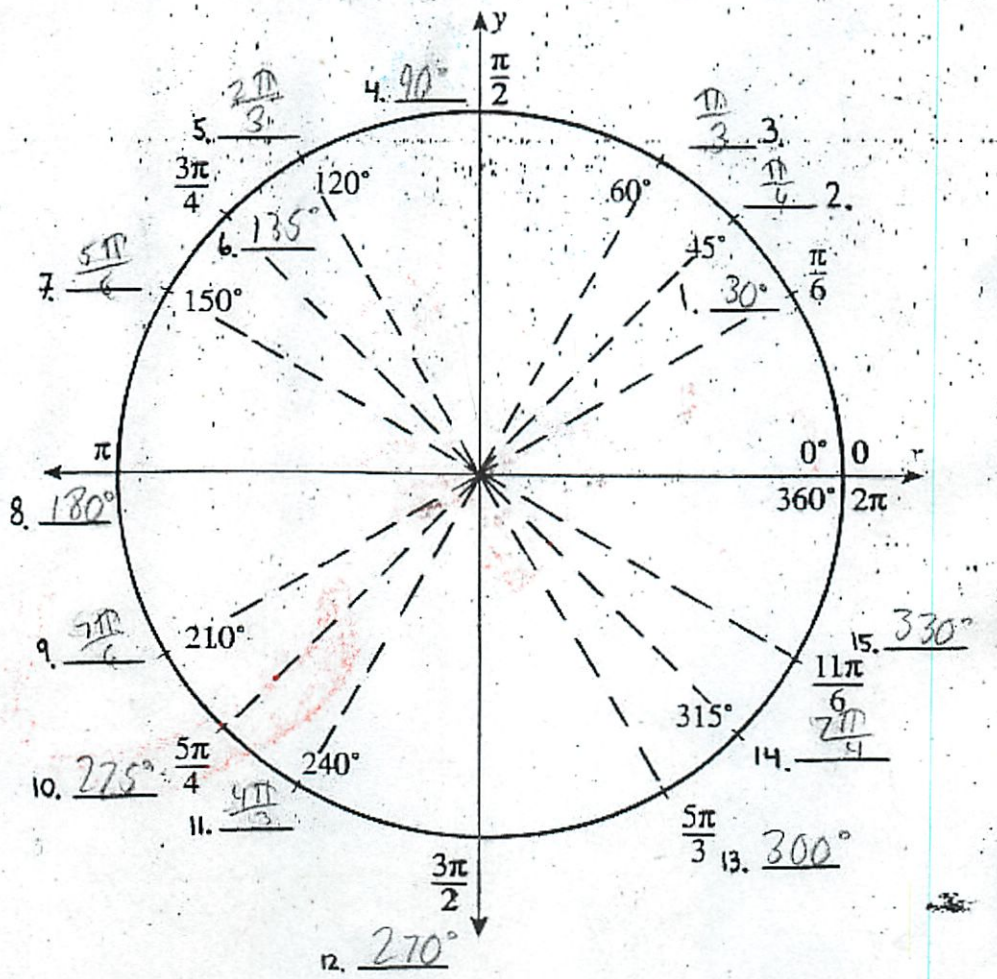
High Dive Formulas

Formula Description	Formula	Variables
Height of Diver Off the **Ground**	$H = 65 + 50 \cdot \sin(9 \cdot W_T)$	*H = height from ground in feet; W = time in seconds until diver is released (after 3:00 position)*
Diver's Fall Time	$F_T = \sqrt{\frac{57 + 50 \cdot \sin(9 \cdot W_T)}{16}}$	*F = time in seconds for diver to fall from release to the cart; W = time in seconds until diver is released (after 3:00 position)*
X-coordinate of Diver when Released	$x = 50 \cdot \cos(9 \cdot W_T)$	*W = time in seconds until diver is released (after 3:00 position)*
X-coordinate of Cart when It Catches the Diver	$x = S + 15 \cdot (W_T + F_T)$	*S = cart's starting x-coordinate; F = time in seconds for diver to fall from release to the cart; W = time in seconds until diver is released (after 3:00 position)*

Name Michael Plasmeier

Unit Circle in terms of Degrees & Radians
(in multiples of 30°, 45°, 60°)

Directions: Convert the radian measure to degrees and convert the given degree measure to radians.



Reciprocal Identities

$$\sin x = \frac{1}{\csc x} \qquad \sec x = \frac{1}{\cos x} \qquad \tan x = \frac{1}{\cot x}$$

$$\csc x = \frac{1}{\sin x} \qquad \cos x = \frac{1}{\sec x} \qquad \cot x = \frac{1}{\tan x}$$

Tangent Identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\csc x\left(\frac{\pi}{2} - x\right) = \sec x \qquad \tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

Negative Angle Identities

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x$$

$$\csc(-x) = -\csc x \qquad \tan(-x) = -\tan x$$

$$\sec(-x) = \sec x \qquad \cot(-x) = -\cot x$$

Sum and Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Double Angle Formulas

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

6 Circular function definitions where θ is any angle

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

$$r = \sqrt{x^2 + y^2}$$

CHART OF THE 3 TRIG FUNCTIONS OF SPECIAL ANGLES

Degrees	Radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Additional Trigonometric Formulas/Identities

Half - Angle Identities:

$$\sin^2 u = \frac{1-\cos 2u}{2} \qquad \cos^2 u = \frac{1+\cos 2u}{2} \qquad \tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$$

Half - Angle Formulas:

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

Alternative Half - Angle Formulas for Tangent:

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} \qquad \tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$$

Product - to - Sum Formulas:

$$\sin u\cos v = \frac{1}{2}\left[\sin(u+v)+\sin(u-v)\right]$$

$$\cos u\sin v = \frac{1}{2}\left[\sin(u+v)-\sin(u-v)\right]$$

$$\cos u\cos v = \frac{1}{2}\left[\cos(u+v)+\cos(u-v)\right]$$

$$\sin u\sin v = -\frac{1}{2}\left[\cos(u+v)-\cos(u-v)\right]$$

Sum - to - Product Formulas:

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

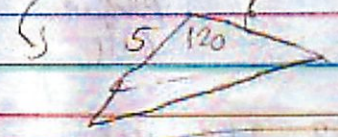
$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

13.6

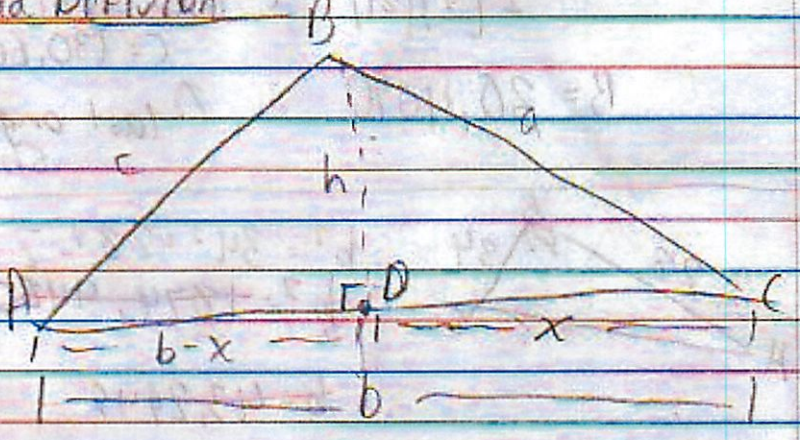
Law of Cosines

3/14

Use when
- given 3 sides + no angles
- given 2 sides + included angle
  - ∠ angle created at intersection of 2 given sides



Formula Derivation



$c^2 = (b-x)^2 + h^2$

$c^2 = (b-x)(b-x) + h^2$

$c^2 = b^2 - 2bx + x^2 + h^2$ ← $h^2 + x^2 = a^2$

$c^2 = -2bx + a^2$ ← $x = a\cos C$ ← $\cos C = \frac{x}{a}$

$c^2 = b^2 - 2b(a\cos C) + a^2$

or

$c^2 = a^2 + b^2 - 2ab\cos C$

↑ solving for a side

Solve for angle

$-(a^2+b^2) \quad -(a^2+b^2)$

$\cos C = \frac{c^2 - a^2 - b^2}{-2ab}$

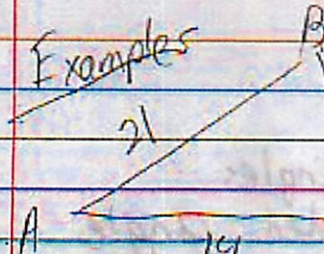
convert

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

3/26

Examples

1.



$\cos A = \frac{14^2 + 21^2 - 9^2}{2(14)(21)}$

just plug in Calc

$\cos A = .9455$

$\cos^{-1}$ $\cos^{-1}$

$A = 18.9895$

$\cos B = \frac{9^2 + 21^2 - 14^2}{2(9)(21)}$

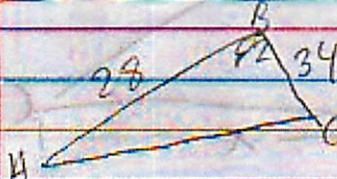
$B = 30.4090$

$C = 180 - (18.9895 + 30.4090)$

$C = 130.6015$

last angle can be short cutted

2.



$b^2 = 34^2 + 28^2 - 2(34)(28)\cos(92)$

$b^2 = 1874.4486$

~~$b = 43.2449$~~ Calc error

$\cos A = \frac{43.2449^2 + 28^2 - 34^2}{2(43.2449)(28)}$

$\cos A = .6197$

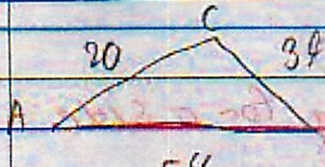
$\cos^{-1}$ $\cos^{-1}$

$A = 51.7064$

$C = 180 - (92 + 51.7064)$

$C = 36.2935$

3.



$\cos A = \frac{20^2 + 54^2 - 39^2}{2(20)(54)}$

$A = 33.7965°$

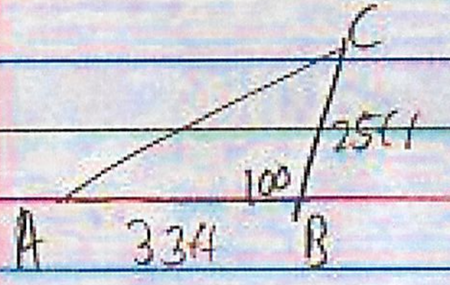
$\cos B = \frac{39^2 + 54^2 - 20^2}{2(39)(54)}$

$B = 16.5740°$

$C = 180 - (33.7965 + 16.5740)$

$C = 129.6295°$

4.



$b^2 = 25^2 + 33^2 - 2(25)(33)\cos(100)$

$b^2 = 2000.5194$

$b = 44.7272\ ft$

$\cos A = \frac{44.7272^2 + 33^2 - 25^2}{2(44.7272)(33)}$

$\cos A = .8249$

$\cos^{-1} \quad \cos^{-1}$

$A = 33.3901°$

$C = 180 - (33.3901 + 100)$

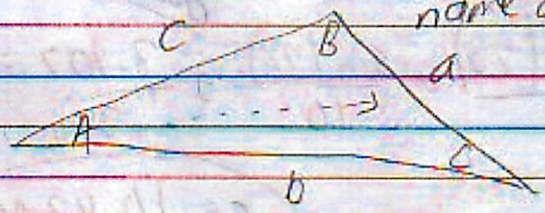
$C = 46.6099$

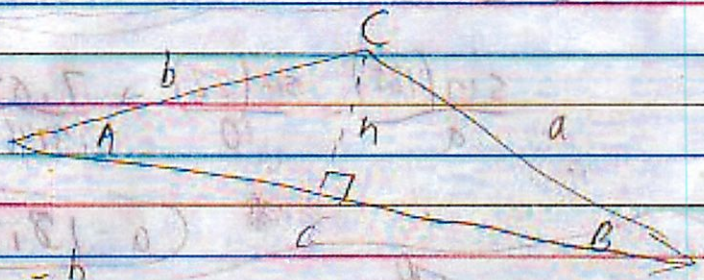
13.5

Law of Sines

3/15

* used to solve oblique angles (triangles w/o right angles)
* to solve a triangle - find the measurements of the missing sides or angles
* must be given 3 pieces of info (sides or angles) with at least one side
* capital letters = angles
  lowercase " = sides





$\sin A = \frac{h}{b}$ or $\sin B = \frac{h}{a}$

$h = \sin(A)\, b$, $h = \sin(B)\, a$

$\sin(A)\, b = \sin(B)\, a$

$\frac{b \sin(A)}{a} = \frac{\sin B}{b}$

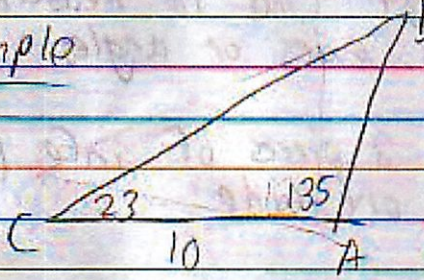
and also

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Law of sines

Use if given 2 angles + a side
or 2 sides + an angle opposite a given side

Example



$B = 180 - (23 + 135)$

$B = 22°$

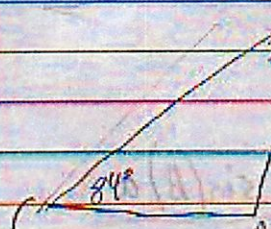
$\frac{\sin(23)}{c} = \frac{\sin(22)}{10}$ → cross multiply → $\frac{3.907}{.3746} = \frac{(.3746)c}{.3746}$

$c = 10.4304$

B = 56°
C = 84°
c = 15ft

$\frac{\sin(135)}{a} = \frac{\sin(22)}{10} \rightarrow \frac{7.071}{.3746} = \frac{.3746a}{.3746}$

$a = 18.8760$



$A = 180 - (84 + 56)$

$A = 40°$

$\frac{\sin(40)}{a} = \frac{\sin(84)}{15}$ → cross x → $\frac{9.6418}{.9945} = \frac{.9945a}{.9945}$

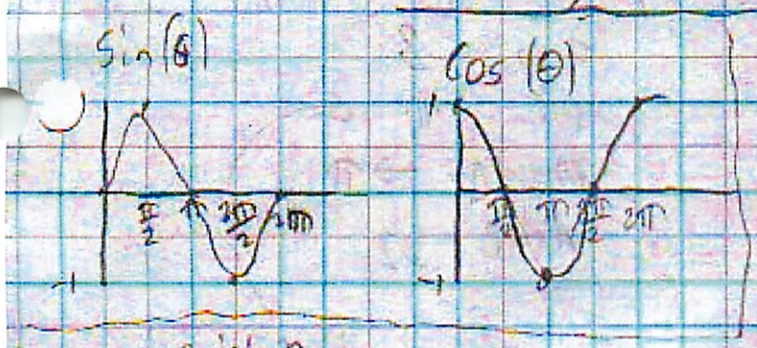
$a = 9.6944$ ft

$\frac{\sin(56)}{b} = \frac{\sin(84)}{15} \rightarrow \frac{12.4356}{.9945} = \frac{.9945c}{.9945}$

$c = 12.5041$

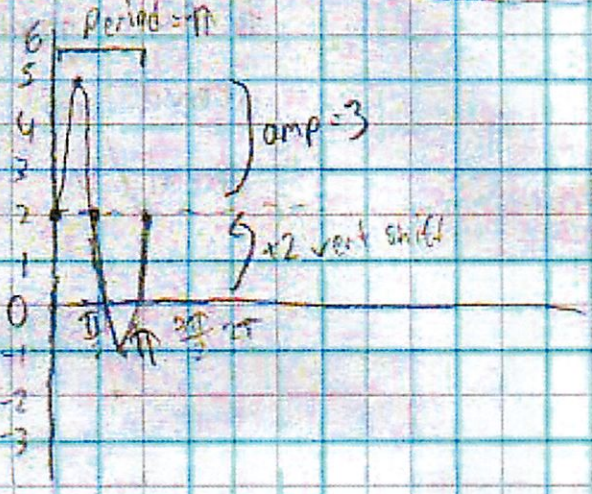
Graphing Sin + Cos Curves

2/28



$Y = 3\sin(2x) + 2$

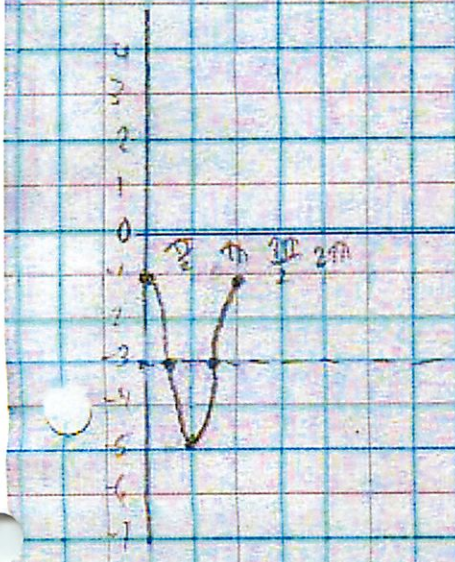
amp = 3
pd = π
phase Δ = 0
vert Δ = ↑2



Not Test

$Y = 2\cos(2x) - 3$

amp = 2
pd = π
phase Δ = 0
vert Δ = ↓3



$y = 3\sin(2x - 2\pi) + 2$

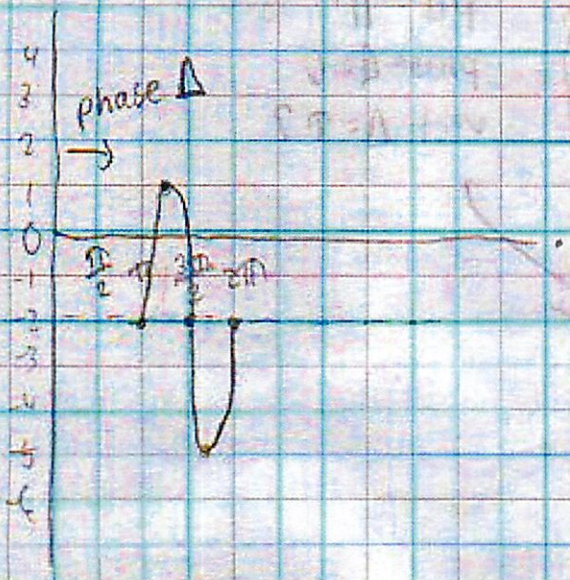
amp = 3

pd → π

phase Δ → π

vert Δ → up 2

to find graph interval do period / 4



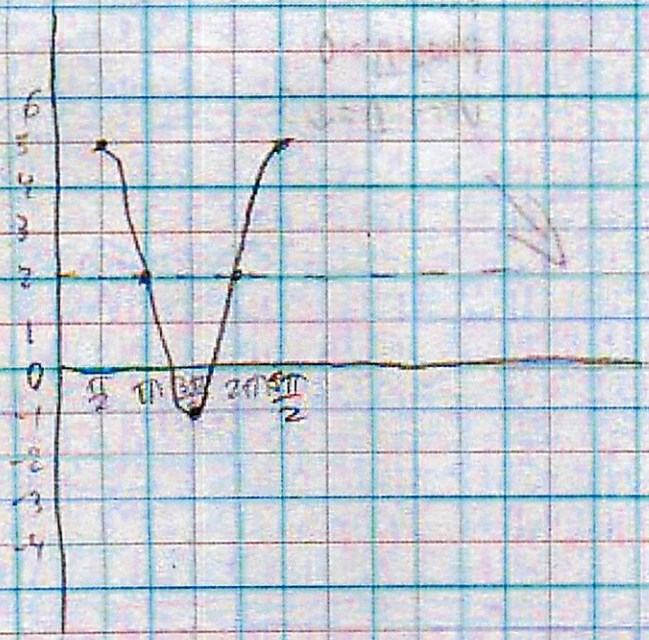
$y = 3\cos\left(x - \frac{\pi}{2}\right) + 2$

amp = 3

pd = 2π

phase Δ → $\frac{\pi}{2}$

vert Δ = ↑ 2



High Dive/Trig
PORTFOLIO (L1)

Name: Michael Plasmeier
Date: 5/11

Your portfolio for High Dive/Trig should include all of the following:

ALL OF THE FOLLOWING PAPERS MUST BE ARRANGED IN THE FOLLOWING ORDER. (IF THEY ARE NOT, POINTS WILL BE DEDUCTED FROM YOUR OVERALL GRADE.)

Worksheets/Notes (1 point each) (Each should be complete and correct.) **POINTS**

- [x] "Ferris Wheeler's Day Off"
- [x] "High Divin' Revivin'"
- [x] "Mixed Review: Law of Sines and Cosines"
- [x] Packet: Applications of Trigonometry to Navigation and Surveying
- [x] "Moving Cart, Turning Ferris Wheel" Group Activity
- [x] "How Many Acres?" POW
- [x] Clswk: Simplifying Trig Expressions and Proving Trig Identities

18 pts ~ 4/17 _____ (7)

Independent Studies/Tests (1 point each)

- [x] Ind. Study #1: Linear Functions
- [x] Test for Ind. Study #1 went
- [x] Ind. Study #2: Factoring & missing 1
- [ ] Test for Ind. Study #2
- [ ] Ind. Study #3: Powers, Roots and Radicals (Sam)
- [ ] Test for Ind. Study #3

_____ (6)

Hot Sheets (1 point each)

- [x] Hot Sheet(s) X X XX ?
- [ ] Identity Sheet (Eli, Sam)
- [x] Unit Circle

_____ (3)

Test/Quizzes (1 point each)

- [x] High Dive/Chpt. 13 & 14 Quiz #1 ✓ print graph afterwards
- [x] High Dive/Chpt. 13 & 14 Quiz #2 Graphing + Solving Equations - Inverse Trig / 64 pts
- [x] High Dive Unit Test
- [x] Law of Sines and Cosines Test (Triange Trig Quiz)
- [ ] Trig Identities Quiz - Simp. Trig Expressions + Identities
- [x] Trig Equations Test - Equations + Identies
- [ ] Double/Half Angle & Sum and Diff. Quiz
- [ ] RETEST for Double/Half Angle & Sum and Diff. Quiz

_____ (8)

TOTAL _____ **(24)**

Reciprocal Identities

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Negative Angle Identities

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x$$

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$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

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$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

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$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

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$$r = \sqrt{x^2 + y^2}$$

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$45°$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
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Additional Trigonometric Formulas/Identities

Half - Angle Identities:

$$\sin^2 u = \frac{1-\cos 2u}{2} \qquad \cos^2 u = \frac{1+\cos 2u}{2} \qquad \tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$$

Half - Angle Formulas:

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

Alternative Half - Angle Formulas for Tangent:

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} \qquad \tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$$

Product - to - Sum Formulas:

$$\sin u\cos v = \frac{1}{2}\left[\sin(u+v)+\sin(u-v)\right]$$

$$\cos u\sin v = \frac{1}{2}\left[\sin(u+v)-\sin(u-v)\right]$$

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Sum - to - Product Formulas:

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

Name Nicholas Rivera

Date 2/14

Unit Laws of Sines + Cosines

IAG 3 HOT SHEET

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cos

Standard
Find side

$a^2 = b^2 + c^2 - 2bc\cos(A)$

$b^2 = a^2 + c^2 - 2ac\cos(B)$

$c^2 = a^2 + b^2 - 2ab\cos(C)$

Alt form
Find angle

$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos(B) = \frac{a^2 + c^2 - b^2}{2ac}$

$\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$

Triangle

When to use

Sines - 2 angles + side
- 2 sides + angle opposite that side

Cos - 3 sides no angles
- 2 sides + included angle

Area Triangle

$\frac{1}{2}ab\sin(C)$

= area

Ambiguous Case

only if 2 sides + angle opposite a side

Find an angle and do $180 - A$, A_2 if A_2 + other angle < 180 for 2 answers

Find other angle equally

Find last side equally