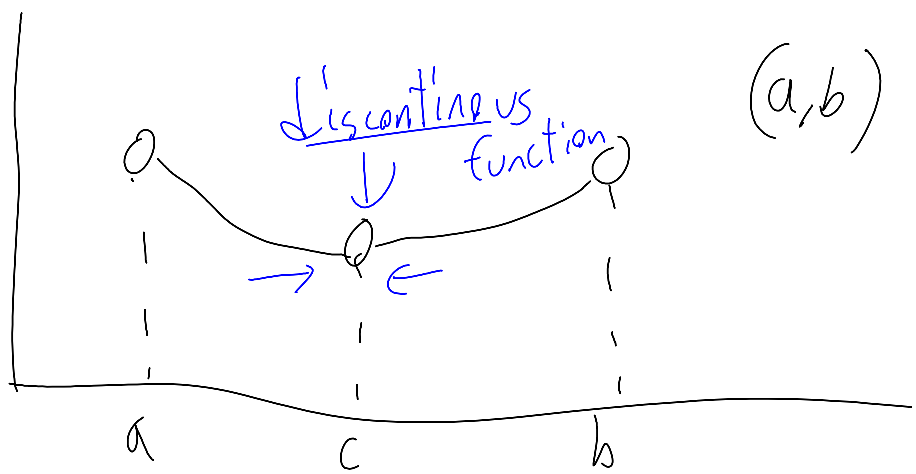
Section 1.4 Continuity & 1 sided Limits

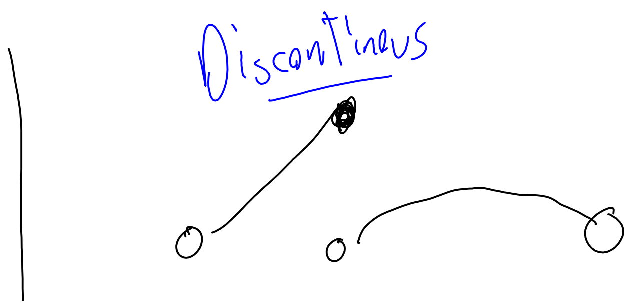
Monday, September 15, 2008
1:21 PM

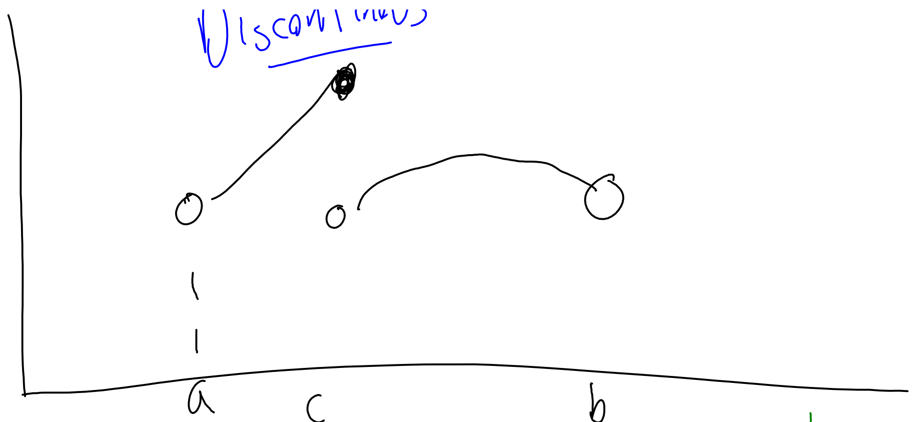
Continous function - no interruptions of the graph of $f(x) = c$
- no holes
- no gaps
- no jumps

Continuity on an <u>open</u> <u>interval</u>



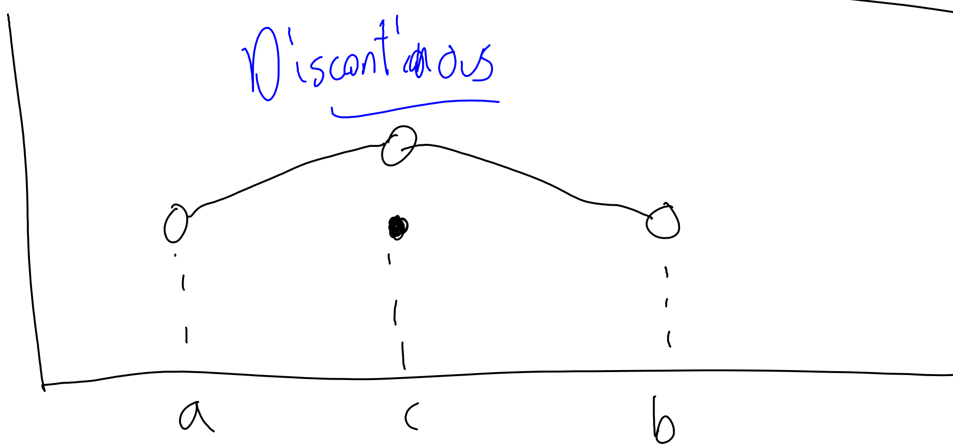
The function is not defined at $x = c$ even though <u>the limit exists</u> at $x = c$





different y values

Limit of $f(x)$ does not exist at $x=c$ but $f(c)$ is defined



$F(c)$ is defined + limit of $f(x)$ still exists but the $\lim_{x\to c}$ of $f(x) \neq f(c)$ so discontinuous

3 Conditions

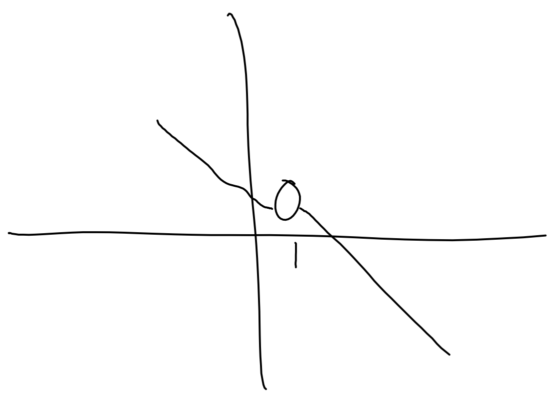
A function is continuous at $x=c$ if

all of the following conditions exist

1. $f(c)$ must exist
2. $\lim_{x \to c} f(x)$ must exist
3. $\lim_{x \to c} f(x) = f(c)$

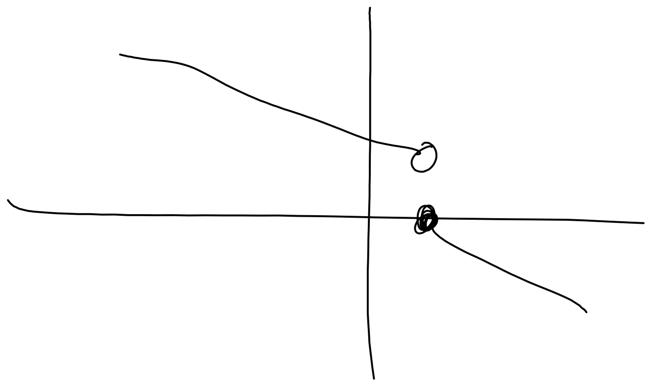
If anythis is not true - function is discontinous

Is $f(x)$ continous at $x = 1$?



No since it is not defined at $x = 1$
so not continous

(2)



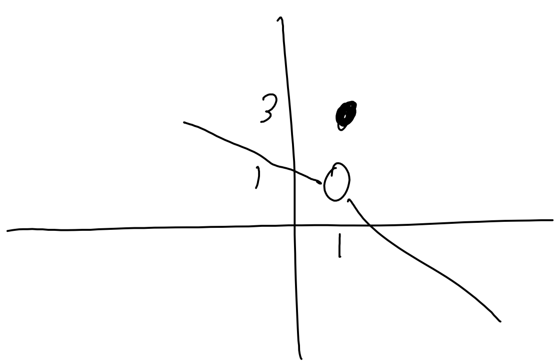
No - it is defined - but the limit does not exist Not contin'ous

function splits

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} -x+1 = 0$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} -x+2 = 1$

(3) Is $f(x)$ continious at $x=1$



$f(x) = 3$ → defined

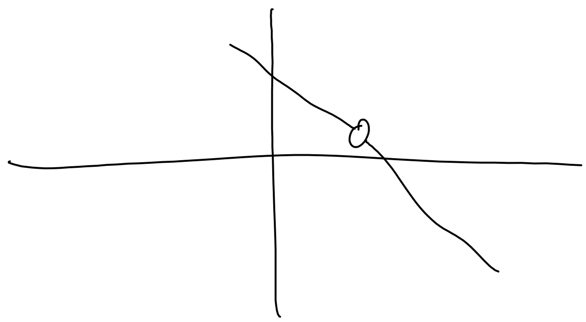
$\lim_{x\to 1} f(x) = 1$ → defined

) but don't =

Not continous

If (1,3) is removed and replaced with (1,1) it would fill the graph then f(x) would be continous

Removable vs Non Removable Discontinuities



Removable discontinuity at $x=1$
Can redefine $f(1)=1$ ← fill the hole
then $f(x)$ would be continious

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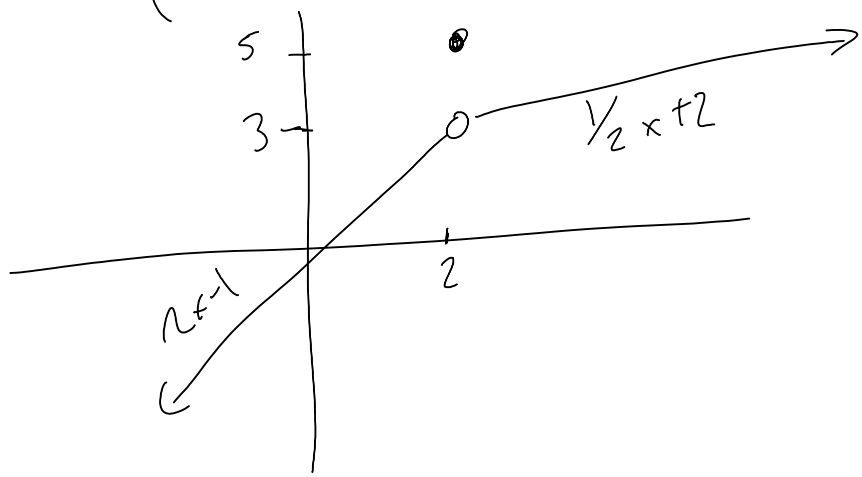
ex $f(x)=\dfrac{x^2-5x+6}{x-3}$ $\dfrac{\cancel{(x-3)}(x-2)}{\cancel{x-3}}$ $x-2$

Is a removable discontinuity at $x=3$

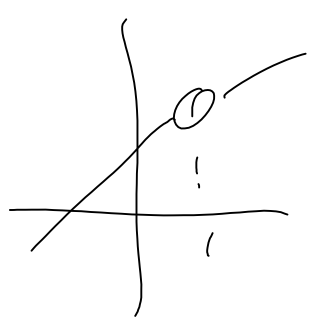
$$f(x) = \begin{cases} 2x-1, & x<2 \\ 5, & x=2 \quad (2,5) \\ \frac{1}{2}x+2, & x>2 \end{cases}$$



Not continuous at $x=2$ but can redefine the function

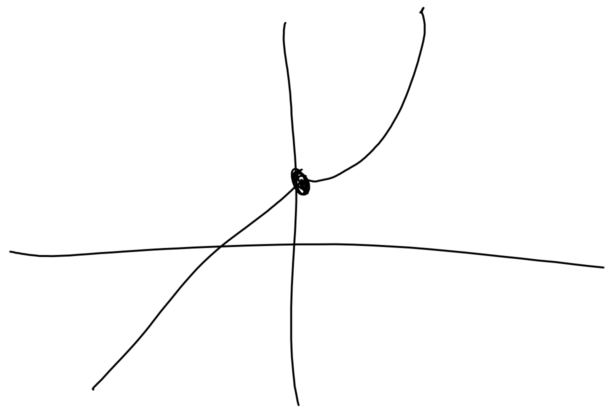
$$g(x) = \frac{x^2-1}{x-1} \rightarrow \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}} \rightarrow x+1$$

→ yes removable discontinuity at $x=1$



New function $f(x)$ is defined at $x=1$

$$h(x) = \begin{cases} x+1, & x \le 0 \\ x^2+1, & x > 0 \end{cases}$$



Domain $= \cancel{\mathbb{R}}$ $\cancel{(-\infty, \infty)}$ $(-\infty, 0] \cup (0, \infty)$

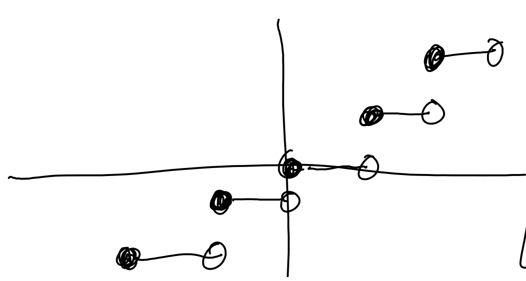
Is continious

$\cancel{f}(x)$ $\sin(x)$

continious everywhere

Non removable discontinuities

- step functions
- rational functions
- and some trig functions



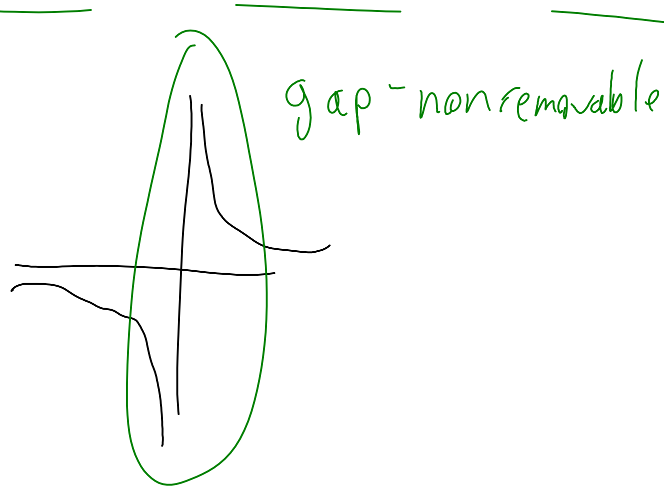
$[[2.6]] = 2$

● — ○ ← always have gaps

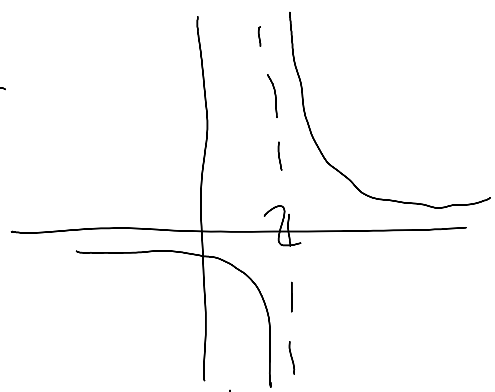
y is always a whole #

Can not redefine it

$f(x) = \frac{1}{x}$



$f(x) = \frac{1}{x-2}$



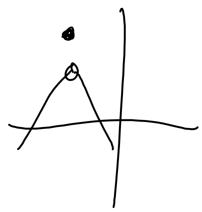
Non removable discontinuity at $x = 2$

domain $(-\infty, 2) \cup (2, \infty)$

P. 78 + 79

#4, 26, 28, 38, 42, 54

4.



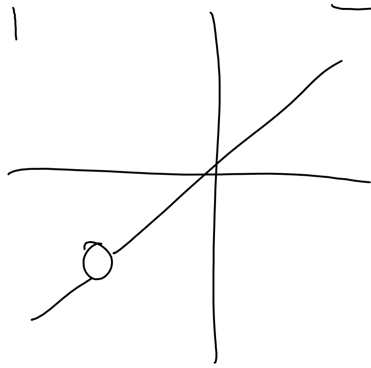
$\lim_{x \to -2} = 2$

$f(x) = 3$

not the same

discontinious

26. $\frac{x^2-1}{x+1} \to \frac{\cancel{(x+1)}(x-1)}{\cancel{x+1}} \to x-1$

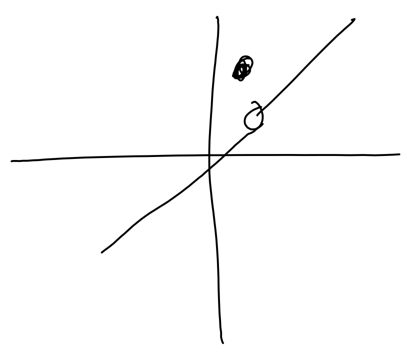


discontinous

$f(-1)$ does not exist

but removable

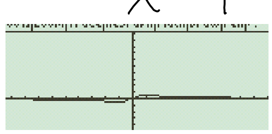
28. $\begin{cases} x, & x<1 \\ 2, & x=1 \\ 2x-1, & x>1 \end{cases}$



$f(x) \neq \lim_{x \to c} f(c)$

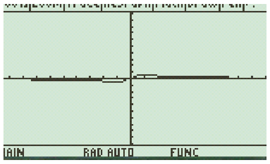
discontinous, removable

38. $\frac{x}{x^2-1}$



?

removable ? discontinous at 1



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42. $\dfrac{x-1}{x^2+x-2} \rightarrow \dfrac{\cancel{x-1}}{(x+2)\cancel{(x-1)}}$ removblale discontinous

54. $f(x) = 3 - [[x]]$

step function

non removable discontinous