Question 1

1 Variables

- N = staffing level; goal
- ρ = utilization = $\frac{\lambda}{N*\mu}$
  - λ = throughput = # of callers per unit time (hour)
  - service time = 5 min = $\frac{5}{60} = \frac{1}{20}$ hour
  - μ = service rate = $\frac{1}{\text{service time}} = \frac{\text{total time unit(hour)}}{\text{time per job}} = \frac{60}{5} = 12$
- C_A = coefficient of variation for interarrival times = $\frac{\sigma}{\mu} = \frac{\text{st dev}}{\text{mean}} = \frac{\sqrt{\text{var}}}{\text{E[ ]}}$; not told; assume 1
- C_S = coefficient of variation for service time; told in the problem deterministic; = 0
- W = wait time = $\frac{L}{\lambda}$ = 1 minute = $\frac{1}{60}$ hour
- L = avg # of customers in the queue

2 Equations

$$L = \frac{\lambda}{N * \mu} \tag{1}$$

$$L = W\rho \tag{2}$$

$$\rho = \frac{\lambda}{N * \mu} \tag{3}$$

3 Substitute

$$\frac{\rho\sqrt{2(N+1)}}{1-\rho} * \frac{C_A^2 + C_S^2}{2} = W\lambda \tag{4}$$

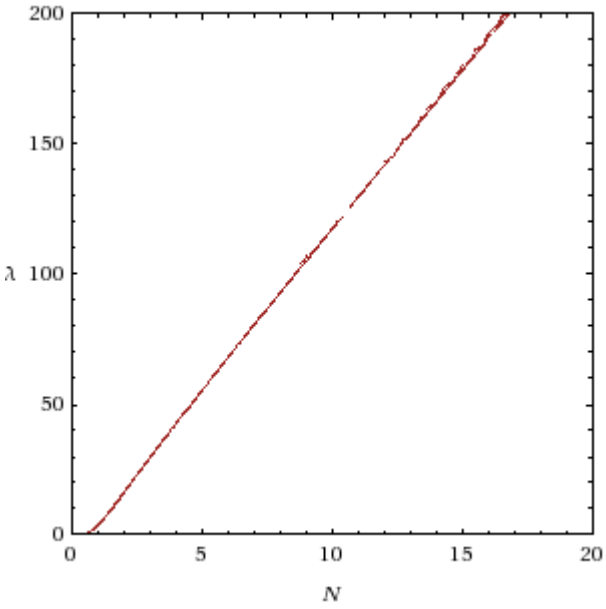
$$\frac{\frac{\lambda}{N*\mu}\sqrt{2(N+1)}}{1-\frac{\lambda}{N*\mu}} * \frac{C_A^2 + C_S^2}{2} = W\lambda \tag{5}$$

$$\frac{\frac{\lambda}{N*12}\sqrt{2(N+1)}}{1-\frac{\lambda}{N*12}} * \frac{1^2 + 0^2}{2} = \frac{1}{60}\lambda \tag{6}$$

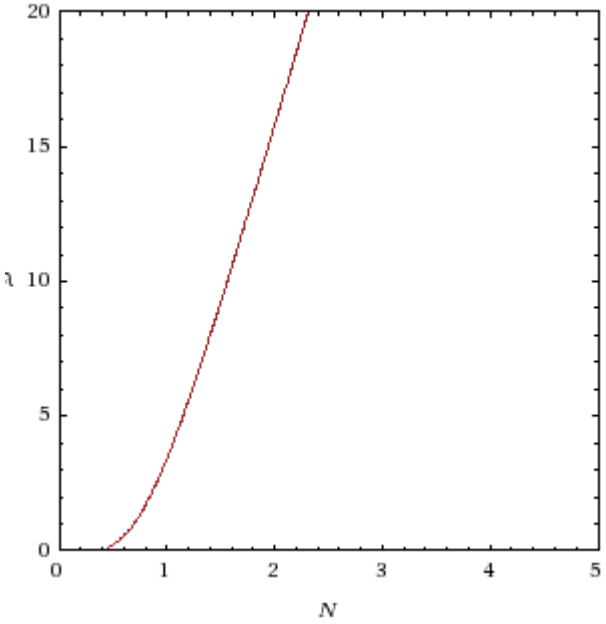
4 Solve

Solve for N in terms of λ

This model suggests that not as many operators are needed during peak periods.



This model is particularly interesting when very few callers are expected.



5 Results

λ (hour)	N
1	.604463
2	.790237
3	.940942
4	1.07524
5	1.19979
6	1.3178
7	1.4311
8	1.54082
9	1.64775
10	1.75241
11	1.85521
12	1.95645
13	2.05635
14	2.15511
15	2.25287
16	2.34976
17	2.44586
18	2.54128
19	2.63607
20	2.7303
21	2.82402
22	2.91728
23	3.01012
24	3.10257
25	3.19465
26	3.28641
27	3.37786
28	3.46902
29	3.55992
30	3.65057
31	3.74099
32	3.83119
33	3.92118
34	4.01098
35	4.1006
36	4.19004
37	4.27932
38	4.36845
39	4.45743
40	4.54627
41	4.63498
42	4.72356
43	4.81202
44	4.90036
45	4.9886
46	5.07673
47	5.16475
48	5.25269
49	5.34052
50	5.42827

λ (hour)	N
51	5.51594
52	5.60352
53	5.69102
54	5.77845
55	5.8658
56	5.95308
57	6.04029
58	6.12743
59	6.21452
60	6.30154
61	6.3885
62	6.4754
63	6.56224
64	6.64903
65	6.73577
66	6.82246
67	6.9091
68	6.99569
69	7.08233
70	7.16872
71	7.25518
72	7.34159
73	7.42795
74	7.51428
75	7.60057
76	7.68682
77	7.77303
78	7.8592
79	7.94534
80	8.03144
81	8.11751
82	8.20355
83	8.28955
84	8.37552
85	8.46146
86	8.54737
87	8.63326
88	8.71911
89	8.80493
90	8.89073
91	8.9765
92	9.06224
93	9.14796
94	9.23365
95	9.31932
96	9.40496
97	9.49058
98	9.49058
99	9.57617
100	9.66175

λ (hour)	N
101	9.83283
102	9.91833
103	10.0038
104	10.0893
105	10.1747
106	10.2602
107	10.3456
108	10.4309
109	10.5163
110	10.6017

Question 2

This question uses the same formula as the previous question, except you should input the sum of calls to of all 3 BTCs as λ