Michael Plasmeier

Haverford High School
Course Overview
Integrated Algebra/Geometry IV Level 1
Miss Kelly

Course Description/Overview

This course extends the trigonometric functions to all angles, defines, and graphs these functions and their inverses. Trigonometric identities are discovered and defined. The motion of falling objects in terms of vertical and horizontal components is studied with quadratic expressions being developed for their height over time. Students build on previous work of functions by exploring some basic families of functions (linear, quadratic, exponential, sine, logarithmic, and rational functions) in terms of the tables, their graphs, their algebraic representations, and the situations they represent. Additional topics included in this course are factoring, radian measure, Law of Sines and Cosines, and complex number operations. **Technology is used to enrich problem solving skills and to develop graphing techniques through use of the TI graphing calculator. The purchase of a TI-83 or TI-83 Plus is required!!**

Course Content: The following is a list of the units which will be studied in this course and their objectives.

High Dive

- Understanding the basics of circles, angles, and right triangle trigonometry.
- Learn the concept of periodicity.
- Develop expressions for circular motion
- Extend the sine and cosine functions to all angles.
- Graph the sine and cosine functions.
- Define inverse sine and cosine functions and principal values.
- Review the concept of instantaneous velocity and its estimation.
- Compare average and instantaneous velocity.
- Develop quadratic expressions for the height of free-falling objects.
- Explain right triangle identities.

Trigonometry

- Perform right triangle trigonometry.
- Learn about angles of rotation and radian measure.
- Evaluate trigonometric functions.
- Perform inverse trig functions.
- Manipulate triangles using Law of Sines and Law of Cosines.
- Learn more about the graphs of Sine and Cosine functions.
- Discover trigonometric identities.
- Solve trigonometric equations.
- Learn about the sum and difference formulas.
- Learn about the double and half angle formulas.

Small World

- One week review of key concepts

Rational Functions

- One week review of key concepts

World of Functions

- Formally define functions.
- Graph functions based on situations.
- Review basic families of functions.
- Find, describe, and prove patterns in the tables of functions.
- Study direct and inverse proportionality.
- Determine vertical and horizontal asymptotes.
- Use calculator regression to find a function that fits data.
- Use absolute value and step functions to model problem situations.
- Define arithmetic operations on functions.
- Work with composition of functions.
- Perform arithmetic operations on functions.
- Find inverses of different functions.

Grades:
Tests & Quizzes
Homework & class work (daily homework assignments)
Problems of the Week/ IAG Powers
Presentations (in class)

Grading policy: Grades are based on a point system. Averages are calculated by dividing the total points earned by the student by the total number of possible points. Letter grades will be assigned in accordance with school guidelines.

Course grade: Marking period 1 – 20%
Marking period 2 – 20%
Marking period 3 – 20%
Marking period 4 – 20%
Final Exam – 20%

Notebooks:
The notebook is an essential part of the course. Taking notes in class is very important. The textbook does not contain the process to solve a particular problem. In class, the students discuss and develop the process. Their notebook becomes their resource when solving problems. A good notebook should have each type of problem solved along with an explanation (steps taken to solve the problem).

Classroom Policies:
As a student in this class, you will be expected to follow and uphold the following policies:

- Arrive ON TIME to class; lateness will not be tolerated
- Be prepared – have ALL materials and assignments **WITH YOU** and **READY**.
- No food or drink is permitted in class (except for water)… **EVER**(unless with special permission).
- Treat everyone the way you would like to be treated; respect everyone.
- Bring a positive attitude everyday.

If you have any questions or concerns about your child's progress in this course please feel free to contact me at (610)853-5900 ext. 2742. Your encouragement and support at home will be a great asset to your child's success in this course!

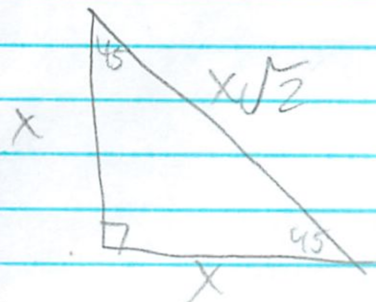
Parent/Guardian Signature ______________________
(indicates that this course overview has been reviewed)

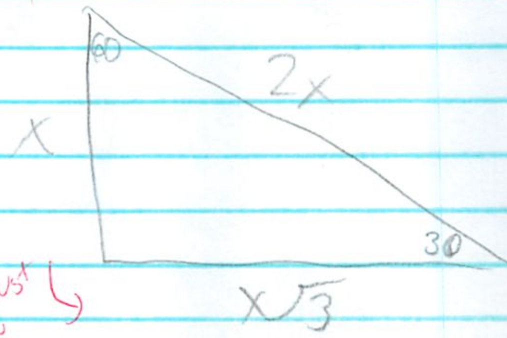
Expectations for Students

- Students are expected to attend class regularly and BE ON TIME.
- Students are expected to be prepared for class every day with the proper tools (including a pencil and eraser) necessary for the course. Failure to be prepared will affect your overall participation.
- Every student will be expected to demonstrate appropriate self control during the entire class period. Interference with classroom instruction or student achievement will not be tolerated.
- Homework will be checked periodically during each week. Homework must be <u>clearly attempted</u> every night to receive full credit. If there was a problem that you did not understand a question that will help clarify it must be written in your homework. If no attempt is made or there are no questions, credit will not be awarded.
- Student textbooks must be covered at all times and must be brought to class every day.
- Every student must keep a neat and organized notebook. A three ring binder is required because papers can easily be arranged. Notebooks are to be brought to class every day.
- All students are expected to take daily notes (which includes copying examples, rules and definitions), complete practice problems and participate in class discussions and group work.
- Missed work and assignments must be made up in a timely manner, ACCORDING TO THE STUDENT HANDBOOK. I will not remind you of what assignments you need to show me – THIS IS YOUR RESPONSIBILITY. Check the homework folder AND black box for any assignments/worksheets that you missed while you were absent.
- It is the STUDENT'S RESPONSIBILITY to make arrangements to make up missed tests, quizzes, and projects.
- In order to be successful in math, you must review and practice concepts each night. When studying for a mat h test, redo problems done in class to reinforce the concepts.
- If you fall behind or do not understand a topic or concept, after school help is always available. Make an appointment to see me.

Special Right Triangles Review

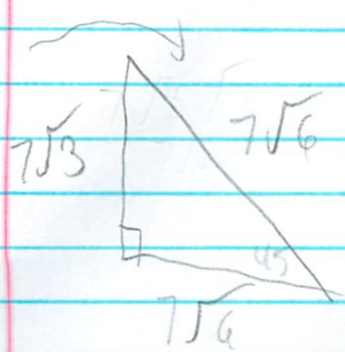
1/30

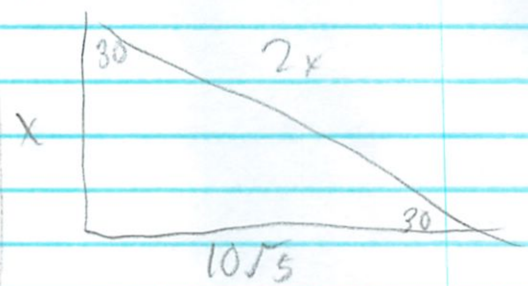




must be longer

$\frac{7\sqrt{3}\sqrt{2}}{6}$





$10\sqrt{5} = x\sqrt{3}$

$\frac{10\sqrt{5}}{\sqrt{3}}$

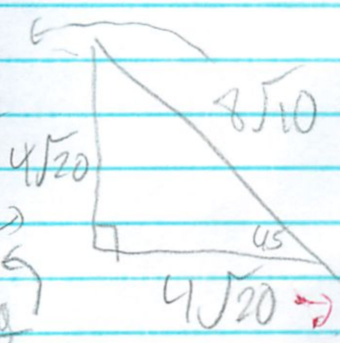
$\frac{10\sqrt{5}(\sqrt{3})}{\sqrt{3}(\sqrt{3})}$

$\frac{10\sqrt{15}}{3}$ x

$\frac{20\sqrt{15}}{3}$ 2x

$\frac{8\sqrt{10}}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}}$

$\frac{8\sqrt{20}}{2}$



$4\sqrt{20} \rightarrow 4\sqrt{4 \cdot 5}$

Rationalizing The Denominator

can go farther

$4\sqrt{4}\sqrt{5}$

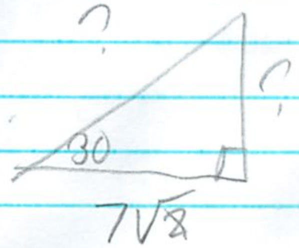
$4 \cdot 2\sqrt{5}$

$8\sqrt{5}$

Warmup 1/31

1/31

1.



$\frac{7\sqrt{8}\,(\sqrt{3})}{\sqrt{3}\,(\sqrt{3})}$ $\frac{7\sqrt{24}}{3}$

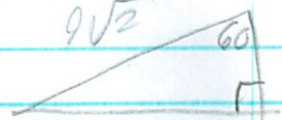
$\frac{7\sqrt{4}\cdot\sqrt{3}}{3}$

$\frac{7\cdot 2\sqrt{3}}{3}$

$x = \frac{14\sqrt{6}}{3}$

$2x = \frac{28\sqrt{6}}{3}$

2.



$4.5\sqrt{2} = x$ or $\frac{9\sqrt{2}}{2}$

$4.5\sqrt{2}\sqrt{3}$

$4.5\sqrt{6}$ or $\frac{9\sqrt{6}}{2}$

3. $\sin = \frac{OPP}{HYP}$

$\cos = \frac{ADJ}{HYP}$

$\tan = \frac{OPP}{Adj}$

More Trig Functions
Notes

1/31

cosecant $\Rightarrow$ $\csc() = \frac{hyp}{opp}$

secant $\Rightarrow$ $\sec() = \frac{hyp}{adj}$

cotangent $\Rightarrow$ $\cot() = \frac{adj}{opp}$

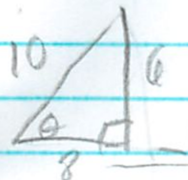
Section 13.1

Right Δ Trig

New Functions Practice

1/31

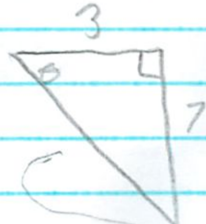
Evaluating Trig Functions w/o finding θ



$6^2 + x^2 = 10^2$
$-36 \qquad -36$
$x^2 = 64$
$x = 8$

$\sin\theta = \frac{6}{10} = \frac{3}{5}$
$\cos\theta = \frac{8}{10} = \frac{4}{5}$
$\tan\theta = \frac{6}{8} = \frac{3}{4}$

$\csc\theta = \frac{10}{6} = \frac{5}{3}$
$\sec\theta = \frac{10}{8} = \frac{5}{4}$
$\cot\theta = \frac{8}{6} = \frac{4}{3}$



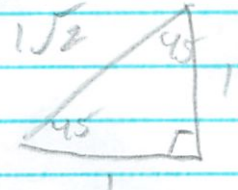
$\sin\theta = \frac{7}{\sqrt{58}} \cdot \frac{\sqrt{58}}{\sqrt{58}}$ $\frac{7\sqrt{58}}{58}$

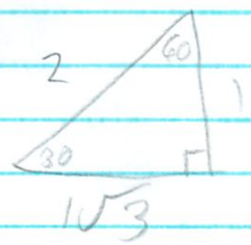
$3^2 + 7^2 = x^2$
$x = \sqrt{58}$ or 7.615

$\cos\theta = \frac{3}{\sqrt{58}} \cdot \frac{\sqrt{58}}{\sqrt{58}}$ $\frac{3\sqrt{58}}{58}$
$\tan\theta = \frac{7}{3}$

$\csc\theta = \frac{\sqrt{58}}{7}$
$\sec\theta = \frac{\sqrt{58}}{3}$
$\cot\theta = \frac{3}{7}$

Evaluating Trig Functions w/ θ





Smallest

$\sin(60) = \frac{1\sqrt{3}}{2}$ $\quad \sec(30) = \frac{2}{1\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ $\frac{2\sqrt{3}}{3}$ $\quad \cot(60) = \frac{1}{1\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ $\frac{\sqrt{3}}{3}$

To find on calculator

$\csc(45) = \frac{1}{\sin(45)} = 1.414$

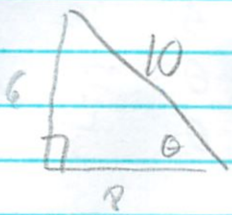
$\sec(200) = \frac{1}{\cos(200)} = -1.064$

$\cot(310) = \frac{1}{\tan(310)} = -.8390$

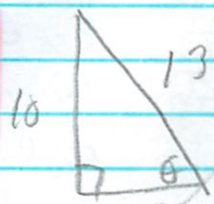
Homework 1/31

Ved: 2 I didn't reduce

1/31

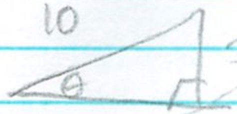
7. 

$\sin\theta = \frac{3}{5}$ $\csc\theta = \frac{5}{3}$
$\cos\theta = \frac{4}{5}$ $\sec\theta = \frac{5}{4}$
$\tan\theta = \frac{3}{4}$ $\cot\theta = \frac{4}{3}$

9. 

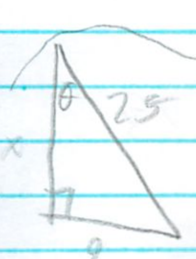
$10^2 + x^2 = 13^2$
$-100 \quad -100$
$x^2 = 69$
$x = \sqrt{69}$ or 8.3

$\sin\theta = \frac{10}{13}$ $\csc\theta = \frac{13}{10}$
$\cos\theta = \frac{\sqrt{69}}{13}$ $\sec\theta = \frac{13}{\sqrt{69}}$ $\frac{13\sqrt{69}}{69}$
$\tan\theta = \frac{10}{\sqrt{69}}$ $\frac{10\sqrt{69}}{69}$ $\cot\theta = \frac{\sqrt{69}}{10}$

11. 

$3^2 + x^2 = 10^2$
$-9 \quad -9$
$x^2 = 91$
$x = \sqrt{91}$ or 9.53

$\sin\theta = \frac{3}{10}$ $\csc\theta = \frac{10}{3}$
$\cos\theta = \frac{\sqrt{91}}{10}$ $\sec\theta = \frac{10}{\sqrt{91}}$ $\frac{10\sqrt{91}}{91}$
$\tan\theta = \frac{3}{\sqrt{91}}$ $\frac{3\sqrt{91}}{91}$ $\cot\theta = \frac{\sqrt{91}}{3}$

13. 

$x^2 + 9^2 = 25^2$
$-81 \quad -81$
$x^2 = 544$
$x = \sqrt{554}$ or 23.32

$\sin\theta = \frac{9}{25}$ $\csc\theta = \frac{25}{9}$
$\cos\theta = \frac{\sqrt{544}}{25}$ $\frac{4\sqrt{34}}{25}$ $\sec\theta = \frac{25}{\sqrt{544}}$ $\frac{25\sqrt{544}}{544}$
$\tan\theta = \frac{9}{\sqrt{544}}$ $\frac{9\sqrt{544}}{544}$ $\cot\theta = \frac{\sqrt{544}}{9}$

or $36\sqrt{34}/544$

$\sqrt{544} \to \sqrt{16}\sqrt{34}$
$4\sqrt{34}$

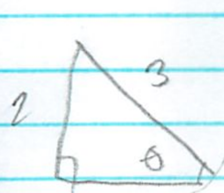
reduce

or

$\frac{100\sqrt{34}}{544}$

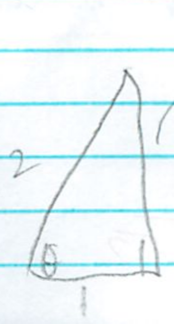
or

$\frac{4\sqrt{34}}{9}$

15. 

$x^2 + 2^2 = 3^2$
$-4 \quad -4$
$x^2 = 5$
$x = \sqrt{5}$ or 2.23

$\sin\theta = \frac{2}{3}$ $\csc\theta = \frac{3}{2}$
$\cos\theta = \frac{\sqrt{5}}{3}$ $\sec\theta = \frac{3}{\sqrt{5}}$ $\frac{3\sqrt{5}}{5}$
$\tan\theta = \frac{2}{\sqrt{5}}$ $\frac{2\sqrt{5}}{5}$ $\cot\theta = \frac{\sqrt{5}}{2}$

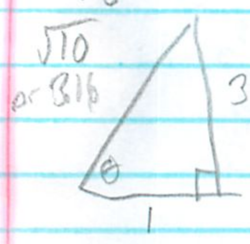
17. 

$x^2 + 1^2 = 2^2$
$-1 \quad -1$
$x^2 = 3$
$x = \sqrt{3}$ or 1.73

$\sin\theta = \frac{\sqrt{3}}{2}$ $\csc\theta = \frac{2}{\sqrt{3}}$ $\frac{2\sqrt{3}}{3}$
$\cos\theta = \frac{1}{2}$ $\sec\theta = \frac{2}{1}$ 2
$\tan\theta = \frac{\sqrt{3}}{2}$ $\cot\theta = \frac{2}{\sqrt{3}}$ $\frac{2\sqrt{3}}{3}$

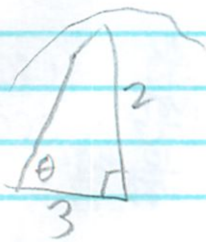
$\sqrt{3^2+1^2}$

19, $\sqrt{10}$ or 3.16



$\sin\theta$	$\frac{3}{\sqrt{10}}$	$\frac{3\sqrt{10}}{10}$	$\csc\theta$	$\frac{\sqrt{10}}{3}$	
$\cos\theta$	$\frac{1}{\sqrt{10}}$	$\frac{1\sqrt{10}}{10}$	$\sec\theta$	$\frac{\sqrt{10}}{1}$	$\sqrt{10}$
$\tan\theta$	$\frac{3}{1}$	3	$\cot\theta$	$\frac{1}{3}$	

21,

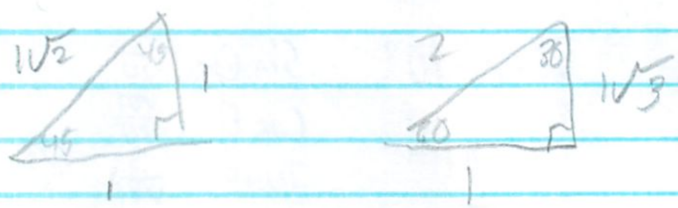


$\sqrt{2^2+3^2}$

$\sqrt{13}$ or

3.06

$\sin\theta$	$\frac{2}{\sqrt{13}}$	$\frac{2\sqrt{13}}{13}$	$\csc\theta$	$\frac{\sqrt{13}}{2}$
$\cos\theta$	$\frac{3}{\sqrt{13}}$	$\frac{3\sqrt{13}}{13}$	$\sec\theta$	$\frac{\sqrt{13}}{3}$
$\tan\theta$	$\frac{2}{3}$		$\cot\theta$	$\frac{3}{2}$



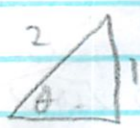
23. $\frac{\sqrt{2}}{1}$ $\sqrt{2}$

27. $\frac{1}{1}$ 1

25 $\frac{2}{1\sqrt{3}}$ $\frac{2\sqrt{3}}{1}$ $\frac{2\sqrt{3}}{3}$

29. $\frac{1}{1\sqrt{3}}$ $\frac{1\sqrt{3}}{1}$ $\sqrt{3}$

31,



$1^2+x^2=2^2$

$x^2=3$

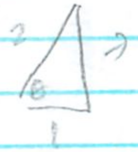
$\sqrt{3}$

$\frac{\sqrt{3}}{2}$

33

$\frac{9}{1}$ 9

35



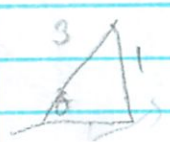
$1^2+x^2=2^2$

$x^2=3$

$\sqrt{3}$

$\frac{1}{\sqrt{3}}$ $\frac{\sqrt{3}}{3}$ remember bottom

37.

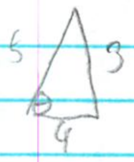


$1^2+x^2=3^2$

$x^2=8$

$\sqrt{8}$

$\frac{3}{\sqrt{8}}$ $\frac{3\sqrt{8}}{8}$ reduce 1 more $\frac{3}{2\sqrt{2}}$

39. 

$\sqrt{3^2+4^2}$

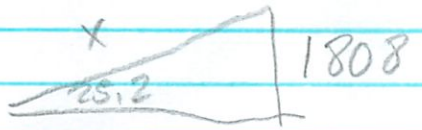
$\sqrt{25}$

5

$\frac{3}{5}$

2x Odd

41. .174
45 .900
49 1.167
53 2.136
5
57

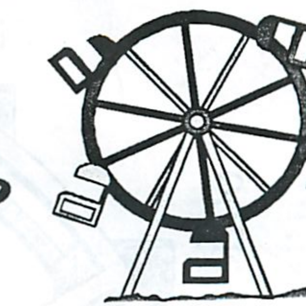


$\sin(25.2) = \frac{1808}{x}$

$\frac{.426x}{.426} = \frac{1808}{.426}$

$x = 4246.331$

What Math Might Be Involved in this Circus Act?



Make a list of things you feel may be important when trying to solve this problem.

- how fast does the wheel turn
- how high is the perpendicular place which the diver jumps
- how fast does the cart move
  - does it accelerate
- what is the reaction time of the assistant
- what is the altitude the feris wheel is at for air resistance
  - for diver + cart
- how large is the cart — diameter of wheel
  - l · w ?
- ayrodynamics of cart
- Speed feris wheel
- lateral wind speed

2/1

Ferris wheel (1)

2/1

1. a. 20ft
b. 35ft
c. 20 ft
d. 5ft

2. $\cos(30) = \frac{15}{x}$
.8660x = 15
.8660 .8660
17.3205 = x
32.99ft

~~$\tan(30) = \frac{x}{15}$~~
~~.57735(15) = x~~
~~8.6603 = x~~
32.99ft

don't need

should also be 15

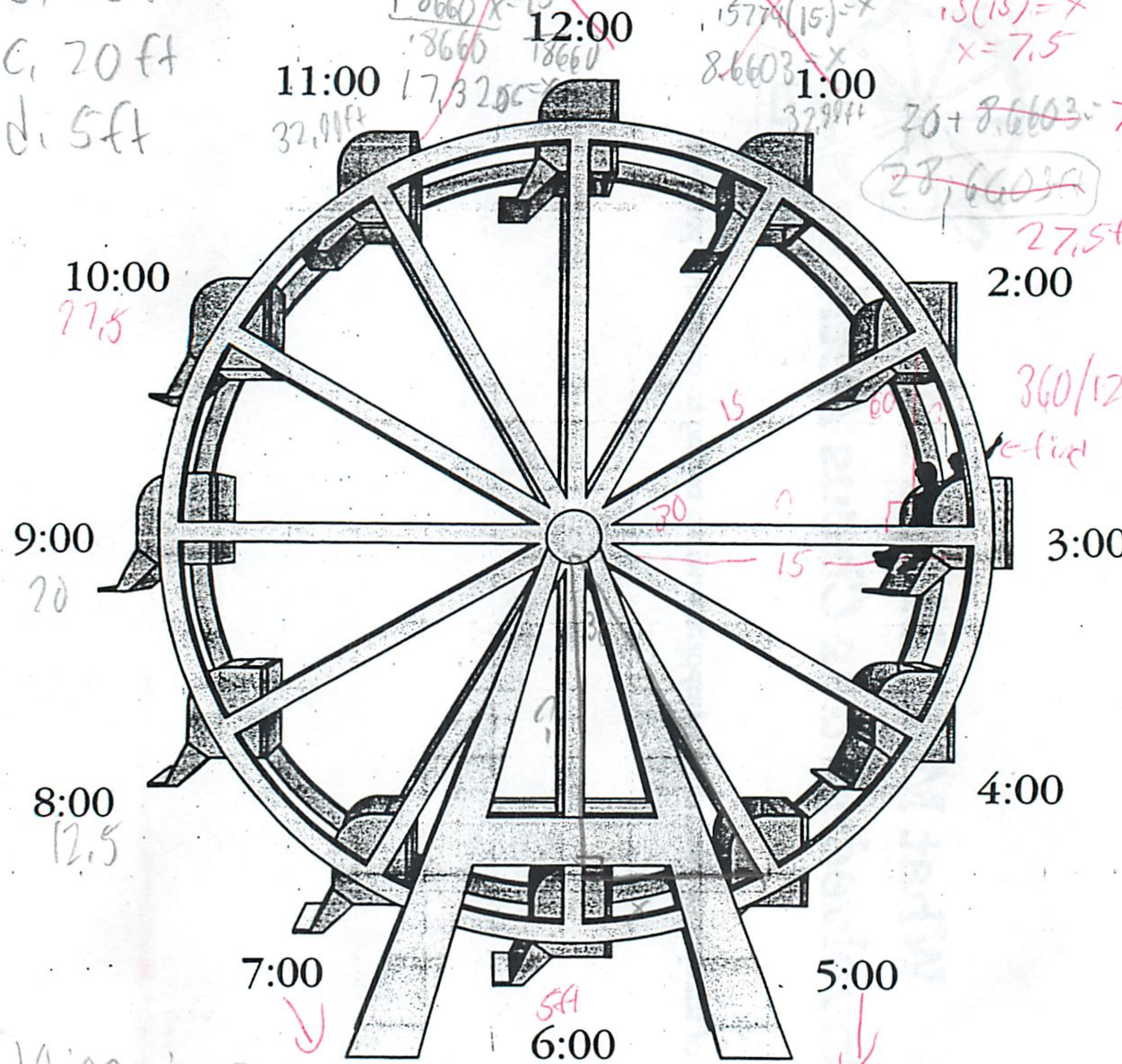
not 15, less

$\sin(30) = \frac{x}{15}$
.5(15) = x
x = 7.5

20 + ~~8.6603~~ 7.5
~~28.66030ft~~
27.5ft

360/12 = 30°

find



3. 10:00 is same as 2:00 so 27.5

5:00 ;

~~$\sin(30) = \frac{x}{15}$~~
~~.5(15) = x~~
~~x = 7.5~~

$\cos 30 = \frac{?}{15}$
.8660(15) = ?
12.9904 = ?

20 − 12.9904 =
7.0096ft

As the Wheel Turns ②

2/1

1. $C = 2\pi r$

$C = 314.1593 / 40 = 7.8540$ ft/sec

or $100\pi / 40 = 2.5\pi$ ft/sec

2. $360° / 40 = 9°$/sec

or $360° / 12 = 30° / 9° = 3.333$ sec   same

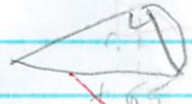
3. 1 section $314.1593 / 12 = 26.1799$ ft $/ 7.8540 = 3.333$ sec

a 4 sections: $3.333 \cdot 4 = 13.3333$ sec

b 8 sections: $3.333 \cdot 8 = 26.6667$ sec

c 11 sections: $3.333 \cdot 11 = 36.6667$ sec

4. see back

a.  7.8540 ft ×2  $C = 5\pi$

$D = \cancel{2.5}$ ft 5

$R = \cancel{1.25}$ 2.5

not really arc radius

$\sin 60 = \frac{x}{\cancel{2.5}\ 5}$

$.8660(\cancel{2.5}\ 5) = x$

$\cancel{2.1651} = x$

4.3301

4.3301

$\cancel{2.1651} + 65 =$

~~67.1675 ft~~

69.3301 ft

b.  $6(2.5\pi)$ ×2 D ~~7.5~~ ~~15~~ 30 ft

$\sin 60 = \frac{x}{\cancel{7.5}\ \cancel{15}\ 30}$

$.8660(7.5) = x$

$x = \cancel{6.4952}$ ~~12.9904~~ 25.9808

25.9808 ~~12.9904~~ ~~6.4952~~ + 65 = ~~71.4952 ft~~ ~~77.9904~~ 90.9808

c. $C = \frac{2 \cdot 10 \cdot 2.5\pi}{2\pi}$

$D = 25$ ft 50

$\sin 60 = \frac{x}{\cancel{25}\ 50}$

$.8660(\cancel{25}\ 50) = x$

$x = \cancel{21.6506}$ 43.3013 $+65 =$ ~~86.6506~~ 108.3013

d. $C = \frac{2 \cdot 14 \cdot 2.5 \cdot \pi}{2\pi}$

$D = 70$ ft

$\sin(60) = \frac{x}{70}$

$.8660(70)$

$60.6218 + 65 = 125.6218$

e. $\frac{2 \cdot 23 \cdot 2.5 \cdot \pi}{\pi}$

$D = 115$

$\sin(60) = \frac{x}{115}$

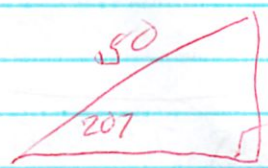
$.8660(115) = x$

$99.5929 + 65 = 164.5929ft$

?shouldn't it go down ?

e

$\sin 207 = -22.7 + 65 = 42.3ft$



f $49 - 9 = 40$ sec

$\frac{20 \cdot 9 \cdot 2.5 \cdot \pi}{\pi}$

$D = 45$

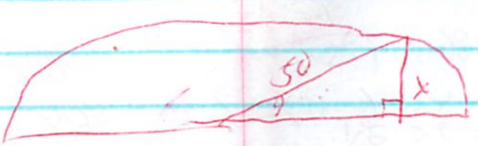
$\sin(60) = \frac{x}{45}$

$.8660(45) = x$

$38.9711 + 65 = 103.9711ft$

Do w/o circumferenc

a

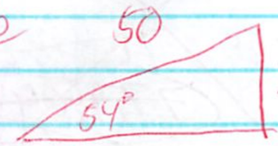


$\sin(9) = \frac{x}{50}$

$7.82 + 65 =$

$72.8ft$

b



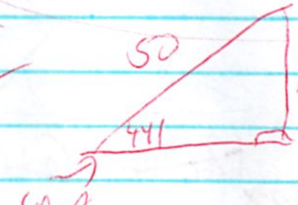
$\sin 54 = \frac{x}{50}$

$.8090(50) = x$

$40.4504 + 65 =$

$105.54°$

f



$\sin 441 = \frac{x}{50}$

$7.82 + 65 = 72.8$

Clear View ③

2/1

1. ~~$C = 2\pi(15)$~~
~~94.2478 / 24 = 3.9270 ft/sec~~
~~or 30π or 1.25π ft/sec~~

~~360°/24 = 15° /sec~~

don't
need - find % ↑n of fence

vertical Δ decrease

30ft 35ft 13ft 8ft

- subtract pedistal

not exactly that 8ft is 8ft of the time

8ft/30ft = ~~26.67%~~ of time

below time

2. The percent would not change just the actual time

see

see

or ~~2~~% above

see what are asking

7 / 15 / 27.8

find

$\sin^{-1}\left(\frac{7}{15}\right)$

27.8°

So above: 180°
27.8
+ 27.8
235.636°

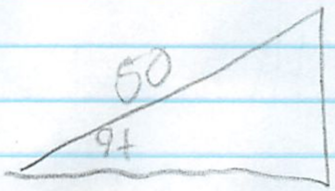
235.636/360° =

65.45%

At Certain Points in Time

2/6

1.



$\sin(9t) = \frac{x}{50}$

$50(\sin(9t)) = x$

$+65$

$50(\sin 9t) + 65 = h$

— or $h(t) = r\left(\sin\left[\text{angular speed} \cdot t\right]\right) + \text{height center}$

2. $t = 1$ $50(\sin 9(1)) + 65 = h$

$h = 72.82$

$t = 6$ $50(\sin(9(6))) + 65 = h$

$h = 105.45$

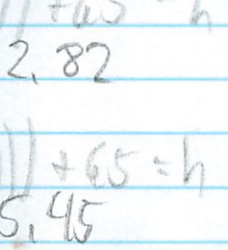
Co function identy

$\sin\theta = \cos(90 - \theta)$

$\sin\theta = \sin\theta_R$

reference angle

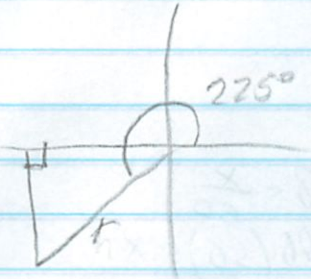
$\sin(441) = \sin(81)$



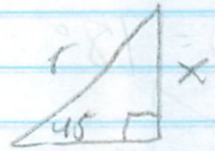
$441 = 360 + 81$

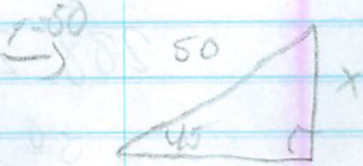
Testing the Definition

2/6

1.a 

$225 - 180 = 45°$





$\sin(45) = \frac{x}{50}$

$.7071(50) = x$

$x = 35.3553$

from here

or

$\left(\frac{-3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$

too many steps

find y too

b $50(\cancel{\sin 225}) + 65$

$50(-\sin 45) + 65$

$50\left(\frac{-\sqrt{2}}{2}\right) + 65$

29.644 ft

$\sin 225 = \dfrac{\frac{-3\sqrt{2}}{2}}{\frac{3}{1}} = \frac{-3\sqrt{2}}{2} \cdot \frac{1}{3}$

$\frac{-3\sqrt{2}}{6}$

also = $\frac{-\sqrt{2}}{2}$

$-(\sin 45)$

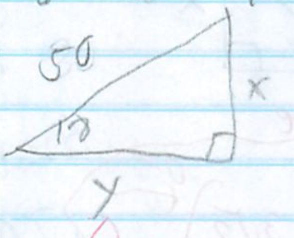
c It does make sense if you draw it + put #s in

d $\sin(225) = -.7071 = \frac{-\sqrt{2}}{2} = -\sin(45)$

2. $t = 32$

$32 \cdot 9 \text{ sec} = 288°$

$288 - 270 = 18°$



$\sin 18 = \frac{x}{50}$

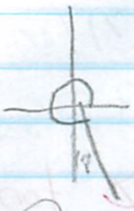
$.3090(50) = x$

$15.45 = x$

$\cos 18 = \frac{y}{50}$

$.9510(50) = y$

$47.5528 = y = -\sin 288$



rotate

$(15.45, -47.55)$

$50(-\cos 18) + 65$

17.4471 ft

c. Does make sense - almost at ground

or

d. $50(\sin 288) + 65 = 17.447$

Yes it works

IAG 4
High Dive

Name ______________________

A Ferris Wheel whose center is 80 feet off the ground with a radius of 60 feet takes 36 seconds to complete one revolution.

a) Write the equation that represents the diver's height at time 't' .

60(sin [10t]) + ~~65~~ 80

b) Determine the diver's height for the following times after 3:00. position

2 seconds 100.52 60(sin 20) + ~~65~~ 80

10 seconds 139.08 60(sin 100) + 80

20 seconds 59.478 60(sin 200) + 80

40 seconds 118.567 60(sin 400) + 80

(80-60)

20 seconds

c) What is the diver's maximum height? 140 and minimum height? ~~31.914~~

must have hit max not min

plug into y =, graph, find max min

d) At what two times (between 0 and 36 seconds) will the diver's height be 100 feet above the ground?

Time #1: 1.9471 Time #2: 16.052

e) At what two times (between 0 and 36 seconds) will the diver's height be 45 feet above the ground?

Time #1: 21.568 Time #2: 32.431

graph and find intersection

y=100

y=45

IAG 4 Name ______________________

High Dive

A Ferris Wheel whose center is 80 feet off the ground with a radius of 60 feet takes 36 seconds to complete one revolution.

a) Write the equation that represents the diver's height at time 't'.

$60(\sin[10t]) + \cancel{65}\ 80$

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graph and find intersection

y=100

y=45

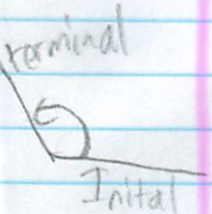
13.2 Notes

Angles of Rotation + Radian Measure

2/7

Vocab

Angle of rotation - determained by rotating a ray about its vertex



Inital side: starting position of a ray (3:00)

terminal side: ending position

Acute ∠s: Betwen 0° + 90° ($0° < \theta < 90°$)

Obtuse ∠s: Between 90 + 180 ($90° < \theta < 180°$)

Positive ∠s: counter clockwise rotation 

Negitive ∠s: clock wise rotation 

Radians a portion of the circumference of a circle in terms of π

Practice

Arc lenght

$\theta = 60°$, $r = 1$ → $\frac{60}{360}(2\pi 1)$ → $\frac{1}{3}\pi$ = $\frac{\pi}{3}$

$\theta = 90°$, $r = 1$ → $\frac{90}{360}(2\pi 1)$ → $\frac{1}{2}\pi$ → $\frac{\pi}{2}$

$\theta = 180$, $r = 1$ → $\frac{180}{360}(2\pi 1)$ → 1π → π

$\theta = 270$, $r = 1$ → $\frac{270}{360}(2\pi 1)$ → 1.5π → $\frac{3}{2}\pi$ → $\frac{3\pi}{2}$

$\theta = 360$
$r = 1$

$\frac{360}{360}(2\pi 1) = 2\pi$

Converting Degrees → Radians

Can do simple;

$\frac{90}{360}(2\pi 1) = \frac{\pi}{2}$

or reduce

$\frac{90(2\pi)}{360} = \frac{\pi}{2}$

fancy way

$90 \cdot \frac{\pi}{180} = \frac{\pi}{2}$

$$\text{Degree measure} \cdot \frac{\pi}{180} = \text{radian measure}$$

Radians → Degrees

$\frac{3\pi}{2} / \frac{\pi}{180} = 270$ or $\frac{3\pi}{2} \cdot \frac{180}{\pi} = 270°$

$$\text{Radian measure} \cdot \frac{180}{\pi} = \text{Degree measure}$$

Radian Conversion Practice + More Notes

2/7

$240°$ $\quad \frac{240}{1} \cdot \frac{\pi}{180} = \cancel{\frac{3\pi}{2}}$ $\frac{4\pi}{3}$ | ✓: $\frac{180}{\pi} = 4 \cdot 60 = 240°$ ✓

thought 1.333 was $\frac{3}{2}$
mental math error

$-80°$ $\quad \frac{-80}{1} \cdot \frac{\pi}{180} = -\frac{4\pi}{9}$

$\frac{7\pi}{6}$ $\quad \frac{7\pi}{6} \cdot \frac{180}{\pi} = 7 \cdot 30 = 210°$

$\frac{16\pi}{5}$ $\quad \frac{16\pi}{5} \cdot \frac{180}{\pi} = 16 \cdot 36 = 576°$

Complementry Angles - Add to $90°$ or $\frac{\pi}{2}$

Supplementry Angles - Add to $180°$ or π

Complement: $24°$ - $90 - 24 = 66°$

Complement: $\frac{\pi}{5}$ - $\frac{\pi}{2} - \frac{\pi}{5} \rightarrow \frac{5\pi}{10} - \frac{2\pi}{10} = \frac{3\pi}{10}$

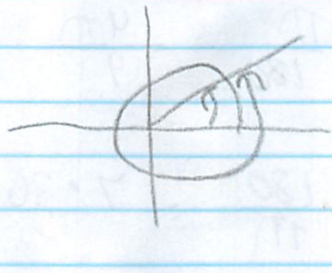
Common denominators

Complement: $\frac{3\pi}{8}$ $\quad \frac{\pi}{2} - \frac{3\pi}{8}$ $\quad \frac{4\pi}{8} - \frac{3\pi}{8} = \frac{\pi}{8}$

Supplement: $96° = 180 - 96 = 84°$

Supplement: $\frac{3\pi}{10}$ $\quad \pi - \frac{3\pi}{10} \rightarrow \frac{10\pi}{10\pi} - \frac{3\pi}{10} = \frac{7\pi}{10}$

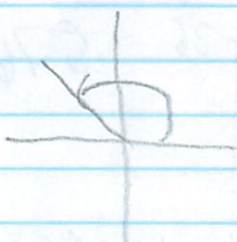
Coterminal angles - 2 angles which have the same initial and terminal ray but have different measures (multiple of 360° difference)



$\underline{30°} + 360° = \underline{390°}$

Coterminal

also, $\underline{-330°}$ works



$\frac{5\pi}{8}$

or $2 \cdot \frac{5\pi}{8}$. or $\frac{21\pi}{8}$

or $2\pi - \frac{5\pi}{8} \rightarrow \frac{16\pi}{8} - \frac{5\pi}{8} = -\frac{11\pi}{8}$

Orrange it $\frac{5\pi}{8} - 2\pi$

13.2 Angles of Rotation practice p689

15. Complement 30; $90 - 30 = 60°$
17. Complement 52°; $90 - 52 = 38°$
19. Complement $\frac{2\pi}{9}$; $\frac{\pi}{2} - \frac{2\pi}{9} \to \frac{9\pi}{18} - \frac{4\pi}{18} = \frac{5\pi}{18}$
21. Complement $\frac{\pi}{6}$; $\frac{\pi}{2} - \frac{\pi}{6} \to \frac{3\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{6} \to \frac{\pi}{3}$
23. Supplement 100; $180 - 100 = 80°$
25. Supplement 24; $180 - 24 = 156°$
27. Supplement $\frac{2\pi}{9}$; $\pi - \frac{2\pi}{9} = \frac{9\pi}{9} - \frac{2\pi}{9} \to \frac{7\pi}{9}$
29. Supplement $\frac{\pi}{3}$; $\pi - \frac{\pi}{3}$ $\frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3}$

Degrees → Radians

39. 20° $\frac{20}{1} \cdot \frac{\pi}{180} = \frac{\pi}{9}$

41. 150 $\frac{150}{1} \cdot \frac{\pi}{180} = \frac{5\pi}{6}$

43. −110 $\frac{-110}{1} \cdot \frac{\pi}{180} = -\frac{11\pi}{18}$

45. 320° $\frac{320}{1} \cdot \frac{\pi}{180} = \frac{16\pi}{9}$

Radians → Degrees

47. $\frac{5\pi}{3}$ $\frac{5\pi}{3} \cdot \frac{180}{\pi}$ $\cancel{3} \cdot 60 = \cancel{180}$ 5 ← copy error 300°

49. ~~$-\frac{3\pi}{2}$ $\frac{-3\pi}{2} \cdot \frac{180}{\pi}$ $-3 \cdot 90 = -270$~~ 18° copy problem from book wrong $\frac{\pi}{10}$

51. $\frac{7\pi}{15}$ $\frac{7\pi}{15} \cdot \frac{180}{\pi}$ $7 \cdot 12 = 84$

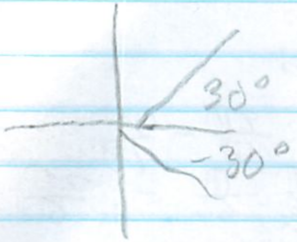
53. $-\frac{7\pi}{6}$ $-\frac{7\pi}{6} \cdot \frac{180}{\pi}$ $-7 \cdot 30 = -210$

55. $\frac{5\pi}{4}$ $\frac{5\pi}{4} \cdot \frac{180}{\pi} = 5 \cdot 45 = 225$ – a

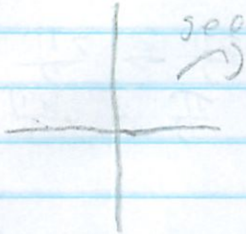
56. 460 c

57. $\frac{7\pi}{3}$ $\frac{7\pi}{3} \cdot \frac{180}{\pi} = 7 \cdot 60 = 420$ b

58. $-\frac{13\pi}{3}$ $\frac{-13\pi}{3} \cdot \frac{180}{\pi}$ $-13 \cdot 60 = -780$ d

8. 

No

11. 

$\frac{\pi}{3}$

$\frac{\pi}{3} - 2\pi \quad \frac{\pi}{3} - \frac{6\pi}{3} = \frac{-5}{3}\pi$ yes

31. $50°$ $\quad 50 + 360 = 410$

$50 - 360 = -310°$

34 did wrong problem

35. $840°$ $\quad 840 + 360 = 1200$

$840 - 360 = 480$

36. $\frac{17\pi}{4}$ $\quad \frac{17\pi}{4} + 2\pi \rightarrow \frac{17\pi}{4} + \frac{8\pi}{4} = \frac{25\pi}{4}$

~~$\frac{17\pi}{4} - 2\pi \rightarrow \frac{17\pi}{4} - \frac{8\pi}{4} = \frac{9\pi}{4}$~~

do something ending in negitive $\quad \frac{17\pi}{4} - 5\pi \rightarrow \frac{17\pi}{4} - \frac{20\pi}{4} = \frac{-3\pi}{4}$

Formula Review

$r = 25$ ft
center → ground = 45 ft
one cycle = 90 sec

write formula — copy error

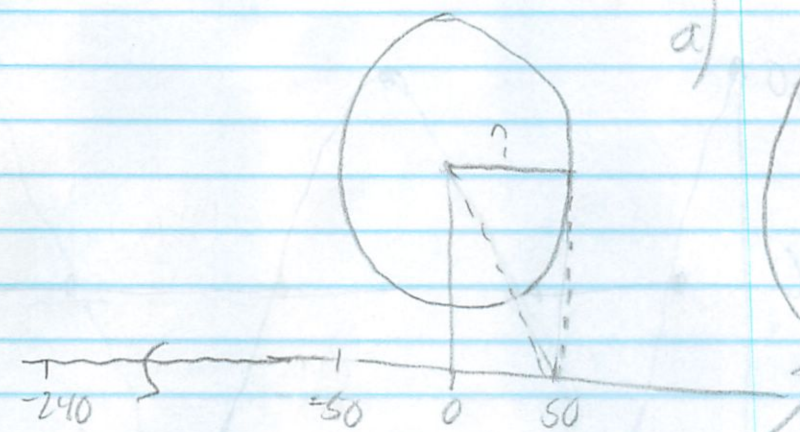
looked up → $25(\sin[360/90]t) + 45$

$t = 12$ sec · $25(\sin 4\cancel{0} \cdot 12) + 45$ · ~~66.6506~~ 63.5786
$t = 56$ sec · $25(\sin 4\cancel{0} \cdot 56) + 45$ · ~~69.6201~~ 27.635

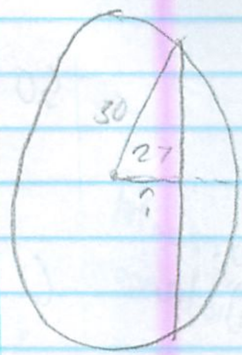
Where does he land (11)

2/8

r = 50ft
Center to ground = 65ft
40sec = cycle



a)



a) $\cos(27) = \frac{?}{50}$
$.8910(50) = ?$
$44.5503 = ?$ ← so 44 ft away

b) $\cos(9\cdot7) = \frac{?}{50}$
$.4539(63) = ?$
28.6014 ft away from center

Formula isn't it just modified vertical height

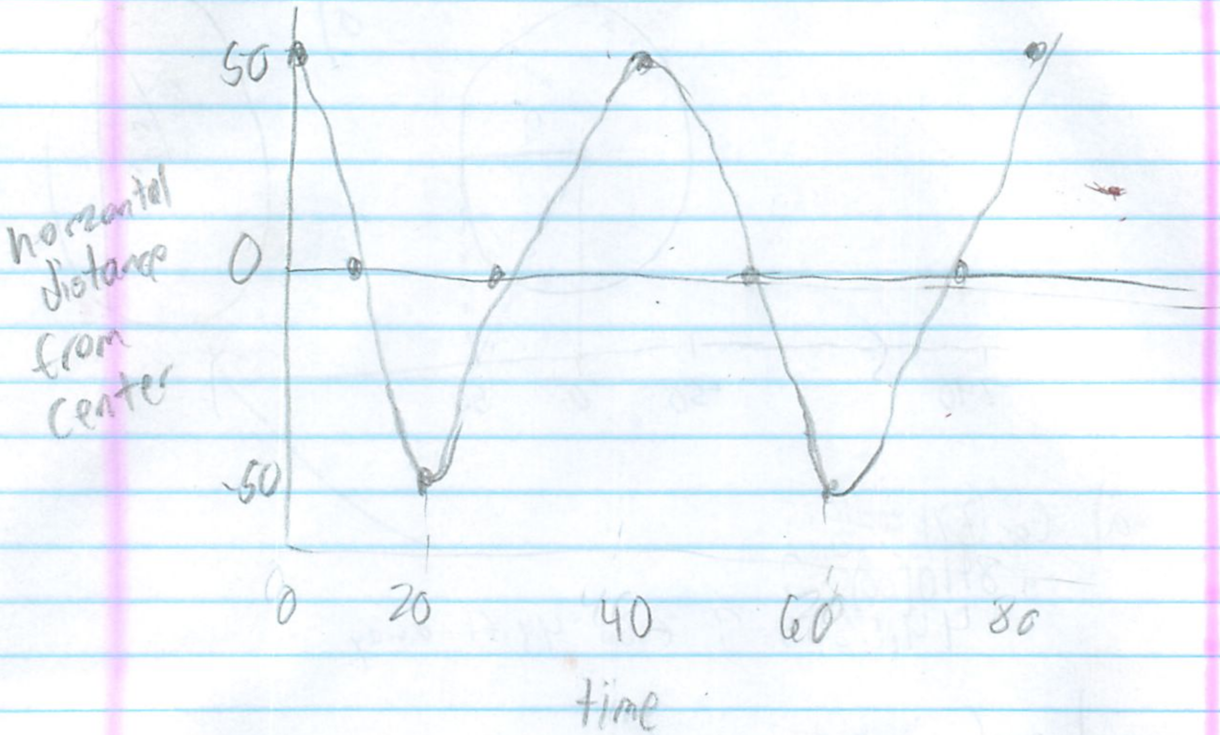
$50(\cos 9t)$

c 12 = −15.45
d 26 = −29.39
e 37 = 44.55

) calc table w/ formula

2/20

Graph

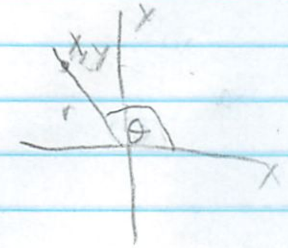


vertical shift = 0
midline = 0
amplitude = 50
period = 40sec

x-int - 10, 30, 50, 70
x max - 0, 40, 80
x mins - 20, 60

Coordinate Tangents (78)

1.

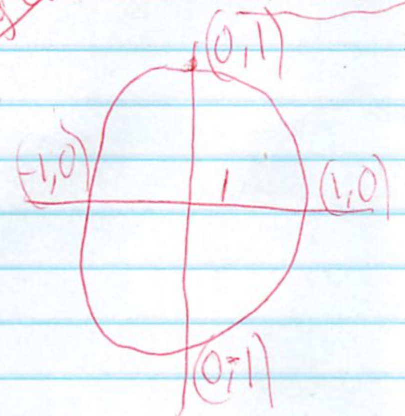


$\tan\theta = \frac{y}{x}$

? Its the change in y divided by the change in x

?? totally confused

Something else



← undefined at 90 and 270°

$\tan\theta = \frac{y}{x}$

$\tan(90) = \frac{1}{0}$ → also → 450

$\tan(270) = \frac{-1}{0}$ → also → 630

every 180° undefined

−270, −90, 90, 270, 450, 630 = undefined

every coterminal of 90 + 270°

2. $\tan\theta = \frac{y}{x}$

↓

$\tan\theta = \frac{\sin\theta\, r}{\cos\theta\, r}$

↓

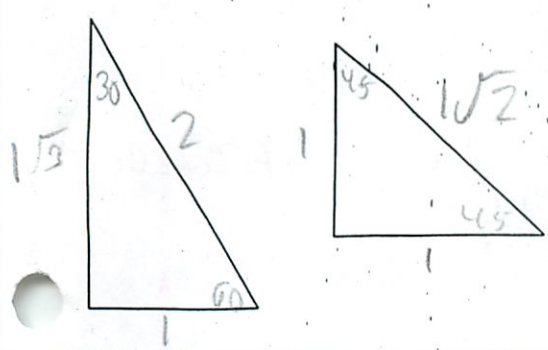
$\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Name: Michael Plasmeier

Activity: Mini Wheel – part 1

Reference:

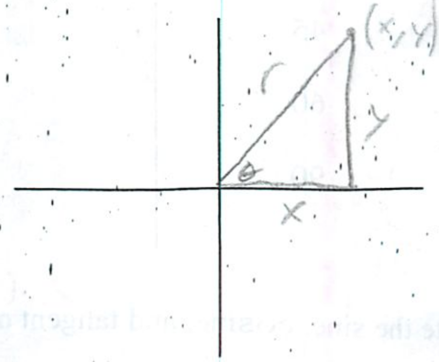
Label the Diagrams below to illustrate the special triangle ratios so that you can refer to them later if needed. Let the shortest leg be 1.



remember

Define sine, cosine, and tangent in terms of x, y, & r.

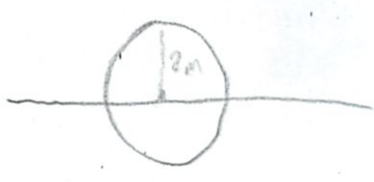
$\sin\theta = \frac{y}{r}$
$\cos\theta = \frac{x}{r}$
$\tan\theta = \frac{y}{x}$



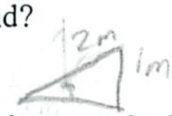
The Situation:

For this activity, consider a very small Ferris Wheel where the center is at ground level and the radius is only 2 meters. Suppose you get on the ride at ground level and rotate upward counter-clockwise.

1. a. Sketch the Mini Wheel below:



b. How many degrees will you have rotated if the seat of the Mini Wheel has moved to a height of 1 meter off the ground?



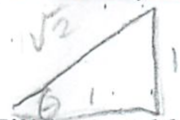
$\sin(\theta) = \frac{1}{2} \rightarrow \sin\theta = 30°$

c. Use your sketch to help you calculate the sine, cosine, and tangent of this angle.

30°
$\sin\theta = \frac{1}{2}$
$\cos\theta = \frac{\sqrt{3}}{2}$
$\tan\theta = \frac{1}{\sqrt{3}}$

60°
$\sin\theta = \frac{\sqrt{3}}{2}$
$\cos\theta = \frac{1}{2}$
$\tan\theta = \frac{\sqrt{3}}{3}$ $(= \cot(30))$

2. Use one of your originals sketches (1st box on front) to help calculate the sin, cos, and tan of 45 degrees.



$Sin = \frac{1}{\sqrt{2}}$ $Tan = \frac{1}{1} = 1$
$Cos = \frac{1}{\sqrt{2}}$

3. Fill in the table for the sine, cosine, and tangent values for the first quadrant special angles.

not undefined

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	~~Undefined~~ 0
30	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{1}$
90	1	0	undefined

$r=1$ $(1,0)$

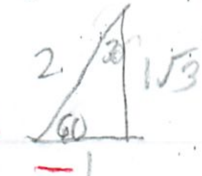
So $\sin 0 = \frac{y}{r} = \frac{0}{1} = 0$

$\cos 0 = \frac{x}{y} = \frac{1}{1} = 1$

See Mini Wheel #2 worksheet before preceeding

4. Calculate the sine, cosine, and tangent of 120 degrees.

$\theta_r = 60^\circ$



$Sin(120) = \frac{1\sqrt{3}}{2}$
$Cos(120) = \frac{-1}{2}$
$Tan(120) = \frac{\sqrt{3}}{-1}$

make sure to keep neg

5. Repeat for 225°

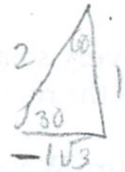
$\theta_r = 45^\circ$

$Sin(225) = \frac{-1}{\sqrt{2}}$
$Cos(225) = \frac{-1}{\sqrt{2}}$
$Tan(225) = \frac{-1}{-1} = 1$

$\neq Cos(45) = -Cos(45)$

6. Do the same for 150°

$\theta_r = 30^\circ$

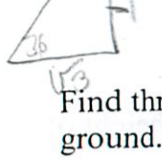


$Sin\ 150 = \frac{1}{2}$
$Cos\ 150 = \frac{-1\sqrt{3}}{2}$
$Tan\ 150 = \frac{1}{-1\sqrt{3}}$

rationalize

7. Now try 330°

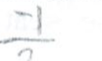
$\theta_r = 30^\circ$



$Sin\ 330 = \frac{-1}{2}$
$Cos\ 330 = \frac{\sqrt{3}}{2}$
$Tan\ 330 = \frac{-1}{\sqrt{3}}$ rationalize?

8. Ok, see if you can do 750°

$750 - 720 = 30^\circ$
first quad



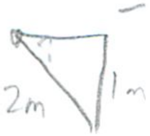
$Sin\ 750 = Sin\ 30 = \frac{1}{2}$
$Cos\ 750 = Cos\ 30 = \frac{\sqrt{3}}{2}$
$Tan\ 750 = Tan\ 30 = \frac{1}{\sqrt{3}}$ rationalize

9. Find three angles of rotation such that the seat of the Mini Wheel will be exactly 1 meter below the ground.

$Sin\ ? = \frac{-1}{2}$

$Sin\ \theta = -30^\circ$

$-30 + 360 = 330$
$-30 - 360 = -390$
30° too



other side

$Sin\ ? = \frac{-1}{2}$
$Sin\ ? = 30^\circ$
?

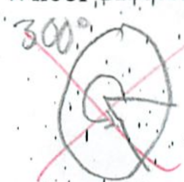
Name: ____________________

Return of the Mini Wheel: part 2 "The Sequel"

Reference Angles

'or this activity, continue to consider a very small Ferris Wheel where the center is at ground level and the adius is only 2 meters. Suppose you get on the ride at ground level and rotate upward counter-clockwise.

1. Suppose you rotate 300°.
   a. The cosine value will be the same as for what first quadrant angle? Support your answer with a Mini Wheel sketch.



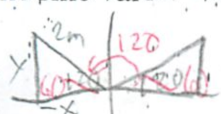
~~sin(60)~~ $\cos(300) = \cancel{\cos(300-270)}$ both = each other but wrong

Do it her way — move the y and x values

   b. How do the y-coordinates of these two angles compare?

same but opposet value (−y for 300°
y for 60° is pos

2. Suppose you rotate 120°.
   a. The sine value will be the same as for what first quadrant angle?



$\sin \cancel{120} = \frac{y}{2}$
60

180 − 120

   b. How do the x-coordinates of these two angles compare?

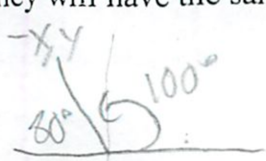
x value for 120 is −
y value for 60 is +

Definition: Reference Angle:

A positive acute angle thats created by using the terminal side of θ and the closest side of x axis

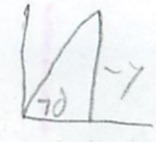
For example, the reference angle for 100° is the first quadrant angle 80° and the reference angle for 250° is 70°.

3. Sketch these pairs of angles in the coordinate plane in a circle so that you see the symmetry that explains why they will have the same trig values.



$\sin 100 = \frac{y}{r}$

$\sin 80 = \frac{y}{r}$



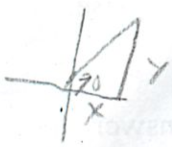
[well not really but in 3rd it is

4. Find the reference angle for . . . .

a. 135°

b. 200°

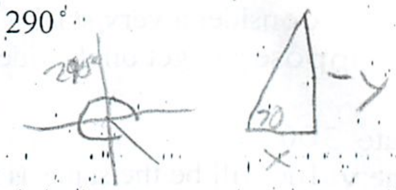
c. 70°



¿same isn't it —

yes 70°

d. 290°



5. Explain in general how one find the reference angle for a . . .

a. Second quadrant angle

$\theta_r = 180° - \theta$

b. *Third* quadrant angle

$\theta_r = \theta - 180°$

note different

c. Fourth quadrant angle

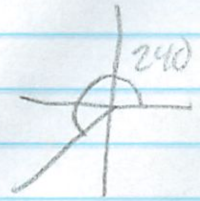
$\theta_r = 360 - \theta$

13.3

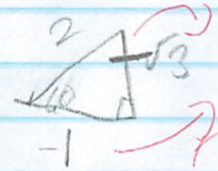
Eval, Any Trig Functn
Practice

2/1

1, tan 240



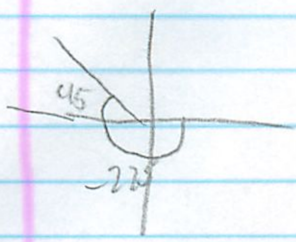
$\theta - 180 = 60°$



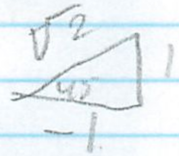
$\frac{-\sqrt{3}}{-1} = \sqrt{3}$

Remember the negitives

2, cot(-225)

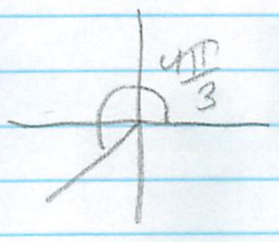


$180 - -225 = 405 - 360 = 45°$



$\frac{-1}{1} = -1$

3, $\cos\left(\frac{4\pi}{3}\right)$



$\frac{4\pi}{3} - \frac{3\pi}{3} = \frac{1\pi}{3} \rightarrow (60°)$

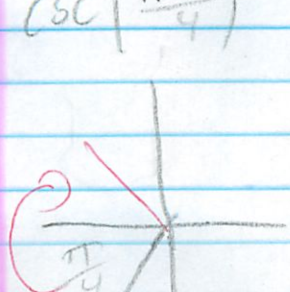
$\frac{-1}{2}$

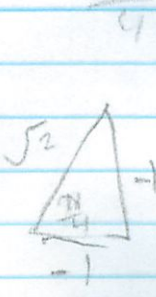
4, $\csc\left(\frac{11\pi}{4}\right)$

Convert error

$\frac{11\pi}{4} - \frac{8\pi}{4} = \frac{3\pi}{4}$ (~~[illegible]~~) 135

$\frac{4\pi}{4} - \frac{3\pi}{4} = \frac{\pi}{4}\ (45°)$





$\frac{\sqrt{2}}{-1}$ $\rightarrow$ $-\sqrt{2}$

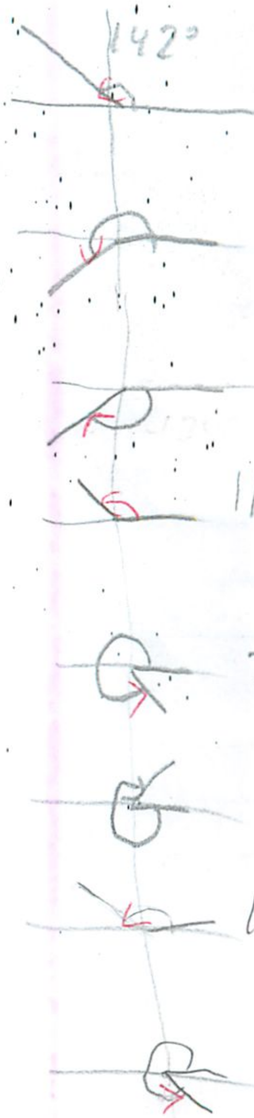
Simplify

Name: Michael Plasmeier

Name that Angle!

Directions: Given the following θ find its reference angle, one positive coterminal angle, and one negative coterminal angle. Also, draw a picture and label θ & θ_R. If the original angle is given in terms of radians, your answers and work must also be completed in radians.

	θ	Reference Angle	Positive Coterminal Angle	Negative Coterminal Angle	Picture
1.	142°	38°	502°	−218°	142°
2.	$\frac{7\pi}{6}$	$\frac{7\pi}{6} - \frac{6\pi}{6} = \frac{\pi}{6}$	$\frac{19\pi}{6}$	$-\frac{5\pi}{12}$	210°
3.	−134°	46°	226°	−494°	
4.	$\frac{5\pi}{8}$	$\frac{8\pi}{8} - \frac{5\pi}{8} = \frac{3\pi}{8}$	$\frac{21\pi}{8}$	$\frac{-11\pi}{8}$	112.5°
5.	$\frac{27\pi}{15}$	$\frac{30\pi}{15} - \frac{27\pi}{15} = \frac{3\pi}{15}$ or $\frac{\pi}{5}$ reduce	$\frac{57\pi}{15} \to \frac{19\pi}{5}$	$\frac{-3\pi}{15}$	324°
6.	−308°	360 − −308 = 668 − 360 = 308; 360 − 308 = 52	52°	−668°	
7.	$\frac{4\pi}{7}$	$\frac{7\pi}{7} - \frac{4\pi}{7} = \frac{3\pi}{7}$	$\frac{18\pi}{7}$	$\frac{-10\pi}{7}$	102.85°
8.	289°	71°	649°	−71°	

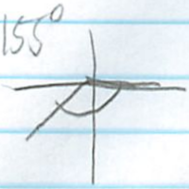


always draw angle arrow

13.3 / Evaluating Trig Functions
Practice

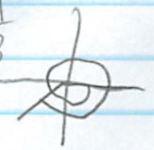
2/10

pg 97 23. $-155°$



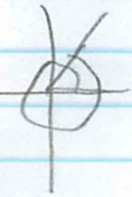
$180 - -155 = 335°$ $360 - 335 = 25°$

25. $-\frac{8\pi}{3}$



$-\frac{8\pi}{3} + \frac{6\pi}{3} = -\frac{2\pi}{3}$ $\frac{3\pi}{3} - \frac{2\pi}{3} = \frac{1\pi}{3}$

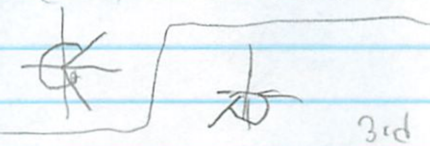
27. $\frac{12\pi}{5}$



$\frac{12\pi}{5} - \frac{10\pi}{5} = \frac{2\pi}{5}$

---

41. $\cos(300) = \cos 60 = \frac{x}{r} = \frac{1}{2}$ (4th)



43. $\csc(-120) = \csc 60 = \frac{r}{-x} = \frac{2}{-\sqrt{3}} \cdot \frac{-\sqrt{3}}{-\sqrt{3}} = -\frac{2\sqrt{3}}{3}$ (3rd)

45. $\sec(750) = \sec(30) = \cancel{x} = \cancel{\frac{\sqrt{3}}{2}}$ $\frac{r}{x} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ $\frac{2\sqrt{3}}{3}$ (1st quad)

→ remember to flip

47. $\cos(-240) = \cos(60) = \frac{-x}{r} = -\frac{1}{2}$ (2nd)

preserve neg

49. $\cot\left(\frac{5\pi}{6}\right) = \cot \frac{6\pi}{6} - \frac{5\pi}{6} = \frac{\pi}{6} \rightarrow -\frac{\sqrt{3}}{1} \rightarrow -\sqrt{3}$ (2nd)

2nd so x is neg

51. $\sin\left(-\frac{7\pi}{6}\right) = \sin \frac{2\pi}{6} - \frac{6\pi}{6} = \frac{\pi}{6} \rightarrow \frac{1}{2}$ (2nd)

1x around + back in 4th quad

do → 53. $\sin\left(-\frac{13\pi}{6}\right) = \cancel{2nd} \rightarrow \frac{\pi}{6} \rightarrow$ 2 $= -\frac{1}{2}$

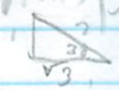
→ 55. $\cos\left(\frac{10\pi}{3}\right) = \frac{10\pi}{3} - \frac{6\pi}{3} = \frac{4\pi}{3}$ 3rd quad $\frac{\pi}{3}$ $\frac{-1}{\cancel{x}2}$ $\frac{-1}{\cancel{x}2}$ $= -\frac{1}{2}$

never negitive

2/12

Warmup

2/12



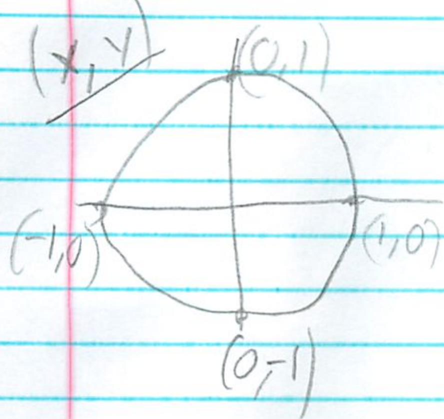
1. $\sin\left(\frac{-7\pi}{6}\right)$ - 2nd quad $\frac{\pi}{6}$ $-\sin = \frac{1}{2}$

2. $\cot\left(\frac{5\pi}{3}\right)$ - 4th quad $\frac{\pi}{3}$ $\frac{1}{-\sqrt{3}} \cdot \frac{-\sqrt{3}}{-\sqrt{3}}$ $\frac{-\sqrt{3}}{3}$

3. $\sec\left(\frac{-2\pi}{3}\right)$ - 3rd quad $\frac{3\pi}{3} - \frac{\pi}{3}$ $\frac{2}{-1} = -2$

Special Cases Review

+ Point Finding Notes 2/12



$\cos(180) = \frac{x}{r} = \frac{-1}{1} = -1$

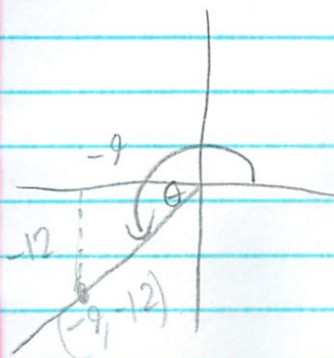
know these   plug in

$\csc\left(\frac{3\pi}{2}\right) = \frac{r}{y} = \frac{1}{-1} = -1$

$\cot(2\pi) = \frac{x}{y} = \frac{1}{0} =$ Undefined

---

given the point (-9, -12) which is on terminal side of θ, write ratio of trig functions



$hyp = \sqrt{9^2+12^2} = \sqrt{225} = \boxed{15}$

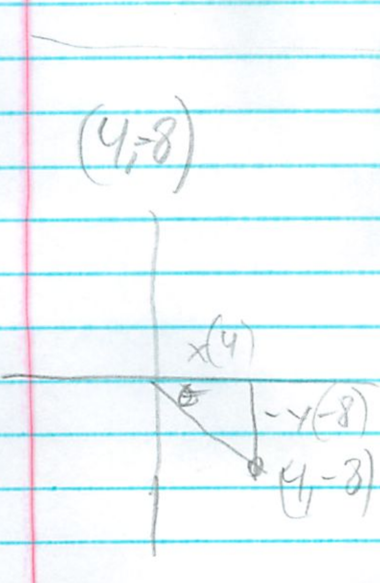
$\sin\theta = \frac{-12}{15}$ $\csc\theta = \frac{15}{-12}$

$\cos\theta = \frac{-9}{15}$ $\sec\theta = \frac{15}{-9}$

$\tan\theta = \frac{-12}{-9} = \frac{12}{9}$ $\cot\theta = \frac{-9}{-12} = \frac{9}{12}$

can reduce all as 3-4-5 triangle

(4, -8)



$hyp = \sqrt{4^2+8^2} = \sqrt{80} = 8.944$ reduce → $4\sqrt{5}$

$\sin\theta = \frac{-8}{\sqrt{80}} \cdot \frac{\sqrt{80}}{\sqrt{80}} = \frac{-8\sqrt{80}}{80}$ $\cancel{\frac{-\sqrt{80}}{10}} \to \cancel{-\sqrt{8}}$ → $\frac{-2\sqrt{5}}{5}$

$\cos\theta = \frac{4}{\sqrt{80}} \cdot \frac{\sqrt{80}}{\sqrt{80}} = \frac{4\sqrt{80}}{80}$ $\cancel{\frac{\sqrt{80}}{20}}$ $\cancel{\sqrt{40}}$ → $\frac{\sqrt{5}}{5}$

$\tan\theta = \frac{-8}{4} \to -2$

$\csc\theta = \frac{\sqrt{80}}{-8} = \frac{4\sqrt{5}}{-8} \to \frac{\sqrt{5}}{-2}$

$\sec\theta = \frac{\sqrt{80}}{4} = \frac{4\sqrt{5}}{4} = \sqrt{5}$

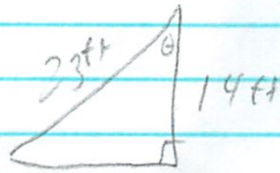
$\cot\theta = \frac{4}{-8} \to -\frac{1}{2}$

not allowed to divide $\sqrt{\ }$ regular

Warmup 2/15

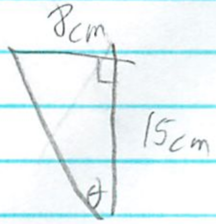
2/15

Find θ



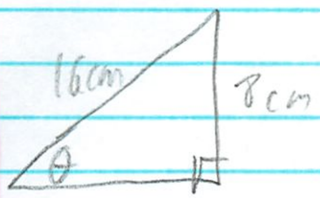
$\cos\theta = \frac{14}{23}$

$\cos^{-1}(14/23) = 52.50448°$



$\tan\theta = \frac{8}{15}$

$\tan^{-1}(8/15) = 28.07249°$



$\sin\theta = \frac{8}{16}$

$\sin^{-1}(8/16) = 30°$

Study Guide : * possibly a calculator + non-calc. section *

High Dive Quiz 1 2/13/07

- Writing ratios of sides for 6 trig functions given a pt. on the terminal side of θ in any quadrant.

- Evaluating csc θ, sec θ + cot θ on calc.

- Horizontal distance + vertical distance of platform after t seconds.

- some application of vocab.

- converting degrees ⟷ radians + vice versa

- complementary, supplementary + coterminal ∠'s.

- reference ∠'s +draw

- evaluating trig functions

- def. of sin θ, cos θ, + tan θ in terms of x, y, r in words

- word problems involving trig.

49.5/54

High Dive/Chpt. 13 Quiz #1
54 points total
Calculator Section : 42/54 points

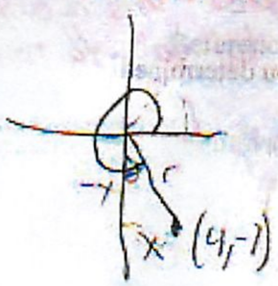
91.6%

Name Michael Plasencia
Date 12/13

*** SHOW ALL WORK!!**
** Remember to rationalize denominators and reduce all fractions where applicable.
*** Make sure your calculator is in the correct mode for the appropriate questions.

1. Find the exact values of the six trig functions of θ if the point (4, -7) lies on the terminal side of θ (1 pt. each)

$r = \sqrt{4^2+7^2} = \sqrt{65} = 8.0622$



$\sin\theta = \frac{y}{r} = \frac{-7}{\sqrt{65}} \cdot \frac{\sqrt{65}}{\sqrt{65}} \quad \frac{-7\sqrt{65}}{65}$ $\qquad \csc\theta = \frac{r}{y} = \frac{\sqrt{65}}{-7}$

$\cos\theta = \frac{x}{r} = \frac{4}{\sqrt{65}} \cdot \frac{\sqrt{65}}{\sqrt{65}} \quad \frac{4\sqrt{65}}{65}$ $\qquad \sec\theta = \frac{r}{x} = \frac{\sqrt{65}}{4}$

$\tan\theta = \frac{y}{x} = \frac{-7}{4}$ $\qquad \cot\theta = \frac{x}{y} = \frac{4}{-7}$

2. Determine the value of the following angles (round to the nearest hundredth). (1 pt. each)

a) $\cot\left(\frac{-4\pi}{7}\right)$ abs value b) $\csc 26^\circ$

$-\frac{1}{2}$ ~~+~~ .23 $\qquad$ 2.28

see sort of how

3. Complete this chart. Show all work to the right of it. ~~Answers must be in the same form as the given angle.~~ (2 pts. each)

	Degrees	Radians	
a)	280°	$\frac{14\pi}{9}$	$280 \cdot \frac{\pi}{180} = \frac{14\pi}{9}$
b)	450°	$\frac{10\pi}{4}$	$\frac{10\pi}{4} \cdot \frac{180}{\pi} = 450^\circ$
c)	100°	$\frac{5\pi}{9}$	$\frac{5\pi}{9} \cdot \frac{180}{\pi} = 100^\circ$
d)	-205°	$-\frac{41\pi}{36}$	$-205 \cdot \frac{\pi}{180} = -\frac{41\pi}{36}$

15.5

4. A Ferris wheel has a radius of 25 feet and its center is 30 feet off the ground. The Ferris wheel turns at a constant speed, makes one complete turn every 60 seconds, and moves in a counterclockwise direction.

   a) Write a function that would show the vertical height off the ground of a person riding on the Ferris wheel. [H(t) represents the height off the ground, t represents the number of seconds past the 3 o'clock position]. (3 points)

$h(t) = 25(\sin[360/60 \cdot t]) + 30$

-1/2 ? resolves to 6 (angular speed)

Should be accepted, because you should see where every variable goes - this is what you need to do w/o re doing

   b) What would be the height of a person after 13 seconds. Explain how you determined your answer. (2 points)

round hundredth

I plugged in 13 as t in the above formula and solved. $25(\sin[360/60 \cdot 13]) + 30$ $= 54.45$ ft

5. Find the complement and supplement of each angle below. Give your answer in the same form as the given angle. (2 pts. each)

   a) 62°

   complement: 28°

   supplement: 118°

   b) $\frac{3\pi}{10}$

   $\frac{5\pi}{10} - \frac{3\pi}{10} = \frac{2\pi}{10} = \frac{\pi}{5}$

   $\frac{10\pi}{10} - \frac{3\pi}{10} = \frac{7\pi}{10}$

   complement: $\frac{\pi}{5}$

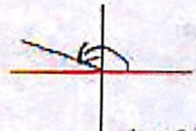
   supplement: $\frac{7\pi}{10}$

6. **Draw** a diagram of the angle given below. (1 pt.)
Give a positive and a negative angle which is coterminal to the given angle. Give your answer in the same form as the given angle. All answers must be between −720° and 720° or -4π and 4π. (2 pts. each) show direction

   a) −236°

   Coterminal <: 124°, −596°

   b) $\frac{5\pi}{6}$



   $\frac{5\pi}{6} + \frac{12\pi}{6} = \frac{17\pi}{6}$

   $\frac{5\pi}{6} - \frac{12\pi}{6} = \frac{-7\pi}{6}$

   Coterminal <: $\frac{17\pi}{6}$, $\frac{-7\pi}{6}$

14/15

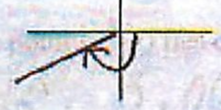
7. **Draw a diagram of the angle given below. (1 pt.)**
Find the reference angle of the given angle (1 pt.)

Show direction

a) $\frac{12\pi}{7}$ $\qquad \frac{14\pi}{7} - \frac{12\pi}{7} = \frac{2\pi}{7}$

Reference <: $\frac{2\pi}{7}$

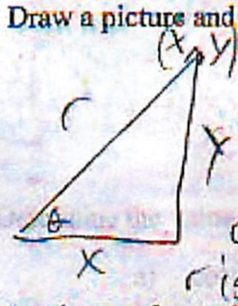
b) -152° $\qquad 180 - 152 = 28^\circ$



Reference <: 28°

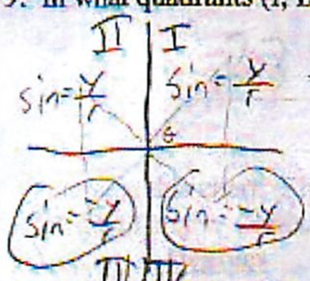
8. Draw a picture and explain why the $\cos\theta = \frac{x}{r}$. (3 pts.)

$\frac{adj}{hyp}$ correspond to $\frac{x}{r}$



Cosine is the adjacent over the hypotenuse, In this example, as on a grid the horizontal distance is the x coordinate while the r is the hypotenuse. Looking at the diagram you can see how?? cosine (x/r)

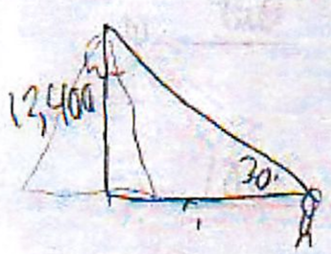
9. In what quadrants (I, II, III, IV) is the value of sine negative? Explain your answer. (2 pts.)



Because sin is $\frac{y}{r}$, and r is always positive you must look at the values where the y is negitive, this is in quadrant's III and IV

always by defintion

10. Mt. Fuji in Japan is approximately 12,400 feet high. A trigonometry student, several miles away, notes that the angle between the level ground and the top of the mountain is 30 degrees. Estimate the distance of the student to the mountain to the nearest foot. (2 pts.)



$\tan(30) = \frac{12400}{?}$

$\frac{.5774(?)}{.5774} = \frac{12400}{.5774}$

$? = \boxed{21{,}477 \text{ ft}}$ however if the mountains hight is only approximate, you would not be able to measure within a foot of accuracy.

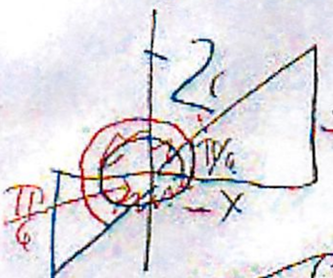
4

Non-calculator Section: 12/54 pts. Name Michael Plasmeier

WITHOUT USING YOUR CALCULATOR, evaluate the following expressions. Answers must be exact. ALL WORK MUST BE SHOWN!! Reduce all fractions completely (if necessary) and rationalize the denominators (if necessary). Use proper notation!! (3 pts. each)

1) $\sec\left(\frac{7\pi}{6}\right)$ $\frac{6\pi}{6}$ is only $\frac{1}{2}$ way around not one around

$\sec\frac{7\pi}{6} - \frac{6\pi}{6} = \sec\frac{\pi}{6} = \frac{r}{-x} = \frac{2}{-\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$



$\boxed{\frac{-2\sqrt{3}}{3}}$

I know that, but brain lapse

2) $\csc 315°$

$360 - 315 = 45°$

$\csc 45 = \frac{r}{y} = \frac{\sqrt{2}}{-1}$

$-\sqrt{2}$

3) $\sin\left(\frac{3\pi}{2}\right)$

$\frac{3\pi}{2} - \frac{2\pi}{2} = \frac{\pi}{2}$ yeah y

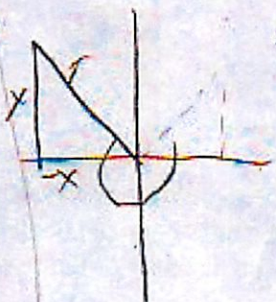
$\sin\frac{3\pi}{2} = \frac{\cancel{x}}{r} = \frac{\cancel{0}}{1} = \frac{-1}{1}$

y/r -1

4) $\tan\left(\frac{-4\pi}{3}\right)$

$-\frac{4\pi}{3} - \frac{3\pi}{3} = \frac{\pi}{3}$

$\tan\frac{\pi}{3} = \frac{y}{-x} = \frac{\sqrt{3}}{-1}$

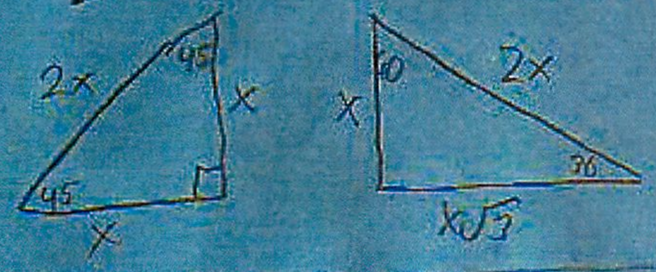


$-\sqrt{3}$

8.5

IAG 4
HOT SHEET

Name *Michael Plasrila*
Date *1/31/07*
Unit *High Dive - Trig*



Cosecant = $\frac{hyp}{opp}$
Secant = $\frac{hyp}{adj}$
Cotangent = $\frac{adj}{opp}$

sin = $\frac{opp}{hyp}$
cos = $\frac{adj}{hyp}$
tan = $\frac{adj}{opp}$

Reciprocal

$\sin\theta = \frac{1}{\csc\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

$\cos\theta = \frac{1}{\sec\theta}$ $\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{1}{\cot\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

Tangent + Cotangent

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean

$(\sin\theta)^2 + (\cos\theta)^2 = 1$

$1 + (\tan\theta)^2 = (\sec\theta)^2$

$1 + (\cot\theta)^2 = (\csc\theta)^2$

Vertical Height off Ground

$h = 50(\sin 9t) + 65$

$h(t) = r\left(\sin\left[\text{angular speed} \cdot t\right]\right) + \text{height center}$

Reference Angle

$\sin\theta = \sin\theta_R$

$\sin\theta = \sin(360 + \theta)$

Cofunction Identity

$\sin\theta = \cos(90 - \theta)$

2

3

Radian Measure

$90° = \frac{\pi}{2}$

$-270° = -\frac{\pi}{2}$

$180° = \pi$

$-180° = -\pi$

$360° = 2\pi$

$-360° = -2\pi$

$-90° = -\frac{\pi}{2}$

$270° = \frac{3\pi}{2}$

Tangent is undefined at any coterminal angle of 90 + 270° (pos and neg)

Radian Conversion

Degree measure $\cdot \frac{\pi}{180}$ = radian measure

Radian measure $\cdot \frac{180}{\pi}$ = Degree measure

Horizontal Distance

$d = 50(\cos 9t)$

$d(t) = r\left(\cos\left[\text{angular speed} \cdot t\right]\right)$

?

$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$

$\csc\theta = \frac{r}{y}$

$\sec\theta = \frac{r}{x}$

$\cot\theta = \frac{x}{y}$

II: $\sin\theta$ = pos, $\cos\theta$ = neg	I: $\sin\theta$ = pos, $\cos\theta$ = pos
III: $\sin\theta$ = neg, $\cos\theta$ = neg	IV: $\sin\theta$ = neg, $\cos\theta$ = pos

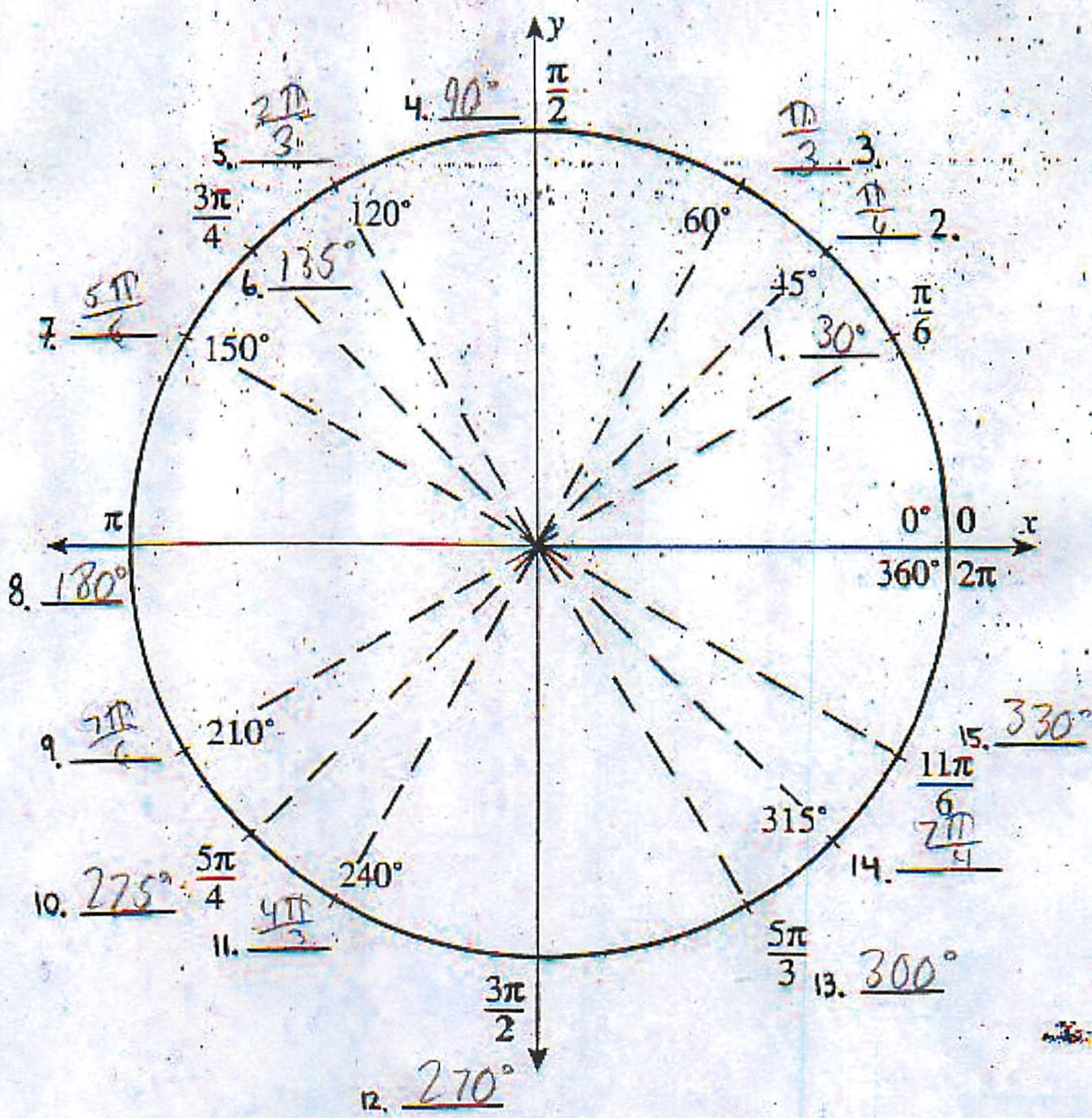
Reference Angle - θ_r

II: $180 - \theta$	I: θ
III: $\theta - 180$	IV: $360 - \theta$

Name Michael Plasket

Unit Circle in terms of Degrees & Radians
(in multiples of 30°, 45°, 60°)

Directions: Convert the radian measure to degrees and
Convert the given degree measure to radians.



13.4 Inverse Trig Functions Notes

2/15

Notation

$\sin^{-1}\theta = \arcsin\theta$ $\quad$ $\csc^{-1}\theta = \operatorname{arccsc}\theta$

$\cos^{-1}\theta = \arccos\theta$ $\quad$ $\sec^{-1}\theta = \operatorname{arcsec}\theta$

$\tan^{-1}\theta = \arctan\theta$ $\quad$ $\cot^{-1}\theta = \operatorname{arccot}\theta$

can only be 1 y value for every x value →

If $\sin^{-1}(a) = \theta$ where $-1 \le a \le 1$,
then $\sin\theta = a$, where $-90° \le \theta \le 90°$
- 1st + 4th quad

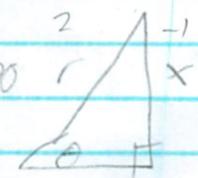
If $\cos^{-1}(a) = \theta$ where $-1 \le a \le 1$,
then $\cos\theta = a$ where $0° \le \theta \le 180°$
- 1st + 2nd Quad

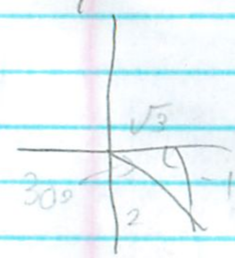
If $\tan^{-1}(a) = \theta$ where a = any real #
then $\tan\theta = a$ where $-90° < \theta < 90°$
- 1st + 4th quad

Experments (no calc) 2/16

$\sin^{-1}\left(-\frac{1}{2}\right) = \cancel{30°}$ in the 4th b/c neg

30,60,90

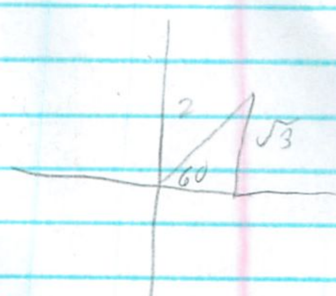




$\theta = 330$

why → so say → $-30°$ or $-\frac{\pi}{6}$
→ $-90 \le \theta\ 90°$

$\arcsin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60°$ or $\frac{\pi}{3}$



$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \cos^{-1} = \left(\frac{-1}{\sqrt{2}}\right)$

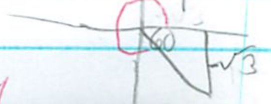
say $135°$ 1st or $\frac{3\pi}{4}$

~~$45°$ $\frac{\pi}{4}$~~ ← don't need to reduce

$\tan^{-1}(-\sqrt{3}) = -\frac{\sqrt{3}}{1}$

$-60°$ $\frac{\pi}{3}$

say $300°$ but its actually

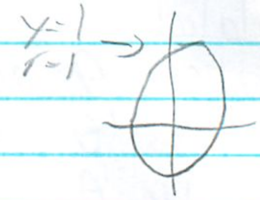


Special Case

$\sin^{-1}(1) = \frac{y}{r} = \frac{1}{1}$

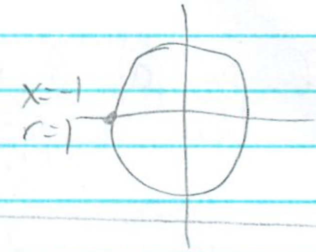
$= 90^\circ$ or $\frac{\pi}{2}$

Unit circle



$\cos^{-1}(-1) = \frac{x}{r} = \frac{-1}{1}$

180° or π



13.4 Inverse HW Trig Practice

2/16

p707

Angle?

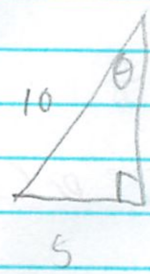
5. 

$\tan\theta = \frac{4}{6}$

$\tan\theta^{-1}(4/6) = 33.6900$

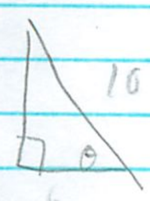
~~$\times \frac{\pi}{180} = .18716\pi$~~

0.588 radians

7. 

$\sin\theta = \frac{5}{10}$

$\sin^{-1}(5/10) = 30 - \frac{\pi}{6}$

9. 

$\cos\theta = \frac{6}{10}$

$\cos^{-1}(6/10) = 53.1301$

~~$\cdot \frac{\pi}{180} = .29516\pi$~~

.927

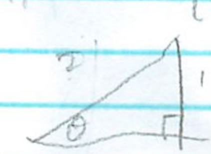
13. $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

C

$\theta = 45°$

$\frac{\pi}{4}$

14. $\sin^{-1}\left(\frac{1}{2}\right)$



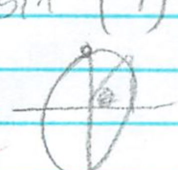
~~$\theta = 60°$~~

~~$\frac{\pi}{3}$~~

? 30°

15. $\sin^{-1}\left(\frac{1}{1}\right)$

C

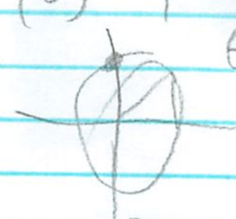


$\theta = 90°$

$\frac{\pi}{2}$

16. $\cos^{-1}(0) = \frac{0}{1}$

C

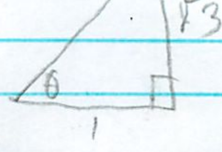


$\theta = 90°$

$\frac{\pi}{2}$

17. $\tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

C



$\theta = 60°$

$\frac{\pi}{3}$

18. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{1}{\sqrt{3}}$

C

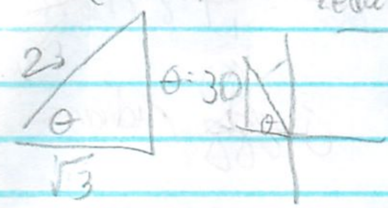
$\theta = 30°$

$\frac{\pi}{6}$

I convert badly — am I doing something wrong — need to flip?

19. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\cancel{\cos^{-1}\left(\frac{1.5}{\sqrt{3}}\right)}$ doesn't need reducing

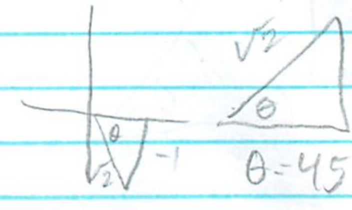


$\theta = 150°$

$\cancel{\frac{6\pi}{5}}$ $\frac{5\pi}{6}$

20. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

$\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

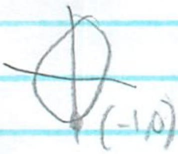


$\theta = 315$

$\theta = -45°$

or $-\frac{\pi}{4}$

21. $\sin^{-1}(-1) = \frac{-1}{1}$



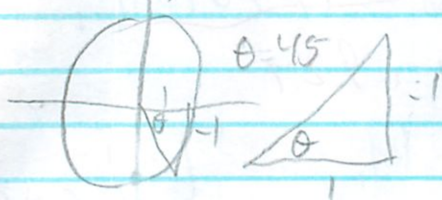
$\theta = 270°$

$\theta = -90°$ or $\frac{-\pi}{2}$

22. $\tan^{-1}(0) = \frac{0}{0}$

$\theta = 0°$ or 0 radians

23. $\tan^{-1}(-1) = \frac{-1}{1}$



$\theta = 315°$

$\theta = -45°$

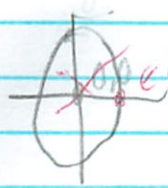
or $-\frac{\pi}{4}$

2/20 Warmup

2/20

$\tan^{-1}(0) = \frac{y}{x} = 0°$

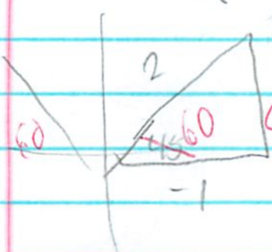
still



do at (1,0)

give radians

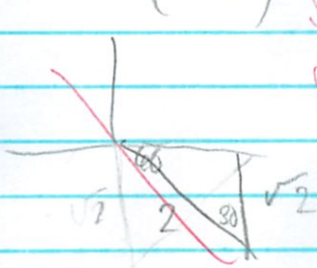
$\cos^{-1}\left(\frac{-1}{2}\right) = \frac{x}{r} = \cancel{135°}\ 120°$ or $\frac{2\pi}{3}$



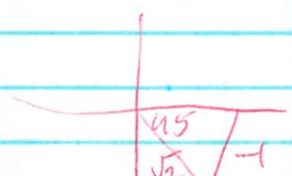
would be $\sqrt{2}$ not 2
lapsed on special triangles

$\sin^{-1}\left(\frac{-\sqrt{2}}{2}\right) = \frac{y}{r} = 3\cancel{0}0°$ or $\cancel{\frac{5\pi}{3}}$

down convert $\frac{-1}{\sqrt{2}}$



so $315°$ or $\frac{7\pi}{4}$



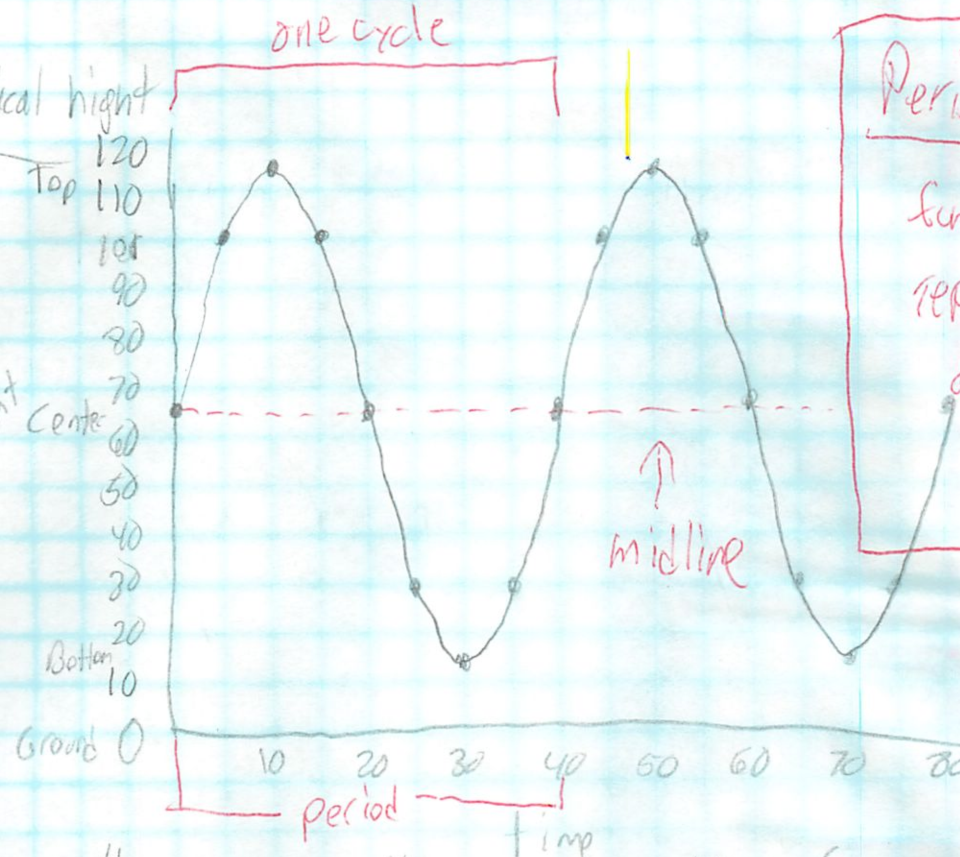
but need $-90° \leq \theta \leq 90$

so $-45°$ or $-\frac{\pi}{4}$

Graphing the Ferris Wheel (4)

2/16

t	h(t) vertical hight
0	65
10	115
20	60
30	15
40	65
50	115
60	65
70	15
80	65



Periodic Function

function which repeats its value at regular intervals

2a, If the radius was smaller, the amplitude (or hight from center or midline would be smaller) max lower min higher

b If it would go faster, the graph would be ~~more~~ squashed together. (shorter period = smaller cycle)

c If the center of the wheel was at the ground, the mid line or the graph would be the x axis

Amplitude	Period	Vertical shift
distance from the midline to the max/min (positive only) (radius of wheel)	time it takes for one cycle to occur (related to angle speed)	the up or down movement of the entire graph

Graph Variations (5)

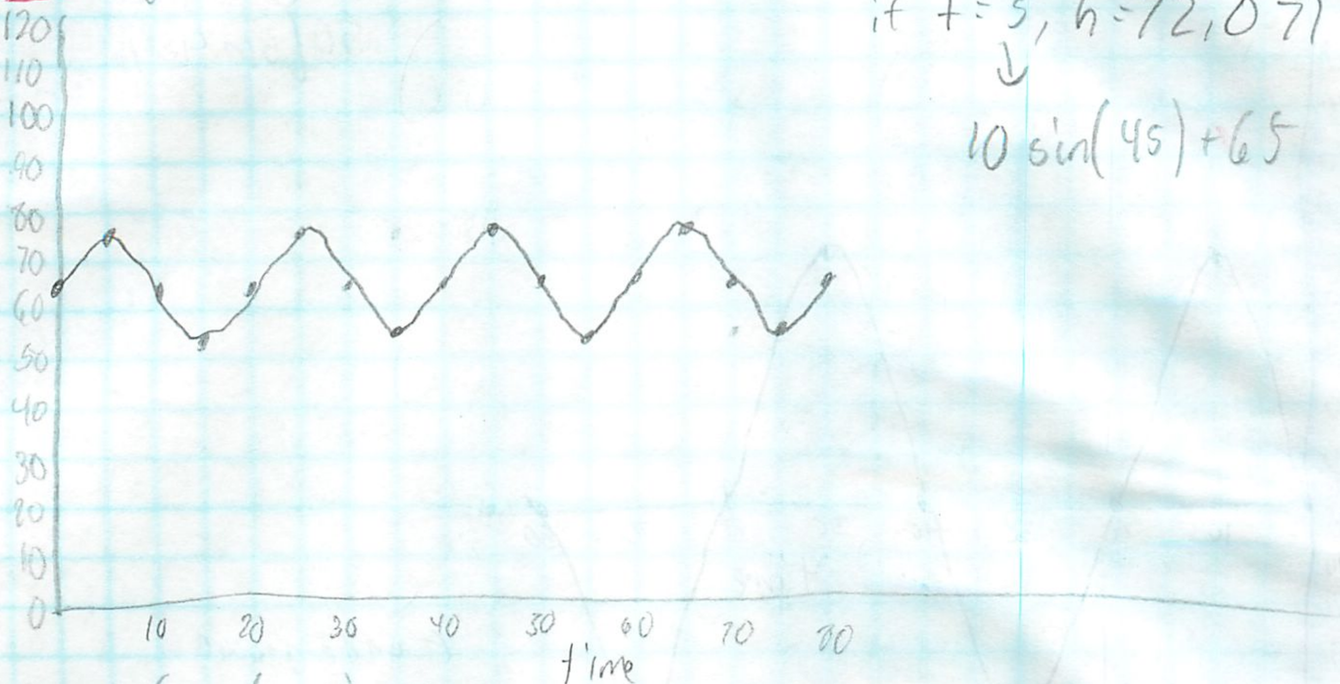
2/16

1. $10\sin(9t)+65$ - smaller radius

if $t=5$, $h=72.071$

$\downarrow$

$10\sin(45)+65$



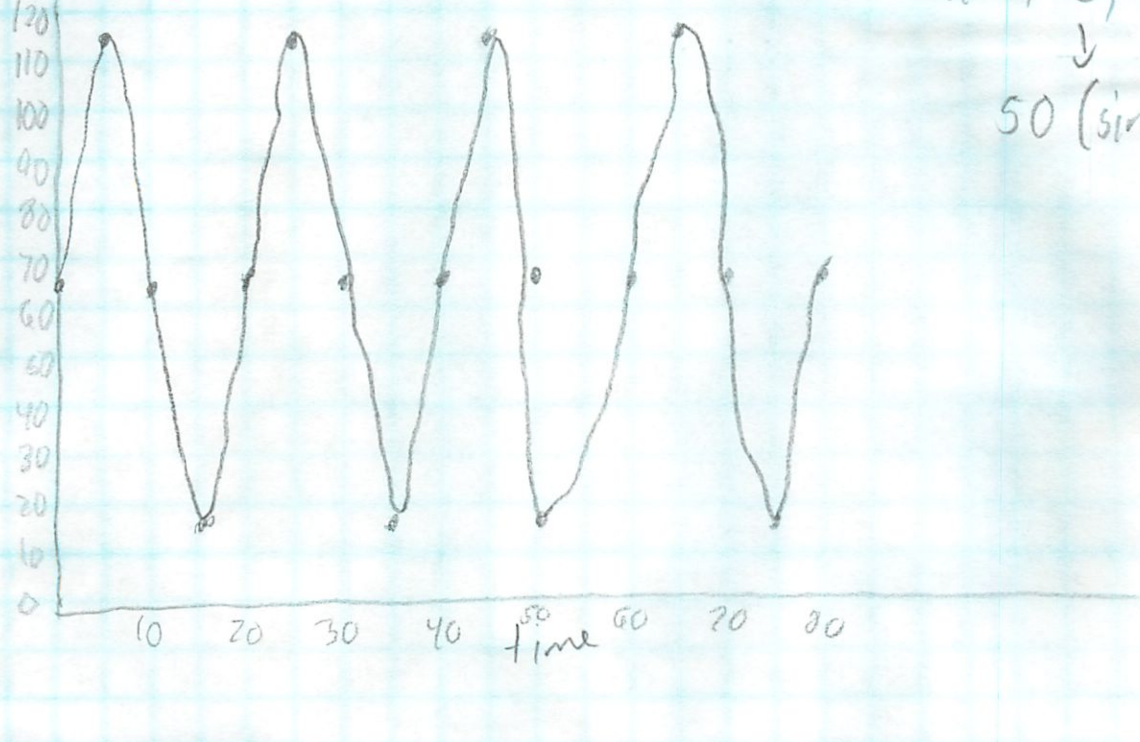
2. $50\sin(360/20\,t)+65$ - faster around

$50\sin(18t)+65$

if $t=5$, $h=115$

$\downarrow$

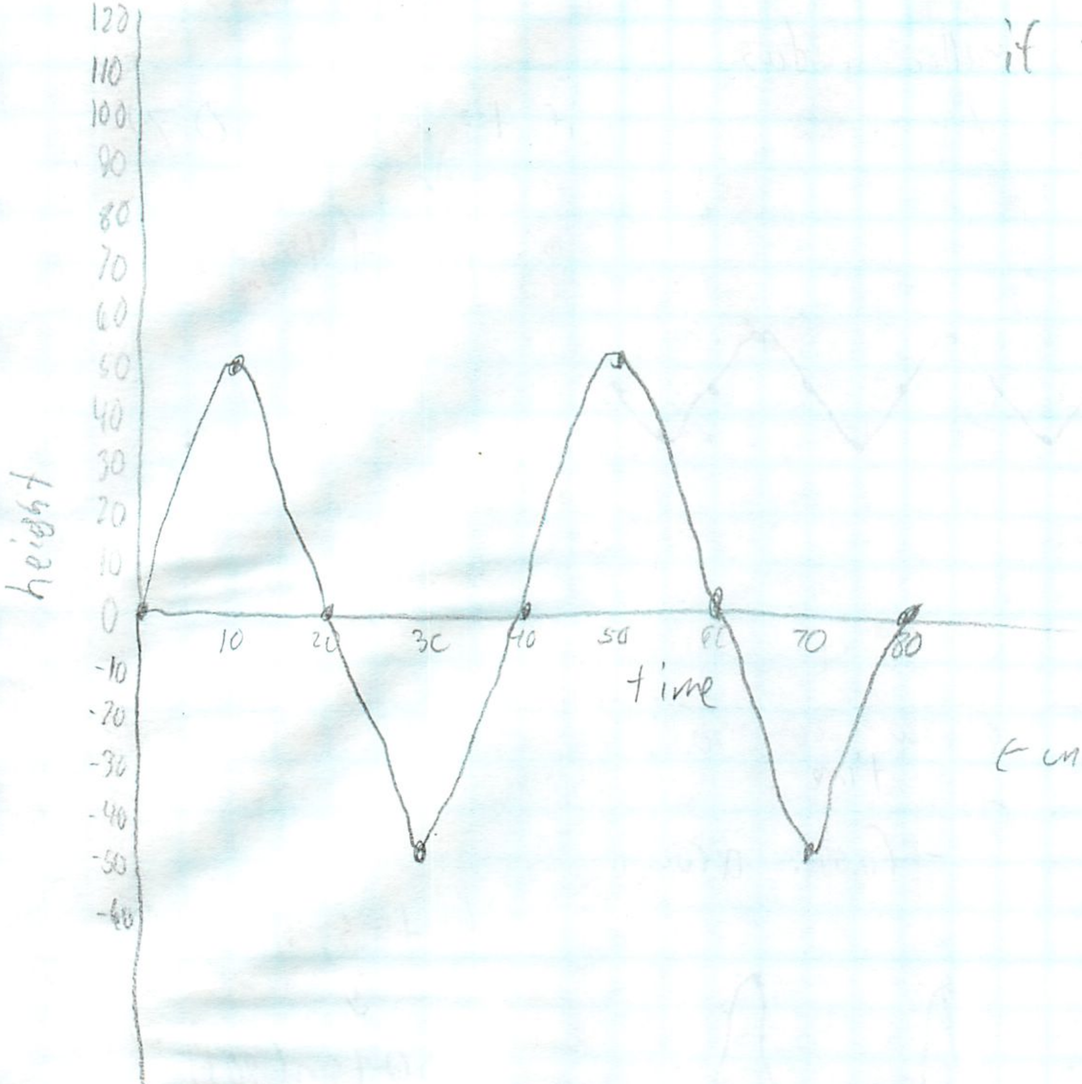
$50(\sin(90))+65$



3. $50(\sin 9t) \underline{+0}$ - center at ground

if $t=5$, $h=35.3553$

$50(\sin 45)$

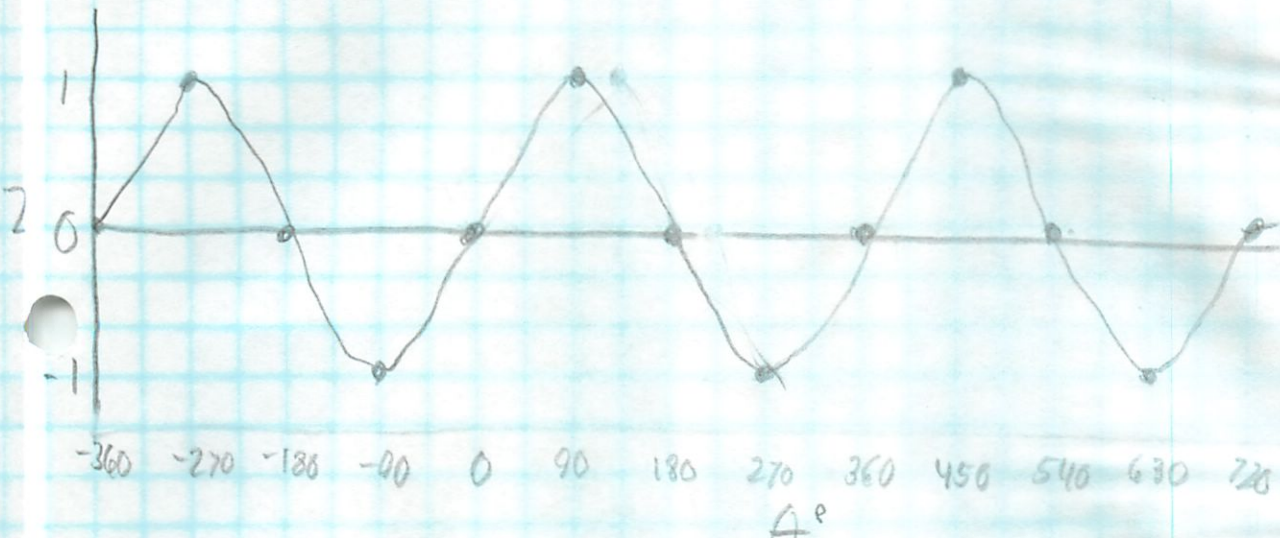


← underground

"Plain" Sine Graph p23 2/10

θ	-360	-270	-180	-90	0	90	180	270	360	450
2	0	1	0	-1	0	1	0	-1	0	1

θ	540	630	720
2	0	-1	0

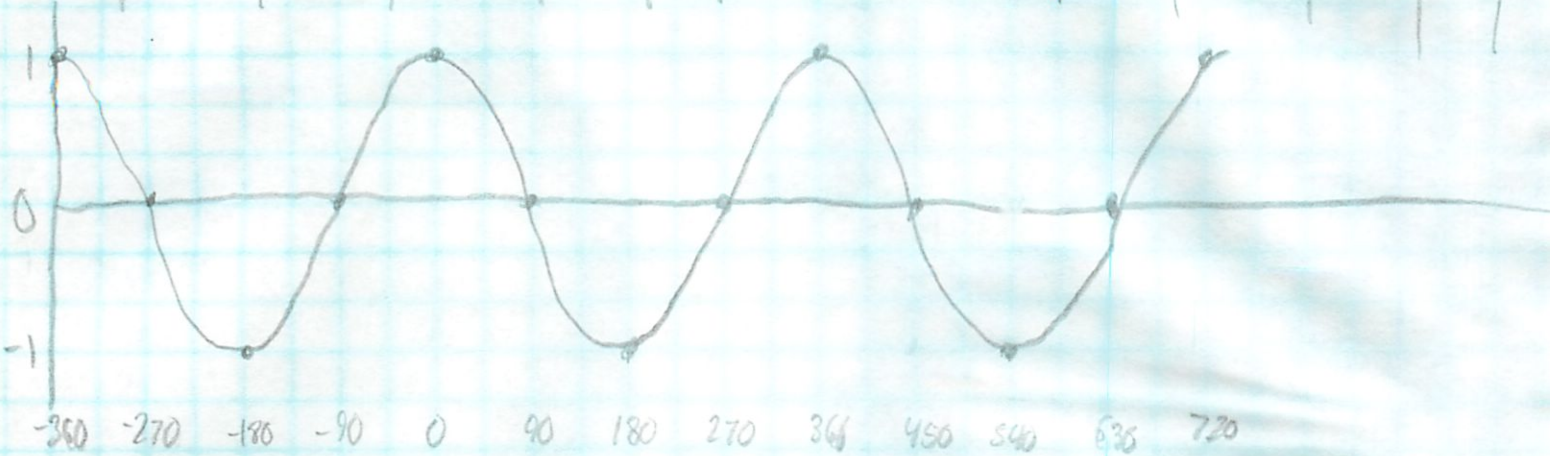


2. Amplitude = 1
3. Period = 360° - ya just keep going around the wheel and going 2x arand is the same as 1st time around
4. X-intercepts = positive and negitive multipals of 180°
5. X max - positive + negitive multiples following the pattern -270, 90, 270° co-terminal 90°
   X min - positive + negitive multiples following the pattern -90, 270, 630° coterminals 270°
6. radius = 1 unit
   center at ground
   ~~Spin time = 360 sec a cycle~~ or 2π don't know

What's Your Cosine 15 2/21

1.

θ	-360	-270	-180	-90	0	90	180	270	360	450	540	630	720
z	1	0	-1	0	1	0	-1	0	1	0	-1	0	1



a. amplitude = 1

b = period = 360° or 2π - it repeats over and over again as wheel spins

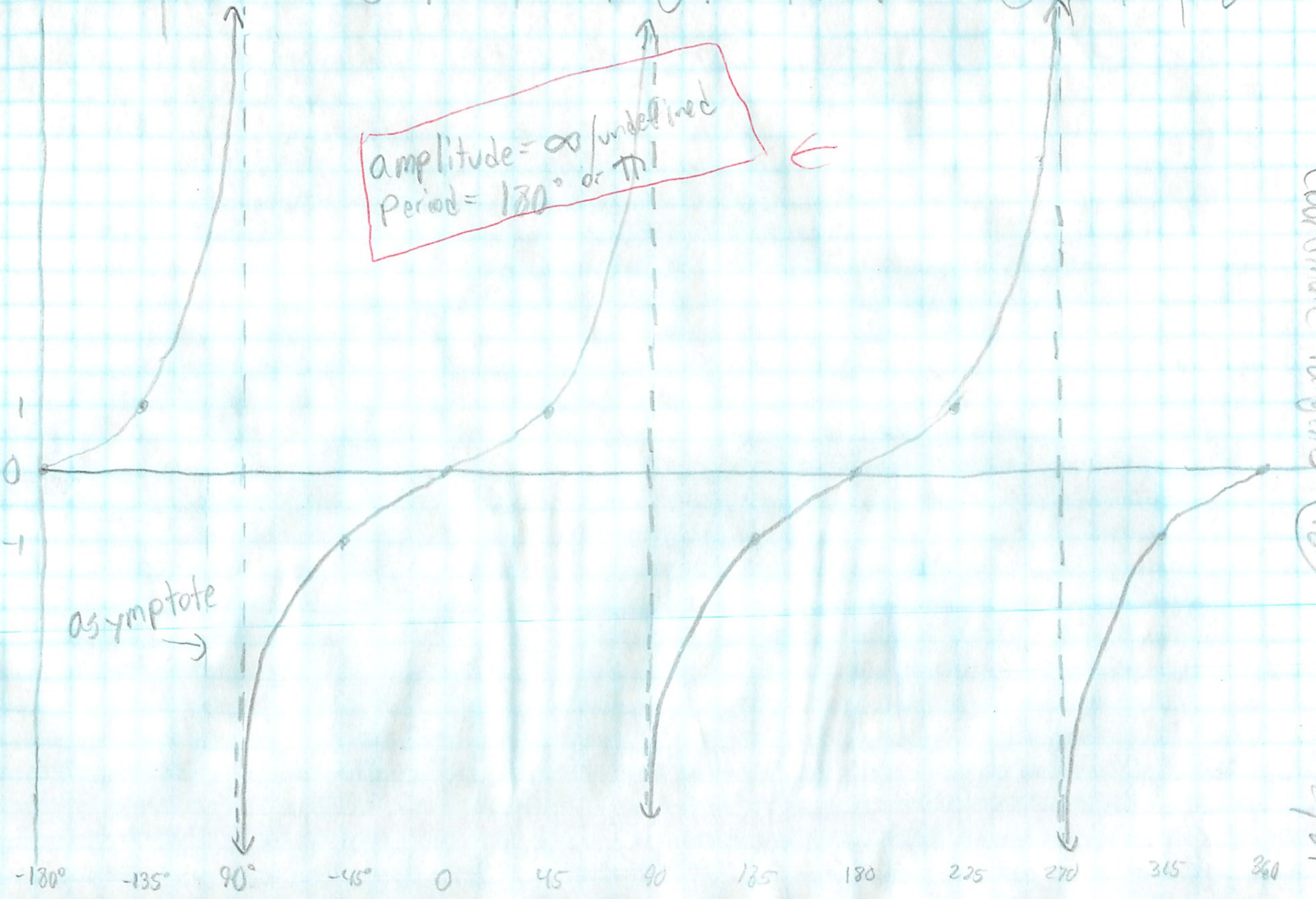
c - x intercepts = -270, -90, 90, 270, 450, 630

d - x max = -360, 0, 360, 720

x min = -180, 180, 540

4. tan θ

θ	-180	-135	-90	-45	0	45	90	135	180	225	270	315	360
z	0	1	E	-1	0	1	E	-1	0	1	E	-1	0

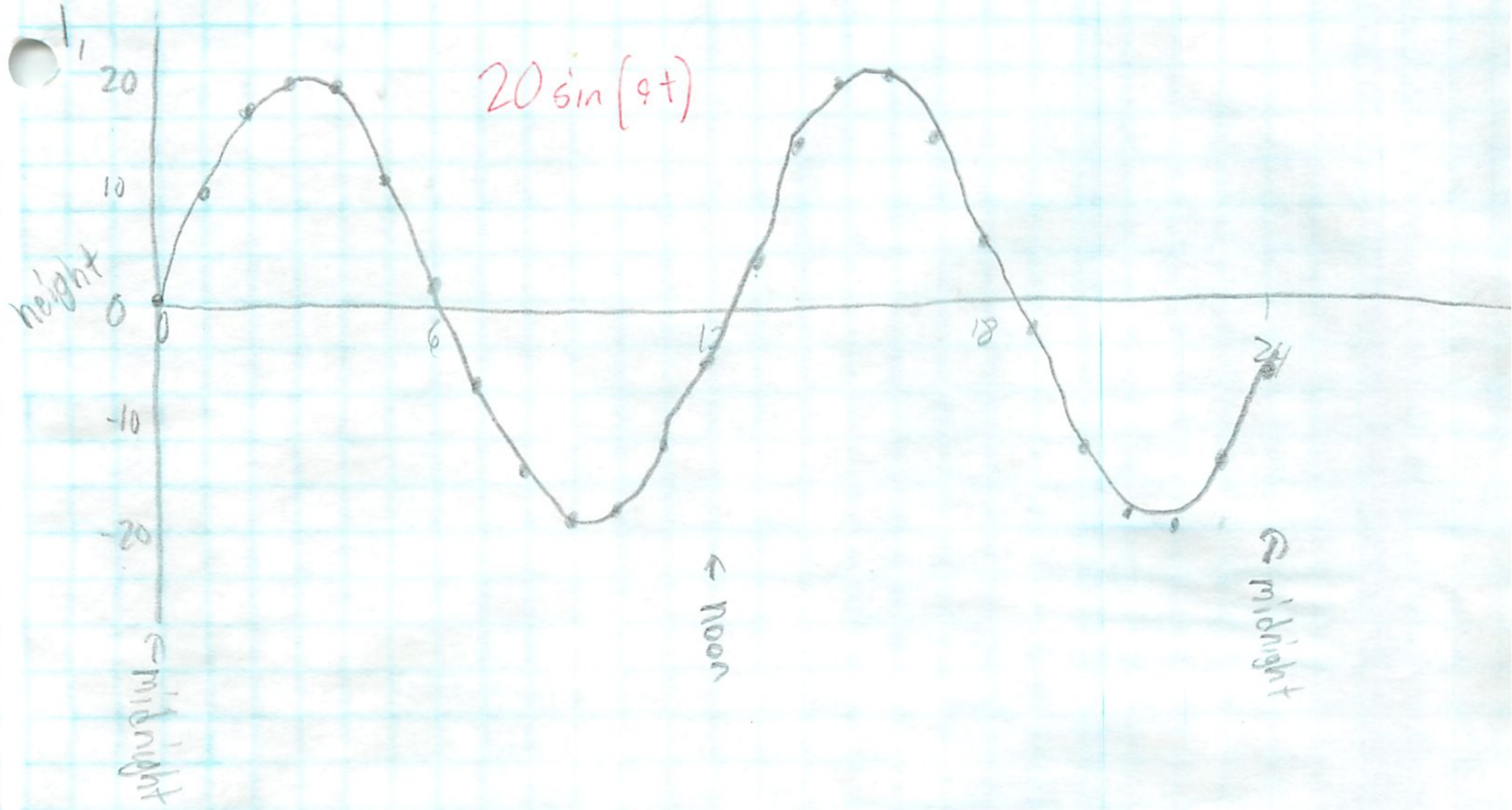


Coordinate tangents

18

2/21

Sand Castles 6 2/21



2.a High tide is 20 ft above normal
Low " " " " below "

3. She has ~6 hours at midline

calc zeros
6.2068
12.4137

6.2069 hours
~or~
6 hrs 12 min
24.84 sec

4. She has ~3.5 hrs at -10 ft below midline

5. About ~~18 ft~~ below midline will get her 2 hrs of time
— thats 1.79 hrs time

17.5 ft gets her ~1.99 hrs of time
8.31 — 10.30

better choice/way
~9.31
find center (min) then go 1 hr before + after and see what that point is - 8.31 and 10.31 which is 17.49 ft

Calc intersection exactly
7.24 = 4.14 hrs
11.38 4 hrs 8 min

Name Michael Plasmeier Date 2/22

totally wrong

See next page

In Which Quadrants. . . ?

1. On your last quiz, you were asked. . . "***In which quadrants*** (I, II, III, IV) is the value of sine negative?" Explain your answer below.

Sin - 1 + 4
Cos - 1 + 2
tan - 1 + 4

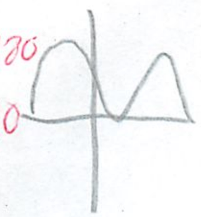
Sin is in 1st ~~2nd~~ 4th quad - so its negitive in #2 4

mem error

2. What about cosine? ***In which quadrants*** (I, II, III, IV) is the value of cosine negative? Explain your answer below and support it with a picture.

Sin $-90 \le \theta \le 90$
Cos $0 \le \theta \le 180$
tan $-90 \le \theta \le 90$

#2



3. What about tangent? ***In which quadrants*** (I, II, III, IV) is the value of tangent negative? Explain your answer below and support it with a picture.

4

4. Now that you've made conclusions about sine, cosine, and tangent, where (***in which quadrants***) will cosecant, secant, and cotangent be negative?

Cos - #1
Sec - #1
tan - #1

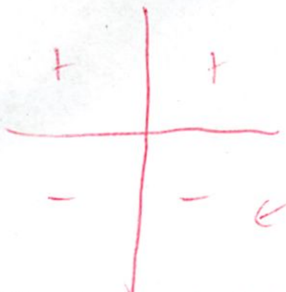
still the same

#4
#2
#4

Name ____________________ 1e 20 Date __________

In Which Quadrants. . . ?

1. On your last quiz, you were asked. . . "***In which quadrants*** (I, II, III, IV) is the value of sine negative?" Explain your answer below.

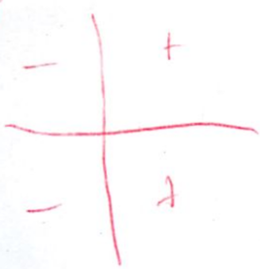


$\frac{y}{r}$ ← always (+)

← y neg

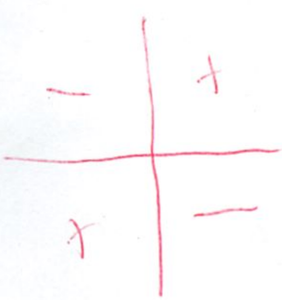
2. What about cosine? ***In which quadrants*** (I, II, III, IV) is the value of cosine negative? Explain your answer below and support it with a picture.

x neg



$\frac{x}{r}$ ← always (+)

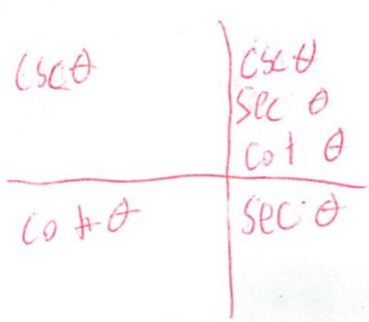
3. What about tangent? ***In which quadrants*** (I, II, III, IV) is the value of tangent negative? Explain your answer below and support it with a picture.



$\frac{y}{x}$ ← if both (−) its pos

4. Now that you've made conclusions about sine, cosine, and tangent, where (***in which quadrants***) will cosecant, secant, and cotangent be negative?

Same places



14.4 Solving Trig Equations

Notes

2/22

$2\sin\theta - 1 = 0$ Equation, not expression

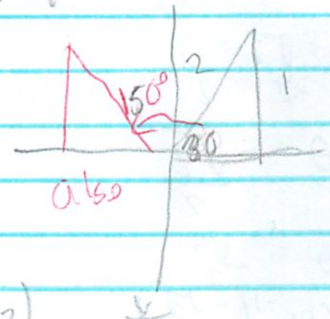
$+1 \quad +1$

$\dfrac{2\sin\theta}{2} = \dfrac{1}{2}$

$\sin\theta = \frac{1}{2}$

$-1 \quad -1$

$\theta = \sin^{-1}(1/2)$

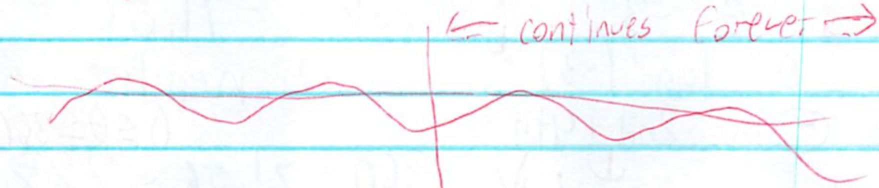


answers in 1+2nd quad

$\theta = 30°$ and $\frac{\pi}{6}$

also $\theta =$ ~~30°~~ $150°$ or $\frac{5\pi}{6}$

all possible values needed with equations



x intercepts = 30, 150, 390, 510 and so on...

Co terminal

recursive function

$30 + 360n$

$\frac{\pi}{6} + 2\pi n$ where n = # rotations

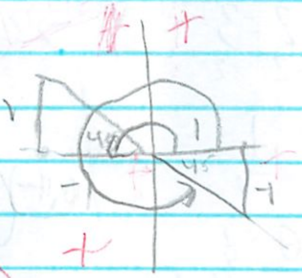
$7\tan\theta + 9 = 2$

$-9 \quad -9$

$\dfrac{7\tan\theta}{7} = \dfrac{-7}{7}$

$\tan\theta = -1$

$\theta = \tan^{-1}(-1)$



$135° + 360n$

$\frac{3\pi}{4} + 2\pi n$

$315° + 360n$

$\frac{7\pi}{4} + 2\pi n$

$1/\sqrt{2}$

$-\frac{\sqrt{2}}{2} - \frac{1}{\sqrt{2}}$

$\cos\theta + \sqrt{2} = -\cos\theta$

$+\cos\theta \quad +\cos\theta$

$2\cos\theta + \sqrt{2} = 0$

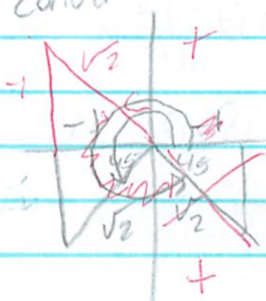
$-\sqrt{2} \quad -\sqrt{2}$

$\frac{2\cos\theta}{2} = \frac{-\sqrt{2}}{2}$

$\cos\theta = \frac{-\sqrt{2}}{2}$

$\theta = \cos^{-1}\left(\frac{-\sqrt{2}}{2}\right)$

convert



cos + in 1 + 4

$225 + 360n$

$\frac{5\pi}{4} + 2\pi n$

~~$315 + 360n$~~

~~$\frac{7\pi}{4} + 2\pi n$~~

$135 + 360n$

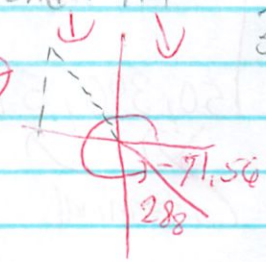
$\frac{3\pi}{4} + 2\pi n$

$\tan\theta = -3$

$\theta = \tan^{-1}(-3)$

tan − in 2nd + 4th

also in 2nd



calc = −71.56

negitive doesn't work

$0 \le \theta \le 360°$

$360 - 71.56 = 288.43$

$\cdot\left(\frac{\pi}{180}\right) = 5.03$ radians

type all in calc

$180 - 71.56 = 108.44$

or 1.89 radians

$9\cos\theta + 2 = 3$

$-2 \quad -2$

$\frac{9\cos\theta}{9} = \frac{1}{9}$

$\cos\theta = \frac{1}{9}$

$\theta = \cos^{-1}(1/9)$

cos + in 1 + 4

calc : 83.6206° or 1.46 radians

$360 - 83.6206 =$

$276.379°$ or 4.82 r

$$\frac{4\sec\theta}{4} = \frac{5}{4}$$

$\sec\theta = \frac{5}{4}$

yes, flip it

$\theta = \sec^{-1}(5/4)$

$\theta = \cos(4/5)$ → 36.87 or .6435 r

$360 - 36.87 = 323.1301°$ or

5.64 radians

sec (+): 1st + 4th

√: Plug in:

$4(1/\cos(36.87))$ → should = 5

√

degree answer

Identifying Amplitude, Period, Phase Shift + Vertical Shift

2/23

Review

$y = 4\cos(x) + 10$
amp = 4
period = 360° or 2π
vertical: ↑10

$y = -7\sin(x) - 5$ always positive
amp = 7
period = 360° or 2π
vertical shift = ↓5

$y = 2\sin x - 6$
amp = 2
period = 360° or 2π
v. shift = ↓6

New

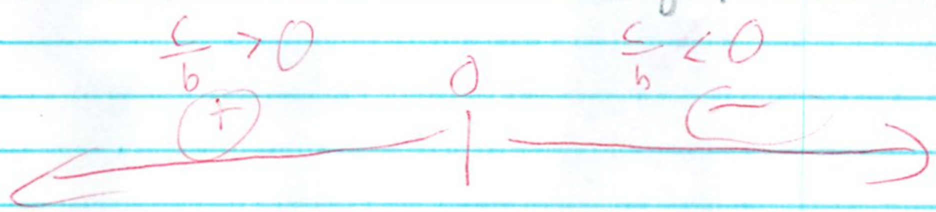
$y = \cos(x) \rightarrow$ period = 360° or 2π
$y = \cos(2x) \rightarrow$ period = 180° or π
$y = \cos(4x) \rightarrow$ period = 90° or $\frac{\pi}{2}$
$y = \cos(-2x) \rightarrow$ period = ~~-180° or $-\pi$~~ abs value
180° or π

$$\boxed{\text{all: } 360/(\text{x co-efficent}) \text{ abs}}$$

2/27

Phase Shift

The horizontal movement of the graph



if $(2x + 3\pi)$
take $\frac{3}{2} = \leftarrow 1.5$ left
or $\frac{c}{b}$
if $(cx + b)$

General Form

$y = a\,\text{trig}(bx + c) + d$

radius amplitude | sin cos | helps find period $\left(\frac{360}{|b|}\right)$ | helps find phase shift $\left(\frac{c}{b}\right)$ | vert shift

For the first marking period, you will be studying the concepts of linear equations. The content for each topic is located in your Algebra 2 Textbook. At the end of your studying of these topics, you should be prepared to do the following:

Linear Functions (1.3, 1.5, 1.6, 2.2)	Linear Functions (2.3, 2.4, 3.2)
▪ How to solve a linear equations ▪ Use linear equations to solve word problems ▪ How to solve a literal equation ▪ How to solve simple and compound inequalities ▪ How to find the slope of a line	▪ How to find the intercepts of a line ▪ How to find the slope-intercept form of an equation ▪ How to the write the equation of line ▪ How to use algebraic methods to solve linear systems

For the assignments below:

- **Write the section and problem #'s at the top of your paper**
- **Copy the problem**
- ***Show all work***
- **Each section should be a new page**
- **Neatness and organization count**

Assignments are for the following sections:

- Section 1.3: p. 21 - #27, 31, 35, 39, 43
- Section 1.5: p. 35 - #12, 15, 17, 19, 21
- Section 1.6: p. 41 - #21, 24, 23, 27, 29, 31, 37, 40
- Section 2.2: p. 74 - #10, 13, 14, 16, 19, 25, 41
- Section 2.3: p. 81 - #9, 11, 17, 18, 31, 35, 43
- Section 2.4: p. 90 - #11, 13, 17, 19, 30
- Section 3.2: p. 134 - #9, 11, 15, 18, 19, 21, 26, 27

Good Luck.

This assignment is due on: 2/22/07

Name: Michael Plasmeier

Score: 18/23

Section 1.3

Solving Linear Equations 2/4

Independent Study: Linear Functions

p21 - #27, 31, 35, 39, 43

Solve + ✓

27. $\frac{1}{2}x - 12 = 4$

$+12 \quad +12$

$\frac{1}{2}x = 16$

$\frac{\frac{1}{2}x}{\frac{1}{2}} = \frac{16}{\frac{1}{2}}$

$x = 32$

Check: $\frac{1}{2}(32) - 12 = 4$

$4 = 4$ ✓

31. $3(x-2) = 6(5+x)$

$3x - 6 = 30 + 6x$

$+6 \quad +6$

$3x = 36 + 6x$

$-6x \quad -6x$

$-3x = 36$

$\frac{-3x}{-3} = \frac{36}{-3}$

$x = -12$

Check: $3(-12-2) = 6(5+-12)$

$-42 = -42$ ✓

35. $3(x-2) + 6 = 4(2-x)$

$3x - 6 + 6 = 8 - 4x$

$3x = 8 - 4x$ ✓

$+4x \quad +4x$

$7x = 8$

$\frac{7x}{7} = \frac{8}{7}$

$x = 1\frac{1}{7}$

Check: $3(1\frac{1}{7} - 2) + 6 = 4(2 - 1\frac{1}{7})$

$\frac{24}{7} = \frac{24}{7}$ ✓

39. $-(x+2) - 2x = -2(x+1)$

$-x - 2 - 2x = -2x + -2$

$-3x - 2 = -2x - 2$

$+2 \quad +2$

$-3x = -2x$

$x = 0$

can't be done

can't ✓

sometimes things work with 0

$+2x \quad +2x$

$-x = 0$

$x = 0$

43. You have 2 summer jobs

The first: 40 hrs/week @ \$6.25/hour

The second: \$5.50/hour for as many hours a week

If you earn \$316/week - how long do you work at your 2nd job

$316 = 6.25(40) + 5.50(x)$

$316 = 250 + 5.50(x)$

$-250 \quad -250$

$66 = 5.50(x)$

$\frac{66}{5.50} = \frac{5.50(x)}{5.50}$

$x = 12$

12 hours

Section 1.5
Literal Equations + Formulas

2/4

Independent Study: Linear Functions

p35 # 12, 15, 17, 19, 21

Solve for that variable

12. Perimeter of rectangle - solve for w

$P = 2l + 2w$

$-2l \quad -2l$

$\frac{2w}{2} = \frac{P - 2l}{2}$

$w = \frac{P - 2l}{2}$ Simplify "2"

$w = \frac{P}{2} - l$

15. Area trapezoid - solve for b_2

$A = \frac{1}{2}(b_1 + b_2)h$

$\cdot 2 \quad \cdot 2$

$\frac{2A}{2h} = \frac{(b_1 + b_2)2h}{2h}$

$\frac{2a}{h} = b_1 + b_2$

$-b_1 \quad -b_1$

$\frac{2a}{h} - b_1 = b_2$

Solve + Evaluate

17. Convert °F → °C - solve for f

$\frac{C}{5/9} = \frac{\frac{5}{9}(f - 32)}{5/9}$

$C/5/9 = f - 32$

$+32 \quad +32$

Simplify $\frac{C}{5/9} + 32 = f$

Swap $\frac{9}{5}C + 32 = f$

19. Investment at simple interest - solve for p

$A = P + Prt$ ~~p is common factor~~

$-Prt \quad -Prt$

$P = A - Prt$ $\quad p = \frac{A}{1 + rt}$

21. Area of ring $= 2\pi p w$

Solve for p. Find p given $A = 22\,cm^2$ + $w = 2\,cm$

$\frac{A}{2\pi w} = \frac{2\pi p w}{2\pi w}$

$p = \frac{A}{2\pi w}$

$p = \frac{22}{2\pi(2)}$

$p = \frac{22}{4\pi}$

$p = 1.750704$

Section 1.6

Solving Linear Inequalities 2/4

(S) whenever $x \div$ by neg number

Independent Study: Linear Functions
p41 #21, 23, 24, 27, 29, 31, 37, 40

Solve

21. $6t + 7 > 11$
$-7 \quad -7$
$6t > 4$
$\frac{6t}{6} > \frac{4}{6}$
$t > \frac{2}{3}$

23. $9 - k < 4$
$-9 \quad -9$
$-k < -5$
flip
$k > 5$

24. $3 - 2x \geq 15$
$-3 \quad -3$
$-2x \geq 12$
$\frac{-2x}{-2}$ (S) $\frac{12}{-2}$
$x \leq -6$

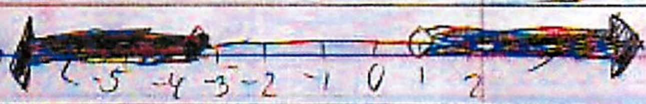
27. $-3 \leq x - 3 \leq 6$
$+3 \quad +3 \quad +3$
$0 \leq x \leq 9$

29. $4 \leq -3x - 1 \leq 9$
$+1 \quad +1 \quad +1$
$5 \leq -3x \leq 10$
$\frac{5}{-3} \quad \frac{}{-3} \quad \frac{}{-3}$
$-1\frac{2}{3} \leq x \leq -3\frac{1}{3}$

other way
$-\frac{10}{3} \leq x \leq -\frac{5}{3}$

Solve

31. $-1 < -2x + 1 \leq 5$ switch sign
$-1 \quad -1 \quad -1$
$-2 < -2x \leq 4$
$\frac{-2}{-2}$ (S) $\frac{-2x}{-2}$ (S) $\frac{4}{-2}$
$1 > x \geq -2$ } switch order
$-2 \geq x > 1$



Solve + sketch

37. $m - 3 > -2$
$+3 \quad +3$
$m > 1$

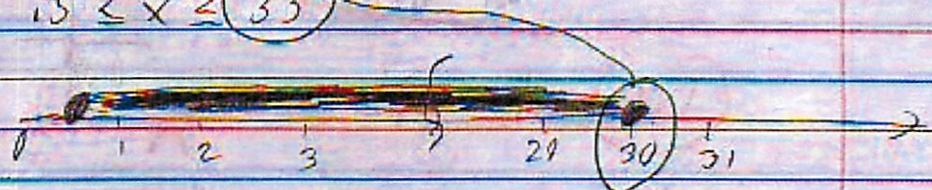
$m + \frac{1}{2} \leq -3$
$-\frac{1}{2} \quad -\frac{1}{2}$
$m \leq -3\frac{1}{2}$

Solve + sketch

40. Longest centipede: 33 cm
Shortest: .5 cm

opps

$.5 \leq x \leq 33$



Section 2.2

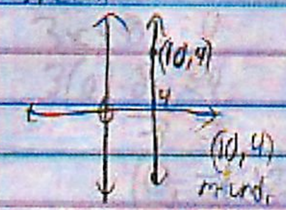
Slope and Rate of Change 2/4

Independent Study: Linear Functions

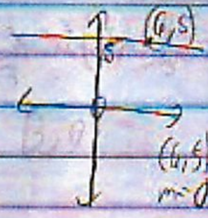
p 74 # 10, 13, 14, 16, 19, 25, 41

Sketch

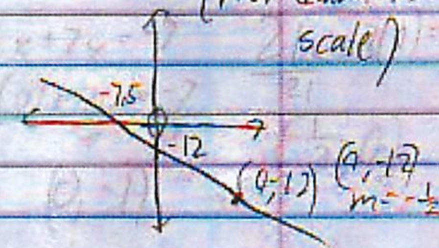
10.



13.

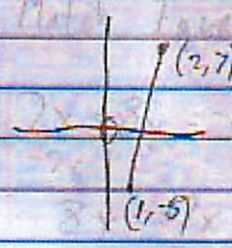


14. (Not drawn to scale)



Sketch + Find Slope

16.



(1,-5)(2,7)

$\frac{7 - -5}{2 - 1} = \frac{12}{1}$

slope = 12

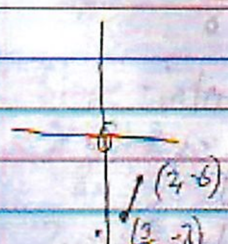
19.

(4,5)(-1,9)

$\frac{9 - 5}{-1 - 4} = \frac{4}{-5}$

slope = $-\frac{4}{5}$

25.



$(\frac{3}{4}, -8)(2, -5)$

$\frac{-8 - -5}{\frac{3}{4} - 2} = \frac{-3}{-1.25}$

slope = 2.4 or $\frac{12}{5}$

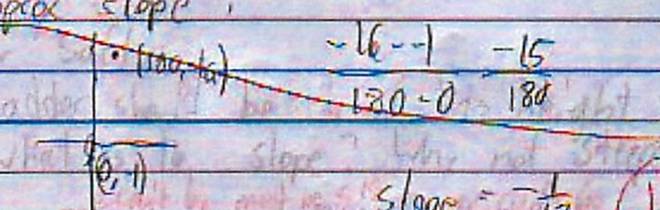
41. Leaning Tower was 180 ft tall

One side sunk 1 foot

Causing top 16 ft off center

Approx slope?

not on quiz



$\frac{-16 - -1}{180 - 0} = \frac{-15}{180}$

slope = $-\frac{1}{12}$

11.2

Section 2.3

Quick Graphs: Linear Equations 2/9

Independent Study: Linear Functions
p81 #9, 11, 17, 18, 31, 35, 43

Find x + y intercepts

9. $3x-8y=9$ $\quad 3(0)-8y=9$ $\quad \frac{-8y}{-8}=\frac{9}{-8}$ $\quad (0,-1.125)$

$\frac{3x}{3}-8(0)=\frac{9}{3}$ $\quad (3,0)$

11. $21x+7y=-7$ $\quad 21(0)+\frac{7y}{7}=\frac{-7}{7}$ $\quad (0,-1)$

$\frac{21x}{21}+7(0)=\frac{-7}{21}$ $\quad (-\frac{1}{3},0)$

Match Equation w/ graph

17. $2x+8y=-24$
$-2x \quad -2x$
$8y=-2x-24$
$y=\frac{-2x-24}{8}$ → calc + graph

Graph A matches

18. $3x-6y=18$
$-3x \quad -3x$
$-6y=-3x+18$
$y=\frac{-3x+18}{-6}$ → Calc + graph

Matches D

Slope-intercept form

31. $8x+2y=1$
$-8x \quad -8x$
$2y=-8x+1$
$y=\frac{-8x+1}{2}$
$y=-4x+\frac{1}{2}$
slope $=-4$
y int $=(0,\frac{1}{2})$

35. $-x+2y=-8$
$+x \quad +x$
$\frac{2y}{2}=\frac{x-8}{2}$
$y=\frac{x}{2}-4$
slope $=\frac{1}{2}$
y int $(0,-4)$

oh well.
not a big deal!

43. Ladder Safty

Ladder should be $\frac{1}{4}$ of its height from the wall
What is the slope? Why not steeper?

can't be must be < 12 more

$\frac{12-0}{0-3}$ slope $=4$ ← can't be

If it was steeper might tip backwards and fall on top of ya

Section 2.4

Writting Equations of Lines 2/4

Independent Study: Linear Functions

p90 # 11, 13, 17, 19, 30

Write equation

11. $(0,3)$ $m = -\frac{1}{2}$ 13. $(-8,1)$ $m = 10$

$y = -\frac{1}{2}x + 3$ $y = 10x$: $y = 80$ at -8 so $+10 +1$

$y = 10x + 81$

Write Equation

17. $(7,6)(10,5)$ 19. $(-9,9)(0,1)$

$\frac{6-5}{7-10} = \frac{1}{-3} = m$ copy error $\frac{9-1}{-9-0} = \frac{8}{-9}$

$y = -\frac{1}{3}$ at $7 = -2\frac{1}{3}$ so $+2\frac{1}{3} + 6$ $y = -\frac{8}{9}x + 1$

$y = -\frac{1}{3}x + 8\frac{1}{3}$ forgot x

$y = 3x - 15$

30. Running the IRS

Using chart, write linear model for adv. cost of running IRS

Let $t = 0$ be cost in 1980 and estimate cost in 1995

0, 2.3

8, 5

10, 5.5

stat edit → $y = .325x + 2.31$

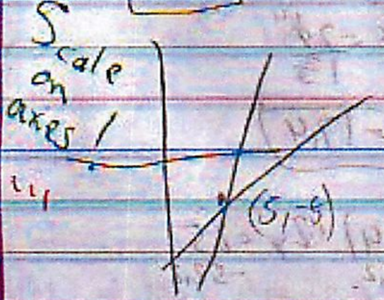
$y = .325(15) + 2.31$ ← in 1995

$y = 7.19$ billion \$

Sketch + Solve

19. $x - y = 10$
$3x - 2y = 25$
$x = 10 + y$
$3(10 + y) - 2y = 25$
$30 + 3y - 2y = 25$
$-30 \qquad -30$
$\boxed{y = -5}$
$x - (-5) = 10$
$-5 \qquad -5$
$\boxed{x = 5}$

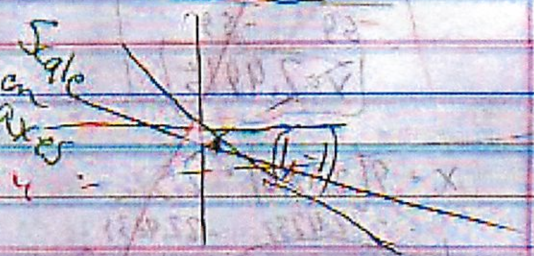
Scale on axes!



21. $4x + 3y = 1$
$-3x + 6y = 3 \;)\cdot 2$
$8x + 6y = 2$
$5x = 5$
$\boxed{x = 1}$
$4(1) + 3y = 1$
$-4 \qquad -4$
$3y = -3$
$\boxed{y = -1}$

was mental division error

Scale on axes



How many solutions

26. $6x - y = 5$
$12x - 2y = 3 \;)\cdot 2$
$12x - 2y = 10$
can't be done

0 solutions

27. $x - 2y = 1$
$3x - 6y = 2 \;)\cdot 3$
$3x - 6y = 3$
can't be done

0 solutions

Both have 2 different integers equal to same equation half.

all when

$0 = 0$

or $3 = 0$

Derived equation is false

Section 2.4

Writing Equations of Lines 2/4

Independent Study: Linear Functions

p90 # 11, 13, 17, 19, 30

Write equation

11. $(0,3)$ $m = -\frac{1}{2}$

$y = -\frac{1}{2}x + 3$

13. $(-8,1)$ $m = 10$

$y = 10x = y = -80$ at -8 so $+80 + 1$

$y = 10x + 81$

Write Equation

17. $(7,6)(10,5)$

$\frac{6-5}{7-10} = \frac{1}{-3} = h$

copy error

$y = -\frac{1}{3}$ at $7 = -2\frac{1}{3}$ so $+2\frac{1}{3} + 6$

~~$y = -\frac{1}{3}x + 8\frac{1}{3}$~~

$y = 3x - 15$

19. $(-9,9)(0,1)$

$\frac{9-1}{-9-0} = \frac{8}{-9}$

$y = -\frac{8}{9}X + 1$

forgot x

30. Running the IRS

Using chart, write linear model for adv. cost of running IRS
Let $t=0$ be cost in 1980 and estimate cost in 1995

0, 2.3
8, 5
10, 5.5

stat edit → $y = .325x + 2.31$

$y = .325(15) + 2.31$ ← in 1995

$y = 7.19$ billion \$

Section 3.2

Solving Linear Systems Algebraically 2/4

Sketch + Solve

19. $x - y = 10$
$3x - 2y = 25$
$x = 10 + y$
$3(10 + y) - 2y = 25$
$30 + 3y - 2y = 25$
$-30 \qquad -30$
$\boxed{y = -5}$

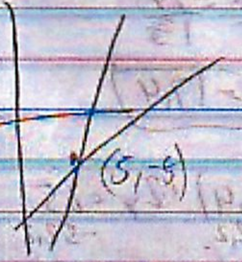
$x - (-5) = 10$
$\boxed{x = 5}$

21. $4x + 3y = 1$
$-3x + 6y = 3$) $\cdot 2$
$-8x + 6y = 2$
$5x = 5$
$\boxed{x = 1}$

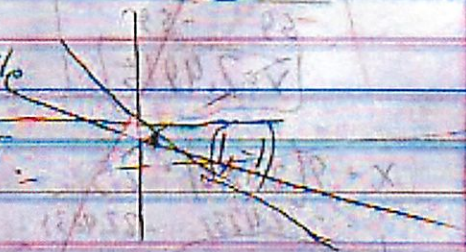
$4(1) + 3y = 1$
$-4 \qquad -4$
$3y = -3$
$\boxed{y = -1}$

was mental division error

Scale on axes !



Scale on axes



How many solutions

26. $6x - y = 5$) $\times 2$
$12x - 2y = 3$
$12x - 2y = 10$
can't be done
0 solutions

27. $x - 2y = 1$) $\cdot 3$
$3x - 6y = 2$
$3x - 6y = 3$
can't be done
0 solutions

Both have 2 different integers equal to same equation half.

all when
$0 = 0$
or
$3 = 0$

derived equation is false

Name: Michael Plosnder 53/55 ★ Independent Study

Linear Functions

Directions: Show all work that is necessary to receive credit. This test is worth **55 points.** Good Luck.

1. Solve the linear equation: $2(3x-1)=5-(x+3)$. **[4 points]**

$6x-2=5-x-3$

$6x=4-x$

$7x=4$

Answer: $x=\frac{4}{7}$

Mostly Everyone got wrong

2. Solve for s: $r^2s-5s=7$. **[2 points]**

Can't solve

$5-5s=7-r^2$

$-4s=7-r^2$

$s=-1.75+\frac{r^2}{4}$

$5-5s=\frac{7}{r^2}$

$-4s=\frac{7}{r^2}$

$s=\frac{\frac{7}{r^2}}{-4}$

wrong prob

GCF for s

$s(r^2-5)=7$

$s=\frac{7}{r^2-5}$

Answer: $s=\frac{\frac{7}{r^2}}{-4}$

3. Solve for F: $C=\frac{5}{9}(F-32)$. **[3 points]**

$\frac{9}{5}C=F-32$

$F=\frac{9}{5}C+32$

Answer: $F=\frac{9}{5}C+32$

know

4. Solve the inequality $-7\le 5x-2<3$ **[3 points]**

$-5\le 5x<5$

$-1\le x<1$

Answer: $-1\le x<1$

10

5. Solve the inequality $-3 < 2x - 3 \le 17$ then sketch its graph. **[5 points]**

+3 +3 +3

$\frac{0}{2} < \frac{2x}{2} \le \frac{20}{2}$

$0 < x \le 10$

-2 0 2 4 6 8 10 12

Answer: $0 < x \le 10$

6. Find the slope of the line the passes through (-3,4) and (1,5). **[3 points]**

$\frac{5-4}{1--3} = \frac{1}{4}$

I don't need

$y = \frac{1}{4}x + 4.75$

Answer: slope $= \frac{1}{4}$

7. Determine whether the following lines are parallel, perpendicular, or neither. **[4 points]**

$3x = 2y - 7$

$6x - 4y = 9$

$-6x \quad -6x$

$\frac{-4y}{-4} = \frac{-6x+9}{-4}$

$y = 1.5x - 2.25$

$3x = 2y - 7$

$+7 \quad +7$

$\frac{2y}{2} = \frac{3x+7}{2}$

$y = 1.5x + 3.5$

Answer: parallel

12

8. Find the x and y intercepts of $3x + 2y = 15$ **[3 points]**

$3(0) + 2y = 15$ $\quad$ $3x + 2(0) = 15$

$\frac{2y}{2} = \frac{15}{2}$ $\quad$ $\frac{3x}{3} = \frac{15}{3}$

$\boxed{y = 7.5}$ $\quad$ $\boxed{x = 5}$

✓

Answer: (5,0) (0,7.5)

9. Write the slope –intercept form of the equation $3x + 2y = 15$. **[3points]**

$-3x \quad -3x$

$\frac{2y}{2} = \frac{-3x + 15}{2}$

$y = -1.5x + 7.5$

✓

Answer: ____________

10. Write an equation for the line that has a slope of 3 and a y-intercept of 5. **[4 points]**

1/2

Answer: $y = 3x + 5$

11. Write an equation of the line that passes through (4,11) and (5,9). **[5 points]**

$\frac{11-9}{4-5} = \frac{2}{-1} = -2$

-1 5, 9 +2

4, 11

3, 13

2, 15

1, 17

0, 19

✓

Answer: $y = -2x + 19$

15

12. Write an equation of the line containing the point (3,-4) and has a slope of $\frac{2}{5}$. **[5 points]**

$\frac{2}{5} = \frac{x}{2}$

$3, -4$ -2

-5 $-2, -6$ $+\frac{4}{5}$

$+2$ $0, -5.2$

$\frac{4}{5} = \frac{5x}{5}$

$\frac{4}{5} = x$

Answer: $y = \frac{2}{5}x - 5.2$

13. Use the **substitution method** to solve the linear system below: **[6 points]**

$$\begin{cases} -x+3y=18 \\ 4x-2y=8 \end{cases}$$

$-x + 3(8) = 18$

$-24 \quad -24$

$\frac{-x}{-1} = \frac{-6}{-1}$

$\boxed{x = 6}$

$-3y \quad -3y$

$\frac{-x}{-1} = \frac{-3y+18}{-1}$

$\boxed{x = 3y - 18}$

$4(3y-18) - 2y = 8$

$12y - 72 - 2y = 8$

$10y - 72 = 8$

$+72 \quad +72$

$\frac{10y}{10} = \frac{80}{10}$

$\boxed{y = 8}$

Answer: $x = 6, y = 8$

14. Use the **linear combination/elimination method** to solve the system below: **[5 points]**

$$\begin{cases} 7x+20y=11 \\ 3x+10y=5 \end{cases}$$

$\cdot -2$ $-6x - 20y = -10$

$1x = 1$

$\boxed{x = 1}$

$7(1) + 20y = 11$

$-7 \quad -7$

$\frac{20y}{20} = \frac{4}{20}$

$\boxed{y = .2}$

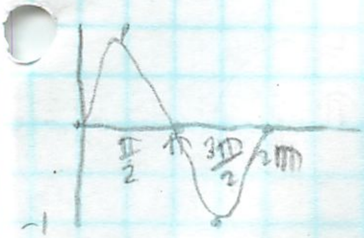
Answer: $x = 1, y = .2$

16

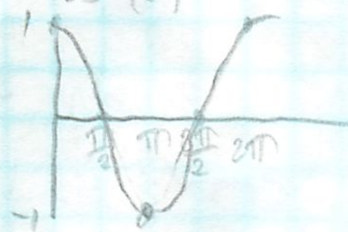
Graphing Sin + Cos Curves

2/28

Sin(θ)



Cos(θ)



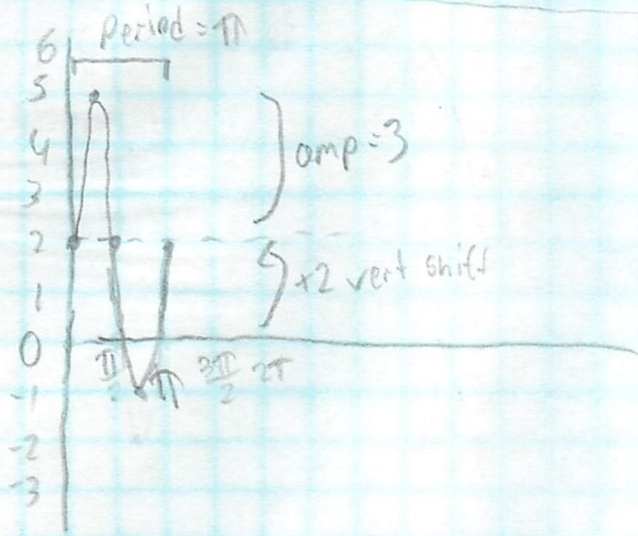
$y = 3\sin(2x) + 2$

amp = 3

pd = π

phase Δ = 0

vert Δ = ↑2



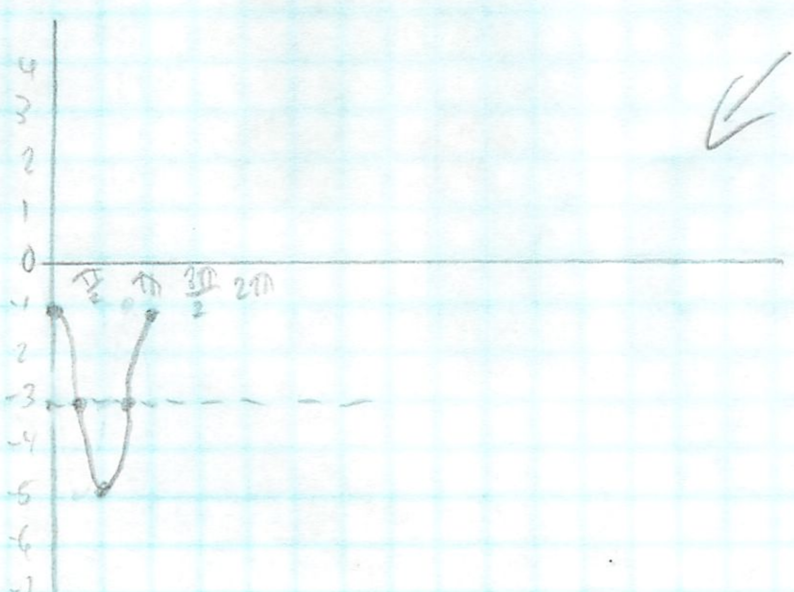
$y = 2\cos(2x) - 3$

amp = 2

pd = π

phase Δ = 0

Vert Δ = ↓3



$y = 3\sin(2x - 2\pi) - 2$

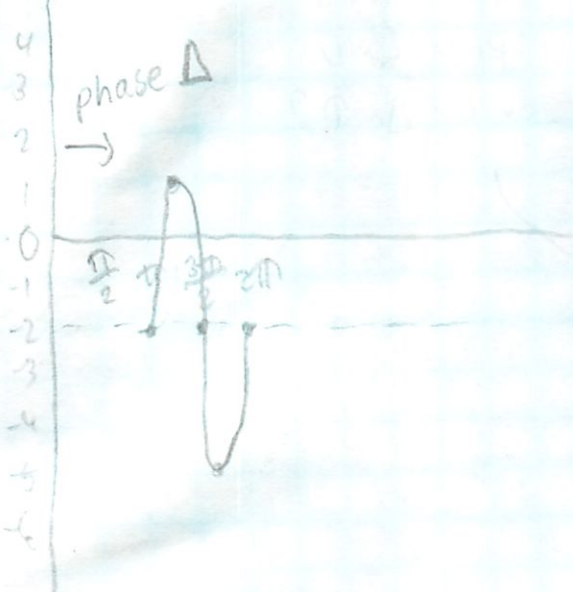
amp = 3

pd = π

phase Δ = $\rightarrow \pi$

vert Δ = $\downarrow 2$

to find graph interval do period / 4



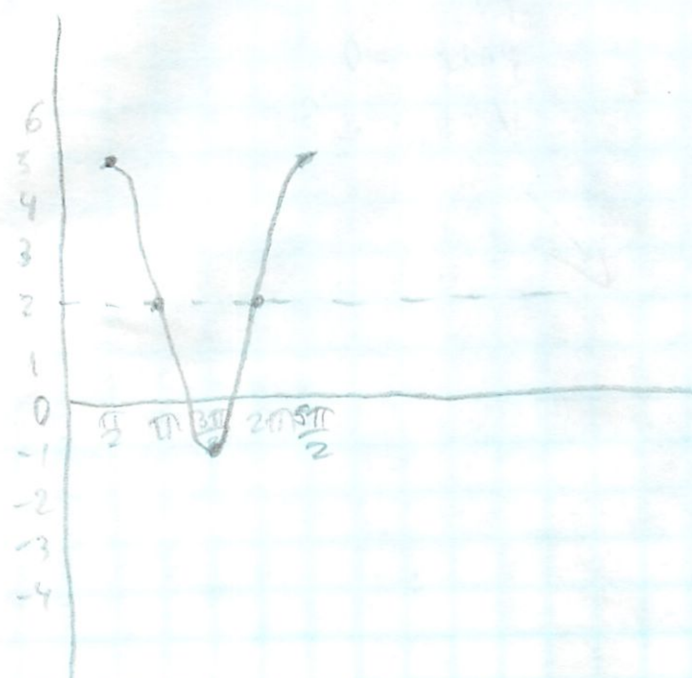
$y = 3\cos\left(x - \frac{\pi}{2}\right) + 2$

amp = 3

pd = 2π

phase Δ $\rightarrow \frac{\pi}{2}$

vert Δ = $\uparrow 2$



Sketching cosine curves

Homework

Name:____________________

Date: 2/28

For each equation find the amplitude, period, phase shift, and vertical shift.

1. $y = 2\cos(2x + 2\pi) - 3$

amp. 2

period π

phase shift $\leftarrow \pi$

vertical shift $\downarrow 3$

2. $y = \cos(x - 90^\circ) - 1$ $\frac{\pi}{4}$

amp. 1

period 2π

phase shift $\rightarrow \frac{\pi}{4}$

vertical shift $\downarrow 1$

3. $y = 3\cos(2x - \pi) + 2$

amp. 3

period π

phase shift $\rightarrow \frac{\pi}{2}$

vertical shift $\uparrow 2$

4. $y = 3\cos(x + 360^\circ)$ π

amp. 3

period 2π

phase shift $\leftarrow \frac{\pi}{2}$

vertical shift 0

On a piece of graph paper sketch and label each equation.

SOLVING SIMPLE TRIG EQUATIONS

WRITTEN EXERCISES

○ = 1st time

□ = 2nd time

A **Solve for $0° \leq \theta < 360°$. Give answers to the nearest tenth of a degree.**

1. $\sin\theta = -0.7$
2. $\cos\theta = 0.42$
3. $\tan\theta = 1.2$
4. $\cot\theta = -0.3$
5. $\sec\theta = -5$
6. $\csc\theta = 14$
7. $3\cos\theta = 1$
8. $4\sin\theta = 3$
9. $5\sec\theta + 6 = 0$
10. $2\tan\theta + 1 = 0$
11. $6\csc\theta - 9 = 0$
12. $4\cot\theta - 5 = 0$

Solve for $0 \leq x < 2\pi$. Give answers to the nearest hundredth of a radian.

13. $\tan x = -1.5$
14. $\sec x = 2.5$
15. $\csc x = -1.4$
16. $\cos x = -0.8$
17. $\cot x = 6$
18. $3\sin x + 2 = 4$
19. $8 = 9\cos x + 2$
20. $\frac{5\csc x}{3} = \frac{9}{4}$
21. $\frac{3\cot x}{4} + 1 = 0$

Find the slope and equation of each line described. Sketch the line.

22. inclination = 45°
    y-intercept = 4
23. inclination = 120°
    contains (2, 3)
24. inclination = 158°
    contains (−3, 5)

2. $\cos\theta = 0.42$

$\theta = \cos^{-1}(.42)$

calc: 65.165

1st + 4th

1st: 65.17° or 1.137 r

4th: 294.83° or 5.146 r

v: ✓

6. $\csc(14) = \theta$

$\theta = \sin^{-1}(1/14)$

calc: 4.096

1st + 2nd

1st: 4.10° or .0715 r

2nd: 179.93 or 3.14

? 175 3.01

Correct?

8. $\frac{4\sin\theta}{4} = \frac{3}{4}$

$\sin\theta = \frac{3}{4}$

$\theta = \sin^{-1}(3/4)$

calc: 48.59

1st + 2nd

1st: 48.59° or .848

2nd: 131.41° or 2.294

v: ✓

12. $4\cot\theta - 5 = 0$

$+5 \quad +5$

$\frac{4\cot\theta}{4} = \frac{5}{4}$

$\cot\theta = \frac{5}{4}$

$\theta = \tan^{-1}(4/5)$

calc: 38.66

1st + 3rd

1st: 38.66° or .675 r

3rd: 218.66° or 3.816

1. $\sin\theta = -.7$

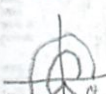
$\theta = \sin^{-1}(-.7)$

calc = -44.42

~~1st + 2nd~~ 3rd or 4th

4th ~~1st~~: 315.57 or 5.508

~~4th~~ 3rd: 224.43 or



3.917

3. $\tan\theta = 1.2$

$\theta = \tan^{-1}(1.2)$

calc: ~~.0209~~

1st + 3rd

50.19

1st: ~~.0209~~ or .0003 r

3rd: ~~180.02~~ or ~~3.141 r~~

230.19

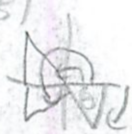
were too small

Calc type error

5. $\sec\theta = -5$

$\theta = \cos^{-1}(-\frac{1}{5})$

calc: 78.46



θ = 2nd + 3rd

2nd: 101.53 or 1.77 r

3rd: 258.46 or 4.511 r

7. $\frac{3\cos\theta}{3} = \frac{1}{3}$

$\cos\theta = \frac{1}{3}$

$\theta = \cos^{-1}(1/3)$

calc: 70.528

1st + 4th

1st: 70.53 or 1.23 r

4th: 289.471 or 5.05 r

v: ✓ 4th

9. $5\sec\theta + 6 = 0$

$-6 \quad -6$

$\frac{5\sec\theta}{5} = \frac{-6}{5}$

$\sec\theta = -\frac{6}{5}$

$\theta = \cos^{-1}(-\frac{5}{6})$

calc: 146.44

2nd + 3rd

2nd: 146.44 or 2.555 r

3rd: ~~236.44~~ or ~~4.126 r~~

213.55

~~v 3rd~~ ✗

error don't add 90°

Otherwise bad

do 360 − 146.4

All Correct
good job

11. $6\csc\theta - 9 = 0$
$+9 \quad +9$
$\frac{6\csc\theta}{6} = \frac{9}{6}$
$\csc\theta = \frac{9}{6}$
$\theta = \sin^{-1}(6/9)$
calc = 41.81
1st + 2nd
1st: 41.81 or .7297 r
2nd: 138.19 or 2.411 r
V1st: V

13. $\tan\theta = -1.5$
$\theta = \tan^{-1}(-1.5)$
calc: -56.3099
2nd + 4th
2nd 123.69 or 2.159 r
4th 303.69 or 5.300 r

15. $\csc\theta = -1.4$
$\theta = \sin^{-1}(\frac{1}{-1.4})$
calc: -45.58
(-) 3rd + 4th
3rd, 225.58 or 3.937 r
4th: 314.42 or 5.488 r

17. $\cot\theta = 6$
$\theta = \tan^{-1}(1/6)$
calc: 9.4623
1st + 3rd
1st: 9.46 or .165
3rd: 189.46 or 3.307
V1st: V

19. $8 = 9\cos\theta + 2$
$-2 \quad -2$
$\frac{9\cos\theta}{9} = \frac{6}{9}$
$\cos\theta = 6/9$
$\theta = \cos^{-1}(6/9)$
calc: 48.19
1st + 4th
1st: 48.19 or .841 r
4th: 311.81 or 5.442 r
V1st: V

21. $3\cot\theta + 1 = 0$
$-1 \quad -1$
$3\cot\theta = -1$
$\cdot 4 \quad \cdot 4$
$\frac{3\cot\theta}{3} = \frac{-4}{3}$
$\cot\theta = -\frac{4}{3}$
$\theta = \tan^{-1}(-3/4)$
calc: -36.86
V2nd: V
2nd + 4th
2nd: 143.13 or 2.498 r
4th: 323.13 or 5.64 r

Warm up 2/26

2/26

1. $y = -5\cos(3x) - 9$
   - amp = 5 ← always pos
   - pd = 120 (360/3) or → $\text{period} = \frac{2\pi}{|v|}$ ← abs value of vertical speed
   - vert shift = ↓9

2. $y = -3\sin\left(\frac{1}{2}x\right) + 4$
   - amp = 3
   - pd = 720
   - ver. = ↑4 ← always show arrow

3. $y = \sin x - 5$
   - amp = 1
   - pd = 360
   - vert. = ~~-5~~ ↓5

arrow not sign + or −

---

w/ Phase Shift Work

2/27

$y = 3\sin(4x + \pi) + 4$
- amp = 3
- phas = $\frac{\pi}{4}$ ← } don't forget arrows
- vert = ↑4
- period = 90° or $\frac{\pi}{2}$

$y = 2\cos(2x + 2\pi) + 3$
- amp = 2
- phaΔ = → π
- vert Δ = ↑3
- pd = 180° or π

Write the Equation

2/28

amp = 5
pd = 2π
phase Δ = $\frac{\pi}{4}$ →
vert Δ = ↑3
sin x

$5\sin\left(1x - \frac{\pi}{4}\right) + 3$

amp = 2
pd = π
phase Δ = ← $\frac{3\pi}{2}$
vert Δ = ↓4
cos

$2\cos\left(2x + 3\pi\right) - 4$

IDENTIFYING AMPLITUDE, PERIOD, VERTICAL SHIFT + PHASE SHIFT

Written Exercises

a

In Exercises 1–8, graph each function for $0 \le x \le 2\pi$. Find the amplitude, period, and phase shift of each function.

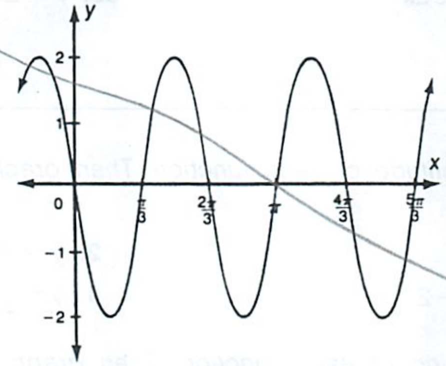
1. $y = 2 \sin\left(x + \frac{\pi}{4}\right)$
2. $y = -2 \sin\left(x - \frac{\pi}{2}\right)$
3. $y = \frac{1}{2} \cos\left(x + \frac{\pi}{4}\right)$
4. $y = -\cos\left(x - \frac{\pi}{3}\right)$
5. $y = \sin 3\left(x - \frac{\pi}{4}\right)$
6. $y = \cos 3\left(x + \frac{\pi}{4}\right)$
7. $y = -2 \cos(2x - \pi)$
8. $y = -\frac{1}{2} \sin(3x + \pi)$

In Exercises 9–16, find the amplitude, period, and phase shift and vertical shift *of each of the given functions.*

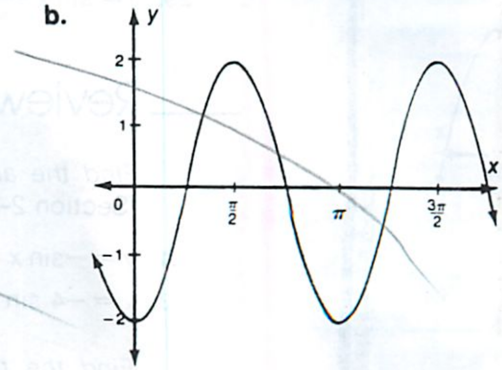
9. $y = \frac{1}{2} \sin(3x - 2\pi) + 2$
10. $y = -\frac{1}{2} \cos\left(\frac{1}{2}x + 4\pi\right) - 3$
11. $y = -3 \sin\left(x - \frac{3\pi}{2}\right) + 6$
12. $y = -3 \sin\left(4x - \frac{3\pi}{2}\right) + 1$
13. $y = \frac{3}{4} \cos(-3x - \pi) - 4$
14. $y = \frac{2}{3} \sin\left(-\frac{1}{2}x + \pi\right) + 7$
15. $y = 4 \sin(-2x - 2\pi) - 6$
16. $y = -7 \cos(-5x + \pi)$

In Exercises 17–20, find the amplitude, period, and phase shift of each of the given functions. ~~Then match each function with one of the four graphs below.~~

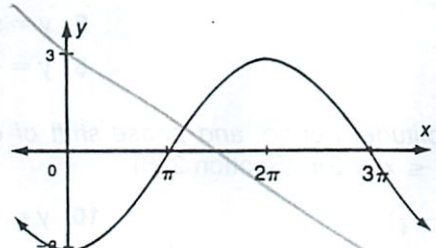
a.



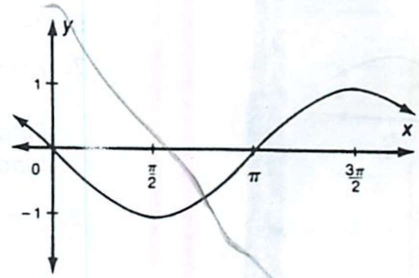
b.



c.



d.



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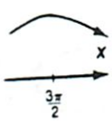
below.



	Function	Amplitude	Period	Phase Shift	Vertical Shift [redacted]
17.	$y=-2\sin\left(2x+\frac{\pi}{2}\right)-3$	?	?	?	?
18.	$y=\cos\left(x+\frac{\pi}{2}\right)-1$	?	?	?	?
19.	$y=2\cos\left(3x+\frac{\pi}{2}\right)+4$	?	?	?	?
20.	$y=3\sin\left(\frac{1}{2}x-\frac{\pi}{2}\right)-8$	?	?	?	?

b

In Exercises 21–23, find a function of the form $y = A \sin B(x - C)$ with the given properties.

21. Amplitude: 5; Period: 2π; Phase Shift: $\frac{\pi}{3}$ units to the right Vertical: down 2

22. Amplitude: 3; Period: $\frac{2\pi}{3}$; Phase Shift: $\frac{\pi}{3}$ units to the right Vertical: up 3

23. Amplitude: $\frac{2}{3}$; Period: $\frac{\pi}{4}$; Phase Shift: $\frac{\pi}{8}$ units to the left Vertical: up 1

In Exercises 24–26, find a function of the form $y = A \cos B(x - C)$ with the given properties.

24. Amplitude: $\frac{1}{5}$; Period: $\frac{\pi}{5}$; Phase Shift: 2π units to the right Vertical: down 4

25. Amplitude: $\frac{7}{3}$; Period: $\frac{5\pi}{6}$; Phase Shift: π units to the left Vertical: down 2

26. Amplitude: 1; Period: $\frac{3\pi}{4}$; Phase Shift: $\frac{3\pi}{4}$ units to the right Vertical: up 2

In Exercises 27–30, graph each function for $-2\pi \le x \le 2\pi$.

27. $y = 2\sin\left(2x+\frac{\pi}{3}\right) - 3$

28. $y = 3\sin\left(\frac{1}{2}x-\frac{\pi}{2}\right) + 2$

C

29. $y = \sin x + \cos 2x$

30. $y = 2\sin\frac{1}{2}x - \cos x$

Review

Find the amplitude of each function. Then graph the function for $0 \le x \le 2\pi$. (Section 2–6)

1. $y = -\sin x$

2. $y = 2\cos x$

3. $y = -4\sin x - 2$

4. $y = \frac{1}{2}\cos x + 1$

Find the period of each function. Then graph the function for $0 \le x \le 2\pi$. (Section 2–7)

5. $y = \cos 3x$

6. $y = \sin\frac{1}{3}x$

7. $y = \frac{1}{4}\sin 4x$

8. $y = -2\cos x$

Find the amplitude, period, and phase shift of each function. Then graph each function for $0 \le x \le 2\pi$. (Section 2–8)

9. $y = 2\cos\left(x-\frac{\pi}{4}\right)$

10. $y = -\cos\left(x+\frac{\pi}{6}\right)$

big mistake

2/27

9. $y = \frac{1}{2}\sin(3x - 2\pi) + 2$

amp = $\frac{1}{2}$

pd = 120° or $\cancel{\frac{3\pi}{2}}$ $\frac{2\pi}{3}$

vert Δ = ↑2

phase Δ = → $\cancel{\frac{2}{3}}$ → $\frac{2\pi}{3}$

11. $y = -3\sin\left(x - \frac{3\pi}{2}\right) + 6$

amp = 3

pd = 360° or 2π

vert Δ = ↑6

phase Δ = → $\cancel{\frac{3}{2}}$ $\frac{3\pi}{2}$

keep π in answer

13. $y = -\frac{3}{4}\cos(-3x - \pi) - 4$

amp = 3/4

pd = 120° or $\cancel{\frac{3\pi}{2}}$ $\frac{2\pi}{3}$

vert Δ = ↓4

Phase Δ = ← $\frac{\pi}{3}$

15. $4\sin(-2x - 2\pi) - 6$

amp = 4

pd = 180° or π

vert Δ = ↓6

Phase Δ = ← 1π

17. $y = -2\left(\sin 2x + \frac{\pi}{2}\right) - 3$

amp = 2

pd = 180° or π

vert Δ = ↓3

phase Δ = ← $\frac{\pi}{4}$

19. $y = 2\cos\left(3x + \frac{\pi}{4}\right) + 4$

amp = 2

pd = 120° or $\cancel{\frac{3\pi}{2}}$ $\frac{2\pi}{3}$

vert Δ = ↑4

phase Δ = ← $\cancel{\frac{\pi}{12}}$ $\frac{\pi}{6}$

2/28

---

21. amp = 5

pd = 2π

phase Δ = → $\frac{\pi}{3}$

vert Δ = ↓2

$5\sin\left(x - \frac{\pi}{3}\right) - 2$

23. amp = $\frac{2}{3}$

pd = $\frac{\pi}{4}$

phase Δ = $\frac{\pi}{8}$ ←

vert Δ = ↑1

$\frac{\pi}{4} = \frac{2\pi}{b}$

$\frac{\pi b}{\pi} = \frac{8\pi}{\pi}$

$b = 8$

$\frac{2}{3}\sin(8x + \pi) + 1$

---

25. amp = $\frac{7}{3}$

pd = $\frac{5\pi}{6}$

phase Δ = ← π

vert Δ = ↓2

$\frac{5\pi}{6} = \frac{2\pi}{b}$

$\frac{5\pi b}{5\pi} = \frac{12\pi}{5\pi}$

$b = 2.4$

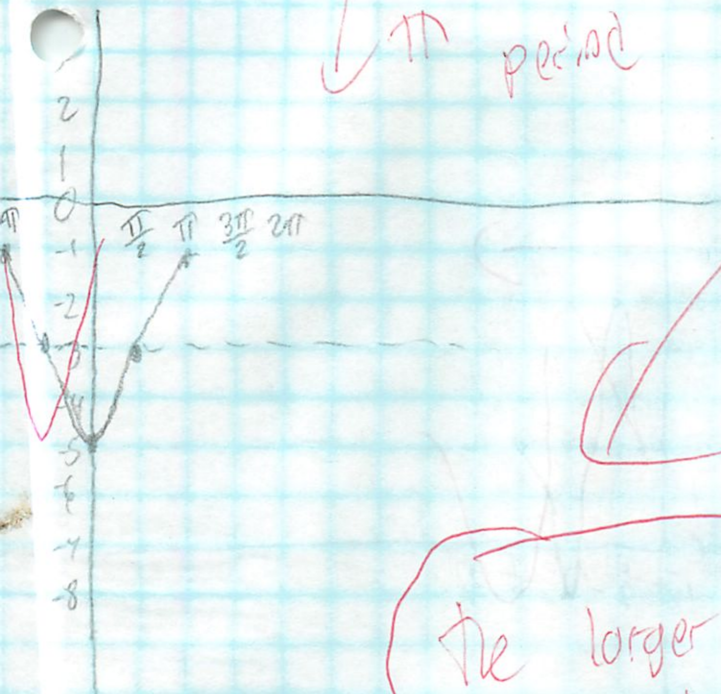
$\frac{7}{3}\cos(2.4x + 2.4\pi) - 2$

← don't forget

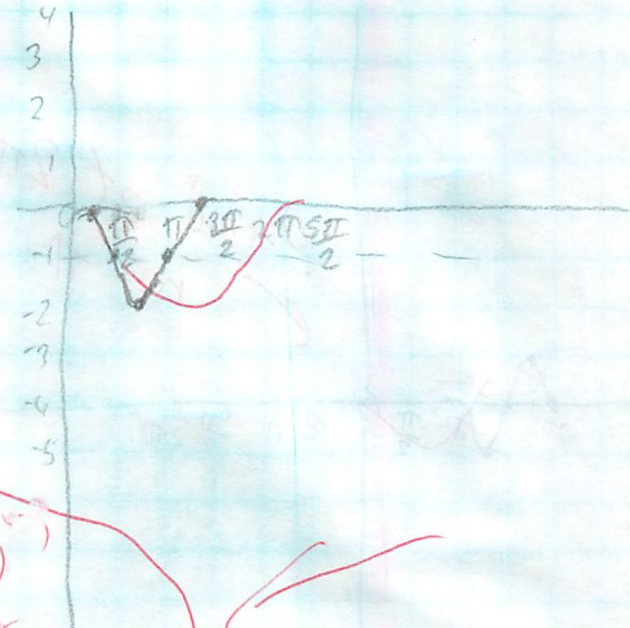
Cos Curves

$y = 2\cos(2x + 2\pi) - 3$

$\downarrow \pi$ period



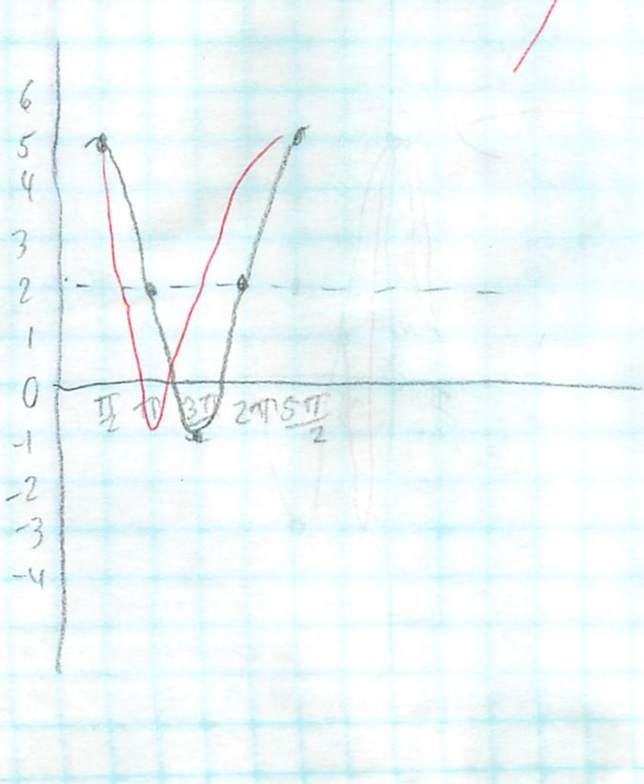
$y = \cos(x - 90°) - 1$



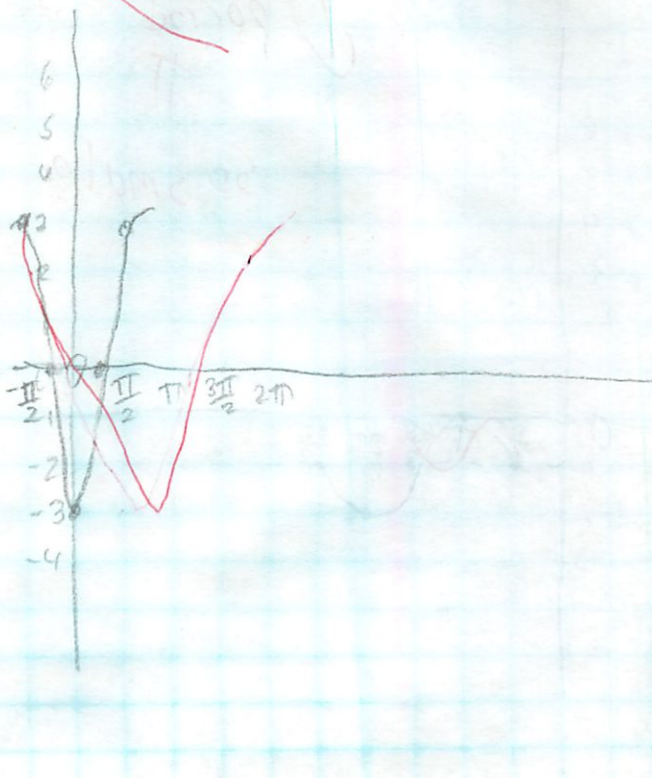
the larger the "b" number, the thiner the graph

$y = 3\cos(2x - \pi) + 2$

$\downarrow \pi$ period



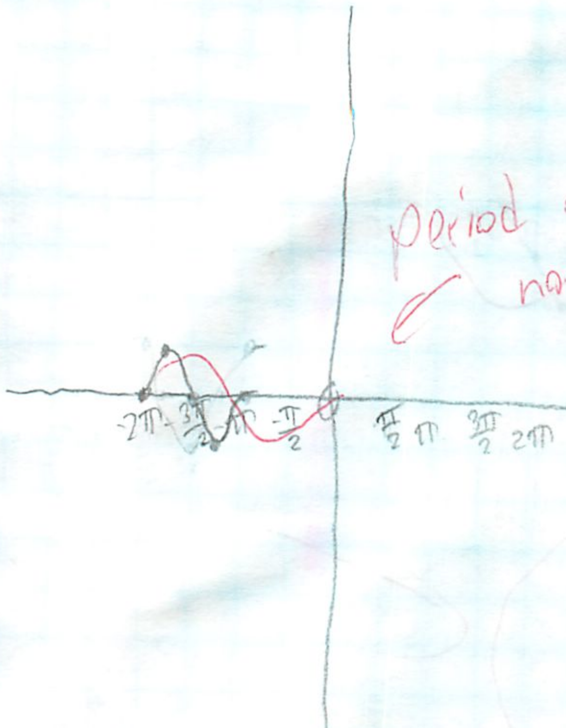
$y = 3\cos(x + 360)$



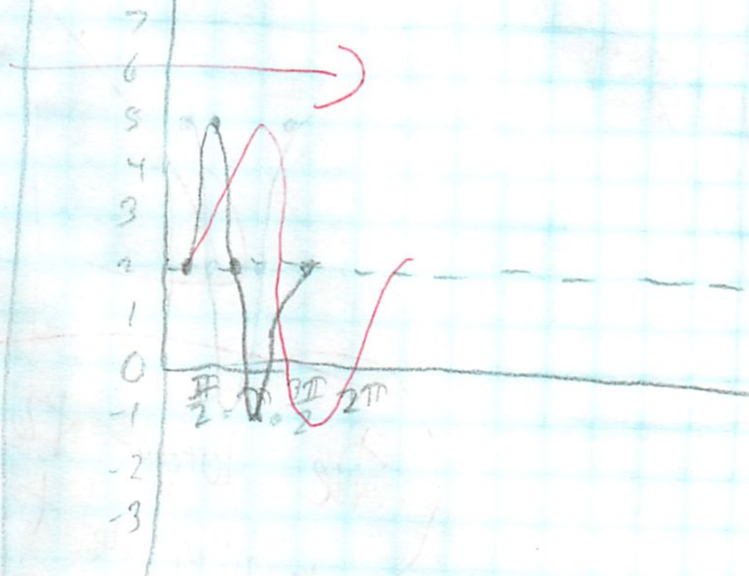
Sin Graphs

$y = \sin(x + 2\pi)$

period of 2π not π

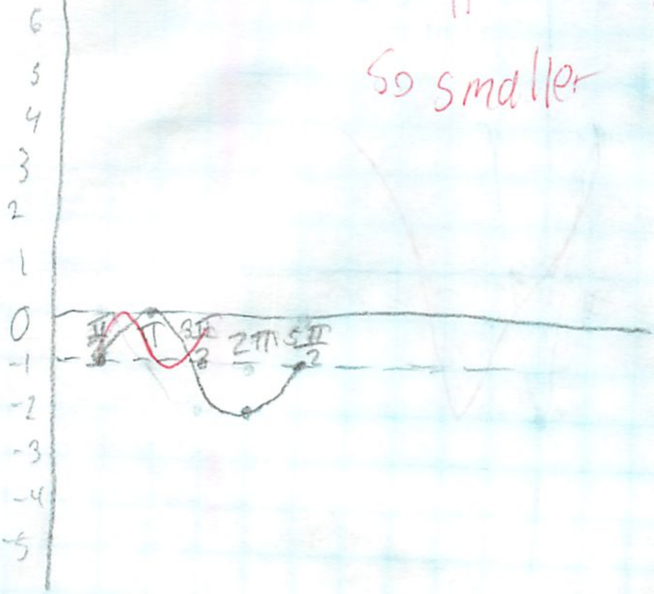


$y = 3\sin(x - 90) + 2$

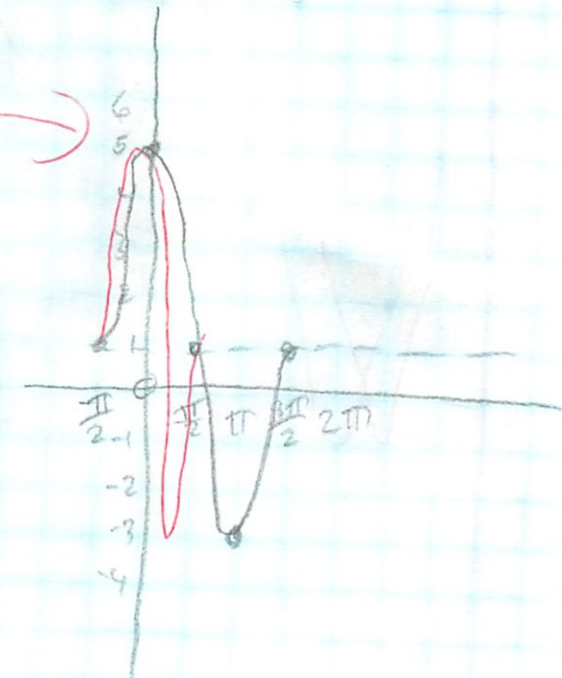


$y = \sin(2x - 180°) - 1$

period of π so smaller



$y = 4\sin(2x + \pi) + 1$



Sketching sine curves

Name: ______________________________

Date: ____________

For each equation find the amplitude, period phase shift, and vertical shift.

1. $y = \sin(4x)$

amp. ___________

period __________

phase shift __________

vertical shift ___________

2. $y = 3\sin(2x)$

amp. ___________

period __________

phase shift __________

vertical shift ___________

3. $y = \sin(2x) - 1$

amp. ___________

period __________

phase shift __________

vertical shift ___________

4. $y = \sin(2x + 180^o) + 1$

amp. ___________

period __________

phase shift __________

vertical shift ___________

On a piece of graph paper sketch and label one period of each equation.

2/28

$-\frac{\pi}{4}$

5. $y = \sin(x + 2\pi)$

amp. 1

period 2π

phase shift $\leftarrow 2\pi$

vertical shift 0

6. $y = 3\sin(x - 90^o) + 2$

amp. 3

period 2π

phase shift $\rightarrow \frac{\pi}{4}$

vertical shift $\uparrow 2$

π

7. $y = \sin(2x - 180^o) - 1$

amp. 1

period π

phase shift $\rightarrow \frac{\pi}{2}$

vertical shift $\downarrow 1$

8. $y = 4\sin(2x + \pi) + 1$

amp. 4

period π

phase shift $\leftarrow \frac{\pi}{2}$

vertical shift $\uparrow 1$

On a piece of graph paper sketch and label one period of each equation.

Warmup

3/1

3/1

1. $y = 2\cos(2x - \pi) + 3$

amp = 2

pd = π

vert Δ = $\uparrow 3$

phase Δ = $\rightarrow \frac{\pi}{2}$

2. $y = 1 + \sin(x + \pi)$

amp = 1

pd = 2π

vert Δ = $\uparrow 1$

phase Δ = $\leftarrow \pi$

3. $y = 5 + 10\sin\left(2x + \frac{3\pi}{2}\right)$

amp = 10

pd = π $\leftarrow 360/16$

vert Δ = $\uparrow 5$

phase Δ = $\leftarrow \cancel{\frac{1.5\pi}{2}}$ $\leftarrow \frac{3\pi}{4}$

4. amp = 3

pd = 2π trig = cos(x)

phase Δ = $\leftarrow \frac{3\pi}{4}$

vert Δ = $\downarrow 2$

$y = 3\cos\left(x + \frac{3\pi}{4}\right) - 2$

5. amp = 1

pd = π sin(x)

phase Δ = $\rightarrow \pi$

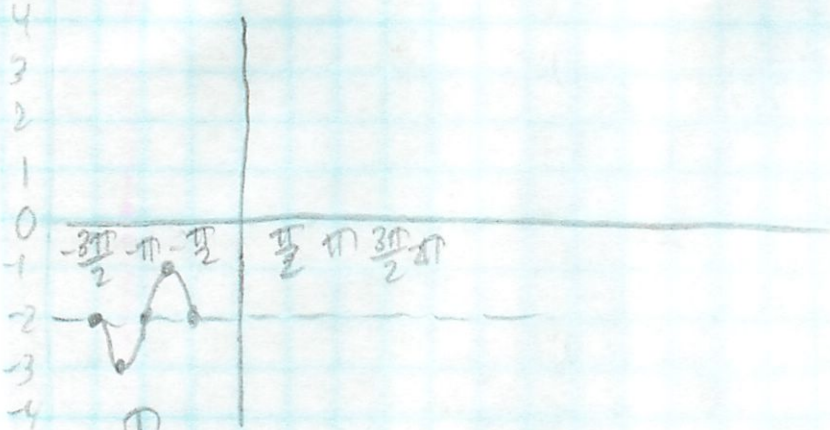
vert Δ = $\uparrow 3$

$y = \sin(2x - 2\pi) + 3$

Graph Reflection

3/1

if amplitude value is negitive, graph is flipped



$y = -\sin(2x + 3\pi) - 2$

amp = 1

pd = π smaller

vert Δ = ↓2

phase Δ = $\frac{3\pi}{2}$ ←

flipped

go down first

$y = -3\cos\left(4x - \frac{3\pi}{2}\right) + 1$

interval

pd/4 →

$\frac{\pi}{2}/4 = \frac{\pi}{2} \cdot \frac{1}{4}$

amp = 3

pd = $\frac{\pi}{2}$ → $\frac{2\pi}{1} \cdot \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

$\frac{2\pi}{|b|}$

vert Δ = ↑1

phase Δ = → $\frac{3\pi}{8}$ ← $\frac{\frac{3\pi}{2}}{4}$

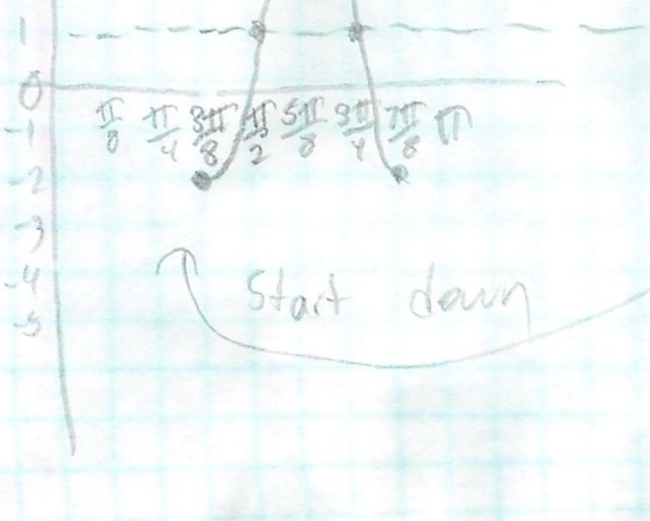
flipped

$\frac{3\pi}{2} \div \frac{4}{1}$

$\frac{3\pi}{2} \cdot \frac{1}{4}$

multiply accross

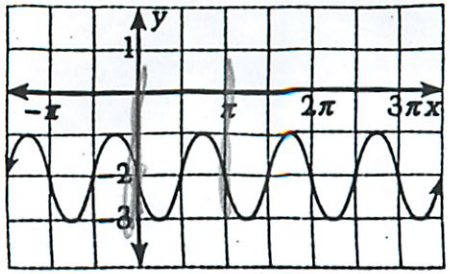
$\frac{3\pi}{8}$



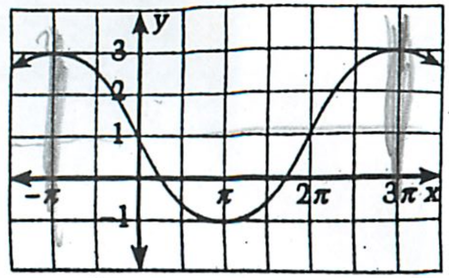
Start down

Reading The Graph

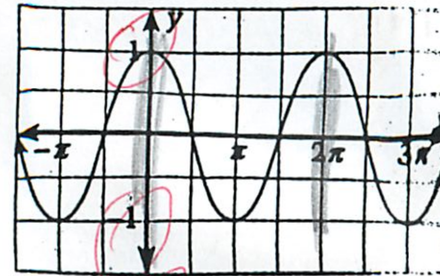
a.



b.

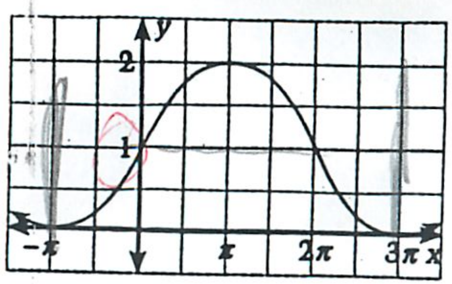


c.

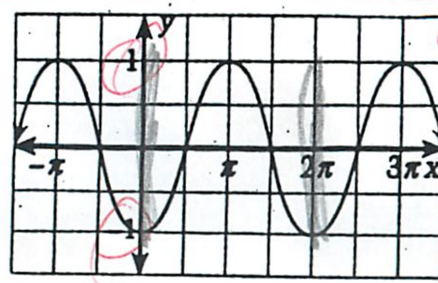


scaling weird

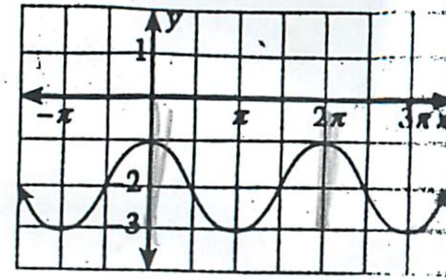
d.



e.



f.



Directions: On the lines below, write the trigonometric equation that describes each of the above graphs.

a: $y = -\sin(2x) - 2$

b: $y = 2\cos(\frac{1}{2}x + \cancel{\frac{1}{2}\pi}) + 1$

say $\frac{\pi}{2}$

c: $y = \cancel{1}\cos(x)$

d: $y = -\cancel{1}\cos(\frac{1}{2}x + \cancel{\frac{1}{2}\pi}) + \cancel{1}$

say $\frac{\pi}{2}$

e: $y = -\cancel{1}\cos(x)$

f: $y = \cos(x) - 2$

watch for weird scaling

IAG 34
HOT SHEET

Name ______________
Date 2/15
Unit High Dive 2

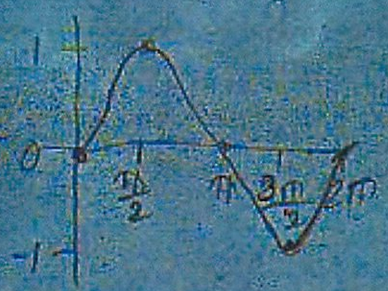
Inverse Trig Functions

$\sin^{-1}\theta$ - 1st + 4th quad
$-90° \le \theta \le 90°$

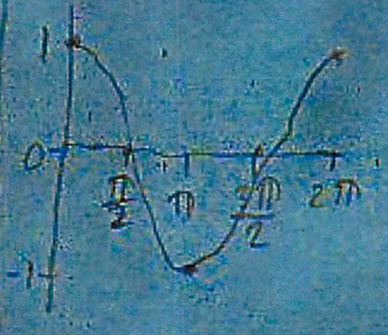
$\cos^{-1}\theta$ - 1st + 2nd quad
$0° \le \theta \le 180°$

$\tan^{-1}\theta$ - 1st + 4th quad
$-90° < \theta < 90°$

Sin θ



Cos θ



Where Positive

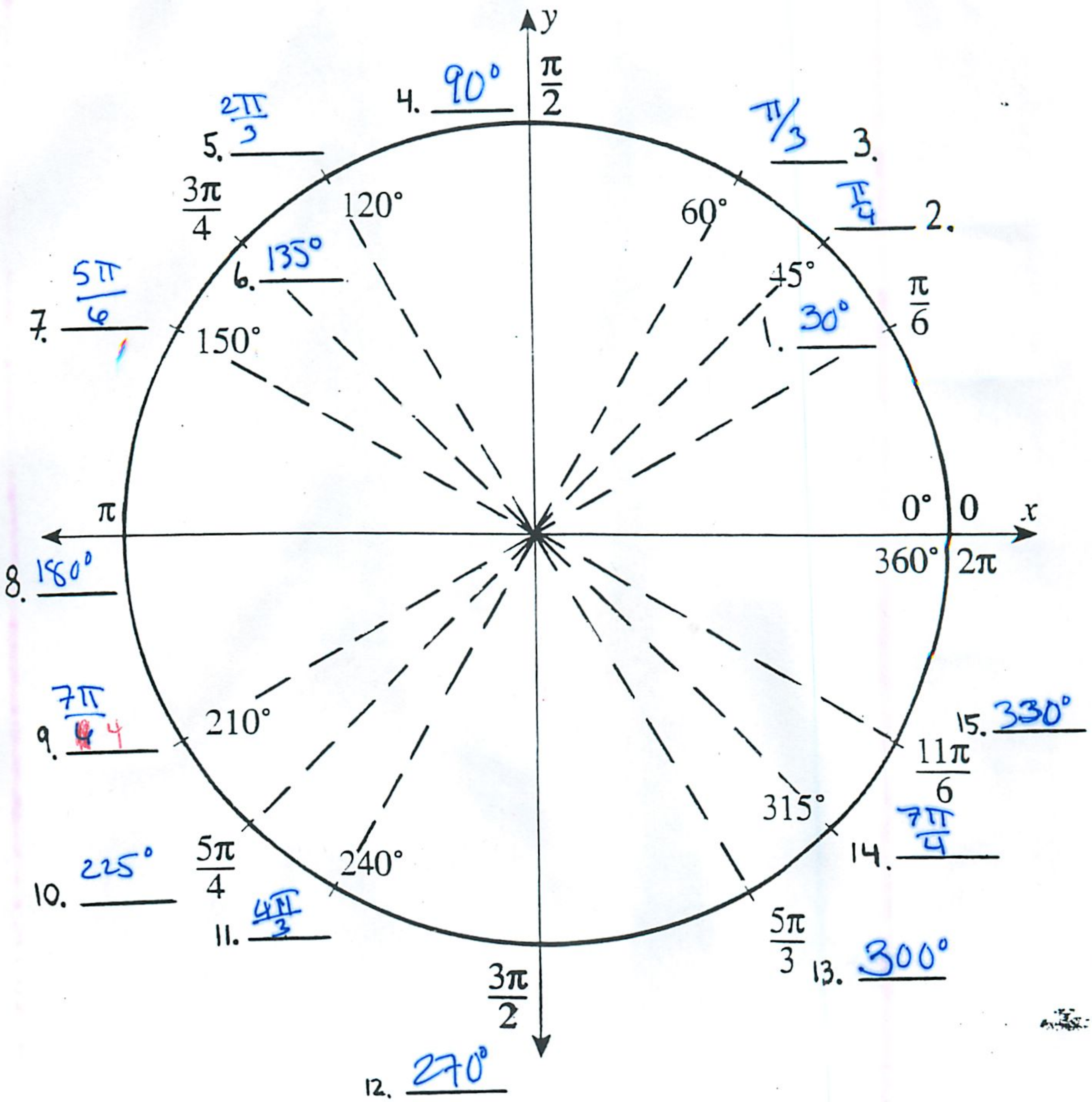
$\sin\theta$	$\sin\theta$ $\cos\theta$ $\tan\theta$
$\tan\theta$	$\cos\theta$

Reciprocal in same places

Name ______________________

Unit Circle in terms of Degrees & Radians
(in multiples of 30°, 45°, 60°)

Directions: Convert the radian measure to degrees and convert the given degree measure to radians.



Quiz 2 Topics
3/2/07

3/1

- calc + non-calc section
- ~~given equation → graph it~~ — or graph
- given equation → write amp, pd, phase Δ, vert Δ
- given amp, pd, vert Δ, phase Δ → write equation
- solving simple trig equations (13.4)
- simplifying trig expressions (14.4)
- applying definition $\sin^{-1}(x)$, $\cos^{-1}(x)$, $\tan^{-1}(x)$

different / harder

non calc
- probably solving trig equation with standard (30-60-90) triangles

Expression

$\tan^{-1}\left(-\frac{1}{12}\right)$
$\cos^{-1}(\sqrt{3})$
) between 0-360°

sin = $-90 \le \theta \le 90$
cos = $0 \le \theta \le 180$
tan = $-90 \le \theta \le 90$

$\sin^{-1}(1/4)$

only 1 answer

think

Equation

answers in — + — quad

S	A
T	C

$\theta = \sin^{-1}(.1/4)$

2 answers $+ 2\pi n$

$4\sin\theta \cancel{+}1 = 0$

Quiz 2 ~~Retest~~ at

Graphing & Solving Equations Name Michael Plasmeier

Inverse Trig Expressions 64/64 Date 3/2

37/64 points

NICE TRY!

yes!

Complete each of the following. (2 points each)

1. If the graph of $y=-5\sin(2x)-4$ was shifted to the left π units, the resulting equation would be. . .

   Equation: $y=-5\sin(2x+2\pi)-4$

2. If the graph of $y=3\cos(x-2\pi)$ was shifted up 4 units, the resulting equation would be. . .

   Equation: $y=3\cos(x-2\pi)+4$

3. The equation of $y=-7\sin\left(6x-\frac{\pi}{2}\right)+3$ has an amplitude of 7 and a period of $\frac{\pi}{3}$.

   ← flipped

   $2\pi/6 = \frac{2\pi}{1}\cdot\frac{1}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$

4. The equation of $y=\cos\left(\frac{1}{3}x+\frac{\pi}{4}\right)-5$ has a phase shift of ← $\frac{3\pi}{4}$ and a vertical shift of ↓5.

   $\frac{\pi}{4}\Big/\frac{1}{3} = \frac{\pi}{4}\cdot\frac{3}{1} = \frac{3\pi}{4}$

For the following, write an equation that is representative of the given specifications. (4 points each)

5. Amplitude: 4
   Period: π
   Phase Shift: left $\frac{\pi}{4}$
   Vertical Shift: down 5
   Trig: $\sin x$

   Equation: $y=4\sin\left(2x+\frac{\pi}{2}\right)-5$

   $\frac{\pi}{2}\cdot\frac{1}{2}=\frac{\pi}{4}$

6. Amplitude: 6
   Period: $\pi/2$
   Phase Shift: right $\frac{5\pi}{8}$
   Vertical Shift: up 3
   Trig: $\cos x$

   Equation: $y=6\cos\left(4x-\frac{5\pi}{2}\right)+3$

   $\frac{5\pi}{2}\cdot\frac{1}{4}=\frac{5}{8}$

16

Solve the following for $0 \le \theta \le 2\pi$. Write your answer in terms of degrees and radians and round to the nearest hundredth, if applicable. SHOW ALL WORK!!!
(#7 - #9 -> 3 pts. each and #10 & #11 -> 4 pts. each)

7. $4\sin\theta = -2$

$\sin\theta = \frac{-2}{4}$

$\theta = \sin^{-1}(-2/4)$

θ: 3rd + 4th $\quad \theta = \sin^{-1}(-1/2)$

$\theta = -30°$

3rd: 210° ~~or~~ $\frac{7\pi}{6}$ $+\frac{1}{2}$

4th: 330° or $\frac{11\pi}{6}$

8. $\cot\theta = 8$

$\tan\theta = \frac{1}{8}$

$\theta = \tan^{-1}(1/8)$

calc: 7.13

θ: 1st + 3rd

1st: 7.13° or .12r

3rd: 187.13° or 3.27r

9. $6\tan\theta - 12 = 18$

$+12 \quad +12$

$\frac{6\tan\theta}{6} = \frac{30}{6}$

$\tan\theta = 5$

$\theta = \tan^{-1}(5)$ $\quad \theta$: 1st + 3rd

calc: 78.69

1st: 78.69 or 1.37r

3rd: 258.69 or 4.51r

10. $5\cos x + \sqrt{3} = 3\cos x$

$-5\cos x \quad -5\cos x$

fixed

$\frac{-2\cos x}{-2} = \frac{\sqrt{3}}{-2}$

$\cos x = \frac{\sqrt{3}}{-2}$

θ: 2nd + 3rd

$x = \cos^{-1}(\sqrt{3}/-2)$

2nd: 150° or $\frac{5\pi}{6}$

3rd: 210° or $\frac{7\pi}{6}$ $+\frac{1}{2}$

Unit circle I looked at was wrong

11. $\frac{-4\sec x}{3} - 2 = 6$

$+2 \quad +2$

$\frac{-4\sec x}{3} = 8$

$\cdot 3 \quad \cdot 3$

θ: 2nd + 3rd

$-4\sec x = 24$

$\sec x = -6$

$\cos x = \frac{1}{-6}$

$x = \cos^{-1}(1/-6)$

calc: 99.59

2nd: 99.59° or 1.74r

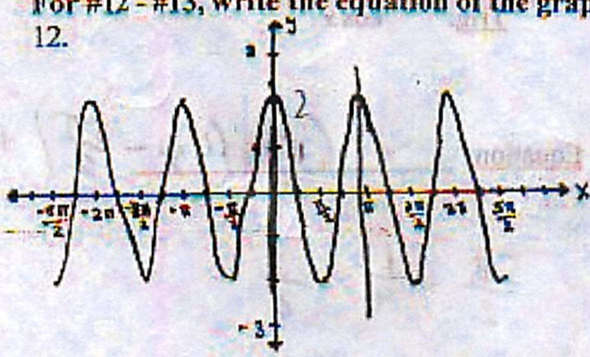
3rd: 260.41° or 4.54

Rounding $\frac{1}{2}$

its right if its pure from calc

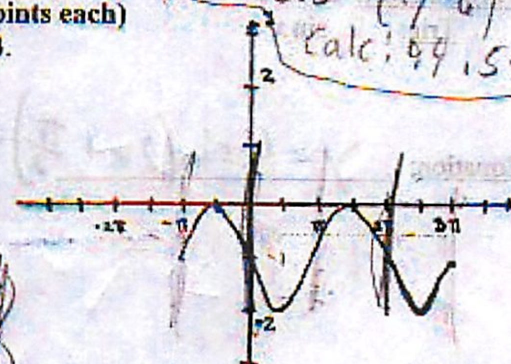
For #12 - #13, write the equation of the graph. (2 points each)

12.



Equation: $y = 2\cos(2x)$

13.



Equation: $y = -\sin(x) - 1$

21

Graphing Equations Name Michael Plesniar

27/64 points

For each of the following, evaluate the expressions and write your answers in both degree measure and radian measure. SHOW ALL WORK & USE PROPER NOTATION!!
(3 points each)

$-90 < \theta < 90°$ $0 \le \theta \le 180°$ $-90 \le \theta \le 90°$

1. arctan(1) $\frac{1}{1}$

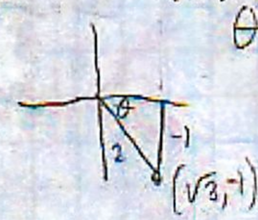
$\theta = 45°$ or $\frac{\pi}{4}$

1st

2. arccos(−1) $\frac{-1}{1}$ 2nd

$\theta = 180°$ or π

3. $\sin^{-1}\left(\frac{-1}{2}\right)$ 4th



$\theta = -30°$ or $-\frac{\pi}{6}$

$\frac{y}{x}$ $\frac{x}{r}$ $\frac{y}{r}$

For each equation find the amplitude, period, phase shift, and vertical shift. Then sketch and label a graph of one period of the function on the graph paper. Make sure your graph is accurate and label the five critical points. Label the axes and the scale on each.
(6 points each)

4. $y = \sin(2x - 2\pi) - 2$

$2\pi/2x$

Amp: 1

Period: π

$-2/2 = -1$

Phase Shift: → π

Vertical Shift: ↓2

ticks $= 4/\pi = \frac{4}{1}\frac{1}{\pi} = \frac{1}{4}\pi$ or $\frac{\pi}{4}$

5. $y = -2\sin\left(x + \frac{\pi}{2}\right) - 3$

Amp: 2

$2\pi/x$

Period: 2π

Phase Shift: ← $\frac{\pi}{2}$

Vertical Shift: ↓3

ticks: $4/2\pi = \frac{4}{1} \cdot \frac{1}{2\pi} = \frac{4}{2\pi} = \frac{1}{2}\pi$ or $\frac{\pi}{2}$

flipped

21

6. $y = 3\cos(4x + \pi) + 3$

Amp: 3

$2\pi/4x$ Period: $\frac{\pi}{2}$

Phase Shift: $\leftarrow \frac{\pi}{4}$

Vertical Shift: $\uparrow 3$

ticks $= 4/\frac{\pi}{2} = \frac{4}{1} \cdot \frac{2}{\pi} = \frac{8}{\pi}$ or $\frac{1}{8}\pi$

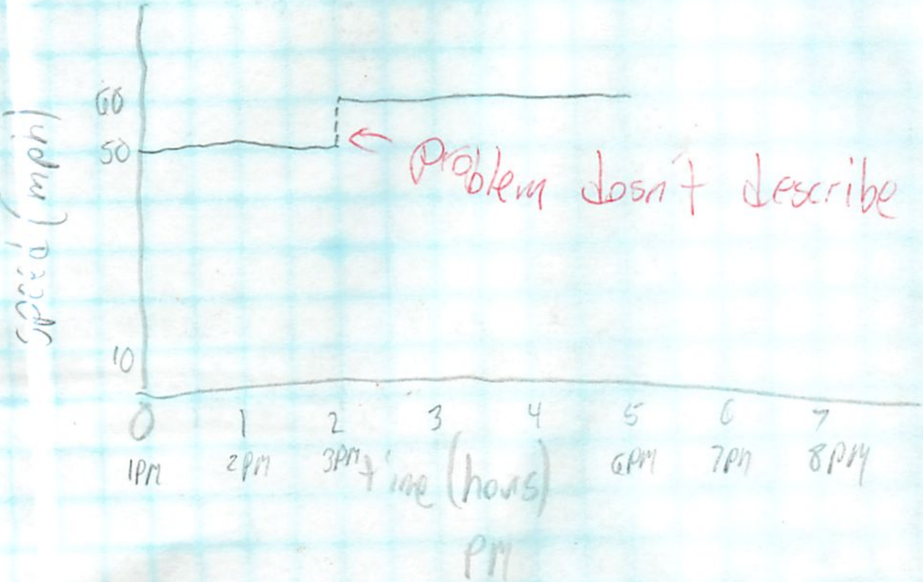
Distance w/ ΔSpeed p31

3/2

1. 1PM - 3PM @ 50 mph
3PM - 6 PM @ 60 mph

$50 \text{ mph} \cdot 2 \text{ hrs} = 100 \text{ miles}$
$60 \text{ mph} \cdot 3 \text{ hrs} = 180 \text{ miles}$
280 miles

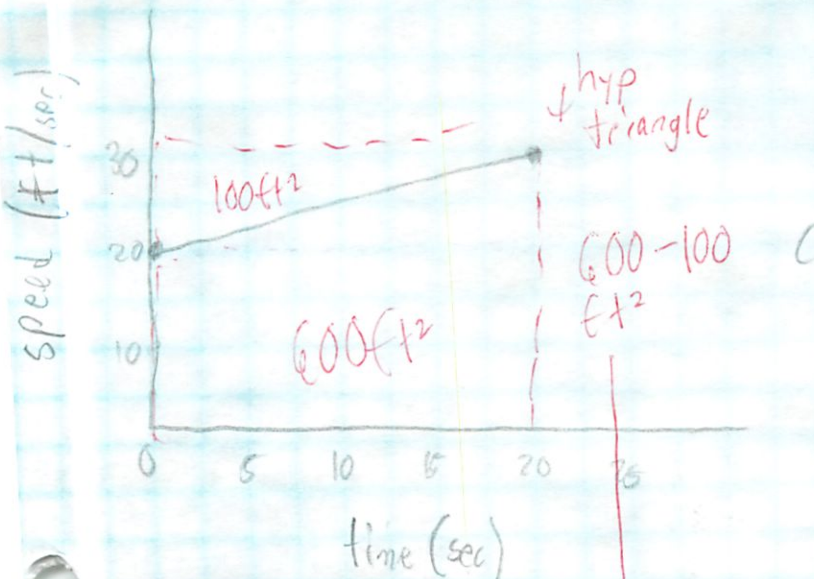
$280/5 = 56 \text{ mph}$



b. the area of x axis is the $\frac{\text{distance}}{\text{time}}$

don't get!

2. 20 ft/sec
20sec: 30 ft/sec



they ran 500 f ??

b. At 10 sec (middle) going 25 ft/sec which is the median (middle) time.

c. find the middle of the area

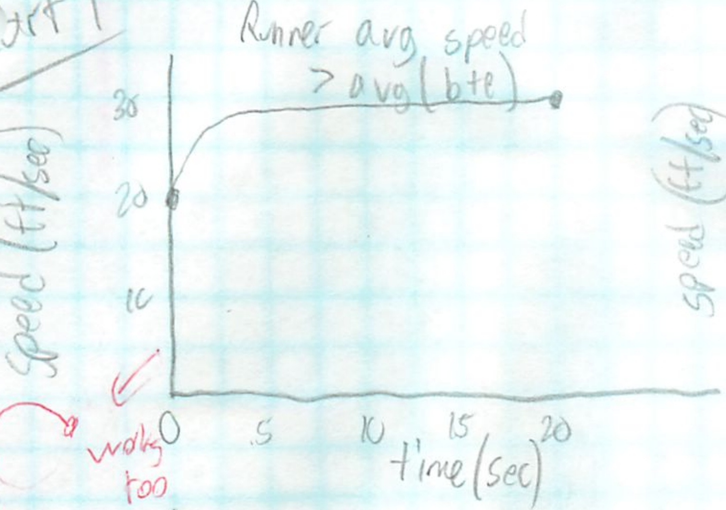
?? take avg of start and end point
- diff method, but same answer

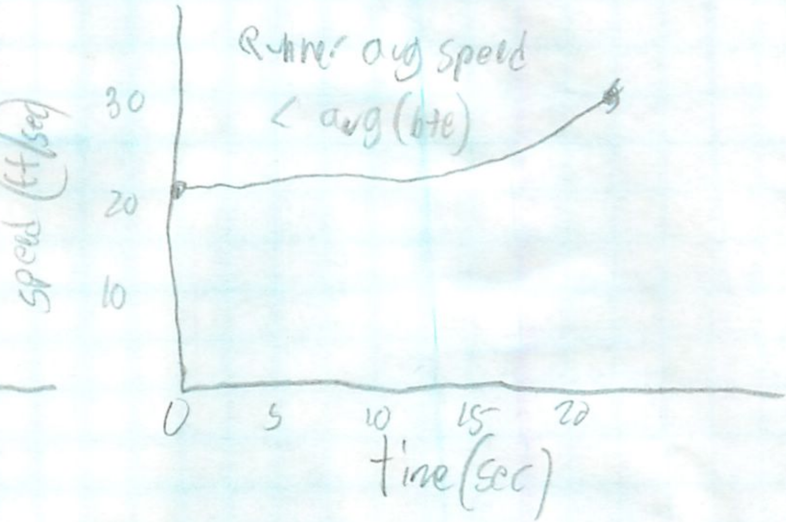
$\frac{20+30}{2} = 25 \text{ ft/sec}$

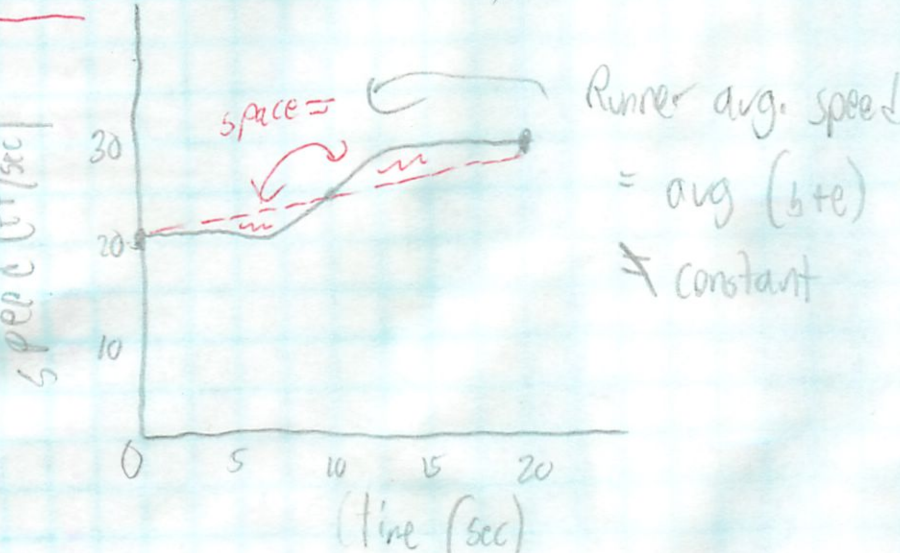
Acceleration Variation (8)

3/2

Part 1







$\frac{\text{speed}}{\text{time}} \neq \frac{\text{distance}}{\text{time}}$

graph

Vocab

Acceleration - The rate at which speed changes

Speed - The rate at which the position changes

Avg. Speed - $\frac{\text{inital + final speed}}{2}$) constant acc

Free Fall

p34 3/5

Instantanous Speed - the rate an object is moving at a certain very small period in time

time (sec)	Instant Speed (ft/sec)	Avg Speed (ft/sec)	Distance (ft)
0	0	0	0
1	32	16	~~32~~ 16
2	64	32	64
3	96	48	144
4	128	64	256
5	160	80	400
n	32n	$\frac{0+32n}{2}$ -or- 16n	$\frac{1}{2}32n^2$ -or- $V_{avg}\, n$

constant acc $= \frac{V_f - V_i}{t}$

← ?? ← d = rate × time

1. $t=5$, $V_{instant} = 160$ ft/sec distance = 400 ft

2. Distance $= \frac{1}{2}32t^2 \rightarrow 16t^2$

3. $h_i - 16t^2 = h$ ← would be 0, b/c prob say it reaches ground

4. $-h_i \qquad -h_i$

$$\frac{-16t^2}{-16} = \frac{h-h_i}{-16}$$

$$t^2 = \frac{h-h_i}{-16}$$

$$t = \sqrt{\frac{h-h_i}{-16}}$$ ← would be 0

why neg? $\frac{\sqrt{h-h_i}}{4} \rightarrow \frac{\sqrt{h_i}}{4} = t$

✓ if $h_i = 0$, use this for real simple answer

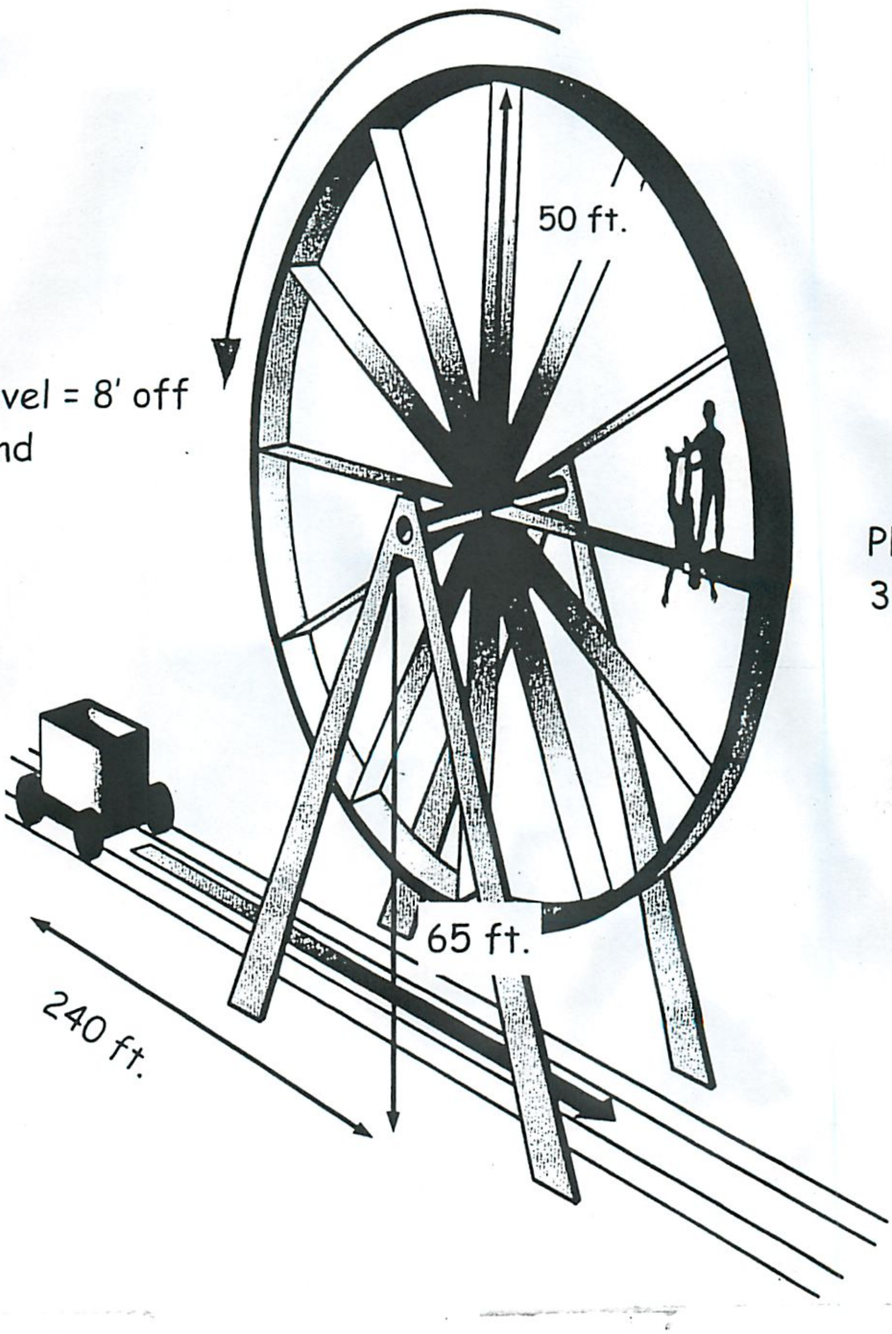
5. $t = \sqrt{\frac{8-90\text{ ft}}{-16}}$

$t = \sqrt{\frac{-82}{-16}}$

$t = \sqrt{5.125}$

$t = 2.26$

1 complete turn every 40 seconds



Not So Spectacular 9

3/5

Algbra

$h = 50(\sin 9t) + 65$

$50(\sin 9t) + 65 \leq 25$

$-65 \qquad -65$

$\frac{50(\sin 9t)}{50} \leq \frac{-40}{50}$

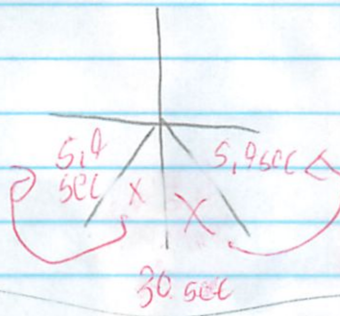
$\sin(9t) \leq -.8$

$\sin^{-1} \qquad \sin^{-1}$

$\frac{9t}{9} \leq \frac{-53.13}{9}$

$t \leq -5.903''$ ← sec not °

almost had in, needed 1 step more



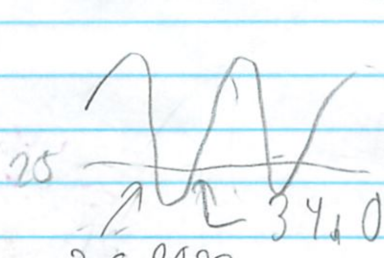
also its here

?? didn't work

Guess + ✓: About 25.9 to 34.1

Graph and find intersects

close



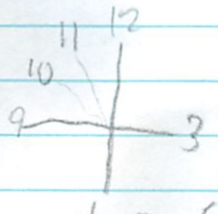
sec

← between 50(sin 9t) + 65 & 25

$25.9033 \leq \text{drop} \leq 34.096$

Practice Jump 10

3/6

1. 

1/4 turn = 10 sec

$\frac{1}{3}$ of that = 13.33 sec

or $\frac{360}{12} = 30 \cdot 4 = 120°$

$\frac{120°}{9} = 13.33$ sec

2. vert = $50 \sin(9 \cdot 13.33) + 65 = 108.3012$ ft

horz = $50 \cos(9 \cdot 13.33) = -25$ ft

3. $\sqrt{\frac{108.3012 - 8}{16}} = \sqrt{6.2688} = 2.5038$ sec

4. $\sqrt{\frac{(50\sin(9 W_t) + 65) - 8}{16}} = F_T$ ← fall time

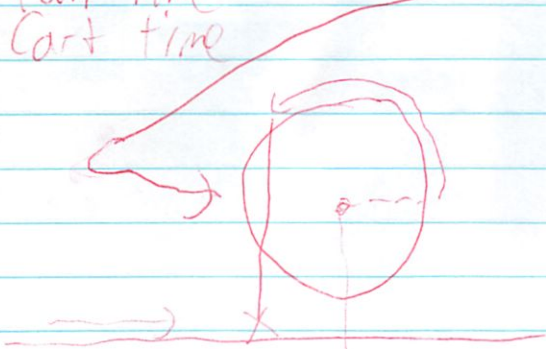
times

W_t = wheel time

F_t = fall time

C_t = Cart time

forget that diver will move tanget outward tow wheel



cart time

must release cart a certain time

$$\boxed{C_T = \sqrt{\frac{[50\sin(9 t_4) + 65] - 8}{16}} + W_T}$$

Cart Time
p38

3/6

$$C_T = F_T + W_T$$

$$C_T = \sqrt{\frac{[60\sin(aW_T)+65]-8}{16}} + W_T$$

The cart time must equal to fall time and the initial wheel time. This means it arrives just in time.

When to let go of the cart can derived from this

Carts + Periodic Problems 12

3/6

Part 1 Where's the cart

not finding actual #

t (s)	xcord
0	-240
1	-225
2	-210
3	-195
4	-180
5	-165
6	-150
7	-135

t	x-cord
8	-120
9	-105
10	-90
11	-75
12	-60
13	-45
14	-30
15	-15
16	0
17	15
18	30
19	45
20	

← 18.666 sec

~~The cart should be let go 18.666 sec before it will be under t=0, the diver needs to be let go at ~16.78 sec after cart w/ wheel not turning~~

Part 2

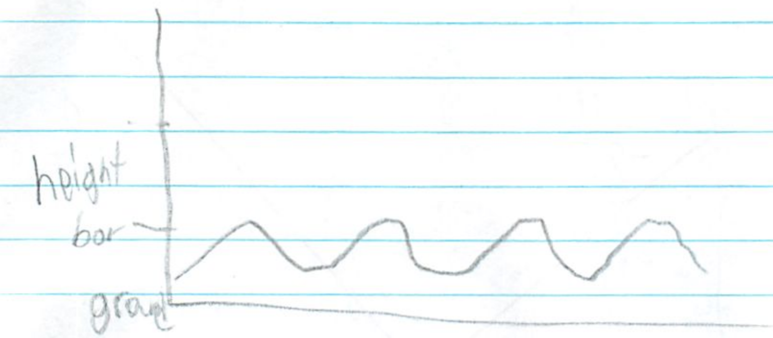
happens over + over again

must repete max + min at reg intervals

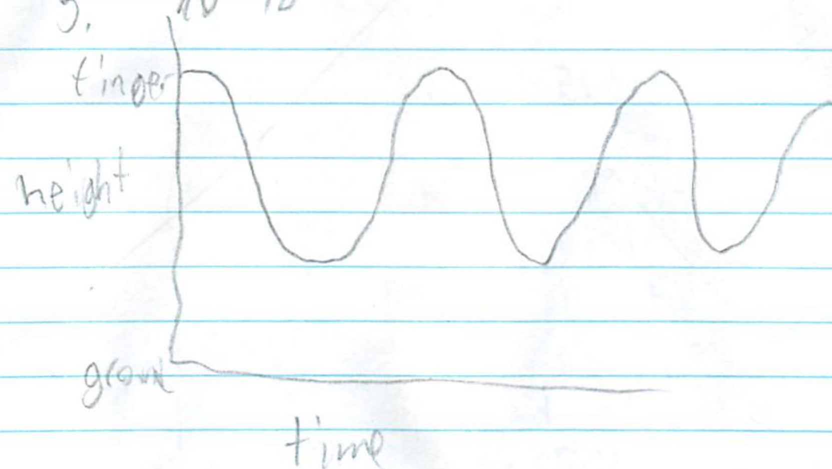
1. A person on a swing with out friction or air restance
2. Clock hands related to x or y axis
   ^minute + hour + second
3. A yo-yo
4. Phases of moon

5. tides

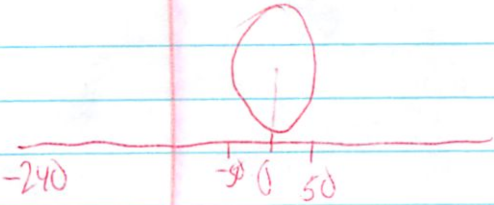
2. Swing



3. Yo-Yo



Part 1 redo



Cart pos. $= -240 + 15(W_T + F_T)$

$\cancel{\sqrt{\frac{[50\sin(9W_T)]+65-8}{16}} + W_T = -240+15}$

$$-240 + 15\left(\sqrt{\frac{[50\sin 9W_T]+65-8}{16}} + W_T\right)$$

Cart Before Wheel (14)

3/7

1. at 25 sec horz pos = 50 cos (9 · 25)
   −35.355
   ← means to the left

2. fall time (25 sec) = 1.1630 sec $\frac{\sqrt{50 \sin(9 \cdot 25) + 65 - 7}}{4}$

   Cart time = fall + wheel times
   25 + 1.1630 = 26.1630 sec

   cart travels 15 ft/sec

   $\overset{sec}{26.1630} \cdot \overset{ft}{15}$ = 392.45 ft cart should travel
   + 35.355 ← needs to stop this much to the left
   427.8 ft away from center of wheel to left

   left where dive is

Find the Wheel 16

3/7

1. $25 \cot 10t$ amp = 25 (radius)
pd = $\cancel{2x}$ 360/10 = 36 sec
angular speed = 10°
∴ just 360/ ang speed sec

$100 \cos 3t$ amp = 100 (radius)
pd = $\cancel{2x}$ $\cancel{2\pi/3}$ = 120 sec
ang speed = 3

2. $10 \cot 20t$

b. The graph of the function would not reach as high and it would be squashed together more

✓

360/10 = 36 sec } less squashed together
360/20 = 18 sec
period

smaller angular speed

Planning for Formulas (13)

3/7

diver's

vertical height $50(\sin(9W_t) + 65 - 8$ ← height of cart water

↓ center's height off ground

distance (center to top of cart

radius; 360/40 cycle time (angular speed)

horizontal position $50\cos(9W_t)$

↓ vertical height

fall time $\dfrac{\sqrt{50\sin(9W_t)+65-8}}{4}$ ← acceleration $\sqrt{16} \to 4$ constant

P34 #4

Cart's pos when diver enters $-240 + 15\left(\dfrac{\sqrt{50\sin(9W_t)+65-8}}{4} + W_t\right)$

↑ cart's start position

↑ feet it moves per sec

↑ fall time

↑ wheel time

50 - radius
65 - center to ground
40 sec per turn around - or 9°/sec
240 ft - cart's start position
15 ft → - cart's constant speed
8 ft → water level in cart - must subtract when calculating falling distance
3 o'clock - start at start of 1st quad

← 3 o'clock

Cart pos = horz pos

when he enters water

Moving Cart, Turning Ferris Wheel **Group Names** P/a 2 Jimmy kelly

23 points

On page 49 of your textbook please complete Moving Cart, Turning Ferris Wheel. Show all equations used and all work clearly. Any reader should be able to follow your work.

1) Determine the correct value of W.

a) Write the equation used to find the value of W. (2 points)

Cart pos = horz pos

$$-240 + 15\left(\frac{\sqrt{50\sin(9W_T)+65-8}}{4} + W_T\right) = 50\cos(9W_T)$$

b) Explain why you used the above equation. (3 points)

She told us that cart's pos must = the wheels' horiz pos, when the two meet. So you can find where the two intersect to find the proper time needing to be elapsed before the assistant lets go (aka W_T)

c) Show how you solved the equation for W (nearest hundredth of a second). (3 points)

Put both in calculator as $Y_1 + Y_2$
Find the intersection point of the two curves

Intersection

(12.28, -17.54)

time, horz pos

Find intersection
[Graph]
[Calc] → [2nd][Trace]
[5]
[Enter]
[Enter]
[Enter]

Show ALL work for #2 - #6!!!!!

2) Where will the platform be in the Ferris Wheel's cycle when the diver is dropped? Answer in degrees (to the nearest hundredth). (3 points)

We thaght way too far into this

$9 \times 12.282855 = 110.55°$

3) Where is the cart when the diver is released from the platform? Include if it's to the left or the right of the center of the ferris wheel in your answer. (to the nearest hundredth of a foot) (3 points)

$-240 + 15(12.282855) = -55.76$ ft

means to the left

4) Where is the cart when the diver hits the water? Include if it's to the left or the right of the center of the ferris wheel in your answer. (to the nearest hundredth of a foot) (3 points)

$-240 + 15\left(\dfrac{\sqrt{50\sin(9 \cdot 12.282855) + 65 - 8}}{4} + 12.282855\right)$

$= -17.55$ ft

means to left

5) How far does the diver fall? (to the nearest hundredth of a foot) (3 points)

$50\sin(9 \times 12.282855) + 65 - 8$

103.82 ft fall to water

6) How long will the diver be in the air? (to the nearest hundredth of a second) (3 points)

$\dfrac{\sqrt{50\sin(9 \times 12.282855) + 65 - 8}}{4}$

$= 2.55$ sec

Name: Michael Plasmeier

Date: 3/8

IAG 4

High Dive

Ferris Wheeler's Day 'Off'

Directions: Using your High Dive Formula's and your growing knowledge of trigonometry, complete each of the problems below. For all problems, we will assume the usual Ferris Wheel units – radius is 50 feet, the center is 65 feet off the ground, and the wheel has an angular speed of 9° per second. Show all work. Good Luck.

Ferris Wheel Height Problems

1. Find the height of any Ferris Wheel diver who decides to jump at the following times after the diving platform passes the 3:00 position:

(a) After 3 seconds $50\sin(9W_t)+65$ *Height:* 87.7 ft

(b) After 9 seconds *Height:* 114.38 ft

(c) After 24 seconds *Height:* 35.61 ft

(d) After 46 seconds *Height:* 105.45 ft

(e) After 1 minute and 10 seconds 70sec *Height:* 15 ft

2. Suppose the diver will only dive from heights that are at or below 45 feet. Between what two times will the diver have to jump in order keep his dives at or below 45 feet? (Keep your times limited to times in one period...which is 40 seconds).

$45 = 50\sin(9W_t)+65$
$-65 \quad -65$
$-20 = 50\sin(9W_t)$
$\frac{-20}{50} = \frac{50\sin(9W_t)}{50}$
$-2/5 = \sin(9W_t)$

$\sin^{-1}(-2/5) = 9W_t$
$\frac{-23.54}{9} = \frac{9W_t}{9}$
$-2.62 = W_t$ need to convert
3rd: $20+2.62 = 22.62$
4th: $40-2.62 = 37.38$

Times: $22.62 \le x \le 37.38$

3. Flying Felipe is the most daring Ferris Wheel Diver in the world. Suppose he performs his jumps only from a height of 100 feet. At what two times should he be released in order to be able to jump from 100 feet? exactly 100 ft, not interval

mental math error

$100 = 50\sin(9W_t)+65$
$-65 \quad -65$
35 $\cancel{45} = 50\sin(9W_t)$
$\frac{}{50} \quad \frac{}{50}$
$\frac{9}{10} = \sin(9W_t)$
$9W_t = \sin^{-1}(\frac{9}{10})$

$\frac{9W_t}{9} = \frac{64.152}{9}$
$W_t = \cancel{7.1287}$

~~2nd: 20 - 7.12~~

Times:
1st: ~~7.1287~~ 4.94
2nd: ~~12.87~~ 15.1

Diver's Fall Time Problems

4. Find the diver's "Fall" time if he/she is released at the following times.

(a) After 10 seconds — Fall Time: 2.587 sec

(b) After 15 seconds — Fall Time: 2.4025 sec

$\frac{\sqrt{50\sin(9t)+65-8}}{4}$

(c) After 33 seconds — Fall Time: 1.88 sec

(d) After 40 seconds — Fall Time: 1.887 sec

Diver's X-Coordinate Problems

5. Find a diver's x-coordinate if he/she is released after the following times:

(a) After 11 seconds — X-Coordinate: (-7.82, 0)

(b) After 22 seconds — X-Coordinate: (-47.55, 0)

(c) After 35 seconds — X-Coordinate: (35.35, 0)

(d) After 1 minute — X-Coordinate: (-50, 0)

negitive means to left

6. After what times (between 0 and 40 seconds) does the diver have the following x-coordinates:

(a) X-Coordinate = 40 feet

$\frac{40}{50}=\frac{50\cos(9W_t)}{50}$

$\frac{4}{5}=\cos(9Wt)$

$\cos^{-1}\left(\frac{4}{5}\right)=9Wt$

$\frac{36.8699}{9}=\frac{9Wt}{9}$

$W_t = 4.097$

Times: 1st: 4.097 sec

4.09

4.09

4th: 40-4.09 = 35.9033 sec

(b) X-Coordinate = -25 feet

$\frac{-25}{50}=\frac{50\cos(9W_t)}{50}$

$-\frac{1}{2}=\cos(9W_t)$

$\cos^{-1}\left(-\frac{1}{2}\right)=9wt$

$\frac{120}{9}=\frac{9wt}{9}$

2nd: Times: $13\frac{1}{3}$ sec

3rd: $20+7\frac{1}{3}$ $27\frac{1}{3}$ sec

(c) X-Coordinate = 4 feet

$\frac{4}{50}=\frac{50\cos(9W_t)}{50}$

$\frac{2}{25}=\cos(9W_t)$

$\cos^{-1}\left(\frac{2}{25}\right)=9Wt$

$\frac{85.411}{9}=\frac{9Wt}{9}$

$9.49 = W_t$

Times: 1st: 9.49 sec

4th: 40-9.49 = 30.51 sec

Cart's X-Coordinate Problems

7. Find the x-coordinate of the cart when it catches the diver if it is positioned 240 feet to the left of the center of the Ferris Wheel when it starts moving; use the diver release times to calculate your answer. Indicate whether or not it will be a 'Splash' or a 'Splat.'

(a) Diver is released after 12 seconds — *X-Coordinate:* (-21.655, 0)

Platform = -15.45
Cart = -21.655

Splash — Splat

(b) Diver is released after 20 seconds — *X-Coordinate:* (88.31, 0)

Platform = -50
Cart = 88.311

Splash — Splat

(c) Diver is released after 57 seconds — *X-Coordinate:* (648.48, 0)

Platform = 87.7
Cart = 648.48

Splash — Splat

neg means to the left

8. If the diver is released 24 seconds after passing the 3:00 position, and the water cart moves at 15 feet per second from its starting position, then find the following:

(a) The diver's x-coordinate for the fall — *X-Coordinate:* (-40.45, 0)

$50\cos(\theta t)$

-40.45

(b) The cart's starting position coordinate (for 'Splash') — *X-Coordinate:* 420.19 to the left

$-40.45 - 15\left(\frac{\sqrt{50\sin\theta t + 65 - 8}}{4} + W_t\right)$

$-40.45 - 15(1.316 + 24)$

-420.19

(-420.19, 0)

The PUT-IT-ALL-TOGETHER Problem

9. Flying Felipe is going for it all – he wants to set the Ferris Wheel diving record by diving from a height which is 105 feet off of the ground. In order to perform the jump, his cart crew has to re-figure the positioning of the cart, etc. They will need to gather some important information.

(a) At what two times (between 0 and 40 seconds) will Felipe need to be released in order to dive from 105 feet?

$105 = 50\sin(9W_t) + 65$
$-65 \qquad -65$

$\frac{40}{50} = \frac{50\sin(9W_t)}{50}$

$\frac{4}{5} = \sin(9wt)$

$9wt = \sin^{-1}\left(\frac{4}{5}\right)$

$\frac{9wt}{9} = \frac{53.13}{9}$

1st $W_t = 5.9033$ sec

2nd: $20 - 5.90$ 14.0966 sec

(b) What are Felipe's x-coordinates at the two times you found in part (a)?

$50\cos(9t)$ $\quad 30.000 \rightarrow (30, 0)$

$-29.999 \rightarrow (-30, 0)$

(c) What will Felipe's "Fall" time be when he dives from 105 feet? Remember that the water level in the tub is 8 feet above the ground.

$\frac{\sqrt{50\sin(9t)+65-8}}{4}$ $\quad 2.4622$ sec

or just $\frac{\sqrt{105-8}}{4}$

(d) At what x-coordinates should Felipe's cart crew start the cart in order to catch Felipe (assuming that the cart is moving at 15 feet per second)? You will need to give two answers here, one for each of the x-coordinates where the diver could land.

$30 - 15(2.4622 + 5.9033) = (-95.48, 0)$ 1st

$-30 - 15(2.4622 + 14.0966) = (-278.38, 0)$ 2nd

Needs to stop at

(e) Suppose that the cart crew refused to start the cart at any location other than the one 240 feet from the left of the center of the Ferris Wheel. However, they were capable of adjusting the speed of the cart in order to catch Felipe in time. At what two different speeds will the cart need to travel in order to catch Felipe at the two possible x-coordinates at which he could land?

$30 = -240 + v(2.4622 + 5.9033)$
$+240 \qquad +240$

$\frac{270}{8.3721} = \frac{v(8.3721)}{8.3721}$

~~$v = 28.66$ ft/sec~~

$v = 32.25$ ft/sec

$-30 = -240 + v(2.4622 + 14.0966)$
$+240 \qquad +240$

$\frac{210}{16.5588} = \frac{v(16.5588)}{16.5588}$

~~$v = 14.4938$ ft/sec~~

$v = 12.68$ ft/sec

Name: Michael Ploesmeier Date: 3/4

IAG 4

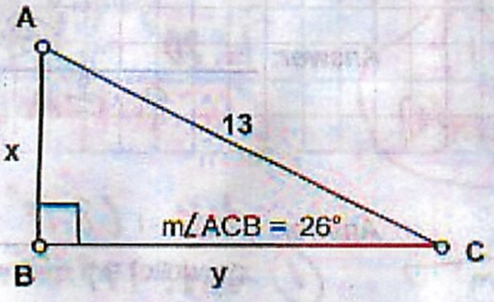
High Dive

High Divin' Revivin'

Directions: Show all work to complete each of the problem below. Good Luck.

Find the missing angle or side in each of the triangles below.

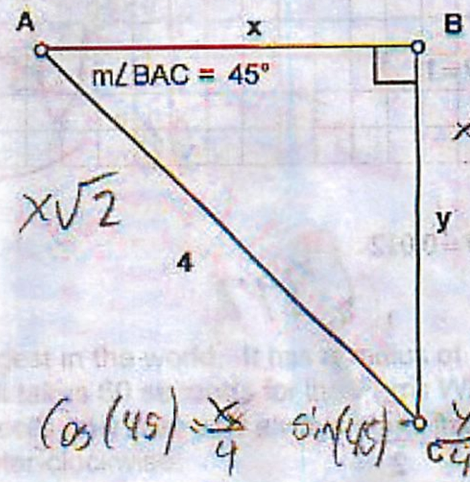
1.



$\sin(26) = \frac{x}{13}$ $\cos(26) \frac{y}{13}$

x = 5.70 y = 11.7

2.



x

$x\sqrt{2}$

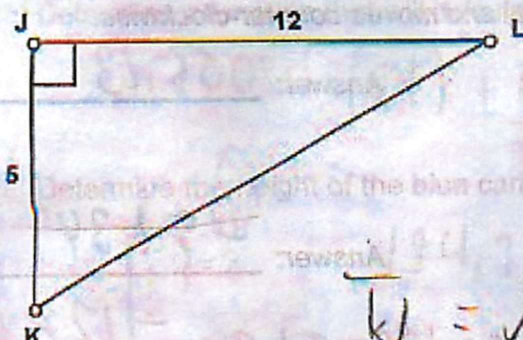
$\cos(45) = \frac{x}{4}$ $\sin(45) = \frac{y}{4}$

x = 2.828 y = 2.828

$\sqrt{8}$ or $2\sqrt{2}$ $\sqrt{8}$ $2\sqrt{2}$

use special designations

3.



$m\overline{KL}$ = 13

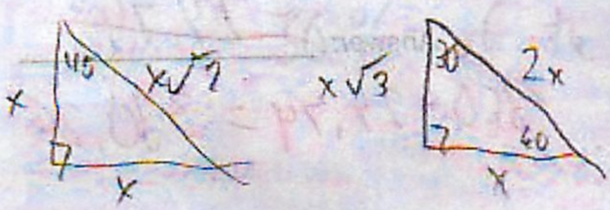
$m\angle JLK$ = 22.62°

$m\angle LKJ$ = 67.38°

$\overline{KL} = \sqrt{5^2+12^2} = 13$

$\cos^{-1}(12/13)$

$\cos^{-1}(5/13)$



sin

how to do?

Find the values for θ between 0° and 360°, given the values for $\sin\theta$. 4,512

4. $\sin\theta = -0.98$

$\theta = \sin^{-1}(-.98)$ -78.52

Answer: 3rd: 258.21 or 4.51r

4th: 281.47° or 4.91r

5. $\sin\theta = -0.5$

$-30°$

Answer: 3rd: 210° or $\frac{7\pi}{4}$

4th: 330° or $\frac{11\pi}{6}$

6. $\sin\theta = 1$

$90°$

Center $\sin^{-1}(1)$ not $\sin(1)$

Answer: 1st: 90° or $\frac{\pi}{4}$

? = no total answer

7. $\sin\theta = 0.012$

$.6877$

check term

Answer: 1st: .6877° or .012r

2nd: 179.31° or 3.13r

8. $\sin\theta = -\frac{\sqrt{2}}{2}$

-45

Answer: 3rd: 225° or $\frac{5\pi}{4}$

4th: 315° or $\frac{7\pi}{4}$

Can check

Find the measure of the angle that passes through the points below. Remember that the angle begins on the first-quadrant side of the x-axis, and moves counter-clockwise.

9. (3, 4)

$\tan^{-1}(4/3)$

opp / adj

Answer: 55.13°

10. (-4, 5)

$\tan^{-1}(5/-4)$

$\theta = 180 - 51.34 = 128.66$

Answer: ~~-51.34°~~

have it move counter clock wise

11. (-2, -1)

$\tan^{-1}(-1/-2)$

Answer: ~~26.57°~~ 206.57

12. (7, -4)

ref angle

$\tan^{-1}(-4/7)$

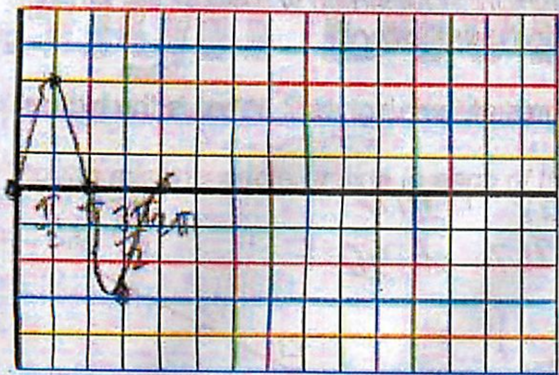
Answer: ~~-29.74°~~

$360 - 29.74 = 330.26$

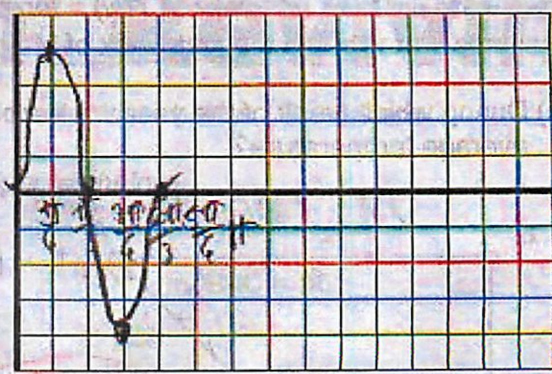
Graph the following sine functions on the graphs below. Graph only from 0° to 360°, and use an appropriate scale.

Period: $2\pi/3 = \frac{2}{3}\pi$

13. $y = 3 \cdot \sin(x) + 1$



14. $y = 4 \cdot \sin(3x)$



ticks: $4 / \frac{2}{3}\pi = \frac{4}{1} \cdot \frac{3}{2\pi} = \frac{12\pi}{2}$ $\frac{6\pi}{6}$

Complete the following.

15. The Ferris Wheel at Super Park is one of the biggest in the world. It has a radius of 100 feet, and its center is 120 feet from the ground. It takes 60 seconds for the Ferris Wheel to make one complete revolution. The Ferris Wheel's only blue cart always starts from the 3:00 position, and the wheel always rotates counter-clockwise.

(a) How many degrees/seconds is the wheel turning?

$360/60 = 6°/\text{sec}$

(b) Determine a function that will calculate the height of the blue cart at any time *t*.

$100 \sin(6t) + 120$

(c) Determine the height of the blue cart after 8 seconds

$t = 8$ 144.31 ft

(d) Determine the height of the blue cart after 42 seconds

$t = 42$

24.89 ft

(e) What percentage of the time is the blue cart 120 feet above the ground?

50 % of the time

16. Len "Tornado" Quartz is a local meteorologist for Channel 11. He has been asked to derive a mathematical function that will predict the average temperatures for the entire year in Philadelphia, PA. After doing some excellent research (which in no way involved some radar that lets you 'fly' into the storm, etc.), Len derived the equation $T(w) = 32 \cdot \sin(7 \cdot w) + 58$, where *T(w) = temperature in °F based on w weeks.* His function assumes that *w = 0* is the first week of March (NOT JANUARY).

(a) During which week of the year are temperatures at their highest? What is the highest average temperature?

Use calc to find max + min

max: week 12.857 at 90°F

~~min: week 38.57 at 26°~~

avg = 58° ← midline

(b) During which week of the year are temperatures at their lowest? What is the lowest average temperature?

average means the data is already avg temp

min: week 38.57 at 26°F

avg = 58° ← midline ← shows that work

(c) During which weeks of the year are temperatures above 75°?

Use calc + find intersect

temp > 75° at 4.58 weeks

temp < 75° at 21.13 weeks

$4.58 \le x \le 21.13$ weeks

$75 = 32\sin(7w) + 58$

$-58 \qquad -58$

$\frac{17}{32} = \sin(7w)$

$\sin^{-1}\left(\frac{17}{32}\right) = 7w$

$\frac{32.09}{7} = \frac{7w}{7}$

$w = 4.58$

(d) During which weeks of the year are temperatures below 40°?

temp < 40° at 30.60 weeks

temp > 40° at 46.54 weeks

$30.60 \le x \le 46.54$ weeks

4.58 4.58

2nd: $\frac{26 - 4.58}{}$ 1st

21.42 weeks

26 weeks = ½ way around

IAG 34
HOT SHEET

Name

Date

Unit

Fall Time

$$F_T = \frac{\sqrt{50\sin(9W_T) + 65} + 8}{4}$$

Carts X position

$$C_{pos} = -240 + 15\left(\frac{\sqrt{50\sin(9W_T) + 65} + 8}{4} + W_T\right)$$

3/19 Makeup FT

70/73

Name: Plaz. Date: 3.16.07

IAG 4 High Dive

52/~~70~~ 73 Points

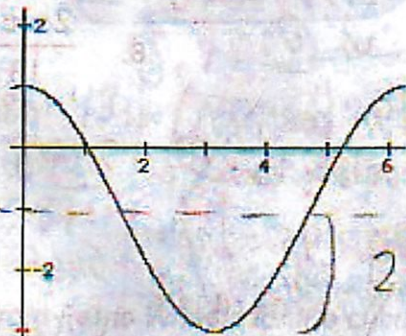
High Dive Test

Directions: Show all work to complete each of the problems below. Read all directions carefully. This test is worth ~~70~~ 73 **points.** Good Luck.

Choose the best answer for each of the following. **[2 points each]**

1. Find the amplitude of the graph shown below.

(a) 2
(b) 1
(c) 3
(d) –3



2. In which quadrants is the sine function negative?

(a) I and II
(b) II and III
(c) I and IV
(d) III and IV

S | A
T | C

+ | +
− | −

II | I
III | IV

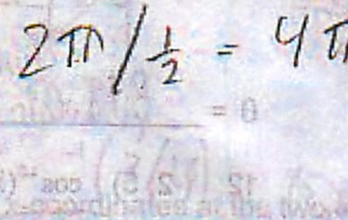
3. What is the period of the trig function $y = 4 \cdot \sin\left(\frac{1}{2}x\right) - 3$

(a) $\frac{\pi}{4}$
(b) $\frac{\pi}{2}$
(c) 4π
(d) π

$2\pi / \frac{1}{2} = 4\pi$

4. During a running race, Jess increased her speed from 18 feet per second to 22 feet per second over a span of 15 seconds. How far did she run over that time period?

(a) 270 feet
(b) 300 feet
(c) 330 feet
(d) 420 feet



$18 + 22 = 40 / 2 = 20$ avg ft/s

20 avg ft/sec • 15 sec =

300 ft

8

S | A
T | C

only degrees needed

Solve the following equations for $0^\circ \le \theta < 360^\circ$. Write your answers in terms of degrees and round to the nearest hundredth. ~~SHOW ALL~~ *WORK!!* **[#5-7 -> 3 pts. each; #8 – 4pts].**

5. $4\sin\theta = 3$

$\frac{4\sin\theta}{4} = \frac{3}{4}$

$\sin\theta = \frac{3}{4}$

$\theta = \sin^{-1}(3/4)$

calc: 48.59

+: 1st + 2nd

1st: 48.59 or ~~.85 radians~~

✓, ✓

2nd: 180 − 48.59 = 131.41° or ~~2.29 radians~~

✓, ✓

6. $2\cos\theta - \sqrt{3} = 0$

$+\sqrt{3}$ $+\sqrt{3}$

$\frac{2\cos\theta}{2} = \frac{\sqrt{3}}{2}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\theta = \cos^{-1}(\sqrt{3}/2)$

calc: 30°

+: 1st + 4th

1st: 30° or ~~.52 radians~~ or ~~$\frac{\pi}{6}$~~ ✓

4th: 360 − 30° = 33[0] ~~or 5.76 radians~~ or ~~$\frac{11\pi}{6}$~~ ✓

7. $6\cot\theta - 11 = -5$

$+11$ $+11$

$\frac{6\cot\theta}{6} = \frac{6}{6}$

+: 1st + 3rd

$\cot\theta = 1$

$\tan\theta = \frac{1}{1}$

$\theta = \tan^{-1}(1)$

calc: 45°

1st: 45° ~~or .79 radians~~ ~~or $\frac{\pi}{4}$~~ ✓

3rd: 180 + 45 = 225° ~~or 3.93 radians or $\frac{5\pi}{4}$~~ ✓

8. $\frac{2\sec x}{13} = \frac{-1}{4}$

$\cdot 13$ $\cdot 13$

$\frac{2\sec\theta}{2} = \frac{-13/4}{2}$

$\sec\theta = \frac{-13}{8}$

$\cos\theta = \frac{8}{-13}$

$\theta = \cos^{-1}(8/-13)$

calc: 127.98

+: 1st + 4th

1st: 127.98 ~~or 2.23 radians~~ ½

4th: 360 − 127.98 = 232.02 ~~or 4.05 radians~~ ½

Using the stated inverse trig function, determine the value of the angle between 0° and 360° whose terminal side passes through the points given below. *NOTE:* *If an inverse trig functions is not stated, you can use any appropriate one. Round to the nearest hundredth.*
[3 points each]

9. (6, 4) ; $\sin^{-1}(x)$

(6, 4) $\sqrt{6^2+4^2}$

$\sqrt{52}$

$\sin = \frac{y}{r} = \frac{4}{\sqrt{52}} \to \frac{4\sqrt{52}}{52}$

$\sin^{-1}\left(\frac{4\sqrt{52}}{52}\right)$

θ = 33.69°

10. (10, -9)

$\tan = \frac{y}{x} = \frac{-9}{10} = -\frac{9}{10}$

(10, -9) $\tan^{-1}(-9/10)$

$-41.99 + 360$

θ = 318.01

11. (-3, -3)

$\tan = \frac{y}{x} = \frac{-3}{-3} = 1$

$\tan^{-1}(1)$

$45 + 180$

(-3, -3)

θ = 225°

12. (-2, 5) ; $\cos^{-1}(x)$

(-2, 5) $\sqrt{5^2+2^2} = \sqrt{29}$

$\cos = \frac{x}{r} = \frac{-2}{\sqrt{29}} \to \frac{-2\sqrt{29}}{29}$

$\cos^{-1}\left(\frac{-2\sqrt{29}}{29}\right)$

θ = 118.80°

copy error? 111.80 I guess

24

Complete each of the following.

13. Sweaty Sam and his brother have a routine when they go fishing in Ocean City – they like to fish from one hour before high tide until one hour after high tide (remember that 'high tide' is the instant at which the tide is at its highest height). The tides in Ocean City can be modeled by the function $h(t) = 7 \cdot \sin(30.2 \cdot t) + 4$, where $h(t)$ is the height of the tide in feet and t is the time in hours. Given that $t = 0$ is midnight, when should Sweaty Sam and his brother go fishing if they want to uphold their tradition? Give your answer in **ACTUAL TIME UNITS AND INCLUDE ALL POSSIBLE TIMES**. You may find your answer algebraically or graphically. **[3 points]**

Using calculator to graph the function and find the maximum which is (2.98, 11). One hour before and after this point is 1.98 and 3.98 hours. 1.98 hours is actually 118 min 48 sec (1.98·60) after midnight or 1:58:48 AM, 3.98 is actually 238 min 48 sec after midnight or 3:58:48 AM

Times: 1:58:48 ≤ x ≤ 3:58:48 AM

-1 another interval? oh like half way through the day 1:54PM 3:54PM

14. *Jeff Drown loves to leap from Ferris Wheels for the circus. For his latest stunt, he wants to dive into a cart that is filled with a Wendy's Frosty. He would like to be released from a height that is 100 feet off of the ground. In order to perform the jump, his cart crew has to re-figure the positioning of the cart, etc. They will need to gather some important information. Round to the nearest hundredth in parts a – d. (Assume radius = 50 ft., the distance from the center to the ground is 65 ft.)*

(a) At what two times (between 0 and 40 seconds) will Jeff need to be released in order to dive from 100 feet? **[4 points]**

$100 = 50\sin(9W_t) + 65$
$-65 \qquad -65$
$\frac{35}{50} = \frac{50\sin(9W_t)}{50}$
$\frac{7}{10} = \sin(9W_t)$
$9W_t = \sin^{-1}(7/10)$

$\frac{9W_t}{9} = \frac{44.43}{9}$
$W_t = 4.94$ sec

and $20 - 4.94 = 15.06$ sec

Time #1: 4.94 sec

Time #2: 15.06 sec

C

(b) What are Jeff's x-coordinates at the two times you found in part (a)? **[4 points]**

$50\cos(9 \cdot 4.94) = 35.71$

$50\cos(9 \cdot 15.06) = -35.71$

Coordinate #1: (35.71, 0) C

Coordinate #2: (-35.71, 0)

10

(c) What will Jeff's "Fall" time be when he dives from 100 feet? Remember that the "Frosty" level in the tub is 8 feet above the ground. **[2 points]**

$\sqrt{\dfrac{50\sin\left(\frac{\pi}{10}4.94\right)+65-8}{16}}$ $\sqrt{\dfrac{50\sin\left(\frac{\pi}{10}15.06\right)+65-8}{16}}$ Just $\dfrac{\sqrt{92}}{4}$

2.40

Fall Time: 2.40 sec

(d) At what x-coordinates should Felipe's cart crew start the cart in order to catch Felipe (assuming that the cart is moving at 15 feet per second)? You will need to give two answers here, one for each of the x-coordinates where the diver could land. **[6 points]**

$35.71 = S + 15(4.94 + 2.40)$

$35.71 = S + 110.1$

$-35.71 \quad -35.71$

$S = 74.39$

↓ b/c its to the left

stupid mistake

Cart Coordinate #1: $(-74.39, 0)$

Cart Coordinate #2: $(297.61, 0)$

$-35.71 = S + 15(15.06 + 2.40)$

$-35.71 = S + 261.9$

$+35.71 \quad +35.71$

$S = 297.61$

assuming some latitude in the width of the cart

Problem checks out on FERRIS

√/4

7

Non-Calculator Section All correct **Name** Plaz

24/76 points
21/73

For each equation, find the amplitude, period, phase shift, and vertical shift. Then sketch and label a graph of one period of the function on the graph paper. Make sure your graph is accurate and label the five critical points. Label the axes. **[6 points each]**

1. $y = 3 \cdot \sin(x - \pi) + 1$

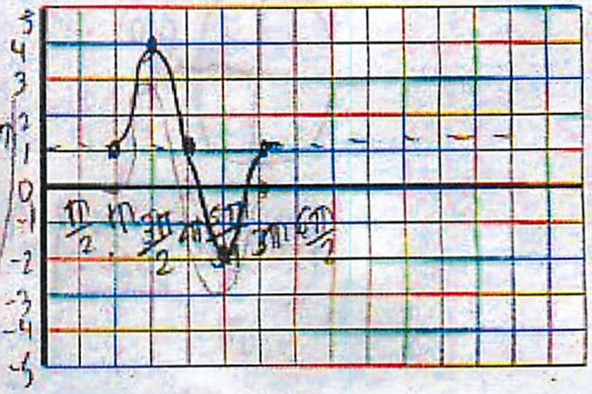
Amplitude: 3

Period: 2π   2π/1 = 2π

Phase Shift: → π   π/x = π

Vertical Shift: ↑ 1

Interval: 2π/4 = ½π



2. $y = 2 \cdot \cos(2x) - 1$

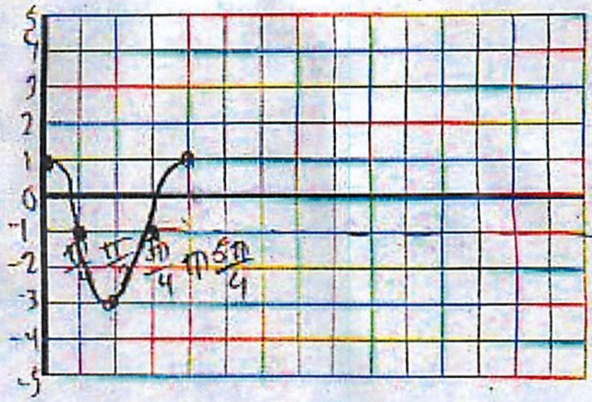
Amplitude: 2

Period: π   2π/2 = π

Phase Shift: –

Vertical Shift: ↓ 1

Interval: π/4 = ¼π



3. $y = 2 \cdot \sin(4x - \pi) + 3$

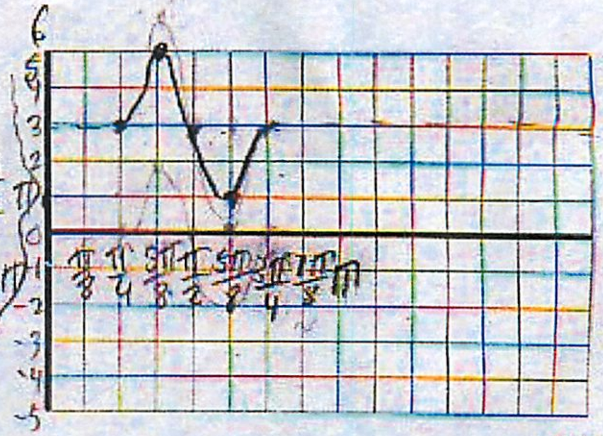
Amplitude: 2

Period: π/2   2π/4 = ½π

Phase Shift: → π/4   π/4 = ¼π

Vertical Shift: ↑ 3

Interval: ½π/4 = ⅛π

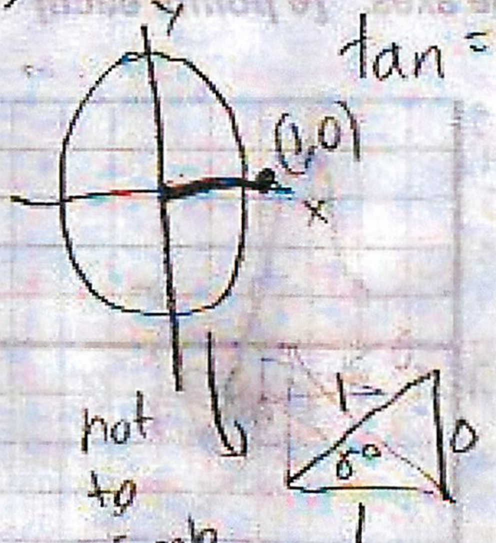


18

For #4 & #5, *evaluate the expressions and write your answers in both degree measure and radian measure.* **SHOW ALL WORK AND USE PROPER NOTATION!!!** **[3 pts. each]**

4. $\cancel{\cos^{-1}\left(-\frac{\sqrt{3}}{3}\right)}$

5. $\tan^{-1}(0)$



not to scale

$\tan = \frac{y}{x} = \frac{0}{1} = 0$

$0°$ or 0 radians

Factor by Grouping

3/12

4 terms not 2 or 3

$2x^3+3x^2+10x+15$ ← normally first two then last two but not always

$x^2(2x+3)+5(2x+3)$

$(2x+3)(x^2+5)$

do w/ third

x^3+4x^2-2x-8

$x^2(x+4)-2(x+4)$

$(x+4)(x^2-2)$

$x^3-2x+4x^2-8$

$x(x^2-2)+4(x^2-2)$

$(x^2-2)(x+4)$

could do to √

$5x^3-20x^2+3x-12$

$5x^2(x-4)+3(x-4)$

$(5x^2+3)(x-4)$

make sure parenthese here or else its $5x^2+[3\cdot(x-4)]$

multiply just the three

bad

Name Michael Plasmeier

Factoring By Grouping

28-38 even
48, 49, 50

In Exercises 15–50, factor completely with respect to the integers.

15. $9x^2 - 4$
16. $3x^2 - 48$
17. ~~$x^3 - 8$~~
18. ~~$x^3 + 64$~~
19. ~~$216x^3 + 1$~~
20. ~~$125x^3 - 1$~~
21. $200x^2 - 50$
22. ~~$32x^3 - 4$~~
23. ~~$128x^3 - 2$~~
24. ~~$8x^3 - 64$~~
25. ~~$3x^3 + 81$~~
26. ~~$40x^3 - 5$~~
27. $x^3 + x^2 + x + 1$
28. $30x^3 + 40x^2 + 3x + 4$
29. $x^3 + 2x^2 + 5x + 10$
30. $x^3 - 2x^2 + 4x - 8$
31. $9x^3 + 18x^2 + 7x + 14$
32. $-2x^3 - 4x^2 - 3x - 6$
33. $2x^3 + 4x^2 + 4x + 8$
34. $18x^3 + 30x^2 + 3x + 5$
35. $2x^3 - 2x^2 + 5x - 5$
36. $2x^3 + 3x^2 - 32x - 48$
37. $5x^3 - 20x^2 + 3x - 12$
38. $18x^3 - 2x^2 + 27x - 3$
39. $7x^3 + 14x^2 + 7x$
40. $3x^2 - 24x + 48$
41. $2x^3 - 4x^2 - 3x + 6$
42. $6x^3 - 18x^2 - 2x + 6$
43. ~~$3x^3 - 24$~~
44. ~~$x^3 + 125$~~
45. $3x^4 - 300x^2$
46. ~~$28x^3 - 7x$~~
47. $3x^4 + 3x^3 + 6x^2 + 6x$
48. $x^4 + 12x^3 + 4x^2 + 48x$
49. $10x^3 - 20x^2 - 2x + 4$
50. $18x^3 - 9x^2 - 18x + 9$

In Exercises 51–62, find all real-number solutions.

51. $x^2 - 3x = 0$
52. $2x^3 - 6x^2 = 0$
53. $x^2 + 8x + 16 = 0$
54. $x^2 - 4x + 4 = 0$
55. $x^3 - 15x^2 + 75x - 125 = 0$
56. $x^3 + 3x^2 + 3x + 1 = 0$
57. $2x^2 - x = 3$
58. $x^2 + 3x = 10$
59. $x^3 + 3x^2 - 2x - 6 = 0$
60. $x^4 + 7x^3 - 8x - 56 = 0$
61. $x^3 + 2x^2 - x = 2$
62. $x^3 - x^2 + 2x = 2$

Factor by Grouping Worksheet

3/12

28. $30x^3 + 40x^2 + 3x + 4$

$10x^2(3x+4) + 1(3x+4)$

$(10x^2)(3x+4)$

+1 ← need the +1

30. $x^3 - 2x^2 + 4x - 8$

$x^2(x-2) + 4(x-2)$

$(x^2+4)(x-2)$

32. $-2x^3 - 4x^2 - 3x - 6$

$-2x^2(-x-2) + 3(-x-2)$ ← watch the neg signs

$(-2x^2+3)(-x-2)$

34. $18x^3 + 30x^2 + 3x + 5$

$6x^2(3x+5) + 1(3x+5)$

$(6x^2)(3x+5)$

+1 ← need +1

36. $2x^3 + 3x^2 - 32x - 48$ ← try rearranging

$2x^3 - 32x + 3x^2 - 48$

$2x(x^2-16) + 3(x^2-16)$

$(2x+3)(x^2-16)$ ← do more?

38. $18x^3 - 2x^2 + 27x - 3$

$2x^2(9x-1) + 3(9x-1)$

$(2x^2+3)(9x-1)$

48. $x^4 + 12x^3 + 4x^2 + 48x$

$x^3(x+12) + 4x(x+12)$

$(x^3+4x)(x+12)$ ↓ go further

$x(x^2+4)(x+12)$

take x out

49. $10x^3 - 20x^2 - 2x + 4$

$10x^2(x-2) - 2(x-2)$

take further gcf

$(10x^2-2)(x-2)$

$2(5x^2-1)(x-2)$

50. $18x^3 - 9x^2 - 18x + 9$

$9x^2(2x-1) - 9(2x-1)$

$(9x^2-9)(2x-1)$

gcf further

$9(x^2-1)(2x-1)$

diff of 2 sq.

further still

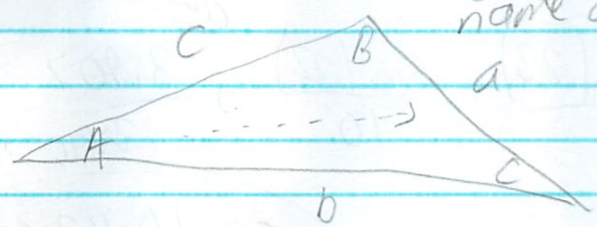
$9(x+1)(x-1)(2x-1)$

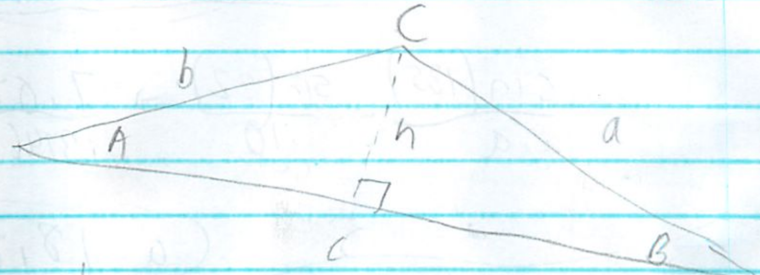
13.5 Law of Sines

3/15

* used to solve oblique angles (triangles w/o right angles)
* to solve a triangle - find the measurements of the missing sides or angles
* must be given 3 pieces of info (sides or angles) with at least one side
* capital letters = angles
  lowercase " = sides

side name opposet





$\sin A = \frac{h}{b}$ or $\sin B = \frac{h}{a}$

$h = \sin(A)\, b$

$h = \sin(B)\, a$ → $\frac{\sin(A)\, b}{a} = \frac{\sin(B)\, a}{a}$

$$\frac{\frac{b\sin(A)}{a}}{b} = \frac{\sin B}{b}$$

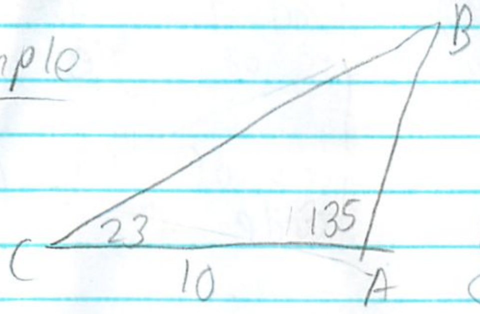
and also

$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$

Law of sines

✱. Use if given 2 angles + a side
or 2 sides + an angle oppset a given side.

example



$B = 180 - (23 + 135)$

$B = 22°$

cross multiply

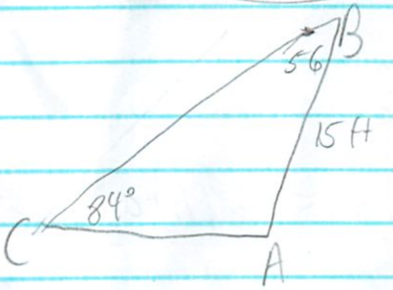
$\frac{\sin(23)}{c} = \frac{\sin(22)}{10} \rightarrow \frac{3.907}{.3746} = \frac{.3746\,c}{.3746}$

$c = 10.4304$

B = 56°
C = 84°
c = 15 ft

$\frac{\sin(135)}{a} = \frac{\sin(22)}{10} \rightarrow \frac{7.071}{.3746} = \frac{.3746\,a}{.3746}$

$a = 18.8760$



$A = 180 - (84 + 56)$

$A = 40°$

cross x

$\frac{\sin(40)}{a} = \frac{\sin(84)}{15} \rightarrow \frac{9.6418}{.9945} = \frac{.9945\,a}{.9945}$

$a = 9.6949$ ft

$\frac{\sin(56)}{b} = \frac{\sin(84)}{15} \rightarrow \frac{12.4356}{.9945} = \frac{.9945\,c}{.9945}$

$c = 12.5041$

Makeup

Ambigious Case

3/15

only if (b+c) $C_1 = C_2$

2 sides + angle (C) oppset a side

$\rightarrow \frac{\sin C}{c} = \frac{\sin B_1}{b}$

from review board demo

Find an angle (B) normally

180 − B_1 = B_2 & corresponding other thing

if $B_2 + C < 180$

then other triangle

find 2 ans for all values

See practice #23 on p 712

find other angle

find A_1 normally

$180 - (C + B_1)$

find A_2 seperatly using B_2 value

$180 - (C + B_2)$

find other side (a) using the 2 values seperatly

$\frac{\sin C_1}{c_1} = \frac{\sin A_1}{a_1}$

2 seperate

$\frac{\sin C_1}{c_1} = \frac{\sin A_2}{a_2}$

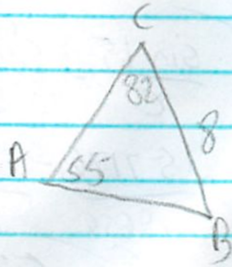
Makeup
Not checked

13.5 Law of Sines

Homework p 712

3/16

C 7.



$B = 180 - (82 + 55)$

$43°$

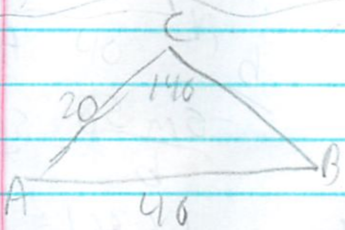
$\frac{\sin(43)}{b} = \frac{\sin(55)}{8}$ $\frac{5.4560}{.8191} = \frac{.8191b}{.8191}$

$b = 6.6605$

$\frac{\sin(82)}{c} = \frac{\sin(55)}{8}$ $\frac{7.922}{.8191} = \frac{.8191c}{.8191}$

$c = 9.6711$

8.



$\frac{\sin(B)}{20} = \frac{\sin(140)}{40}$ $\frac{12.86}{40} = \frac{\sin(B)\,40}{40}$

$.3214 = \sin B$

$\sin^{-1}$ $\sin^{-1}$

$B = 18.747°$

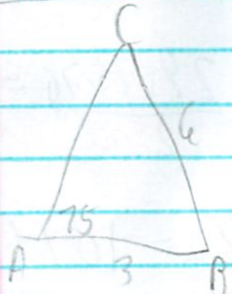
$A = 180 - (140 + 18.74)$

$A = 21.253$

$\frac{\sin(21.253)}{a} = \frac{\sin(140)}{40}$ $\frac{14.489}{.6428} = \frac{.6478a}{.6478}$

$a = 22.557$

11.



C/ $\frac{\sin 75}{6} = \frac{\sin C}{3}$

$\frac{2.898}{6} = \frac{6 \sin C}{6}$

$.48296 = \sin C$

$\sin^{-1}$ $\sin^{-1}$

$C = 28.88$

$152 + 75 \not< 180$ ⓧ

B/ $180 - (75 + 28.88)$

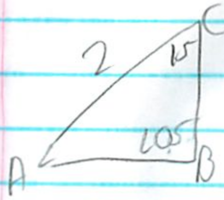
$76.12 = B$

b/ $\frac{\sin 76.12}{b} = \frac{\sin 75}{6}$

$\frac{5.8248}{.9659} = \frac{.9659b}{.9659}$

$6.030 = b$

13.

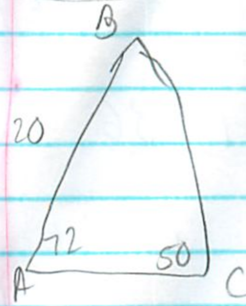


A
$180-(105+15)$
$A=60$

a
$$\frac{\sin 60}{a}=\frac{\sin 105}{2}$$
$$\frac{1.73}{.9659}=\frac{.9659a}{.9659}$$
$$.76612=a$$

c
$$\frac{\sin 15}{c}=\frac{\sin 105}{2}$$
$$\frac{.57176}{.9659}=\frac{.9659c}{.9652}$$
$$c=.5352$$

15.

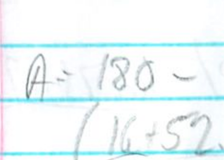


B
$180-(72+50)$
$B=58°$

a
$$\frac{\sin 72}{a}=\frac{\sin 50}{20}$$
$$\frac{19.02}{.7660}=\frac{.7660a}{.7660}$$
$$24.8303=a$$

b
$$\frac{\sin 58}{b}=\frac{\sin 50}{20}$$
$$\frac{16.961}{.7660}=\frac{.7660b}{.7660}$$
$$b=22.141$$

23.



done w/ board demo

$A=180-(16+52.42)$
$=111.58°$

$$B=\frac{\sin 16}{32}=\frac{\sin(B)}{92}$$
$$\frac{32\sin B}{32}=\frac{\sin(16)\,92}{32}$$
$$B=\sin^{-1}\left(\frac{\sin(16)92}{32}\right)$$
$$B=52.42°$$

$$A=\frac{\sin(16)}{32}=\frac{\sin(111.58)}{a}$$
$$\frac{29.7576}{.2756}=\frac{.2756a}{.2756}$$
$$a=107.9576$$

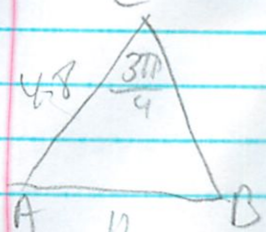
Sin pos in 1st + 2nd
if $B_2+C<180$ 2nd quad
2nd: $180-52.42=127.58$

2nd $180-(16+127.58)=36.42°$

$$\frac{\sin(16)}{32}=\frac{\sin(36.42)}{a}$$
$$a=68.93$$

use other A

25.



↓ use actual 135° or put calc in radians

B_1 $\dfrac{\sin\left(\cancel{\frac{3\pi}{4}}\right)}{12} = \dfrac{\sin B}{48}$

$\dfrac{33.941 \;\cancel{1.9733}}{12} = \dfrac{12\sin B}{12}$

2.828 $\cancel{.1644} = \sin(A)$

$\sin^{-1}$ $\sin^{-1}$

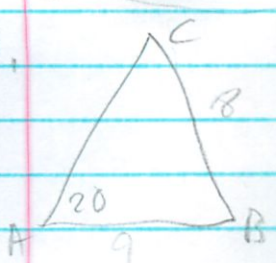
$\cancel{A_1 = 9.469}$ Domain error

$\cancel{180 - \left(9.469 + \frac{3\pi}{4}\right)}$ can't be done

$\cancel{A_2 = 168.18}$

$\angle 180 - 2\text{ans}$

30.



C_1 $\dfrac{\sin(20)}{8} = \dfrac{\sin C}{9}$

$\dfrac{3.0781}{8} = \dfrac{8\sin C}{8}$

$.3848 = \sin C$

$\sin^{-1}$ $\sin^{-1}$

$C_1 = 22.6296$

$B_1 = 180 - (22.6296 + 20)$

$B_1 = 137.37$

$C_2 = 180 - 22.63 + \cancel{20}$ not here

$C_2 = 157.37$

$\angle 180$ ✓

$B_2 = 180 - (137.37 + 20)$

$B_2 = 22.6246$

b_1 $\dfrac{\sin 20}{8} = \dfrac{\sin 137.37}{b_1}$

$\dfrac{5.418}{.3420} = \dfrac{.3420 b_1}{.3420}$

$15.052 = b_1$

b_2

$\dfrac{\sin 20}{8} = \dfrac{\sin 22.629}{b_2}$

$\dfrac{3.078}{.3420} = \dfrac{.3420 b_2}{.3420}$

$b_2 = 9$

B 37.4
C 22.6
b 15.83

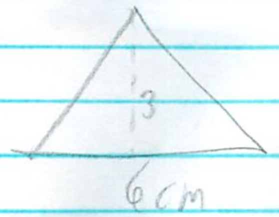
B 2.6
C 157.4
b 1.06

Warmup 3/19

Area Triangles

3/19

1.

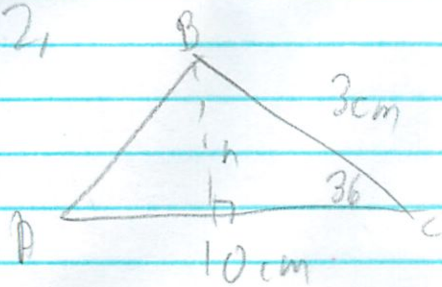


$A = \frac{1}{2}(bh)$

$A = \frac{1}{2}(6 \cdot 3)$

$A = 9 cm^2$

2.



$\sin 36 = \frac{h}{3}$

$.5878(3) = h$

$1.7633 = h$

$A = \frac{1}{2}(bh)$

$A = \frac{1}{2}(10 \cdot 1.7633)$

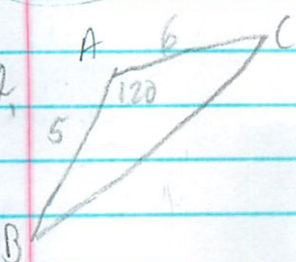
$A = 8.817 cm^2$

$\boxed{A = \frac{1}{2} b a \sin(C)}$

must be included angle

Just use formula

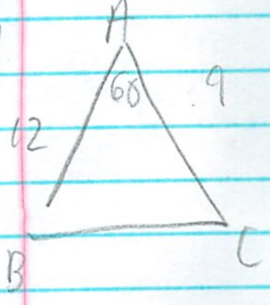
p713 48.



$A = \frac{1}{2}(5)(6)\sin(120) \rightarrow A = \frac{1}{2}(b)(c)\sin(A)$

$A = 12.99 cm^2$

51



give formula

$A = \frac{1}{2}(12)(9)\sin(60) \rightarrow A = \frac{1}{2}(b)(c)\sin(A)$

$A = 46.77 cm^2$

round

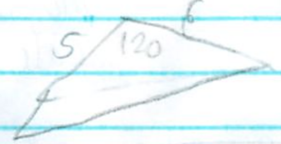
13.6

Law of Cosines

3/14

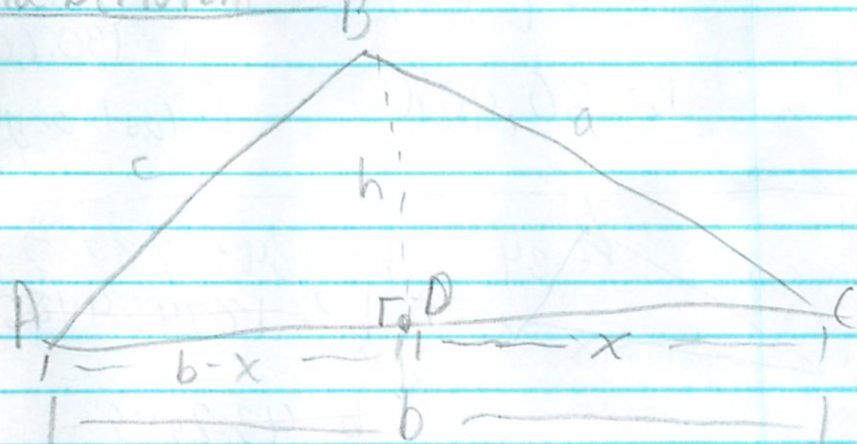
<u>Use when</u>

- given 3 sides + no angles
- given 2 sides + included angle



← angle created at intersection of 2 given sides

<u>Formula Derivation</u>



$c^2 = (b-x)^2 + h^2$

$c^2 = (b-x)(b-x) + h^2$

$c^2 = b^2 - 2bx + x^2 + h^2 \leftarrow h^2 + x^2 = a^2$

$c^2 = b^2 - 2bx + a^2 \leftarrow x = a\cos C \leftarrow \cos C = \frac{x}{a}$

$c^2 = b^2 - 2b(a\cos C) + a^2$

or $\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$

solving for a side

solve for angle

$-(a^2+b^2) \quad -(a^2+b^2)$

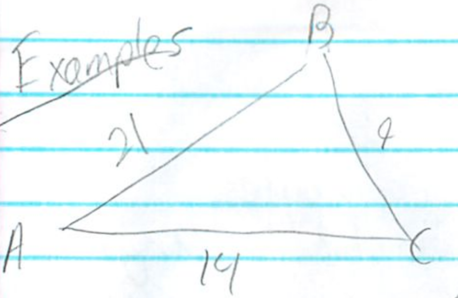
$\cos C = \frac{c^2 - a^2 - b^2}{-2ab}$

convert $\boxed{\cos(C) = \frac{a^2+b^2-c^2}{2ab}}$

3/26

Examples

1.



$\cos A = \dfrac{14^2 + 21^2 - 9^2}{2(14)(21)}$

just plug in calc

$\cos A = .9455$

$\cos^{-1}$ $\cos^{-1}$

$A = 18.9895$

$\cos B = \dfrac{9^2 + 21^2 - 14^2}{2(9)(21)}$

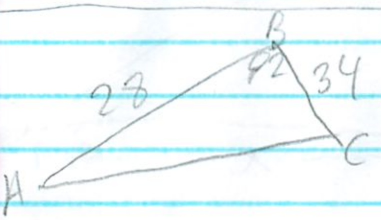
$B = 30.4090$

$C = 180 - (18.9895 + 30.4090)$

$C = 130.6015$

↑ last angle can be short cutted

2.



$b^2 = 34^2 + 28^2 - 2(34)(28)\cos(92)$

$b^2 = \cancel{1874.4486}$

$\cancel{b = 43.2949}$ calc error

$\cos A = \dfrac{43.2949^2 + 28^2 - 34^2}{2(43.2949)(28)}$

$\cos A = .6197$

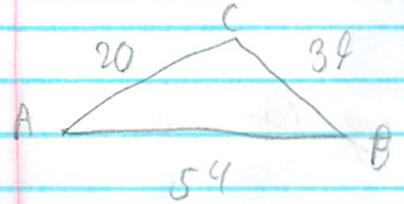
$\cos^{-1}$ $\cos^{-1}$

$A = 51.7064$

$C = 180 - (92 + 51.7064)$

$C = 36.2935$

3.



$\cos A = \dfrac{20^2 + 54^2 - 39^2}{2(20)(54)}$

$A = 33.7965°$

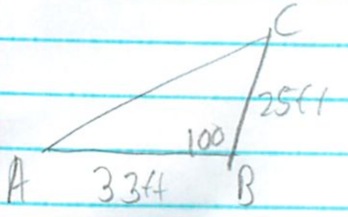
$\cos B = \dfrac{39^2 + 54^2 - 20^2}{2(39)(54)}$

$B = 16.5740°$

$C = 180 - (33.7965 + 16.5740)$

$C = 129.6295°$

4.



$b^2 = 25^2 + 33^2 - 2(25)(33)\cos(100)$

$b^2 = 2000.5194$

$b = 44.7272 \text{ ft}$

$\cos A = \dfrac{44.7272^2 + 33^2 - 25^2}{2(44.7272)(33)}$

$\cos A = .8349$

$\cos^{-1} \quad \cos^{-1}$

$A = 33.3901°$

$C = 180 - (33.3901 + 100)$

$C = 46.6099$

Applications of Law of Sines and the Law of Cosines

Name Michael Plosmeier

See work page

Directions: Draw a picture and solve each of the following problems using the Law of Sines or Cosines.

1. A boat race runs along a triangular course marked by buoys A, B, and C. The race starts at point A with the boats heading west for 2500 meters. The other two sides of the course lie to the north of the first side, and their lengths are 1100 meters and 2000 meters, respectively. Draw a diagram to represent the problem and find the angles in the triangular course.

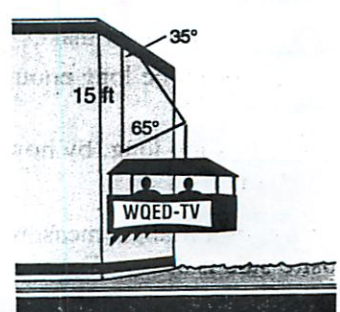
2. The angles of elevation to an airplane from two points A and B on level ground are 55° and 72°, respectively. The point A and B are 2.2 miles apart. Find the altitude of the plane.

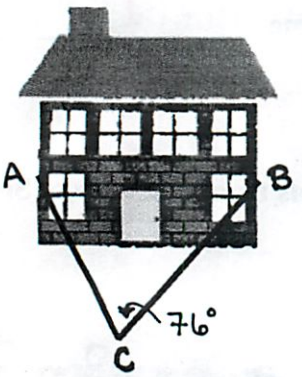
3. A tree near a school was struck by lightning and you're wondering whether or not it will hit the school when it falls. Suppose you are standing 50 feet from the base of the tree. The angle of elevation from you to the top of the tree is 35°. You then move 10 feet closer to the base of the tree and the new angle of elevation to the top of the tree is 42°. If the school sits 75 feet from the base of tree (and on the other side of the tree from where you are), is the school safe?

4. You want to find the distance across a lake near your house so that you know how long the bridge must be in order to cross it. To do this, you measure the distance from a point **on the north shore** of the lake to a point several steps east of the lake: 100 feet. You then measure the distance from this point to a point **on the south shore** of the lake: 85 feet. You finally measure the angle between the two straight line paths that you walked: 65°. How long must the bridge be?

5. A triangular field is 102.4 yds. on one side and 113.2 yds. on another. The measure of the angle between them is 63.6°. Determine the length of the third side.

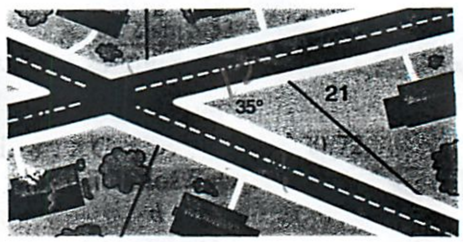
6. Two steel braces are attached to a building to support a perch for two television personalities who will broadcast the 4th of July parade. The braces are attached to the building 15 ft. apart vertically and at angles of 65° and 35°. How long is the longer brace?





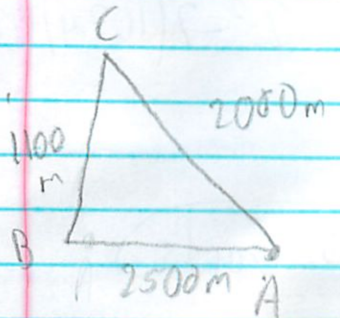
7. A surveyor wishes to measure the width of a building. The distances from A to B to the surveyor at C are 400 and 520 ft., respectively. The measure of the angle at C is 76° . Determine the width of the building.

8. A monument 21 ft. long is to be placed across a triangular lot formed by two streets, Elm and Oak. The angle between the streets is 35°, and the monument must be 12ft. from the corner along Elm Street. What is the least distance that the monument could be from the corner along Oak Street?



Law of Sines
Application Word Problems 3/20

1.



$\cos A = \dfrac{2000^2 + 2500^2 - 1100^2}{2(2500)(2000)}$

$\cos A = .904$

$\cos^{-1}$ $\cos^{-1}$

$A = 25.311°$

$\cos B = \dfrac{2500^2 + 1100^2 - 2000^2}{2(2500)(1100)}$

$\cos B = .6291$

$\cos^{-1}$ $\cos^{-1}$

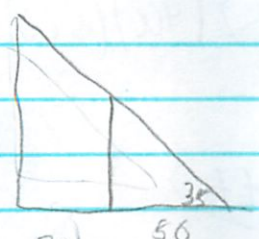
$B = 51.017°$

$C = 180 - (25.311 + 51.017)$

$C = 103.6720°$

#2

3.



$H_{Tree} = \tan(35) = \frac{x}{50}$

$.7002(50) = x$

$x = 35.0107$

$H_{Tree} = \tan(42) = \frac{x}{40}$

~~x = 45.0202~~

calc error

36.016

typed 50 not 40

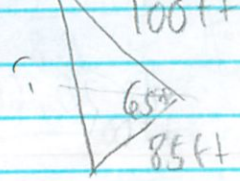
why diff?

The school would be safe

-? ~~too easy missing something~~?

~~Should be same~~ - numbers have been rounded in problem

4.

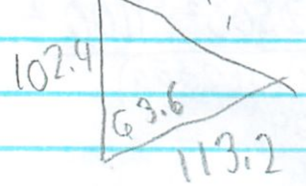


$c^2 = 100^2 + 85^2 - 2(100)(85)\cos(85)$

$c^2 = 14743$

$c = 125.4725$ ft

5.



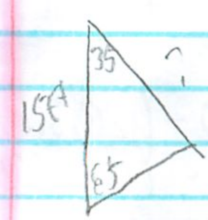
✓

$c^2 = 102.4^2 + 113.2^2 - 2(102.4)(113.2)\cos(63.6)$

$c^2 = 12991$

$\boxed{c = 113.9815 \text{ yards}}$

6.



✓

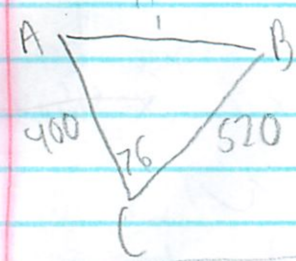
$180 - (35 + 65)$
$80°$

$\frac{\sin 80}{15} = \frac{\sin 65}{x}$

$\frac{13.5946}{.9848} = \frac{.9848x}{.9848}$

$\boxed{x = 13.8043 \text{ ft}}$

7.

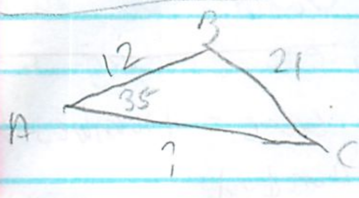


$c^2 = 400^2 + 520^2 - 2(400)(500)\cos(76)$

$c^2 = 329760$

$\boxed{c = 574.24 \text{ ft}}$

8.



✓

D + F sort of off

$\frac{\sin 35}{21} = \frac{\sin C}{12}$

$\frac{6.8829}{21} = \frac{21 \sin C}{21}$

$.3278 = \sin C$
$\sin^{-1} \quad \sin^{-1}$

$\boxed{C = 19.1327°}$

$B = 180 - (19.1327 + 35)$

$\boxed{B = 125.867}$

$\frac{\sin 35}{21} = \frac{\sin 125.867}{b}$

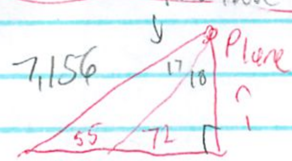
$\frac{17.0179}{.5738} = \frac{.5738b}{.5738}$

$\boxed{29.668 \text{ ft} = c}$

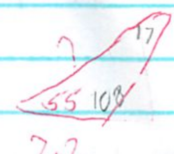
Find these 1st

#2.

Do as class



→



$\frac{\sin 108}{?} = \frac{\sin 17}{2.2}$

$\frac{2.0923}{.2924} = \frac{.2924x}{.2924}$

$\boxed{x = 7.156 \text{ miles}}$

$\sin 55 = \frac{?}{7.156}$

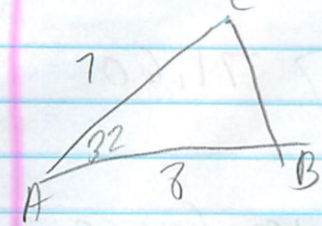
$.8191(7.156)$

$\rightarrow \boxed{? = 5.86 \text{ miles}}$

13.6 Law of Cos Practice

3/20

9.



$a^2 = 7^2 + 8^2 - 2(7)(8)\cos(32)$

$a^2 = 18.018$

$a = 4.2448$

√ w/ Triangle PRGM ✓

$\cos B = \dfrac{4.24^2 + 8^2 - 7^2}{2(4.24)(8)}$

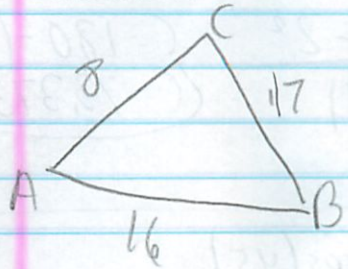
$\cos B = .4881$

$B = 60.1992°$ close - sle prob rounded

$C = 180 - (60.1992 + 32)$

$C = 87.0884°$

11.



$\cos A = \dfrac{8^2 + 16^2 - 17^2}{2(8)(16)}$

$\cos A = .1211$

$A = 83.0448°$

$\cos B = \dfrac{16^2 + 17^2 - 8^2}{2(16)(17)}$

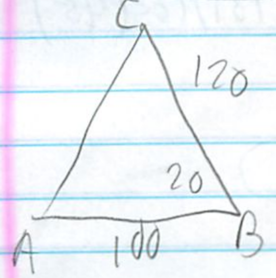
$\cos B = .8842$

$B = 27.848°$

$C = 180 - (83.0448 + 27.848)$

$C = 69.1074°$

13.



$b^2 = 120^2 + 100^2 - 2(120)(100)\cos(20)$

$b^2 = 1847.38$

$b = 42.9811$

$\cos A = \dfrac{42.9811^2 + 100^2 - 120^2}{2(100)(42.9811)}$

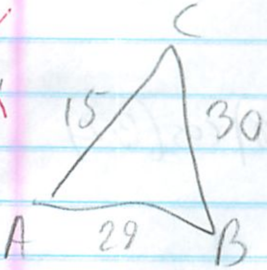
$A = 107.2743°$

$C = 180 - (107.27 + 20)$

$C = 52.7257°$

Wrong

~~15~~.



?

$$\cos A = \frac{15^2+29^2-30^2}{2(29)(30)}$$

~~A = 84.5255°~~ 71.60

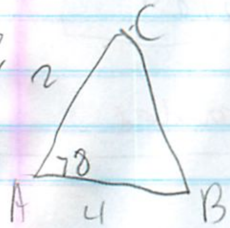
$$\cos B = \frac{29^2+30^2-15^2}{2(29)(30)}$$

B = 29.3941° ← right

C = 180 − (84.53 + 29.39)

~~C = 66.0803°~~ 79.00

17.



$a^2 = 2^2+4^2-2(2)(4)\cos(78)$

a = 4.0833

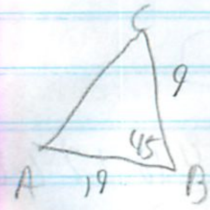
$$\cos B = \frac{4.0833^2+4^2-2^2}{2(4.0833)(4)}$$

B = 28.6263°

C = 180 − (28.62 + 78)

C = 73.3737°

19.



$b^2 = 19^2+9^2-2(19)(9)\cos(45)$

b = 14.148

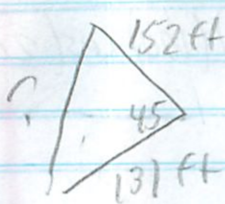
$$\cos A = \frac{19^2+14.15^2-9^2}{2(14.15)(19)}$$

A = 26.7314°

C = 180 − (26.73 + 45)

C = 108.2685°

49.



$c^2 = 152^2+131^2-2(152)(131)\cos(45)$

$c^2 = 12105$

c = 110.0235 ft

acccross pond

15/16

Classwork

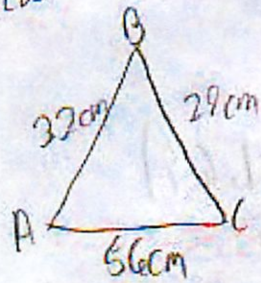
Mixed Review: Law of Sines and Cosines

Name Michael Plasucior
Tom Powell

DIRECTIONS: For each of the following, use the Law of Sines or Cosines to *solve* the triangles. Please create a box that includes all of the missing sides and angles.

1. In ΔABC, a = 29 cm, b = 56 cm, c = 32 cm

Cos



$\cos A = \frac{56^2 + 32^2 - 29^2}{2(56)(32)}$

$\cos A = .9261$

$\cos^{-1}$ $\cos^{-1}$

$A = 22.17°$

$\cos B = \frac{29^2 + 32^2 - 56^2}{2(29)(32)}$

$\cos B = -.6848$

$\cos^{-1}$ $\cos^{-1}$

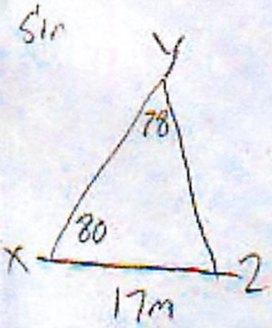
$B = 133.22$

$C = 180 - (22.17 + 133.22)$

$C = 24.608°$

2. In ΔXYZ, <X = 80°, <Y = 78°, y = 17 m

Sin



$Z = 180 - (78 + 80)$

$Z = 22°$

$x = \frac{\sin 78}{17} = \frac{\sin 80}{x}$

$\frac{16.74}{.9781} = \frac{.9781x}{.9781}$

$x = 17.1158m$

$z = \frac{\sin 78}{17} = \frac{\sin 22}{z}$

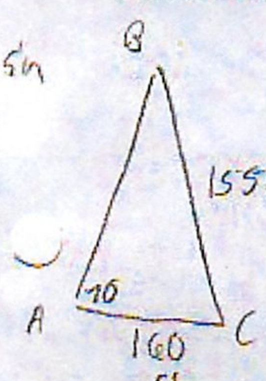
$\frac{6.368}{.9781} = \frac{.9781z}{.9781}$

$z = 6.51m$

−1 Wrong letters used

3. In ΔABC, < A = 70°, a = 155 ft., c = 160 ft.

Sin



$C_1 = \frac{\sin 70}{155} = \frac{\sin C}{160}$

$\frac{150.35}{155} = \frac{155 \sin C}{155}$

$\sin C = .9700$

$\sin^{-1}$ $\sin^{-1}$

$C_1 = 75.93°$

Amb yes

$180 - 75.93 =$

$104.07° = C_2$

$104.07 + 70 > 180$

C_2 $180 - (104.07 + 70)$

$B_2 = 5.9314°$

C_1 $180 - (75.93 + 70) = 34.0686° = B_1$

b_1

$\frac{\sin 70}{155} = \frac{\sin 34.068}{c_1}$

$\frac{88.89}{.9397} = \frac{.9397c_1}{.9397}$

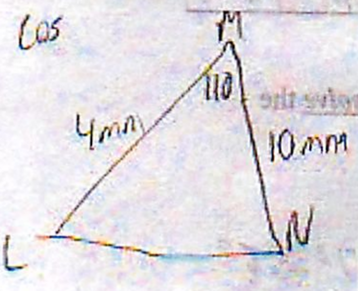
$92.40 ft = b_1$

11

(11/15)

4. In ΔLMN, $<M = 110°$, n = 4 mm, l = 10mm

cos



$m^2 = 10^2 + 4^2 - 2(4)(10)\cos(110)$

$m^2 = 143.3616$

$m = 11.97\text{ mm}$

$\cos L = \dfrac{4^2 + 11.97^2 - 10^2}{2(4)(11.97)}$ ← opposite one

$\cos L = .6192$

$\cos^{-1}$ $\cos^{-1}$

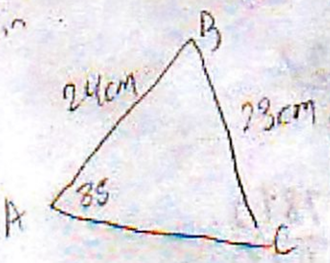
$L = 51.69°$

$N = 180 - (110 + 51.69)$

$N = 18.3088°$

5. In ΔABC, A = 85°, a = 23 cm, c = 24 cm

sin



Find another angle 1st

C_1: $\dfrac{\sin 85}{23} = \dfrac{\sin C_1}{24}$

$\dfrac{23.9087}{23} = \dfrac{23 \sin C_1}{23}$

$1.0395 = \sin C_1$

$\sin^{-1}$ $\sin^{-1}$

error → No Solution!

means

b. $\dfrac{\sin 70}{165} = \dfrac{\sin 5.9314}{C_2}$

(4)

$\dfrac{16.02}{.9397} = \dfrac{.9397 C_2}{.9397}$

#13

$C_2 = 17.045\text{ ft}$

9-5 Applications of Trigonometry to Navigation and Surveying

Objective *To use trigonometry to solve navigation and surveying problems.*

As shown below, the *course* of a ship or plane is the angle, measured clockwise, from the north direction to the direction of the ship or plane.

course of ship = 110°　　course of plane = 270°

As shown at the right, the *compass bearing* of one location from another is measured in the same way. Note that compass bearings and courses are given with three digits, such as 060° rather than 60°.

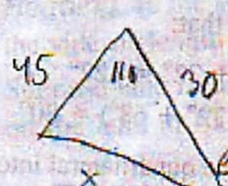
bearing of B from A = 060°
bearing of A from B = 240°

Example 1 A ship proceeds on a course of 300° for 2 hours at a speed of 15 knots (1 knot = 1 nautical mile per hour). Then it changes course to 230°, continuing at 15 knots for 3 more hours. At that time, how far is the ship from its starting point?

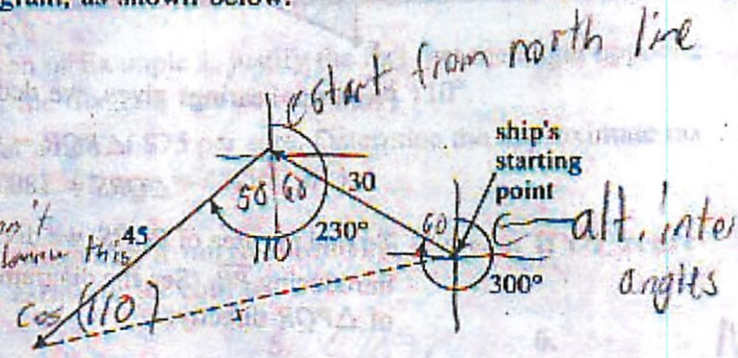
Solution Make a diagram, as shown below.

deme

start from north line



don't know this



alt. interior angles

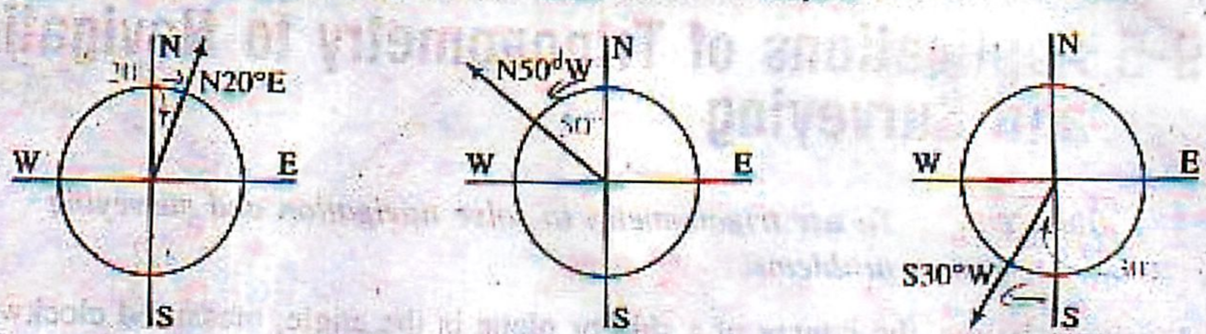
$x^2 = 30^2 + 45^2 - 2(45)(30)\cos(110)$

The ship travels first along a path of length $2 \cdot 15 = 30$ knots and then along a path of length $3 \cdot 15 = 45$ knots. The angle between the two paths is 110°. (You can find this angle by drawing north-south lines and using geometry.) To find x, the distance of the ship from its starting point, use the law of cosines:

$$x^2 = 30^2 + 45^2 - 2 \cdot 30 \cdot 45 \cdot \cos 110° \approx 3848$$

Thus, $x \approx \sqrt{3848} \approx 62.0$ nautical miles.

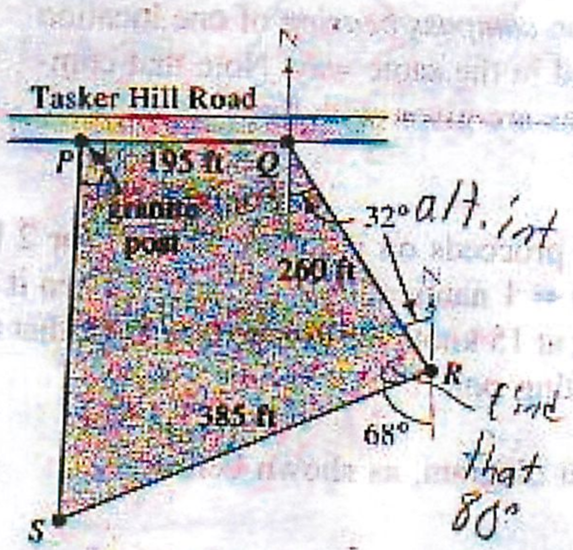
In surveying, a compass reading is usually given as an acute angle from the north-south line toward the east or west. A few examples are shown below.

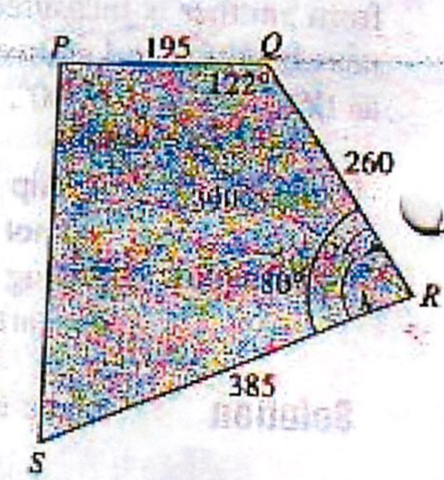


Example 2 Very often a plot of land is taxed according to its area. Sketch the plot of land described. Then find its area.

From a granite post, proceed 195 ft east along Tasker Hill Road, then along a bearing of S32°E for 260 ft, then along a bearing of S68°W for 385 ft, and finally along a line back to the granite post.

Solution We first sketch the plot of land, one side at a time and in the sequence described. (See the diagram at the left below.)





From the bearings given, we deduce that:

$$\angle PQR = 90° + 32° = 122°$$

$$\angle QRS = 180° - (32° + 68°) = 80°$$

To find the area of $PQRS$, we divide the quadrilateral into two triangles by introducing $\overline{PR}$. (See the diagram at the right above.) We can find the area of ΔPQR directly:

$$\text{Area of } \Delta PQR = \frac{1}{2} \cdot PQ \cdot QR \cdot \sin Q$$

$$= \frac{1}{2} \cdot 195 \cdot 260 \cdot \sin 122° \approx 21{,}500 \text{ ft}$$

To find the area of $\triangle PRS$, we must first find PR and $\angle PRS$.

To find PR, we use the law of cosines:

$$PR^2 = 195^2 + 260^2 - 2 \cdot 195 \cdot 260 \cdot \cos 122^\circ$$
$$\approx 159{,}000$$

Therefore, $PR \approx \sqrt{159{,}000} \approx 399$ ft.

To find $\angle PRS$, we find $\angle PRQ$ by the law of sines:

$$\frac{\sin PRQ}{195} = \frac{\sin 122^\circ}{399}$$

$$\sin PRQ = \frac{195 \sin 122^\circ}{399} = 0.4145$$

$$\angle PRQ \approx 24.5^\circ$$

Therefore, $\angle PRS = \angle QRS - \angle PRQ \approx 80^\circ - 24.5^\circ = 55.5^\circ$.

Knowing that $PR = 399$ ft and $\angle PRS = 55.5^\circ$, we have:

$$\text{Area of } \triangle PRS = \tfrac{1}{2} \cdot PR \cdot RS \cdot \sin PRS$$

$$\approx \tfrac{1}{2} \cdot 399 \cdot 385 \cdot \sin 55.5^\circ \approx$$

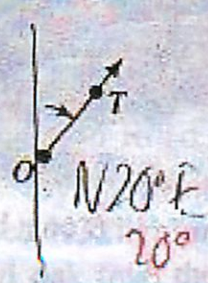
Thus, we have:

$$\text{Area of quadrilateral } PQRS = \text{area of } \triangle PQR + \text{area of } \triangle PRS$$
$$\approx \qquad + \qquad$$
$$= 84{,}800 \text{ ft}^2$$

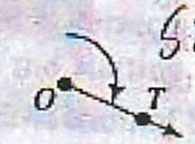
CLASS EXERCISES

1. *Reading* In the solution of Example 1, justify the fact that the angle opposite the side of length x in the diagram has a measure of 110°.
2. Suppose land is taxed at a rate of \$75 per acre. Determine the approximate tax on the land in Example 2. (1 acre = 43,560 ft^2)

Visual Thinking In each diagram, a north-south line is given. If $\overrightarrow{OT}$ represents the path of a ship, estimate its course.

3. 

N20°E
20°

4. 

S80°E
110°

5. S20°W 

200°

6. N80°W 

300°

7. *Visual Thinking* Suppose point X is directly east of point Y.
   a. Give the bearing of X from Y.
   b. Give the bearing of Y from X.

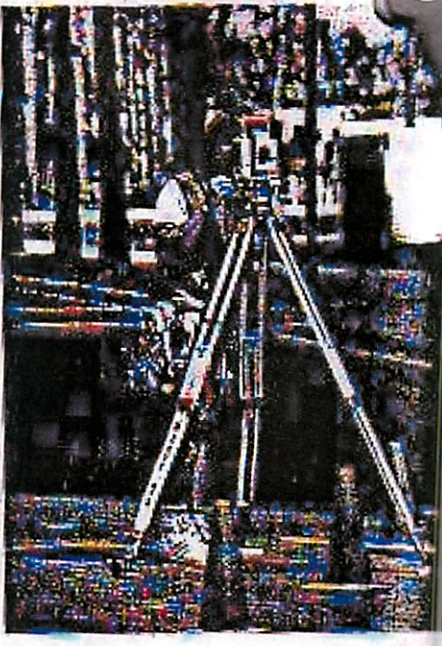
For Exercises 8–11, make a sketch of each compass reading.

8. N70°E 9. N10°W 10. S15°E 11. S40°W

8.
9.
10.
11.

12. Match each compass direction with a course.

Direction	*Course*
northeast	135°
southeast	225°
northwest	315°
southwest	045°



13. A famous Alfred Hitchcock movie is *North by Northwest.* This direction is midway between north and northwest. What course corresponds to this direction?

14. From one corner of a triangular plot of land, a surveyor determines the directions to the other two corners to be N32°E and S76°E. What is the measure of the angle formed by the edges of the plot of land at the corner where the surveyor is?

14. 32 76 72°

WRITTEN EXERCISES

In Exercises 1–4, draw a diagram like those in Class Exercises 3–6 to show the path of a ship proceeding on each given course.

S45°W

4.

A 1. 070° 2. 150° 3. 340° 4. 225°

5. Ship A sights ship B on a compass bearing of 080°. Make a sketch and give the compass bearing of ship A from ship B.

6. Ship X sights ship Y on a bearing of 308°. What is the bearing of X from Y?

7. An airplane flies on a course of 110° at a speed of 1200 km/h. How far east of its starting point is it after 2 h?

angle error

110°

70

1200 km/h

2400 km

and didn't see qu

$\sin 70 = \frac{x}{2400}$

$x = 2255$ km

8. A hunter walks east for 1 h and then north for $1\frac{1}{2}$ h. What course should the hunter take to return to his starting point? What assumptions do you make to answer the question?

9. Point *B* is 10 km north of point *A*, and point *C* is 10 km from *B* on a bearing of 060° from *B*. Find the bearing and distance of *C* from *A*.

10. Point S is 4 km west of point R, and point T is 4 km southwest of S. Find the bearing and distance of R from T.

11. *Navigation* Traveling at a speed of 10 knots, a ship proceeds south from its port for $1\frac{1}{2}$ h and then changes course to 130° for $\frac{1}{2}$ h. At this time, how far from port is the ship?

12. *Navigation* A sailboat leaves its dock and proceeds east for 2 mi. It then changes course to 205° until it is due south of its dock. How far south is this?

13. *Navigation* Two ships, A and B, leave port at the same time. Ship A proceeds at 12 knots on a course of 040°, while ship B proceeds at 9 knots on a course of 115°. After 2 h, ship A loses power and radios for help. How far and on what course must ship B travel to reach ship A?

14. *Navigation* A ship leaves port and sails northwest for 1 h and then northeast for 2 h. If it does not change speed, find what course the ship should take to return directly to port. Also find how long this return will take.

In Exercises 15–18, sketch each plot of land described and find its area.

B 15. *Surveying* From an iron post, proceed 500 m northeast to the brook, then 300 m east along the brook to the old mill, then 200 m S15°E to a post on the edge of Wiggin's Road, and finally along Wiggin's Road back to the iron post.

16. *Surveying* From a cement marker, proceed 260 m southwest to the river, then 240 m south along the river to the bridge, then 280 m N40°E to a sign on the edge of Sycamore Lane, and finally along Sycamore Lane back to the cement marker.

17. *Surveying* From the southeast corner of the cemetery on Burnham Road, proceed S78°W for 250 m along the southern boundary of the cemetery until a granite post is reached, then S15°E for 180 m to Allard Road, then N78°E along Allard Road until it intersects Burnham Road, and finally N30°E along Burnham Road back to the starting point.

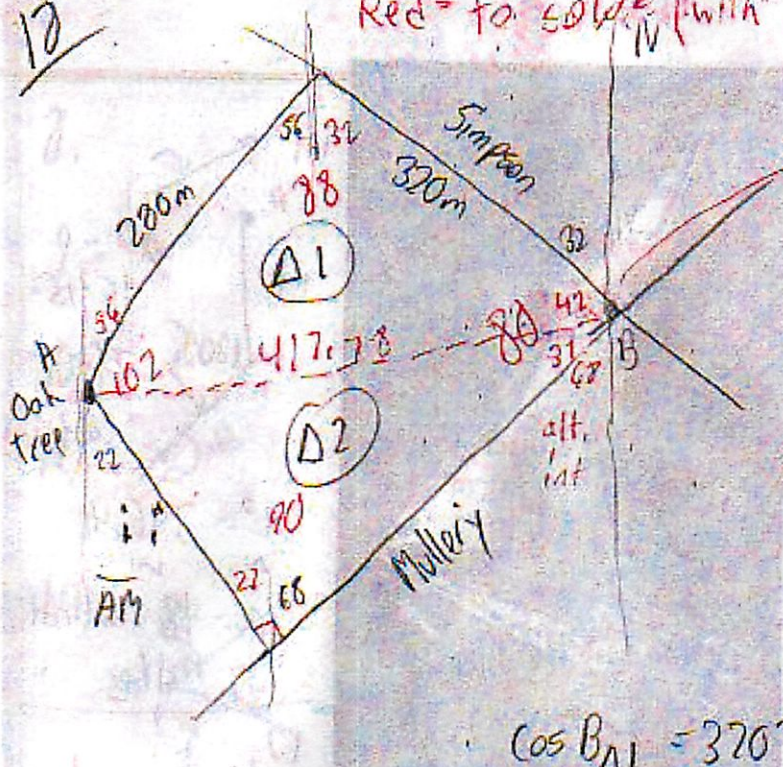
18. *Surveying* From the intersection of Simpson's Road and Mulberry Lane, proceed N32°W for 320 m along Simpson's Road, then S56°W for 280 m to the old oak tree, then S22°E until Mulberry Lane is reached, and finally N68°E along Mulberry Lane back to the starting point.

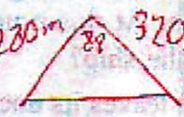
11.
p
15k
130°
5k
$a^2 = 15^2 + 5^2 - 2(15)(5)\cos(130)$
$a^2 = 346.41$
$a \approx 18.61$ nat. miles

12.
2
65
25
$\tan(25) = \frac{?}{2}$
? = 4.28
~~.9326~~ miles
angle error

12

Red = to solve (with pencil)





$a^2 = 280^2 + 320^2 - 2(280)(320)\cos(88)$

$a^2 = 174546$

$\boxed{a = 417.787m}$

$\cos B_{\Delta 1} = \dfrac{320^2 + 417.78^2 - 280^2}{2(320)(417.78)}$

$A_{\Delta_1} = \frac{1}{2}(280)(320)\sin(88)$

$\boxed{A_{\Delta 1} = 44772.70}$

$\cos B = .7426$

$\cos^{-1}$ $\cos^{-1}$

$\boxed{B_{\Delta_1} = 42.05°}$

$A_{\Delta 2} = \frac{1}{2}(333)(251)$

$\boxed{A_{\Delta 2} = 41946.1}$

$B_2 = 80 - 42.05 = \boxed{37.95 = B_2}$

$\overline{AM} = \sin 37 = \dfrac{\overline{AM}}{417.78}$

$.6018(417.78) = \overline{AM}$

$\boxed{251.4305 = \overline{AM}}$

$A_T = A_{\Delta 1} + A_{\Delta 2}$

$\boxed{A_\Delta = 86718.81 m^2}$

$\text{Mullbery} = \cos 37 = \dfrac{\text{Mullbery}}{417.78}$

$\boxed{\text{Mullberry} = 333.6596}$

Chapter Test

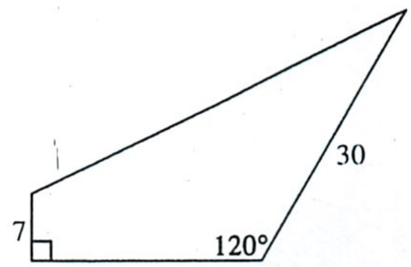
Where appropriate, give angle measures to the nearest tenth of a degree and lengths of sides in simplest radical form or to three significant digits.

1. The sides of an isosceles triangle have lengths 5, 10, and 10. What are the measures of its angles? **9-1**

2. At a distance of 100 m, the angle of elevation to the top of a fir tree is 28°. About how tall is the tree?

3. A regular pentagon is inscribed in a circle of radius 4 in. Find the area of the pentagon. **9-2**

4. Find the area of the quadrilateral shown at the right.



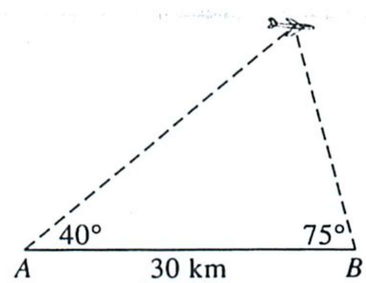
Ex. 4

5. How many different triangles PQR can be constructed using the given information? **9-3**

a. $p = 5$, $q = 4$, $\angle Q = 74°$

b. $p = 9$, $q = 8$, $\angle P = 23°$

6. Observers at points A and B, 30 km apart, sight an airplane at angles of elevation of 40° and 75°, respectively, as shown in the diagram. How far is the plane from each observer?



Ex. 6

7. A triangle has sides of length 5, 8, and 10. What is the measure of its largest angle? **9-4**

8. Two hikers are following a trail which, at a certain point, separates into two forks. Each hiker takes a different fork. If the forks diverge at an angle of 67° and each hiker walks at a speed of 3.5 mi/h, how far apart are the hikers after 1 h?

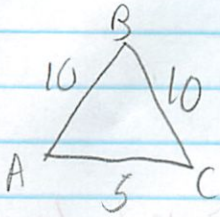
9. ***Writing*** When only three parts of a triangle are known, the law of sines and the law of cosines can be used to find the unknown parts. Write a paragraph in which you discuss the specific circumstances for using each law.

10. After leaving an airport, a plane flies for 1.5 h at a speed of 200 km/h on a course of 200°. Then, on a course of 340°, the plane flies for 2 h at a speed of 250 km/h. At this time, how far from the airport is the plane? **9-5**

Michael Ploszaj[?]

Laws of Sin + Cos Review

3/22

1. 

$\cos(A) = \dfrac{5^2+10^2-10^2}{2(5)(10)}$

$\cos A = .25$

$\cos^{-1}$ $\cos^{-1}$

$A = 75.5225°$

$\cos(B) = \dfrac{10^2+10^2-5^2}{2(10)(10)}$

$\cos B = .875$

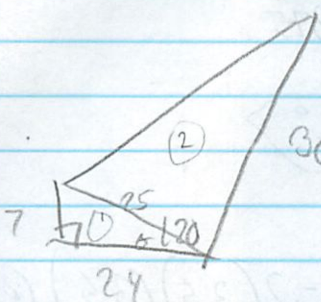
$\cos^{-1}$ $\cos^{-1}$

$B = 28.9550°$

$C = 180-(75.2525+28.9550)$

$C = 75.5225°$

✓

4. 

$A_{\Delta 1} = \frac{1}{2}(7)(24)$

$A_{\Delta 1} = 84$

$\text{line} = \sqrt{7^2+24^2}$

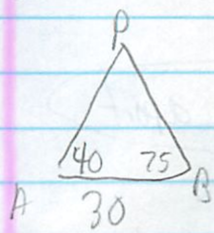
$\text{line} = 25$

$A_{\Delta 2} = \frac{1}{2}(25)(30)$

$A_{\Delta 2} = 375$

$84+375$

459

6. 

$P = 180-40-75$

$P = 65°$

$b = \dfrac{\sin 65}{30} = \dfrac{\sin 75}{b}$

$\dfrac{28.9778}{.9063} = \dfrac{.9063b}{.9063}$

$b = 31.9734$ km

$a = \dfrac{\sin 65}{30} = \dfrac{\sin 40}{a}$

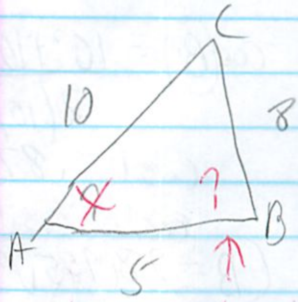
$\dfrac{19.2836}{.9063} = \dfrac{.9063a}{.9063}$

$a = 21.27712$ km

X why?

→

7.



was thinking this was 90 when thinking what is biggest angle

$\cos A = \frac{10^2+5^2-8^2}{2(10)(5)}$

$\cos A = .61$

$\cos^{-1} \quad \cos^{-1}$

~~A = 52.41050°~~ not longest angle

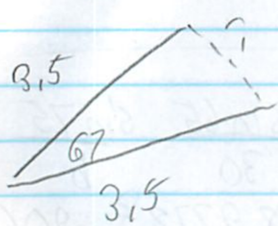
$\cos B = \frac{5^2+8^2-10^2}{2(5)(8)}$

$\cos B = -.1375$

$\cos^{-1} \quad \cos^{-1}$

$B = 97.9032°$

8.



$c^2 = 3.5^2 + 3.5^2 - 2(3.5)(3.5)\cos 67$

$c^2 = 14.9270$

$c = 3.8636$ miles apart

4

IAG 3
HOT SHEET

Name Michael Moore

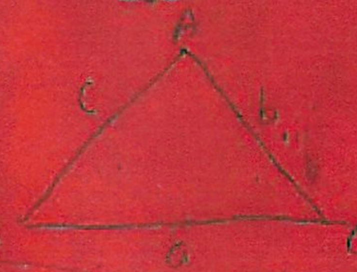
Date 3/11

Unit Laws of Sines + Cosines

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Triangle



Law of Cos

Standard

Find sides

$a^2 = b^2 + c^2 - 2bc\cos(A)$

$b^2 = a^2 + c^2 - 2ac\cos(B)$

$c^2 = a^2 + b^2 - 2ab\cos(C)$

When to use

Sines - 2 angles + side

- 2 sides + angle opposit that side (ambiguous)

Cos - 3 sides no angles

- 2 sides + included angle

Ambiguous Case

only for 2 sides + angle opposit a side

Find an angle and do $180 - A_1 = A_2$ if $A_2 +$ given angle < 180 then 2 answers

Find other angle separatly

Find last side separtly

Alt form

Find angle

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos(B) = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$$

Area Triangle

$\frac{1}{2} ba \sin(C)$

Triangle Trigonometry Quiz 50/52

Name: Michael Plasmeier

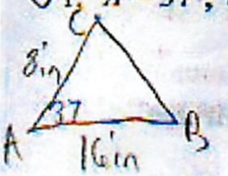
Points **VERSION B**

Date: 3/23

Read ALL DIRECTIONS very carefully. SHOW ALL WORK!! Good Luck!!

Find the area of the triangles below. Round your answer to the nearest unit.

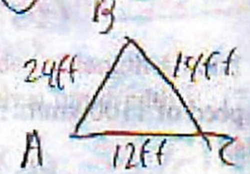
1. $A = 37°$, b = 8 in., c = 16 in. (3 pts.)



$A = \frac{1}{2}(8)(16)\sin(37)$

$A = 39 \text{ in}^2$

2. a = 14 ft., b = 12 ft., c = 24 ft. (5 pts.)



$\cos A = \frac{12^2+24^2-14^2}{2(12)(24)}$

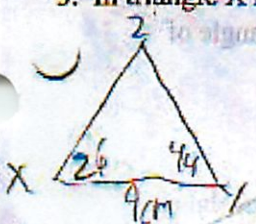
$\boxed{\angle A = 24.533°}$

$A = \frac{1}{2}(12)(24)\sin(24.533)$

$A = 60 \text{ ft}^2$

Solve each of the following triangles below using either the Law of Sines or Law of Cosines. Round all measurements to the nearest hundredth and be sure to indicate a second solution if there is one. (6 pts. each)

3. In triangle XYZ, $<X = 26°$, $<Y = 44°$, z = 9 cm.



$Z = 180 - (26+44)$

$Z = 110°$

<Z = 110°
x = 4.20cm
y = 6.65cm

$x = \frac{\sin 110}{9} = \frac{\sin 26}{x}$

$\frac{3.9453}{.9397} = \frac{.9397x}{.9397}$

$x = 4.20 \text{ cm}$

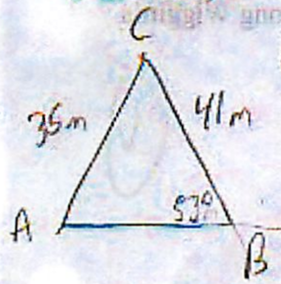
$y = \frac{\sin 110}{9} = \frac{\sin 44}{x}$

$y = \frac{6.2519}{.9397} = \frac{.9397}{.9397}$

$x = 6.65$ cm

thought I got it wrong

4. In ΔABC, $a = 41m, b = 35m, B = 53°$



$A = \frac{\sin 53}{35} = \frac{\sin A}{41}$

$\frac{32.7444}{35} = \frac{35 \sin A}{35}$

$\sin A = .9355$

$\sin^{-1}$ $\sin^{-1}$

$A = 69.32°$

Amb v:
180 - 69.32 =
110.68 + 53 =
163.68 = Amb

$C = 180 - (69.32 + 53)$

$C = 57.68°$

$c = \frac{\sin 53}{35} = \frac{\sin 57.68}{c}$

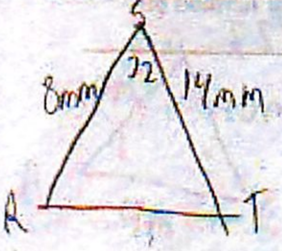
$\frac{c = 29.5788}{.7986} = \frac{.7986c}{.7986}$

$c = 37.04m$

<A = 69.32°, 16.32
<C = 57.68°, 110.68
c = 37.04m, ~~?~~ m

18

5. In ΔRST, r = 14 mm, t = 8 mm, S = 22°



$S^2 = 8^2 + 14^2 - 2(8)(14)\cos(22)$

$S^2 = 52.31$

$\boxed{S = 7.23 cm}$

$\cos R = \dfrac{8^2 + 7.23^2 - 14^2}{2(8)(7.23)}$

$\cos R = -.6884$

$\cos^{-1}$ $\cos^{-1}$

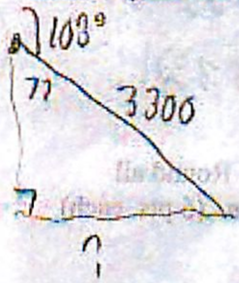
$\boxed{R = 133.52°}$

$T = 180 - (133.52 + 22)$

$\boxed{T = 24.48°}$

<R = 133.52°
<T = 24.48°
s = 7.23 cm

6. An airplane flies on a course of 103° at a speed of 1100 km/h. ~~How far east of its starting point is it after 3 hours?~~ (Answer to the nearest km.) (3 pts.)



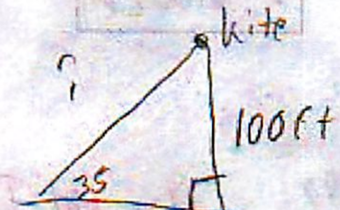
$1100 \cdot 3 = 3300$

$\sin(77) = \dfrac{?}{3300}$

$.9744(3300) = ?$

3215 km

7. A kite is 100 ft above the ground. How many feet of string (to the nearest foot) are used if the angle of elevation is 35°? (3 pts.)



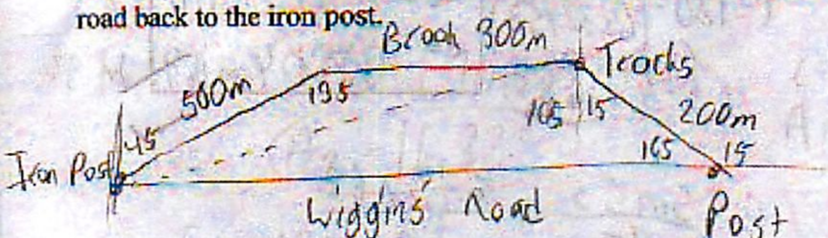
$\sin 35 = \dfrac{100}{?}$

$\dfrac{.5736\,? = 100}{.5736 \quad .5736}$

$? = 174 ft$

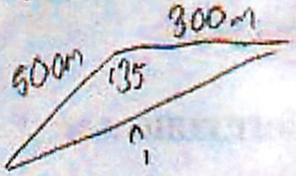
8. Sketch the plot of land (2 pts.) and answer the following questions.

From an iron post, proceed 500m northeast to the brook, then 300 m east along the brook to the train tracks, then 200m $S15°E$ along the train tracks to a post on the edge of Wiggin's Road, and finally along Wiggin's road back to the iron post.



14

What is the distance from the iron post to the northern end of the train tracks? Round to the nearest hundredth. (3 pts.)

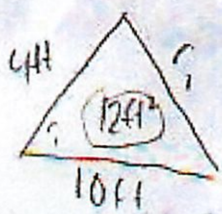


$c^2 = 500^2 + 300^2 - 2(500)(300)\cos(135)$

$c^2 = 552132$

$c = 743.06$ m

9. An obtuse triangle with area 12 ft^2 has two sides of length 4ft and 10 ft. Find the length of the third side. (There are two answers.) (Round to the nearest hundredth.) (7 pts.)



$\frac{12}{20} = \frac{\frac{1}{2}(4)(10)\sin(?)}{20}$ must divide

$.6 = \sin ?$

$\sin^{-1}$ $\sin^{-1}$

$\boxed{\angle ?_1 = 36.87°}$

$180 - 36.87 =$

$\boxed{143.13 = \angle ?_2}$

$?_1^2 = 4^2 + 10^2 - 2(4)(10)\cos(36.87)$

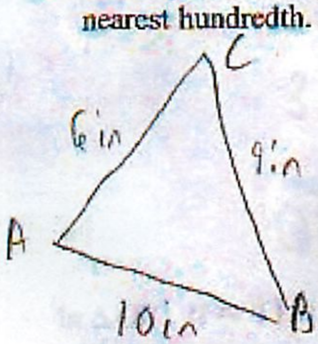
$?_1^2 = 52$

$?_1 = 7.21$ ft

$?_2^2 = 4^2 + 10^2 - 2(4)(10)\cos(143.13)$

$?_2 = 13.42$ ft

10. A triangle has sides of length 6 in., 9 in., and 10 in. What is the measure of its largest angle? Round to the nearest hundredth. (4 pts.)



$\cos C = \frac{6^2 + 9^2 - 10^2}{2(6)(9)}$

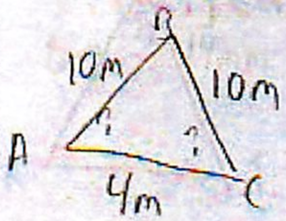
$\cos C = .1574$

$\cos^{-1}$ $\cos^{-1}$

$C = 80.94°$

The largest angle is the one between the two smallest sides.

11. The sides of an isosceles triangle have lengths 10 m, 10 m, and 4 m. Find the measure of its base angles to the nearest tenth of a degree. (4 pts.)



$$\cos A = \frac{10^2+4^2-10^2}{2(10)(4)}$$

$$\cos A = .2$$

$$\cos^{-1} \quad \cos^{-1}$$

$$A = 78.5°$$

√ The other base angle will be the same because its on isoscles tringle,

---

#4. $A_2 = 180 - (110.6837+53)$

$A_2 = 16.32°$

$C_2 = 180 - (16.32+53)$

$C_2 = 110.68°$

$c = \frac{\sin 53}{35} = \frac{\sin 110.68}{c}$

$\frac{32.744}{.7986} = \frac{.7986c}{.7986}$

$c = 41m$

Phythagorean Trig

p60

3/22

1. $\sin = \frac{y}{1}$ $\quad r=1 \quad$ $\cos = \frac{x}{1}$

Phytagorean Identy

$$(\cos\theta)^2 + (\sin\theta)^2 = 1$$

suppose to find this

like this

2. $1 = \sqrt{x^2+y^2}$

$1^2 = x^2 + y^2$

$\boxed{1 = x^2 + y^2}$

also

$y = r\sin\theta$

$x = r\cos\theta$

3. $-x^2 \quad -x^2$

$1^2 - x^2 = y^2$

$\sqrt{1^2 - x^2} = y \rightarrow \sin = \frac{\sqrt{1^2-x^2}}{1} \rightarrow \sin\sqrt{1-x^2}$

$\frac{\sin}{1} = \frac{y}{1} \quad y = \sin\theta$

$-y^2 \quad -y^2$

$1^2 - y^2 = x^2$

$\sqrt{1-y^2} = x \rightarrow \cos\frac{\sqrt{1-y^2}}{1} \rightarrow \cos\sqrt{1-y^2}$

I mean these are $=$ $\quad \cos = \frac{x}{1} \quad x = \cos\theta$

$\rightarrow r = \sqrt{x^2+y^2}$

$r = \sqrt{\sin^2 + \cos^2}$

$1 = \sqrt{\sin^2 + \cos^2}$

3. $(0,1)$ $\quad \sin\left(\frac{1}{1} = 1\right) \quad \cos\frac{0}{1} = 0$

$\sin\sqrt{1-0^2} = 1 \quad \cos\sqrt{1-1^2} = 0 \quad \theta = 650$

$(\cos 650)^2 + (\sin 650)^2 = 1$

$(1,0)$ $\quad \sin\frac{0}{1} = 0 \quad \cos\frac{1}{1} = 1$

$\sin\sqrt{1-1^2} = 0 \quad \cos = \sqrt{1-0^2} = 1$

$(-1,0)$ $\quad \sin\frac{0}{1} = 0 \quad \cos\frac{-1}{1} = -1$

$\sin\sqrt{1-0^2} = 0 \quad \cos\sqrt{1-0^2} = 1$ $\quad$ same answer when cos'ed

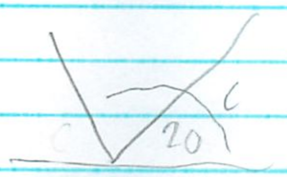
$(0,-1)$ $\quad \sin\frac{-1}{1} = -1$ $\quad$ not the same $\quad \cos\frac{0}{1} = 0$

$\sin\sqrt{1-0^2} = 1 \quad \cos\sqrt{1--1^2} = \sqrt{2}$ $\quad$ not the same

Position on the Wheel

p62

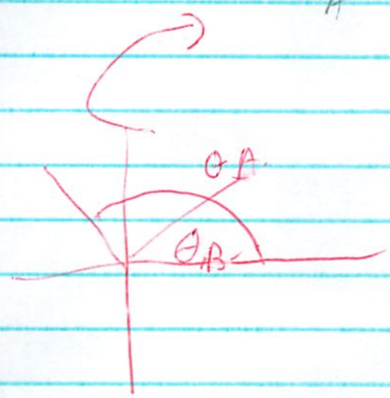
3/22

1. 

$\theta_B = 90 + \theta_A$

$\sin\theta_A = \sin(\cancel{90+}\ 180-\theta_B)$

didn't even work



$\frac{y}{r} = \frac{y}{r}$

More Positions on the Wheel (19)

3/26

1. a. They are both at the same X-coord. because they are on top of each other. In a circle, an X-coord. is always touched twice, once while moving to the left, and once while moving to the right (if the wheel is going in the same direction.)

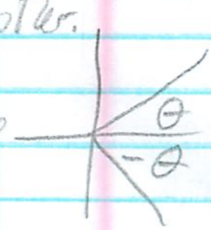
to the top and bottom

$\cos(\theta) = \cos(-\theta)$

b. That they are the same X-coord.

2. a. See 1a. The points are on top of each other.

b. Look at it, the X-coords are the same



Part 2

3. $\sin(-\theta) = -\sin\theta$

$\sin(-0) = -\sin 0$

$\sin(-90) = -\sin 90$

$\sin(-360) = -\sin 360$

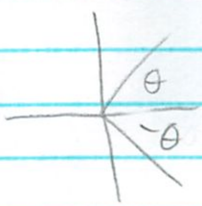
$\sin(-420) = -\sin 420$

$\sin(--50) = -\sin -50$

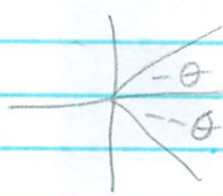
$\sin(-\theta) = \frac{-y}{r}$

$-\sin(\theta) = \frac{-y}{r}$

4.+5.



with - subistuted in or



The Ferris wheel now normally moves clockwise (backwards) So when it goes "backwards" from that (or forward) thats a 2x negitive

For this marking period, you will be studying the concepts of factoring. The content for each topic is located in your Algebra 2 Textbook and on supplemental sheets. At the end of your studying of these topics, you should be prepared to do the following:

Factoring (9.3)	
▪ How to factor polynomial expressions ▪ How to factor with a GCF ▪ How to factor trinomials ▪ How to factor difference of two squares	▪ How to factor by grouping ▪ How to factor the sum or difference of perfect cubes

For the assignments below:

- ~~**Copy the problem**~~
- ***Show all work***
- **Each section should be a new page**
- **Neatness and organization count**

Assignments are for the following sections:

- Complete the following worksheets (with ALL WORK SHOWN): Factor 1, Factor 2, Factor 3, Factor 4, Factor 5.

Good Luck.

This assignment is due on: 3/27/07

Name: Michael Plasmeier

Score: 19/25

76%

Test 3/29 3/30

FACTOR – 1

Factor the following problems using GCF-Factoring. Good Luck.

1. $bc^2 - 2b$ Answer: $b(c^2-2)$

2. $ax + ax^2$ Answer: $ax(1+x)$

3. $3ab^2 - 6a^2b$ Answer: $3ab(b-2a)$

4. $12x + 8xy$ Answer: $4x(3+2y)$

5. $2x^2 + 8x - 4$ ← no x Answer: $2(x^2+4x-2)$

6. $5x^2 - 20x + 10$ Answer: $5(x^2-4x+2)$

7. $n^3 + n^2 + n$ Answer: $n(n^2+n+1)$

8. $y^3 + y^2$ Answer: $y^2(y+1)$

9. $\pi \cdot r^2 + \pi \cdot r \cdot h$ Answer: $\pi r(r+h)$

$a^1 \cdot a^2 = a^{1+2} = a^3$

10. $y^{(x+1)} - y^x$ Answer: $y^x(y^1-1)$

$y^x \cdot y^1 - y^x$

10

$y^x(y^1-1)$

FOIL diamond

FACTOR – 2

Factor the following problems using trinomial factoring. If an expression is not factorable, write "non-factorable." Good Luck.

1. $a^2 - 6a + 5$ Answer: $(a-5)(a-1)$

2. $x^2 + 8x + 12$ Answer: $(x+2)(x+6)$

3. $x^2 + 16x + 39$ Answer: $(x+3)(x+13)$

4. $3x^2 - 2x - 5$ Answer: $(3x-5)(x+1)$

5. $6m^2 - 17m + 5$ Answer: $(2x-5)(3x-1)$

6. $3x^2 - 11x + 6$ Answer: $(3x-2)(x-3)$

7. $6x^2 + 13x - 28$ Answer: ~~non factorable~~ $(2x+7)(3x-4)$

8. $5a^2 - 11a - 12$ Answer: $(5x-4)(x+3)$

9. $5x^2 - 15x - 21$ Answer: non-factorable

10. $20d^2 + 27d - 8$ Answer: $(5d+8)(4d-1)$

9

FACTOR – 3

Factor the following problems using difference of squares factoring. If an expression is not factorable, write "non-factorable." ~~Good Luck~~

1. $a^2 - 49$ Answer: $(a+7)(a-7)$

2. $4x^2 - 49$ Answer: $(2x+7)(2x-7)$

3. $9 - 16m^2$ ← change one Answer: $(+4m+3)(-4m+3)$ ← switches ← always pos

4. $16x^4 - 81$ Answer: $(4x^2+9)(4x^2-9)$ diff 2 sq. $(2x+3)(2x-3)$

5. $169c^2 - 100$ Answer: $(13c+10)(13c-10)$

6. $4x^2 - 25y^2$ Answer: $(2x+5y)(2x-5y)$

7. $x^6 - 16$ Answer: $(x^3+4)(x^3-4)$

8. $x^{2a} - 1$ Answer: $(x^a+1)(x^a-1)$

9. $4a^2 - 225$ Answer: $(2a+15)(2a-15)$

10. $-x^2 + 1$ Answer: $(1+x)(1-x)$

8

a^3+b^3 I don't

FACTOR – 4

Factor the following problems using sum and difference of cubes factoring. If an expression is not factorable, write "non-factorable." Good Luck.

same, inverse, always +, didn't really know

1. $y^3 - 125$ y^3-5^3 Answer: $(y-5)(y^2+5y+25)$

2. $x^3 + 64$ Answer: $(x+4)(x^2-4x+16)$

vert shift

3. $a^3 - 1$ a^3-1^3 Answer: $(a-1)(a^2$ ... midline

amp, phase shift

4. $d^3 + e^3$ d^3+e^3 Answer: $(d+e)(d^2-de+e^2)$

5. $27a^3 - 64$ $(3a)^3-4^3$ Answer: $(3a-4)(9a^2+12a+16)$

6. $216x^3 + y^3$ $6x^3+y^3$ Answer: $(6x+y)(36x^2-6xy+y^2)$

7. $x^6 - 8$ phase shift $(x^2)^3\ 2^3$ Answer: $(x^2+2)(x^4-2x^2+4)$

8. $x^3y^3 + 27$ $(xy)^3+3^3$ Answer: $(xy+3)(x^2y^2-3xy+9)$ can't reverse $(3-4x^4)$

9. $27 - 64x^{12}$ $(4x^4)^3\ 3^3$ Answer: $(4x^4-3)(16x^8+12x^4+9)$

10. $125 + (abc)^3$ $(abc)^3+5^3$ Answer: $(abc+5)(a^2b^2c^2-5abc+25)$

vert shift, amp, midline, period

might not always work w/ (a+b)+(c+d)
try (a+c)(b+d)

FACTOR – 5

Factor the following problems using factoring by grouping. If an expression is not factorable, write "non-factorable." Good Luck.

1. $30x^3 + 40x^2 + 3x + 4$ Answer: $(10x^2+1)(3x+4)$ need
$(30x^3+40x^2)+(3x+4)$
$10x^2(3x+4)(3x+4)$

2. $9x^3 + 18x^2 + 7x + 14$ Answer: $(9x^2+7)(x+2)$
$9x^2(x+2)+7(x+2)$

3. $18x^3 + 30x^2 + 3x + 5$ Answer: $(6x^2+1)(3x+5)$ need
$6x^2(3x+5)+1(3x+5)$

4. $5x^3 - 20x^2 + 3x - 12$ Answer: $(5x^2+3)\cancel{(x+4)}(x-4)$ not x^3, so no diff 2 cubes
$5x^2(x-4)+3(x-4)$
$(5x^2+3)(x-4)$

5. $-2x^3 - 4x^2 - 3x - 6$ Answer: $(-2x^2-3)(x+2)$
$-2x^2(x+2)+-3(x+2)$

6. $3x^4 + 3x^3 + 6x^2 + 6x$ Answer: $3x^2(x+2)(x+1)$
$3x^3(x+1)\ 6x^2(x+1)$
$(3x^3+6x^2)(x+1)$

7. $x^4 + 12x^3 + 4x^2 + 48$ Answer: non-factorable
$x^3(x+12)+4(x^2+12)$ rearrange
$x^4+4x^2+12x^3+48$
$x^2(x^2+4)+12(x^3+4)$

8. $x^4 + 7x^3 - 8x - 56$ Answer: $\cancel{(x^3-8)}(x+7)$
diff 2 cubes
$(x-2)(x^2+2x+4)$
$x^3(x+7)-8(x+7)$

9. $18x^3 - 9x^2 - 18x + 9$ Answer: $9(x+1)(x-1)(2x+1)$
$9x^2(2x-1)+9(2x+1)$ $9(x^2-1)(2x+1)$
further
$(9x^2-9)(2x+1)$

10. $18x^3 - 2x^2 + 27x - 3$ Answer: $(2x^2+3)(9x-1)$
$2x^2(9x-1)+3(9x-1)$

8

IAG 3
HOT SHEET

Name ________________

Date ________________

Unit ________________

14.3 Simplfing

Trig Identities

3/27

Simplify

- might help if substitude given expression into terms of sin() and cos(),
  - dosn't always work

Ex

$\sec(x) \cdot \cot(x)$

$\frac{1}{\cos(x)} \cdot \frac{1}{\tan x}$

$\frac{1}{\cos x} \cdot \frac{1}{\frac{\sin x}{\cos x}}$

$\frac{1}{\cos(x)} \cdot \frac{\cos x}{\sin x}$ or

$\frac{1}{\sin x} = \boxed{\csc(x)}$

$\sec(x) \cdot \cot(x)$

$\frac{1}{\cos x} \cdot \frac{\cos(x)}{\sin(x)}$

$\frac{1}{\sin x} = \csc(x)$

Ex 2

$\tan x \cdot \cot x$

$\frac{\tan x}{1} \cdot \frac{1}{\tan x}$

$\boxed{1}$

$\tan x \cdot \cot x$

$\frac{\sin x}{\cos x} \cdot \frac{\cos x}{\sin x}$

$\boxed{1}$

Ex 3

$\sec^2 x + 2\tan^2 x$

$1 + \tan^2 x + 2\tan^2 x$

$1 + 3\tan^2 x$

Ex 4

$\cos(-x) \cdot \cot(\frac{\pi}{2} - x)$

$\cos x \cdot \tan x$

$\cos x \cdot \frac{\sin x}{\cos x}$

$\boxed{\sin x}$

$\longrightarrow$

5. $\cos x - \cos x \sin^2 x$

$\cancel{\sin^2 x}$ ← can't do

$\cos x - \cos x(1-\cos^2 x)$

$\cos x - \cos x + \cos^3 x$

$\cos^3 x$

~~or~~ GCF

$\cos x - \cos x \sin^2 x$

$\cos x(1-\sin^2 x)$

$\cos x \cdot \cos^2 x$

$\cos^3 x$

6. $\sin\left(\frac{\pi}{2} - x\right)\tan x$

$\cos x \tan x$

$\frac{\cos x}{1} \cdot \frac{\sin x}{\cos x} = \sin x$

go farther

7. $\dfrac{1-\cos^2 x}{\sec^2 x - \tan^2 x}$

$\dfrac{1-\cos^2 x}{1+\tan^2 x - \tan^2 x}$

$\dfrac{1-\cos^2 x}{1}$) can't do $\cancel{\dfrac{1-\cos^2 x}{1} \to -\cos^2 x}$

$\dfrac{\sin^2 x}{1} \to \sin^2 x$

8. $\cos x(1+\tan^2 x)$

$\cos x \sec^2 x$

$\frac{1}{\sec x} \cdot \frac{\sec^2 x}{1} = \frac{\sec^2 x}{\sec x} \to \sec x$

14.3 Proving Trig Identities

3/27

- Goal transform the left side of identity into the right
- may need to use factoring, distributive property, adding or subtracting fractions, etc.

Ex $\frac{\cot^2 x}{\csc x} = \csc - \sin x$

$\frac{\csc^2 x - 1}{\csc x}$

$\frac{\csc^2 x}{\csc x} - \frac{1}{\csc x}$ say $\csc x - \sin x = \csc x - \sin x$

$\csc x - \sin x$

Ex 2 $\cos^2 x - \sin^2 x = 2\cos^2 x - 1$ $\leftarrow \cos^2 x - (1 - \cos^2 x)$

$\cos^2 x - 1 + \cos^2 x$

$2\cos^2 x - 1 = 2\cos^2 x - 1$

4/12

Ex 3 $\frac{1 - \cos^2 x}{\sec^2 x - \tan^2 x} = \sin^2 x$

$\frac{\sin^2 x}{(1 + \tan^2 x) - \tan^2 x}$

$\frac{\sin^2 x}{1} = \sin^2 x$

Ex 4 $\frac{\cos^2 x}{\cos(\frac{\pi}{2} - x)} + \sin x = \csc x$

$\frac{\cos^2 x}{\sin x} + \frac{\sin x}{1}$ ← common denometer

$\frac{\cos^2 x}{\sin x} + \frac{\sin x^2}{\sin x}$

$\frac{\cos^2 x + \sin x^2}{\sin x}$

$\frac{1}{\sin x} = \csc x$

14.3 Trig Identities

Practice

3/27

Simplify

p. 47

6. $\dfrac{\cot(-x)}{\csc(-x)}$

$\dfrac{\left(\dfrac{\cos x}{-\sin x}\right)}{\dfrac{1}{-\sin x}}$ $\dfrac{\cos x}{-\sin x} \times \dfrac{-\sin x}{1} = \cos x$

8. $\cos x\,(1+\tan^2 x)$

$\cos x \sec^2 x$

$\cancel{\cos x} \cdot \dfrac{1}{\cos^2 x}$

$\dfrac{\cancel{\cos x}}{\cos^2 x} = \frac{1}{2}\cos x = \sec x$

do oter thing convert

$\dfrac{1}{\sec x}\ \dfrac{\sec^2 x}{1} = \dfrac{\sec^2 x}{\sec x} = \sec x$

← same way but different

9. $\sec^2\left(\frac{\pi}{2}-x\right)-1$

$\csc^2 x - 1$

$\cot^2 x$ ← pythagorean identities

10. $\dfrac{\cos^2 x \cot^2\left(\frac{\pi}{2}-x\right)-1}{\cos^2 x}$

$\dfrac{\cos^2 x \tan^2 x - 1}{\cos^2 x}$

can't break up

$\cancel{\dfrac{\cos^2 x}{\cos^2 x}} \times \dfrac{\tan^2 x}{\cos^2 x} - \dfrac{1}{\cos^2 x}$

cancle out

$\cancel{\dfrac{\frac{\sin^2 x}{\cos^2 x}}{\cos^2 x}} + \dfrac{-1}{\cos^2 x}$

$\dfrac{\cos^2 x \tan^2 x}{\cos^2 x} - \dfrac{1}{\cos^2 x}$ ← now cancle out

$\tan^2 x - \sec^2 x$

$1+\tan^2 x = \sec^2 x$

$-\sec^2 x \quad \sec^2$

$1+\tan^2 x - \sec^2 x = 0$

$-1 \quad -1$

-1

$\cancel{\dfrac{\sin^2 x}{\cos^2 x} \quad \dfrac{\cos^2 x}{1}}$

$\cancel{1 - \dfrac{\sec x}{\sin x}}$

12. $\cot\left(\frac{\pi}{2}-x\right)\cot x$

$\tan x \cot x$

$\frac{\sin x}{\cos x} \cdot \frac{\cos x}{\sin x} = 1$

18. $\frac{\cos^2(-x)}{\cot^2 x}$

$\frac{\cos^2 x}{\cot^2 x}$

$\frac{\cos^2 x}{\frac{\cos^2 x}{\sin^2 x}} \qquad \frac{\cos^2 x}{1} \cdot \frac{\sin^2 x}{\cos^2} = \sin^2 x$

22. $\tan x \cot\left(\frac{\pi}{2}-x\right) - \sec^2 x$

$\tan x \tan x - \sec^2 x$

$\tan^2 x - \sec^2 x$

$\cancel{\frac{\sin^2 x}{\cos^2 x}} \cdot \cancel{\frac{1}{\cos^2 x}} \qquad \cancel{\sin^2 x - 1 \rightarrow \cos^2 x}$

? ~~keep denom~~?

$\tan^2 x - (1+\tan^2 x)$

$\tan^2 x - 1 - \tan^2 x$

-1

WRITTEN EXERCISES

Simplify.

A

foil

1. a. $\cos^2\theta + \sin^2\theta$ b. $(1 - \cos\theta)(1 + \cos\theta)$ c. $(\sin\theta - 1)(\sin\theta + 1)$
2. a. $1 + \tan^2\theta$ b. $(\sec x - 1)(\sec x + 1)$ c. $\tan^2 x - \sec^2 x$
3. a. $1 + \cot^2 A$ b. $(\csc A - 1)(\csc A + 1)$ c. $\dfrac{1}{\sin^2 A} - \dfrac{1}{\tan^2 A}$
4. a. $\dfrac{1}{\cos(90° - \theta)}$ b. $1 - \dfrac{\sin^2\theta}{\tan^2\theta}$ c. $\dfrac{1}{\cos^2\theta} - \dfrac{1}{\cot^2\theta}$
5. a. $\cos\theta\cot(90° - \theta)$ b. $\csc^2 x(1 - \cos^2 x)$ c. $\cos\theta(\sec\theta - \cos\theta)$
6. a. $\cot A\sec A\sin A$ b. $\cos^2 A(\sec^2 A - 1)$ c. $\sin\theta(\csc\theta - \sin\theta)$
7. $\sin A\tan A + \sin(90° - A)$
8. $\csc A - \cos A\cot A$
9. $(\sec B - \tan B)(\sec B + \tan B)$
10. $(1 - \cos B)(\csc B + \cot B)$
11. $(\csc x - \cot x)(\sec x + 1)$
12. $(1 - \cos x)(1 + \sec x)\cos x$

Simplify each expression.

13. $\dfrac{\sin x\cos x}{1 - \cos^2 x}$
14. $\dfrac{\tan x + \cot x}{\sec^2 x}$
15. $(\sin x + \cos x)^2 + (\sin x - \cos x)^2$
16. $(\sec^2\theta - 1)(\csc^2\theta - 1)$
17. $\dfrac{\cot^2\theta}{1 + \csc\theta} + \sin\theta\csc\theta$
18. $\dfrac{\tan^2\theta}{\sec\theta + 1} + 1$
19. $\cos^3 y + \cos y\sin^2 y$ cos y(
20. $\dfrac{\sec y + \csc y}{1 + \tan y}$

B

21. $\dfrac{\cos\theta}{1 + \sin\theta} + \dfrac{1 + \sin\theta}{\cos\theta}$
22. $\dfrac{\sin\theta\cot\theta + \cos\theta}{2\tan(90° - \theta)}$
23. $\dfrac{\sin^4\theta - \cos^4\theta}{\sin^2\theta - \cos^2\theta}$
24. $\dfrac{\sin^2\theta}{1 + \cos\theta}$ (*Hint:* See Exercise 1(b).)
25. Use the equation $\sin^2\theta + \cos^2\theta = 1$ to prove that $\tan^2\theta + 1 = \sec^2\theta$.
26. Use the equation $\sin^2\theta + \cos^2\theta = 1$ to prove that $\cot^2\theta + 1 = \csc^2\theta$.

In Exercises 27 and 28, use a graphing calculator or a computer to graph the given functions.

27. Graph the function $y = \sin^4 x + 2\sin^2 x\cos^2 x + \cos^4 x$. What is the domain of this function? What other function could have this same graph? Use trigonometric relationships to verify the suggested identity.
28. Graph the function $y = (\sin x \div \cos x) \div \tan x$. What is the domain of this function? What other function could have this same graph? Use trigonometric relationships to verify the suggested identity.

In Exercises 29–36, prove the given identity.

29. $\cot^2\theta + \cos^2\theta + \sin^2\theta = \csc^2\theta$

30. $\dfrac{\cot\theta - \tan\theta}{\sin\theta\cos\theta} = \csc^2\theta - \sec^2\theta$

31. $\dfrac{\sin\theta}{\csc\theta} + \dfrac{\cos\theta}{\sec\theta} = \sin\theta\csc\theta$

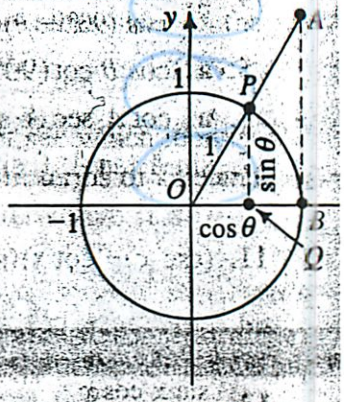
32. $\dfrac{1-\sin^2\theta}{1+\cot^2\theta} = \sin^2\theta\cos^2\theta$

33. $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$

34. $\dfrac{\tan^2 x}{1+\tan^2 x} = \sin^2 x$

35. $\dfrac{\sin\theta}{\sin\theta + \cos\theta} = \dfrac{\tan\theta}{1+\tan\theta}$

36. $\dfrac{\tan x}{1+\sec x} + \dfrac{1+\sec x}{\tan x} = 2\csc x$



Ex. 37

37. $\overline{AB}$ is tangent to the unit circle at $B(1, 0)$.

a. Why is $\triangle OPQ \sim \triangle OAB$?

b. Use part (a) to explain why

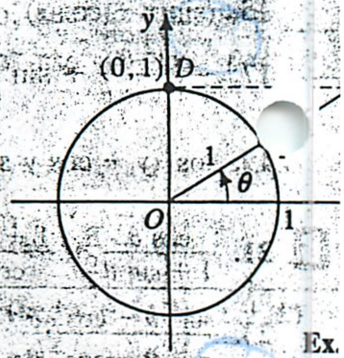
$$\frac{PQ}{OQ} = \frac{AB}{OB} \text{ and } \frac{OP}{OQ} = \frac{AO}{BO}.$$

c. Use part (b) to show that $AB = \tan\theta$ and $AO = \sec\theta$.

d. *Visual Thinking* Use the diagram to explain why the name *tangent* is given to the expression $\dfrac{\sin x}{\cos x}$ and the name *secant* is given to the expression $\dfrac{1}{\cos x}$.

e. Use right triangle AOB to prove that $\sec^2\theta = 1 + \tan^2\theta$.

f. Extend $\overline{AO}$ to intersect the circle at C. A theorem from geometry states that $(AB)^2 = AP \cdot AC$. Use this fact to prove $\tan^2\theta = (\sec\theta - 1)(\sec\theta + 1)$.



Ex.

38. $\overline{CD}$ is tangent to the unit circle at $D(0, 1)$. Show that

$$CD = \cot\theta \text{ and } CO = \csc\theta.$$

(*Hint:* See Exercise 37.)

39. *Writing* Jon expected that the graph of $y = \sqrt{1+\tan^2 x}$ would be the same as the graph of $y = \sec x$, but his graphing calculator showed that this was not the case. Write a paragraph explaining why this happened.

40. Express $\tan\theta$ in terms of $\cos\theta$ only.

41. Express $\sec\theta$ in terms of $\sin\theta$ on

C **42.** Prove $\sqrt{\dfrac{1-\sin x}{1+\sin x}} = |\sec x - \tan x|$. For what values of x is this identity true?

COMPUTER EXERCISE

Imagine that your computer can calculate only the sine function. Write a program for which the input is any x, where $0 \le x \le \frac{\pi}{2}$, and the outputs are the six trigonometric functions of x. (*Hint:* See Exercise 41.)

Simplifying Trig Identites
Written Exercises

4/9

Circled Problems

Blue

1a $\cos^2\theta + \sin^2\theta$

phy identity

1

1b. $(1-\cos\theta)(1+\cos\theta)$

$(1\cdot 1)+(1\cdot\cos\theta)+(-\cos\theta\cdot 1)+(-\cos\theta\cdot\cos\theta)$

$-\cos^2\theta + 1$

$\cancel{1+\sin^2\theta+1}$ $\quad 1-\cos^2\theta$

$\cancel{2+\sin^2\theta}$ $\quad \sin^2 x$

3a $1+\cot^2\theta$

$\csc^2 x$

3b $(\csc x+1)(\csc x-1)$

$(\csc x\cdot\csc x)(\csc x\cdot -1)(1\cdot\csc x)(1\cdot -1)$

$\csc x^2 - 1$

$1+\cot x^2 - 1$

$\cot x^2$

4a $\dfrac{1}{\cos(90°-\theta)}$ $\quad \dfrac{1}{\sin x} = \csc x$

$90 = \frac{\pi}{2}$

$\cancel{\dfrac{1}{\cos\theta}} \quad \cancel{\sec\theta}$

b. $1 - \dfrac{\sin^2\theta}{\tan^2\theta}$ keep sin $\quad \dfrac{1}{\cancel{\sin^2 x}}$ $\quad \cancel{1-\csc x}$

$1 - \dfrac{\cancel{1-\cos^2 x}\ \sin^2}{\dfrac{\sin x}{\cos x}} \rightarrow \dfrac{\cancel{1-\cot^2 x}}{1} \cdot \cancel{\dfrac{\cos x}{\sin x}} \rightarrow \dfrac{1}{\sin x}$

$1 - \left(\dfrac{\sin^2 x}{1} \cdot \dfrac{\cos^2}{\sin^2}\right) \rightarrow 1-\cos^2$

$\sin^2 x$

5a $\cos\theta\cot(90°-\theta)$

~~error~~

$\cos\theta \quad \tan\theta$

$\dfrac{\cos x}{1} \quad \dfrac{\sin x}{\cos x} \rightarrow \sin x$

b. $\csc^2 x(1-\cos^2 x)$

$\csc^2 x \sin^2 x$

$\dfrac{1}{\sin^2 x} \quad \dfrac{\sin^2 x}{1} \rightarrow \dfrac{\sin^2 x}{\sin^2 x} \rightarrow 1$

13. $\frac{\sin x \cos x}{1-\cos^2 x}$ $\to$ $\frac{\sin x \cos x}{\sin^2 x}$ $\to$ $\frac{\cos x}{\sin x}$ $\Rightarrow$ $\cot x$

$\cancel{\frac{1-\cos x(\cos x)}{1-\cos^2 x}}$?

$\cancel{\frac{1}{\sin^2 x} = \csc^2 x}$

~~$1-\cos \cdot \cos x$ cancle out~~ ?

18. $\frac{\tan^2\theta}{\sec\theta+1}+1$

$\cancel{\frac{\tan^2 x}{1+\tan x+1}+1}$

$\cancel{\frac{\tan^2 x}{2}+\tan x+1}$

$\cancel{\frac{\tan^2 x}{2}+\sec^2 x}$

?

$\frac{\sec^2\theta-1}{\sec^2\theta+1}+1$

$\frac{(\sec\theta+1)(\sec\theta-1)}{\sec^2\theta+1}+1$

$\sec\theta-1+1$

$\sec\theta$

21. $\frac{\cos\theta}{1+\sin\theta}+\frac{1+\sin\theta}{\cos\theta}$

$2\sec\theta$

~~recipical~~

1

?, ~~addition not multiplication~~

retry on new paper

red squares

4/10

what is FOILing

$(a+b)(c+d)$

$(a\cdot c)+(a\cdot d)+(b\cdot c)+(b\cdot d)$

9. $(\sec B-\tan B)(\sec B+\tan B)$

$(\sec B\times\sec B)+(\sec B\times\tan B)+(-\tan B\times\sec B)+(-\tan B\times\tan B)$

$\sec B^2-\tan B^2$

pyt identity

1

10. $(1-\cos B)(\csc B+\cot B)$

$(1\times\csc B)+(1\times\cot B)+(-\cos B\cdot\csc B)+(-\cos B\cdot\cot B)$

change every thing to sin + cos

don't reduce

$\frac{1}{\sin B}+\frac{\cos B}{\sin B}+\left(-\cos B\cdot\frac{1}{\sin B}\right)+\left(-\cos B\cdot\frac{\cos B}{\sin B}\right)$

$\frac{1}{\sin B}+\frac{\cos B}{\sin B}-\frac{\cos B}{\sin B}-\frac{\cos B^2}{\sin B}$

$\frac{1}{\sin B}-\frac{\cos B^2}{\sin B}$ keep deneromenator $\frac{1-\cos B^2}{\sin B}\rightarrow\frac{\sin B^2}{\sin B}\Rightarrow \sin B$

$\cancel{\frac{1-\cos B^2}{\sin B^2}}$

11. $(\csc x-\cot x)(\sec x+1)\rightarrow$ foil lol $\Rightarrow(\csc x\cdot\sec x)+(\csc x\cdot 1)+(-\cot\cdot\sec x)+(-\cot)$

$\frac{1}{\sin x}\cdot\frac{1}{\cos x}+\frac{1}{\sin x}-\frac{\cos x}{\sin x}\cdot\frac{1}{\cos x}-\frac{\cos x}{\sin x}$

$\frac{1}{\sin x\cos x}-\frac{\cos x}{\sin x\cos x}+\frac{1}{\sin x}-\frac{\cos x}{\sin x}$

find common denemeater

$\frac{1}{\sin x\cos x}-\frac{1}{\sin x}+\frac{1}{\sin x}-\frac{\cos x}{\sin x}$

$\frac{1}{\sin x\cos x}-\frac{\cos x}{\sin x}\rightarrow\frac{1}{\sin x\cos x}-\frac{\cos x^2}{\sin x\cos x}$

$\frac{1-\cos x^2}{\sin x\cos x}$

$\frac{\sin x^2}{\sin x\cos x}$

$\frac{\sin x}{\cos x}$

$\tan x$

~~sec x~~

$\tan x$

12. $(1-\cos x)(1+\sec x)\cos x$

$[(1\cdot 1)+(1\cdot\sec x)+(-\cos x\cdot 1)+(-\cos\cdot\sec x)]\cos x$

add

$1+(\sec x)(-\cos x\cdot\sec x)$

$1-\cos x\ \sec^2 x$

doesn't work w/o power

over

12 cont.

$(1 + \sec x - \cos x + -\cos \cdot \sec x) \cos x$

$\left(1 + \frac{1}{\cos x} - \frac{\cos x}{1} + \frac{-\cos x}{1} \cdot \frac{1}{\cos x}\right) \cos x$

$\left(1 + \frac{1}{\cos x} - \frac{\cos x}{1} + \frac{-\cos x}{\cos x}\right) \cos x$

$\left(1 - 1 + \frac{1}{\cos x} - \frac{\cos x}{1}\right) \cos x$

$\left(\frac{1}{\cos x} - \frac{\cos x^2}{\cos x}\right) \cos x$

$\left(\frac{1 - \cos x^2}{\cos x}\right) \cos x$

$\frac{\sin x}{\cos x} \cos x$

$\tan x \cos x$

$(\sec x - \cos x) \cos x$

$\sec x \cos x - \cos x^2$

$\frac{1}{\cos x} \cdot \frac{\cos x}{1} - \cos x^2$

$1 - \cos x^2$

$\sin x^2$

Pencil squares + 21

21. $\frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta}$

Foil → $\frac{\cos\theta^2}{(1+\sin\theta)(\cos\theta)} + \frac{(1+\sin\theta)\cancel{(\cos\theta)}(1+\sin\theta)}{\cancel{\cos\theta^2}\ \cos\theta(1+\sin\theta)}$

~~what help was that?~~

don't foil → $\frac{\cos\theta^2}{\cos\theta(1+\sin\theta)} + \frac{(1\cdot 1)+(1\cdot\sin\theta)+(\sin\theta\cdot 1)(\sin\theta^2)}{\cos\theta(1+\sin\theta)}$

can combine when denomonators are =

$\frac{\cos\theta^2 + 2\sin\theta + \sin\theta^2 + 1}{\cos(1+\sin\theta)}$

$\frac{2\sin\theta+2}{\cos(1+\sin\theta)} \rightarrow \frac{2(\sin\theta+1)}{\cos\theta(\sin\theta+1)} \rightarrow \frac{2}{\cos\theta} \rightarrow 2\sec\theta$

1c. $(\sin\theta - 1)(\sin\theta+1)$ ← $\boxed{\text{Foil}}$ ← remember

+ ← make sure

$(\sin\theta\cdot\sin\theta) + (\sin\theta\cdot 1) + (-1\cdot\sin\theta) + (-1\cdot 1)$

$\sin\theta^2 - 1$

~~+ leave~~

$-\cos^2\theta$

5c $\cos\theta(\sec\theta - \cos\theta)$

$\cos\theta\left(\frac{1}{\cos\theta} - \frac{\cos\theta}{1}\right)$ → common denometer

$\cos\theta\left(\frac{1}{\cos\theta} - \frac{\cos\theta^2}{\cos\theta}\right)$

$\cos\theta\left(\frac{1-\cos\theta^2}{\cos\theta}\right)$

$1-\cos\theta^2$

$\sin\theta^2$

6a. $\cot A \sec A \sin A$

$\frac{\cos A}{\sin A} \cdot \frac{1}{\cos A} \cdot \frac{\sin A}{1}$

doesn't work like that :(

$\cancel{\sin A \cdot \sin A}$

$\cancel{\sin A^2}$

$\frac{\cos A}{\sin A} \cdot \frac{\sin A}{\cos A}$

1

7. $\sin A \tan A + \sin(90 - A)$

$\frac{\sin A}{1} \cdot \frac{\sin A}{\cos A} + \frac{\cos A}{1}$

$\frac{\sin A^2}{\cos A} + \frac{\cos A}{1}$ common denom.

$\frac{\sin A^2}{\cos A} + \frac{\cos A^2}{\cos A}$

$\frac{\sin A^2 + \cos A^2}{\cos A}$

$\frac{1}{\cos A}$

$\sec A$

16. $(\sec^2\theta - 1)(\csc^2\theta - 1)$

$\tan^2\theta \cot^2\theta$

$\frac{\sin^2\theta}{\cos^2\theta} \cdot \frac{\cos^2\theta}{\sin^2\theta}$

1

19. $\cos^3 y + \cos y \sin^2 y$ ← GCF

$\cos y(\cos^2 y + \sin^2 y)$

$\cos y(1)$

$\cos y$

14.3 Trig Identies

Independent Practice 4/12

p747

23 $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

$1 - \sin^2 x - \sin^2 x$

$1 - 2\sin^2 x = 1 - 2\sin^2 x$

25. $\cot^2 x(\sec^2 x - 1) = 1$

$\cot^2 x \cdot \tan^2 x$

$\frac{\cos^2 x}{\sin^2 x} \cdot \frac{\sin^2 x}{\cos^2 x}$

$1 = 1$

27. $\sin x \csc x = 1$

$\sin x \cdot \frac{1}{\sin x}$

$1 = 1$

29. $\tan^2 x + 4 = \sec^2 x + 3$

$\sec^2 x - 1 + 4$

$\sec^2 x + 3 = \sec^2 x + 3$

33. $\frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = 2\sec x$

$\frac{(1+\sin x)^2}{\cos x(1+\sin x)} + \frac{\cos x^2}{(1+\sin x)\cos x}$

$\frac{(1+\sin x)^2 + \cos x^2}{\cos x(1+\sin x)}$

keep - this doesn't disappear!

FOIL that

$\cancel{1+\sin x + \cos x}$

$1 + \frac{1}{\csc x} + \frac{1}{\sec x}$

$$\frac{(1\cdot 1)+(1\cdot \sin x)+(\sin x\cdot 1)+(\sin x^2)+\cos x}{\cos x\,(1+\sin x)}$$

$$\frac{1+2\sin x+\sin x^2+\cos x}{(1+\sin x)\cos x}$$

$$\frac{2+2\sin x}{(1+\sin x)\cos x}$$

GCF

$$\frac{2(1+\sin x)}{\cos x\,(1+\sin x)}$$

$$\frac{2}{\cos x} \rightarrow 2\sec x$$

say again

$$2\sec x = 2\sec x$$

37. $\cos\left(\frac{\pi}{2}-x\right)\csc x = 1$

$\sin x \csc x$

$\frac{1}{\csc x}\cdot\frac{\csc x}{1}$

$1 = 1$

14.3

Answers

Section 14.3: Trigonometric Identities

pg 747 #23-37 odd (not #31, 35)

23) $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

$(1-\sin^2 x) - \sin^2 x$

$1 - 2\sin^2 x = 1 - 2\sin^2 x$

25) $\cot^2 x\,(\sec^2 x - 1) = 1$

$\cot^2 x\,(\tan^2 x) = 1$

$\frac{1}{\tan^2 x} \cdot \frac{\tan^2 x}{1} = 1$

$1 = 1$

27) $\sin x \csc x = 1$

$\sin x \cdot \frac{1}{\sin x} = 1$

$1 = 1$

29) $\tan^2 x + 4 = \sec^2 x + 3$

$(\sec^2 x - 1) + 4$

$\sec^2 x - 1 + 4$

$\sec^2 x + 3 = \sec^2 x + 3$

~~33~~

(33) $\frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} = 2\sec x$

$$\frac{(1+\sin x)(1+\sin x)}{\cos x(1+\sin x)} + \frac{\cos x \cdot \cos x}{\cos x(1+\sin x)}$$

$$\frac{1+2\sin x+\sin^2 x + \cos^2 x}{\cos x(1+\sin x)}$$

$$\frac{1+2\sin x+1}{\cos(1+\sin x)}$$

$$\frac{2+2\sin x}{\cos x(1+\sin x)}$$

$$\frac{2(1+\sin x)}{\cos x(1+\sin x)}$$

$$\frac{2}{\cos x}$$

$$2\sec x = 2\sec x$$

(37) $\cos\left(\frac{\pi}{2}-x\right)\csc x = 1$

$$\sin x \cdot \csc x$$

$$\frac{1}{\csc x} \cdot \frac{\csc x}{1}$$

$$1 = 1$$

Warmup 4/13

4/13

p747

28. $\tan y \cot y = 1$

$\frac{\sin y}{\cos y} \cdot \frac{\cos x}{\sin x}$

$1 = 1$

38. $\frac{\cos\left(\frac{\pi}{2} \cdot x\right)}{\sin\left(\frac{\pi}{2} - x\right)} = \tan x$

$\frac{\sin x}{\cos x}$

$\tan x = \tan x$

Proving Trig Identities

For Problems 1–26, show the steps in transforming the expression on the left to the one on the right.

1. $\cos x \tan x$ to $\sin x$
2. $\csc x \tan x$ to $\sec x$
3. $\sec A \cot A \sin A$ to 1
4. $\csc B \tan B \cos B$ to 1
5. $\sin^2 \theta \sec \theta \csc \theta$ to $\tan \theta$
6. $\cos^2 \alpha \csc \alpha \sec \alpha$ to $\cot \alpha$
7. $\cot R + \tan R$ to $\csc R \sec R$
8. $\cot D \cos D + \sin D$ to $\csc D$
9. $\csc x - \sin x$ to $\cot x \cos x$
10. $\sec x - \cos x$ to $\sin x \tan x$
11. $\tan x(\cot x \cos x + \sin x)$ to $\sec x$
12. $\cos x(\sec x + \cos x \csc^2 x)$ to $\csc^2 x$
13. $(1 + \sin B)(1 - \sin B)$ to $\cos^2 B$
14. $(\sec E - 1)(\sec E + 1)$ to $\tan^2 E$
15. $(\cos \phi - \sin \phi)^2$ to $1 - 2 \cos \phi \sin \phi$
16. $(1 - \tan \phi)^2$ to $\sec^2 \phi - 2 \tan \phi$
17. $(\tan n + \cot n)^2$ to $\sec^2 n + \csc^2 n$
18. $(\cos k - \sec k)^2$ to $\tan^2 k - \sin^2 k$
19. $\dfrac{\csc^2 x - 1}{\cos x}$ to $\cot x \csc x$
20. $\dfrac{1 - \cos^2 x}{\tan x}$ to $\sin x \cos x$
21. $\dfrac{\sec^2 \theta - 1}{\sin \theta}$ to $\tan \theta \sec \theta$
22. $\dfrac{1 + \cot^2 \theta}{\sec^2 \theta}$ to $\cot^2 \theta$

23. $\dfrac{\sec A}{\sin A} - \dfrac{\sin A}{\cos A}$ to $\cot A$
24. $\dfrac{\csc B}{\cos B} - \dfrac{\cos B}{\sin B}$ to $\tan B$
25. $\dfrac{1}{1 - \cos x} + \dfrac{1}{1 + \cos x}$ to $2 \csc^2 x$
26. $\dfrac{1}{\sec D - \tan D} + \dfrac{1}{\sec D + \tan D}$ to $2 \sec D$

For Problems 27–36, prove algebraically that the given equation is an identity.

27. $\sec x(\sec x - \cos x) = \tan^2 x$
28. $\tan x(\cot x + \tan x) = \sec^2 x$
29. $\sin x(\csc x - \sin x) = \cos^2 x$
30. $\cos x(\sec x - \cos x) = \sin^2 x$
31. $\csc^2 \theta - \cos^2 \theta \csc^2 \theta = 1$
32. $\cos^2 \theta + \tan^2 \theta \cos^2 \theta = 1$
33. $(\sec \theta + 1)(\sec \theta - 1) = \tan^2 \theta$
34. $(1 + \sin \theta)(1 - \sin \theta) = \cos^2 \theta$
35. $(2 \cos x + 3 \sin x)^2 + (3 \cos x - 2 \sin x)^2 = 13$
36. $(5 \cos x - 4 \sin x)^2 + (4 \cos x + 5 \sin x)^2 = 41$
37. Confirm that the equation in Problem 33 is an identity by plotting the two graphs.
38. Confirm that the equation in Problem 34 is an identity by plotting the two graphs.

Proving Trig Identites Worksheet

4/13

Variables are used interchangably

1. $\cos x \tan x = \sin x$

$\frac{\cos x}{1} \quad \frac{\sin}{\cos x}$

$\sin x = \sin x$

2. $\csc x \tan x = \sec x$

$\frac{1}{\sin x} \cdot \frac{\sin}{\cos x}$

$\frac{1}{\cos}$

$\sec x = \sec x$

3. $\sec A \cot A \sin A = 1$

$\frac{1}{\cos A} \cdot \frac{\cos A}{\sin A} \cdot \frac{\sin A}{1}$

$\frac{\cos A \sin A}{\cos A \sin A}$

$1 = 1$

4. $\csc B \tan B \cos B = 1$

$\frac{1}{\sin x} \quad \frac{\sin x}{\cos x} \quad \frac{\cos x}{1}$

$\frac{\sin x \cos x}{\sin x \cos x}$

$1 = 1$

5. $\sin^2\theta \sec\theta \csc\theta = \tan\theta$

$\frac{\sin^2 x}{1} \quad \frac{1}{\cos x} \quad \frac{1}{\sin x}$

$\frac{\sin^2 x}{\cos x \sin x}$

$\frac{\sin x}{\cos x}$

$\tan\theta = \tan\theta$

6. $\cos^2 x \csc x \sec x = \cot x$

$\frac{\cos^2 x}{1} \quad \frac{1}{\sin x} \quad \frac{1}{\cos x}$

$\frac{\cos^2 x}{\sin x \cos x}$

$\frac{\cos x}{\sin x}$

$\cot x = \cot x$

7. $\cot R + \tan R = \csc R \sec R$

$\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$

$\frac{\cos x^2}{\sin x \cdot \cos x} + \frac{\sin^2 x}{\sin x \cdot \cos x}$

$\frac{\cos x^2 + \sin^2 x}{\sin x \cdot \cos x}$

$\frac{1}{\sin x \cos x}$

$\frac{1}{\sin x} \cdot \frac{1}{\cos x}$

$\csc R \sec R = \csc R \sec R$

8. $\cot D \cos D + \sin D = \csc D$

$\frac{\cos x}{\sin x} \quad \frac{\cos D}{1} + \frac{\sin D}{1}$

$\frac{\cos x^2}{\sin x} + \frac{\sin D}{1}$

$\frac{\cos x^2}{\sin x} + \frac{\sin D^2}{\sin x}$

$\frac{\cos x^2 + \sin x^2}{\sin x}$

$\frac{1}{\sin x}$

$\csc D = \csc D$

9. $\csc x - \sin x = \cot x \cos x$

$\frac{1}{\sin x} - \frac{\sin x}{1}$

$\frac{1}{\sin x} - \frac{\sin x^2}{\sin x}$

$\frac{1 - \sin x^2}{\sin x}$

$\frac{\cos^2 x}{\sin x} \rightarrow \frac{\cos x}{\sin x} \cdot \frac{\cos x}{1}$

$\cot x \cdot \cos x = \cot x \cdot \cos x$

10. $\sec x - \cos x = \sin x \tan x$

$\frac{1}{\cos x} - \frac{\cos x}{1}$

$\frac{1}{\cos x} - \frac{\cos x^2}{\cos x}$

$\frac{1 - \cos x^2}{\cos x}$

$\frac{\sin x^2}{\cos x} \rightarrow \frac{\sin x}{\cos x} \cdot \frac{\sin x}{1}$

$\sin x \tan x = \sin x \tan x$

11. $\tan x (\cot x \cos x + \sin x) = \sec x$

$\tan x \cot x \cos x + \sin x \tan x$

$\frac{\sin x}{\cos x} \frac{\cos x}{\tan x} \frac{\cos x}{1} + \frac{\sin x}{1} \frac{\sin x}{\cos x}$

$\frac{\cos x}{1} + \frac{\sin x^2}{\cos x}$

$\frac{\cos x^2}{\cos x} + \frac{\sin x^2}{\cos x}$

$\frac{1}{\cos x}$

$\sec x = \sec x$

12. $\cos x(\sec x + \cos x \csc^2 x) = \csc^2 x$

$\cos x \sec x + \cos x \cos x \csc^2 x$

$\frac{\cos x}{1} \cdot \frac{1}{\cos x} + \frac{\cos x^2}{1} \cdot \frac{1}{\sin^2 x}$

doesn't disappear turns to a 1

$1 + \frac{\cos x^2}{\sin x^2}$

$1 + \cot^2 x$

$\csc^2 x = \csc^2 x$

13. $(1+\sin B)(1-\sin B) = \cos^2 B$

$(1 \cdot 1) + (1 \cdot -\sin B) + (\sin B \cdot 1) + (\sin B \cdot -\sin B)$

$-\sin B^2 + 1$

$1 - \sin B^2$

$\cos^2 B = \cos^2 B$

14. $(\sec E - 1)(\sec E + 1) = \tan^2 E$

$(\sec E \cdot \sec E) + (\sec E \cdot 1) + (-1 \cdot \sec E) + (-1 \cdot 1)$

$\sec E^2 - 1$

$\tan^2 E = \tan^2 E$

15. $(\cos x - \sin x)^2 = 1 - 2\cos x \sin x$

$(\cos x - \sin x)(\cos x - \sin x)$

$(\cos x \cdot \cos x) + (\cos x \cdot -\sin x) + (-\sin x \cdot \cos x) + (-\sin x \cdot -\sin x)$

$\cos x^2 - \sin x \cos x - \sin x \cos x + \sin x^2$

$1 - 2\cos x \sin x = 1 - 2\cos x \sin x$

16. $(1-\tan x)^2 = \sec x - 2\tan x$

$(1-\tan x)(1-\tan x)$

$(1 \cdot 1) + (1 \cdot -\tan x) + (-\tan x \cdot 1) + (-\tan x \cdot -\tan x)$

$1 - 2\tan x + \tan x^2$

$\sec x - 2\tan x = \sec x - 2\tan x$

17. $(\tan x + \cot x)^2 = \sec^2 x + \csc^2 x$

$(\tan x + \cot x)(\tan x + \cot x)$

$(\tan x \cdot \tan x) + (\tan x \cdot \cot x) + (\cot x \cdot \tan x) + (\cot x \cdot \cot x)$

$\tan x^2 + \underbrace{\frac{\sin x}{\cos x} \cdot \frac{\cos x}{\sin x}}_{1} + \underbrace{\frac{\cos x}{\sin x} \cdot \frac{\sin x}{\cos x}}_{1} + \cot x^2$

$1 + \tan x^2 + 1 + \cot x^2$

$\sec^2 x + \csc^2 x = \sec^2 x + \csc^2 x$

18. $(\cos k - \sec k)^2 = \tan^2 k - \sin^2 k$

$(\cos k - \sec k)(\cos k - \sec k)$

$(\cos x \cdot \cos x) + (\cos x \cdot -\sec x) + (-\sec x \cdot \cos x) + (-\sec x \cdot -\sec x)$

$\cos x^2 - \sec x \cos x - \sec x \cos x + \sec x^2$

$\cos x^2 - \underbrace{\frac{1}{\cos x} \frac{\cos x}{1}}_{1} - \underbrace{\frac{1}{\cos x} \frac{\cos x}{1}}_{1} + \sec x^2$

$\cos x^2 - 1 - 1 + \sec x^2$

$-\sin^2 k + \tan^2 x = \tan^2 x - \sin^2 x$

19. $\frac{\csc^2 x - 1}{\cos x} = \cot x \csc x$

$\frac{\cot^2 x}{\cos x} \rightarrow \frac{\cot x}{\cos x} + \frac{\cot x}{1}$

$\frac{\frac{\cos}{\sin}}{\cos x} + \frac{\cot x}{1}$

$\frac{\cos}{\sin} \cdot \frac{1}{\cos x}$

$\frac{1}{\sin x} + \cot x$

$\csc x + \cot x = \csc x + \cot x$

20. $\frac{1-\cos^2 x}{\tan x} = \sin x \cos x$

$\frac{\sin^2 x}{\tan x} \rightarrow \frac{\sin x}{\tan x} + \frac{\sin x}{1}$

$\frac{\sin x}{1} \cdot \frac{\cos x}{\sin x} \leftarrow \frac{\sin x}{\frac{\sin}{\cos}} + \sin x$

$\cos x \sin x = \cos x \sin x$

21. $\frac{\sec^2\theta - 1}{\sin\theta} = \tan\theta \sec\theta$

$\frac{\tan^2\theta}{\sin\theta} \rightarrow \frac{\tan\theta}{\sin\theta} + \frac{\tan\theta}{1}$

$\frac{\sin}{\cos} \cdot \frac{1}{\sin} \leftarrow \frac{\frac{\sin}{\cos}}{\sin\theta} + \tan\theta$

$\frac{1}{\cos\theta} + \tan\theta$

$\sec\theta + \tan\theta = \sec\theta + \tan\theta$

22. $\frac{1+\cot^2\theta}{\sec^2\theta} = \cot^2\theta$

Copy error

$\frac{\csc^2\theta}{\sec^2\theta} \rightarrow \cancel{\frac{\csc\theta}{\sec\theta} + \frac{\csc\theta}{1}}$

$\frac{\csc}{1} \; \frac{\cos}{1} \quad \frac{\csc^2\theta}{\frac{1}{\cos^2 x}}$

$\csc^2 \cos^2$

$\cos^2 \frac{1}{\sin^2}$

$\frac{\cos^2}{\sin^2}$

$\cot^2 x = \cot^2 x$

this is actually $\frac{\sec A}{1} \cdot \frac{1}{\sin A}$

$\frac{1}{\cos} \cdot \frac{1}{\sin}$

23. $\frac{\sec A}{\sin A} - \frac{\sin A}{\cos A} = \cot A$

$\cancel{\frac{\frac{1}{\cos}}{\sin A}} - \frac{\sin A}{\cos A}$

$\cancel{\frac{1}{\cos x} \frac{\sin x}{1}}$

$\frac{1}{\sin}$ ← divided by - not equaling

$\cancel{\frac{\sin x}{\cos x} - \frac{\sin A}{\cos A}}$

$\cancel{0 \quad \in ?}$

$\frac{1}{\cos} \cdot \frac{1}{\sin}$

$\frac{1}{\cos x \sin x} - \frac{\sin x^2}{\cos x \sin x}$

$\frac{1 - \sin x^2}{\cos x \sin x}$

$\frac{\cos^2 x}{\cos x \sin x} = \frac{\cos x}{\sin x} = \cot x = \cot x$

~~24.~~ $\frac{\csc B}{\cos B} - \frac{\cos B}{\sin B} = \tan B$

$\frac{\csc B}{1} \cdot \frac{1}{\cos B} - \frac{\cos A}{\sin A}$

$\cancel{\frac{1}{\sin} \frac{\cos}{1}}$

doesn't work see 23

← $\cancel{\frac{\frac{1}{\sin x}}{\cos x}} - \cot x$

$\cancel{\frac{\cos x}{\sin x} - \frac{\cos x}{\sin x}}$

$\cancel{0 \quad \in ?}$

$\frac{1}{\sin} \cdot \frac{1}{\cos B} \quad \frac{\cos A}{\sin A (\cos A)}$

$\frac{1}{\sin x \cos x} - \frac{\cos^2 x}{\sin x \cos x}$

$\frac{1 - \cos^2 x}{\sin x \cos x} \rightarrow \frac{\sin^2}{\sin x \cos x} \rightarrow \frac{\sin x}{\cos x} \rightarrow \tan x = \tan x$

25. $\frac{1}{1 - \cos x} + \frac{1}{1 + \cos x} = 2\csc^2 x$

common denometers

$\frac{1(1+\cos x)}{(1-\cos x)(1+\cos x)} + \frac{1(1-\cos x)}{(1+\cos x)(1-\cos x)}$

$\cancel{\frac{1}{\sin x} + \frac{1}{1+\cos x}}$

$\cancel{\csc x + \frac{1}{1+\cos x}}$

?

$\frac{1 + \cos x + 1 - \cos x}{(1+\cos x)(1-\cos x)}$

foil ↙

$\frac{2}{(1-\cos\theta^2)} \rightarrow \frac{2}{\sin^2 x} \rightarrow 2 \cdot \frac{1}{\sin^2} \rightarrow 2\csc^2 x = 2\csc^2 x$

31. $\csc^2\theta - \cos^2\theta \csc^2\theta = 1$

$\frac{1}{\sin^2} - \frac{\cos^2\theta}{1} \cdot \frac{1}{\sin^2\theta}$

$\frac{1}{\sin^2} - \frac{\cos^2\theta}{\sin^2\theta}$

$\frac{1-\cos^2\theta}{\sin^2\theta}$

$\frac{\sin^2\theta}{\sin^2\theta}$

$1 = 1$

Name Michael Plasmeier

Date 4/17

Block ____________

Section 14.4 – Solving Trigonometric Equations

Directions: Solve the following equations for the variable given.

Review Problems

1. $-6+3x=-20$

$+6 \quad +6$

$\frac{3x}{3}=\frac{-14}{3}$

$x=\frac{-14}{3}$

2. $-3(x-2)+6=5(3-2x)$

$-3x+6+6=15-10x$

$+3x \quad +3x$

$12=15-7x$

$-15 \quad -15$

$\frac{-3}{-7}=\frac{-7x}{-7}$

$\frac{3}{7}=x$

3. $-x+3=7x+8$

$+x \quad +x$

$3=8x+8$

$-8 \quad -8$

$\frac{-5}{8}=\frac{8x}{8}$

$x=\frac{-5}{8}$

4. $3(1-x)-(3+x)=8$

$3-3x-3-x=8$

$\frac{-4x}{-4}=\frac{8}{-4}$

$x=-2$

Solving Trigonometric Equations

1. Solve $2\sin x-1=0$

$+1 \quad +1$

$\frac{2\sin x}{2}=\frac{1}{2}$

$\sin x=\frac{1}{2}$

$\sin^{-1} \quad \sin^{-1}$

x = $30^\circ \quad \frac{\pi}{6}$

also works $150^\circ \quad \frac{5\pi}{6}$

When you solve for **x**, what type of trig function will you have to use ?

Convert all your answers into radian measure.

Is this the only possible answer?

$x^2=16$

$\sqrt{\ } \quad \sqrt{\ }$

$x=4$

but also

$x=-4$

2. Solve by collecting like terms and simplify.

Solve $\cos x+\sqrt{2}=-\cos x$

$+\cos x$

$2\cos x+\sqrt{2}=0$

$-\sqrt{2} \quad -\sqrt{2}$

$\frac{2\cos x}{2}=\frac{-\sqrt{2}}{2}$

$\cos x=-\frac{\sqrt{2}}{2} \rightarrow \frac{1}{\sqrt{2}}$

$\cos^{-1} \quad \cos^{-1}$

2nd + 3rd

$x=135^\circ$ or $\frac{3\pi}{4}$ $\quad 225 \quad \frac{5\pi}{4}$

3. Solve $3\tan^2 x-1=0$

$+1 \quad +1$

$\frac{3\tan^2 x}{3}=\frac{1}{3}$

$\sqrt{\ }$ 1st $\tan^2 x=\frac{1}{3}$

$\sqrt{\ } \quad \sqrt{\ }$ ← make sure

$\tan x=\pm\sqrt{\frac{1}{3}}$

$\tan^{-1} \quad \tan^{-1}$

(+) = 1st + 3rd

1st $x=30^\circ \quad \frac{\pi}{6}$

3rd $=210^\circ \quad \frac{7\pi}{6}$

Write 2nd: $150^\circ \quad \frac{5\pi}{6}$ $\quad$ 4th: $330^\circ \quad \frac{11\pi}{6}$

4. $4\tan(x) - \sqrt{3} = \tan x$
$-\tan x \qquad -\tan x$

$3\tan x - \sqrt{3} = 0$
$+\sqrt{3} \quad +\sqrt{3}$

$\frac{3\tan x}{3} = \frac{\sqrt{3}}{3}$

$\tan x = \frac{\sqrt{3}}{3}$
$\tan^{-1} \quad \tan^{-1}$

1st + 3rd

$x \overset{1st}{=} 30° \quad \frac{\pi}{6}$

3rd $= 210° \quad \frac{7\pi}{6}$

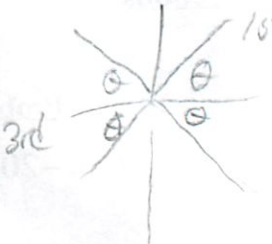
5. $2\csc^2 x - 4 = 0$
$+4 \quad +4$

$\frac{2\csc^2 x}{2} = \frac{4}{2}$

$\csc^2 x = 2$
$\sqrt{\ } \quad \sqrt{\ }$

$\csc x = \pm\sqrt{2}$

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) \leftarrow$ type



1st $= 45, \ \frac{\pi}{4}$
2nd $= 135°, \ \frac{3\pi}{4}$
3rd $= 225°, \ \frac{5\pi}{4}$
4th $= 315° \ \frac{7\pi}{4}$

these $-$

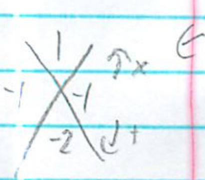
More Examples

4/18

6. $\sin^2 x - 2\sin x = -1$

$\quad +1 \quad +1$

$\sin^2 x - 2\sin x + 1 = 0$



$(\sin x - 1)(\sin x - 1) = 0$

$\sin x - 1 = 0$

$\quad +1 \quad +1$

$\sin x = 1$

$\sin^{-1} \quad \sin^{-1}$

$x = 90^\circ \qquad x = 90^\circ$

1 only 1 answer

7. $\sin^2 x - \cos^2 x = 0$

$(1 - \cos^2 x) - \cos^2 x = 0$

$1 - 2\cos^2 x = 0$

$-1 \qquad -1$

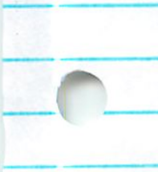
$\frac{-2\cos^2 x}{-2} = \frac{-1}{-2}$

$\cos^2 x = \frac{1}{2}$

$\sqrt{\ } \qquad \sqrt{\ }$

$\cos x = \pm\sqrt{\frac{1}{2}}$

$\cos^{-1} \qquad \cos^{-1}$



(+) 1st = 45°, $\frac{\pi}{4}$

(−) 2nd = 135° $\frac{3\pi}{4}$

(−) 3rd = 225° $\frac{5\pi}{4}$

(+) 4th = 315° $\frac{7\pi}{4}$

13.3 Trig Equations

Independent Practice

2/17

p757

S | A
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13. $\sqrt{2}\cos x - 1 = 0$

$+1 \quad +1$

$\frac{\sqrt{2}\cos x}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\cos x = \frac{1}{\sqrt{2}}$

$\cos^{-1} \quad \cos^{-1}$

1st: $45°$ $\frac{\pi}{4}$

4th: $315°$ $\frac{7\pi}{4}$

14. $7\sec x - 7 = 0$

$+7 \quad +7$

$\frac{7\sec x}{7} = \frac{7}{7}$

$\sec x = 1$

$\sin^{-1} = \left(\frac{1}{1}\right)$

1st: ~~90°~~ $\frac{\pi}{2}$

: others

0°

360°

15. $3\sin x = \sin x + 1$

$-\sin x \quad -\sin x$

$\frac{2\sin x}{2} = \frac{1}{2}$

$\sin x = \frac{1}{2}$

$\sin^{-1} \quad \sin^{-1}$

1st: $30°$ $\frac{\pi}{6}$

2nd: $150°$ $\frac{5\pi}{6}$

16. $5\cos x - \sqrt{3} = 3\cos x$

$-5\cos x \quad -5\cos x$

$\frac{-\sqrt{3}}{\cancel{-8}\,-2} = \frac{\cancel{-8}\,-2\cos x}{\cancel{-8}\,-2}$ ← subtraction error

$\frac{\sqrt{3}}{\cancel{8}\,2} = \cos x$

$\cos^{-1} \quad \cos^{-1}$ 30° ↓ $\cdot\left(\frac{\pi}{180}\right)$

Why is ans ( ) nice when mine's not Subtraction error

1st: ~~77.4961~~ or ~~1.35r~~

4th: ~~282.5639~~ or ~~4.93r~~

330°

17. $2\csc x + 17 = 15 + \csc x$

$-\csc x \quad -17 \quad -17 \quad -\csc x$

$\csc x = \cancel{-17}\ 2$ ← subtraction error

$\sin^{-1}\left(\frac{1}{-17}\right)$

confused w/θ

3rd: ~~183.37~~ or ~~3.20r~~

4th: ~~356.63~~ or ~~6.22r~~

210°

330°

18. $\cos x - 1 = -\cos x$

$-\cos x \quad -\cos x$

$\frac{-1}{-2} = \frac{-2\cos x}{-2}$

$\frac{1}{2} = \cos x$

$\cos^{-1} \quad \cos^{-1}$

1st: $60°$ or $\frac{\pi}{3}$

4th: $300°$ $\frac{5\pi}{3}$

19 $4\sin^2 x - 2 = 0$

$+2 \quad +2$

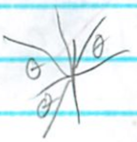
$\frac{4\sin^2 x}{4} = \frac{2}{4}$

$\sin^2 x = \frac{1}{2}$

$\sqrt{\ } \quad \sqrt{\ }$

$\sin x = \pm\sqrt{\frac{1}{2}}$

$\sin^{-1} \quad \sin^{-1}$



$+$ 1st: $45°$ $\frac{\pi}{4}$ ✓

$+$ 2nd: $135°$ $\frac{3\pi}{4}$ ✓

$-$ 3rd: $225°$ $\frac{5\pi}{4}$ ✓

$-$ 4th: $315°$ $\frac{7\pi}{8}$ ✓

bit confused w/ radians

22. $3\sec^2 x - 4 = 0$

$+4 \quad +4$

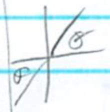
$\frac{3\sec^2 x}{3} = \frac{4}{3}$

$\sec^2 x = \frac{4}{3}$

$\sqrt{\ } \quad \sqrt{\ }$

$\sec x = \pm\sqrt{\frac{4}{3}}$

$\cos^{-1}\left(\sqrt{\frac{3}{4}}\right)$



1st $= 30°$ $\frac{\pi}{6}$ ✓

2nd $= 150°$ $\frac{5\pi}{6}$ ✓

3rd $= 210°$ or $\frac{7\pi}{6}$ ✓

4th $= 330°$ or $\frac{11\pi}{6}$ ✓

Check them

20. $\tan^2 x - 3 = 0$

$+3 \quad +3$

$3 = \tan^2 x$

$\sqrt{\ } \quad \sqrt{\ }$

$\pm\sqrt{3} = \tan x$

$\tan^{-1} \quad \tan^{-1}$

$+$ 1st: $60°$ $\frac{\pi}{3}$ ✓

2nd: $120°$ $\frac{2\pi}{3}$ ✓

$+$ 3rd: $240°$ $\frac{4\pi}{3}$ ✓

4th: $300°$ $\frac{5\pi}{3}$ ✓

why did I get clean # now and wrong # before?

21. $3\tan^2 x - 1 = 0$

$+1 \quad +1$

$\frac{3\tan^2 x}{3} = \frac{1}{3}$

$\tan^2 x = \frac{1}{3}$

$\sqrt{\ } \quad \sqrt{\ }$

$\tan x = \pm\sqrt{\frac{1}{3}}$

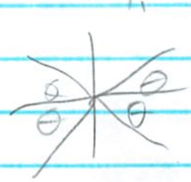
$\tan^{-1} \quad \tan^{-1}$

$+$ 1st: $30°$ $\frac{\pi}{6}$

$-$ 2nd: $150°$ $\frac{5\pi}{6}$

$+$ 3rd: $210°$ $\frac{7\pi}{6}$

$-$ 4th: 330 $\frac{11\pi}{6}$



Classwork: Simplifying Trig Expressions and Proving Trig Identities (18 pts)

Name: Michael Plasmeier
Name: Eric Jacobs

14/18

Simplify the following Trigonometric Expressions: (3 points each)

1. $\sec x \cos(-x) - \sin^2 x$

$\sec x \cos x - \sin^2 x$

$\frac{1}{\cos x} \cdot \frac{\cos x}{1} - \sin^2 x$

$\frac{\cos x}{\cos x} - \sin^2 x$

$1 - \sin^2 x$

$\cos^2 x$

2. $\cot\left(\frac{\pi}{2} - x\right)\cos x$

$\tan x \cos x$

$\sin x$

3. $\dfrac{\sec x \tan x}{\sin(-x)\cot\left(\frac{\pi}{2} - x\right)}$

$\dfrac{\frac{1}{\cos} \cdot \tan x}{-\sin x \tan x}$ cancle out

$\dfrac{-\sin x \cos x}{\cos x - \sin x}$

$\dfrac{\frac{1}{\cos x}}{-\sin x}$ put like this

$\frac{1}{\cos x} \cdot \frac{1}{-\sin x}$ → multiplying fractions → don't need a common denominator

$\frac{-\sin x}{\cos x(-\sin x)} \quad \frac{\cos x}{\cos x(-\sin x)}$ can just combine

$\sec x \cdot \csc(-x)$ -2 ← but ans is simple

7

Prove the following trigonometric identities: (3 points each).

4. $\dfrac{1}{\sin x} - \sin x = \cot x \cos x$

$\csc x - \sin x$

$\frac{1}{\sin x} - \frac{\sin x}{1}$

$\frac{1}{\sin x} - \frac{\sin x^2}{\sin x}$

$\frac{1 - \sin x^2}{\sin x}$

$\frac{\cos^2 x}{\sin x} \rightarrow \frac{\cos x}{\sin x}, \frac{\cos x}{1}$

$\cot x \cos x = \cot x \cos x$

5. $(1+\sin x)(1+\sin(-x)) = \cos^2 x$

$(1+\sin x)(1-\sin x)$

$(1\cdot 1) + (1\cdot -\sin x) + (\sin x \cdot 1) + (\sin x \cdot -\sin x)$

$1 - \sin^2 x$

$\cos^2 x = \cos^2 x$

6. $\cos\left(\dfrac{\pi}{2} - x\right) + \cos x \tan\left(\dfrac{\pi}{2} - x\right) = \csc x$

$\sin x + \cos x \cot x = \csc x$

$\sin x + \frac{\cos x}{1} \frac{\cos x}{\sin x}$

$\sin x + \frac{\cos x^2 \sin x}{\sin x}$? Copy error

??

$\sin x + \cos^2 x$

??

$\csc x = \csc x$

-2

7

4/18 Warmup

4/18

Find roots/solutions

1. $0 = 2x^2 - 3x - 20$

$+20 \qquad +20$

$20 = 2x^2 - 3x$

forget how to do

diamond + area rug method

ac, b, $\uparrow \times$, $d+$

-40, -8, 5, -3

place in here

	x	-4
$2x$	$2x^2$	$-8x$
$+5$	$5x$	-20

$(x-4)(2x+5) = 0$

set each = 0

$x - 4 = 0$

$+4 \quad +4$

$x = 4$

$(4, 0)$

$2x + 5 = 0$

$-5 \quad -5$

$\frac{2x}{2} = \frac{-5}{2}$

$x = \frac{-5}{2}$

$(-\frac{5}{2}, 0)$

2. $x^3 + 2x^2 - 24x = 0$

$x(x^2 + 2x - 24) = 0$ GCF

-24, $\uparrow \times$, -4, 6, 2, $\leftarrow +$

$x(x+6)(x-4) = 0$

$x = 0$

$(0,0)$

$x + 6 = 0$

$-6 \quad -6$

$x = -6$

$(-6, 0)$

$x - 4 = 0$

$+4 \quad +4$

$x = 4$

$(4, 0)$

$\leftarrow$ 3 answers

3. $0 = 12x^2 - 11x - 5$

-60, $\uparrow \times$, 4, -15, -11, $d+$

	$3x$	$+1$
$4x$	$12x^2$	$4x$
-5	$-15x$	-5

$(3x+1)(4x-5) = 0$

$3x + 1 = 0$

$-1 \quad -1$

$\frac{3x}{3} = \frac{-1}{3}$

$x = -\frac{1}{3}$

$(-\frac{1}{3}, 0)$

$4x - 5 = 0$

$+5 \quad +5$

$\frac{4x}{4} = \frac{5}{4}$

$x = \frac{5}{4}$

$(\frac{5}{4}, 0)$

14.4 Solving Trig Equations

More Practice

4/18

23
24
26

23. $\sec x \csc x - 2\csc x = 0$ ↓ GCF

$\csc x(\sec x - 2) = 0$

$\csc x = 0$ $\quad$ $\sec x - 2 = 0$

$\sin^{-1}\left(\frac{1}{0}\right)$ $\quad$ $+2 \; +2$

error $\quad$ $\sec x = 2$

not 0 $\quad$ $\cos^{-1}\left(\frac{1}{2}\right)$

(+) 1st $= 60°$ or $\frac{\pi}{3}$ ✓

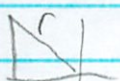
~~(−) 2nd $= 120°$ or $\frac{2\pi}{3}$~~ ← don't check $\quad$ not ^{2}

~~(−) 3rd $= 240°$ or $\frac{4\pi}{3}$~~ ←

(+) 4th $= 300°$ or $\frac{5\pi}{3}$ ✓

24. $\sqrt{2}\cos x \sin x - \cos x = 0$

$\cos x(\sqrt{2}\sin x - 1) = 0$



x

x=0

$\cos x = 0$ $\quad$ $\sqrt{2}\sin x - 1 = 0$

$\cos^{-1}(0)$ $\quad$ $+1 \; +1$

1st $= 90°$ ✓ $\quad$ $\sqrt{2}\sin x = 1$

$270°$ ✓ $\quad$ $\frac{\sqrt{2}\sin x}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

or $\frac{\pi}{2}$, $\frac{3\pi}{2}$ $\quad$ $\sin x = \frac{1}{\sqrt{2}}$

$\sin^{-1}$ $\quad$ $\sin^{-1}$

1st $= 45°$ or $\frac{\pi}{4}$ ✓

2nd $= 135°$ or $\frac{3\pi}{4}$ ✓

$\boxed{\text{no 3rd or 4th not}^2}$

→

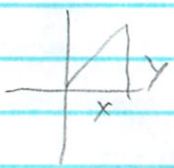
check them!

26. $3\tan^3 - \tan x = 0$

$\tan x\,(3\tan^2 - 1) = 0$

$\tan x = 0$ $\qquad$ $3\tan^2 - 1 = 0$

$\tan^{-1}$ $\tan^{-1}$ $\qquad$ $+1$ $\quad$ $+1$



$x = 0$ or 0 ✓ $\qquad$ $\frac{3\tan^2}{3} = \frac{1}{3}$

$x = \cancel{90}$ $\cancel{\frac{\pi}{2}}$ X

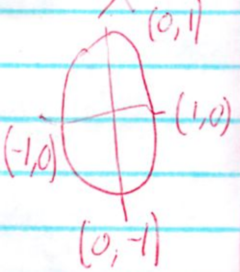
180 π ✓ $\qquad$ $\tan^2 = \frac{1}{3}$

$\cancel{270}$ $\cancel{\frac{3\pi}{4}}$ X $\qquad$ $\sqrt{\ }$ $\quad$ $\sqrt{\ }$

$\frac{Y}{X}$

don't work
forget special
cases $\qquad$ $\tan x = \pm\sqrt{\frac{1}{3}}$

$\tan^{-1}$ $\quad$ $\tan^{-1}$



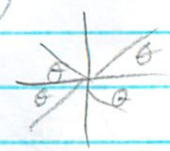
← rules

⊕ 1st = $30°$ or $\frac{\pi}{6}$ ✓

⊖ 2nd = $150°$ $\frac{5\pi}{6}$ ✓

⊕ 3rd = $210°$ or $\frac{7\pi}{4}$

⊖ 4th = $330°$ $\frac{11\pi}{6}$



14.4 Solving Trig Equations
HW

4/18

25. $4\sin^2 x - 3 = 0$

$+3 \quad +3$

$\frac{4\sin^2 x}{4} = \frac{3}{4}$

$\sin^2 x = \frac{3}{4}$

$\sqrt{\ } \quad \sqrt{\ }$

$\sin x = \pm\sqrt{\frac{3}{4}}$

$\sin^{-1} \quad \sin^{-1}$

(+) 1st $= 60°$ $\frac{\pi}{3}$

(+) 2nd $= 120°$ $\frac{2\pi}{3}$

(−) 3rd $= 240°$ $\frac{4\pi}{3}$

(−) 4th $= 300°$ $\frac{5\pi}{3}$ ✓

27. ~~$\cos^3 x = \cos x$~~

~~$-\cos x \quad -\cos x$~~

~~$\cos^2 x = 0$~~

~~$\sqrt{\ } \quad \sqrt{\ }$~~

~~$\cos x = \pm\sqrt{0}$~~

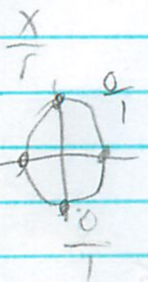
~~$\cos^{-1} \quad \cos^{-1}$~~

~~1st: $90°$ $\frac{\pi}{2}$~~

~~4th: $180°$ π~~

doesn't work like that $x^3 - x \neq x^2$

$0°$
$360°$ also work
$270°$ why?



GCF $\cos^3 x - \cos x = 0$

$\cos x(\cos^2 x - 1)$

$\cos x = 0$

$\cos^{-1} \quad \cos^{-1}$

$90°$ $\frac{\pi}{2}$

$270°$ $\frac{3\pi}{2}$

Same answers

$\cos^2 x - 1 = 0$

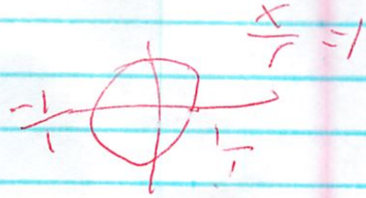
$+1 \quad +1$

$\cos^2 x = 1$

$\sqrt{\ } \quad \sqrt{\ }$

$\cos x = \pm\sqrt{1}$

$\cos^{-1} \quad \cos^{-1}$



$0°$ 0 $+360°$ 2π

$180°$ π

29. $\sqrt{3}\tan = 3\tan x \csc x - \sqrt{3}\tan x$

28. $3\sin x \sec x - 2\sqrt{3}\sin x = 0$ GCF

$\sin x(3\sec x - 2\sqrt{3})$

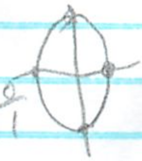
$\sin x = 0$ $\qquad$ $3\sec x - 2\sqrt{3} = 0$

$\sin^{-1}$ $\sin^{-1}$ $\qquad$ $+2\sqrt{3}$

$\frac{y}{r}$

$x = 0°$ 0 $\qquad$ $\frac{3\sec x}{3} = \frac{2\sqrt{3}}{3}$

$180°$ π ✓



$\frac{0}{1}$ $\frac{0}{1}$

$\sec x = \frac{2\sqrt{3}}{3}$

$\cos^{-1}\left(1/\frac{2\sqrt{3}}{3}\right)$ e type right

1st: $30°$ $\frac{\pi}{6}$ ✓

4th: $330°$ $\frac{11\pi}{6}$

29. $\sqrt{3}\tan x = 3\tan x \csc x - \sqrt{3}\tan$

$-\sqrt{3}\tan$ $\qquad$ $+\sqrt{3}\tan$

~~$0 = 3\tan x \csc x$~~

$0 = 3\tan x \csc x - \sqrt{3}\tan x - \sqrt{3}\tan x$

GCF

$\frac{3\tan x}{3} = \frac{0}{3}$ $\qquad$ ~~$\csc x = 0$~~

~~$\sin^{-1} = \frac{1}{0}$~~

~~error~~

$0 = \tan x(3\csc - \sqrt{3} - \sqrt{3})$

$0 = \tan x(3\csc - 2\sqrt{3})$

$\frac{y}{x}$

$\tan x = \frac{0}{3}$

$\tan^{-1}$ $\tan^{-1}$

$x = 0°$ 0

$180°$ π

similar

same answer

$\frac{0}{-3}$ $\frac{0}{3}$

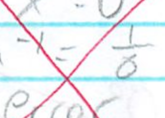
$3\csc - 2\sqrt{3} = 0$

$+2\sqrt{3}$ $+2\sqrt{3}$

$\frac{3\csc}{3} = \frac{2\sqrt{3}}{3}$

$\csc = \frac{2\sqrt{3}}{3}$

$\sin^{-1}\left(1/\frac{2\sqrt{3}}{3}\right)$



1st: $60°$ $\frac{\pi}{3}$

2nd: $120°$ $\frac{2\pi}{3}$

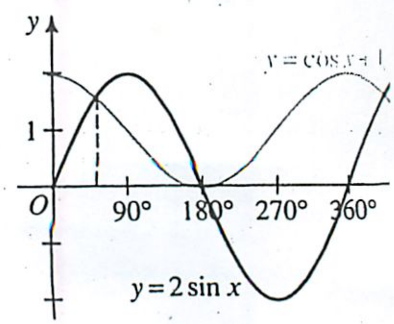
Method 2

Use a computer or graphing calculator to graph the equations

$$y = 2 \sin x \text{ and } y = \cos x + 1.$$

(If you can't use your computer in the degree mode, use the radian moc and then convert your answers to degrees.) For the graphs, choose th intervals

$0° \le x < 360°$ and $-2 \le y \le 2$.



The graphs are shown at the right. Notice that for values of x between 0° and 360°, the graphs intersect in two points. By using a zoom (or trace) feature, you can find that the solutions are about 53.1° and 180°.

WRITTEN EXERCISES

Solve for $0° \le \theta < 360°$. Give answers to the nearest tenth of a degree.

A

1. $\sec^2 \theta = 9$
2. $\tan^2 \theta = 1$
3. $1 - \csc^2 \theta = -3$
4. $8 \cos^2 \theta - 3 = 1$
5. $6 \sin^2 \theta - 7 \sin \theta + 2 = 0$
6. $2 \tan^2 \theta = 3 \tan \theta - 1$
7. $6 \sin^2 \theta = 7 - 5 \cos \theta$
8. $\cos^2 \theta - 3 \sin \theta = 3$

Solve for $0 \le x < 2\pi$. Give answers to the nearest hundredth of a radian when necessary.

9. $\cos x \tan x = \cos x$
10. $\sec x \sin x = 2 \sin x$
11. $\sin^2 x = \sin x$
12. $\tan^2 x = \tan x$
13. $2 \cos^2 x = \cos x$
14. $3 \sin x = \cos x$
15. $\sin x + \cos x = 0$
16. $\sec x = 2 \csc x$

Solve each equation for $0 \le x < 2\pi$ algebraically by using identities or graphically by using a graphing calculator or computer. Give answers to the nearest hundredth of a radian when necessary.

B

17. $\tan^2 x = 2 \tan x \sin x$
18. $2 \sin x \cos x = \tan x$ $=0$
19. $2 \csc^2 x = 3 \cot^2 x - 1$
20. $2 \sec^2 x + \tan x = 5$
21. $\sin^2 x + \sin x - 1 = 0$
22. $\cos^2 x - 2 \cos x - 1 = 0$
23. $3 \cos x \cot x + 7 = 5 \csc x$
24. $2 \sin^3 x - \sin^2 x - 2 \sin x + 1$
25. $2 \cos^2 x - \cos x = 2 - \sec x$
26. $\csc^2 x - 2 \csc x = 2 - 4 \sin x$

Trig Equations

Worksheet

4/19

3. $1-\csc^2\theta=-3$

$-1 \qquad -1$

$-\csc^2\theta=-4$

$\div -1 \qquad \div -1$

$\csc^2\theta=4$

$\sqrt{\ } \qquad \sqrt{\ }$

$\csc\theta=\pm\sqrt{4}$

$\sin^{-1}\left(\frac{1}{\sqrt{4}}\right)$

(+) 1st: $30°$ $\frac{\pi}{6}$

(+) 2nd: $150°$ $\frac{5\pi}{6}$

(−) 3rd: $210°$ $\frac{7\pi}{4}$

(−) 4th: $330°$ $\frac{11\pi}{6}$

5. $6\sin^2\theta-7\sin\theta+2=0$

$-2 \qquad -2$

$\cancel{6\sin^2\theta-7\sin\theta=-2}$

$\cancel{\sin x(6\sin\theta-7)=-2}$

$\cancel{\sin x=-2}$ $\cancel{6\sin\theta-7=-2}$

$\cancel{\sin^{-1} \quad \sin^{-1}}$ $\cancel{+7 \quad +7}$

~~error~~ $\cancel{6\sin\theta}$

method doesn't work here - must be =0

12

−4 −3

−7

	$2\sin\theta$	-1
$3\sin\theta$	$6\sin^2\theta$	$-3\sin\theta$
-2	$-4\sin\theta$	2

$(2\sin\theta-1)(3\sin\theta-2)$

$2\sin\theta-1=0$ | $3\sin\theta-2=0$

$+1 \quad +1$ | $+2 \quad +2$

$\frac{2\sin\theta}{2}=\frac{1}{2}$ | $\frac{3\sin\theta}{3}=\frac{2}{3}$

$\sin\theta=\frac{1}{2}$ | $\sin\theta=\frac{2}{3}$

$\sin^{-1} \quad \sin^{-1}$ | $\sin^{-1} \quad \sin^{-1}$

1st: $30°$ $\frac{\pi}{6}$ $\left(\frac{\pi}{180}\right)$ | 1st: $41.81°$

2nd: $150°$ $\frac{5\pi}{6}$ | 2nd: $138.19°$

or $.73r$ / $2.41r$

6. $2\tan^2\theta = 3\tan\theta - 1$

$-2\tan^2\theta$

$-2\tan^2\theta + 3\tan\theta - 1 = 0$

	$-\tan\theta$	$+1$
$+2\tan\theta$	$-2\tan^2\theta$	$2\tan\theta$
-1	$1\tan\theta$	-1

$(2\tan x - 1)(-\tan x + 1) = 0$

$2\tan x - 1 = 0$ | $-\tan x + 1 = 0$
$+1 \quad +1$ | $-1 \quad -1$

$2\tan x = 1$, $\frac{2\tan x}{2} = \frac{1}{2}$ | $-\tan x = -1$, $\div -1 \quad \div -1$

$\tan x = \frac{1}{2}$ | $\tan x = 1$

$\tan^{-1} \quad \tan^{-1}$ | $\tan^{-1} \quad \tan^{-1}$

1st: $26.57°$ or $.46r$ | 1st: $45°$ $\frac{\pi}{4}$

3rd: $206.57°$ or $3.61r$ | 3rd: $225°$ $\frac{5\pi}{4}$

do it other way don't try to gate stuff to convert, just convert, get extra info, distribute and combine

7. $6\sin^2\theta = 7 - 5\cos\theta$

$-6\sin^2\theta \quad -6\sin^2\theta$

$+6\sin^2\theta - 5\cos\theta + 7$

foil cos

$+6(1-\cancel{6}\cos^2\theta) - 5\cos\theta + 7$ how distribute? (confused)

$+6\cos^2\theta - 5\cos\theta + 1$

$6 - 6\cos^2\theta + 5\cos\theta$

	$\cancel{-\cos\theta + 1}$	
$6\cos$	$+6\cos^2\theta$	$6\cos\theta$
$+1$	$-\cos\theta$	1

$(2\cos\theta - 1)$

$(3\cos\theta - 1)$

$60°$ | $70.53°$

$300°$ | $289.47°$

was − then errased, did I misread it?

~~$(6\cos x + 1)(-\cos\theta + 1)$~~

~~$6\cos x + 1 = 0$~~ | ~~$-\cos\theta + 1 = 0$~~

~~$-1 \quad -1$~~ | ~~$-1 \quad -1$~~

~~$\frac{6\cos x}{6} = \frac{-1}{6}$~~ | ~~$-\cos\theta = -1$~~

~~$\cos x = \frac{1}{6}$~~ | ~~$\cos\theta = 1$~~

~~$\cos^{-1} \quad \cos^{-1}$~~ | ~~$\cos^{-1} \quad \cos^{-1}$~~

~~1st: $80.41°$~~ | ~~1st: $0°$~~

~~$1.40r$~~ | ~~4th: $90°$~~

~~4th: $279.59°$~~

~~$4.88r$~~

$\frac{x}{r}$

4. $8\cos^2\theta - 3 = 1$

$+3 \quad +3$

$$\frac{8\cos^2\theta}{8} = \frac{4}{8}$$

$\cos^2\theta = \frac{1}{2}$

$\sqrt{\ } \quad \sqrt{\ }$

$\cos\theta = \pm\sqrt{\frac{1}{2}}$

$\cos^{-1} \quad \cos^{-1}$

1st = $45°$ $\frac{\pi}{4}$

2nd = $135°$ $\frac{3\pi}{4}$

3rd = $225°$ $\frac{5\pi}{4}$

4th = $315°$ $\frac{7\pi}{4}$

8. $\cos^2 x - 3\sin\theta = 3$

$-3 \quad -3$

$\cos^2 x - 3\sin\theta - 3 = 0$

$1 - \cos^2 x - 3\sin\theta - 4 = 0$

$\sin^2\theta - 3\sin\theta - 4 = 0$

-4 ↑x

-4 +1

-3 ↓+

	$\sin\theta$	-4
$\sin\theta$	$\sin^2\theta$	$-4\sin\theta$
$+1$	$\sin\theta$	-4

$(\sin\theta - 4)(\sin\theta + 1)$

$\sin\theta - 4 = 0$ $\quad$ $\sin\theta + 1 = 0$

$+4 \quad +4$ $\quad$ $-1 \quad -1$

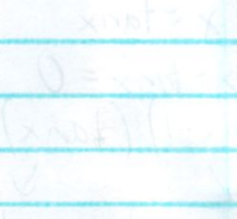
$\sin\theta = 4$ $\quad$ $\sin\theta = -1$

$\sin^{-1} \quad \sin^{-1}$ $\quad$ $x = -90°$ $\quad$ $\frac{y}{r}$

error $\quad$ ~~180°~~ ⊗

(no solution) $\quad$ $270°$ ✓ ← guess + ✓

~~90°~~ ⊗

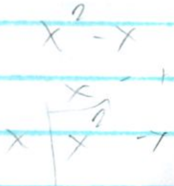


11. $\sin^2 x = \sin x$

$-\sin x \quad -\sin x$

$\sin^2 x - \sin x = 0$

$(\sin x - 1)(\sin x)$

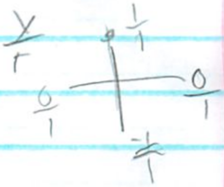


$\sin x - 1 = 0$ $\qquad$ $\sin x = 0$

$+1 \quad +1$ $\qquad$ $\sin^{-1} \quad \sin^{-1}$

$\sin x = 1$ $\qquad$ $x = 0°$

$\sin^{-1} \quad \sin^{-1}$ $\qquad$ $180°$ π



$x = 90°$ $\frac{\pi}{4}$

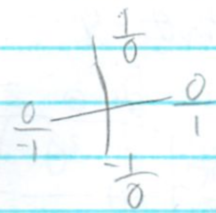
12. $\tan^2 x = \tan x$

$\tan^2 x - \tan x = 0$

$(\tan x - 1)(\tan x)$

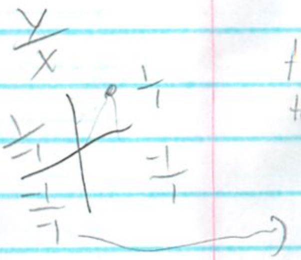
$\tan x - 1 = 0$ $\qquad$ $\tan x = 0$

$+1 \quad 11$ $\qquad$ $\tan^{-1} \quad \tan^{-1}$



$\tan x = 1$ $\qquad$ $x = 0, 0$

$\tan^{-1} \quad \tan^{-1}$ $\qquad$ $180°$ π



$x = 45°$, $\frac{\pi}{4}$

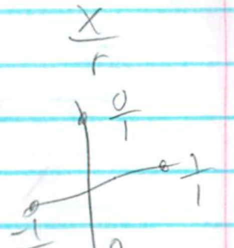
$225°$ $\frac{5\pi}{4}$

13. $2\cos^2 x = \cos x$

$-\cos x \quad -\cos x$

$2\cos^2 x - \cos x = 0$

$(2\cos x - 1)(\cos x)$



$\cos x = 0$

$\cos^{-1} \quad \cos^{-1}$

$x = 90°$; $\frac{\pi}{2}$

$270°$ $\frac{3\pi}{2}$

$2\cos x - 1 = 0$

$+1 \quad +1$

$\frac{2\cos x}{2} = \frac{1}{2}$

$\cos x = \frac{1}{2}$

$\cos^{-1} \quad \cos^{-1}$

1st $x = 60°$ $\frac{\pi}{3}$

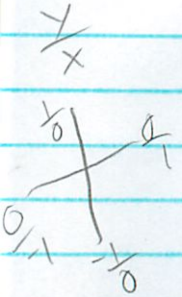
4th $= 300°$ $\frac{\pi}{?}$

17. $\tan^2 x = 2\tan x \sin x$

$-2\tan x \sin x$

$\tan^2 x - 2\tan x \sin x = 0$ GCF

$\tan x(\tan x - 2\sin x) = 0$



$\tan x = 0$

$\tan^{-1}$ $\tan^{-1}$

$x = 0°, 0$

$180°, \pi$

$\tan x - 2\sin x = 0$

$\frac{\sin x}{\cos x} - \frac{2\sin x}{1} = 0$

$\frac{\sin x}{\cos x} - \frac{2\sin x \cos x}{\cos x} = 0$

$\frac{\sin x - 2\sin x \cos x}{\cos x}$

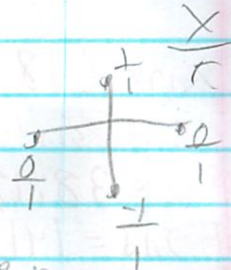
$-\sin x = 0$

$\div -1$ $\div -1$

$\sin x = -0$

$\sin^{-1}$ $\sin^{-1}$

$x = 0$



$0°, 0$

$180°, \pi$

~~19.~~ $2\csc^2 x = 3\cot^2 x - 1$

$-(3\cot^2 x - 1) - (3\cot^2 x - 1)$

$2\csc^2 x - 3\cot^2 x - 1 = 0$

$2\cot^2 x - 3\cot^2 x = 0$

$-\cot^2 x = 0$

$\div -1$ $\div -1$

$\cot^2 x = -0$

$\sqrt{}$ $\sqrt{}$

$\cot^2 x = \pm\sqrt{-0}$

$\tan^{-1}(1/\pm\sqrt{-0})$

error

converting error w/ the -1 things

$\rightarrow 2(1+\cot^2 x) - 3\cot^2 x - 1$ distribute

$\rightarrow 3\cot^2 x - 2 - 2\cot^2 x - 1$

$\cot^2 x - 3 = 0$

$+3$ $+3$

$\cot^2 x = 3$

$\sqrt{}$ $\sqrt{}$

$\cot x = \pm\sqrt{3}$

$\tan^{-1}(1/\pm\sqrt{3})$

(+) 1st $= 30°$ $\frac{\pi}{6}$

(−) 2nd $= 150°$ $\frac{5\pi}{6}$

(+) 3rd $= 210°$ $\frac{7\pi}{6}$

(−) 4th $= 330°$ $\frac{11\pi}{6}$

21. $\sin^2 x + \sin x - 1 = 0$

$x^2 + x - 1$

$-x + 1$

	x^2	x
$-x$	x^2	x
-1	x	-1

quadratic formula

~~(.6180, 0)(-1.6180, 0)~~ don't do decimal yet

$\sin x = \frac{-1 \pm \sqrt{5}}{2}$ set =

$\sin x = .618$ | $\sin x = -1.6180$

$\sin^{-1}$ $\sin^{-1}$ | $\sin^{-1}$ $\sin^{-1}$

(+) 1st $x = 38.1727$ or $.666$ r | x = error

(+) 2nd = 141.827 or 2.475 r | no more solutions

write this for credit

22. $\cos^2 x - 2\cos x - 1 = 0$

quadratic formula $\rightarrow$ $\frac{-(-2\cos x) \pm \sqrt{(-2\cos x) - 4(\cos^2 x)(-1)}}{2(\cos^2 x)}$

$\frac{2\cos x \pm \sqrt{1}}{2\cos^2 x}$

+ -

$\cos x = 2.4142$ | $\cos x = -.4142$

$\cos^{-1}$ $\cos^{-1}$ | $\cos^{-1}$ $\cos^{-1}$

error undefined | (+) 1st $x = 114.469°$ or 2 r

(+) 4th = 245.53° or 4.29 r

24. $2\sin^3 x - \sin^2 x - 2\sin x + 1 = 0$

$\sin^2 x(2\sin x - 1) - 1(-2\sin x + 1)$

$(\sin^2 x - 1)(2\sin x - 1)$

$\sin^2 x - 1 = 0$ $\qquad$ $2\sin x - 1 = 0$

$+1 \quad +1$ $\qquad$ $+1 \quad +1$

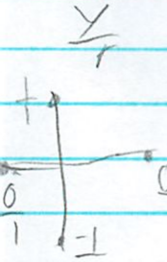
$\sin^2 x = 1$ $\qquad$ $\frac{2\sin x}{2} = \frac{1}{2}$

$\sqrt{\ } \quad \sqrt{\ }$

$\sin x = \pm\sqrt{1}$ $\qquad$ $\sin x = \frac{1}{2}$

$\sin^{-1} \quad \sin^{-1}$ $\qquad$ $\sin^{-1} \quad \sin^{-1}$

write radians



the negative value

(+) 1st: $90°$ $\frac{\pi}{2}$ $\qquad$ (+) $x = 30°$ $\frac{\pi}{6}$

(+) 2nd: $\qquad$ (+) 2nd $= 150°$ $\frac{5\pi}{6}$

(−) 3rd: 270 $\frac{\pi}{6}$

(−) 4th:

~~Section 14.4:~~
Solving Trig Equations

Name ____________________

Finding ALL The Solutions

Directions: Find the solutions of the following equations for $0 \le \theta < 2\pi$.

diamond + area ~~not~~ rooten!

1. $2\cos^2\theta + 3\cos\theta + 1 = 0$

diamond

2. $2\sin^2\theta + \sin\theta - 6 = 0$

idenity

3. $2\tan x - \sec^2 x = 0$

solve for 0 → grouping

4. $2\tan x \csc x + 2\csc x = -\tan x - 1$

solve for 0 → diamond + area

5. $2\sin^2\theta = 1 - \sin\theta$

diamond + area

6. $12\sin^2\theta - 5\sin\theta - 2 = 0$

solve for 0 → grouping

7. $2\sin\theta\csc\theta - \csc\theta = 4\sin\theta - 2$

simple solve

8. $\sqrt{3} + 2\sin\theta = 0$

Finding All The Solutions Worksheet

4

1. $2\cos^2\theta + 3\cos\theta + 1 = 0$

2, 2, 1, 3, $\uparrow \times$, $\downarrow +$

	$\cos\theta$	$+1$
$2\cos\theta$	$2\cos^2\theta$	$2\cos\theta$
$+1$	$\cos\theta$	1

$(\cos\theta + 1)\ (2\cos\theta + 1)$

$\cos\theta + 1 = 0$ $\quad$ $2\cos\theta + 1 = 0$

$-1 \ -1$ $\quad$ $-1 \ -1$

$\cos\theta = -1$ $\quad$ $\dfrac{2\cos\theta}{2} = \dfrac{-1}{2}$

$\cos^{-1}$ $\cos^{-1}$ $\quad$ $\cos\theta = -\frac{1}{2}$

$\theta = 180°, \pi$ $\quad$ $\cos^{-1}$ $\cos^{-1}$

remember radians

$\ominus$ 2nd; $120, \frac{2\pi}{3}$ ← remember if neg

$\ominus$ 3rd; $240, \frac{4\pi}{3}$ ←

2. $2\sin^2\theta + \sin\theta - 6 = 0$

-12, -3, 4, 1, $\uparrow \times$, $\downarrow +$

	$\sin x$	$+2$
$2\sin x$	$2\sin^2\theta$	$4\sin\theta$
$\cancel{+}\ 3$	$-3\sin\theta$	-6

copy error

$(\sin\theta + 2)\ (2\sin x \cancel{+} 3)$

$\sin\theta + 2 = 0$ $\quad$ $2\sin x \cancel{+} 3 = 0$

$-2 \ -2$ $\quad$ $+3 \ +3$

$\sin\theta = -2$ $\quad$ $\dfrac{2\sin x}{2} = \dfrac{\cancel{+}3}{2}$

$\sin^{-1}$ $\sin^{-1}$ $\quad$ $\sin x = \cancel{+}\frac{3}{2}$

$\sin^{-1}$ $\sin^{-1}$

error
undefined

still undefined

, identity - aim to replace 1 term
not gather + replace multiple
terms

Just use cosine w/ pho
therom - eaiser

1

1

tan-1

-0 $\frac{\pi}{4}$

5° $\frac{5\pi}{4}$

- tan x - 1

+ (- tan x - 1)

x + 1 = 0

tan x + 1) = 0

-1)

x + 1 = 0
-1 -1

an x = -1

tan-1 tan-1

= -45°

135° $\frac{3\pi}{4}$) like #3

315° $\frac{7\pi}{4}$

5. $2\sin^2 x = 1 - \sin x$

$-(1-\sin x) \quad -(1-\sin x)$

$2\sin^2 x - 1 + \sin x$

-2 ↑x, -1, 2, 1 ↙+

	$2\sin x$	-1
$\sin x$	$2\sin^2 x$	$-1\sin x$
$+1$	$2\sin x$	-1

not 0!

$(\sin x + 1)(2\sin x - 1)$

$\sin x + 1 = 0 \qquad 2\sin x - 1 = 0$

$-1 \;\; -1 \qquad\qquad +1 \;\; +1$

$\sin x = -1 \qquad \frac{2\sin x}{2} = \frac{1}{2}$

$\sin^{-1} \; \sin^{-1}$

$x = -90°$ $\qquad \sin x = \frac{1}{2}$

$270° \; \frac{3\pi}{2}$ $\qquad \sin^{-1} \; \sin^{-1}$

(+) 1st $= 30°$ $\frac{\pi}{6}$

(+) 2nd $= 150°$ $\frac{5\pi}{6}$

$\frac{y}{r}$ $\frac{1}{1}$ θ $\frac{-1}{1}$

6. $12\sin^2 x - 5\sin x - 2 = 0$

-24 ↑x, -8, 3, -5 ↙+

	$3\sin x$	-2
$4\sin x$	$12\sin^2 x$	$-8\sin x$
$+1$	$3\sin x$	-2

$(4\sin x + 1)(3\sin x - 2)$

$4\sin x + 1 = 0 \qquad 3\sin x - 2 = 0$

$-1 \;\; -1 \qquad\qquad +2 \;\; +2$

$\frac{4\sin x}{4} = \frac{-1}{4} \qquad \frac{3\sin x}{3} = \frac{2}{3}$

$\sin x = -\frac{1}{4} \qquad \sin x = \frac{2}{3}$

$\sin^{-1} \; \sin^{-1} \qquad \sin^{-1} \; \sin^{-1}$

$-\theta$

$x = -14.4775 \qquad x \; 41.81$

(−) 3rd $= 194.4775$ (−) $\qquad$ (+) 1st: $41.81°$

or 3.34 r $\qquad$ or $.7279$ r

(−) 4th $= 345.5225$ $\qquad$ (+) 2nd: $138.19°$

or 6.03 r $\qquad$ or 2.412 r

7. $2\sin\theta\csc\theta - \csc\theta = 4\sin\theta - 2$

$-(+4\sin\theta - 2) \qquad -(+4\sin\theta - 2)$

$2\sin\theta\csc\theta - \csc\theta - 4\sin\theta + 2 = 0$

$\csc\theta(2\sin\theta - 1) - 2(2\sin\theta - 1) = 0$

$(\csc\theta - 2)(2\sin\theta - 1)$

$\csc\theta - 2 = 0 \qquad 2\sin\theta - 1 = 0$

$+2 \quad +2 \qquad +1 \quad +1$

$\csc\theta = 2 \qquad \frac{2\sin\theta}{2} = \frac{1}{2}$

$\sin^{-1}(1/2) \qquad \sin\theta = \frac{1}{2}$

same answers $\qquad \sin^{-1} \quad \sin^{-1}$

(+) 1st: $30°$ $\frac{\pi}{6}$

(+) 2nd: $150°$ $\frac{5\pi}{6}$

8. $\sqrt{3} + 2\sin\theta = 0$

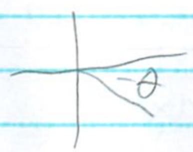
$-\sqrt{3} \qquad -\sqrt{3}$

$\frac{2\sin\theta}{2} = -\frac{\sqrt{3}}{2}$

$\sin\theta = -\frac{\sqrt{3}}{2}$

$\sin^{-1} \quad \sin^{-1}$

$x = -60°, -\frac{\pi}{3}$



(−) 3rd: $240°$ $\frac{4\pi}{3}$ (v)

(−) 4th: $300°$, $\frac{5\pi}{3}$

4/24/07 Quiz Review

Solving Trig Equations

4/23

Simplify

p773 Algebra 2

28. $\cos^2 x + \sin(-x)$

$\cos^2 x + -\sin x$

$1 - \sin^2 x - \sin x$

leave like that

29. $\tan(-x)\cos(-x)$

$-\tan x \cos x$

$-\frac{\sin x}{\cos x} \quad \frac{\cos x}{1}$

don't need = denomenators for multiplication

$\frac{-\sin x \cos x}{\cos x}$

$-\sin x$

30. $\sin^2\left(\frac{\pi}{2} - x\right) - 2\sin^2 x + 1$

$\cos^2 x - 2\sin^2 x + 1$

ib $1 - \sin^2 x - 2\sin^2 x + 1$

$-3\sin^2 x + 2$

done

31. $\cot^2\left(\frac{\pi}{2} - x\right) - \sec^2(-x)$

$\tan^2 x - \sec^2 x$

$\frac{\sin^2 x}{\cos^2 x} - \frac{1}{\cos^2 x}$

$\frac{\sin^2 x - 1}{\cos^2 x}$

$\frac{-\cos^2 x}{\cos^2 x}$

-1

32. $\csc^2(-x)\cos^2\left(\frac{\pi}{2}-x\right)$

$-\csc^2 x \sin^2 x$

$-\frac{1}{\sin^2 x} \quad \frac{\sin^2 x}{1}$

$\frac{-\sin^2 x}{\sin^2 x} \quad -1$

33. $-1+\tan(-x)\frac{\left(\cos\frac{\pi}{2}-x\right)}{\left(\sin\frac{\pi}{2}-x\right)}$

$-1-\tan x \frac{\sin x}{\cos x}$

$-1-\tan^2 x$

$\cancel{-1 \quad -1}$

$\cancel{1+\tan^2 x}$

$-\sec^2 y$

bad at neg conversion

can't do that

Verify

34. $\cot^2 x(\tan^2 x+1)=1+\cot^2 x$

$\cot^2 x(\sec^2 x)$

$\frac{\cos^2 x}{\sin^2 x} \quad \frac{1}{\cos^2 x}$

$\frac{\cos^2 x}{\sin^2 x \cos^2} \rightarrow \frac{1}{\sin^2 x} \rightarrow \csc^2 x \rightarrow 1+\cot^2 x$

35. $\cot x+\tan x=\sec x\csc x$

$\frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}$

$\frac{\cos^2 x}{\sin x\cos x}+\frac{\sin^2 x}{\sin x+\cos x}$

$\frac{\sin^2 x\cos^2 x}{\sin x\cos x}$

$\frac{1}{\sin x\cos x}$

Name: Michael Plasreier 47/50

IAG 4-1

GREAT JOB!

Date: 4/24/01

50 points

Test: Equations [illegible] tities

SHOW ALL WORK THROU[illegible] TEST!!

1. Solve the following equations for $0 \le \theta < 360°$ and $0 \le x \le 2\pi$. DO NOT round any of your **radian or degree** measures. *Please circle all of your answers.* (3 points each)

a) $\sqrt{2}\sec x = 2$

$\frac{\sqrt{2}\sec x}{\sqrt{2}} = \frac{2}{\sqrt{2}}$

$\sec x = \frac{2}{\sqrt{2}}$

$\cos^{-1}\left(1/\frac{2}{\sqrt{2}}\right)$

(+) 1st: $45°, \frac{\pi}{4}$ ✓

(+) 4th: $315°, \frac{7\pi}{4}$ ✓

b) $2\csc x + 17 = 15 + \csc x$

$-\csc x - 17 \quad -17 \quad -\csc x$

$\csc x = -2$

$\sin^{-1}(1/-2)$

$x = -30°$ which = s

(−) 3rd: $210°, \frac{7\pi}{4}$ ✓

(−) 4th: $330°, \frac{11\pi}{6}$ ✓

2. Solve the following equations for $0 \le \theta < 360°$ and $0 \le x \le 2\pi$. Give answers to the nearest hundredth of a degree and radian, if necessary. *Please circle all of your answers.*

a) $2\tan^2\theta - 6 = 0$ (4 pts)

$+6 \quad +6$

$\frac{2\tan^2\theta}{2} = \frac{6}{2}$

$\tan^2\theta = 3$

$\tan\theta = \pm\sqrt{3}$

$\tan^{-1} \quad \tan^{-1}$

(+) 1st: $60°, \frac{\pi}{3}$ ✓ (+) 3rd: $240°, \frac{4\pi}{3}$ ✓

(−) 2nd: $120°, \frac{2\pi}{3}$ ✓ (−) 4th: $300°, \frac{5\pi}{3}$ ✓

b) $2\cos^2\theta + 3\sin\theta = 3$ (7 pts)

-3

$2\cos^2\theta + 3\sin\theta - 3 = 0$

$2(1-\sin^2\theta) + 3\sin\theta - 3 = 0$

$2 - 2\sin^2\theta + 3\sin\theta - 3 = 0$

$-2\sin^2\theta + 3\sin\theta - 1 = 0$

Back →

2, 2, 1, 3, ↑x, d+

	$\sin\theta$	-1
$-2\sin\theta$	$-2\sin^2\theta$	$2\sin\theta$
$+1$	$1\sin\theta$	-1

c) $\sin\theta\tan\theta = 2\sin\theta$ (5 pts)

$-2\sin\theta \quad -2\sin\theta$

would have to be Multiplied

~~$\sin\theta\tan\theta$~~

$\sin\theta\tan\theta - 2\sin\theta = 0$ GCF $\sin\theta(\tan\theta - 2) = 0$

$\sin\theta = 0$ $\tan\theta - 2 = 0$

$\tan\theta = 2$

$\tan^{-1}$ 63.43°, 1.11r

$\sin\theta = 0$

$\sin^{-1}$

(+) 1st: ~~0~~ ✓

(+) 3rd: ~~180~~ 243.43° ✓ 4.25r

(+) 1st: $0°, 0$ ✓

(+) 2nd: $180°, \pi$ ✓

d) $\cot^2\theta - 2\cot\theta - 15 = 0$ (6 pts)

-15, -5, 3, -2, ↑x, d+

$(\cot\theta - 5)(\cot\theta + 3)$

$\cot\theta - 5 = 0$ $\cot\theta + 3 = 0$

$+5 \quad +5$ $-3 \quad -3$

$\cot\theta = 5$ $\cot\theta = -3$

$\tan^{-1}(1/5)$ $\tan^{-1}(1/-3)$

$x = -18.43$

(+) 1st: 11.31° or .20r ✓

(−) 2nd: 161.57° or 2.82r ✓

(+) 3rd: 191.31° or 3.34r ✓

(−) 4th: 341.57° or 5.96r ✓

25

b)

$(-2\sin\theta + 1)(\sin\theta - 1)$

$-2\sin\theta + 1 = 0$

$-1 \quad -1$

$\frac{-2\sin\theta}{-2} = \frac{-1}{-2}$

$\sin\theta = \frac{1}{2}$

$\sin^{-1} \quad \sin^{-1}$

(+) 1st: $30°, \frac{\pi}{6}$ ✓

(+) 2nd: $150°, \frac{5\pi}{6}$ ✓

$\sin\theta - 1 = 0$

$+1 \quad +1$

$\sin\theta = 1$

$\sin^{-1} \quad \sin^{-1}$

(+) 1st: $90°, \frac{\pi}{2}$ ✓

$\frac{y}{r}$

(×) 180, 0, 360, 270

c. $2\sin^3 x+\sin^2 x+2\sin x+1=0$ (7 pts)

$\sin^2 x(2\sin x+1)+1(2\sin x+1)$

$(\sin^2 x+1)(2\sin x+1)$

$\sin^2 x+1=0$ — $2\sin x+1=0$

$\sin^2 x=-1$ — $2\sin x=-1$

$\sin x=\pm\sqrt{-1}$ — $\sin x=-\frac{1}{2}$

$\sin^{-1}$ $\sin^{-1}$ — $\sin^{-1}$ $\sin^{-1}$

$x=-30°$

(+) 1st: undefined
(+) 2nd: undefined
(−) 3rd: undefied
(−) 4th: undefined

(−) 3rd: 210°, $\frac{7\pi}{6}$
(−) 4th: 330°, $\frac{11\pi}{6}$

$\frac{7\pi}{6}$ unit circle copy error

3. Simplify the following expression. **SHOW ALL WORK!! ALL OF IT!!**

$\sin\theta(\csc\theta-\sin\theta)$ (3 pts)

$\sin\theta\left(\frac{1}{\sin\theta}-\frac{\sin\theta}{1}\right)$

$\sin\theta\left(\frac{1-\sin^2\theta}{\sin\theta}\right)$

$\sin\theta\left(\frac{\cos^2\theta}{\sin\theta}\right)$

$\frac{\sin\theta}{1}\cdot\frac{\cos^2\theta}{\sin\theta}$

$\frac{\cancel{\sin\theta}\cos^2\theta}{\cancel{\sin\theta}}$

$\cos^2\theta$

4. Prove the following identities. **SHOW ALL WORK!! ALL OF IT!!** (6 pts each)

a. $\cos x(\sec x+\cos x\csc^2 x)=\csc^2 x$

$\cos x\left(\frac{1}{\cos x}+\frac{\cos x}{1}\frac{1}{\sin^2 x}\right)$

$\cos x\left(\frac{1}{\cos x}+\frac{\cos x}{\sin^2 x}\right)$

$\cos x\left(\frac{(\sin^2 x)}{\cos x(\sin^2 x)}+\frac{\cos^2 x}{\sin^2 x(\cos x)}\right)$

$\cos x\left(\frac{\sin^2 x+\cos^2 x}{\cos x\sin^2 x}\right)$

$\cos x\left(\frac{1}{\cos x\sin^2 x}\right)$

$\frac{\cos x}{\cos x\sin^2 x}$

$\frac{1}{\sin^2 x}\rightarrow$ $\csc^2 x=\csc^2 x$

b. $(1-\cos\theta)(\csc\theta+\cot\theta)=\sin\theta$ FOIL

$(1\cdot\csc\theta)+(1\cdot\cot\theta)+(-\cos\theta\cdot\csc\theta)+(-\cos\theta\cdot\cot\theta)$

$\csc\theta+\cot\theta+\left(\frac{-\cos\theta}{1}\frac{1}{\sin\theta}\right)+\left(\frac{-\cos\theta}{1}\frac{\cos\theta}{\sin\theta}\right)$

$\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}-\frac{\cos\theta}{\sin\theta}+\frac{-\cos^2\theta}{\sin\theta}$

0

$\frac{1}{\sin\theta}-\frac{\cos^2\theta}{\sin\theta}$

$\frac{1-\cos^2\theta}{\sin\theta}$

$\frac{\sin^2\theta}{\sin\theta}\rightarrow$ $\sin\theta=\sin\theta$

22

14.5 Sum + Difference Formula

4/23

$\sin(u+v) = \sin u \cos v + \cos u \sin v$

$\sin(u-v) = \sin u \cos v - \cos u \sin v$

$\cos(u+v) = \cos u \cos v - \sin u \sin v$ signs switched

$\cos(u-v) = \cos u \cos v + \sin u \sin v$

to add or subtract angles exactly

better stated

to use known values of certain angles to evaluate a trig expression

30°: 30°, 150°, 330°

45°: 45°, 135°, 225°, 315°

60°: 60°, 120°, 240°, 300°

ex/ Find the exact value of 75°

multiple ways: $\sin 75 = \sin(30+45) = \sin(120-45)$

use chart

$$\sin(30+45) = \sin 30 \cos 45 + \cos 30 \sin 45$$
$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\cdot\left(\frac{\sqrt{3}}{2}\right)$$
$$\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$
$$\frac{\sqrt{2}+\sqrt{6}}{4}$$ ← answer

$$\sin(120-45) = \sin 120 \cos 45 - \sin 45 \cos 120$$
$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right)$$ ← cos is − in 2nd + 3rd quadrants
$$\left(\frac{\sqrt{6}}{4}\right) - \left(\frac{-\sqrt{2}}{4}\right)$$
$$\frac{\sqrt{2}+\sqrt{6}}{4}$$

Ex2 $\sin(15) = \sin(45-30)$

$\sin(45-30) = \sin 45 + \cos 30 - \sin 30 \cos 45$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\left(\frac{\sqrt{6}}{4}\right) - \left(\frac{\sqrt{2}}{4}\right)$

$\frac{\sqrt{6}-\sqrt{2}}{4}$

Ex3

$\csc(345) = \csc(315+30)$

convert + reciprical answer

different

$\sin(315+30) = \sin 315 \cos 30 + \sin 30 \cos 315$

$\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

be careful → $\ominus$: 3rd + 4th $\oplus$: 1st + 4th

$\left(\frac{-\sqrt{6}}{4}\right) + \left(\frac{\sqrt{2}}{4}\right)$

$\frac{\sqrt{2}-\sqrt{6}}{4}$

reciprical, flip

$\boxed{\frac{4}{\sqrt{2}-\sqrt{6}}}$ ← need to get rid of

ex4

$\cos\left(\frac{\pi}{12}\right) = \cos 15$

$\cos(45-30)$

$\cos 45 \cos 30 + \sin 45 \sin 30$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} + \frac{1}{2}$

$\frac{\sqrt{6}+\sqrt{2}}{4}$

COPY from board

$\frac{4}{\sqrt{2}-\sqrt{6}} \cdot \frac{\sqrt{2}+\sqrt{6}}{\sqrt{2}+\sqrt{6}}$ ← this positive too

$\frac{4(\sqrt{2}+\sqrt{6})}{2+\sqrt{12}-\sqrt{12}-6}$

$\frac{4\sqrt{2}+4\sqrt{6}}{-4}$

$-\sqrt{2}-\sqrt{6}$

✓

ex5

$\sec(195) = \frac{1}{\cos(195)}$

1

$\cos(195) = \cos(225-30) = \cos(135+60) = \cos(45+150)$

$\cos(45)\cos 150 - \sin 45 \sin 150$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{-\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$\cos 195 = \frac{-\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$\sec 195 = \boxed{\frac{4}{-\sqrt{6}-\sqrt{2}}}$

multiply by the Conigate

$\frac{4}{(-\sqrt{6}-\sqrt{2})} \cdot \frac{(-\sqrt{6}+2)}{(-\sqrt{6}+\sqrt{2})}$ oppset sign

$(\sqrt{a}-\sqrt{b}) \cdot (\sqrt{a}+\sqrt{b})$

FOIL

$\times$

$\frac{4(-\sqrt{6}+\sqrt{2})}{6-\sqrt{12}+\sqrt{12}-2} \rightarrow \frac{-4\sqrt{6}+4\sqrt{6}}{4}$

$\div 4$

$-\sqrt{6}+\sqrt{2}$

ex6

$\tan 285 = \tan(240+45) = \tan(225+60)$

$\frac{\tan(240)+\tan(45)}{1-\tan 240 \tan 45}$

oppset

$\frac{\sqrt{3}+1}{1-\sqrt{3}\cdot 1} \rightarrow \frac{\sqrt{3}+1}{1-\sqrt{3}}$

$\frac{\sqrt{3}+1}{1-\sqrt{3}} \cdot \frac{(1+\sqrt{3})}{(1+\sqrt{3})}$

$\frac{\sqrt{3}+3+1+\sqrt{3}}{1-\sqrt{3}+\sqrt{3}-3}$

$\frac{4+2\sqrt{3}}{-2} \rightarrow -2-\sqrt{3}$

Check by comparing this to problem or interm solution or problem ex 7

$\cos 50 \cos 100 - \sin 50 \sin 100$

$\cos(50 + 100)$

$\cos 150$

$-\frac{\sqrt{3}}{2}$

ex 8

$\frac{\tan 128 - \tan 98}{1 + \tan 128 \tan 98}$ sign same

$\tan(128 \cancel{+} 98)$

$\tan(\cancel{226})\ (30)$

$\frac{\sqrt{3}}{3}$

ex 9 Solve for x

$\sin\left(x + \frac{\pi}{4}\right) + 1 = \sin\left(\frac{\pi}{4} - x\right)$

$(\sin x \cos 45 + \cos x \sin 45) + 1 = (\sin 45 \cos x - \cos 45 \sin x$

$-\sin 45 \cos x + \cos 45 \sin x$ Cancle out $-\sin 45 \cos x + \cos 45 \sin x$

$2 \cos 45 - \sin x + 1 = 0$

$2 \cdot \frac{\sqrt{2}}{2} \sin x + 1 = 0$

$\frac{2}{1} \cdot \frac{\sqrt{2}}{2} \sin x + 1 = 0$

$\sqrt{2} \sin x + 1 = 0$

$-1 \quad -1$

$\frac{\sqrt{2} \sin x}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$

$\sin x = \frac{-1}{\sqrt{2}}$

$\sin^{-1} \quad \sin^{-1}$

~~(+) 1st ; 45°, $\frac{\pi}{4}$~~

~~(+) 2nd ; 135°, $\frac{3\pi}{4}$~~

$x = -45^\circ$

(−) 3rd ; 225°, $\frac{5\pi}{4}$

(−) 4th ; 315°, $\frac{7\pi}{4}$

14.5 Sum + Difference Formulas Homework

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Find exact value

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5. $\cos 105° = \cos(60+45)$

$\cos(60+45) = \cos 60 \cos 45 - \sin 60 \sin 45$

$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2}$

$\frac{\sqrt{2}-\sqrt{6}}{2}$

7. $\tan(75°) = \tan(45+30)$

$\frac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30}$

$\frac{1 + \frac{\sqrt{3}}{3}}{1 - 1 \cdot \frac{\sqrt{3}}{3}}$

convert to $\frac{3}{3}$

makes it easier

$\frac{\frac{3}{3} + \frac{\sqrt{3}}{3}}{\frac{3}{3} - \frac{\sqrt{3}}{3}} \rightarrow \frac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}} \rightarrow \frac{3+\sqrt{3}}{\cancel{3}} \cdot \frac{\cancel{3}}{3-\sqrt{3}}$

$\frac{3+\sqrt{3}}{3-\sqrt{3}} \cdot \frac{3+\sqrt{3}}{3+\sqrt{3}} \rightarrow \frac{9 + 3\sqrt{3} + 3\sqrt{3} + 3}{9 + 3\sqrt{3} - 3\sqrt{3} - \cancel{9}\ 3}$

$\frac{12 + 6\sqrt{3}}{6} \rightarrow 2 + \sqrt{3}$

? 9. ~~$\cos 225°$~~ $\cos 255$

~~$-\frac{\sqrt{2}}{2}$~~

$\frac{\sqrt{2}-\sqrt{6}}{4}$

copy problem wrong

? 11. $\tan \frac{11\pi}{12}$

~~$\tan 270°$~~

~~0~~

$\sqrt{3}-2$

13. $\sec \frac{19\pi}{12}$

$\sec 285$

$\frac{\pi}{12} = 15°$

$1 / (\cos(225+60) = \cos 225 \cos 60 - \sin 225 \sin 60$

$\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) \rightarrow \frac{\sqrt{6}-\sqrt{2}}{\cancel{2}\ 4}$

remember

$-\frac{\sqrt{2}}{\cancel{2}\ 4} + \frac{\sqrt{6}}{\cancel{2}\ 4}$

13. cont, Flip → $\boxed{\frac{4}{\sqrt{6}-\sqrt{2}}} \cdot \frac{(\sqrt{6}+\sqrt{2})}{(\sqrt{6}+\sqrt{2})}$

$$\frac{4\sqrt{6} \cdot 4\sqrt{2}}{6 \cdot \sqrt{12} \cdot -\sqrt{12} \cdot -2}$$

$$\frac{4\sqrt{6} \cdot 4\sqrt{2}}{4}$$

$$\sqrt{6} \cdot \sqrt{2}$$

Simplify

17, $\cos 35 \cos 15 - \sin 35 \sin 15$

$\cos(35+15)$

$\cos(50)$

19. $\sin 4 \cos 2.2 - \cos 4 \sin 2.2$

$\sin(4-2.2)$

$\sin(1.8)$

Evalate

21, $\sin 30 \cos 45 + \cos 30 \sin 45$

$\sin(30+45)$

$\sin(75)$

$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

remember $\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

x here

$\frac{\sqrt{2}+\sqrt{6}}{4}$ ← but not here

23, $\sin 240 \cos 45 - \cos 240 \sin 45$

$\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$-\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$\frac{\sqrt{2}-\sqrt{6}}{4}$

25, $\cos\frac{5\pi}{3} \cos\frac{\pi}{4} + \sin\frac{5\pi}{3} \sin\frac{\pi}{4}$

$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$ ← That is negative

$\left(\frac{\sqrt{2}}{4}\right) \cancel{+} \left(\frac{\sqrt{6}}{4}\right)$

$\boxed{\frac{\sqrt{2} \cancel{+} \sqrt{6}}{4}}$

Solve

27. $\sin\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{3}\right)=1$

$\underbrace{\sin x\cos\frac{\pi}{3}+\cos x\sin\frac{\pi}{3}+\sin x\cos\frac{\pi}{3}-\cos x\sin\frac{\pi}{3}}=1$

$2\sin x\cos\frac{\pi}{3}=1$

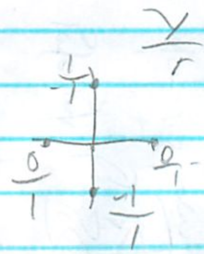
↓ convert out

$2\sin x\frac{1}{2}=1$

$\sin x=1$

1st $= 90, \frac{\pi}{2}$

~~2nd =~~



29. $\cos\left(x+\frac{\pi}{4}\right)+\cos\left(x-\frac{\pi}{4}\right)=1$

$\cos x\cos\frac{\pi}{4}-\underbrace{\sin x\sin\frac{\pi}{4}+\cos x\cos\frac{\pi}{4}+\sin x\sin\frac{\pi}{4}}_{0}=1$

$2\cos x\cos\frac{\pi}{4}=1$

$2\cos x\frac{\sqrt{2}}{2}=1$

$\frac{\sqrt{2}\cos x}{\sqrt{2}}=\frac{1}{\sqrt{2}}$

$\cos x=\frac{1}{\sqrt{2}}$

$\cos^{-1}$ $\cos^{-1}$

(+) 1st $= 45°$ $\frac{\pi}{4}$

(+) 4th $= 315°, \frac{7\pi}{4}$

Extra Problems

1. $\tan(195) = \tan(150+45) = \tan(135+60) = \tan(225-30)$

$$\frac{\tan 150 + \tan 45}{1 - \tan 150 \tan 45}$$

$$\boxed{\frac{-\frac{\sqrt{3}}{3} + 1}{1 + \frac{\sqrt{3}}{3} \cdot 1}}$$

convert 1 to $\frac{3}{3}$

$$\frac{\frac{3}{3} - \frac{\sqrt{3}}{3}}{\frac{3}{3} + \frac{\sqrt{3}}{3}} \rightarrow \frac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}} \rightarrow \frac{3-\sqrt{3}}{\cancel{3}} \cdot \frac{\cancel{3}}{3+\sqrt{3}}$$

cross out like this

$\leftarrow$ x conjigate

$$\frac{3(3-\sqrt{3})}{3(3+\sqrt{3})} \rightarrow \frac{9-3\sqrt{3}}{9+3\sqrt{3}} \quad \frac{3-\sqrt{3}}{3+\sqrt{3}} \cdot \frac{(3-\sqrt{3})}{(3-\sqrt{3})}$$

FOIL

$$\frac{9-3\sqrt{3}-3\sqrt{3}+3}{9-\underbrace{3\sqrt{3}+3\sqrt{3}}_{0}-3} \rightarrow \frac{12-6\sqrt{3}}{6} \rightarrow 2-\sqrt{3}$$

31. $\tan(x+\pi) + 2\sin(x+\pi) = 0$

$$\frac{\tan x + \tan 180}{1 - \tan x \tan 180} + 2(\sin x \cos 180 + \sin 180 \cos x) = 0$$

$$\frac{\tan x + 0}{1 - \tan x \cdot 0} + 2(\sin x \cdot -1 + 0 \cdot \cos x) = 0$$

$$\frac{\tan x}{1} + 2(-\sin x) = 0$$

$$\tan x - 2\sin x = 0$$

$$\tan x - 2(\cos x \tan x) = 0$$

$$\tan x(1-2\cos x)$$

$1 - 2\cos x = 0$

$-1 \quad -1$

$\frac{-2\cos x}{-2} = \frac{-1}{-2}$

$\cos x = \frac{1}{2}$

$\cos^{-1} \quad \cos^{-1}$

(+) 1st $= 60°, \frac{\pi}{3}$

(+) 4th $= 300°, \frac{5\pi}{3}$

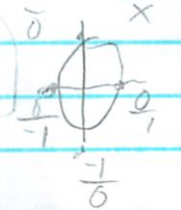
$\tan x = 0$

$\tan^{-1} \quad \tan^{-1}$

(+) 1st $= 0°, 0$

(+) 3rd $= 180°, \pi$

$\frac{y}{x}$



14.6 Double + Half Angle Formulas

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1. Find the exact value of $\tan\left(\frac{\pi}{8}\right)$

$\frac{1}{2}\cdot\frac{\pi}{4}=\frac{\pi}{8}$

$\frac{\theta}{2}=\frac{\pi}{8}$

$\theta=\frac{\pi}{4}$

$$\tan\left(\frac{\theta}{2}\right)=\frac{1-\cos\theta}{\sin\theta}$$

$$\tan\left(\frac{\pi}{8}\right)=\frac{1-\cos\frac{\pi}{4}}{\sin\frac{\pi}{4}}$$

$$\frac{1-\frac{\sqrt2}{2}}{\frac{\sqrt2}{2}}\rightarrow\frac{\frac{2}{2}-\frac{\sqrt2}{2}}{\frac{\sqrt2}{2}}\rightarrow\frac{\frac{2-\sqrt2}{2}}{\frac{\sqrt2}{2}}$$

$$\frac{2-\sqrt2}{2}\cdot\frac{2}{\sqrt2}\rightarrow\frac{2(2-\sqrt2)}{2\sqrt2}\rightarrow\frac{4-2\sqrt2}{2\sqrt2}$$

$$\frac{4-2\sqrt2\cdot\sqrt2}{2\sqrt2\cdot\sqrt2}\quad\frac{4\sqrt2-4}{4}\quad\sqrt2-1$$

2. $\sin\left(\frac{\pi}{12}\right)$

$\frac{1}{2}\cdot\frac{\pi}{6}=\frac{\pi}{12}$

$\frac{\theta}{2}=\frac{\pi}{12}$

$\theta=\frac{\pi}{6}$

$$\sin\left(\frac{\theta}{2}\right)=\pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\sin\left(\frac{\pi}{12}\right)=\sqrt{\frac{1-\cos\frac{\pi}{6}}{2}}$$

$$\sqrt{\frac{1-\frac{\sqrt3}{2}}{2}}\quad\sqrt{\frac{\frac{2}{2}-\frac{\sqrt3}{2}}{2}}\rightarrow\sqrt{\frac{\frac{2-\sqrt3}{2}}{2}}$$

$$\sqrt{\frac{2-\sqrt3}{2}\cdot\frac{1}{2}}\rightarrow\cancel{\sqrt{\frac{4-2\sqrt3}{2}}}\rightarrow\cancel{\sqrt{2}\ \sqrt{3}}$$

$$\sqrt{\frac{2-\sqrt3}{4}}\rightarrow\frac{\sqrt{2-\sqrt3}}{\sqrt4}\rightarrow\frac{\sqrt{2-\sqrt3}}{2}$$

3. Use $\cos 72° = \frac{-1+\sqrt{5}}{4}$ to find an exact value of $\cos 36°$

$36° = \frac{\theta}{2}$

$72° = \theta$

$\cos 36 = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$4-1\sqrt{5}$

$\pm\sqrt{\frac{1+\frac{-1+\sqrt{5}}{4}}{2}} \rightarrow \pm\sqrt{\frac{\frac{4}{4}+\frac{-1+\sqrt{5}}{4}}{2}} \rightarrow \sqrt{\frac{3+\sqrt{5}}{4}\cdot\frac{1}{2}}$

$\sqrt{\frac{3+\sqrt{5}}{8}} \rightarrow \frac{\sqrt{3+\sqrt{5}}}{\sqrt{8}} \rightarrow \sqrt{4}\cdot\sqrt{2} \rightarrow \frac{\sqrt{3+\sqrt{5}}}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$\frac{\sqrt{2}\left(\sqrt{3+\sqrt{5}}\right)}{4}$

4. use $\cos 72° = \frac{-1+\sqrt{5}}{4}$ to find an exact value of $\sin 36°$

$36° = \frac{\theta}{2}$

$72° = \theta$

$\sin 36 = +\sqrt{\frac{1-\cos\theta}{2}}$

$\sqrt{\frac{1-\left(\frac{-1+\sqrt{5}}{4}\right)}{2}} \rightarrow \sqrt{\frac{\frac{4}{4}-\left(\frac{-1+\sqrt{5}}{4}\right)}{2}} \rightarrow \sqrt{\frac{\frac{5-\sqrt{5}}{4}\cdot\frac{1}{2}}{2}}$

$\sqrt{\frac{5-\sqrt{5}}{8}} \xrightarrow{\text{split}} \frac{\sqrt{5-\sqrt{5}}}{\sqrt{8}} \rightarrow \sqrt{4}\cdot\sqrt{2} \rightarrow \frac{\sqrt{5-\sqrt{5}}}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$\frac{\sqrt{2}\left(\sqrt{5-\sqrt{5}}\right)}{4}$

14.6 Double + Half Angles
Homework

4/26

Rewrite w/o the 2x angles + simplify

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Algebra2

7. $\tan 2x(1+\tan x)$

$\frac{2\tan x}{1-\tan^2 x} \cdot 1+\cancel{\tan x}$

$1-\tan^2 x \Rightarrow (1-\tan x)(1+\cancel{\tan x})$ I factors out

$\frac{2\tan x}{1-\tan x}$

9. $\cot x \tan 2x$

$\cot x \cdot \frac{2\tan x}{1-\tan^2 x}$

$\frac{1}{\cancel{\tan x}} \cdot \frac{2\cancel{\tan x}}{1-\tan^2 x}$

$\frac{2}{1-\tan^2 x}$ or $\frac{2}{(1-\tan x)(1+\tan x)}$

11. $\frac{\cos 2x}{\cos^2 x}$

$\frac{2\cos^2 x-1}{\cos^2 x}$ split $\rightarrow$ $\frac{2\cancel{\cos^2 x}}{\cancel{\cos^2 x}} - \frac{1}{\cos^2 x}$

$2 - \frac{1}{\cos^2 x}$

$2-1$

1

$2 - \sec^2 x$

Find the exact value

25. $\tan \frac{\pi}{12}$

$\frac{1}{2} \cdot \frac{\pi}{6} = \frac{\pi}{12}$

$\frac{\theta}{2} = \frac{\pi}{12}$

$\theta = \frac{\pi}{6}$

$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta}$

$\tan\left(\frac{\pi}{12}\right) = \frac{1-\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}} \rightarrow \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}} \rightarrow \frac{\frac{2-\sqrt{3}}{2}}{\frac{1}{2}} \rightarrow$

$\frac{2-\sqrt{3}}{2} \cdot \frac{2}{1} \rightarrow \frac{4-2\sqrt{3}}{2} \rightarrow 2-\sqrt{3}$

26. $\sin \frac{7\pi}{12}$

$\frac{1}{2} \cdot \frac{7\pi}{6} = \frac{7\pi}{12}$

$\frac{\theta}{2} = \frac{7\pi}{12}$

$\theta = \frac{7\pi}{6}$

$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\sin\left(\frac{7\pi}{12}\right) = \pm\sqrt{\frac{1-\cos\frac{7\pi}{6}}{2}}$ ← 3rd quad so its neg

?

$\sqrt{\frac{1-\frac{-\sqrt{3}}{2}}{2}} \to \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} \to \sqrt{\frac{2+\sqrt{3}}{2} \cdot \frac{1}{2}}$

$\sqrt{\frac{2+\sqrt{3}}{4}}$ $\frac{\sqrt{2+\sqrt{3}}}{\sqrt{4}} \to \frac{\sqrt{2+\sqrt{3}}}{2}$

28. $\cos \frac{7\pi}{8}$

$\frac{1}{2} \cdot \frac{7\pi}{4} = \frac{7\pi}{8}$

$\frac{\theta}{2} = \frac{7\pi}{8}$

$\theta = \frac{7\pi}{4}$ write

is this ⊕ or ⊖ in that quad

$\cos\left(\frac{\theta}{2}\right) = \ominus\sqrt{\frac{1+\cos\frac{7\pi}{4}}{2}}$

$-\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} \to \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}} \cdot \frac{1}{2}$

$-\sqrt{\frac{2+\sqrt{2}}{4}}$ split up $\to -\frac{\sqrt{2+\sqrt{2}}}{\sqrt{4}} \to -\frac{\sqrt{2+\sqrt{2}}}{2}$

29. $\tan \frac{3\pi}{8}$

$\frac{1}{2} \cdot \frac{3\pi}{4} = \frac{3\pi}{8}$

$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\frac{3\pi}{4}}{\sin\frac{3\pi}{4}}$

thats ⊕ with sin

$\frac{1+\frac{\sqrt{2}}{2}}{+\frac{\sqrt{2}}{2}}$ $\frac{\frac{2+\sqrt{2}}{2}}{1\frac{\sqrt{2}}{2}}$ $\frac{2+\sqrt{2}}{\cancel{2}} \cdot \frac{+\cancel{2}}{\sqrt{2}}$ remember cancle out

$\frac{\cancel{4}+2\sqrt{2}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ $\cancel{-\frac{4\sqrt{2}-2\sqrt{4}}{2}}$

$\cancel{-2\sqrt{2}-\sqrt{4}}$

$\frac{2\sqrt{2}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ $\frac{\sqrt{2}(2+\sqrt{2})}{2}$ $\frac{2\sqrt{2}+2}{2} \to \sqrt{2}+1$

Double Angle Notes

4/28

1. $\cos x + \cos 2x = 0$ ↓ use formula to switch

$\neq \cos 3x$

$\cos x + 2\cos^2 x - 1 = 0$

-2
2 -1 ↑×
1 ↓+

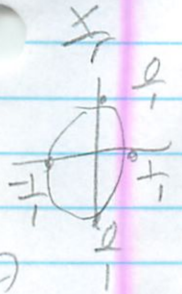
$2\cos x - 1$

	$2\cos^2 x$	$-\cos x$
$\cos x$	$2\cos^2 x$	$-\cos x$
$+1$	$2\cos x$	-1

↓ as normal

$(\cos x + 1)$ $(2\cos x - 1)$

$\cos x + 1 = 0$
$-1 \quad -1$
$\cos x = -1$
$\cos^{-1} \quad \cos^{-1}$
$180°, \pi$

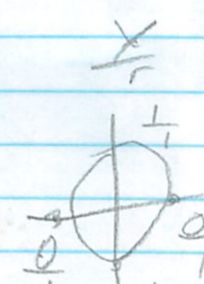


$2\cos x - 1 = 0$
$+1 \quad +1$
$\frac{2\cos x}{2} = \frac{1}{2}$
$\cos x = \frac{1}{2}$
$\cos^{-1} \quad \cos^{-1}$
(+) 1st ; $60°, \frac{\pi}{3}$
(+) 4th ; $300°, \frac{5\pi}{3}$

2. $2\cos x + \sin 2x$
$2\cos x + 2\sin x \cos x$
$\cos x (2 + 2\sin x)$

$\cos x = 0$
$90°, \frac{\pi}{2}$
$270°, \frac{3\pi}{2}$

$2 + 2\sin x = 0$
$-2 \quad -2$
$\frac{2\sin x}{2} = \frac{-2}{2}$
$\sin x = -1$
$\sin^{-1} \quad \sin^{-1}$
$270°, \frac{3\pi}{2}$



14.6 More Double + Half Angle Formulas

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Solve

13. $\cos 2x = -1$

$2\cos^2 x - 1 = -1$

$+1 \quad +1$

$\frac{2\cos^2 x}{2} = \frac{0}{2}$

✓

$\cos^2 x = 0$

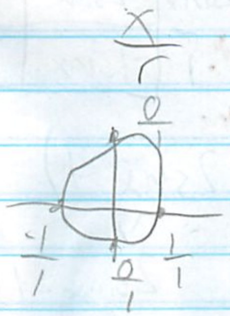
$\sqrt{\ } \quad \sqrt{\ }$

$\cos x = \pm\sqrt{0}$

$\cos^{-1} \quad \cos^{-1}$

$x_1 = 90°, \frac{\pi}{2}$

$x_2 = 270°, \frac{3\pi}{2}$



15. $\sin 2x \sin x = \cos x$

$2\sin x \cos x \sin x = \cos x$

$-\cos x \quad -\cos x$

squares togeter

GCF ($\cancel{3\sin x \cos x - \cos x = 0}$ → $2\sin^2 x \cos x - \cos x$

$\cos x(\cancel{3\sin x - 1}) = 0$

$(2\sin x^2 - 1)$

$\cos x = 0$ (see #15)

$90°, \frac{\pi}{2}$

$270°, \frac{3\pi}{2}$

~~$3\sin x - 1 = 0$~~

~~$+1 \quad +1$~~

~~$\frac{3\sin x}{3} = \frac{1}{3}$~~

~~$\sin x = \frac{1}{3}$~~

~~$\sin^{-1} \sin^{-1}$~~

~~(+) 1st = 19.47°, .34r~~

~~(+) 2nd = 160.53°, 2.80r~~

$2\sin^2 x - 1 = 0$

$+1 \quad +1$

$\frac{2\sin^2 x}{2} = \frac{1}{2}$

$\sin^2 x = \frac{1}{2}$

$\sqrt{\ } \quad \sqrt{\ }$

$\sin x = \pm\sqrt{\frac{1}{2}}$

(+) 1st : $45°$ $\frac{\pi}{4}$

(+) 2nd : $135°, \frac{3\pi}{4}$

(−) 3rd : $225°, \frac{5\pi}{4}$

(−) 4th : $315°, \frac{7\pi}{4}$

21. $\cos 2x - 3\sin x = 2$

$1 - 2\sin^2 x - 3\sin x = 2$

$-2 \qquad\qquad\qquad\qquad -2$

$-2\sin^2 x - 3\sin x - 1 = 0$

	$\sin x$	$+1$
$-2\sin x$	$-2\sin^2 x$	$-2\sin x$
-1	$-1\sin x$	-1

$(\sin x + 1)(-2\sin x - 1)$

$\sin x + 1 = 0$ $\qquad$ $-2\sin x - 1 = 0$

$-1 \quad -1$ $\qquad$ $+1 \quad +1$

$\sin x = -1$ $\qquad$ $-2\sin x = 1$

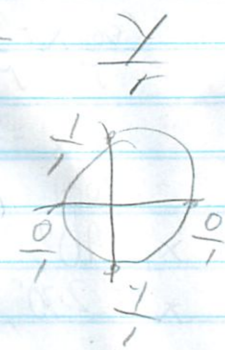
$\sin^{-1} \quad \sin^{-1}$ $\qquad$ $\frac{-2\sin x}{-2} = \frac{1}{-2}$

$x = -90°$ $\qquad$ $\sin x = -\frac{1}{2}$

$270°, \frac{3\pi}{2}$ $\qquad$ $\sin^{-1} \quad \sin^{-1} \rightarrow x = -30°$

$\ominus$ 3rd $= 210°$ $\frac{7\pi}{6}$

$\ominus$ 4th $= 330°$ $\frac{11\pi}{6}$



Use given to find

31. $\cos 195° = -\frac{1}{2}\sqrt{2+\sqrt{3}}$ $\qquad$ $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(195)}{2}}$

$\sin 97.5 = ?$

$\sqrt{\frac{1-\left(-\frac{1}{2}\sqrt{2+\sqrt{3}}\right)}{2}} \rightarrow \sqrt{\frac{\frac{2}{2}+\frac{\sqrt{2+\sqrt{3}}}{2}}{2}}$

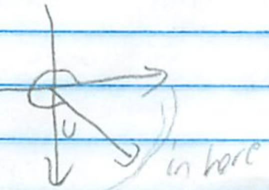
$\sqrt{\frac{\frac{2+\sqrt{2+\sqrt{3}}}{2}}{2}} \cdot \frac{1}{2} \rightarrow \sqrt{\frac{2+\sqrt{2+\sqrt{3}}}{4}}$

$\frac{\sqrt{2+\sqrt{2+\sqrt{3}}}}{\sqrt{4}} \rightarrow \frac{\sqrt{2+\sqrt{2+\sqrt{3}}}}{2}$

More 2x Angle Notes

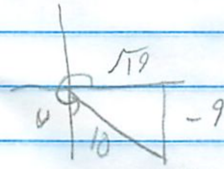
4/30

7. If $\sin v = \frac{-9}{10}$ and $\frac{3\pi}{2} \le v \le 2\pi$, find $\sin\frac{v}{2}, \cos\frac{v}{2}, \tan\frac{v}{2}$



$\sin = \frac{y}{r}$

Phy therom



$10^2 = -9^2 + x^2$

$\sqrt{19} = x$

$\sin\frac{v}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$ adj/hyp

half of what is in 4th quad

$$\frac{\frac{3\pi}{2}}{2} \le \frac{v}{2} \le \frac{2\pi}{2}$$

$$\frac{3\pi}{4} \le \frac{v}{2} \le \pi$$

$$+\sqrt{\frac{1-\frac{\sqrt{19}}{10}}{2}}$$

$$\sqrt{\frac{\frac{10}{10}-\frac{\sqrt{19}}{10}}{2}},\ \sqrt{\frac{\frac{10-\sqrt{19}}{10}}{2}} \cdot \frac{1}{2},\ \sqrt{\frac{10-\sqrt{19}}{20}}$$

$$\frac{\sqrt{10-\sqrt{19}}}{\sqrt{20}},\ \frac{\sqrt{10-\sqrt{19}}}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}\ \frac{\sqrt{5}\left(\sqrt{10-\sqrt{19}}\right)}{10}$$

$$\cos\frac{v}{2} = \pm\sqrt{\frac{1+\cos\frac{\sqrt{19}}{10}}{2}} - \sqrt{\frac{\frac{10+\sqrt{19}}{10}}{2}} \cdot \frac{1}{2}$$

carry ←

$$\sqrt{\frac{10+\sqrt{19}}{20}} - \frac{\sqrt{10+\sqrt{19}}}{\sqrt{20}},\ \frac{\sqrt{10+\sqrt{19}}}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} - \frac{\sqrt{5}\left(\sqrt{10+\sqrt{19}}\right)}{10}$$

$$\tan\frac{v}{2} - \frac{1-\cos\theta}{\sin\theta} = \frac{\frac{10}{10}-\frac{\sqrt{19}}{10}}{\frac{-9}{10}} = -\frac{\frac{10-\sqrt{19}}{\cancel{10}}}{\frac{9}{\cancel{10}}} \cdot \frac{-\cancel{10}}{9}$$

← cancles ~~out~~

← don't look at

$$\cancel{\frac{+10\left(10-\sqrt{19}\right)}{90}}\ \cancel{\frac{+100+10\sqrt{19}}{90}}$$ more

$$\frac{10-\sqrt{19}}{-9}$$

However, the following formulas, which can be derived by simplifying the radical expression in formula (12), may be more useful.

$$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x} \qquad \tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$$

Notice that these formulas don't need the ambiguous sign ±. (Why?) You are asked to prove these formulas in Exercises 37 and 38.

The following table summarizes the double-angle and half-angle formulas.

Double-Angle and Half-Angle Formulas

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}} \qquad \cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \frac{\sin\alpha}{1+\cos\alpha} = \frac{1-\cos\alpha}{\sin\alpha}$$

p 770 Find exact value $\sin\frac{v}{2}$, $\cos\frac{v}{2}$, $\tan\frac{v}{2}$

35. $0 \le v < \frac{\pi}{2}$, $\tan v = 2$

ans $\sqrt{\frac{\sqrt{5}-1}{2\sqrt{5}}}$, $\sqrt{\frac{\sqrt{5}+1}{2\sqrt{5}}}$, $\frac{\sqrt{5}-1}{2}$

$\frac{2}{\sqrt{5}+1}$ or $\sqrt{\frac{\sqrt{5}-1}{\sqrt{5}+1}}$

39. $\sin v = \frac{4}{5}$, $0 \le v < \frac{\pi}{2}$

$\sqrt{\frac{1}{5}}$, $\sqrt{\frac{4}{5}}$, $\frac{1}{2}$ ans

In Exercises 1–10, simplify the given expression.

1. $2\sin 10^\circ\cos 10^\circ$
2. $\cos^2 15^\circ - \sin^2 15^\circ$
3. $1 - 2\sin^2 35^\circ$
4. $2\cos^2 25^\circ - 1$
5. $\dfrac{2\tan 50^\circ}{1-\tan^2 50^\circ}$
6. $\dfrac{2\tan 40^\circ}{1-\tan^2 40^\circ}$
7. $1 - \sin^2 x$
8. $1 - 2\sin^2 x$
9. $2\sin 3\alpha\cos 3\alpha$
10. $\cos^2 5\theta - \sin^2 5\theta$
11. *Discussion* Given that $\cos 70^\circ \approx 0.3420$, explain how you can find $\cos 35^\circ$.

WRITTEN EXERCISES

In Exercises 1–12, simplify the given expression.

A

1. $2\cos^2 10^\circ - 1$
2. $2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$
3. $\dfrac{4\tan\beta}{1-\tan^2\beta}$
4. $1 - 2\sin^2 20^\circ$
5. $2\sin 35^\circ\cos 35^\circ$
6. $\cos^2 4A - \sin^2 4A$

7. $\dfrac{2\tan 25^\circ}{1-\tan^2 25^\circ}$

8. $2\cos^2 3\alpha - 1$

9. $1 - 2\sin^2 \dfrac{x}{2}$

10. $\cos^2 40^\circ - \sin^2 40^\circ$

11. $\sqrt{\dfrac{1-\cos 80^\circ}{2}}$

12. $\sqrt{\dfrac{1+\cos 70^\circ}{2}}$

In Exercises 13–18, find the exact value of the given expression.

13. $2\cos^2 \dfrac{\pi}{8} - 1$

14. $\dfrac{2\tan \frac{\pi}{8}}{1-\tan^2 \frac{\pi}{8}}$

15. $\cos^2 \dfrac{\pi}{12} - \sin^2 \dfrac{\pi}{12}$

16. $1 - 2\sin^2 \dfrac{7\pi}{12}$

17. $\sin 15^\circ \cos 15^\circ$

18. $4\sin \dfrac{2\pi}{3}\cos\dfrac{2\pi}{3}$

In Exercises 19–24, $\angle A$ is acute.

19. If $\sin A = \dfrac{5}{13}$, find $\sin 2A$ and $\cos 2A$.

20. If $\tan A = \dfrac{1}{2}$, find $\cos 2A$ and $\tan 2A$.

21. If $\sin A = \dfrac{3}{5}$, find $\sin 2A$ and $\sin 4A$.

22. If $\cos A = \dfrac{1}{3}$, find $\cos 2A$ and $\cos 4A$

23. If $\cos A = \dfrac{1}{5}$, find $\cos 2A$ and $\cos \dfrac{A}{2}$.

24. If $\cos A = \dfrac{1}{4}$, find $\sin 2A$ and $\sin \dfrac{A}{2}$.

25. Find $\cos 105^\circ$ using **(a)** an addition formula and **(b)** a half-angle formula.

26. Find $\sin 75^\circ$ using **(a)** an addition formula and **(b)** a half-angle formula.

Use a graphing calculator or computer to sketch the graph of each function. Then give the range and period of the function.

B

27. $y = \sin x + \cos 2x$

28. $y = \sin 3x + 2\cos x$

29. $y = 2\sin 2x + 4\cos 4x$

30. $y = 3\cos \dfrac{x}{2} - 2\cos 3x$

In Exercises 31–38, prove that the given equation is an identity.

31. $\dfrac{\sin 2A}{1-\cos 2A} = \cot A$

32. $\dfrac{1-\cos 2A}{1+\cos 2A} = \tan^2 A$

33. $\left(\sin\dfrac{x}{2} + \cos\dfrac{x}{2}\right)^2 = 1 + \sin x$

34. $\sin 4x = 4\sin x\cos x\cos 2x$

35. $\dfrac{1-\tan^2 x}{1+\tan^2 x} = \cos 2x$

36. $\dfrac{1+\sin A - \cos 2A}{\cos A + \sin 2A} = \tan A$

37. $\tan\dfrac{x}{2} = \dfrac{\sin x}{1+\cos x}$

38. $\tan\dfrac{x}{2} = \dfrac{1-\cos x}{\sin x}$

Simplify the given expression.

39. $\dfrac{1+\cos 2x}{\cot x}$

40. $\dfrac{(1+\tan^2 x)(1-\cos 2x)}{2}$

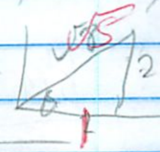
Double + Half Angle Worksheet

4/30

p770 Find the exact value of $\sin\frac{u}{2}, \cos\frac{u}{2}, \tan\frac{u}{2}$

35. $\tan u = 2$ if $0 \leq u < \frac{\pi}{2}$

$\tan\frac{y}{x}$



$y^2 = 2^2 + 2^2$

$y^2 = 8$

$y = \sqrt{8}$

$\frac{2}{2} \neq \frac{2}{1}$

It is

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \rightarrow +\sqrt{\frac{1-\frac{2}{\sqrt{8}}}{2}}$$

$$\sqrt{\frac{\frac{\sqrt{8}}{\sqrt{8}} - \frac{2}{\sqrt{8}}}{2}} \quad \sqrt{\frac{\frac{\sqrt{8}-2}{\sqrt{8}}}{2}} \cdot \frac{1}{2} \rightarrow \sqrt{\frac{\sqrt{8}-2}{2\sqrt{8}}}$$

$$\frac{\sqrt{\sqrt{8}-2}}{\sqrt{2\sqrt{8}}} \cdot \frac{\sqrt{8}}{\sqrt{8}} \quad \frac{\sqrt{8}\left(\sqrt{\sqrt{8}-2}\right)}{\sqrt{16}} \quad \frac{\sqrt{8}\left(\sqrt{\sqrt{8}-2}\right)}{4}$$

$$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \rightarrow +\sqrt{\frac{1+\frac{2}{\sqrt{8}}}{2}}$$

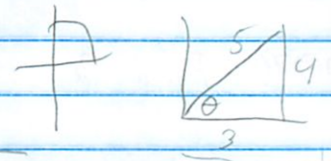
$$\sqrt{\frac{\frac{\sqrt{8}}{\sqrt{8}} + \frac{2}{\sqrt{8}}}{2}} \cdot \frac{1}{2} \quad \sqrt{\frac{\sqrt{8}+2}{2\sqrt{8}}}$$

$$\frac{\sqrt{\sqrt{8}+2}}{\sqrt{2\sqrt{8}}} \cdot \frac{\sqrt{8}}{\sqrt{8}} \quad \frac{\sqrt{8}\left(\sqrt{\sqrt{8}+2}\right)}{\sqrt{16}} \quad \frac{\sqrt{8}\left(\sqrt{\sqrt{8}+2}\right)}{4}$$

$$\tan\frac{u}{2} = \frac{1-\cos\theta}{\sin\theta} \quad \frac{1-\frac{2}{\sqrt{8}}}{\frac{2}{\sqrt{8}}} \quad \frac{\frac{\sqrt{8}}{\sqrt{8}} - \frac{2}{\sqrt{8}}}{\frac{2}{\sqrt{8}}} \quad \frac{\frac{\sqrt{8}-2}{\sqrt{8}}}{\frac{2}{\sqrt{8}}} \cdot \frac{\sqrt{8}}{2}$$

$$\frac{\sqrt{8}(\sqrt{8}-2)}{2\sqrt{8}} \cdot \frac{\sqrt{8}}{\sqrt{8}} \quad \frac{\sqrt{8}\left(\sqrt{8}(\sqrt{8}-2)\right)}{16}$$

39. $\overset{\sin}{v} = \frac{4}{5}$ $\quad 0 \le v \le \frac{\pi}{2}$



$5^2 = 4^2 + x^2$
$-16 \quad -16$
$9 = x^2$
$x = 3$

$\sin\frac{v}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\sqrt{\frac{\frac{5}{5}-\frac{3}{5}}{2}}$ $\quad \sqrt{\frac{\frac{2}{5}}{2}\cdot\frac{1}{2}}$ $\quad \sqrt{\frac{2}{10}}$ $\quad \sqrt{\frac{1}{5}}$

go farther

$\frac{\sqrt{1}}{\sqrt{5}} \rightarrow \frac{\sqrt{5}}{\sqrt{5}}$ $\quad \frac{\sqrt{5}}{5}$

$\cos\frac{v}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$\sqrt{\frac{\frac{5}{5}+\frac{3}{5}}{2}} \rightarrow \sqrt{\frac{\frac{8}{5}}{2}\cdot\frac{1}{2}}$ $\quad \sqrt{\frac{8}{10}} \rightarrow \sqrt{\frac{4}{5}}$

$\frac{\sqrt{4}}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}$ $\quad \frac{\sqrt{4}\cdot\sqrt{5}}{5} \rightarrow \frac{2\sqrt{5}}{5}$

go farther

Check by entering $\sin^{-1}(4/5) / 2$ & $\cos(\;)$ should = ans

$\tan\frac{v}{2} = \frac{1-\cos\theta}{\sin\theta} \rightarrow \frac{\frac{5}{5}-\frac{3}{5}}{\frac{4}{5}}$ $\quad \frac{\frac{2}{5}}{\frac{4}{5}}$

$\frac{2}{5}\cdot\frac{5}{4} = \frac{10}{20} = \frac{1}{2}$

All correct

Class Exercises

Simplify

1. $2\sin 10 \cos 10$

$\sin 2(10^\circ)$ $\sin(20^\circ)$

3. $1-\sin^2(35^\circ)$

$\cos 2(35^\circ)$ $\cos(70^\circ)$

5. $\dfrac{2\tan(50^\circ)}{1-\tan^2(50^\circ)}$

$\tan 2(50^\circ)$ $\tan(100^\circ)$

7. $1-\sin^2 x$

$\cos^2 x$

9. $2\sin 3x\cos 3x$

$\sin 2(3x)$ $\sin(6x)$

Written Exercises

1. $2\cos^2(10^\circ)-1$

$\cos 2(10^\circ)$ $\cos(20^\circ)$

2. $2\sin\frac{a}{2}\cos\frac{a}{2}$

$\sin 2(\frac{a}{2})$ $\sin(a)$

4. $1-2\sin^2(20^\circ)$

$\cos 2(20^\circ)$ $\cos(40^\circ)$

5. $2\sin(35^\circ)\cos(35^\circ)$

$\sin 2(35^\circ)$ $\sin(70^\circ)$

All correct

Simplify

7. $\dfrac{2\tan(25^\circ)}{1-\tan^2(25^\circ)} = \tan 2(25)^\circ \quad \tan(50^\circ)$

9. $1-2\sin^2\frac{x}{2} = \cos 2\left(\frac{x}{2}\right) = \cos(x)$

11. $\sqrt{\dfrac{1-\cos 80^\circ}{2}} \quad \sin\left(\frac{80}{2}\right) = \sin(40^\circ)$

Give exact answer

13. $2\cos^2\frac{\pi}{8} - 1$

$\cos 2\left(\frac{\pi}{8}\right) = \cos\left(\frac{\pi}{4}\right)$

$\frac{\sqrt{2}}{2}$

15. $\cos^2\frac{\pi}{12} - \sin^2\frac{\pi}{12}$

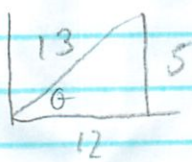
$\cos 2\left(\frac{\pi}{12}\right)$

$\cos\left(\frac{\pi}{6}\right)$

$\frac{\sqrt{3}}{2}$

19. $\angle A$ is acute

is $\sin a = \frac{5}{13}$ find $\sin 2A + \cos 2A$



$13^2 = 5^2 + x^2$

$-25 \quad -25$

$144 = x^2$

$12 = x$

$\sin 2A = 2\sin\theta\cos\theta$

$\left(\frac{2}{1}\right)\left(\frac{5}{13}\right)\left(\frac{12}{13}\right)$

$\frac{120}{169}$

$\cos 2A = 2\cos^2\theta - 1$

$2\left(\frac{12}{13}\right)^2 - 1$

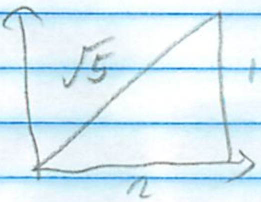
$\frac{2}{1}\left(\frac{144}{169}\right) - 1$

$\frac{288}{169} - \frac{169}{169}$

$\frac{119}{169}$

A is acute

20. if $\tan A = \frac{1}{2}$ find $\cos 2A + \tan 2A$



$x^2 = 1^2 + 2^2$

$x^2 = 5$

$x = \sqrt{5}$

$\cos 2A = 2\cos^2 x - 1$

$2\left(\frac{2}{\sqrt{5}}\right)^2 - 1$

$2\left(\frac{2}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}}\right) - 1$

$2\left(\frac{4}{\sqrt{25}}\right) - 1$

$\left(\frac{2}{1}\right)\left(\frac{4}{5}\right) - 1$

$\frac{8}{5} - \frac{5}{5}$

$\frac{3}{5}$

$\tan 2A = \frac{2\tan x}{1 - \tan^2 x}$

$\frac{2\left(\frac{1}{2}\right)}{1 - \left(\frac{1}{2}\right)^2}$

$\frac{1}{\frac{4}{4} - \frac{1}{4}}$

$\frac{1}{\frac{3}{4}} \to \frac{4}{3}$

X fractions

$\frac{1}{5} \cdot \frac{2}{10} \quad \frac{2}{50} = \frac{1}{25}$

multiply accross

Prove

Use identies

31. $\frac{\sin 2A}{1 - \cos 2A} = \cot A$

$\frac{2\sin A \cos A}{1 - (1 - 2\sin^2 A)} \to \frac{2\sin A \cos A}{2\sin^2 A}$

$\frac{\cos\theta}{\sin\theta}$

~~$\tan\frac{\theta}{2} = \frac{\sin\theta}{1-\cos\theta}$~~

~~$(\theta = 2A)$~~

$\frac{\cos\theta}{\sin\theta} = \cot A = \cot A$

32. $\frac{1 - \cos 2A}{1 + \cos 2A} = \tan^2 A$

$\frac{1 - (1 + 2\sin^2 x)}{1 + 2\cos^2 x - 1}$

$\frac{\cancel{2}\sin^2 x}{\cancel{2}\cos^2 x}$

$\frac{\sin^2 x}{\cos^2 x} = \tan^2 x = \tan^2 x$

35. $\frac{1-\tan^2 x}{1+\tan^2 x} = \cos 2x$

$$\frac{1-\tan^2 x}{\sec^2 x}$$

$$\frac{1}{\sec^2 x} - \frac{\tan^2 x}{\sec^2 x}$$

$$\cos^2 x - \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}}$$

$$\cos^2 x - \sin^2 x$$

$$\cos 2x = \cos 2x$$

5/3 Quiz

5/3

section 14.5 + 14.6

- simplifying expressions
- proving identities
- solving equations

) useing double + half angle identies

Review Worksheet

Chapter Test

1. Simplify the given expression. **10-1**
   a. $\cos 75° \cos 15° + \sin 75° \sin 15°$
   b. $\sin 75° \cos 15° + \cos 75° \sin 15°$
   c. $\cos (30° + x) + \cos (30° - x)$ - 2 double angles
   d. $\sin (45° - x) - \sin (45° + x)$

2. Find the exact value of $\cos 15°$.

3. Find $\tan \left(\frac{5\pi}{4} - \theta\right)$ when $\tan \theta = -\frac{1}{3}$. **10-2**

4. If $\tan \alpha = \frac{4}{3}$ and $\tan \beta = -\frac{1}{2}$, show that $\tan (\alpha + \beta) = \tan (\pi - \beta)$.

5. ***Writing*** Consider the lines $y = 2x + 1$ and $y = 4 - 3x$. Write a paragraph explaining how it is possible to find two different angles that are formed by the intersection of these lines. What is the relationship between these angles?

6. Suppose $\angle A$ is acute and $\cos A = \frac{4}{5}$. Find each of the following: **10-3**
   a. $\sin A$
   b. $\cos 2A$
   c. $\sin 2A$
   d. $\sin 4A$

7. Simplify the given expression.
   a. $\dfrac{\sin 2x}{1 - \cos 2x}$
   b. $(1 + \tan^2 y)(\cos 2y - 1)$
   c. $\dfrac{\tan t}{\sec t + 1}$
   d. $\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}$

8. Evaluate the given expression.
   a. $2 \cos^2 \frac{\pi}{12} - 1$
   b. $4 \sin \frac{\pi}{6} \cos \frac{\pi}{6}$

9. Prove that the given equation is an identity.
   a. $(1 + \cot^2 x)(1 - \cos 2x) = 2$
   b. $\dfrac{\sin \theta \sec \theta}{\tan \theta + \cot \theta} = \cos^2 \theta - \cos 2\theta$

10. a. On the same set of axes, sketch the graphs of $y = \cos 2\theta$ and $y = \sin \theta$ for $0° \le \theta < 360°$. **10-4**
    b. Determine where the graphs intersect by solving $\cos 2\theta = \sin \theta$.

11. Solve the equation $\cos 2x = \cos x + 2$ either graphically or algebraically for $0 \le x < 2\pi$.