 d / 2\hat{i}$ below the slab, $z < -d/2$.

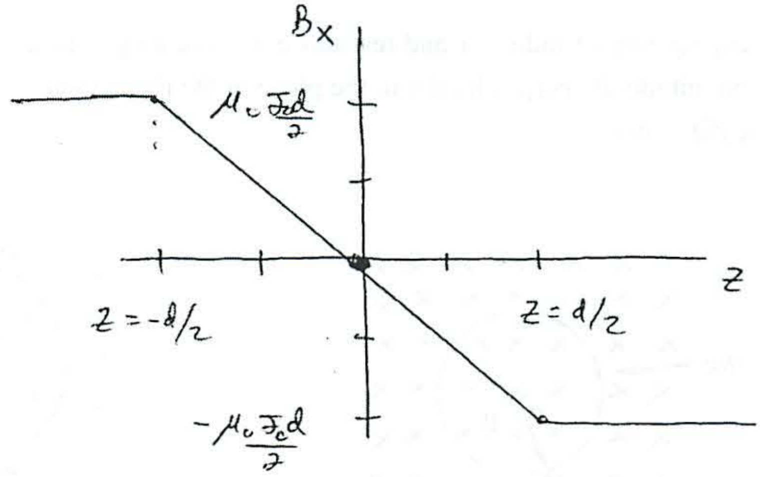
Inside the slab, the magnetic field is zero at $z = 0$, so we choose an Amperian loop with one leg at $z = 0$ as shown in the figure below.



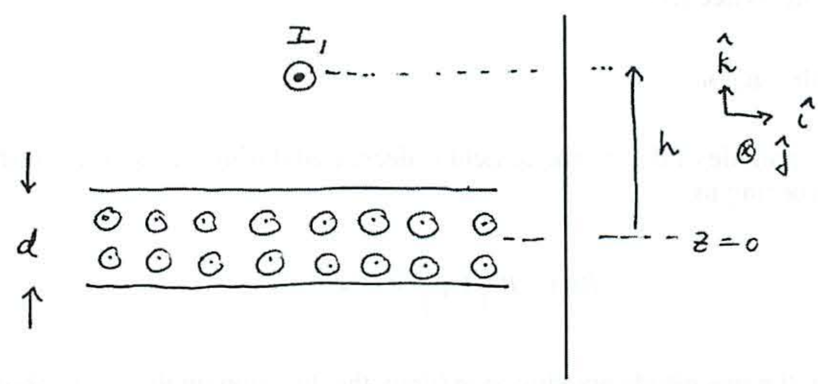
Therefore with our choice of circulation direction the left-hand-side of Ampere's Law $\oint \vec{B} \cdot d\vec{s} = \mu_0 \iint \vec{J} \cdot d\vec{a}$ is now $\oint \vec{B} \cdot d\vec{s} = Bl$. The right-hand side of Ampere's Law becomes $\mu_0 \iint \vec{J} \cdot d\vec{a} = \mu_0 J_0 l z$. Equate the two sides of Ampere's Law, we have that $Bl = \mu_0 J_0 l z$ which we can solve for the magnitude of the magnetic field $B = \mu_0 J_0 |z|$. For positive z such that $0 < z < d/2$, the direction of the magnetic field is in the $-\hat{\mathbf{j}}$-direction and for negative z such that $-d/2 < z < 0$, the direction of the magnetic field is in the $+\hat{\mathbf{j}}$-direction. Thus

iii) $\vec{B} = -\mu_0 J_0 z\hat{i}$ for $-d/2 < z < d/2$.

b) Make a carefully labeled graph showing your results for the dependence of the field components upon position.



c) A very long wire is now placed at a height $z = h$ above the slab. The wire carries a current I_1 pointing out of the page in the diagram below. What is the direction and magnitude of the force per unit length on the wire?



Solution: The force on a small length ds of the wire is given by

$$d\vec{F} = I_1 d\vec{s} \times \vec{B} = -I_1 \hat{j} \times -\frac{\mu_0 J_0 d}{2}\hat{i} = -\frac{(ds) I_1 \mu_0 J_0 d}{2}\hat{k}$$

Therefore the direction and magnitude of the force per unit length on the wire is

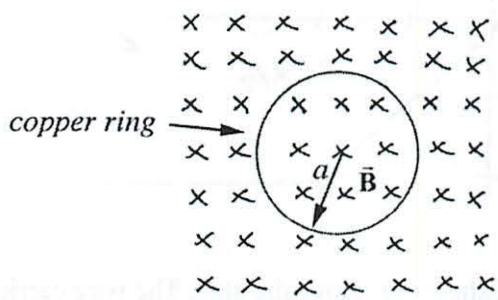
$$\frac{d\vec{F}}{ds} = -\frac{I_1 \mu_0 J_0 d}{2}\hat{k}.$$

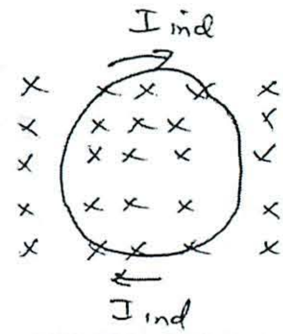
The current is the wire and the current in the slab are in the same direction so the force is attractive.

Problem 3 (25 points)

NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!) .

Consider a copper ring of radius a and resistance R. The loop is in a constant magnetic field $\vec{\mathbf{B}}$ of magnitude B_0 perpendicular to the plane of the ring (pointing into the page, as shown in the diagram).





(a) What is the magnetic flux Φ through the ring? Express your answer in terms of B_0, a, R, and μ_0 as needed.

Solution: $\Phi = B_0 \pi a^2$

Now, the magnitude of the magnetic field is decreased during a time interval from $t = 0$ to $t = T$ according to

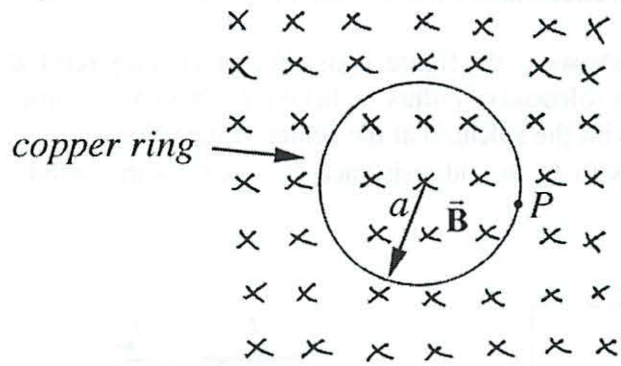
$$B(t) = B_0\left(1 - \frac{t}{T}\right), \text{ for } 0 < t \le T$$

(b) What are the magnitude and direction (draw the direction on the figure above) of the current I in the ring? Express your answer in terms of B_0, T, a, R, t, and μ_0 as needed.

Solution: The external flux is into the page and decreasing so the induced current is in the clockwise direction producing flux into the page through the ring opposing the change. The magnitude of the induced current is non-zero during the interval $0 < t \le T$ and is equal to

$$I = \frac{1}{R}\left|\frac{d\Phi}{dt}\right| = \frac{1}{R}\left|\frac{d}{dt}\left(B_0\left(1 - \frac{t}{T}\right)\pi a^2\right)\right| = \frac{1}{R}\left|\frac{d}{dt}\left(B_0\left(1 - \frac{t}{T}\right)\pi a^2\right)\right| = \frac{B_0 \pi a^2}{TR}, \textbf{ for } 0 < t \le T$$

(c) What is the total charge Q that has moved past a fixed point P in the ring during the time interval that the magnetic field is changing? Express your answer in terms of B_0, T, a, R, t, and μ_0 as needed.



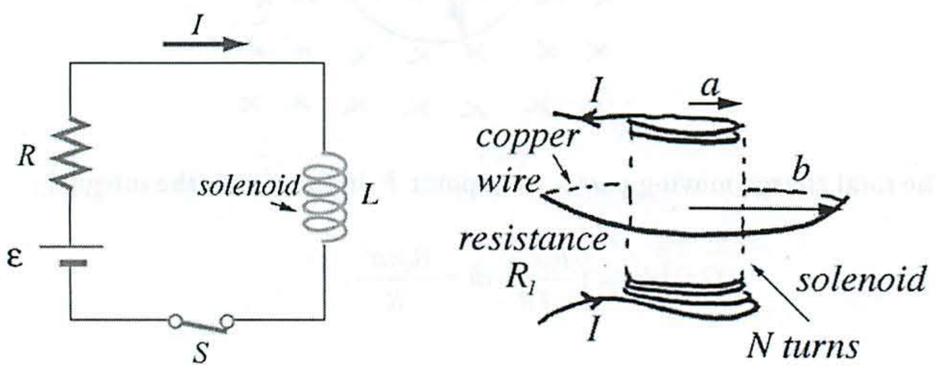
Solution: The total charge moving past a fixed point P in the ring is the integral

$$Q = \int_0^T I dt = \int_0^T \frac{B_0 \pi a^2}{TR} dt = \frac{B_0 \pi a^2}{R}.$$

Problem 4 (25 points)

NOTE: YOU MUST SHOW WORK in order to get any credit for this problem. Make it clear to us that you understand what you are doing (use a few words!) .

Consider the circuit shown in the figure, consisting of a battery (emf ε), a resistor with resistance R, a long solenoid of radius a, height H that has N turns and a switch S. Coaxial with the solenoid at the center of the solenoid is a circular copper ring of wire of radius b with $b > a$ and resistance R_l. At $t = 0$ the switch S is closed.



(a) What is the rate that the current is changing the instant the switch is closed at $t = 0$? Express your answer in terms of R, ε, and L, the self-inductance of the solenoid, as needed.

Solution: At $t = 0$, the current in the circuit is zero so the emf is related to the changing current by

$$\varepsilon = L\frac{dI}{dt}(t=0).$$

Thus

$$\frac{dI}{dt}(t=0) = \frac{\varepsilon}{L}$$

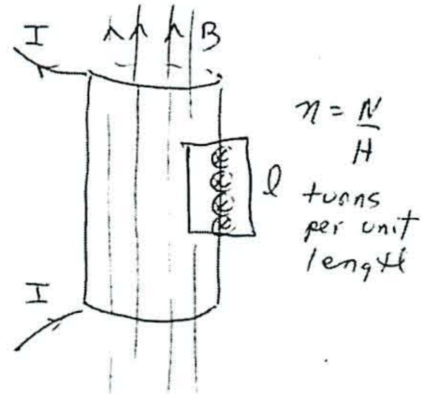
Alternatively, the loop equation is given by $\varepsilon - IR - L\frac{dI}{dt} = 0$. Thus at $t = 0$, the current in the circuit is zero and so $\varepsilon = L\frac{dI}{dt}(t=0)$. The current is the circuit is given by $I(t) = \frac{\varepsilon}{R}(1 - e^{-tR/L})$.

So

$$\frac{dI}{dt}(t=0) = \frac{\varepsilon}{R}\frac{R}{L}e^{-tR/L}(t=0) = \frac{\varepsilon}{L}.$$

(b) What is the self-inductance L of the solenoid? You may assume that the solenoid is very long and so can ignore edge effects. Express your answer in terms of μ_0, a, b, H, N, R_1, R, and ε as needed. **Answers without any work shown will receive no credit.**

Solution: The direction of the magnetic field upwards (see figure).



Choose an Amperian loop shown in the figure below, then Ampere's Law becomes $Bl = \mu_0 nlI$. Therefore the magnitude of the magnetic field in the solenoid is

$$B = \mu_0 nI = \frac{\mu_0 NI}{H}.$$

The self inductance through the solenoid is

$$L = \frac{N\Phi_{loop}}{I} = \frac{NB\pi a^2}{I} = \frac{\mu_0 N^2 \pi a^2}{H}.$$

c) What is the induced current in the copper ring at the instant the switch is closed at $t=0$? Express your answer in terms of μ_0, a, b, H, N, R_1, R, and ε as needed.

Solution: The induced current is noting that the relevant area where the magnetic field is non-zero is πa^2

$$I_{ind} = \frac{1}{R_1}\frac{d\Phi}{dt} = \frac{1}{R_1}\frac{dB}{dt}\pi a^2 = \frac{1}{R_1}\frac{\mu_0 N}{H}\pi a^2 \frac{dI}{dt}(t=0)$$

$$\frac{1}{R_1}\frac{\mu_0 N}{H}\pi a^2 \frac{\varepsilon}{L} = \frac{1}{R_1}\frac{\mu_0 N}{H}\pi a^2 \frac{H\varepsilon}{\mu_0 N^2 \pi a^2} = \frac{\varepsilon}{NR_1}.$$

Review Day 28+30

5/1

So this class before exam always not enough attention paid + never get it

So <u>Poynting Vector</u>

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

How much energy passes through a given area per unit time
Points in direction of energy flow

points <u>into</u> resistor → means energy used up

<u>Displacement curren</u>

changing E field → "make" a current from that

name is historic

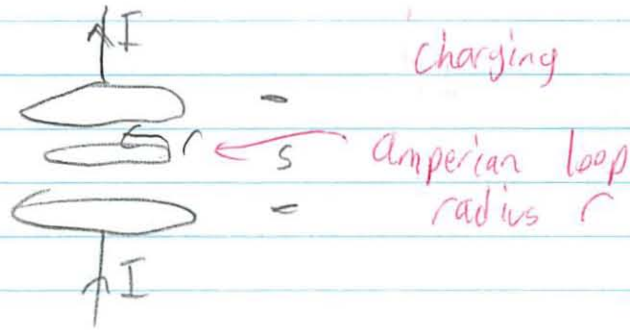
$$\oint_c B \cdot ds = \mu_0 (I_{enc} + I_d)$$
↑ displacement

I_d : ⊕ → in dir $\vec{E}$
⊖ opposite dir $\vec{E}$

$$I_d = \epsilon_0 \frac{d\Phi_E}{dE}$$

- Should have this reexplained

So for a capicator



Line integral of magnetic field. arond loop is (+)

← displacement current same dir as current if charging / discharging

B field in dir integrating

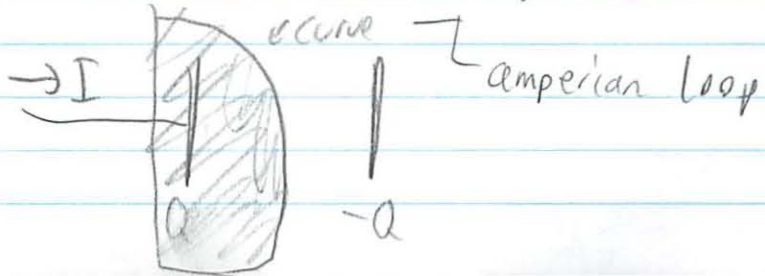
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi}{dt}$$

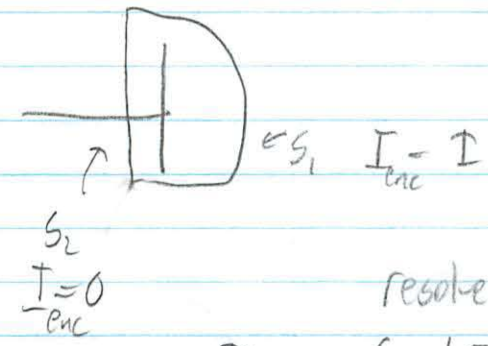
↑ normal current ↑ flux changing

If same radius as plate
- larger loop = larger integral

That all makes sense - but it does not really explain

Read Course notes chap 13





resolve ambugity

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\oint \vec{B}\,d\vec{s} = \mu_0 (I + I_d)$$

$$\Phi = \oiint \vec{E} \cdot d\vec{A} = EA = \frac{Q}{\varepsilon_0}$$

$$I_d = \varepsilon_0 \frac{d\Phi}{dt} = \frac{dQ}{dt}$$

↑ related to ↑ of charge

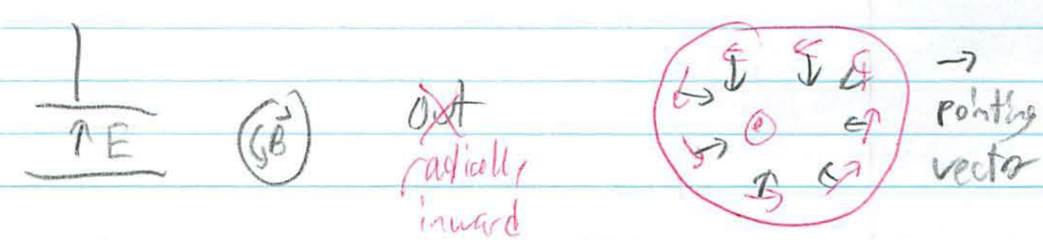
simply I

So $I = I_d$

- so surface does not matter

I still don't get why this is really needed - but ok just add it

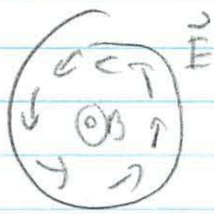
Poynting Vector



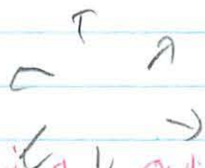
in capicator → inward

- so electric field inside is ↑
- so Q is ↑

inductor



so out



energy flowing ↓ outward

so energy in $\vec{B}$ ↓

so I ↓

$I \propto \vec{B}$

↑ proportional to

Review problem solving ? sometime

Day 30 wave review

all the different components

→ there is that weird plane representation of a wave

Wave equations

Will want to see more specifics on waves
- from next few days
- do MP now

Topics: Maxwell's Equations, EM Radiation & Energy Flow
Related Reading: Course Notes: Sections 13.3-13.4, 13.6-13.8.1, 13.10

Topic Introduction

Today's class continues the discussion of electromagnetic waves from last week. We will also show that the Poynting vector applies to situations other than just EM waves, in particular to the flow of energy in circuits.

Maxwell's Equations

(1) $\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$ (2) $\oiint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$

(3) $\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$ (4) $\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$

(1) Gauss's Law states that electric charge creates diverging electric fields.
(2) Magnetic Gauss's Law states that there are no magnetic charges (monopoles).
(3) Faraday's Law states that changing magnetic fields induce electric fields (which curl around the changing flux).
(4) Ampere-Maxwell's Law states that magnetic fields are created both by currents and by changing electric fields, and that in each case the field curls around its creator.

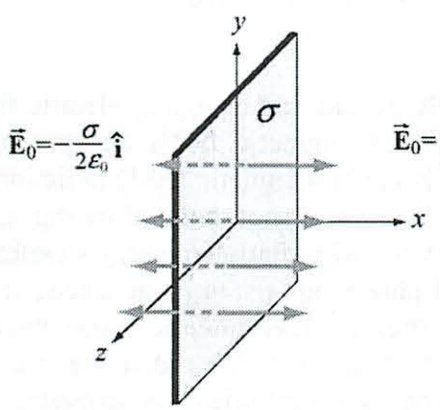
Electromagnetic Radiation

The fact that changing magnetic fields create electric fields and that changing electric fields create magnetic fields means that oscillating electric and magnetic fields can propagate through space (each pushing forward the other). This is electromagnetic (EM) radiation. It is the single most useful discovery we discuss in this class, not only allowing us to understand natural phenomena, like light, but also to create EM radiation to carry a variety of useful information: radio, broadcast television and cell phone signals, to name a few, are all EM radiation. In order to understand the mathematics of EM radiation you need to understand how to write an equation for a traveling wave (a wave that propagates through space as a function of time). Any function that is written f(x-vt) satisfies this property. As t increases, a function of this form moves to the right (increasing x) with velocity v. You can see this as follows: At t=0 f(0) is at x=0. At a later time t=t, f(0) is at x=vt. That is, the function has moved a distance vt during a time t.

Sinusoidal traveling waves (plane waves) look like waves both as a function of position and as a function of time. If you sit at one position and watch the wave travel by you say that it has a period T, inversely related to its frequency f, and angular frequency, $\omega\left(T = f^{-1} = 2\pi\omega^{-1}\right)$. If instead you freeze time and look at a wave as a function of position, you say that it has a wavelength λ, inversely related to its wavevector k $\left(\lambda = 2\pi k^{-1}\right)$. Using this notation, we can rewrite our function f(x-vt) = $f_0$sin(kx-ωt), where v = ω/k.
We typically treat both electric and magnetic fields as plane waves as they propagate through space (if you have one you must have the other). They travel at the speed of light (v=c). They also obey two more constraints. First, their magnitudes are fixed relative to each other:

$E_0 = cB_0$ (check the units!) Secondly, E & B always oscillate at right angles to each other and to their direction of propagation (they are *transverse* waves). That is, if the wave is traveling in the z-direction, and the E field points in the x-direction then the B field must point along the y-direction. More generally we write $\hat{\mathbf{E}} \times \hat{\mathbf{B}} = \hat{\mathbf{p}}$, where $\hat{\mathbf{p}}$ is the direction of propagation.

Energy and the Poynting Vector
As EM Waves travel through space they carry energy with them. This is clearly true – light from the sun warms us up. It also makes sense in light of the fact that energy is stored in electric and magnetic fields, so if those fields move through space then the energy moves with them. It turns out that we can describe how much energy passes through a given area per unit time by the Poynting Vector: $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$. Note that this points in the direction of propagation of the EM waves (from above) which makes sense – the energy is carried in the same direction that the waves are traveling. The Poynting Vector is also useful in thinking about energy in circuit components. For example, consider a cylindrical resistor. The current flows through it in the direction that the electric field is pointing. The B field curls around. The Poynting vector thus points radially *into* the resistor – the resistor consumes energy. We will repeat this exercise for capacitors and inductors in class.



Generating Plane Electromagnetic Waves: How do we generate plane electromagnetic waves? We do this by shaking a sheet of charge up and down, making waves on the electric field lines of the charges in the sheet. We discuss this process quantitatively in this lecture, and show that the work that we do to shake the sheet up and down provides exactly the amount of energy carried away in electromagnetic waves.

Important Equations

Maxwell's Equations:

(1) $\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$ (2) $\oiint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$

(3) $\oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$ (4) $\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc} + \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$

EM Plane Waves:

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},t) = E_0 \sin(k\hat{\mathbf{p}} \cdot \vec{\mathbf{r}} - \omega t)\hat{\mathbf{E}}$$
$$\vec{\mathbf{B}}(\vec{\mathbf{r}},t) = B_0 \sin(k\hat{\mathbf{p}} \cdot \vec{\mathbf{r}} - \omega t)\hat{\mathbf{B}}$$

with $E_0 = cB_0$; $\hat{\mathbf{E}} \times \hat{\mathbf{B}} = \hat{\mathbf{p}}$; $\omega = ck$

Poynting Vector: $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$

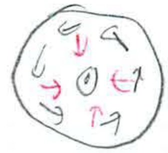
5/3

Class 32: Outline

Hour 1:
Energy Flow in EM Waves

Hour 2:
Generating EM Waves

P32- 1



radially inward
= charging

outward → discharging

Wave propagating
- like pointing vector
- dir of energy = dir wave traveling

Review:
Traveling EM Waves

P32- 2

$\vec{E}$

- electric charges	Gauss
- time changing $\vec{B}$ field	Faraday

$\vec{B}$

- moving electric	Ampere's
time changing $\vec{E}$ field	Maxwell's addition

Maxwell's Equations

$\oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$ → 0	(Gauss's Law)
$\oint_S \vec{B} \cdot d\vec{A} = 0$	(Magnetic Gauss's Law)
$\oint_C \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$	(Faraday's Law)
$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$ ($\mu_0 I_{enc}$ → 0)	(Ampere-Maxwell Law)

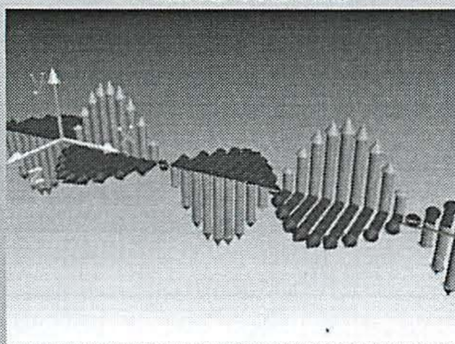
Solve in free space (no charge/current) to get…

P32- 3

→0 when empty space
- no charges/currents

(can say) Charges create $\vec{E}$ fields
✱ or if have charges must have $\vec{E}$ fields

Electromagnetic Radiation: Plane Waves



propagating

traveling in phase

* have same mag everywhere on line
* huge sheet moving

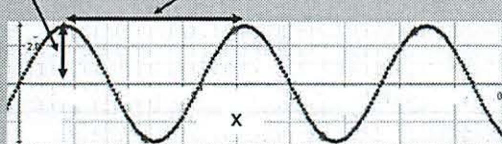
Traveling EM Wave: Space

$$E = E_0 \sin\left(k\left(x \pm ct\right)\right) = E_0 \sin\left(kx \pm \omega t\right)$$

At a fixed time (e.g. t=0): $E = E_0 \sin(kx)$

Amplitude (E_0)

Wavelength $(\lambda) = \dfrac{2\pi}{\text{wavenumber } (k)}$



know all of the wave terms

$\frac{E}{B} = V = C$

$c = 3 \cdot 10^8$ m/s in vacuum

$\frac{1}{\sqrt{\mu_0 \epsilon_0}}$ in other spaces

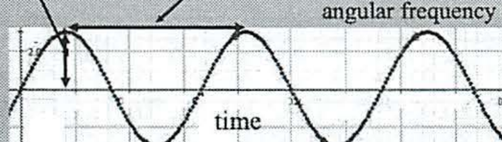
Traveling EM Wave: Time

$$E = E_0 \sin\left(k\left(x \pm ct\right)\right) = E_0 \sin\left(kx \pm \omega t\right)$$

At x=0, just a function of time: $E = E_0 \sin(kvt) \equiv E_0 \sin(\omega t)$

Period $(T) = \dfrac{1}{\text{frequency } (f)} = \dfrac{2\pi}{\text{angular frequency } (\omega)}$

Amplitude (E_0)



here is wave eq

Traveling E & B Waves

Wavelength: λ
Frequency : f

$$\vec{\mathbf{E}} = \hat{\mathbf{E}} E_0 \sin(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)$$

Wave Number: $k = \frac{2\pi}{\lambda}$

Angular Freq.: $\omega = 2\pi f$

Period: $T = \frac{1}{f} = \frac{2\pi}{\omega}$

Speed: $v = \frac{\omega}{k} = \lambda f$

Direction: $+\hat{\mathbf{k}} = \hat{\mathbf{E}} \times \hat{\mathbf{B}}$

$$\frac{E}{B} = \frac{E_0}{B_0} = v$$

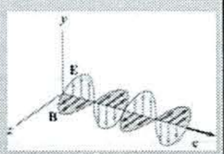
In vacuum...

$$= c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \frac{m}{s}$$

Properties of EM Waves

Travel (through vacuum) with speed of light

$$v = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \frac{m}{s}$$



At every point in the wave and any instant of time, E and B are in phase with one another, with

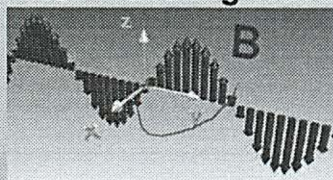
$$\frac{E}{B} = \frac{E_0}{B_0} = v$$

E and B fields perpendicular to one another, and to the direction of propagation (they are **transverse**):

Direction of propagation = Direction of $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$

PRS Questions: Traveling Wave

PRS: Traveling Wave



The B field of a plane EM wave is $\vec{B}(z,t) = \hat{k}B_0 \sin(ky - \omega t)$

The electric field of this wave is given by

- 0% 1. $\vec{E}(z,t) = \hat{j}E_0 \sin(ky - \omega t)$
- 0% 2. $\vec{E}(z,t) = -\hat{j}E_0 \sin(ky - \omega t)$
- 0% 3. $\vec{E}(z,t) = \hat{i}E_0 \sin(ky - \omega t)$
- 0% 4. $\vec{E}(z,t) = -\hat{i}E_0 \sin(ky - \omega t)$
- 0% 5. I don't know

:20

dir of propagation — traveling in y → ĵ; if ky + ωt → −ĵ

Most common qu on this

if get this → set

E B

$\hat{i} \times \hat{k} = -\hat{j}$

so $-\hat{i} \times \hat{k} = \hat{j}$

$F = q(E + vB)$

$E_0 = cB$

should be able to figure out

Energy & the Poynting Vector

$\vec{E} = \hat{j} E_0 \sin(kz + \omega t)$

$\vec{B} =$

E B

$\hat{j} \times \_ = -\hat{k}$

↑ ✓ got it

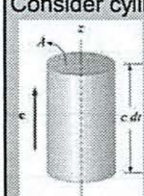
$-\hat{k}\, B_0 \sin(kz + \omega t)$ — copy

slides out of order today

Energy in EM Waves

Energy densities: $u_E = \frac{1}{2}\varepsilon_0 E^2$, $u_B = \frac{1}{2\mu_0}B^2$

Consider cylinder:



$$dU = (u_E + u_B)A\,dz = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right)Ac\,dt$$

What is rate of energy flow per unit area?

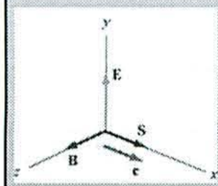
$$S = \frac{1}{A}\frac{dU}{dt} = \frac{c}{2}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right) = \frac{c}{2}\left(\varepsilon_0 cEB + \frac{EB}{c\mu_0}\right)$$ ← in terms of EB

$$= \frac{EB}{2\mu_0}\left(\varepsilon_0\mu_0 c^2 + 1\right) = \frac{EB}{\mu_0}$$

pointing vector per unit area

Poynting Vector and Intensity

Direction of energy flow = direction of wave propagation



$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$: Poynting vector

units: Joules per square meter per sec

Intensity I:

$$I \equiv \langle S \rangle = \frac{E_0 B_0}{2\mu_0} = \frac{E_0^2}{2\mu_0 c} = \frac{cB_0^2}{2\mu_0}$$

P32-13

time average

intensity =
time average of Poynting Vector

Momentum & Radiation Pressure

EM waves transport energy: $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$

They also transport momentum: $p = \frac{U}{c}$

And exert a pressure: $P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt} = \frac{1}{cA}\frac{dU}{dt} = \frac{S}{c}$

when wave hits

This is only for hitting an absorbing surface. For hitting a perfectly reflecting surface the values are doubled:

Momentum transfer: $p = \frac{2U}{c}$; Radiation pressure: $P = \frac{2S}{c}$

P32-14

Light carries energy + momentum
can cut w/ a laser
only true of light

2x as much pressure

$S = \frac{P}{A} = \frac{4 \cdot 10^{26} \text{ watts}}{\text{sphere surrounding sun}}$

In Class Problem: Catchin' Rays

As you lie on a beach in the bright midday sun, approximately what force does the light exert on you?

The sun:

Total energy output of ~ 4 x 10^{26} Watts

Distance from Earth 1 AU ~ 150 x 10^6 km

Speed of light c = 3 x 10^8 m/s

P32-15

Watts is power
person ≈ 1 m²
$P = \frac{F}{A} = \frac{U}{t} = SA$
How much of sun's energy hits you?
– sphere radius of sphere
= the area $= 4\pi d^2$

$= \frac{4 \cdot 10^{26}}{4\pi (1.5 \cdot 10^{11})^2}$

$\approx 10^4 \text{ W/m}^2$ ← good # to know for solar power – the max energy you can get

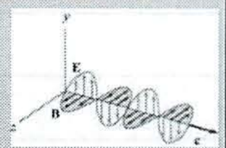
$F = PA = \frac{S}{c}A = \frac{10^4 \frac{W}{m^2} \cdot 1 m^2}{3 \cdot 10^8 m/s} = 10^{-4} N$

less than a paper clip

Properties of EM Waves

Travel (through vacuum) with speed of light

$$v = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times 10^8 \frac{m}{s}$$



At every point in the wave and any instant of time, E and B are in phase with one another, with

$$\frac{E}{B} = \frac{E_0}{B_0} = v$$

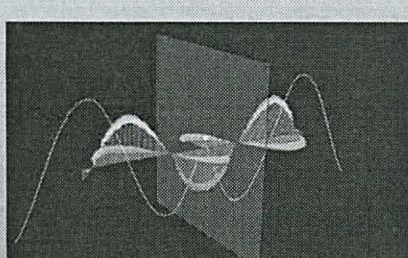
E and B fields perpendicular to one another, and to the direction of propagation (they are **transverse**):

Direction of propagation = Direction of $\vec{\mathbf{E}}\times\vec{\mathbf{B}}$

Generating Plane Electromagnetic Radiation

How do you actually generate EM radiation?

Shake a Sheet of Charge



Link

current out of pg

bottom view

Just consider right side of sheet

$E = vB$

$v = \frac{d}{t}$

$v = \frac{ct}{t} = c$

$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$

$2Bl = \mu_0 \sigma l v$

$|B| = \frac{\mu_0 \sigma v}{2}$

$B(x,t) = \frac{\mu_0 \sigma y_0 \omega \cos\left(\frac{\omega}{c}x - \omega t\right)}{2} \hat{k}$

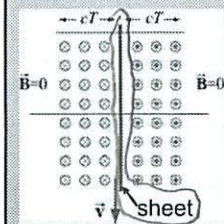
derivative of

$E(x,t) = \frac{\mu_0 \sigma y_0 \omega c \cos\left(\frac{\omega}{c}x - \omega t\right)}{2} \hat{j}$

for right side

flip signs for left

Group Problem: B Field Generation



Sheet (blue) has uniform charge density σ

Starting time T ago pulled down at velocity v

1) What is B field?

(HINT: Change drawing perspective)

2) If sheet position is $y(t) = y_0 \sin(\omega t)$

What is B(x,t)?

What is E(x,t)? What Direction?

first pull it down

then shake it sinusoidally

$k = \frac{2\pi}{\lambda}$

$\lambda = \frac{c}{f}$

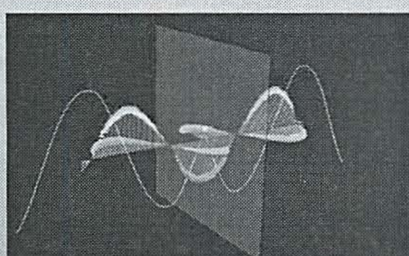
$k = \frac{2\pi f}{c} = \frac{\omega}{c} = \frac{2\pi}{\text{freq}}$ = wave #

↑ relation b/w $k + \omega$

$k = \frac{1}{\text{meters}}$

away from sheet

You Made a Plane Wave!



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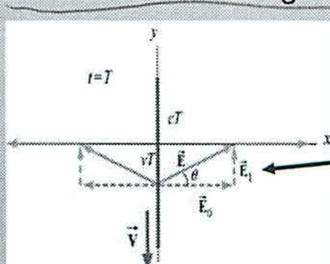
gold = $\vec{E}$ vectors

How to Think About E-Field

E-Field lines like strings tied to plane



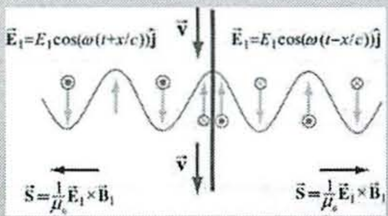
This is the field you calculated & that propagates

$\vec{E}$ straight out when still

then when shake it propagates

did not plug in velocity for sheet

Group Problem: Energy in Wave



You Found: $B_1 = \mu_0 \sigma v / 2$

1) What is total power per unit area radiated away?
2) Where is that energy coming from?
3) Calculate power generated to see efficiency

energy need to put in compared w/ energy radiated away

calculate $E_1 = cB = \frac{c\mu_0 \sigma V}{2}$

$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$

$S = \frac{2\left(\frac{\mu_0 \sigma V}{2}\right)^2 c}{\mu_0}$ (2: both dir)

$\vec{S} = \frac{\mu_0 \sigma^2 V^2 c}{2}$

Exam 3 Results

$F = qE = \sigma A E$

$\frac{P}{A} = \frac{FV}{A} = \sigma E V$

$= \frac{\mu_0 \sigma^2 V^2 c}{2}$

C = speed of light
V = speed pulling sheet

both = so 100% efficient
- all energy into radiation

magnetic force wrong dir (⊥)

Good, hard exam qu

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In Class W13D1_4 Solutions: Catchin' Rays

Problem:
As you lie on a beach in the bright midday sun, approximately what force does the light exert on you?

The sun:
Total energy output of ~ 4 x 10^{26} Watts
Distance from Earth 1 AU ~ 150 x 10^6 km
Speed of light c = 3 x 10^8 m/s

Solution:
The power per unit area of the sun at the Earth is found by assuming the power goes out uniformly in all directions:

$$S = \frac{P}{4\pi R^2_{Sun-Earth}} \sim \frac{4\times 10^{26}\,\text{Watts}}{4\pi\left(\frac{3}{2}\times 10^{11}\,\text{m}\right)^2} \sim \frac{4}{27}\times 10^4 \frac{\text{W}}{\text{m}^2} \sim 1500\frac{\text{W}}{\text{m}^2}$$

That's actually a good number to know – the average solar constant above the atmosphere. More accurately, it is 1366 W m^{-2}.

To find the force on a sunbather, we assume a sunbather has an area of about 1 m^2 (2 m x 0.5 m) and multiply that by the pressure:

$$F = \text{Pressure}\times\text{Area} = \frac{S}{c}\cdot A \sim \frac{1500\frac{\text{W}}{\text{m}^2}}{3\cdot 10^8\frac{\text{m}}{\text{s}}}\cdot 1\,\text{m}^2 \sim 5\cdot 10^{-6}\frac{\text{Ws}}{\text{m}} = 5\cdot 10^{-6}\,\text{N}$$

Of course, if you were really shiny that might as much as double (completely reflecting the light doubles the force).

So, is that a reasonable force? It corresponds roughly to a 0.5 μg mass sitting on you. You aren't going to feel it. But you don't feel the weight of sunlight either, so that's reasonable.

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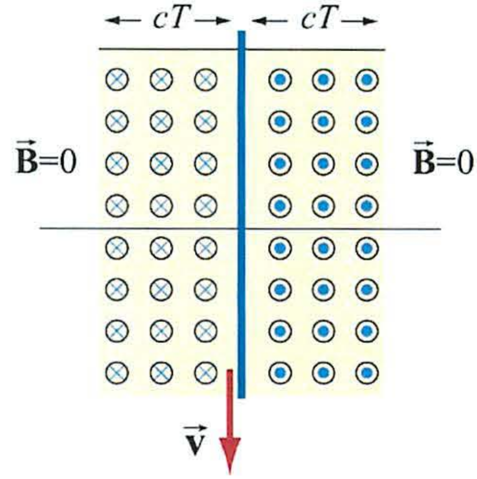
Solutions: B Field Generation



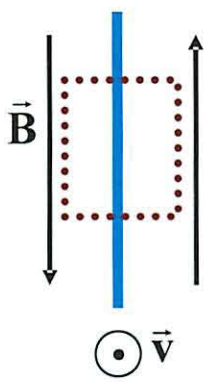
Problem: The charged sheet at right has a uniform charge density σ and is being pulled downward at a velocity **v**.

1) What is the B field that is generated?
2) If the sheet position oscillates as $y(t) = y_0 \sin(\omega t)$, what are E(x,t) and B(x,t)?

Solution:

1) What is the B field that is generated?

Its always best to redraw so that the magnetic field lies in the plane of the page:



So we need to do Ampere's law around the loop. The current is just the moving charge density:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = 2Bl = \mu_o I_{enc} = \mu_o \sigma v l \quad \Rightarrow \quad \boxed{B = \mu_o \sigma v / 2}$$

2) If the sheet position oscillates as $y(t) = y_0 \sin(\omega t)$, what are E(x,t) and B(x,t)?

$$y(t) = y_0 \sin(\omega t) \quad \Rightarrow \quad v = y_0 \omega \cos(\omega t)$$

$$\vec{\mathbf{B}} = \frac{\mu_o \sigma}{2} y_0 \omega \cos\left(\frac{\omega}{c} x - \omega t\right) \hat{\mathbf{k}}$$

Sheet moves in y, wave travels in x

$$\vec{\mathbf{E}} = \frac{\mu_o \sigma c}{2} y_0 \omega \cos\left(\frac{\omega}{c} x - \omega t\right) \hat{\mathbf{j}}$$

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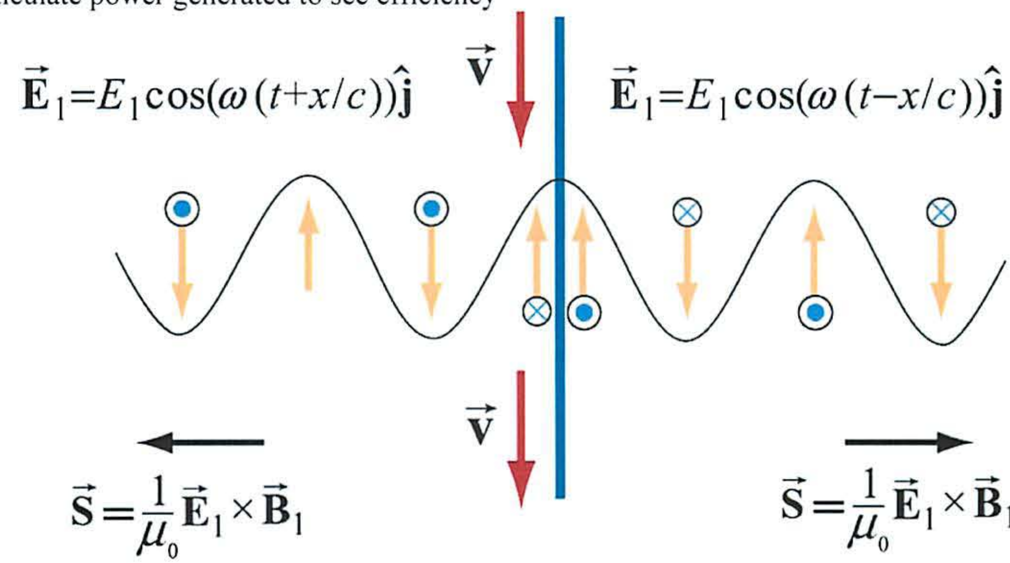
Solutions: B Field Generation

Problem: For the wave pictured below, where you calculated that $B_1 = \mu_o \sigma v/2$:

1) What is total power per unit area radiated away?
2) Where is that energy coming from?
3) Calculate power generated to see efficiency



Solution:

1) What is total power per unit area radiated away?

$$\frac{P_{\text{total}}}{\text{Area}} = \underbrace{2S}_{\text{two sides}} = 2\frac{E_1 B_1}{\mu_o} = 2\frac{cB_1^2}{\mu_o} = 2\frac{c\left(\mu_o \sigma v/2\right)^2}{\mu_o} = \frac{\mu_o c \sigma^2 v^2}{2}$$

2) Where is that energy coming from?

It is coming from the moving sheet.

3) Calculate power generated to see efficiency

The electric field exerts a force on the charges, and they are moving, so

$$\frac{P}{A} = \frac{Fv}{A} = \frac{qEv}{a} = \sigma E v = \sigma c B v = \sigma c v\left(\mu_o \sigma v/2\right) = \frac{\mu_o c \sigma^2 v^2}{2}$$

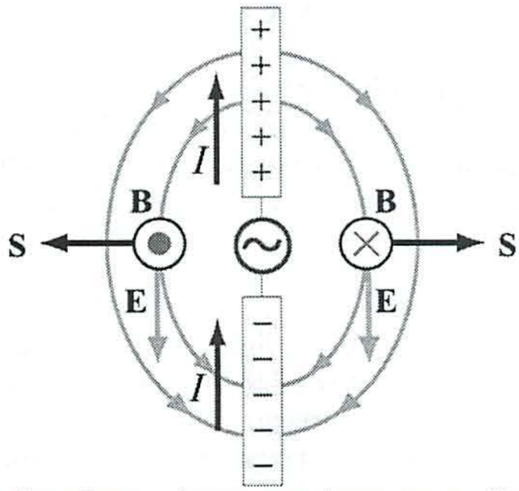
This is the same as the power radiated, so this is 100% efficient!

5/5

Topics: Dipole Radiation, Polarization and Interference
Related Reading: Course Notes: Sections 13.8, 14.1-14.3, 14.11.1-14.11.3
Experiments: (10) Microwave Generator

Topic Introduction

Today we will talk about polarization and interference of electromagnetic waves. We will also discuss and do a lab using a spark-gap transmitter.

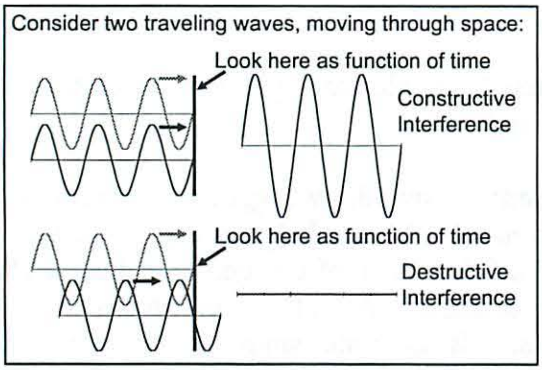


Antenna: How do we generate electric dipole radiation? Again, by shaking charge, but this time not an infinite plane of charge, but a line of charge on an antenna. At left is an illustration of an antenna. It is quite simple in principle. An oscillator drives charges back and forth from one end of the antenna to the other (at the moment pictured the top is positive the bottom negative, but this will change in half a period). This separation of charge creates an electric field that points from the positive to the negative side of the antenna. This field also begins to propagate away from the antenna (in the direction of the Poynting vector **S**). When the charge changes sides the field will flip directions – hence you have an oscillating electric field that is propagating away from the antenna. This changing E field generates a changing B field, as pictured, and you thus have an electromagnetic wave. The length of each part of the antenna above (e.g. the top half) is about equal in length to ¼ of the wavelength if the radiation that it produces. Why is that? The charges move at close to the speed of light in the antenna so that in making one complete oscillation of the wave (by moving from the top to the bottom and back again) they move about as far as the wave has itself (one wavelength).

Polarization
As mentioned in the last class, EM waves are transverse waves – the E & B fields are both perpendicular to the direction of propagation $\hat{\mathbf{p}}$ as well as to each other. Given $\hat{\mathbf{p}}$, the E & B fields can thus oscillate along an infinite number of directions (any direction perpendicular to $\hat{\mathbf{p}}$). We call the axis that the E field is oscillating along the polarization axis (often a "polarization direction" is stated, but since the E field oscillates, sometimes E points along the polarization direction, sometimes opposite it). When light has a specific polarization direction we say that it is polarized. Most light (for example, that coming from the sun or from light bulbs) is unpolarized – the electric fields are oscillating along lots of different axes. However, in certain cases light can become polarized. A very common example is that when light scatters off of a surface only the polarization which is parallel to that surface survives. This is why Polaroid sunglasses are useful. They stop all light which is horizontally polarized, thus blocking a large fraction of light which reflects off of horizontal surfaces (glare). If you happen to own a pair of Polaroid sunglasses, you can find other situations in which light becomes polarized. Rainbows, for example, are polarized. So is the

sky under the right conditions (can you figure out what the conditions are?) This is because the blue light that you see in the sky is scattered sun light.

Interference



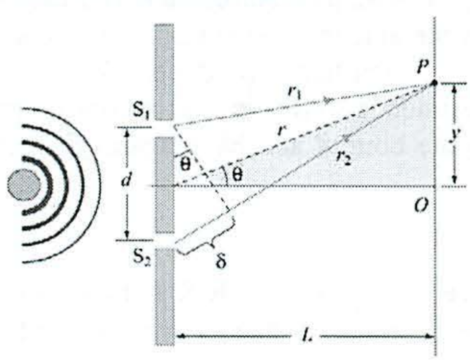
The picture at left forms the basis of all the phenomena we will discuss today. Two different waves (red & blue) arrive at a single position in space (at the screen). If they are in phase then they add constructively and you see a bright spot. If they are out of phase then the add destructively and you see nothing (dark spot).

The key to creating interference is creating phase shift between two waves that are then brought together at a single position. A common way to do that is to add extra path length to one of the waves relative to the other. We will look at a variety of systems in which that happens.

Thin Film Interference

The first phenomenon we consider is thin film interference. When light hits a thin film (like a soap bubble or an oily rain puddle) it does two things. Part of the light reflects off the surface. Part continues forward, then reflects off the next surface. Interference between these two different waves is responsible for the vivid colors that appear in many systems.

Two Slit Interference



Light from the laser hits two very narrow slits, which then act like in-phase point sources of light. In traveling from the slits to the screen, however, the light from the two slits travel different distances. In the picture at left the light from the bottom slit travels further than the light from the top slit. This extra path length introduces a phase shift between the two waves and leads to a position dependent interference pattern on the screen.

Here the extra path length is $\delta = d \sin(\theta)$, leading to a phase shift ϕ given by $\dfrac{\delta}{\lambda} = \dfrac{\phi}{2\pi}$. Realizing that phase shifts that are multiples of 2π give us constructive interference while odd multiples of π lead to destructive interference leads to the following conditions: Maxima: $d \sin(\theta) = m\lambda$; Minima: $d \sin(\theta) = \left(m + \frac{1}{2}\right)\lambda$

Important Equations

Maxwell's Equations:

$$(1) \oint\!\!\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0} \qquad (2) \oint\!\!\oint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

$$(3) \oint_C \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt} \qquad (4) \oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

EM Plane Waves:

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},t) = E_0 \sin(k\hat{\mathbf{p}} \cdot \vec{\mathbf{r}} - \omega t)\hat{\mathbf{E}}$$
$$\vec{\mathbf{B}}(\vec{\mathbf{r}},t) = B_0 \sin(k\hat{\mathbf{p}} \cdot \vec{\mathbf{r}} - \omega t)\hat{\mathbf{B}}$$

with $E_0 = cB_0$; $\hat{\mathbf{E}} \times \hat{\mathbf{B}} = \hat{\mathbf{p}}$; $\omega = ck$

Interference Conditions

$$\frac{\Delta L}{\lambda} = \frac{\phi}{2\pi} = \begin{cases} m & \text{constructive} \\ m + \frac{1}{2} & \text{destructive} \end{cases}:$$

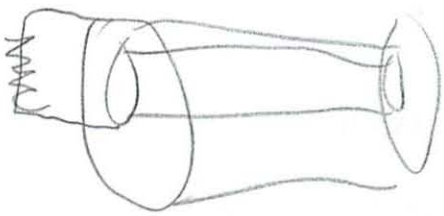
Two Slit **Maxima**: $d \sin(\theta) = m\lambda$

Experiment 10: Microwaves

Preparation: Read pre-lab and answer pre-lab questions

In today's lab you will create microwaves (EM radiation with a wavelength of several centimeters) using a spark gap transmitter. This is a type of quarter wavelength antenna that works on the principles described above. You will measure the polarization of the produced EM waves, and try to understand the intensity distribution created by such an antenna (where is the signal the strongest? The weakest?)

PRS



← capicator

What use to measure $\vec{E}$ b/w inner outer cylinder

1. Grass (from static charge) – is current, not static charges

~~2. Ampere (Maxwell's Addition)~~ – creation of B Field

3. ~~Faraday's Law (from B field)~~ $\vec{B}$ not changing, since I constant

4. ~~$\frac{b}{b-a}$~~ Field not constant

resistor in parallell w/ capicator

Charge does build up

both a capicator and wire

Pointing vector – how much power disipated

5/5

Class 33: Outline

Hour 1:
- Generating Electromagnetic Waves
- Electric Dipole EM Waves
- Experiment 9: Microwaves

Hour 2:
- Interference and Diffraction

P33- 1

Traveling E & B Waves

Wavelength: λ

Frequency : f

$$\vec{\mathbf{E}} = \hat{\mathbf{E}} E_0 \sin(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)$$

Wave Number: $k = \frac{2\pi}{\lambda}$

Angular Freq.: $\omega = 2\pi f$

Period: $T = \frac{1}{f} = \frac{2\pi}{\omega}$

Speed: $v = \frac{\omega}{k} = \lambda f$

Direction: $+\hat{\mathbf{k}} = \hat{\mathbf{E}} \times \hat{\mathbf{B}}$

$$\frac{E}{B} = \frac{E_0}{B_0} = v$$

In vacuum...

$$= c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \frac{m}{s}$$

P33- 2

reminder

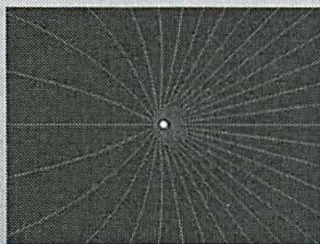
plane wave

how do know sin + cos
- phase, depends what they tell you at $x=0$
- in class either since not working w/ phase much

Generating Electric Dipole Electromagnetic Waves

P33- 3

Generating Electric Dipole Radiation Applet



Link to applet

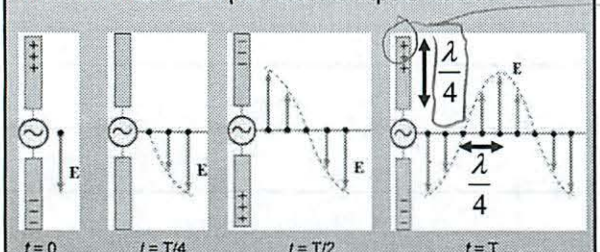
P33- 4

a charge, not an ∞ sheet

like a fountain kinda

Quarter-Wavelength Antenna

Accelerated charges are the source of EM waves.
Most common example: Electric Dipole Radiation.



P33- 5

charge ossilates

charge at extream

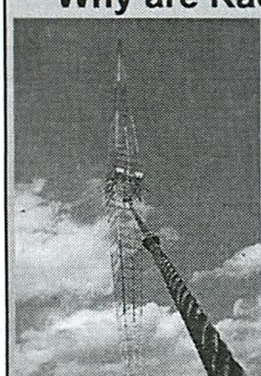
flips

field →

charge information near the speed of light

quarter wave lenght antenna
- well 2 together = half wave lenght

Why are Radio Towers Tall?



AM Radio stations have frequencies 535 – 1605 kHz. WLW 700 Cincinnati is at 700 kHz.

$$\lambda = \frac{c}{f} = \frac{3\times10^8\,\text{m/s}}{700\times10^3\,\text{Hz}} = 429\,\text{m}$$

$$\lambda/4 \approx 107\text{m} \approx 350\text{ft}$$

Tower is 747 ft tall

P33- 6

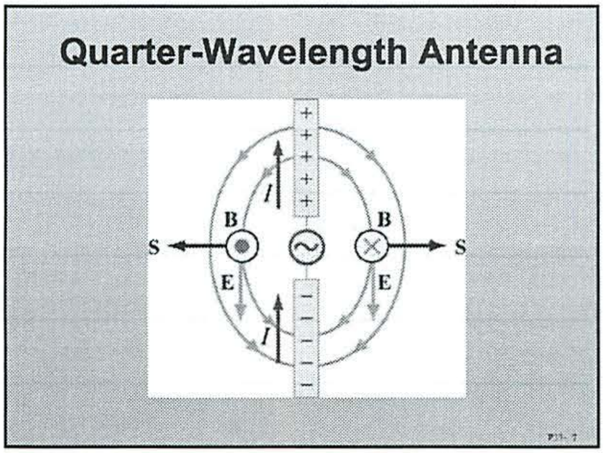
double

AM signals are long

FM ↑ freq

mHz → about 100 ↑

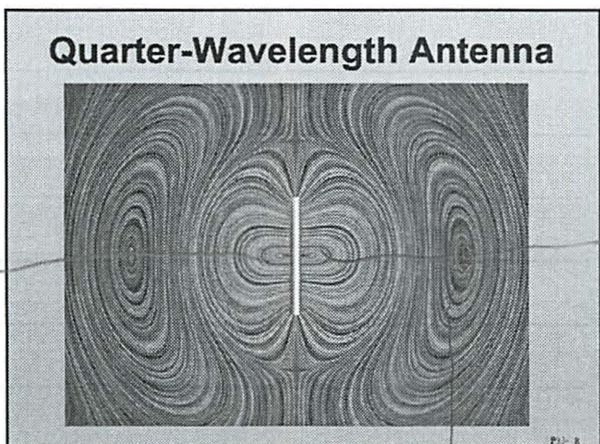
wave lenght is shorter



charge Seperation → $\vec{E}$

current moving → $\vec{B}$

like Mon's sheet of charge problem



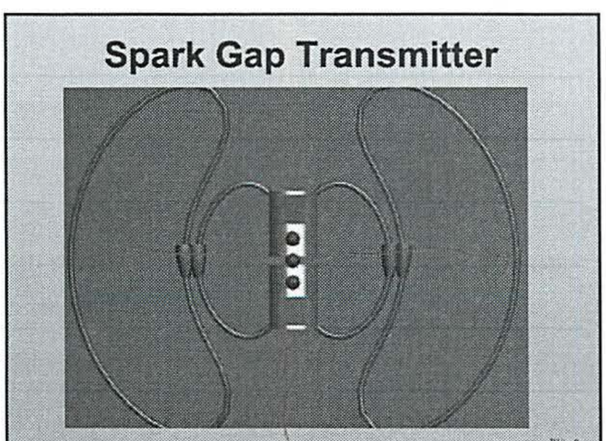
really cool

field lines are like loops

up → down → up → down → up ...

watch

sit there and it is more or less a plane wave



making a diapole wave

← first antenna made

$\vec{E}$ fields going away

clothes pin

mega Ω resistor

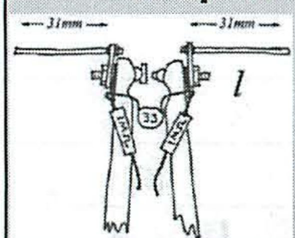
capicator

wires going out → inductor

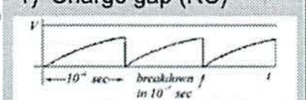
Spark Gap Generator: An LC Oscillator

P13- 10

Our spark gap antenna

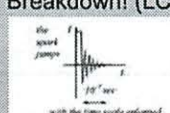


1) Charge gap (RC)



$\tau = RC = (4.5\times10^{6}\,\Omega)(33\times10^{-12}\,\mathrm{F}) = 1.5\times10^{-4}\,\mathrm{s}$

2) Breakdown! (LC)



$$f_{rad} = \frac{1}{T} = \frac{c}{4l} = \frac{3\times10^{10}\ \mathrm{cm/s}}{12.4\,\mathrm{cm}} = 2.4\times10^{9}\ \mathrm{Hz} = 2.4\,\mathrm{GHz}$$

3) Repeat

P13- 11

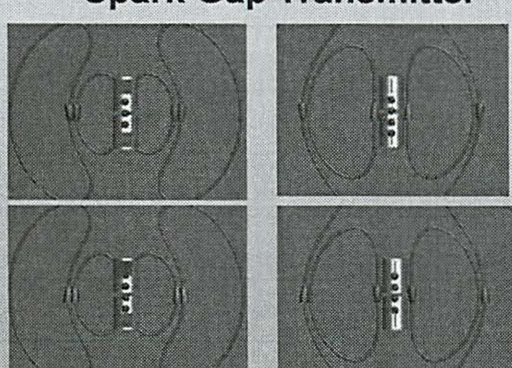
↳ cordless phones

air in gap going to break down –

← oscillations

charge rushing back + forth

Spark Gap Transmitter

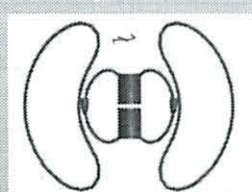


P13- 12

PRS Question: Spark Gap Antenna

PRS: Spark Gap

At the time shown the charge on the top half of our ¼ wave antenna is positive and at its maximum value. At this time the current across the spark gap is



- 0% (1.) Zero
- 0% 2. A maximum and downward
- 0% (3.) A maximum and upward
- 0% 4. Can't tell from the information given
- 0% 5. I don't know

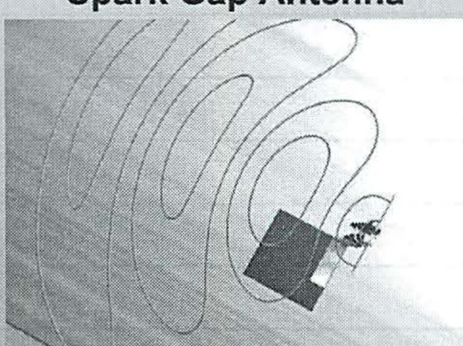
20

LC circuit

max charge → current 0

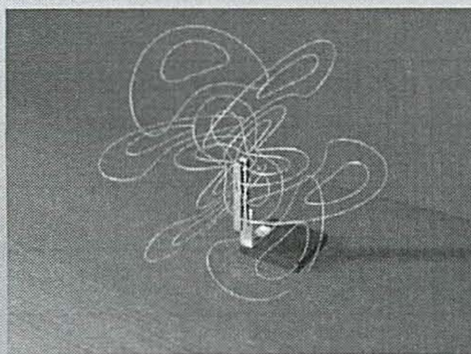
will be max ↓ ¼ cycle later

Spark Gap Antenna



shape of radiation pattern

Spark Gap Antenna



parallel to antenna

Demonstration: Antenna

Polarization

Polarization of TV EM Waves

Why oriented as shown?

Why different lengths?

P33-19

AM radio
towers |
so waves —
so antenna must be |

Very complex - so do little quantatively
w/ amplifyer can get good
signal w/ little strenght

TV towers —
waves ||
antenna —
different lenghts = different channels
b/c no good amplifers back then

Demonstration: Microwave Polarization

P33-20

signal not lost when rotate
antena b/c of bouncing off
of stuff

Experiment 10: Microwaves

P33-21

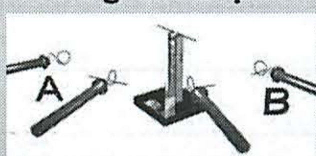
How do signals go through walls?
- why not?
- hand blocks signal
- textbook does not block
- metal bars |||||| block
" " ☰ not block

PRS Questions: Angular Distribution & Polarization of Radiation

P33- 22

hand asorbs waves ← conductive
book does not ↑
hand asorbs the wave

PRS: Angular Dependence



As you moved your receiving antenna around the spark gap transmitting antenna as above, you saw

- 0% 1. Increased power at B compared to A
- 0% 2. Decreased power at B compared to A
- 0% 3. No change in power at B compared to A
- 0% 4. I don't know

:20

polarization |
so TTTTT stops it
electrons have a lot of space to move up + down
not much to asorb E since not much up + down

metal sheet any direction asorbs

the longer the wavelenght the harder it is to get through walls

PRS: Polarization

20

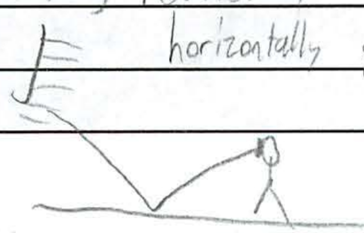
When located as shown, your receiving antenna saw maximum power when oriented such that

- 0% 1. Its straight portion was parallel to the straight portion of the transmitter
- 0% 2. Its straight portion was perpendicular to the straight portion of the transmitter
- 0% 3. I don't know

P33- 24

polarized sun glasses
- block light in a certain direction
- helpful on water, snow
- cutting reflection

horizontally polarized light



blocks glare off of horizontal surfaces

Interference

How in the world do we measure 1/10,000 of a cm?

Visible (red) light:

$$f_{red} = 4.6\times10^{14}\ Hz \quad \lambda_{red} = \frac{c}{f} = 6.54\times10^{-5}\ cm$$

How do waves interact?

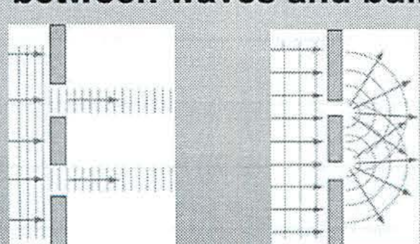
We Use Interference

This is also how we know that light is a wave phenomena

Brief Comment: What is light?

Have more than one source
How do they interact?

Interference: The difference between waves and bullets



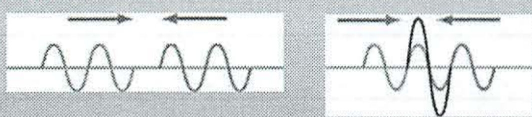
No Interference: if light were made up of bullets

Interference: If light is a wave we see spreading and addition and subtraction

Interference

Interference: Combination of two or more waves to form composite wave – use superposition principle.

Waves can add *constructively* or *destructively*



Conditions for interference:

1. **Coherence**: the sources must maintain a constant phase with respect to each other
2. **Monochromaticity**: the sources consist of waves of a single wavelength

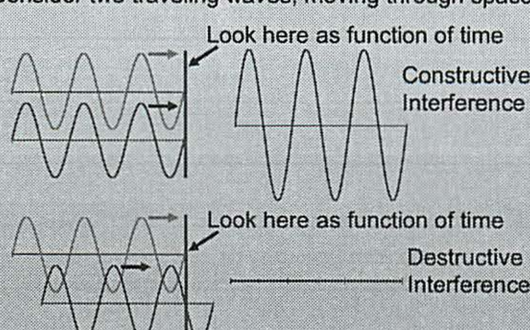
P33- 28

Demonstration: Microwave Interference

P33- 29

Interference – Phase Shift

Consider two traveling waves, moving through space:



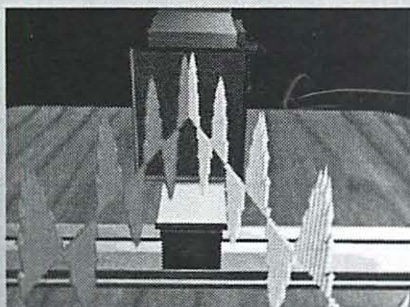
P33- 30

in phase

← add in nice way → double

← add to 0

Microwave Interference



as more constructive sometimes destructive other times

Interference – Phase Shift

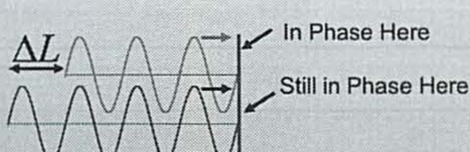
What can introduce a phase shift?

1. From different, out of phase sources → very difficult
2. Sources in phase, but travel different distances
   1. Thin films
   2. Coming from different locations → different distances

Microwave demonstration

Double slit or diffraction grating

Extra Path Length



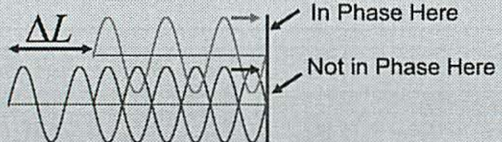
$$\Delta L = m\lambda \quad (m=0, \pm 1, \pm 2\ldots)$$

$$\Downarrow$$

Constructive Interference

adds phase shift

Extra Path Length



$$\Delta L = \left(m + \tfrac{1}{2}\right)\lambda$$

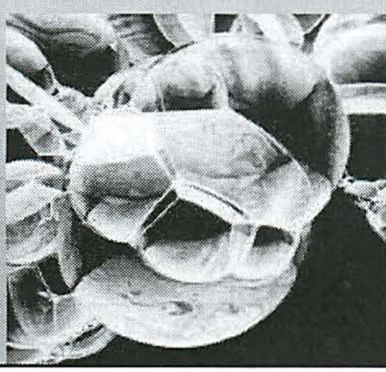
(m=0, ±1, ±2...)

⇓

Destructive Interference

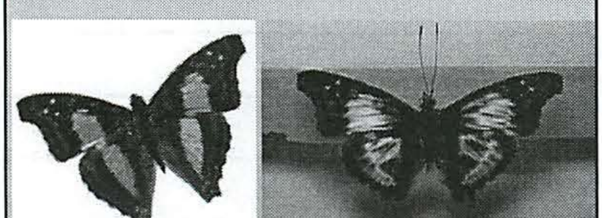
π phase shift

will always talk about the 2 extreame case



soap bubble

different thickness = different color



oil slick
diff colors = diff thickness.

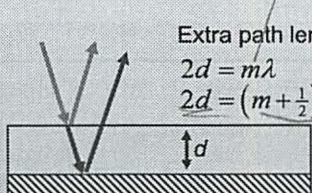
integral

Thin Film: Extra Path

Extra path length ~ $2d$

$2d = m\lambda \Rightarrow$ Constructive

$2d = (m+\frac{1}{2})\lambda \Rightarrow$ Destructive



Oil on concrete, non-reflective coating on glass, etc.

R O Y G B I V

Red $\lambda \sim 700$ nm (long) — Violet $\lambda \sim 400$ nm (short)

diffraction

Waves gone different distances

oil on concrete
soap bubble

gets too thick → becomes white
thin → see one color

Phase Shift = Extra Path?

What is exact relationship between ΔL & ϕ?

$$\sin(k(x+\Delta L)) = \sin(kx + k\Delta L)$$

$$= \sin(kx + \frac{2\pi}{\lambda}\Delta L) \equiv \sin(kx+\varphi)$$

$$\boxed{\frac{\Delta L}{\lambda} = \frac{\phi}{2\pi}} = \begin{cases} m & \text{constructive} \\ m+\frac{1}{2} & \text{destructive} \end{cases}$$

2π over → no change

Where is an oil puddle deeper?

shorter λ shallower

red → blue → red

red blue red

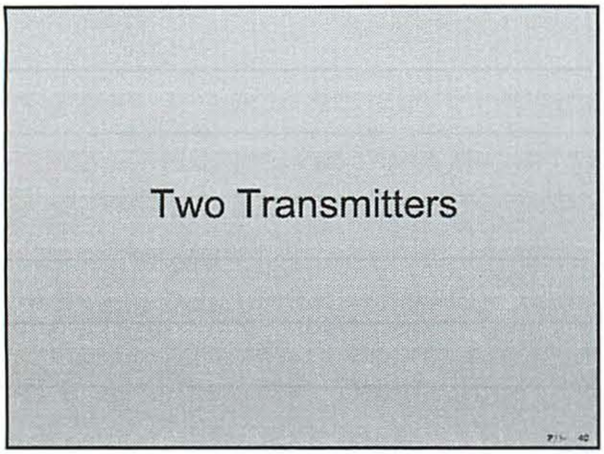
2h 2d

short shift

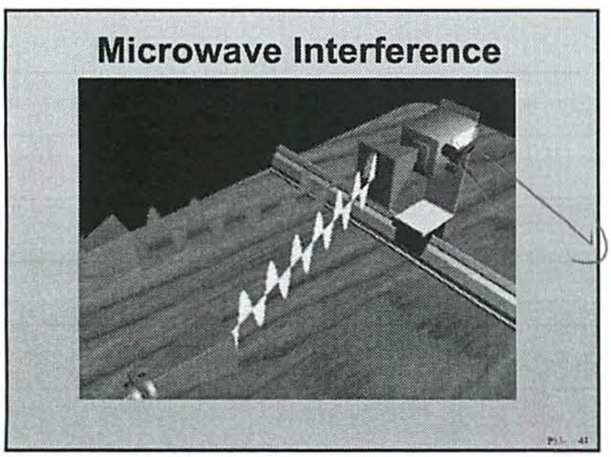
$2d = m(700nm)$

$2h = m(400nm)$

$2h = (m-1)\,700nm$

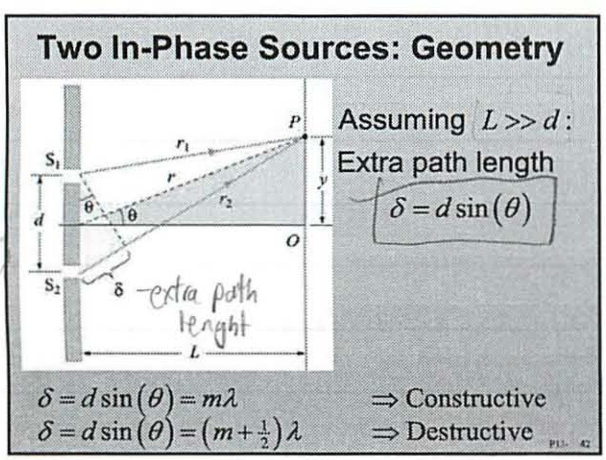


Video



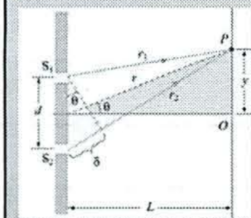
distances are changing (transmitter → reciever)

Slits or transmitter



δ - extra path lenght

Two Sources in Phase



Assume $L >> d >> \lambda$

$$y = L\tan\theta \approx L\sin\theta$$

$$\Rightarrow \delta = d\sin\theta = dy/L$$

(1) Constructive: $\delta = m\lambda$

$$y_{constructive} = m\frac{\lambda L}{d} \quad m = 0,1...$$

(2) Destructive: $\delta = (m + 1/2)\lambda$

$$y_{destructive} = \left(m + \frac{1}{2}\right)\frac{\lambda L}{d} \quad m = 0,1,...$$

P33- 43

integer

$\frac{1}{2}$ integer

1. Identify extra path lenght
2. Is it m or $\frac{m}{2}$ shift?

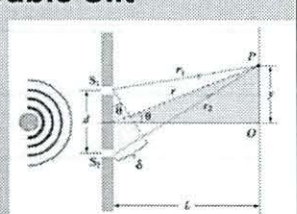
PRS Question
Two Slits with Width

P33- 44

* Different distances change phase *

PRS: Double Slit

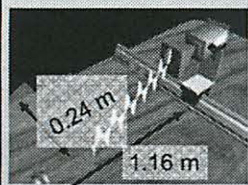
Coherent monochromatic plane waves impinge on two apertures separated by a distance d. An approximate formula for the path length difference between the two rays shown is



0% 1. d sin θ
0% 2. L sin θ
0% 3. d cos θ
0% 4. L cos θ
0% 5. Don't have a clue.

:20

Group Problem: Lecture Demo



We just found that

$$y_{destructive} = \left(m + \frac{1}{2}\right)\frac{\lambda L}{d} \quad m = 0, 1, \ldots$$

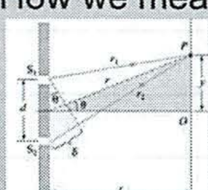
For m = 0 (the first minimum):

$$y_{destructive} = \lambda L / 2d$$

From our lecture demo, estimate the wavelength & frequency of our microwaves.

The Light Equivalent: Two Slits

How we measure 1/10,000 of a cm



Question: How do you measure the wavelength of light?

Answer: Do the same experiment we just did (with light)

First $y_{destructive} = \lambda L / 2d$

λ is smaller by 10,000 times.

But d can be smaller (0.1 mm instead of 0.24 m)

So y will only be 10 times smaller – **still measurable**

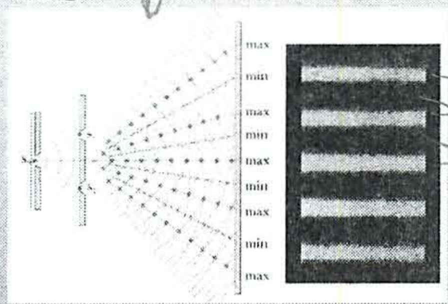
light → 4-500 nm
microwave lower freq
So longer wave lenght

first used for radar

very small slits close together

did not print well

Young's Double-Slit Experiment



Bright Fringes: Constructive interference
Dark Fringes: Destructive interference

P33-49

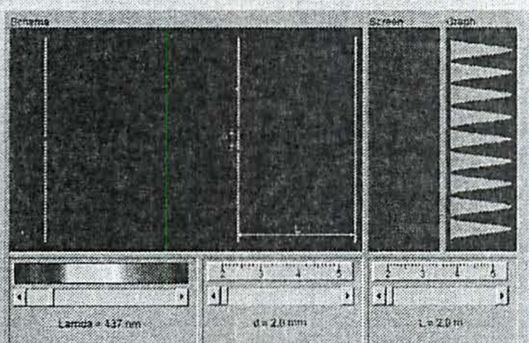
laser on wall

max = visible

min = not visible

lab - measure wave lenght of light w/ slits
- read pre lab

Double Slit Interference



Source of Applet:

P33-50

Is light particles or wave
quantum mechanics now says both

Lecture Demonstration:
Double Slit

P33-51

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Experiment 10: Microwaves

OBJECTIVES

1. To observe the polarization and angular dependence of radiation from a microwave generator

PRE-LAB READING

INTRODUCTION

Heinrich Hertz first generated electromagnetic waves in 1888, and we replicate Hertz's original experiment here. The method he used was to charge and discharge a capacitor connected to a spark gap and a quarter-wave antenna. When the spark "jumps" across the gap the antenna is excited by this discharge current, and charges oscillate back and forth in the antenna at the antenna's natural resonance frequency. For a brief period around the breakdown ("spark"), the antenna radiates electromagnetic waves at this high frequency. We will detect and measure the wavelength λ of these bursts of radiation. Using the relation $f\lambda = c = 3\times10^{10}\,\text{cm/s}$, we will then deduce the natural resonance frequency of the antenna, and show that this frequency is what we expect on the basis of the very simple considerations given below.

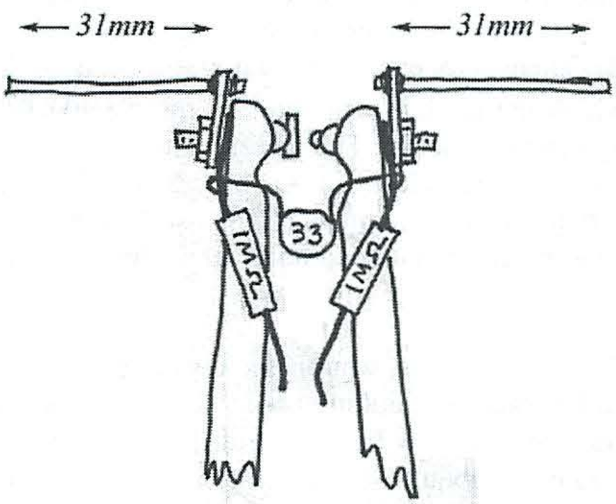


Figure 1 Spark-gap transmitter. The "33" is a 33 pF capacitor. It is responsible for storing energy to be rapidly discharged across a "spark gap," formed by two tungsten cylinders pictured directly above it (one with a vertical axis, one horizontal). Two MΩ resistors limit current off of the capacitor and back out the leads, protecting the user from shocks from the 800 V to which the capacitor will be charged. They also limit radiation at incorrect frequencies.

The 33-pF capacitor shown in fig. 1 is charged by a high-voltage power supply on the circuit board provided. This HVPS voltage is typically 800 V, but this is safe because the current from the supply is limited to a very small value. When the electric field that this voltage generates in the "spark gap" between the tungsten rods is high enough (when it exceeds the breakdown field of air of about 1000 V/mm) the capacitor discharges across the gap (fig. 2a). The voltage on the capacitor then rebuilds, until high enough to cause another spark, resulting in a continuous series of charges followed by rapid bursts of discharge (fig. 2b).

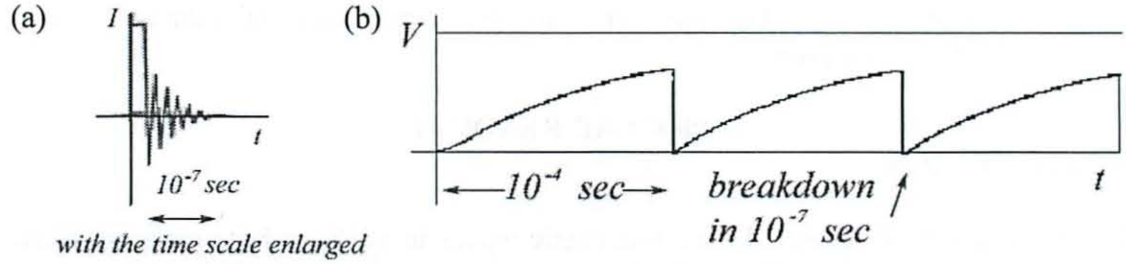


Figure 2 Charging and Discharging the Capacitor. The capacitor is slowly charged (limited by the RC time constant, with R = 4.5 MΩ) and then (a) rapidly discharges across the spark gap, resulting in (b) a series of slow charge/rapid discharge bursts. This is an example of a "relaxation oscillator."

The radiation we are seeking is generated in this discharge.

Resonant Frequency of the Antenna

The frequency of the radiation is determined by the time it takes charge to flow along the antenna. Just before breakdown, the two halves of the antenna are charged positive and negative (+,−) forming an electric dipole. There is an electric field in the vicinity of this dipole. During the short time during which the capacitor discharges, the electric field decays and large currents flow, producing magnetic fields. The currents flow through the spark gap and charge the antenna with the opposite polarity. This process continues on and on for many cycles at the resonance frequency of the antenna. The oscillations damp out as energy is dissipated and some of the energy is radiated away until the antenna is finally discharged.

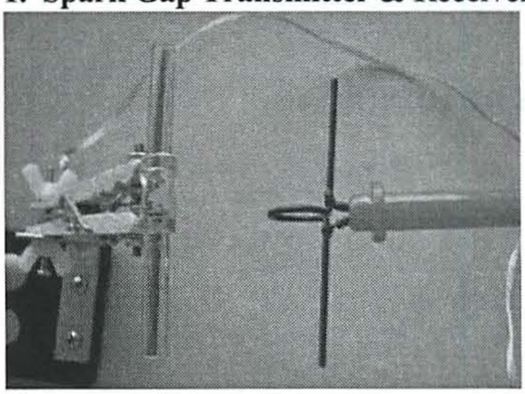
How fast do these oscillations take place – that is, what is the resulting frequency of the radiated energy? An estimate can be made by thinking about the charge flow in the antenna once a spark in the gap allows charge to flow from one side to the other. If l is the length of one of the halves of the antenna (about $l = 31$ mm in our case), then the distance that the charge oscillation travels going from the (+,−) polarity to the (−,+) polarity and back again to the original (+,−) polarity is $4l$ (from one tip of the antenna to the other tip and back again). The time T it takes for this to happen, assuming that information flows at the speed of light c, is $T = 4l / c$, leading to electromagnetic radiation at a frequency of $1/T$.

Detecting (Receiving) the Radiation

In addition to generating EM radiation we will want to detect it. For this purpose we will use a receiving antenna through which charge will be driven by the incoming EM radiation. This current is rectified and amplified, and you will read its average value on a multimeter (although the fields come in bursts, the multimeter will show a roughly constant amplitude because the time between bursts is very short

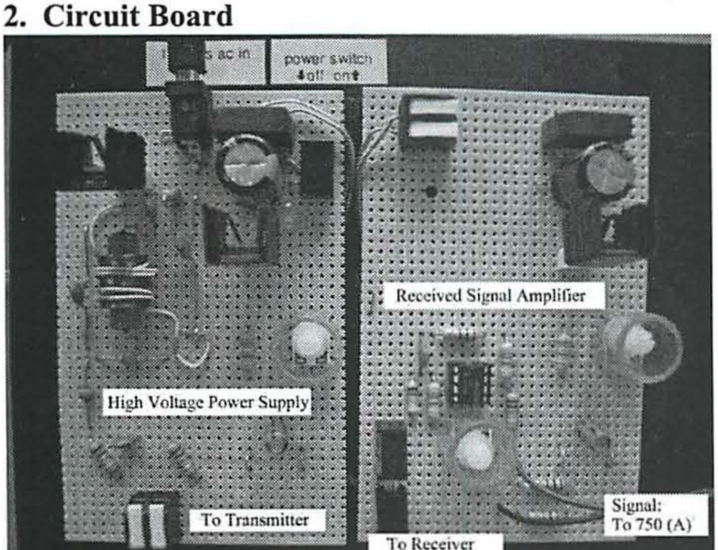
APPARATUS

1. Spark Gap Transmitter & Receiver



These have been described in detail above. The spark gap of the transmitter (pictured left) can be adjusted by turning the plastic wing nut (top). It is permanently wired in to the high voltage power supply on the circuit board. The receiver (pictured right) must be plugged in to the circuit board.

2. Circuit Board



↑ seperate board

This board contains a high voltage power supply for charging the transmitter, as well as an amplifier for boosting the signal from the receiver. It is powered by a small DC transformer that must be plugged in (AC in). When power is on, the green LED (top center) will glow.

3. Science Workshop 750 Interface and Voltage Probe

We read the signal strength from the receiver – proportional to the radiation intensity at the receiver – by connecting the output (lower right of circuit board) to a voltage probe plugged in to channel A of the 750.

GENERALIZED PROCEDURE

In this lab you will turn on the transmitter, and then, using the receiver, measure the intensity of the radiation at various locations and orientations. It consists of three main parts.

Part 1: Polarization of the Emitted Radiation
In this part you will measure to see if the produced radiation is polarized, and if so, along what axis.

Part 2: Angular Dependence of the Emitted Radiation
Next, you will measure the angular dependence of the radiation, determining if your position relative to the transmitter matters.

END OF PRE-LAB READING

IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

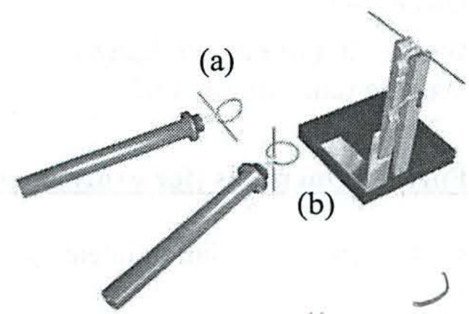
1. Download the LabView file from the web and save the file to your desktop. Start LabView by double clicking on this file.
2. Plug the power supply into the circuit board
3. Plug the receiver into the input jack on the circuit board
4. Turn on the transmitter – a LED will light indicating it is on
5. Adjust the spark gap using the wing nut on the clothespin antenna. Start with a large gap, and close the gap until a steady spark is observed. You should observe a small, steady bright blue light and hear the hum of sparking.
6. Use the receiver to measure the intensity of the radiation as described below

MEASUREMENTS

Part 1: Polarization of the Emitted Radiation

In this part we will measure the polarization of the emitted radiation.

1. Press the green "Go" button above the graph to perform this process.
2. Rotate the receiver between the two orientations (a & b) pictured at right



voltage implies $\vec{E}$

Question 1: DC Volts

Which orientation, if either, results in a larger signal in the receiver?

vertical a

Question 2:

Is the electric field polarized? That is, is it oscillating along a certain direction, as opposed to being unpolarized in which case it points along a wide variety of directions? If it is polarized, along which axis?

polarized along vertical axis

Question 3: see visualization

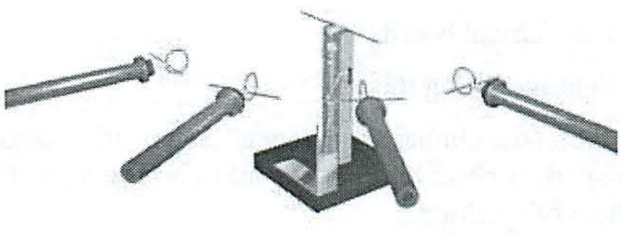
Is the magnetic field polarized? If so, along which axis? How do you know?

yes horizontal axis

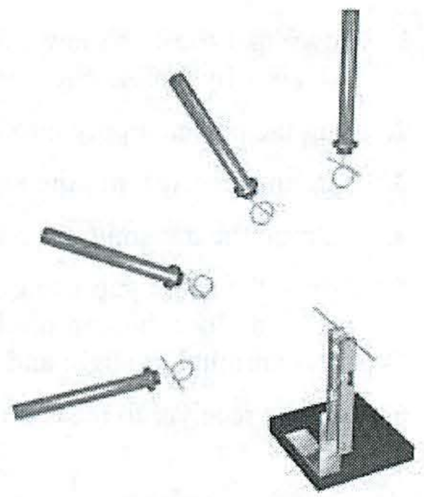
E-M wave

Part 2: Angular Dependence of the Emitted Radiation

1. Now measure the angular dependence of the radiation intensity by moving the receiver along the two paths indicated in the below figures.



Angular dependence - Horizontal



Angular dependence - Vertical

Question 4:

Which kind of motion, horizontal or vertical, shows a larger change in radiation intensity over the range of motion? ~~No angular dependence~~

Further Questions (for experiment, thought, future exam questions…)

- Is there any radiation intensity of any polarization off the ends of the antenna?
- An antenna similar to this was used by Marconi for his first transatlantic broadcast. What issues would you face to receive such a broadcast?

~~Voltage reading same everywhere~~

horizontal very little change

vertical decreased

light on

light off

walk away gets dimmer
but then comes back
– nodes + maxima

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Experiment 9 *Solutions*: Microwaves

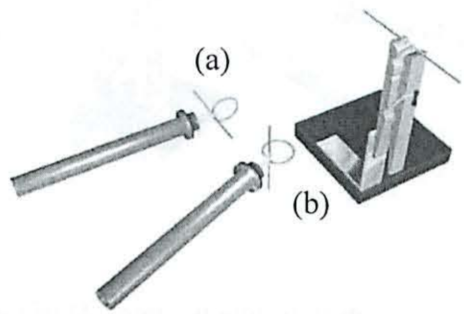
IN-LAB ACTIVITIES

MEASUREMENTS

Part 1: Polarization of the Emitted Radiation

In this part we will measure the polarization of the emitted radiation.

1. Press the green "Go" button above the graph to perform this process.
2. Rotate the receiver between the two orientations (a & b) pictured at right



Question 1:

Which orientation, if either, results in a larger signal in the receiver?

The reception is largest in orientation (a), where the receiver is parallel to the antenna.

Question 2:

Is the electric field polarized? That is, is it oscillating along a certain direction, as opposed to being unpolarized in which case it points along a wide variety of directions? If it is polarized, along which axis?

Yes, the electric field is polarized along the axis of the antenna.

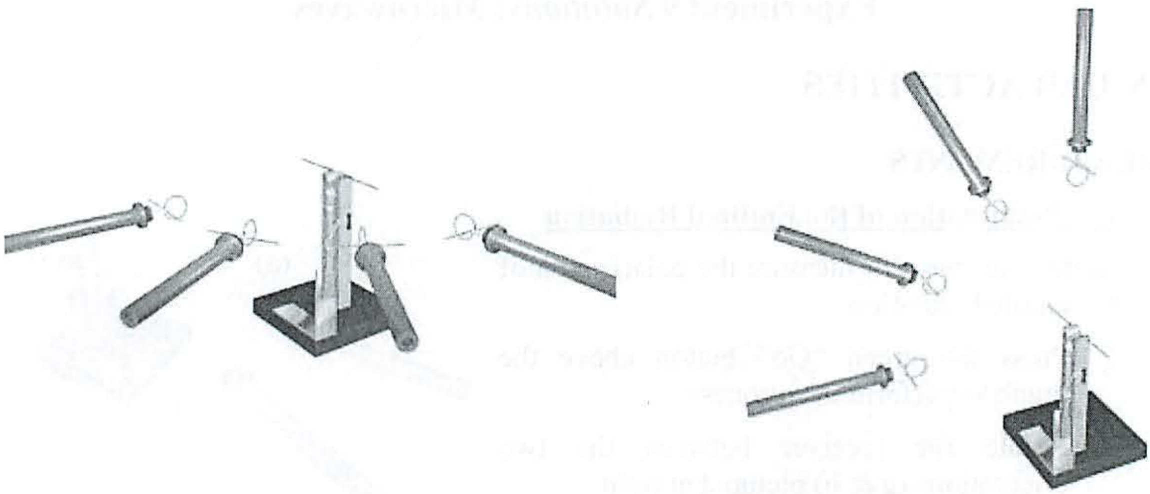
Question 3:

Is the magnetic field polarized? If so, along which axis? How do you know?

Yes, if the electric field is polarized the magnetic field must also be polarized, perpendicular to both the electric field and the direction of propagation (that is, along the axis of the receiver pictured in orientation b).

Part 2: Angular Dependence of the Emitted Radiation

1. Now measure the angular dependence of the radiation intensity by moving the receiver along the two paths indicated in the below figures.



Angular dependence - Horizontal　　　Angular dependence - Vertical

Question 4:

Which kind of motion, horizontal or vertical, shows a larger change in radiation intensity over the range of motion?

The horizontal motion shows a large drop in radiation intensity as the receiver moves towards being perpendicular to the antenna. The vertical motion does not show any noticeable change in radiation intensity.

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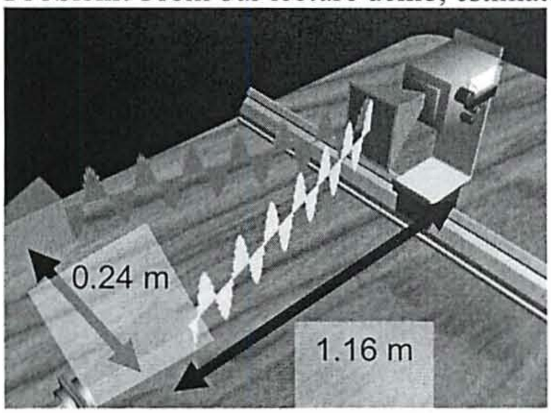
In Class W13D2_2 Solutions: Microwave Lecture Demo

Problem: From our lecture demo, estimate the wavelength & frequency of our microwaves



Solution:
We are able to measure the location of the first minimum, $y_{destructive} \sim x$ cm, which we have previously calculated to be at:

$$y_{destructive} = \lambda L / 2d$$

$$\lambda = \frac{2 d y_{destructive}}{L} \approx \frac{2(0.24\,\text{m})(x\,\text{cm})}{(1.16\,\text{m})} \approx \text{ cm}$$

Dormaskin Review Waves

Waves Review (for final)

- not P-set really

Tue: Poying Vector basic cases

Wed: RLC

~~Explain~~ Core Basic Case

$$\vec{E}(x,t) = E_{y0} \sin(kx - \omega t)\,\hat{j}$$

$$\vec{B}(x,t) = \frac{E_{y0}}{c} \sin(kx - \omega t)\,\hat{k}$$

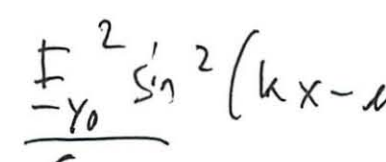
$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} = \frac{E_{y0}^2}{c\mu_0} \sin^2(kx - \omega t)\,\hat{i}$$

Details of deriving wave eq not critical

Emphize solution to wave eq

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \iint \vec{B} \cdot d\vec{a}$$

↓ deemphize



becomes partial diff eq

$$= \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$ (involves E + B)

don't worry about deviation
- go back to Tve if question

Ampere's Law (generalized)
- Tve will be covered

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \frac{d}{dt} \iint \vec{E} \cdot d\vec{s}$$

#

$$-\frac{\partial B_z}{\partial x} = \mu_0 \epsilon_0 \frac{\partial E_y}{\partial t}$$

d → ∂

↑ space ↑ time

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

Put 2 ☐ boxed eq together

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial^2 E_z}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 B_z}{\partial t^2}$$

specific case

(3)

$$\frac{\partial^2 E_y}{\partial z^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial x^2} = \frac{1}{c^2}\frac{\partial E_y}{\partial t^2}$$ wave eq

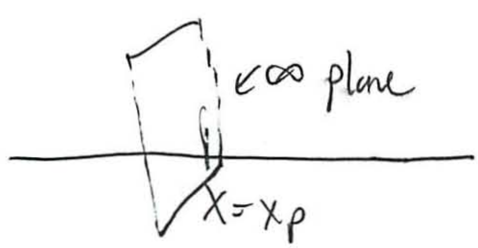
Look at wave solutions

Combining 2 of maxwell's eq

← could prove, but did not

Special case: plane wave

- easy to work with
- lots of types of waves
- only depends on X
- independent of Y, Z

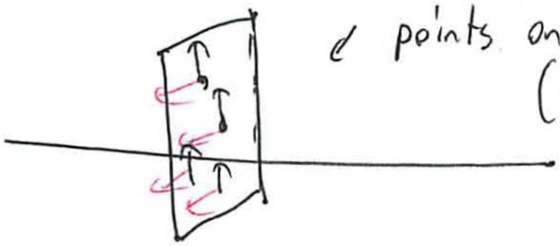


Electric field → vector field
- direction
- magnitude

$E_y(x, t)$

only Y component

(4)



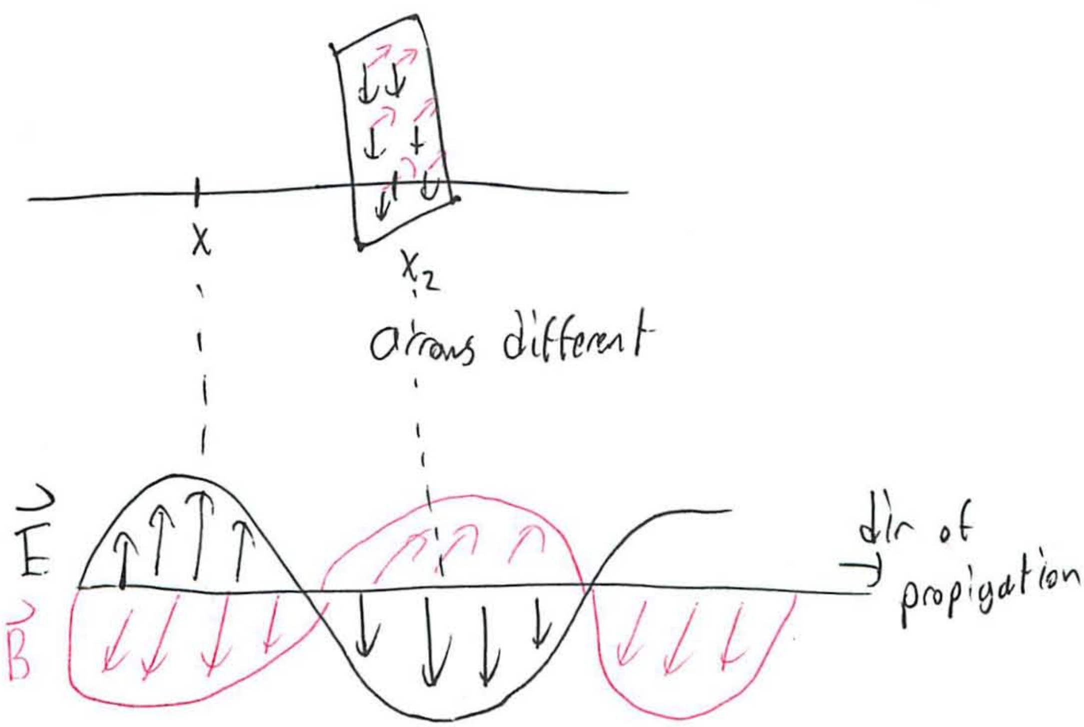
points on a plane
(x_p, y, z)

↑ at different pl arrow is same
Only dependent on x

arrows same everywhere on plane
same $\vec{B}$ field everywhere too

every point
same force

((fake wave, can't generate, ignore generation)



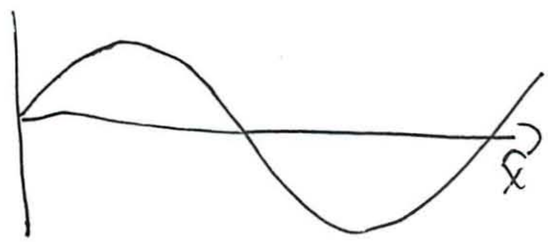
(5)

Direction of Propigation

$E_y(x,t) = E_{y0} \sin(kx - \omega t)$

Draw a picture at $t=0$

$E_y(x, t=0) = E_{y0} \sin(kx)$



$t=0$

$\sin(0) = 0$ at $x=0$
$\sin(kx) = 0$ at $t=0$
) all arrows 0 at origin

Now let t be positive
- where does plane of waves go?

$\sin(kx - \omega t) = 0$

$\sin(0) = 0$

$kx - \omega t = 0$

$x = \frac{\omega}{k} t$

$$\boxed{\frac{\omega}{k} = C}$$

6

Write solution

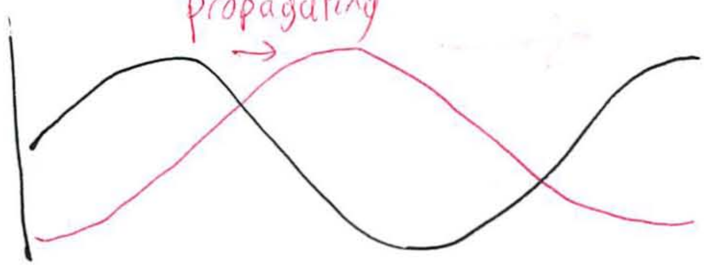
$$E_y = E_{y0} \sin(kx - \omega t)$$

$$-k^2 E_{y0} \sin(kx - \omega t) = \frac{1}{c^2}(-\omega)^2 E_{y0} \sin(kx - \omega t)$$

try taking 2 derivs

$$= \quad k^2 = \frac{\omega^2}{c^2} \rightarrow \boxed{\frac{\omega}{k} = c}$$

nessarry to solve maxwell's eq



$\vec{E}$ pointing $\pm \hat{y}$

propagating $+\hat{x}$

$E_{y0} \sin(-kx - \omega t)\ \hat{j}$

$E_{y0} \sin(kx + \omega t)\ \hat{j}$ } same, traveling $-\hat{x}$

$E_{y0} \sin(kx - \omega t)\ \hat{j}$

$E_{y0} \sin(-kx + \omega t)\ \hat{j}$ } same, $+\hat{x}$

remember $\sin(-a) = -\sin(a)$

7)

remember 2 dir → dir of field
dir of propagation

Some trials

$E(y,t) = E_{0z} \sin(-ky - \omega t)\, \hat{k}$

pointing $\pm \hat{k}$

propagating $-\hat{y}$ minus since signs same

associated in $\vec{B}$ field

~~B(y,t)~~

$B(y,t) = \frac{E_{z0}}{c} \sin(-ky - \omega t)$

pointing - assume both are ⊕
- same for both

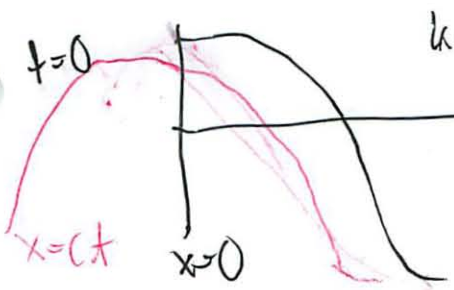
dir $\vec{E}$ × dir $\vec{B}$ = dir propagating

$\hat{k} \times \underline{\quad} = -\hat{j}$

$-\hat{i}$ ← direction $\vec{B}$ pointing

$E_y \cos(-kx - \omega t) = 0$

$E_{y0} \cos(-kx) = E_y \cos(kx)$



$kx + 70$

$\cos(0) = 1$

$-kx - \omega t = 0$

$x = \frac{-\omega}{k} t = -c\,t$

(8)

← traveling x (dir propigation)

$\vec{E} = E_{y0} \sin(kx - \omega t)\,\hat{j} + E_{y0}(kx + \omega t)\,\hat{j}$

- 2 waves traveling opposite dir

$= E_{y0}\left(\sin(kx - \omega t) + \sin(kx + \omega t)\right)\hat{j}$

- standing wave
- like tlw
-

Write $\vec{B}$ field associated w/ each

Do the "thing" we talked about

$\vec{B} = \frac{E_{y0}}{c} \sin(kx - \omega t)\,\hat{k} - \frac{E_{y0}}{c} \sin(kx + \omega t)\,\hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$ $\quad \hat{j} \times (-\hat{k}) = -\hat{i}$

↑ fill in

$\vec{E} \times \vec{B}$ = dir propigation

$= \frac{E_{y0}}{c}\left(\sin(kx - \omega t) - \sin(kx + \omega t)\right)\hat{k}$

Waves traveling in opposite dir creates standing wave

Waves bounce off ends

w/ nodes not interfearing

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 E_y}{\partial t^2}$$ wave eq

* plug in eq to see if it satisfies

Guess $E_y = E_{y0} \sin(kx - \omega t)$

(9)

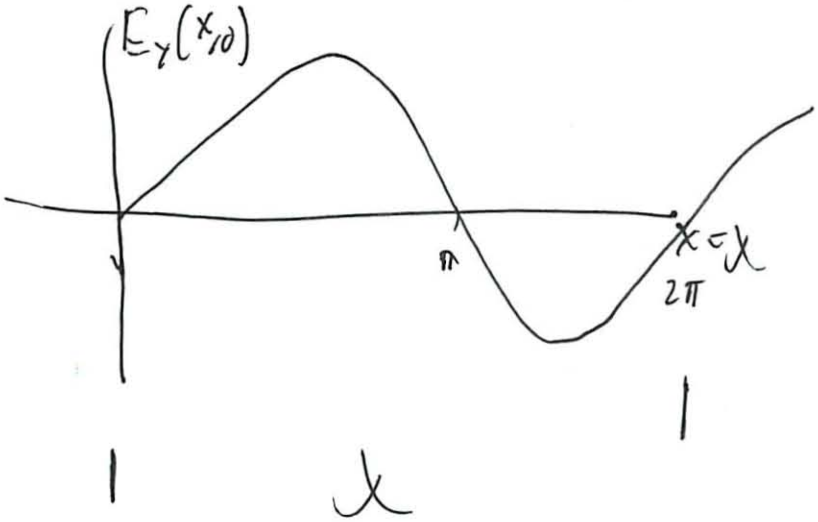
What k mean?

- ask about peridicity

$\tilde{E} = E_{y0} \sin(kx - \omega t)\,\hat{j}$

- Set $t=0$ (take picture/snapshot)

$\vec{E}(x,0) = E_{y0} \sin(kx)\,\hat{j}$



has a periodicity - repeats ↶ distance = λ

$\sin(kx) = ?$

$\sin(0) = 0$

$\sin(kx = \pi) = 0$

$\sin(kx = 2\pi) = 0$

$\boxed{k\lambda = 2\pi}$ $k = \frac{2\pi}{\lambda}$ "angular wave #"

or angular freq in space

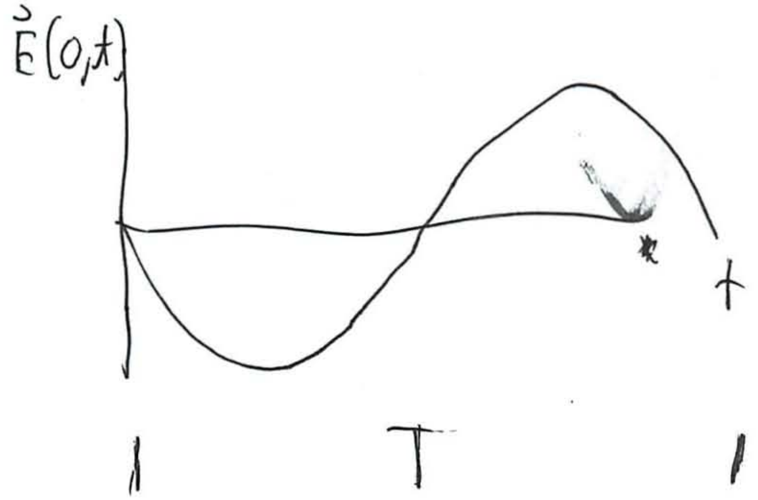
(10)

What is ω mean ?

You are sitting in ocean
waves going by
you are bobbing up + down

$\hat{x} = 0$ plane

$$\vec{E}(0, t) = E_{y_0} \sin(-\omega t)\,\hat{\jmath}$$
$$= -E_{y_0} \sin(\omega t)\,\hat{\jmath}$$



$\omega T = 2\pi$
$\sin(2\pi) = 0$

$$\boxed{\omega = \frac{2\pi}{T}}$$ angular freq

(11)

Summary

$$f = \frac{1}{T}$$

$$\frac{w}{k} = c \qquad w = \frac{2\pi}{T} = 2\pi f$$

$$k = \frac{2\pi}{\lambda}$$

$$\frac{\frac{2\pi}{T}}{\frac{2\pi}{\lambda}} = c$$

$$\frac{\lambda}{T} = c \rightarrow \lambda = Tc = \frac{c}{f}$$

$$\lambda f = c$$

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8.02 **Spring 2010**

Problem Set 12

Due: Friday, May 7 at 5 pm.

Hand in your problem set in your section slot in the boxes outside the door of 32-082. Make sure you clearly write your name and section on your problem set.

Text: Liao, Dourmashkin, Belcher; Introduction to E & M MIT 8.02 Course Notes.

Week Fourteen Maxwell's Equations

Class 32 W14D1 M/T May 3/4	Generating EM Waves
Reading:	Course Notes: Sections 13.3-13.4, 13.6-13.8.1, 13.10
Class 33 W14D2 W/R May 5/6	Dipole Radiation; Expt. 10 MW Polarization; Interference
Reading:	Course Notes: Sections 13.8, 14.1-14.3, 14.11.1-14.11.3
Experiment:	Expt. 10 MW Polarization
Class 34 W14D3 F May 7	PS10 E&M Waves
Reading:	Course Notes: Sections 13.11, 14.1-14.3, 14.11.1-14.11.3

Week Fifteen Interference and Diffraction; Final Review

Class 35 W15D1 M/T May 10/11	Diffraction; Expt. 11: Interference and Diffraction
Experiment:	Expt. 11: Interference and Diffraction
Reading:	Course Notes: Chapter 14
Class 36 W15D2 W/R May 12/13	Final Exam Review

Final Exam Johnson Athletic Center
Monday Morning May 17 from 9 am-12 noon

Problem 1: Read Experiment 9: Interference and Diffraction.

1. Measuring the Wavelength of Laser Light

In the first part of this experiment you will shine a red laser through a pair of narrow slits ($a = 40$ μm) separated by a known distance (you will use both $d = 250$ μm and 500 μm) and allow the resulting interference pattern to fall on a screen a distance L away ($L \sim 40$ cm). This set up is as pictured in Fig. 2 (in the "Two Slit Interference" section above).

(a) Will the center of the pattern (directly between the two holes) be an interference minimum or maximum?

(b) You should be able to easily mark and then measure the locations of the interference maxima. For the sizes given above, will these maxima be roughly equally spaced, or will they spread out away from the central peak? If you find that they are equally spaced, note that you can use this to your advantage by measuring the distance between distance maxima and dividing by the number of intermediate maxima to get an average spacing. If they spread out, which spacing should you use in your measurement to get the most accurate results, one close to the center or one farther away?

(c) Approximately how many interference maxima will you see on one side of the pattern before their intensity is significantly reduced by diffraction due to the finite width a of the slit?

(d) Derive an equation for calculating the wavelength λ of the laser light from your measurement of the distance Δy between interference maxima. Make sure that you keep a copy of this equation in your notes! You will need it for the lab.

(e) In order to most accurately measure the distance between maxima Δy, it helps to have them as far apart as possible. (Why?) Assuming that the slit parameters and light wavelength are fixed, what can we do in order to make Δy bigger? What are some reasons that can we not do this ad infinitum?

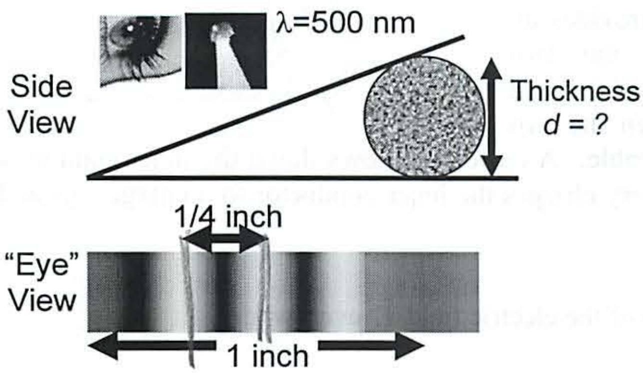
2. Single Slit Interference

Now that you have measured the wavelength λ of the light you are using, you will want to measure the width of some slits from their diffraction pattern. When measuring diffraction patterns (as opposed to the interference patterns of problem 1) it is typically easiest to measure between diffraction minima.

(a) Derive an equation for calculating the width a of a slit from your measurement of the distance Δy between diffraction minima. Make sure that you keep a copy of this equation in your notes! You will need it for the lab. Note that this same equation will be used for measuring the thickness of your hair.

(b) What is the width of the central maximum (the distance on the screen between the m=-1 and m=1 minima)? How does this compare to the distance Δy between other adjacent minima?

3. Another Way to Measure Hair



In addition to using hair as a thin object for diffraction, you can also measure its thickness using an interferometer. In fact, you can use this to measure even smaller objects. Its use on a small fiber is pictured at left. The fiber is placed between two glass slides, lifting one at an angle relative to the other. The slides are illuminated with green light from above, and when the set-up is viewed from above, an interference pattern, pictured in the "Eye View", appears.

What is the thickness d of the fiber?

4. CD

In the last part of this lab you will reflect light off of a CD and measure the resulting interference pattern on a screen a distance $L \sim 5$ cm away.

(a) A CD has a number of tracks, each of width d (this is what you are going to measure). Each track contains a number of bits, of length $l \sim d/3$. Approximately how many bits are there on a CD? In case you didn't know, CDs sample two channels (left and right) at a rate of 44100 samples/second, with a resolution of 16 bits/sample. In addition to the actual data bits, there are error correction and packing bits that roughly double the number of bits on the CD.

(b) What, approximately, must the track width be in order to accommodate this number of bits on a CD? In case you don't have a ruler, a CD has an inner diameter of 40 mm and an outer diameter of 120 mm.

(c) Derive an equation for calculating the width d of the tracks from your measurement of the distance Δy between interference maxima. Make sure that you keep a copy of this equation in your notes! You will need it for the lab.

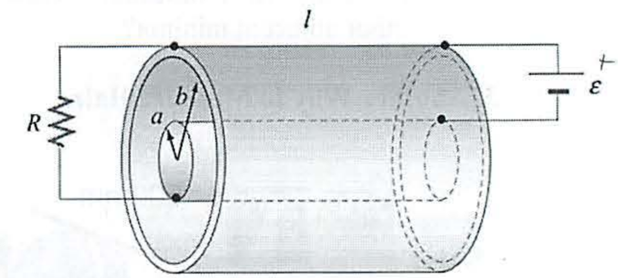
(d) Using the previous results, what approximately will the distance between interference maxima Δy be on the screen?

diffraction

radius does not really matter
factor of 3 $\frac{1}{r}$
relatively small

Problem 2: Coaxial Cable and Power Flow

A coaxial cable consists of two concentric long hollow cylinders of zero resistance; the inner has radius a, the outer has radius b, and the length of both is l, with $l >> b$, as shown in the figure. The cable transmits DC power from a battery to a load. The battery provides an electromotive force ε between the two conductors at one end of the cable, and the load is a resistance R connected between the two conductors at the other end of the cable. A current I flows down the inner conductor and back up the outer one. The battery charges the inner conductor to a charge $-Q$ and the outer conductor to a charge $+Q$.



(a) Find the direction and magnitude of the electric field $\vec{\mathbf{E}}$ everywhere.

(b) Find the direction and magnitude of the magnetic field $\vec{\mathbf{B}}$ everywhere.

(c) Calculate the Poynting vector $\vec{\mathbf{S}}$ in the cable.

(d) By integrating $\vec{\mathbf{S}}$ over appropriate surface, find the power that flows into the coaxial cable.

(e) How does your result in (d) compare to the power dissipated in the resistor?

Problem 3: ***Standing Waves*** The electric field of an electromagnetic wave is given by the superposition of two waves

$$\vec{\mathbf{E}} = E_0 \cos(kz - \omega t)\,\hat{\mathbf{i}} + E_0 \cos(kz + \omega t)\,\hat{\mathbf{i}}\,.$$

You may find the following identities and definitions useful

$$\cos(kz + \omega t) = \cos(kz)\cos(\omega t) - \sin(kz)\sin(\omega t)$$
$$\sin(kz + \omega t) = \sin(kz)\cos(\omega t) + \cos(kz)\sin(\omega t)$$

a) What is the associated magnetic field $\vec{\mathbf{B}}(x, y, z, t)$.

b) What is the energy per unit area per unit time (the Poynting vector $\vec{\mathbf{S}}$) transported by this wave?

c) What is the time average of the Poynting $\langle \vec{\mathbf{S}} \rangle$ vector? Briefly explain your answer. Note the time average is given by

$$\langle \vec{\mathbf{S}} \rangle \equiv \frac{1}{T}\int_0^T \vec{\mathbf{S}}\,dt\,.$$

Problem 4: ***Radiation Pressure*** You have designed a solar space craft of mass m that is accelerated by the force due to the 'radiation pressure' from the sun's light that fall on a perfectly reflective circular sail that it is oriented face-on to the sun. The time averaged radiative power of the sun is P_{sun}. The gravitational constant is G. The mass of the sun is m_s. The speed of light is c. Model the sun's light as a plane electromagnetic wave, traveling in the +z direction with the electric field given by

$$\vec{\mathbf{E}}(z,t) = E_{x,0} \cos(kz - \omega t)\,\hat{\mathbf{i}}\,.$$

You may express your answer in terms of the symbols m, $\langle P \rangle$, c, m_s, G, k, and ω as necessary.

a) What is the magnetic field $\vec{\mathbf{B}}$ associated with this electric field?

b) What is the Poynting vector $\vec{\mathbf{S}} = \dfrac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$ associated with this wave? What is the time averaged Poynting vector $\langle \vec{\mathbf{S}} \rangle = \dfrac{1}{T}\int_0^T \vec{\mathbf{S}}\,dt$ associated with this superposition, where T is the period of oscillation. What is the amplitude of the electric field at your starting point?

c) What is the minimum area for the sail in order to exactly balance the gravitational attraction from the sun?

Problem 5. ***Electromagnetic Waves***

The magnetic field of a plane electromagnetic wave is described as follows:

$$\vec{\mathbf{B}} = B_0 \sin(kx - \omega t)\,\hat{\mathbf{j}}$$

a) What is the wavelength λ of the wave?

b) Write an expression for the electric field $\vec{\mathbf{E}}$ associated to this magnetic field. Be sure to indicate the direction with a unit vector and an appropriate sign (+ or –).

c) What is the direction and magnitude Poynting vector associated with this wave? Give appropriate units, as well as magnitude.

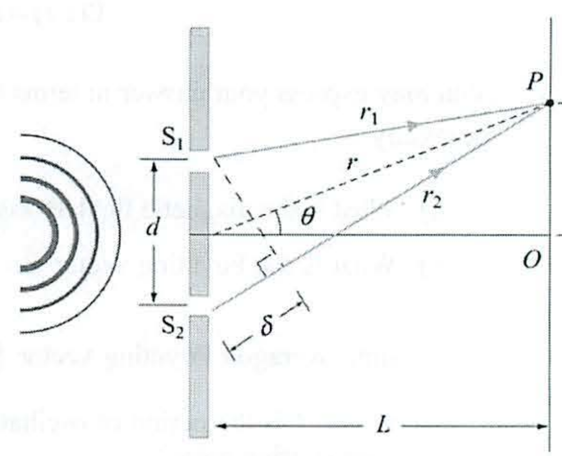
d) This wave is totally reflected by a thin conducting sheet lying in the *y-z* plane at $x = 0$. What is the resulting radiation pressure on the sheet? Give appropriate units, as well as magnitude.

f) The component of an electric field parallel to the surface of an ideal conductor must be zero. Using this fact, find expressions for the electric and magnetic fields for the reflected wave? What are the total electric and magnetic fields at the conducting sheet, $x = 0$. Check that your answer satisfies the condition on the electric field at the conducting sheet, $x = 0$.

g) An oscillating surface current $\vec{\mathbf{K}}$ flows in the thin conducting sheet as a result of this reflection. Along which axis does it oscillates? What is the amplitude of oscillation?

Problem 6: Phase Difference (cf. Section 14.2 of the *Course Notes*)

In the double-slit interference experiment shown in the figure, suppose $d = 0.100\,\text{mm}$ and $L = 1.20\,\text{m}$, and the incident light is monochromatic with a wavelength λ=600 nm.



(a) What is the phase difference between the two waves arriving at a point *P* on the screen when $\theta = 0.800°$?

(b) What is the phase difference between the two waves arriving at a point *P* on the screen when $y = 4.00\,\text{mm}$?

(c) If the phase difference between the two waves arriving at point *P* is ϕ=1/3 rad, what is the value of θ?

(d) If the path difference is $\delta = \lambda/4$, what is the value of θ?

(e) In the double-slit interference experiment, suppose the slits are separated by $d = 1.00\,\text{cm}$ and the viewing screen is located at a distance $L = 1.20\,\text{m}$ from the slits. Let the incident light be monochromatic with a wavelength λ=500 nm. Calculate the spacing between the adjacent bright fringes on the viewing screen.

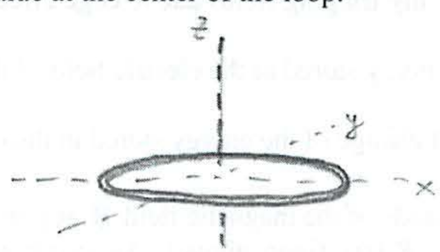
(f) What is the distance between the third-order fringe and the center line on the viewing screen?

Problem 7: ***Loop Antenna***

An electromagnetic wave propagating in air has a magnetic field given by

$$B_x = 0 \qquad B_y = 0 \qquad B_z = B_0 \cos(\omega t - kx).$$

It encounters a circular loop antenna of radius a centered at the origin $(x, y, z) = (0, 0, 0)$ and lying in the x-y plane. The radius of the antenna $a << \lambda$ where λ is the wavelength of the wave. So you can assume that at any time t the magnetic field inside the loop is approximately equal to its value at the center of the loop.



a) What is the magnetic flux, $\Phi_{mag}(t) \equiv \iint_{disk} \vec{B} \cdot d\vec{a}$, through the plane of the loop of the antenna?

The loop has a self-inductance L and a resistance R. Faraday's law for the circuit is

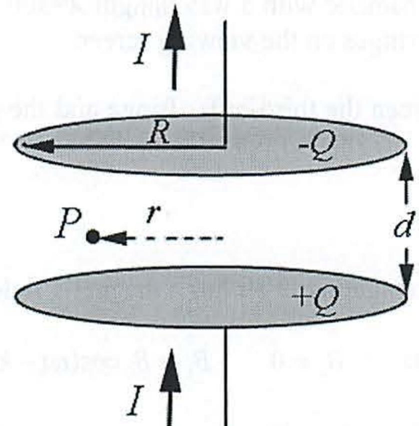
$$IR = -\frac{d\Phi_{mag}}{dt} - L\frac{dI}{dt}.$$

b) Assume a solution for the current of the form $I(t) = I_0 \sin(\omega t - \phi)$ where ω is the angular frequency of the electromagnetic wave, I_0 is the amplitude of the current, and ϕ is a phase shift between the changing magnetic flux and the current.. Find expressions for the constants ϕ and I_0.

c) What is the magnetic field created at the center of the loop by this current $I(t)$?

Problem 8: Charging Capacitor (10 points)

A parallel-plate capacitor consists of two circular plates, each with radius R, separated by a distance d. A steady current I is flowing towards the lower plate and away from the upper plate, charging the plates.



a) What is the direction and magnitude of the electric field $\vec{\mathbf{E}}$ between the plates? You may neglect any fringing fields due to edge effects.

b) What is the total energy stored in the electric field of the capacitor?

c) What is the rate of change of the energy stored in the electric field?

d) What is the magnitude of the magnetic field $\vec{\mathbf{B}}$ at point P located between the plates at radius $r < R$ (see figure above). As seen from above, is the direction of the magnetic field *clockwise* or *counterclockwise*. Explain your answer.

e) Make a sketch of the electric and magnetic field inside the capacitor.

f) What is the direction and magnitude of the Pointing vector $\vec{\mathbf{S}}$ at a distance $r = R$ from the center of the capacitor.

g) By integrating $\vec{\mathbf{S}}$ over an appropriate surface, find the power that flows into the capacitor.

h) How does your answer in part g) compare to your answer in part c)?

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

8.02 **Spring 2010**

Problem Set 12 Solutions

Problem 1: Read Experiment 9: Interference and Diffraction.

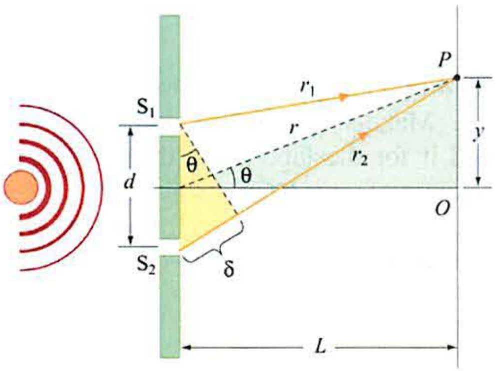
1. Measuring the Wavelength of Laser Light

In the first part of this experiment you will shine a red laser through a pair of narrow slits (a = 40 μm) separated by a known distance (you will use both d = 250 μm and 500 μm) and allow the resulting interference pattern to fall on a screen a distance L away ($L \sim 40$ cm). This set up is as pictured in Fig. 2 (in the "Two Slit Interference" section above).

(a) Will the center of the pattern (directly between the two holes) be an interference minimum or maximum?

The center of the pattern will be a maximum because the waves from both slits travel the same distance to get to the center and hence are in phase.

(b) You should be able to easily mark and then measure the locations of the interference maxima. For the sizes given above, will these maxima be roughly equally spaced, or will they spread out away from the central peak? If you find that they are equally spaced, note that you can use this to your advantage by measuring the distance between distance maxima and dividing by the number of intermediate maxima to get an average spacing. If they spread out, which spacing should you use in your measurement to get the most accurate results, one close to the center or one farther away?



Looking at the picture at left, we get a maximum every time that the extra path length is an integral number of wavelengths:

$$d\sin\theta = m\lambda$$

The spacing is the distance between these locations, $y_{m+1} - y_m$. We can get y_m from θ:

$$\sin\theta_m = \frac{y_m}{\sqrt{L^2 + y_m^2}} = \frac{m\lambda}{d} \equiv \alpha_m \Rightarrow \frac{y_m^2}{L^2 + y_m^2} = \alpha_m^2$$

$$y_m^2\left(1-\alpha_m^2\right) = \alpha_m^2 L^2 \Rightarrow y_m = \frac{\alpha_m L}{\sqrt{1-\alpha_m^2}} \approx \alpha_m L\left(1-\frac{\alpha_m^2}{2}\right)$$

We have made the approximation that $\alpha_m << 1$, which is valid for the wavelengths and slit separations of this lab (it is order 10^{-3}). As long as this approximation is valid, we can also ignore the term that goes like $(\alpha_m)^2$, and hence we find the maxima are equally spaced: $$y_{m+1} - y_m \approx \frac{\lambda L}{d}$$

(c) Approximately how many interference maxima will you see on one side of the pattern before their intensity is significantly reduced by diffraction due to the finite width *a* of the slit?

The first single slit minimum appears at $a\sin\theta = \lambda$. So when we approach:

$m = \frac{d}{\lambda}\sin\theta \approx \frac{d}{\lambda}\frac{\lambda}{a} = \frac{d}{a}$ we will lose signal due to the diffraction minimum.

(d) Derive an equation for calculating the wavelength λ of the laser light from your measurement of the distance Δy between interference maxima. Make sure that you keep a copy of this equation in your notes! You will need it for the lab.

Using what we derived for part b,

$$\Delta y = y_{m+1} - y_m \approx \frac{\lambda L}{d} \quad \Rightarrow \quad \lambda = \frac{d\Delta y}{L}$$

(e) In order to most accurately measure the distance between maxima Δy, it helps to have them as far apart as possible. (Why?) Assuming that the slit parameters and light wavelength are fixed, what can we do in order to make Δy bigger? What are some reasons that can we not do this ad infinitum?

We can increase the distance to the screen and measure the distance between distant interference maxima (e.g. m = 1 and m = 4), which increases distances, making them easier to measure, and then allows us to divide down any measurement errors.

2. Single Slit Interference

Now that you have measured the wavelength λ of the light you are using, you will want to measure the width of some slits from their diffraction pattern. When measuring diffraction patterns (as opposed to the interference patterns of problem 1) it is typically easiest to measure between diffraction minima.

(a) Derive an equation for calculating the width *a* of a slit from your measurement of the distance Δy between diffraction minima. Make sure that you keep a copy of this equation in your notes! You will need it for the lab. Note that this same equation will be used for measuring the thickness of your hair.

Single slit minima obey the relationship $a\sin\theta = m\lambda$, which is the same formula as two slit maxima. So we can calculate the slit width from what we derived in 1b (replacing the distance between the slits *d* with the width of the single slit *a*):

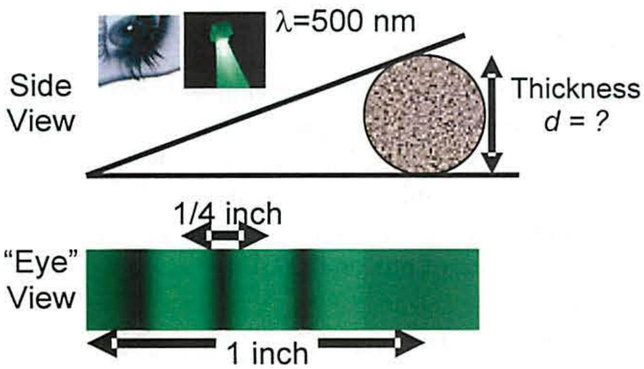
$$a = \frac{\lambda L}{\Delta y}$$

(b) What is the width of the central maximum (the distance on the screen between the m=-1 and m=1 minima)? How does this compare to the distance Δy between other adjacent minima?

The central minimum is twice as wide as the distance between other minima. It is:

$$\Delta y_{\text{central}} = 2\frac{\lambda L}{a}$$

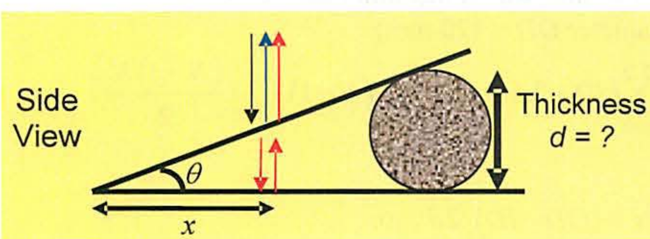
3. Another Way to Measure Hair



In addition to using hair as a thin object for diffraction, you can also measure its thickness using an interferometer. In fact, you can use this to measure even smaller objects. Its use on a small fiber is pictured at left. The fiber is placed between two glass slides, lifting one at an angle relative to the other. The slides are illuminated with green light from above, and when the set-up is viewed from above, an interference pattern, pictured in the "Eye View", appears.

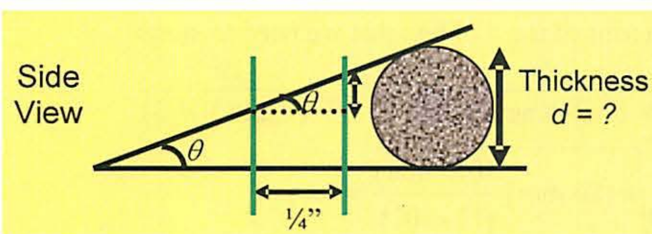
What is the thickness *d* of the fiber?

The interference comes about because there are two paths the light can take. In the first light goes straight down, reflects off the glass, and goes straight back (we ignore the slight angle). In the second light goes down, passes through the glass and reflects off the lower glass, then goes straight back up. Let's redraw the picture as follows:



The light comes in (black arrow) and splits into two parts: immediate reflection (blue) and pass through then reflection (red). They eventually meet up to interfere. The extra path length taken by the second wave (red) is twice the height at that location, or $\delta = 2x\tan(\theta)$

Now consider two adjacent maxima, which apparently are about ¼ inch apart:



Notice that the extra height from the first to the second max (as indicated by the vertical arrow) is related to the distance between the successive maxima by:

$$\Delta h = \frac{\lambda}{2} = \frac{1}{4}\text{inch}\cdot\tan(\theta)$$

Why λ/2? Because the extra path (which is twice Δh) must be λ – one extra wavelength moves from one constructive maximum to the next. So:

$$d = 1\text{ inch}\cdot\left(\frac{\lambda}{2}\Big/\frac{1}{4}\text{inch}\right) = 2\lambda = 1000\,\text{nm} = 1\,\mu\text{m}$$

4. CD

In the last part of this lab you will reflect light off of a CD and measure the resulting interference pattern on a screen a distance $L \sim 5$ cm away.

(a) A CD has a number of tracks, each of width d (this is what you are going to measure). Each track contains a number of bits, of length $l \sim d/3$. Approximately how many bits are there on a CD? In case you didn't know, CDs sample two channels (left and right) at a rate of 44100 samples/second, with a resolution of 16 bits/sample. In addition to the actual data bits, there are error correction and packing bits that roughly double the number of bits on the CD.

A CD can store about 74 minutes of music, so:

$$\#\text{bits} \approx (74\text{ min})\left(60\frac{\text{s}}{\text{min}}\right)\left(44100\frac{\text{samp}}{\text{sec}}\right)\left(16\frac{\text{data bits}}{\text{samp}\cdot\text{chan}}\right)(2\text{ chan})\left(2\frac{\text{bits}}{\text{data bits}}\right)$$

$$\approx 12\times 10^9\text{ bits}$$

(b) What, approximately, must the track width be in order to accommodate this number of bits on a CD? In case you don't have a ruler, a CD has an inner diameter of 40 mm and an outer diameter of 120 mm.

The track width d controls the number of tracks we end up with. What really matters is the overall length L of the tracks though. This is going to be a sum over the length of each track, starting with the inner most one (which has inner diameter ID = 40 mm) and going to the outer one (with outer diameter OD = 120 mm).

$$L = \sum_{\text{all tracks}} \ell_{\text{track}} = \sum_{n=0}^{N-1}\pi D_n = \pi\sum_{n=0}^{N-1}(ID + 2dn) = \pi\left(ID\cdot(N-1) + 2d\frac{(N-1)N}{2}\right)$$

$$= \pi(N-1)(ID + dN)$$

The number of tracks N is given by $N = (OD - ID)/2d$, so:

$$L = \pi\left(\frac{OD - ID}{2d} - 1\right)\left(\frac{OD + ID}{2}\right) \equiv \pi D_{ave}\left(\frac{\Delta r}{d} - 1\right) \approx \pi D_{ave}\frac{\Delta r}{d}$$

which makes sense – it's just the average diameter times the number of tracks.

No we can solve for the width d in terms of the # of bits that we need to store:

$$d \approx \pi D_{ave}\frac{\Delta r}{L} = \pi D_{ave}\frac{\Delta r}{(\#\text{ bits})(\text{length } l/\text{bit})} = \pi D_{ave}\frac{\Delta r}{(\#\text{ bits})(d/3)}$$

$$d \approx \sqrt{\pi D_{ave}\frac{3\Delta r}{(\#\text{ bits})}} = \sqrt{\pi(80\text{ mm})\frac{3(40\text{ mm})}{(12\times 10^9)}} \approx 1.6\ \mu\text{m}$$

I should comment that calling this distance the track width is a bit of a misnomer. More accurately, it is the distance between the tracks, which are only a few hundred nanometers wide.

(c) Derive an equation for calculating the width d of the tracks from your measurement of the distance Δy between interference maxima. Make sure that you keep a copy of this equation in your notes! You will need it for the lab.

The derivation is just what we did in problems 1 and 2, yielding:

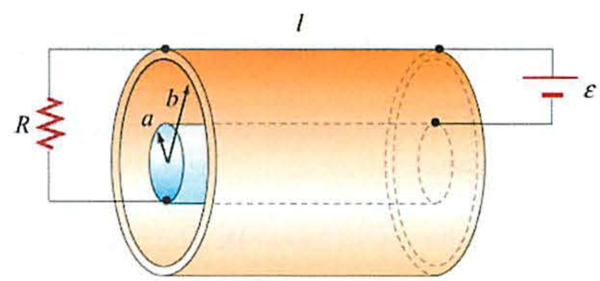
$$d = \frac{\lambda L}{\Delta y}$$

(d) Using the previous results, what approximately will the distance between interference maxima Δy be on the screen?

I didn't tell you the wavelength of the light we will be using, but it's red so it's around $\lambda = 600$ nm, so

$$\Delta y = \frac{\lambda L}{d} \approx \frac{(600\text{ nm})(5\text{ cm})}{(1.6\ \mu\text{m})} \approx 2\text{ cm}$$

Problem 2: Coaxial Cable and Power Flow



A coaxial cable consists of two concentric long hollow cylinders of zero resistance; the inner has radius a, the outer has radius b, and the length of both is l, with $l >> b$, as shown in the figure. The cable transmits DC power from a battery to a load. The battery provides an electromotive force ε between the two conductors at one end of the cable, and the load is a resistance R connected between the two conductors at the other end of the cable. A current I flows down the inner conductor and back up the outer one. The battery charges the inner conductor to a charge $-Q$ and the outer conductor to a charge $+Q$.

(a) Find the direction and magnitude of the electric field $\vec{\mathbf{E}}$ everywhere.

Consider a Gaussian surface in the form of a cylinder with radius r and length l, coaxial with the cylinders. Inside the inner cylinder ($r<a$) and outside the outer cylinder ($r>b$) no charge is enclosed and hence the field is 0. In between the two cylinders ($a<r<b$) the charge enclosed by the Gaussian surface is $-Q$, the total flux through the Gaussian cylinder is

$$\Phi_E = \oiint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E(2\pi r l)$$

Thus, Gauss's law leads to $E(2\pi r l) = \frac{q_{\text{enc}}}{\varepsilon_0}$, or

$$\vec{\mathbf{E}} = \frac{q_{\text{enc}}}{2\pi r l}\hat{\mathbf{r}} = -\frac{Q}{2\pi\varepsilon_0 r l}\hat{\mathbf{r}} \text{ (inward) for } a < r < b, \text{ 0 elsewhere}$$

(b) Find the direction and magnitude of the magnetic field $\vec{\mathbf{B}}$ everywhere.

Just as with the E field, the enclosed current I_{enc} in the Ampere's loop with radius r is zero inside the inner cylinder ($r<a$) and outside the outer cylinder ($r>b$) and hence the field there is 0. In between the two cylinders ($a<r<b$) the current enclosed is $-I$.

Applying Ampere's law, $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_0 I_{\text{enc}}$, we obtain

$$\vec{\mathbf{B}} = -\frac{\mu_0 I}{2\pi r}\hat{\boldsymbol{\varphi}} \text{ (clockwise viewing from the left side) for } a<r<b, \ 0 \text{ elsewhere}$$

(c) Calculate the Poynting vector $\vec{\mathbf{S}}$ in the cable.

For $a<r<b$, the Poynting vector is

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}} = \frac{1}{\mu_0}\left(-\frac{Q}{2\pi\varepsilon_0 rl}\hat{\mathbf{r}}\right)\times\left(-\frac{\mu_0 I}{2\pi r}\hat{\boldsymbol{\varphi}}\right) = \left(\frac{QI}{4\pi^2\varepsilon_0 r^2 l}\right)\hat{\mathbf{k}} \quad \text{(from right to left)}$$

On the other hand, for $r<a$ and $r>b$, we have $\vec{\mathbf{S}} = 0$.

(d) By integrating $\vec{\mathbf{S}}$ over appropriate surface, find the power that flows into the coaxial cable.

With $d\vec{\mathbf{A}} = (2\pi r\, dr)\hat{\mathbf{k}}$, the power is

$$P = \iint_S \vec{\mathbf{S}}\cdot d\vec{\mathbf{A}} = \frac{QI}{4\pi^2\varepsilon_0 l}\int_a^b \frac{1}{r^2}(2\pi r dr) = \frac{QI}{2\pi\varepsilon_0 l}\ln\left(\frac{b}{a}\right)$$

(e) How does your result in (d) compare to the power dissipated in the resistor?

Since

$$\varepsilon = \int \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = \int_a^b \frac{Q}{2\pi r l\varepsilon_0}dr = \frac{Q}{2\pi l\varepsilon_0}\ln\left(\frac{b}{a}\right) = IR$$

the charge Q is related to the resistance R by $Q = \dfrac{2\pi\varepsilon_0 lIR}{\ln(b/a)}$. The above expression for P becomes

$$P = \left(\frac{2\pi\varepsilon_0 lIR}{\ln(b/a)}\right)\frac{I}{2\pi\varepsilon_0 l}\ln\left(\frac{b}{a}\right) = I^2 R$$

which is equal to the rate of energy dissipation in a resistor with resistance R.

Problem 3: ***Standing Waves***

The electric field of an electromagnetic wave is given by the superposition of two waves

$$\vec{\mathbf{E}} = E_0 \cos(kz - \omega t)\,\hat{\mathbf{i}} + E_0 \cos(kz + \omega t)\,\hat{\mathbf{i}}\,.$$

You may find the following identities and definitions useful

$$\cos(kz + \omega t) = \cos(kz)\cos(\omega t) - \sin(kz)\sin(\omega t)$$
$$\sin(kz + \omega t) = \sin(kz)\cos(\omega t) + \cos(kz)\sin(\omega t)$$

a) What is the associated magnetic field $\vec{\mathbf{B}}(x, y, z, t)$.
b) What is the energy per unit area per unit time (the Poynting vector $\vec{\mathbf{S}}$) transported by this wave?
c) What is the time average of the Poynting $\langle \vec{\mathbf{S}} \rangle$ vector? Briefly explain your answer. Note the time average is given by

$$\langle \vec{\mathbf{S}} \rangle \equiv \frac{1}{T}\int_0^T \vec{\mathbf{S}}\,dt\,.$$

Solution

The electric field is given by

$$\vec{\mathbf{E}} = E_0 \cos(kz - \omega t)\,\hat{\mathbf{i}} + E_0 \cos(kz + \omega t)\,\hat{\mathbf{i}}$$

The first term describes an electric field propagating in the $+x$-direction. The associated magnetic field is given by

$$\vec{B}_1 = E_0 \cos(kz - \omega t)\,\hat{\mathbf{i}} + E_0 \cos(kz + \omega t)\,\hat{\mathbf{i}}$$

$$\vec{E} = E_0 \cos(kz - \omega t)\hat{i} + E_0 \cos(kz + \omega t)\hat{i}$$

$$\vec{B} = -\int (\vec{\nabla} \times \vec{E})\, dt$$

$$= -\int \frac{\partial E_x}{\partial z}(+\hat{j})\, dt$$

$$= -\int -k E_0 \sin(kz - \omega t)\, dt\, \hat{j} + -\int -k E_0 \sin(kz + \omega t)\, dt\, \hat{j}$$

$$= -E_0 k \frac{\cos(kz - \omega t)}{(-\omega)}\hat{j} - E_0 \frac{\cos(kz + \omega t)}{(+\omega)}\hat{j}$$

$$= \frac{k}{\omega} E_0 \cos(kz - \omega t)\hat{j} - \frac{k}{\omega} E_0 \cos(kz + \omega t)\hat{j}$$

$$\frac{\partial^2 E_x}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E_x}{\partial t^2} \Rightarrow -k^2 E_x = -\frac{\omega^2}{c^2} E_x$$

$$\Rightarrow \quad \frac{\omega}{k} = c$$

$$\vec{B} = \frac{E_0}{c}\cos(kz - \omega t)\hat{j} - \frac{E_0}{c}\cos(kz + \omega t)\hat{j}$$

note: $\vec{E} = E_0(\cos(kz)\cos(\omega t) - \sin(kz)\sin(-\omega t)\ \hat{i}$

$+ E_0(\cos(kz)\cos(\omega t) - \sin(kz)\sin(\omega t))\ \hat{i}$

$= 2E_0 \cos(kz)\cos(\omega t)\ \hat{i}$

$\vec{B} = \frac{+2E_0}{c} \sin(kz)\sin(\omega t)\ \hat{j}$

$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} = \frac{4E_0^2}{c} \sin(kz)\cos(kz)\cos(\omega t)\sin(\omega t)\ \hat{k}$

$\vec{S} = \frac{E_0^2}{c} \sin(2kz)\cos(2\omega t)\ \hat{k}$

$\langle \vec{S} \rangle = 0$ since

$\frac{1}{T}\int_0^T \cos(2\omega t)\, dt = 0$

this is a standing wave so it
does not transport power.

Problem 4: ***Radiation Pressure***

You have designed a solar space craft of mass m that is accelerated by the force due to the 'radiation pressure' from the sun's light that fall on a perfectly reflective circular sail that it is oriented face-on to the sun. The time averaged radiative power of the sun is P_{sun}. The gravitational constant is G. The mass of the sun is m_s. The speed of light is c. Model the sun's light as a plane electromagnetic wave, traveling in the +z direction with the electric field given by

$$\vec{\mathbf{E}}(z,t) = E_{x,0}\cos(kz - \omega t)\hat{\mathbf{i}}.$$

You may express your answer in terms of the symbols m, $\langle P \rangle$, c, m_s, G, k, and ω as necessary.

a) What is the magnetic field $\vec{\mathbf{B}}$ associated with this electric field?

b) What is the Poynting vector $\vec{\mathbf{S}} = \dfrac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$ associated with this wave? What is the time averaged Poynting vector $\langle \vec{\mathbf{S}} \rangle = \dfrac{1}{T}\int_0^T \vec{\mathbf{S}}\, dt$ associated with this superposition, where T is the period of oscillation. What is the amplitude of the electric field at your starting point?

c) What is the minimum area for the sail in order to exactly balance the gravitational attraction from the sun?

(a) $\vec{E} = E_{x,0} \cos(kz - \omega t)\, \hat{i}$

$\vec{B} = \frac{E_{x,0}}{c} \cos(kz - \omega t)\, \hat{j}$

note $\quad dir(\vec{E} \times \vec{B}) = dir(\overrightarrow{propagation})$

$(\pm \hat{i}) \times (\pm \hat{j}) = \hat{k}$

(b) $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} = \frac{1}{\mu_0} \frac{E_{x0}^2}{c} \cos^2(kz - \omega t)\, \hat{k}$

time averaged Poynting vector

$|\langle \vec{S} \rangle| = \frac{1}{2} \frac{E_{x0}^2}{\mu_0 c}$

time average ↗ (the $\frac{1}{2}$)

time averaged power $\langle P \rangle$ of sun

$\frac{\langle P \rangle}{4\pi r^2} = |\langle \vec{S} \rangle| = \frac{1}{2} \frac{E_{x0}^2}{\mu_0 c}$

$\Rightarrow \quad E_{x,0} = \left(\frac{2 \langle P \rangle \mu_0 c}{4\pi r^2} \right)^{1/2}$

(c) radiation pressure for a perfectly reflecting sail

$\langle P_{rad} \rangle = \frac{2 |\langle \vec{S} \rangle|}{c} = \frac{|\langle \vec{F}_{rad} \rangle|}{Area} \quad \Rightarrow$

$|\langle \vec{F}_{rad} \rangle| = \frac{2 \langle |\vec{S}| \rangle\, Area}{c} = \frac{2 \langle P \rangle (Area)}{4\pi r^2 c}$

$$\vec{F}_{rad} + \vec{F}_{grav} = \frac{m d^2 \hat{r}}{dt^2}$$

when $|\vec{F}_{rad}| = |\vec{F}_{grav}|$

$$\frac{2 \langle P \rangle (Area)_{min}}{4\pi r^2 c} = \frac{G m m_s}{r^2}$$

$$\Rightarrow \quad (Area)_{min} = \frac{G m m_s \, 4\pi c}{2 \langle P \rangle}$$

$$c = \frac{\omega}{k} \quad \Rightarrow$$

$$(Area)_{min} = \frac{G m m_s \, 2\pi}{\langle P \rangle} \frac{\omega}{k}$$

Problem 5. *Electromagnetic Waves*

The magnetic field of a plane electromagnetic wave is described as follows:

$$\vec{\mathbf{B}} = B_0 \sin(kx - \omega t)\,\hat{\mathbf{j}}$$

a) What is the wavelength λ of the wave?

b) Write an expression for the electric field $\vec{\mathbf{E}}$ associated to this magnetic field. Be sure to indicate the direction with a unit vector and an appropriate sign (+ or –).

c) What is the direction and magnitude Poynting vector associated with this wave? Give appropriate units, as well as magnitude.

d) This wave is totally reflected by a thin conducting sheet lying in the *y*-*z* plane at $x = 0$. What is the resulting radiation pressure on the sheet? Give appropriate units, as well as magnitude.

f) The component of an electric field parallel to the surface of an ideal conductor must be zero. Using this fact, find expressions for the electric and magnetic fields for the reflected wave? What are the total electric and magnetic fields at the conducting sheet, $x = 0$. Check that your answer satisfies the condition on the electric field at the conducting sheet, $x = 0$.

g) An oscillating surface current $\vec{\mathbf{K}}$ flows in the thin conducting sheet as a result of this reflection. Along which axis does it oscillates? What is the amplitude of oscillation?

Solution:

(a) $\lambda = \frac{2\pi}{k}$

(b) $\vec{E} = c B_0 \sin(kx - \omega t)(-\hat{k})$

Since dir $\vec{E}$ × dir $\vec{B}$ = dir $\overrightarrow{(\text{propagation})}$

$$\pm(-\hat{k}) \times \pm(\hat{j}) = \hat{i}$$

(c) $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} = \frac{c B_0^2}{\mu_0} \sin^2(kx - \omega t)\,\hat{i}$

(d) $\langle P_{rad} \rangle = \frac{2|\langle \vec{S} \rangle|}{c}$

time averaged $|\langle \vec{S} \rangle| = \frac{c B_0^2}{2\mu_0}$

$$\langle P_{rad} \rangle = \frac{c B_0^2}{\mu_0 c}$$

Boundary Condition:

$$\vec{E}_{reflected}(x=0) + \vec{E}_{incident}(x=0) = 0$$

$$\vec{E}_{reflected}(x=0) = -\vec{E}_{incident}(x=0)$$

Let $\vec{E}_{reflected}(x=0) = E_{r,0} \sin(kx + \omega t)\Big|_{x=0} (-\hat{k})$

$$= E_{r,0} \sin \omega t \,(-\hat{k})$$

$$\vec{E}_{incident}(x=0) = cB_0 \sin(-\omega t)(-\hat{k})$$

$$= cB_0 \sin \omega t \; (\hat{k})$$

$$\vec{E}_{reflected}(x=0) = -\vec{E}_{incident}(x=0)$$

$$\Rightarrow \quad E_{r,0} \sin \omega t(-\hat{k}) = -cB_0 \sin \omega t\,(\hat{k})$$

$$\Rightarrow \quad E_{r,0} = +cB_0$$

$$\Rightarrow \vec{E}_{reflected} = +cB_0 \sin(kx+\omega t)(-\hat{k})$$

$$= -cB_0 \sin(kx+\omega t)\,\hat{k}$$

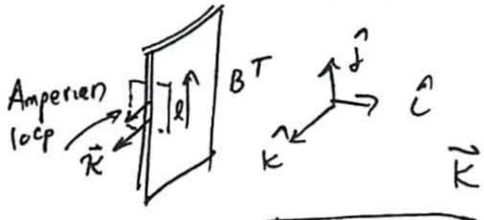
$$\Rightarrow \vec{B}_{reflected} = -B_0 \sin(kx+\omega t)\,\hat{j}$$

$$\vec{E}_{total} = \vec{E}_{reflected} + \vec{E}_{incident}$$

$$= -cB_0 \sin(kx+\omega t)\,\hat{k} + cB_0 \sin(kx-\omega t)(-\hat{k})$$

$$\vec{B}_{total} = -B_0 \sin(kx+\omega t)\,\hat{j} + B_0 \sin(kx-\omega t)\,\hat{j}$$

at $x=0$ $\quad \vec{B}_{Total} = -2B_0 \sin(\omega t)\,\hat{j}$



$$\oint \vec{B} \cdot d\vec{r} = \mu_0 \iint \vec{J} \cdot d\vec{a}$$

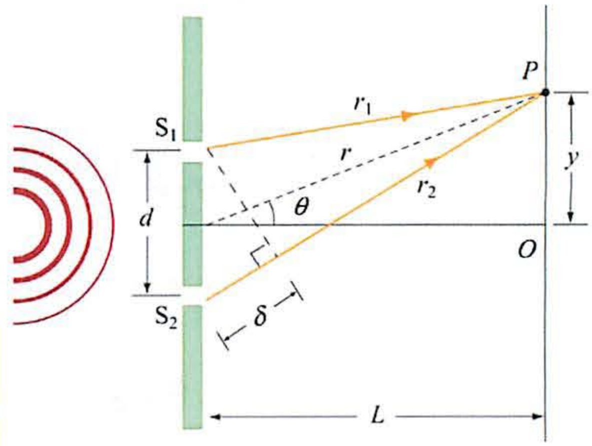
$$B^T \ell = \mu_0 K \ell$$

$$\vec{K} = \frac{B^T}{\mu_0}\hat{k} = \frac{2Bc \sin(\omega t)}{\mu_0}\hat{k}$$

$$\Rightarrow \boxed{K_0 = \frac{2Bc}{\mu_0}}$$

Problem 6: Phase Difference (cf. Section 14.2 of the *Course Notes*)

In the double-slit interference experiment shown in the figure, suppose $d = 0.100\,\text{mm}$ and $L = 1.20\,\text{m}$, and the incident light is monochromatic with a wavelength λ=600 nm.



(a) What is the phase difference between the two waves arriving at a point *P* on the screen when $\theta = 0.800°$?

$$\begin{aligned}\phi &= 2\pi\frac{\delta}{\lambda}\\ &= 2\pi\frac{d\sin\theta}{\lambda}\\ &= 2(3.14)\frac{(1.00\times10^{-4}\ m)\sin 0.8°}{6.00\times10^{-7}\ m}\\ &= 14.6\,rad\end{aligned}$$

(b) What is the phase difference between the two waves arriving at a point *P* on the screen when $y = 4.00\,\text{mm}$?

$$\begin{aligned}\phi &= 2\pi\frac{dy}{\lambda L} \quad (\because \sin\theta \approx \frac{y}{L})\\ &= 2(3.14)\frac{(1.00\times10^{-4}\ m)(4.00\times10^{-3}\ m)}{(6.00\times10^{-7}\ m)(1.20\,m)}\\ &= 3.49\,rad\end{aligned}$$

(c) If the phase difference between the two waves arriving at point *P* is ϕ=1/3 rad, what is the value of θ?

$$\phi = \frac{1}{3}\,rad = 2\pi\frac{d\sin\theta}{\lambda} \;\Rightarrow\; \theta = \sin^{-1}\left(\frac{\lambda\phi}{2\pi d}\right) = 3.18\times10^{-4}\,rad = 0.0182°$$

(d) If the path difference is $\delta = \lambda/4$, what is the value of θ?

$$\delta = d\sin\theta \;\Rightarrow\; \theta = \sin^{-1}\left(\frac{\delta}{d}\right) = \sin^{-1}\left(\frac{\lambda}{4d}\right) = 1.50\times10^{-3}\,rad = 0.0860°$$

(e) In the double-slit interference experiment, suppose the slits are separated by $d = 1.00\,\text{cm}$ and the viewing screen is located at a distance $L = 1.20\,\text{m}$ from the slits. Let the incident light be monochromatic with a wavelength λ=500 nm. Calculate the spacing between the adjacent bright fringes on the viewing screen.

Since $y_b = m\dfrac{\lambda L}{d}$, the spacing between adjacent bright fringes is

$$\begin{aligned}
\Delta y_b &= y_b(m+1) - y_b(m) \\
&= (m+1)\frac{\lambda L}{d} - m\frac{\lambda L}{d} \\
&= \frac{\lambda L}{d} \\
&= \frac{(5.00\times 10^{-7}\,m)(1.20\,m)}{(1.00\times 10^{-2}\,m)} \\
&= 6.00\times 10^{-5}\,m \\
&= 60.0\,\mu m
\end{aligned}$$

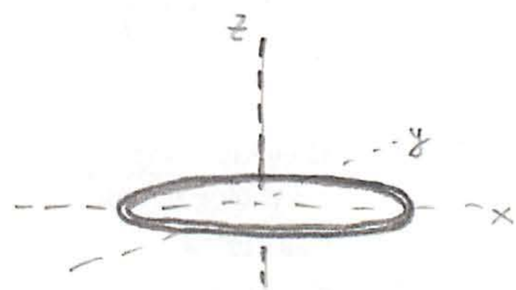
(f) What is the distance between the third-order fringe and the center line on the viewing screen?

$$\begin{aligned}
\Delta y_b &= y_b(3) - y_b(0) \\
&= (3)\frac{\lambda L}{d} - 0 \\
&= 3\frac{\lambda L}{d} \\
&= 3\frac{(5.00\times 10^{-7}\,m)(1.20\,m)}{(1.00\times 10^{-2}\,m)} \\
&= 1.80\times 10^{-4}\,m \\
&= 180\,\mu m
\end{aligned}$$

Problem 7: *Loop Antenna*. An electromagnetic wave propagating in air has a magnetic field given by

$$B_x = 0 \qquad B_y = 0 \qquad B_z = B_0 \cos(\omega t - kx).$$

It encounters a circular loop antenna of radius a centered at the origin $(x,y,z) = (0,0,0)$ and lying in the x-y plane. The radius of the antenna $a \ll \lambda$ where λ is the wavelength of the wave. So you can assume that at any time t the magnetic field inside the loop is approximately equal to its value at the center of the loop.



a) What is the magnetic flux, $\Phi_{mag}(t) \equiv \iint_{disk} \vec{\mathbf{B}} \cdot d\vec{\mathbf{a}}$, through the plane of the loop of the antenna?

The loop has a self-inductance L and a resistance R. Faraday's law for the circuit is

$$IR = -\frac{d\Phi_{mag}}{dt} - L\frac{dI}{dt}.$$

b) Assume a solution for the current of the form $I(t) = I_0 \sin(\omega t - \phi)$ where ω is the angular frequency of the electromagnetic wave, I_0 is the amplitude of the current, and ϕ is a phase shift between the changing magnetic flux and the current.. Find expressions for the constants ϕ and I_0.

c) What is the magnetic field created at the center of the loop by this current $I(t)$?

a) $\Phi_{mag} = \iint \vec{B} \cdot d\vec{a} \simeq B_0 \cos(\omega t - kz)\, \pi a^2$

b) $\frac{d\Phi_{mag}}{dt} = -IR - L\frac{dI}{dt}$

$-B_0 \omega \sin(\omega t - kz)\, \pi a^2 = -IR - L\frac{dI}{dt}$

$B_0 \omega \pi a^2 \sin(\omega t - kz) = IR + L\frac{dI}{dt}$

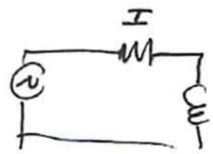
Set $x = 0$ we solved this equation for the case

$$V(t) = V_0 \sin(\omega t - kx) \quad , \quad kx \simeq \text{constant}$$

$$V(t) = V_0 \sin \omega t \quad , \quad I(t) = I_0 \sin(\omega t - \phi)$$

$$V_0 = B_0 \omega \pi a^2$$

$$V(t) = IR + L \frac{dI}{dt} \quad , \quad Z^T = R + i\omega L = |Z^T| e^{i\delta} \quad , \quad \delta = \tan^{-1}\left(\frac{\omega L}{R}\right), \; |Z^T| = (R^2 + (\omega L)^2)^{1/2}$$



$$I = \mathrm{Im}\, I_0 \, e^{i\omega t} e^{-i\phi}$$

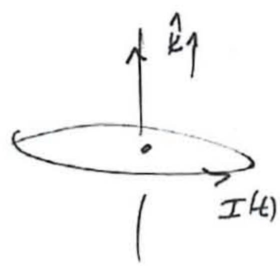
$$V = \mathrm{Im}\, V_0 \, e^{i\omega t}$$

$$V_0 e^{i\omega t} = Z^T I = I_0 |Z^T| e^{i\delta} e^{i\omega t} e^{-i\phi}$$

$$\tan^{-1}\left(\frac{\omega L}{R}\right) = \delta = \phi \quad , \quad I_0 = \frac{V_0}{|Z^T|} = \frac{V_0}{(R^2 + (\omega L)^2)^{1/2}}$$

$$I(t) = \frac{B_0 \omega \pi a^2}{(R^2 + (\omega L)^2)^{1/2}} \sin(\omega t - \phi)$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) \qquad \hat{e}_x \times \hat{r} = \hat{k}$$



From Biot Savart

$$\vec{B}_{center} = \frac{\mu_0}{4\pi} \int \frac{I \, d\vec{l} \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}$$

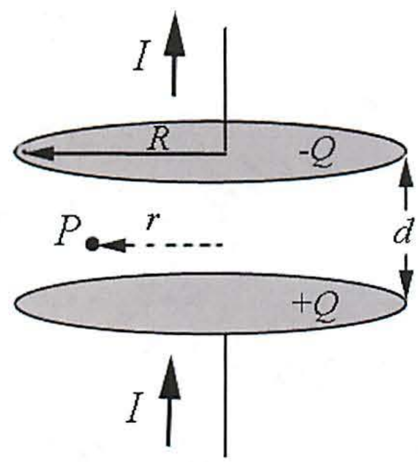
$$= \frac{\mu_0}{4\pi} I \int \frac{a \, d\theta \, \hat{e}_x (-a \hat{r})}{a^3}$$

$$\vec{B} = \frac{\mu_0 I}{2a} \hat{k}$$

$$\vec{B}_{center} = \frac{\mu_0}{2a} \frac{B \omega \pi a^2}{(R^2 + (\omega L)^2)^{1/2}} \sin(\omega t - \phi) \; \hat{k}$$

Problem 8: Charging Capacitor (10 points)

A parallel-plate capacitor consists of two circular plates, each with radius R, separated by a distance d. A steady current I is flowing towards the lower plate and away from the upper plate, charging the plates.



a) What is the direction and magnitude of the electric field $\vec{\mathbf{E}}$ between the plates? You may neglect any fringing fields due to edge effects.

Solution: If we ignore fringing fields then we can calculate the electric field using Gauss's Law,

$$\oiint_{\substack{closed\\surface}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{a}} = \frac{Q_{enc}}{\varepsilon_0}.$$

By superposition, the electric field is non-zero between the plates and zero everywhere else. Choose a Gaussian cylinder passing through the lower plate with its end faces parallel to the plates. Let A_{cap} denote the area of the endface. The surface charge density is given by $\sigma = Q / \pi R^2$. Let $\hat{\mathbf{k}}$ denote the unit vector pointing from the lower plate to the upper plate. Then Gauss' Law becomes

$$\left|\vec{\mathbf{E}}\right| A_{cap} = \frac{\sigma A_{cap}}{\varepsilon_0}$$

which we can solve for the electric field

$$\vec{\mathbf{E}} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{k}} = \frac{Q}{\pi R^2 \varepsilon_0}\hat{\mathbf{k}}.$$

b) What is the total energy stored in the electric field of the capacitor?

Solution: The total energy stored in the electric field is given by

$$U_{elec} = \frac{1}{2}\varepsilon_0 \int_{volume} E^2 dV = \frac{1}{2}\varepsilon_0 E^2 \pi R^2 d .$$

Substitute the result for the electric field into the energy equation yields

$$U_{elec} = \frac{1}{2}\varepsilon_0 \left(\frac{Q}{\pi R^2 \varepsilon_0}\right)^2 \pi R^2 d = \frac{1}{2}\frac{Q^2 d}{\pi R^2 \varepsilon_0}.$$

c) What is the rate of change of the energy stored in the electric field?

Solution: The rate of change of the stored electric energy is found by taking the time derivative of the energy equation

$$\frac{d}{dt}U_{elec} = \frac{Qd}{\pi R^2 \varepsilon_0}\frac{dQ}{dt}.$$

The current flowing to the plate is equal to

$$I = \frac{dQ}{dt}.$$

Substitute the expression for the current into the expression for the rate of change of the stored electric energy yields

$$\frac{d}{dt}U_{elec} = \frac{QId}{\pi R^2 \varepsilon_0}.$$

d) What is the magnitude of the magnetic field $\vec{\mathbf{B}}$ at point P located between the plates at radius $r < R$ (see figure above). As seen from above, is the direction of the magnetic field *clockwise* or *counterclockwise*. Explain your answer.

Solution: We shall calculate the magnetic field by using the generalized Ampere's Law,

$$\oint_{closed\ path} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \iint_{open\ surface} \vec{\mathbf{J}} \cdot d\vec{\mathbf{a}} + \mu_0 \varepsilon_0 \frac{d}{dt} \iint_{open\ surface} \vec{\mathbf{E}} \cdot d\vec{\mathbf{a}}$$

We choose a circle of radius $r < R$ passing through the point P as the Amperian loop and the disk defined by the circle as the open surface with the circle as its boundary. We choose to circulate around the loop in the counterclockwise direction as seen from above. This means that flux in the positive $\hat{\mathbf{k}}$-direction is positive.

The left hand side (LHS) of the generalized Ampere's Law becomes

$$LHS = \oint_{circle} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \left|\vec{\mathbf{B}}\right| 2\pi r\,.$$

The conduction current is zero passing through the disk, since no charges are moving between the plates. There is an electric flux passing through the disk. So the right hand side (RHS) of the generalized Ampere's Law becomes

$$RHS = \mu_0 \varepsilon_0 \frac{d}{dt} \iint_{disk} \vec{\mathbf{E}} \cdot d\vec{\mathbf{a}} = \mu_0 \varepsilon_0 \frac{d\left|\vec{\mathbf{E}}\right|}{dt} \pi r^2\,.$$

Take the time derivative of the expression for the electric field and the expression for the current, and substitute it into the RHS of the generalized Ampere's Law:

$$RHS = \mu_0 \varepsilon_0 \frac{d\left|\vec{\mathbf{E}}\right|}{dt} \pi r^2 = \frac{\mu_0 I \pi r^2}{\pi R^2}$$

Equating the two sides of the generalized Ampere's Law yields

$$\left|\vec{\mathbf{B}}\right| 2\pi r = \frac{\mu_0 I \pi r^2}{\pi R^2}$$

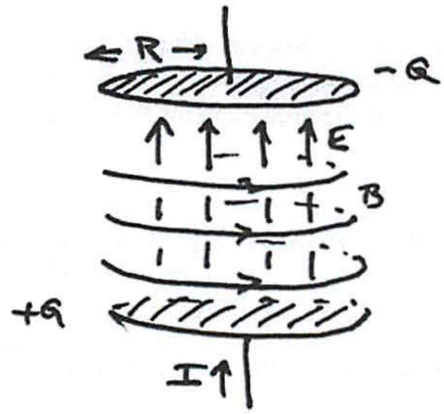
Finally the magnetic field between the plates is then

$$\left|\vec{\mathbf{B}}\right| = \frac{\mu_0 I}{2\pi R^2} r\,;\ 0 < r < R\,.$$

The sign of the magnetic field is positive therefore the magnetic field points in the counterclockwise direction (consistent with our sign convention for the integration direction for the circle) as seen from above. Define the unit vector $\hat{\boldsymbol{\theta}}$ such that is it tangent to the circle pointing in the counterclockwise direction, then

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi R^2} r\,\hat{\boldsymbol{\theta}}\,;\ 0 < r < R\,.$$

e) Make a sketch of the electric and magnetic field inside the capacitor.



f) What is the direction and magnitude of the Pointing vector $\vec{\mathbf{S}}$ at a distance $r = R$ from the center of the capacitor.

Solution: The Poynting vector at a distance $r = R$ is given by

$$\vec{\mathbf{S}}(r = R) = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}\Big|_{r=R}.$$

Substituting the electric field and the magnetic field (setting $r = R$) into the above equation, and noting that $\hat{\mathbf{k}} \times \hat{\boldsymbol{\theta}} = -\hat{\mathbf{r}}$, yields

$$\vec{\mathbf{S}}(r = R) = \frac{1}{\mu_0}\frac{Q}{\pi R^2 \varepsilon_0}\hat{\mathbf{k}} \times \frac{\mu_0 I}{2\pi R}\hat{\boldsymbol{\theta}} = \frac{Q}{\pi R^2 \varepsilon_0}\frac{I}{2\pi R}(-\hat{\mathbf{r}}).$$

So the Poynting vector points inward with magnitude

$$\left|\vec{\mathbf{S}}(r = R)\right| = \frac{Q}{\pi R^2 \varepsilon_0}\frac{I}{2\pi R}.$$

g) By integrating $\vec{\mathbf{S}}$ over an appropriate surface, find the power that flows into the capacitor.

Solution: The power flowing into the capacitor is the closed surface integral

$$P = \oiint_{closed\ surface} \vec{\mathbf{S}}(r = R) \cdot d\vec{\mathbf{a}}.$$

The Poynting vector points radially inward so the only contribution to this integral is from the cylindrical body of the capacitor. The unit normal associated with the area vector for a closed surface integral always points outward, so on the cylindrical body $d\vec{\mathbf{a}} = da\,\hat{\mathbf{r}}$. Use this definition for the area element and the power is then

$$P = \iint_{\substack{cylindrical \\ body}} \vec{\mathbf{S}}(r=R)\cdot d\vec{\mathbf{a}} = \iint_{\substack{cylindrical \\ body}} \frac{Q}{\pi R^2 \varepsilon_0}\frac{I}{2\pi R}(-\hat{\mathbf{r}})\cdot da\,\hat{\mathbf{r}}$$

The Poynting vector is constant and the area of the cylindrical body is $2\pi Rd$, so

$$P = \iint_{\substack{cylindrical \\ body}} \frac{Q}{\pi R^2 \varepsilon_0}\frac{I}{2\pi R}(-\hat{\mathbf{r}})\cdot da\,\hat{\mathbf{r}} = -\frac{Q}{\pi R^2 \varepsilon_0}\frac{I}{2\pi R}2\pi Rd = -\frac{QId}{\pi R^2 \varepsilon_0}.$$

The minus sign correspond to power flowing into the region.

h) How does your answer in part g) compare to your answer in part c)?

Solution: The two expressions for power are equal so the power flowing in is equal to the change of energy stored in the electric fields.

Topics: EM Radiation
Related Reading: Course Notes: Sections 13.11, 14.1-14.3, 14.11.1-14.11.3

Topic Introduction

Today you will work through analytic problems related to EM Waves and the Poynting vector.

Electromagnetic Radiation

The fact that changing magnetic fields create electric fields and that changing electric fields create magnetic fields means that oscillating electric and magnetic fields can propagate through space (each pushing forward the other). This is electromagnetic (EM) radiation. It is the single most useful discovery we discuss in this class, not only allowing us to understand natural phenomena, like light, but also to create EM radiation to carry a variety of useful information: radio, broadcast television and cell phone signals, to name a few, are all EM radiation. In order to understand the mathematics of EM radiation you need to understand how to write an equation for a traveling wave (a wave that propagates through space as a function of time). Any function that is written f(x-vt) satisfies this property. As t increases, a function of this form moves to the right (increasing x) with velocity v. You can see this as follows: At t=0 f(0) is at x=0. At a later time t=t, f(0) is at x=vt. That is, the function has moved a distance vt during a time t.

Sinusoidal traveling waves (plane waves) look like waves both as a function of position and as a function of time. If you sit at one position and watch the wave travel by you say that it has a period *T*, inversely related to its frequency *f*, and angular frequency, $\omega\left(T = f^{-1} = 2\pi\omega^{-1}\right)$. If instead you freeze time and look at a wave as a function of position, you say that it has a wavelength λ, inversely related to its wavevector k $\left(\lambda = 2\pi k^{-1}\right)$. Using this notation, we can rewrite our function f(x-vt) = $f_0$sin(kx-ωt), where v = ω/k.

We typically treat both electric and magnetic fields as plane waves as they propagate through space (if you have one you must have the other). They travel at the speed of light (v=c). They also obey two more constraints. First, their magnitudes are fixed relative to each other: $E_0 = cB_0$ (check the units!) Secondly, E & B always oscillate at right angles to each other and to their direction of propagation (they are *transverse* waves). That is, if the wave is traveling in the z-direction, and the E field points in the x-direction then the B field must point along the y-direction. More generally we write $\hat{\mathbf{E}} \times \hat{\mathbf{B}} = \hat{\mathbf{p}}$, where $\hat{\mathbf{p}}$ is the direction of propagation.

Energy and the Poynting Vector

The Poynting Vector $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$ describes how much energy passes through a given area per unit time, and points in the direction of energy flow. Although this is commonly used when thinking about electromagnetic radiation, it generically tells you about energy flow, and is particularly useful in thinking about energy in circuit components. For example, consider a cylindrical resistor. The current flows through it in the direction that the electric field points. The B field curls around. The Poynting vector thus points radially *into* the resistor – the resistor consumes energy. In today's problem solving session you will calculate the Poynting vector in a capacitor, and will find that if the capacitor is charging then **S** points in towards the center of the capacitor (energy flows into the capacitor) whereas if the capacitor is discharging **S** points outwards (it is giving up energy).

Important Equations

Maxwell's Equations:

(1) $\oint\!\!\oint_S \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \dfrac{Q_{in}}{\varepsilon_0}$ (2) $\oint\!\!\oint_S \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}} = 0$

(3) $\oint_C \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = -\dfrac{d\Phi_B}{dt}$ (4) $\oint_C \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \mu_0 I_{enc} + \mu_0\varepsilon_0 \dfrac{d\Phi_E}{dt}$

EM Plane Waves:

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},t) = E_0 \sin\left(k\hat{\mathbf{p}}\cdot\vec{\mathbf{r}} - \omega t\right)\hat{\mathbf{E}}$$
$$\vec{\mathbf{B}}(\vec{\mathbf{r}},t) = B_0 \sin\left(k\hat{\mathbf{p}}\cdot\vec{\mathbf{r}} - \omega t\right)\hat{\mathbf{B}}$$

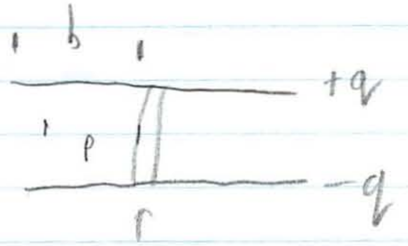
with $E_0 = cB_0$; $\hat{\mathbf{E}}\times\hat{\mathbf{B}} = \hat{\mathbf{p}}$; $\omega = ck$

Poynting Vector: $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}}$

Old replacement Problem Solving

instead

Problem Solving
Answer



circular
tube resistor

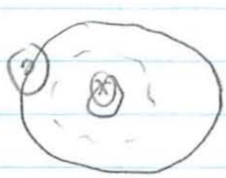
Gauss' Law
- pillbox

$$\oint \oint E \cdot dA = \frac{Q_{enc}}{\epsilon_0}$$

$$E A = \frac{Q \frac{A}{\pi b^2}}{\epsilon_0}$$

$$E = \frac{Q}{\pi b^2 \epsilon_0} \quad \text{down}$$

Ampere's Law
- current must be into or out of page



$$\oint B \cdot ds = \mu_0 (I + I_d)$$

↑ changing displacement current

$$I_d = \epsilon \frac{d\Phi}{dt}$$

$$\Phi_E = E \pi r^2$$

ampèrian loop

$$\oint B \cdot ds = \mu_0 \left(I + \epsilon \pi r^2 \frac{dE}{dt} \right)$$

$$B = \frac{\mu_0}{2\pi r} \left(-\frac{dQ}{dt} + \frac{dQ}{dt} \frac{r^2}{b^2} \right)$$ ← Fraction of displacement current

↑ minus sign or write dir

*I + displacement current ~~must have opposit signs~~

$\vec{E}\uparrow$ same dir I, I_d

$\vec{E}\downarrow$ opposit dir I, I_d

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0} = \frac{Q}{\pi b^2 \epsilon_0 2\pi r} \frac{dQ}{dt} \left(\frac{r^2}{b^2} - 1 \right)$$ inward

Integrate over relevant area

No pointing vector at outer edge $\left(\frac{r^2}{r^2} - 1 = 0 \right)$

- flowing toward center
- no pointing vector outside

Power = pointing flux ⊥ to area

Integrate over smaller circle

- should get $I^2 R$
- but don't
- pointing vector inside small circle outward

Use time average when they ask for it

- waves

Final

5/7

Maxwell

Gauss

Ampere

Fariday

Combine ideas

capicator just did

LRC will be on!

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Department of Physics

Problem Solving 10: Interference

OBJECTIVES

1. To understand the meaning of constructive and destructive interference
2. To understand how to determine the interference conditions for double slit interference
3. To understand how to determine the intensity of the light associated with double slit interference

REFERENCE: Sections 14-1 through 14-3 Course Notes.

Introduction

The *Huygens Principle* states that every unobstructed point on a wavefront will act a source of a secondary spherical wave. We add to this principle, the *Superposition Principle* that the amplitude of the wave at any point beyond the initial wave front is the superposition of the amplitudes of all the secondary waves.

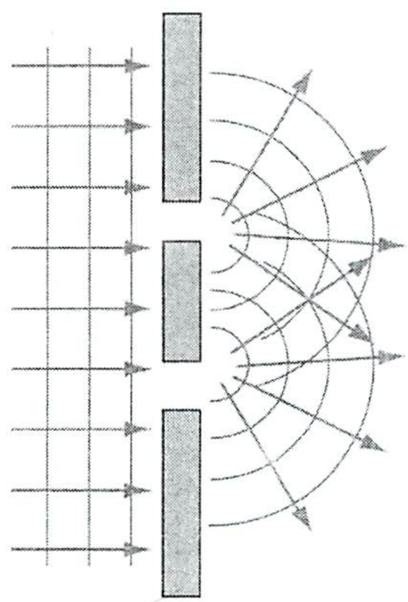

Figure 1: Huygens-Fresnel Principle applied to double slit

When ordinary light is emitted from two different sources and passes through two narrow slits, the plane waves do not maintain a constant phase relation and so the light will show no interference patterns in the region beyond the openings. In order for an interference pattern to develop, the incoming light must satisfy two conditions:

- The light sources must be coherent. This means that the plane waves from the sources must maintain a constant phase relation.

- The light must be monochromatic. This means that the light has just one wavelength.

When the coherent monochromatic laser light falls on two slits separated by a distance d, the emerging light will produce an interference pattern on a viewing screen a distance D away from the center of the slits. The geometry of the double slit interference is shown in the figure below.

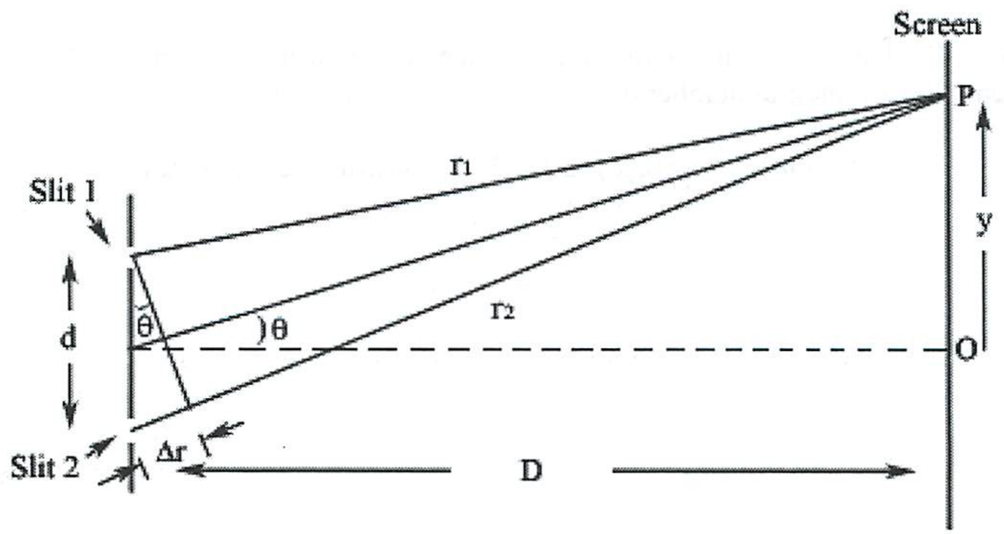

Figure 2: Double slit interference

Consider light that falls on the screen at a point P a distance y from the point O that lies on the screen a perpendicular distance D from the double slit system. The light from the slit 2 will travel an extra distance $r_2 - r_1 = \Delta r$ to the point P than the light from slit 1. This extra distance is called the path length.

Question 1: Draw a picture of two traveling waves that add up to form *constructive interference*.

Answer:

Question 2: Draw a picture of two traveling waves that add up to form *destructive interference*.

Answer:

Question 3: Explain why constructive interference will appear at the point P when the path length is equal to an integral number of wavelengths of the monochromatic light.

$$\Delta r = m\lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots \text{ constructive interference}$$

Answer:

Question 4: Based on the geometry of the double slits, show that the condition for constructive interference becomes

$$d \sin\theta = m\lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots \text{ constructive interference.}$$

Answer:

Question 5: Explain why destructive interference will appear at the point P when the path length is equal to an odd integral number of half wavelengths

$$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots \text{ destructive interference.}$$

Answer:

Question 6: Let y be the distance between the point P and the point O on the screen. Find a relation between the distance y, the wavelength λ, the distance between the slits d, and the distance to the screen D such that a constructive interference pattern will occur at the point P.

Answer:

Question 7: Find a similar relation such that destructive interference fringes will occur at the point P.

Answer:

Intensity of Double Slit Interference:

Suppose that the waves are emerging from the slits are sinusoidal plane waves. The slits are located at the plane $x = -D$. The light that emerges from slit 1 and slit 2 at time t are in phase. Let the screen be placed at the plane $x = 0$. Suppose the component of the electric field of the wave from slit 1 at the point P is given by

$$E_1 = E_0 \sin(\omega t).$$

Let's assume that the plane wave from slit 2 has the same amplitude E_0 as the wave from slit 1. Since the plane wave from slit 2 has to travel an extra distance to the point P equal to the path length, this wave will have a phase shift ϕ relative to the wave from slit 1,

$$E_2 = E_0 \sin(\omega t + \phi).$$

Question 8: Why are the phase shift ϕ, the wavelength λ, the distance between the slits, and the angle related θ by

$$\phi = \frac{2\pi}{\lambda} d \sin\theta.$$

As a hint how are the ratio of the phase shift ϕ to 2π and the ratio of the path length $\Delta r = d \sin\theta$ to wavelength λ, related?

Answer:

Question 9: Use the trigonometric identity

$$\sin A + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right).$$

To show that the total component of the electric field is

$$E_{total} = E_1 + E_2 = 2E_0 \sin\left(\omega t + \frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right).$$

Answer:

The *intensity* of the light is equal to the time-averaged Poynting vector

$$I = \langle \vec{\mathbf{S}} \rangle = \frac{1}{\mu_0}\langle \vec{\mathbf{E}} \times \vec{\mathbf{B}} \rangle.$$

Since the amplitude of the magnetic field is related to the amplitude of the electric field by $B_0 = E_0/c$. The intensity of the light is proportional to the time-averaged square of the electric field,

$$I \propto \langle E_{total}{}^2 \rangle = 4E_0{}^2\cos^2\left(\frac{\phi}{2}\right)\left\langle \sin^2\left(\omega t + \frac{\phi}{2}\right)\right\rangle = 2E_0{}^2\cos^2\left(\frac{\phi}{2}\right),$$

where the time-averaged value of the square of the sine function is

$$\left\langle \sin^2\left(\omega t + \frac{\phi}{2}\right)\right\rangle = \frac{1}{2}.$$

Let $I_{\max}$ be the amplitude of the intensity. Then the intensity of the light at the point P is

$$I = I_{\max}\cos^2\left(\frac{\phi}{2}\right)$$

Question 10: Show that the intensity is maximal when $d \sin\theta = m\lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots$.

Answer:

Question 11: Graph the intensity pattern on the screen as a function of distance y from the point O for the case that $D >> d$ and $d >> \lambda$.

Question 12: Since the energy of the light is proportional to the square of the electric fields, is energy conserved for the time-averaged superposition of the electric fields i.e. does the following relation hold,

$$\left\langle (E_1 + E_2)^2 \right\rangle = \left\langle E_1^{\,2} \right\rangle + \left\langle E_2^{\,2} \right\rangle$$

Answer:

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Department of Physics

Tear off this page and turn it in at the end of class !!!!

Note:
Writing in the name of a student who is not present is a Committee on Discipline offense.

Problem Solving 10: Interference

Group ______________________________ **(e.g. 6A Please Fill Out)**

Names ______________________________

____

____

Question 1: Draw a picture of two traveling waves that add up to form *constructive interference.*

Answer:

Question 2: Draw a picture of two traveling waves that add up to form *destructive interference.*

Answer:

Question 3: Explain why constructive interference will appear at the point P when the path length is equal to an integral number of wavelengths of the monochromatic light.

$$\Delta r = m\lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots \text{ constructive interference}$$

Answer:

Question 4: Based on the geometry of the double slits, show that the condition for constructive interference becomes

$$d \sin \theta = m\lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots \text{ constructive interference.}$$

Answer:

Question 5: Explain why destructive interference will appear at the point P when the path length is equal to an odd integral number of half wavelengths

$$d \sin \theta = \left(m + \frac{1}{2} \right) \lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots \text{ destructive interference.}$$

Answer:

Question 6: Let y be the distance between the point P and the point O on the screen. Find a relation between the distance y, the wavelength λ, the distance between the slits d, and the distance to the screen D such that a constructive interference pattern will occur at the point P.

Answer:

Question 7: Find a similar relation such that destructive interference fringes will occur at the point P.

Answer:

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$$E_{total} = E_1 + E_2 = 2E_0 \sin\left(\omega t + \frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right).$$

Answer:

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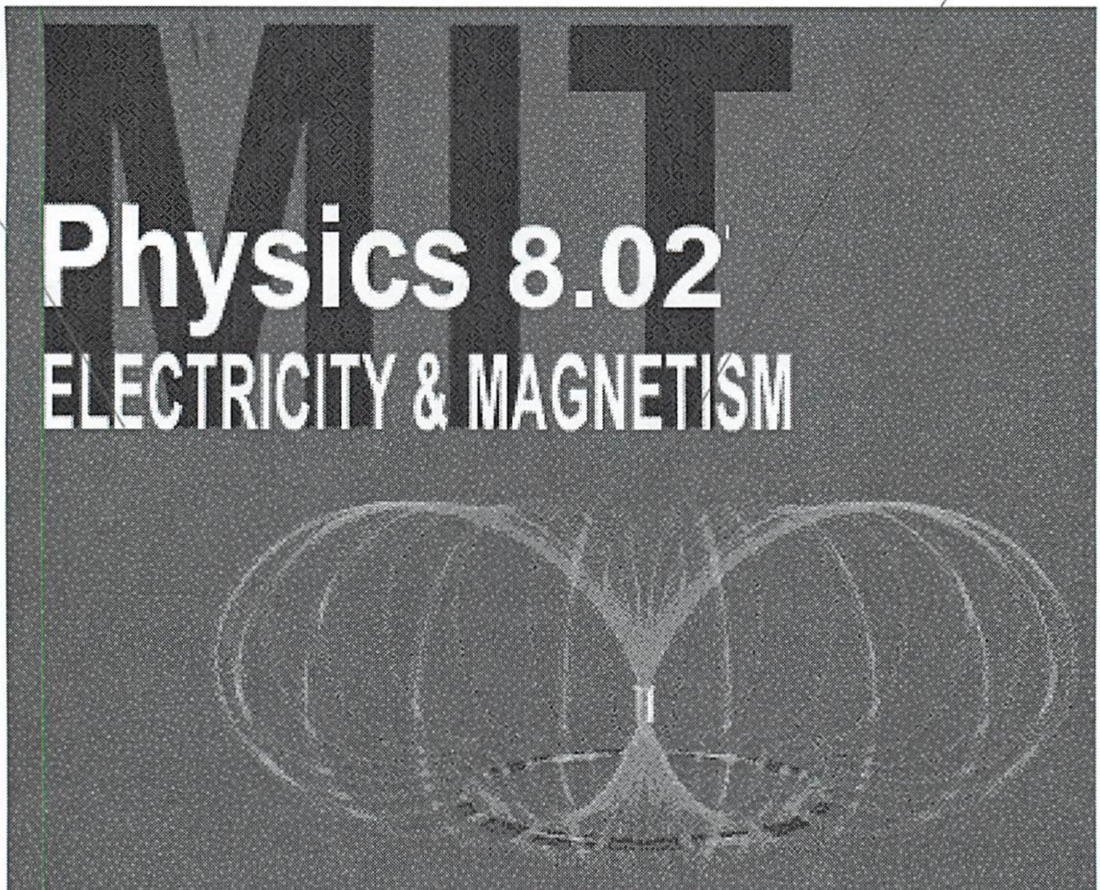
Answer:



Experiments

Website: http://web.mit.edu/8.02t/www/

Scrap pape

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Department of Physics
8.02

Problem Solving 10 Solutions: Interference and Diffraction

OBJECTIVES

1. To understand the meaning of constructive and destructive interference

2. To understand how to determine the interference conditions for double slit interference

3. To understand how to determine the intensity of the light associated with double slit interference

REFERENCE: Sections 14-1 through 14-3 Course Notes.

Introduction

The *Huygens Principle* states that every unobstructed point on a wavefront will act a source of a secondary spherical wave. We add to this principle, the *Superposition Principle* that the amplitude of the wave at any point beyond the initial wave front is the superposition of the amplitudes of all the secondary waves.

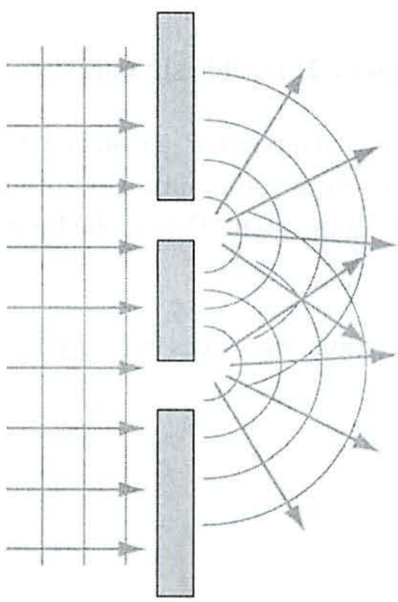


Figure 1: Huygens-Fresnel Principle applied to double slit

When ordinary light is emitted from two different sources and passes through two narrow slits, the plane waves do not maintain a constant phase relation and so the light will show no interference patterns in the region beyond the openings. In order for an interference pattern to develop, the incoming light must satisfy two conditions:

- The light sources must be coherent. This means that the plane waves from the sources must maintain a constant phase relation.

- The light must be monochromatic. This means that the light has just one wavelength.

When the coherent monochromatic laser light falls on two slits separated by a distance d, the emerging light will produce an interference pattern on a viewing screen a distance D away from the center of the slits. The geometry of the double slit interference is shown in the figure below.

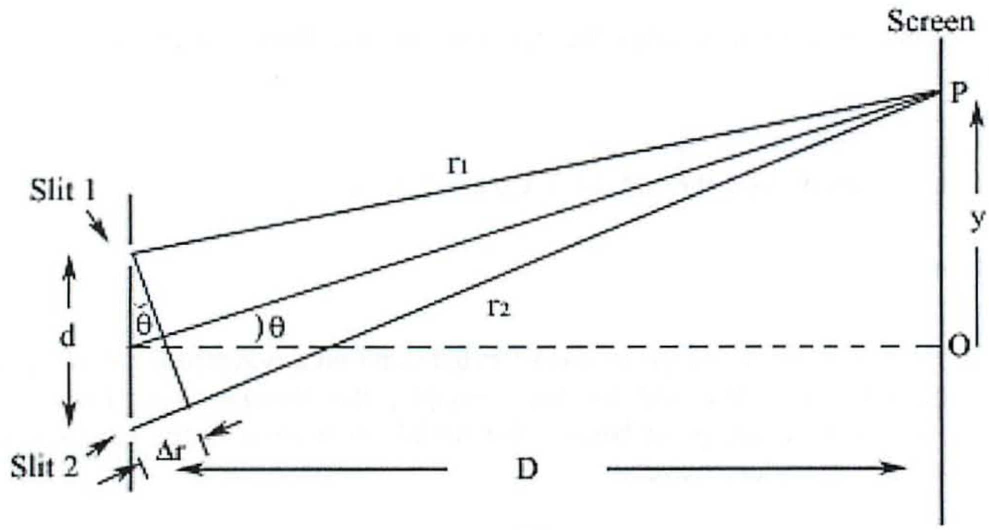
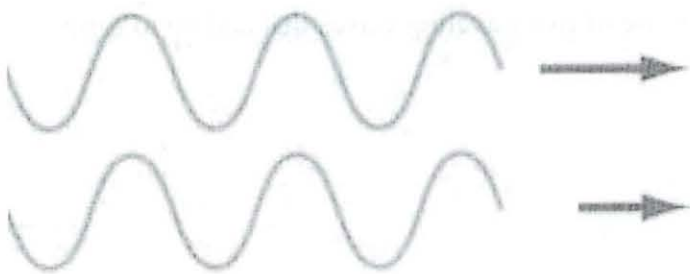


Figure 2: Double slit interference

Consider light that falls on the screen at a point P a distance y from the point O that lies on the screen a perpendicular distance D from the double slit system. The light from the slit 2 will travel an extra distance $r_2 - r_1 = \Delta r$ to the point P than the light from slit 1. This extra distance is called the path length.

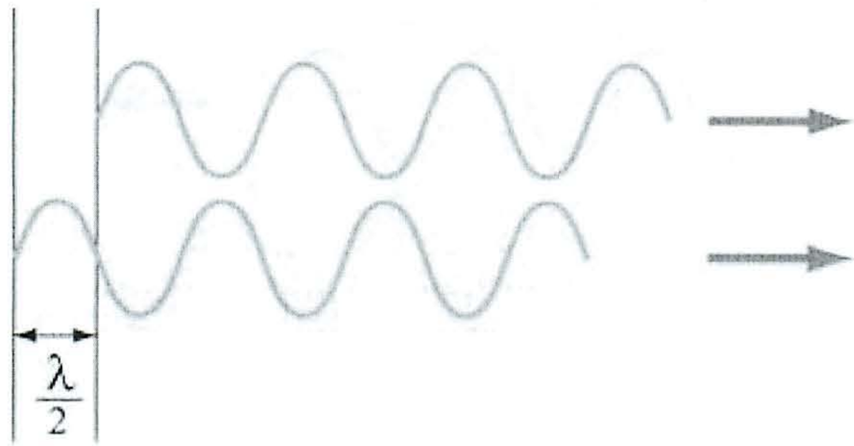
Question 1: Draw a picture of two traveling waves that add up to form *constructive interference*.

Answer:



Question 2: Draw a picture of two traveling waves that add up to form *destructive interference*.

Answer:



Question 3: Explain why constructive interference will appear at the point P when the path length is equal to an integral number of wavelengths of the monochromatic light.

$$\Delta r = m\lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots \text{ constructive interference}$$

Answer: The wavefront that emerges from slit 2 travels a further distance to reach the point P than the wavefront from slit 1. The extra distance is the path length Δr. When this extra distance is an integral number of wavelengths, the two wavefronts line up as in the figure in the answer to Question1 and so constructive interference occurs. The negative values of m correspond to the case when the slit 2 is closer to the point P then the slit 1.

We place the screen so that the distance to the screen is much greater than the distance between the slits, $D >> d$. In addition we assume that the distance between the slits is much greater than the wavelength of the monochromatic light, $d >> \lambda$.

Question 4: Based on the geometry of the double slits, show that the condition for constructive interference becomes

$$d \sin\theta = m\lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots \text{ constructive interference.}$$

Answer: From the geometry of the slits, the path length is related to the distance d between the slits according to $\Delta r = d\sin\theta$. This establishes the condition for constructive interference.

Question 5: Explain why destructive interference will appear at the point P when the path length is equal to an odd integral number of half wavelengths

$$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots \text{ destructive interference.}$$

Answer: When the path length is an odd integral number of half wavelengths, the wavefront is shifted as in the answer to Question 2, so the maximum and minimum line up producing destructive interference. (The negative values of m correspond to the case when the slit 2 is closer to the point P then the slit 1.)

Question 6: Let y be the distance between the point P and the point O on the screen. Find a relation between the distance y, the wavelength λ, the distance between the slits d, and the distance to the screen D such that a constructive interference pattern will occur at the point P.

Answer: Since the distance to the screen is much greater than the distance between the slits, $D >> d$, the angle θ is very small, so that

$$\sin\theta \approx \tan\theta = y/D .$$

Then the constructive interference fringe patterns will occur at the distances,

$$y \approx m\frac{D\lambda}{d}, \quad m = 0, \pm 1, \pm 2, \pm 3, ... \quad \text{constructive interference}$$

Question 7: Find a similar relation such that destructive interference fringes will occur at the point P.

Answer: The destructive interference fringes will occur at

$$y \approx \left(m + \frac{1}{2}\right)\frac{D\lambda}{d}, \quad m = 0, \pm 1, \pm 2, \pm 3, ...$$

Intensity of Double Slit Interference:

Suppose that the waves are emerging from the slits are sinusoidal plane waves. The slits are located at the plane $x = -D$. The light that emerges from slit 1 and slit 2 at time t are in phase. Let the screen be placed at the plane $x = 0$. Suppose the component of the electric field of the wave from slit 1 at the point P is given by

$$E_1 = E_0 \sin(\omega t) .$$

Let's assume that the plane wave from slit 2 has the same amplitude E_0 as the wave from slit 1. Since the plane wave from slit 2 has to travel an extra distance to the point P equal to the path length, this wave will have a phase shift ϕ relative to the wave from slit 1,

$$E_2 = E_0 \sin(\omega t + \phi) .$$

Question 8: Why are the phase shift ϕ, the wavelength λ, the distance between the slits, and the angle related θ by

$$\phi = \frac{2\pi}{\lambda} d \sin\theta .$$

As a hint how are the ratio of the phase shift ϕ to 2π and the ratio of the path length $\Delta r = d\sin\theta$ to wavelength λ, related?

Answer: The ratio of the phase shift ϕ to 2π is the same as the ratio of the path length $\Delta r = d\sin\theta$ to wavelength λ,

$$\frac{\phi}{2\pi} = \frac{\Delta r}{\lambda} .$$

Therefore the phase shift ϕ is given by

$$\phi = \frac{2\pi}{\lambda} d \sin\theta .$$

The total electric field at the point P is the superposition of the these two fields

$$E_{total} = E_1 + E_2 = E_0\left(\sin(\omega t) + \sin(\omega t + \phi)\right).$$

Question 9: Use the trigonometric identity

$$\sin A + \sin(B) = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right).$$

To show that the total component of the electric field is

$$E_{total} = E_1 + E_2 = 2E_0 \sin\left(\omega t + \frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right).$$

Answer: So the electric field is given by

$$E_{total} = E_0\left(\sin(\omega t) + \sin(\omega t + \phi)\right) = 2E_0 \sin\left(\frac{\omega t + \omega t + \phi}{2}\right)\cos\left(\frac{\omega t - \omega t + \phi}{2}\right)$$

Thus the total component of the electric field is

$$E_{total} = E_1 + E_2 = 2E_0 \sin\left(\omega t + \frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right).$$

The *intensity* of the light is equal to the time-averaged Poynting vector

$$I = \langle \vec{\mathbf{S}} \rangle = \frac{1}{\mu_0} \langle \vec{\mathbf{E}} \times \vec{\mathbf{B}} \rangle .$$

Since the amplitude of the magnetic field is related to the amplitude of the electric field by $B_0 = E_0/c$. The intensity of the light is proportional to the time-averaged square of the electric field,

$$I \propto \langle E_{total}{}^2 \rangle = 4E_0{}^2 \cos^2\left(\frac{\phi}{2}\right) \left\langle \sin^2\left(\omega t + \frac{\phi}{2}\right) \right\rangle = 2E_0{}^2 \cos^2\left(\frac{\phi}{2}\right),$$

where the time-averaged value of the square of the sine function is

$$\left\langle \sin^2\left(\omega t + \frac{\phi}{2}\right) \right\rangle = \frac{1}{2}.$$

Let $I_{\max}$ be the amplitude of the intensity. Then the intensity of the light at the point P is

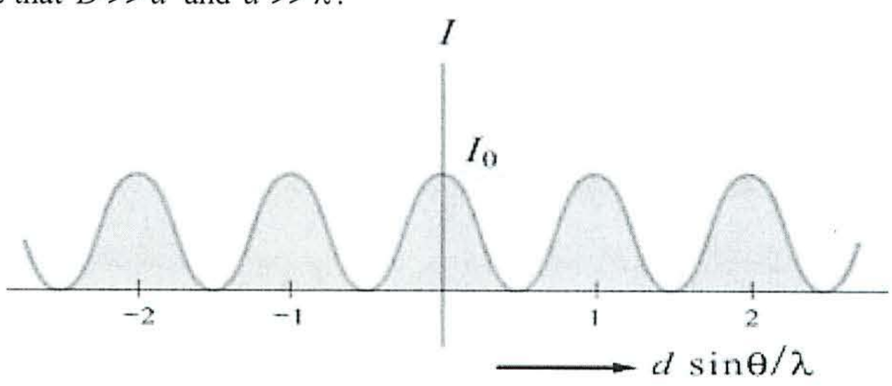
$$I = I_{\max} \cos^2\left(\frac{\phi}{2}\right)$$

Question 10: Show that the intensity is maximal when $d \sin\theta = m\lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots$.

Answer: The intensity has a maximum when the argument of the cosine is an integer number of multiples of π, $\phi/2 = \pm m\pi$. Since the phase shift is given by $\phi = \frac{2\pi}{\lambda} d \sin\theta$, we have that $\frac{\pi}{\lambda} d \sin\theta = \pm m\pi$. Thus we have the condition for constructive interference,

$$d \sin\theta = \pm m\lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots$$

Question 11: Graph the intensity pattern on the screen as a function of distance y from the point O for the case that $D >> d$ and $d >> \lambda$.



Question 12: Since the energy of the light is proportional to the square of the electric fields, is energy conserved for the time-averaged superposition of the electric fields i.e. does the following relation hold,

$$\left\langle (E_1 + E_2)^2 \right\rangle = \left\langle E_1^2 \right\rangle + \left\langle E_2^2 \right\rangle$$

Answer: The time-averaged square of the electric field is

$$\left\langle (E_1 + E_2)^2 \right\rangle = 4E_0^2 \cos^2\left(\frac{\phi}{2}\right)\left\langle \sin^2\left(\omega t + \frac{\phi}{2}\right)\right\rangle = 2E_0^2 \cos^2\left(\frac{\phi}{2}\right).$$

If we now average this over all phases,

$$2E_0^2 \frac{1}{2m\pi}\int_0^{2m\pi} \cos^2\left(\frac{\phi}{2}\right)d\phi = \frac{E_0^2}{m\pi}\int_0^{2m\pi}(1+\cos\phi)d\phi = \frac{E_0^2}{2m\pi}(2m\pi + \sin\phi|_0^{2\pi}) = E_0^2 .$$

The time-average $\left\langle E_1^2 \right\rangle = E_0^2 \left\langle \sin^2(\omega t)\right\rangle = E_0^2/2$.

The time-average $\left\langle E_2^2 \right\rangle = E_0^2 \left\langle \sin^2(\omega t + \phi)\right\rangle = E_0^2/2$.

Therefore only when we average over all possible phases is

$$\left\langle (E_1 + E_2)^2 \right\rangle = \left\langle E_1^2 \right\rangle + \left\langle E_2^2 \right\rangle.$$

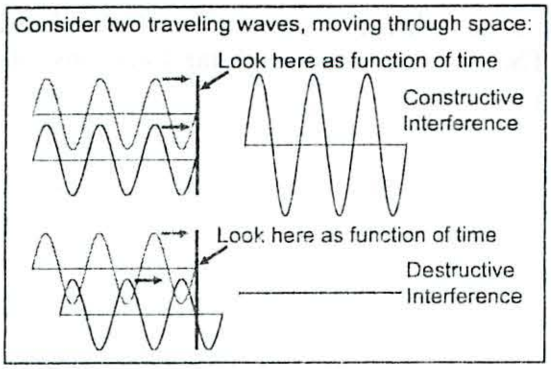
But this is precisely what we must do in order to conserve energy.

Topics: Interference
Related Reading: Course Notes: Chapter 14
Experiments: (11) Interference

Topic Introduction

Today we will continue our investigation of the interference of EM waves with a discussion about diffraction, and then we will conduct our final experiment.

The General Picture



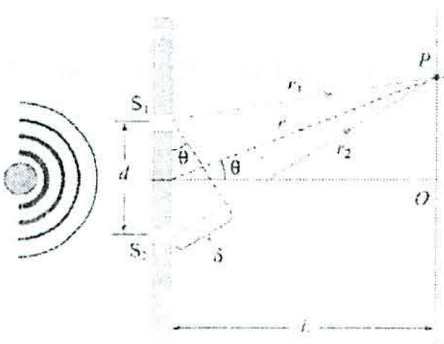
The picture at left forms the basis of all the phenomena we will discuss today. Two different waves (red & blue) arrive at a single position in space (at the screen). If they are in phase then they add constructively and you see a bright spot. If they are out of phase then the add destructively and you see nothing (dark spot).

The key to creating interference is creating phase shift between two waves that are then brought together at a single position. A common way to do that is to add extra path length to one of the waves relative to the other. We will look at a variety of systems in which that happens.

Thin Film Interference

The first phenomenon we consider is thin film interference. When light hits a thin film (like a soap bubble or an oily rain puddle) it does two things. Part of the light reflects off the surface. Part continues forward, then reflects off the next surface. Interference between these two different waves is responsible for the vivid colors that appear in many systems.

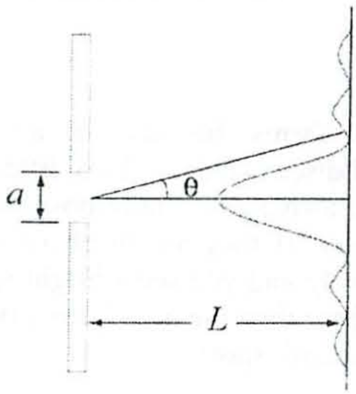
Two Slit Interference



Light from the laser hits two very narrow slits, which then act like in-phase point sources of light. In traveling from the slits to the screen, however, the light from the two slits travel different distances. In the picture at left the light from the bottom slit travels further than the light from the top slit. This extra path length introduces a phase shift between the two waves and leads to a position dependent interference pattern on the screen.

Here the extra path length is $\delta = d\sin(\theta)$, leading to a phase shift ϕ given by $\frac{\delta}{\lambda} = \frac{\phi}{2\pi}$. Realizing that phase shifts that are multiples of 2π give us constructive interference while odd multiples of π lead to destructive interference leads to the following conditions: Maxima: $d\sin(\theta) = m\lambda$; Minima: $d\sin(\theta) = \left(m + \frac{1}{2}\right)\lambda$

Diffraction



The next kind of interference we consider is light going through a single slit, interfering with itself. This is called diffraction, and arises from the finite width of the slit (a in the picture at left). The resultant effect is not nearly as easy to derive as that from two-slit interference (which, as you can see from above, is straight-forward). The result for the anglular locations of the minima is $a\sin(\theta) = m\lambda$.

Important Equations

Interference Conditions $\frac{\Delta L}{\lambda} = \frac{\phi}{2\pi} = \begin{cases} m & \text{constructive} \\ m + \frac{1}{2} & \text{destructive} \end{cases}$

Two Slit **Maxima**: $d\sin(\theta) = m\lambda$

Single Slit (Diffraction) **Minima**: $a\sin(\theta) = m\lambda$

Experiment 11: Interference

Preparation: Read pre-lab and answer pre-lab questions

The lab investigates interference of laser light going through slits, diffracting off of hair and reflecting off of a CD.

5/10

Class 35: Outline

Hour 1 & 2:

- Diffraction
- Experiment 11: Interference and Diffraction

Blue laser
- maxima closer together

Spacing on CD ~ 1 micron
- must be at least 1 wave lenght plus some for saftey

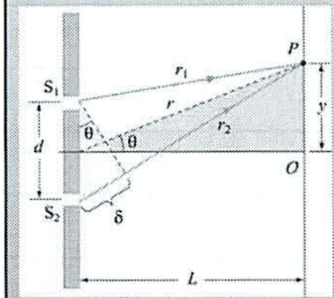
How in the world do we measure 1/10,000 of a cm?

Visible (red) light:

$$f_{red} = 4.6\times10^{14}\ Hz \quad \lambda_{red} = \frac{c}{f} = 6.54\times10^{-5}\ cm$$

Use Interference

Two In-Phase Sources: Geometry



Assuming $L >> d$:

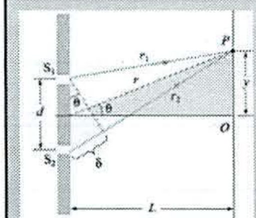
Extra path length

$$\delta = d\sin(\theta)$$

$\delta = d\sin(\theta) = m\lambda$ $\Rightarrow$ Constructive

$\delta = d\sin(\theta) = (m+\frac{1}{2})\lambda$ $\Rightarrow$ Destructive

Two Sources in Phase



Assume $L >> d >> \lambda$

$$y = L\tan\theta \approx L\sin\theta$$

$$\Rightarrow \delta = d\sin\theta = dy/L$$

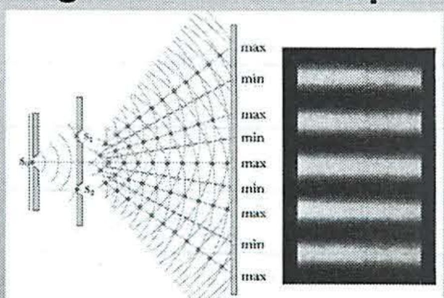
(1) Constructive: $\delta = m\lambda$

$$y_{constructive} = m\frac{\lambda L}{d} \quad m = 0, 1...$$

(2) Destructive: $\delta = (m+1/2)\lambda$

$$y_{destructive} = \left(m+\frac{1}{2}\right)\frac{\lambda L}{d} \quad m = 0, 1, ...$$

Young's Double-Slit Experiment



Bright Fringes: Constructive interference

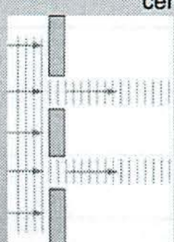
Dark Fringes: Destructive interference

Diffraction

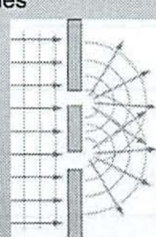
P35- 7

Diffraction

Diffraction: The bending of waves as they pass by certain obstacles



No Diffraction
No spreading after passing though slits



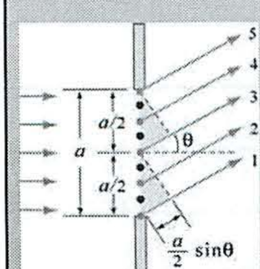
Diffraction
Spreading after passing though slits

P35- 8

interfears w/ itself

Single-Slit Diffraction

"Derivation" (Motivation) by Division:



Divide slit into two portions:

$$\delta = r_1 - r_3 = r_2 - r_4 = \frac{a}{2}\sin\theta$$

Destructive interference:

$$\delta = \frac{a}{2}\sin\theta = \left(m + \tfrac{1}{2}\right)\lambda$$

$$a\sin\theta = m\lambda \qquad m = \pm 1, \pm 2, \ldots$$

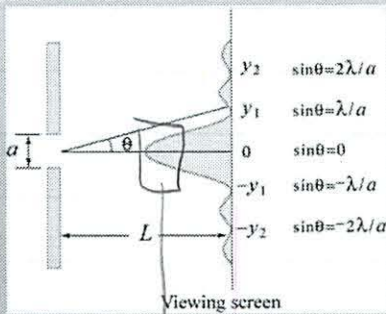
Don't get confused – this is DESTRUCTIVE!

P35- 9

top of hole interfearing w/ bottom hole

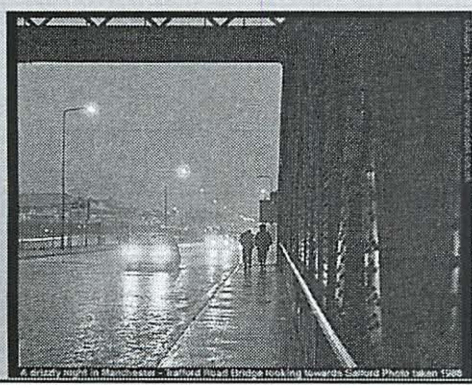
Intensity Distribution

Destructive Interference: $a\sin\theta = m\lambda \qquad m = \pm 1, \pm 2, \ldots$



P15- 10

broad central maxima



P15- 11

headlights r streetlights
-through camera's eye

eye limits ability to seperate objects
-spreads light out

Diffraction in Everyday Life: Rayleigh Criterion

For circular apertures of diameter *D* (like pupils, optics…)

$$\sin\theta_{\min} = 1.22\,\lambda/D$$

Point-like light sources become "airy disks" after diffraction:

The apparent size of the object depends on the size *D* of the aperture (lens, pupil)

To resolve two objects, they need to be separated by more than the critical angle:

$$\alpha_{critical} = 1.22\,\lambda/D$$

P15- 12

Group Problem: Headlights

- Headlight separation:
  - d ~ 1.5 m
- Pupil Diameter:
  - D ~ 4 mm
- Wavelength:
  - λ ~ 550 nm

About how close must a car be before you can distinguish the two headlights?

measure critical angle

Plug into critical angle eq

PRS: Headlight Resolution

Is it easier to resolve two headlights at night or during the day?

:20

0% 1. At night
0% 2. During the day
0% 3. It doesn't matter
0% 4. I don't know

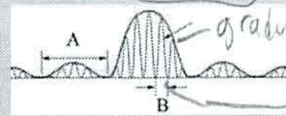
eyes dilate
- pupils bigger
- critical angle smaller
  ↳ less diffraction bigger holes
- easier to distinguish far away

Interference & Diffraction Together

2 different aspects

PRS: Interference & Diffraction

Coherent monochromatic plane waves impinge on two long narrow apertures (width a) that are separated by a distance d (d >> a).



gradual change

specific dots

The resulting pattern on a screen far away is shown above. Which structure in the pattern above is due to the finite width a of the apertures?

0% 1. The distantly-spaced zeroes of the envelope, as indicated by the length A above. — diffractions, width of slit

0% 2. The closely-spaced zeroes of the rapidly varying fringes with length B above. — interference, distance b/w slits

0% 3. I don't know

20

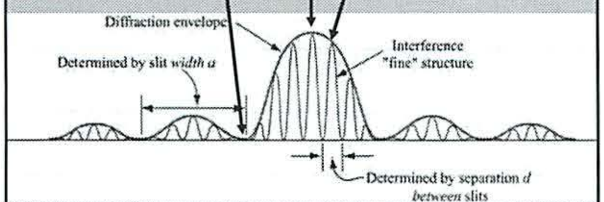
P35-16

Two Slits With Finite width a

Zero Order Maximum

First Diff. Minimum $a\sin\theta = \lambda$

First Order Maximum $d\sin\theta = \lambda$



P35-17

← this picture likely on final

Understand it completly

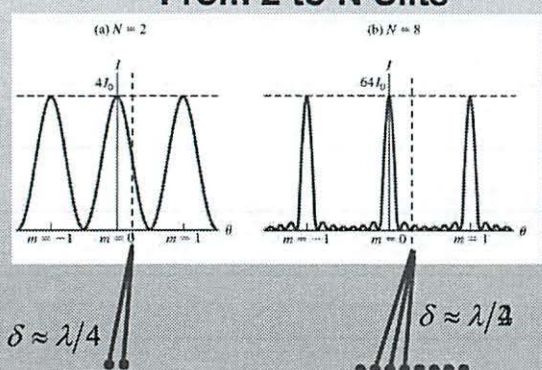
Lecture Demonstration: Double Slits with Width

P35-18

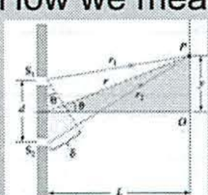
Experiment 11, Part I: Measure Laser Wavelength

$$y_{constructive} = m\frac{\lambda L}{d} \quad m = 0,1\ldots$$

From 2 to N Slits



How we measure 1/10,000 of a cm



Question: How do you measure the wavelength of light?

Answer: Do the same experiment we just did (with light)

First $y_{destructive} = \lambda L/2d$

λ is smaller by 10,000 times.

But d can be smaller (0.1 mm instead of 0.24 m)

So y will only be 10 times smaller – **still measurable**

Experiment 10, Part I: Measure Laser Wavelength

$$y_{constructive} = m\frac{\lambda L}{d} \quad m = 0, 1 \ldots$$

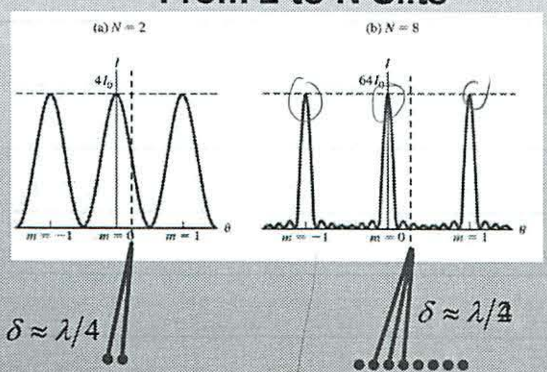
PRS: Changing Colors

You just observed an interference pattern using a red laser. What if instead you had used a blue laser? In that case the interference maxima you just saw would be

0% 1. Closer Together
0% 2. Further Apart
0% 3. I Don't Know.

:20

From 2 to N Slits



Very broad opportunity for diffraction

sharp points to pull out

Experiment 10, Part II: Diffraction Grating: CD

$$y_{constructive} = m\frac{\lambda L}{d} \quad m = 0,1\ldots$$

P35-25

Measuring Human Hair

$A + B = 0$ so $A = -B$

Babinet's Principle

Case I: Put in a slit, get diffraction

Case II: Fill up slit, get nothing

Case III: Remove slit, get diffraction

By superposition, the E field with the slit and the E field with just the filling must be opposites in order to cancel:

$$E_{\text{filling}} = -E_{\text{slit}}$$

So the intensities are identical: $I_{\text{filling}} = I_{\text{slit}}$

P35-26

Experiment 10, Part III: Measure Hair Thickness

$$y_{destructive} = m\frac{\lambda L}{a} \quad m = 1,2\ldots$$

P35-27

PRS: Lower Limit?

Using diffraction seems to be a useful technique for measuring the size of small objects. Is there a lower limit for the size of objects that can be measured this way?

0% 1. Yes – but if we use blue light we can measure even smaller objects
0% 2. Yes – and if we used blue light we couldn't even measure objects this small
0% 3. Not really
0% 4. I Don't Know

Yes – (1)s

Can't measure smaller than wavelenght

– unless trick, fiberoptic wire

buildings are smaller than AM radio wavelenght
– so ignores it

higher freq = more data, less building pass through

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics
8.02

Experiment 11: Interference and Diffraction

OBJECTIVES

1. To explore the diffraction of light through a variety of apertures
2. To learn how interference can be used to measure small distances very accurately. By example we will measure the wavelength of the laser, the spacing between tracks on a CD and the thickness of human hair

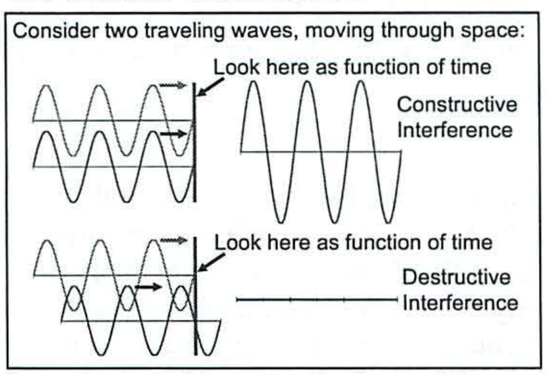
WARNING! The beam of laser pointers is so concentrated that it can cause *real* damage to your retina if you look into the beam either directly or by reflection from a shiny object. Do NOT shine them at others or yourself.

PRE-LAB READING

INTRODUCTION

Electromagnetic radiation propagates as a wave, and as such can exhibit interference and diffraction. This is most strikingly seen with laser light, where light shining on a piece of paper looks speckled (with light and dark spots) rather than evenly illuminated, and where light shining through a small hole makes a pattern of bright and dark spots rather than the single spot you might expect from your everyday experiences with light. In this lab we will use laser light to investigate the phenomena of interference and diffraction and will see how we can use these phenomena to make accurate measurements of very small objects like the spacing between tracks on a CD and the thickness of human hair.

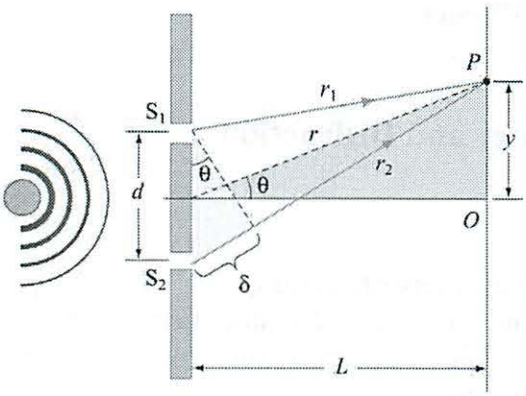
The Details: Interference



The picture at left forms the basis of all the phenomena you will observe in the lab. Two different waves arrive at a single position in space (at the screen). If they are in phase then they add constructively and you see a bright spot. If they are out of phase then they add destructively and you see nothing (dark spot).

The key to creating interference is creating phase shift between two waves that are then brought together at a single position. A common way to do that is to add extra path length to one of the waves relative to the other. In this lab the distance traveled from source to screen, and hence the relative phase of incoming waves, changes as a function of lateral position on the screen, creating a visual interference pattern.

Two Slit Interference

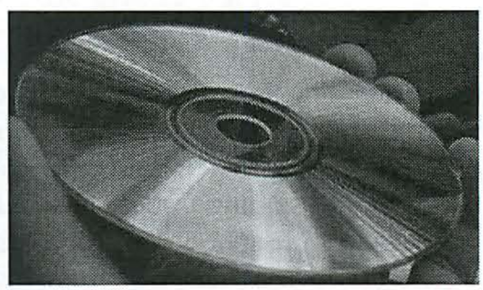


The first phenomenon we consider is two slit interference. Light from the laser hits two very narrow slits, which then act like in-phase point sources of light. In traveling from the slits to the screen, however, the light from the two slits travel different distances. In the picture at left light hitting point P from the bottom slit travels further than the light from the top slit. This extra path length introduces a phase shift between the two waves and leads to a position dependent interference pattern on the screen.

Here the extra path length is $\delta = d\sin(\theta)$, leading to a phase shift ϕ given by $\frac{\delta}{\lambda} = \frac{\phi}{2\pi}$. Realizing that phase shifts that are multiples of 2π give us constructive interference while odd multiples of π lead to destructive interference leads to the following conditions: Maxima: $d\sin(\theta) = m\lambda$; Minima: $d\sin(\theta) = (m+\frac{1}{2})\lambda$

constructive destructive

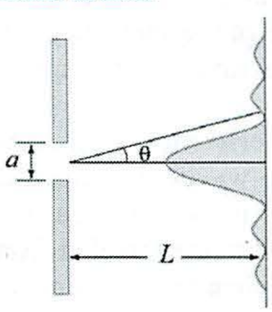
Multiple Slit Interference



If instead of two identical slits separated by a distance d there are multiple identical slits, each separated by a distance d, the same effect happens. For example, at all angles θ satisfying $d\sin(\theta) = m\lambda$ we find constructive interference, now from all of the holes. The difference in the resulting interference pattern lies in those regions that are neither maxima or minima but rather in between. Here, because more incoming waves are available to interfere, the interference becomes more destructive, making the minima appear broader and the maxima sharper. This explains the appearance of a brilliant array of colors that change as a function of angle when looking at a CD. A CD has a large number of small grooves, each reflecting light and becoming a new source like a small slit. For a given angle, a distinct set of wavelengths will form constructive maxima when the reflected light reaches your eyes.

cool

Diffraction

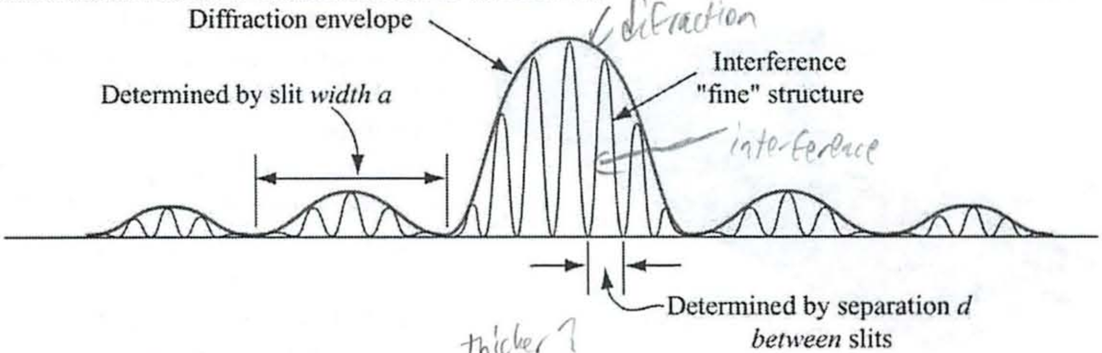


The next kind of interference we consider is light going through a single slit, interfering with itself. This is called diffraction, and arises from the finite width of the slit (a in the picture at left). The resultant effect is not nearly as easy to derive as that from two-slit interference (which, as you can see from above, is straight-forward). The result for the anglular locations of the minima is $a\sin(\theta) = m\lambda$.

both were on MP
that I did not understand
- this reading much better

Putting it Together

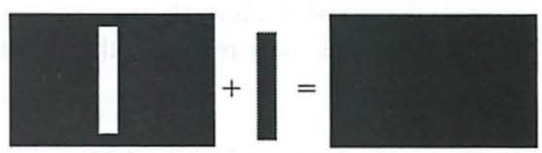
If you have two wide slits, that is, slits that exhibit both diffraction and interference, the pattern observed on a distant screen is as follows:



Here the amplitude modulation (the red envelope) is set by the diffraction (the width of the slits), while the "individual wiggles" are due to the interference between the light coming from the two different slits. You know that this must be the case because d must be larger than a, and hence the minima locations, which go like $1/d$, are closer together for the two slit pattern than for the single slit pattern.

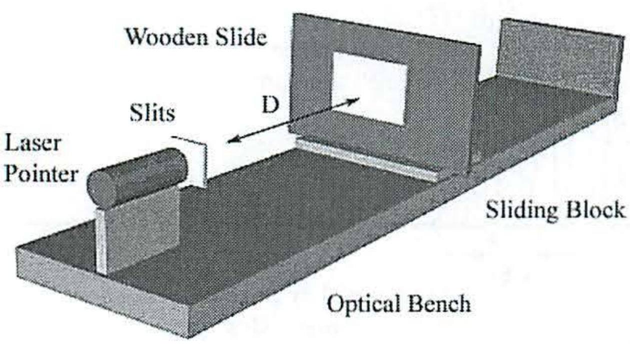
The Opposite of a Slit: Babinet's Principle

So far we have discussed sending light through very narrow slits or reflecting it off of small grooves, in each case creating a series of point-like "new sources" of light that can then go on and interfere. Rather amazingly, light hitting a small solid object, like a piece of hair, creates the same interference pattern as if the object were replaced with a hole of the same dimensions. This idea is Babinet's Principle, and the reason behind it is summed up by the pictorial equation at right. If you add an object to a hole of the same size, you get a filled hole. EM waves hitting those objects must add in the same fashion, that is, the electric fields produced when light hits the hole, when added to the electric fields produced by the small object, must add to the electric fields produced when light hits the filled hole. Since no light can get through the filled hole, $E_{hole} + E_{object} = 0$. Thus we find that the electric fields coming out of the hole are equal and opposite to the electric fields diffracting off of the small object. Since the observed interference pattern depends on intensity, the square of the electric field, the hole and the object will generate identical diffraction patterns. By measuring properties of the diffraction pattern we can thus measure the width of the small object. In this lab the small object will be a piece of your hair.



APPARATUS

1. Optical bench



The optical bench consists of a holder for a laser pointer, a mount for slides (which contain the slits you will shine light through), and a sliding block to which you will attach pieces of paper to mark your observed interference patterns. Note that a small ring can be slid over the button of the laser pointer in order to keep it on while you make your measurements.

2. Slit Slides

You will be given two slides, each containing four sets of slits labeled a through d. One slide contains single slits with widths from 20 μm to 160 μm. The other slide contains double slits with widths of 40 μm or 80 μm, separated by distances of 250 μm or 500 μm.

GENERALIZED PROCEDURE

In this lab you will shine the light through slits, across hairs or off of CDs and make measurements of the resulting interference pattern.

Part 1: Laser Wavelength
In this part you will measure the wavelength of the laser using the two narrow double slits.

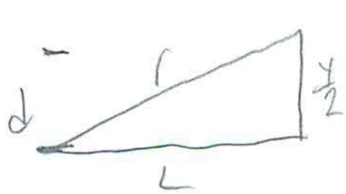
Part 2: Interference from a CD
Next, you will measure the width of tracks on a CD by reflecting laser light off of it and measuring the resulting multi-slit interference pattern.

Part 3: Thickness of Human Hair
Finally, you will discover the ability to measure the size of small objects using diffraction, by measuring the width of a human hair.

Cool!

END OF PRE-LAB READING

P-Set $y = \frac{\lambda L}{4D}$



IN-LAB ACTIVITIES

EXPERIMENTAL SETUP

1. Download the LabView file from the web and save the file to your desktop. Start LabView by double clicking on this file.

MEASUREMENTS

Part 1: Laser Wavelength

In this part you will measure the wavelength λ of the laser light you are using

1. Set up the optical bench as pictured in the apparatus diagram.
    a. Clip paper onto the wooden slide, and place some distance away from the slide holder (is it better to be farther away or closer?)
    b. Place the double slit slide in the slide holder and align so that light from the laser goes through slit pattern *a*. as close as possible
    c. Turn the laser on (lock it with the clip that slides around the on button)
    d. Adjust the location of the wooden slide so that the pattern is visible but as large as possible
2. Mark the locations of the intensity maxima. If they are too close to measure individually, mark of a set of them and determine the average spacing.

Question 1:

What distance between the slide and the screen did you use? What was the average distance Δy between maxima?

$L = 110 - 16.7 = 93.3$ cm $= .933$ m

$\Delta y = 4$ mm $= .004$ m or $1.4/5 = .28$ mm $= .0028$ m

Question 2:

Using $\lambda = \frac{d\Delta y}{L}$, what do you calculate to be the wavelength of the laser light? Does this make sense?

$d = .125$ mm $= .25 \cdot 10^{-3}$ m $= .00025$

$\lambda = 7.5 \cdot 10^{-7}$, still a little high

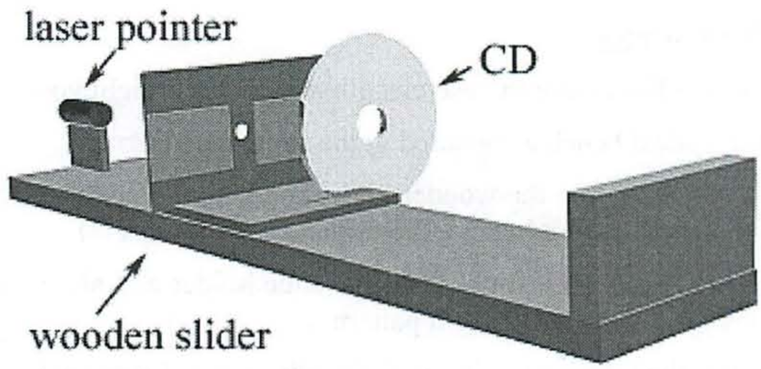
~500 - 700 nm

real sci - measure a few times

Part 2: Interference from a CD

In this part you will determine the track width on a CD by measuring the distance between interference maxima generated by light reflected from it.

1. Remove the slide from in front of the laser pointer
2. Clip a card with a hole in it to the back of the wooden slide.
3. Place a CD in the groove in the back of the wooden slider. Light will pass through the hole in the slider and card, reflect off the CD, and land on the card.
4. Turn on the laser and measure the distance between interference maxima.



Question 3:

Using $d = \frac{\lambda L}{\Delta y}$, what is the width of the tracks? Does this make sense? Why are they that size?

d should be smaller than tracks

$\Delta y = 1.5\,cm = .015\,m$

$L = 3\,cm = .03\,m$

$\lambda = 7.5 \cdot 10^{-7}\,m$

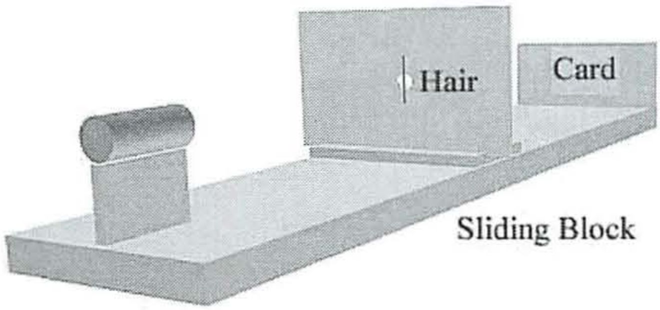
$d = 1.5 \cdot 10^{-6}\,m$

1 micron

Part 3: Thickness of Human Hair

Now you will measure the thickness of a human hair using diffraction.

1. Remove the CD and card from the wooden slide, and tape some hair across the hole (the hair should run vertically as pictured below).
2. Clip a card to the block at the end of the apparatus.
3. Shine the laser on the hair, and adjust the distance between the hair and the card so that you obtain a useable diffraction pattern.



Question 4:

What is the thickness of the hair that you measure? Does this seem reasonable? Is it the same for all members of your group?

$\Delta y = 5\text{ mm} = .005\text{ m}$

$L = 110 - 68.2 = 41.8 = .418$

$\lambda = 7.5 \cdot 10^{-7}\text{ m}$

$d = 6.27 \cdot 10^{-5}\text{ m}$

67 microns

~100 microns

Further Questions (for experiment, thought, future exam questions...)

- Instead of measuring the wavelength of light from the two slit patterns, you could have instead used single slits. Would that have been more or less accurate? Why?
- Why did you use two slit pattern *a* to measure the light wavelength rather than *d*?
- Where does most of the measurement error come from? How would you improve this in future labs?
- If we redid these experiments with a blue laser instead of red, what changes would you have needed to make? Would it have affected the accuracy of the measurements?
- Does the track width change as a function of location on the CD? If so, is it larger or smaller near the outside?
- What is the ratio of the track size to the wavelength of the light that you used (which is very similar to the wavelength of light used in commercial players)?
- What would happen to the diffraction pattern if the track width was smaller?
- Why is Blu-Ray an improvement over older CD/DVD technology?

Final Exam Date and Room TBA (Most Likely Monday Morning May 17 from 9 am-12 noon) Location: Johnson Track (upstairs).

Material Covered & Exam Format:

1. All material covered in the course through the end of the course (through interference) will be fair game for the final exam.
2. The exam will be slightly less than twice the length of your first three exams, with analytic and conceptual questions.
3. This will be a closed book exam. There will be a formula sheet given on the exam. You should have plenty of time to finish the exam in the three hours allotted.

What We Expect From You In Particular On The Final

(1) An understanding of Maxwell's equations, including Maxwell's addition to Ampere's Law (displacement current). You should be able to produce and identify each of Maxwell's equations, as well as give brief explanations of the meaning and use of each of them (don't be surprised by a question like "State each of Maxwell's equations and briefly explain their meaning and typical use.") In particular:

(a) The ability to use Gauss's Law to obtain electric fields from highly symmetric distributions of charge.

(b) An ability to use Ampere's Law to obtain magnetic fields in magnetostatics for highly symmetric distributions of current.

(c) An ability to do analytic problems related to the displacement current. That is, you should be able to calculate the magnetic field anywhere inside a charging capacitor, and so on.

(d) An understanding of how to use Faraday's Law in problems involving the generation of induced EMFs. You should be able to formulate quantitative answers to questions about energy considerations in Faraday's Law problems, e.g. the power going into ohmic dissipation comes from the decreasing kinetic energy of a rolling rod, etc.

(2) An understanding of the concept of electric field and electric potential difference, an ability to calculate those in specific circumstances (e.g. given V(x,y,z) find **E**(x,y,z), or given **E**(x,y,z) find V(x,y,z), and so on). This includes the ability to calculate capacitance for highly symmetric situations.

(3) An understanding of the concept of an electric dipole and the forces and torques on such a dipole in an external electric field.

(4) An ability to use the Biot-Savart Law to obtain magnetic fields in magnetostatics for any distribution of current.

(5) An understanding of how to calculate the forces and torques on a current element in an external magnetic field or on a charge moving in an external magnetic field, including the characteristics of cyclotron motion.

(6) An understanding of the concept of a magnetic dipole and the forces and torques on such a dipole in an external magnetic field.

(7) An understanding of inductance and the ability to calculate it for simple geometries.

(8) An understanding of the behavior of DC and AC circuits involving resistors, capacitors, inductors and any combination thereof.

(9) An understanding of the concepts of energy in electric and magnetic fields, and of energy flow in the Poynting vector.

(10) An ability do to analytic and conceptual problems related to plane electromagnetic waves—e.g. obtain **E** given **B** and vice versa, determine the direction of propagation, and so on.

(11) An understanding of the concepts of interference and diffraction, and the ability to do simple (conceptual) problems related to these concepts.

(12) An ability to calculate the Poynting flux vector and integrals of that vector over surfaces to show energy conservation in situations involving, for example, a charging or discharging capacitor, a resistor, a charging or discharging battery, and an inductor where the current is increasing or decreasing. This means that you should both be able to do the analytic calculations and explain their physical significance.

email in 9U

Topics

- Online past exams are good
- Maxwell's Eq → $\vec{E}$ + $\vec{B}$ Fields
- Gauss
- Ampere
  - displacement current
- Colombs
- B-S
  (may be on prob not)
- Faraday's Law
- Waves
- Poynting Vector
- Capicator problem (P-Set 12 #2, 8 w/ Solonoid)
- Integrate Poynting vector over area
- RLC
  - under represented on finals
- EM waves
- Interfearence - little math, just concepts
  - phase shift
  - #5 on last P-set

5/12

Class 36: Outline

Final Exam Review

A Final Topic

Before Starting…

All of your grades should now be posted (with possible exception of last problem set). If this is not the case contact your grad TA immediately.

-1 or Exc means excused

Maxwell's Equations

$$\oint\!\!\oint_S \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0} \quad \text{(Gauss's Law)}$$

$$\oint_C \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's Law)}$$

$$\oint\!\!\oint_S \vec{B}\cdot d\vec{A} = 0 \quad \text{(Magnetic Gauss's Law)}$$

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0 I_{enc} + \mu_0\varepsilon_0\frac{d\Phi_E}{dt} \quad \text{(Ampere-Maxwell Law)}$$

$$\vec{F} = q(\vec{E} + \vec{v}\times\vec{B}) \quad \text{(Lorentz force Law)}$$

inbetween plates of capicator if $\vec{E}$ changing

Know really well

Parallel plate capicator

PS 12 #2

Guass → Ampere → S → Integrate

↳current flowing in

Solenoid

- B from Ampere $L = \frac{\Phi}{I}$
- E from Faraday S, P

Coxaial cable ($\frac{1}{r}$, S F get ln)

Waves $E = E_0 \sin(kx - \omega t)$ → B=?

Series RLC driven → lead/lag

T-Shirts are in differential form

Green + Guass to get ride of S

really study

Based on observation

$\vec{E}$ + $\vec{B}$ are actually same thing

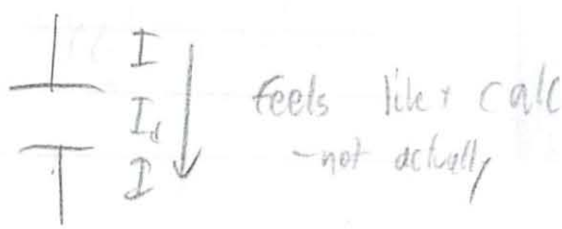
different ways of viewing same thing

$I_d = \oint B \cdot ds = \mu_0 (I + I_d)$

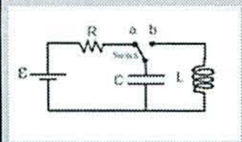
if electric fields charging must use I_d

- waves where no current
- capicator ← most likly
  - charging / discharging
- PSet 12 #8

*need to look at more physical extension of current

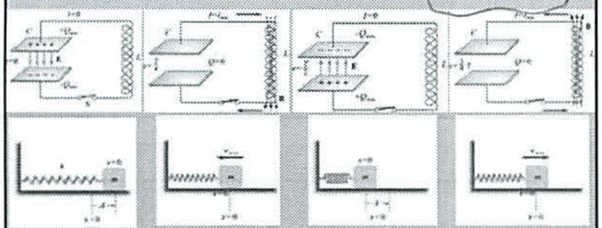


Undriven LC Circuit



Oscillations: From charge on capacitor (Spring) to current in inductor (Mass)

$$\omega_0 = \frac{1}{\sqrt{LC}}$$ natural freq



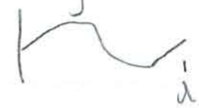
P36 - 4

Occilates

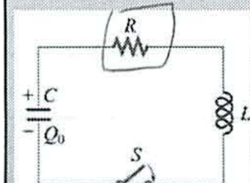
Conservation of Energy

Max charge = Current

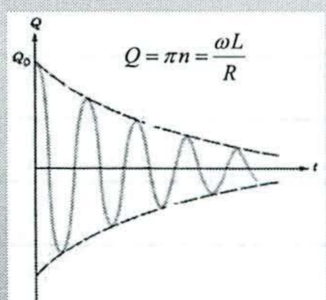
How long wait until...



Damped LC Oscillations



Resistor dissipates energy and system rings down over time



P36 - 5

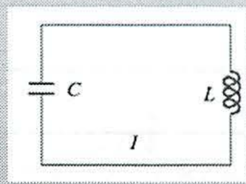
PRS Questions: Undriven RLC Circuits

Classes 27 & 28

P36 - 6

PRS: LC Circuit

Consider the LC circuit at right. At the time shown the current has its maximum value. At this time



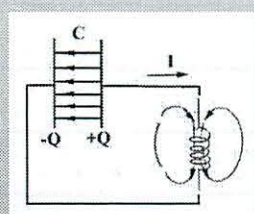
0%	1. The charge on the capacitor has its maximum value
0%	2. The magnetic field is zero
0%	3. The electric field has its maximum value
0%	4. The charge on the capacitor is zero
0%	5. Don't have a clue

:20

PRS: LC Circuit

20

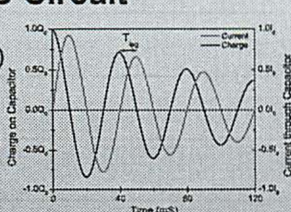
In the LC circuit at right the current is in the direction shown and the charges on the capacitor have the signs shown. At this time,



0%	1. I is increasing and Q is increasing
0%	2. I is increasing and Q is decreasing
0%	3. I is decreasing and Q is increasing
0%	4. I is decreasing and Q is decreasing
0%	5. Don't have a clue

PRS: LC Circuit

The plot shows the charge on a capacitor (black curve) and the current through it (red curve) after you turn off the power supply. If you put a core into the inductor what will happen to the time T_{Lag}?

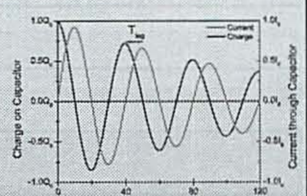


0%	1. It will increase
0%	2. It will decrease
0%	3. It will stay the same
0%	4. I don't know

20

PRS: LC Circuit

If you increase the resistance in the circuit what will happen to rate of decay of the pictured amplitudes?



0%	1. It will increase (decay more rapidly)
0%	2. It will decrease (decay less rapidly)
0%	3. It will stay the same
0%	4. I don't know

:20

AC Circuits: Summary

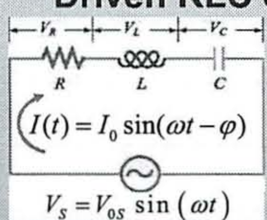
Element	V vs I_0	Current vs. Voltage	Resistance-Reactance (Impedance)
Resistor	$V_{0R} = I_0 R$	In Phase	$R = R$
Capacitor	$V_{0C} = \frac{I_0}{\omega C}$	Leads (90°)	$X_C = \frac{1}{\omega C}$
Inductor	$V_{0L} = I_0 \omega L$	Lags (90°)	$X_L = \omega L$

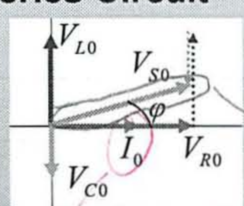
P36 - 11

low freq } works hard "dominates"
high freq }

to think about in Driven RLC

Driven RLC Series Circuit





$$V_{S0} = \sqrt{V_{R0}^2 + (V_{L0} - V_{C0})^2} = I_0 \sqrt{R^2 + (X_L - X_C)^2} \equiv I_0 Z$$

$I_0 = \frac{V_{S0}}{Z}$ $\quad$ $Z = \sqrt{R^2 + (X_L - X_C)^2}$ Impedance $\quad$ $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

P36 - 12

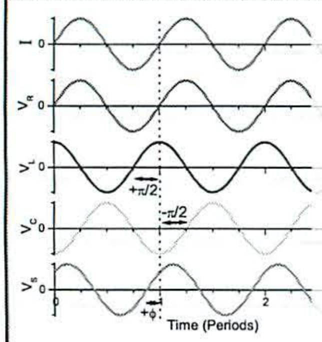
draw a phasor diagram

not always in this position
can be $-\frac{\pi}{2}$ to $\frac{\pi}{2}$

I_0 is always →

L ↑ high freq
C ↓ low freq

Plot I, V's vs. Time



$I(t) = I_0 \sin(\omega t)$

$V_R(t) = I_0 R \sin(\omega t)$

$V_L(t) = I_0 X_L \sin\left(\omega t + \frac{\pi}{2}\right)$ leads

$V_C(t) = I_0 X_C \sin\left(\omega t - \frac{\pi}{2}\right)$ lags

$V_S(t) = V_{S0} \sin(\omega t + \varphi)$ the most common mistake

$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

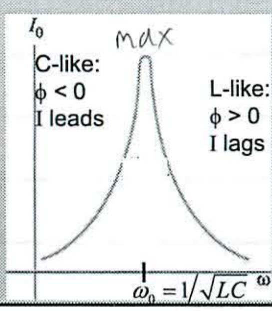
P36 - 13

\+ leads

− lags

Resonance

$$I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + (X_L - X_C)^2}}; \quad X_L = \omega L, \quad X_C = \frac{1}{\omega C}$$



On resonance:
I_0 is max; $X_L = X_C$; Z=R;
ϕ=0; Power to R is max

P36 - 14

Average Power: Resistor

$$\begin{aligned} \langle P \rangle &= \langle I^2(t) R \rangle \\ &= \langle I_0^2 \sin^2(\omega t - \varphi) R \rangle \\ &= I_0^2 R \langle \sin^2(\omega t - \varphi) \rangle \\ &= I_0^{\,2} R\left(\tfrac{1}{2}\right) \end{aligned}$$

P36 - 15

PRS Questions: Driven RLC Circuits

Classes 27 & 28

current always ⟶

V_c and I 90° out of phase

Capicator lags

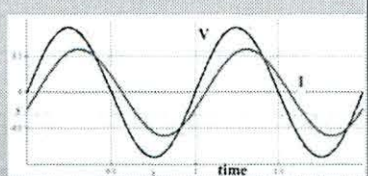
capicator dominating → below ω_0

* need to study more

be able to tell the traces!

PRS: Leading or Lagging?

The plot shows the driving voltage V (black curve) and the current I (red curve) in a driven RLC circuit. In this circuit,



0% 1. The current leads the voltage

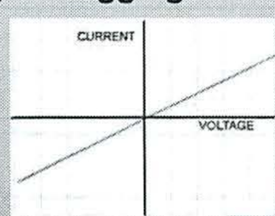
0% 2. The current lags the voltage

0% 3. Don't have a clue

:20

PRS: Leading or Lagging?

The graph shows current versus voltage in a driven RLC circuit at a given driving frequency. In this plot



0% 1. The current leads the voltage by about 45°

0% 2. The current lags the voltage by about 45°

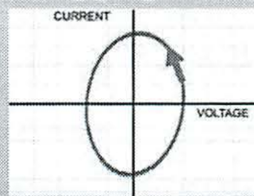
0% 3. The current and the voltage are in phase

0% 4. Don't have a clue.

20

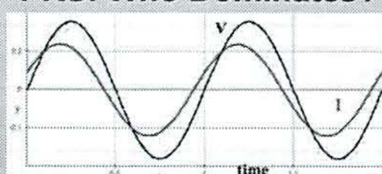
20 PRS: Leading or Lagging?

The graph shows current versus voltage in a driven RLC circuit at a given driving frequency. In this plot



0% 1. Current lags voltage by ~90°
0% 2. Current leads voltage by ~90°
0% 3. Current and voltage are almost in phase
0% 4. Not enough info (but they aren't in phase!)
0% 5. I don't know

PRS: Who Dominates?

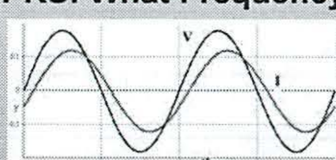


The graph shows current & voltage vs. time in a driven RLC circuit at a particular driving frequency. At this frequency, the circuit is dominated by its

0% 1. Resistance
0% 2. Inductance
0% 3. Capacitance
0% 4. I don't know

20

PRS: What Frequency?



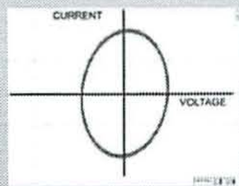
The graph shows current & voltage vs. time in a driven RLC circuit at a particular driving frequency. Is this frequency above or below the resonance frequency of the circuit?

0% 1. Above the resonance frequency
0% 2. Below the resonance frequency
0% 3. I don't know

:20

PRS: Leading or Lagging

The graph shows the current versus the voltage in a driven RLC circuit at a given driving frequency. In this plot

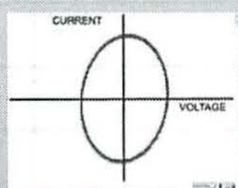


0% 1. Current lags voltage by ~90°
0% 2. Current leads voltage by ~90°
0% 3. Current and voltage are almost in phase
0% 4. We don't have enough information (but they aren't in phase!)
0% 5. I don't know

20

PRS: Leading or Lagging

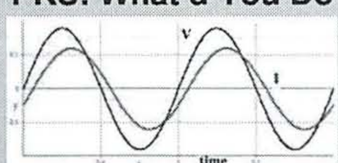
Answer: 4. Can't Tell



Without the direction you can't tell whether the current or voltage is leading or lagging. You can only tell that you aren't in phase (in fact, you are out of phase by ~90°)

PRS: What'd You Do?

20

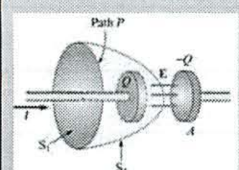


The graph shows current & voltage vs. time in a driven RLC circuit. We had been in resonance a second ago but then either put in or took out the core from the inductor. Which was it?

0% 1. Put in the core
0% 2. Took out the core
0% 3. I don't know

Displacement Current

Displacement Current



$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

Direction is same as E field (opposite if negative)

So we have to modify Ampere's Law:

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 (I_{encl} + I_d)$$

EM Waves

know this entire slide
- should be on eq sheet

Traveling E & B Waves

Wavelength: λ
Frequency : f

$$\vec{\mathbf{E}} = \hat{\mathbf{E}} E_0 \sin(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)$$

Wave Number: $k = \frac{2\pi}{\lambda}$

Angular Freq.: $\omega = 2\pi f$

Period: $T = \frac{1}{f} = \frac{2\pi}{\omega}$

Speed: $v = \frac{\omega}{k} = \lambda f$

Direction: $+\hat{\mathbf{k}} = \hat{\mathbf{E}} \times \hat{\mathbf{B}}$

$$\frac{E}{B} = \frac{E_0}{B_0} = v$$

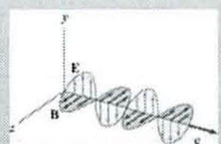
In vacuum...

$$= c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \frac{m}{s}$$

EM Waves

Travel (through vacuum) with speed of light

$$v = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \frac{m}{s}$$



At every point in the wave and any instant of time, E and B are in phase with one another, with

$$\frac{E}{B} = \frac{E_0}{B_0} = v$$

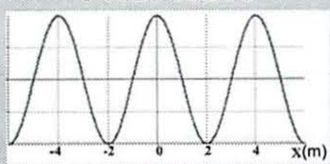
E and B fields perpendicular to one another, and to the direction of propagation (they are **transverse**):

Direction of propagation = Direction of $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$

PRS Questions: EM Waves

Classes 30, 31 & 33

PRS: Wave



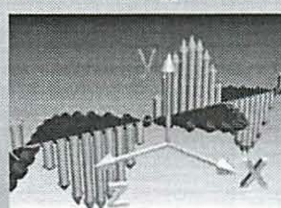
The graph shows a plot of the function $y = \cos(k\,x)$. The value of k is

- 0% 1. $\frac{1}{2}\ m^{-1}$
- 0% 2. $\frac{1}{4}\ m^{-1}$
- 0% 3. $\pi\ m^{-1}$
- 0% 4. $\pi/2\ m^{-1}$
- 0% 5. I don't know

:20

PRS: Direction of Propagation

The figure shows the E (yellow) and B (blue) fields of a plane wave. This wave is propagating in the



- 0% 1. +x direction
- 0% 2. –x direction
- 0% 3. +z direction
- 0% 4. –z direction
- 0% 5. I don't know

20

PRS: Traveling Wave

The B field of a plane EM wave is $\vec{B}(z,t) = \hat{k}B_0 \sin(ky - \omega t)$

The electric field of this wave is given by

- 0% 1. $\vec{E}(z,t) = \hat{j}E_0 \sin(ky - \omega t)$
- 0% 2. $\vec{E}(z,t) = -\hat{j}E_0 \sin(ky - \omega t)$
- 0% 3. $\vec{E}(z,t) = \hat{i}E_0 \sin(ky - \omega t)$
- 0% 4. $\vec{E}(z,t) = -\hat{i}E_0 \sin(ky - \omega t)$
- 0% 5. I don't know

:20

forgot how to distinguise

+y dir propigation

$\hat{k} \times \underline{\ \ \ } = \hat{y}$

$-\hat{i}$

PRS EM Wave

The E field of a plane wave is:

$$\vec{\mathbf{E}}(z,t) = \hat{\mathbf{j}} E_0 \sin(kz + \omega t)$$

The magnetic field of this wave is given by:

- 0% 1. $\vec{\mathbf{B}}(z,t) = \hat{\mathbf{i}} B_0 \sin(kz + \omega t)$
- 0% 2. $\vec{\mathbf{B}}(z,t) = -\hat{\mathbf{i}} B_0 \sin(kz + \omega t)$
- 0% 3. $\vec{\mathbf{B}}(z,t) = \hat{\mathbf{k}} B_0 \sin(kz + \omega t)$
- 0% 4. $\vec{\mathbf{B}}(z,t) = -\hat{\mathbf{k}} B_0 \sin(kz + \omega t)$
- 0% 5. I don't know

:20

Energy Flow

Poynting vector: $\vec{\mathbf{S}} = \dfrac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0}$

Intensity: $I \equiv \langle S \rangle = \dfrac{E_0 B_0}{2\mu_0} = \dfrac{E_0^2}{2\mu_0 c} = \dfrac{c B_0^2}{2\mu_0}$

Radiation pressure: $P_{absorb} = \dfrac{S}{c}$; $P_{reflect} = \dfrac{2S}{c}$

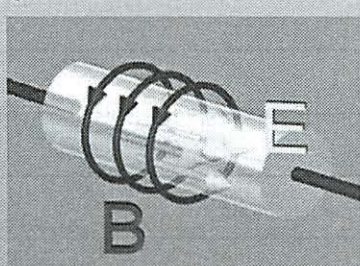
P38 - 35

direction of propagation

I know how to do
but prob not have to do

Also in Circuit Elements…

$\vec{\mathbf{S}} = \dfrac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0}$ On surface of resistor is INWARD

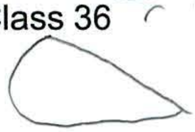


P38 - 36

Microwave → vibrates Food
occilating $\vec{E}$ field
charges feel a force
move → by Friction
get heat

metal will spark
– conducting
strong $\vec{E}$ field

$\frac{kq_1}{r} = \frac{kq_2}{R}$



$\vec{E}$ field enhanced at sharp point – metal is not a problem
sharp metal is the problem

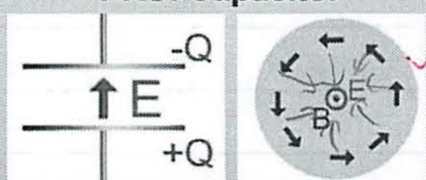
$\frac{kq^2}{r^2}$

PRS Questions: Poynting Vector

Class 33

P36-31

PRS: Capacitor



radially inward

The figures above show a side and top view of a capacitor with charge Q and electric and magnetic fields E and B at time *t*. At this time the charge Q is:

- 0% 1. Increasing in time ✓
- 0% 2. Constant in time.
- 0% 3. Decreasing in time.
- 0% 4. I don't know

20

only think about 1st order

PRS: Inductor

radially outward

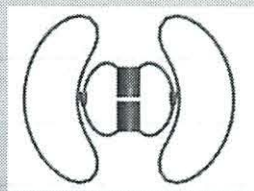
The figures above show a side and top view of a solenoid carrying current I with electric and magnetic fields E and B at time t. In the solenoid, the current I is:

- 0% 1. Increasing in time
- 0% 2. Constant in time.
- 0% 3. Decreasing in time. ✓
- 0% 4. I don't know

:20

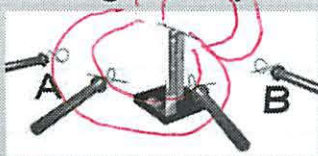
PRS: Spark Gap

At the time shown the charge on the top half of our ¼ wave antenna is positive and at its maximum value. At this time the current across the spark gap is



0% 1. Zero
0% 2. A maximum and downward
0% 3. A maximum and upward
0% 4. Can't tell from the information given
0% 5. I don't know

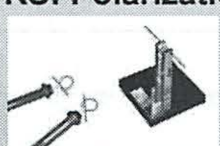
PRS: Angular Dependence



As you moved your receiving antenna around the spark gap transmitting antenna as above, you saw

0% 1. Increased power at B compared to A
0% 2. Decreased power at B compared to A
0% 3. No change in power at B compared to A
0% 4. I don't know

PRS: Polarization

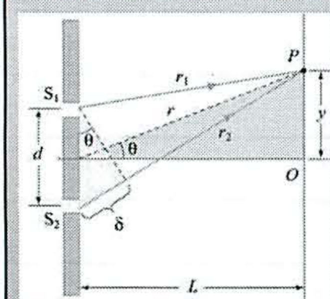


When located as shown, your receiving antenna saw maximum power when oriented such that

0% 1. Its straight portion was parallel to the straight portion of the transmitter
0% 2. Its straight portion was perpendicular to the straight portion of the transmitter
0% 3. I don't know

Interference

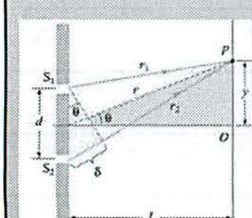
Two In-Phase Sources: Geometry



Assuming $L >> d$:
Extra path length
$\delta = d\sin(\theta)$

$\delta = d\sin(\theta) = m\lambda \quad \Rightarrow$ Constructive
$\delta = d\sin(\theta) = (m+\frac{1}{2})\lambda \quad \Rightarrow$ Destructive

Two Sources in Phase



Assume $L >> d >> \lambda$
$y = L\tan\theta \approx L\sin\theta$
$\Rightarrow \delta = d\sin\theta = dy/L$

(1) Constructive: $\delta = m\lambda$

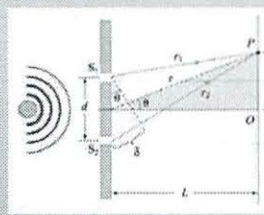
$$y_{constructive} = m\frac{\lambda L}{d} \quad m = 0,1...$$

(2) Destructive: $\delta = (m+1/2)\lambda$

$$y_{destructive} = \left(m+\frac{1}{2}\right)\frac{\lambda L}{d} \quad m = 0,1,...$$

PRS: Double Slit

Coherent monochromatic plane waves impinge on two apertures separated by a distance d. An approximate formula for the path length difference between the two rays shown is



- 0% 1. d sin θ
- 0% 2. L sin θ
- 0% 3. d cos θ
- 0% 4. L cos θ
- 0% 5. Don't have a clue.

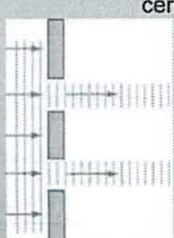
:20

Diffraction

P36-47

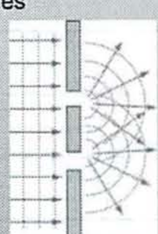
Diffraction

Diffraction: The bending of waves as they pass by certain obstacles



No Diffraction

No spreading after passing though slits



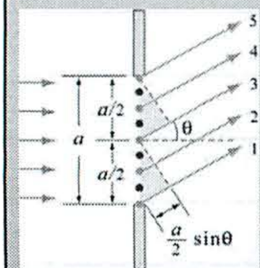
Diffraction

Spreading after passing though slits

P36-48

Single-Slit Diffraction

"Derivation" (Motivation) by Division:



Divide slit into two portions:

$$\delta = r_1 - r_3 = r_2 - r_4 = \frac{a}{2}\sin\theta$$

Destructive interference:

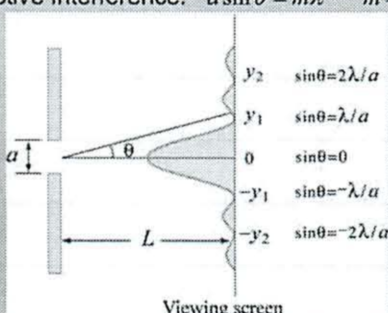
$$\delta = \frac{a}{2}\sin\theta = \left(m+\tfrac{1}{2}\right)\lambda$$

$$a\sin\theta = m\lambda \qquad m = \pm 1, \pm 2, \ldots$$

Don't get confused – this is DESTRUCTIVE!

Intensity Distribution

Destructive Interference: $a\sin\theta = m\lambda \qquad m = \pm 1, \pm 2, \ldots$

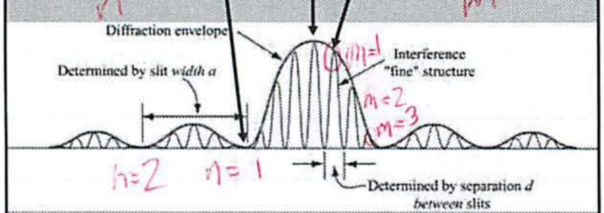


Two Slits With Finite width *a*

Zero Order Maximum

First Diff. Minimum $a\sin\theta = \lambda$ n

First Order Maximum $d\sin\theta = \lambda$ m



2 slits where slits have some width

PRS: Headlight Resolution

Is it easier to resolve two headlights at night or during the day?

0% 1. At night

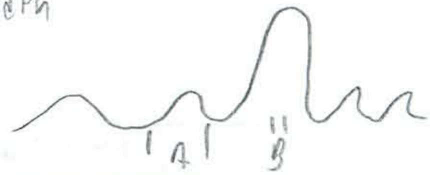
0% 2. During the day

0% 3. It doesn't matter

0% 4. I don't know

:20

distance ↓ $d >> a$ ↓ slit width



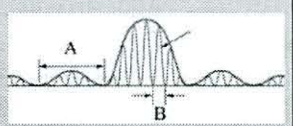
$d \approx 4a$

$m = 4$ interference peak is killed by $n = 1$ diffraction minimum

$\sin\theta = \frac{m\lambda}{d} = \frac{n\lambda}{a}$

PRS: Interference & Diffraction

Coherent monochromatic plane waves impinge on two long narrow apertures (width a) that are separated by a distance d (d >> a).



The resulting pattern on a screen far away is shown above. Which structure in the pattern above is due to the finite width a of the apertures?

0% 1. The distantly-spaced zeroes of the envelope, as indicated by the length A above.

0% 2. The closely-spaced zeroes of the rapidly varying fringes with length B above.

0% 3. I don't know

20

PRS: Changing Colors

You just observed an interference pattern using a red laser. What if instead you had used a blue laser? In that case the interference maxima you just saw would be

0% 1. Closer Together

0% 2. Further Apart

0% 3. I Don't Know.

:20

PRS: Lower Limit?

:20

Using diffraction seems to be a useful technique for measuring the size of small objects. Is there a lower limit for the size of objects that can be measured this way?

0% 1. Yes – but if we use blue light we can measure even smaller objects

0% 2. Yes – and if we used blue light we couldn't even measure objects this small

0% 3. Not really

0% 4. I Don't Know

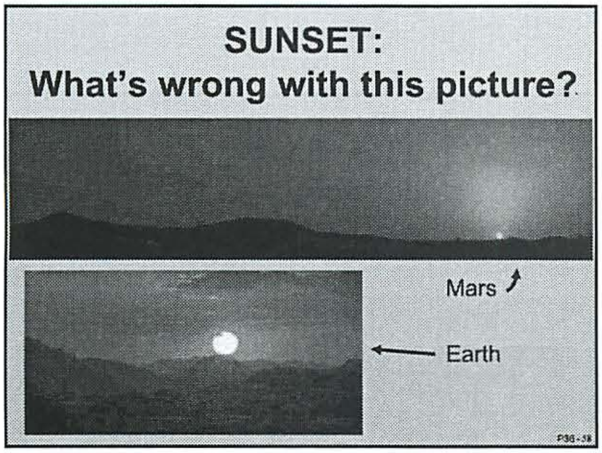
P36 - 35

SAMPLE EXAM:

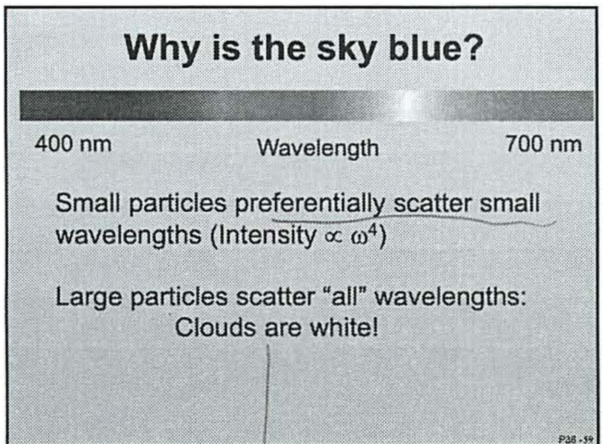
P36 - 36

One Last Topic...

P36 - 37



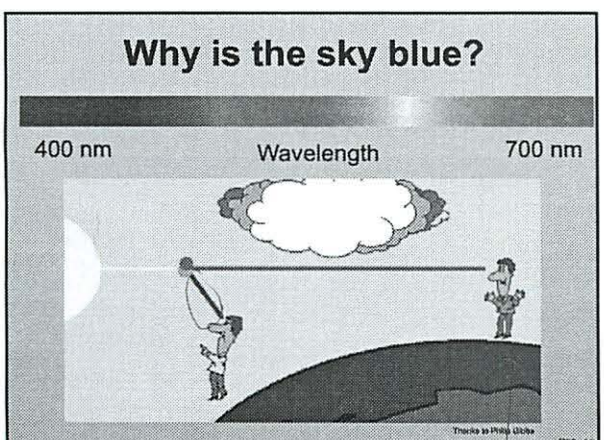
sun is blue, not red



blue more likely to scatter off of

lots of small particles in atmosphere

scatter a bunch of wavelenghts



early in day

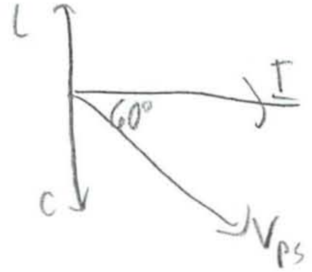
later in day - sunsets are red
only long wave lenghts left

In Class Problems

5/12

08 exam #4 black box

ω always same so one or oter



← capicator like

~~So capicator~~

both

ω at 1 sec → $\frac{\pi}{3}$ I leads V_s

2 sec → $\frac{\pi}{3}$ I **lags** V_s

? did not see this

$I = ?$
$C = ?$

~~$V = IR + \frac{Q}{C}$~~

~~$I = \frac{V - \frac{Q}{C}}{R}$~~

~~$C \Rightarrow X_C = \frac{1}{\omega C}$~~

~~$\frac{\pi}{3} = \frac{1}{\omega C}$?~~

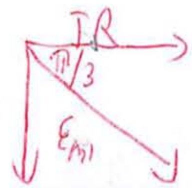
pull it both up + down

Can find resonance

I ← without L or C at each time sepertly

$I = \frac{V}{Z}$ ← $\sqrt{R^2 + (X_L - X_C)^2}$ ← don't have those vales

know V_s has magnitude of E_m



$\cos\frac{\pi}{3} = \frac{IR}{E_m}$ ← see in phasor diagram

Plug in #

$$I_0 = \frac{\mathcal{E}_m \cos\frac{\pi}{3}}{R} = \frac{1 \cos\frac{\pi}{3}}{2} = \frac{1}{4} \text{ Amp}$$

Now at $\omega = 2$

- same

Asks for magnitude of current
- same if not dampening

If wants $I(t)$

What is $\sin(\omega t)$? V_{ps}?

$$I(t) = \frac{1}{4} A \cdot \sin\left(\omega t + \frac{\pi}{3}\right) \text{ at } \omega = 1$$

$$I(t) = \frac{1}{4} A \sin\left(\omega t - \frac{\pi}{3}\right) \text{ at } \omega = 2$$

$$\tan\frac{\pi}{3} = \underbrace{\frac{X_L - X_C}{R}}_{\omega = 2} = \underbrace{\frac{X_C - X_L}{R}}_{\omega = 1}$$ divide out current

$$\omega_2 L - \frac{1}{\omega_2 C} = \frac{1}{\omega_1 C} - \omega_1 L$$

Solve for L and C
all algebra
- not worth many points

What drive at for max current?
- Resonance - b/w 1 and 2 $\omega = \frac{1}{\sqrt{LC}}$ ← do it when they ask

$V_L(t)$ = when current is max

$\omega = \frac{1}{\sqrt{LC}}$

$= I_0 \omega L \sin(\omega t + \frac{\pi}{2})$

→ are 90° apart V_L and V_{ps}

Dormashkin Review 2

5/11

First 15 min big overview
25 min examples + strategy

How do you use the Maxwell Equations to Calculate $\vec{E}$ and $\vec{B}$ fields?

Find $\vec{E}$ field
- 2 maxwell eq
- Gauss $\oiint E \cdot dA = \frac{1}{\epsilon_0} \iiint \rho dv$
  - closed surface integral ($\oiint$)
  - q_{enc} ($\iiint \rho dv$)
  - for each region!

Find $\vec{E}$ constant in time

- Varrying $\vec{E}(t)$ field
  Faradays Law $\oint \vec{E} \cdot ds = -\frac{d}{dt} \iint \vec{B} \cdot d\vec{A}$
  - Closed path ($\oint$)
  - ?B field not constant like Sollidnoid know $B(t)$

- If not enough symmatry: Brute Force Columb
  - - must actually $\int$
  - charged ring

(2)

$\vec{B}$ fields

Ampere's Law (generalized)

$$\oint_{\substack{\text{closed}\\\text{path}}} \vec{B} \cdot d\vec{s} = \underbrace{\mu_0 \iint \vec{J}\, d\vec{a}}_{\text{current through the path, open surface } S} + \mu_0 \epsilon_0 \underbrace{\frac{d}{dt} \iint \vec{E} \cdot d\vec{a}}_{\text{open surface } S}$$

- Statics:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

- non constant $\vec{E}(t)$
  - ↳ can choose Amperean Loop w/ no current going through (capicator is usual case)

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \frac{d}{dt} \iint \vec{E} \cdot d\vec{a}$$

No symatry: Brute Force: Biot-Savart Law

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

$$F = m\vec{a}$$

$$q(\vec{E} + \vec{v} \times \vec{B}) = ma$$

2nd law

Once get $\vec{E} + \vec{B}$ use Force law

(more to this than covering tonight)

③

Pointing Vector + Energy Conservation

- people entering + leaving room
  - like flux
- if people enter room, # people in room ↑

- charge

$\oint\oint \vec{J} \cdot d\vec{a}$ stuff = charge

$$J = \frac{I}{a} = \frac{\frac{dQ}{dt}}{A} = \frac{\frac{d\,stuff}{dt}}{A}$$

$$d\vec{a} = \hat{n}_{out}\, da$$

stuff flowing in → ⊖ integral

$$= -\frac{d}{dt}\iiint \rho\, dV$$

↑ charge in volume changes w/ time

⊖ because stuff entering

- energy

stuff now = electromagnetic energy

$$\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0} = \frac{power}{area} = \frac{\frac{d}{dt}energy}{area}$$

4

1. Always get $\vec{E}$ + $\vec{B}$ w/ Maxwell's eq

2. $\oiint \vec{S} \cdot d\vec{a}$

Closed surface
always $\hat{n}$ out

The rate
↑ power is entering the region $\frac{dE}{dt}$

Energy stored in volume can ↑
(more people in room)
Stored in $\vec{E}$ and $\vec{B}$ fields

$$\oiint \vec{S} \cdot d\vec{a} = -\frac{d}{dt} \iiint \left(\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2\right) dV$$

↑ sign adding stuff

Change in energy stored in field

- energy could go elsewhere (coaxial cable problem)

$\vec{E}$ fields don't just store energy, they do work
Wire to potential diff → $\vec{E}$ field in wire →
exerts a force on charges

Force · displacement = work

$\vec{B}$ fields don't do work

- always $\perp$ to velocity

$$\oiint \vec{S}\cdot d\vec{a} = -\frac{d}{dt}\iiint\left(\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}B^2\right)dv - \frac{dW}{dt}$$

↑ rate that the $\vec{E}$ field is doing work on charges inside the volume

- Charging capicator
- Solidnoid
- current in a wire
- wave

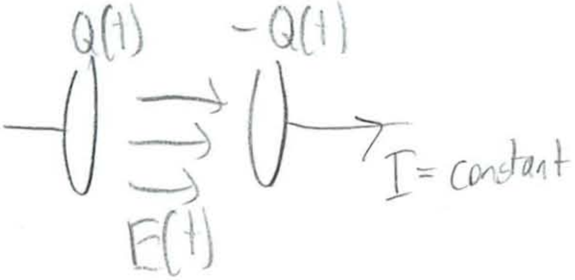
Look at each of the 4 cases

Charging Capicator

1. $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$ must calc $\vec{E},\vec{B}$ in either order

2. Calc S
3. Conceptually where is $\vec{E}$ flowing
   - which terms 0 or non 0
4. $\oiint \vec{E}\cdot d\vec{a} = -\frac{d}{dt}\int(u_E + u_B)\,dV - \frac{dW_E}{dt}$

(e)



If I constant, $\vec{B}$ constant, but $\vec{E}$ not constant

Storing energy up to $\frac{Q^2}{C}$

$\vec{S}$ is flowing in

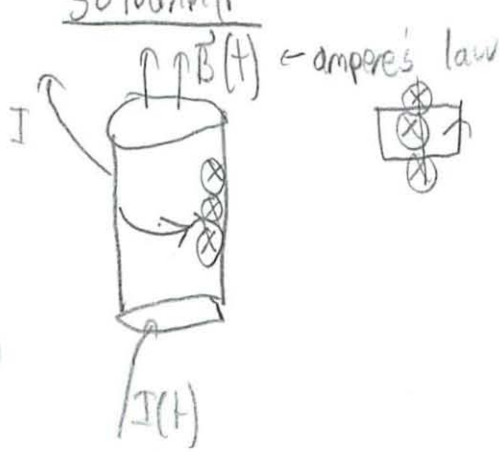
are no charges inside

→ nothing to do work on

→ no $\frac{dW_E}{dt}$

u_B not changing

Solodneid



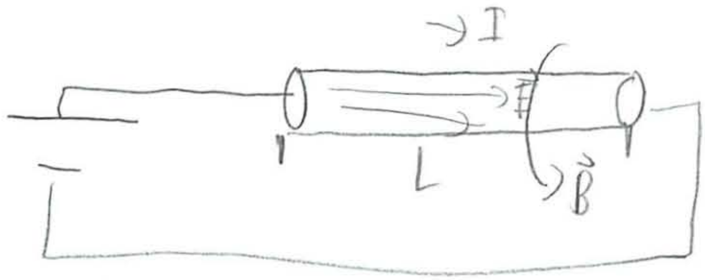
If $\vec{B}$ changing w/ time, can calculate $\vec{E}$ field

If $\frac{dI}{dt}$ is a constant, $\vec{E}$ is constant

(7)

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}, \quad \oint \vec{S} \cdot d\vec{a}$$

Current in a wire



$\vec{E} = \frac{V}{L}$ ← voltage difference   constant → steady current

Use Ampere's Law to get $\vec{E}$ of wire

$\vec{B}$ constant as well

~~$\vec{S}$ → constant as well~~

- all static, no change in time

But there is an S flow in from the sides

$\vec{E}$ field doing work

- accelerating electrons → collision
- power dissipated I^2R

$$\oint \vec{S} \cdot d\vec{a} = -I^2R$$

energy conservation

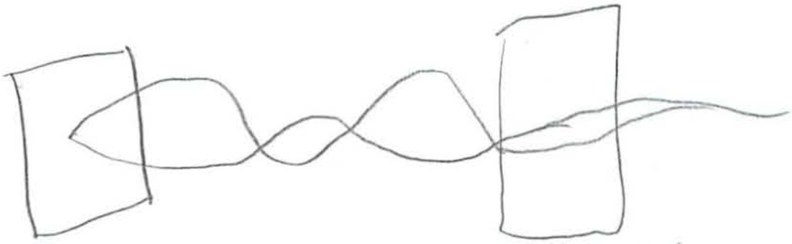
---

(8)

$\vec{E} = E_{y0} \sin(kx - \omega t)\,\hat{j}$

$\vec{B} = \frac{E_{y0}}{c} \sin(kx - \omega t)\,\hat{k}$

$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$



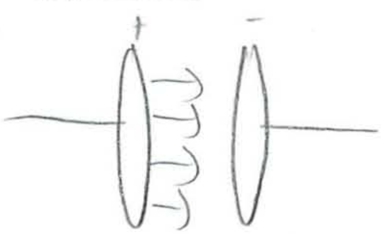
pointing vector on a particular surface

$= S A$

solor radiation

Now calculating $\vec{E}$ and $\vec{B}$ then $\vec{S}$

Capicator



$\vec{E} \rightarrow$ Guass

$$\iint \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon_0} \iiint \rho \, dV$$

$$E A = \frac{1}{\epsilon_0} \sigma A$$

$$E = \frac{\sigma}{\epsilon_0} \hat{k} \quad \text{inside}$$

$$= 0 \quad \text{outside}$$

(9)

$$\frac{dE}{dt} = \frac{1}{\epsilon_0}\frac{dQ}{dt}\frac{1}{A_{plate}}$$

$$= \frac{1}{\epsilon_0 A_{Plate}}$$

$\underline{\vec{B}}$

Choose Ampere's Law

↑ choose circle of radius r

$$\oint \vec{B} \cdot ds = \mu_0 \epsilon_0 \frac{d}{dt}\iint \vec{E} \cdot d\vec{a}$$

only $\vec{B}$ field tangental

$$B\,2\pi r = \mu_0 \epsilon_0 \cdot \frac{dE}{dt} \cdot A$$

↑ uniform ↑ flux integral

not constant

$$B = \mu_0 \epsilon_0 \frac{dE}{dt} \frac{\pi r^2}{2\pi r} \hat{\theta}$$

↑ tangental

$$= \mu_0 \epsilon_0 \left(\frac{1}{A_{plate}\epsilon_0} \frac{dQ}{dt}\right) \frac{r}{2} \hat{\theta}$$

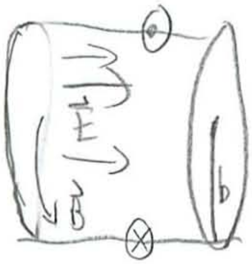
$$= \frac{\mu_0}{A_{plate}} I \frac{r}{2} \hat{\theta}$$

No way to get around Complexity

$\vec{B}$ field related to changing $\vec{E}$ field

(10)

Now can calculate Poynting Vector



← Cylindral surface of radius $r=b$

$$\vec{S} = \frac{\frac{Q}{\epsilon_0 A_{plate}} \vec{E} \times \mu_0 \frac{I}{A_{plate}} \frac{b}{2} \hat{\theta}}{\mu_0}$$

$$\hat{z} \times \hat{\theta} = -\hat{r}$$

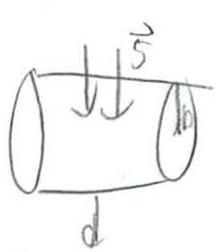
$$\vec{S} = \frac{Q}{\epsilon_0 A_{plate}^2} I \frac{b}{2} (-\hat{r})$$

How much power into region?

Surface S

but vector ⊥ to surface, so easier

$$\oiint \vec{S} \cdot d\vec{a} = \pm |\vec{S}| \text{ area of surface}$$



$\vec{S}$ flowing in so ⊖

$$\oiint \vec{S} \cdot d\vec{a} = -\left(\frac{Q}{\epsilon_0 A_{plate}} \frac{I}{A_{plate}} \frac{b}{2}\right) 2\pi b d$$

↑ what is area of surface

(11)

$$= -\frac{Q}{\epsilon_0 A_{plate}} \frac{I}{A_{plate}} (\pi b^2 d)$$

↑ volume

Not simple

Calc Poynting Vector

Maxwell for $\tilde{E}$ and $\tilde{B}$

- use one to get another
- solidnoid B first → then $\tilde{E}$

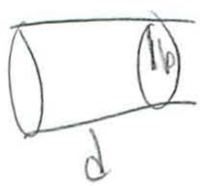
Then Cross product

One more calculation

Rate of change of E inside capicator

$$-\frac{d}{dt}\int u_E \, dV = -\frac{d}{dt}\int \left(\frac{1}{2}\epsilon_0\right) E^2 \, dV$$

Stored energy



$$= -\frac{d}{dt}\left(\frac{1}{2}\epsilon_0 E^2\right)\pi b^2 d$$

$$= \frac{1}{2}\epsilon_0 \, 2E \frac{dE}{dt} \pi b^2 d$$

(12)

$$E = \epsilon_0 \frac{Q}{A_{plate}\,\epsilon_0}$$

$$\frac{dE}{dt} = \frac{1}{A_{plate}\,\epsilon_0} \frac{\partial Q}{\partial t} \pi b^2 d$$

$$= \frac{1}{A_{plate}\,\epsilon_0} I \pi b^2 d$$

$$= \frac{-Q}{A_{plate}\,\epsilon_0} \frac{I}{A_{plate}} \pi b^2 d$$

Same as poynting vector

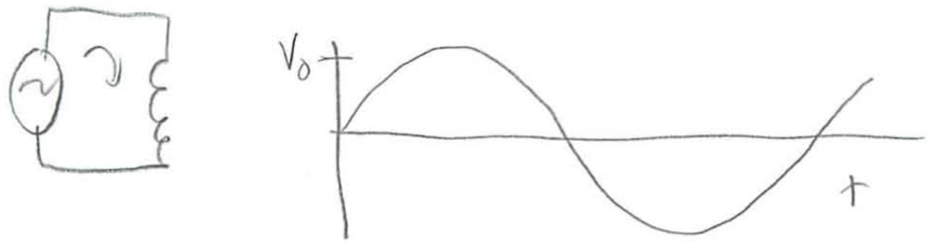
Hardest part is what surface is $\vec{S}$ flowing through?

feel good about this topic - good to [illegible] anyway

Dormaskin Review 3

5/12

Driver circuits



$v(t) = V_0 \sin(\omega t)$

↑ can be filled in w/ a #

ω = radians/sec but dimensionless so say sec^{-1}

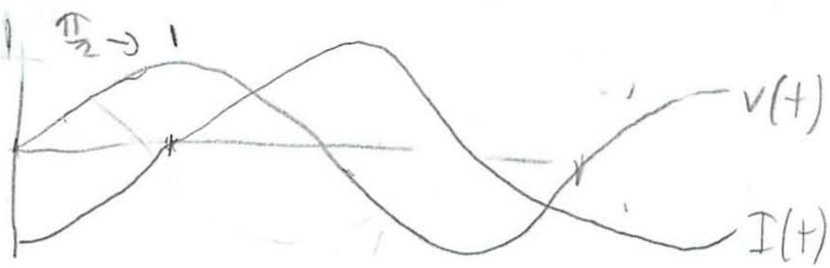
$$V(t) = L \frac{dI}{dt}$$

$$I(t) = I_0 \sin(\omega t - \text{phase shift})$$

phase shift ϕ

$\phi = \frac{\pi}{2}$

$$I(t) = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$



current leads voltage

know $\sin(0) = 0$

$\omega t - \pi/2 = 0$

$\omega t = \frac{\pi}{2}$

to the right

also $I(t) = -I_0 (\cos \omega t)$

another way to write

Check differential eq

$V(t) = V_0 \sin \omega t$

$I(t) = -I_0 \cos \omega t = I_0 \sin(\omega t - \pi/2)$

$\frac{dI}{dt} = \omega I_0 \sin \omega t$

$V(t)$	$= L \frac{dI}{dt}$
$V_0 \sin \omega t$	$L \omega I_0 \sin \omega t$

$I_0 = \frac{V_0}{\omega L} \quad \phi = \frac{\pi}{2}$

Drive it w/ voltage (depends on time)

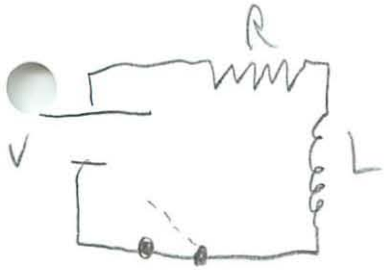
Current is the response

$I(t) = \frac{V_0}{\omega L} \sin(\omega t - \frac{\pi}{2})$

Most important thing

V_0 and I_0 90° out of phase

(3)

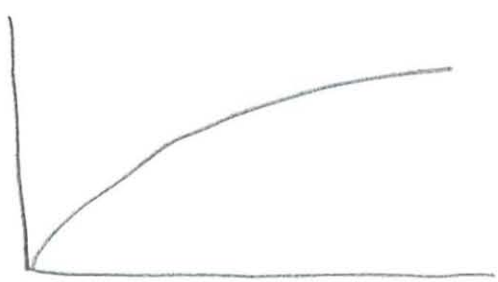


at $t=0$ close switch

$V \neq 0$

Inductor wants to keep things same

$I = 0$



this is why voltage leads current

be careful to look at what peaks first

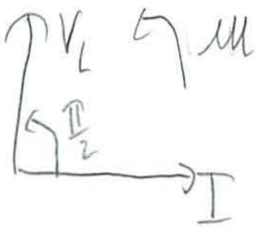
remember $\frac{dI}{dt}$ = slope of I

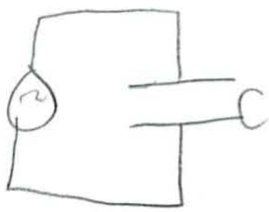
the formula actually checks out

$$V = L\frac{dI}{dt}$$

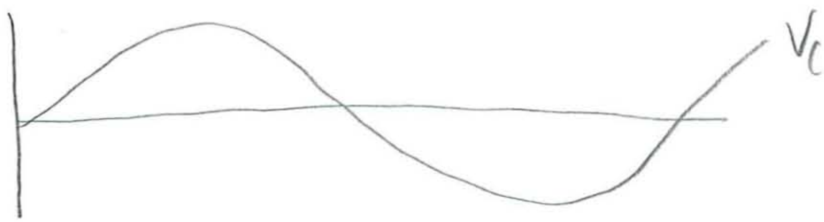
Voltage proportional to deriv of current

4





$$V = \frac{Q}{C} \qquad \frac{dV}{dt} = \frac{dQ}{dt}\frac{1}{C} = \frac{I}{C}$$

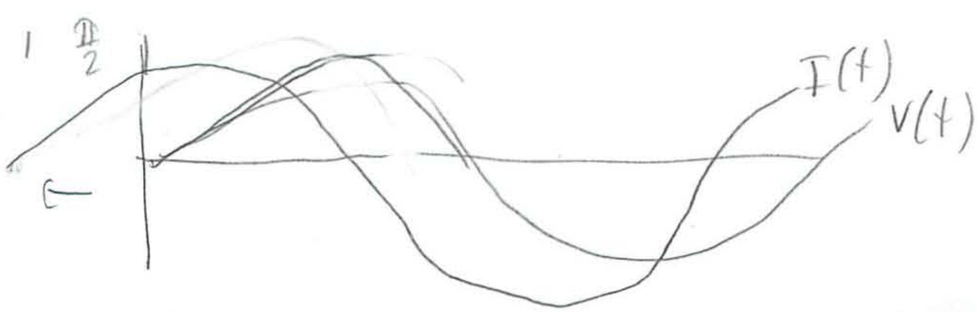


$$I(t) = I_0 \sin(\omega t - \phi)$$

$$\phi = -\frac{\pi}{2}$$

$$I(t) = I_0 \sin\left(\omega t - \left(-\frac{\pi}{2}\right)\right)$$

$$= I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$



(5)

$\sin(0) = 0$

$\omega t + \frac{\pi}{2} = 0$

$\omega t = -\frac{\pi}{2}$

$I(t)$ also $= \cos(\omega t - 0)$

Is this really the solution

$V(t) = V_0 \sin \omega t$

$I(t) = V_0 \cos \omega t$

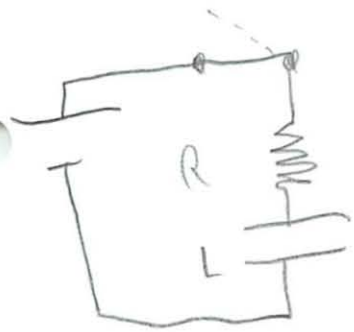
$\phi = -\frac{\pi}{2}$

$\frac{dV}{dt}$	$= \frac{I}{C}$
$\omega V_0 \cos \omega t$	$\frac{I_0}{C} \cos \omega t$

$I_0 = \omega C V_0$

$V_0 = \frac{I_0}{\omega C}$

$I(t) = \omega C V_0 \sin\left(\omega t + \frac{\pi}{2}\right)$



at $t = 0$

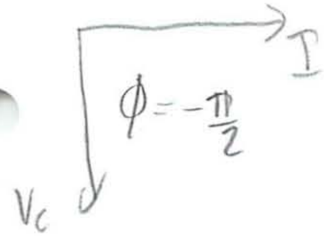
close switch

$$V_C = \frac{Q}{C}$$

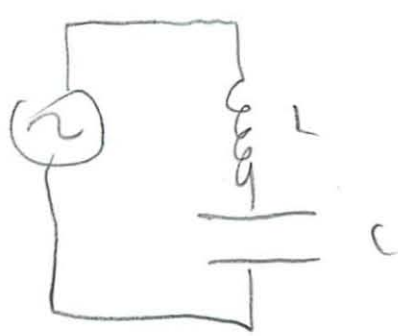
capicator has no voltage yet

I starts to charge up capicator

current <u>leads</u> voltage



$$V_C = \frac{I_0}{\omega C}$$



$$V(t) = V_0 \sin(\omega t)$$

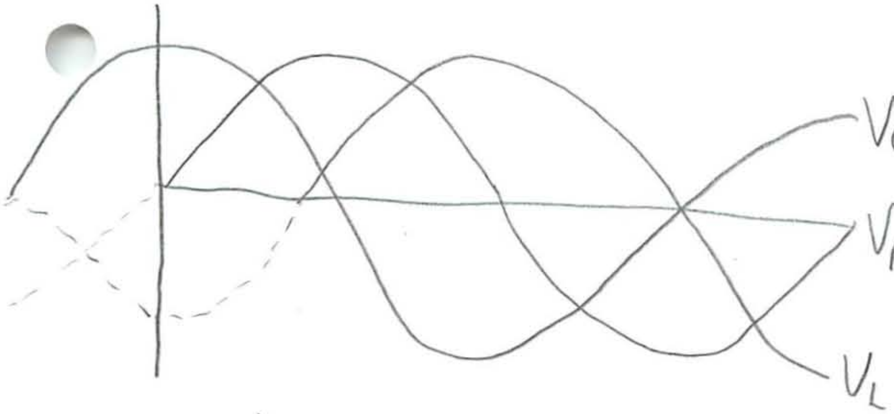
$$I(t) = I_0 \sin(\omega t - \phi)$$

$$\phi =$$

$$V = L\frac{dI}{dt} + \frac{Q}{C}$$

$$\frac{dV}{dt} = L\frac{d^2I}{dt^2} + \frac{I}{C}$$

(1)



$$V_L = \omega L I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$V_C = \frac{I_0}{\omega C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

V_L and V_C 180° out of phase

Can add as vectors

$V_L = \omega L I_0$

$V_C = \frac{1}{\omega C} I_0$

Which is bigger?

if $\omega L = \frac{1}{\omega C}$

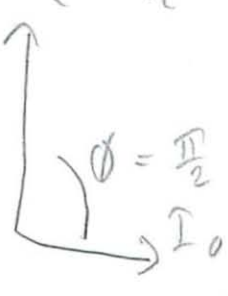
$\Downarrow$

$\omega^2 = \frac{1}{LC} \rightarrow \omega = \frac{1}{\sqrt{LC}}$ resonance

they cancle each oter out

if $\omega L > \frac{1}{\omega C}$

$V = V_L + V_C = \left(\omega L - \frac{1}{\omega C}\right) I_0$



$V = V_C + V_L$

$V = V_0 \sin \omega t$

$I = I_0 \sin(\omega t - \phi)$

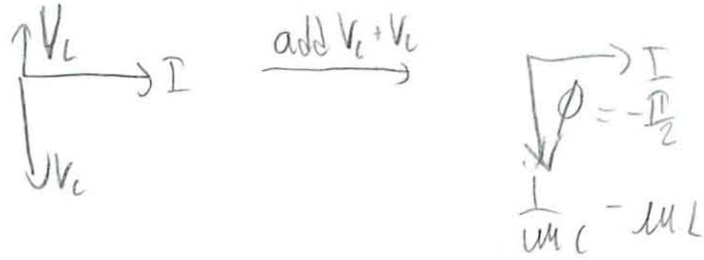
↑ go to phasor diagram
add vectors V_L and V_C
get amplitude

Voltage leading current

so $\phi = \frac{\pi}{2}$

(which is $\omega t - \frac{\pi}{2}$)
↑remember

if $\omega L < \frac{1}{\omega C}$



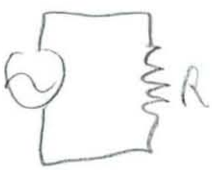
Current leads voltage

$V_0 = \left(\omega C - \frac{1}{\omega L}\right) I_0$

$$I_0 = \frac{V_0}{\left(\omega C - \frac{1}{\omega L}\right)} \qquad \phi = -\frac{\pi}{2}$$

$I(t) = I_0 \sin\left(\omega t - - \frac{\pi}{2}\right)$

Now put a resistor in $\underline{RLC}$



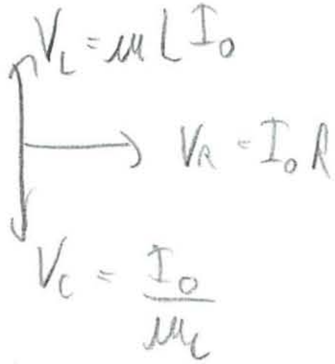
$V(t) = IR$

$I(t) = \frac{V_0}{R} \sin \omega t$

in phase

$V(t) = V_0 \sin(\omega t)$

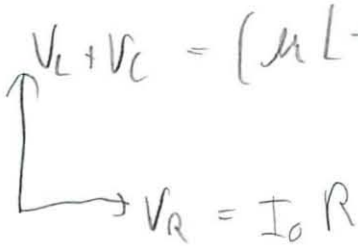
$I(t) = I_0 \sin(\omega t - \phi)$



$V = V_R + V_C + V_L$

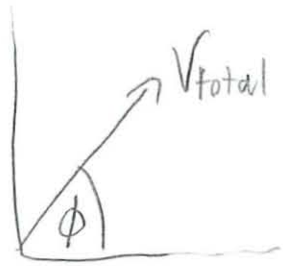
add as vectors

$\omega L > \frac{1}{\omega C}$ $\omega^2 > \frac{1}{LC}$ above resonance



(10)

$V_{total} =$



← here ϕ is weird angle

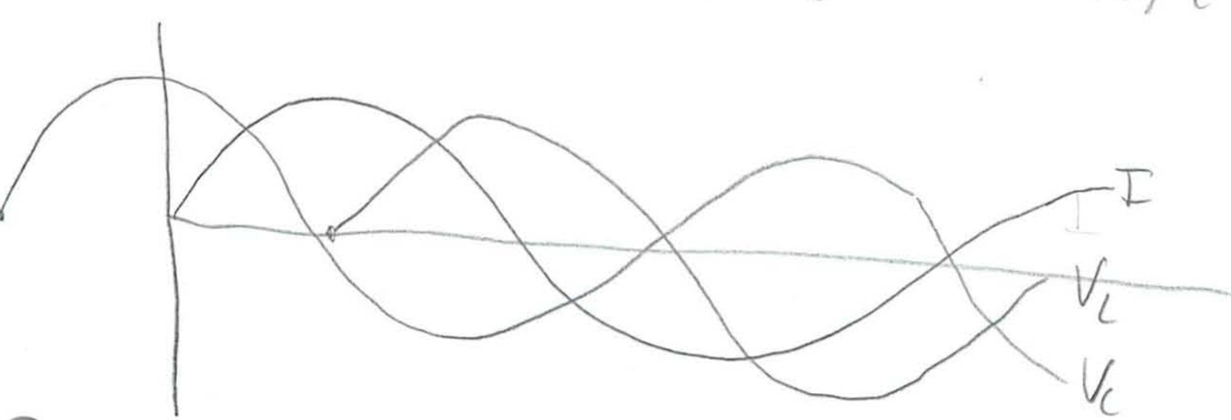
$$V(t) = V_0 \sin(\omega t)$$
$$I(t) = I_0 \sin(\omega t - \phi)$$

$$V_0 = \left(\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2\right)^{1/2}$$

$$I_0 = \frac{V_0}{\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right)^{1/2}}$$

$$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

Now draw all 4 graphs V_L, V_C, V, I

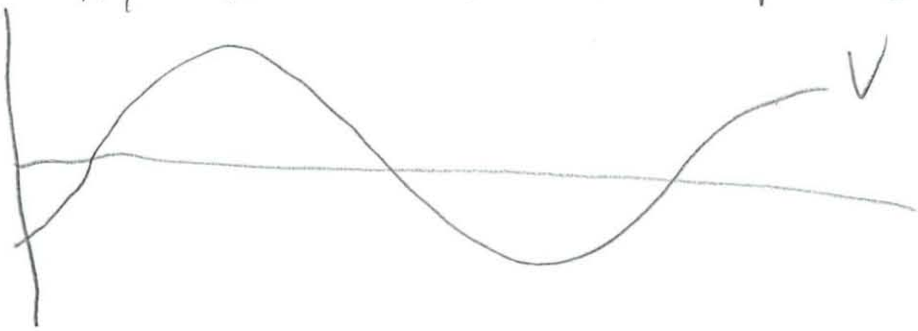


eaisest to start current at 0

(11)

Now last is V

- which is not a nice ϕ
- try to draw as nice as possible



in RLC V is the weird one

when you have to identify lines

What would happen if less than ?

V would be on other side of current

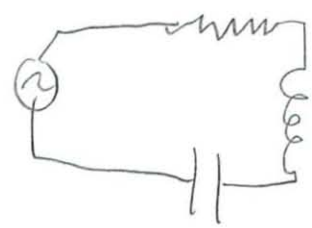
Capicative

I leads V (I peaks First)

Can take equations

in series

$$V(t) = V_0 \sin \omega t$$
$$I(t) = I_0 \sin(\omega t - \phi)$$
$$I_0 = \frac{V_0}{\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right)^{1/2}}$$



(12)

$\tan \phi = \infty$

$\frac{\pi}{2}$

Set $R = 0$ → eliminate R

set $L = 0$ → eliminate L

eliminate C → set $C \to \infty$

Graphical anaylsis just as important.