## Linear Nim

## POW \# 6

## 1. Problem Statement: Not necessary to do.

2. Process (Playing): I started playing Linear Nim with Michael Lewis. We started out just playing to try and find strategies. Our tallies are recorded on the attached page. The numbers are recorded as follows:


On the $3^{\text {rd }}$ try, I let my opponent go first. He picked 3 which was the mistake I made on my $1^{\text {st }}$ turn. I ended up winning.

On the fourth try, I tried to pick 1 first. I lost by being stuck with 4. I could have won by watching more closely. If my opponent picks 1 on his first turn there are 8 lines left. If I pick 1 , my opponent picks 1 , I can pick 1, 2, or 3. Either way my opponent picks the opposite (you 1-he 3) or [(maximum per turn)-(his pick)+1] and it's my choice with 4 left. He wins. If he picks 2 , I can pick 3, and he picks with 4 and looses. If he picks 3, then I can pick 2, and leave him with 4 . Therefore, picking 1 first lets your opponent control your fate.

The fifth time around I developed the winning strategy. See below.
I did not do anymore playing where I don't go first, because I found my answer.
3. Solution (of Original): I found out in my $5^{\text {th }}$ term the winning strategy for the original game. If I went first, and picked 2, I could win. If my opponent picked 3, thin I would pick 1 on my next term. There would be 4 left and it would be his turn. If it's your turn and you have 4 (maximum per turn)+1) left, you lose. Now when I picked 2 first, and my opponent picks 2 , I would take 2 also to leave him with 4 . If he takes 1 , I take 3 and He is again left with 4 . This is my strategy.

If the other person goes first, hope they won't pick 2, if so, your done. If he picks 1 first, pick 2 , or 3 and you might lose. If he picks 3 , pick 3 and force him with 4.
4. Extension (Generalizations): I am also seeing a pattern that leaving him with 4 [(maximum per turn $)+1]$ requires I can also leave him with 8 , which happens to be [ $2 x$ (maximum per turn +1 )] When I start at 10 , I need to take 2 to get him to [ $2 x$ (maximum per turn +1 )]. I did not know that at the time, so we tried playing with 15 marks and keeping 3 at a time. I found that if you got the game down to 10 and your turn, it is just like playing the original game. I now also know that if I got him down to picking from 8 [ $2 x$ (maximum per turn +1 )] and my turn, I could also win. I suppose this will also work with all variations of $\mathrm{X}:\left[\boldsymbol{X}^{*}(\right.$ maximum per turn +1$\left.)\right]$ in all variations of liner nim. I can also try and get him to 12 [3x(maximum per turn +1 )], by going first and taking 3. I can now know that if I can go first, I will always win, unless [ $X$ *(maximum per turn +1 )] is the starting number, where I would want him to go first.. This is because I can always take (initial number)-[ $X^{*}$ (maximum per turn +1 )], and catch him in a loop picking the opposite of what he picks, which equals, [(maximum per turn)-(his pick)+1].

I found out that trapping people at 4 was the same as (maximum per turn) +1 when I to increased the maximum per turn to 4 and played with the initial number of 10 . I ended up winning, however it would have been better to have known the strategy above.
5. Evaluation: Not necessary to do.

## If initial number equals [ X *(maximum per turn+1)]; <br> Then

Label A
Have Partner Go
Take (maximum per turn)-(his pick)+1
Goto A
Else
Go First
Take (initial number)-[X*(maximum per turn+1)
Have Partner Go
Take (maximum per turn)-(his pick)+1
Goto A

During Problem: X is equal to the number closest to < (initial number)


